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Rethinking Map Literacy and an Analysis of Quantitative Map Literacy

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Rethinking Map Literacy and an Analysis of Quantitative Map Literacy

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy
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Maps are increasingly being used in traditional and virtual media, and civic discourse on political, social, and environmental issues, among others, is more and more becoming influenced by them. The often-used expression of a “picture tells a 1000 words” has never been so apt in our progressively more visual world. Despite this increased role and importance of maps, map literacy, as a field of research, remains rather underdeveloped. This is especially so for thematic maps, the very type of map that is finding increasing currency in discourse. As part of this underdeveloped nature of map literacy, the quantitative skills used in map reading and interpretation have not been systematically investigated, and previous commentary on the subject has been limited to listings of relatively low-level skills. As modern technologies, such as GIS, enable the more sophisticated production of maps, their interpretation can come to depend on more advanced quantitative literacy. The quantitative literacy required for map interpretation can also be expected to vary significantly with the type of map, and while map literacy studies generally recognize the broad distinction of reference and thematic maps, they do not provide a more nuanced framework for investigating how quantitative literacy may vary both within these broad categories and for maps which overlap these categories.

This dissertation represents a first attempt to address these issues, and at least provide conceptual frameworks for their investigation. For the first conceptual framework, the dissertation introduces a three-set Venn model to discuss the content and relationships of three “literacies”: map literacy, quantitative literacy, and background information. As part of this, the
field of Quantitative Map Literacy (QMP) is introduced and defined as the knowledge (concepts, skills and facts) required to accurately read, use, interpret, and understand the quantitative information embedded in geographic backgrounds. It is conceptualized as the intersection of the Map Literacy and Quantitative Literacy “sets”. The dissertation also introduces the conceptual framework of a compositional triangle based on the ratio of reference to thematic map purpose and the level of generalization/distortion within maps. This framework allows for any type of map to be located within the triangle and then related to the type and level of quantitative literacy they demand. Finally, based on these two frameworks, the dissertation uses the pedagogical tool of “word problems” to explore the variability of map reading skills and knowledge, and does this for specific map examples.
CHAPTER ONE:
INTRODUCTION

The advent and increasingly widespread use of desktop Geographical Information Systems (GIS) and other mapping software over the past 30 years has revolutionized and democratized the production of maps to where now they are commonplace in newspapers, periodicals, televisions, and other media. Maps now greatly impact the overall literacy of our citizenry on major issues. Many maps include large amounts of quantitative data, and there is the danger that citizens without the relevant knowledge and skills to read and interpret such maps could be easily confused or misled.

Meanwhile, the notion of map literacy is currently vaguely defined and can mean different things to different people. Traditionally, studies of map reading have focused on low-level tasks and skills. As a result, map reading has perhaps come to be perceived as an “easy task,” and many researchers and map makers probably take it for granted that map readers can understand the maps they produce. Studies of the map reading process, especially those involving higher-level skills, are still very limited.

Many new concepts and methods of mapping have been developed for different purposes over the history of cartography, and especially within the 20th Century. Introduction of these new concepts and methods in map production has brought an increasing need for quantitative skills and knowledge to the domain of map literacy. As the graphical tool for “visualizing geospatial data” (Wilke 2019, chapter 15), maps have sufficient reason for study in the domain of
quantitative literacy. Thus there is, in fact, an area of overlap of map literacy and quantitative literacy, namely quantitative map literacy (Xie et al. 2017).

**Organization and Scope of This Dissertation**

This dissertation comprises three main chapters between this brief introductory chapter and the final concluding remarks. The three are arranged in order from broader perspective to highly focused – from map literacy (ML, Chapter 2), to quantitative map literacy (QML, Chapter 3), and then to the quantitative skills and knowledge of QML (Chapter 4). All three chapters are written to be stand-alone papers for separate publication in the professional literature. Chapter 3 has already been published (Xie et al. 2017), and conceiving of Chapter 4 led to the ideas found in Chapter 2. In other words, the dissertation does not represent a chronological ordering but rather the culmination of considerable interaction of thought across all aspects. I have decided to order the chapters in this sequence to bring out the logic of starting with the entire set (ML), moving to a characterization of how maps vary across a subset (QML), and ending with a detailed analysis of particular QML features.

The dissertation introduces two new visualization methods to the subject of ML (broadly defined). Chapter 2 develops a three-circle Venn diagram to discuss the relation of ML, QL, and background information. Chapter 3 uses a compositional triangle (Wainer 1995; Vacher 2005) to sketch out the domains of various maps, such as reference maps, thematic maps, topographic maps and cartograms. Chapter 4 applies both types of visualizations to the analysis of how to read maps quantitatively.
CHAPTER TWO: RETHINKING MAP LITERACY

The “Understandings of Literacy” chapter of UNESCO’s Education for All Global Monitoring Report 2006 opens with “At first glance, ‘literacy’ would seem to be a term that everyone understands. But at the same time, literacy as a concept has proved to be both complex and dynamic, continuing to be interpreted and defined in a multiplicity of ways” (UNESCO 2006, p. 147). Later, in a section labeled “Literacy as skills” (p. 149-151), the report elaborates under three subheadings: “Reading, writing and oral skills,” “Numeracy skills,” and “Skills enabling access to knowledge and information.” In the latter category, the report lists information literacy, visual literacy, media literacy, and scientific literacy as examples. Clearly, map literacy, the subject of this chapter, can be classified in this third category of literacy skills that enable access to knowledge and information.

Xie et al. (2017) introduced the term “quantitative map literacy” (QML) and defined it as “the knowledge (concepts, skills and facts) required to accurately read, use, interpret, and understand the quantitative information embedded in a geographic background.” Conceptually, they envisioned QML to be a cross between map literacy (ML) and quantitative literacy (QL), the latter being the name by which “numeracy” is commonly known in the United States. Although they did not draw the diagram, it is safe to say they clearly conceptualized QML to be the intersection of two sets, QL and ML (Fig. 1).
The purpose of this chapter is to explore the two intersecting sets of Figure 1. Because the QL is the modifier of the ML, I will start with a brief, selective review of quantitative literacy, and then give a comprehensive literature review of map literacy. This unequal treatment of the two sets is a priori reasonable because one can anticipate there to be much content in the subset QL\ML (i.e., the content of QL that does not intersect with ML), whereas it is open to question how much (or how little, really) content there is in the ML\QL subset (the content of ML that does not intersect with QL). However, the literature review will reveal that ML intersects in fundamental ways with other literacies that also intersect with QL. I will conclude, therefore, with a proposal of two, three-set Venn diagram models to characterize the scope of map literacy: (1) QL, ML, and geographic literacy for reference maps and (2) QL, ML, and thematic literacy for thematic maps.

Quantitative Literacy: A Link in a Concept Chain from Literacy to Maps

The concept chain elaborated here is from literacy, to numeracy, to quantitative literacy, to graph literacy, to graphicacy and maps.
From Literacy to Numeracy

The link between literacy and numeracy is spelled out in the literacy of skills section of UNESCO (2006):

“Numeracy – and the competencies it comprises – is usually understood either as a supplement to the set of skills encompassed by ‘literacy’ or as a component of literacy itself.” (p. 149)

As indicated by UNESCO (2006) among many other sources, the link was made 60 years ago by the Crowther Report to the UK Ministry of Interior (1959) of an inquiry about considering the education of teenagers, especially the balance at various levels of studies and how it is related to the social and industrial need. The Crowther Report was the first publication to use the term, although a definition of “numeracy” is not explicitly stated in the report. It is clear from the following passage in a section labeled “LITERACY AND ‘NUMERACY’” that numeracy was seen as a sort of counterweight to literacy within a rounded education:

“In schools where the conditions we have described in the last paragraph prevail, little is done to make science specialists more "literate" than they were when they left the Fifth Form and nothing to make arts specialists more "numerate", if we may coin a word to represent the mirror image of literacy.” (Crowther, 1959, paragraph 398, p. 269)

Also clear in the Crowther Report was the notion that what the two had in common is the ability to communicate. Thus (from paragraph 401, p. 271):

“Just as by "literacy", in this context, we mean much more than its dictionary sense of the ability to read and write, so by "numeracy" we mean more than mere ability to manipulate the rule of three. When we say that a scientist is "illiterate", we mean
that he is not well enough read to be able to communicate effectively with those who have had a literary education. When we say that a historian or a linguist is "innumerate" we mean that he cannot even begin to understand what scientists and mathematicians are talking about. The aim of a good Sixth Form should be to send out into the world men and women who are both literate and numerate.”

The long paragraph added a couple of specifics:

“It is perhaps possible to distinguish two different aspects of numeracy that should concern the Sixth Former. On the one hand is an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand, there is the need in the modern world to think quantitatively, to realize how far our problems are problems of degree even when they appear as problems of kind. Statistical ignorance and statistical fallacies are quite as widespread and quite as dangerous as the logical fallacies which come under the heading of illiteracy.”

Thus the founding document of numeracy framed the question, “What is numeracy for all?” in the following way. At the very least, it is the ability to do the arithmetic of numbers, but what else? The answer was: have some understanding of the scientific method and some ability to think quantitatively about data.

Some two decades after the Crowther Report came a second UK Government report that today serves as a second benchmark in the evolving meaning of numeracy. The Cockcroft Report (1982), titled Mathematics Counts, was a select Committee’s response to

... (a decision by Parliament, March 1978) to "establish an Inquiry to consider the teaching of mathematics in primary and secondary schools in England and Wales, with particular regard to its
effectiveness and intelligibility and to the match between the
mathematical curriculum and the skills required in further
education, employment and adult life generally”. (Cockroft, 1982,
p. ix)

In the prefatory introduction, the report outlined the work of the Inquiry: meetings and visits,
submissions of evidence, commissioning of complementary research studies, Governmental
publications and announcements, and statistical information. Under “submissions of evidence,” it
reported (p. x):

“Throughout our work we have been greatly encouraged by the ...
helpful response which we have received to our requests for
information and written evidence. We have received written
submissions, many of them of considerable length, from 930
individuals and bodies of many kinds. 73 individuals and groups
have met members of the Committee for discussion.”

These submissions provided much useful evidence specifically about the question of
numeracy in regard to “adult life generally.” Thus (p.10, paragraph 35):

“The words ‘numeracy’ and 'numerate' occur in many of the
written submissions which we have received….. (We) believe that it
is appropriate to ask whether or not an ability to cope confidently
with the mathematical needs of adult life ... should be thought to
be sufficient to constitute 'numeracy'”.

The first finding on the subject of numeracy was negative (p. 11, paragraph 37)

“In none of the submissions which we have received are the words
‘numeracy’ or numerate’ used in the sense in which the Crowther
Report defines them. Indeed, we are in no doubt that the words, as
commonly used, have changed their meaning considerably in the
last twenty years. The association with science is no longer present
and the level of mathematical understanding to which the words
refer is much lower. This change is reflected in the various dictionary definitions of these words. Whereas the Oxford Dictionary defines 'numerate' to mean "acquainted with the basic principles of English mathematics and science", Collins Concise Dictionary gives "able to perform basic arithmetic operations".

The second finding was more definite, but troubling (paragraph 38):

“The second of these definitions reflects the meaning which seems to be intended by most of those who have used the word in submissions to us. However, if we are to equate numeracy with an ability to cope confidently with the mathematical demands of adult life, this definition is too restricted because it refers only to ability to perform basic arithmetic operations and not to ability to make use of them with confidence in practical everyday situations.”

Then the Report states its preference on the meaning of numeracy (paragraph 39):

“We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. .... We are, in fact, asking for more than is included in the definition in Collins but not as much as is implied by that in the Oxford dictionary-though it will, of course, be the case that anyone who fulfils the latter criteria will be numerate. Our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skill of computation.”
In summary, the Crowther Report opened consideration of the numeracy needs of adult life, and, together, the two benchmark reports seemed to imply levels of numeracy. At the lowest – below which a person would be considered innumerate – is a competence with arithmetic computations. At a second level, there is, additionally, an “at-homeness” with using numbers in everyday life, including the willingness and ability to read data presentations such as graphs and tables. At a third level, there is, additionally, a basic appreciation of empirical science, including some statistics.

**From Numeracy to Quantitative Literacy**

After the Cockroft Report, the numeracy concept came to the U.S., changed its name to Quantitative Literacy, and became a movement. The quantitative literacy movement in the U.S. is most commonly identified with the names of two deeply networked members of the Mathematical Association of America – Lynn Arthur Steen and Bernard L. Madison. Bibliographic information for these two prolific authors is available in two Numeracy-citation indexes in the journal, *Numeracy* (Vacher, 2015, and Grawe and Vacher, 2016, respectively). The following highlights provide a view of the progress, reach and scope of what became the numeracy / quantitative literacy / quantitative reasoning triad that evolved in the U.S. as seen through a selection of their writings.

1990. “Numeracy” (Steen, article in *Daedalus*)
1992. *Heeding the call for change: Suggestions for curricular action* (Steen, edited volume)
1999. “Numeracy: The new literacy for a data-drenched society” (Steen, article in *Educational Leadership*)
2000. “Reading, writing, and numeracy” (Steen, article in *Liberal Education*)
2001. “Mathematics and numeracy: Two literacies, one language” (Steen article in *The Mathematics Educator*)
2001. “Quantitative literacy: Everybody’s orphan.” (Madison, article in *MAA Focus*)
2004. “Two mathematics: Ever the twain shall meet?” (Madison, article in *Peer Review*)
2007. “Every teacher is a teacher of mathematics” (Steen, article in *Principal Leadership*)
2007. “How mathematics counts” (Steen, article in *Educational Leadership*)
2008. “Evolution of numeracy and the National Numeracy Network” (Madison and Steen, article in *Numeracy*)
2009. “All the More Reason for QR across the Curriculum” (Madison, article in *numeracy*)
2014. “How Does One Design or Evaluate a Course in Quantitative Reasoning?” (Madison, article in *Numeracy*)
2015. “Quantitative Literacy and the Common Core Standards in Mathematics.” (Madison, article in Numeracy)

2019. “Quantitative literacy: An orphan no longer (Madison, article in Tunstall et al.)

The above list illustrates an important point made by Madison and Steen (2008) in the inaugural issue of the journal Numeracy. Much of numeracy’s pathway from the UK Crowder and Cockroft Reports to Numeracy was due to a project shepherded by Robert Orrill through, first, the College Board, and then, the National Council on Education and the Disciplines (NCED), which he founded and directed. The first volume of the project (Why Numbers Count, 1997) was published by the College Board. The second and third volumes (Mathematics and Democracy, 2001, and Why Numeracy Matters, 2003) were published by the NCED. The fourth volume in the series (Achieving Quantitative Literacy, 2004) completed the project and was published by the Mathematical Association of America (MAA). It was the QL Design Team for the Mathematics and Democracy volume (MAD) that formed the core of the NCED outreach group, under the leadership of Susan Ganter of the MAA that ultimately formed the National Numeracy Network (NNN).

Orrill, in his preface to MAD (Orrill, 2001, p. xiv), drew attention to the writings of historian Lawrence Cremin (1988), specifically the distinction between “inert literacy” and “liberating literacy.” NNN’s vision statement – “The National Numeracy Network envisions a society in which all citizens possess the power and habit of mind to search out quantitative information, critique it, reflect upon it, and apply it in their public, personal, and professional lives” – “draws on the wording by Cremin (1988) in describing what he calls ‘liberating’ literacy” (Madison and Steen 2008, p. 6). This statement may be a restatement of the “at-homeness” level of numeracy suggested by the Cockroft Report.
There are also numerous footprints of the Mathematics Association of America in the list of Steen and Madison references on numeracy, quantitative literacy and quantitative reasoning. Not only was the 2004 *Achieving Quantitative Literacy* volume published by the MAA; so were the 1992 *Heeding the Call for Change* volume and the 2008 *Calculation vs. Context* volume. *Heeding the Call for Change* is #22 in the MAA Notes series, which also includes two edited volumes specifically on quantitative literacy: #70, *Current Practices in Quantitative Literacy* (Gillman, 2006) and #88, *Shifting Contexts, Stable Core: Advancing Quantitative Literacy in Higher Education* (Tunstall et al. 2019). *Current Practices* was a direct outgrowth of *Quantitative Reasoning for College Students: A Complement to the Standards* (Sons 1994), which was a product of the MAA’s Committee on the Undergraduate Program (CUPM) (Sons 2019). *Current Practices*, in turn, was the product of the MAA’s then-new quantitative literacy special interest group (SIGMA-QL), which effectively replaced the Quantitative Literacy Subcommittee of the CUPM (Gillman, 2019). The 2019 *Shifting Context, Stable Core* volume carries on the tradition of collecting and disseminating the curricular and institutional experiences of the SIGMAA-QL community. The many threads – MAA, NCED, NNN – are pulled together by Ganter (2019) in her forward to *Shifting Context, Stable Core*.

From the start of work of the QL Subcommittee of the CUPM in 1989 to the publication of the *Shifting Context, Stable Core* volume is a period of 30 years. For a definition of quantitative literacy, Ganter (2019, p ix) settles on “an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work” from the International Life Skills Survey (2000).
Meanwhile, Sons (2019, p. 4) still likes the definition implicit in the “Quantitative Literacy: Goals” section of *Quantitative Reasoning for College Students* (Sons 1994): “In short, every college graduate should be able to apply simple mathematical methods to the solution of real-world problems.” In detail, the five goals in the report were as follows (Sons 2019, Table 2):

A quantitatively literate college graduate should be able to:

1) Interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them.
2) Represent mathematical information symbolically, visually, numerically, and verbally.
3) Use arithmetical, algebraic, geometric and statistical methods to solve problems.
4) Estimate and check answers to mathematical problems in order to determine reasonableness, identify alternatives, and select optimal results.
5) Recognize that mathematical and statistical methods have limits.

Regarding the semantics of numeracy, vs. quantitative literacy, vs. quantitative reasoning, Madison and Steen (2008) say:

“In discussions of US education, the term quantitative literacy is much more common than numeracy, especially in recent years, although both terms continue to be used as synonyms. Some view quantitative literacy as the more inclusive term, while others (perhaps fearing the association of quantitative with mathematical) prefer the alternative expression quantitative reasoning. Robert Orrill has described QL as a cultural field where language and quantitative constructs merge and are no longer one or the other. From this perspective, "quantitative literacy" is a more inclusive term than the narrower word "numeracy". Others view QL as part of a portfolio of literacies
(e.g., historical, information, communicative, scientific, document, financial, and quantitative). In recent years quantitative literacy has received increasing attention, in part because it is most notably lacking and most critically needed.”

More recently, Vacher (2014) has argued some nuanced differences among the three word forms (numeracy, QL and QR) from an analysis using the online relational lexical database, WordNet, and his familiarity with all the papers in Numeracy at the time. He posited that there are four word senses involving the three word forms: (1) where all three are used interchangeably (as synonyms); (2) where QL and QR are used interchangeably; (3) where numeracy and QL are used interchangeably; and (4) where numeracy stands alone. Then, amongst them, these four word senses sit on three distinct branches diverging from the word sense representing the concept of cognition and knowledge. Specifically the first word sense sits on the “mental attitude” branch; the second sits on the “cognitive process” branch; and the third and fourth sit on the “cognitive skill and ability” branch.

In the same vein, Karaali et al. (2016) did a critical analysis of the definitions of the three terms (numeracy, QL and QR) along with several others including mathematical literacy and statistical literacy. Those authors established a hierarchy from numeracy through QR in terms four dimensions: quality of desired outcome, mathematical knowledge domain, display of expertise, and use of context. As summarized by Piercey (2017), who used the Karaali et al. (2016) analysis to frame a conceptual approach for his year-long algebra course, those dimensions grade from numeracy to QR in the following way (Piercey 2017, Table 1):

1) Quality of desired outcome: basic skills in numeracy; basic skills and habit of mind in QL; habit of mind in QR
2) **Knowledge domain:** arithmetic, mathematics, and logic in numeracy; preceding plus data (descriptive statistics) in **QL**; preceding plus inferential statistics in **QR**.

3) **Display of expertise:** understand and appreciate in all three; cope in numeracy vs. analyze, decide, and use in **QL**; plus critique in **QR**; (hence, passive and reactive in numeracy; not passive but reactive in **QL**; active but also proactive in **QR**).

4) **Context:** information and practical situations in numeracy; preceding plus active citizenship in both **QL** and **QR**.

The numeracy – quantitative literacy – quantitative reasoning hierarchy of Karaali et al. (2016) and Piercey (2016) are consistent with the scheme of Vacher (2014), and they both broadly agree with the three “levels” of numeracy implied by the Cockroft Report, as discussed in the previous link in the chain.

It is obvious now that there is considerable flexibility in the semantics of the three terms, and there’s ample literature to support a myriad of definitions, depending on the purpose of the project at hand. With that freedom in mind, I have selected the most updated definition proposed by Ganter (2019) in the International Life Skills Survey (2000) for the purpose of this study.

**From Quantitative Literacy to Graph Literacy**

Mathematical operations and quantitative analysis based on graphs have long been an important aspect of **QL**. Graphs were mentioned along with “at-homeness” in the Cockroft Report (1982, paragraph 39). They were prominently included in the Sons Report (1994) in an important listing of 24 **QL** topics, which were classified into five categories (arithmetic, geometry, algebra, statistics, and other). Of the 24 topics, at least three are directly related to graph comprehension:
1) “Algebra in graphs and tables, construction, reading, interpreting, extrapolating quantitative information from graph and tables,”

2) “Graphical display of data, including pie and bar charts, frequency polygons, visual impact of scale changes,”

3) “Graphical and computational methods of problem solving.”

Understanding and answering questions about graphs has been an important element of assessing levels of quantitative literacy. As a recent example, the Quantitative Reasoning for College Science (QuaRCS) assessment developed by Follette et al. (2015), has graph reading and table reading as two of its ten major categories of QR skills.

Because comprehension of graphs is becoming ever more important in processing information in an increasingly highly technological society (Curcio 1987), and because graphs seem to make quantitative information easier to understand (MacDonald-Ross 1977, Winn 1987), many researchers have studied the process of understanding graphs, and developed the research topic of graph literacy (graphic literacy). Freedman and Shah (2011) defined the term of “Graphic Literacy” as the ability to understand the information presented in graphic form.

Extracting information from graphs and making inferences based on graphs are the two major aspects concerned in their definition. Freedman and Shah used the understanding of bar and line graphs as examples to illustrate this definition in their later studies (Freedman and Shah 2011).

Wood (1968) summarized the three kinds of behavior involved in the comprehension of information in written or symbolic form as:

1) Translation (e.g., describe the content of a table/graph, comment on the specific structure of the graph),

2) Interpretation (e.g., look for relationships among specified or labeled axes in a graph),
3) Extrapolation and interpolation (e.g. note the trend perceived in data, specify implications).

Although this early study was not conducted specifically for graph comprehension, it provided a structure for the graphic reading process for studies in graphic literacy that followed.

Based on these three kinds of behavior, Curcio (1987) constructed three levels of graph comprehension: “reading the data,” corresponding to Wood’s (1968) “translation”; “reading between the data,” corresponding to Wood’s (1968) “interpretation;” and “reading beyond the data”, corresponding to Wood’s (1968) “extrapolation and interpolation.” Curcio (1987) also designed an assessment instrument based on these three levels. An example of a question at the level of “reading the data” would be: “What was the value of Stock X on June 15?” An example at the level of “reading between the data” would be: “Compare the change of value of Stock X and Stock Y between June 15 and June 16”. An example of a question at the level of “reading beyond the data” would be: “How would you predict the trend for the value of Stock X?” Obviously there is a hierarchy of complexity in these three levels of questions.

Friel et al. (2001) reviewed the studies in graph reading, and found a similar pattern of the three levels of skills tested in evaluation scales. In a similar review of graph comprehension research conducted by Shah and Hoeffner (2002), the same three levels in graph reading were identified and explained in detail through task analysis of graph comprehension. The first level dealt with encoding the visual array and identifying the important visual features (title, labeled axis, curved line, etc.). Skills at this level could be the literal reading of graph content or background knowledge of graph making. The second level dealt with relating visual features to conceptual relations represented by those features and correlating the visual features (comparison and correlation of graph features). Skills at this level could be comparing and contrasting visual features. The third level dealt with determining the referent of concepts being quantified and
associating those referents to the encoded functions (corresponding knowledge beyond the graph). Skills at this level involve synthesis beyond the graph itself.

Another framework of interpreting tables and graphs was proposed by Kemp and Kissane (2010). This framework was composed of five steps as shown in Table 1. Although Kemp and Kissane did not directly refer to the three hierarchical levels of components in graph comprehension proposed by Curcio (1987), the steps included in their framework indicated those components. Steps 1 and 2 are directly reading data and text, in other words, directly “reading the data.” Steps 3 and 4 are comparing the differences; they are “reading between data”. Step 5 is connecting differences with other knowledge, i.e., “reading beyond data”.

Table 1. Five Step Framework for Interpreting Tables and Graphs (Kemp and Kissane, 2010)

<table>
<thead>
<tr>
<th>Step 1: Getting Started</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look at the title, axis, headings, legends, footnotes and source to find out the context and expected quality of the data. Take into account information on the questions asked in surveys and polls, sample size, sampling procedures and sampling error.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2: WHAT do the numbers mean?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make sure you know what all numbers (percentages, ‘000s, etc.) represent. Look for the largest and smallest value in one or more categories or years to get an impression of the data.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3: HOW do they differ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look at the differences in the values of the data in a single data set, a row or column or part of a graph. This may involve changes over time, or comparison within a category, such as male and female at any time.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4: WHERE are the differences?</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the relationships in the table that connect the variables? Use information from Step 3 to help you make comparisons across two or more categories or time frames.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 5: WHY do they change?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why are they differences? Look for reasons for the relationships in the data that you have found by considering social environmental and economic factors. Think about sudden or unexpected changes in terms of state, national and international policies.</td>
</tr>
</tbody>
</table>

With this structure of hierarchical levels of graph reading skills, several assessment instruments have been designed by researchers. For example, a recent study by Bolch and
Jacobbe (2019) combined Curcio’s (1987) three levels of graph comprehension and the Levels of Conceptual Understanding in Statistics (LOCUS) assessments, to develop the “Levels of Graphical Comprehension.” Some assessment tools have also been developed for specific types of graphs, such as line graphs (Boote 2014) and box plots (Pfannkuch 2006).

When it comes to “graph literacy,” it seems that researchers have focused mostly on statistical graphs, such as bar graphs, scatter plots, or bar charts. Some researchers have also classified the studies of graph comprehension into the field of “statistical literacy” (Ben-Zvi and Garfield 1997, Gal 2004, Nolan and Perrett 2016). Gal (2004) pointed out that one aspect of statistical literacy skills is document literacy, which requires people to be able to identify, interpret, and use information from a variety of media including graphical displays. Furthermore, among the five key parts of statistical knowledge that form the basis of statistical literacy summarized by Gal (2004), two of them considered the knowledge and skills in processing graphical and tabular displays of statistical data. When the term “graph” is mentioned in these studies, it usually means statistical graphs.

**From Graph Literacy to Graphicacy and Maps**

Maps, as a special type of graph, have drawn the attention of researchers in the field of graphicacy study. Aldrich and Sheppard (2000) defined the term “Graphicacy” as the ability to understand and present information in various graphical forms, including but not limited to sketches, photographs, diagrams, maps, charts. Before that, Balchin (1976) had described the content of graphicacy as the “visual-spatial aspect of human intelligence and communication.” He specifically emphasized “visual-spatial” in this definition and argued that maps are one of the
spatial documents that are the tools of graphicity studies. He also argued that the skills of graphicity are best communicated through geography.

There are some studies on the topics of quantitative literacy and graph literacy that have mentioned maps. Tufte (1992, 2001) did a series of studies on the graphic visualization of quantitative information that created principles for displaying quantitative data in graphical backgrounds. Among the various statistical graphs included in his studies, Tufte discussed geo-spatial quantitative data, which he referred to as “data maps.” He gave a very high appraisal for data maps based on their advantages for storing geo-spatial data.

One of the earliest and probably most well-known applications of a map in displaying quantitative information and mentioned in Tufte’s work would be the so-called “Ghost Map” (Fig.2). It was produced by Dr. John Snow to depict the distribution patterns of death from cholera cases in central London in 1854. This “Ghost Map” worked like a dot map, and proved its ability in revealing the underlying relationship between the variables displayed in the map (in this case, the occurrence of cholera and water pump distribution). The idea of showing quantitative data on a spatial related background was definitely an innovation in Snow’s time. Furthermore, the process of analyzing the spatially distributed victim data in this example also illustrated the importance of map literacy in detecting patterns and drawing conclusions from spatially related quantitative data.
Figure 2. Clusters of Cholera Cases in the London Epidemic of 1854 (Snow 1855).

Jungck (2012) studied the quantitative information in the “Ghost Map,” including the geometry (spatial distribution of geographic features), statistics (density of geographic features, distance, area, etc.), modeling (correlation between geographic features), and networks (critical thinking and reasoning of the correlation between geographic features). He noted the usefulness of using this map in a core course in public health, and teaching the students about the QR procedures in reading and interpreting this map. His paper illustrates the possibility of improving QL and QR skills through the medium of maps.

In synthesizing the studies conducted on the “Ghost Map,” it seems that although map reading was not considered in the graph comprehension studies mentioned earlier, the “Ghost Map” example does share commonalities with the interpretation of traditional statistical graphs. The three components of graph comprehension proposed by Curcio (1987), for example, could also be applied in map interpretation. In the “Ghost Map,” identifying the symbols of cholera
cases and water pumps belong to the category of “reading the data;” comparing the spatial distribution pattern of cholera cases and water pumps belongs to the category of “reading between data;” inference of the correlation between occurrence of cholera cases and locations of water pumps belongs to the category of “reading beyond the data.”

Similar to Tufte’s books on the visualization of quantitative data, Wilke (2019) has presented a list of helpful principles for making graphs, in a striking book, *Fundamentals of Data Visualization*. Notably, Wilke devotes a chapter specifically to geo-spatial data. He discusses the importance and hazards of projection systems for reference maps, as well as some tricks and traps involving thematic mapping (choropleth maps and cartograms are two examples in his study). His is a semi-detailed overview of principles for map making, especially in a book for general graphical literacy study. It is also notable that he included thematic mapping in recognition of modern mapping emphases because thematic maps, especially cartograms, are not usually mentioned in any book on general graphical literacy.

As a conclusion to this section of the literature review it seems that graph comprehension is an important sub-topic in quantitative literacy studies. The components of graph reading skills have been well articulated and documented through previous studies, and assessment instruments to evaluate graph users’ ability to interpret quantitative information in graphs have also been developed for several different types of graphs. However, as a special type of graphic representation of quantitative data, *maps* are usually overlooked in the graphic literacy studies. This is partly due to the differences in producing maps relative to traditional statistical graphs, as well as the background knowledge to interpret them. It may be unrealistic to discuss the interpretation of maps and traditional statistical graphs together. In fact, the interpretation of
maps has usually been studied specifically by cartographers under the topic of “Map Literacy,” and this will be reviewed and discussed in the next section.

That said, although maps are not considered in most graphic literacy studies, some of the ideas and concepts from such studies could be applied to map reading. For example, the three hierarchical levels of graph comprehension (“reading the data,” “reading between the data,” and “reading beyond the data”) would seem a useful construct to apply in map reading.

Furthermore, Graphicacy is only one branch of the scope of QL, and the scope of QL continues to expand (e.g., Craig and Guzman 2018; Fisher 2019; Craig et al. 2019). Vacher (2019) enumerated a list of 44 types of literacy (digital literacy, media literacy, data literacy, visual literacy, etc.) that had their own pages on Wikipedia. Many of them are closely related to QL. Yet the list is by no mean exhaustive. In a recent Numeracy editorial, Vacher (2019) used the term “Seas of Literacy” as a metaphor for the many types of literacy (rather like the “portfolio of literacies” mentioned by Madison and Steen in their 2018 paper introducing Numeracy). It is not unreasonable to think that quantitative literacy, map literacy, and even quantitative map literacy should be mapped out within the Literacy World Ocean.

Map Literacy

Dent et al. (2009) identified two general types of map, including general purpose (reference) maps, which focus on displaying locational information about geographic features, and thematic maps, which focus on displaying attribute information or data. Because of the differences in map reading and processes producing them, map literacy studies are usually conducted separately for these two types of maps. In the following section, previous studies on map literacy will also be reviewed, in sequence, for these two types of maps.
Map Literacy Studies for Reference Maps

Cartographers did not really start thinking formally about map literacy until the 1970s. In reviewing the history of cartography in Canada, Ruggles (1974) referred to the term “Map Literacy” as “knowing how to interpret cartographic conventions,” and he claimed a general lack of “Map Literacy” in the public. Ruggles’ (1974) definition of map literacy can only be described as rather vague. Even an updated one by Clarke (2003) regarding what he terms functional map literacy, “the ability to understand and use maps in daily life, for work and in the community,” shares the same characteristic of vagueness. Although lacking more precise formal definitions, map literacy has nevertheless been a topic of academic study, often through an emphasis on map reading skills.

