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# Correction to "El Niño–Southern Oscillation-related ocean-atmosphere coupling in the western equatorial Pacific" by D. A. Mayer and R. H. Weisberg

In the paper "El Niño–Southern Oscillation-related ocean-atmosphere coupling in the western equatorial Pacific" by D. A. Mayer and R. H. Weisberg (*Journal of Geophysical Research*, *103*(C9), 18,635–18,648, 1998), the appendix contained several errors. The corrected appendix follows.

## Appendix: Principal Components for Single-Field and Joint-Field (SVD) Analyses

Let the fields of SST and SLP be represented by the matrices **X** and **Y** of dimension $m_x \times N$ and $m_y \times N$, respectively, where $m_x$ and $m_y$ are the spatial dimensions for these fields and $N$ (=516 months) is their common time dimension. We define **X** as the left-field variable and **Y** as the right-field variable. The combined temporal covariance matrix is

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_{xx} & \mathbf{C}_{xy} \\ \mathbf{C}_{yx} & \mathbf{C}_{yy} \end{pmatrix}$$

where $\mathbf{C}_{xx} = \mathbf{XX}'/N$ has dimension $m_x \times m_x$, $\mathbf{C}_{yy} = \mathbf{YY}'/N$ has dimension $m_y \times m_y$, and $\mathbf{C}_{xy} = \mathbf{XY}'/N$ has dimension $m_x \times m_y$. Generally, $m_x \neq m_y$, but for our analyses, $m_x = m_y$. If each time series is standardized (normalized by its standard deviation), then covariances become correlations. Results using this approach were found to be similar to those using covariances, so our discussions are limited to the latter. The diagonal elements of **C** relate to the autoanalysis or single-field analyses of the left and right fields and either $\mathbf{C}_{xy}$ or $\mathbf{C}_{yx}$ relates to the cross analysis or joint analysis.

For the autoanalysis the results consist of eigenvectors, spatial patterns, and their associated principal components (PCs) or time series. Using the left field, for example, a SVD of the data matrix directly provides the spatial eigenvectors and temporal coefficients in one step, i.e., $\mathbf{U}_x\mathbf{S}_x\mathbf{V}_x' = \mathbf{X}$, where $\mathbf{V}_x'$ denotes the transpose of $\mathbf{V}_x$. The square orthogonal matrix $\mathbf{U}_x$ consists of $m_x$ orthonormal column vectors and is the set of spatial patterns where the first column is the first-mode pattern. Each element is a spatial weight at its corresponding position in the data field. The diagonal matrix $\mathbf{S}_x$ is the set of singular values that contains the units and thus the scaling of the data field. The orthogonal matrix $\mathbf{V}_x$ of dimension $N \times m_x$ consists of $m_x$ orthonormal PCs where the first column is the first-mode time series. The invariant is the sum of all squared elements of **X**. It is also equal to the sum of each squared element ($s_{x_i}^2$) of $\mathbf{S}_x$ or trace ($\mathbf{S}_x^2$) = $N$[trace ($\mathbf{C}_{xx}$)] = $N$[trace ($\mathbf{\Lambda}_x$)]. The diagonal matrix $\mathbf{\Lambda}_x$ with eigenvalue elements $\lambda_{x_i}$ is the eigenvalue matrix obtained by computing EOFs (a special case of SVD) of $\mathbf{C}_{xx}$, i.e., $\mathbf{U}_x\mathbf{\Lambda}_x\mathbf{U}_x' = \mathbf{C}_{xx}$. The trace ($\mathbf{\Lambda}_x$) is the total temporal variance of **X**, so the singular value matrix is related to the eigenvalue matrix by $\mathbf{S}_x = \sqrt{N\mathbf{\Lambda}_x}$. Each element of $\mathbf{\Lambda}_x$ is $\lambda_{x_i}$ and is the temporal variance accounted for by the $i$th mode and is organized as a hierarchy from first mode (maximum) to last mode (minimum). The first mode consists of the first column of $\mathbf{U}_x$ (spatial pattern) and the first row of $\mathbf{S}_x\mathbf{V}_x'$ (dimensional time series), so $\mathbf{U}_x'\mathbf{X} = \mathbf{\chi} = \mathbf{S}_x\mathbf{V}_x'$. Note that $\mathbf{\Lambda}_x = \mathbf{\chi\chi}'/N$. The invariant quantity, trace ($\mathbf{S}_x^2$), is therefore the same whether spatial or temporal covariances are computed for $\mathbf{C}_{xx}$, but the eigenvalues are not. For spatial covariances, $\mathbf{S}_x = \sqrt{m_x\mathbf{\Lambda}_x}$, and the first mode consists of the first column of $\mathbf{U}_x$ (time coefficients) and the first row of $\mathbf{S}_x\mathbf{V}_x'$ (spatial pattern). All of the above relationships are equally applicable to the **Y** (right) field.

For the cross analysis the joint variability between **X** and **Y** can be characterized by an SVD of $\mathbf{C}_{xy}$, i.e., $\mathbf{U}_{xy}\mathbf{S}_{xy}\mathbf{V}_{xy}' = \mathbf{C}_{xy}$. The square orthogonal matrices $\mathbf{U}_{xy}$ and $\mathbf{V}_{xy}$ are the left ($m_x \times m_x$) and right ($m_y \times m_y$) set of orthonormal column vectors that are the left- and right-singular vectors or spatial patterns, respectively. For each of these the first column is the first-mode singular vector or spatial pattern. The PCs are obtained from $\mathbf{U}_{xy}'\mathbf{X} = \mathbf{\chi}_{xy}$ for the left field and $\mathbf{V}_{xy}'\mathbf{Y} = \mathbf{\Psi}_{xy}$ for the right field, and the first row of each represents the first-mode time coefficients. The diagonal matrix $\mathbf{S}_{xy}$ is the singular-value matrix where each element ($s_{xy_i}$) along the diagonal is the singular value associated with the $i$th mode and is the cross covariance of the left and right $i$th-mode PCs, i.e., $\mathbf{S}_{xy} = \mathbf{\chi}_{xy}\mathbf{\Psi}_{xy}'/N$. The invariant quantity is the sum of each squared element ($s_{xy_i}^2$) of $\mathbf{S}_{xy}^2$ or trace ($\mathbf{S}_{xy}^2$) and is the total squared cross covariance between the fields. It is also equal to trace ($\mathbf{C}_{xy}\mathbf{C}_{xy}'$) = trace ($\mathbf{C}_{xy}'\mathbf{C}_{xy}$), where $\mathbf{C}_{xy}'\mathbf{C}_{xy} = \mathbf{U}_{xy}\mathbf{S}_{xy}^2\mathbf{U}_{xy}'$ and $\mathbf{C}_{xy}\mathbf{C}_{xy}' = \mathbf{V}_{xy}\mathbf{S}_{xy}^2\mathbf{V}_{xy}'$. Each squared element ($s_{xy_i}^2$) is the maximum possible squared cross covariance common to both fields for the $i$th mode and is organized as a hierarchy from first mode (maximum) to last mode (minimum).

Canonical correlation analysis entails a similar sequence of calculations except that $\mathbf{C}_{xy} = \mathbf{\chi\Psi}'/N$. Thus the cross covariance between the two fields is computed with the left and right single-field PCs, and each squared element ($s_{xy_i}^2$) is the maximum possible correlation squared between these two fields.



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