3.7 Switching Networks: A Problem-Solving Tool

GETTING STARTED

Computer Circuits

Do you know how a computer works? All computers use a logic system devised by George Boole. The switches inside a computer chip can be arranged into a switching network, or a system of gates, delivering logical results (see Figure 1). The most fundamental logic gates are called AND, OR, and NOT gates. The application of logic to electric circuits was pioneered by Claude E. Shannon of the Massachusetts Institute of Technology and Bell Telephone Laboratories. The idea is to simplify circuits as much as possible by finding equivalent circuits in much the same way as we find equivalent statements. At Bell Laboratories, after several days' work, a group of engineers once produced a circuit with 65 contacts. Then an engineer, trained in symbolic logic, designed an equivalent circuit with 47 contacts in only three hours. In this section you will learn about these circuits and simplify several yourself in problems 16–20 of Exercises 3.7.

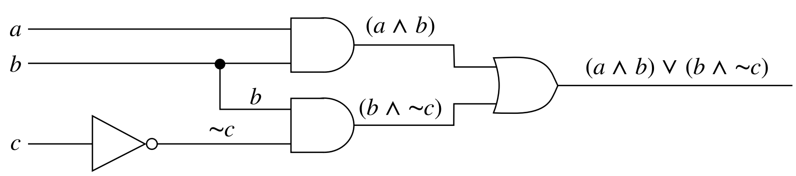


FIGURE 1
This logic diagram shows the inputs and outputs for a series of logic gates. From left to right, the gates shown are a NOT gate, two AND gates, and an OR gate. The lines that connect the gates represent the physical wires that connect "decision-making" devices on a circuit board or in an integrated circuit. For more information on logic gates see the Discovery section of Exercises 3.7. ▶

A. Switching Networks

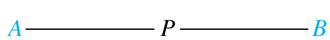


FIGURE 2

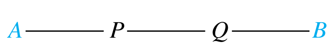


FIGURE 3
Switches in series.

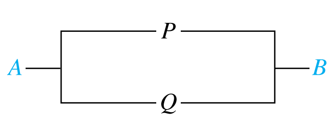


FIGURE 4
Switches in parallel.

The theory of logic discussed in this chapter can be used to develop a theory of simple switching networks. A **switching network** is an arrangement of wires and switches that connects two terminals. A **closed** switch permits the flow of current, whereas an **open** switch prevents the flow. One can also think of a switch as a drawbridge over a river controlling the flow of traffic along a road (see Figure 2).

Two switches can be connected in **series** (in a line from left to right), as in Figure 3. In this network, the current flows between **terminals** A and B only if both switches P and Q are closed.

Can we use logic to describe circuits? In considering such a problem, let p be the statement "Switch P is closed" and let q be the statement "Switch Q is closed." If p is true, then the switch is closed, and current flows. If p is false, then the switch is open, and current does not flow. Thus, the circuit of Figure 2 can be associated with the statement p.

When two switches are connected in series, as in Figure 3, the current will flow only when both switches are closed. Thus, the circuit is associated with the statement $p \wedge q$. On the other hand, in Figure 4, current will flow when either switch P or switch Q is closed. The switches are connected in **parallel** and the statement corresponding to this circuit is $p \vee q$. What about the statement $\sim p$? This statement corresponds to P', a switch that is open if P is closed and vice versa. Switches P and P' are called **complementary.**

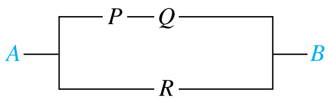


FIGURE 5

Series and parallel circuits can be combined to form more complicated networks, as shown in Figure 5. The network there corresponds to the statement $(p \wedge q) \vee r$. If we think of the switches as drawbridges, we can see that we are able to go from A to B when both P and Q are down (closed) or when R is down (closed). Now, all compound statements can be represented by switching networks. When switches open and close simultaneously, the switches will be represented by the same letter and will be called **equivalent.**

PROBLEM SOLVING

Switching Networks

Construct a network corresponding to the statement $(p \vee q) \wedge r$.

1. **Read** the problem and select the unknown.

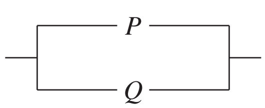
Design a circuit starting at A, ending at B, and corresponding to $(p \vee q) \wedge r$.

2. **Think** of a plan. Find the components of $(p \vee q) \wedge r$ and the switches corresponding to each of the components.

The components of the conjunction $(p \vee q) \wedge r$ are $(p \vee q)$ and r. The parallel circuit containing the switches P and Q corresponds to the statement $(p \vee q)$, and the switch R corresponds to r.

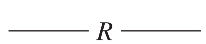
3. **Use** your knowledge to carry out the plan.

Draw the switch corresponding to $(p \vee q)$.



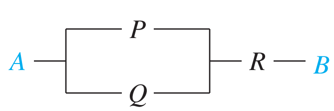
corresponds to $(p \vee q)$

Draw the switch corresponding to r.



corresponds to r

Start at A and connect the two circuits in series to correspond to the connective $\wedge$. End at B.



corresponds to $(p \vee q) \wedge r$

TRY EXAMPLE 1 NOW.