Some pioneering studies of map reading processes and skills were conducted even before the definition of “Map Literacy.” Board (1975), for example, completed an early work on map reading tasks. He grouped the main reference map reading tasks under three main domains (see Table 2): 1) navigation, which deals with directly identifying and searching geographic features on a map (for example, find a landmark on the map, or find direction between two locations); 2) measurement, which deals with extracting data or information, as well as further analysis based on the extracted information (for example, search for and count certain type of geographic features, measure different routes between two locations and compare or the distances); 3) visualization, which is a comprehensive evaluation of the map display (for example, describe the content or purpose of the map, evaluating the map display or mapping methods). This was one of the first attempts of classifying map reading skills, and Board’s work built a framework for the ensuing research on map reading skills related to reference maps.
Table 2. Map Reading Tasks (Board 1975)

<table>
<thead>
<tr>
<th>Navigation</th>
<th>Measurement</th>
<th>Visualization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search</td>
<td>Search</td>
<td>Search</td>
</tr>
<tr>
<td>Identify and locate own position on map</td>
<td>Identify</td>
<td>Identify</td>
</tr>
<tr>
<td>Orient map</td>
<td>Count</td>
<td>Describe</td>
</tr>
<tr>
<td>Search for optimum routine on map</td>
<td>Compare</td>
<td>Compare</td>
</tr>
<tr>
<td>Search for landmark on route</td>
<td>Contrast</td>
<td>Contrast</td>
</tr>
<tr>
<td>Recognize landmark on route</td>
<td>Estimate</td>
<td>Discriminate</td>
</tr>
<tr>
<td>Search for destination</td>
<td>Interpolate</td>
<td>Delimit</td>
</tr>
<tr>
<td>Identify destination</td>
<td>Measure</td>
<td>Verify</td>
</tr>
<tr>
<td>Verify</td>
<td></td>
<td>Generalize</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prefer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Like</td>
</tr>
</tbody>
</table>

However, as a pioneering study on map reading skills, Board’s (1975) work had obvious limitations. Several map reading skills are duplicated in the three categories (such as search, identify, compare features). This creates vague boundaries between the categories of map reading skills. Furthermore, the complexity of the map reading skills in each category was not discussed. As an improvement on Board’s (1975) work, Morrison (1978) offered an alternative arrangement of map reading skills into four different categories. Morrison expanded the navigation skills in Board’s (1975) study to include detection, and discrimination and recognition skills; he also changed measurement skills to estimation skills, and visualization skills to attitudes on map style. Morrison also added a new type of skill as pre-map reading which notably refers to the background knowledge about maps before using a map; however, he did not provide a detailed discussion on it. The logic of his rearrangement was based on the order of the map reading process: pre-map reading skills as a pre-requisite of map reading; detection and recognition of map symbols as the first step; measurement and comparison of geographic
features from the map symbols as the second step; and appreciation of the map (evaluate and comment on the mapping method) as the final step. In terms of the difficulty of the map reading skills, Morrison indicated that the skills mentioned in the steps are rather simple, and often the more complex skills are composed of the combination of these elementary skills.

Olson (1976), meanwhile, had claimed that map reading skills should have a structure that includes a hierarchy of complexity, defined in three levels. The first level focused on the recognizing and understanding of individual symbols (for example, read and compare the shape, relative size, importance of different symbols). The second level focused on the comparison and relations among the whole set of symbols (for example, summarize a spatial distribution pattern of symbols). In the third level, map symbols are no longer directly the focus of map reading and it is more important to use “the map as a decision-making or content-knowledge-building device through integration of the symbols with other information.” In other words, the third level includes the ability to apply maps to solve real world problems, and skills could involve sets of different maps and other information. Olson’s study was the first study that pointed out the differences in complexity within map reading skills.

Although Olson (1976) pointed out the hierarchy of complexity in map reading skills, he didn’t try to list and specify specific skills. A more comprehensive study on map reading skills with a hierarchy of complexity was conducted by Golledge et al. (2008) where map reading skills were conceived in a broader sense and the term “spatial thinking” was used (instead of “map literacy”). The hierarchical set of spatial thinking concepts consisted of five levels of difficulty developed as shown in Table 3. Instead of classifying the skills and tasks based on their functions, as Board (1975) had, or stage of map reading, as Morrison (1978) had, Golledge et al. directly classified skills based on the level of difficulty. However, similar to the three levels of
complexity presented by Olson (1976), this hierarchical set of spatial thinking concepts was also developed based on subjective ideas and the study lacks a theoretical basis.

Table 3. Hierarchical Set of Spatial Thinking Concepts (Golledge et al. 2008)

<table>
<thead>
<tr>
<th>Primitive level</th>
<th>Simple level</th>
<th>Difficult Level</th>
<th>Complicated level</th>
<th>Complex level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity</td>
<td>Arrangement</td>
<td>Adjacency</td>
<td>Buffer</td>
<td>Area association</td>
</tr>
<tr>
<td>Location</td>
<td>Distribution</td>
<td>Angle</td>
<td>Connectivity</td>
<td>Interpolations</td>
</tr>
<tr>
<td>Magnitude</td>
<td>Shape</td>
<td>Classification</td>
<td>Gradient</td>
<td>Map projection</td>
</tr>
<tr>
<td>Space-time</td>
<td>Boundary</td>
<td>Coordinate</td>
<td>Profile</td>
<td>Subjective space</td>
</tr>
<tr>
<td></td>
<td>Distance</td>
<td>Grid pattern</td>
<td>Representation</td>
<td>Virtual reality</td>
</tr>
<tr>
<td>Reference frame</td>
<td>Sequence</td>
<td>Polygon</td>
<td>Scale</td>
<td></td>
</tr>
</tbody>
</table>

Clarke (2003) also agreed with Olson’s (1976) idea of a hierarchy of difficulty in map reading skills. Clarke confirmed and improved Olsen’s three levels by using the theoretical basis of Bloom’s Taxonomy of Learning (Anderson et al. 2001), which indicates a hierarchical level of learning including (low to high): knowledge, comprehension, application, analysis, synthesis, evaluation. He discussed three skill levels to mirror the three hierarchical levels of Olson (1976): an entry level of getting information from single or simple symbols; a second level of recognizing properties of symbol groups on the map as a whole and analyzing spatial patterns; a third level of understanding the meaning of spatial phenomena for knowledge enhancement. In reference to his definition of map literacy noted earlier, Clark claimed that a person can be considered as functionally map literate at the second level.

Clarke also summarized a list of map reading skills from recognizing map elements to forming spatial mental-models but he didn’t explicitly specify which of his skill levels they belonged to. These map reading skills included:

1). Recognition (*searching, locating and identifying geographic features*).
2). Orienting map, recalling (from memory).
3). Detecting (geographic features).
4). Reorganization (classify, outline, summarize, generalize, synthesize geographic information).
5). Inferential comprehension (including prediction and interpretation).
6). Evaluation (including judgment).
7). Appreciation (comments on mapping method).
8). Decoding the perceived visual patterns.
9). Symbol group recognition and lexical interpretation.
10). Parsing a spatial relationship into its meaning constituents and establishing a local coherence of meaning.
11). Comparing, describing, contrasting, discriminating, forming spatial mental-model or message (such as forming a mental 3-D model of morphology based on elevation contour).

Clarke (2003) did indicate that this list was by no means exhaustive, and was in no particular order.

Another important point made by Clarke (2003) was that the knowledge and skills involved in map reading were more than what involved the map itself. When Board (1975), Olson (1976) and Morrison (1978) tried to summarize map reading skills, they all focused just on the map itself (such as identifying map symbols, reading and using map elements). Clarke noted that there is knowledge, not directly related to maps, but still necessary for map reading. For example, geological background knowledge is necessary to interpret a geological map, and understanding the rules by which elections are decided is necessary to interpret a map of election results. Clarke called this prior knowledge.
In terms of the use of QL and QR skills in map reading, Mark Monmonier (1996) specifically summarized the quantitative information displayed in maps. In his well-known book *How to Lie with Maps*, he claimed that “not only is it easy to lie with maps, it's essential.” For reference maps, they inevitably distort the original data when transforming it from the three-dimensional real world to a flat sheet of paper or video screen as a scaled model of the Earth. For thematic maps, Monmonier provided various examples of how data could be “distorted” in the map making process. Some examples include manipulating the data classification method; using a non-linear scale but present the size of different categories linearly. Monmonier warned that map users generally trust map makers too much: “they understand the need to distort geometry and suppress features, and they believe the cartographer really does know where to draw the line.” As a result, map users can easily be misled by the map maker. Map users need the knowledge and ability to discern and unravel such depictions (or deceptions!).

Some basic mathematic operations in map reading were summarized by Innes (2003). She classified spatial analysis skills into four hierarchical levels: level 1 includes identifying boundaries and describing direction; level 2 includes using or describing absolute location (using geographic coordinates); level 3 includes assessing altitude or height and measuring distance and calculating area; level 4 includes calculating gradients and drawing or interpreting profiles (to identify landforms). Innes’s study focused only on the simple mathematic operations for reference maps, and therefore the listed map reading skills were mostly at a relatively low level. For the same reason, these levels didn’t confirm with the levels of graph comprehension proposed by Curcio (1987). For example, the component of “reading beyond the data” is not covered in these four levels.
Evaluation scales for map literacy have also been a theme in map literacy studies. Borrowing the concept of “spatial thinking” proposed by Golledge et al. (2008), Lee and Bednarz (2012) developed a spatial thinking ability test (STAT) that could be applied to assess a student’s map literacy level. This assessment tool consisted of eight different components of map reading skills including:

1) Comprehending orientation and direction;
2) Comparing map information to graphic information;
3) Choosing the best location based on several spatial factors;
4) Imagining a slope profile based on a topographic map;
5) Correlating spatially distributed phenomena;
6) Mentally visualizing 3-D images based on 2-D information;
7) Overlaying and dissolving maps;
8) Comprehending geographic features represented as point, line or polygon.

These specific tasks indicate some of the map reading tasks and skills listed in Tables 1 through 3. A similar assessment tool for map literacy was developed by Koç and Demir (2014). Their assessment tool was composed of several different map reading tasks that can be classified into four major types, including:

1) Reading and interpreting maps (for example, understanding the information presented with the help of the legend, making sense of the relationship between geographical formations and land by using topography maps, etc.).
2) Using maps (for example, making use of road maps during journeys.)
3) Carrying out procedures in maps (for example, calculating the distance, area, slope between two locations).
4) Sketching maps (for example, drawing a topography sketch using contour lines, isobaric charts using isobars, or precipitation maps using isohyets).

Obviously these tasks and skills cover mostly reference maps (such as topographic maps and street maps). Koç and Demir (2014) did mention other types of maps that can be classified as thematic maps (such as weather maps, land use maps), but they did not discuss the skills and knowledge for such maps. There is a common limitation in the evaluation scales developed by Lee and Bednarz (2012) and Koç and Demir (2014): they didn’t distinguish between the complexity of map reading tasks in their evaluation scales – in other words, all of the map reading skills were weighted the same in the overall assessment score.

Rautenbach et al. (2017) developed an assessment tool specifically based on topographic maps. The taxonomy defines six levels of tasks with increasing difficulty and complexity:

1) Recognizing symbology (for example, naming the phenomenon represented by the symbol, describing the difference in characteristics of phenomenon based on the symbols or patterns, etc.);

2) Orienting maps (determine direction, azimuth and bearing);

3) Locating features (locate features and exhibit its specific relationship to another feature);

4) Measuring and estimating (for example, determine length or area of features, estimate altitude/height/volume of features);

5) Calculating and explaining (including produce and reproduce features, explain patterns of occurrence or features);

6) Extracting knowledge (for example, analyzing spatial distribution pattern, infer knowledge of interrelationship of features or patterns, etc.).
Although this assessment tool is constructed only for topographic maps, these six sets of map reading tasks did follow an increasing level of difficulty, which confirmed with the hierarchical levels of map reading tasks proposed by Olson’s (1976) and Clarke’s (2003) studies.

**Map Literacy Studies for Thematic Maps**

The knowledge and skills involved in thematic map reading have often been overlooked in map literacy research. A recent dissertation by Phillips (2013) investigated adolescents’ interpretation and production of thematic maps. By reviewing the research on the teaching and learning of thematic maps, Phillips (2013) found that although thematic maps have proven to be a powerful tool to present spatial data, the knowledge and skills for thematic map reading are seldom taught systematically in education, and researchers “know very little about how they (thematic maps) are interpreted, understood and read.” Phillips argued that the map users’ ability to read thematic maps has been taken for granted. Similarly, Wiegand (2006), who studied the mental representations of thematic maps, as well as cartographic problem solving, claimed that almost all the teaching and learning for thematic maps focused on simple skills and knowledge (e.g., reading map element or data value, identifying features), rather than critical thinking about how maps convey information or the conclusions that might be based on using thematic maps. He asserted: “interpretation (of thematic maps) may be more problematic than has hitherto been recognized” (Wiegand 2006).

In terms of the map reading studies that have been done with respect to thematic maps, Kulhavy (1992) categorized five criteria for thematic map reading tasks as:

1) *Reading names, which is the subject of reading the name of both geographic features and thematic contents;*
2) Describing, which refers to the ability of describing the characteristics of a feature or theme in terms of physical properties, space, and function;

3) Relations, which refers to the ability of summarizing two or more units in terms of either their relation to one another, or the pattern they formed on the map;

4) Counting, which refers to the ability of using quantitative terms to describe map unit or thematic information;

5) Map context, which refers to the ability of reading map elements, such as map legend, coordinate system, etc.

Perhaps under the influence of previous map literacy studies that were biased to reference maps, the five criteria summarized by Kulhavy (1992) covered map reading tasks that processed both locational information and thematic information in the map. He also developed an assessment tool based on his criteria and used it with eight high school and college students to evaluate their understanding of thematic maps, evaluating the results separately for geographic and thematic aspects for the five criteria. It should be noted that the five categories of thematic map reading tasks Kulhavy established mostly belong to the lower levels of map reading skills.

Other map reading studies involving thematic maps have focused on evaluating map users’ understanding of specific aspects of thematic map elements, and then either verifying or criticizing the map design, or proposing a better design. For example, Nelson (2000) studied the bivariate symbol design in cartography. She studied the effectiveness of bivariate map symbols by testing map users’ performance in a series of tasks comparing variable values. The comparing tasks included a baseline task (the comparison of multiple symbols based on only one variable when the other variable is the same); a filtering task (a grouping task of multiple symbols according to only one variable); a redundancy task (a comparison of multiple symbols based on two variables); and a condensation task (a grouping and comparing task based on two variables).
Nelson tested the effectiveness of twelve different symbolization methods (Fig. 3) based on the reaction time of subjects when dealing with these four sets of tasks. Although there is an increasing difficulty in these four sets of tasks, the skills involved in completing these tasks remain mostly low level—i.e., mostly reading and comparing quantitative values.

**Figure 3.** Different Symbolization Scheme (Nelson 2000)
Similar studies to that of Nelson (2000) have also been performed by other authors: graduated circle/square symbols (Flannery 1971; Cox 1976; Brewer and Campbell 1998; Edwards and Nelson 2001); colored line symbols (Gill 1988); color schemes and effect on perception (Garner 1977).

Another research direction in thematic map literacy has focused on individual differences among map readers. Similar to the studies focused on map elements, cognitive experiments have often been used to test map readers accuracy in obtaining information from maps. Rieger (1999) conducted a set of experiments testing the map reading process based on a variety of GIS images. The GIS images used in the experiment included a collection of categorical maps, land-use maps and isopleth maps. Rieger designed a questionnaire which consisted of 34 questions that tested map reading tasks including legend reading, feature identification, location determination, the interpretation of 2D and 3D spatial patterns, data analysis and synthesis, and spatial distribution analysis. Rieger’s study explored and tested several potential factors that could affect map readers’ understanding toward thematic maps based on their questionnaire scores. Such factors included map user’s gender, level of GIS experience, and the method of map presentation (hard copy, electronic version on screen). However, the study did not discuss the actual skills being utilized to answer the questions in the questionnaire.

Lloyd and Bunch conducted a series of studies on the topic of thematic map reading. Similar to Rieger (1999), Lloyd and Bunch (2005) developed a series of questions that tested a map user’s ability to read map elements, correlate symbols, and summarize simple spatial distribution patterns. A comprehensive assessment system was then developed by assessing three different parameters of map user’s performance on the test:

1) *Reaction time, which is the time that map users spend in answer the questions;*
2) Accuracy, which is the percent correct rate of the answer;
3) Confidence, which is a self-evaluation map users gave to themselves after the test.

Based on this assessment system, Lloyd and Bunch (2008) conducted several cognitive experiments on the map reading process. They tested the influence of individual differences in gender, working memory capacity, and brain lateralization on map reading.

Nursat et al (2018) compared the effectiveness of thematic map reading for cartograms. Similar to studies mentioned above, questionnaires were applied in this study as evaluation tools. Figure 4 shows several questions quoted from the questionnaires produced by Nusrat et al (2018). These questions tested whether map users are able to correctly obtain information from the maps by looking at the accuracy of the answers provided by map users. They also presented a list of questions they used to test different map reading tasks. The map reading tasks included:

1) Identifying and locating features, of which the example question could be showing a cartogram of the U.S.A. and asking subjects to locate a certain State;
2) Finding values, of which the example question could be showing a cartogram of U.S. population and asking subjects to find the state with highest/second highest population;
3) Comparing, of which the example question could be showing a cartogram of U.S. population with 2 state highlighted, and asking the subject which state has more population;
4) Summarizing, of which the example questions could be giving a cartogram of Germany GDP and ask which part of the country contributes more to GDP, or two separate U.S. population cartogram and asking the subjects to summarize the trend in population growth.
Similar studies have been done using different types of thematic maps, such as two-variable maps (Wainer and Francolini 1980; Olson 1981; Eyton 1984; Nelson 2000), unclassed choropleth maps (Peterson 1979), sequenced choropleth maps (Slocum et al. 1990), and cartograms (Sun and Li 2010). These studies of thematic map literacy have some common drawbacks. One of them is that they tend to be map-type-based. In other words, each of the studies focused on map users’ understanding toward a single type of map. Studies over a range of thematic maps are very limited, and somewhat superficial (Mosenthal and Kirsch 1990).

![The figure shows a cartogram with two states highlighted, one state in red, another in blue. Which state is bigger?](image1.jpg)

![The following cartogram shows the GDP (Gross Domestic Product) of Germany. Which part of the country contributes more to GDP?](image2.jpg)

**Figure 4.** Questions in the Questionnaire Produced Based on Maps (Nusrat et al. 2018).

*The “bigger” in the left figure means bigger in value, not the geographic size.*

Another limitation of previous research on thematic map literacy is that they have tended to test map reading abilities using only low-level skills dealing with simple map reading tasks. By examining the questions asked in the questionnaires from studies cited above, we can have an idea of what kinds of map reading skills were tested. Referring back to the questions applied in the Nusrat’s (2018) study, in order to solve these questions, map users often only need basic knowledge of the maps. Except for the questions on the task of summarizing, which require
some analysis skills, other questions can be solved by simple map reading tasks, such as identifying features and reading values. This is partly because low-level map reading skills, such as identifying features or reading values, can be easily evaluated, and directly tell the researchers whether subjects obtain quantitative data from the map.

However, obtaining quantitative data is perhaps only a small part of thematic map interpretation. To fully understand the phenomena illustrated in thematic maps, perhaps map users need more than just reading values correctly; perhaps they also need spatial analysis skills and knowledge about map making. MacEachren (1994) proposed a model of cartography (Fig. 5) that alludes to this greater complexity in thematic map reading. MacEachren chose three components to describe thematic mapping: 1) whether the focus of spatial data exploration is presenting “known” or revealing “unknown” 2) whether the human-map interaction is high or low; 3) whether map users explore spatial data in a private realm or a public realm. He defined map communication as an approach to presenting “known” data that involves less human-map interaction in the public domain (such as a newspaper or television), while map visualization is an approach to revealing “unknown” phenomena that involves more human-map interaction in the private domain (such as scientific studies or reports).

In this sense, the map communication to map visualization continuum represents a gradient of increasing complexity. If quantitative data is substituted for the ‘presenting known’ – ‘revealing unknown’ axis, and level of skills/knowledge is substituted for the level of human-map interaction then, in the context of this dissertation, we can think of map communication as reflecting the use of low-level skills/knowledge to extract straightforward numeric data from maps, whereas map visualization involves the use of higher-level skills/knowledge to interpret additional data or information based on the quantitative data presented.
In conclusion to the literature review, then, map literacy has been studied for both reference maps and thematic maps, but with a distinct bias to the former in terms of both history and number of studies. There are some limitations in previous map literacy studies as follows.

1) Researchers have tended to focus on map design (map element, mapping method, etc.) and individual differences (gender, racial group etc.) in map reading abilities, while overlooking the actual skills involved in map reading, especially for thematic maps. In the first chapter, QML was defined as “the knowledge (concepts, skills and facts) required to accurately read, use, interpret, and understand the quantitative information embedded in a geospatial representation of data on a geographic background.”(Xie et al. 2018). In terms of this definitions, previous studies of map literacy have focused primarily on the second half of the definition, that is on whether map users are able to “accurately read, use interpret and understand the quantitative information”
in the maps and put less focus on the first half of the definition, which is “the knowledge (concepts, skills and facts)” involved in map reading.

2) Map reading skills and tasks have been mostly at a low level (e.g., being able to identify geographic feature and correctly read data value), especially indicated by the questionnaires applied in the experiments or assessment tools. Such studies have focused on the low-level map communication end of the complexity that MacEachren envisaged. As claimed before, this drawback has likely led to the apparent assumption that map reading is an easy task.

3) Researchers have focused on single types of map or symbolization. Comprehensive studies over a collection of different types of maps, especially across reference maps and thematic maps, are lacking. This void is partly because of the differences in reading and producing different types of maps. Researchers tend to focus on single types of map or symbolization to ensure depth of study.

This dissertation aims to explore the whole continuum of the map communication to map visualization spectrum, including the higher-level skills and knowledge required for some interpretations/uses of some maps, whether those maps are reference maps or thematic maps. To me, and to borrow MacEachren’s terminology, map literacy to date has been too focused on how well map readers understand presented knowns and too little attention has been focused on how map readers can reveal unknowns from their use of maps.

A Proposed Venn Model for Quantitative Map Literacy

This dissertation will classify the knowledge and skills for map reading based on multiple domains of literacy. As used in this dissertation, these domains are defined as:
Map Literacy (ML): the knowledge and skills involving the map directly. It should be noted that the term “Map Literacy” in the past has been used both in a broad sense and in a narrow sense. Ruggles’ (1975) initial definition, for example, was a narrow one, whereas the definition by Clarke (2003) was broad. For the purposes of the Venn model proposed here, “Map Literacy” includes the skills that one would need to work with maps, such as the knowledge and skills involving map elements (graticules, legends, color meanings, etc.) or map concepts (orientation, projection, type). Such skills could be at a high or low level as classified by Olson (1976) or Golledge et al. (2008). It is the usual meaning when most researchers mentioned “Map Literacy,” such as the one defined by Clarke (2003).

Quantitative Literacy (QL): as claimed in the previous section, Ganter’s (2019) definition of QL as “an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work” is accepted for this study. In the full range of potential applications, this set of knowledge and skills exceeds what is used to read maps, of course, which is why a Venn diagram as shown in Figure 1 is appropriate.

Geographic or Thematic Literacy (GL or TL): background knowledge that maybe necessary to read and interpret maps. Thematic Literacy predominates for thematic maps and refers to knowledge of the subject matter being mapped (e.g., cancer rates, gross domestic product, racial groups, election results, etc.). Geographic Literacy, which predominates for reference maps, mostly involves knowledge about locations (e.g., at world scale, where India is; at regional scale, where the City of Orlando is; at USF campus scale, where the parking facility is), or a property of the Earth (e.g., its shape; its size; its graticule). Like quantitative literacy,
background knowledge may or may not be directly related to the map. Some of the knowledge can be considered prerequisite, in the sense that map readers need to have it before using a map.

Geographic and Thematic background knowledge has often been overlooked, or given scant attention, in previous studies of map literacy (in the broad sense). In the map reading skills summarized by Board (1975), Morrison (1978), and Olson (1976), none of these authors mention background knowledge. Clarke (2003) did note that there is more to map reading skills than map elements and map concepts, although he didn’t include the relevant knowledge in the list of map reading skills summarized in his study.

The three domains of literacy overlap. Graticules, for example, were noted above as an example for both map literacy and geographic literacy (background knowledge). As the grid of latitudes and longitudes, the graticule is a basic concept of geography and therefore an element of the $GL$ set (note, italicized because it’s a set). As parallels and meridians are routinely shown on regional-, small-, and often large-scale reference maps, the graticule is an element of the set, $ML$, since a map user needs to be able to identify the symbology of these elements and interpret their meaning. Therefore it is knowledge that belongs in the intersection: $ML \cap GL$. There is an additional overlap, of course. Latitudes and longitudes are defined as angles, which mean they are quantities. Whenever they are used as quantities (numbers with units attached), they are elements of quantitative literacy as well. In cases where they are used without reference to a map, they are elements of $QL \cap GL$. Whenever they are used with a map, for example, to determine scale, distance or direction (e.g., azimuth), they are an element of the three-way intersection $QL \cap ML \cap GL$. In such cases, they are in $QL \cap ML$, because $QL \cap ML \cap GL$ is a subset of $QL \cap ML$; note, $QL \cap ML \cap GL$ is a subset of $QL \cap GL$ as well.
The concept of map scale may be a more familiar example. Map scale, as its name implies, is clearly an element of ML. Map scale is very specifically a ratio, and ratios are elements of QL. Thus map scale is an element of both QL and ML, i.e., QL ∩ ML. There are many other ratios, of course, which have nothing to do with maps, and these ratios are in QL\ML (verbalized as QL without ML). QL without ML is the same set as QL ∩ ML̅, the intersection of QL and the complement of ML, which can be also written as QL ∩ ML̅̅̅̅ or QL − ML. Some of these ratios may be basic knowledge about the Earth (without maps); the ratio of the Earth’s equatorial radius to the polar radius is an example. Such ratios would be in both QL and GL, hence QL ∩ GL; they would also be in QL ∩ GL ∩ ML̅̅̅̅, which is a subset of QL ∩ GL. Also, some ratios may not have anything to do with either maps or geography (e.g., the conversion ratio of kilograms to metric tons). Such ratios would be in QL\GL\ML (QL without GL and without ML), which can also be written as QL ∩ GL̅ ∩ ML̅. In the context of this dissertation, this set could be termed “pure quantitative literacy,” in contrast to QL for “full quantitative literacy”, or “total QL”, or “QL in its entirety”.

Similarly, there are skills/knowledge that are directly applied in the map reading process, but do not involve quantitative operation. For example, being able to orient the map (finding the north based on north arrow) belongs to ML\QL, i.e., ML without QL, (ML ∩ QL̅). Moreover, that activity does involve the concept of north, which is in GL, and so in the context of the three sets the action of orienting the maps is in ML ∩ QL̅ ∩ GL. However, if the action involves the calculation of azimuth or bearing, it then belongs to the overlap of all three, ML ∩ QL ∩ GL.

In the same way, thematic background knowledge comes into play in thematic maps, such as a map showing the distribution of cancer clusters. Knowing about cancer in general belongs in thematic literacy, TL, where the theme may be health or cancer, in general. It is
background knowledge about the theme of the map before actually using a map and without quantitative reasoning. When the issue narrows down to focus on a quantity such as cancer rates, it becomes an element of \( TL \cap QL \). More to the point of proactive literacies, when answering a question about cancer by calculating cancer rates, the person is using skills in \( TL \cap QL \). If the quantity is mapped and the map appropriately symbolizes the data, and then someone reads the map symbols for cancer clusters, the map reader is using skills in \( TL \cap ML \). If the map reader goes on and does some proactive analytical thinking such as making quantitative comparisons that test or investigate a hypothesis about the distribution of cancer clusters based on some characteristics of the map, the map reader is using skills in \( TL \cap ML \cap QL \) (at a level comparable to that of QR in the scheme of Piercey 2017 discussed in the review of QL above).

Obviously, a three-set Venn diagram (Fig. 6) can be used to visualize these relations between QL, ML, and GL and between QL, ML, and TL. Not counting the set that is the complement of them all – i.e., \( U \setminus (QL \cup ML \cup GL) \) for reference maps and \( U \setminus (QL \cup ML \cup TL) \) for thematic maps – there are seven subsets involved. From outside to inside, they are

- \( QL \setminus ML \setminus GL \) (pink)
- \( ML \setminus QL \setminus GL \) (blue)
- \( GL \setminus ML \setminus QL \) (yellow)
- \( (QL \cap ML) \setminus GL \) (purple)
- \( (ML \cap GL) \setminus QL \) (green)
- \( (QL \cap GL) \setminus ML \) (orange)
- \( QL \cap ML \cap GL \) (brown)

As defined by Xie et al. (2017), Quantitative Map Literacy (QML) is the union of purple and the brown. For the purposes of this dissertation, the first three subsets may, in places, be called “pure QL”, “pure ML”, and “pure GL” or “pure TL” (in the sense of uncombined with one or more of the other sets).
With respect to the purpose of this chapter, namely rethinking map literacy, it is relevant to note that ML is composed of four subsets here: the blue, the green, the purple and the brown of Figure 6. With a Venn diagram approach, it is thus possible to show the overlaps between different sets of knowledge and skills, and thereby more systematically represent the complexity in map reading that previous map literacy research has either not focused on or merely alluded to. In that vein, it worthwhile now to review previous studies involving map literacy relative to this Venn model.

![Three-set Venn Diagram Literacy Model](image)

**Figure 6.** Three-set Venn Diagram Literacy Model

Recall that Ruggles (1975) used the term “Map Literacy” to describe the ability of “knowing how to interpret cartographic conventions”. This definition is strictly limited to the
map reading skills that work with map concepts and map elements only. Therefore this definition belongs to the subset “pure ML” (ML\QL\GL).

For the map reading skills summarized by Morrison (1978) and Olson (1976), they included some quantitative skills. Although these quantitative skills were mostly at a very low level (such as counting and measuring), they did expand the scope of “Map Literacy” to include some knowledge and skills that belong to the intersection of quantitative literacy and map literacy. However, these authors did not mention background geographic or thematic knowledge. Therefore the “map literacy” defined by these researchers belongs to the subset of map literacy minus background knowledge (ML\TL) in the Venn diagram representation.

Clarke’s (2003) definition of “Functional Map Literacy” as "the ability to understand and use maps in daily life, for work and in the community” is as a broad and vague definition. Judged by the map reading skills summarized in his study, his definition at least was meant to include the quantitative skills noted by Morrison (1978) and Olson (1976). Furthermore, Clarke (2003) realized the role of background knowledge in interpreting maps. Although he didn’t explicitly include background knowledge in the map reading skills summarized in his study, his recognition of the importance of background knowledge does expand the scope of “Map Literacy” to include the overlap between background knowledge and map literacy. Despite his recognition of all domains, it is difficult to discern whether Clarke (2003) considered the interaction between the three domains of literacy (the center eye of the Venn diagram) based on the vagueness of the description of the map skills he reviewed.
Table 4. The Scope of Different Definitions of “Map Literacy” Shown by the Venn Model

<table>
<thead>
<tr>
<th>Definitions</th>
<th>Domain of Literacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Map Literacy” as “knowing how to interpret cartographic conventions” (Ruggles 1974)</td>
<td><img src="image1" alt="Venn Diagram 1" /></td>
</tr>
<tr>
<td>“Map Literacy” defined by the map reading skills that are summarized by Board (1975), Morrison (1978) and Olson (1976)</td>
<td><img src="image2" alt="Venn Diagram 2" /></td>
</tr>
<tr>
<td>“Functional Map Literacy” as &quot;the ability to understand and use maps in daily life, for work and in the community” (Clarke 2003)</td>
<td><img src="image3" alt="Venn Diagram 3" /></td>
</tr>
<tr>
<td>“Quantitative Map Literacy” as “the knowledge (concepts, skills and facts) required to accurately read, use, interpret, and understand the quantitative information embedded in geographic background” (Xie et al. 2018)</td>
<td><img src="image4" alt="Venn Diagram 4" /></td>
</tr>
</tbody>
</table>
Table 4 shows the evolving definition of “Map Literacy” and what kind of map reading knowledge and skills are included in these definitions according to the Venn Diagram Model. In this dissertation, QML is defined as an interdisciplinary study involving both quantitative literacy and map literacy, and it belongs to the subset or overlap between quantitative literacy and map literacy.

As mentioned before, the skills and knowledge involved in the map reading process can be quite different for different types of maps. Therefore the contents in each domain (sets of the Venn model) will change for different types of maps. In order to show how this proposed new model works in detail for different types of maps, the model can be discussed for two extreme conditions: pure reference maps and pure thematic maps.

