Formation of Gas Jets and Vortex Rings from Bursting Bubbles: Visualization, Kinematics, and Fluid Dynamics

Ali A. Dasouqi

University of South Florida

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Formation of Gas Jets and Vortex Rings from Bursting Bubbles: Visualization, Kinematics, and Fluid Dynamics

by

Ali A. Dasouqi

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering
Department of Mechanical Engineering
College of Engineering
University of South Florida

Major Professor: David W. Murphy, Ph.D.
Rasim Guldiken, Ph.D.
Andres Tejada-Martinez, Ph.D.
Gary Mitchum, Ph.D.
Damian Grundle, Ph.D.

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Dedication

This dissertation is dedicated to my parents whose prayers helped me with every hardship during my graduate studies, to my aunt Ms. Mahasen Dasouqi for her continued support all over the years, to my wife Dr. Saja who always encouraged me to pursue my dreams and finish my dissertation, to all my family whom I love the most.
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# Table of Contents

List of Tables ........................................................................................................................................iii

List of Figures........................................................................................................................................iv

Abstract................................................................................................................................................vi

Chapter 1: Introduction..........................................................................................................................1

Chapter 2: Literature Review ..................................................................................................................5
  2.1 The Fluid Dynamics of Bubble Bursting .......................................................................................5
  2.2 Film Rupture and Retraction..........................................................................................................6
  2.3 Droplet Formation..........................................................................................................................8
    2.3.1 Jet Drops .................................................................................................................................8
    2.3.2 Film Drops .............................................................................................................................9
  2.4 Gas Escape ....................................................................................................................................11
  2.5 Vortex Rings ................................................................................................................................13

Chapter 3: Gas Escape Behavior from Bursting Bubbles ......................................................................15
  3.1 Abstract .......................................................................................................................................15
  3.2 Visualization Study .......................................................................................................................15

Chapter 4: Bursting Bubbles and the Formation of Gas Jets and Vortex Rings ....................................19
  4.1 Introduction .................................................................................................................................19
  4.2 Experimental Methods ..................................................................................................................23
  4.3 Results ........................................................................................................................................30
    4.3.1 Flow Visualization ................................................................................................................30
    4.3.2 Gas Jet Characterization .......................................................................................................35
    4.3.3 Vortex Ring Characterization ...............................................................................................41
  4.4 Discussion ....................................................................................................................................47
  4.5 Conclusions ..................................................................................................................................48

Chapter 5: The Effect of Liquid Properties on the Release of Gas from Bursting Bubbles .................50
  5.1 Introduction ..................................................................................................................................50
  5.2 Experimental Methods ..................................................................................................................53
  5.3 Results ........................................................................................................................................56
    5.3.1 Flow Visualization ................................................................................................................56
    5.3.2 Gas Jet Characterization .......................................................................................................67
  5.4 Conclusions ...................................................................................................................................74
List of Tables

Table 4.1 A comprehensive list of the filming characteristics for each of the two cameras used in the experiments .................................................................26

Table 5.1 Values of physical properties of liquids used in the experiments at 20 °C ..............56

Table 5.2 Values of parent bubble Bond number and film Ohnesorge number for the differently sized bubbles bursting at four different working fluid surfaces (Figures 5.1-3 (a-d)) ................................................................................................58
List of Figures

Figure 2.1 Sketch showing the bursting of a “large” cm-scale bubble ........................................5

Figure 2.2 Time series of images showing the spontaneous bursting of similarly-sized bubbles comprised of air and water (top row) and of air and oily water (bottom row) .................................................................7

Figure 3.1 Sample images from a time series showing the bursting of an 2R=3.91 mm smoke-filled bubble floating on the water surface .............................................................17

Figure 3.2 Sample images from a time series showing the bursting of an 2R=22.7 mm smoke filled bubble floating on the water surface .............................................................18

Figure 4.1 Setup for high speed stereophotogrammetric visualization experiments ..................24

Figure 4.2 (a) Experimentally measured bubble diameter (2R) for small bubbles .....................29

Figure 4.3 Sample images from a time series showing the bursting of an 2R=41.0 mm smoke-filled bubble floating on the water surface .......................................................31

Figure 4.4 Sample images from a time series showing the bursting of an 2R=19.3 mm smoke-filled bubble floating on the water surface .......................................................33

Figure 4.5 Sample images from a time series showing the bursting of an 2R=2.01 mm smoke-filled bubble floating on the water surface .......................................................35

Figure 4.6 Sample images from a time series showing the bursting of an 2R= 691 µm smoke-filled bubble floating on the water surface .......................................................36

Figure 4.7 Gas jet front speed V_{jet} as a function of bubble equivalent diameter 2R measured at three time points (t=0.4 ms, t=1 ms, t=2 ms) after hole nucleation, which occurs at t=0 ms ........................................................................36

Figure 4.8 Film retraction speed V_{fr} and the calculated film thickness h at t=0.4 ms after hole nucleation as a function of bubble equivalent diameter 2R ........................................37

Figure 4.9 (a) Ratio of the cross-sectional area of the opening in the bubble cap A_{h} at t=0.4 ms after bursting to the surface area of the bubble cap prior to bursting A_{c} as a function of bubble equivalent diameter 2R ...........................................39
Figure 4.10 Decay in jet speed $V_{jet}$ normalized by initial jet speed $V_o$ over time for bubble equivalent diameters of 2R = 41 mm, 19 mm, and 2 mm ........................................41

Figure 4.11 (a) Temporal growth of the major (D$_{VR}$) and minor diameters ($d_{VR}$) of the primary vortex ring ejected from bursting bubbles with bubble equivalent diameters of 2R = 41 mm, 19 mm, and 2 mm ........................................................43

Figure 4.12 Vortex ring travel distance TD$_{VR}$ as a function of bubble equivalent diameter 2R ............................................................................................................45

Figure 4.13 The number of the initially formed vortex rings as a function of bubble equivalent diameter 2R ............................................................................................................46

Figure 5.1 Sample images from a time series showing the bursting of 2R= 26.6 mm, 23.5 mm, 29.9 mm, and 26.0 mm smoke-filled bubbles floating on water, soapy water, engine oil, and glycerin surfaces (left-right) .................................................62

Figure 5.2 Sample images from a time series showing the bursting of 2R= 6.4 mm, 6.8 mm, 7.1 mm, and 6.9 mm smoke-filled bubbles floating on water, soapy water, engine oil, and glycerin surfaces (left-right) .................................................63

Figure 5.3 Sample images from a time series showing the bursting of 2R= 1.78 mm, 1.55 mm, 1.28 mm, and 1.51 mm smoke-filled bubbles floating on water, soapy water, engine oil, and glycerin surfaces (a-d) .................................................................66

Figure 5.4 Gas jet front speed $V_{jet}$ as a function of bubble equivalent diameter 2R measured at $t=0.4$ ms after hole nucleation ($t=0$ ms) for the different liquids........68

Figure 5.5 Film retraction speed $V_f$ as a function of bubble equivalent diameter 2R measured at $t=0.4$ ms after hole nucleation ($t=0$ ms) for the different liquids......70

Figure 5.6 The calculated film thickness $h$ at $t=0.4$ ms after hole nucleation as a function of bubble equivalent diameter 2R for the different liquids.................72

Figure 5.7 Jet speed $V_{jet}$ as a function of film retraction speed $V_f$ at $t=0.4$ ms for the different liquids..................................................................................................73

Figure 5.8 Reynolds number of the emerging jet at $t=0.4$ ms as a function of the parent bubble Bond number for the different liquids..................................................73

Figure B.1 APS DFD GFM 2019 Video Winner Award...........................................................................94

Figure B.2 AAAS Science Magazine Article ..........................................................................................95

Figure B.3 Short Web Video on DW Science .........................................................................................96
Abstract

The bursting of bubbles at an air-liquid interface is an important physical process in many environmental and industrial applications. Bubble bursting has implications for climate (e.g. marine aerosol formation; Veron 2015), human respiratory health (e.g. aerosolization of harmful substances; Prather et al 2013), food science (e.g. beer and champagne bubbles; Liger-Belair et al 2009), industry (e.g. processes such as gas fluxing and electrowinning; Zhang et al 2011), and volcanology (e.g. Strombolian eruptions; Chojnicki et al 2015). Much of the previous research into bubble bursting has focused on the fluid dynamics of the liquid component, namely the bubble cap film rupture and retraction and the generation of liquid droplets (e.g. jet drops and film drops). However, the fluid dynamics of the pressurized gas escaping from inside the bursting bubble is not well understood. Prior work and preliminary flow visualization results presented here show that this gas forms a high speed jet which rolls up into a vortex ring that may travel a long distance (Rogers 1858), but the fluid dynamics of this process as a function of bubble properties (e.g. size, liquid viscosity, surface tension) is not known.

In this study, I aim to explore the fluid dynamics of the gas escape from bursting bubbles using a stereophotogrammetric high speed camera system. This system allows to quantitatively visualize in three dimensions the bursting of smoke-filled bubbles resting atop a liquid surface and the subsequent release of smoke jets and vortex rings. Further, I intend to vary the size of these single bubbles and the surface tension and viscosity of their comprising fluid in order to perform a scaling analysis. More specifically, I present measurements of parent bubble characteristics and its emerging smoke jet in high spatial and temporal resolutions to better interpret the
nondimensionally-scaled relationship. In a different vein, I show measurements of the parameters of the emitted vortex ring at early formation and its subsequent growth, entrainment, and travel distance. Finally, I investigate the interaction between the emanating smoke jet and the film retraction process to further explain the role of droplet ejection on the formation of additional vortices.

Understanding the fluid dynamics of gas jets and vortex rings released from bursting bubbles could lead to novel applications in industry or food science. For example, bubble properties could be tuned to enhance transport of pleasurable odors released from bursting champagne bubbles or decrease mixing of harmful gases released from bubbles while gas fluxing molten aluminum. Further, the gas jets and vortex rings created from bursting bubbles could carry newly or previously formed aerosol particles far from their generation site. These flows are thus likely important in the lofting of marine aerosol particles from near the ocean surface into the atmosphere, but the interaction between these air flows and fine droplets is not well understood. This study will thus establish a new understanding of a fundamental flow important in myriad applications.
Chapter 1: Introduction

Bubbles are ubiquitous in industrial and environmental processes, and bubble bursting is a widely studied and highly important physical process. MacIntyre (1972) estimated that $10^{18}$-$10^{20}$ bubbles burst every second on the ocean surface, and the tiny marine aerosol droplets ejected into the atmosphere through bursting have a significant impact on climate processes including cloud formation and precipitation (Mason 1957, Blanchard 1963). In addition, marine aerosol droplets ejected from bursting bubbles significantly contribute to the exchange and circulation of organic and chemical materials between water bodies and the atmosphere (Lewis, E. R. and Schwartz 2004). These droplets also may present a respiratory health risk as they may carry brevetoxins from algal blooms or crude oil and chemical dispersants from oil spills (Cheng et al. 2005, Prather et al. 2013, Ehrenhauser et al. 2014, Murphy et al. 2015). Many industrial applications, including gas fluxing and copper electrowinning, also involve bubble bursting in which the release of hazardous aerosols and gases need to be controlled (Fjeld 2006, Zhang et al. 2011, Al Shakarji et al. 2012, Mortensen et al. 2017). Furthermore, bubble bursting is of interest to food scientists. The tiny droplets ejected from bursting champagne bubbles evaporate on their upward trajectory, lifting the aroma and releasing the flavor to the consumer (Liger-Belair et al. 2009, Ghabache et al. 2014, Ghabache and Séon 2016, Séon and Liger-Belair 2017). Other examples include the stability and bursting of beer and espresso foams (Bamforth 1985, Illy and Luciano 2011).

Due to its tremendous importance and its diversity of applications, various aspects of the fluid dynamics of bubble bursting have long been studied. Much of the early work on the fluid dynamics of bubble bursting was motivated by a desire to understand the mechanisms of marine
aerosol production and thus it focused on the liquid component, namely the bubble cap film rupture and retraction and the generation of liquid droplets (e.g. jet droplets and film drops) (Stuhlman 1932; Kientzler et al. 1954; Newitt et al. 1954; Mason 1957; Day and Lease 1968). Jet droplets form from the stem of fluid comprising the unstable water column which undergoes a capillary instability whereas film drops are generated by the violent destruction of the thin film comprising the bubble cap. Further research also has focused on the dynamics of the Worthington jet produced by the upward collapse of tiny bubbles (Krishnan et al. 2017; Deike et al. 2018), the mechanisms of hole initiation in bursting bubbles (Poulain et al. 2018), and the role of viscosity in the retraction of bubble cap films (Savva and Bush 2009; Ghabache et al. 2014). In contrast, the fluid dynamics of the pressurized gas escaping from inside the bursting bubble is not well understood. Several early studies noted the formation of a vortex ring released from the bursting bubble (Rogers 1858; Swinton and Beale 1917), and Newitt et al (1954) first photographed this phenomenon. Other researchers examined this flow as a possible mechanism for lofting tiny marine aerosol droplets up into the atmosphere (Newitt et al. 1954; Blanchard 1963; Iacono and Blanchard 1987).

The goal of this study is to understand the fluid dynamics of gaseous release from single bubbles bursting at an air-liquid interface as a function of bubble diameter and liquid properties, namely surface tension, viscosity, and density.

- Specific Aim 1: Investigate the role of bubble size on the formation of gas jets and vortex rings emanating from bursting bubbles resting at the air-water interface.

The formation of gas jets and vortex rings emanating from bursting bubbles of different sizes resting at the air-water interface was examined using high speed visualization and stereophotogrammetry. Such flows may be important in lofting fine marine aerosol droplets from near the sea surface into the atmosphere and may also affect the air-sea momentum flux. Further,
jets and vortex rings emanating from bubbles in carbonated beverages may be important in carrying odor molecules to the nose of the consumer. We thus focus on the fluid mechanics of the pressurized gas escaping through the expanding hole in the bubble cap. This flow is driven by a pressure difference $\Delta P$ and is known \textit{a priori} by $\Delta P = 4\sigma / R_c$, where $\sigma$ is the surface tension and $R_c$ is the bubble film radius of curvature. Therefore, bubbles of different sizes may produce significantly different flows. 3D measurements of the gas jet and vortex ring characteristics are presented, and relations between these quantities and the bubble Bond number and jet Reynolds number, respectively, are considered.

- Specific Aim 2: Examine the effects of fluid properties (e.g. liquid density $\rho$, viscosity $\mu$, and surface tension $\sigma$) on the formation of gas jets and vortex rings emerging from bursting bubbles of different sizes at an air-liquid interface.

The emerging gas jets produced by bursting bubbles in four liquids with varying surface tensions, viscosities, and densities were studied using a single high-speed camera. Since both increased liquid viscosity and reduced surface tension will likely decrease film retraction speed, we thus expect to see variation in the speed of the emerging jet. Here, we vary the bubble size and its film properties to consider the effects of increasing film viscosity characterized by Ohnesorge number $Oh$ and deceasing film surface tension. 2D measurements of the gas jet and characteristics of retraction of parent bubble cap film are presented and these quantities were related to the jet Reynolds number and bubble nondimensional numbers, namely Bond number, Laplace number, and Froude number.

Results of this study will lead to a better understanding of the gaseous release from a bursting bubble. Kinematic and flow measurements associated with bubble bursting lead to better estimates of the physics of fizziness and effervescence. In addition, understanding the fluid
dynamics of gas jets and vortex rings released from bursting bubbles could lead to novel applications in industry or food science. The gas escape behavior and its interaction with the liquid component of bubble bursting (e.g. Jet droplets and film drops) are key in understanding the entire phenomena.

This thesis is organized as follows: Background information concerning both of liquid and gas components of bubble bursting will be presented in Chapter 2. An introductory visualization study which compares the bursting behavior between a small and a large bubble will be shown in Chapter 3. In Chapter 4, an investigation on the formation of gas jets and vortex rings for air-water bubbles of different sizes based on high speed stereophotogrammetrical measurements will be described. In Chapter 5, a more detailed study on the effects of liquid properties (e.g. surface tension, viscosity, and density) of the bubble cap film on the emerging gas jet and subsequent formation of vortex rings will be explained. Chapter 6 will draw conclusions based on this work, draw out some of the limitations of the current study, and present suggestions for future work.
Chapter 2: Literature Review

2.1 The Fluid Dynamics of Bubble Bursting

Due to its tremendous importance and its diversity of applications, various aspects of the fluid dynamics of bubble bursting have long been studied. We initially consider the model of a “large” (e.g. cm scale) air bubble resting atop the air-water interface. “Small” (mm scale) bubbles are mostly spherical and almost completely submerged in the water (Lhuissier and Villermaux 2011; Teixeira et al 2015). As shown in a sketch in Figure 2.1, the bubble comprises an essentially hemispherical liquid film joined to the bulk fluid by the curved meniscus, and the bubble cap encloses an air cavity that is at a higher pressure than the surrounding atmosphere (Lhuissier and Villermaux 2011). Figure 2.1 also provides a framework for thinking about the processes comprising bubble bursting. These three processes, rupture and film retraction, droplet production, and gas escape, will be reviewed. We note that most of the prior work has focused on the liquid component of bubble bursting (e.g. film rupture and retraction and droplet formation). In contrast, this work focuses on the escaping gas, how the smallest droplets are formed, and how the ejected gas interacts with and potentially transports those droplets.

