An Example: The Evolution of a Key Assignment in Calculus I

For several years, the mathematics faculty teaching Calculus I have enjoyed using a classic allometry problem from Sir D'Arcy Wentworth Thompson's *On Growth and Form* (Thompson 1992), written in 1917, revised by the author in 1942, and published as a paperback in 1992. Professor Thompson describes the growth, measured by the short diameter of the shell, of the window-pane oyster from Sri Lanka (then Ceylon): he provides data and a graph of the shell diameter over time. We ask the students to find a mathematical model for the data, a function which describes and predicts the growth as accurately as possible. Here we describe how the assignment itself has changed over the years in order to better reflect learning outcomes.

Stage 1: An exam question for program assessment

In the earliest form of this problem, we included it as a question on the final exam. We gave the students the data and three graphs: a Cartesian graph with linear scales on each axis, a semilog graph (logarithmic scale on the y-axis), and a log-log graph. Using that information, they were to choose the type of function that would best fit the data and then find such a function. (Students had access to a calculator, but not necessarily to a computer or graphics software.) Note that in this course we did not allow students to use built-in least-squares regression tools. We focus throughout the course on estimating parameters and improving the model by adjusting them.

We could then ask the students to use their model to predict the oyster's diameter after ten years, for example. The question tested the students' knowledge of linear, exponential and power functions and their behavior; the students' computational fluency and accuracy in finding an appropriate model; and their ease in using the model to make predictions. Students have learned to evaluate the fit of the model both graphically and numerically. They might also have been asked to briefly explain their answers but, as one question on a timed exam, there was not much time to write thoughtfully or carefully. We used this question as part of our program assessment for the mathematics major for several years, as it incorporated several important skills we hoped our majors would master.

Stage 2: A homework assignment for QL assessment

Calculus I is also a course which satisfies the QL Core Curriculum requirement, and we quickly realized that, with slight changes, we could use this assignment to assess every one of the QL learning outcomes. We changed the assignment from an exam question to a homework problem and gave students access to a computer, including the use of Maple[1] software for computation, graphics, and text. We continued to avoid using built-in regression tools so that students would have to focus on estimating parameters. In this incarnation of the assignment, used in 2015 and 2016, we addressed all of the QL learning outcomes:

- Students use **quantitative data** from a real problem in mathematical biology and must interpret it.
- Students use **technology** for **computation**; they must develop an accurate mathematical model.
- Students must decide what kind of function fits the data best and tell why: they **create arguments based on the data given them.**
- This time we just give students the data in a table. And so they must **create appropriate graphs themselves and communicate their arguments using them**: Cartesian, semilog, log-log to choose the type of model, and then a Cartesian graph showing their model alongside the original data to illustrate the accuracy of their model. They are also using **a data table and mathematical expressions** to develop and evaluate their model.
- Now that this is a homework assignment, we can ask students to take more time to describe in words what they have done – so they are indeed **communicating their arguments through the narrative analysis.**

Both the assignment and a typical (good) student submission are included in Appendix A of this document. The assignment consisted of a list of step-by-step instructions, asking students to enter the data in Maple and create a scatterplot, determine an appropriate function to model the data, graph the model along with the original data, and predict the diameter of an oyster shell after 10 and 20 years. Finally, students were asked to write a paragraph explaining what they had done.

In general, students dutifully followed the instructions and developed reasonable models. Thus, the instructor for the course felt obliged to assign the student whose work appears in Appendix A a Level 4 (Advanced) score in each category. However, after reviewing the student submissions, the instructor realized that this apparently exemplary submission was still not quite what we wanted. Many students still didn’t seem to “get” what they were being asked to do. The instructor wanted to make that clearer, but at the same time she also wanted to encourage them to think a little more for themselves and figure out what to do. She says:

> *I wanted to try to get away, a bit, from "step by step" instruction where I tell them to make a graph, then find a model, then answer these questions, and instead try to move more towards "Here is some data and here is a question I want you to try and answer. Go." I didn't, by any means, go all the way there, as you'll see in the 2017 instructions, mainly because I wanted to give the students some idea of my expectations so that I wouldn't end up grading rubbish. For example, I told them explicitly that work in Maple*

[1] https://www.maplesoft.com/

was required and that they should make an effort to look for a model. (I was picturing having folks just guessing what the diameter would be in 10 years.) I think I could push this a little more and say even less to the students. Maybe I'll be brave enough to leave out those explicit instructions in the future. This may be something we can discuss.

