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Bayesian Reliability Analysis for Optical Media Using Accelerated Degradation Test Data

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by

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Abstract

ISO (the International Organization for Standardization) 10995:2011 is the international standard providing guidelines for assessing the reliability and service life of optical media, which is designed to be highly reliable and possesses a long lifetime. A well-known challenge of reliability analysis for highly reliable devices is that it is hard to obtain sufficient failure data under their normal use conditions. Accelerated degradation tests (ADTs) are commonly used to quickly obtain physical degradation data under elevated stress conditions, which are then extrapolated to predict reliability under the normal use condition. This standard achieves the estimation of the lifetime of recordable media, such as Magneto-Optical media, via an accelerated degradation test for measuring the error rate of these hard devices under elevated temperature and relative humidity levels. The observed degradation measures are modeled with regression analysis to predict the unobserved failure time, which is then used as observed failure time for estimating the lifetime distribution and predict the device quantile/median lifetime. However, the ISO 10995:2011 analysis fails to consider the uncertainty of the predicted failure times, as well as the heterogeneity of the test units, and hence could lead to imprecise and overconfident estimation. This thesis presents a Bayesian method to analyze the ISO degradation data, which (1) provides more accurate quantification of uncertainty through the use of a hierarchical degradation path model, (2) includes random effects to capture the unit-to-unit variation for improving analysis of heterogeneity, and (3) offers more straightforward implementation for estimating reliability and its associated uncertainty based on general ADT data.

1 Introduction

There is a common consensus that predicting the archival life of optical media plays an increasingly important role in its market and industry. Existing standards have documented methods for predicting the lifetime of recordable media, which is high reliable product. The aim of the ISO 10995:2011[1] is to provide guidance on current practice based on conducting accelerated degradation tests for understanding the underlying failure mechanism and predicting the archival life of optical media products.

Optical media are designed to be long-life devices. It is often impossible to observe sufficient failure data under its normal use condition during a reasonable test period. Accelerated degradation tests (ADTs) become popular in testing highly reliable products to quickly inquire the sufficient failure data. In ADTs, the test units are assigned to elevated stress levels or exposed to harsher use conditions to speed up the failing process, and measures on characteristics related to the physical degradation process are obtained, analyzed, and extrapolated to predict the product lifetime under normal operating conditions. In ISO 10995:2011, four different test conditions were used with different combinations of the temperature (T) and relative humidity (RH) levels. It was believed that elevated stress conditions would accelerate the rate of chemical reactions, which will then lead to the degradation of the ideal material properties that ultimately causes disk failure.

1.1 Background

The worldwide standardization system consists of both the ISO (International Organization for Standardization) and the IEC (International Electrotechnical Commission).

ISO or IEC participates in the development of international standards through technical committees established by their respective organizations to deal with technical activities in specific areas. The purpose of ISO/IEC is to develop and maintain standards in the fields of information technology and communications technology. ISO 10995:2011 [1] uses the ADTs to estimate and predict the lifetime of optical media. Particularly, it uses the degradation data obtained from an ADT experiment to estimate the life expectancy of the searchability of information stored on a recordable optical disc.

In the ISO 10995:2011 experiment, several different types of optical media formats were tested, such as DVD-R/-RW/-RAM, +R/+RW. The outlined procedure can be adapted to other disc formats with appropriate specifications as desired. This international standard includes assumptions, stress conditions, ambient conditions, evaluation system description, test units preparation, degradation data description procedure, and data interpretation. The documented ADT experiment used the temperature and relative humidity as the accelerating factors. It has no specific procedures to simulate degradation due to the dynamics of complex failure mechanisms, nor has it consider other factors such as changes in light, corrosive gases, pollutants, processing, and playback subsystems. Disks exposed to higher temperature and RH levels are expected to have a shorter life. In ISO 10995:2011, the experimenter recorded the degradation measurements periodically from each unit tested under varied conditions. The observed degradation data were fitted to a regression model to determine the time-to-failure of each test unit as the first time that its predicted degradation level exceeds the failure threshold value at MAX PI Sum 8 = 280. In ISO 10995:2011, the time-to-failure was determined based on linking the disk errors with the material degradation. By assuming an exponential lifetime distribution [1], the predicted time-to-failure for all the test units were fitted in an accelerated failure time model and extrapolated to predict product median lifetime under the normal use condition.

The method used in ISO 10995:2011 has some problems. For example, it believes what they have from the recorded failure time is as same as the actual failure time and com-

pletely ignores the uncertainty of the predicted failure time. Second, the model used in ISO 10995:2011 assumes a homogeneous distribution across the population and completely ignores the unit-to-unit variation among the test samples, which can be prominently observed from Figure 1.1 below.

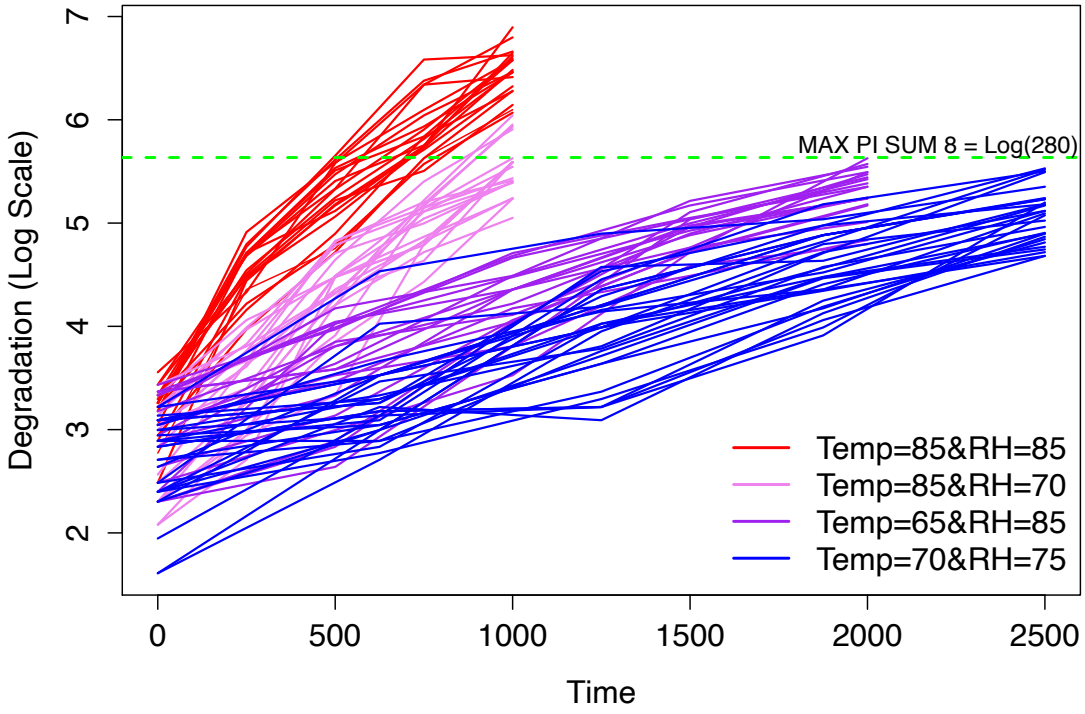

Figure 1.1: Degradation Paths From the ISO 10995:2011 ADT Data

Figure 1.1 shows the degradation paths (measured on the log scale) of all the 90 test units from the ISO accelerated degradation test. Different colors present the test units under the four different test conditions. A few patterns can be observed. First, there exist different degradation rates across different test conditions. For example, the units tested under the highest temperature (at $85°C$) and RH (at 85) levels underwent the highest degradation rates (shown as the red paths in Figure 1.1). On the other hand, the units tested under the lower temperature and RH levels had lower degradation rates (such as the blue paths tested at $70°C$ and $RH = 75$). Second, the degradation rate does not necessarily stay constant over

time. For example, we can observe some degree of curvature for many paths. This indicates the degradation paths may not necessarily be a linear function of time for all the test units, and a polynomial regression or nonlinear regression model might be needed to flexibly model the variation of the degradation trend. Third, there is an apparent unit-to-unit variation in terms of both the initial degradation levels and the degradation rates. For example, at the initial time point (time 0) of the study, each degradation path starts with a different initial value which suggests varied starting conditions of the test units. In addition, we observe some degree of positive correlation between the initial degradation condition and the degradation rate. For example, the red paths which generally have higher initial degradation values are also associated with higher degradation rates over time. This indicates an apparent unit-to-unit variation with potential positive correlation between the initial condition and the temporal degradation rate. Therefore, it is necessary to include bivariate random effects for both the individual initial condition and the temporal degradation rate.

The horizontal green dash line in Figure 1.1 shows the soft failure threshold (at $log(280)$) for this type of recordable media used in the test. The test unit is considered to fail when its degradation level passes this threshold value. We can see that all the test units shown in the red paths which were tested at 85°C and 85% RH (the harshest test condition among all 4 test conditions) failed during the test period. Under test conditions (65°C, 85%) and (70°C, 75%), the degradation paths are shown as purple and blue solid lines. With the test duration (2500 hours) being 2.5 times longer than the harshest condition (1000 hours), no failure was observed during the test period. This is reassuring both temperature and RH are effective acceleration factors for this type of optical media. And it is expected that under the normal use condition at (25°C, 50%), it would take a very long time before any failure can be observed. Therefore, the ADT test plays an important role in accelerating the degradation process and collecting experimental data in a timely fashion for studying high reliability products.

From Figure 1.1, we can see unit-to-unit variation presents an important source of hetero-

geneity in the ISO 10995:2011 data. It can cause variations in both the shape of degradation curves and the predicted failure times. Below we summarize the key sources of unit-to-unit variation in ADT tests:

- Initial conditions. Individual units will vary with different initial levels of degradation depends on the amount of material use. For example, on our raw data in ISO 10995:2011, at time 0, the degradation measurement varied from each other, indicating each unit has its own initial degradation.

- Material properties. The unit-to-unit variations in the material natures parameters C and the initial size of products would affect the degradation rate, which differs from unit to unit. Material properties parameters C is the degradation rate which is defined as $dD(t)/dt$, where $D(t)$ is the amount of degradation measurement at time t. For example, with the different material properties, products with higher initial degradation may reach out to the failure time threshold later than the product with lower initial degradation.

- Component geometry or dimensions. Different sizes, shapes, or forms of the products can also cause unit-to-unit variability.

- Within-unit variation. Spatial variation of material properties within a unit can often happen and causes a variation.

Thus, we need to take those uncertainties into account and we conducted new data analysis using a different method, which considered both unit-to-unit variation and failure time uncertainties.

Our raw data is from ISO 10995:2011, later we will name this dataset ISO data. As shown at the Appendix, ISO data consists of (1) four different test conditions; which is the combination of temperature and relative humidity, specifically the four test conditions combination are (85°C, 85%), (85°C, 70%), (65°C, 85%) and (70°C, 75%) (2) The number of test disks within each test condition, with 20 test disk specimens in the first three conditions

and 30 test disk specimens in the last conditions, in total, there are 90 test units in this experiment. We can later rearrange this 90 test units each by time points to let the dataset has 450 test units. See changes in below data compared with the original data from the Appendix B; (3) test time in hours, each disk was tested during a time period with 5 recorded time points, and recorded the (4) failure rate measurement, also called as degradation data measurement. Therefore, our response variable D_{ijk} in ISO data has a length of 450 because each of the 90 test units repeated measured at 5 different time points. After wrangling the data, part of our ISO data now shown as follow. In this table, it only shows the observed degradation measurements for the first 3 test specimens (A1, A2, A3) under one of the test conditions (85°C, 85%) with repeated time in (0,250,500,750,1000) hours.

ISO_data

Disk #	Deg. data	Hours	Temp.	RH
A1	16	0	85	85
A1	78	250	85	85
A1	116	500	85	85
A1	278	750	85	85
A1	445	1000	85	85
A2	25	0	85	85
A2	64	250	85	85
A2	134	500	85	85
A2	342	750	85	85
A2	532	1000	85	85
A3	26	0	85	85
A3	94	250	85	85
A3	190	500	85	85
A3	335	750	85	85
A3	642	1000	85	85

Figure 1.2: A Part of ISO Data after Data Processing

1.2 Goal and Motivation

The lifetime prediction of high reliable products is of major importance in reliability analysis. Reliability is the probability that a material, component, or system will perform

its intended function under defined operating conditions for a specified period of time[2]. The subject we are about to predict is optical media, such high reliable products' lifetime can be last a long period under normal life use condition. It is often hard to observe a failure under the normal use condition over a reasonable testing period. ISO 10995:2011 introduced ADTs to obtain the degradation data by putting the product under more severe environmental stress. Because many failure mechanisms can be traced to an underlying degradation process and degradation would eventually lead to a weakness that can cause failure.