**Literacy Model for Reference Maps**

For reference maps, the map literacy domain involves such concepts as knowledge and skills that relate to map elements and concepts such as map orientation, geographic coordinates, scales, projection systems. The knowledge and skills under the domain of map literacy are listed in Table 5, and classified into several subjects, from reading basic map elements to the application and evaluation of maps. The map reading skills and knowledge are summarized and rearranged from previous studies (Board 1975; Morrison 1978; Golledge et al. 2008). The subjects are listed in an order of increasing complexity based on Bloom’s Taxonomy (Anderson et al. 2001). Because the previous studies on map reading skills and knowledge didn’t describe the skills at the higher levels in detail, some of the knowledge and skills at these levels were taken from the body of knowledge in Geographic Information Science and Technology (GIS&T) summarized by DiBiase et al. (2006).
Table 5. Map Literacy for Reference map

<table>
<thead>
<tr>
<th>Map concept:</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>Orientation (being able</td>
<td>true north, magnetic north, grid north)</td>
</tr>
<tr>
<td>to distinguish true</td>
<td>north, magnetic north, grid north)</td>
</tr>
<tr>
<td>north, magnetic north,</td>
<td>Grid North)</td>
</tr>
<tr>
<td>Grid North)</td>
<td></td>
</tr>
<tr>
<td>Scale (being able to</td>
<td>read and use RF scale and bar scale, knowing how they affect</td>
</tr>
<tr>
<td>read and use RF scale</td>
<td>the map range)</td>
</tr>
<tr>
<td>and bar scale, knowing</td>
<td>how they affect the map range)</td>
</tr>
<tr>
<td>how they affect the map</td>
<td>range)</td>
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<tr>
<td>range)</td>
<td></td>
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<tr>
<td>Projections(knowing</td>
<td>what they are and how they work; knowing the categories of</td>
</tr>
<tr>
<td>what they are and how</td>
<td>projection systems; knowing some typical projection systems and</td>
</tr>
<tr>
<td>they work; knowing the</td>
<td>their characteristics; inferring the projection system of a</td>
</tr>
<tr>
<td>categories of projection</td>
<td>given map)</td>
</tr>
<tr>
<td>systems; knowing some</td>
<td>typical projection systems and their characteristics; inferring</td>
</tr>
<tr>
<td>typical projection</td>
<td>the projection system of a given map)</td>
</tr>
<tr>
<td>systems and their</td>
<td>characteristics; inferring the projection system of a given</td>
</tr>
<tr>
<td>characteristics;</td>
<td>map)</td>
</tr>
<tr>
<td>inferring the</td>
<td>projection system of a given map)</td>
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<tr>
<td>projection system of a</td>
<td>given map)</td>
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<tr>
<td>given map)</td>
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<td></td>
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<tr>
<td>Map symbol:</td>
<td></td>
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<tr>
<td>Legend (being able to</td>
<td>read legend and identify features on the map)</td>
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<tr>
<td>read legend and identify</td>
<td>features on the map)</td>
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<tr>
<td>features on the map)</td>
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<tr>
<td>Familiarity with common</td>
<td>common map symbols and color schemes.</td>
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<tr>
<td>map symbols and color</td>
<td>Familiarity with common map symbols and color schemes.</td>
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<tr>
<td>schemes.</td>
<td></td>
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<tr>
<td>Reference map symbolism</td>
<td>(understand common reference map symbolization schemes, such as</td>
</tr>
<tr>
<td>(understand common</td>
<td>land cover, elevation contour, etc.)</td>
</tr>
<tr>
<td>reference map symbolization</td>
<td></td>
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<tr>
<td>schemes, such as land</td>
<td>cover, elevation contour, etc.)</td>
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<tr>
<td>cover, elevation contour,</td>
<td></td>
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<tr>
<td>etc.)</td>
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<td></td>
<td></td>
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<tr>
<td>Map Interpretation:</td>
<td></td>
</tr>
<tr>
<td>Comparing and contrasting</td>
<td>the interpretation of landscape, geomorphic features.</td>
</tr>
<tr>
<td>the interpretation of</td>
<td>Matching features on a map to corresponding features in the</td>
</tr>
<tr>
<td>landscape, geomorphic</td>
<td>world.</td>
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<tr>
<td>features.</td>
<td>Identifying the landforms represented by specific patterns in</td>
</tr>
<tr>
<td>Matching features on a</td>
<td>contours on topographic maps)</td>
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<tr>
<td>map to corresponding</td>
<td></td>
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<td>features in the world.</td>
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<tr>
<td>Identifying the</td>
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<td>landforms represented by</td>
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<td>specific patterns in</td>
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<td>contours on topographic</td>
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<td>maps)</td>
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<td></td>
<td></td>
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<tr>
<td>Spatial Analysis:</td>
<td></td>
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<tr>
<td>Basic spatial pattern,</td>
<td>such as describing distributions (e.g. the morphological</td>
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<tr>
<td>such as describing</td>
<td>pattern on a topographic map), recognizing clusters (e.g.</td>
</tr>
<tr>
<td>distributions (e.g.</td>
<td>distributions of schools/hospital map in a city map).</td>
</tr>
<tr>
<td>the morphological pattern</td>
<td></td>
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<tr>
<td>on a topographic map),</td>
<td></td>
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<tr>
<td>recognizing clusters</td>
<td>Advanced spatial pattern: being able to tell the spatial</td>
</tr>
<tr>
<td>(e.g. distributions of</td>
<td>correlation between the features plotted on maps (e.g. the</td>
</tr>
<tr>
<td>schools/hospital map in</td>
<td>distribution correlation between attractions and hotels).</td>
</tr>
<tr>
<td>a city map).</td>
<td></td>
</tr>
<tr>
<td>Advanced spatial pattern:</td>
<td>being able to tell the spatial correlation between the features</td>
</tr>
<tr>
<td>being able to tell the</td>
<td>plotted on maps (e.g. the distribution correlation between</td>
</tr>
<tr>
<td>spatial correlation</td>
<td>attractions and hotels).</td>
</tr>
<tr>
<td>between the features</td>
<td></td>
</tr>
<tr>
<td>plotted on maps (e.g.</td>
<td></td>
</tr>
<tr>
<td>the distribution</td>
<td></td>
</tr>
<tr>
<td>correlation between</td>
<td></td>
</tr>
<tr>
<td>attractions and hotels).</td>
<td></td>
</tr>
<tr>
<td>Evaluation and Testing:</td>
<td></td>
</tr>
<tr>
<td>Knowing the</td>
<td></td>
</tr>
<tr>
<td>generalization and</td>
<td></td>
</tr>
<tr>
<td>distortion in reference</td>
<td></td>
</tr>
<tr>
<td>maps, being able to</td>
<td></td>
</tr>
<tr>
<td>explain why and how</td>
<td></td>
</tr>
<tr>
<td>that happens.</td>
<td></td>
</tr>
<tr>
<td>Being able to choose the</td>
<td>correct map for certain purpose (e.g. using Mercator Projection</td>
</tr>
<tr>
<td>the correct map for</td>
<td>Map for compass based navigation, while Gnomonic Projection</td>
</tr>
<tr>
<td>certain purpose (e.g.</td>
<td>Map for aviation navigation).</td>
</tr>
<tr>
<td>using Mercator Projection</td>
<td></td>
</tr>
<tr>
<td>Map for compass based</td>
<td></td>
</tr>
<tr>
<td>navigation, while</td>
<td></td>
</tr>
<tr>
<td>Gnomonic Projection Map</td>
<td></td>
</tr>
<tr>
<td>for aviation navigation).</td>
<td></td>
</tr>
<tr>
<td>Being able to criticize</td>
<td>a map.</td>
</tr>
<tr>
<td>a map.</td>
<td></td>
</tr>
</tbody>
</table>

Quantitative literacy contributes an overall framework for problem solving through quantitative reasoning, which brings to bear critical and creative thinking, calculation, and communication. Quantitative literacy has been well defined and studied in many aspects. Recall that Sons (2019) summarized 24 subjects in the quantitative literacy studies, but not all of them are closely related or applied in map reading. Some of the topics that are closely related to map reading and interpretation are listed in Table 6.
Table 6. Quantitative Literacy for Reference Maps

<table>
<thead>
<tr>
<th>Arithmetic:</th>
<th>Calculation of ratio, commonly applied in calculating distance with scales, or slope with contour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculation of distance and area based on geographic coordinates</td>
</tr>
<tr>
<td></td>
<td>Estimation of distance with scale bar, slopes with contours</td>
</tr>
</tbody>
</table>

| Geometry:  | Measurement (length and area)                                                                    |
|           | Units and conversion                                                                             |
|           | Angles: calculation and conversion between bearing and azimuth.                                   |
|           | Shapes (triangle, rectangle, circle) and relevant calculation on maps.                            |
|           | Error and distortion calculation from projections                                                |

| Statistics:| Counting geographic features.                                                                    |
|           | Spatial correlation, measuring and evaluating the relationship between two geographic features.  |

Geographic literacy is a broad term concerning the problem solving and decision making based on the understanding towards Earth system. In this study, it is generally referred as the geographic background knowledge that helps map users better understand the map. Such knowledge includes the concepts of latitude and longitude, the shape of the Earth, and familiarity with the location of places, with a more complete list given as Table 7.

Table 7. Background Knowledge for Reference Maps

<table>
<thead>
<tr>
<th>Geographic Concept:</th>
<th>Spherical Earth, and the relevant knowledge based on it (radius, perimeter, etc.).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Background knowledge of important geographic concepts (equator, North and South Pole, Tropics of Cancer/Capricorn, etc.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Locations:</th>
<th>Being able to describe the location of certain geographic feature (e.g. where is China?).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Being able to tell the relative location of two geographic features (e.g. is India south or north of China)</td>
</tr>
<tr>
<td></td>
<td>Being able to estimate the geographic coordinate of certain geographic feature (e.g. what is the latitude and longitude of Greenwich?)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geographic Coordinates:</th>
<th>Latitude and longitude.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UTM coordinates, State Plane etc.</td>
</tr>
</tbody>
</table>

| Geographic Background: | Necessary background knowledge that could help interpret the map, such as culture, economy, population, climate, topography, landmark, etc. |
In the proposed Venn model, and as noted above, some map reading tasks require map reading knowledge and skills from more than one domain. For example, calculating the distance based on a scale bar would require quantitative skills about ratio and scale, which belong to the domain of quantitative literacy, and relevant knowledge about the map element itself, which belongs to the domain of map literacy. Similarly, using geographic coordinates (latitude/longitude, UTM, etc.) to calculate distances or areas would require arithmetic.
calculation, which belongs to the domain of quantitative literacy, but also relevant knowledge about the structure of the coordinate grid, which belongs to the domain of geographic literacy, and the reading of coordinate grid lines through a legend, which belongs to the domain of map literacy. The Venn diagram model for reference maps is illustrated in Figure 6. In order to clearly indicate the map reading knowledge and skills across the Venn model, especially those within the overlaps between different domains of literacy, some examples are shown in Table 8 in conjunction with Figure 7. This table is by no means exhaustive, and more detailed examples for specific maps can be found in Chapter 4 of this dissertation.

### Table 8. Examples of Map Reading Tasks and Skills for Reference Map

<table>
<thead>
<tr>
<th>Task Description</th>
<th>Map Literacy</th>
<th>Quantitative literacy</th>
<th>Geographic Literacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading map element based on legend.</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculating distance based on maps scale, or slope based on contour line on map.</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Calculating distance based on given coordinate</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Estimating the radius of the Earth. Calculating the length of given parallel.</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Finding routes between two locations and then comparing their distances</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Looking for certain landscape or geomorphology on map</td>
<td>×</td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Inferring about the projection system applied to the map and knowing its pros and cons</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculating the distortion rate along parallels on the World Map</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Read the map area on a topographic map, calculating the length and area on earth based on latitude and longitude</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>
Literacy Model for Thematic Maps

The map reading skills and knowledge can also be classified into three domains of literacy for thematic maps. As indicated in the literature review, studies of the map reading knowledge and skills for thematic maps are very limited, and so the map reading knowledge and skills listed in this section are inferred from the research topics of thematic literacy discussed in the literature review, or from similar skills found within reference map literacy.

For thematic maps, some map reading skills for traditional reference maps, such as map scales and map projection systems, are less important than before. This is because in thematic maps, the sizes or shapes of geographic units are not as important as they are in reference maps, as long as geographic units can be recognized. In fact, in the extreme case, cartograms, the sizes and shapes of geographic units may be totally distorted based on data values. As indicated from the studies on thematic map types (Peterson 1979; Wainer and Francolini 1980; Olson 1981; Eyton 1984; Slocum, et al. 1990; Nelson 2000; Sun and Li 2010; Nusrat et al. 2018) and thematic map symbolizations (Flannery 1971; Cox 1976; Garner 1977; Gill 1988; Brewer and Campbell 1998; Nelson 2000; Edwards and Nelson 2001), map concepts and map symbols are still an important subject in thematic map literacy. However, the skills and knowledge under the domains of thematic map literacy deal with thematic map elements and symbolizations, such as thematic map symbols, statistical methods in map making. Some of the skills and knowledge can be pure ML (not intersecting with QL or TL), such as simply reading the color on choropleth map. Some may belong to the subset within QL or TL, such as statistical analysis on the thematic data ($QL \cap ML$) or identifying and analysis of the location or clusters based on the character of
thematic data. Specifically speaking, the topics in the domain of map literacy for thematic maps are listed in Table 9:

Table 9. Map Literacy for Thematic Maps

<table>
<thead>
<tr>
<th>Map concept:</th>
<th>Map symbol:</th>
<th>Evaluation and Testing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Types of thematic maps (choropleth maps, isopleths maps, dot maps, etc.): knowing what they are, what they are good for, and what their limitations are.</td>
<td>Legend (being able to read legend and distinguish the range of data value) Familiar with common thematic map symbols and color schemes. Thematic map symbolization, (understand common thematic map symbolization scheme, such as proportional circle, graduate circle, etc.) being able to read them on the map;</td>
<td>Knowing the generalization in thematic map data, knowing the dangers of interpreting the data. Being able to criticize a thematic map.</td>
</tr>
</tbody>
</table>

In terms of the domain of quantitative literacy involved in thematic map reading, the skills shift from arithmetic and geometric calculation of geographic features to the statistical processes involved in using thematic data. Some of the skills may better fit in the domain of statistical literacy (Schield 2004, 2010). As indicated in the literature review, the quantitative skills involved in traditional statistical graph reading have been well studied and documented. But these studies seldom included the statistical data in thematic maps, and quantitative skills involved in thematic map reading are also an apparent blind spot in cartography-based studies. In this study, the quantitative skills in thematic map reading are summarized in two different ways: 1) by examining the quantitative process in thematic map reading studies; 2) by applying similar quantitative skills involved in statistical graph reading to the map domain. Some skills are listed as follow in Table 10. This list is by no mean exhaustive, since new quantitative processes could be applied to thematic mapping.
Table 10. Quantitative Literacy for Thematic Map

<table>
<thead>
<tr>
<th><strong>Arithmetic:</strong></th>
<th>Counting.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic calculation of thematic data, such as subtracting the data value of two features to find difference, calculating the amount of data value per unit area</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Statistics:</strong></th>
<th>Knowing how thematic data it is classified through data classification methods, being able to calculate the breaks, being able to infer about the data classification method applied in the map by reading the legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing how thematic data is processed through advanced statistical method in thematic mapping (data smoothing method, kriging method).</td>
<td></td>
</tr>
<tr>
<td>Estimation of data distribution with map by reading the values and comparing with data classification methods</td>
<td></td>
</tr>
<tr>
<td>Spatial correlation, measuring and evaluating the relationship between two variables.</td>
<td></td>
</tr>
<tr>
<td>Knowledge of linear regression.</td>
<td></td>
</tr>
</tbody>
</table>

The domain of background knowledge changes dramatically for thematic maps, because the geographic knowledge of locations and shapes of geographic features is less important in thematic map reading, while the knowledge about the theme or topic presented in the map becomes very important. For example, map users need to know about the rules of elections to correctly interpret an election outcome map, or the knowledge about certain diseases to interpret maps of cases or rates.

Table 11. Background Knowledge for Thematic Maps

<table>
<thead>
<tr>
<th><strong>Locations:</strong></th>
<th>Being able to identify the geographic features in thematic maps</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th><strong>Knowledge of the Theme:</strong></th>
<th>Necessary background knowledge about the theme of the thematic map, this knowledge would change based on the theme of the map, examples include but not limited to: rules of the election as to election map; definition of sudden mortality (SMR) rate as to a SMR map; knowledge about the cause of disease as to a cluster map of certain disease; climate pattern its causes as to global climate map; origin and migration of population as to a racial group map; origin and preach of religions as to religions group map</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th><strong>Knowledge Related to the Theme:</strong></th>
<th>Background knowledge that is not exactly the theme of the map, but is related to the theme and is helpful to interpret the map. For example, knowing the population, racial distribution, education level is helpful to interpret a household income map.</th>
</tr>
</thead>
</table>
Obviously, there are also overlaps between different domains of literacy. For example, understanding the advantages and limitations of different types of data classification methods or thematic mapping methods belongs to the domain of thematic map literacy. A breakdown of skills within each domain of literacy and the interrelationships between the three domains of literacy are shown as the three-set Venn diagram in Figure 7. Similarly, some typical examples of map reading tasks within the overlap sections between different domains of literacy are shown in Table 12 in conjunction with Figure 8. This table is by no means exhaustive, and detailed examples for specific maps can be found in Chapter 4 of this dissertation.

![Three-set Venn Diagram Literacy Model for Thematic Map](image)

**Figure 8.** Three-set Venn Diagram Literacy Model for Thematic Map
<table>
<thead>
<tr>
<th>Task</th>
<th>Map Literacy</th>
<th>Quantitative literacy</th>
<th>Thematic Literacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading data value with legend</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the theme of the map? What might cause spatial variability?</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Judging by the legend, what is the data classification method? How</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>is it calculated?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What is the thematic data presented in the map? How is it calculated?</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>What is the thematic symbolization used in the map? Is it appropriate</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>to present the thematic data?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe the spatial correlation between variables in a multi-variable</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>map. What is the cause of the correlation?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count the value of an attribute on categorical map (e.g. election</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>map) to make inference about the thematic data (which party is likely</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>to win the election</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe the spatial distribution pattern of the thematic data. Infer</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>about the reason for the spatial distribution pattern.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Discussion**

Although the two types of three-set Venn diagram models are illustrated separately for reference maps and thematic maps in this section, these two models are not irrelevant to each other. As indicated in the triangular plot for different types of maps that forms the content of the next chapter and which is published as Xie et al., 2018, there is no clear boundary between
thematic maps and reference maps. As the focus of maps switch from locational information to thematic information, the knowledge and skills in the literacy model switch from processing geographic features to thematic data. One way to understand this relation is that "Geographic Literacy" is the "Thematic Literacy" for reference maps because the theme or topics for reference maps are the geographic features.

In the triangular plot of various types of maps, there are also maps that locate in the intermediate zone between reference maps and thematic maps (such as land use maps, geological maps, etc.). Here, the two types of information, locational and thematic, co-mingle and the importance of each relative to the task at hand will determine the types of knowledge and skills required. In the literacy model for these kinds of maps, the domain of literacy represented by the top set shown in the Venn diagram should be a combination of "Geographic Literacy" and "Thematic Literacy".

In the next chapter, the model of the triangular plot that is used to describe different types of maps will be introduced in detail. Further, in Chapter 4, the map reading skills and knowledge associated with different types of maps will be examined relative to both the Venn-diagram based literacy model developed in this chapter, as well as the triangular plot of map types from chapter 3. Essentially, the Venn model introduced in this chapter is a tool to describe different types of map reading skills and knowledge, while the triangular plot described in the next chapter is a tool to describe different types of maps. With the help of the both models, we can develop a clear perspective of the knowledge and skills involved in the map reading process for different types of maps.
CHAPTER THREE:

TRIANGULAR PLOT OF MAPS¹

Introduction

This paper is the first in a series in which we discuss the concept of quantitative map literacy and how it applies to quantitative literacy (QL) in general. We define *quantitative map literacy* as the knowledge (concepts, skills and facts) required to accurately read, use, interpret, and understand the quantitative information embedded in a geospatial representation of data on a geographic background. The advent and increasingly widespread use of desktop geographical information systems (GIS) and other mapping software over the past 20 years has revolutionized and democratized the production of maps to the point where they are now commonplace in newspapers and periodicals. Maps greatly impact the overall literacy of our citizenry on major issues. As such, we believe a systematic consideration of quantitative map literacy is both significant and timely. Of course, there is a long history of studying the communication and understanding of quantitative information in graphics (e.g., Tufte 1992, 2001), but, as “graphics” that represent *geographic* or *geospatial information*, maps have specific attributes that cry out for separate study. Not surprisingly, quantitative reasoning with maps has started to draw the attention of others in the QL community (e.g., Jungck 2012; Perez et al. 2015).

¹ Portion of this chapter has been previously published in *Numeracy*, 2018, Volume 11, Issue 1, Article 4, and have been reproduced with the permission included in Appendix E.
A foundational – and self-evident – concept of this series of articles is that the constitution of quantitative map literacy varies with the characteristics of a map. These characteristics are linked to map purpose. The characteristics and purposes of maps lead to a diversity of map types. As we began our research into quantitative map literacy, we became aware that, to the best of our knowledge, a systematic map classification scheme for map types, on which we might base our research, is lacking. Cartographers have certainly defined various types of maps, but generally they fall into broad categories that fail to accommodate the nuances of different map characteristics that we think are significant. Accordingly, our goal in this first paper is to produce a general framework within which key map characteristics can be used as the basis for discussion in subsequent papers of how quantitative map literacy varies with the balance and level of these key characteristics as we see them. We wish to emphasize that we are not proposing a new map classification per se, but rather a system for thinking about quantitative information communicated by maps; more specifically, we envisage maps positioned within a bi-directional continuum based on three characteristics (locational information, thematic information, and generalization/distortion) that, we believe, will help us think about quantitative map literacy.

**Map Classification**

In cartography, map types are generally defined and named based on purpose and theme. Some example quotations illustrating these definitions are (italics added):

1) “Another name commonly applied to the general purpose map is reference map” (Robinson 1975). “Such maps customarily display objects (both natural and man-made) from the geographic environment. The emphasis is on location, and the purpose is to
show a variety of features of the world or a portion of it” (quotation from Robinson and Petchenik 1975, as found in Dent et al. 2009, 6).

2) “Maps that show the shape and elevation of terrain are generally called topographic maps” (Campbell 1993, 9).

3) “Cadastral maps show how land is divided into real property, and sometimes the kinds of built improvements” (Harvey 2008, 13).

4) “Thematic maps (or statistical maps) are used to emphasize the spatial pattern of one or more geographic attributes (or variables), such as population density, family income, and daily temperature maximums” (Slocum et al. 2009). The thematic map’s “main objective is specifically to communicate geographical concepts such as the distribution of densities, relative magnitudes, gradients, spatial relationships, movements, and all the myriad interrelationships and aspects among the distributional characteristics of the earth’s phenomena” (Robinson and Sale 1969, 10-11).

5) “The choropleth map is a common map type for mapping data collected in enumeration units. Each unit, such as a county, state, country, province, is shaded according to a variable or attribute, such as population density” (Dent et al. 2009, 8). “Choropleth mapping is performed by mapping spatial data that are constrained to lie within a bona fide administrative unit. The administrative unit may be based on political jurisdictions such as cities, counties, states, countries, school districts, emergency response districts, and tax zones, etc.” (Jensen and Jensen 2013, 306).

6) “Cartograms, both contiguous and non-contiguous, show quantitative difference by altering the size of the geographic units according to the relative proportion of the geographic unit’s
property” (Harvey 2008, 213). “Cartograms are created by substituting a different standard of measurement (time or cost, for example) for the distance measurements customarily used. When this is done, sizes, shapes and distances as we normally think of them are modified or distorted” (Campbell 1993).

According to the prominent cartographer B.D. Dent, maps can first be classified according to the media used (Dent et al. 2009). He identifies mental maps, tangible maps, and virtual maps (Fig. 9). Mental maps are “developed in our minds over time by the accumulation of many sensory inputs, including tangible or virtual maps” (Dent et al. 2009). Within the realm of maps produced on tangible or virtual media (our focus), two major types of maps are identified: general purpose (reference maps) and thematic maps (Dent et al. 2009). Reference maps focus on displaying location information about geographic features, i.e., geographic features that are tangibly located on Earth (such as rivers, roads, and political boundaries). In contrast, thematic maps are made to display attribute information or data, i.e., quantitative or categorical data that cannot be directly observed on Earth (such as population, income, house prices, electoral voting, and the like; i.e., the types of maps increasingly found in the print and online media through the democratization of map production enabled by GIS).

![Figure 9. A Taxonomy of Map Types (redrawn from Dent et al. 2009)](image-url)
Obviously there is a tradition of classifying maps into two major types. This classification of maps as either “reference” or “thematic,” however, generally reflects their primary purpose, and so what becomes obscured through such a classification is that maps generally contain a mixture of locational information and thematic information. We argue that the types and balances of quantitative concepts and skills vary across this mixture and, therefore, a more nuanced organizational framework that recognizes a continuum is needed. Also, and just as importantly, we point out that maps occupying similar locations along this continuum between locational and thematic information endpoints may vary substantially in terms of accuracy and precision. This variability in accuracy and precision is caused by effects of geographic scale, map projection, locational and thematic data generalization, and locational and thematic data distortion. Such concepts also impact the types and balances of quantitative concepts and skills required and, therefore, we propose that a second dimension to our organizational framework for maps, for the purpose of quantitative map literacy, be added to represent the amount of generalization and/or distortion present in the map product.

**Triangular Plot and Its Application**

Given these two dimensions – (1) a ratio of locational information to thematic information, and (2) a level of generalization and/or distortion – we propose to adopt a triangular, or ternary, plot to visualize our organizational framework. Similar plots are commonly found in fields such as physical chemistry, petrology, mineralogy, pedology/edaphology, and metallurgy, among other natural and applied sciences (Pettijohn, 1957; Derringh, 1998), where the plots are used to graph the mixtures of three different
components (or endmembers). Figure 10a shows a classic example of a triangular plot applied to sedimentary deposits (e.g., as in a beach or a river bar). The three endmembers are mud, sand, and gravel. Locations and boundaries within the triangle reflect, in this case, the two dimensions of sand-to-mud ratio and percent gravel.

![Figure 10](image)

**Figure 10.** Examples of Triangular Plots: (a) Folk's Classification System of Sediment Types (Folk 1974) (from Poppe et al. 2005); (b) A Typical Triangular Plot with Edge-parallel Grid Lines.

By way of explanation, every location inside a standard triangular plot represents the percentage composition of the three endmembers shown at the vertices. The featured dot in Figure 10b, for example, has the composition 60% A, 10% B, and 30% C. The three legs of the triangle are each a binary mixture; for example, points along the BC leg are mixtures of B and C. The grid lines are labeled 0% to 100% along each of the legs of the triangle from one endmember to the other, generally in an overall clockwise direction. Thus along the BC leg, the labels go from 0 at C to 100 at B, meaning that the percentage of endmember B in the binary mix increases from 0% at C to 100% at B. The three legs each represent the 0% contour for the percentage of the opposite-vertex endmember in the three-way mix. Each of the other grid lines
is a contour of the percentage of the opposite-vertex endmember; for example, the three highlighted gridlines in Figure 10b are the (horizontal) A=60% contour, the (upper-left to lower-right) B=10% contour, and the (lower-left to upper-right) C=30% contour. (For more details, see Wainer 1997 and Vacher 2005). The contour lines discussed above can be termed iso-percentage lines. In addition to these contour lines, iso-ratio contours (Fig. 11) can be useful. A line from one vertex to the opposite leg (from A to BC in Figure 11) represents a constant ratio of the percentages of the two endmembers of the opposite leg (i.e., the line is a contour of B:C). In the case of the highlighted iso-ratio contour shown in Figure 11, the B:C ratio is 60% to 40%, or 3:2, which can be read on the scale along BC. The same ratio applies at the two featured dots on the same figure. Thus the composition at the upper dot is 50% A, 30% B, and 20% C, and the composition at the lower dot is 30% A, 42% B, and 28% C.

Figure 11. An Iso-ratio Contour in a Triangular Plot (B:C = 3:2).
These two types of contours provide two schemata for dividing a triangular plot into regions. Iso-percentage lines, which run parallel to an edge of the triangle (Fig. 12a), emphasize the percentage of a single endmember; iso-ratio lines, which extend from a vertex to an opposite edge (Fig. 12b), emphasize the ratio of two endmembers. In the case of Figure 10a, both these types of line were used to demarcate regions, which then produced a classic typology of sedimentary rocks (Folk 1974).

**Figure 12.** Two Ways of Dividing Triangular Plots Corresponding to the Two Different Types of Contours: Iso-percentage Lines (left); Iso-ratio Lines (right).

**Proposed Triangular Plot for Maps**

In order to design a triangular plot for maps, we propose three conceptual endmembers: “locational information,” “thematic information,” and “generalization and/or distortion” (see Fig. 13). Similar to the Folk (1974) plot (Fig. 10a), we also propose two dimensions (hereafter referred to as parameters). The first is the ratio between locational information and thematic information (the “locational-to-thematic” or “L/T-ratio parameter”), and the second is the level
of generalization and/or distortion (the “generalization-distortion” or “G-D parameter”). Any map can be plotted within our triangle based on an assessment of these two parameters, with placement using the iso-ratio lines for the L/T-ratio parameter and the iso-level lines for the G-D parameter. The assessment proceeds by first considering the L/T-ratio and then, subsequently, the G-D level.

![Proposed Triangular Plot for Thinking about Maps](image)

**Figure 13.** Proposed Triangular Plot for Thinking about Maps.

We are proposing this framework for maps only because it meets our needs for consideration of quantitative map literacy. We have not formally considered the more general usefulness of our scheme within cartography, although we would certainly welcome scholarly consideration of that by others. In this regard, and unlike the Folk (1974) plot of Figure 10a, we note that we are not proposing to identify and label regions of our triangle for specific map types based on iso-ratio and iso-level line boundaries, although we do recognize that it would be an interesting (and, no doubt, provocative) exercise for future work. For the purpose of thinking about quantitative map literacy, we are more interested in examining how maps and attendant
QL skills vary as one moves across the triangle defined by our two parameters (and three endmembers).

We also do not intend to be dogmatically prescriptive as to where any particular map may be placed in the triangle. We anticipate that different individuals would place maps in different locations, perhaps reflecting their own intentions and use of map products based upon their own (or cultural) biases and context. We do suggest, however, where we believe some maps types may lie in the triangle in the subsequent section to this paper, but even as authors we had slightly different viewpoints as we worked through our examples.

Regarding the L/T-ratio parameter, the key aspect to consider is the purpose of the map, and that issue can be complicated. The question of map purpose involves an interplay between the mapmaker’s intended purpose for the map and the actual interpretation or purpose bestowed on it by the user. Because thematic maps actively communicate a message, the intention (of the mapmaker) and the interpretation (by the user) are typically more congruent with regard to purpose (though not necessarily effectiveness). With locational maps, the intent of the mapmaker is to present message-free reference data which the user can put to many different purposes, including ones that the mapmaker may not have thought of (although, by the very nature of such maps, these purposes are largely constrained to involve the locational properties of the features presented).

Tufte (1992, 2001) states that a graph should display meaningful information, and all other graphic features are “graph junk.” For example, readers are probably familiar with historic exploration maps where presumed, but unexplored, lands or seas were often adorned with elaborate symbols such as serpents, dragons, and the like. Although in some cases such symbols were suggestive of what “themes” lay beyond (e.g., a buffalo in the American West) and
therefore somewhat informative, in many cases they would qualify as “graph junk.” Similarly, a modern thematic map could include a large amount of locational information in the form of geographic features, such as administrative boundaries, highways, river systems and so on, which provide reference, but which are not germane to the main theme (purpose) of the map. We may not consider such features as “graph junk” necessarily, but the ratio of locational to thematic information would certainly be tilted towards the thematic. Our point, in other words, is that it is not the sheer quantity of raw information presented that is important (i.e., the “ink” in Tufte’s vernacular) but rather the importance of that information to the map’s purpose (the “meaning”).

It is also important to note that the G-D level considers the generalization and distortion in both the locational information and the thematic information simultaneously. For locational data, generalization can take the form of simplified feature representation, feature selection, feature classification, and so on. Distortion of locational data derives from such processes as map projection and the deliberate offset of features to accommodate their display without conflicting with other map components. For thematic data, on the other hand, generalization may reflect more-reductionist data-classification schemes, such as aggregation of data to coarser geographic features, as in spatial hierarchies of political or administrative units, or simply coarser data classes. Distortion in thematic data may result from data simplification or the intentional choice of a certain thematic data-classification scheme over another. In this latter regard, it should be noted that distortion may be benign and simply result from well-intentioned processes such as accommodating coincident spatial features or simplifying data for easier interpretation (e.g., rounding numbers), or it may be quite intentional as in the case of the manipulation of a thematic data-classification scheme to influence the interpretation of the map (e.g., Monmonier 1996).
Similarly, it should also be noted for the locational data case that the choice of map projection, though often benign or simply uninformed, is sometimes intentional. A good, historically familiar example would be the production in the United States of politically expedient (read propaganda) “Cold War” maps using the World Mercator projection – a projection which “conveniently” exaggerates the areal extent of high-latitude land countries (such as the “menacing” Soviet Union) and relatively diminishes the areal extent of lower-latitude countries such as the United States and countries that lie in the region of the equator.

In some cases, generalization and distortion are related. One example already mentioned is where thematic data may be generalized to different levels of (say) areal units (counties, states) and, yet, this generalization obviously changes the portrayal of locational data too. An even more-stark example would be the cartogram. In a cartogram, locational data are deliberately distorted so that some geometric attribute of the locational data is made directly proportional (thus sensitive) to the value of a thematic attribute (e.g., where the geographic areas of states directly reflect their numerical populations). In a cartogram, therefore, locational data are intentionally distorted to give a more-accurate (i.e., less generalized) portrayal of the thematic attribute. An interesting question in this regard then becomes how would we assess the overall G-D parameter for a cartogram? We argue that this assessment needs to take place with the purpose of the map in mind. Since the main purpose of a cartogram is to better communicate the thematic attribute in a non-generalized way, the fact that we may greatly distort the locational data is comparatively less important. Consequently, in our view, the cartogram has a low level of generalization-distortion given the relative weighting of thematic vs. locational importance. We take this position despite how counter-intuitive it may seem at first sight (which, shall we say, was one of our own internal points for lively discussion during our analysis).
Given these observations, we propose that “assessment” of the level of the G-D parameter use the following conceptual equation (a weighted average):

\[ Z = Z_L \cdot \frac{L}{L + A} + Z_T \cdot \frac{A}{L + A} \]

where \( Z \) is the “assessment” of overall generalization-distortion; \( Z_L \) is the generalization-distortion “assessment” for the locational information; \( Z_T \) is the generalization-distortion “assessment” for the thematic information; \( L \) is the “assessment” of the importance of the locational information to the map’s purpose; and \( T \) is the “assessment” of the importance of the thematic information to the map’s purpose. Determination of \( Z \) thus depends on the prior assessments of \( L \) and \( T \) for the first parameter of our two-dimensional (two-parameter) scheme discussed above. Obviously we do not advocate strict quantitative evaluation of the variables in this equation (which is why we use the quotation marks for “assessment”). The equation is meant to be a conceptual model to have in mind as one thinks about the location of a given map in the triangular plot we are proposing. (Not surprisingly, perhaps, the equation itself was another locus of lively discussion).

**Maps across the Triangle**

As stated, maps can be located on the triangular plot by considering two parameters: the ratio of locational information to thematic information (the “L/T ratio”) and the level of map generalization and/or distortion (the “G-D level”). We contend that similar maps tend to be located within similar areas on the triangular plot. Moreover, the differences and connections between maps can be instructively discussed by examining trends across the triangular plot.