She also wanted to emphasize the narrative analysis part of the assignment:

With the old version of the assignment, I found it difficult to score students on their communication skills. Although we were explicitly telling the students that they were going to be assessed in their communication skills, I found that many of them (anecdotally speaking) didn't take this seriously. I thought that if I asked the students to write a report, then at least they knew I was serious about considering their communication skills carefully. I also built up to the QL assessment report-writing with more intentional instruction in report-writing in class, feedback on previous reports, etc. None of what I did in class leading up to the assessment was any different than previous semesters in terms of the materials I used, but I, personally, put more emphasis on the mechanics of report-writing in my own class.

Completing the assessment cycle: An improved homework assignment

And so we come to a new, improved Key Assignment, deployed for the first time in Fall 2017. Instead of being given explicit step-by-step instructions ("Enter these data into Maple, create a scatterplot, be sure to label the axes, describe what kind of model you think might fit the data…"), students were told:

A team of biologists is studying the growth of the shells of the window-pane oyster (found in Sri Lanka). The biologists have asked for the help of MATH 201 students at Hood College. *They would like you to give them a reasonable prediction for the diameter of an oyster's shell after 10 years and after 20 years.*

And then they were pretty much turned loose to figure out what to do. The final product was a report, typed in Microsoft Word, describing their work and defending their results. It no longer looked like a solution to a textbook homework problem, with answers to Part a, Part b, etc. without any context; it was a more sophisticated product that focused more on narrative analysis, while still addressing the other student outcomes. The assignment itself and an exemplary student submission may be found in Appendix B of this document.

The instructor also assigned this student's work a Level 4 in each category. While it provides basically the same information as the 2015 student work, it is much more polished, and it illustrates the type of report writing we encourage in our students. The instructor, and the Department, were much happier with the new assignment and the quality of the submissions we got from students.

Appendix A: MATH 201 Calculus I, Fall 2015

Part 1: The Assignment

Open a new Maple file and write your name and date at the top. You will use Maple to answer the following questions.

The table below contains data describing the growth of the shells of window-pane oysters (found in Sri Lanka) from page 166 of **ON GROWTH AND FORM** by D'Arcy Wentworth Thompson:

Time (Years)	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
Diameter (mm)	50	75	90	100	110	118	125	132	139	146	152	158	164	170

a) Enter these data into Maple and create a scatterplot. Include appropriate axis labels and a title for the graph.
b) What kind of function do you think will model these data? Use what you have learned in this class to determine an appropriate model. Show all of your work, and include comments in text to explain what you are doing.
c) Graph your model along with the original data and comment on how well your model captures the trend of the data. Make sure your graph has appropriate labels and a title.
d) Use your model to predict an oyster shell's diameter after 10 years; after 20 years.
e) In text mode in Maple, write a paragraph about what you have discovered by completing this assignment. You will be assessed on your communication skills as well as on the accuracy of your mathematics.

☞ ***Export your file as a pdf and submit that file via the Chalk and Wire link on Blackboard.***

Do not delete the files from your network drive until we let you know that the assessment is completed.

Part 2: Student Submission, Fall 2015

a)