An ADT test for the prediction can be involved with two models, one is fit the degradation data into a regression model and the other one is failure distribution model for certain degradation parameters[3]. The simple test steps can be done by:

1. Perform acceleration test and record the degradation measurements under each specific stress conditions, normally the test conditions are set by increase the level of temperature and relative humidity compared with products normal use condition;

2. For each test unit, fit the degradation data into a regression analysis, and calculate the failure time as the first crossing time when the degradation path reaches the performance threshold value;

3. Fit the predicted failure times from each test unit into a failure distribution model;

4. The failure time can be extrapolated from the test conditions to the ambient (normal use) condition (here is under 25°C, 50%) by fitting the acceleration degradation model.

This thesis study is based on the resources from both ISO 10995:2011 and an article which written by Fang et al.[3], based on Fang et al., it used the frequentist approach which is maximum likelihood estimation (MLE) method on hierarchical model parameters estimation, then designed an algorithm based on Monte Carlo simulation to predict the median lifetime of optical media. In terms of my works, instead of using MLE to estimated parameters, I use the Bayesian approach on parameter estimation. Specifically, using JAGS(Just

Another Gibbs Sampler) which is based on the Gibbs Sampler algorithm, and STAN which is based on No-U-Turn Sampler (NUTS). More specific in using Markov chain Monte Carlo (MCMC) simulation algorithm and reliability analytical method. A Markov chain is a random process that has the property that the future depends only on the current state of the process and not the past, which is memoryless. With MCMC, one can draw samples from some probability distribution. The motivation for using MCMC simulation is because for approximating the intractable posterior distribution, MCMC provides an advanced algorithm on computing process. In addition, the MCMC parameter estimation makes the inference a faster computation since it based on the Gibbs Sampling which contains models with many unobserved variables, then the likelihood function of these models is very hard to take integrals using maximum likelihood estimation. However, with Bayesian approach using MCMC, we only have to consider the likelihood function conditional on the unobserved variables, thus make the computation faster than frequentist approach. More explanations will be in section 3.2.

The frequentist approach and the Bayesian approach have different starting points and standpoints when they discuss the issue of "uncertainty". The biggest difference between the frequentist and the Bayesian actually comes from the cognition of parameter space. The parameter space is the range of possible values of the parameter one cares about. In a Bayesian framework, we model the data probabilistically as well as the parameters that govern the distribution of the data. In a frequentist framework, the data is modeled probabilistically, but the parameters are not.

This thesis aims to use degradation data from ISO 10995:2011 to predict optical media lifetime and analyze reliability using Bayesian data analysis. More specific in using Markov chain Monte Carlo (MCMC) simulation algorithm and reliability analytical method.

Another motivation is that the original standard failed to consider the uncertainties in the failure time, which was assumed that the actual failure time is as the observed projected failure time. We found that in fact the true failure time is not directly observed in many

cases, therefore, in addition to predict the lifetime by degradation data, this thesis also draws uncertainty boundaries for both reliability and quantile lifetime, from which manufactures could have more insight views on how does the product fail and on how much percentile it fails during periodic intervals. What's more in this thesis is in section 3.5, where gives a short discussion on sensitivity analysis, which tells how posterior inferences vary for estimands and predictive quantities of interest when trying a variety of different distributions for likelihood and prior models.

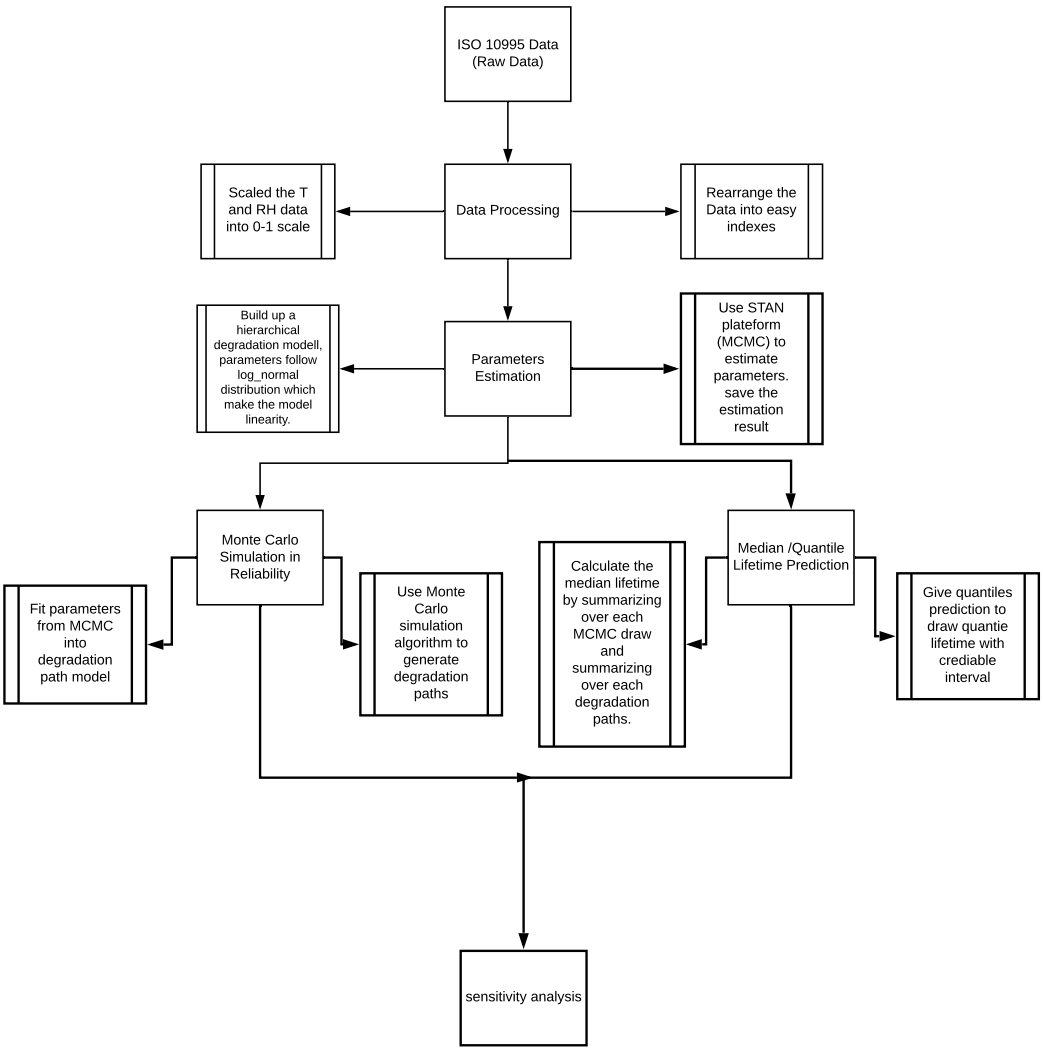

Figure 1.3: A Flow Chart of this Study

1.3 Overview

The remaining of this thesis is going to follow this format. Chapter 3 provides the model and methodology details, such as general degradation path model[4] explanation; the parameters estimation part using MCMC algorithm based on STAN[5] platform to generate posteriors draw of model parameters; Chapter 4 talks about the reliability analysis[2] using simulation data to predict the median lifetime of optical media and draw credible intervals for both reliability and quantile lifetime; in Chapter 5, we will give a short view on sensitivity analysis[6], which tells how posterior inferences vary for estimands and predictive quantities of interest when trying a variety of different distributions for likelihood and prior models, it provides a comparison between different prediction based on different priors we use when estimating parameters. The above flow chart in Figure 1.3 is a simple process of the whole study.

2 Literature Review

2.1 Previous Works

Degradation analysis relates to product reliability. Through an underlying degradation process, it eventually leads to a weakness that can cause failure. When measuring degradation is possible, degradation data can provide considerably more reliable information than traditional data of failure-time. Accelerated degradation tests are used to obtain failure information more quickly. Therefore ISO 10995:2011 introduced the acceleration degradation test to obtain testing data for later analysis works.

Acceleration degradation test has been widely used to obtain data from high reliability products more quickly. Since it is impossible to obtain and measure the actual degradation in some reliability studies, while measures of product performance degradation may be available, modeling performance degradation may be useful but could be complicated because performance may be affected by more than one underlying degradation process. Depending on the application, degradation data may be available continuously or at specific points in time where measurements are taken. Besides, Direct observation of the physical degradation process provides more credible on reliability estimatation and a firmer basis for often-needed extrapolation.

In ISO 10995:2011, researchers trying to use the Eyring[7] acceleration model (Eyring Method) to predict the median lifetime of Optical media. Henry Eyring developed Eyring Equation in 1935, which updated the description of relationship between reaction rate and temperature. It was five decades later than Arrhenius[8] model, which also describes the

temperature dependence of reaction rates. However, whereas Arrhenius Equation[9] can be applied only with the relationship with temperature, while Eyring method can be applied in both temperature and other factors. Using the Eyring model, the following equation which handle condition variables of both temperature and relative humidity can be written as:

$$t = AT^a e^{\Delta H/kT} e^{(B+C/T)RH} \tag{1}$$

where

t is the time to failure

A is the pre-exponential time constant;

T^a is the pre-exponential temperature factor;

ΔH is the activation energy per molecule;

k is the Boltzmann's constant ($13807 \times 10^{-23} J$/molecule degree K);

T is the temperature (in Kelvin);

B, C are the RH exponential constants;

RH is the relative humidity;

And based on the Eyring mode, an acceleration factor (AF) can be written as

$$\text{AF} = AT^a exp[\Delta H/kT + (B + C/T) log(RH)] \tag{2}$$

Based on the temperature range that used in ISO 10995:2011 test method, "a" and "C" would be better to set to zero[1]. Then here is the reduced Eyring model equation:

$$t = Ae^{\Delta H/kT} e^{B*RH} \tag{3}$$

By calculating different Acceleration Factors in all test conditions, ISO 10995:2011 conducted the lifetime prediction for each stress condition, then given the ambient used condition, a lifetime in normal use condition can be extrapolated.

In addition to conducting the ADTs, Meeker, Escobar, and Lu(1998)[10] give a comprehensive discussion on degradation modeling. Nelson(1981)[11] reviews the degradation literature, surveys applications, and describes basic information on accelerated degradation models. Tseng and Wen(2000)[12] describes the use of step-stress ADTs for assessing the reliability of light-emitting diodes (LEDs). Fang et al.(2018)[3] uses frequency analysis to estimate the parameters of Eyring model, gives a median lifetime prediction on optical media base on the data provided from ISO 10995:2011.

3 Methodology

3.1 Model for Degradation Data

Based on the ISO 10995:2011 standard, there are two stress variables that affect the lifetime of optical media, one is temperature (T) and the other one is relative humidity (RH), The standard therefore makes four different combinations of T and RH as testing conditions; in each condition, standard sets up 20-30 number of specimens to record their performance under test conditions. For each specimen, at each testing period point, a non-linear hierarchical model of degradation measurement is recorded to predicte the time-to-failure. The obtained degradation measurements are assumed to exhibit a lognormal distribution, and the location parameter of the lognormal distribution will be the only parameter that affected by stress variables. Model details would be provided later in section 3.1.2 Hierarchical Model, and data details would be described in section 3.1.1 Degradation Data from ISO 10995:2011.

3.1.1 Degradation Data From ISO 10995:2011

In ISO 10995:2011 standard, the experiment is conducted in 4 testing conditions, 20 specimens in condition 1 which is (85°C, 85%), condition 2: (85°C, 70%), and condition 3: (65°C, 85%), 30 specimens in condition 4: (70°C, 75%). Also, in conditions 1 and 2, the specimens are recorded at time 0, 250, 500, 750, and 1000 hours from the duration of test; in condition 3 specimens degradation are measured at 0, 500, 1000, 1500, and 2000 hours; in condition 4 are measured at 0, 625, 1250, 1875, and 2500 hours. The experiment was set different time duration under each stress condition because a longer time period is used

under less severe stress condition, the products are assumed to spend more time until failure is reached when the test condition is less stressful. The original degradation test data table from ISO 10995:2011 is shown in the Appendix. The four stress conditions are listed in Table 3.1.

Table 3.1: ISO Testing Conditions

Conditions	Temperature, (°C)	Relative Humidity (%)	Number of Specimens
1	85	85	20
2	85	70	20
3	65	85	20
4	70	75	30

As the raw data are shown in Appendix, there are 7 columns in total, which the first column represent the specimens' number, followed 5 columns are the time points in different intervals, the last column is named projected failure, it represents the corresponding time when the recorded data first across the threshold. According to ISO 10995:2011, the performance threshold is the parity inner error[13] (PIE) which has a value of 280 for Optical Media. Since not all test units in this experiment been observed as fail during the test time period, the projected failure was calculated by fitting the degradation measurements into a linear regression, where the ln of the error rate as the response variable and time in log scaled as the predictor variable. Then the projected failure time is calculated from the linear regression as the time at which the degradation measurement would have reached the threshold which is Max PI Sum 8 of 280[1]. Notice that when later predicting reliability, this threshold should also be taken log-transform since our model was built under the log scale.

The experiment conducted in ISO 10995:2011 was under the instruction of Acceleration Degradation Tests (ADTs), optical media often designed for high-reliability systems, which contain vary high level reliability, even after long periods of time. Using stress conditions can make the products fail quickly compared with under normal use condition. Thus, a

relationship between failure and degradation measurements can be use to make inferences and predictions on failure time. Next, we will introduce model used in our analysis.