Figure 2.1 Sketch showing the bursting of a “large” cm-scale bubble
2.2 Film Rupture and Retraction

The liquid in the film of a bubble emerging from a bulk fluid surface immediately begins to drain due to gravity and capillary forces within the film. Film thickness $h$ thus decreases with time, and, for tap water, $h$ decreases from approximately 40 μm directly after emergence to 2 μm after at 20 s (Lhuissier and Villermaux 2011). Bubbles of extremely pure water burst almost immediately, but in water containing even trace amounts of surfactants, a vertical surface tension gradient is established that stabilizes the bubble over time periods of up to tens of seconds (Lhuissier and Villermaux 2011). Bubble bursting begins when a hole nucleates in the bubble cap film. In bubbles comprised of air and water, hole nucleation usually occurs near the meniscus (at the base of the bubble cap). The process by which this occurs is still not well understood but is thought to be linked with unstable convection cells within the bubble film itself and to the presence of impurities such as particles and microbubbles (Poulain et al. 2018). In contrast, in soap bubbles, hole nucleation usually occurs at the top of the bubble via thermal fluctuations (Lhuissier and Villermaux 2011).

Once a hole nucleates in the bubble film, surface tension drives the film to rapidly retract, thus enlarging the hole. After a short transient time, the film retracts at a constant speed known as the Taylor-Culick velocity $V = \sqrt{2\sigma/\rho h}$, in which $\sigma$ is the surface tension, $\rho$ is the fluid density, and $h$ is the film thickness (Taylor 1959, Culick 1960, Savva and Bush 2009). The behavior of the film depends on the Ohnesorge number $Oh = \mu/\sqrt{2h\rho\sigma}$, in which $\mu$ is the viscosity of the fluid comprising the film (Brenner and Gueyffier 1999, Savva and Bush 2009). At low $Oh$ ($Oh < 0.01$) and moderate $Oh$ ($0.01 < Oh < 10$), fluid collects in a thickened rim on the edge of the retracting film. However, at low $Oh$, a capillary wave develops in front of the retracting rim, whereas at moderate $Oh$, no such capillary waves form. At high $Oh$ ($Oh > 10$), fluid does not collect in a rim
because of the dominating force of viscosity (Debrégeas et al 1995, Debrégeas et al 1998, Brenner and Gueyffier 1999, Savva and Bush 2009). Most situations of practical importance fall in the moderate $Oh$ range. For example, Murphy et al (2015) examined bubbles formed by the impact of raindrops on a pool of artificial seawater and on artificial seawater with a floating layer of crude oil. Figure 2.2 shows sample images of a time series of the bursting of a seawater bubble ($Oh=0.013$, top row) and of one contaminated with crude oil ($Oh=0.15$, bottom row). The time $t=0$ corresponds to the instant before a hole nucleates in the bubble film. In both cases, it is apparent that the rim of the retracting film accumulates fluid. Furthermore, in accordance with the Taylor-Culick velocity equation, the oil-contaminated bubble film retracts much more slowly. Repeated tests showed that the receding rim speed of the oily case was approximately $1\pm0.2$ m/s whereas that of the seawater case was $1.8\pm0.4$ m/s. The lesser speed of the oily bubble corresponds with the lower surface tension of the crude oil (27 mN/m) versus that of seawater (72 mN/m).

Figure 2.2 Time series of images showing the spontaneous bursting of similarly-sized bubbles comprised of air and water (top row) and of air and oily water (bottom row). Film retraction speed and droplet formation are reduced with oil.
2.3 Droplet Formation

Much of the early work on the fluid dynamics of bubble bursting was motivated by a desire to understand the mechanisms of marine aerosol production and to determine the size distributions of produced droplets. In this process, air entrained by breaking waves at sea forms bubbles which rise to the surface and burst, producing aerosol droplets by at least two different mechanisms (Lewis and Schwartz 2004). Researchers thus have classified these droplets as ‘film drops’ or ‘jet drops’ depending on their mechanism of production.

2.3.1 Jet Drops

Bubbles smaller than approximately 2 mm in diameter remain largely submerged upon reaching the air-sea interface, with only a small bubble cap protruding above the surface. The rupture of this film creates a void which collapses upward, due to high surface tension forces and gravity, to create a high-speed jet extending upward beyond the surrounding sea surface. The stem of fluid comprising the upward jet undergoes a capillary instability and produces a small number of ‘jet drops’ ejected vertically at high speeds, a phenomenon first photographed by Kientzler et al (1954). Jet drops are less commonly generated when the bubble is greater than 2 mm in diameter because larger bubbles are less submerged and do not produce an upwards jet upon bursting. The size, ejection height, and number of generated droplets (always less than ten) as a function of bubble size has been a matter of great interest (Hayami and Toba 1958, Resch et al 1986, Blanchard 1989a,b, Spiel 1997, Veron 2015). For example, Wu (2002) found that jet drops are in the range of 0.13-0.15 times the size of the parent bubble. Blanchard (1989a,b) found that the ejection height of jet drops depended on bubble size, with 2 mm diameter bubbles producing droplets reaching 20 cm and 200 μm bubbles producing droplets reaching only 1 cm. Ejection speeds of the uppermost jet drop may exceed 8 m/s (Spiel 1995), and Lee et al (2011) found jet drops ejected by bubbles
with diameters as small as 26.5 μm. More recently, Deike et al (2018) found a universal scaling for the velocity of the jet which generates jet drops as a function of the Laplace and Bond numbers, and Braz et al (2018) used high-speed imaging and simulations to show that, for very small bubbles, the minimum size for the top jet drop is approximately 1% of the parent bubble’s size.

2.3.2 Film Drops

The violent destruction of the thin film comprising the bubble cap is thought to form film drops, which range in size from 10 nm up to several hundred microns in diameter (Blanchard 1963, 1983, Afeti and Resch 1990, Veron 2015). Submicron-scale droplets are more numerous than larger droplets and are important as cloud nucleation sites (Veron 2015). However, it is unclear whether the same mechanism is responsible for producing film drops over this entire size range. While large droplets can be optically characterized via photography or holography, submicron droplets cannot and require specialized particle sizing equipment. Thus, early researchers used cloud chambers to rapidly grow submicron-scale film drops into a size which could be visualized (Mason 1954, 1957, Blanchard 1963, Day 1964, Day and Lease 1968, Patterson and Spillane 1969). Their drawings and photographs reveal these droplets being carried upwards in a gas jet or vortex ring emanating from the burst bubble. Later researchers, however, have focused on retraction and breakup of the bubble cap film as the source of much larger (tens of microns) film droplets (Resch et al 1986, Afeti and Resch 1990, Resch and Afeti 1991, Spiel 1997, 1998, Lhuissier and Villermaux 2011). At low Oh (characteristic of seawater and air), the film retracts at a high speed from the point of bursting and collects fluid in its thickened rim. This thickened rim centripetally accelerates as it follows the bubble cap curvature and undergoes a Rayleigh-Taylor instability, forming elongating ligaments extending from the rim. As shown for a bursting air-water bubble at the \( t=8 \) ms timepoint in Figure 2.2, these ligaments then develop Plateau-
Rayleigh (capillary) instabilities and shed μm-scale film drops from their tips (Spiel 1997, 1998). The majority of these droplets are ejected horizontally or downwards to impact the water surface (Spiel 1998). Larger bubbles produce larger and more numerous film drops (Blanchard 1963, Day 1964, Afeti and Resch 1990, Spiel 1998). Largely submerged small bubbles (less than approximately 2.4 mm diameter) are thought not to form film drops because the caps of these bubbles do not allow enough angular acceleration to develop to eject droplets (Spiel 1998). Film drop formation also can be reduced or eliminated in lower surface tension or more viscous fluid because the speed of film retraction and likelihood of instability development are reduced, as seen in Figure 2.2 for oily water (Paterson and Spillane 1969, Modini et al 2013, Murphy et al 2015). The discrepancy in size and ejection direction between the large droplets (ejected horizontally) and the sub-micron droplets (ejected vertically) points towards different production mechanisms for these two size classes. In addition, researchers have found multiple modes in the size distributions of sub-micron droplets, suggesting that multiple production mechanisms may be at work to produce film drops in the sub-micron range (Tyree et al 2007, Modini et al 2013). These production mechanisms are currently unknown, but several hypotheses have been put forward. Spiel (1998) posited that the tiny film drops are formed by the impact of larger film droplets, moving at speeds up to 20 m/s, on the surrounding water surface. The impact of droplet-producing ligaments against each other also has been hypothesized to produce these droplets (Lhuissier and Villermaux 2011, Modini et al 2013). Kem et al (2017) hypothesized that the tiny film drops were satellite drops created when the larger film drops separate from ligaments. However, no firm evidence has emerged to support any hypothesis, and more research is needed to determine the production mechanism of these fine droplets.
2.4 Gas Escape

It is well known that the pressure difference across an interface is given by the Young-Laplace equation: \( \Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \), where \( \sigma \) is the surface tension and \( R_1 \) and \( R_2 \) are the principal radii of curvature. For a spherical bubble with interior and exterior surfaces, the Young-Laplace equation simplifies to \( \Delta P = \frac{4\sigma}{R} \), where \( R \) is the bubble radius. Owing to surface tension within the film cap, the gas within a bubble is therefore pressurized relative to the surrounding atmosphere. Thus, when a hole nucleates in the bubble cap film or the film is punctured, gas escapes from the enlarging hole in the film as a jet which then may roll up into a vortex ring. W. B. Rogers, the founder of MIT, first noted this phenomenon in 1858 by filling soap bubbles with smoke (Rogers 1858). Swinton and Beale (1917) also visualized the formation of smoke-filled bubbles in resin oil. Stuhlman (1932) later hypothesized that the high-speed vortex ring sucked the column jet (which forms jet drops) upwards from the water surface, but his hypothesis was shown to be incorrect by the photographs of Kientzler et al (1954). Newitt et al (1954) first photographed the vortex rings emanating from bursting bubbles via smoke visualization and speculated that the gas jet may help transport film drops upwards from the bubble. This hypothesis seemed to be confirmed by Blanchard’s (1963) experiments using a diffusion cloud chamber, in which he visualized a mushroom-shape distribution of film droplets carried upwards from a cm-scale bursting bubble, a pattern which was interpreted to indicate the presence of a vortex ring. Smaller mm-scale bursting bubbles produced a vertical line of droplets indicative of a vertical jet without a vortex ring. Some later experiments replicated these results (Day 1964, Patterson and Spillane 1969) but other experiments found that film droplets moved in parabolic trajectories unaffected by the outrush of air from the bubble. Iacano and Blanchard (1987) performed the most in-depth study of the gas jet produced from surface bubbles bursting to date in an effort to investigate how these
vortices could help transport film drops into the atmosphere more effectively (but did not actually measure the motion of film drops). These researchers visualized the bursting of 2-6 mm diameter bubbles in distilled water using a layer of dry ice fog laying atop the water surface and found vortex vertical travel distances of 1-5 cm. Vortex speed could not be measured directly because of the dry ice fog and was (under-) estimated at 80 cm/s. Finally, other have found leapfrogging vortex rings produced by bursting hemispherical soap bubbles (Buchholz and Sigurdson 2000, 2002) or have investigated the bursting of much larger (4-6 cm) spherical soap bubbles (with lower surface tension) filled with inert gas for firefighting purposes (Jaw et al 2007, Torikai et al 2011, Kim et al 2014). In this latter case, the pressure difference between the bubble interior and the atmosphere is small and no gas jet is created. Instead, the rapid film retraction creates a series of Kelvin-Helmholtz instabilities around the perimeter of the gas inside the bubble. Finally, Ingram et al (2015) investigated the bursting and flammability of cm-scale hydrogen bubbles at the water surface using a Schlieren video system and found vortex ring-like puffs traveling up to 10 cm above the water surface and dispersing over a few hundred ms.

In sum, the gas jets and vortex rings produced by bursting bubbles are likely important in the production and transport of the film drops comprising a majority of marine aerosol (MacIntyre 1972, Spiel 1998), but many of the relevant studies concern this gas flow only incidentally or contradict one another. In addition, the gas released from bubbles bursting at the ocean surface is expected to have a high water vapor content and a temperature close to that of the seawater due to its potentially prolonged submergence. Just as the evaporation of marine aerosol droplets forms a cool and moist Droplet Evaporation Layer (Andreas et al 1995), the spatial distribution and vertical extent of this gas released from a bursting bubble could therefore be important in the air-sea flux of heat and moisture. In industrial processes, the spatial distribution and vertical extent of
hazardous gas released from bursting bubbles also could be important if the gas needs to be controlled. Alternatively, in food sciences, tuning the bubble properties to increase the lofting distance of pleasurable smells released from bubbles upon bursting to the consumer may be a possibility. However, little is truly known about the fluid dynamics of this important flow.

2.5 Vortex Rings

Vortex rings are one of the most fundamental and fascinating phenomena in fluid mechanics. Vortex flow is characterized by the localization of vorticity (curl of velocity) in bounded regions of a space beyond which the vorticity is absent. Concentrated vortices are usually seen in nature (e.g. atmospheric cyclones (Syono et al. 1951; Syono and Yamasaki 1966), tornados and whirlwinds (Lugt 1989), explosive outburst of volcanoes (Chojnicki et al. 2015)), and many biological creatures (Dabiri and Gharib 2005; Russell 2011)). Vortex rings can be generated by several methods. One way a vortex ring may be formed is by injecting a compact mass of fast-moving fluid into another mass of stationary fluid. Viscous friction at the interface between the two fluids slows down the outer layers of the moving fluid relative to its core (Rogers 1858; Helmholtz 1867). Those outer layers then slip around the mass of the flowing fluid and collect at the rear where they re-enter the mass in the wake of the faster-moving inner part. Thus, a poloidal flow in the moving fluid evolves into a vortex ring. This mechanism is commonly seen, for example, when a drop of colored liquid falls into a cup of water. It is also often seen at the leading edge of a plume or jet of fluid as it enters a stationary mass. The mushroom-like head (i.e. starting plume) that develops at the tip of the jet has a vortex-ring structure. A vortex ring is also formed when a mass of fluid is impulsively pushed from an enclosed space through a narrow opening (Rogers 1858). In this case the poloidal flow is set in motion, at least in part, by interaction between the outer parts of the fluid mass and the edges of the opening. Of particular interest is vortex rings
which form from the bursting of a gas bubble (Rogers 1858; Iacono and Blanchard 1987; Buchholz and Sigurdson 2000). In fact, the gas does not simply rush out of the bubble in all directions, instead it rises up in delicate vortex rings of which one often constitutes a vortex tube closed into a toroidal ring moving in a surrounding fluid.

A great volume of research has been published on vortex rings. In particular, the conditions of vortex formation (Rogers 1858; Glezer 1988; Mohseni and Gharib 1998; Dabiri and Gharib 2005), the motion of a vortex ring in a viscous medium (Maxworthy 1972&1974; Sullivan et al. 2008), and the vortex entrainment of surrounding fluid (Gharib et al. 1998) have all been well investigated. In these previous studies, vortex rings have always been characterized by their parameters at the moment of complete formation which allows to consider vortex flow as an initial value problem with the parameters at formation as the initial conditions. These parameters include the initial diameter of the ring $D_0$, the diameter of the core $d_0$ which characterizes the degree of vorticity concentration in the vortex ring, the translational velocity $u_0$, and the vortex ring circulation $\Gamma_0$ which determines the structure of the completely formed vortex ring. Further, the impulse $P = \rho \Gamma \pi R^2$ (Sullivan et al. 2008), which is an integral of the vortex ring motion, is conserved in a viscous medium and considered the most fundamental characteristic of a vortex ring.
Chapter 3: Gas Escape Behavior from Bursting Bubbles

3.1 Abstract

This paper is associated with a video winner of a 2019 American Physical Society’s Division of Fluid Dynamics (DFD) Gallery of Fluid Motion Award for work presented at the DFD Gallery of Fluid Motion. The original video is available online at the Gallery of Fluid Motion, https://gfm.aps.org/meetings/dfd-2019/5d7fe8e4199e4c429a9b30e4.