On the left side of the triangle, maps are generally focused on the provision of locational information. The maps portrayed in Figure 14 occupy an “iso-ratio wedge” of similar, relatively
high, L/T ratios. However, as the map scale decreases (large-scale [small area of coverage] to small-scale [large area of coverage]), the maps move upwards within the wedge to reflect higher G-D levels of generalization-distortion. Note that although the maps in Figure 14 have similar L/T ratios that reflect the map purpose, the two larger-scale maps (c, d) are located more to the right within the wedge. This shift reflects the fact that the thematic content, and importance, is greater in the larger-scale maps.

**Figure 14.** Examples of Maps at Different G-D Levels in a High-L/T Wedge of the Map Triangle: (a) World Map (United Nations 2010); (b) Political Map of Europe (Ssolbergj 2009); (c) 15×15 minute USGS Topographic Map of Dragoon, AZ. (U. S. Geological Survey 1958); and (d) 7.5×7.5 minute USGS Topographic Map of Steele Hills, AZ. (U. S. Geological Survey 1996).
Conversely, maps on the right side of the triangle (relatively low L/T ratios) are generally focused on the communication of thematic information (Fig. 15). The amount of generalization and/or distortion in thematic maps is mostly driven by choices in how the data are represented (e.g., data classification). For example, the three maps in Figure 15 are made with the same data: the population of counties in the state of New York. The two choropleth maps (Figs. 15a and 15b) differ in the granularity of the classes: three classes in Figure 15a (high G-D), and seven classes in Figure 15b (lower G-D). The map with seven classes obviously displays a less-reductionist representation of the thematic data.

**Figure 15.** Examples of Maps at Different G-D Levels in a Low-L/T Wedge of the Map Triangle: (a) Population of New York State by County, 2010 (Three Data Classes); (b) Population of New York State by County, 2010 (Seven Data Classes); (c) Population of New York State by County, 2010 (Cartogram). (Data source: United States Census Bureau)
Figure 15c, on the other hand, is a cartogram of the county population data, where the geometries of the geographic features (counties) are highly distorted so that their areas are proportional to the values of a specific “attribute” (the GIS term for the thematic data). Although the locational information is highly distorted, the representation of the thematic data is now a direct translation of the numeric value of the attribute so it is less generalized than the data representation of the choropleth maps of Figures 15a and 15b. Therefore, according to our conceptual equation, the cartogram is placed lower in the L/T wedge because the distortion in the locational information has little weight for maps with a low L/T ratio, and, as stated, there is minimal generalization of the thematic data. Note that although the maps in Figure 15 have similar L/T ratios that reflect the map purpose, the choropleth maps do communicate the location of the thematic data in a more direct way, and so they place a little more to the left within the wedge. Meanwhile, in addition to its proportional representation of population by geographic area, this particular cartogram also represents the same data using five classes. This double representation affords different uses and interpretations of the thematic data and so increases its importance relative to locational information.
Maps located at a medial position in the L/T spectrum (Fig. 16) represent situations where the communication of locational and thematic data is more equally weighted. An example would be a land use map, in which the location information (location and geometry of land parcels) and thematic attribute information (land use types) are both potentially important. Figure 16a is a land use map of downtown Tallahassee, and Figure 16b is a land use map of Tallahassee-Leon County. Relative to the city-scale map, the county-scale map has geographic boundaries that are more generalized and distorted; additionally, the land use data are

**Figure 16.** Examples of Maps at Different G-D Levels in a Medial-L/T Position the Map Triangle: (a) Land Use Map of Downtown Tallahassee; (b) Land Use Map of Tallahassee-Leon County. (Data Source: Tallahassee County Planning Department.)
generalized into fewer categories. As a consequence, the county-scale map is placed at a higher G-D level.

Note that we have placed these two maps directly on the same iso-ratio line of locational to thematic information. This placement reflects the fact that, regardless of scale, the relative importance of the locational and thematic information is likely to be more determined by the immediate (and changing) user-defined uses of the maps rather than the inherent design characteristics of the maps themselves. In this regard, recall our earlier discussion of map purpose as an interplay between the intent of the mapmaker and the use and interpretation of the map by the map user. The positioning of maps along the medial line of the L/T cross-triangle spectrum should therefore be interpreted as a “pivot zone” where in actual usage of the map sways the map to either the locational or the thematic side. This greater focus on the users and usage contrasts somewhat with the maps discussed previously, where the content itself (locational or thematic) more clearly impacts the purposes and interpretation for users.

Figure 17 shows a cross-L/T band for maps of a similar G-D level. The subway map (Fig. 17a), although highly distorted geographically, remains a predominantly navigational (locational) tool, albeit one that relies on the topology of features (the connectivity of stations/lines) rather than their geometric properties of absolute location. The airline route map (Fig. 17b) is similarly, though relatively less, distorted; it is not a navigational tool but, rather, a means to convey the thematic information of flights between cities. To be sure, there is the implication of connections,” particularly if “hub” cities are identified, but the central purpose of the map is not for someone to be able to plan a navigable route across country, in the same way a subway map user does to cross a city. The purpose of the driving-time map (Fig. 17c), on the other hand, is to convey the thematic information of “driving time.” The map is perhaps more
generalized than distorted, and the locational generalization takes the form of the cities chosen for inclusion and the thematic generalization is the fact that times will be approximate and not reflective of different driving conditions (e.g., time of day, traffic, weather).

**Figure 17.** Examples of Maps across a G-D Band of the Triangle: (a) Subway Map of New York City (Calcagno2010); (b) Route Map of United Airlines (United Airlines 2017); (c) Driving Distances/time Map for Massachusetts (Massachusetts Office of Travel and Tourism 2016).

Seemingly similar maps may well be placed in different locations on the triangular plot, based on their own characteristics and purpose. For example, the barometric map in Figure 18a is an isopleth map of air pressure, with its location information in the form of contours; it provides rich locational information about differential high- and low-pressure extremes, pressure gradients, and wind speeds. Meanwhile, the weather map in Figure 18b has generalized the barometric contours through selection and/or re-categorization; has generalized the pressure extremes into symbolic labels (“H” and “L”); has highlighted the fronts; and has added
precipitation data. These differences shift this map to the right of (smaller L/T) and above (higher G-D) the barometric map.

**Figure 18.** Examples of Topically Similar Maps Differing in both L/T and G-T: (a) Barometric map of United States (HPC Surface Analysis 2010); (b) Weather map of United States (The Weather Channel 2008).

We contend that all maps can be located on the triangular plot based on the two parameters we have identified (viz., three conceptual endmembers but fundamentally two dimensions). To further evidence our contention, we can take a somewhat “three-corner approach” and consider three fairly extreme examples (Fig. 19). Figure 19a shows a “cartoon map” of theme parks near Orlando FL. The map’s purpose is largely locational (i.e., navigational), showing approximate theme park locations and the major routes around them. The locational information is greatly generalized and/or distorted in this map and so it places to the upper left part of the triangular plot. In contrast, an engineering-survey plot (Fig. 19b) shows
highly accurate and detailed locational information at a large scale. This type of map, therefore, locates at the lower left corner of the triangle. Finally, a map communicating multiple thematic variables such as both county wide population and household income (Fig. 19c) is heavy on thematic information. The map locates firmly towards the right side of the triangle, and its vertical position depends very much on the degree of generalization-distortion in the thematic data representation.

**Figure 19.** Examples of Maps near the Corners of the Map Triangle: (a) Cartoon Map of City of Orlando (Middleton 2010); (b) Survey Plot Map (Veatch, 1995); (c) Multi-variable Map Showing the Population and Household income of Florida by county. (Data Source: United States Census Bureau).

**Discussion**

We argue that because maps vary so widely in their content, design, purpose, and scale of representation, a systematic framework for considering the features of maps that are most pertinent to quantitative map literacy is a necessary first step before we can discuss how and
what quantitative concepts, skills, and facts are involved in quantitative map literacy. In the second paper of this series our focus will shift to that very discussion, but based on what we have reported here we can now provide a brief survey of the general landscape.

Our triangular plot framework emphasizes two parameters: the ratio of locational information to thematic information (L/T), and the level of generalization and/or distortion (G-D); the latter depends on the former to some extent. The L/T ratio is fundamental, for it divides the triangle into significantly different sides: a left side (L/T > 1), more aligned with data and experiences in STEM-type disciplines, continuous variables, and measurement; and a right side (L/T < 1), more aligned with data and experiences in social-science disciplines, categorical classifications, counting, and the need for statistical literacy (e.g., social construction of statistics) of the kind championed by Joel Best (2001, 2004, 2008) and Milo Schield (2004, 2010).

For maps with a high L/T and of a scale where G-D is at a minimum for locational representation (i.e., lower part of the left side of the triangle), the quantitative knowledge and skills are more apt to be algebraic and of a routine nature familiar to lower-division STEM majors. Examples might include being able to use scale to calculate areas and distances, use ratios to calculate gradients, calculate angles for bearings and direction. Such calculations may be as precise as allowed for by the map or may be approximate (e.g., relative slopes) depending on the purpose. For maps with high L/T and at scales where G-D is relatively high (upper part of the left side), knowledge of the quantitative aspects of the methods of map projection and map generalization and how those aspects affect such calculations as those discussed above are all important. These questions are technically more advanced and often require a feeling for calculus to understand them (e.g., equal-area vs. equal-angle projections, rhumb lines, and the like). For
maps with a low L/T ratio and of a scale where generalization and/or distortion are at a minimum for thematic representation (lower part of the right area), the quantitative concepts and skills are more likely to range across, as examples, comparison of quantitative thematic values, calculations of such comparisons, summarizing and assessing the spatial distribution of quantitative thematic values, and calculating thematic values relative to their geographic units. These topics in application are advanced, and they require more confidence and technical insight and sophistication than are typically attained in elementary statistics courses. Finally, for maps with a low L/T ratio and where thematic data representation is quite reductionist in nature (upper part of the right side), the quantitative concepts and skills are more apt to be descriptive-statistical and relatively low-order in nature (and thus more vulnerable for map users to be manipulated or misled). Examples might include knowledge of data classification methods, probability distributions, data transformations, and how these aspects affect the representation of the thematic data values. We would hope that students would be able to consider such topics with understanding as a result of their courses in elementary statistics (or, perhaps more specifically, statistical literacy).

The reader will note a certain dissymmetry in our discussion. Our current view is that the mathematical and statistical concepts, skills and facts needed for the lower part of the left side and the upper part of the right side of the triangle are ones that should draw upon relatively standard classroom training. Therefore, the “calculations” and “judgments” involved should be fairly familiar and of the kind that Pólya (1957) characterized as “exercises.” Meanwhile, the mathematical and statistical knowledge and skills involved in the upper part of the left side and lower part of the right side of the triangle may require mastery of a greater depth of concepts, skills, and facts as regards aspects such as map projection and generalization, evaluation and
interpretation of the spatial distribution of thematic data, and in general more sophisticated interaction of calculation, analysis and context. These are what we believe Pólya (1957) would have called “problems.” In the medial zone midway between locational concentration and thematic concentration, the maps and the nature of the skills and knowledge are highly dependent on the specific questions the users are asking of them. The same map (a land-use map for example) could require high L/T-ratio thinking for some problems and low L/T-ratio thinking for other problems. Such is life in a transition zone.

For maps which are more locational in nature, basic map literacy in terms of use of symbols is an inherent part of using quantitative knowledge and skills in the lower part of the left-side of the triangle, e.g. the calculation of slope based on the use of contours. For maps which are locational in nature but more generalized/distorted (upper part of the left side of the triangle) more advanced map literacy, in the form of knowledge of map projections and map generalization, is required to successfully apply quantitative knowledge and skills. On the thematic side, and particularly in the upper part of the right side of the triangle, there is perhaps less immediate dependency between map literacy and quantitative literacy, and the latter, often involving statistical literacy, tends to dominate. However, in the lower part of the right side of the triangle, we would argue that the required map literacy, such as the knowledge of how cartograms are created/read or the interpretation of spatial patterns and distributions, is at quite high level and also fundamental to the successful application of quantitative knowledge and skills. In this regard, thematic maps, in particular, may also be good analogs of what Oughton (2018) refers to as “situated numeracy” where, in this case, the mapmaker’s intent to convey a thematic message stimulates the application of QL knowledge and skills in the user.
CHAPTER FOUR:
MAP READING SKILLS ACROSS THE TRIANGULAR PLOT

Introduction

Map users need certain knowledge and skills to correctly obtain and interpret information from maps and avoid misunderstanding, and the knowledge and skills involved in reading different types of maps will be different. As a straightforward example, obtaining information from a topographic map requires a very different map reading process than interpreting a cartogram. In this chapter the different knowledge and skills necessary for reading and interpreting different types of map are explored with reference to where such maps place within the triangular plot framework developed in Chapter 3.

As outlined in Chapter 2, there are generally three types of knowledge and skills involved in map reading. The first type is map literacy, which is knowledge and skills about map concepts and map elements. For reference maps, it’s more about map elements, such as geographic coordinates, scales, and projection systems. For thematic maps, it’s more about thematic mapping methods and symbolization. The second type of knowledge is quantitative literacy, which deals with the quantitative concepts and quantitative reasoning included in map information. For reference maps, it includes the calculation of geographic features, such as distance, area, azimuthal/bearing, etc. For the thematic maps, some statistical skills are routinely needed. The third type of knowledge is the background knowledge, which could be independent from maps, but helpful, or even necessary, for reading the map. For the reference maps, the
background knowledge is geographic, including such issues as the shape of the Earth, and latitude and longitude. This kind of background knowledge involved in reference maps can be referred to as geographic literacy. For the thematic maps, on the other hand, the required background knowledge is more about the theme of the map.

In this chapter, the knowledge and skills within map reading will be explored based on the three domains of literacy from Chapter 2: map literacy, quantitative literacy, and background knowledge, with the latter ranging from geographic literacy in reference maps to dominantly thematic knowledge in thematic maps. The map reading skills/knowledge and the intersections between them, will be identified and illustrated with specific examples of maps, and these maps will be located within the triangular plot of Xie et al. (2018).

The methodology used to identify and discuss the knowledge and skills required for map reading is that of word problems. Word problems have been widely utilized as assessment items and learning vehicles in education and research tools in cognitive psychology studies (Briars and Larkin 1984; Cummins 1988; Miller 2010; Wyndhamn and Saljo 1997). Regarding education, word problems reveal students’ learning process and provide a guide in curriculum design and reform (Reed 1999). Regarding cognitive psychology, the way people approach word problems indicates the cognitive processes involved in recognizing and solving problems (Marcel and Patricia 1977).

Of particular relevance to the disciplinary context of this dissertation, map reading of reference maps has been a steadfast topic in geosciences education. There have been major sections for reference map reading, especially topographic maps, in introductory geology lab manuals for decades (Miller and Scholten 1966, Zumberge and Rutford 1983, Hamblin and Howard 1986). In the following section on reference maps, some existing word problems for
topographic maps will be examined, and word problems for other types of maps will be produced. The quantitative skills that are taught or tested with these word problems indicate the quantitative skills needed to understand the corresponding maps. By looking at what quantitative skills are involved in solving word problems first in reference maps, and then for other interpretive issues in reading thematic maps, we can gain an idea of the range of quantitative skills that are involved in map reading. For thematic maps, there are limited examples of using word problems to teach map reading. However, word problems have been commonly used in the questionnaires to assess map readers’ understanding towards thematic maps. Rieger’s (1999) and Nusrat’s (2018) studies are two examples of using word problems in thematic map reading experiments, and they were covered in Chapter 2 in detail.

As defined in Chapter 2, QML is the intersection between quantitative literacy and map literacy. In the following section, however, example word problems that fall both within and outside the definition of QML will be used to have a fuller picture of what map readers need to understand maps in different ways.

**Knowledge and Skills for Reading Reference Maps**

In Chapter 3, it was argued that scale is an important factor when considering reference maps, because scale directly affects the generalization and distortion level. Therefore, the map reading skills for reference maps will be explored based on their scale. Specifically, the map reading skills for large-scale maps, regional maps, world maps and, lastly, topological maps will be studied in detail in the following section (Note: topological maps are included not based on scale but rather because of their high distortion in geographic features). As indicated in Figure
20, this exploration of map reading skills is from the bottom to the top of the triangular plot along its left side.

![Triangular Plot Diagram]

**Figure 20.** Types of Reference Map and Their Location in the Triangular Plot

**Large-scale Reference Maps**

Snyder and Voxland (1989) categorized reference maps based on scale, and they describe large-scale mapping as mapping at a scale larger than approximately 1:75,000. This kind of map usually covers less than 5 degrees of latitude and longitude. Therefore, areas represented are small enough that curvature of the Earth can be ignored. Thus, for practical purposes, large-scale maps can be considered as maps of a flat Earth, and so the scale is the same across the map and in all directions: a square area on the map represents a square area on the ground. Calculations of distance, area, and slope based on scale are relatively accurate; therefore, such calculations
would be the major part of the quantitative skills involved in reading and using large-scale reference maps.

**Site Maps:** At an extreme, some large-scale maps may represent only a very small area of the Earth’s surface. The campus map in Figure 21 is a typical example, and such maps have only a minor level of distortion and generalization in the geographic features. They locate close to the lower left corner of the triangular plot.

![Figure 21. Campus Map of University of South Florida (University of South Florida Parking and Transportation Service, 2017)](image)

Although this kind of site map is commonly used in everyday life, likely not many map users have thought about "how" they actually use the map (i.e., what knowledge and map reading skills does one actually use?). These knowledge and skills can be identified by considering the
questions that map users ask themselves while reading such maps. These “word problems” are shown below in Table 13, along with the map reading skills and knowledge needed to answer them. The Venn diagram-based literacy model of Chapter 2 is applied to identify the domain of such knowledge and skills. Additional examples of word problems for this campus map can be found in Appendix A.

Table 13. Word Problem Examples for USF Campus Map.

<table>
<thead>
<tr>
<th>Word Problems</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the library and the student dormitory (Juniper Hall) on the map.</td>
<td>Read map labels; find Route</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td>Find the route between them.</td>
<td></td>
<td>Quantitative Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, a map user needs to be able to read map elements and find a route. Therefore it belongs to pure map literacy.)</em></td>
<td></td>
<td>Map Literacy</td>
</tr>
<tr>
<td>The rectangular dimensions of the campus are 1 mile by 1.5 miles. What is the area of the campus in acres? <em>(To solve this word problem, one needs to do pure calculation without need of the map. Therefore it belongs to pure quantitative literacy.)</em></td>
<td>Calculate area; convert units (without map)</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td>Where is the administration office of USF? Where can you find food on the campus? <em>(To solve this word problem, one needs to have background information about USF. Therefore it belongs to pure geographic literacy.)</em></td>
<td>Background knowledge</td>
<td>Quantitative Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Map Literacy</td>
</tr>
</tbody>
</table>
| The campus occupies one and a half sections in the US PLSS. What is the area of the campus?  
(The knowledge about US PLSS belongs to geographic literacy, and calculation of area belongs to quantitative literacy. Therefore it belongs in the intersection of geographic literacy and quantitative literacy minus map literacy.) | Background knowledge about the US PLSS; Calculate area; Unit conversion | ![Diagram](image) |
|---|---|---|
| Given the dimensions of the campus, develop a scale for this map and use it to calculate the distance from the library to Juniper Hall.  
(The knowledge of map scale and finding route belongs to map literacy, and calculation of distance belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus geographic literacy.) | Ratio and scale, calculate distance using map scale | ![Diagram](image) |
| Identify the places where you can find food on campus and find the routes to them from your current location.  
(Knowing where you can find food belongs to geographic literacy, and finding routes belongs to map literacy. Therefore these belong to the intersection of geographic literacy and map literacy minus quantitative literacy.) | Background knowledge; Reading legend; find route | ![Diagram](image) |
| Find the shortest route from your current location to a place where you can find food on the campus  
(Knowing where you can find food belongs to geographic literacy, finding a route belongs to map literacy, and calculation of distance belongs to quantitative literacy. Therefore this belongs to the intersection of the three) | Background knowledge; reading legend; find route; calculate distance using map scale | ![Diagram](image) |
Based on Table 13, finding a route is obviously one of the most common skills involved in map reading of this type of map, because finding locations and routes is its major purpose. As such it is often referred to as a “visitors map,” and, in the specific case of USF, it is posted in strategically located kiosks around campus. Finding a route involves the topological skills of identifying connections between geographic features, and as such could be considered as quantitative skills. In this dissertation, however, such skills are classified as pure map literacy to emphasize the centrality of the map elements that form topological relations. Because of the relatively small distortion in this kind of map, the map is able to provide further information, such as travel time and distance. Of course, corresponding quantitative skills, such as calculating distance based on scales and calculating travel time using proper ratio relations, are necessary to obtain and process such quantitative information.

Skills in the three domains mentioned above are needed to correctly read this map. In terms of the map literacy, map users need to be able to orient maps, identify geographic features and find routes. In terms of quantitative literacy, map users need to be able to use the scale and measure or calculate the distance and area on the map. In terms of geographic literacy, some background knowledge and information about USF helps to guide the problem solving thinking.

Some complex tasks can require a combination of the individual skills mentioned above. An example would be finding several routes between two points and evaluating the routes by distance or travel time. To solve such tasks, map users may need to identify locations, find routes, calculate distances using scale, and even draw on background knowledge about the pedestrian and vehicular traffic (relative to time of day) on the USF campus.
**Topographic Maps:** Another typical example for a large-scale reference map with a low G-D level is the topographic map (such as the standard USGS Topographic Map shown in Figures 14a and 14b). Topographic maps have been widely applied in geological and environmental mapping and other field activities. As noted before, for example, word problems for topographic maps are common in laboratory manuals for physical geology courses. Examples of such word problems are included and analyzed in Table 14 in regard to the map reading skills and knowledge involved and their literacy domains. Some map elements that are included on standard USGS topographic maps, such as use of various north arrows (true north, grid north, and magnetic north) and calculations according to various geographic coordinate systems (UTM and USPLSS) are not typically tested in the lab manuals for introductory courses; examples of these more challenging issues are included in Appendix B.

These word problems for a topographic map indicate that the quantitative skills and knowledge to understand or use a topographic map, such as using ratios to calculate slopes and gradients, or calculating angles for bearings and direction, should mostly be within the capabilities of lower-division STEM majors. A typical example word problem would be: knowing the map scale and distance between A and B in map units calculate the actual distance between them. Because the distortion level of the map is relatively low, the calculation is very straightforward: just divide the map distance by the map scale (the representative fraction, e.g., 1:24,000, or 1/24,000, and convert units).
### Table 14. Word Problem Examples for Topographic Map.

<table>
<thead>
<tr>
<th>Word Problems</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the scale of the map? What is the contour interval of the map? (Zumberge and Rutford 1983) <em>(To solve this word problem, a map user needs to be able to read map elements. Therefore it belongs to pure map literacy.)</em></td>
<td>Read map elements</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td>The scale of the map is 1:24,000. That means 1 inch on the map represents how many feet on the ground? (Zumberge and Rutford, 1983) <em>(To solve this word problem, one needs to be able to work with ratio. Therefore it belongs to pure quantitative literacy.)</em></td>
<td>Scale and ratio; calculate with scales (without map)</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td>Which part of the map has more population? <em>(To solve this word problem, one needs to have background information about the map area. Therefore it belongs to pure geographic literacy.)</em></td>
<td>Geographic background knowledge</td>
<td>Geographic Literacy</td>
</tr>
</tbody>
</table>
| What is the length of 1 degree of latitude (along a meridian, and assuming the Earth is a sphere with an equatorial circumference of 40,000 km)? *(The knowledge about latitude and longitude belongs to geographic literacy, and calculation of distance belongs to quantitative literacy. Therefore it belongs to the intersection of geographic literacy and quantitative literacy minus map literacy.)* | Geographic background knowledge; calculate length and distance | Geographic Literacy }
Table 14 (Continued).

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Skill Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate the distance and slope between two points using the map scale.</td>
<td>Ratio and scale, calculate distance using map scale</td>
</tr>
<tr>
<td><em>(Being able to read map elements belongs to map literacy, and calculation of distance and slopes belongs to quantitative literacy. Therefore it belongs to the intersection of map literacy and quantitative literacy minus geographic literacy)</em></td>
<td><em>Quantitative Literacy</em> <em>Map Literacy</em></td>
</tr>
<tr>
<td>Identify certain landforms in the map area. Why are these landforms present?</td>
<td>Geographic background knowledge; Read map element;</td>
</tr>
<tr>
<td><em>(Being able to identify morphologic features on maps belongs to map literacy, and knowledge about their origin belongs to [physical] geographic literacy. Therefore it belongs to the intersection of map literacy and geographic literacy minus quantitative literacy.)</em></td>
<td><em>Quantitative Literacy</em> <em>Map Literacy</em></td>
</tr>
<tr>
<td>Define the boundaries of the area shown on the map in terms of latitude and longitude. <em>(Miller and Scholten 1966).</em> What is the size of the map area (length, width, area of the quadrangle) assuming the Earth is a sphere with perimeter of 40,000 km? <em>(Knowing about latitudes and longitudes belongs to geographic literacy, reading their map elements belongs to map literacy, and calculation of distance belongs to quantitative literacy. Therefore it belongs to the intersection of the three.)</em></td>
<td>Geographic background knowledge; reading map element; calculate geographic coordinate</td>
</tr>
</tbody>
</table>

As topographic maps are paradigmatic of low-G-D-level reference maps, the quantitative skills involved in the low G-D level reference maps in general are also typically at a low level.

To be sure, there are some higher-level quantitative skills that are used by some with topographic maps, such as forming mental topographic models. However, these skills are not typically used by casual map users, especially in classrooms, except in the pre-professional (or majors) courses.
In other words, these higher-level quantitative skills apply to map users who are very familiar with this type of map and already have a high level of QML for such maps.

In terms of map literacy, standard topographic maps usually provide detailed, nontrivial map elements, and map users usually will need to know about them. For example, topographic maps are produced with formal scales. Map users need to know what they are and how they affect the geographic features displayed on map. Other map elements include orientation (differences between true north, magnetic north, and grid north), and geographic coordinates (UTM, USPLSS, what they are and how they help read the map). In terms of quantitative literacy, topographic maps also involve the calculation of distance and area with scale. Furthermore, with the introduction of elevation contours to the map, users, if able, can calculate, estimate and compare slope. In terms of geographic literacy, map users need to have some idea about the background knowledge about the map area, such as morphology, traffic, culture, or population, to better interpret the map.

Compared with the site map above, topographic map-reading often involves more skills and knowledge. With lower levels of skills and knowledge, map users can still perform some simple tasks with topographic maps, such as orienting maps and finding routes. Quantitative skills, such as calculating distance, slope and area is also more commonly used in topographic maps. In order to complete tasks with more complexity, such as forming a mental topography model, map users need to have higher-level skills.

**Small-Scale Reference Maps**

Reference maps with relatively high G-D level include regional maps and world maps. Compared with large-scale reference maps in the previous section, the distortion and
generalization in regional maps and world maps can no longer be ignored. Thus, although world maps are commonly seen in the everyday life, it is actually not easy to have a good understanding of their spatial fidelity. One of the challenges in these maps is the map projection system. Because the distortion level is so much more significant in small-scale maps than it is in large-scale maps, map users need this knowledge in order to understand what kind of features are preserved or distorted in the map, and therefore avoid misunderstandings and correctly obtain information from the map.

There are many types of projection systems developed for small-scale maps to fulfill different purposes. They can be classified based on either the kind of surface the sphere is projected onto (e.g. cylinders for cylindrical projections; cones for conic projections; and planes for azimuthal projections), or the properties of the geographic features that the projections preserve (e.g. azimuthal projection that preserves direction radially from the center; equal-areal projections preserve areas; conformal projections preserve shape locally). In order to understand the geometry of geographic features shown on small-scale maps, users need to have enough knowledge of projection systems to identify the projection applied in the map and know its influence on the geometry of the geographic features shown on in the map, before they can correctly obtain information from these maps.

Orthographic projection (Fig. 22a) is commonly used for regional maps. Most students have probably seen or used this kind of map, but they probably couldn’t tell you the name of the projection system or, more importantly, describe the distortion it introduces. Orthographic projection projects a hemisphere onto a flat surface. Therefore it can show no more than one hemisphere a time. The great advantage of the orthographic projection is that it provides a straightforward image of how the Earth would look from outer space. The “cost” of this
advantage is that an orthographic projection is neither conformal nor equal-area, which means that it is not able to preserve angle, shape, or area, and it is only free of distortion in the center of the map. The distortion increases rapidly away from the center, and it is very significant at the edge of the map as shown in the corresponding map of Tissot indicatrices\(^2\) below the map.

Figure 22b shows a regional map example of the Gnomonic Projection, which is an azimuthal projection. Compared with the Orthographic Projection shown in Figure 22a, the distortion of the Gnomonic projection also increases rapidly from the center, especially along the meridian (judging by the corresponding map of Tissot indicatrices). The most special feature that the Gnomonic projection has is that all great circles are shown as straight lines in the map (thus the meridians and equator are shown as straight lines; also all straight lines are great circles). Therefore it can help navigators (especially aviators) find the shortest path between two points on the map.

The well-known Mercator Projection is a familiar and historically significant choice of projection for small scale maps (Fig. 22c). The Mercator projection is a cylindrical projection. For the familiar world map, the axis of the projection cylinder is along the axis of the Earth (there are other Mercator projections, such as the Transverse Mercator, which is the projection used for USGS topographic maps). The projection is also conformal. There are other conformal projections: for example, the Stereographic Projection (familiar to students in structural geology for plotting rock fabrics) is a conformal azimuthal projection; for conic projections, there is the Lambert conic projection. For the familiar world map, the Mercator projection preserves the same scale along the equator or along two parallels equidistant from the equator. The scale is

\(^2\)Tissot’s indicatrix, developed by Nicolas Auguste Tissot, describes the distortion introduced by map projections. The indicatrix is an ellipse, in which the lengths of the two major axes show the magnitude and direction of the maximum and minimum scale at a given point on the map.
preserved at the cost of the increasing distortion from equator to the poles. There is so much distortion in the polar area that the poles cannot be shown. Another key feature of Mercator projection is that all the rhumb lines (lines that make equal angles with all meridians; i.e., they maintain a constant azimuth or bearing) are shown as straight lines on the map. Therefore the classic Mercator world map has been useful – historically so – in compass-based navigation. However, it is inappropriate for discerning relative sizes of geographic areas because of the extreme distortion in high-latitude areas.

Figure 22d shows a world map example of the Albers Conic Projection. It has the round shape because the projection surface (metaphorically) has been unrolled from a cone fit to the Earth like a dunce cap fixed against a globe. Cone-based projections are able to preserve the general geometry of the features on the hemisphere that the metaphoric cone covered, so it is commonly used in nation-scale maps, such as the map of Russia, Central Europe, and United States. In the Albers Conic projection, areas of geographic features are preserved at the cost of distorting their shape.

Several word problems are produced based on regional maps and world maps to explore the quantitative skills involved in reading this kind of map (see Table 15). Many word problems are overlapping for regional maps and world maps and so they are listed together in Table 15. Some of the word problems are different and they are noted in Table 15. Word problems produced based on other projection systems can be found in Appendix C.
Figure 22. a) Regional Map, Oblique Orthographic Projection; b) Regional Map, Oblique Gnomonic Projection; c) World Map, Mercator Projection; d) World Map, Lambert Cylindrical Equal-Area Projection. The Tissot Indicatrix Patterns for Each Map are Indicated below. (Snyder and Voxland, 1989).
Table 15. Word Problem Examples for World Map.

<table>
<thead>
<tr>
<th>Word Problems</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the characteristics of the Mercator Projection? What features does it preserve? What kinds of distortion does it introduce? <em>(To solve this word problem, a map user needs to have knowledge about map projection systems. Therefore it belongs to pure map literacy.)</em></td>
<td>Knowledge about map projection system</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>Given the variable scale of a generic Mercator Projection (Fig. 23), how does the scale at 0° latitude compare to the scale at 72° latitude? <em>(To solve this word problem, one needs to compare the value of ratio. Therefore it belongs to pure quantitative literacy.)</em></td>
<td>Scale and ratio; comparison</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td><em>(World map)</em> Where is the U.S. on the Earth? Where is China? Where are the Galapagos Islands? <em>(Regional map): Where is San Francisco? Where is Vancouver?)</em> <em>(To solve these word problems, one needs to know the location of particular places. Therefore it belongs to pure geographic literacy.)</em></td>
<td>Geographic background knowledge</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td><em>(World map)</em> Where is the U.S. on the Earth? Where is China? Where are the Galapagos Islands? <em>(Regional map): Where is San Francisco? Where is Vancouver?)</em> <em>(To solve these word problems, one needs to know the location of particular places. Therefore it belongs to pure geographic literacy.)</em></td>
<td>Geographic background knowledge</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td>Assuming the Earth is a sphere with a great-circle length of 40,000 km, how long are the 45th and 75th parallels? How long is a degree of latitude? <em>(The knowledge about latitude and longitude belongs to geographic literacy, and calculation of length belongs to quantitative literacy. Therefore it belongs to the intersection of geographic literacy and quantitative literacy minus map literacy.)</em></td>
<td>Solid geometry; trigonometric function; knowledge about latitude and longitude</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
</tbody>
</table>
Table 15 (Continued).