3.1.2 Hierarchical Degradation Model

An acceleration model can be used to extrapolate the products' failure time from the elevated acceleration test conditions to the normal use condition. The term "acceleration" has many different meanings within the field of reliability, but it generally implies making "time" go more quickly, so that reliability information can be obtained more rapidly[2]. There are different types of tests, for example, Accelerated Life Tests (ALTs) is one obtains information on the failure time for units that fail and lower bounds for the failure time for units that do not fail; while another is Accelerated Degradation Tests (ADTs), which is one observes, at one or more points in time, the amount of degradation for a unit.

Since our testing conditions are the combination of T and RH, which have more than one accelerating variable, a generalized Eyring Model needs to be used to calculate the acceleration factor which describes the relationship between different sets of conditions T, RH and the normal use condition. In particular, the acceleration factor under test condition associated with T and RH is in equation (2).

Because this model has two levels, we aim to use a hierarchical degradation model which describes the degradation paths under elevated acceleration test conditions that described in the ISO data. The first level is the log scale of dependent variable $log\ y_{ijk}$ for test units i under test conditions j among time point k, and the dependent variable $log\ y_{ijk}$ is assumed to fit the non-linear degradation path model in the transformed time variable $t_{ijk}^{\gamma_i}$ and measurement error ϵ_{ijk}, ie,

$$\begin{aligned} log y_{ijk} &= D_{ijk} + \epsilon_{ijk} \\ &= \beta_{0i} + \beta_{1j} t_{ijk}^{\gamma_i} + \epsilon_{ijk}, \end{aligned} \tag{4}$$

where i=1, ..., 90, j = 1,...,4, and k = 1,...,5. We assume that the measurement errors are

i.i.d. ie, $\epsilon_{ijk} \sim \mathrm{N}(0,\sigma^2)$.

The second level of this model is for random effects, β_{0i} and γ_i which according to unit-to-unit variations, β_{0i} is the initial degradation measurement in log scale varies among units before accelerating under stress conditions; and the scale parameter γ_i which describes the unit's degradation path curve shape also varies from unit to unit. Therefore, β_{0i} , γ_i and β_{1j} are given by

$$\beta_{0i} = \mu_0 + \epsilon_{0i}, \tag{5}$$

$$\gamma_i = \gamma_0 + \epsilon_{1i}, \tag{6}$$

$$\beta_{1j} = exp\left(logA + BlogRH_j + \Delta H \frac{11605}{T_j + 273.15} \right), \tag{7}$$

where μ_0, γ_0, $logA$, B , ΔH and σ are the model parameters that need to be estimated. For the random effects β_{0i} and γ_i, we can rewrite them as a vector that follows the bivariate normal distribution with mean vector of μ_0 and γ_0 because both β_{0i} and γ_i are linear functions of the same two independent normal random variables, their joint PDF takes a special form bivariate normal PDF. Thus, the hyper-parameters σ_0 , σ_{01} and σ_1 from the var-covariance matrix also need to be estimated. The random effect could be write as

$$\begin{bmatrix} \beta_{0i} \\ \gamma_i \end{bmatrix} \sim MVN \left(\begin{bmatrix} \mu_0 \\ \gamma_0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix} \right) \tag{8}$$

3.2 Parameters Estimation

Frequentist and Bayesian approaches use different methods to estimate parameters.

Frequentists don't assign probabilities to possible parameter values and they use point estimates of unknown parameters to predict new data points (Maximum Likelihood Estimation); while Bayesian approach introduces the probability distribution to any uncertain Parameter given a likelihood function of some data, and update the estimation of parameters based on the newly introduced information or evidence.

So far, we already have the time-to-failure dataset and a hierarchical degradation model in Equation (4), we can now use those degradation data to process estimation in model parameters. At the end of the last section, we mention that μ_0, γ_0, $logA$, B , ΔH and σ are the model parameters to be estimated. For those parameters, we will give each of them a prior distribution when using Bayesian to generate the estimation. A scaled inverse Wishart distribution as prior for cov-variance matrix was recommended by Gelman and Hill[14] , where we also need to estimate those hyper-parameters : σ_0 , σ_{01} and σ_1. However, In Stan, the conjugate of the multiple priors has no limitation, based on Gelman and Hill, we take the decomposition of cov-variance matrix prior into a coefficient scalar and a correlation matrix to avoid computation directly on the co-variance matrix. Therefore, our goal in this section is to use one of the MCMC algorithms to simulate those 9 parameters.

Before discussing Bayesian approach, one has to know there is another common approach to estimate parameters, classical frequentist approach. This approach is statistically driven, and defines probability only for a certain event as the limit of its relative frequency in many trials. The methods that frequentist approach used are usually Maximum Likelihood Estimation and Least Squares Estimation. In contrast, Bayesian approach is widely use the probability on both sampling and represent subjective uncertainty. The sampling method Markov Chain Monte–Carlo (MCMC) will throughout the whole section of this chapter.

3.2.1 Gibbs Sampler

Markov Chain Monte–Carlo (MCMC) is a popular method for obtaining information about distributions, especially for estimating posterior distributions in Bayesian inference. Over the last two decades, the use of MCMC sampling has increasingly grown. MCMC has

two parts: Markov Chain and Monte Carlo. The first part, in Markov Chain, time is discrete and a state X at time $t+1$ only depends on the one preceding state X at time t, and it is independent from the past states $X(t-1), \ldots, X(1)$. The second part Monte Carlo (MC) is the practice of estimating the properties of a distribution by examining random samples from the distribution, this approach would be to draw a large number of random samples from some common used priors distributions, such as normal distribution, Beta distribution and uniform distribution, then calculate the sample mean of those.

In the modern Bayesian statistical inference, the research of calculating high dimensional posterior integral by MC sampling method has developed rapidly. Gibbs sampler is one of the common method in MCMC algorithms. It was first proposed by Grenander (1983)[15], while the formal term was introduced by German and German (1984)[16]. The basic idea of Gibbs sampling[17] is to construct Markov chain $\{\theta^{(i)}\}$ through the conditional distribution family of the components of the parameter vector θ when inferring the high-dimensional parameters posteriors, and make its stationary distribution the target distribution. Suppose that $\theta = (\theta_1, \ldots, \theta_p)'$ is the parameter vector of p dimension, and $\pi(\theta|D)$ is the posterior distribution of theta after the observed data set D, then the basic sampling method is as follows:

- step 0 : Arbitraryly picking a starting point $\theta^{(0)} = (\theta_{1,0}, \theta_{2,0}, \ldots, \theta_{p,0})'$, and set $i = 0$;
- step 1 : Generate $\theta^{(i+1)} = (\theta_{1,i+1}, \theta_{2,i+1}, \ldots, \theta_{p,i+1})'$ as follow:

  generating $\theta_{1,i+1} \sim \pi(\theta_1|\theta_{2,i}, \ldots, \theta_{p,i}, D)$,

  generating $\theta_{2,i+1} \sim \pi(\theta_2|\theta_{1,i+1}, \theta_{3,i}, \ldots, \theta_{p,i}, D)$,

  .....

  generating $\theta_{p,i+1} \sim \pi(\theta_p|\theta_{1,i+1}, \theta_{2,i+1}, \ldots, \theta_{p-1,i+1}, D)$,
- step 2 : Then set $i = i+1$, and return to step 1.[18]

In this algorithm process, each component of θ is generated in a natural order, and each loop needs to generate p random variables.

3.2.2 JAGS and STAN

Both JAGS and STAN are the R interface platforms. STAN is a C++ library for Bayesian modeling and inference that primarily uses the No-U-Turn sampler (NUTS) (Hoffman and Gelman 2012[19]) to obtain posterior simulations given a user-specified model and data. JAGS stands for Just Another Gibbs Sampler. It is a programming language for analysis of Bayesian hierarchical models using Markov Chain Monte Carlo (MCMC) simulation. It uses a dialect of the BUGS language, similar but a little different to OpenBUGS and WinBUGS.

STAN and JAGS are quite different probabilistic programming languages but they can be both used for the Bayesian data analysis. JAGS is similar to WinBUGS and OpenBUGS, where a model states just relations between variables. While Stan is a program where a model has clearly defined parts, such as Data, Parameters, and Model blocks. Both STAN and BUGS can be run by themselves, but it has been found that STAN runs most convenient from R. Since JAGS and STAN has different way to estimate parameters, the number of MCMC samples that they need are different too. Stan runs in a more efficient way compared with JAGS, hence needing fewer samples to obtain a posterior result of similar quality with JAGS. From a view based on model development, JAGS (rjags, R2jags) is slightly more time consuming when integrated in R than STAN (RStan), mostly because JAGS models pretend to be R models, which means the editor will help JAGS to take those complicated integrals, while RStan has its own model just in a text vector, which has a better solution in hard compute integrals. In addition, STAN is also a highly organized model coding language.

In the 1940s and 1950s, the Monte Carlo method was founded by Metropolis, Von Neumann and Stanislaw Ulam, after whom STAN was named. STAN is a probabilistic programming language based on C++, mainly used for Bayesian inference. With STAN, users need

to provide data, script models written by STAN code, compile the program are written by Stan, and then run with the data. The posterior simulation process of model parameters is automatically realized. In addition to compiling and running STAN scripts to write models in a command line environment, STAN provides interfaces to other programming languages such as R, Python, Matlab, and so on, making it easy for users familiar with other programming languages to invoke and analyze data. However, unlike such interpreted programming languages as Python and R, STAN code needs to be translated into C++ code and then compiled using a system compiler. If the R language interface is used, the compiled dynamic link library can be loaded into R memory and then executed by other R function calls. Stan's built-in sampler no-u-turn (NUTS) is derived from the Hamiltonian Monte Carlo (HMC) algorithm and was first proposed by Hoffman and Gelman (2014). Software BUGS and JAGS with similar functions to STAN mainly use Gibbs sampler. The former was developed based on Pascal language from 1989 to 2004, while the latter was actively developed based on C++ from 2007 to 2013. In terms of time, STAN has the advantage of late development, especially in terms of flexibility and extensibility. It supports arbitrary target functions, and the model language is simpler and easier to learn. In modeling analysis of large amounts of data, Stan can process complex models more quickly, thanks in part to its efficient algorithm implementation and memory management, and in part to the advanced MCMC algorithm - the HMC algorithm with the NUTS sampler.

3.2.3 STAN Convergence

Once we have the hierarchical degradation model in section 3.1.2, we can use the STAN code to fit into this model. The structure of RStan consists of three fundamental parts, with some of the parts we need to do transformed, which means pretreatment or precalculate. These three parts in STAN code are data block, parameters block and model block.

for example, the simple linear regression model, STAN can be coding as follow into three basic blocks: data, parameters and model blocks. The example R code is shown in Appendix

A-1.

Implement for our model in equation (4), we can write it equivalently into

$$y_{ijk} \sim lognormal(D_{ijk}, \sigma), \tag{9}$$

where

$$D_{ijk} = \beta_{0i} + \beta_{1j} t_{ijk}^{\gamma_i}, \tag{10}$$

Therefore, we can re-write our degradation model as follow:

$$y_{ijk} \sim lognormal(\beta_{0i} + \beta_{1j} t_{ijk}^{\gamma_i}, \sigma), \tag{11}$$

From the above equation, we can write a log-normal with $\beta_{0i} + \beta_{1j} t_{ijk}^{\gamma_i}$ mean and σ standard deviation in the model block using STAN. Before importing the data, it is always helpful when wrangling the raw data first. As shown in appendix and section 1.1 with after wrangling dataset, we can fit this ISO data into our model and using STAN as the computation platform to calculate parameters.

With STAN, we can fit the degradation model in and generate MCMC posteriors draws for each parameter. Two notices should be taken into concern. First, scaling. In most cases, the data set will contain features highly varying in magnitudes, units and range. However, since most MCMC algorithms use Euclidean distance between two data points in the calculation, this problem should be pre-processing via scaling. By doing so, we can bring all features to the same level of magnitudes. In our ISO data, we have to scale the temperatures and relative humidity. The methods we preformed here is using Min-Max Scaling[20], this method will bring the value between 0 and 1, we named the after scaling temperatures and relative humidity as X_1 and X_2, respectively. Secondly, as shown before in equation (7) and (8), β_{1j} and var-covariance matrix need to be declared at transformed parameters block be-

fore enter into the model block. Thus, a transformed data block for X_1 and X_2, transformed parameters block for β_{1j} and var-covariance matrix have to be introduced into our STAN code. When compiling at STAN, we should put the scaled temperatures and relative humidity into transformed data block; and put the β_{1j} and var-covariance into the transformed parameters block. The partial of the STAN code for those transformed processes would be provided in Appendix A-2.

When compiling STAN code, the most important thing is to diagnostic the MCMC chains converged. Here are some key indicators that guide if an MCMC chain is converged, or say the posterior draws are stationary distributed.

