

## **ATTENTION:**

Your grade for this assignment is based on your answers to the end-of-module questions AND the post-test. If you DO NOT take the post-test you will NOT be eligible for full points.

The module itself will take approximately 2 hours to complete. Please do not leave this until the last minute.

## **What to do if you get locked out during a quiz**

If you enter an incorrect answer and are then unable to try to re-enter a new answer do the following to unlock the module: exit the module completely by closing the window, re-open the module, when asked if you would like to start where you left off say 'no'. You will have to re-enter all of your quiz answers again, however this will unlock the module for you.

# Building Mountains: Isostasy



Photo from USGS

## Core Quantitative Literacy Topics

Arithmetic, algebra

## Core Geoscience Subject

Mountain building, isostasy, buoyancy

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# Getting Started

Late on the night of April 14, 1912 the lookouts on the RMS Titanic sounded the alarm that an iceberg lay directly ahead of the ship. Despite this warning and the evasive maneuvers that the ship's First Officer deployed, the Titanic struck the iceberg and eventually sank. This resulted in a massive loss of lives and the complete loss of the ship beneath the waves of the frigid Atlantic Ocean.

The damage done by the iceberg was below water level along the ship's hull and occurred so far beneath the surface of the ocean because the majority of the mass of any iceberg actually lies beneath the ocean, hidden by the waves. This is the result of buoyancy and isostasy, which also both turn out to play a major role in the formation of mountain ranges both on land and under the sea. We will explore these concepts in detail in this module.



After completing this module you should be able to:

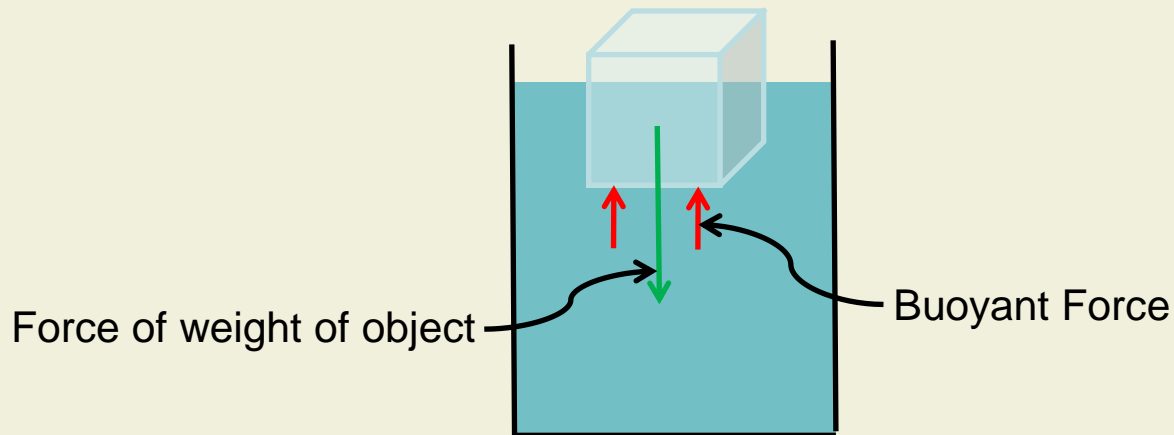
- Calculate the forces responsible for isostasy.
- Calculate the thickness of roots and crust and the heights of mountain ranges.
- Understand the concept of isostasy and what role it plays in mountain building.

# Buoyancy and Isostasy

Imagine taking an ice cube and placing it in a glass of water. The ice cube will float. But what exactly makes it float?? It is a concept called buoyancy, which is an upward acting force exerted by the fluid (in this case water) that exactly equals the downward directed force of the weight of the object (in this case the ice cube) as shown in the figure below. Since the density of the ice is less than the density of the water, a portion of the ice cube remains above the surface of the water while the rest is below the surface of the water.

When you place an ice cube in a glass of water the level of the water in the glass is raised. This is because the ice cube displaces it's own weight in water. This concept was first recognized by the Greek philosopher Archimedes who lived from 287-212 B.C.

These same concepts apply to the Earth and its materials. Mountain ranges for example “float” on the mantle, which is called isostasy. Throughout this module we will be focusing on the concepts of buoyancy and isostasy by using simple block models and more complex and realistic models of isostasy.



# Buoyancy: A balance of forces

To understand how buoyancy works, imagine a block floating in water. Gravity pulls down on the block with a force ( $F_g$ , green arrow in figure below) equal to the weight of the block, which is equal to the block's mass times the gravitational constant  $g = 9.8 \text{ m/s}^2$ .

$$F_g = m * g$$

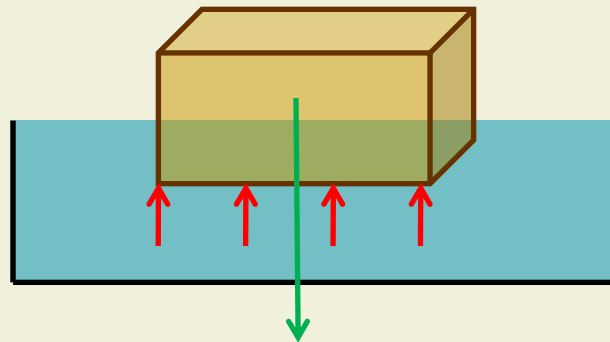
Since the block is floating (and did not sink) we know that some buoyant force is holding the block up. The buoyant force ( $F_B$ , red arrows in figure below) is an upward directed force exerted by the fluid and is equal to the density of the fluid ( $\rho_f$ ) times the volume of fluid displaced by the block ( $V_d$ ) times the gravitational constant ( $g$ ).

$$F_B = \rho_f * V_d * g$$

If the block is floating stationary (i.e., not rising out of the water or sinking into the water) we know that the force of gravity on the block is exactly equal to the buoyant force.