3.2 Visualization Study

The bursting of bubbles at an air-liquid interface is important in air-sea interactions, food science, and industrial processes. Whereas the film retraction and droplet formation processes of bubble bursting have been well studied, the escape of pressurized gas from inside the bubble is not well understood but could have implications for the transport of vapor, odors, and aerosolized microdroplets near the liquid surface (Day 1964; Iacono and Blanchard 1987). Vortex ring formation from bursting bubbles was first reported in 1858 by William Rogers, the founder of the Massachusetts Institute of Technology (Rogers 1858). Here we use high speed smoke visualization and stereophotogrammetry to visualize and measure the gas jet flow and subsequent production of vortex rings released from small (2R = 3.91 mm) and large (2R = 22.7 mm) bubbles bursting at the air-water interface, where 2R is the bubble equivalent diameter (Teixeira et al. 2015). Figure

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Authors: Ali A. Dasouqi and David W. Murphy

Author contributions: A.D. and D.W.M. conceived and designed the experiment. A.D. carried out experimental work and data analysis. A.D. and D.W.M wrote the manuscript.
3.1 shows the bursting of the small bubble, which, by the Young-Laplace equation $\Delta P = \frac{4\sigma}{R}$, has an interior pressure that is 145 Pa greater than the surrounding atmosphere. At $t=0.4$ ms, the bubble cap film has been completely destroyed, retracting at a speed of $V_{fr}=5.19$ m/s (corresponding to a film thickness of $h=5.3$ µm by the Taylor-Culick formula $V_{fr} = \sqrt{\frac{2\sigma}{\rho h}}$, where $\sigma$ and $\rho$ are the water surface tension and density, respectively), and a smoke jet has emerged at a speed of $V_{jet}=2.56$ m/s and at a jet Reynolds number $Re_{jet} = \frac{V_{jet}D_{jet}}{\nu}$ of 106, where $D_{jet}$ is the jet frontal diameter and $\nu$ is the kinematic viscosity of air. The smoke jet rolls up into a primary vortex ring which travels 32 mm before stopping. A secondary vortex ring is formed from the upward impulse of the Worthington jet (e.g. $t=14.8$ ms). Figure 3.2 shows the bursting of the large bubble which has an interior pressure that is 25 Pa greater than that of the surrounding atmosphere. At $t=0.4$ ms, a 2.3 mm diameter hole has opened in the bubble film through which the smoke is being ejected. The film retraction speed for the large bubble is slower (e.g. $V_{fr}=2.73$ m/s), corresponding to a larger film thickness of $h=18.9$ µm. The jet speed at $t=0.4$ ms is $V_{jet}=4.07$ m/s (larger than that of the small bubble), corresponding to $Re_{jet} = 517$. The jet rolls up into a large, oblate primary vortex ring, and the continuing retraction of the film creates a series of Kelvin-Helmholtz vortex-like instabilities in the smoke beneath the primary vortex ring (e.g. $t=2.8 – 6.4$ ms). The primary vortex ring generated from the large bubble travels a much longer distance (173 mm) in comparison to that generated by the small bubble. In comparing the behavior of these gas jets as a function of bubble size, it is remarkable that the large, low pressure bubble generates a higher speed jet. In general, small bubbles have thin films which retract extremely rapidly, leading to relatively wide apertures through which low speed gas jets are ejected. In contrast, large bubbles have thick films which retract more slowly, thus generating higher speed jets released through relatively small openings. These jets and the resulting upward-traveling vortex rings generated by bubbles bursting
at the air-sea interface may be important in transporting tiny marine aerosol droplets into the atmosphere.

Figure 3.1 Sample images from a time series showing the bursting of an 2R=3.91 mm smoke-filled bubble floating on the water surface. The hole in the bubble cap forms at $t=0$ ms. [https://gfm.aps.org/meetings/dfd-2019/5d7fe8e4199e4c429a9b30e4]
Figure 3.2 Sample images from a time series showing the bursting of an $2R=22.7$ mm smoke filled bubble floating on the water surface. The hole in the bubble cap forms at $t=0$ ms. [https://gfm.aps.org/meetings/dfd-2019/5d7fe8e4199e4c429a9b30e4]
Chapter 4: Bursting Bubbles and the Formation of Gas Jets and Vortex Rings

4.1 Introduction

Bubbles are ubiquitous in industrial and environmental processes, and bubble bursting is a widely studied and highly important physical process. MacIntyre (1972) estimated that $10^{18}$ - $10^{20}$ bubbles burst every second on the ocean surface, and the tiny marine aerosol droplets ejected into the atmosphere through bursting have a significant impact on climate processes including cloud formation and precipitation (Mason 1957, Blanchard 1963). In addition, marine aerosol droplets ejected from bursting bubbles significantly contribute to the exchange and circulation of organic and chemical materials between water bodies and the atmosphere (Lewis, E. R. and Schwartz 2004). These droplets also may present a respiratory health risk as they may carry brevetoxins from algal blooms or crude oil and chemical dispersants from oil spills (Cheng et al. 2005, Prather et al. 2013, Ehrenhauser et al. 2014, Murphy et al. 2015). Many industrial applications, including gas fluxing and copper electrowinning, also involve bubble bursting in which the release of hazardous aerosols and gases needs to be controlled (Fjeld 2006, Zhang et al. 2011, Al Shakarji et al. 2012, Mortensen et al. 2017). In addition, in the field of volcanology, Strombolian eruptions involve the bursting of meter-scale magma bubbles and the plume-like release of noxious gases (Blackburn et al. 1976, Vergniolle and Brandeis 1996, Patrick 2007, Kobayashi et al. 2010,

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Authors: Ali A. Dasouqi, Geum-Su Yeom, and David W. Murphy

AD and DM conceived and designed the experiment and analyzed data. AD carried out experimental work. G-SY developed the model. AD, G-SY, and DM wrote the manuscript. All authors approved the final manuscript.
Chojnicki et al. 2015). Furthermore, bubble bursting is of interest to food scientists. The tiny droplets ejected from bursting champagne bubbles evaporate on their upward trajectory, lifting the aroma and releasing the flavor to the consumer (Liger-Belair et al. 2009, Ghabache et al. 2014, Ghabache and Séon 2016, Séon and Liger-Belair 2017). Other examples include the stability and bursting of beer and espresso foams (Bamforth 1985, Illy and Luciano 2011).

Bubble bursting begins when a hole nucleates in the bubble cap film, which, in bubbles comprised of air and water, usually occurs at the base of the bubble cap (Lhuissier and Villermaux 2011). Hole nucleation is still not well understood but is thought to be linked with unstable convection cells within the bubble film itself and to the presence of impurities such as particles and microbubbles (Poulain et al. 2018). Once a hole nucleates in the bubble film, surface tension drives the film to rapidly retract, thus enlarging the hole. After a short transient time, the film retracts at a constant speed known as the Taylor-Culick velocity $V = \sqrt{2\sigma/\rho h}$, in which $\sigma$ is the surface tension, $\rho$ is the fluid density, and $h$ is the film thickness (Taylor 1959, Culick 1960, Savva and Bush 2009). The destruction of the thin film comprising the bubble cap forms film drops, which range in size from 10 nm up to several hundred microns in diameter (Blanchard 1963, Afeti and Resch 1990, Lhuissier and Villermaux 2011, Veron 2015). Specifically, the film retracts at a high speed from the point of bursting and collects fluid in its thickened rim. This thickened rim centripetally accelerates as it follows the bubble cap curvature and undergoes a Rayleigh-Taylor instability, forming elongating ligaments extending from the rim. These ligaments then develop Plateau-Rayleigh (capillary) instabilities and shed μm-scale film drops from their tips (Spiel 1997, 1998, Lhuissier and Villermaux 2011). Largely submerged small bubbles (less than approximately 2.4 mm diameter) are thought not to form film drops because the caps of these bubbles do not allow enough angular acceleration to develop to eject droplets (Spiel 1998). However, these small
bubbles form jet drops as the bubble cavity collapses upwards to form an unstable water column which disintegrates, ejecting up to 10 droplets. The size, ejection height, and number of generated droplets as a function of bubble size is a matter of great interest in oceanography as these droplets form the aforementioned marine aerosol (Hayami and Toba 1958, Resch et al. 1986, Blanchard 1989a,b, 1989, Spiel 1997, Veron 2015, Brasz et al. 2018, Deike et al. 2018).

Whereas the liquid component of the bubble bursting process has been well studied, the process of the pressurized gas escaping from inside the bursting bubble is much less well understood. Rogers (1858) first visualized the phenomenon of an air jet and vortex ring formed from the rupture of a bubble resting atop a liquid surface and examined the conditions under which those rings were produced. Newitt et al. (1954) first photographed the vortex rings emanating from bursting bubbles via smoke visualization and speculated that the gas jet may help transport film drops upwards from the bubble, thereby transporting marine aerosol droplets into the atmosphere. This hypothesis seemed to be confirmed by Blanchard’s (1963) experiments using a diffusion cloud chamber, in which he visualized a mushroom-shape distribution of droplets carried upwards from a cm-scale bursting bubble, a pattern which was interpreted to indicate the presence of a vortex ring. In an investigation of how bubble bursting may transport marine aerosol droplets into the atmosphere, Iacano and Blanchard (1987) performed the most in-depth study to date of the gas jet produced from surface bubbles bursting and measured the ejection speed, orientation, and maximum height of vortex rings generated from 2-6 mm diameter bubbles bursting in distilled water. However, these investigators used a layer of dry ice fog laying atop the water surface to visualize the flow and thus did not observe the actual bubble bursting event. In addition, Buchholz and Sigurdson (2000, 2002) qualitatively analyzed vortex rings formed by the bursting of large (4-6 cm) hemispherical soap bubbles filled with a glycerin-based aerosol and resting on a metal
surface by using high speed photography. The evolution of vortex rings was explained using two 2D numerical blob models. Others have investigated the bursting of large (4-6 cm) spherical soap bubbles (with low surface tension) filled with inert gas for firefighting (Jaw et al. 2007, Torikai et al. 2011, Murashita et al. 2012, Kim et al. 2014). In this case, the pressure difference between the bubble interior and the atmosphere is small and no gas jet is created. Instead, the rapid film retraction creates a series of Kelvin-Helmholtz instabilities around the perimeter of the gas inside the bubble. In addition, in their study of bubble bursting, Lhuissier and Villermaux (2011) made theoretical predictions of the gas jet ejection speed. In a similar vein, Gekle et al (2010) and Peters et al (2013) studied the supersonic airflow escaping from a collapsing liquid cavity generated by the impact of a circular disk (Peters et al. 2013, Gekle et al. 2010). Finally, Dasouqi and Murphy (in press) used high speed smoke visualization and stereophotogrammetry to visualize and measure the gas jet flow and subsequent production of vortex rings released from small and large bubbles bursting at the air-water interface.

Here, we use high speed visualization and stereophotogrammetry to study the formation of gas jets and vortex rings emanating from bursting bubbles of different sizes resting at the air-liquid interface. Such flows may be important in lofting fine marine aerosol droplets from near the sea surface into the atmosphere and may also affect the air-sea momentum flux. Further, jets and vortex rings emanating from bubbles in carbonated beverages may be important in carrying odor molecules to the nose of the consumer. We thus focus on the fluid mechanics of the pressurized gas escaping through the expanding hole in the bubble cap. This flow is driven by a pressure difference $\Delta P$ and is known $a priori$ by $\Delta P = 4\sigma / R_c$, where $\sigma$ is the surface tension and $R_c$ is the bubble film radius of curvature. Therefore, bubbles of different sizes may produce significantly different flows. We present measurements of the gas jet and vortex ring characteristics and relate
these quantities to the bubble Bond number and jet Reynolds number, respectively. We also present a theoretical quasi-one-dimensional nozzle model to predict the velocity of the gas jet generated by the bubble bursting.

4.2 Experimental Methods

Figure 4.1 shows a 3D schematic of the experimental setup. The experimental setup comprised a rectangular acrylic tank (10.2 × 10.2 × 15.2 cm$^3$) with a bulkhead fitting in the bottom through which various nozzle tips could be inserted. The tank was filled to the brim with deionized (DI) water in order to reduce the meniscus. Single bubbles could be injected at a constant rate by using a programmable syringe pump (New Era Pump Systems Inc.). In this way, a single bubble on the still water surface could be obtained. Several different nozzle diameters (gauge 17-25) and a drawn-out capillary tube were used to produce a wide range of bubble sizes (bubble diameters of 440 µm-4 cm). The nozzle tip was generally 5 cm below the water surface, but bubbles greater than 2 cm in diameter were created by raising the nozzle tip 3 cm closer to the water surface, thus allowing multiple smaller bubbles to coalesce into a single large bubble. The tank was mounted on an elevating table (Velmex B29) in order to facilitate visualization. The tank and syringe pump were enclosed inside a transparent acrylic box (63.5 × 63.5 × 76.2 cm$^3$) in order to prevent stray air currents from influencing the air flow emanating from the bubbles, which were allowed to age and burst naturally.
Figure 4.1 Setup for high speed stereophotogrammetric visualization experiments. The labels represent the following: (1) Camera (master), (2) Camera (slave), (3 and 4) LED backlighting, (5) Syringe pump, (6) Acrylic tank, (7) Acrylic box, and (8) a smoke-filled bubble floating on the water surface.

The bursting bubbles were visualized by two synchronized and orthogonally positioned high-speed cameras (Photron FASTCAM SA-Z and Phantom VEO 640S) recording at 5-10 kHz. The spatial resolution of the Photron camera was 1024×1024 pixels while that of the Phantom camera was 768×1024 pixels. As shown in Figure 4.1, backlighting was provided by two LED lights (Nila NINZLB), and additional lighting was provided by halogen lamps. As shown in Table 4.1, the fields of view of the two cameras varied according to the bubble size from 19×19 mm to 203×203 mm, with the spatial resolutions varying accordingly. In order to quantify three-
dimensional phenomena within the measurement volume shared by the two cameras, stereophotogrammetry (i.e. estimating three-dimensional coordinates within a spatially calibrated volume using multiple cameras viewing that volume from different perspectives) was used. Stereophotogrammetry provides more accurate measurements by avoiding projection errors inherent in using only a single camera (e.g. when a bursting bubble ejects gas outside of the plane of focus). For example, measured jet speeds were 7-15% greater using stereophotogrammetry as compared to the use of a single camera. The cameras were spatially calibrated using the bundle adjustment method as implemented in the DLTdv5 and Argus 3D software packages (Hedrick 2008; Lourakis and Argyros 2009, Theriault et al 2014, Jackson et al. 2016). The calibration was accomplished using either a transparent ruler or a 15.2 cm long calibration wand which was moved through the measurement volume in a variety of orientations. The cameras were set to a post-trigger configuration and were manually triggered upon observing a bubble bursting within the focal plane.

Though several hundred bursting events were observed, sixty-six video pairs in which the bubble was well focused were selected for further analysis. In these videos, the first 500 to 1000 frames were saved at full temporal resolution in order to examine the bubble bursting event in detail, and the full video was saved at decreased temporal resolution in order to examine the behavior of the released smoke. The resulting videos then were processed in DLTdv5 software (Hedrick 2008). The bubble size was first characterized. For small (<3mm) bubbles which are essentially spherical and submerged beneath the air-water interface, the horizontal bubble diameter $2R$ was measured in ImageJ software (NIH) beneath the water surface as shown in Figure 4.2(a). Larger bubbles are more hemispherical and float atop the water surface.
Table 4.1 A comprehensive list of the filming characteristics for each of the two cameras used in the experiments

<table>
<thead>
<tr>
<th>Recording parameters</th>
<th>Photron FASTCAM SA-Z (1024×1024)</th>
<th>Phantom VEO 640S (768×1024)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lens</td>
<td>Lens</td>
</tr>
<tr>
<td>Magnification (kHz)</td>
<td>Focal length (mm)</td>
<td>Focal length (mm)</td>
</tr>
<tr>
<td>Magnification (kHz)</td>
<td>Field of view (mm)</td>
<td>Field of view (mm)</td>
</tr>
<tr>
<td>Magnification (kHz)</td>
<td>Spatial resolution (µm/pixel)</td>
<td>Spatial resolution (µm/pixel)</td>
</tr>
<tr>
<td>Magnification (kHz)</td>
<td>Exposure time (s)</td>
<td>Exposure time (s)</td>
</tr>
<tr>
<td>Low 5</td>
<td>50 203×203 198 22.5 8.74</td>
<td>50 203×203 265 7.5 13.7</td>
</tr>
<tr>
<td>Low 5</td>
<td>50 174×174 170 18.75 8.74</td>
<td>50 174×174 227 5 13.7</td>
</tr>
<tr>
<td>Medium 5</td>
<td>105 64×64 62 30 8.74</td>
<td>105 64×64 83 12.5 13.7</td>
</tr>
<tr>
<td>High 10</td>
<td>200 19×19 19 45 4.37</td>
<td>105 19×19 25 15 6.85</td>
</tr>
<tr>
<td>2D Low 10</td>
<td>50 203×406 396 8.75 4.34</td>
<td></td>
</tr>
</tbody>
</table>
Thus, as shown in Figure 4.2(b), the bubble equivalent diameter $2R$, the diameter of a sphere with the same enclosed volume as the hemispherical bubble, was calculated in the following manner (Teixeira 2015). The bubble height $H$ (the vertical distance from the top of the meniscus to the top of the bubble cap film) and base diameter $2r$ (measured at the top of the meniscus) were measured using DLTdv5 software. Then, the bubble contact angle $\theta_b$, which is the inclination at the top of the meniscus, was determined by

$$\theta_b = 2 \tan^{-1} \left( \frac{H}{r} \right). \quad (4.1)$$

The bisector for the bubble contact angle $\theta_b$ passes through the upmost point on the bubble cap. Then, the bubble equivalent radius $R$ and thus the bubble equivalent diameter $2R$ were calculated by

$$R = \frac{r}{\sin \theta_b} \quad (4.2)$$

It is important to correlate the bubble film radius of curvature $R_c$ in the Young-Laplace equation $\Delta P = 4\sigma / R_c$ with bubble radius $R$ in order to calculate the pressure increase inside a bubble. Here, $R_c = 2R$ for bubbles with $2R < 3$ mm and $R_c = R$ for bubbles with $2R > 3$ mm because the cavity radius was directly measured for bubbles less than this size whereas the film radius of curvature was calculated for bubbles greater than this size. In addition, the Bond number was determined for each bubble as

$$Bo = \frac{\rho g R^2}{\sigma} \quad (4.3)$$

The frame at which bursting initiated was determined and set as $t=0$. The azimuthal angle $\xi$ of the rupture in the bubble film above the water surface was measured. Videos in which the rupture was located at $\xi < 60^\circ$ were eliminated because the retracting film often hit the water surface, thus altering the gas emission from inside the bubble and interfering with vortex ring formation. Forty-one videos were thus left for further processing. In these videos, the position of
the front of the gas jet emanating from the expanding hole in the bubble film (marked with an arrow in Figure 4.3) was manually tracked in 3D space over 3-5 ms, thus allowing calculation of jet speed $V_{\text{jet}}$ over time. The parameter $V_{\text{jet}}$ was calculated at 0.4 ms, 1 ms, and 2 ms by dividing the distance the jet front traveled by the time elapsed between the two frames temporally centered on each of these three time points. For example, for data acquired at 5000 Hz, $V_{\text{jet}}$ at $t=1$ ms was calculated between frames at $t=0.8$ ms and $t=1.2$ ms, but for data acquired at 10000 Hz, $V_{\text{jet}}$ at $t=1$ ms was calculated between frames at $t=0.9$ ms and $t=1.1$ ms. For ten videos, the jet length $L_{\text{jet}}$ and jet frontal diameter $D_{\text{jet}}$, shown in Figure 4.2(c), were additionally measured in 3D space at $t=0.4$ ms as inputs to the analytical model described in the appendix. The Reynolds number of the jet was calculated for these videos as

$$Re_{\text{jet}} = \frac{V_{\text{jet}}D_{\text{jet}}}{\nu}$$

(4.4)

where $\nu = 1.5\times10^{-5}$ m$^2$/s is the kinematic viscosity of air at 21°C. The hole diameter $D_h$ (shown in Figure 4.2(c)) also was measured for these videos at $t=0.2$ ms and $t=0.4$ ms as model input.