- Density Plot: Figure 3.1 is the parameters' density plot from the MCMC result, which shows normal/bell shape among the parameters. Those shapes indicate the MCMC simulation chains have been reached for a stationary state where the simulation results have little affected by the priors. The stationary states are also implying that the parameters from this MCMC chain have already converted.

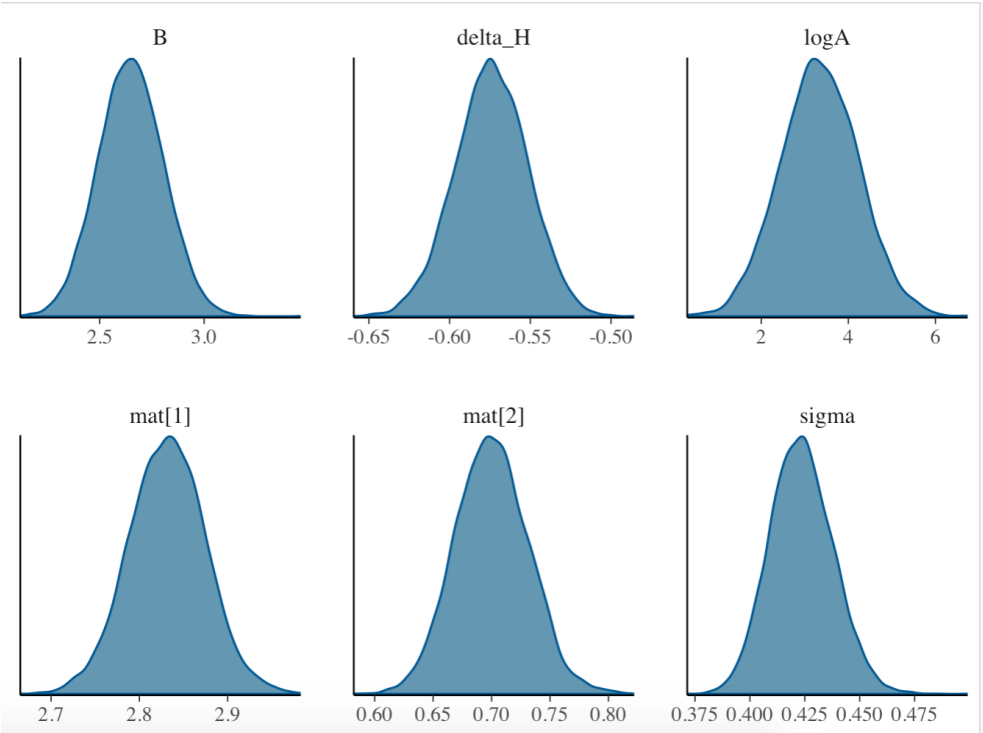

Figure 3.1: Density Plots for Convergence Parameters

- MCMC Trace is a useful diagnostic plot, which is a time series plot of the Markov chains. A trace plot is a line chart with x-axis represents time and y-axis represents the posterior values of the draws. Ideally, a well mixed trace plot that is no apparent anomalies means the sample draw from MCMC chains become stationary. To improve the trace plot, one can increase the number of iterations in each MCMC chain to increase the sample size draw from the MCMC run, and let the chains explore the sample space many times; or one can increase the number of warm-up period, also called burn-in period. With more warm-up, the more likely to get the stationary chains. Here in figure 3.2 is the trace plot for convergence draws, and figure 3.3 is the trace plot for non-converged draws.

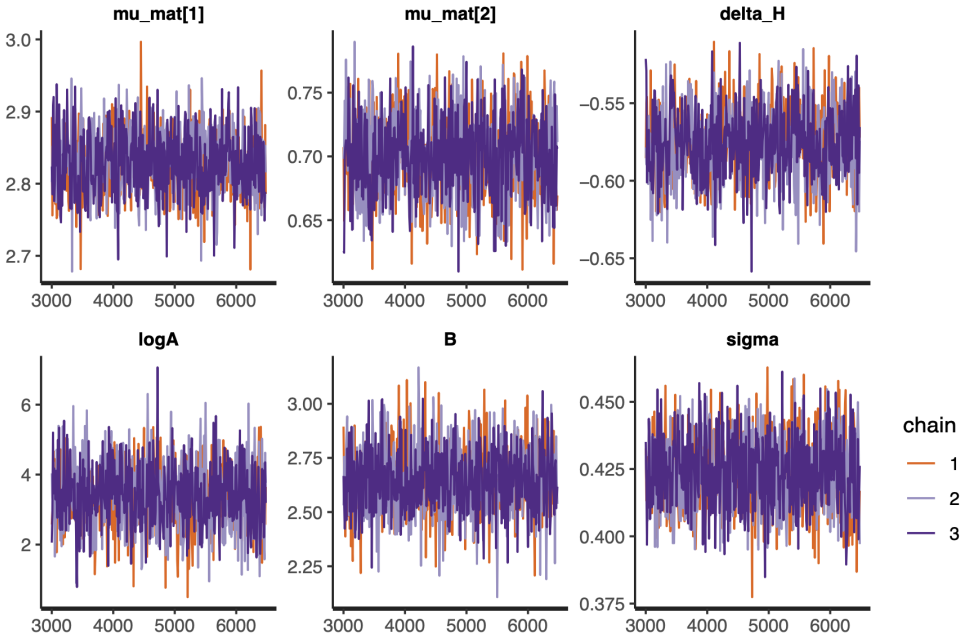

Figure 3.2: Trace Plots for Convergence Parameters

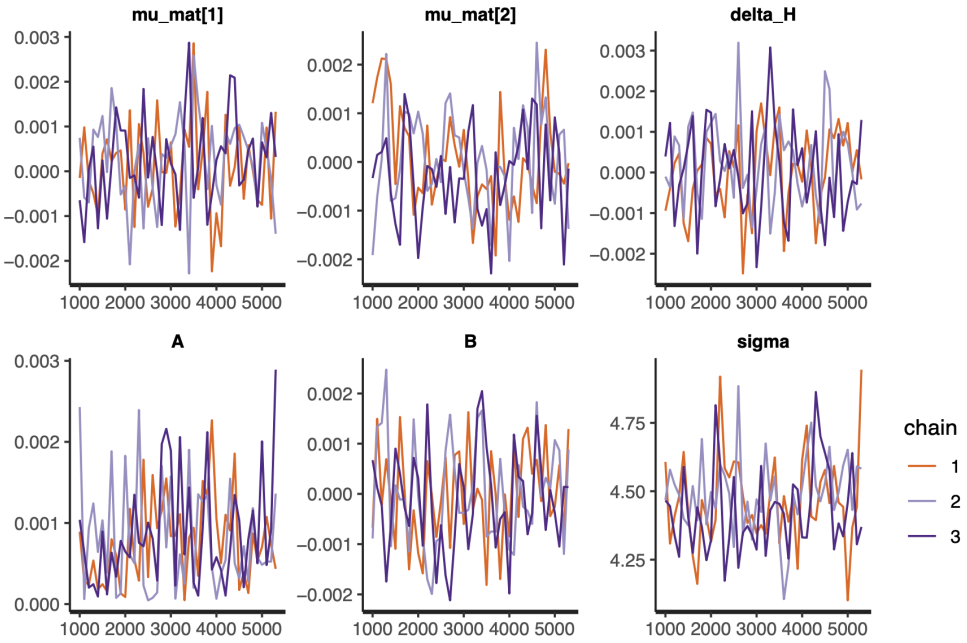

Figure 3.3: Trace Plots for Non-convergence Parameters

- Autocorrelation Function Plots (ACF) is the function that describe the relationship between lags and the autocorrelations. It is a line chart that contains the autocorrelation function, with x-axis represents the number of lags and y-axis represents the autocorrelations corresponding to the lags. By inspecting trace plots, we can get a sense of the degree of auto-correlation from the draws. From these plots, one can determine if the MCMC chain get converged from looking at if the auto-correlation reduced quickly from lag 1. The desired ACF plots should show that large autocorrelation at short lags, but then goes to zero pretty quickly, which means the iterations of each chain are independent samples. To improve the ACF plots, we can thin the MCMC chain, that is we discard n samples for every sample that we keep. Here in Figure 3.4 the desired ACF plot.

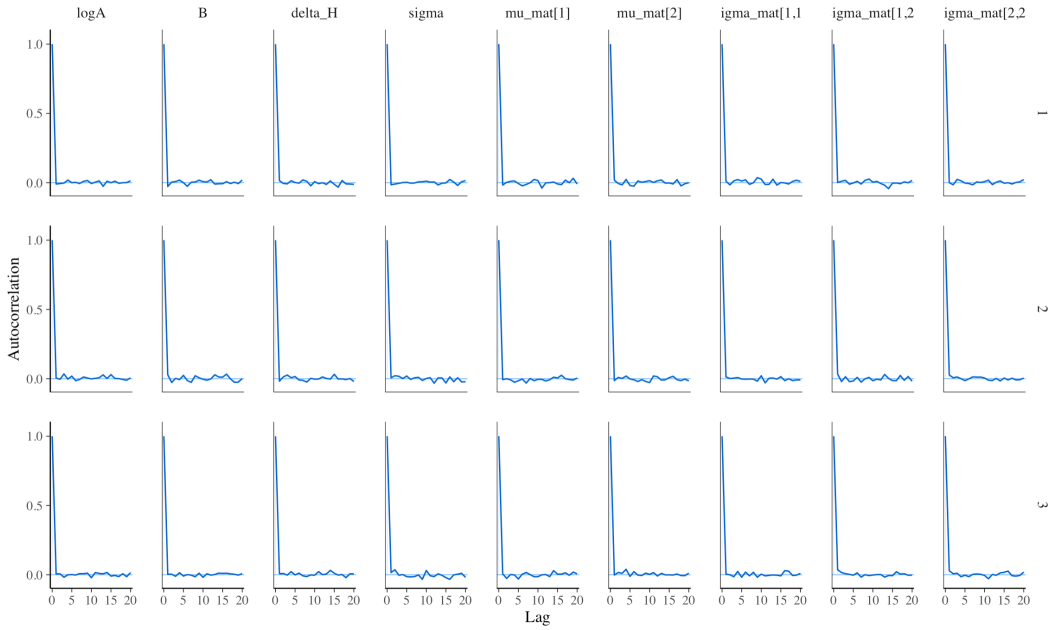

Figure 3.4: ACF Plots for Convergence Parameters

- Rhat from Table 2 usually shows at the MCMC result is another convergence diagnostic indicator that tells if a parameter is converted. It compares the between- and within-chain estimates for model parameters. The not converged chains have R-hat larger than 1, which means not all the MCMC chains mixed very well. It is recommended that a sensitive estimation with R-hat is less than 1.05.

Sometimes we can get an ACF plot shows each chain is independent samples but the trace plot doesn't converge. That happens since ACF shows the auto-correlation for each chain but trace shows the draws changes over time for all chains. Therefore, it is advisable to use density, trace and ACF plots together.

3.2.4 STAN Output

From Stan sampler draws, we could have the posteriors simulation for the model parameters. For each draw, it's a set of parameters combination. The simplest way to show an overview of a Stan result is to print the Stan model fit, as shown below in Table **??**.

Table 3.2: Parameters Estimation From STAN output

Parameters	Mean Estimation	SE Mean	Standard Error	Rhat
$LogA$	3.37	0.032	0.19	1.00
B	2.67	0.01	0.161	1.00
ΔH	-0.57	0.003	0.17	1.00
σ	0.42	0.002	0.0046	1.00
σ_0	0.15	0.000	0.0262	1.00
σ_{01}	-0.01	0.001	0.0017	1.00
σ_1	0.035	0.010	0.0015	1.00
μ_0	2.83	0.001	0.04	1.00
γ_0	0.70	0.004	0.03	1.00

The Stan result in Figure 2 shows the parameters' mean, mean standard error, standard error and Rhat value within each draws. We have to diagnostic for the accuracy of the results by looking at the last two columns neff and Rhat. neff represents the crude measure of an effective sample size from the total post-warmup draws; and Rhat is the potential scale reduction factor on split chains, at convergence, Rhat value is between 0.99 to 1.01.

In our STAN model, we run a total post-warmup draws = 7600 and print the overall results in figure 3.5. The equals 1.00 Rhat value for each parameter is not only a reasonable sign to decide the MCMC chain has already converged. We also have to check the convergence from both plots and Rhat to make sure we have a sensitive enough posteriors estimation.