$$F_g = F_B$$

$$m * g = \rho_f * V_d * g$$



# Buoyancy: A balance of forces

What we really care about in this situation is the height of block above the surface of the water ( $h_m$ ) and the height of block below the surface of the water ( $h_r$ ). Like the case of the iceberg, how much block can we see, and how much is lurking beneath the surface. If we know the exact dimensions of the block (width, length, and height) and the densities of both the block and the fluid then we can derive the equations for both  $h_m$  and  $h_r$ . So we start with the equation from the last slide and derive the equations we need.

$$m * g = \rho_f * V_d * g$$

$$m = \rho_f * V_d$$

$$\rho_b * V_b = \rho_f * V_d$$

$$\rho_b * A_b * h = \rho_f * A_b * h_r$$

$$\rho_b * h = \rho_f * h_r$$

$$h_r = (\rho_b * h) / \rho_f$$

$$F_g = F_B$$

g's cancel out

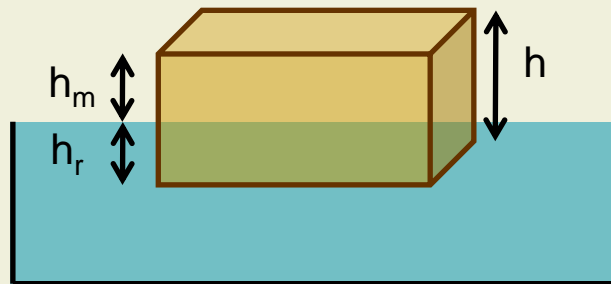
Substitute  $m = \rho_b * V_b$  ( $\rho_b$  = density of block)

$V_b = A_b * h$  ( $V_b$  = volume of block,  $A_b$  = area of block)

$V_d = A_b * h_r$  ( $V_d$  = volume of fluid displaced by block)

$A_b$ 's cancel out

Solve for  $h_r$



Return to [Slide 8](#).

# Buoyancy: A balance of forces

So, now we just have to solve for  $h_m$  and we're set. Looking at the diagram below we can see that the height ( $h$ ) equals the height above the surface ( $h_m$ ) plus the height below the surface ( $h_r$ ). So.....

$$h = h_m + h_r$$

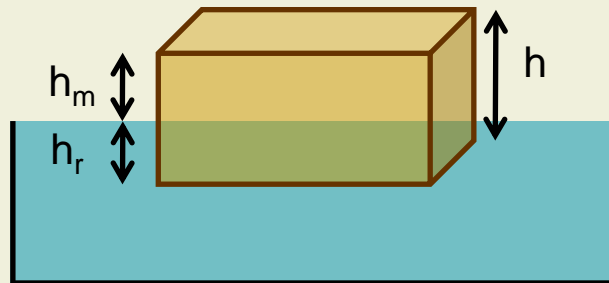
$$h_m = h - h_r$$

rearrange

$$h_m = h - [(\rho_b * h) / \rho_f]$$

Substitute  $h_r = (\rho_b * h) / \rho_f$

And now we're ready to start working through some problems.



Return to [Slide 8](#).

# Problem 1: Blocks in water

To begin we'll look at the very simple case of a block of cork floating in water. For this case we will use the equations we calculated in [Slide 6](#) and [Slide 7](#). Imagine a block of cork that is 20 cm high floating in a container of water, the water has a density of 1 g/cm<sup>3</sup> and the cork is 0.24 g/cm<sup>3</sup>. What is the height of the cork block above the surface of the water? Perform your calculations in Section 1 of the worksheet provided.

	A	B	C	D	E	F	G
1	#	Points		Enter answers in gray areas (text), or orange cells (formulas)			
2							
3				Cork block in water		Pine block in water	
4				Density cork (g/cm <sup>3</sup> )	0.24	Density pine (g/cm <sup>3</sup> )	0.53
5				Density fluid (g/cm <sup>3</sup> )	1	Density fluid (g/cm <sup>3</sup> )	1
6				Height block (cm)	20	Height block (cm)	20
7	1	3		$h_r$ (cm)		$h_r$ (cm)	
8				$h_m$ (cm)		$h_m$ (cm)	
9				Which block sits higher in the water, cork or pine? What does this tell you about the effect of density on the height a block sits above the fluid surface? What would happen to the height if you used a block of oak (a more dense wood)? What about iron?			
10							
11							
12							

See list of equations on [Slide 31](#) or Return to [Slide 13](#).



# Problem 1: Blocks in water

Now let's do the same thing but this time the block is made of pine and the density of pine is  $0.53 \text{ g/cm}^3$ . Repeat the steps from the previous to calculate the heights  $h_r$  and  $h_m$ .

	A	B	C	D	E	F	G
1	#	Points		Enter answers in gray areas (text), or orange cells (formulas)			
2							
3				Cork block in water		Pine block in water	
4				Density cork ( $\text{g/cm}^3$ )	0.24	Density pine ( $\text{g/cm}^3$ )	0.53
5				Density fluid ( $\text{g/cm}^3$ )	1	Density fluid ( $\text{g/cm}^3$ )	1
6				Height block (cm)	20	Height block (cm)	20
7	1	3		$h_r$ (cm)		$h_r$ (cm)	
8				$h_m$ (cm)		$h_m$ (cm)	
9				Which block sits higher in the water, cork or pine? What does this tell you about the effect of density on the height a block sits above the fluid surface? What would happen to the height if you used a block of oak (a more dense wood)? What about iron?			
10							
11							
12							

Which block sits higher in the water, cork or pine? What does this tell you about the effect of density on the height a block sits above the fluid surface? What would happen to the height if you used a block of oak (a more dense wood)? What about iron?