For each bursting bubble, a single point on the retracting film lip was similarly tracked over one to four frames in order to calculate film retraction speed $V_{\text{fr}}$. For the smallest bubbles, the film completely retracted over only 1-2 frames, but the film retraction speed was measured for up to four frames for larger bubbles. Because film retraction speed was found to increase over these time periods (by 0.2% for a 2 mm bubble and 2% for a 35 mm bubble) from the point of bursting, the last-measured film retraction speed was used. The film thickness $h$ was then calculated using the Taylor-Culick equation. In addition, the time from bubble formation to bursting $t_b$ was recorded for those videos in which the instance of formation was also captured. For small bubbles, formation was defined as the point in time at which the bubble reached the water surface. For large bubbles (>2 cm) formed by the merging of smaller bubbles, formation was defined as the point in time at
which the coalesced bubble obtained a hemispherical form. Finally, the number of vortices initially formed by the bursting bubble was counted.

![Diagram](image)

Figure 4.2 (a) Experimentally measured bubble diameter (2R) for small bubbles. (b) Experimentally measured bubble dimensions for larger bubbles: apparent bubble contact angle $\theta_b$, radius $r$ of the top of the meniscus or base of the bubble, and bubble height $H$ measured from the top of the meniscus. The radius $r$ is related to the bubble film curvature radius $R$ through $r = R \sin \theta_b$. (c) Schematic showing a gas jet emitted from a bursting bubble and the parameters used in the theoretical model. (d) Schematic of a vortex ring showing the major ($D_{VR}$) and minor ($d_{VR}$) diameters.

The three-dimensional travel distance $TD_{VR}$ of the smoke jet or primary vortex ring initially released from a bursting bubble was measured at a time point by which the forward progression of the smoke feature (i.e. jet or vortex ring) became negligible. In some cases, the smoke vortex ring left the shared field of view prior to stopping, in which case the observed travel distance was not recorded. Because of this limitation, another set of experiments with a larger field of view...
20.32×40.64 cm (Table 4.1) was conducted with a single camera (FASTCAM) to measure the maximum projected (i.e. two dimensional) travel distance of vortex rings released from the larger (>2cm) bubbles. In these experiments, bubbles bursting within the plane of focus were preferentially selected for analysis in order to more accurately measure the vortex ring travel distance. Finally, for eight videos, 2D projections of the major diameter $D_{VR}$ and minor diameter $d_{VR}$, shown in Figure 4.2(d), of vortex rings emitted from bubbles of different sizes were measured for 5 ms past the time of vortex formation (defined as the point in time when the core of the vortex ring has acquired a measurable minor diameter) in order to investigate vortex ring entrainment and growth.

4.3 Results

4.3.1 Flow Visualization

Figure 4.3 shows a sequence of images illustrating the bursting of a 41.0 mm diameter smoke-filled bubble resting on the water surface. At $t=0.4$ ms, a 3.02 mm diameter hole has opened in the bubble cap film, the rim of which is retracting at a speed of 1.56 m/s, and a smoke jet has emerged at a speed of 4.74 m/s. At $t=3$ ms, the hole in the bubble cap has grown to 17.3 mm in diameter, and the rim has acquired a crown shape of circumferentially-distributed short ligaments extending outwards. In addition, the speed of the gas jet front decreases to 4.16 m/s at this time point. At $t=5.8$ ms, the bubble film has almost completely collapsed into the surrounding water. A few short ligaments extending outwards from the rim pinch off film drops prior to falling into the surrounding water. The major and minor diameters of the primary smoke vortex ring (illustrated in Figure 4.2(d) and marked with an arrow) have grown to 5.48 mm and 0.92 mm, respectively, while the vortex ring travels at a forward speed of 3.41 m/s at this time point ($t=5.8$ ms). Beneath the primary vortex ring, the smoke jet diameter increases towards the water surface. A series of
four to five vortices lies along the edge of this smoke jet (marked with a line). These vortices arise from a Kelvin-Helmholtz instability as the liquid film rapidly retracts over the surface of the smoke within the bubble and thus may not be fully formed vortex rings. A similar Kelvin Helmholtz instability process has been observed in bursting spherical soap bubbles (Buchholz and Sigurdson 2000, Jaw et al. 2007, Kim et al. 2014). At $t=12.6$ ms, the initial vortex ring is traveling at 2.51 m/s and has traveled a distance of 36.8 mm above the water surface. The primary vortex ring has grown substantially to 15.7 mm and 3.22 mm for the major and minor diameters respectively. The Kelvin-Helmholtz vortices also have grown but to a smaller extent. At $t=33.6$ ms, the primary vortex ring continues to grow and travel upwards at a speed of 1.38 m/s and has entrained much of the smoke constituting the now-disorganized secondary Kelvin-Helmholtz vortices.

Figure 4.3 Sample images from a time series showing the bursting of an $2R=41.0$ mm smoke-filled bubble floating on the water surface. The hole in the bubble cap forms at $t=0$ ms. The arrow at $t=3$ ms indicates an example of the front of the smoke jet which was tracked to measure jet speed. The two stationary water droplets are on the acrylic box and remain from a previous experiment.
Figure 4.4 shows a sequence of images illustrating the bursting of a 19.3 mm diameter smoke-filled bubble resting on the water surface. At $t=0.4$ ms, a 2.36 mm hole has opened in the bubble cap film, the rim of which is retracting at a speed of 3.04 m/s, and a smoke jet has emerged at a speed of 4.01 m/s. At $t=2.6$ ms, the jet has rolled up into a vortex ring with major and minor diameters of 4.58 mm and 0.96 mm, respectively. The primary vortex travels upward at a speed of 3.16 m/s. However, the base of the smoke jet is wider since the hole in the bubble film has continued to grow and now has a diameter of 10.2 mm. At $t=4.8$ ms, the bubble film has almost completely collapsed into the surrounding water. A few short ligaments extending outwards from the rim are visible, but these do not pinch off any film drops prior to falling into the surrounding water. The initial smoke vortex ring has grown to 6.91 mm and 1.55 mm for the major and minor diameters, respectively, while travelling at a speed of 2.51 m/s. A second vortex ring which is slightly larger but somewhat less developed is evident at this time point. This vortex ring forms when the hole in the film is much larger than the hole producing the first vortex ring. At $t=9.8$ ms, the initial vortex ring continues to grow and travels at a speed of 1.55 m/s. The second vortex ring has grown substantially, and a third, less well developed vortex ring is visible beneath the second vortex ring. The second and third vortex rings die out quickly, as seen at $t=13.6$ ms, whereas the first vortex ring continues to grow and travel upwards at a speed of 1.02 m/s.

Figure 4.5 shows a sequence of images illustrating the bursting of a 2.01 mm diameter smoke-filled bubble. The bubble initially rests on the water surface ($t=-5.9$ ms) and bursts at $t=0$ ms. The film retracts over three frames at a speed of 5.86 m/s and ejects a spray of film drops nearly horizontal to the water surface (not shown).
At $t=0.4$ ms, the film has fully retracted and the gas jet has started emerging with a speed of 1.44 m/s. However, the cavity has not yet begun to collapse upwards at this time point. At $t=1.7$ ms, the speed of the smoke jet decreases to 1.01 m/s after traveling 1.34 mm above the water surface and its leading edge has begun to roll up into the beginning of a vortex ring. The leading edge of the smoke jet is substantially thinner than its base which has a width of 1.66 mm. Further, the cavity bottom is collapsing upwards at a speed of 1.83 m/s. At $t=4.4$ ms, the tip of the smoke jet continues to roll up into a vortex ring which ascends upwards with a speed of 0.39 m/s. The base of the smoke column continues to expand as smoke is expelled from the initial cavity by the rise of the water surface. Indeed, the subsurface cavity is no longer visible, and the tip of the upward-
projecting jet (i.e. a Worthington jet) reaches above the water surface at this point (Gordillo and Gekle 2010). At $t=6.8$ ms, a primary vortex ring has completely formed which travels at 0.19 m/s and has major and minor diameters of 1.42 and 1.16, respectively. The advancing Worthington jet becomes unstable at a speed of 2.16 m/s, from which a single jet droplet pinches off. A similar vortex ring structure has been observed in bursting bubbles on the water surface by Iacano and Blanchard (1987) for bubbles in the size range of 2-6 mm. At $t=10.6$ ms, the forward speed of the vortex ring decreases to 0.15 m/s, which is less than that of the jet drop traveling at 1.54 m/s. In addition, a secondary, less well developed smoke vortex ring forms below the first vortex ring, the source of which seems to be the upward collapse of the cavity and the formation of the Worthington jet. Also, the water column has begun falling back onto the water surface at this point. At $t=16.6$ ms, the first vortex ring continues to rise at a speed of 0.12 m/s while the secondary vortex ring has separated from the water surface and rises at a speed of 0.53 m/s (faster than the primary vortex ring). The jet drop does not interact with the primary smoke vortex ring in this case, but in some videos the drop travels through the primary vortex ring and carries a thin trail of smoke in its wake. Finally, at $t=27.5$ ms, the speeds of the primary and secondary vortex rings decrease to 0.03 m/s and 0.08 m/s, respectively, as their initial momentum dies away.

Figure 4.6 shows a sequence of images illustrating the bursting of a 691 µm diameter smoke-filled bubble. The bubble initially rests below the water surface ($t=-2.7$ ms) and bursts at $t=0$ ms. The film retracts over two frames at a speed of 6.50 m/s. At $t=0.4$ ms, the film has fully retracted and the gas jet (shown in the inset) has started emerging with a speed of 0.67 m/s. At $t=1.2$ ms, a thin stem-like jet with a width of approximately 45 µm has ejected with a speed of 0.58 m/s after traveling 0.91 mm above the water surface. While no vortex ring is formed, the tip of the jet does curl downwards, as seen at $t=4.4$ ms. At this time point, the upmost portion of the smoke
jet has traveled 1.55 mm and continues to ascend upwards with a speed of 0.19 m/s. Day (1964) and Day and Lease (1968) used long exposure photography to visualize tracks of film droplets ejected from similarly sized bubbles. These tracks appear very similar to the smoke jet patterns observed here.

Figure 4.5 Sample images from a time series showing the bursting of an 2R=2.01 mm smoke-filled bubble floating on the water surface. The hole in the bubble cap forms at \( t=0 \) ms.

4.3.2 Gas Jet Characterization

Figure 4.7 shows the relation between the bubble equivalent diameter and \( V_{\text{jet}} \). Gas jet front speeds are shown at three different time points (\( t=0.4 \) ms, 1 ms, and 2 ms) after hole nucleation in order to show jet speed decay over time and because, in some videos, the gas jet front speed could
not be measured at the earliest time point. In small bubbles, this was because the smoke had not yet emerged from the subsurface bubble cavity. In larger bubbles, the bubble cap sometimes occluded the cameras’ view of the smoke jet.

![Sample images from a time series showing the bursting of an 2R= 691 µm smoke-filled bubble floating on the water surface.](image1)

Figure 4.6 Sample images from a time series showing the bursting of an 2R= 691 µm smoke-filled bubble floating on the water surface. The hole in the bubble cap forms at t=0 ms. The inset at t=0.4 ms shows a magnified region of interest represented by the white box in which the contrast has been increased. The scale bar in the inset is 0.5 mm.

![Graph showing gas jet front speed](image2)

Figure 4.7 Gas jet front speed $V_{jet}$ as a function of bubble equivalent diameter 2R measured at three time points (t=0.4 ms, t=1 ms, and t=2 ms) after hole nucleation, which occurs at t=0 ms. The inset shows the Reynolds number of the emerging jet at t=0.4 ms as a function of the parent bubble Bond number and a line representing the scaling of $Re_{jet}=126Bo^{0.5}$.
Figure 4.7 shows that jet speed increases linearly with bubble equivalent diameter from the smallest bubble (440 µm), which has an ejection speed of 0.28 m/s at \( t = 1 \) ms, to a bubble size of 5.44 mm, which has an ejection speed of 2.53 m/s at \( t = 1 \) ms. These measured speeds are significantly higher than those theoretically estimated by Iacano and Blanchard (1987) based on relations between characteristic dimensions and speeds of vortex rings (e.g. 0.8-0.9 m/s for a 4.9 mm diameter bubble). For larger bubbles, jet speed also increases linearly but at a lower rate, reaching a maximum ejection speed of 4.74 m/s at \( t = 0.4 \) ms for a bubble size of 41 mm. Given that a 4 mm diameter bubble has interior excess pressure of 141 Pa and a 4 cm bubble has a much smaller corresponding interior excess pressure of 14.1 Pa, it is perhaps surprising that the initial speed of the air jet emitted from the large bubble is so much greater than that emitted from the tiny bubble. This seeming contradiction may be partially explained by the corresponding differences in film retraction speed and film thickness shown in Figure 4.8.

Figure 4.8 Film retraction speed \( V_{fr} \) and the calculated film thickness \( h \) at \( t = 0.4 \) ms after hole nucleation as a function of bubble equivalent diameter \( 2R \)
Film retraction speed decreases with increasing bubble equivalent diameter, whereas the film thickness increases with increasing bubble equivalent diameter. Thus, small bubbles had thin films which retracted at extremely high speeds whereas large bubbles had thicker films which retracted more slowly. For example, the cap of the smallest bubble (440 µm) has a film thickness of 3.2 µm and retracts at a speed of 6.65 m/s. In contrast, the cap of the largest bubble (41 mm) acquires a thickness one order of magnitude greater (57.9 µm) and retracts at a lower speed of 1.56 m/s. Considering that both small and large bubbles experience the same surface tension, the lower speeds of the thicker films are due to the greater mass contained in these films as compared to the lower mass contained in the thin films. It should be noted that all bubbles burst within one second of formation and thus drainage played a minimal role in thinning the bubble caps prior to bursting (Lhuissier and Villermaux 2011). The effect of higher film retraction speeds on small bubbles is that the bubble cap film is completely destroyed before any significant airflow escaping from the bubble interior has begun. For example, in Figure 4.5, the bubble cap film is gone by $t=0.4$ ms, but the smoke has not yet emerged from the bubble cavity. Thus, when the smoke jet begins emerging, it does so through a relatively wide aperture. In contrast, the slowly retracting film found in large bursting bubbles means that the air jet is ejected at high speeds through a relatively small aperture. This difference in aperture size relative to bubble size is quantified in Figure 4.9(a), which shows the ratio of the cross-sectional area of the aperture in the bubble cap ($A_h$) at $t=0.4$ ms after bursting to the surface area of the bubble cap prior to bursting ($A_c$). The ratio $A_h/A_c$ decreases exponentially from approximately 1 for a 440 µm bubble to approximately 0.062 for the largest bubble tested. Further, Figure 4.9(b) shows that jet ejection speed decreases linearly with increasing film retraction speed, thus showing that the effect of film retraction speed dominates over the effect of bubble size and excess pressure.
Figure 4.9 (a) Ratio of the cross-sectional area of the opening in the bubble cap $A_h$ at $t=0.4$ ms after bursting to the surface area of the bubble cap prior to bursting $A_c$ as a function of bubble equivalent diameter $2R$. Sketches indicate the relative sizes of expanding hole in the bubble film to the bubble cap size for small and large bubbles at this time point. (b) Jet speed $V_{jet}$ as a function of film retraction speed $V_{fr}$ at $t=0.4$ ms

Returning to Figure 4.7, the change in slope between the two regions in the relationship between $V_{jet}$ and $2R$ is likely due to decreasing submergence and the transition in bubble shape from spherical ($2R<~3$ mm) to ellipsoidal ($~3$ mm$<2R<~9$ mm) to hemispherical ($2R>~9$ mm) as bubble size increases. Indeed, this transition (marked by a dashed line) occurs at a bubble equivalent diameter of $2R=5.44$ mm, which corresponds to $Bo=1$, the point at which gravitational forces are balanced by surface tension forces. Bubbles with $Bo<1$ are small, mostly spherical, and are largely submerged beneath the water surface by the effect of surface tension. In addition to the
effect of relatively large openings caused by rapid film destruction, the relatively lower jet speeds in this region may be a result of continuity, in which some of the air submerged in a cavity (once the bubble film is burst) is ejected only when the cavity collapses upward. In contrast, bubbles with $Bo>1$ are large, hemispherical, float on the water surface under the effect of buoyancy, and emit relatively higher speed jets through relatively narrow apertures. In addition, the inset in Figure 4.7 shows $Re_{jet}$ at $t=0.4$ ms as a function of the parent bubble $Bo$ on a log-log scale plot. Also plotted is a proposed scaling of $Re_{jet}=126Bo^{0.5}$, which corresponds to $Re_{jet}=126 R/a$, where $a = \sqrt{\sigma/\rho g}$ is the capillary length. This simple scaling fits the data well over four orders of magnitude (though it deviates slightly for large bubbles) and encapsulates the effects of bubble size, surface tension, gravity, and density on the inertia of the emerging air jet. Similar types of scaling for the dynamics of the rising liquid jet ejected upon bubble collapse also have been developed (Ghabache et al. 2014; Krishnan et al. 2017; Deike et al. 2018).