Cover the solution, write your own solution, and then check your work.

EXAMPLE 1 ► Constructing Networks

Construct a network corresponding to the statement $(p \vee r) \wedge (q \vee r)$.

Solution

The network associated with the given statement appears in Figure 6. The parallel circuit containing the switches P and R corresponds to the statement $p \vee r$. Similarly, the parallel circuit with the switches Q and R corresponds to the statement $q \vee r$. These two parallel circuits are connected in series to correspond to the connective $\wedge$, which joins the two statements $p \vee r$ and $q \vee r$. ■

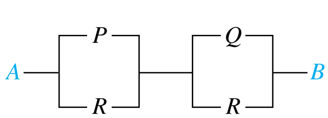


FIGURE 6

TABLE 1

p	q	*Basic Conjunction*
T	T	$p \wedge q$
T	F	$p \wedge \sim q$
F	T	$\sim p \wedge q$
F	F	$\sim p \wedge \sim q$

Notice that in the networks of Figures 5 and 6, current will flow if P and Q are closed or if R is closed. For this reason, these two networks are said to be equivalent. In general, two networks are **equivalent** if their corresponding statements are equivalent. For example, the statement corresponding to the network in Figure 5 is $(p \wedge q) \vee r$ and that to Figure 6 is $(p \vee r) \wedge (q \vee r)$. The truth tables of these two statements are identical, so the networks of Figures 5 and 6 are equivalent.

Finally, we shall consider the design of certain networks having specified properties. An equivalent problem is that of constructing a compound statement having a specified truth table. The procedure used will involve the basic conjunctions given in Table 1. For example, a network associated with a statement having truth table *TTFF* will be the one corresponding to the statement $(p \wedge q) \vee (p \wedge \sim q)$ (see Exercises 3.2, problems 49 and 50).

EXAMPLE 2 ► Designing Toys

A toy designer plans to build a battery-operated kitten with front legs that can be lowered or raised and a purring mechanism. He wants his kitten to purr only when the *right* front leg or *both* front legs are raised; with any other arrangement, the purring mechanism is to be off. Construct a switching circuit that will do this.

TABLE 2

p	q	*Desired Truth Table*
T	T	T
T	F	T
F	T	F
F	F	F

Solution

Let p be the statement "The right front leg is raised" and let q be the statement "The left front leg is raised." The desired truth table is Table 2. We note that the cat will purr when p is true (rows 1 and 2). Hence, our network will correspond to a statement having truth table *TTFF*. We have just seen that one such statement is $(p \wedge q) \vee (p \wedge \sim q)$, so the network associated with this statement (see Figure 9 on page S4) is a possible network for the toy kitten. ■

Online Study Center

To see examples of logic switches used to design something, access link 3.7.1 on this textbook's Online Study Center.

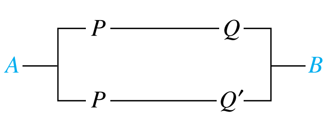


FIGURE 7

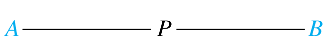


FIGURE 8

Notice that in the network of Figure 7 current will flow when P is closed (because if Q is closed, current will flow through the top branch, and if Q is open, current will flow through the bottom branch). Hence, an equivalent network is the one given in Figure 8.

As indicated in problems 49 and 50 of Exercises 3.2, we can always write a statement corresponding to a given truth table as follows:

1. For each row with a T in the final column, write a conjunction using the variable that has a T in its column and the negation of the variable with an F in its column.
2. Write the disjunction of these conjunctions.

For example, suppose that two rows of the given truth table are as in Table 3 and that all other rows end with an F.

TABLE 3

p	q	r	*Final Column*	*Desired Conjunction*
T	T	F	T	$p \wedge q \wedge \sim r$
T	F	F	T	$p \wedge \sim q \wedge \sim r$

Then the desired conjunctions are as shown in Table 3, and the final desired statement is

$$(p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r)$$

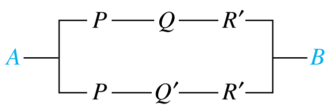


FIGURE 9

The corresponding network is shown in Figure 9. Of course, this procedure does not always give the simplest result for the given truth table. The statement obtained can be simplified to $p \wedge \sim r$, as you can verify by checking the network.

Online Study Center To verify that $(p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r)$ is equivalent to $p \wedge \sim r$, access link 3.7.2 on this textbook's Online Study Center. To further explore equivalent networks, access link 3.7.3.

EXERCISES 3.7

A Switching Networks

In problems 1–6, construct a network corresponding to each statement.

1. $(p \wedge q) \vee p$

2. $p \vee (q \wedge r)$

3. $(\sim p \wedge q) \vee (p \wedge \sim r)$

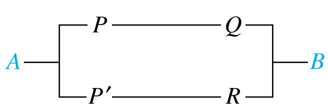
4. $(p \vee q) \wedge \sim r$

5. $[(p \vee \sim q) \vee q \vee (\sim p \vee q)] \vee q$

6. $[(p \vee \sim q) \vee q \vee (\sim p \vee q)] \wedge q$

In problems 7–11, write the compound statement corresponding to each network given.