See list of equations on [Slide 31](#) or Return to [Slide 13](#) or [Slide 32](#).

# Isostasy - Problem 1

Question 1 of 4 ▾

What is the value for  $h_r$  for the cork block?

\*Remember you can always  
go back and review material

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## Problem 2a: Stratified blocks in water

Now let's make the scenario a bit more complicated. What if the blocks were composed of two different types of materials like in the figure below? The materials have densities  $\rho_1$  and  $\rho_2$  and heights  $h_1$  and  $h_2$ . We still have  $h_m$  as the height of the whole block above the surface of the fluid and  $h_r$  the height of the whole block below the surface of the fluid. All we have to do now is modify our equations from Problem 1 to include two different materials as opposed to one.

$$(m_1 * g) + (m_2 * g) = \rho_f * V_d * g$$

$$m_1 + m_2 = \rho_f * V_d$$

$$(\rho_1 * V_1) + (\rho_2 * V_2) = \rho_f * V_d$$

$$(\rho_1 * A_b * h_1) + (\rho_2 * A_b * h_2) = \rho_f * A_b * h_r$$

$$(\rho_1 * h_1) + (\rho_2 * h_2) = \rho_f * h_r$$

$$h_r = [(\rho_1 * h_1) + (\rho_2 * h_2)] / \rho_f$$

$$F_{g \text{ of material 1}} + F_{g \text{ of material 2}} = F_B$$

$g$ 's cancel out

Substitute  $m_1 = \rho_1 * V_1$  and  $m_2 = \rho_2 * V_2$

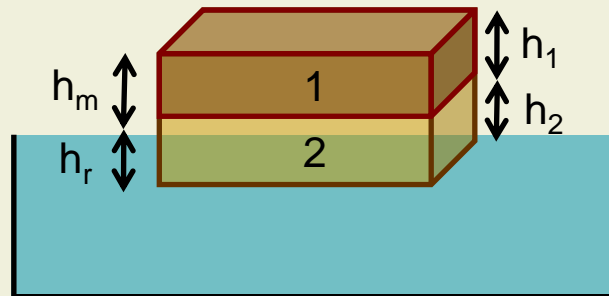
$$V_1 = A_b * h_1 \text{ (volume of material 1)}$$

$$V_2 = A_b * h_2 \text{ (volume of material 2)}$$

$$V_d = A_b * h_r \text{ (volume of fluid displaced by block)}$$

$A_b$ 's cancel out

Solve for  $h_r$



## Problem 2a: Stratified blocks in water

Now we just have to solve for  $h_m$  like before. Looking at the diagram below we can see that the sum of the heights  $h_1$  and  $h_2$  equals the height above the surface ( $h_m$ ) plus the height below the surface ( $h_r$ ). So.....

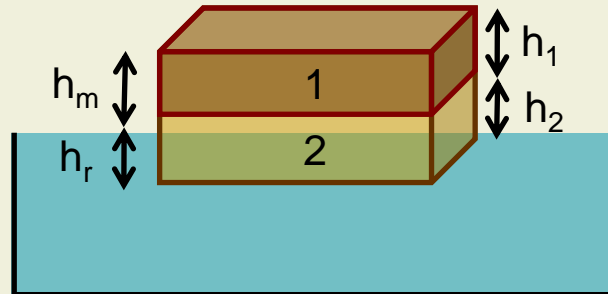
$$(h_1 + h_2) = h_m + h_r$$

$$h_m = (h_1 + h_2) - h_r$$

rearrange

$$h_m = (h_1 + h_2) - [(\rho_1 * h_1) + (\rho_2 * h_2)] / \rho_f$$

Substitute  $h_r = [(\rho_1 * h_1) + (\rho_2 * h_2)] / \rho_f$



Return to [Slide 17](#).

# Problem 2a: Stratified blocks in water

Now that we know how to deal with the stratified block scenario let's test it out. Assume you have two blocks, one is composed of cork and oak, the other pine and oak. The density of the cork is the same as in [Slide 8](#) and the density of the oak is  $0.93 \text{ g/cm}^3$ . The thickness of the cork ( $h_1$ ) is 20 cm and the oak ( $h_2$ ) is 50 cm. Calculate the height ( $h_r$ ) below the water's surface and the height ( $h_m$ ) above the water's surface.

Then do the same for the pine and oak block, where the density of the pine is the same as you used in [Slide 9](#) and  $h_1$  (pine) is 20 cm and  $h_2$  (oak) is 50 cm. Again, perform your calculations in Section 2a of your worksheet.

15			Stratified Block: Cork and Oak		Stratified Block: Pine and Oak	
16			Density cork ( $\text{g/cm}^3$ )	0.24	Density pine ( $\text{g/cm}^3$ )	0.24
17			Density oak ( $\text{g/cm}^3$ )	0.93	Density oak ( $\text{g/cm}^3$ )	0.93
18			Density fluid ( $\text{g/cm}^3$ )	1	Density fluid ( $\text{g/cm}^3$ )	1
19			$h_1$ (cm)	20	$h_1$ (cm)	20
20	2a	4	$h_2$ (cm)	50	$h_2$ (cm)	50
21			$h_r$ (cm)		$h_r$ (cm)	
22			$h_m$ (cm)		$h_m$ (cm)	

See list of equations on [Slide 31](#) or Return to [Slide 32](#).

Which sits higher in the water? The cork and oak block or the pine and oak block? How do the values of  $h_m$  you calculated here compare to those you calculated in [Slide 9](#)? What did adding the 50 cm thick piece of oak do to  $h_m$ ? Explain this.

# Isostasy - Problem 2a

Question 1 of 4 ▾

What is the value for  $h_r$  for the cork and oak block?