$Time := [0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7]$

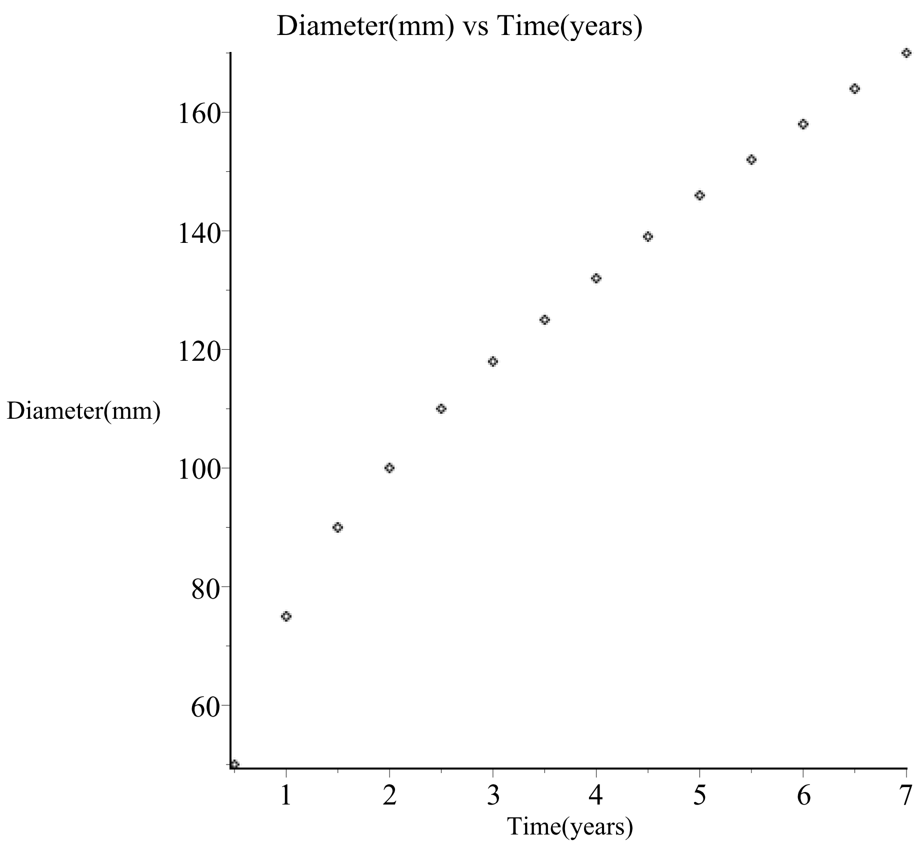
$$[0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7] \quad (1)$$

$Diameter := [50, 75, 90, 100, 110, 118, 125, 132, 139, 146, 152, 158, 164, 170]$

$$[50, 75, 90, 100, 110, 118, 125, 132, 139, 146, 152, 158, 164, 170] \quad (2)$$

$with(plots):$

$plot(Time, Diameter, style = point)$

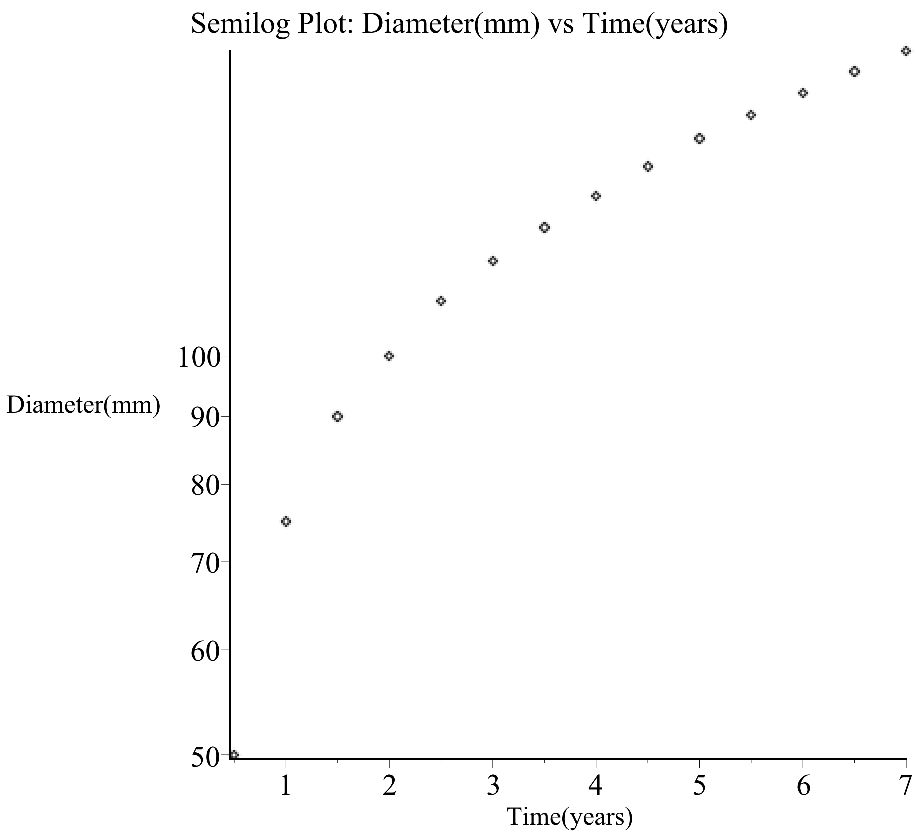


b)

To show what kind of function will model the data I will graph both a semilog plot and a loglog plot of the data. Whichever graph shows a linear display of the data will determine whether the function that models the data is an exponential function or a power function.