7.



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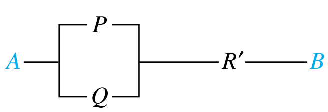

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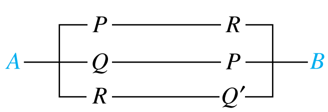

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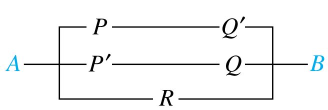

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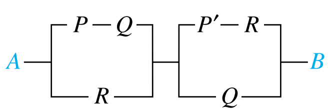


12. A switching network to control the launching of ICBMs is to be designed so that it can be operated by three generals. For safety, the Department of Defense requires that in order to fire the missile, two of the three generals will have to close their switches. Design a network that will do this.

In problems 13–15, draw a pair of switching networks to indicate that the statements are equivalent.

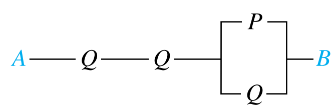
13. $p \vee p$ and p

14. $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$

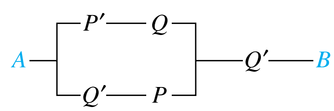
15. $(p \wedge q) \vee p$ and p

In problems 16–20, simplify each network.

16.



17.



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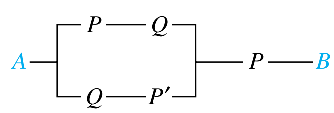

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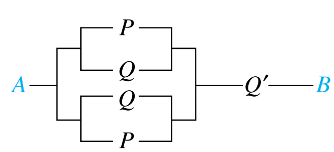

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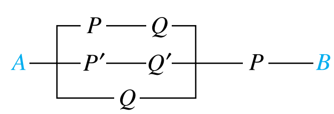


In Other Words

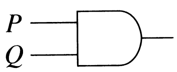
21. Explain the relationship between a series circuit with switches P and Q and the statement $p \wedge q$.

22. Explain the relationship between a parallel circuit with switches P and Q and the statement $p \vee q$.

Discovery

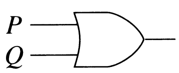
Computer Gates

Digital computers have circuits in which the flow of current is regulated by *gates,* as shown in the figures below. These circuits respond to high (1) or low (0) voltages and can be described by *logic statements.* Keep in mind that 1 and 0 correspond to high and low voltages, respectively.



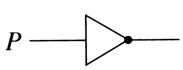
AND gate ($p \wedge q$)

Output 1 if all input voltages (*P* and *Q*) are 1
Output 0 otherwise



OR gate ($p \vee q$)

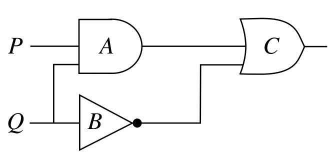
Output 1 if at least one input (*P* or *Q*) is 1
Output 0 otherwise



NOT gate ($\sim p$)

Output 0 if input *P* is 1
Output 1 if input *P* is 0

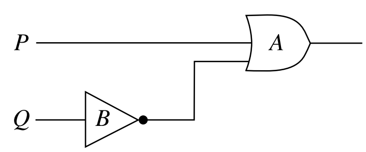
Logic diagrams show how the gates are connected. The outputs in these diagrams can be obtained in the same way in which truth tables are constructed, with 1s replacing the *T*'s and 0s replacing the *F*'s. For example, we can express the outputs for all possible inputs for the logic diagram shown below as follows:



The output of *A* is symbolized by $p \wedge q$, and the output of *B* by $\sim q$. Thus, the inputs of *C* correspond to $p \wedge q$ and $\sim q$. Since *C* is an OR gate, the final output can be symbolized by the statement $(p \wedge q) \vee \sim q$. If we construct the truth table for this statement using 1 for *T* and 0 for *F*, we obtain the following table:

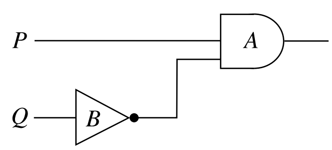
1	*2*	*3*	*5*	*4*
p	q	$(p \wedge q)$	$\vee$	$\sim q$
1	1	1	1	0
1	0	0	1	1
0	1	0	0	0
0	0	0	1	1

The table shows that the final output (column 5) is always a high (1) voltage except in the third row, where the input voltage corresponding to *P* is low (0) and that corresponding to *Q* is high (1). Since we have shown earlier that the statement $(p \wedge q) \vee \sim q$ is equivalent to $p \vee \sim q$, the given circuit can be simplified to one corresponding to the latter statement. The diagram for this circuit is

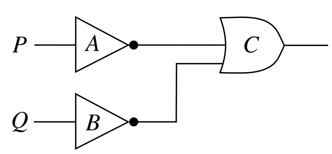


In problems 23–25, determine the final outputs for all possible inputs for each diagram.

23.



24.



25.