4 Reliability Analysis

Reliability Analysis has been developed for several reasons, such as predicting product warranty costs, accessing characteristics of materials over a product's design life and so on. Technically, reliability is often defined as the probability that a system, machine, device, etc. will perform its intended function under operating conditions for a specific period of time. In short, we say reliability is quality over time, and over the past ten decades, there has been an increasing interest in improving quality, productivity, and reliability of manufactured products. Our problem in predicting the lifetime of Optical Media, one should know that with extremely high reliability of Optical Media, reliability analysis with only the degradation data from tests maybe not sensitive. Besides, the history of Optical Media is pretty short, less than 30 years, but the lifetime may extend between 30 to 300 years from previous studies. Based on the ISO 10995:2011 and Fang et al., each of them gave a median lifetime prediction for 35.94 years, which is 31,789,170 hours. (From ISO 10995:2011, the standard time unit is hours, which gives the median lifetime in log scaled is 12.66) and 50.5 years (log scaled 12.99). Here we'd like to use our simulated parameter estimation from STAN to fit a reliability model and give our own prediction on the median lifetime of Optical Media. In our prediction section, we provide richer summaries including reliability and quantile lifetime with uncertainty bounds. With our more detailed prediction, manufactures would have a better understanding of their products' reliability performance and a guide for them to make decisions on how well their products performed during a whole service life as well as under a specific percentage of lifetime period.

4.1 Degradation Path Model

As mentioned previously in ISO 10995, the raw data was collected from an acceleration degradation test, and most failures stem from the potential degradation of the product.In general, three shapes for degradation curves in arbitrary units of degradation and time : liner, convex, and concave. Convex degradation is the models with increasing degradation rate and concave degradation is the model with decreasing degradation rate. Linear degradation is applicable in some simple wear process, take the automobile tire wear for example, say $D(t)$ is the amount of wear at time t and wear rate is $dD(t)/dt = C$. then the linear degradation model is

$$D(t) = D(0) + C \times t. \tag{12}$$

where the parameters $D(0)$ is the initial value of degradation and C is the degradation rate, both of them could be taken as constant for individual units, but random from unit-to-unit.

In our data, there is some degree of unit-to-unit variability, these variabilities which have mentioned in section 1.1 cause the difference in both degradation curves and the failure times. By taking all these into consideration, we build a general degradation path model for ISO data. Here originally, the actual degradation path for each unit over time is denoted by $D(t)$,$t > 0$. The values of $D(t)$ are sampled at discrete time points t_1, t_2, ... Then the observed degradation measurements y_{ij} of unit i at time t_j is

$$y_{ij} = D_{ij} + \epsilon_{ij}, i = 1, \ldots, n, j = 1, \ldots, m_i. \tag{13}$$

While we have a multi-level model for our specific problem. Our degradation path model

can be written as

$$\begin{aligned} logy_{ijk} &= D_{ijk} + \epsilon_{ijk} \\ &= \beta_{0i} + \beta_{1j} t_{ijk}^{\gamma_i} + \epsilon_{ijk}, \end{aligned} \tag{14}$$

Where we took log transformation to make sure the degradation path model is linear. To access the reliability, we need to evaluate a specified model for $D(t)$ and D_f, where D_f usually represents the threshold value for the testing products, and we define the first crossing time with the threshold would be considered as failure time with a distribution of failure time. Generally, this specified distribution can be expressed as a function of the degradation model parameters and time t. Therefore, we defined a unit's failure time is at the time t if the degradation level first crossing D_f at time t, then

$$Pr(T \leq t) = F(t) = F(t; \theta_\beta) = Pr[D(t, \beta_1, \ldots, \beta_k) \geq D_f]. \tag{15}$$

With equation (18), for a fixed D_f, the distribution of T depends on the basic path parameters in θ_β. In some simple cases, it may be easy to calculate a closed-form expression for $F(T)$, however in general, such a closed-form expression is hard to get especially in some practical path models because the integration for random effect terms is hard to get, therefore when more than one of the $\beta_1, \ldots, \beta_k$ is random, then we have to using simulation method to evaluate $F(t)$. In the next subsection, we would discuss the Monte Carlo evaluation on no closed-form $F(t)$ and apply this method to our problem.

4.2 Monte Carlo Simulation to Draw Degradation Paths

Monte Carlo simulation is a commonly used versatile method for evaluating $F(t)$ at most practical reliability cases. The basic idea is to generate a large number of degradation paths from the assumed degradation path model, then using the proportion of path which

pass the threshold D_F at each time t as the evaluation of $F(t)$. The algorithm is as follows:

1. Generate N simulated realizations of parameters β, where N is a large number. In our case, we generate the parameters from STAN;

2. Compute the N simulated failure time corresponding to the N realizations of parameters, finding the number of crossing time for each path at all time points;

3. For any desired values of t, use

$$F(t) \approx \frac{\text{Number of Simulated First Crossing Times} \leq t}{N} \tag{16}$$

as an evaluation of $F(t)$.

Using this method, we can generate $i = N$ sample draws with $j = 200$ paths at each time point t_k. The model function of $F(t)$ can be written into R code, see details in Appendix A-3. Then from the model of $F(t)$, we have the degradation path figures, for example:

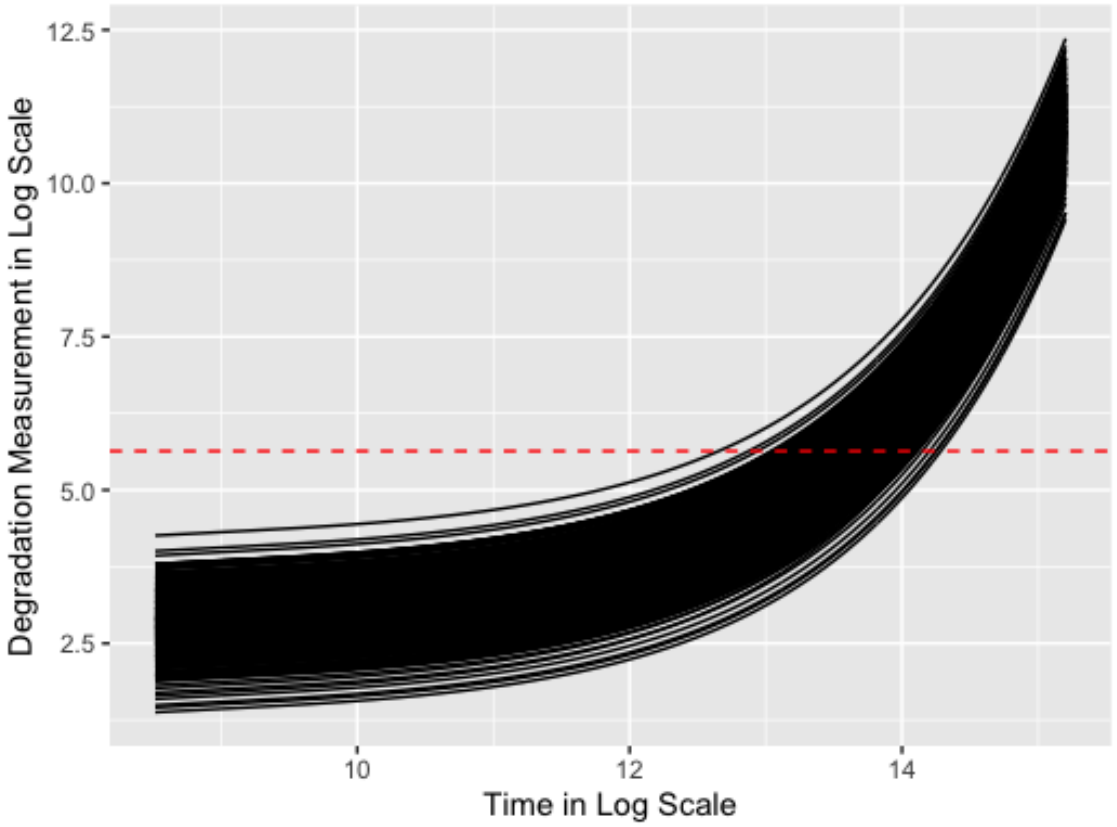

Figure 4.1: 500 Simulated Degradation Paths Cross the Threshold

From above Figure 4.1, we can see in this specific sample draw, it is nearly no paths cross the threshold $log(280)$ in dashed line before time point 12.50, and all the paths have reached the threshold value near time point 14.30. Here we have to note that the time point does not mean the actual time in hours that the product performed during the time, it is the log scaled time points. Then when the products start to fail at time point 12.50 is actually in real time 268,337 hours; and all products failed at time point 14.30 is actually 1,623,346 hours, in terms of years, the lifetime approximately in a range 0f [30.63, 185.31] years. However, our goal is to estimate the product's lifetime with uncertainty bounds, especially in predicting quantile lifetime and 50% as median lifetime with these simulated results, then we also have to summarize over each path, and summarizing over each MCMC draws to get the median lifetime with uncertainty boundaries. Detail explanation would be provided in the following subsection. Figure 4.2 and 4.3 is the reliability curve with 95% credible interval in both real

time hours and log scaled time.

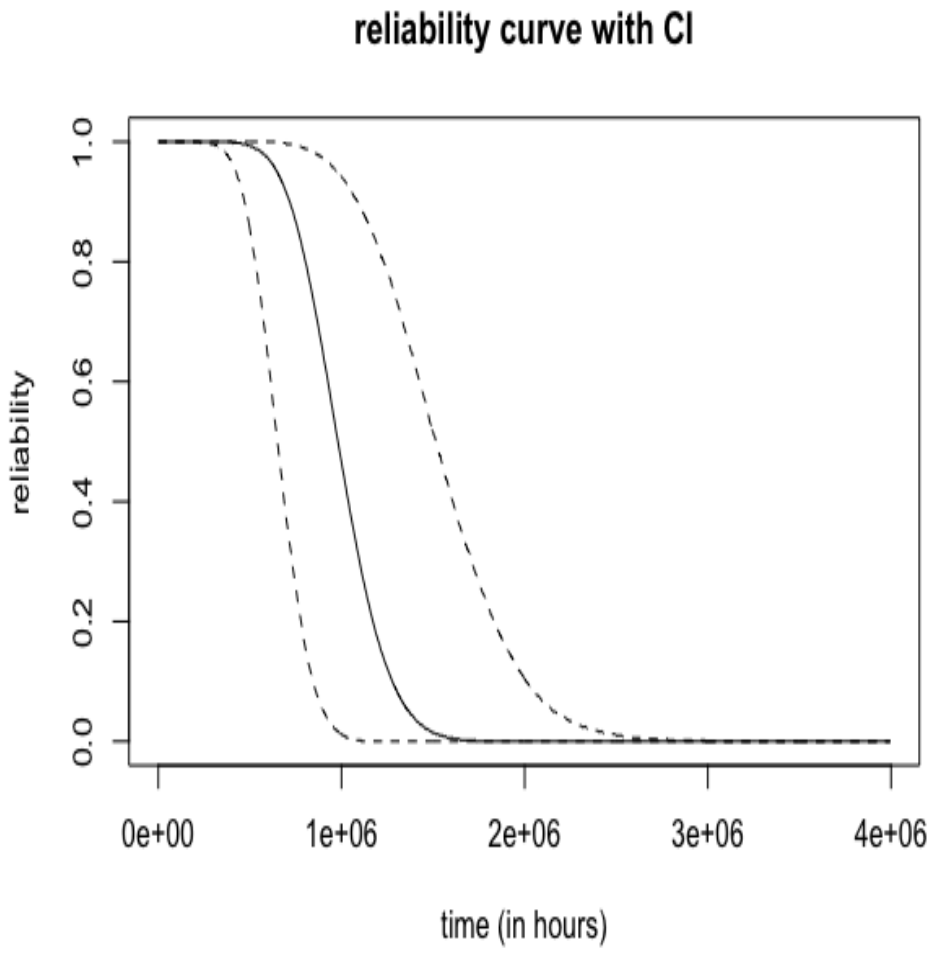

Figure 4.2: Reliability Curve with 95% Credible Interval in Real Time Hours

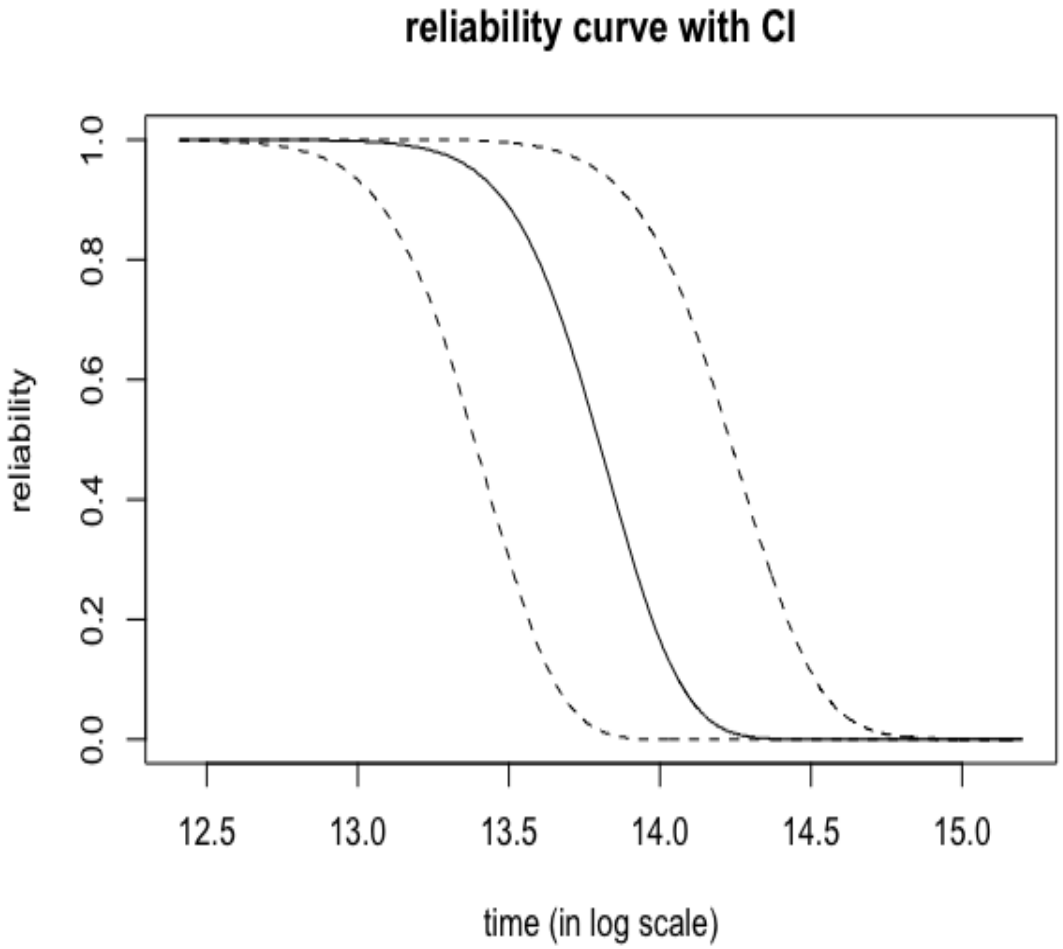

Figure 4.3: Reliability Curve with 95% Credible Interval in Log Scaled Time

As we know from the relationship between the reliability and failure rate function, R(t) = 1 - F(t). The term reliability is a function that gives the probability that an object of interest will survive beyond any specified time. Thus, we can plot our reliability curve as the above figure. The detail procedures would be followed below:

1. Take the computation results from the $F(t)$, which is the number of simulated degradation paths calculated by the MCMC draws. Specifically with $j = 500$ paths for each $i = 2000$ draws.

2. Summarizing over 500 paths within each draws over time, then take the proportion

of the number of degradation measurements greater than $log(280)$ at each time point, this proportion as the failure rate at each time point. Then we can plot one failure rate curve for each draws, in total, we are going to have 2000 different failure rate curves.

3. Within the failure proportion from step 2, summarizing over each MCMC draws at each time points to get the median and 95% credible interval of $F(t)$.

4. Based on the relationship between failure time and reliability function $R(t)$=1-$F(t)$, we have the projected reliability curve with 95% credible interval.

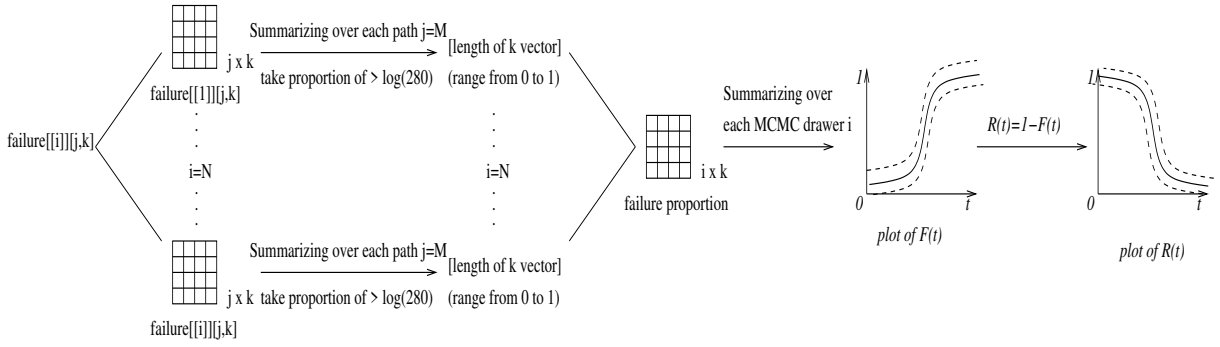

Figure 4.4: Graphical Explanation of Simulating Reliability Curve

4.3 Quantile Lifetime Prediction with Credible Interval

Due to the nature of hierarchical model, products' median lifetime is a challenge to be estimated, at the same time it is very useful to engineers to be able to get some ideas of quantile lifetime because our goals are not only aim to compare the median lifetime with ISO prediction which was estimated without considering uncertainties, but also would like to draw the percentage fails during a whole lifetime prediction. From the standard, we know the failure time is defined as the first crossing time from the threshold time, which is so-called PIE Sum 8 reaches 280 for the product type DVD-R/-RW, +R/+RW. In our model since log transformation has been taken, we need to capture the PIE Sum 8 also in log scale, which makes the threshold becomes $log(280)$. The algorithm to estimate the median lifetime and

95% credible interval from the simulation method mentioned in section 3.3.2 would be used here and the specific steps would follow:

Step 1. From the simulation result $F(t)$ in the last section, we know that $failure$ represents the time-to-failure, with each $failure[1, 2, \ldots, i]$ is the number of simulation MCMC drawers where $i = 2000$, and inside each MCMC draw, is a number of j simulated degradation paths over the total time points k, where $j = 500$, and $k = 800$; based on those information, we can calculate the quantile lifetime t_q, where q represents the percentile, range from 0 to 1; and t represents the predicted lifetime.

Step 2. To compute the median lifetime, we have to summarize over degradation paths first, then summarizing over MCMC draw. Therefore, we work within the MCMC draws first to summarizing over each path, then take the first crossing time greater than $log(280)$ for each path, this first crossing time as the failure lifetime of each degradation path. Then with each MCMC drawers, there would be 500 different failure lifetimes. We take quantile for all 500 lifetimes within each draw. Then each MCMC draw has 500 t_q as t_{q1}, t_{q2}, ..., t_{q500}. Each t_q is a vector of length number of q, in our study, we choose 15 different percentiles as $q = 0.01$,$q = 0.05$, ..., $q = 0.99$. Results in Table **??**.

Step 3. For a total number of 2000 MCMC draws, we have 2000 matrixs of quantile lifetimes prediction. We can take the 95% credible interval among those quantile lifetimes to get the prediction with uncertainty bounds. In particular, the median lifetime with 95% credible interval can also be predicted by setting q=0.5.

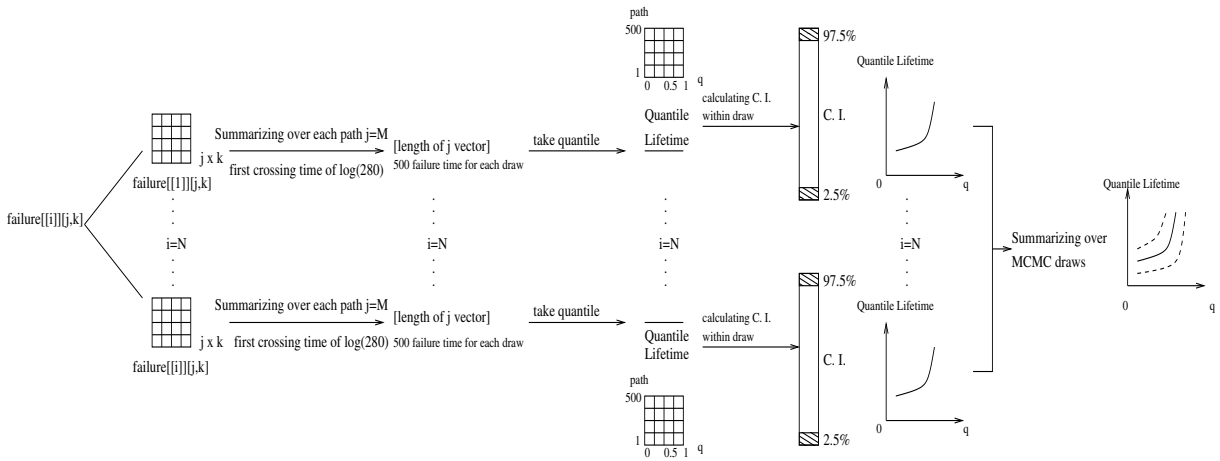

Figure 4.5: Graphical Explanation of Simulating Quantile Lifetime with CI

The above simulation algorithm gives a predicting 95% credible interval of median lifetime under $25°C$ and 50% RH ambient condition is [13.36,14.27] in log scaled years. As compared with the ISO 10995:2011 standard prediction C.I. of median lifetime, [12.63,12.69], our prediction is much wider than the standard's because ISO 10995:2011 fails to take uncertainty into account and it fails assumed the observed failure time is the actual failure time. Besides, our method to generate median lifetime also using Monte Carlo simulation to generate a large number of simulation data, which provides robustness in predicting.

Our study in reliability analysis is not only interested in predicting products' median lifetime, but also in predicting the quantile lifetime, with which the manufacture can design the warranty years for their products, researchers can use the quantile lifetime as a reference when evaluating the products' performance over different time period. As we know, the median lifetime is at 0.5 quantile lifetime, our goal in quantile lifetime is to predict t_q then percentage $q = 0.01,\ 0.1,\ \ldots,\ 0.99$. With those different quantile predictions, the user would have an approximate idea of after some specific years, how many percentages does the product fails. Figure 4.6 is the estimated quantile lifetime prediction.

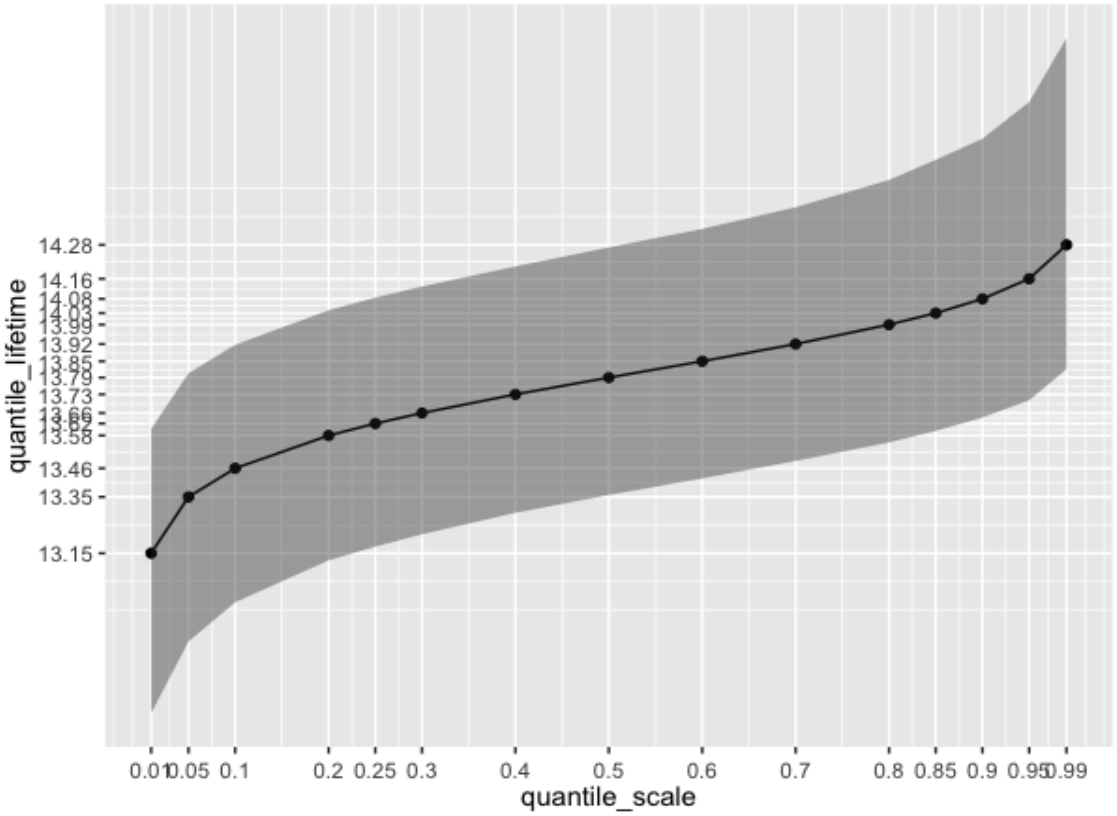

Figure 4.6: Quantile Lifetime Prediction with 95% Credible Interval

The corresponding quantile lifetime prediction in real years is shown in below table. The converting process takes exponential first to convert into real time in hours, then divided by 24 (hours per day) and 365 (days per year) to get the real time in years. Comparing with previous researches, the estimated lifetime of Optical Media under recommended storage conditions, CD-R, DVD-R, and DVD+R discs should have a life expectancy of 100 to 200 years or more; CD-RW, DVD-RW, DVD+RW, and DVD-RAM discs should have a life expectancy of 25 years or more. While it may vary from the different writing and reading process when using the discs.