First the semilog plot:

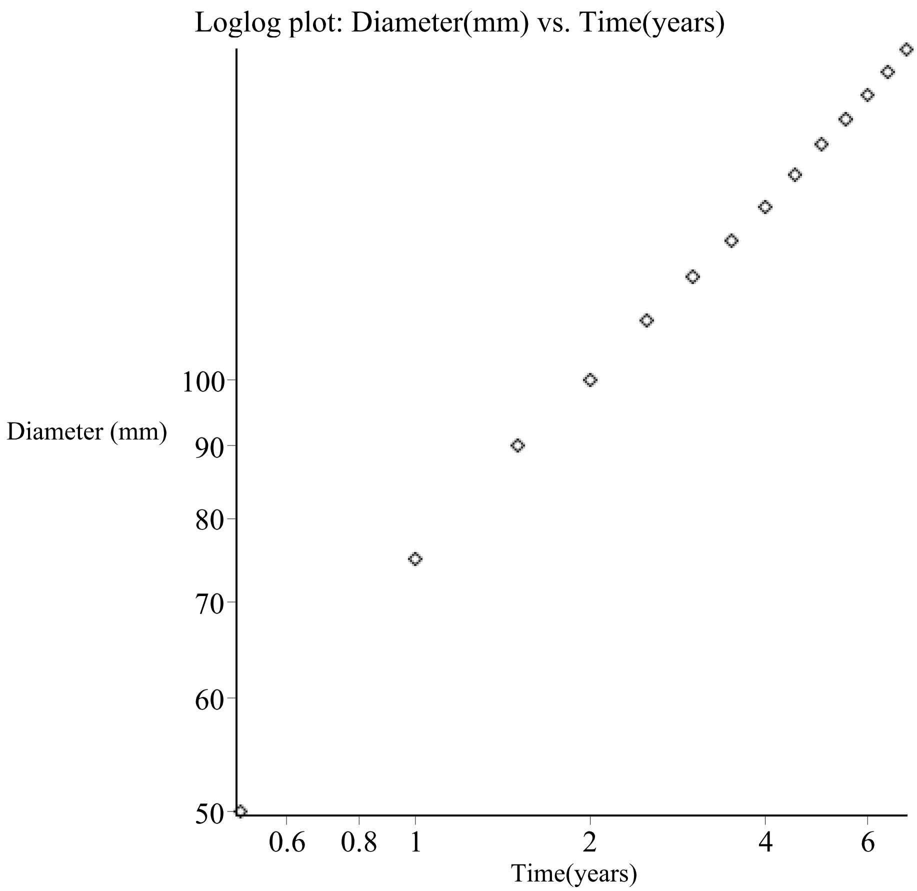
logplot(*Time*, *Diameter*, *style* = *point*)



We can see in this graph that the semilog plot does not show a linear, or straight line, representation of the data, there is a clear curve in the data.

Now the loglog plot:

loglogplot(*Time*, *Diameter*, *style* = *point*)



We can see in this graph that the data looks to be linear and form a straight line. From this we know that the type of function that will model the data will be a power fucntion.

c)