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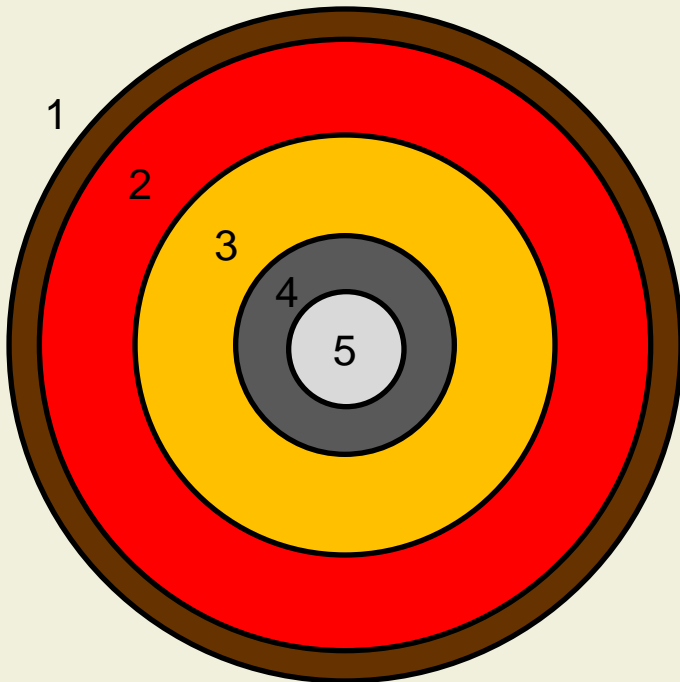
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# Chemically different layers

The Earth consists of two types of distinct layers: chemically distinct layers and physically distinct layers. The chemically distinct layers are the crust, mantle, and core which have distinct chemical compositions. The crust is the topmost layer and is broken into continental crust and oceanic crust. Beneath the crust is the mantle, which makes up the largest volume of the whole Earth by far and consists of an upper and lower mantle. Finally, the core is composed of an outer core and an inner core. The differences in chemical composition also mean that each layer has a very distinct density.



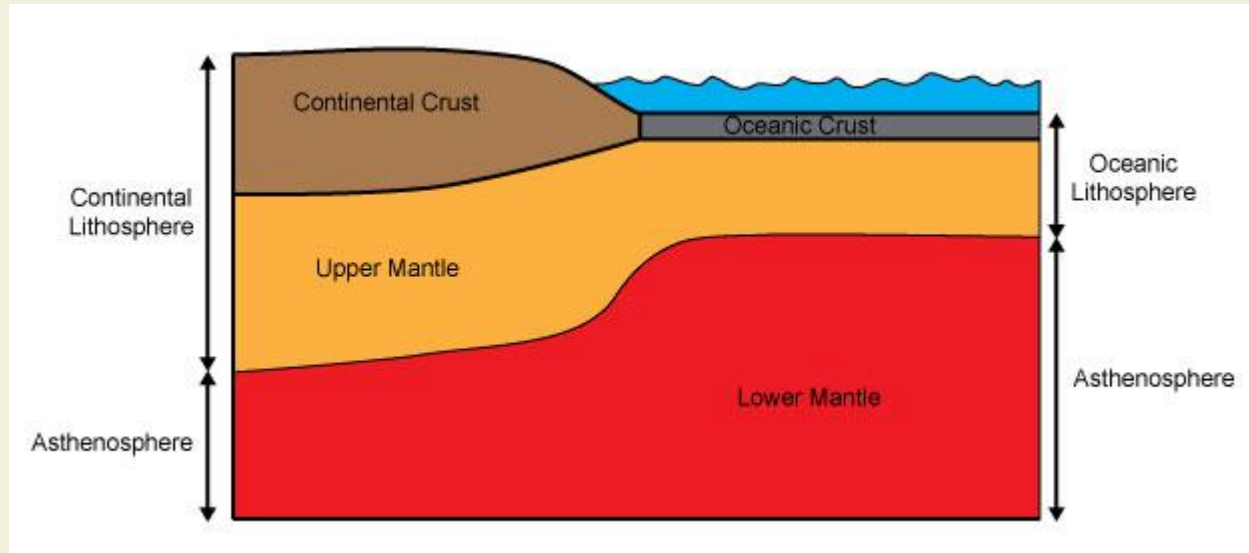
#	Layer	Density
1	Continental Crust	2.5 g/cm <sup>3</sup>
1	Oceanic Crust	3.0 g/cm <sup>3</sup>
2	Upper Mantle	3.3 g/cm <sup>3</sup>
3	Lower Mantle	4.4 g/cm <sup>3</sup>
4	Outer Core	11.3 g/cm <sup>3</sup>
5	Inner Core	16.4 g/cm <sup>3</sup>

Return to [Slide 16](#).

# Physically different layers

The lithosphere and asthenosphere are the two physically distinct layers and vary based on their rigidity, i.e., whether they bend or flow when a load is placed on them. The asthenosphere is the more ductile of the two layers, with a consistency similar to warm wax which will flow when a load is placed on it. The lithosphere is the more brittle layer and will bend when a load is placed on it.

The asthenosphere is composed entirely of lower mantle, while the lithosphere is composed of both crust and upper mantle. Since there are two types of crust, there are also two types of lithosphere: the thicker continental lithosphere (continental crust and upper mantle) and the thinner oceanic lithosphere (oceanic crust and upper mantle).



Remember from [Slide 15](#) that continental crust and oceanic crust have different densities, which when coupled with the different thicknesses causes the continental lithosphere to “float” higher than the oceanic lithosphere. Which leads us to our next problem.