Table 4.1: Quantile Lifetime Prediction with 95% Credible Interval

Quantile	log	years	95% lower bounds(in log)	95% upper bounds(in log)
1%	13.15	58.67	12.82	13.62
5%	13.35	71.66	13.03	13.82
10%	13.46	80.00	13.12	13.92
20%	13.58	90.20	13.24	14.04
25%	13.62	93.88	13.28	14.09
30%	13.66	97.72	13.31	14.12
40%	13.73	104.80	13.33	14.18
50%	13.79	111.28	13.36	14.27
60%	13.85	118.16	13.47	14.29
70%	13.92	126.73	13.53	14.36
80%	13.99	135.92	13.59	14.44
85%	14.03	141.46	13.62	14.48
90%	14.08	148.71	13.66	14.53
95%	14.16	161.04	13.71	14.58
99%	14.28	181.64	13.82	14.73

5 Sensitivity Analysis

Sensitivity analysis is useful for evaluating decision outcomes when situations are different from the key assumptions. It helps to assess the risks of the strategy and to determine how sensitive the output would be to the changes in assumptions or specified input values. In Bayesian data analysis, the inference results are often dependent on the assumptions of the prior distributions. In this section, we examine some alternative choices of the prior distributions to understand the robustness of our predictions.

So far, we have built the model and predicted the reliability and quantile lifetime of the device. When estimating model parameters using Bayesian approach, we chose normal prior distributions for some model parameters. Given that normal distributions are notoriously nonrobust to outliers, a more robust model can be used to assess the sensitivity of posterior inference to the prior choices. The basic idea of a sensitivity analysis is to try a variety of different distributions for priors and see how posterior inferences vary for estimands and predictive quantities of interest. In the section, we are going to draw the posterior from an alternative precision level in the normal distribution that we have applied before.

5.1 Priors Selection

Sensitivity analysis of prior distributions plays an important role in the application of Bayesian analysis. This is especially true for Bayesian hierarchical models, because at deeper levels of the hierarchy, the interpretability of parameters becomes increasingly challenging.

To understand how does the impact of the prior choices inference estimation results, one can explore both non-informative and informative prior choices and to compare the affection on the parameter estimation. Then in our case, different estimation draws from MCMC would make changes in the results of predicted reliability and quantile lifetime. We begin with a brief introduction of the appropriate prior selection in Bayesian analysis.

Generally speaking, there are five levels of priors: 1) Flat prior; 2) Super-vague but proper prior; 3) Weakly informative prior; 4) Generic weakly informative prior; and 5) Specific informative prior. Always note that the terms of "informative" or "weakly informative" depend crucially on what questions are being asked. One should avoid choosing priors after seeing the data since the selected priors could be affected by what we see in the data and do not truly reflect our prior knowledge on possible values of the parameters. Flat and super-vague priors are usually used when there is no prior knowledge about the underlying process and what are possible ranges of the parameter values.

Some principles could be helpful when choosing proper priors in Stan. First is the computational goal for reducing instability typically arising from bad geometry in the posterior distribution. For example, having heavy tails can lead to a generation of a smaller number of effective samples and have heavy tails; for the multivariate normal distribution, generally generate the covariance from Wishart or Inverse Wishart distribution, however in Stan, the computational issue cause it's difficult to realize, since inverting the covariance is numerically unstable and inefficient in Stan, for the computationally advantageous, it is better to use the Cholesky decomposition, which is more numerically stable and efficient than direct matrix inversion and also there has already a consensus that to decompose a covariance matrix into a correlation matrix when giving prior for a covariance matrix.

5.2 Comparison of Parameter Estimation

Given the lack of prior understanding of the parameters, our comparison will focus on be-

tween the non-informative vs. the weakly informative prior distributions, i.e. $normal(0, 1000)$ vs. the weakly informative prior, $normal(0, 100)$ for parameter, Table 5.1 shows the assigned prior distribution of each parameters, and for covariance parameters where named L_Sigma has to be decomposed due to the computational principle in STAN, then we assign priors distribution for the correlation matrix L_Omega and the scalar vector L_std to make sure the covariance matrix can be realized from the multi normal distribution.The R code for implementing this comparison can be found in Appendix A-4.

Table 5.1: Priors Selected in Stan

Parameters in Stan	Selected Priors in Stan
μ_0	$\sim normal(0, sqrt(100))$
$logA$	$\sim normal(0, sqrt(100))$
B	$\sim normal(0, sqrt(100))$
ΔH	$\sim normal(0, sqrt(100))$
γ_0	$\sim normal(0, sqrt(100))$
σ	$\sim gamma(1e-2, 1e-2)$
σ_0	L_Sigma[1,1]
σ_1	L_Sigma[2,2]
σ_{01}	L_Sigma[1,2]
L_Sigma	quad_form_diag(L_Omega,L_std)
L_Omega	$\sim lkj\_corr(2)$
L_std	$\sim cauchy(0, 2.5)$

Table 5.2 summarizes the estimated model parameters with the different choices of prior distributions. As shown, the parameter estimation is similar between the two priors results. The estimation based on the prior $normal(0, 100)$ contains more information than the prior $normal(0, 1000)$, it can cause some of the parameter estimation slightly smaller or more

central than the estimation based on non-information prior, which allows the parameters to be estimated from the original data. Besides, we almost assign the non-informative prior distributions to all the individual parameters, then the model would fit the data very closely. Since as we keep updating the priors with more information by decreasing the variance from prior distribution, but still keep the priors vogue, the information contains in prior distributions would start but not at all to dominate the information from data, then the estimation would fall into a more sensitive range, thus the value of estimated parameters would be more precise.

Table 5.2: Parameters Estimation Comparison using Different Priors

parameters	$normal(0, 1000)$	$normal(0, 100)$
μ_0	2.831686	2.8283952
$logA$	3.3688449	3.353566
B	2.673266	2.656245
ΔH	-0.5702964	-0.5753626
γ_0	0.7012557	0.6980425
σ	0.4238182	0.4229889
σ_0	0.1522418	0.1495703
σ_1	0.0359529	0.0349437
σ_{01}	-0.00902528	-0.0068488

5.3 Comparison of Predicted Reliability and Quantile Lifetime

Table 5.3 summarized the predicted quantile lifetime under the two prior choices. According to the $normal(0, 1000)$ prediction, the predicted lifetime is relatively central than the prediction based on $normal(0, 100)$.

Table 5.3: Comparison of the Predicted Quantile Lifetime using Different Priors

Quantile	$normal(0, 100)$ in log scale	$normal(0, 100)$ in archaeological years	$normal(0, 1000)$ in log scale	$normal(0, 1000)$ in archaeological years
1%	13.21	62.31	13.15	58.67
5%	13.41	76.10	13.35	71.66
10%	13.51	84.10	13.46	80.00
20%	13.62	93.88	13.58	90.20
25%	13.66	97.71	13.62	93.88
30%	13.70	101.7	13.66	97.72
40%	13.76	107.99	13.73	104.80
50%	13.82	114.67	13.79	111.28
60%	13.87	120.55	13.85	118.16
70%	13.93	128.00	13.92	126.73
80%	13.99	135.92	13.99	135.92
85%	14.02	140.06	14.03	141.46
90%	14.07	147.24	14.08	148.71
95%	14.13	156.34	14.16	161.04
99%	14.24	175.52	14.28	181.64

From the above Table 5.3 in comparison of the predicted quantile lifetime under the two prior choices of $normal(0, 1000)$ and $normal(0, 100)$. Results are shown in archaeological years and also in the log scale as in ISO 10995:2011.) we could see t_{q1000} which is quantile lifetime prediction under priors $normal(0, 1000)$ gives a prediction in median lifetime of 13.82 in log scale which is 114.67 years in real time; and t_{q100} quantile lifetime under priors $normal(0, 100)$ gives a prediction in median lifetime of 13.79 in log scale which is 111.28 years, where $q = (0.01, 0.05, \ldots, 0.99)$. The 95% median lifetime credible interval is [13.43,14.24] and [13.36, 14.27] respectively. With the smaller the standard deviation is in the prior distribution, the wider the credible interval lays. The reason we use the "traditional" standard deviation (square root of variance) is because R or STAN has defined using σ inside the normal prior distribution, while RJAGS using precision. Therefore, in terms of precision, $normal(0, 1000)$ and $normal(0, 100)$ are having precision in 0.001 and 0.01 respectively, which represents with larger precision value, the estimation can be narrow down to a small interval. In general, if we had more informative prior, the result would be more precise.

5.4 Overall Prediction Results

In summary from the above analysis, we give the overall prediction result as shown in below Figure 5.1. Compared with the two results from the sensitivity analysis, the quantile lifetime t_{q1000} and t_{q100} were calculated using the parameters that estimated from Bayesian inference given the weakly-informative prior $normal(0, 100)$, and non-informative prior $normal(0, 1000)$, respectively. However, the difference is $normal(0, 100)$ provides more information than $normal(0, 1000)$ because it has the smaller value of variance. As the 95% credible intervals shown from the previous section, the quantile lifetime t_{q100} which contains more information when estimating model parameters has a relatively narrow interval, indicates the predicting result is more precise than the one from t_{q1000} prediction. It appears in the Figure 5.1 that the dashed lines represent 95% credible interval of quantile lifetime from t_{q100} is narrow and it's inside the interval of dot lines which are the 95% credible interval of median lifetime from t_{q1000}. Therefore, we give our prediction based on the result from t_{q100} since it provides a more accurate and precise prediction. The 95% credible interval of median lifetime would be [13.36, 14.27] in log scaled time. The median lifetime prediction is 13.79 from Table 5 which in real time years is 111.28 years. Refers to the Byers[21]'s previous study, the estimated optical media life expectancy can range from 20 to 200 years, our estimation also falls within the same sensitive range prediction. Here we take life expectancy, longevity, or service life of optical media is all considered as "the period of time in which the information recorded on the disc can be retrieved without loss."[13]

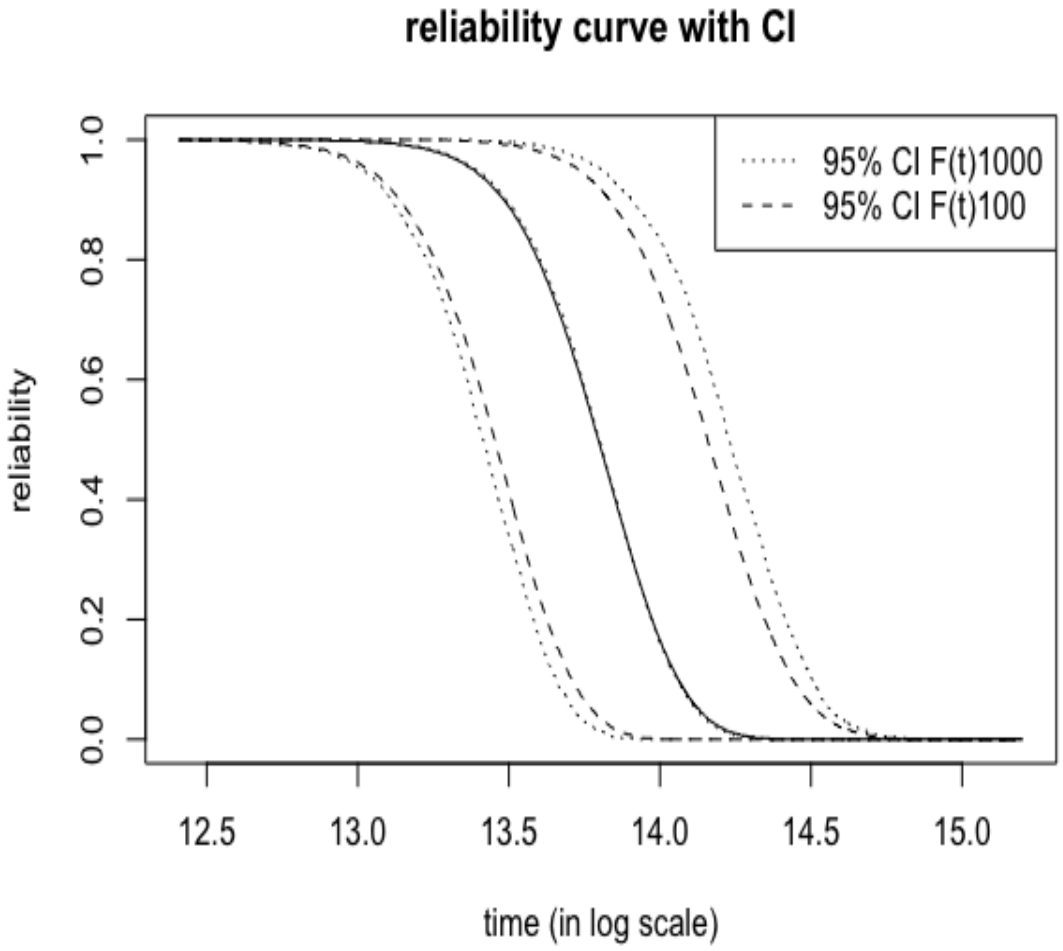

Figure 5.1: Reliability Curve with 95% Credible Interval in Log Scaled Time

6 Concluding Remarks

Bayesian data analysis provides a useful framework for dealing with uncertainty. Particularly in reliability analysis, often a large number of failure data is hard to obtain due to the cost and logistic constraints for conducting the tests and collecting the data. When limited data are available, prior information can be leveraged to provide more precise estimation of the degradation and lifetime distribution. In Bayesian statistics, the latest development of the MCMC approaches makes it much easier to obtain an approximate posterior distribution for complex hierarchical models. In reliability analysis, Monte Carlo simulation is also a particularly versatile method to deal with the integration of a large number of model parameters when there is not a closed-form expression of the reliability functions. By generating a large number of random degradation paths from the general assumed degradation path model, then using the proportion of path crossing the threshold by time t as an evaluation of $F(t)$.