The general power function: f = constant*variable^power

$p := \frac{(\ln(170) - \ln(50))}{(\ln(7) - \ln(0.5))}$

$$\frac{\ln(170) - \ln(50)}{\ln(7) + 0.6931471806} \quad (3)$$

$c := \frac{50}{0.5^p}$

$$\frac{50}{0.5^{\frac{\ln(170)-\ln(50)}{\ln(7)+0.6931471806}}} \tag{4}$$

$$\frac{50}{0.5^{\frac{\ln(170)-\ln(50)}{\ln(7)+0.6931471806}}} \tag{5}$$

$f := t \to c \cdot t^p$

$$t \to c\, t^p \tag{6}$$

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dataGraph := plot(Time, Diameter, style = point) :
modelGraph := plot(f(t), t = 0.5 ..7, thickness = 2, color = blue) :
display(dataGraph, modelGraph)
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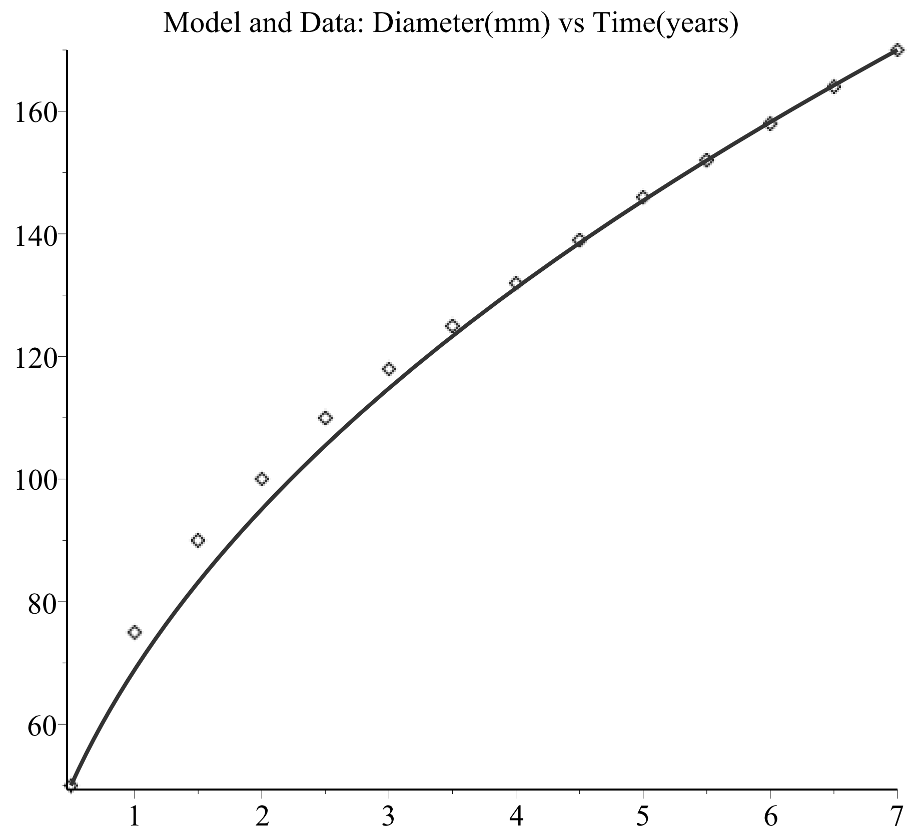


My model captures the trend of the data moderately well. The model goes through a little over half of the points. Through exploration using different numbers to calculate the value of p the ones that I did use modeled the data the best. Although the model could be better it seems to be more accurate as time goes on.

d)
$f(10)$

$$\frac{50\, 10^{\frac{\ln(170) - \ln(50)}{\ln(7) + 0.6931471806}}}{0.5^{\frac{\ln(170) - \ln(50)}{\ln(7) + 0.6931471806}}} \quad \textbf{(7)}$$

$\xrightarrow{\text{at 10 digits}}$

$$200.5762813 \quad \textbf{(8)}$$

$f(20)$

$$\frac{50\, 20^{\frac{\ln(170) - \ln(50)}{\ln(7) + 0.6931471806}}}{0.5^{\frac{\ln(170) - \ln(50)}{\ln(7) + 0.6931471806}}} \quad \textbf{(9)}$$

$\xrightarrow{\text{at 10 digits}}$

$$276.6127954 \quad \textbf{(10)}$$

e)
Through this assignment I have discovered that the type of model that best represents the diameter growth in the shells of oysters is best modeled by a power function. I learned how to better use Maple to my advantage to be able to visualize different type of equations, and understand how to use the semilog and loglog plot of a function to determine what kind of model will besr fit the data. I showed through the use of the Maple software my knowledge on the subject of equations and models, and I effctively show through the use of graphs which type of function best models the data/

I pledge that I have neither given nore recieved unauthorized aid on this assignment.