<table>
<thead>
<tr>
<th>Without using the graphic variable scale on the map, what is the scale of the parallel at the equator? What about the 45th parallel? What about 75th parallel? (Being able to read map elements belongs to map literacy, and calculation of scales belongs to quantitative literacy. Therefore it belongs to the intersection of map literacy and quantitative literacy minus geographic literacy.)</th>
<th>Ratio and scale, knowledge about projection system.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare the size of Greenland and India on this map. What do you think about their real relative areas on the Earth? (The knowledge about map projections belongs to map literacy, and the knowledge about Greenland and India belongs to geographic literacy. Therefore it belongs to the intersection of geographic literacy and map literacy minus quantitative literacy)</td>
<td>Read map element; knowledge about projection system.</td>
</tr>
<tr>
<td>Knowing that Tampa is about 28° N, 83° W; and Seattle is about 48° N, 122° W, what is the bearing of rhumb lines from Tampa to Seattle according to true north? Directly measure it on the map and compare your calculation and measurement. (Knowing the property of the Earth belongs to geographic literacy, reading map elements belongs to map literacy, and calculation of distance belongs to quantitative literacy. Therefore this problem belongs to the intersection of the three domains)</td>
<td>Geographic background knowledge; Reading map elements; calculation with latitude and longitude</td>
</tr>
</tbody>
</table>

As indicated in the word problems listed in Table 15 and included in Appendix C, map users need higher map-reading skills and knowledge concerning projections to use and understand these maps. Thinking of the same word problems that were discussed for large-scale maps but now transferring the mover to small-scale maps: Suppose you measure the distance
between A and B on the map, how do you calculate the real distance on the Earth? Because the distortion is now a factor, the scale varies from place to place and in different directions on the map; in short, the calculation gets more complicated. The shortest distance is generally not able to be calculated directly from a straight line on the map. Variable scales (Fig.23) are used in some small-scale maps that preserve distance along a latitude. That means if the two points are at the same latitude, the distance along the latitude can be calculated with the variable-scale graph. The map users should still know that this distance is simply the distance along the parallel of latitude, and not the shortest distance, unless the two points are on the equator (i.e., a parallel of latitude is a short circle, not a great circle). If the two points are at different latitude, the question becomes more complicated and involves the transformation of coordinate system in order to solve for the angle of between the geocentric position vectors of the two points (Vacher 1999). These quantitative skills are obviously at higher level than those observed in the word problems for large-scale maps.

![Variable Scale Used in World Map](image)

**Figure 23.** Variable Scale Used in World Map

Furthermore, map users will need the knowledge of the projection characteristics to use the appropriate map for specific purposes. This can be also inferred from the word problems. For example, the Mercator projection is a good choice when determining directions between two locations; the Gnomonic Projection is a good choice when trying to find the route of shortest
distance; the Albers Conic Projection is a good choice to compare the area of geographic features.

Of course there are some low-level skills, such as identifying geographic features and reading map elements, involved in the map reading process of high G-D level, small-scale reference maps. For example, when trying to find where Greenland is on the Mercator world map (Fig. 22c), the skills involved in this map reading process would stay at very low level (e.g., orient map, identify features). However, in order to determine the correct size of Greenland, higher-level quantitative skills are needed. Thus, with only lower-level skills, utility of the small-scale map is limited for some purposes, because map users are not able to correctly obtain quantitative information (e.g., distance, area, angle) without considering the projection used.

Overall, then, there is a hierarchy of skills involved in this example: low-level skills include identifying Greenland; medium-level skills include knowing that Greenland is not as big as it appears because of the projection; high-level skills include estimating the real area of Greenland.

**Topological Maps**

If the distortion level keeps increasing it comes to a sort of fault line on the triangle, where maps have been produced without any projection system, and no reliable scales are used. The subway map shown in Figure 24 is an example. The geometry of this map is totally distorted, and only the topology (i.e., adjacency, inclusion, and connection) of the map is reasonably preserved. The generalization and distortion of the geographic features of these maps is too significant to allow seen in the word problems in Table 16:
Figure 24. Part of the Official New York City Subway Map (Metropolitan Transportation Authority of the State of New York 2013).

Table 16. Word Problem Examples for the Subway Map of New York City.

<table>
<thead>
<tr>
<th>Word Problems</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find a route from Jamaica Center Station to South Ferry Station by subway</td>
<td>Read map elements; find route</td>
<td>Geographic Literacy</td>
</tr>
<tr>
<td>according to this map. (To solve this word problem, map users need to be able</td>
<td></td>
<td>Quantitative Literacy</td>
</tr>
<tr>
<td>to read map elements and find route. Therefore it belongs to pure map literacy.)</td>
<td></td>
<td>Geologic background knowledge</td>
</tr>
<tr>
<td>Where is the Statue of Liberty? What about China Town? (To solve these word</td>
<td>Geographic background knowledge</td>
<td></td>
</tr>
<tr>
<td>problems, one needs to have some idea about where the Statue of Liberty or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>China Town is located. Therefore it belongs to pure geographic literacy.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 16 (Continued).

<table>
<thead>
<tr>
<th>Question</th>
<th>Skill Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>What station should you travel to if you want to visit the Statue of Liberty? What about China Town? (To solve these word problems, map users need to have some idea about where Statue of Liberty or China Town are located and then be able to find the closest station on the map. Therefore it belongs to the intersection of geographic literacy and map literacy minus quantitative literacy.)</td>
<td>Geographic Literacy; Read map elements; orient the map; geographic background knowledge</td>
</tr>
<tr>
<td>How many stations are there between Jamaica Center Station and South Ferry Station by subway according to this map? If the average travel time between two stations is 8 minutes, how long would it take to travel between these two stations? (Being able to read map elements belongs to map literacy, and being able to count belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus geographic literacy.)</td>
<td>Geographic Literacy; Read map elements; count</td>
</tr>
<tr>
<td>How is the shape of Central Park distorted on this map? (Knowing the actual dimensions of Central Park belongs to geographic literacy, finding Central Park on the map belongs to map literacy, and determining the length-width ratio on the map and comparing it to the actual ratio belong to quantitative literacy. Therefore this belongs to the intersection of the three)</td>
<td>Background knowledge; reading legend; find route; ratio</td>
</tr>
</tbody>
</table>

One can note that the skills in only a few subsets shown in the Venn-diagram of the literacy model pertain to the subway map (Table 16). This indicates that only very limited quantitative skills are involved in the map reading of this kind of topological map. As basic map reading skills, identifying geographic features and recognizing map elements are still involved in topological maps. Orienting maps is also necessary sometimes, and navigation and route finding
seem to be dominant skills in using topological maps, because topological maps preserve the connectivity and relative location between different geographic features. Comparing the word problems produced based on topological maps and those based on large-scale maps, the differences between the quantitative skills involved in these two different types of maps that usually cover a small part of Earth surface can be distinguished. Although they can both be used to find routes, the calculation of distances and directions is applicable only in the large-scale maps because they are drawn to scale.

**Knowledge and Skills for Reading Thematic Maps**

For thematic maps, scale is no longer the major factor affecting distortion and generalization, and these maps are more affected by how the thematic data is aggregated and presented (i.e., the thematic mapping method). In the following section, three types of thematic maps are considered: (1) two-category choropleth maps of a binary variable; (2) multi-category choropleth maps of continuous variables, and (3) multi-category, multi-variable maps that consider the geographic variation of more than one thematic variable at a time. The locations of these types of maps on the QML triangle of Chapter 3 are shown in Figure 25. Multi-variable maps contain more thematic information and therefore locate to the right side of the other two types of single-variable maps. Multi-category maps have the data classified in more detail. Thus, they better preserve the thematic data and therefore have lower distortion compared with two-category maps.
Two-category, Binary Variable Maps

An election map that denotes the winning candidate, or party, by geographic area is a good example of a two-category, binary variable map. For example, in the presidential election map shown in Figure 26, the data presented represents which of the two presidential candidates, Democratic or Republican, won the plurality of votes in the geographic map units (the 48 conterminous states). This election map has a high G-D level and plots on the QML triangle above other choropleth maps because the raw data underlying the map (votes) has been highly generalized into just two categories.
All three domains of literacy for the literacy model (Venn diagram) – geographic/thematic, quantitative, and map – are needed to correctly read and interpret the results shown on Figure 26. From geographic and thematic literacy, map users need to be able to identify the state, and know the basics about national elections (e.g., the electoral college), respectively. From quantitative literacy, map users need to frame and be able to calculate two sums and compare them to determine who won the election. From map literacy, map users need to know how the map is produced – that the numbers of electors in the various states are not provided in this kind of election map, and the color of a particular state does not mean all the voters of that state voted for the red, or the blue, candidate.

Although a national presidential election-result map, such as this one, is a commonly-seen map, it is one which obscures, through generalization, considerable variability in the underlying data, and it is also often open to misinterpretation. This election map shows which presidential candidate won which state. This is the “raw interpretation” of this map, and requires only low-level skills such as identifying states and reading the color of each state. However, this
map may seem to address a couple of other questions such as: Which of the two presidential candidates got the most votes? Which of the two presidential candidates got the most electoral votes? These questions are more difficult; they require more background thematic information regarding the electoral college. The extreme generalization, through data aggregation, of the raw data underlying this map (votes by voting precinct) needs to be recognized so as to appreciate that considerable variability in voting patterns may exist at the sub-state level. Thus, with only lower level of knowledge and skills, map users could be misled by this map if they were to attempt to use it to answer questions beyond the data it presents in its raw form. With higher-level skills and knowledge, map users know which states they should be paying more attention to, and, with a table showing the numbers of electors from each state, they can calculate who won the election, and which states contribute more to the total than others. However, they would not be able to know from the map, which candidate got the most votes nationally.

Map reading tasks for the presidential election map are listed in Table 17. The map reading skills and knowledge required to perform the map reading tasks and the domains of literacy they belong to are also included in the table. As with topological maps (Table 16), only a few subsets of the literacy model are applied to this map. Referring back to Curcio’s (1987) three levels of graph comprehension discussed in Chapter 2, these skills mostly belong to “reading the data,” which are relatively low-level skills. Although this suggests that not many high-level skills are involved in reading such an election map, it certainly does not mean that election maps are easy to understand. To the contrary, map readers can be easily misled if they attempt to interpret the map beyond the raw data it presents without sufficient higher-level skills and knowledge.
Table 17. Map Reading Tasks for Presidential Election Map

<table>
<thead>
<tr>
<th>Map Reading Tasks</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which states are more influential in determining the winning candidate?</td>
<td>Background knowledge about election</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, one needs to have some background knowledge about the electoral college. Therefore it belongs to pure thematic literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>How is this map produced? What do the colors in the map mean?</td>
<td>Election map making</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, a map user needs to be able to read, and interpret, map elements. Therefore it belongs to pure map literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>How many states did the Democratic candidate win? How many states did the</td>
<td>Read map elements; count</td>
<td></td>
</tr>
<tr>
<td>Republican Party win? <em>(Being able to read map element belongs to map literacy, and being able to count belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus thematic literacy.)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can you tell which presidential candidate won the election through the map?</td>
<td>Background knowledge about election; election map making</td>
<td></td>
</tr>
<tr>
<td><em>(Knowing the limits of the map belongs to map literacy, the knowledge about electoral college belongs to thematic literacy, counting calculation and comparison between the two presidential candidates belongs to the quantitative literacy. Therefore it belongs to the intersection of the three.)</em></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
There is an obvious limitation to presidential election maps: although they show the voter preference in each geographic map unit, they do not on their own show the influence of that map unit in determining the election outcome. In order to provide this missing information in election maps, cartographers have sought various ways to improve the presentation of the data. Cartograms are one such attempt. The idea is to resize the area of the geographic map unit according to the value of a thematic variable, for example, the number of electoral-college votes each State has. Figure 27a is an example of a contiguous cartogram, where the shape of geographic units is distorted, but the adjacency between them is preserved. Sun and Li (2010) studied the visualization of different cartogram methods, and discussed other types of cartograms including: (1) the non-contiguous cartogram (Fig. 27b), which preserves the shape of the geographic map units (the states in this case), at the cost of showing their adjacency, and (2) the Dorling cartogram (Fig. 27c), in which the geographic map units are redrawn as circles.

Compared to the normal national-election map (Fig.25), cartograms are able to convey more thematic data. In terms of the triangular plot from Chapter 3, although cartograms have more distortion in geometry, that kind of distortion carries less weight on the (thematic) right side of the QML triangle than the thematic distortion. The thematic distortion is low for cartograms, specifically because they are able to display thematic information in more detail and more faithfully on account of the proportionality between number of electoral-college votes and area of the colored bins. Therefore, overall, cartograms have relatively low distortion and will typically locate lower on the QML triangular plot than normal choropleth maps. Furthermore, the cartogram allows a second thematic variable to be represented (namely the number of electors, in addition to which candidate won the state). Therefore it is technically multi-variable and should locate to the right of single-variable choropleth maps.
Specifically, for the three types of cartograms, the non-contiguous cartogram (Fig. 27b) preserves the geometry of the geographic map units, and therefore has more locational information. The Dorling cartogram (Fig. 27c) loses geometry, topology, and true shape of geographic features, and therefore has relatively less locational information. Thus the cartogram of Figure 27b would plot to the left of the cartogram of Figure 27c. The contiguous cartogram (Fig. 27a) preserves the topology of geographic map units, but loses the geometry. Therefore it locates between the other two cartograms on the triangular plot.
As claimed earlier in this section, a cartogram is a type of multi-variable map. The interpretation of the first variable (which presidential candidate won which state) is the same as the regular presidential election map shown in Figure 25. The interpretation of the second variable shown in the map (number of electors requires very different types of map reading skills. Cartograms can convey thematic information in an impressive way, but map users need to be familiar with the concepts and purpose of the cartogram. For map users lacking this knowledge/skill set, the great departure from ordinary maps that cartograms represent may possibly become an obstacle to map communication. Dent (1972) claimed that “these cartograms are thought to be confusing and difficult to read.” He also suggested that cartographers should apply helpful communication strategies in making cartograms, such as providing an inset map or labeling the geographic map units on cartograms, in order to accommodate uninitiated map users who need to understand them better.

There are also some differences in the map reading skills involved in the three examples of cartograms. For the non-contiguous cartogram (Fig. 27a), because the shapes of the geographic map units are preserved, it is easier to identify them from their shape. However, it is relatively difficult to compare their sizes when they are nearly equal. Meanwhile, for the Dorling map, which represents the sizes by circles, it is easier to compare the sizes, but it is quite difficult to identify the geographic map units because they have lost their geometry and topology.
### Table 18. Map Reading Tasks for Election Cartogram

<table>
<thead>
<tr>
<th>Map Reading Tasks</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
</table>
| Recognizing the states on the Cartogram. (To solve this word problem, one needs to know about the locations of states. This is especially important in the Cartogram since the geometry or topology is distorted.) | Background knowledge about election                                                            | Thematic Literacy  
Quantitative Literacy  
Map Literacy                                                                 |
| How is this map produced? What does the size of geographic features in the map mean? (To solve this word problem, map user need to be able to read map elements, and how cartograms are made. Therefore it belongs to pure map literacy.) | Election map making                                                                           | Thematic Literacy  
Quantitative Literacy  
Map Literacy                                                                 |
| Which state has the largest size of Democratic electoral college? Which state has the largest size of Republican electoral college? (Being able to read map element belongs to map literacy, and being able to compare values belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus thematic literacy.) | Read map elements; comparison                                                                  | Thematic Literacy  
Quantitative Literacy  
Map Literacy                                                                 |
| Can you tell which party won the election through the map? (Knowing the limit of the map belongs to map literacy, and the knowledge about rule of election belongs to thematic literacy, and comparing the value of thematic data to make better inference belong to quantitative literacy. Therefore it belongs to the intersection of the three) | Background knowledge about election; election map making                                    | Thematic Literacy  
Quantitative Literacy  
Map Literacy                                                                 |
The map reading tasks for cartograms are listed in Table 18, along with the map reading skills/knowledge and the domains of the literacy. Because cartograms indicate the values of thematic variable by the size of geographic map unit, some quantitative skills (such as comparison) can be added to the map reading skills. Map users can make better inferences about the data distribution by comparing the size of geographic map units.

**Multi-Category, Continuous Variable Choropleth Maps**

Figure 28 is an example of a multiple-category choropleth map representation of a continuous variable showing the spatial distribution of infant mortality in Austria (Waldhoer et al. 2008). The thematic variable is the standardized mortality ratio (SMR) of observed counts to statistically expected counts of infant deaths. The geographic map units are administrative districts \((n = 98)\). The statistically expected counts (the denominator of the SMR ratio) were determined for each administrative district using a logistic regression model of individual deaths. Across Austria, the range of values of the SMR variable is 0.83 to 1.21; thus the SMR in the highest unit is almost 50% higher than in the lowest. It is quite evident from the four colors that the distribution of infant mortality (the theme of the map) is not uniform: the lower values of SMR occur in the southeastern part of the country, and the higher values occur in the northern and eastern regions of the country, with border districts showing some of the highest values.
Figure 28. Map of SMR’s for overall infant mortality in Austria (Waldhoer et al. 2008) and Its Location on the Triangular Plot

Obviously, this map has some of the same limitations seen in election maps: the same value represents a whole district and variability across each administration unit in the same map category is not shown. Similarly, it gives an impression that every place in the administration units of the lowest-SMR class has a lower infant mortality rate than all localities in all the other administration units. Thus map users need to know how the choropleth maps are made in order to properly read and understand them. Furthermore, because there are more data classes displayed in the map, the quantitative skills and knowledge about the methods of data classification are among the most important skills in understanding choropleth maps.

Choropleth maps, like the example in Figure 28 show classified (binned) data rather than the raw data. Only by knowing the data classification method can map users have a reasonable inference of the data distribution patterns. The choropleth map shown in Figure 28, as an example, bins its data using quantiles --- specifically quartiles. In other words, the classification method producing Figure 28 aimed to group the 98 “observations” – where an observation is the SMR value of an administration district – into four classes (bins) of an equal number of observations (24-25). As shown in the key of Figure 28, the median value (0.998) is effectively
where the observed counts equal the expected counts. For the 75\textsuperscript{th} percentile, the observed counts exceed the expected counts by 5\% or more, and for the 25\textsuperscript{th} percentile, the observed counts are less than the expected counts by 6.3\%. Thus the range of SMR values varies from class to class (i.e., from bin to bin, and color to color). Specifically, taking into account the extreme geographic map units (0.83 and 1.21), the four ranges are 0.107 for the 1\textsuperscript{st} quartile; 0.061 for the 2\textsuperscript{nd} quartile; 0.052 for the 3\textsuperscript{rd}-quartile; and 0.16 for the 4\textsuperscript{th} quartile.

Thus, a quantile data classification method emphasizes the variability according to the rank (or order) of the observations rather than their absolute values. Plus, because geographic features are evenly distributed in classes according to rank, the map is not overwhelmingly dominated by certain classes containing large numbers of observations (districts). The quantile classification method contrasts with an equal-interval data classification, which divides the data into classes with equal ranges. In the case of Figure 28, the equal-area classification methods would produce four classes, each with a range of 0.095, and these would give categories, or bins, with 0.83, 0.925, 1.02, 1.15, and 1.21 as the limits and cut-points. Obviously, there would be substantially fewer administration districts in the highest and lowest classes. In this way, equal-interval data classification method gives a better view of the absolute distribution pattern of the thematic data.

As a summary of the map reading skills and knowledge involved in understanding simple choropleth maps such as the one in Figure 28, map reading tasks are listed in Table 19, along with corresponding map reading skills/knowledge and domains of literacy. Compared with the map reading skills in election maps, the knowledge and skills are at a higher level, particularly with regard to understanding data-classification methods. The determination of the plotted metric
is also crucial. Specifically for this SMR map, map users need to know what the SMR values shown in the legend (0.937, 0.998, 1.06) mean, and how these attribute values are calculated.

Table 19. Map Reading Tasks for Choropleth Map

<table>
<thead>
<tr>
<th>Map Reading Tasks</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the range of SMR of a certain administration unit?</td>
<td>Thematic map elements</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, a map user needs to be able to read map elements. Therefore it belongs to pure map literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>What is an SMR? What factors might affect its spatial distribution?</td>
<td>Background knowledge about the theme</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, one needs to have some background knowledge about the Sudden Mortality Rate. Therefore it belongs to pure thematic literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>Estimate the median of the thematic data?</td>
<td>Statistical calculation</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, one would need to be able to perform a statistical calculation. Therefore it belongs to pure quantitative literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
</tbody>
</table>
Identify the data classification method used in the map. How is the classification performed? What are the advantages/disadvantages of the data classification method?  
(Being able to read the legend and know about the data classification method belongs to map literacy, and knowing how data classification is performed involves quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus thematic literacy.)

<table>
<thead>
<tr>
<th>Knowledge about data classification method</th>
</tr>
</thead>
</table>

Which part of Austria has higher level of SMR? Which part has lower level of SMR? What could cause this distribution?  
(Being able to read and identify the clusters of high/low value belongs to map literacy, and knowledge about what might cause such a distribution of Sudden Mortality Rate belongs to thematic literacy. Therefore it belongs to the intersection of thematic literacy and map literacy minus quantitative literacy.)

<table>
<thead>
<tr>
<th>Knowledge about data classification method; background knowledge about the theme</th>
</tr>
</thead>
</table>

How is SMR calculated? Estimate the number of cases in each geographic unit in the map.  
(Knowledge about Sudden Mortality Rate belongs to thematic literacy, and calculation of SMR belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and thematic literacy)

<table>
<thead>
<tr>
<th>Background knowledge about the theme; statistical calculation</th>
</tr>
</thead>
</table>

Spatial distribution analysis:  
How many contiguous clusters of districts are there in the highest SMR data category? Where are they located, and what might explain their presence in those locations?  
(The complex spatial distribution analysis involves knowledge about theme, reading data from map, and calculation of thematic data. Therefore it belongs to the intersection of the three)

<table>
<thead>
<tr>
<th>Thematic map element; background knowledge about the theme; statistical calculation</th>
</tr>
</thead>
</table>
As the use of choropleth mapping has evolved, more advanced statistical methods have been introduced into mapping procedures. As a result, map users need corresponding statistical skills and knowledge to appreciate how such maps have been produced. One example of these advanced statistical skills would be “data smoothing,” a method widely applied in mapping disease risk. Disease risk maps convey the spatial patterns of the probability of disease using the ratio of disease cases over the areal unit’s population. However, this ratio is likely to be unreliable in geographic areas with small populations or numbers of disease cases. This situation is often called the "small number problem" (Waller and Gotway 2004).

Empirical Bayes Smoothing is one of the data smoothing methods applied to cope with the small number problem. Instead of directly showing the rate, the Empirical Bayes Smoothing Method “borrows” information from the whole data set (or a regional subset), in order to produce more reliable estimates. The amount of “borrowing” is different depending on the extent of the small number problem for an individual area unit. In other words, the geographic units with more population rely more on their own data values for the risk rate, while those with less population rely more on the surrounding data values for the risk rate. For example, Figure 29 shows a series of two maps of sudden infant death (SID) rate in North Carolina produced by Cressie (1992). Figure 29a is the map made with raw data. Figure 29b is the map made using the Empirical Bayes Smoothing method. In order to fully understand the risk maps produced with smoothed data (Figure 29b) and be able to criticize and evaluate the maps, map readers need a high level of quantitative skills involving understanding the data smoothing approach.
Specifically speaking, map users need to first realize that the map (Fig. 29b) is not directly showing the raw data (29a), but rather smoothed data that relies on data from neighboring (or all) counties. Secondly, map users should possess the knowledge that many parameters applied in the data smoothing process can affect the values portrayed by the map. For example, data smoothing is based on some notion of “neighboring features” about how these are defined and how weights (if any) are applied to reflect the degree of ‘neighborliness,” as well as which geographic units are more affected by the smoothing process. Furthermore, spatial smoothing methods, by definition, remove extreme observations and emphasize the overall spatial distribution of the data set. Therefore, when using a smoothed choropleth map, map users
should focus on the overall spatial distribution pattern, and less on interpretation on the data of individual counties.

An important concept to consider in regards to Figures 29a and 29b is the respective levels of generalization-distortion (G-D). From one perspective, it could be argued that the smoothed map has more G-D since it generalizes (smoothes) data over numerous observations and so “distorts” the value of any one observation. On the other hand, it needs to be recognized that the areal units (the counties in this case) that form the basis for the observations are arbitrary and obfuscate the fact that the “theme” they try to capture actually has a continuous spatial distribution. Given this perspective, it can be argued, and I do, that the smoothed map is actually a closer representation of the spatial distribution of the theme than the raw data map. As such, it has a lower G-D level.

The map reading tasks for the choropleth map based on smoothed data are listed in Table 20. The map reading skills and knowledge required to perform the map reading tasks and the domains of the literacy that the skills and knowledge belong to are also included in the table. Compared with the map reading skills for simple choropleth maps, some of the map reading skills and knowledge are at an obviously higher level.
Table 20. Map Reading Tasks for Choropleth Map with Smoothed Data

<table>
<thead>
<tr>
<th>Map Reading Tasks</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the range of SMR of a certain administration unit? (To solve this word problem, map user need to be able to read map elements. Therefore it belongs to pure map literacy)</td>
<td>Thematic map elements</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>What does SID means? What could cause its spatial distribution? (To solve this word problem, one needs to have some background knowledge about Sudden Infant Death. Therefore it belongs to pure thematic literacy.)</td>
<td>Background knowledge about the theme</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>How is the SID rate calculated? (To solve this word problem, one needs to be able to work with statistical calculation. Therefore it belongs to pure quantitative literacy.)</td>
<td>Statistical calculation</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>What is data smoothing? How is it performed for this map? How does it affect interpretation of the map? (Knowledge about the data smoothing method belongs to map literacy, and being able to calculate it belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus thematic map.)</td>
<td>Knowledge about data smoothing method</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
</tbody>
</table>
### Table 20 (Continued).

<table>
<thead>
<tr>
<th>Question</th>
<th>Knowledge about data classification method; background knowledge about the theme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why should map makers apply data smoothing method to this map?</td>
<td>Knowledge about data classification method; background knowledge about the theme</td>
</tr>
<tr>
<td><em>(Knowledge about the data smoothing method belongs to map literacy, and knowledge about Sudden Infant Death belongs to thematic literacy. Therefore it belongs to the intersection of thematic literacy and map literacy minus quantitative literacy)</em></td>
<td></td>
</tr>
<tr>
<td>How are the smoothed values calculated? Estimate the sudden death cases in each geographic unit in the map after the data is smoothed. <em>(Knowledge about the Sudden Infant Death belongs to thematic literacy, and calculation of the thematic data belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and thematic literacy)</em></td>
<td>Background knowledge about the theme; statistical calculation</td>
</tr>
<tr>
<td>Comparing the map produced with smoothed data and the one produced with the raw data. How do they differ? What kind of changes happened to the data? What are the benefits of using data smoothing method to show SID data? <em>(To solve this question, map users need to read the data, compare between the data. In order to compare and criticize the thematic mapping methods, map users need to have knowledge about the thematic mapping methods as well as the knowledge about the theme of the map. Therefore it belongs to the intersection of the three)</em></td>
<td>Thematic map element; background knowledge about the theme; statistical calculation</td>
</tr>
</tbody>
</table>

Extending the work of Cressie (1992) on the Empirical Bayes smoothing method, Berke (2004) applied the interpolation method of kriging to interpolate the areal data and transform the areal choropleth map produced with smoothed data into an isopleth map (Fig. 30). Interpolation methods, such as kriging, solve the long-existing problem of choropleth maps – namely, that
they seem to imply that the same rate of something like a disease applies throughout the whole geographic unit. This problem can create misunderstandings for map readers who are not knowledgeable about choropleth maps, or in our words, not thematic-map literate.

![Figure 30. a) Choropleth Map of Smoothed SID Rate of the Counties of North Carolina, 1974-1978 (4 Classes); b) Isopleth Map from Kriging the SID Value of the Counties of North Carolina (Berke 2004), and theirs on the Triangular Plot](image)

The general purpose of kriging is to create a continuous surface based on discontinuous point data. Specifically, kriging measures the statistical covariance between the data points separated by a given distance to construct an empirical variogram model of the spatial data sets. This empirical variogram model is then used to predict the values at all locations. In this way, spatial kriging methods transform choropleth maps into isopleth maps, and therefore enable map makers to display (calculated) data continuously though the space (Beyer et al. 2012). As mentioned before, kriging methods further address the areal bias and related problems in
choropleth maps. They have proven to be powerful tools in detecting spatial patterns (such as cancer clusters) in spatial analysis (Lemke et al. 2013).

Different sets of quantitative skills are involved in the map reading, as well as the critique and evaluation, of the isopleth maps produced by kriging methods. Akin to data-smoothed maps, the more skills and knowledge map users have about kriging methods, the more they can critique the map and appreciate its relative strengths and weaknesses.

Although these isopleth maps may provide a better idea of the spatial distribution of a thematic variable, map users need to be aware of the fact that kriging is an interpolation method, so the map, despite its compelling continuous visual, is not in fact showing the actual distribution. Similar to the choropleth map made from smoothed data, the isopleths produced with kriging method maps aim to display the overall spatial distribution pattern, and map users should be cautious when obtaining information from a small part of the map. Also, map users need to realize that the parameters applied in the kriging method have significant influence on the final map. For example, how is the centroid defined? - is it geographic center or population-weighted center?

Theoretically, and as noted above, disease rates vary continuously in space. It was noted above that smoothed maps have less G-D because they produce a closer representation of the underlying continuous distribution than do raw data maps. However, a data-smoothed choropleth map still uses the areal units and so implies a “step” when moving from one areal unit to another. As a result, an isopleth map is a closer representation to reality (less G-D) even if it is only an interpolation of smoothed data, and so not closer to the raw data. This is the reason that it locates lower than the smoothed map in terms of G-D level in the triangular plot in Figure 30. Also, because an isopleth map indicates the locational information about the thematic data in a non-
arbitrary form (through the isopleths) that captures its variation, it actually locates to the left of the choropleth maps in the triangular plot.

**Table 21. Map Reading Tasks for Isopleth Map Produced with Kriging Method**

<table>
<thead>
<tr>
<th>Map Reading Tasks</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the locations of high value clusters. What is the data value there?</td>
<td>Thematic map elements</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, map users need to be able to read map elements. Therefore it belongs to pure map literacy)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>What does SID mean? What could cause its spatial distribution?</td>
<td>Background knowledge about the theme</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, one needs to have some background knowledge about Sudden Infant Death. Therefore it belongs to pure thematic literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>What are the highest and lowest values in the kriged results? What is the contour interval?</td>
<td>Arithmetic calculation</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(To solve this word problem, one needs to be able to work with statistical calculation. Therefore it belongs to pure quantitative literacy)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>What is the kriging method? How is it performed? How would it affect the map?</td>
<td>Knowledge about kriging method</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td><em>(Knowledge about the kriging method belongs to map literacy, and being able to calculate it belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus thematic literacy.)</em></td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>Why should map makers apply data kriging method in map making?</td>
<td>Knowledge about data classification method; background knowledge about the theme</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>(Knowledge about the kriging method belongs to map literacy, and knowledge about Sudden Infant Death belongs to thematic literacy. Therefore it belongs to the intersection of thematic literacy and map literacy minus quantitative literacy)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How is kriging done? Estimate the sudden death cases in a given geographic unit in the map. (Knowledge about Sudden Infant Death belongs to thematic literacy, and calculation of the thematic data belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and thematic literacy)</td>
<td>Background knowledge about the theme; statistical calculation</td>
<td></td>
</tr>
<tr>
<td>Compare the isopleths map produced with kriging method and choropleth map produced with smoothed data. Do they show similar values/clusters? What’s the difference between extracting information from the isopleth map vs. a choropleth map for the same data set? Which map is better at presenting the distribution pattern of SID rate? (The complex spatial distribution analysis involves knowledge about SID, reading data from the map, and calculation and comparison of thematic data. Therefore it belongs to the intersection of the three. Notice how isopleth map presents the clusters in a more straightforward way)</td>
<td>Thematic map element; background knowledge about the theme; statistical calculation</td>
<td></td>
</tr>
</tbody>
</table>

The map reading tasks for this isopleth map produced through kriging method are listed in Table 21. Although the map reading skills are at a higher level, it does not mean that these maps produced with advanced mapping methods are more difficult to understand than election maps. Actually, the advanced thematic mapping methods help avoid some possible misunderstandings in map reading. However, in order to fully understand how these thematic
mapping methods work and how they affect the thematic data, map users will need higher-level skills and knowledge concerning them.

**Multi-Category, Multi-variable Maps**

Multi-variable maps display multiple thematic variables on the same map, and because their role is to communicate more thematic information, they locate to the very right side of the triangular plot. Multi-variable maps serve to compare and correlate different variables directly. The spatial correlation between variables is the main topic in reading and understanding multi-variable maps.

Because multi-variable maps present multiple sets of thematic information on the same map, they require higher map reading skills to correctly interpret them. They often use imaginative and innovative methods of symbolization. Referring back to Curcio’s (1987) three levels of graph comprehension discussed in Chapter 2, the correlation between different variables belongs to the level of “reading between data” (it is worth noting that “reading the data” sometimes also requires higher-level skill because of the more complex symbolization), and the analysis of the reasons for the correlation belongs to “reading beyond data.” These are all high-level skills.

Cartographers have studied the mapping skills of multi-variable maps and developed some principles to communicate multiple sets of thematic information effectively (Wainer and Francolini 1980, Eyton 1984). Olson (1981) used spectrally encoded mapping, which shows two variables in a two-way matrix of colors; see, for an example, his map of educational attainment and per capita income on Figure 31. With the introduction of more than one variable, multi-variable maps are able to show the information about the relations between the two (or more)
variables in addition to the spatial distribution of each variable. Judging by the extreme values, in Figure 31, the dark color (dark blue) denotes counties with high education and high income, while the light color (yellow) denotes counties with low education and low income. These two colors represent a set of counties where there is a positive association between education and income. On the other hand, the green color denotes counties with high education and low income, while red colored notes counties with low education and high income. These two colors represent a set of counties where there is a negative association between the education and income. Map users can get the information of the spatial distribution pattern of education level and income level from this map with the quantitative skills discussed early. Furthermore, map users can get a message about the relation between education and income– specifically that, because the map is generally covered by either dark hues or light hues, it communicates that, overall, there tends to be a positive association between education and income.

Figure 31. Interrelationship of Educational Attainment and Per Capita Income (Olson 1981)
Of course, this inferred positive association is a rough estimation done simply by looking at the extreme values. It involves relatively low-level skills within the domain of map literacy in the literacy model. Another way to look at the relationship between the mapped variables is to do so quantitatively and directly by arraying the data in a contingency table. High-level statistical skills, such as regression model, are necessary to do a comprehensive spatial correlation analysis of the data. Map users could also perform a spatial correlation interpretation using both quantitative literacy and map literacy. The parts of skills within the quantitative literacy part are at relatively low level: just knowing what correlation is. While those within the map literacy part are at higher level: being able to read the map critically and identifying trends, clusters, orientation, patterns by geographic units.