# Problem 2b: Stratified Earth blocks

For our next problem we will build on the stratified block floating in fluid model from [Slide 12](#). The only difference here is that we are using very different materials. We'll start with a block of continental lithosphere (continental crust plus upper mantle). Assume that continental crust has a density of  $2.5 \times 10^{12} \text{ kg/km}^3$ , the upper mantle has a density of  $3.3 \times 10^{12} \text{ kg/km}^3$ , and the lower mantle has a density of  $4.4 \times 10^{12} \text{ kg/km}^3$ . Also assume that the lithosphere has a thickness of 150 km, and the continental crust in our block is 70 km thick. Calculate  $h_2$  (km),  $h_r$  (km), and  $h_m$  (km). Perform your calculations in Section 2b in your worksheet.

		Stratified Earth Blocks: Continental Crust		Stratified Earth Blocks: Oceanic Crust	
29					
30		Density continental crust (kg/km <sup>3</sup> )	2.5E+12	Density oceanic crust (kg/km <sup>3</sup> )	3E+12
31		Density upper mantle (kg/km <sup>3</sup> )	3.3E+12	Density upper mantle (kg/km <sup>3</sup> )	3.3E+12
32		Density lower mantle (kg/km <sup>3</sup> )	4.4E+12	Density lower mantle (kg/km <sup>3</sup> )	4.4E+12
33		$h_1$ (km)	70	$h_1$ (km)	10
34	2b	$h_2$ (km)		$h_2$ (km)	
35	4	$h_r$ (km)		$h_r$ (km)	
36		$h_m$ (km)		$h_m$ (km)	

See list of equations on [Slide 31](#).

# Problem 2b: Stratified Earth blocks

Now do the same for a block of oceanic lithosphere (oceanic crust plus upper mantle). Assume a density of  $3.0 \times 10^{12} \text{ kg/km}^3$  for oceanic crust,  $3.3 \times 10^{12} \text{ kg/km}^3$  for the upper mantle, and  $4.4 \times 10^{12} \text{ kg/km}^3$  for the lower mantle. Remember, that oceanic lithosphere is thinner than continental lithosphere, so assume it is 100 km thick, and the oceanic crust is 10 km thick.

29			Stratified Earth Blocks: Continental Crust	Stratified Earth Blocks: Oceanic Crust
30			Density continental crust ( $\text{kg/km}^3$ ) 2.5E+12	Density oceanic crust ( $\text{kg/km}^3$ ) 3E+12
31			Density upper mantle ( $\text{kg/km}^3$ ) 3.3E+12	Density upper mantle ( $\text{kg/km}^3$ ) 3.3E+12
32			Density lower mantle ( $\text{kg/km}^3$ ) 4.4E+12	Density lower mantle ( $\text{kg/km}^3$ ) 4.4E+12
33			$h_1$ (km) 70	$h_1$ (km) 10
34	2b	4	$h_2$ (km)	$h_2$ (km)
35			$h_r$ (km)	$h_r$ (km)
36			$h_m$ (km)	$h_m$ (km)

Explain what these results mean. What does  $h_r$  represent? What about  $h_m$ ? Do you think the numbers are reasonable based on what you already know about the different types of “floating” lithosphere? Why or why not?

See list of equations on [Slide 31](#) or Return to [Slide 32](#).

# Isostasy - Problem 2b

Question 1 of 6 ▾

What is the value for  $h_2$  for the block of continental crust?

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# Isostasy and mountains

Have you ever wondered why the mountains are high? It turns out it has to do with isostasy. The mountains “float” on top of the mantle. This means that the mountains are in isostatic equilibrium with the denser mantle below.



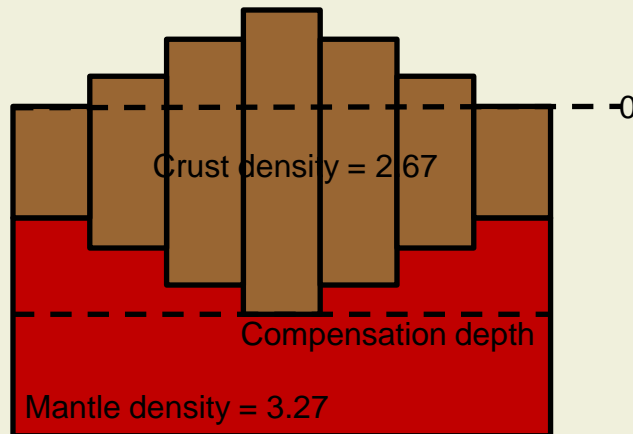
Photo from USGS

But you know from experience (and from the picture to the left) that mountains are not much like perfectly flat blocks. The elevation changes depending upon where you are in the mountain range. This means that our simple model of (and equations for) a block floating in a liquid won't work anymore. We need a better model.

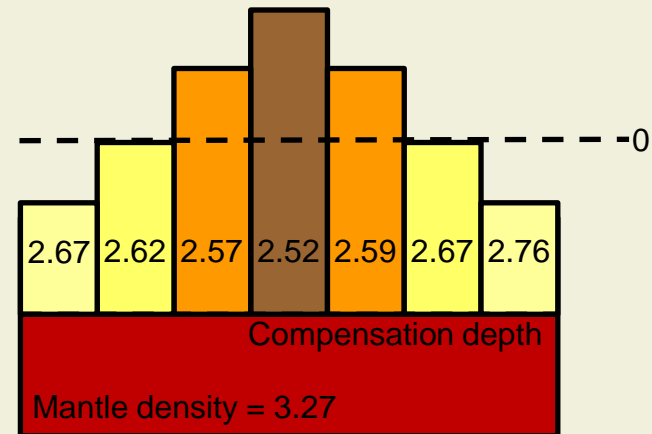
# Models of isostasy

There are two basic models of isostasy: the Airy model and the Pratt Model. The Airy model assumes that the densities of all the rocks that make up the mountain are equal, and that the height of the mountains is compensated for by a crustal root. The Pratt model assumes that the mountains are made of rocks of varying lateral densities (i.e. densities do not change with depth), but there is no crustal root. This means that the rocks at the highest elevations have the lowest densities. For both models the weight of material above some compensation depth is equal everywhere.