Appendix B: MATH 201 Fall 2017

Part 1: The Assignment

A team of biologists is studying the growth of the shells of the window-pane oyster (found in Sri Lanka). The biologists have asked for the help of MATH 201 students at Hood College. *They would like you to give them a reasonable prediction for the diameter of an oyster's shell after 10 years and after 20 years.*

The table below contains data describing the growth of the shells of window-pane oysters. (These data are from page 166 of **ON GROWTH AND FORM** by D'Arcy Wentworth Thompson.)

Time (Years)	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
Diameter (mm)	50	75	90	100	110	118	125	132	139	146	152	158	164	170

Part I: Find an appropriate model for the data in the table above.

Use Maple and show all of the work that you do in Maple and by hand to come up with an appropriate model for the given data. Use your model to make the predictions that were asked for by the biologists – for the diameter of an oyster's shell after 10 years and after 20 years.

- Work in Maple is REQUIRED for this part of the assignment.
- All work, including graphs, must be shown to receive full credit.
- For full credit, your model should fit the data well and you should demonstrate that it fits well.
- Save your Maple file somewhere safe for submission later.

Part II: Write a report on your process and your findings.

<u>Do NOT assume</u> that the person who is reading your report will have read your work

from Part I. Give enough detail in your report so that the person reading it is

- clear on the goal of the project,
- can understand the mathematics you used to solve the problem, and
- is convinced that the answer you have provided to the problem is reasonable.

Your report should be written in Microsoft Word in a similar style to the reports you have been writing for this class all semester, with a similar level of detail expected. You should refer to the Writing subfolder in the Course Documents tab.

Submit: by 10:30am on Wednesday November 8^{th}, 2017

1. All of your Maple work and work by hand for Part I ON PAPER, stapled together
2. Your Maple file submitted electronically via Dropbox.
3. A printed copy of your report for Part II and
4. A copy of your report as a PDF file submitted electronically in Dropbox
5. We may ask you to submit some or all of this work via Chalk & Wire also. Stay tuned for further directions, but in the meantime, save all of your work.

Part 2: Student Submission, Fall 2017

Student name

11/03/17

MATH 201_01

Quantitative Literacy Assignment

Oyster Growth

The purpose of this lab was to find a model that represents the growth of the window-pane oyster shell. Through graphing and analyzing the data, I found that the oyster shell growth is best modeled as a power function which takes the form

$$f(t) = 75.191(t^{0.41288})$$

where 75.191 represents a constant, t represents time in years, and 0.41288 represents the initial size of the oyster shell at time 0.

The shells diameter was measured every half year for seven years beginning at the half year mark.

Table 1: Growth of window-pane oyster over 7-year period

Time (Years)	*Diameter (mm)*
0.5	50
1	75
1.5	90
2	100
2.5	110
3	118
3.5	125
4	132
4.5	139
5	146
5.5	152
6	158
6.5	164
7	170

To begin I graphed the data as a scatterplot and observed that that the data was not linear which ruled out that the data should be modeled linearly.

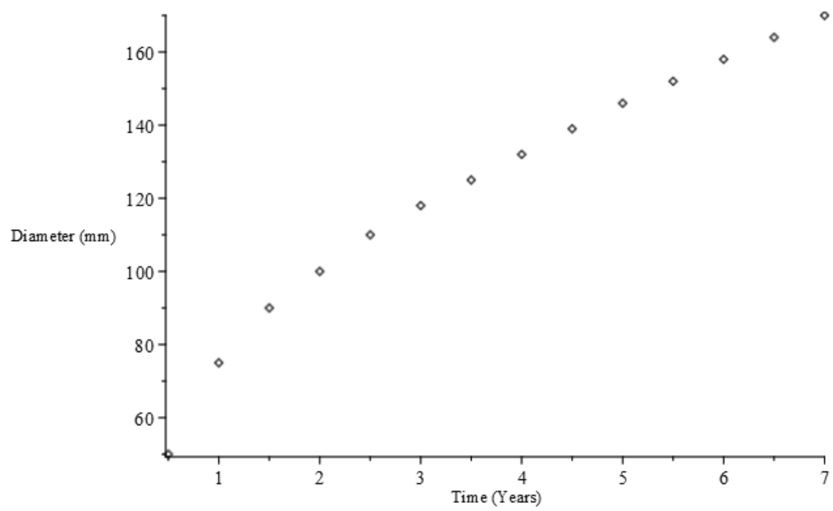


Figure 1: Growth of window-pane oyster over 7-year period

I then graphed both the log of the diameter against the years, and the log of the diameter against the log of the years. I observed that the data was linear when plotted on a log-log scale, which means that the data is best modeled as a power function.