Figure 4.7 also shows that jet speed sharply decreases with time over the first several ms after hole nucleation, a trend that is visualized in the four examples presented in Figures 4.3-4.6. This decrease in jet speed with time contradicts the theoretical expression for jet speed proposed by Lhuissier and Villermaux (2011), in which the decreasing diameter of a bursting bubble cap causes a corresponding increase in jet speed. One possible reason for this discrepancy is that jet speeds were measured at the tip of the advancing jet front in the current study whereas their theoretical jet speed is located at the opening in the bubble film. However, if jet speed at the opening increased with time, the jet speeds measured at the tip also should have increased with time, which the visualizations do not show. Instead, it seems that the time scale of depressurization is much smaller than that of film deflation, leading to jet speeds which decrease over time. Further, the jet speed and corresponding vortex ring speed decrease more sharply with time for small
bubbles as opposed to large bubbles, which have more momentum. Figure 4.10 shows the jet speed $V_{\text{jet}}$ normalized by the initial jet speed $V_o$ for three different bubble sizes. The jet emitted from the 2.0 mm bubble has lost half of its initial speed by 2.4 ms whereas the jet emitted from the 41 mm bubble has lost only 6.1% of its speed at this time point.

Figure 4.10 Decay in jet speed $V_{\text{jet}}$ normalized by initial jet speed $V_o$ over time for bubble equivalent diameters of $2R = 41$ mm, 19 mm, and 2 mm

4.3.3 Vortex Ring Characterization

The characteristics of vortex rings produced by bursting bubbles varied greatly with bubble size and the corresponding jet Reynolds number and are examined here. Figure 4.11(a) shows temporal growth of the major and minor diameters ($D_{VR}$ and $d_{VR}$, respectively) of the primary vortex ring ejected from the bursting of three bubbles (i.e. the same as in Figures 4.3-4.5 and Figure
4.10), beginning from the time of vortex ring formation. These three are shown as representative examples of the eight bubbles for which this analysis was performed. The inset in Figure 4.11(a) shows \( \frac{D_{VR}}{d_{VR}} \) at the time of vortex ring formation as a function of \( Re_{jet} \). The initial shape of the vortex ring varied with \( 2R \) and \( Re_{jet} \). The high \( Re_{jet} \) vortex rings formed from the \( 2R=41 \text{ mm} \) and \( 2R=19 \text{ mm} \) bubbles are highly oblate, with initial values of \( D_{VR} \) that are approximately 6.4 and 4.8 times greater than \( d_{VR} \), respectively. In contrast, the low \( Re_{jet} \) vortex rings produced by smaller bubbles are more spherical, with a ratio of \( D_{VR}/d_{VR}=1.9 \) for the \( 2R=2 \text{ mm} \) bubble. Ahkmetov (2009) similarly found that vortex ring oblateness sharply increased as a function of Reynolds number in the range of \( 20<Re<10,000 \) and that a vortex ring with Reynolds number of 20 was nearly spherical and closely approximated a Hill’s spherical vortex.

As shown in Figure 4.11(a), the vortex ring major and minor diameters increase linearly over time from the time of vortex formation (though the major diameter of the vortex produced by largest bubble increases somewhat faster). The inset in Figure 4.11(b) shows the growth rate \( \omega_{VR} \) of \( D_{VR} \) and \( d_{VR} \) as a function of bubble size as determined from the slope of a line fit to the corresponding vortex ring dimension plotted over time (e.g. Figure 4.11(a)). Minor diameters grew at nearly the same rate for all bubble sizes, but major diameters grew faster with increasing bubble size. In order to examine the volumetric growth rate of the vortex rings, the vortex rings were modeled as oblate ellipsoids with major diameters of \( D_{VR} \), minor diameters of \( d_{VR} \), and volumes of \( V_{VR} = \frac{\pi}{6} D_{VR}^2 d_{VR} \). Figure 4.11(b) shows the volumetric growth rate \( \alpha_{VR} \) of the vortex rings over the first 5 ms past their formation as a function of \( Re_{jet} \). The parameter \( \alpha_{VR} \) increases with \( Re_{jet} \) which indicates higher entrainment of ambient air for vortex rings ejected from larger bubbles. This higher entrainment can be largely attributed to the increase in vortex ring major diameter with increasing bubble size.
Figure 4.11 (a) Temporal growth of the major ($D_{VR}$) and minor diameters ($d_{VR}$) of the primary vortex ring ejected from bursting bubbles with bubble equivalent diameters of $2R = 41$ mm, 19 mm, and 2 mm. The inset shows the ratio $D_{VR}/d_{VR}$ for bubbles of different sizes as a function of $Re_{jet}$. (b) Volumetric growth of the primary vortex ring ejected from bursting bubbles as a function of $Re_{jet}$. The inset shows the growth rates of the major and minor diameters as a function of bubble equivalent diameter.

Figure 4.12 shows the relation between the bubble equivalent diameter and the travel distance of the primary vortex ring. The travel distance of the smoke jet or primary vortex ring
increases linearly with bubble equivalent diameter from 6.5 mm for the smallest bubble (440 µm) up to 336 mm for a 44 mm bubble. Also plotted in Figure 4.12 are the maximum heights of vortex rings ejected from bursting bubbles on the water surface as measured by Iacano and Blanchard (1987) for bubbles in the size range of 2-6 mm. Since their vortex rings largely traveled vertically, little difference is seen between the two different measurements. The observed increase in travel distance with 2R corresponds to the observed increase in initial jet speed with 2R as larger bubbles generate high velocity jets which roll up to form far-traveling vortex rings. The slope of the line in Figure 4.12 shows that the vortex ring travel distance is approximately seven times the bubble equivalent diameter. In sum, large bubbles eject higher Reynolds number jets which roll up into highly oblate vortex rings which grow quickly and travel far whereas small bubbles eject lower Reynolds number jets which roll up into more spherical vortex rings which grow more slowly and travel short distances.

A further factor which likely influences the vortex ring formation is the retraction speed of the film. Owing to the different speeds at which the bubble film retracts as a function of bubble size, bubbles of different sizes will operate as vortex ring generators with ‘exit nozzles’ opening at different speeds. Dabiri and Gharib (2005) used particle image velocimetry to study how temporal changes in the exit nozzle diameter of vortex generators affect vortex ring characteristics. These authors found higher-energy vortex rings with peak vorticity located further from the axis of symmetry (as compared to a static nozzle) when the nozzle exit diameter increased over time during the early stages of vortex ring formation. A direct comparison with the trends they observed is difficult because of a lack of measured flow fields in the current study and because of confounding variables such as different pressures in different sized bubbles. It would be interesting
in the future to study how vortex ring characteristics vary with differences in film retraction speed by varying the fluid viscosity.

Figure 4.12 Vortex ring travel distance $TD_{VR}$ as a function of bubble equivalent diameter $2R$. Black dots indicate three-dimensional travel distances measured by the stereophotogrammetry system whereas red ‘x’ symbols indicate two-dimensional projections measured by one camera. Also plotted are vortex ring travel distance measurements reported by Iacano and Blanchard (1987) for bubbles in the size range of 2-6 mm.

Finally, Figure 4.13 shows the relation between the bubble equivalent diameter and the number of initially formed vortex rings or vortex ring-like instabilities. As shown in Figure 4.6, tiny bubbles (<1.5 mm) eject a thin stem-like jet which does not roll up to form a vortex ring. Slightly larger bubbles form only a primary vortex ring. As bubble size increases further (1.5-7 mm), a second vortex ring may form from one of two mechanisms. In the first mechanism, the
impulse of the advancing Worthington jet creates a second vortex ring at the water surface, as shown in Figure 4.5. In the second mechanism, the rapid retraction of the film and escaping gas create a shear layer which undergoes a Kelvin-Helmholtz instability, giving rise to what appears to be a second vortex ring. However, vortex rings formed in this manner are much less organized than the primary vortex ring and dissipate quickly without traveling a substantial distance. Medium (8-28mm) and large (>29mm) bubbles produce a primary vortex ring and additional vortex ring-like flow structures using this secondary mechanism. As the bubble size increases in this range, the number of these instability-induced vortex rings increases because of lower film retraction speed and larger enclosed volume and surface area. Jaw et al (2007) similarly investigated the bursting of a spherical soap bubble and found comparable Kelvin-Helmholtz instabilities as the film retracts around the bubble periphery. However, owing to the reduced surface tension and corresponding decrease in the pressure difference across the bubble cap, these authors did not observe the ejection jet found here as the flow was dominated by the retracting film.

Figure 4.13 The number of the initially formed vortex rings as a function of bubble equivalent diameter 2R
4.4 Discussion

The observed bubble bursting, gas jet, and vortex ring formation phenomena have potential implications for ocean-atmosphere interactions. The gas jets and vortex rings produced by bursting bubbles may be important in lofting marine aerosol droplets from near the sea surface into the atmosphere (Iacano and Blanchard 1987). Tiny jet and film droplets with low settling speeds would be particularly susceptible to being entrained and carried by these vortex rings. Indeed, cloud chamber experiments by Blanchard (1963) and Day (1964) showed film droplets being advected upwards in apparent vortex rings produced by bubbles larger than about 2 mm in diameter. In the current experiments, the highest spatial resolution (19 µm/pixel) allowed visualization of droplets larger than approximately 38 µm (i.e. 2 pixels). Thus, these tiny film droplets were not visualized. However, in several higher magnification recordings at 80 kHz with spatial resolution of 8.3 µm/pixel, we observed approximately 22 µm diameter droplets which appeared to be advected upwards in the vortex ring’s trailing jet. These droplets appeared to be generated from the retracting film impacting the surrounding water surface. Indeed, droplets entrained by vortex rings need not be formed from the film cap of the bubble ejecting the vortex ring but may form from the bursting of a prior bubble. The experiments by Iacano and Blanchard (1987) showed that 5 µm diameter fog droplets residing in a layer on the water surface were easily entrained and advected by the vortex ring. An analysis of the Stokes number St can reveal how well droplets of various sizes are carried by these vortex rings, where a value of St<0.1 means that the particles faithfully follow the flow. Here \( St = \frac{t_{\text{relaxation}}}{t_{\text{characteristic}}} \), where \( t_{\text{relaxation}} = \frac{\rho_p D_p^2}{18 \mu_p} \) is the particle relaxation time and \( t_{\text{characteristic}} = \frac{D_{VR}}{V_{jet}} \) is the time scale of the flow, where \( \rho_p \) is the particle density, \( D_p \) is the particle diameter, and \( \mu_p = 1.81 \times 10^{-5} \) is the dynamic viscosity of air. It is found that the maximum droplet size for the condition St<0.1 is approximately 6 µm regardless of bubble size. However,
the trajectories of somewhat larger droplets will still be influenced by the airflow. Bubble size also will affect the height to which entrained droplets are carried. We found a linear relationship which indicates that vortex ring travel distance is approximately seven times the parent bubble diameter. Thus, a vortex ring produced from a large 41 mm diameter bubble (though bubbles this size are rare in nature; Iacano and Blanchard 1987), may loft tiny droplets approximately 1/3 meter whereas a vortex ring produced by a more common 4 mm bubble could potentially loft a droplet by 30-40 mm. In a similar vein, Kim et al (2019) recently found that the vortex ring created by raindrop impact on a leaf effectively lofted fungal spores out of the leaf’s boundary layer, thus enabling transport over long distances through the atmosphere. Vortex rings generated by bubble bursting may similarly serve an important role in lofting fine aerosol droplets from near the sea surface into the atmosphere. Vortex rings generated by the bursting of champagne or carbonated beverage bubbles could similarly play an important role in transporting odor molecules to the nose of the consumer (Liger-Belair et al. 2009). Finally, considering that the maximum flow speeds of the high impulse air jets released by bursting bubbles (e.g. up to approximately 4 m s\(^{-1}\) for the largest bubbles and over 1 m s\(^{-1}\) for more common 2 mm diameter bubbles) are comparable to near-surface air speeds when the 10 meter height wind speed \(U_{10}\) is in the range of 5-10 m s\(^{-1}\) (Buckley and Veron 2017, Yousefi et al 2020), it may also be worth considering how bubble bursting could influence the momentum flux at the air-sea interface.

4.5 Conclusions

In conclusion, we have visualized, characterized, and modeled the gas jets and vortex rings released from bubbles with diameters ranging from 440 µm to 4 cm bursting at the air-water interface. The smallest bubbles produced a thin stem-like jet which did not roll up to form a vortex ring. Larger bubbles produced increasing numbers of vortex rings, and the largest bubble produced
an oblate primary vortex ring followed by a series of vortices produced by a Kelvin-Helmholtz instability resulting from film retraction. The initial speed of the gas jet released from the bubbles sharply increased with parent bubble size until the bubble Bond number reached unity, corresponding to a bubble equivalent diameter of 2R=5.44 mm, and subsequently increased at a slower rate. The high film retraction speeds characteristic of smaller bubbles produced correspondingly slower jets emitted through relatively larger openings whereas the low film retraction speeds characteristic of larger bubbles produced correspondingly higher jet speeds emitted through relatively smaller openings. A simple scaling relationship of $Re_{jet} = 126 Bo^{0.5}$ is proposed to relate the Reynolds number of the emerging jet to the Bond number of the parent bubble. It also was found that large bubbles eject slowly decaying, high speed, high Reynolds number jets which roll up into highly oblate vortex rings which grow quickly and travel far whereas small bubbles eject quickly decaying, low speed, low Reynolds number jets which roll up into more spherical vortex rings which grow more slowly and travel short distances. Finally, upward-traveling vortex rings generated by bubbles bursting at the air-sea interface may be important in transporting tiny marine aerosol droplets into the atmosphere.
Chapter 5: The Effect of Liquid Properties on the Release of Gas from Bursting Bubbles

5.1 Introduction

The bursting of bubbles at an air-liquid interface is an important physical process in many environmental and industrial applications. Bubble bursting has implications for climate (e.g. marine aerosol formation; Veron 2015), human respiratory health (e.g. aerosolization of harmful substances; Prather et al. 2013), food science (e.g. beer and champagne bubbles; Liger-Belair et al. 2009), industry (e.g. processes such as gas fluxing and electrowinning; Zhang et al. 2011), and volcanology (e.g. Strombolian eruptions; Chojnicki et al. 2015). Due to its tremendous importance and its diversity of applications, various aspects of the fluid dynamics of bubble bursting have long been studied. Much of the early work on the fluid dynamics of bubble bursting was motivated by a desire to understand the mechanisms of marine aerosol production and thus it focused on the liquid component, namely the bubble cap film rupture and retraction and the generation of liquid droplets (e.g. jet droplets and film drops) (Stuhlman 1932; Kientzler et al. 1954; Newitt et al. 1954; Mason 1957; Day and Lease 1968). Jet droplets form from the stem of fluid comprising the unstable water column which undergoes a capillary instability whereas film drops are generated by the violent destruction of the thin film comprising the bubble cap. Further research also has focused on the dynamics of the Worthington jet produced by the upward collapse of tiny bubbles (Krishnan et al. 2017; Deike et al. 2018), the mechanisms of hole initiation in bursting bubbles (Poulain et al. 2018), and the role of viscosity in the retraction of bubble cap films (Savva and Bush 2009; Ghabache et al. 2014).
In contrast, the fluid dynamics of the pressurized gas escaping from inside the bursting bubble is not well understood. Several early studies noted the formation of a vortex ring released from the bursting bubble (Rogers 1858; Swinton and Beale 1917), and Newitt et al (1954) first photographed this phenomenon. Other researchers examined this flow as a possible mechanism for lofting tiny marine aerosol droplets up into the atmosphere (Newitt et al. 1954; Blanchard 1963; Iacono and Blanchard 1987). Recently, Dasouqi et al. (2020) used high speed stereophotogrammetry to study this gas escape behavior for a wide range of bubble sizes (e.g. 440 µm-4 cm) bursting at an air-water surface and presented 3D measurements of the gas jet and vortex ring characteristics. Indeed, their 3D measurements showed that the initial speed of the gas jet released from the bubbles sharply increases with parent bubble size until the bubble Bond number reaches unity and subsequently increases at a slower rate. In that investigation, these researchers also presented flow visualization results which explain how film retraction speed, which is correlated with bubble size, affects the initial speed of the gas jet. Specifically, the high film retraction speeds characteristic of smaller bubbles produced correspondingly slower jets emitted through relatively larger openings whereas the low film retraction speeds characteristic of larger bubbles produced correspondingly higher jet speeds emitted through relatively smaller openings. In addition, a simple scaling relationship of \( \text{Re}_{\text{jet}} = 126 \text{Bo}^{0.5} \) was proposed to relate the Reynolds number of the emerging jet to the Bond number of the parent bubble. They concluded that large bubbles eject high speed, high Reynolds number jets which slowly decay and which roll up into fast-growing, highly oblate vortex rings which travel far. In contrast, small bubbles eject low speed, low Reynolds number jets which quickly decay and which roll up into slow-growing, spherical vortex rings which travel short distances.
However, the effect of fluid properties (e.g. liquid density, viscosity, and surface tension) on the behavior of the gas jet and vortex ring emitted from bursting bubbles is not well understood. Dasouqi et al (2020) used only water as a working fluid but did note the importance of film retraction speed in controlling jet behavior. In addition, Buchholz and Sigurdson (2000, 2002) examined vortex rings emitted from large (4-6 cm) hemispherical soap bubbles resting on a solid surface, and other researchers (Jaw et al. 2007; Torikai et al. 2011; MURASHITA et al. 2012; Kim et al. 2014) have examined the bursting of large, spherical soap bubbles filled with inert gas for fire-fighting purposes. These authors observed lower film retraction speeds owing to the lower surface tension but did not investigate how bubble size may also affect the jet behavior. Further, other studies (Debregeas et al. 1995; Savva and Bush 2009) have investigated how highly viscous films retract as expanding circular holes and found two distinct features that only emerge when viscosity effects are significant. Specifically, they found the absence of a rim on the toroidal edge of the retracting film and a delay in the transient acceleration caused by viscous forces, after which the film attains a constant retraction speed. The effects of these viscosity-related phenomena on gas escape from bursting bubbles also is not known.