**Airy Model**



**Pratt Model**



So which one is “correct”?? We are about to find out!

# The equations: Airy model

First we have to derive the equations we will use. We'll start with the a simplified Airy model, shown in the figure below. We have a mountain made of three blocks of rock, two with height  $h_c$  and one with a height equal to  $h_m + h_c + h_r$ . The density of the mountains is  $\rho_c$  and the density of the mantle is  $\rho_m$ . Remember that the weight of material above the compensation depth is equal everywhere, so we can chose two columns (light gray columns in figure below) of mountain (column 2) or mountain + mantle (column 1) with equal thicknesses anywhere and know that they have equal weights.

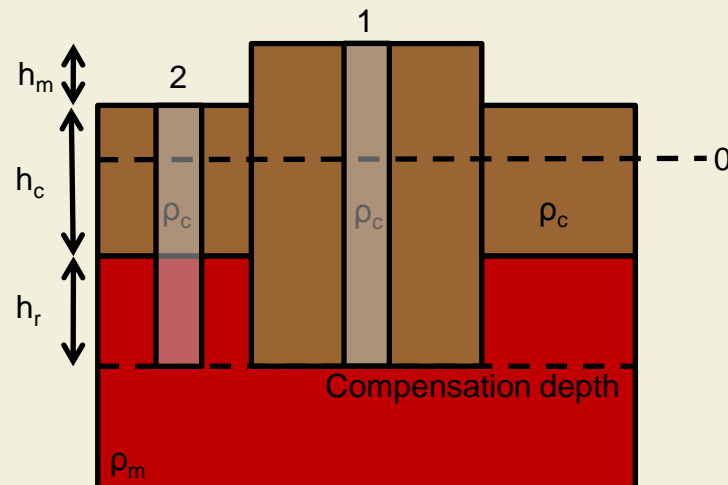
$$(m_1 * g) = (m_2 * g)$$

$$m_1 = m_2$$

Weight of column 1 and column 2 are equal

$g$ 's cancel out

Hmmm....but we don't know  $m_1$  OR  $m_2$ ! So what do we know? Ahh, the densities and heights!



# The equations: Airy model

Remember that  $m = \rho * V$  (mass = density \* volume). So...

$$m_1 = \rho_c * V_{c1}$$

$$m_2 = \rho_c * V_c + \rho_m * V_{m2}$$

$$\rho_c * V_{c1} = (\rho_c * V_{c2}) + (\rho_m * V_{m2})$$

$$\rho_c * A_1 * (h_m + h_c + h_r) = (\rho_c * A_2 * h_c) + (\rho_m * A_2 * h_r)$$

$$\rho_c * (h_m + h_c + h_r) = (\rho_c * h_c) + (\rho_m * h_r)$$

$$(\rho_c * h_m) + (\rho_c * h_c) + (\rho_c * h_r) = (\rho_c * h_c) + (\rho_m * h_r)$$

$$(\rho_c * h_m) = (\rho_c * h_c) + (\rho_m * h_r) - (\rho_c * h_c) - (\rho_c * h_r)$$

$$(\rho_c * h_m) = (\rho_m * h_r) - (\rho_c * h_r)$$

$$(\rho_c * h_m) = h_r (\rho_m - \rho_c)$$

$$h_m = h_r [(\rho_m - \rho_c) / \rho_c]$$

$$h_r = (\rho_c * h_m) / (\rho_m - \rho_c)$$

For column 1

For column 2

$$m_1 = m_2$$

Substitute  $V_{c1} = A_1 * (h_m + h_c + h_r)$

$$V_{c2} = A_2 * h_c$$

$$V_{m2} = A_2 * h_r$$

As columns 1 and 2 are same thickness  
 $A_1 = A_2$  so the A's cancel out

Now we just simplify

Solve for  $h_m$

Solve for  $h_r$

Return to [Slide 26](#).

# The equations: Pratt model

Now onto the Pratt model. We'll use the same idea that the columns of mountain have equal weights, but in this case we have rock with very different densities making up our columns but no mantle involved. Refer to the figure below for the variables we will use. The only difference is that we will use the height of the crust ( $h_c$ ) instead of the height of the root ( $h_r$ ) since the Pratt model does not include a root.

$$(m_A * g) = (m_B * g)$$

$$m_A = m_B$$

$$\rho_A * V_A = \rho_B * V_B$$

$$\rho_A * A_A * (h_m + h_c) = \rho_B * A_B * h_c$$

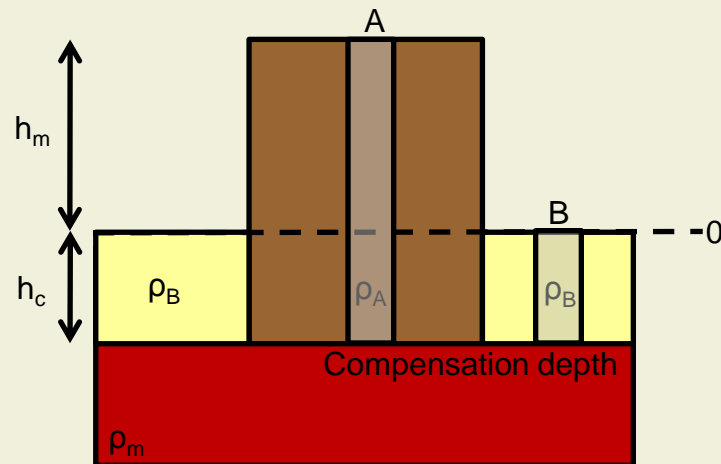
Weight of column 1 and column 2 are equal