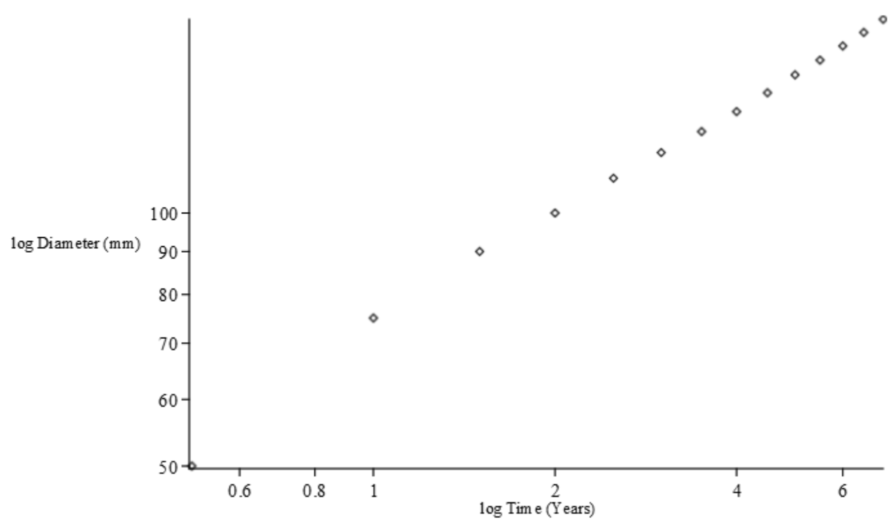


Figure 2: Growth of window-pane oyster over 7-year period on a log-log scale

The basic formula of a power function is:

$$f(t) = c(t^q)$$

where c represents a constant, t represents time, and q represents the initial diameter of the shells at time 0. Since the data doesn't give the diameter of the shell at time 0, I first had to find the value of q by choosing two data points and finding the difference of their natural logs

$$q = \frac{\ln 170 - \ln 90}{\ln 7 - \ln 1.5}$$

which is 0.41288. I then solved for c by choosing another point of data and plugging it into the power function formula to solve for c.

$$152 = c(5.5^{0.41288})$$

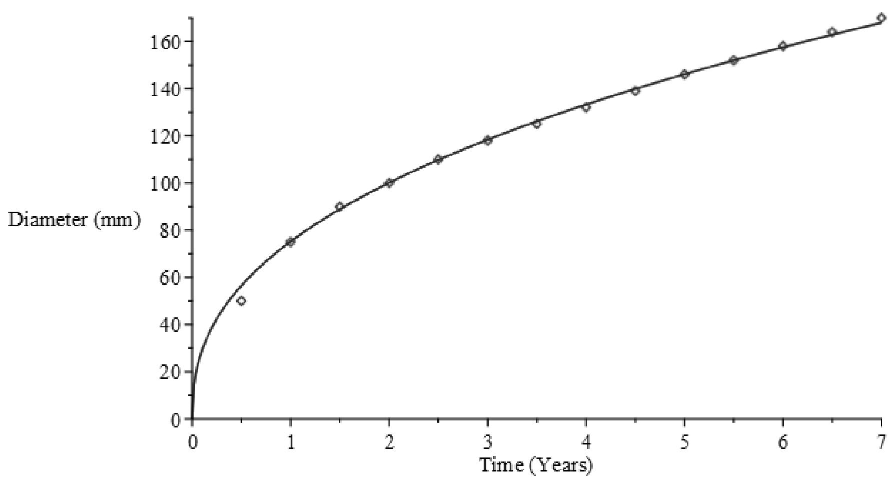
Which means that

$$c = \frac{152}{5.5^{0.41288}}$$

This means that the model that best suits the growth of the window-pane oyster is

$$f(t) = 75.191(t^{0.41288})$$

When this model is graphed along with values the researchers initially observed they appear almost identical.



With this model, I'm confident in predicting that after 10 years the oyster shell will have a diameter of 194.556 mm, and after 20 years, 259.021 mm. I'm confident my model would be accurate to the tenths place due to how well it fits the original data points.