Here, we use a two-dimensional high-speed visualization system to examine the effects of bubble size and fluid properties (e.g. liquid density ρ, viscosity µ, and surface tension σ) on the formation of gas jets and vortex rings emerging from bubbles bursting at an air-liquid interface. Since both increased liquid viscosity and reduced surface tension will likely decrease film retraction speed, we thus expect to observe a corresponding variation in the speed of the emerging gas jet. Dasouqi et al (2020) used water only and showed that the initial speed of the gas jet released from the bubbles increased with parent bubble size until the Bond number reached unity and subsequently increased more slowly. In the present study, we vary the bubble size and film
properties to consider the effects of increasing film viscosity \((0.001<Oh<40.3)\) and deceasing film surface tension \((32.2 \text{ mN/m}<\sigma<72.6 \text{ mN/m})\) on the formation of gas jets and vortex rings. We present measurements of the gas jet and characteristics of retraction of parent bubble cap film and relate these quantities to the jet Reynolds number, bubble Bond number, and film Ohnesorge number, respectively.

5.2 Experimental Methods

An experimental setup similar to that of Dasouqi et al (2020) was used to study the emerging gas jets produced by bursting bubbles in four liquids with varying surface tensions, viscosities, and densities. Whereas that setup comprised a stereophotogrammetry system with two synchronized and orthogonally positioned high-speed cameras used to measure gas jet parameters in three dimensions, an alternative setup comprising a single high-speed camera was used here to measure two dimensional (projected) parameters. Briefly, a rectangular acrylic tank \((10.2 \times 10.2 \times 15.2 \text{ cm}^3)\) was filled to the brim with one of four liquids, namely deionized water, soapy water (at a concentration of 0.4 mg soap/ml water using Alconox powder soap), engine oil (5W-30, Pennzoil), or glycerol (Nature’s Oil). Single bubbles ranging in diameter from 500 µm-4 cm were produced at a constant rate by a programmable syringe pump (New Era Pump Systems Inc.) injecting smoky air through nozzle tips of various diameters inserted through a bulkhead fitting in the bottom of the acrylic tank. The tank was lifted by an elevating table (Velmex B29) in order to facilitate visualization. A transparent acrylic box \((63.5 \times 63.5 \times 76.2 \text{ cm}^3)\) enclosed the tank to prevent stray air currents from affecting the air flow emanating from the bubbles, which were allowed to age and burst naturally. Smoke for visualization was produced by the combustion of wood chips (PolyScience Breville Smoking Gun) and was captured in a 60 ml syringe prior to injection.
The liquid viscosity and surface tension of the fluid significantly influence the bursting behavior and were thus measured. The pendant drop method (Song and Springer 1996) was used to quantify the air-liquid surface tension for each liquid by measuring the surface tension of 5 separate pendant droplets and averaging the results. The liquid viscosity was measured using various Cannon Fenske viscometers. The physical properties of the fluids are listed in Table 5.1. It is worth noting that the surface tension of the soapy water solution (38.9 ± 0.84 mN/m) is somewhat higher than values found in the literature. This higher value can be attributed to the low soap concentration which was chosen to limit the formation of coalesced foamy bubbles on the water surface. Density values for engine oil and glycerol were taken from manufacturer specifications and were measured for the water and soapy water with a densitometer (Anton-Par-Str.20). Table 5.1 also shows the Capillary length \( a \) of each fluid, where \( a = \frac{\sigma}{\sqrt{\rho g}} \).

The bursting bubbles were visualized by a single high-speed camera (Photron FASTCAM SA-Z) recording at 10 kHz with a spatial resolution of 1024×1024 pixels. Backlighting was provided by two LED lights (Nila NINZLB). Several fields of view (from 15×15 mm to 78×78 mm) were employed according to the bubble size, with the spatial resolutions varying accordingly from 15-76 µm/pixel. The camera was calibrated both above and below the liquid surface using a transparent microslide ruler with the focal plane positioned at the center of the tank. The camera was set to a post-trigger configuration, and, upon observing a bubble bursting within the focal plane, was manually triggered.

Approximately a hundred bursting events were observed for each liquid, and about forty videos for each liquid in which the bubble was well focused were selected for further analysis. The recorded videos then were processed in ImageJ software (NIH) following the method of Dasouqi et al (2020). Briefly, the bubble size was first characterized following a similar approach to that of
The bubble equivalent diameter $2R$, the diameter of a sphere with the same enclosed volume as the hemispherical bubble, is used to represent bubble size for large bubbles. For small bubbles residing mostly beneath the surface, the bubble diameter was directly measured and is reported as $2R$. In addition, the bubble Bond number $Bo$ was determined for each bubble as

$$Bo = \frac{\rho g R^2}{\sigma}. \quad (5.1)$$

For each recorded event, the frame at which bursting initiated was set as time $t=0$. For each bursting bubble, a single point on the tip of the retracting film was manually tracked over $t=0.3-0.5$ ms from which a measured distance could be divided by the elapsed time to calculate film retraction speed $V_{fr}$. However, for the smallest bubbles, the last-measured film retraction speed was used due to complete destruction of the film prior to $t=0.5$ ms. The film thickness $h$ was theoretically calculated using the Taylor-Culick equation $V = \sqrt{2\sigma/\rho h}$ (Brenner and Gueyffier 1999; Savva and Bush 2009; Pierson et al. 2020). Further, the forward displacement of the gas jet front after emerging from the enlarging hole in the bubble film was similarly tracked over $t=0.3-0.5$ ms in order to calculate jet speed $V_{jet}$. Finally, the jet frontal diameter $D_{jet}$ was measured at $t=0.4$ ms, thus allowing calculation of the jet Reynolds number

$$Re_{jet} = \frac{V_{jet}D_{jet}}{v}. \quad (5.2)$$

where $v=1.5\times10^{-5}$ m$^2$/s is the kinematic viscosity of air at 20°C. It is important to note that measurements for water were acquired using a 3D stereophotogrammetry system (Dasouqi et al 2020) whereas those presented here for soapy water, engine oil, and glycerol were acquired using a single camera. The bubbles formed by engine oil and glycerol largely burst vertically in the plane of focus and thus did not require 3D measurements. The soapy water bubbles did not necessarily burst vertically, and thus more scatter is seen in these data.
Table 5.1 Values of physical properties of liquids used in the experiments at 20 °C

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Water</th>
<th>Soapy Water</th>
<th>Engine Oil</th>
<th>Glycerol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5W30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water</td>
<td>Soapy Water</td>
<td>Engine Oil</td>
<td>Glycerol</td>
</tr>
<tr>
<td>Surface Tension (σ)</td>
<td>mN/m</td>
<td>70.7</td>
<td>38.9</td>
<td>32.2</td>
<td>63.9</td>
</tr>
<tr>
<td>Dynamic Viscosity (μ)</td>
<td>mPa.s</td>
<td>1</td>
<td>1</td>
<td>55</td>
<td>944</td>
</tr>
<tr>
<td>Density (ρ)</td>
<td>Kg/m³</td>
<td>998</td>
<td>998</td>
<td>859</td>
<td>1259</td>
</tr>
<tr>
<td>Capillary Length (a)</td>
<td>mm</td>
<td>2.69</td>
<td>1.99</td>
<td>1.95</td>
<td>2.28</td>
</tr>
</tbody>
</table>

5.3 Results

5.3.1 Flow Visualization

Flow visualization of the bursting of large, medium, and small bubbles is now presented. The diameters, Bond numbers, and Ohnesorge numbers of the presented bubbles are shown in Table 5.2. Here, the Ohnesorge number compares viscous resistance to surface tension forces on the bubble cap film and is defined as $Oh = \frac{\mu}{\sqrt{2h\rho\sigma}}$. Figure 5.1(a-d) shows a sequence of images illustrating the bursting of 26.6 mm, 23.5 mm, 29.9 mm, and 26.0 mm diameter smoke-filled bubbles resting on deionized water, soapy water, engine oil, and glycerin fluid surfaces, respectively. Prior to bursting ($t=2$ ms), the bubble shape is governed by its Bond number which describes bubble size, fluid density, gravity, surface tension, and the pressure difference across its film interface $\Delta P$. For example, the engine oil bubble in Fig. 5.1(c) which has the lowest surface tension acquired a more flattened shape (i.e. low curvature) due to its high Bond number ($Bo=58.5$). In contrast, the water, soapy water, and glycerin bubbles in Fig. 5.1(a, b, & d) have higher film curvatures, which reflect their higher surface tensions and pressure differences and
their correspondingly lower Bond numbers \((Bo=24.4, Bo=34.6, \text{ and } Bo=32.6)\), respectively. At \(t=0.5 \text{ ms}\), a single, small (e.g. 2-3 mm) diameter hole has opened in each bubble cap film, the edge of which is retracting at speeds of 2.51 m/s for water, 0.57 m/s for soapy water, 0.81 m/s for engine oil, and 1.68 m/s for glycerin. Simultaneously, a smoke jet has emerged through the expanding hole at speeds of 4.13 m/s, 2.75 m/s, 4.12 m/s, and 3.60 m/s, respectively. At \(t=2 \text{ ms}\), the holes in the bubble caps have grown to 13.4 mm, 9.45 mm, 6.62 mm, and 10.5 mm in diameter, respectively, and the retracting edges have acquired various shapes according to their respective film Ohnesorge numbers. For instance, a crown shape of circumferentially-distributed short ligaments extending outwards is seen for the water bubble, which has the lowest Ohnesorge number of \(Oh=0.001\). In fact, this thickened rim centripetally accelerates as it follows the bubble cap curvature and undergoes a Rayleigh-Taylor instability while facing a negligible damping effect because of the low viscosity. In contrast, in the soapy water bubble \((Oh=0.007)\), the slower film retraction due to a lower surface tension reduced the extent of formation of those ligaments because the lower speed resulted in a lower centripetal acceleration along the bubble cap curvature which limited the development of a Rayleigh-Taylor instability. Both of these film retraction events fall into the low Ohnesorge number regime as described by Savva and Bush (2009), in which film retraction is inertia-dominated and capillary wave disturbances are generated ahead of the retracting rim. As Ohnesorge number increased to 0.69 for the engine oil bubble, the instabilities on the receding rim disappeared owing to the higher viscosity of the fluid. This film retraction event corresponds to the medium Ohnesorge number regime \((Savva \text{ and } Bush 2009)\) in which capillary waves disappear and the rim diffuses in towards the bulk of the film (e.g. Fig 5.1c). Finally, in the glycerin bubble, no fluid seems to collect in the receding edge because of the dominating force of viscosity \((Oh=12.3)\). This film retraction event corresponds to the high
Ohnesorge number regime (Savva and Bush 2009) in which the retraction is dominated by viscosity and no rim forms at all (e.g. Fig 5.1d). In addition, the speed of the gas jet front, which has already started rolling up into a vortex ring for water, engine oil, and glycerin at this time point (t=2ms), decreases to 3.52 m/s, 2.07 m/s, 2.39 m/s, and 2.91 m/s for the water, soapy water, engine oil, and glycerin bubbles, respectively.

Table 5.2 Values of parent bubble Bond number and film Ohnesorge number for the differently sized bubbles bursting at four different working fluid surfaces (Figures 5.1-3 (a-d))

<table>
<thead>
<tr>
<th>Size</th>
<th>Parameter</th>
<th>Fluid</th>
<th>Water</th>
<th>Soapy Water</th>
<th>Engine Oil</th>
<th>Glycerin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>2R (mm)</td>
<td></td>
<td>26.6</td>
<td>23.5</td>
<td>29.9</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>Bo</td>
<td></td>
<td>24.4</td>
<td>34.6</td>
<td>58.5</td>
<td>32.6</td>
</tr>
<tr>
<td></td>
<td>Oh</td>
<td></td>
<td>0.001</td>
<td>0.007</td>
<td>0.691</td>
<td>12.33</td>
</tr>
<tr>
<td>Medium</td>
<td>2R (mm)</td>
<td></td>
<td>6.4</td>
<td>6.8</td>
<td>7.1</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>Bo</td>
<td></td>
<td>1.34</td>
<td>2.93</td>
<td>3.32</td>
<td>2.31</td>
</tr>
<tr>
<td></td>
<td>Oh</td>
<td></td>
<td>0.031</td>
<td>0.033</td>
<td>1.815</td>
<td>24.08</td>
</tr>
<tr>
<td></td>
<td>2R (mm)</td>
<td></td>
<td>1.78</td>
<td>1.55</td>
<td>1.28</td>
<td>1.51</td>
</tr>
<tr>
<td>Small</td>
<td>Bo</td>
<td></td>
<td>0.12</td>
<td>0.15</td>
<td>0.10</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>Oh</td>
<td></td>
<td>0.042</td>
<td>0.039</td>
<td>3.282</td>
<td>40.33</td>
</tr>
</tbody>
</table>

At t=5 ms, the bubble film has almost completely collapsed into the surrounding fluid in the water and soapy water cases but is still slowly retracting for the engine oil bubble. However, the more viscous film of the glycerin bubble tends to collapse downward under the influence of gravity as it slowly retracts at this time point. This slow retraction speed is likely due to the high fluid viscosity dominating over the surface tension. The gas jet has not yet started rolling up into
a vortex ring in the soapy water case because of lower film retraction speed and the lower pressure difference driving the gas out of the bubble. The flow thus resembles the spherical soapy water bubble bursting flows visualized by Kim et al (2014) where no significant jet was formed. In contrast, the primary vortex ring continues to grow at this time point and travels at lower speeds of 2.56 m/s for water, 1.57 m/s for engine oil, and 2.18 m/s for glycerin. The size of the primary vortex ring in the glycerin case is noticeably smaller than that of the other fluids. Here the size of the primary vortex ring is characterized by its major diameter, namely the axial distance between vortex centers (Dasouqi et al. 2020). This difference is attributed to variations in the time of vortex formation which also is related to film retraction speed. The slowly retracting glycerin film creates a relatively small opening through which the gas escapes, leading to a relatively small jet and vortex ring. Further, in each case, the smoke jet diameter increase moving downwards from the primary vortex ring, if present, towards the bursting bubble. A series of three vortices along the edge of the smoke jet is clearly visible for the water bubble whereas these vortices are absent at this time point in the smoke jets produced by the other working fluids. These vortices in the water bubble arise from a Kelvin-Helmholtz instability as the liquid film rapidly retracts over the surface of the smoke within the bubble. Thus, these vortices are likely not fully formed vortex rings. In contrast, the film retracts more slowly for the other fluids, leading to smooth jet boundaries at this time point. At \( t=30 \) ms, the smoke jet produced by the soapy water bubble has finally rolled up into a somewhat disorganized vortex ring which travels towards the camera. All initial vortex rings have grown substantially while traveling at 1.27 m/s, 0.38 m/s, 0.61 m/s, and 1.05 m/s for the water, soapy water, engine oil, and glycerin cases, respectively. These different vortex ring speeds lead to travel distances of 47.9 mm, 23.5 mm, 23.9 mm, and 31.5 mm above the liquid surface for the water, soapy water, engine oil, and glycerin cases, respectively. In addition, the vortex ring
produced from the water bubble has almost completely pinched off from its trailing flow. In contrast, a series of 1-3 vortices are now visible along the edge of the smoke jets produced by the bursting bubbles of other fluids due to the slower retraction of the corresponding bubble cap films.

