

## Appendix B: Detailed Examples of Items 5, 8 and 14 as Illustrative of the Process Used to Construct the Strategy Items

Examples 1 to 3 illustrate the connection between the think-aloud protocol transcriptions used for the development and presentation of the solution strategies produced for the assessment instrument.

### Example 1: Think-Aloud Transcriptions, Solution Strategies, and Regression Plots for Item 5

Question: Divide and simplify  $\frac{7}{4} \div 7$

- a) You would have to multiply by  $\frac{1}{4}$ . So you get  $\frac{7}{4} \cdot \frac{7}{4}$  divided by  $\frac{7}{4}$  equals 1.

*Student 1* said, “And then,  $\frac{7}{4} \div 7$ , so you would have to get this; you have to multiply by  $\frac{1}{4}$  and 'cause that's by 1, so you get, uh,  $\frac{7}{4}$  divided by  $\frac{7}{4}$  equals 1.”

- b) You start out by changing  $\frac{1}{4}$  to  $1\frac{3}{4}$  and then dividing by 7.

*Student 3* said, “Divide and simplify; make them both a whole number. So it would be  $1\frac{3}{4}$  and to divide you have to make 7 into a fraction too, so you have to take 7 and make a  $6\frac{4}{4}$  and you subtract the two. So, you get rid of the 1 and make it all  $\frac{4}{4}$  and then you're left with 6.”

- c) You should start out by changing to a multiplication problem:  $\frac{7}{4}$  times  $\frac{1}{7}$ .

*Student 14* said, “I think you have to do ‘keep change flip’ and change to a multiplication problem after changing 7 to  $\frac{1}{7}$ .”

- d) After you have  $\frac{7}{4}$  times  $\frac{1}{7}$  you cross-multiply to get  $\frac{49}{4}$  or  $12\frac{1}{4}$ .

*Student 17* said “I'm not sure if I am supposed to cross multiply but I'm going to cross multiply 7 times 7 and divide by 4 times 1 and get  $\frac{49}{4}$  or  $12\frac{1}{4}$ .”

Extracting the solution strategies for item 5 was not straightforward because the interviews of the five students who thought aloud about the solution to this problem did not indicate a complete solution strategy. Solution strategy C was deemed correct even though the student only stated correctly how the solution started out and did not provide a complete solution. Solution strategy D was considered a because the strategy incorrectly stated “cross-multiply.”

Example 2: Think-Aloud Transcriptions, Solution Strategies, and Regression Plots for Item 8

Question: Subtract and simplify.  $\frac{7}{10} - \frac{13}{25}$

- a) I need to get the denominator to be 100. Then I get  $\frac{70}{100}$  and  $\frac{52}{100}$ , which is  $\frac{18}{100}$  and simplified to  $\frac{9}{50}$ .

*Student 10 said*, “Subtract: I am just going to get the denominator to 100, I times the bottom one by 10 and the top by 10 and give me  $\frac{70}{100}$ . I times the bottom by 4 and the top by 4 and give me  $\frac{52}{100}$ .  $70 - 52 = 18$ . Gives me  $\frac{18}{100}$ , which can be simplified to  $\frac{9}{50}$ .”

- b) I have to find the lowest common denominator, which is 250.

*Student 19 said*, “I’m not sure but I have to find the lowest common denominator by multiplying denominators, which is 250.

- c) First, I cross-multiply and get  $(10)(13)$  and  $(7)(25)$ . It gives me  $\frac{30}{175}$ . Then I subtract to get  $\frac{1}{45}$ .

*Student 16 said*, “First, I gotta cross multiply and get  $(10)(13)$  and  $(7)(25)$ . Then the next step is to subtract 175 minus 130 to get  $\frac{1}{45}$ .”

- d) Cannot be done.  $\frac{13}{25}$  is greater than  $\frac{7}{10}$ . The answer could be negative.

*Student 21 said*, “This problem cannot be done because  $\frac{13}{25}$  is bigger than  $\frac{7}{10}$ . The answer could be negative.” Example 3: Think-Aloud Transcriptions, Solution Strategies, and Regression Plots for Item 14

Divide. Write a mixed numeral for the answer:  $12 \div 1\frac{1}{13}$

- a) I change 12 to  $\frac{12}{1}$  and  $1\frac{1}{13}$  to  $\frac{14}{13}$  and then cross-multiply.

*Student 14 said*, “I change 12 to  $\frac{12}{1}$  and I did ‘keep change flip’ to get  $\frac{13}{14}$  and then I cross multiplied.”

- b) I’m not sure what a mixed numeral is.

*Student 14 said*, “I don’t remember what a mixed numeral is.”

c) I change  $1\frac{1}{13}$  to  $\frac{14}{13}$  then I multiply 12 by  $\frac{13}{14}$ .

*Student 8* said, “:  $\frac{12}{1} \div 1\frac{1}{13}$ .  $13 \times 1 = 13 + 1 = 14\frac{14}{13}$ . Keep, change, and that becomes  $\frac{13}{14}$ .  $12 \times 14 = 168$ ,  $168 \times 13$ . I’m sick of math. I know how to do up to a certain point.”

d) The answer is  $11\frac{1}{7}$

*Student 22* said, “I’m not sure if this is right, but this is the answer I got.”