$g$ 's cancel out

Once again we use  $m = \rho * V$

Substitute  $V_A = A_A * (h_m + h_c)$

$V_B = A_B * h_c$





# The equations: Pratt model

$$\rho_A * (h_m + h_c) = \rho_B * h_c$$

$$\rho_A * h_m + \rho_A * h_c = \rho_B * h_c$$

$$\rho_A * h_m = \rho_B * h_c - \rho_A * h_c$$

$$\rho_A * h_m = h_c (\rho_B - \rho_A)$$

$$h_m = h_c [(\rho_B - \rho_A) / \rho_A]$$

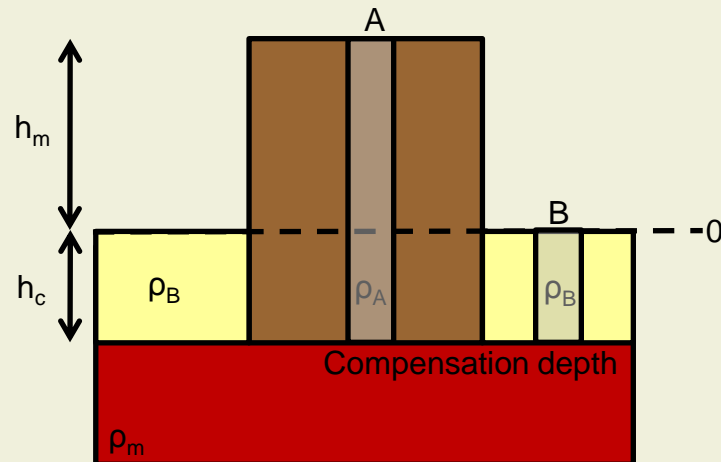
$$h_c = h_m [\rho_A / (\rho_B - \rho_A)]$$

Again columns A and B are the same thickness so  $A_A = A_B$  and the A's cancel out

Now simplify

Solve for  $h_m$

Solve for  $h_c$



Return to [Slide 26](#).

## Problem 3a: Airy model vs. Pratt model

Now that we have the equations we can plug in some realistic values for the density of the crust and mantle and the length of the root of a mountain range ( $h_r$ ). We'll begin with the Airy model. Imagine that we have a crust with a density of  $2.7 \times 10^{12} \text{ kg/km}^3$ , a mantle density of  $3.3 \times 10^{12} \text{ kg/km}^3$ , and a root length ( $h_r$ ) of 50 km. Now calculate the height of the mountains ( $h_m$ ) using the equation from [Slide 23](#). Perform your calculations in Section 3a of your worksheet.

Then do the same thing using the equations we derived on [Slide 25](#) for the Pratt model, and  $2.3 \times 10^{12} \text{ kg/km}^3$  for density A,  $2.7 \times 10^{12} \text{ kg/km}^3$  for density B, 50 km for  $h_c$ .

43			Airy Model		Pratt Model	
44			Density crust ( $\text{kg/km}^3$ )	2.7E+12	Density A ( $\text{kg/km}^3$ )	2.3E+12
45			Density mantle ( $\text{kg/km}^3$ )	3.3E+12	Density B ( $\text{kg/km}^3$ )	2.7E+12
46	3a	2	$h_r$ (km)	50	$h_c$ (km)	50
47			$h_m$ (km)		$h_m$ (km)	

How do the mountain heights ( $h_m$ ) differ for the Airy and Pratt models? Which one do you think is "correct"? Airy, Pratt, both, or neither?

See list of equations on [Slide 31](#) or Return to [Slide 32](#).

# Isostasy - Problem 3a

Question 1 of 2 ▾

What is the value for  $h_m$  for the Airy model?

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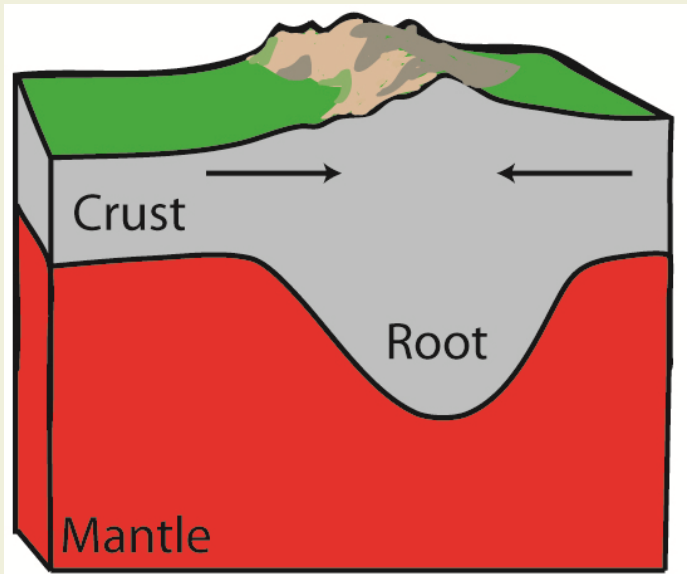
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# The truth about the Airy and Pratt models

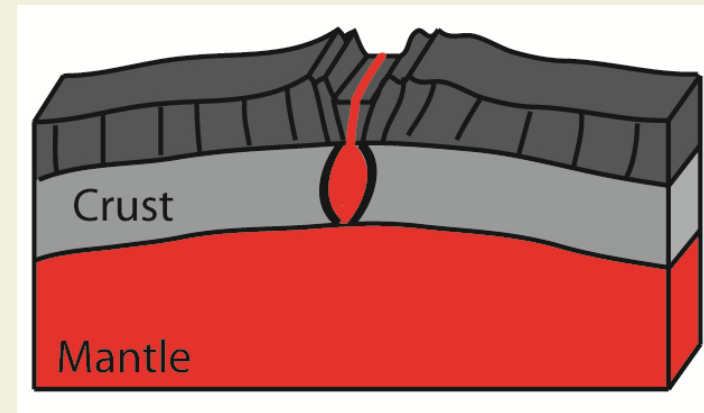
The truth is that both models are correct. However, they apply to very different situations. The Airy model best describes continental mountain ranges like the Appalachians or the Himalayas. Geophysical studies have shown that these types of mountain ranges do indeed have crustal roots as the Airy model predicts.