Figure 5.2(a-d) shows a sequence of images illustrating the bursting of 6.4 mm, 6.8 mm, 7.1 mm, and 6.9 mm diameter smoke-filled bubbles resting atop water, soapy water, engine oil, and glycerin surfaces, respectively. Prior to bursting ($t=-2$ ms), the bubble shapes are similar among the four working fluids owing to small variations in bubble Bond number. At $t=0.5$ ms, the water bubble film has almost completely collapsed into the surrounding water, with a few extending ligaments pinching off film droplets due to the high film retraction speed. However, the nucleation hole has only grown to 2.49 mm, 2.02 mm, and 3.07 mm diameter in soapy water, engine oil, and glycerin, respectively. Similar to larger bubbles, the edge of the retracting film demonstrated three distinct regimes according to the bubble film Ohnesorge number. Specifically, an unstable rim was observed in the low Ohnesorge number regime (e.g. water and soapy water (both at $Oh=0.03$)), whereas the instabilities on the receding rim disappeared in the medium Ohnesorge number regime owing to the higher viscosity of engine oil ($Oh=1.82$). In the high Ohnesorge number regime, the highly viscous film of the glycerin bubble ($Oh=24.1$) formed no rim at all. In addition, the receding edge of the film retracts at a speed of 4.37 m/s for water, 1.83 m/s for soapy water, 2.13 m/s for engine oil, and 3.27 m/s for glycerin at this time point. Simultaneously, a smoke jet emerges through the expanding hole at corresponding forward speeds of 3.12 m/s, 1.01 m/s, 2.14 m/s, and 1.67 m/s, respectively. In addition, the shape of the smoke jet is affected by the retraction behavior of the bubble film. For example, the conical shape acquired by the smoke jet emitted from soapy water and engine oil bubbles is due to the slower opening of their holes whereas the nipple-like shape acquired by the smoke jet emitted from the glycerin
bubble may be attributed to a downward collapse of the bubble cap. At \( t = 2 \) ms, the holes in all bubble caps have disappeared as the films have largely been destroyed at this time point. Further, the upward collapse of the bubble cavity additionally forces the smoke jet to be vertically ejected. The speed of the gas jet front decreases to 2.16 m/s, 0.75 m/s, 1.89 m/s, and 1.37 m/s, respectively. In addition, the smoke jet has already rolled up into a primary vortex ring, at this time point, beneath which a secondary wider vortex is seen for all liquids except for engine oil, for which this secondary vortex ring forms later. The source of the secondary vortex may be related to the fast upward collapse of the cavity (e.g. for water) and to the interaction of the retracting film and the gas jet for the other cases (Dasouqi et al 2020). At \( t = 5 \) ms, the bubble cavity continues to collapse upward which drives the smoke jet and its preceding vortex ring to maintain their upward advance. Further, outward moving waves on the surface of the bath are observed in all but the most viscous liquid (i.e. glycerin). At \( t = 11 \) ms, the primary vortex ring has grown substantially because it entrained smoke from that constituting the now-disorganized secondary Kelvin-Helmholtz instability as seen in water and soapy water or from the trailing jet as seen in engine oil. Finally, the tip of the upward-projecting jet (i.e. a Worthington jet) reaches above the surface in the water and soapy water cases at this point (Gekle and Gordillo 2010). In contrast, the slower collapse of the bubble cavity inhibited the formation of a Worthington jet in engine oil and glycerin due to the damping effects of viscosity.
Figure 5.1 Sample images from a time series showing the bursting of $2R = 26.6$ mm, 23.5 mm, 29.9 mm, and 26.0 mm smoke-filled bubbles floating on water, soapy water, engine oil, and glycerin surfaces (left-right). The hole in the bubble cap forms at $t=0$ ms. The scale bar in all images is 5 mm.
Figure 5.2 Sample images from a time series showing the bursting of $2R=6.4$ mm, 6.8 mm, 7.1 mm, and 6.9 mm smoke-filled bubbles floating on water, soapy water, engine oil, and glycerin surfaces (left-right). The hole in the bubble cap forms at $t=0$ ms. The scale bar in all images is 3 mm.
Figure 5.3(a-d) shows a sequence of images illustrating the bursting of 1.78 mm, 1.55 mm, 1.28 mm, and 1.51 mm diameter smoke-filled bubbles resting atop water, soapy water, engine oil, and glycerin surfaces, respectively. Prior to bursting (\(t=-2\) ms), smaller bubbles obtained a more spherical shape with much lower Bond numbers (\(Bo<0.15\)) and a higher pressure difference (\(\Delta P>200\) Pa). Of particular note is that the soapy water bubble, though a similar to the water bubble is submerged significantly further owing to the decreased surface tension. In addition, the glycerin bubble protrudes significantly due to its relatively high surface tension and high density. The engine oil bubble has the lowest liquid density and thus is highly submerged. After bursting (\(t=0\) ms), the film retracts over three frames at a speed of 5.94 m/s for the water bubble, 3.03 m/s for the soapy water bubble, 3.84 m/s for the engine oil bubble, and 5.46 m/s for the glycerin bubble. At \(t=0.5\) ms, the bubble film has fully retracted in all bubbles and a gas jet has started emerging with speeds of 1.31 m/s, 0.66 m/s, 1.06 m/s, and 1.07 m/s for the water, soapy water, engine oil, and glycerin cases, respectively. An ejection of a spray of film drops nearly horizontal to the surface is seen for the water bubble due to the film’s low Ohnesorge number. At \(t=2\) ms, the speed of the smoke jet decreases to 0.71 m/s for water case, 0.33 m/s for soapy water case, 0.59 m/s for engine oil case, and 0.57 m/s for glycerin case. Further, the leading edge has begun to roll up into the beginning of a vortex ring at this time point. In all cases, the vortex rings are substantially smaller than the width of the jet where it emerges from the bubble. The base jet widths are 1.46 mm, 0.93 mm, 1.03 mm, 0.86 mm for the water, soapy water, engine oil, and glycerin cases, respectively. In addition, the cavity bottom has already started collapsing upwards at a speed of 1.97 m/s for the water bubble. In addition, the soapy water cavity has already collapsed upwards and the tip of the resulting, upward-projecting jet (i.e. a Worthington jet) reaches above the soapy
water surface at this point (Gekle and Gordillo 2010). The cavities of the viscous fluids collapse upwards more slowly (e.g. 0.35 m/s for the engine oil bubble).

At \( t = 5 \) ms, the tip of the smoke jet has already rolled up into a vortex ring which travels upward with a speed of 0.46 m/s for the water case, 0.11 m/s for the soapy water case, 0.28 m/s for the engine oil case, and 0.19 m/s for the glycerin case. The primary vortex produced by glycerin bubble is no longer traveling upwards at this time point but has hooked to the left. As will become apparent, the fast-moving smoke being expelled from the upwards collapsing cavity seems to overtake the primary vortex ring and subsequently forms a secondary vortex ring comparable in size to the primary vortex ring. In each case, the base of the smoke column continues to expand as smoke is expelled from the initial cavity by the rise of the liquid surface. Indeed, the subsurface cavity is no longer visible for water, and the tip of the Worthington jet reaches above the water surface. In the soapy water case, Worthington jet has become unstable and a single jet droplet has pinched off and is ejected upwards at a speed of 1.16 m/s. The subsurface cavity is still collapsing upward for engine oil and glycerin at this time point. At \( t = 29 \) ms, the speed of the primary vortex ring has become negligible as its initial momentum dies away after traveling 5.35 mm, 2.84 mm, 5.62 mm, and 3.89 mm for water, soapy water, engine oil, and glycerin cases, respectively. In addition, a secondary, less well developed smoke vortex ring forms below the initial vortex ring in glycerin, the source of which seems due to the change in direction of the initial vortex. Finally, in the water case, a jet drop (larger than that produced by the soapy water) has pinched off while the unstable water column has fallen back onto the water surface at this point.
Figure 5.3 Sample images from a time series showing the bursting of $2R= 1.78 \text{ mm}$, $1.55 \text{ mm}$, $1.28 \text{ mm}$, and $1.51 \text{ mm}$ smoke-filled bubbles floating on water, soapy water, engine oil, and glycerin surfaces (a-d). The hole in the bubble cap forms at $t=0 \text{ ms}$. The scale bar in all images is $1 \text{ mm}$.
5.3.2 Gas Jet Characterization

Figure 5.4 shows the relation between the bubble equivalent diameter and $V_{jet}$ at $t=0.4$ ms after hole nucleation for water, soapy water, engine oil, and glycerin, where data for water are taken from Dasouqi et al (2020). This plot will be discussed in relation to Figures 5.5 and 5.6, which show the film retraction speed and film thickness as a function of bubble size and in relation to Figure 5.7, which shows the jet speed as a function of film retraction speed. As described by Dasouqi et al (2020), the gas jet speed for water bubbles increases linearly with bubble size until $2R=5.44$ mm and then increases at a lower rate. These authors showed that this break in slope occurs at $Bo=\sim 1$, at which the gravitational forces balance surface tension, and that this break corresponds to the decreased submergence of larger bubbles beneath the water surface. Further, as shown in Table 1, this break occurs when the bubble radius of curvature equals the capillary length. Further, film retraction speeds decreased and film thickness increased with increasing bubble size (Figures 5.5-5.6), and a negative correlation was found between initial jet speed and film retraction speed (Figure 5.7). Taking the water case as the baseline, we will first examine soapy water, which has similar viscosity and density and a reduced surface tension (Table 1). Figure 5.4 shows that the jet speeds for the soapy water bubbles were lower than those of the water bubbles. For example, in Figure 5.2 the water bubble ($2R=6.4$ mm) produced a jet speed of 3.12 m/s whereas a soapy water bubble of approximately the same size ($2R=6.8$ mm) produced a lower jet speed of 1.01 m/s. This decrease in jet speed occurs even though, as shown in Figure 5.5, the film retraction speed is lower for soapy water than for water owing to the lower surface tension and more massive (i.e. thicker) films (e.g. Figure 5.6). Considering that Dasouqi et al (2020) found that lower film retraction speeds correlated with higher jet speeds, it is thus perhaps surprising that the jet speed for soapy water bubbles decreased. These lower jet speeds reflect the lower pressures sustained
inside of the soapy water bubbles owing to the lower liquid surface tension. Similar to the water case, the soapy water jet speeds also show a decrease in slope with increasing bubble size. This break occurs close to Bo=1 (corresponding to 2R=3.98 mm or approximately twice the capillary length), though scatter in the data make it difficult to pinpoint the break’s exact location. Figure 5.7 shows that, as with water, jet speed similarly decreases fairly linearly with increasing film retraction speed, though the magnitudes are substantially less than those of the water case owing to the lower surface tension involved. Finally, as compared to all other liquids tested, the largest soapy water bubbles (2R=26.8 mm) produce the lowest speed jets, lowest film retraction speeds, and the thickest films.

Figure 5.4 Gas jet front speed $V_{jet}$ as a function of bubble equivalent diameter $2R$ measured at $t=0.4$ ms after hole nucleation ($t=0$ ms) for the different liquids
We now examine the behavior of engine oil bubbles, for which the density decreases slightly, viscosity increases by an order of magnitude, and surface tension is cut by more than half (Table 1). As shown in Figure 5.4, the trend in jet speed with increasing engine oil bubble size is similar to that of the water bubbles, with a similar break in slope occurring at approximately 2R=3 mm, which again closely corresponds to the bubble size where Bo=1 (2R=3.90 mm, or approximately twice the capillary length). However, the jet speed magnitudes show important differences. Below the break (2R<~3 mm) and for large bubble sizes (2R>~22 mm), jet speeds from engine oil and water bubbles are similar. This similarity may be explained by the fact that the engine oil bubbles have both lower film retraction speeds (owing to lower surface tension, increased viscosity, and thicker films) and lower initial pressure difference (owing to the lower surface tension) as compared to the water bubbles, which have proportionally higher film retraction speeds and a greater initial pressure difference (e.g. Figures 5.5-5.7). This proportional difference leads to similar jet speeds. For example, engine oil and water bubbles with similar sizes (29.9 mm and 26.6 mm, respectively) had initial pressure differences of 4.32 Pa and 10.6 Pa, respectively, and corresponding film retraction speeds of 0.81 m/s and 2.51 m/s, respectively (e.g. Figure 5.1). These very different initial conditions nonetheless produced almost identical jet speeds of 4.12 m/s and 4.13 m/s for engine oil and water bubbles, respectively. In the region 3 mm <2R < 22 mm, however, engine oil jet speeds are significantly lower than those produced by water bubbles. As explained in Dasouqi et al (2020), these lower jet speeds begin at bubble sizes corresponding to Bo=1 and are caused by decreasing bubble submergence and the resulting change in bubble shape from spherical to hemispherical. This transition occurs for smaller diameter bubbles in engine oil (2R=3.91 mm) as compared to those in water (2R=5.44 mm). The lower jet speeds in this region may also be linked with the sharp increase in film thickness within this size
range (Figure 5.6). This difference also seems to be linked to the non-linearity in the relationship between jet speed and film retraction speed observed for $1 \text{ m/s} < V_{fr} < 2.5 \text{ m/s}$, though it is not clear why both jet speeds and film retraction speeds are lower than expected within this size range.

Figure 5.5 Film retraction speed $V_{fr}$ as a function of bubble equivalent diameter $2R$ measured at $t=0.4 \text{ ms}$ after hole nucleation ($t=0 \text{ ms}$) for the different liquids.

We turn now to the behavior of glycerin bubbles, for which the density is slightly greater than that of water, viscosity increases by two orders of magnitude, and surface tension is only slightly lower than that of water (Table 1). Figure 5.4 shows that the trend is similar to that seen in other fluids. However, no distinct break in the slope was seen in jet speeds produced from glycerin bubbles, though this break would be expected to occur at $2R=4.74 \text{ mm}$ (corresponding to $Bo=1$). This may be because of the sparsity of data acquired for the smallest bubbles due to the
difficulty in producing tiny smoke-filled bubbles that would burst in a reasonable amount of time in this highly viscous fluid. Jet speeds from the glycerin bubbles were lower than those produced from water bubbles over most of their size range. However, at bubble sizes greater than 2R= 32.9 mm, jet speeds from glycerin bubbles slightly exceeded those produced from water bubbles, reaching speeds up to 5.19 m/s for a 2R= 43.5 mm bubble. These similar jet speeds can be explained by the similar film retraction speeds found in the large water and glycerin bubbles as shown in Figure 5.5 and by the similar surface tensions between the two liquids. In addition, Figure 5.5 shows that film retraction speeds in glycerin bubbles were slightly lower than those in water bubbles over most of their size range because of lower surface tension and higher density. These small differences in density and surface tension values between water and glycerin also explain the slightly thicker films of the latter as seen in Figure 5.6. Further, as seen in Figure 5.7, the relationship between film retraction speed and jet speed is much less linear for glycerin than it is for water, which explains why the glycerin jet speeds are lower than those of water in the intermediate size range.

Figure 5.8 shows the scaling relationship between the parent bubble Bond number and the Reynolds number of the resulting gas jet released from the bursting bubble for all four investigated fluids. Here data for water are taken from Dasouqi et al (2020), as is the scaling relationship of $Re_{jet}=126Bo^{0.5}$ (equivalent to $Re_{jet}=126R/a$) found previously by the same authors for water bubbles. The inset in Figure 5.8 shows the measured jet diameters used to calculate the jet Reynolds number. The data for all fluids generally follow the previously obtained scaling relationship, with a few exceptions. For example, Reynolds numbers for all fluids are somewhat lower than predicted for the highest Bond number bubbles (e.g. $Bo>\sim10$), a trend also found for water by Dasouqi et al (2020). This departure from the scaling trend is particularly evident for
soapy water owing to the extremely low pressure difference between the bubble interior and the atmosphere. In addition, the jet Reynolds numbers for engine oil are systematically lower than predicted by the scaling line. Nonetheless, despite this small offset, the trend is the same for all fluids. This scaling relationship shows that the momentum of the emerging jet can be adequately described in terms of the fluid surface tension, density, gravity, and bubble size.

It is interesting to note that fluid viscosity, which varied by almost a thousand fold, did not significantly affect this scaling relationship. As evident in the Taylor-Culick equation, the film retraction speed is proportional to surface tension, fluid density, and film thickness and is thus not significantly affected by viscosity. The Ohnesorge number, which varied over three orders of magnitude, did affect the morphology of the retracting film since low Oh number films produced thickened rims with ligaments which shed droplets whereas the high Oh number films had thin edges which did not form droplets. However, these alterations in edge shape did not translate into differences in retraction speed or jet Reynolds number.

![Figure 5.6](image.png)

Figure 5.6 The calculated film thickness $h$ at $t=0.4$ ms after hole nucleation as a function of bubble equivalent diameter $2R$ for the different liquids
Figure 5.7 Jet speed $V_{jet}$ as a function of film retraction speed $V_{fr}$ at $t=0.4$ ms for the different liquids.

Figure 5.8 Reynolds number of the emerging jet at $t=0.4$ ms as a function of the parent bubble Bond number for the different liquids. The inset shows measured values of the jet frontal diameter $D_{jet}$ at $t=0.4$ ms as a function of bubble equivalent diameter $2R$. 
Transient behavior may also have affected the results and scaling fit. For example, the Taylor-Culick equation predicts the steady state film retraction speed, but the film speeds measured here may not have reached a steady state by 0.4 ms after hole initiation. Dasouqi et al (2020) found that in bursting water bubbles the film speed increased by up to 2% over this time period, thus potentially introducing error into the film thickness calculations in Figure 5.6. Variations in the initial film retraction speed among the various tested fluids owing to differences in surface tension, density, or film thickness also may have affected the hole area and thus the emanating jet speeds. Further, transient behavior in the collapse of the film may have affected jet speeds by shaping the liquid aperture through which the gas jet is emitted. For example, the downward collapse of the dense glycerin film (e.g. Fig 5.2d at t=0.5 ms) produces a nipple-shaped gas jet which is not observed in the other fluids. This behavior seems to be present mainly in intermediate-sized bubbles and thus may be responsible for the non-linearity observed in the relationship between film retraction speed and jet speed (e.g. Fig 5.7). Nonetheless, the relationship between the parent bubble Bond number and emerging jet Reynolds number is still well described by the previously stated scaling formula, indicating that these potential error sources are relatively minor.

