1 Supplementary material to "The radius of the umbrella cloud helps characterize large explosive 2 volcanic eruptions " 3 4 *Robert Constantinescu¹, Aurelian Hopulele-Gligor², Charles B. Connor¹, Costanza Bonadonna³, Laura J. Connor¹, Jan Marie Lindsay⁴, Sylvain Charbonnier¹, Alain C. M. Volentik^{5,1} 5 6 7 ¹ School of Geosciences, University of South Florida, Tampa, Fl, U.S.A. 8 ² Independent software engineer, Clui-Napoca, Romania 9 ³ Department of Earth Sciences, University of Geneva, Geneva, Switzerland ⁴ School of Environment, University of Auckland, Auckland, New Zealand 10 11 ⁵ ExxonMobil, Spring, Tx, U.S.A. 12 13 **Corresponding author:** Robert Constantinescu | *email*: robert.constantinescu00@gmail.com 14 15 **Supplementary Note 1** 16 Summary of the 2450BP Pululagua eruption. Pululagua (3358 m.a.s.l.) is a dacitic caldera in the 17 Northern Volcanic Zone of the Andean chain, just 15km north of Quito, Ecuador. Its eruptive history has been well documented by Papale and Rosi ¹, Andrade and Molina ² and Volentik et al. ³. Here we model 18 19 the deposit of the 2450 B.P. Plinian eruption that occurred in no-wind or negligible wind conditions ^{1,3}. 20 The eruption was dacitic in composition and the stratigraphic sequence indicates that the event started 21 with a series of small phreatomagmatic explosions (BGA) that lead to the initiation of the Plinian activity 22 (BF1). The fallout deposit of the climactic phase (BF2 – modeled here) was followed by the deposition of 23 a thinner deposit (BF3) and the end of the eruption was marked by the deposition of a thin white-ash layer (WA) ³. Except BF2 layer that displays near-circular isopachs, all tephra layers associated with the other 24 25 phases of the eruption display a NW dispersal axis.

The best previous estimates for the ESPs for the climactic phase (BF2) have been conducted by Volentik et al. (2010) using inversion techniques with the Tephra2 model 4,5 , without the umbrella cloud source. The results indicated a total erupted mass of $4.5 \times 10^{11} \, \mathrm{kg}$ and a column height of 27 - $29 \, \mathrm{km}$. Previous estimates using statistical methods suggest a mass for BF2 of $3 \times 10^{11} \, \mathrm{kg}$ (exponential-fit 6) and $5 \pm \times 10^{11} \, \mathrm{kg}$ (power-law fit 7), with a column height of $28 \, \mathrm{km}$ and $36 \, \mathrm{km}$ based on the methods of Pyle 6 and Carey and Sparks 8 .

Supplementary Table 1. Summary of the data used to plot the range of total erupted mass calculated with the umbrella cloud model in figure 2 of the main text. We also report the parameters estimated by other models for the same tephra deposit. The erupted mass (kg) refers to the mass of tephra of the climactic phase of the eruption. The column height (km) refers to the volcanic plume height estimated by other models and the height of the umbrella cloud in our model.

Method	Erupted mass (kg)			Column height (km)		
TTl1114 4-1	Minimum	Best fit	Maximum	Minimum	Best fit	Maximum
Umbrella cloud model	1.5×10^{11}	2.5×10^{11}	5 x 10 ¹¹	20	25	30
Tephra2 inversion ³	4.5 x 10 ¹¹			10-30km*		
Exponential fit ⁶	3×10^{11}			-		
Power law ⁷	$5 \pm 1.5 \times 10^{11}$			-		
C&S 8	-			36		
Exponential fit ⁶	-			28		

Supplementary Table 2. Summary of the parameters used in the simulations with different source term geometries. The results are plotted in figure 3 of the main text and show the variations in deposit thickness with distance according to each source term.

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Source term model	Erupted mass (kg)	Column height (km)	Diffusion coefficient (m ² s ⁻¹)	Radius (km)
Disk source	2.5 x 10 ¹¹	25	9500	10
Point source	2.5×10^{11}	25	9500	-
Line source	2.5×10^{11}	20 – 25*	9500	-

^{*}the line source term was described by a series of stacked point sources along a vertical line between 20 and 25 km above the vent. The erupted mass was divided equally along the line.

^{*}Volentik et al. 3 used inversion with Tephra2 model to obtain a range of column heights by inverting for individual grainsizes.

48 **Supplementary References** 49 50 1 Papale, P. & Rosi, M. A case of no-wind plinian fallout at Pululagua caldera (Ecuador): 51 implications for models of clast dispersal. Bulletin of Volcanology 55, 523, 52 doi:10.1007/bf00304594 (1993). 53 2 Andrade, D. & Molina, I. Pululahua caldera: dacitic domes and explosive volcanism. Field guide 54 for the COV4 meeting in Quito (2006). (2006). 55 3 Volentik, A. C. M., Bonadonna, C., Connor, C. B., Connor, L. J. & Rosi, M. Modeling tephra 56 dispersal in absence of wind: Insights from the climactic phase of the 2450BP Plinian eruption of 57 Pululagua volcano (Ecuador). Journal of Volcanology and Geothermal Research 193, 117-136, 58 doi:https://doi.org/10.1016/j.jvolgeores.2010.03.011 (2010). 59 Connor, L. J. & Connor, C. B. in Statistics in Volcanology Vol. 231-242 (eds H. Mader, S.C. 4 60 Coles, C.B. Connor, & L.J. Connor) (Geological Society, 2006). 61 5 Bonadonna, C. et al. Probabilistic modeling of tephra dispersal: Hazard assessment of a 62 multiphase rhyolitic eruption at Tarawera, New Zealand. Journal of Geophysical Research: Solid 63 Earth 110, doi:10.1029/2003JB002896 (2005). 64 6 Pyle, D. M. The thickness, volume and grainsize of tephra fall deposits. Bulletin of Volcanology 65 51, 1-15, doi:10.1007/BF01086757 (1989). 66 7 Bonadonna, C. & Houghton, B. F. Total grain-size distribution and volume of tephra-fall 67 deposits. Bulletin of Volcanology 67, 441-456, doi:10.1007/s00445-004-0386-2 (2005). 68 8 Carey, S. & Sparks, R. S. J. Quantitative models of the fallout and dispersal of tephra from 69 volcanic eruption columns. Bulletin of Volcanology 48, 109-125, doi:10.1007/BF01046546 70 (1986).

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