Multi-variable maps have been applied and interpreted in research of various subjects to study the spatial correlations of multiple variables. As an example, Younus et al. (2007) studied the outbreak of salmonella vs. educational attainment and racial group using a multivariable Poisson regression model, as well as two sets of multi-variable maps (Fig. 32). Unlike Olson (1981), who used a joint table of spectral codes to present the two different variables on the same map, Younus et al. (2007) used the brightness of geographic unit (black, dark grey, light grey and white) to present one variable, and the color of the boundaries of geographic features to present the other variable (notice also that Younus did not apply any data smoothing method to the map - the geographic units with small population are marked with dot to avoid the “small number problem”). In Younus’s study, multi-variable maps are used to detect the spatial distribution pattern of each variable. After that, an inference of a possible association between two variables is hypothesized, and then tested with statistical analysis methods.
Figure 32. Salmonella Incidence by Race at the Block Group Level (Younus 2007).
<table>
<thead>
<tr>
<th>Map Reading Tasks</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where do counties that are low on both variables tend to locate? Where is the largest concentration of block groups that are predominantly Black and have the highest rates of Salmonella? <em>(To solve this word problem, map user need to be able to read multi-variable symbology. Therefore it belongs to pure map literacy.)</em></td>
<td>Multi-variable symbology</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td>Knowledge about the theme of the map: What does per capita income mean? How is it calculated? What is Salmonella? What could be the cause of this disease? <em>(To solve this word problem, one needs to have some background knowledge about capital income/Salmonella. Therefore it belongs to pure thematic literacy.)</em></td>
<td>Background knowledge about the theme</td>
<td>Thematic Literacy</td>
</tr>
<tr>
<td>Compare values of variables for certain geographic units. How does one calculate the correlation of two variables? <em>(To solve this word problem, one needs to be able to work with statistical calculation. Therefore it belongs to pure quantitative literacy.)</em></td>
<td>Statistical calculation</td>
<td>Quantitative Literacy, Map Literacy</td>
</tr>
<tr>
<td>Is there a correlation between variables? Confirm your estimation with multi-variable analysis methods. <em>(Being able to read the multi-variable symbology belongs to the map literacy, and being able to calculate the correlations belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus thematic literacy.)</em></td>
<td>Statistical calculation; multi-variable symbology</td>
<td>Quantitative Literacy, Map Literacy</td>
</tr>
</tbody>
</table>
Table 22 (Continued).

<table>
<thead>
<tr>
<th>Question</th>
<th>Requires Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criticize the multi-variable symbology for the thematic data. How effective is it in presenting multiple variables on a same map? Is there any problem caused by it?</td>
<td>Knowledge about the multi-variable symbology belongs to map literacy, and knowledge about capital income/Salmonella belongs to thematic literacy. Therefore it belongs to the intersection of thematic literacy and map literacy minus quantitative literacy.</td>
</tr>
<tr>
<td>What is the average/median per capita income in the western U.S.? What about eastern U.S.?</td>
<td>Knowledge about capital income/Salmonella belongs to thematic literacy, and calculation of the thematic data belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and thematic literacy minus map literacy.</td>
</tr>
<tr>
<td>What is the spatial correlation between the variables? What could be the reason for that?</td>
<td>Being able to read the multi-variable symbology to the map literacy, being able to calculate the correlations belongs to quantitative literacy, and inference on the cause of the correlation need background knowledge about the theme. Therefore it belongs to the intersection of the three</td>
</tr>
</tbody>
</table>

In the multi-variable map reading, to recap, the first set of quantitative skills involved is the interpretation of each variable separately; the second step is the detection of associations between variables. The abilities to critique and evaluate the maps also consist of two aspects. The first aspect is the critique and evaluation of each variable separately, in which the quantitative skills involved are similar to those we discussed earlier for the single-variable choropleth maps. The other aspect is the knowledge of multi-variable correlations. Statistically,
the correlation between two variables can be illustrated through 2-D scatter plots. Multi-variable maps work like a graphical version of scatter plots, which additionally describe the spatial distribution of the variables. However, the drawback of multi-variable maps compared with 2-D scatter plots is that they show the value of variables in classes, which is not very accurate for statistical analysis. In order to have an accurate measure of the correlation between variables, skills of statistical correlation analysis, such as regression models, is commonly used in addition to the multi-variable map, as illustrated by Younus (2007). Some map reading tasks are listed in Table 22 above. The correlation between variables becomes an important component of the map reading skills listed in the table.

**Knowledge and Skills for Reading Maps in the Median Area**

Besides the reference maps that locate at the left side of the triangular plot and the thematic maps that locate at the right side of the triangular plot, the following section will shed some light on the quantitative skills and knowledge involved in map reading of maps within the median area in terms of the L/T ratio of the triangular plot. Maps with median L/T ratio present both locational information and thematic information, and these two elements are often of similar importance when the map is read. Various maps locate in this part of the triangular plot, such as land use and land cover maps, geological maps, weather maps and so on. For example, when map users read a land use map, they often may need to look at both the geometry (locational information) and the category (thematic information) of land use polygons to answer a question or accomplish a task. Also as indicated in Chapter 3, the balance between locational and thematic information partly depends on the question one seeks to answer when reading the map. The same
map may actually move around within the median domain of the triangle depending on that question.

The generalization and distortion of maps with median L/T ratio is also valued in both locational and thematic information. In terms of the distortion in the locational information, it works the same way as it does for the left side of the triangular plot. As claimed here previously, usually large-scale maps have a low G-D level in the locational information while small-scale maps have a high G-D level. In terms of the distortion in the thematic information, it works the same way as it does for the right side of the triangular plot. The G-D level in thematic information is determined by the accuracy with which thematic data is represented by the map. Assuming the thematic data is correctly plotted in the map, the scale of the map can also affect the G-D level of thematic information for the map in the median area of the triangular plot. This is because when the map is plotted at a smaller scale, often some geographic map units become combined or generalized. If those geographic features actually have different thematic values, the thematic values would also get combined or generalized. For example, a land use map of a city can show the type of land use in very detailed units (such as blocks). It can also show the data in very detailed categories; for example, particular residential areas may be classified it into “single family,” “multi-family,” “mobile home,” etc. However, if the land use map is plotted at a state-wide scale, it will generalize the geographic features and typically display the thematic data in much less detail as well. For instance all the various types of residential area mentioned above are likely to be classified as simply “residential.” As a result, the G-D level of thematic information is likely to be higher for small-scale map.
Figure 33 shows a regional land use map of downtown Tallahassee. It has relatively low G-D levels in both geographic information and thematic information, so the map appears in the middle-low part of the triangular plot.

**Figure 33.** Land-Use Map of Downtown Tallahassee (Data Source: Tallahassee County Planning Department), and Its Location in the Triangular Plot

In order to explore the map reading skills and knowledge involved in understanding this type of land use map, some word problems are produced based on it as before. The map reading skills and knowledge needed to solve these problems and the domains of literacy that the skills belong to are listed in Table 23. More examples of word problems for this land use map of Downtown Tallahassee can be found in Appendix D.
Table 23. Word Problem Examples for Tallahassee Land-use Map.

<table>
<thead>
<tr>
<th>Word Problems</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the land use type that covers most of the area in downtown Tallahassee? (To solve this word problem, map user need to be able to read map elements. Therefore it belongs to pure map literacy.)</td>
<td>Read map element</td>
<td></td>
</tr>
<tr>
<td>What are the factors that affect the land-use distribution of Tallahassee? (To solve this word problem, one needs to have some background knowledge about the geography or history of Tallahassee. Therefore it belongs to pure background knowledge—i.e., background knowledge minus quantitative literacy minus map literacy.)</td>
<td>Background knowledge</td>
<td></td>
</tr>
<tr>
<td>Where is the City Hall of Tallahassee located? What kind of land cover type is there? (Background knowledge about the locations belongs to background knowledge, and reading map elements belongs to map literacy. Therefore it belongs to the intersection of background knowledge and map literacy minus quantitative literacy.)</td>
<td>Background knowledge; read map element</td>
<td></td>
</tr>
<tr>
<td>What is total area of the land use type that covers most of the area in downtown Tallahassee? (Reading map elements belongs to map literacy, and calculation of area with scale belongs to quantitative literacy. Therefore it belongs to the intersection of quantitative literacy and map literacy minus background knowledge)</td>
<td>Read map element; ratio and scale, calculate area using map scale</td>
<td></td>
</tr>
</tbody>
</table>
Table 23 (Continued).

<table>
<thead>
<tr>
<th>Land use planning: if you plan to to build a shopping center within downtown Tallahassee, which would occupy at least 0.5 square miles of area and meet the following requirement?:</th>
<th>Background knowledge; read legend; calculate area and distance using map scale;</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Must be built on “Open Space Undesignated” or “Open Space Common Areas”</td>
<td></td>
</tr>
<tr>
<td>b. Must be within 1.5 to 2.5 mile from the center of the city.</td>
<td></td>
</tr>
</tbody>
</table>

(*Complex land-use design need map reading skills in all three domains. Therefore it belongs to the intersection of the three.*)

Obviously some basic map reading skills involved in large-scale topographic maps are also involved in large-scale land-use maps, such as recognizing categories, calculating distances and areas. Besides these basic skills, some spatial analysis methods (such as, overlaying) are involved in the map reading (the land use planning question). These spatial analysis methods are typically used for planning purpose.

In terms of median L/T ratio maps with high G-D level, an example would be a global land cover map (Fig. 34) because the generalization and distortion in both geographic and thematic information is at a high level. Word problems and their corresponding map reading skills/knowledge for such a map are listed in Table 24. More examples of word problems for this global land cover map can be found in Appendix D.
Figure 34. Global land cover map (Boston University and NASA GSFC, 2002), and Its Location on the Triangular Plot.

<table>
<thead>
<tr>
<th>Word Problems</th>
<th>Map Reading Skills Involved</th>
<th>Domain of Literacy that the Skills Belong to</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the land use type that covers the most area of Australia?</td>
<td>Read map element</td>
<td>Background Knowledge</td>
</tr>
<tr>
<td>(To solve this word problem, map user need to be able to read map elements.</td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>Therefore it belongs to pure map literacy)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How is land cover related to climate?.</td>
<td>Background knowledge</td>
<td>Background Knowledge</td>
</tr>
<tr>
<td>(To solve this word problem, one needs to have some background knowledge about</td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>the interrelation between climate and land cover. Therefore it belongs to pure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>background knowledge)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Where on the Earth is Barren/Sparsely Vegetated area mostly located? Why?</td>
<td>Read map element; background</td>
<td>Background Knowledge</td>
</tr>
<tr>
<td>(Background knowledge about the locations belongs to background knowledge, and</td>
<td></td>
<td>Quantitative Literacy Map Literacy</td>
</tr>
<tr>
<td>reading map elements belongs to map literacy. Therefore it belongs to the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>intersection of background knowledge and quantitative literacy)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 24 (Continued).

<table>
<thead>
<tr>
<th>Estimate the proportion of the Earth’s land area that is covered by Barren/Sparsely Vegetated land cover. (Reading map elements and knowing the effects of projection system on the area of geographic features belongs to map literacy, calculate the de Therefore it belongs to the intersection of the three.)</th>
<th>Read map element; percentage calculation, estimate area on maps; knowledge about projection system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate the proportion of the Earth’s land area that is ideal for human to live. What do you need to know when you try to make this estimation (Knowing which kind of land cover is ideal for human to live belong to background knowledge. Reading map elements and find the area for Barren/Sparsely Vegetated area belongs to map literacy, calculate the de Therefore it belongs to the intersection of the three.)</td>
<td>Read map element; percentage calculation, estimate area on maps; knowledge about projection system</td>
</tr>
</tbody>
</table>

By looking at the word problems made based this map, it is found that some basic map reading skills, such as identifying geographic features and reading categories are still involved in map reading. Because the distortion level in the geographic feature is relatively high, arithmetic calculation of the distance, angle, and area is not as important as they are in the large-scale land-use map. However, it is still important to have the knowledge of projection systems to make correct inferences. The knowledge concerning the thematic information, such as the relation between land cover and climate type becomes important in understanding these maps.

Variability of Map Reading Skills across the Triangular Plot

Examining the map reading skills and knowledge discussed in the previous sections, there seems to be significant differences involved across the various types of maps. Therefore it
would be instructive to consider the types and levels of skills/knowledge relative to the concept of the triangular plot.

Generally speaking, the quantitative skills involved in the map reading process for reference maps (located on the left side of the triangular plot) are more aligned with data and experiences in STEM-type disciplines and measurement. These quantitative skills typically involve concepts like distances, slopes, directions, routes or other geometries of locational features. As indicated in the additional word problems in the appendix, calculations based on scales, ratio, and angles (trigonometric functions) are commonly utilized in the reference map reading process.

G-D level also influences the quantitative skills involved in map reading. For the reference maps for which the G-D level is relatively low (i.e., lower part of the left side of the triangle), the quantitative skills are most likely to be algebraic and of a routine nature familiar to lower-division STEM majors, such as being able to use scale to calculate areas and distances, use ratios to calculate gradients, calculate angles for bearings and direction. The calculation is mostly straightforward and no higher skill is involved.

As the G-D level increases for reference maps, the major challenge in map reading becomes dealing with generalization and distortion relating to the Earth’s curvature. To correctly understand these small-scale maps, then, knowledge and skills related to projection systems must be utilized. For example, many calculations are no longer straightforward. They are technically more advanced and often require relevant knowledge of calculus (e.g., equal-area vs. equal-angle projections) or transformation of coordinate systems. There are also other aspects of changing scale that affect G-D level, such as the deliberate offsetting of the spatial locations of features
when the map scale becomes smaller. This would require relevant knowledge within the domain of map literacy to correctly interpret the map.

Although it appears as though the level of quantitative skills increases as the G-D level increases, when the G-D level reaches a point where the geometry of the map is totally distorted and only topology is preserved (e.g., the topology map in Fig. 24), the map reading skills return to basic skills like identifying geographic features and finding routes. Even low-level skills like arithmetic calculation over distance and angle are not applicable for this type of map because of the excessive distortion. The differences between the word problems produced based on the large-scale reference map and those produced based on topological maps reinforce this conclusion.

By combining the previous discussion and the framework of the triangular plot, it indicates a clear view of how quantitative skills and knowledge involved in map reading vary based on the G-D level for different types of reference maps. As shown in Figure 35, there is an increasing level of quantitative skills involved in the map reading with the increasing of G-D level. This is because of the increasing relevance of comprehending the various projection systems applied in the small-scale reference map. This increasing level of quantitative skills stops and returns to low-level map reading skills when the G-D level reaches the point of topological maps, where the projection system is not applied in the map making.
On the other hand, map reading skills and knowledge vary quite differently on the right side of the triangular plot. Generally speaking, thematic map reading shares some similar lower-level skills with reference maps, such as searching and identifying geographic features, reading map elements (title, legend, etc.), counting and comparing counts. Except for these basic skills, however, in order to fully understand a thematic map, map users need very different quantitative skills than those used in reference map reading. The quantitative skills involved in thematic map reading usually deal with the statistics of thematic information rather than the geometry of geographic features.

The ability to evaluate the “message” conveyed in a thematic map and to recognize its shortcomings is an important skill for almost all types of thematic maps. The difficulty of this task varies for different types of thematic maps. For maps with a high G-D level, such as the election map (Fig. 26) located in the upper right part of the triangular plot, it is not reliable to conduct any detailed quantitative analysis based on the map. But map users still need to know the concept and characteristics of choropleth maps to avoid being misled by the generalized
information from the map. When the thematic data is classified in more detail, like the choropleth map shown in Figure 28, the major task of map reading is to realize the data uncertainty introduced with data classification methods, and to be able to critically obtain quantitative information from the map. The statistical methods, such as smoothing methods and kriging, applied in map making, allow map users to get closer to the likely underlying continuity of the thematic variable, but map users would need more advanced quantitative skills and knowledge concerning these statistical methods to fully understand the advantages and shortcomings of them.

Combining the previous discussion and the framework of triangular plot gives a clear view of how quantitative skills involved in the map reading vary based on the G-D level for different types of thematic maps. As shown in Figure 36, there is an increasing level of quantitative skills involved in the map reading with the decrease of G-D level. This is because, although the introduction of advanced mapping methods and statistical methods help make the map convey the quantitative information closer to the underlying continuity of the theme (less G-D), the map reading itself requires higher quantitative skills and knowledge.

However, it is important to understand that, although some thematic maps require lower-level skills to read, that doesn't mean they are easy to read. Actually they could easily be misunderstood without the necessary background knowledge. For example, a person looking at the election map (e.g., Fig. 26) in a newspaper could make easily make an incorrect inference about the result of the election by looking at the size of each color, without realizing that this map is not appropriate for that purpose.
As a special type of thematic map, multi-variable maps involve more thematic information and therefore have lower L/T ratios. As a result of more variables, different kinds of quantitative skills are introduced – those relating to spatial correlation of the variables. Map users need the quantitative skills in understanding single-variable choropleth map to understand the individual variables, as well as the ability of spatial correlation to find the interrelation between different variables. Based on the three-levels of graph reading skills proposed by Curcio (1987), the latter of these is “reading beyond the data,” and therefore it belongs to a type of higher-level skills.
CHAPTER FIVE: CONCLUDING REMARKS

Looking Back

As indicated in its title, this dissertation aims to rethink map literacy and conduct some foundational work for a newly defined part of it, Quantitative Map Literacy (QML). The field of QML is envisioned as an interdisciplinary subject involving the intersection of quantitative literacy and map literacy (Fig. 1). Graph literacy, in turn, is regarded as a bridge that links the field of quantitative literacy and map literacy, because graphs (especially statistical graphs) have been an important part of the study of quantitative literacy. In particular, maps can be regarded as a special type of graphs. In the case of thematic maps, they display quantitative or categorical data on a spatial background. In the case of reference maps, the display geographic features that inherently possess quantitative attributes themselves or can yield quantitative information by measurement. As a result, a concept chain can be formed starting with the general concept of literacy, followed by numeracy, quantitative literacy, graph literacy, graphicacy, and finally, map literacy.

As claimed in Crowther’s Report (1959), “literacy” means more than being able to read and write, and “numeracy” should also mean more than being able to calculate or work with numbers. Specifically, there is a hierarchical concept chain from numeracy to quantitative literacy to quantitative reasoning, where the scope expands from narrow (numerical operations) to broad (complex quantitative reason processes). As this scope grows, graphs appear in the
study field of quantitative literacy. Graphs, especially statistical graphs (such as box plots, scatter plots, etc.), are greatly valued for their ability to display complex quantitative information in a concise form. At the same time, whether users can correctly understand graphs has become a concern of researchers. Curcio (1987) pointed out there should be a hierarchy of complexity in reading graphs, which he partitioned into three levels as “reading the data,” “reading between the data,” and “reading beyond the data.”

This idea of hierarchical levels in graph reading is reflected in other studies in the field of graph literacy -- and even map literacy -- in many ways. As a special type of graph that presents geo-spatial data, maps are mentioned in some general studies of graph literacy (e.g., Tufte 1992, Wainer 1997). The deeper and more specific studies on how map users understand maps are conducted by geographers and cartographers. The knowledge and skills involved are explored and summarized, for example, by Board (1975) and Morrison (1978). Olson (1976) pointed out a hierarchical complexity in map reading skills, which is very similar to Curcio’s (1987) three levels of graph reading skills. Based on these levels of knowledge and skills, the domain of map literacy has been developed and defined. Despite these achievements in the field of map literacy, however, several shortcomings are noted: (1) in the early studies, cartographers tended to focus on reference maps due to the limited use of thematic maps, and even when thematic maps became more prominent, map literacy studies tended to focus on either reference maps or thematic maps rather than approach map literacy in a holistic way; (2) researchers have tended to focus on low-level skills in map reading, and this has had the consequence that map reading has come to be regarded as an “easy task;” (3) in thematic map literacy studies, the skills and knowledge involved in map reading have not been summarized or documented. Rather, researchers have tended to look at factors, such as gender, education levels, map media, etc., that
may affect the map reading process, while paying scant attention to the process (the skills and knowledge required) itself.

A Venn model is constructed in this dissertation in order to systematically analyze map reading skills and knowledge. In this Venn model, the knowledge and skills involved in map reading are classified into three domains of literacy: map literacy, quantitative literacy, and geographic/thematic literacy. A three-set Venn diagram is applied to illustrate the relations among and between these three domains of literacy. It also provides a framework for discussion of how the scope of map literacy study has been developing.

It is noted that the content in each subset of the Venn model varies according to types of map. The most intuitive variability is that geographic literacy is an important domain of knowledge in reference map reading, while for thematic map reading, knowledge is primarily about the theme of the map (called thematic literacy). This fundamental difference indicates the importance of map type to map literacy and why it is necessary to know the type of map before considering map interpretation. Therefore, a triangular plot model has been proposed as a conceptual framework to discuss the variation among maps. In the triangular plot model, the critical features of maps (locational information, thematic information, and generalization and/or distortion) are chosen as the vertices of a compositional triangle. A variety of example maps are located on this triangular plot to illustrate its ability to distinguish different types of maps.

Based on the Venn model for map reading skills/knowledge and the triangular plot system for differentiating maps, it is possible to explore and discuss different quantitative skills involved in map reading for different types of maps. Generally speaking, for reference maps, the quantitative skills mostly deal with the location or geometry of geographic features. The G-D level (generalization-distortion level) also has impact on the quantitative skills involved in
reference map reading. Within a limited range, the level of quantitative skills involved in the reference map reading increases as the G-D level increases. This is because while doing the quantitative reasoning based on maps higher in the triangle, map users need to consider the influence of distortion and generalization, and use higher order skills and call on higher level knowledge. For example, in this part of the triangle, the knowledge and skills concern such topics as map projections, which involve some nontrivial mathematics to apply to solving problems quantitatively. However, when the generalization and distortion exceeds a certain level and reaches the level of topological maps, where the map is not made based on any projection system (geometry of the geographic features are totally distorted and only the topology is preserved), the map reading skills involved in the map reading return to the basic skills such as identifying features and finding routes.

Of course, the foregoing summary of the variability of quantitative skills is applied only to the reference maps (left side of the triangular plot). Word problems for the maps in the median L/T ratio zone indicate that the background knowledge of the thematic information becomes important in map reading. This is especially the case for the maps with median L/T ratio and high G-D level, where the distance, area and angle cannot be calculated directly.

On the right side of the triangular plot, generally speaking, map reading tasks involved in thematic map reading focuses on the interpretation of thematic information. A lot of the skills involved in this type of map reading belong to the subjects of statistical analysis and relevant knowledge of the statistical processes used in the map making.

The complexity of quantitative skills involved in the thematic map reading seems to have direct correlation with that of the statistical methods applied in the mapping. Choropleth maps require knowledge of statistical classification schemes that are not required for binary categorical
maps for instance. Meanwhile, advanced statistical methods often reduce the data uncertainty in thematic data because they can give a representation that is more continuous than implied by the typical use of choropleth maps. Therefore they shift maps that use such methods downward in terms of the G-D level in the triangular plot, and so they require advanced quantitative skills and knowledge for the map users to fully understand the maps, as well as to be able to criticize and evaluate the maps. The above points can be summarized in diagrammatic form as shown in Figure 37.

![Figure 37](image)

**Figure 37.** Variability of Quantitative Skills involved in the Map Reading within the Triangular Plot

As claimed at the beginning of the dissertation, this study aims to rethink the definition of map literacy and establish the new field of QML. The definition of QML is given verbally, and it is graphically illustrated by the intersection subset in the Venn model. The developing scope of map literacy study is also indicated through the Venn model. In term of the quantitative skills involved in map reading, the triangular plot is introduced to describe their variability for
different types of maps. These two models proved their ability to describe maps and map reading skills. They represent my view of the fundamental frameworks needed for QML study, and provide powerful tools when researchers want to systematically describe different types of map and the relevant map reading skills/knowledge involved for their interpretation.

Map reading is not as easy as it has often thought to be, and as reflected in the literature review of Chapter 2. As indicated in the previous sections, map reading has often been considered an “easy task” because researchers have tended to focus on low-level skills, rather than the complex spatial analysis that involves knowledge from multiple domains of literacy and calls on higher-level skills. This is especially the case for thematic maps, since there hasn’t been any systematic curriculum development involving thematic map reading until college-level education. And even at the college level, the relevant knowledge about thematic map is rarely covered except for perhaps geography majors. That means a large proportion of people who read thematic maps and make decisions based on thematic maps lack appropriate training on how to interpret them. My study is not the first to note this. Wiegand (2006) claimed that thematic map reading is likely to be more problematic than researchers have thought it to be, and Phillips (2013) argued that the map users’ ability to understand thematic maps is taken for granted.

Finally, in terms of the variability of map literacy for different types of maps, it is expected that, because the type of knowledge and map reading skills needed vary by type of map, a person’s ability to read and interpret any map depends on the types of quantitative skills that person possesses. Thus, one who is proficient at interpreting one type of map (e.g., a topographic map) doesn’t necessarily possess the map literacy to correctly obtain information from another type of map (e.g., cartogram).
Looking Forward

The content of this dissertation is only a starting point for a field of QML, but hopefully it creates a framework for studies on other topics that would fall under this field. I will discuss these briefly below.

**Development of Assessment Tools**

Given the frameworks developed in this dissertation and their use in organizing the concept of quantitative skills/knowledge over various types of maps, a next logical step would be to develop assessment tools to evaluate map users’ QML level by testing their proficiency in such knowledge and skills.

A potential way of developing this assessment tool is to construct an assessment instrument based on Bloom’s Taxonomy. The revised Bloom’s Taxonomy (Anderson et al. 2001) is a taxonomy of cognitive processes with increasing complexity for teaching, learning and assessing. There are six categories of cognitive actions listed in Bloom’s Taxonomy – namely (from low level to high level) remembering, understanding, applying, analyzing, evaluating, and creating. A detailed listing of the cognitive actions classified in these six categories is shown in Table 25.

<table>
<thead>
<tr>
<th>Remember</th>
<th>Understand</th>
<th>Apply</th>
<th>Analyze</th>
<th>Evaluate</th>
<th>Create</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognizing</td>
<td>Interpreting</td>
<td>Executing</td>
<td>Differentiating</td>
<td>Checking</td>
<td>Generating</td>
</tr>
<tr>
<td>Recalling</td>
<td>Exemplifying</td>
<td>Implementing</td>
<td>Organizing</td>
<td>Critiquing</td>
<td>Planning</td>
</tr>
<tr>
<td>Classifying</td>
<td>Attributing</td>
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<td></td>
<td>Producing</td>
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<td>Summarizing</td>
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<td>Inferring</td>
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<td>Comparing</td>
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<td>Explaining</td>
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</tbody>
</table>
Bloom’s Taxonomy has been widely applied in education and proven to be a powerful tool in course design (Betts 2008), exam design (Kim et al. 2012), and progress evaluation (McNeil 2012). As discussed in Chapter 4, there are different levels of skills/knowledge associated with different degrees of difficulty in map reading. By introducing Bloom’s Taxonomy, the quantitative skills involved in map reading could potentially be classified into this hierarchy of difficulties. An example classification of quantitative skills involved in both reference maps and thematic maps, based on Bloom’s Taxonomy, is shown in Table 26.

Table 26. A Tentative Classification of Quantitative Skills Involved in Map Reading Using Bloom’s Taxonomy

<table>
<thead>
<tr>
<th>Bloom's Taxonomy</th>
<th>Reference Map</th>
<th>Thematic Map</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Remember</strong></td>
<td>Search, locate, identify features. Orient map</td>
<td>Search, locate, identify features. Reading values of attributes</td>
</tr>
<tr>
<td><strong>Understand</strong></td>
<td>Count, calculate, compare, measure features</td>
<td>Count, calculate, compare attributes</td>
</tr>
<tr>
<td><strong>Apply</strong></td>
<td>Find route Problem solving (focus on problems related to location)</td>
<td>Problem solving (focus on problems related to data) Statistical analysis</td>
</tr>
<tr>
<td><strong>Analyze</strong></td>
<td>Identify projection system Classify, outline, summarize, generalize, synthesize features Analyze spatial distribution pattern of features.</td>
<td>Identify data classification method Analyze spatial distribution pattern of attributes.</td>
</tr>
<tr>
<td><strong>Evaluate</strong></td>
<td>Judgment on the preserved and distorted features in the projection system</td>
<td>Estimate data uncertainty Judgment of data classification method</td>
</tr>
<tr>
<td><strong>Create</strong></td>
<td>Forming the spatial mental-model from the elevation contour</td>
<td>Estimate the data distribution pattern of the thematic data shown in map</td>
</tr>
</tbody>
</table>
Map Communication Model Using the Triangular Plot

Frank Fearing (1953) stated that human communication is composed of four basic components, namely: (a) the communicator, (b) the interpreter, (c) the communication content, and (d) the communication situation. A map is a specific type of communication (Dent, 1972): the communicator is the map maker; the interpreter is the map reader; the communication content is the map and its symbols; and the communication situation is the purpose of map. Communication situations include "finding direction" (for a navigation map) or showing the distribution of some thematic attribute, perhaps a disease rate, across a set of areal units (in a choropleth map).

As stated in the previous section, the triangular plot model is a powerful tool to describe the different type of maps. Therefore it could be useful to construct a model for map communication. In the high-L/T ratio side (mostly reference maps), the maps usually don’t have a specific theme. Maps are made by the map maker based on a standard. The communication usually happens from the map reader when they have a specific need requiring the map, for example, “I want to find the way to the airport.” The map maker provides various kinds of information in the map without specifically emphasizing any of them, to potentially accommodate many different types of question. The type and level of question the map reader can answer from the map depends on their level of map literacy (or QML) for this type of map.

In the low-L/T ratio side (mostly thematic maps), however, there is usually a specific theme for each map. The communication could happen in either of two opposing ways: the map reader looks for thematic maps that convey the information that is needed for the question at hand, or the map makers want to send a message through the maps to potential map readers. Because it is extremely easy for map makers to manipulate map symbols, data classification
methods, or advanced statistical techniques, map readers need to have good quantitative skills (typically statistical skills) to correctly obtain information from such maps.

These are only initial thoughts about a model for map communication. Map communication processes for different types of map need to be explored in detail to construct such a model. Nevertheless, the triangular plot model proposed in this study could potentially serve as an important tool to describe the types of maps, and so communication processes, involved in a model.

Factors Affecting the Level of Quantitative Map Literacy

As listed in the literature review section in Chapter 2, there has been research on the factors that might affect the understanding of thematic maps (Rieger 1999; Lloyd and Bunch 2008). Several factors (e.g., gender, education background, map media) have been tested through cognitive experiments. Similar research, though to a lesser extent, has been conducted for reference maps. Gilmartin and Patton (1984) conducted a series of cognitive experiments examining the influence of gender and map representation (on hard copy, or computer screen) towards the understanding of large-scale reference maps.

Individual research studies on factors that might affect the level of QML have, similar to map literacy research in general, tended to focus on either reference maps or thematic maps, more than likely reflecting the significant difference in the map reading skills involved in these two different types of maps. However, as indicated earlier, the map reading knowledge and skills for reference map and thematic maps can be very different. Thus, a person proficient in reading reference maps isn’t necessarily skilled or accurate at reading and interpreting thematic maps.
However, it is still unclear what, if any, interrelation there is between the understanding of reference maps and thematic maps. How would one factor influence the map reading level of different types of maps? Would the understanding of one type of map affect that of another type of map? These are the research questions waiting to be addressed under this topic.

The potential method to solve these research questions is to conduct a comprehensive cognitive experiment over different types of maps, as well as different factors. A scoring system would need to be developed to quantitatively evaluate map users’ understanding towards the maps. A questionnaire like the one used by Rieger (1999) or Nusrat et al. (2018) could be applied in this evaluation tool. However, it is important to include questions that require high-level skills to answer. This would avoid the drawbacks in previous studies which have predominantly focused only on low-level skills. The word problems included in the Appendix of this dissertation may be reasonable examples for the questions used in such a questionnaire. After that, a multivariable statistical regression analysis could be conducted to examine the potential interrelations between the factor’s influences over the map reading process for different type of maps.

**Improvement of Quantitative Map Literacy through Education Practice**

This topic of research would follow the completion of the assessment tool for QML described earlier. An approach that seems to be reasonable under this research topic, initially, would be to look at the influence of geoscience education on map users’ QML level. A pre-and-post cognitive experiment on a group of students, without prior specific training on map reading, and taking a geosciences course could be a practical way of doing this research. By examining students’ understanding toward various types of maps before and after taking the course, one
could evaluate whether and how the relevant education is helping improve students’ QML level. The advantage of choosing college students as the subjects of this study is that they are the group that is likely to obtain education in such topic. Thus, they are easy to access, and they are more familiar with assessments based on word problems.

However, the drawback of this choice is that the subjects are necessarily appropriate for the task at hand. Since we are more interested in how education practice could improve the QML level of general public, it is dangerous to use college students as representative of general public. By choosing the students without specific training on map reading, it may be possible to simulate the general public, which has not usually obtained education on map reading. However, students as a class may still not be the appropriate group of subjects, since college students are different from the general public in many ways, such as experience and pace of life, study ability and motivation, and so on.

