Density and the layers of the Earth

Core Quantitative Literacy Topics
Percentages, ratios, weighted average

Supporting Quantitative Literacy Topics
Unit Conversions

Core Geoscience Subject
Layers of the Earth, density

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In 1772, Nevil Maskelyne was pondering Sir Isaac Newton’s theory of gravitation, in which it was proposed that ANY object should exert a gravitational attraction towards any other object. Maskelyne surmised that this should mean that one could measure the gravitational attraction of, for instance, a large mountain and that it should be proportional to the gravitational attraction of the entire Earth. So he set to work measuring the gravitational attraction that a mountain named Schiehallion exerted on his plumb bob (a kind of weight on a string used by surveyors). Six years later a geologist named Charles Hutton used the measurements taken by Maskelyne to estimate the mean density of the Earth at 4.5 g/cm³, which is pretty close to the modern estimate of 5.5 g/cm³!
The Earth consists of several distinct layers. The crust is the topmost layer, which we live on. The crust has an average composition similar to granite and consists of both the ocean basins and the continents. Beneath the crust is the mantle, which has the composition of peridotite. It makes up the largest volume of the whole Earth by far. Finally, the core is composed of an outer core and an inner core. Both the inner core and outer core are composed of an iron-nickel alloy but the outer core is liquid while the inner core is solid. The liquid outer core is the source of Earth’s magnetic field.

Looking closer: The crust and upper mantle also make up two physically distinct layers called the lithosphere and the asthenosphere.

- The lithosphere is composed of the crust and uppermost mantle and is a rigid material which will break when deformed.
- The asthenosphere is composed only of upper mantle and will flow like warm wax when deformed.
The density of an object is the ratio of its mass to its volume and in general describes how tightly packed the atoms in a material are. The formula for density is:

\[
\rho = \frac{m}{v} = \text{mass/volume}
\]

A good way to understand density is to think of two cubes with the exact same volume, one cube is made of foam the other of iron as shown in the figure below. Which cube would “feel” heavier?? The iron cube has the larger density (i.e., a larger mass of iron atoms per the same volume cube) and therefore is heavier than the foam cube. If we were to look at those cubes under a microscope we would see that the atoms that make up the iron are much more closely packed into the cube than those of the foam. Therefore we can fit many more iron atoms into the same volume than we can atoms that make up the foam. Since each atom has a certain mass (known as the atomic mass) the more atoms we have in the cube the more mass we have in the cube and thus the larger the density of the cube.
Each layer of Earth has a specific density. However, rocks found at the surface of Earth have a much lower density than that of the whole Earth. What does this mean? Where are the high density materials that increase the average density of the whole Earth? The answer is that the interior layers (mantle, outer core and inner core) become denser with depth. This makes sense if we think about what density is.

As we go deeper into the Earth the pressure increases because the amount of material above increases. This increased pressure helps to pack the atoms of Earth’s inner materials more tightly together thus increasing the density! So the densest layer of Earth is the inner core.
As you should have noticed, the equation for density requires a volume. For the volume of the whole earth this is calculated by using the equation for the volume of a sphere.

\[ V_{sphere} = \left(\frac{4}{3}\right) \pi r^3, \]

Where \( r \) is the radius of the sphere. However, for the volumes of the individual layers the calculation is a little bit more complicated as we must now calculate the volumes of spherical shells. Each spherical shell now has an inner \( (r_1) \) and outer \( (r_2) \) radius, as shown below. The equation for calculating the volume of a spherical shell takes both of these radii into account.

\[ V_{shell} = \left(\frac{4}{3}\right) \pi \left(r_2^3 - r_1^3\right). \]

Where \( r_1 = \) inner radius
\[ r_2 = \text{outer radius} \]
In the next few slides you will use the equations from the previous slides to determine the densities of Earth's layers. You can go back through the module to review the slides you have just read but you will not be able to progress in this module until you have passed this test, however you have an unlimited number of tries. It is a good idea to keep track of your answers as you go along because some questions use values from previous questions.

PROPERTIES
On passing, 'Finish' button: Goes to Next Slide
On failing, 'Finish' button: Goes to Slide
Allow user to leave quiz: At any time
User may view slides after quiz: After passing quiz
User may attempt quiz: Unlimited times
1. To calculate the density of the whole Earth in Slide 13 you used the following equation:

\[ \rho = \frac{\sum M_{\text{shells}}}{\sum V_{\text{shells}}} \]

(the sum of the masses of each layer divided by the sum of the volumes of each layer), which is a weighted average. Now go back to your spreadsheet and calculate the average of the layer densities from Column G (excluding the whole Earth) using Excel’s AVERAGE() function. How do the two densities you calculated compare? Explain this discrepancy.

2. Now calculate the density of the Moon using the table of information given below. How does the density of the moon compare to that of the Earth? Does this make sense, given how the Moon was formed?

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (km)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Moon</td>
<td>1738</td>
<td>7.35 x 10^{22}</td>
</tr>
<tr>
<td>Crust</td>
<td>65</td>
<td>11</td>
</tr>
<tr>
<td>Mantle</td>
<td>1673</td>
<td>89</td>
</tr>
</tbody>
</table>

3. Another example of a weighted average is the calculation of a weighted grade. Imagine that in a course the grade is based on 4 exams and 4 assignments. The exams are worth 60% and the assignments are worth 40%. On the exams you receive a 45/50, 47/50, 20/50, and 49/50. On the assignments you receive a 10/20, 20/20, 15/20, and 18/20. Calculate the final grade you will receive in the course using the following equation, where \( w_i \) is the weighting factor and \( x_i \) is the average grade for either the exams or assignments.

\[ \text{Grade} = \frac{\sum w_i x_i}{\sum w_i} \]