Conversely, the Pratt model describes mid-ocean ridges, such as the Mid-Atlantic Ridge. In these cases there is no crustal root and the densities do, in fact, change the farther you are from the axial rift valley. So it turns out that both the Airy and Pratt models of isostasy are correct depending on which geologic setting you are studying.

Airy Model



Pratt Model



## Problem 3b: Airy model vs. Pratt Model

Now let's apply what we have learned to a real world problem. Calculating the thickness of the root of the Himalayas using the Airy model and calculating the thickness of the oceanic crust using the Pratt model. The density of the rocks that make up the Himalayas is  $2.6 \times 10^{12}$  kg/km<sup>3</sup>, density of the mantle is  $3.3 \times 10^{12}$  kg/km<sup>3</sup>, and the height of the range is 10 km.

For the thickness of the oceanic crust we will assume a two density model, i.e. the oceanic crust is composed of basalt of density A ( $2.5 \times 10^{12}$  kg/km<sup>3</sup>) when it is close to the mid-ocean ridge and increases to density B ( $3.5 \times 10^{12}$  kg/km<sup>3</sup>) when it is far from the ridge. The height of the ridge above the ocean bottom is 2 km. Perform the calculations in Section 3b of your worksheet.

53			Root of himalayas		Thickness of oceanic crust	
54			Density crust (kg/km <sup>3</sup> )	2.6E+12	Density A (kg/km <sup>3</sup> )	2.5E+12
55	3b	2	Density mantle (kg/km <sup>3</sup> )	3.3E+12	Density B (kg/km <sup>3</sup> )	3.5E+12
56			$h_m$ (km)	10	$h_m$ (km)	2
57			$h_r$ (km)		$h_c$ (km)	

See list of equations on [Slide 31](#).

# Isostasy - Problem 3b

Question 1 of 2 ▾

What is the size of the root ( $h_r$ ) for the Himalayas?

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# Equations and Variables

Here is a list of the equations and variables used in this module for reference.

Equations	Variables
Single layer block $h_r = (\rho_b * h) / \rho_f$ $h_m = h - [(\rho_b * h) / \rho_f]$	$h_r = \text{height of root}$ $\rho_b = \text{density of block}$ $h = \text{height of block}$ $\rho_f = \text{density of fluid}$ $h_m = \text{height of block/mountains above fluid surface}$ $\rho_A = \text{density material A}$ $\rho_B = \text{density material B}$ $\rho_1 = \text{density material 1}$ $\rho_2 = \text{density material 2}$ $h_1 = \text{thickness material 1}$ $h_2 = \text{thickness material 2}$ $\rho_c = \text{density crust}$ $\rho_m = \text{density mantle}$
Stratified block $h_r = [(\rho_1 * h_1) + (\rho_2 * h_2)] / \rho_f$ $h_m = (h_1 + h_2) - [(\rho_1 * h_1) + (\rho_2 * h_2)] / \rho_f$	
Airy model $h_m = h_r [(\rho_m - \rho_c) / \rho_c]$ $h_r = (\rho_c * h_m) / (\rho_m - \rho_c)$	
Pratt model $h_m = h_c [(\rho_B - \rho_A) / \rho_A]$ $h_c = h_m [\rho_A / (\rho_B - \rho_A)]$	

Return to [Slide 8](#), [Slide 9](#), [Slide 13](#), [Slide 17](#), [Slide 18](#), [Slide 26](#), or [Slide 29](#).

## End-of-module assignments

1. Answer questions on slides [9](#), [13](#), [18](#), and [26](#).
2. Calculate the root ( $h_r$ ) of the Appalachian mountains and the Sierra Nevada mountains if the highest point in each range is 2.0 km and 4.4 km, respectively. The average density of the rocks in the Appalachian mountains is  $2.9 \times 10^{12} \text{ kg/km}^3$  and  $2.6 \times 10^{12} \text{ kg/km}^3$  in the Sierra Nevada. Assume the density of the mantle is the same as in previous problems.
3. The Himalayas are currently being uplifted at a rate of 1 cm/yr. If this rate stays constant calculate what  $h_r$  and  $h_m$  will be in 100 years, 10,000 years, and 1,000,000 years.
4. However, the Himalayas are actually at the maximum height possible for a mountain range on Earth. So what must the rate of erosion be per year? What if I increase the rate of erosion by a factor of 3, what will happen to  $h_r$  and  $h_m$  after 100 years? 10,000 years? 1,000,000 years? Explain your results.
5. Calculate the thickness of the oceanic crust on a planet that is exactly like Earth, but is composed of very different material. Assume the oceanic crust is instead made of a rock called coasterite which has a density of  $1.5 \times 10^{12} \text{ kg/km}^3$  close to the rift valley and a density of  $5.0 \times 10^{12} \text{ kg/km}^3$  far from the rift and  $h_m$  is 2 km.
6. Based on newspaper stories published after the Titanic sank the iceberg was reported to have been between 50-100 ft tall. Calculate the height of the root ( $h_r$ ) using the Airy model and densities of  $9.2 \times 10^5 \text{ g/m}^3$  for ice and  $1.02 \times 10^6 \text{ g/m}^3$  for sea water.