Closing Thought

As can be seen from these future directions, there is much work to be done on the topic of quantitative map literacy, as there is in general in the “seas of literacy,” the metaphor used by Vacher (2019). This dissertation has shown that map literacy, in general, has perhaps been one of the more overlooked of these “seas,” perhaps because it has traditionally been a specialized field associated with cartographers. With the democratization of the map-making process, the availability of technologies like GIS, the rapidly increasing use of statistical graphics (including maps) in traditional and virtual media outlets, the time is ripe for a much more systematic and rigorous approach to the study of map literacy. My hope is that this dissertation has provided useful frameworks and ways of thinking to guide such study.
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APPENDIX A:

WORD PROBLEMS EXAMPLES (BASED ON CAMPUS MAP OF USF)³

Campus Maps of USF

Figure A1. USF Campus Visitor Map.⁴

³ The content of Appendix A, B, C and D consist of word problems selected from a project that I work with Dr. Vacher to develop sets of QMP word problems for GLY 3866 (Computational Geology) for publication in Scholar Commons later this year. These word problems are included here with his permission.
1.1. **Sizing up the map.** The campus is the rectangular area bounded on the north by Fletcher Ave., on the south by Fowler Ave., on the west by Bruce B. Downs Blvd., and on the east by 50th St. The red box outlines the map area in Fig. A2.

a. If you want to measure the dimensions and area of the USF campus from this map, you will need what map element that is now missing?

   A map scale is missing. A bar scale would do. That could be achieved by some indication of the on-the-ground length of some distance shown on the map.

b. The western boundary of the campus is 1.00 mile long. Given that you are looking at a PDF version of this problem set on your computer screen, use your Snipping Tool to make a picture of the map shown in Figure 1.1, and then Paint to measure the length of the on-the-map distance from the southwest corner of campus (intersection of Bruce B. Downs Blvd and Fowler Ave.) to the northwest corner of campus (intersection of Bruce B. Downs Blvd. and Fletcher Ave.). Then draw a horizontal bar of this length in a convenient place in the lower left portion of your map. Label the 0, ¼, ½, and 1.0 mile distances. What would be the representative fraction (RF)?

   We got a length of 4.2 in. on our copy, which means that we would mark our bar scale at 0, 1.05, 2.1, 3.15, and 4.2 in. for the quarter-mile lengths. (These lengths would be 0, 440, 880, 1320, and 1760 for a bar scale marked in yards.) The RF would be \( \frac{4.2 \text{ in.}}{1 \text{ mi}} \left( \frac{1760 \text{ yd}}{1 \text{ mi}} \right) \left( \frac{36 \text{ in}}{1 \text{ yd}} \right) = 0.000663 = \frac{1}{15,086} \), which is a larger scale than the classic USGS 1:24,000, 7.5-minute quadrangle sheet.

1.2. **Sizing up the campus.** Based on these considerations of scale of the campus map, how large is the campus?

a. What is the length of the south boundary of campus (i.e., the distance from the intersection of Bruce B. Downs Blvd. and Fowler Ave. to the intersection of Fowler Ave. and 50th St.)?

   The answer should be 1.5 mi.

b. What is the perimeter of the campus (in miles and kilometers)?

   The perimeter is \( (2 \times 1.5 \text{ mi}) + (2 \times 1.0 \text{ mi}) = (5 \text{ mi}) \left( \frac{1.609 \text{ km}}{1 \text{ mi}} \right) = 8 \text{ km} \)

c. What is the area of the campus (in square miles and acres)?

   The area is \( (1.5 \text{ mi})(1.0 \text{ mi}) = (1.5 \text{ mi})^2 \left( \frac{640 \text{ ac}}{\text{sq mi}} \right) = 960 \text{ acres} \)

d. If the campus was covered by water to an average depth of 1 ft, how many gallons would that be?

   The volume is \( (960 \text{ ac}) \times (1 \text{ ft}) = (960 \text{ ac-ft}) \left( \frac{325.851 \text{ gal}}{\text{acre-ft}} \right) = 2.6 \text{ million gallons (US)} \)
1.3. **Traffic lights per mile on campus boundaries.** For example, along 50th Street, there are two traffic lights, one at the beginning and one at the end of the 1 mile stretch. The density then is 1 per mile (don’t count the one at the start; you will see why in the next question).

a. Given these distances along the four boundary roads, what are the densities of the traffic lights (in number per mile)?

   Along Bruce B. Downs: 4 traffic lights, including the one at the start: so 3 per mile.
   Along Fowler Avenue: 5 including the one at the start so 4 per 1.5 mi, or 2.7 per mile
   Along Fletcher Avenue: 6 including the one at the start, so 5 per 1.5 mi, or 3.3 per mile

b. What is the density of traffic lights in a circuit around the periphery of campus?

   Counterclockwise from the southwest corner, you encounter 4 along Fowler, 1 along 50th St., 5 along Fletcher, and then 3 along Bruce B. Downs (and here you do count the one at the southwest corner where you began). That makes a total of 13 in the 5 mile circuit of 2.6 per mile (or one per 677 yards).

c. The third traffic light east of Bruce B. Downs Blvd. on Fowler Ave. is the main entrance to campus. It is Leroy Collins Blvd, which runs north/south past the Library to the campus flagpole (and what used to be the main administration building). How far is the traffic light at the entrance from the southwest and southeast corners of campus?

   The answer should be 0.75 mi.

1.4. **Zooming in on part of campus.** Get familiar with the map shown in Figure. A2. Use what you know from Figure A1.

a. What is the scale?

   First measure the EW length of library in Figure 1.1, and it is 0.24 inches. Therefore the real dimension is: 0.24 in / (1:15,086) = 3620.6 in

   And then measure the EW length of library in Figure 1.2, and it is 0.62 inches. Therefore the scale is: 0.62 in / 3620.6 in = 1:5840

b. What is the zoom factor?
The zoom factor is $\frac{1.5840}{1:1,586} = 2.58$

c. What are the dimensions of the map area (length and height)?

The length is 6.5 inches. That is: $(6.5 \text{ in})/(1:5840)=37960 \text{ in}=1054.4 \text{ yd}=0.5991 \text{ mi}$ in real Earth.
The height is 1.65 inches. That is: $(1.65 \text{ in})/(1:5840)=9636 \text{ in}=267.7 \text{ yd}=0.1521 \text{ mi}$ in real Earth.

1.5. Estimating the footprint of the Library. What is the footprint of the Library (i.e., for our purposes, “footprint of a building” is the area of the smallest rectangle that encloses the perimeter of the structure).

In Word Problem 1-4, we have measured the EW length of library in Figure 1.2, and it is 0.62 inches. And it is 3620.6 in real earth.

Similarly, measure the NS height of library, and it is 0.33 inches. That is $\frac{0.33 \text{ in}}{1:5840}=1927.2 \text{ in}$.

Therefore the footprint of the library is $3620.6 \text{ in} \times 1927.2 \text{ in} = 6.9776 \times 10^6 \text{ in}^2 = 1.1156 \text{ Acre}$

1.6. Estimating walking time to the Library. Juniper Hall (JPH, the red building at the northwest corner of Fig.1.2) is a student residence hall. Suppose you are a student living in Juniper Hall and need to get to class in Cooper Hall (CPR, next building east of LIB) at 2:00 PM. Suppose you walk at 5 miles per hour and want to get to the classroom 3 minute prior to the class time. You have to get through JPH, cross LeRoy Collins Blvd where there is a crossing guard directing traffic and pedestrians, and each of them would take 1 minute. Your classroom is on the third floor, and it would take 2 minute to walk up the steps to there. When should you leave your room?

The distance between the JPH building and the CPR building is about 2400 feet according to the known scale. The walking speed is 5 mi/hr=$440 \text{ ft/min}$. The walking time is $2400 \text{ ft} \div \frac{440 \text{ ft}}{\text{min}} = 5.45 \text{ min}$.

There are three road to cross, two on LeRoy Collins Blvd and one on USF Board Dr. This will take 3 min. Plus 2 minute to walk up the steps, you will spend totally: $5.45 \text{ min} + 3 \text{ min} + 2 \text{ min} = 10.45 \text{ min} = 10 \text{ min} 27 \text{ sec}$

Therefore you should leave before 1:49:33 PM.
2.1. Scale of the map. Recall that the campus is the 1.5 mi × 1.0 mi rectangular area bounded on the north by Fletcher Ave, on the south by Fowler Ave, on the west by Brue B. Downs Blvd (south of Fowler, it’s known as 30th St.), and on the east by 50th St.

a. What is the scale of the map of the campus as shown in Figure 1.3?

Measure the distance of Bruce B. Downs Blvd between Fletcher Ave. and Fowler Ave., and it is 2.74 inches. Therefore the scale is $\left(\frac{2.74 \text{ in}}{1 \text{ mi}} \right) \left(\frac{1 \text{ mi}}{63360 \text{ in}}\right) = 1:23,124$, which is a little bit larger scale than the classic USGS 1:24,000, 7.5-minute quadrangle sheet.

b. By what zoom factor is the map of the campus shown in Figure 1.3 magnified or shrunk relative to that of the map shown in Figure 1.1?

The zoom factor is $\frac{1:23,124}{1:15,086} = 0.65$

c. Develop a bar scale for this map (valid for campus area).

---

One option of the bar scale would be drawing a line equal to the distance of the west boundary of campus, and then mark it as 1 mile. The length of the bar scale may vary, for example, drawing a line equal to half of the distance of the west boundary of campus, and mark it as 2640 ft.

2.2. Directions on the main map.

a. What is the direction from the southwest corner (Fowler and Bruce B. Downs) of campus to the northeast corner of campus (Fletcher and 50th). Use the ground distances. Give answer in azimuth and bearing.

Let $\alpha$ be the counterclockwise angle from the positive $x$-axis (E) to the positive $y$-axis (N). Then $\alpha = \arctan\left(\frac{1\text{ mi}}{1.5\text{ mi}}\right) = 34^\circ$, and so the azimuth is $56^\circ$, and the bearing is N56E. Note, if the question asked for the direction from the northeast corner to the southwest corner, the tangent would have the same value, i.e., $-\frac{1\text{ mi}}{1.5\text{ mi}} = -\frac{1\text{ mi}}{1.5\text{ mi}} = 0.667$, but the direction would be different by $180^\circ$. Thus $\alpha = \arctan\left(-\frac{1\text{ mi}}{-1.5\text{ mi}}\right) = 214^\circ$, and the azimuth is $-124^\circ = 236^\circ$, and the bearing is S56W. It helps to draw a diagram.

b. Confirm that the scale you have calculated in the preceding question is the same in both NS and EW directions. That is, what is the length of the EW boundary and the NS boundary in the same units (in. or cm), and then use those values to answer part a again.

2.3. Determining anisotropy. In order to communicate about maps with anisotropic scales (i.e., where scale depends on direction), we will refer to NS as “vertical” and EW as “horizontal.” Then, with that terminology in mind:

a. We can define an anisotropy ratio as $\frac{\text{vertical scale}}{\text{horizontal scale}}$ and (in analogy with geological and topographic cross-sections) we can call it $VE$ for vertical exaggeration. Using the values you measured on the map for 1.14, what are the two scales, and hence the $VE$ for the map of the campus shown in Figure 1.3? It should be noted that $VE > 1$ for vertically exaggerated maps, and $VE < 1$ for horizontally exaggerated maps.

The vertical scale is $\left(\frac{2.74\text{ in}}{1\text{ mi}}\right)\left(\frac{1\text{ mi}}{63360\text{ in}}\right) = 1:23,124$ The horizontal scale is $\left(\frac{3.97\text{ in}}{1.5\text{ mi}}\right)\left(\frac{1\text{ mi}}{63360\text{ in}}\right) = 1:23,939$. Thus $VE = 1.03$.

A map is isotropic when $VE = 1$. 1.03 is very close to 1 and the difference can be considered as measurement error.

b. The anisotropy ratio can be visualized by a graphic showing perpendicularly intersecting bar scales in form of a plus (+). The plus may be stretched in one or the other direction (vertically or horizontally) depending on anisotropy. If the plus is stretched vertically, the vertical scale is larger than the horizontal scale; the map is vertically exaggerated. If the plus is stretched horizontally, the horizontal scale is larger than the vertical scale; the map is horizontally exaggerated. Describe the VE plus-graphic for the campus area assuming that you have the vertical and horizontal bars each represent 220 yards (1/8 mile).
The length on map would be: \(220 \text{ yd} \times \frac{1}{23\frac{1}{24}} = 0.00951 \text{ yd} = 0.342 \text{ in} = 0.870 \text{ cm}\)

The two bars would each be 0.342 in (or 0.870 cm) long.

2.4a. Anisotropy case 1 (Fletcher Ave - Skipper Road): anisotropy ratio. The upper boxed rectangle shows the excursion of the east-bound bus on Fletcher, north on 42\(^{nd}\) Street, then east on Skipper Ave, and then back to Fletcher on 46\(^{th}\) St. (and the reverse for buses west-bound on Fletcher).

a. The intersections at Fletcher and 42\(^{nd}\) and Fletcher and 46\(^{th}\) are the 3\(^{rd}\) and 5\(^{th}\) traffic lights east of Fletcher as shown on Figure 1.1. Using the scale on that figure, what is the distance between the two intersections?

The distance should be ¼ mile.

b. The north-south distance from Fletcher to Skipper is ¾ mile. Using the foregoing information, what is the vertical (i.e., NS) scale of the inset map? Does that agree with the distance you measure on the map?

The length is 1.13 inches. That is: \(\frac{1.13 \text{ in}}{1.23\frac{1}{24} \text{ in}} = 26130 \text{ in} = 0.412 \text{ mi} \text{ in real Earth}. Therefore it doesn’t agree with the distance.

c. What is the anisotropy ratio of the inset map?

Using Paint, I make the rectangle to be 446 pixels high by 323 pixels wide. So the vertical scale is \(\frac{446 \text{ px}}{0.75 \text{ mi}} = 595 \text{ px/mi}\), and the horizontal scale is \(\frac{323 \text{ px}}{0.25 \text{ mi}} = 1292 \text{ px/mi}\). So \(VE = \frac{595 \text{ px/mi}}{1292 \text{ px/mi}} = 0.461 = \frac{1}{2.17}\).

This means the map in the inset is horizontally exaggerated. The NS distance is drawn to a reduced scale.

d. Describe how the anisotropy graphic would look.

The horizontal bar would be about 2.2\(\times\) as long as the vertical bar. Rather than a circle, the outline would be that of an ellipse lying on its side. If the graphic of the main part of the map was a circle 1 cm in diameter, the graphic for the northern inset would be that circle stretched to where the horizontal diameter was about 2.2 cm long.

2.4b. Anisotropy case 1(Fletcher Ave - Skipper Road): determining directions. Directions on the northern inset map. On an anisotropic map, angles measured on the map are generally not the same as on the ground. The reason is that one of the scales is stretched relative to the other. Imagine a rhombus: if it is stretched vertically, the sloping sides are steeper; if it is stretched horizontally, the sloping sides are not as steep. The relation of the angles is:

\[VE = \left(\frac{\text{vertical scale}}{\text{horizontal scale}}\right) = \frac{\tan(\alpha_s)}{\tan(\alpha_0)},\]

the angles (\(\alpha\)) refer to the angle measured counterclockwise from the positive x-axis; and \(\alpha_s\) and \(\alpha_0\) are the stretched and pre-stretched angles, respectively. Then:

a. What is the direction on the ground from the intersection at Fletcher and 42\(^{nd}\) to Skipper and 46\(^{th}\)?

The given on-the-ground distances (displacements) are ¼ mile and ¾ mile for the EW leg from 42\(^{nd}\) to 46\(^{th}\) and the NS leg from Fletcher to Skipper, respectively. So, the angle on the ground is
\[ \alpha_0 = \tan\left(\frac{3/4 \text{ mi}}{1/4 \text{ mi}}\right) = \tan(3) = 72^\circ \] (with both the x-direction and y-direction positive, the angle is in the first, i.e., ne, quadrant). The azimuth is 18°, and the bearing is N18E.

b. What is the direction on the ground from the intersection at Fletcher and 46th to Skipper and 42nd?

The on-the-ground distances the same magnitude as before, but now the EW displacement is in the reverse direction (negative). So, the angle on the ground is \( \alpha_0 = \tan\left(-\frac{3/4 \text{ mi}}{1/4 \text{ mi}}\right) = \tan(-3) = -72^\circ + 180^\circ = 108^\circ \) (with the x-displacement negative and the y-displacement positive, the angle is in the second, i.e., nw, quadrant; algorithmically, add +180° if the x-displacement is negative). Then the azimuth is 198° (or -162°), and the bearing is S18W.

c. What is the direction on the ground from the intersection at Skipper and 46th to Fletcher and 42nd?

The on-the-ground distances are again the same magnitude as before, but now both of them are in the negative direction. So, the angle on the ground is \( \alpha_0 = \tan\left(-\frac{3/4 \text{ mi}}{-1/4 \text{ mi}}\right) = \tan(-3) = -72^\circ + 180^\circ = 252^\circ \) (with both displacements negative, the angle is in the third quadrant; algorithmically, add +180° if the x-displacement is negative). Then the azimuth is 198° (or -162°), and the bearing is S18W.

d. What is the direction on the ground from the intersection at Skipper and 42nd to Fletcher and 46th?

The on-the-ground distances are again the same magnitude as before, but now the y-displacement is in the negative direction. The angle on the ground is \( \alpha_0 = \tan\left(-\frac{3/4 \text{ mi}}{1/4 \text{ mi}}\right) = \tan(-3) = -72^\circ \) (with the x-direction positive and the y-direction negative, the angle is in the fourth, i.e., se, quadrant). Then the azimuth is 162°, and the bearing is S18E.

e. Using your previously calculated VE ratio, what is the “apparent” direction on the map for each of the four on-the-ground directions we just calculated?

For (a), the direction was N18E, and so \( \alpha_0 = 72^\circ \). Then \( \alpha_s = \tan(VE \tan \alpha_0) = \tan(0.461 \tan(72^\circ)) = \tan(1.42) = 55^\circ \). On the map, then, the direction would appear to be N35E.

For (b), the on-the-ground bearing was N18W and so, \( \alpha_0 = 108^\circ \). Then \( \alpha_s = \tan(VE \tan \alpha_0) = \tan(0.461 \tan(108^\circ)) = \tan(-1.42) = -55^\circ + 180^\circ = 125^\circ \). On the map, then, the direction would appear to be N35W.

For (c), the on-the-ground bearing was S18W and so, \( \alpha_0 = 252^\circ \). Then \( \alpha_s = \tan(VE \tan \alpha_0) = \tan(0.461 \tan(252^\circ)) = \tan(1.42) = 55^\circ + 180^\circ = 235^\circ \). On the map, then, the direction would appear to be S35W.

For (d), the on-the-ground bearing was S18E and so, \( \alpha_0 = -72^\circ \). Then \( \alpha_s = \tan(VE \tan \alpha_0) = \tan(0.461 \tan(-72^\circ)) = \tan(-1.42) = -55^\circ \). On the map, then, the direction would appear to be S35E.

f. How do these results compare with the angles measured using lengths measured on the maps along Fletcher and along 46th?

Previously we determined that the on-the-map lengths from Fletcher Ave to Skipper Rd and from 42nd St to 46th St were 466 and 323 pixels, respectively. Then a “vertical” right triangle with height 466 px and width 323 px, would diverge from the x-axis at an angle of \( \frac{466 \text{ px}}{323 \text{ px}} = \tan(1.44) = 55^\circ \). Depending on how the angle opens, the directions would vary: (i) if it opened up from the east-pointing x-axis, it would have a bearing of N35E (Az35°); (ii) if it opened up from the west-pointing x-axis, it would have a bearing of N35W(Az325°); (iii) if it opened down
from the west-pointing x-axis, it would have a bearing of S35W (Az 215°); and (iv) if it opened down from the x-east-pointing axis, it would have a bearing of S35E(Az145°).

2.4c. Anisotropy case 1 (Fletcher Ave - Skipper Road): determining distance (along diagonals). On an anisotropic map, the distance along diagonals measured on the map are generally not the same as on the ground. The reason is that one of the scales is stretched relative to the other. The relation of the distance along the diaganals and VE is:

\[ D = \text{horizontal scale} \times \sqrt{a^2 + (VE \times b)^2}, \]

where \( D \) is the distance on the map, \( a \) and \( b \) are the distance on the ground along the two right angle.

a. What is the distance on the ground of the diagonal from the intersection at Fletcher and 42nd to Skipper and 46th (or Fletcher and 46th to Skipper and 42nd)?

The distance is: \( \sqrt{(0.25 \text{ mi})^2 + (0.75 \text{ mi})^2} = 0.79 \text{ mi} \)

b. Using your previously calculated VE ratio, what is the “apparent” distance on the map for each of the on-the-ground distance we just calculated?

The apparent distance is: \( \frac{3.97 \text{ in}}{1.5 \text{ mi}} \times \sqrt{(0.25 \text{ mi})^2 + (0.461 \times 0.75 \text{ mi})^2} = 1.17 \text{ in.} \)

c. How do these results compare with the length measured on the maps along the diagonal?

The measured distance on the map is about 1.18 in. There difference could be cause by measurement error.

2.5a. Anisotropy case 2 (Fowler Ave - Bougainville Road): anisotropy ratio. The left boxed rectangle shows the bus on McKinley Dr., Bougainvilles Ave, and N 50th St.

a. What is the distance between the intersections at Fowler and McKinley and Fowler and N 50th?

Measure the distance on the big map, The length is 2.38 inches. That is: \( \frac{2.38 \text{ in}}{12.3124} = 55035 \text{ in} = 0.8686 \text{ mi} \text{ in real Earth.} \)

b. Measure the same distance on the small map in the left red box. What is the horizontal scale of the small map in the left red box?

The length is 1.25 inches. Therefore the scale is \( \frac{\frac{1.25 \text{ in}}{55035 \text{ in}}}{1:50,567} \)

c. The north-south distance from Fowler to Bougainville is ¾ mile. Using the foregoing information, what is the vertical (i.e., NS) scale of the inset map? Does that agree with the distance you measure on the map?

The length is 0.57 inches. That is: \( \frac{0.57 \text{ in}}{1:50,567} = 28,846 \text{ in} = 0.397 \text{ mi} \text{ in real Earth. Therefore it doesn’t agree with the distance.} \)

d. What is the anisotropy ratio of the inset map?

According to Word Problem 1.17.b, the horizontal scale is 1:50,567. The vertical scale is \( \frac{0.57 \text{ in}}{0.75 \text{ mi}} \times \frac{1 \text{ mi}}{63360 \text{ in}} = 1:83,368. \text{So} \) \( VE = \frac{1.83368}{1:50,567} = 0.607 = \frac{1}{1.65}. \) This means the map in the inset is horizontally exaggerated. The NS distance is drawn to a reduced scale.

d. Describe how the anisotropy graphic would look.
The horizontal bar would be about 1.65\times as long as the vertical bar. Rather than a circle, the outline would be that of an ellipse lying on its side. If the graphic of the main part of the map was a circle 1 cm in diameter, the graphic for the northern inset would be that circle stretched to where the horizontal diameter was about 1.65 cm long.

2.5b. Anisotropy case 2 (Fowler Ave - Bougainville Road): determining directions.

Directions on the left inset map. On an anisotropic map, angles measured on the map are generally not the same as on the ground. The reason is that one of the scales is stretched relative to the other. Imagine a rhombus: if it is stretched vertically, the sloping sides are steeper; if it is stretched horizontally, the sloping sides are not as steep. The relation of the angles is:

\[ VE = \left( \frac{\text{vertical scale}}{\text{horizontal scale}} \right) = \frac{\tan(\alpha_s)}{\tan(\alpha_0)}, \]

the the angles (\( \alpha \)) refer to the angle measured counterclockwise from the positive x-axis; and \( \alpha_s \) and \( \alpha_0 \) are the stretched and pre-stretched angles, respectively. Then:

a. What is the direction on the ground from the intersection at Bougainville and McKinley to Fowler and N 50\(^{th}\)?

The given on-the-ground distances (displacements) are 0.87 mile and 0.75 mile for the EW leg from McKinley to N 50\(^{th}\) and the NS leg from Bougainville to Fowler, respectively. So, the angle on the ground is \( \alpha_0 = \arctan \left( \frac{0.75 \text{ mi}}{0.87 \text{ mi}} \right) = \arctan(0.86) = 41^\circ \) (with both the x-direction and y-direction positive, the angle is in the first, i.e., ne, quadrant). The azimuth is 49°, and the bearing is N49E.

b. What is the direction on the ground from the intersection at Bougainville and N 50\(^{th}\) to Fowler and McKinley?

The on-the-ground distances the same magnitude as before, but now the EW displacement is in the reverse direction (negative). So, the angle on the ground is \( \alpha_0 = \arctan \left( \frac{-0.75 \text{ mi}}{-0.87 \text{ mi}} \right) = \arctan(-0.86) = -41^\circ + 180^\circ = 139^\circ \) (with the x-displacement negative and the y-displacement positive, the angle is in the second, i.e., nw, quadrant; algorithmically, add +180° if the x-displacement is negative). Then the azimuth is -49° (or 311°), and the bearing is N49W.

c. What is the direction on the ground from the intersection at Fowler and N 50\(^{th}\) to Bougainville and McKinley?

The on-the-ground distances are again the same magnitude as before, but now both of them are in the negative direction. So, the angle on the ground is \( \alpha_0 = \arctan \left( \frac{-0.75 \text{ mi}}{-0.87 \text{ mi}} \right) = \arctan(0.86) = 41^\circ + 180^\circ = 221^\circ \) (with both displacements negative, the angle is in the third quadrant; algorithmically, add +180° if the x-displacement is negative). Then the azimuth is 229° (or -131°), and the bearing is S49W.

d. What is the direction on the ground from the intersection at Fowler and McKinley to Bougainville and N 50\(^{th}\)?

The on-the-ground distances are again the same magnitude as before, but now the y-displacement is in the negative direction. The angle on the ground is \( \alpha_0 = \arctan \left( \frac{-0.75 \text{ mi}}{0.87 \text{ mi}} \right) = \arctan(-0.86) = -41^\circ \) (with the x-direction positive and the y-direction negative, the angle is in the fourth, i.e., se, quadrant). Then the azimuth is 131°, and the bearing is S49E.
e. Using your previously calculated VE ratio, what is the “apparent” direction on the map for each of the four on-the-ground directions we just calculated?

For (a), the direction was N49E, and so $\alpha_0 = 41^\circ$. Then $\alpha_s = \text{atan}(VE \tan \alpha_0) = \text{atan}(0.607 \tan(41^\circ)) = \text{atan}(0.528) = 28^\circ$. On the map, then, the direction would appear to be N62E.

For (b), the on-the-ground bearing was N49W and so, $\alpha_0 = 139^\circ$. Then $\alpha_s = \text{atan}(VE \tan \alpha_0) = \text{atan}(0.607 \tan(139^\circ)) = \text{atan}(-0.528) = -28^\circ + 180^\circ = 152^\circ$. On the map, then, the direction would appear to be N62W.

For (c), the on-the-ground bearing was S49W and so, $\alpha_0 = 221^\circ$. Then $\alpha_s = \text{atan}(VE \tan \alpha_0) = \text{atan}(0.607 \tan(221^\circ)) = \text{atan}(0.528) = 28^\circ + 180^\circ = 208^\circ$. On the map, then, the direction would appear to be S62W.

For (d), the on-the-ground bearing was S49E and so, $\alpha_0 = -41^\circ$. Then $\alpha_s = \text{atan}(VE \tan \alpha_0) = \text{atan}(0.607 \tan(-41^\circ)) = \text{atan}(-0.528) = -28^\circ$. On the map, then, the direction would appear to be S62E.

f. How do these results compare with the angles measured using lengths measured on the maps along Bougainville and along N 50th?

Previously we determined that the on-the-map lengths from Bougainville Ave to Fowler Ave and from McKinley St to N 50th St were 0.57 inches and 1.25 inches, respectively. Then a “vertical” right triangle with height 0.57 in and width 1.25 in, would diverge from the x-axis at an angle of $\alpha = \text{atan}(\frac{0.57}{1.25}) = \text{atan}(0.45) = 28^\circ$. Depending on how the angle opens, the directions would vary:

(i) if it opened up from the east-pointing x-axis, it would have a bearing of N62E (Az62º); (ii) if it opened up from the west-pointing x-axis, it would have a bearing of N62W(Az298º); (iii) if it opened down from the west-pointing x-axis, it would have a bearing of S62W (Az 242º); and (iv) if it opened down from the east-pointing axis, it would have a bearing of S62E(Az118º).

2.5c. Anisotropy case 2 (Fowler Ave - Bougainville Road): determining distance (along diagonals). On an anisotropic map, the distance along diagonals measured on the map are generally not the same as on the ground. The reason is that one of the scales is stretched relative to the other. The relation of the distance along the diagonals and VE is:

$$D = \text{horizontal scale} \times \sqrt{a^2 + (VE \times b)^2},$$

where $D$ is the distance on the map, $a$ and $b$ are the distance on-the-ground along the two right angle. Then:

a. What is the distanceon the groundof the diagonal from the intersection at Fletcher and 42nd to Skipper and 46th (or Fletcher and 46th to Skipper and 42nd)?

The distance is: $\sqrt{(0.87 \text{ mi})^2 + (0.75 \text{ mi})^2} = 1.15 \text{ mi}$

b. Using your previously calculated VE ratio, what is the “apparent” distance on the map for each of the on-the-ground distance we just calculated?

The apparent distance is: $\frac{1.25 \text{ in}}{0.87 \text{ mi}} \times \sqrt{(0.87 \text{ mi})^2 + (0.607 \times 0.75 \text{ mi})^2} = 1.39 \text{ in}.$

c. How do these results compare with the length measured on the maps along the diagonal?

The measured distance on the map is about 1.38 in. There difference could be cause by measurement error.

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APPENDIX B:

WORD PROBLEMS EXAMPLES (BASED ON TOPOGRAPHIC MAP)

Sulphur Springs Quadrangle

Figure B1. Sulphur Springs, FL 1995 (HTMC, 1999 ed.)

Scale and the size of the map area
1.1. Geographic coordinates. The map area is defined by the latitudes 28° and 28.16° and longitudes −82.3° and −82.12°. What are the coordinates of the corners of the quadrangle, in degrees-minutes format?

<table>
<thead>
<tr>
<th>North East Corner</th>
<th>North West Corner</th>
<th>South East Corner</th>
<th>South West Corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>(28° 7.5′N / 82°30′W)</td>
<td>(28° 7.5′N / 82°22.5′W)</td>
<td>(28° N / 82°30′W)</td>
<td>(28° N / 82°22.5′W)</td>
</tr>
</tbody>
</table>

1.2. Estimating quad size. Assume a spherical Earth with circumference 40,000 km,

a. What is the “height” dimension (i.e., NS) of the quadrangle on the ground?

On the ground: NS edges = \((\frac{0.125°}{90°})\) (10,000 km) = 13.889 km

b. What is the “width” (i.e., EW length) of the northern edge of the map area?

On the ground: northern edge = \((\frac{0.125°}{90°})\) (10,000 km) cos (28.125°) = 12.249 km

c. What is the “width” of the southern edge of the map area?

southern edge = \((\frac{0.125°}{90°})\) (10,000 km) cos (28°) = 12.263 km

d. How much shorter is the northern edge than the shorter edge.

14 m.

e. How much longer are the vertical edges than the horizontal edges.

1.623 and 1.640 km (or a little more than a mile)

Location of the map area

1.3. Nearby quadrangles. Return to your live view of the Get Maps | topoView interface seeking the Sulphur Springs 24K quadrangle as shown in Figure 3.2. Scroll out to where the scale is large enough that the names of the quadrangles appear. What are the names of the surrounding eight quadrangles?

<table>
<thead>
<tr>
<th>Odessa</th>
<th>Lutz</th>
<th>Wesley Chapel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Citrus Park</td>
<td>Sulphur Springs</td>
<td>Thonotosassa</td>
</tr>
<tr>
<td>Gandy Bridge</td>
<td>Tampa</td>
<td>Brandon</td>
</tr>
</tbody>
</table>

1.4. Functionality of the topoView map. While answering the previous question, you likely noticed that there were changes in the scale bars (lower left of map image), the representative fraction scale (RF) and the lat/long information (lower center), and the small-scale index map (lower right) as you moved the mouse and rolled the scroll wheel. Specifically,

a. What happens when you move the mouse?

The mouse icon moves around on the map, and the lat/long information changes accordingly.
b. What happens when you roll the scroll bar?

The scale of the map changes, and hence the RF, bar scales, quad sizes, area of coverage and detail such as place names all change.

c. What is the step between scales as you zoom in? And what changes occur with a single step?

The scale doubles with each step, and so the lengths and widths of all the quads double, their areas quadruple, and the area of coverage decreases to a fourth of its previous size.

d. What is the largest scale permitted? What are the corresponding indicated lengths of the bars?

1:18,056. 300 m and 1000 ft.

e. What is the smallest scale for which the boundaries of the quads are still shown?

1:4,622,325. 100 km and 50 mi.

f. How many steps are there between the preceding two scales?

\[
\log_2 \left( \frac{4,622,325}{18,056} \right) = \frac{\log(256)}{\log(2)} = 8, \text{ which means there are nine scales counting the two extremes.}
\]

1.5. Smaller scales. Stay with the Get Maps | topoView interface to explore what can be seen at other scales.

a. While the blue pointer is still marking the location of the Sulphur Springs, double-click on the 100K circle in the navigation column to reveal a grid of 100K maps. Describe the location of the Sulphur Springs 7.5-min quad in this grid of 100K maps.

The Sulphur Springs quad is the fourth of 19 7.5-min quads, from Dunedin on the west to Melbourne East on the east. At 12.2-km/quad, the width of the peninsula at the latitude of USF is about (19 quads) \( \left( \frac{12.2 \text{ km}}{\text{quad}} \right) = \sim 230\text{km} \). The distance from USF to the Atlantic coast is about 4× as long as the distance from USF to the Gulf coast (about 200 km vs. 50 km)

h. How many 7.5-min quads are there across the peninsula along the row in which the Sulphur Springs occurs?

The Tarpon Spring map area, which includes the Sulphur Springs map area, is bounded by latitudes 28° N and 28.5° N and longitudes 82° W and 83° W.
b. Now double-click on the 250 circle to reveal a grid of 100K maps. Describe the location of the Sulphur Springs 7.5-min quad in this grid of 100K maps.

The 250K maps are $1^\circ \times 2^\circ$ quadrangles.