This thesis uses a Bayesian method to analyze the ISO 10995:2011 data. By using a nonlinear hierarchical degradation model with random effects included to capture unit-to-unit variation, a more straightforward and realistic assessment of uncertainty is achieved to avoid overconfidence in the predicted reliability in ISO 10995:2011. The method used this thesis provides a general framework for reliability analysis based on ADT data. The MCMC approach can be applied to a variety of linear and nonlinear degradation path models. The Monte Carlo simulation-based method for assessing reliability and quantile lifetime can be adapted to broad degradation models with analytically intractable reliability functions.

Especially for MCMC improved a lot on computing the complicated integration that the frequentist approach is hard to realize.

In addition, ISO 10995:2011 and other existing work only estimates median lifetime of the optical media. This thesis provides richer inference on product reliability and quantile lifetime metrics, which offers versatile summaries to facilitate informative decision-making for system management and maintenance.

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Appendix A Three basic blocks in RStan (data, parameters, and model)

```
data {
  int<lower=0> N;
  vector[N] x;
  vector[N] y;
}
parameters {
  real alpha; //intercept parameter
  real beta;  //slope parameter
  real <lower=0> sigma; // declare parameter
                        // and add constrain for sigma
}
model {
  y ~ normal(alpha + beta * x, sigma);
}
```

Appendix B Example of transformed data and transformed parameters

```
transformed data {
  real x1[(N-1)*5+K]; // x1 and x2 are scaled data,
  real x2[(N-1)*5+K]; // see at appendix for initial value.
  for(k in 1:K){
    for(n in 1:N){
      x1[(n-1)*5+k] = log(RH[(n-1)*5+k]);
      x2[(n-1)*5+k] = 11605/(T[(n-1)*5+k]+273.15);
    }
  }
}

transformed parameters {
   // Random effects
  row_vector[(N-1)*5+K] beta0;
  row_vector[(N-1)*5+K] gamma0;
  row_vector[(N-1)*5+K] beta1;
  row_vector[(N-1)*5+K] mu;
  matrix[2,2] L_Sigma;
    // Cholesky factor for covariance matrix
  L_Sigma =  quad_form_diag(L_Omega,L_std);

  for(n in 1:N){
    for(k in 1:K){
      beta0[(n-1)*5+k] = mu_mat[1];
```

```
      gamma0[(n-1)*5+k] = mu_mat[2];
      beta1[(n-1)*5+k] = exp(logA + B*x1[(n-1)*5+k]
                          + delta_H*x2[(n-1)*5+k]);
      mu[(n-1)*5+k] = beta0[n] + beta1[(n-1)*5+k] *
                        pow(t_ijk[(n-1)*5+k],gamma0[n]);
    }
  }
}
model {
   for (n in 1:N){
    for (k in 1:K){
      y_ijk[(n-1)*5+k] ~ lognormal(mu[(n-1)*5+k], sigma);
    }
  }
}
```

Appendix C The R code for model function of $F(t)$

```
re_func <- function(t, par){
  failure <- rep(list(matrix(NA,200,100)),10000)
  for(i in 1:10000){ ## i represents number of sample draws
    for(j in 1:200) ## j represents number of degradation paths
    {
      beta0_gamma <- mvrnorm(1,c(par[i,1],par[i,5]),
       matrix(c(par[i,7],par[i,9],par[i,9],par[i,8]),2,2))
      beta1 <- exp(par[i,2]+par[i,3]*log(50)
       +par[i,4]*(11605/(25+273.15)))
      for(k in 1:100){  ## k represents number of time points
        failure[[i]][j,k] <- beta0_gamma[1]
        +beta1*(t[k]^(beta0_gamma[2]))
      }
    }
  }
  return(failure)
}
failure <-re_func(t=t,par=Stan_sampler_draws)
```

Appendix D R code for sensitive analysis using different priors

$normal(0, sqrt(1000))$ as the priors

```
parameters {
  // Cholesky factor for the correlation matrix
  corr_matrix[2] L_Omega;
  // Sqrt of variances for each variate
  vector<lower=0>[2] L_std;
  real<lower=0> sigma;
  // Hyperparameters
  vector[2] mat;
  vector[2] mu_mat;
  real logA;
  real B;
  real delta_H;
}

//Parameters processing before the postier is computed
transformed parameters{
  // Random effect
  row_vector[N] beta0;
  row_vector[N] gamma0;
  row_vector[(N-1)*5+K] beta1;
  row_vector[(N-1)*5+K] mu;
  matrix[2,2] L_Sigma;
    // Cholesky factor for covariance matrix
  L_Sigma =  quad_form_diag(L_Omega,L_std);
```

```
  for(n in 1:N){

    for(k in 1:K){
      beta0[n] = mat[1];
      gamma0[n] = mat[2];
      beta1[(n-1)*5+k] = exp(logA + B*x1[(n-1)*5+k] + delta_H*x2[(n
          -1)*5+k]);
      mu[(n-1)*5+k] = beta0[n] + beta1[(n-1)*5+k] * pow(t_ijk[(n-1)
          *5+k],gamma0[n]);
      // print("mu:", mu);
    }
  }
}

model {
  sigma ~ gamma(1e-4,1e-4);
  mu_mat[1] ~ normal(0, sqrt(1000));
  mu_mat[2] ~ normal(0, sqrt(1000));
  logA ~ normal(0, sqrt(1000));
  B ~ normal(0, sqrt(1000));
  delta_H ~ normal(0, sqrt(1000));
  // Prior on LKJ Cholesky decomposition of correlation matrix
  L_Omega ~ lkj_corr(2);
  // Prior on standard deviations for each variate
  L_std ~ cauchy(0, 2.5);
  // Set mu_mat follow MVN with 0 mean and L_Sigma as covariance
  mu_mat ~ multi_normal_cholesky_lpdf(rep_vector(0,2), L_Sigma);
  // L_Sigma is calculated at transformed parameter block
```

```
  for (n in 1:N){
    for (k in 1:K){
      y_ijk[(n-1)*5+k] ~ lognormal(mu[(n-1)*5+k], sigma);
    }
  }
}
```

$normal(0, sqrt(100))$ as the priors

```
parameters {
  // Cholesky factor for the correlation matrix
  corr_matrix[2] L_Omega;
  // Sqrt of variances for each variate
  vector<lower=0>[2] L_std;
  real<lower=0> sigma;
  // Hyperparameters
  vector[2] mat;
  vector[2] mu_mat;
  real logA;
  real B;
  real delta_H;
}

//Parameters processing before the postier is computed
transformed parameters{
  // Random effect
  row_vector[N] beta0;
  row_vector[N] gamma0;
  row_vector[(N-1)*5+K] beta1;
  row_vector[(N-1)*5+K] mu;
  matrix[2,2] L_Sigma;
    // Cholesky factor for covariance matrix
  L_Sigma =  quad_form_diag(L_Omega,L_std);



  for(n in 1:N){
```

```
    for(k in 1:K){
      beta0[n] = mat[1];
      gamma0[n] = mat[2];
      beta1[(n-1)*5+k] = exp(logA + B*x1[(n-1)*5+k] + delta_H*x2[(n
         -1)*5+k]);
      mu[(n-1)*5+k] = beta0[n] + beta1[(n-1)*5+k] * pow(t_ijk[(n-1)
         *5+k],gamma0[n]);
      // print("mu:", mu);
    }
  }
}

model {
  sigma ~ gamma(1e-3,1e-3);
  mu_mat[1] ~ normal(0, sqrt(100));
  mu_mat[2] ~ normal(0, sqrt(100));
  logA ~ normal(0, sqrt(100));
  B ~ normal(0, sqrt(100));
  delta_H ~ normal(0, sqrt(100));
   // Prior on LKJ Cholesky decomposition of correlation matrix
  L_Omega ~ lkj_corr(1);
  // Prior on standard deviations for each variate
  L_std ~ cauchy(0, 2.5);
  // Set mu_mat follow MVN with 0 mean and L_Sigma as covariance
  mu_mat ~ multi_normal_cholesky_lpdf(rep_vector(0,2), L_Sigma);

  for (n in 1:N){
    for (k in 1:K){
```

```
      y_ijk[(n-1)*5+k] ~ lognormal(mu[(n-1)*5+k], sigma);
    }
  }
}
```

Appendix E Original Data Table from ISO 10995:2011[1]

Table E.1: Original Degradation Data Test Condition A

TEMP=85°C, RH=85%

	Hours					Project
Disk	0	250	500	750	1000	Failure
A1	16	78	116	278	445	788
A2	25	64	134	342	532	743
A3	26	94	190	335	642	685
A4	26	111	247	343	718	647
A5	27	89	185	246	466	762
A6	21	111	207	567	896	607
A7	26	121	274	589	781	588
A8	31	108	223	315	745	654
A9	24	118	285	723	754	654
A10	12	85	178	312	988	669
A11	28	111	167	312	771	671
A12	24	136	267	444	719	614
A13	35	76	265	567	610	626
A14	19	53	112	278	534	778
A15	28	88	158	308	654	704
A16	27	68	120	263	432	807
A17	18	87	176	302	558	723
A18	26	109	238	421	641	645
A19	26	111	253	378	638	649
A20	31	91	206	367	728	656

Table E.2: Original Degradation Data Test Condition B

TEMP=85°C, RH=70%

	Hours					Project
Disk	0	250	500	750	1000	Failure
B1	10	20	67	112	156	1117
B2	8	20	47	84	188	1118
B3	12	26	72	185	421	880
B4	20	43	120	166	219	999
B5	32	45	76	103	267	1126
B6	21	37	104	222	368	870
B7	21	30	89	155	221	1035
B8	22	26	72	125	267	1043
B9	25	46	124	182	224	994
B10	17	38	67	179	378	911
B11	28	58	88	120	268	1065
B12	8	15	36	144	189	1059
B13	10	27	89	175	385	880
B14	23	54	111	148	221	1037
B15	28	39	125	172	278	959
B16	25	53	88	130	188	1149
B17	20	43	75	166	256	999
B18	22	26	50	172	229	1058
B19	13	38	78	124	189	1078
B20	10	19	28	121	268	1046

Table E.3: Original Degradation Data Test Condition C

TEMP=65°C, RH=85%

	Hours					Project
Disk	0	500	1000	1500	2000	Failure
C1	14	23	58	112	278	2057
C2	10	17	55	165	263	1948
C3	11	56	88	138	189	2078
C4	18	28	78	117	243	2106
C5	17	45	78	143	189	2167
C6	10	14	45	154	231	2031
C7	31	53	111	156	211	2151
C8	29	54	106	154	218	2128
C9	22	32	65	89	126	2799
C10	29	36	78	145	188	2297
C11	21	38	89	148	227	2075
C12	24	45	68	134	211	2236
C13	28	57	78	132	190	2352
C14	19	47	61	117	150	2486
C15	25	65	89	184	256	1972
C16	10	18	57	113	178	2189
C17	21	34	45	98	121	2845
C18	12	20	34	122	176	2308
C19	28	56	108	176	243	2001
C20	29	36	57	143	238	2207

Table E.4: Original Degradation Data Test Condition D

TEMP=70°C, RH=75%

	Hours					Project
Disk	0	625	1250	1875	2500	Failure
D1	25	34	64	92	167	3240
D2	25	93	134	154	211	2596
D3	7	23	97	103	178	2615
D4	10	20	56	89	155	2920
D5	5	20	78	132	187	2496
D6	5	15	52	112	167	2644
D7	22	34	67	132	188	2851
D8	12	17	56	78	108	3318
D9	22	34	67	132	189	2847
D10	23	27	54	121	152	3129
D11	11	20	41	87	115	3249
D12	15	18	43	88	118	3343
D13	19	21	38	82	135	3435
D14	18	22	86	178	245	2456
D15	22	26	73	145	252	2582
D16	18	18	29	66	127	3649
D17	22	26	93	145	178	2761
D18	18	27	56	88	134	3316
D19	11	32	44	97	143	3051
D20	12	56	66	124	249	2550
D21	14	34	54	77	112	3500
D22	20	23	25	50	181	3593
D23	11	16	27	54	160	3275
D24	17	24	25	58	108	4034
D25	11	25	22	62	130	3488
D26	17	24	25	70	123	3707
D27	21	39	63	78	163	3304
D28	20	28	45	111	243	2787
D29	15	21	38	65	134	3453
D30	10	34	54	96	176	2841