5.4 Conclusions

In conclusion, we have visualized and quantified the release of gas from bubbles with diameters ranging from 500 µm to 44 mm bursting on top of water, soapy water, engine oil, and glycerin surfaces. Similar to our previous finding, the initial gas jet speed ejected from bursting bubbles increases with parent bubble size until the bubble Bond number reaches unity, and subsequently increases at a slower rate for all investigated fluids. However, these tested liquids ejected gas jets with different speeds within intermediate bubble sizes due to a dissimilar transition
in bubble shape, decreasing bubble submergence, and the downward collapsing behavior of the bubble cap film in the highly viscous liquids. Further, the film retraction speed, which was hardly affected by a thousand-fold increase in liquid viscosity, significantly decreased among the investigated liquids by reducing liquid surface tension by a half. It is remarkable that all working fluids obey the trend described by our scaling relationship of $Re_{jet}=126Bo^{0.5}$ (equivalent to $Re_{jet}=126R/a$), which shows that the momentum of the emerging jet can be adequately described in terms of the fluid surface tension, density, gravity, and bubble size. In addition, slight changes in liquid density was found to affect bubble submergence and film retraction speed and thus may have affected the speed of the emanating gas jets. A transient behavior in the collapse of the viscous films might decrease jet speeds by shaping the liquid aperture while gas jet is emerging. Finally, the role of liquid surface tension in the ejection of gas jets and subsequent formation of vortex rings from bursting bubbles dominate over that of density and viscosity.
Chapter 6: Conclusions and Future Work

6.1 Conclusions

The results and analysis of a wide range of bubble sizes bursting on top of four different liquid surfaces with varying physical properties show that bubble size and its cap film properties govern the speed of the emerging gas jet and subsequent formation of vortex rings. Gas escape behavior corresponded strongly with the bubble cap radius of curvature and its film surface tension, with small bubbles ejecting the slowest jets, large bubbles ejecting the fastest jets, and the medium-sized bubbles ejecting gas jets at intermediate speeds. Further, the low speed jets characteristic of small bubbles are attributed to high retraction speeds of their thinner films whereas the high speed jets characteristic of large bubbles are attributed to low retraction speeds of their thicker films. The lowest speed jets ejected from tiny bubbles formed thin stem-like jets which did not roll up into a vortex ring. In contrast, higher speed jets rolled up into a primary vortex ring beneath which an increasing number of additional vortices might form due to a lower film retraction speed. The primary vortex ring may travel nearly seven times its parent bubble diameter, and the shape of this vortex ring also depended on bubble size and the film retraction process.

The bursting behavior of air bubbles of different sizes (e.g. 440 µm - 41 mm) on a DI water surface was observed using a 3D stereophotogrammetrical system in order to study the effect of bubble size on the formation of gas jets and vortex rings. In this vein, it was found that the initial speed of the gas jet released from the bubbles sharply increased with parent bubble size until the bubble Bond number reached unity, corresponding to a bubble equivalent diameter of $2R=5.44$ mm, and subsequently increased at a slower rate. The visualization results of this investigation
explained how film retraction speed, which is correlated with bubble size, affects the initial speed of the gas jet. Specifically, the high film retraction speeds characteristic of smaller bubbles produced correspondingly slower jets emitted through relatively larger openings whereas the low film retraction speeds characteristic of larger bubbles produced correspondingly higher jet speeds emitted through relatively smaller openings. Further, a simple scaling relationship of $Re_{jet}=126Bo^{0.5}$ was proposed to relate the Reynolds number of the emerging jet to the Bond number of the parent bubble. This study concluded that large bubbles eject high speed, high Reynolds number jets which slowly decay and which roll up into fast-growing, highly oblate vortex rings which travel far. In contrast, small bubbles eject low speed, low Reynolds number jets which quickly decay and which roll up into slow-growing, spherical vortex rings which travel short distances.

A more detailed investigation was conducted using a 2D high speed visualization system in order to examine the effects of bubble properties on the formation of gas jets and vortex rings. In a similar way, gas jet speed increased with bubble size for all working fluids with important differences in jet speed magnitudes due to a group of dissimilar initial conditions like pressure difference and corresponding film retraction speed. Surprisingly, we found that increasing the liquid viscosity by nearly a thousand times without affecting its surface tension resulted in a minimal effect on both jet speed and film retraction speed. In contrast, reducing the surface tension by a half resulted in a significant decrease in film retraction speed and thus affected the magnitude of jet speed to a greater extent. Here, the same scaling relationship of $Re_{jet}=126Bo^{0.5}$, which relates the Reynolds number of the emerging jet to the Bond number of the parent bubble was found to apply to the other tested liquids. This scaling relationship showed that the momentum of the emerging jet can be adequately described in terms of the fluid surface tension, density, gravity,
and bubble size. Finally, the film retraction speed is proportional to surface tension, fluid density, and film thickness and is thus not significantly affected by viscosity.

6.2 Unique Contributions

Unique contributions of this work include:

- Quantification of the emerging gas jet parameters and vortex ring characteristics for a wide range of bubble sizes (e.g. 440 µm – 41 mm) bursting on a water surface via high-speed stereophotogrammetry.
- Quantification of the emanating gas jet parameters for a wide range of bubble sizes (e.g. ~500 µm – 45 mm) bursting on three other liquid surfaces using 2D high-speed visualization.
- Discovery of the role of the film retraction process on the speed of a gas jet which override the role played by initial pressure difference as shown by the Young-Laplace pressure equation.
- The novel finding that the vortex ring shape is a function of bubble size, with small bubbles producing spherical vortex rings and large bubbles producing oblate vortex rings.
- The novel finding that the travel distance of a vortex ring released from a bursting bubble on the water surface linearly increases with bubble size and can be expressed as $TD_{VR}=7.1*2R + 7.2$.
- Development of a scaling relationship of $Rejet=126Bo^{0.5}$ that well describes the momentum of the emerging jet in terms of the fluid surface tension, density, gravity, and bubble size. Surprisingly, this scaling relationship can also be expressed as a simple
ratio of two length scales as $Re_{jet}=126R/a$, where $R$ is the bubble cap radius and $a$ is the capillary length.

- Development of ultra high-speed stereophotogrammetry system to measure the three dimensional kinematics of gas jets and vortex rings emitted from bursting bubbles on a resolved time scale.

### 6.3 Broader Impact

The exchange of gases between the atmosphere and ocean is important in climate and weather. For example, the absorption of excess carbon dioxide by the oceans is thought to mitigate the anthropogenic greenhouse effect. In addition, the transfer of dimethyl sulfide (DMS) from the sea to the atmosphere is important because of its role in cloud formation and in radiative forcing. The penetration of oxygen through the sea surface also can affect ecosystem and organism health by diminishing the effects of hypoxia in coastal and estuarine environments. Finally, on local and regional scales, gas exchange between air and water affects the fate of volatile pollutants (Wanninkhof et al. 2009).

The rate at which gas is transferred across the air-sea interface, known as the gas transfer velocity $k$, depends on a number of environmental forcing factors. These factors include the wind speed, the presence of bubbles near the water surface, turbulence levels, the existence of surfactants in the sea surface microlayer, and precipitation such as rainfall or snowfall (Wanninkhof et al. 2009). Wind speed is thought to play the dominant role in determining gas transfer velocity over the global ocean, and many studies have attempted to determine an empirical relationship between the gas transfer velocity and $U_{10}$, the wind speed at a height of 10 m above the ocean surface (Wanninkhof and McGillis 1999, Nightingale et al. 2000, Ho et al. 2006).
Bubbles entrained by breaking waves and subsequently bursting at the water surface may affect the transfer rate of gas and other constituents between the ocean and atmosphere. Gases within bubbles that have been entrained may dissolve into the seawater. Further, these bubbles may gain or lose heat while entrained. Thus, the air released upon bursting may have a different temperature than the ambient air, thus affecting the heat transfer across the ocean surface. Further, marine aerosol droplets may carry brevetoxins from algal blooms or crude oil and chemical dispersant from oil spills, thus endangering the respiratory health of the public (Cheng et al. 2005, Prather et al. 2013, Ehrenhauser et al. 2014, Murphy et al. 2015). Bubble scavenging, which known as the enrichment of bubbles with biological and chemical materials throughout rising and until reaching the ocean surface, may lead to ejecting jet and film drops that are highly enriched with bacteria and other microbes (Blanchard 1975). Organic-laden drops ejected from the ocean may provide a clue to the mechanism for the formation of marine rain.

Finally, bubbles may arise from methane released from the sea floor during storms in a process known as ebullition (Shakhova et al. 2014). These bubbles then will burst as the sea surface, releasing this potent greenhouse gas. The upward momentum imparted by the bursting bubble may enhance mixing in a way that is not accounted for in current models. In particular, the high-speed gas jets released from bursting bubbles may increase the upward velocity of gasses released from the ocean while reaching higher altitudes which approximate to seven times the parent bubble diameter.

6.4 Error Analysis

This study used both a 3D stereophotogrammetry system and a 2D high speed imaging setup that recorded bubble bursting events with high spatial and temporal resolution. However, the utilized equipment might have involved some systematic errors that slightly affected the results.
A common source for these errors may be the sensor of the camera, lens aberrations, and the imaging software. To better address those systematic errors, a systemic approach was used to calibrate both sensors of the camera and its attached lens. The sensors of the camera and attached lens were calibrated using a high resolution calibration plate or wand filmed at different orientations within the share field of view to obtain a camera profile. A camera profile provided values of the offset (i.e. systematic) errors which varied from 0.67-3.4% between the two high speed cameras. In addition, this calibration technique quantified 11 inherent parameters of which calibration coefficients were automatically calculated using Argus 3D and subsequently plugged into MATLAB software for flow tracking.

To further account for systematic errors, the bundle adjustment method as implemented in the DLTdv5 and Argus 3D software packages (Hedrick 2008; Lourakis and Argyros 2009, Theriault et al 2014, Jackson et al. 2016) also assisted in flow tracking. For example, flow tracking was fairly easy when the emerging smoke jet was clearly visible at the early time points. In contrast, flow tracking became relatively harder as the smoke density degraded with the smallest bubble sizes. In the latter case, tracking the gas flow was cross validated using the epipolar line which served as an assistant tool. In addition, as described in Chapter 5, a 10-15% human error was introduced into the measurements of the bubble base radius and bubble height to estimate the sensitivity of the bubble equivalent diameter, as the main control parameter, to those human errors. Finally, a sensitivity analysis was conducted and showed that the values of those errors are minimal.

6.5 Limitations and Challenges

This work presented many novel contributions on the formation of gas jets and vortex rings from bursting bubbles such as 3D kinematics and flow visualizations and analysis for a wide range
of bubble sizes on top of four different liquids. However, there are some challenges and limitations on conducting further research. One of the big challenges is the difficulty of producing bubbles of all sizes with a high enough smoke density to track the emerging flow accurately. Specifically, the small bubbles have low corresponding volumes which sustain less amount of smoke inside of the bubble cap which makes the tracking of the emanating gas flow at the very early stages even more challenging. A related challenge was finding a visualization method in which a particle-laden air flow could be injected as bubbles into the various fluids without affecting the surface tension of that fluid. Fog particles produced from dry ice pellets submerged in water were initially used because the dense tracer was good for visualization but were abandoned as bubble size could not be controlled. Subsequently, aerosolized glycerin particles produced from a commercial fog generator were used, but this idea was also abandoned because the particles became suspended in the liquid interior (and on the bubble boundary) and thus reduced the surface tension. A TSI particle generator (9302) also was used in an attempt to produce particles but was abandoned because the particle concentration inside of a bubble was too low. The solution of using a cold smoke generator, as described in Chapter 4, finally solved these problems by producing a dense concentration of tiny smoke particles that did not significantly affect the fluid surface tension.

Another challenge in this research is the necessarily small field of view, which makes it difficult to get the entire bubble bursting event in focus in both camera views due to the unpredicted nature of bursting. This problem required that a very large number of bubble bursting events to be captured of which a lower number of videos would be useful for further analysis. Because some of these bubbles are tiny and it is necessary to reconstruct the bursting kinematics accurately, a fine spatial resolution is needed which requires a small field of view. This small field of view made it such that the rising bubble prior to it reaching the liquid surface could not simultaneously be
imaged. This made subsequent measurements of the bubble volume somewhat more difficult. Besides these challenges, the emerging gas jet and the primary vortex ring have several orientations and very dissimilar shapes which travel into very different heights for each individual bursting event. These different orientations and shapes, the complex entrainment process, and motion of the primary vortex ring make it prohibitively difficult to employ automatic tracking for digitization. Thus, the digitization process is very time and labor consuming and slows down the overall research process. Finally, interpreting the results posed many questions throughout the course of analysis because the visualizations supported the results and outcomes and showed good agreement but both were counterintuitive as we have long thought that the small bubbles would produce a high speed jet compared to a large bursting bubble with a much lower pressure.

6.6 Future Work

This study contributes quantified kinematic data for three orders of magnitude of bubble sizes bursting on top of four different liquid surfaces with varying physical properties for the first time. The flow tracking via manual digitization steps is very time consuming and open to subjective decisions between person to person and even frame to frame. To obtain more accurate kinematic parameters both in fine temporal and 3D spatial resolutions, it is vital to develop more efficient digitization methods. The current automatic tracking algorithms need to be improved by implementing some advanced machine learning algorithms to track the emerging gas flow and the growing primary vortex ring instantaneously with high accuracy. There are some opensource software that use deep neural networks to track the points already available to use, for example DeepLabCut (Mathis et al. 2018). In the future, a similar software should be utilized or, if necessary, should be modified to use for the tracking of escaping gas behavior. Deep learning is the direction the 3D kinematics research should be heading.
In the future, measurements of the flow field of the gas escaping from the bubble while bursting would be very interesting. These measurements could reveal the flow velocities at the opening in the bubble film, which could not be measured in the current study. These measurements also would reveal the circulation characteristics of the vortex rings emitted from bursting bubbles. In addition, these measurements would detect the roles of buoyancy and flow shear on the stability of the emerging gas jet and the subsequent formation of vortex rings or arising Kelvin-Helmholtz instabilities. These roles could possibly be characterized using the Richardson number (Ri) which compares buoyant suppression of turbulence to shear generation of turbulence. However, two dimensional particle image velocimetry (PIV) measurements would be difficult owing to the highly curved bubble film, which would refract the laser sheet. Alternatively, a volumetric flow measurement technique such as 4D particle tracking velocimetry (PTV) could possibly be used. A microscope-based particle image velocimetry system using an extra long working distance microscope could also be used with backlighting to investigate the flows produced by the smallest bubbles. Finally, x-ray velocimetry (such as that available at Argonne National Laboratory) could be used to better define the shape of the bubble cavity while it is bursting.

Furthermore, it would be interesting to better investigate the possible interaction between liquid and gas components, namely the possibility of the gas jets advecting marine aerosol droplets up into the atmosphere. These droplets may or may not be produced from the bursting bubble itself but could be produced by a bubble that previously burst. Nonetheless, aerosol droplets in a layer near the water surface would experience an overall upward momentum from the bursting of bubbles at the ocean surface.

Finally, the formation of gas jets and vortex rings from bursting bubbles is a very complicated phenomenon that includes unsteady fluid dynamics in three dimensions and thus
simulating such flows can be extremely challenging. The number of variables to be considered in a comprehensive study that would accurately model the flow is relatively high due to the random nature of bursting and to major differences in bursting behavior between small and large bubbles. In the current study, some effort was devoted to developing two simple mathematical models, namely a Constant Pressure and Volume model and a Constant Volume model, that would set upper and lower bounds of the gas jet speed emitted from a bursting bubble as a function of bubble size. However, many assumptions were made due to the asymmetric pattern found in bursting bubbles, the highly unsteady nature of gas jets and vortex rings, and a possible interaction between the gas and liquid components involved in bubble bursting. Those assumptions affected the overall performance of the models. Nonetheless, the outcomes when comparing both models confirmed the role of the film retraction speed on the depressurization process of the bubbles. In this work, a quasi-one-dimensional nozzle model was developed by Dr. Geum-Su Yeom to theoretically predict the velocity of the gas jet generated by the bursting bubble. It would be really helpful to develop three dimensional mathematical and computational models that mimic the actual bubble bursting phenomena in regard to the gas component as there exist plenty of models that well describe the liquid part.
References


Appendix A: Copyright Permissions

A.1 Copyright Permission for the Materials Used in Chapter 3

Gas escape behavior from bursting bubbles
All A. Deshpande and David W. Murphy
Phys. Rev. Fluids 5, 114502 (2020) - Published 12 November 2020

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Appendix B: Supplementary Materials

This research is a winner of a 2019 Gallery of Fluid Motion (GFM) Award associated with a video presented at the American Physical Society’s Division of Fluid Dynamics (APSDFD) of the same year.

Watch the video here: https://gfm.aps.org/meetings/dfd-019/5d7fe8e4199e4c429a9b30e4

Figure B.1 APS DFD GFM 2019 Video Winner Award
This work was highlighted in Science Magazine (AAAS) in November 2019 by Adrian Cho. Watch the video and read the article here:


Figure B.2 AAAS Science Magazine Article
This work was also highlighted in Deutsche Welle (DW Science), a German International Broadcaster, as a short Web video in December 2019 by Sophia Wagner. Watch the video here:

https://twitter.com/dw_scitech/status/1208846670588366849

Figure B.3 Short Web Video on DW Science