<table>
<thead>
<tr>
<th>Gainesville</th>
<th>Daytona Beach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant City</td>
<td>Orlando</td>
</tr>
<tr>
<td>St. Petersburg</td>
<td>Fort Pierce</td>
</tr>
</tbody>
</table>

The Plant City map area, which includes the Sulphur Springs map area, is bounded by latitudes $28^\circ N$ and $29^\circ N$ and longitudes $82^\circ W$ and $84^\circ W$.

Scale and size of the map itself

1.6. RF vs. verbal scales. The normative scale of the map is 1:24,000. Both numbers must have the same units, so that the representative fraction says, for example, “1 inch on map represents 24,000 inches on the ground,” or “1 cm on the map represents 24,000 cm on the ground.” It is common, however, to express or think of the ratio with numbers of dissimilar units. Such “verbal scales,” as such ratios are sometimes called, have the form $[1$ unit : _____ different units]. Common verbal scales for the 1:24,000 scale are “[1 inch : (number)_(non-inches)]” and “[1 cm : (number) (non-centimeters)],” for Imperial and metric versions, respectively. What are they?

- $1$ inch : $2000$ feet
- $1$ cm : $240$ m or $1$ cm : $0.24$ km

1.7. Lengths via the digital map. Rather than estimate the lengths that way, we can determine them on the map itself by measuring the length of one of the bar scales and dividing by the scale.

a. What is the measured length of the 1-km bar scale?

$8.78$ cm on our map image.

b. What is the apparent RF scale (meaning the scale corresponding to our measured length of the bar scale on the map image)?

The apparent RF scale is $\left(\frac{0.78\text{ cm}}{1\text{ km}}\right) \left(\frac{1\text{ km}}{100,000\text{ cm}}\right) = 1:11,340$ (or $1$ cm : $0.1134$ km)

c. What are the “height” and “widths” measured on the map?

- Height = $119.96$ cm ($47.23$ in.)
- Short width = $106.49$ cm ($41.93$ in.)
- Long width = $106.63$ cm ($41.98$ in.)

d. What are the corresponding lengths on the ground?

- NS edges: $\frac{119.96\text{ cm}}{1\text{ cm}/0.1134\text{ km}} = 13.603$ km (vs. $13.889$ km estimate)
- EW length at the northern edges: $\frac{106.49\text{ cm}}{1\text{ cm}/0.1134\text{ km}} = 12.076$ km (vs. $12.249$ km estimate)
- EW length at the southern edges: $\frac{106.63\text{ cm}}{1\text{ cm}/0.1134\text{ km}} = 12.092$ km (vs. $12.263$ km estimate)

e. How do the estimated values compare with the actual values?
NS edges: estimate is longer than the actual by 286 m (or 2.1%)

EW edges: estimates are longer than actual values by 173 m (1.4%) and 171 m (1.4%)

1.8. Size of the map area at nominative scale. Imagine you wish to cover a wall with a large-scale map of central Florida by making a mosaic of 7.5-min. quadrangle maps

a. What are the length of the edges of the Sulphur Springs map area on an actual paper map where the RF scale really is 1:24,000, i.e., the nominative scale?

This is an exercise in scaling. How does the image change when you enlarge or reduce the scale by a given percentage? Here, we know the lengths on the map at our apparent RF scale. The nominative scale is 52.75% smaller; i.e., \( \frac{1}{24,000} = 0.4725 \). Therefore lengths “on the wall” would be smaller than on the map with the apparent RF, namely:

- NS edge: \( (119.96 \text{ cm})(0.4725) = 56.7 \text{ cm} = 22.3 \text{ in.} \)
- Upper EW edge: \( (106.49 \text{ cm})(0.4725) = 50.32 \text{ cm} = 19.81 \text{ in.} \)
- Lower EW edge: \( (106.63 \text{ cm})(0.4725) = 50.38 \text{ cm} = 19.84 \text{ in.} \)

b. How many such maps could you tile together to fit on a wall 10-ft high?

Round down to integer value: \( \frac{10 \text{ ft}}{22.3 \text{ in.}} = 5.4 \) maps = 5 whole maps

c. How much horizontal length of the wall would your map mosaic take?

Computationally: \( \text{Wall width} = (5 \text{ maps}) \left( \frac{20 \text{ in.}}{\text{map}} \right) \left( \frac{\text{ft}}{12 \text{ in.}} \right) = 8.3 \text{ ft} \)

d. Describe the geographic coverage of the mosaic. Assume the Sulphur Springs quad (and hence USF) is at the center of the mosaic?

Going back to topoView, the mosaic would consist of the following 25 quads:

<table>
<thead>
<tr>
<th>Port Richey</th>
<th>Firvay Junction</th>
<th>Ehren</th>
<th>San Antonio</th>
<th>Dade City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elfers</td>
<td>Odessa</td>
<td>Lutz</td>
<td>Wesley Chapel</td>
<td>Zephyrhills</td>
</tr>
<tr>
<td>Oldsmar</td>
<td>Citrus Park</td>
<td><strong>Sulphur Springs</strong></td>
<td>Thonotosassa</td>
<td>Plant City West</td>
</tr>
<tr>
<td>Safety Harbor</td>
<td>Gandy Bridge</td>
<td>Tampa</td>
<td>Brandon</td>
<td>Dover</td>
</tr>
<tr>
<td>Saint Petersburg</td>
<td>Port Tampa</td>
<td>Gibsonton</td>
<td>Riverview</td>
<td>Lithia</td>
</tr>
</tbody>
</table>

The mosaic would be bounded by latitudes \( 27°45′N \) and \( 28°22′30″N \) and by longitudes \( 82°7′30″W \) and \( 82°45′W \). On the west, it would not reach the Gulf except for on the Elfers and Port Richey quads. It would barely reach the southwest edge of the Green Swamp in the northeast corner of the mosaic. It would include most of Tampa Bay, down to about 10 km short of the southern tip of the Pinellas peninsula.

1.9. Scales of page-size maps. Given pages with dimensions of 8.5 inches × 11 inches, and 1-inch margins, reasonable sizes would be 6.5-in. wide for a two-column figure, and 3.0-in. wide for a one-column figure. What would be the apparent scale of the Sulphur Springs quadrangle
for these widths, assuming the map area takes up the whole figure width (i.e., not allowing for margins)? Assume values for the quadrangle at nominative scale: a scale of 1:24,000, and a on-the-ground width at the southern edge 12.092 km (7.515 mi).

a. For the 6.5-inch map

\[
(RF)_a = \frac{6.5 \text{ in.}}{(7.515 \text{ mi}) \left( \frac{5280 \text{ ft}}{\text{mi}} \right) \left( \frac{12 \text{ in.}}{\text{ft}} \right)} = 0.00001365 = 1:73,254
\]

\[
(RF)_a = \frac{1}{73,254} = 0.328 = 1:3.05
\]

Thus the 6.5-in wide map is a 67.2% reduction of the 1:24000-scale map, which is to say it is a 1:3.05-scale map of the printed nominative-scale map

b. For the 3.0-inch map

\[
(RF)_a = \frac{3 \text{ in.}}{(7.515 \text{ mi}) \left( \frac{5280 \text{ ft}}{\text{mi}} \right) \left( \frac{12 \text{ in.}}{\text{ft}} \right)} = 0.000006301 = 1:158,717
\]

\[
(RF)_a = \frac{1}{158,717} = 0.006413 = 1:15.87
\]

The 3-in wide map is an 84.9% reduction of the 1:24,000-scale map, which is to say it is a 1:15.87-scale map of the printed nominative-scale map

1.10. Points on the map to feet on the ground. Collect the various scales considered in these problems concerning the map area of Map 3, and arrange them from largest scale to smallest. For each scale, answer two questions about the sizes on the ground that correspond to various marks on the map, given the conversions that 72 points = 1 inch, and 2.54 mm = 1 inch.

For the nominative scale question (a), the 0.5-mm line width:

\[
\text{Length on ground} = \frac{0.5 \text{ mm}}{1/24,000} = \left( \frac{3.281 \text{ ft}}{\text{m}} \right) = 39.3 \text{ ft}
\]

For the nominative scale question (b), the 12-point font:

\[
\text{Length on ground} = \frac{12 \text{ points}}{1/24,000} = \left( \frac{288,000 \text{ pts}}{\text{inch}} \right) \left( \frac{\text{ft}}{12 \text{ in.}} \right) = 333 \text{ ft}
\]

<table>
<thead>
<tr>
<th>Context (WP)</th>
<th>Scale</th>
<th>Length corresponding to 0.5-mm line width (ft)</th>
<th>Length corresponding to a 12-point M. (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital map (3-7b)</td>
<td>1:11,340</td>
<td>19</td>
<td>157</td>
</tr>
<tr>
<td>Nominative scale</td>
<td>1:24,000</td>
<td>39</td>
<td>333</td>
</tr>
<tr>
<td>6.5-inch column (3-9a)</td>
<td>1:73,254</td>
<td>120</td>
<td>1004</td>
</tr>
<tr>
<td>3-inch column (3-9b)</td>
<td>1:158,717</td>
<td>260</td>
<td>2204</td>
</tr>
</tbody>
</table>
The US PLSS

1.11. Section numbers. Open the Sulphur Springs 1:24,000 sheet again, either as the PDF from the Supplemental Files or on topoView. Magnify the scale to where you can see detail and scroll to the northern edge of the map area. Notice the red numbers and the vertical and horizontal red lines halfway between them. (Note: two versions, 1995 (1999 ed) and 1956 (1957 ed)

a. What is the succession of red numbers left to right across the top row of numbers? How many numbers are there?

22, 23, 24, 19, 20, 21, 22, 23. 8 numbers

b. Write out the succession of red numbers, north to south, down the east most column. How many numbers are there?

23, 26, 35, 2, 11, 14, 23, 26, 35. 9 numbers

c. Recall the dimensions of the quadrangle (in miles) from 3.2?

7.6 mi west-east and 8.6 mi north-south.

d. Explain the relationship between the numbers in (a) and (b), and the distances in (c)

The numbers are each a repeated succession of six numbers: 19, 29, 21, 22, 23, 24 in (a) and 2, 11, 14, 23, 26, 35 in (b). In each case they are spaced about a mile apart. The red horizontal and vertical lines represent the surveyed boundaries of nominally one-mile squares (called sections) in what’s known as the USPLSS grid.

![Map of the US PLSS grid of Hillsborough County](https://www.hcpafl.org/Portals/HCPAFL/pdfs/str_map.pdf)

Figure B2. The US PLSS grid of Hillsborough County.

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The grid consists of columns (called ranges) and rows (called townships). Both the columns and rows are nominatively 6 mi wide; therefore, they intersect in 36-mi² squares (also called townships, unfortunately). Each of the square townships is subdivided into 36 1-mi² squares (called sections). Sections and their fractional parts (such as quarter sections, and quarter-quarter sections) played an important role in defining properties and their boundaries as the U.S. expanded outward from the original colonies. Hillsborough County is crossed by six ranges and eight townships. The northwest township is T27S, R17E. The southeast township is T32S, R22E

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6 For background see [https://nationalmap.gov/small_scale/a_plss.html](https://nationalmap.gov/small_scale/a_plss.html) and for tutorial see [https://dnr.wi.gov/topic/forestmanagement/documents/plsstutorial.pdf](https://dnr.wi.gov/topic/forestmanagement/documents/plsstutorial.pdf).

7 [https://www.hcpafl.org/Portals/HCPAFL/pdfs/str_map.pdf](https://www.hcpafl.org/Portals/HCPAFL/pdfs/str_map.pdf)
1.12. Counting sections. The green, purple, and tan areas on the map are incorporated towns, The uncolored areas are unincorporated, The green and purple areas are clearly Tampa and Temple Terrace, respectively.

a. What is the tan area?

Plant City

b. Estimate the area of Temple Terrace.

According to the map, Temple Terrace covers 9 sections. One section is nominally 1 mi$^2$, therefore the area of Temple Terrace is about 9 mi$^2$.

c. Estimate the land area of Hillsborough County.

Again, the area can be estimated by counting the sections (and in this case 36 mi$^2$ townships). There are 26 full townships; 1 township with 32 sections; 1 town with 18 sections; 1 township with 21 sections; 1 township with 28 sections; 1 township with 2 sections. The total area is about $26 \times 36 \, \text{mi}^2 + 32 \, \text{mi}^2 + 18 \, \text{mi}^2 + 21 \, \text{mi}^2 + 28 \, \text{mi}^2 + 2 \, \text{mi}^2 + 24 \, \text{mi}^2 = 1061 \, \text{mi}^2$.

1.13. Location of the USF campus in the US PLSS.

a. Identify the township which is in the northwest corner of the map of Figure 3.3.

According to the figure caption, it is T27S, R18E

b. Identify the township in which USF occurs (Fig. 3.3).

USF is just west of Temple Terrace, which is colored purple on the map of Figure 3.3. In fact, with good eyes and some magnification, you might see a little label in the section next to the northwest corner of Temple Terrace. It says USF, and is visible if you zoom in to about 400%. Since that township is 2 ranges over and one township down from T27S, R17E, it is safe to say that USF is in T28S, R19E, which you can confirm from the labels of the rows of townships and the columns of ranges in the figure.

c. Identify the section(s) in which USF occurs.

The label occurs in Section 9 of T28S, R19E.

But we can say more than that. Recall our discussion of Figure 1.1, the USF Campus Visitor Map. The campus is bounded by Fletcher and Fowler Avenues which are 1 mile apart, and by Bruce B. Downs Blvd and 50th Street which are 1.5 miles apart. It certainly looks reasonable from those two maps that USF occupies all of Section 9 and the eastern half of Section 8. In the language of US PLSS, the USF campus is in S9, T28S, R19E and E 1/2 S8, T28S, R19E.

Confirm this location of the USF Tampa campus from the Sulphur Springs quadrangle map. To do so, recall that the section lines are red (do not confuse them with the black straight lines. To see the section lines bordering the campus, look first at the northeast corner of campus where the section line extends north into the swamp north of Fletcher (the section line is overprinted by 50th St south of Fletcher). Similarly for the northern and southern bounding section lines, follow Fletcher and Fowler Avenues west to where they either stray off the section line (Fletcher) or end (Fowler), in both cases because of those many (karst) lakes in Carrollwood.

d. Locate Lake Behnke on Figure 1.1 and state its location using the US PLS System by looking at the Sulphur Springs quard sheet.

Lake Behnke is the lake east of Bruce B. Downs Blvd between the two traffic lights between Fowler and Fletcher. Knowing that Bruce B. Downs is the western boundary of campus, and that the western third of campus is the E 1/2 S8, T28S, R19E, then we know immediately that the lake is in the western half of the eastern half of the Section, i.e., W1/2 E1/2 S8, T28S, R19E.
However, we can be more precise than that. The trick is in knowing about quarter sections, and where the boundary between Sections 8 and 9 is. So, go to the Sulphur Springs quad, and you will see that the section line cuts right through campus in a very obvious way. (North of Fletcher, it is overprinted by 37th Street. South of Fowler it is obvious again as it cuts through the industrial area where an old airfield still can be seen on the map. It continues through Busch Gardens, and is hard to follow south of Busch Blvd. It is evident again as it crosses the big meander in the Hillsborough River south of which it is overprinted again by 37th St. in a residential area.) Now, visually trace out the eastern half of Section 9 from its northeast corner where the section line crosses Fletcher, to the intersection of Fletcher and Bruce B. Downs, then south to Fowler, then east to that section line, and back up the section line through campus. Next visually divide that half-section into north and south halves. Those are two quarter sections, which would be called the NE ¼ and the SE ¼ of Section 9, the latter of which contains Lake Behnke (and the section number, 8, is just to the right of the section center; the presence of the road prevents its being placed at the center) Using the centerpoint 8 as a guide, you can visually trace out the boundaries of the quarter section, and then visually quarter that quarter section, whereupon you can see that Lake Behnke fits well into western half of that quarter-quarter section. So, from this exercising of eyeballing it, we can say that Lake Behnke approximately coincides with the W ½ NW ¼ SE ¼ S9, T28S, R19E.

1.14. Using PLSS to estimate acreage. Sections are nominally a square mile in area, and an acre is defined so that there will be 640 acres in a square mile.
   a. What is the area of Behnke Lake in acres?
      Going with the result of W ½ NW ¼ SE ¼ S9, T28S, R19E, the answer is 20 acres, because a section is 640 acres, a quarter section is 160 acres, and a quarter-quarter section is 40 acres. So a half of a quarter-quarter section is 20 acres.
   b. What proportion of the USF campus is Lake Behnke?
      The USF campus is 1.5 square miles, so it is 960 acres. Lake Behnke then is \( \frac{20 \text{ acres}}{960 \text{ acres}} = 21\% \) of the campus.

1.15. Road grids and the PLSS. No doubt you have noticed the propensity for many roads to run north-south and east-west. There is a reason for that: they tend to follow the grid of PLSS, where it exists. Thus important through-roads tend to be spaced at 1-mile, or ½-mile, or even ¼-mile steps. Check it out on the Sulphur Springs map (and Google for some of the road names).
   a. Relative to Fowler Avenue, where are each of the east-west following:
      i. Fletcher Ave. _____________ 1 mi north
      ii. 131st Ave (formerly Atlantic Blvd) _______ ¾ mi north
      iii. 127th Ave (not labeled on quad sheets; see Google map) _____ ½ mi north
      iv. 122nd Ave (not labeled on quad sheets; see Google map) ______ ¼ mi north
      ---- Fowler Ave ----
      v. 109th Ave. _______________ ½ mi south
      vi. Bougainvillea Ave _________ ¾ mi south
      vii. Linebaugh Ave. ___________ 1 mi south (find the section line in Busch Gardens)
      viii. Annie St. ________________ 1¼ mi south
      ix. Busch Blvd. _______________ 1½ mi south
      x. Yukon St. ___________________ 1¾ mi south
      xi. Waters Ave. ________________ 2 mi. south
      xii. Kirby St. ___________________ 2 ½ mi south
xiii. Sligh Ave. ______________ 3 mi. south (find the section line in the River)
xiv. Hanna Ave. ______________ 3 ½ mi. south
xv. Hillsborough Ave. _________ 4 mi south (next quad sheet)

b. East from Nebraska Ave. along Fowler Ave.
   i. N 15th St. ________________ ½ mi
   ii. N 22nd St. ________________ 1 mi.
   iii. N 30th St. (Bruce B. Downs Blvd) _____ 1 ½ mi
   iv. N 37th St. (see south part of map) _____ 2 mi
   v. N 46th St. ________________ 2 ½ mi.
   vi. N 50th St. ________________ 3 mi.
   vii. N 56th St _________________ 3 ½ mi.

c. West from Nebraska Ave. along Waters Ave.
   i. Florida Ave. ______________ ½ mi.
   ii. N Boulevard ______________ 1 mi
   iii. N Rome Ave. _____________ 1 ½ mi.
   iv. N Armenia Ave. ___________ 2 mi.
   v. N Habana Ave. ____________ 2 ¼ mi.

1.16. **Section corners.** Section line cross at so-called section corners, which are important to knowing one’s way around a grid. The number-labels for the four sections that meet up at a section corner take considerable getting used to. For example, the intersection of Nebraska and Fowler is a section corner; the four sections that meet at that point are (clockwise from the northwest): S12, T29S, R18E; S7, T29S, R19E; S18, T29S, R19E; and S13, T29S, R18E. Using the results of the preceding question and the PLSS map Hillsborough County (Fig. 3.3), what are the four sections that meet at each of the following road intersections:

   a. Armena and Waters.
      S22, T29S, R18E; S23, T29S, R18E; S26, T29S, R18 E; and S27 T29S, R18E
      (2 mi south and 2 mi west of Nebraska and Fowler)

   b. 22nd and Fletcher
      S6, T29S, R19E; S5, T29S, R19E; S8, T29S, R19E; and S7, T29S, R19E
      (1 mi north and 1 mi east of Nebraska and Fowler)

   c. 50th and Linebaugh
      S16, T29S, R19E; S15, T29S, R19E; S22, T29, R19E; and S21 T29S, R 19E
      (1 mi south and 3 mi east of Nebraska and Fowler)

1.17. **Distances and directions between points on the grid.** The PLSS can be used to set up a rough Cartesian coordinate system with a local origin, conveniently on the map. Points of interest can be given x- (east) and y- (north) coordinates as miles from the local origin. Distances between them can be calculated by vector subtraction, and so can the azimuths.

   a. Using the intersection of Nebraska and Fowler as the local origin, what are the x,y coordinates to each of the following points, and write out the expression of the vector, in components form, from the origin to the point in question.

      i. the flagpole north of the main (Fowler) entrance to USF
coordinates: (2.3, 0.4)  \text{ vector: } \mathbf{v}_1 = 2.3\mathbf{i} + 0.4\mathbf{j}

direction and magnitude (azimuth, AZ) of each of the positions

i. the intersection of Bruce B. Downs Blvd and Busch Gardens.

coordinates: (1.5, -1.5)  \text{ vector: } \mathbf{v}_2 = 1.5\mathbf{i} - 1.5\mathbf{j}

ii. Section corner at S34, T28S, R18E; S35, T28S, R18E, S2, T29S, R18E; and
S3, T29S, R18E (nw shore of Lake Magdalene)
coordinates: (-2, 2)  \text{ vector: } \mathbf{v}_3 = -2\mathbf{i} + 2\mathbf{j}

iii. Section corner at S27, T29S, R18E; S26, T29S, R18E, S35, T29S, R18E; and
S34, T29S, R18E (Armenia and Sligh, near Egypt Lake)
coordinates: (-2, -3)  \text{ vector: } \mathbf{v}_3 = -2\mathbf{i} - 3\mathbf{j}

iv. Section corner at S27, T29S, R18E; S26, T29S, R18E, S35, T29S, R18E; and
S27, T28S, R19E (swamp far northeast out Bruce B. Downs Blvd)
coordinates: (4, 4)  \text{ vector: } \mathbf{v}_4 = 4\mathbf{i} + 4\mathbf{j}

v. Section corner at S28, T29S, R19E; S27, T29S, R19E; S34, T29S, R19E; and
S33, T29S, R19E (Sligh Ave between 40th St and 56th St., near Shady Grove
Cemetery)
coordinates: (3, -1)  \text{ vector: } \mathbf{v}_5 = 3\mathbf{i} - 3\mathbf{j}

b. What is the magnitude (length) and direction (azimuth, AZ) of each of the positions
vectors (i.e., the vectors from the origin at (0,0) to the point in question);

i. For \( \mathbf{v}_1 = 2.3\mathbf{i} + 0.4\mathbf{j} \)

\[ |\mathbf{v}_1| = \sqrt{(2.3)^2 + (0.4)^2} = 2.33 \]

\[ \theta_{v1} = \tan\left(\frac{0.4}{2.3}\right) = 9.9^\circ, \text{ where } \theta \text{ is the angle measured counter-clockwise from the} \]

x-axis. When the x-displacement is negative, 180° must be added to reverse the direction of the
vector. This can be done in Excel with a simple logic function such as

\[ =\text{IF}(B28>0, \text{ATAN}(C28/B28)*180/\pi(),\text{ATAN}(C28/B28)*180/\pi()+180), \]

where Cells B28 and C28 contain the x-displacement and the y-displacement, respectively.

\[ AZ_{v1} = 90 - \theta_{v1} = \text{MOD}(90, 360) = 80.1^\circ \text{ To prevent negative azimuths one can use modulo} \]

operation. The Excel equation, following the equation for \( \theta_{v1} \) in cell E28, is

\[ =\text{MOD}(E28, 360), \text{ which produces the desired result (always positive and less than 360°).} \]

ii. For \( \mathbf{v}_2 = 1.5\mathbf{i} - 1.5\mathbf{j} \)

\[ |\mathbf{v}_2| = \sqrt{(1.5)^2 + (-1.5)^2} = 2.12; \theta_{v2} = \tan\left(-\frac{1.5}{1.5}\right) = -45^\circ; AZ_{v2} = 90^\circ - (-45^\circ) = 135^\circ \]

iii. For \( \mathbf{v}_3 = -2\mathbf{i} + 2\mathbf{j} \)

\[ |\mathbf{v}_3| = \sqrt{(-2)^2 + (1.5)^2} = 2.83; \theta_{v3} = \tan\left(\frac{2}{1.5}\right) = -45^\circ + 180^\circ = 135^\circ; \]

\[ AZ_{v3} = 90^\circ - 135^\circ = -45^\circ + 360^\circ = 315^\circ \]

iv. For \( \mathbf{v}_4 = -2\mathbf{i} - 3\mathbf{j} \)

\[ |\mathbf{v}_4| = \sqrt{(-2)^2 + (-3)^2} = 3.61; \theta_{v4} = \tan\left(-\frac{3}{-2}\right) = 56^\circ + 180^\circ = 236^\circ; \]

\[ AZ_{v4} = 90^\circ - 236^\circ = -146^\circ + 360^\circ = 214^\circ \]

v. For \( \mathbf{v}_5 = 4\mathbf{i} + 4\mathbf{j} \)

\[ |\mathbf{v}_5| = \sqrt{(4)^2 + (4)^2} = 5.66; \theta_{v5} = \tan\left(\frac{4}{4}\right) = 45^\circ; \]

\[ AZ_{v5} = 45^\circ \]

vi. For \( \mathbf{v}_6 = 3\mathbf{i} - 3\mathbf{j} \)

\[ |\mathbf{v}_6| = \sqrt{(3)^2 + (-3)^2} = 4.24; \theta_{v6} = \tan\left(\frac{3}{-3}\right) = -45^\circ; AZ_{v6} = 90^\circ - (-45^\circ) = 135^\circ \]
c. What are the magnitudes and directions of the vectors from the USF flagpole to the other three points in question

i. For flagpole to Busch Gardens corner
\[ \mathbf{v}_{1-2} = \mathbf{v}_2 - \mathbf{v}_1 = (1.5i - 1.5j) - (2.3i + 0.4j) = -0.8i - 1.1j \]
\[ |\mathbf{v}_{1-2}| = \sqrt{(-0.8)^2 + (-1.1)^2} = 1.36 \text{ mi}; \]
\[ \theta_{1-2} = \arctan \left( \frac{-1.1}{-0.8} \right) = 54^\circ + 180^\circ = 234^\circ; \]
\[ AZ_{1-2} = 90^\circ - 234^\circ = -144^\circ + 360^\circ = 216^\circ \]
\[ Bearing(1 \to 2) = S(216^\circ - 180^\circ)W = S26^\circ W \]

ii. For flagpole to Lake Magdalene corner
\[ \mathbf{v}_{1-3} = \mathbf{v}_3 - \mathbf{v}_1 = (-2.0i + 2.0j) - (2.3i + 0.4j) = -4.3i + 1.6j \]
\[ |\mathbf{v}_{1-3}| = \sqrt{(-4.3)^2 + (1.6)^2} = 4.59 \text{ mi}; \]
\[ \theta_{1-3} = \arctan \left( \frac{1.6}{-4.3} \right) = 38^\circ + 180^\circ = 218^\circ; \]
\[ AZ_{1-3} = 90^\circ - 218^\circ = -128^\circ + 360^\circ = 232^\circ \]
\[ Bearing(1 \to 3) = S(232^\circ - 180^\circ)W = S52^\circ W \]

iii. For flagpole to Armenia and Sligh corner
\[ \mathbf{v}_{1-4} = \mathbf{v}_4 - \mathbf{v}_1 = (-2.0i - 3.0j) - (2.3i + 0.4j) = -4.3i - 3.4j \]
\[ |\mathbf{v}_{1-4}| = \sqrt{(-4.3)^2 + (-3.4)^2} = 5.48 \text{ mi}; \]
\[ \theta_{1-4} = \arctan \left( \frac{-3.4}{-4.3} \right) = 38^\circ + 180^\circ = 218^\circ; \]
\[ AZ_{1-4} = 90^\circ - 218^\circ = -128^\circ + 360^\circ = 232^\circ \]
\[ Bearing(1 \to 4) = S(232^\circ - 180^\circ)W = S52^\circ W \]

iv. For flagpole to far northeast section corner in the swamp
\[ \mathbf{v}_{1-5} = \mathbf{v}_5 - \mathbf{v}_1 = (4.0i + 4.0j) - (2.3i + 0.4j) = 1.7i + 3.6j \]
\[ |\mathbf{v}_{1-5}| = \sqrt{(1.7)^2 + (3.6)^2} = 3.98 \text{ mi}; \]
\[ \theta_{1-5} = \arctan \left( \frac{3.6}{1.7} \right) = 65^\circ; \]
\[ AZ_{1-5} = 65^\circ \]
\[ Bearing(1 \to 5) = N52^\circ E \]

v. For flagpole to section corner near Shady Grove Cemetery
\[ \mathbf{v}_{1-6} = \mathbf{v}_6 - \mathbf{v}_1 = (3.0i - 3.0j) - (2.3i + 0.4j) = 0.7i - 3.4j \]
\[ |\mathbf{v}_{1-6}| = \sqrt{(0.7)^2 + (-3.4)^2} = 3.47 \text{ mi}; \]
\[ \theta_{1-6} = \arctan \left( \frac{-3.4}{0.7} \right) = -78^\circ + 180^\circ = 102^\circ; \]
\[ AZ_{1-6} = 90^\circ - 102^\circ = 168^\circ \]
\[ Bearing(1 \to 6) = S(180^\circ - 168^\circ)E = S12^\circ E \]

1.19. Vectors to track the Hillsborough River

a. Using the same local origin (at Fowler and Nebraska), what are the coordinates and position vectors for each of the following 10 locations along the river.

i. the bridge on Fletcher over the River \((4.5, 1.0)\) \(\mathbf{v}_1 = 4.5i + 1.0j\)

ii. the bridge on Fowler over the River (on the Thonotosassa sheet) \((5.2, 0)\) \(\mathbf{v}_2 = 5.2i\)

iii. the right angle bend of the River at the section corner across from Florida College (on the Thonotosassa sheet). \((5.0, -1.0)\) \(\mathbf{v}_3 = 5i - 1j\)

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iv. the bridge on Busch Blvd (actually Temple Terrace Highway) over the River  
\( (4.2, -1.5) \quad v_5 = 4.2i - 1.5j \)

v. the section line where it crosses the River at the southern end of Temple Terrace.  
\( (4.0, -2.8) \quad v_5 = 4i - 2.8j \)

vi. the bridge on 56th over the River  
\( (3.5, -2.0) \quad v_5 = 3.5i - 2j \)

vii. the bridge on 40th over the River  
\( (2.25, -3.1) \quad v_7 = 2.25i - 3.1j \)

viii. the River at the dam near Rowlett Park  
(1.4, -2.0)  \quad v_8 = 1.4i - 2j

ix. the right angle bend in the River just north of Sligh Jr High  
(0.5, -2.7)  \quad v_9 = 0.5i - 2.7j

x. the bridge on Nebraska Ave over the River (at Sulphur Springs)  
(0, -2.4)  \quad v_{10} = -2.4j

xi. the bridge on Florida Ave. over the River  
(−1.0, -2.3)  \quad v_{11} = -1i - 2.3j

xii. the bridge on Sligh Ave. over the River  
(−1.3, -3.0)  \quad v_{12} = -1.3i - 3j

b. Now, without looking at the map this time, just at the coordinates, how would you describe the downstream route of the Hillsborough River, from Fletcher to Sligh.

Pretty much, it’s north to south from Fletcher to the south end of Temple Terrace with some but not much sinuosity. Thus, from (i) to (v), \( \Delta y_{\text{total}} = y_5 - y_1 = -3.8\text{ mi} \) and \( \Delta x_{\text{total}} = x_5 - x_1 = -0.5\text{ mi} \), while \( \sum_{i=1}^{4}\left|y_{i+1} - y_i \right| = 3.8\text{ mi} \) and \( \sum_{i=1}^{4}\left|x_{i+1} - x_i \right| = 1.9\text{ mi} \). So the river moved a total of 3.8 miles north and south to get a distance of 3.8 miles south, and it moved a total of 1.9 miles east-west (back and forth) to get 0.5 miles west.

In contrast, from (v) to (xii), it’s steadily westward with considerable meanderings. Thus \( \Delta x_{\text{total}} = x_{12} - x_5 = -5.3\text{ mi} \), and \( \sum_{i=5}^{11}\left|y_{i+1} - y_i \right| = 5.3\text{ mi} \), while \( \Delta y_{\text{total}} = -0.2\text{ mi} \) and \( \sum_{i=5}^{11}\left|x_{i+1} - x_i \right| = 4.8\text{ mi} \). So the river moved a total of 5.3 miles east and west to get a distance of 5.3 miles west, and it moved a total of 4.8 miles north and south to get 0.2 miles south.

c. From the 12 vectors in part (a) compile the 11 station-to-station downstream distances and azimuths from station 1 to station 12.

<table>
<thead>
<tr>
<th>stations</th>
<th>Vector</th>
<th>Distance (mi)</th>
<th>Theta</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>i → ii</td>
<td>( v_2 - v_1 = 0.7i - 1.0j )</td>
<td>1.22 mi</td>
<td>-55°</td>
<td>145°</td>
</tr>
<tr>
<td>ii → iii</td>
<td>( v_3 - v_2 = -0.2i - 1.0j )</td>
<td>1.02 mi</td>
<td>79°</td>
<td>191°</td>
</tr>
<tr>
<td>iii → iv</td>
<td>( v_4 - v_3 = -0.8i - 0.5j )</td>
<td>0.94 mi</td>
<td>32°</td>
<td>238°</td>
</tr>
<tr>
<td>iv → v</td>
<td>( v_5 - v_4 = -0.2i - 1.3j )</td>
<td>1.32 mi</td>
<td>81°</td>
<td>189°</td>
</tr>
<tr>
<td>v → vi</td>
<td>( v_6 - v_5 = -0.5i + 0.8j )</td>
<td>0.94 mi</td>
<td>-58°</td>
<td>328°</td>
</tr>
<tr>
<td>vi → vii</td>
<td>( v_7 - v_6 = -1.25i - 1.1j )</td>
<td>1.67 mi</td>
<td>41°</td>
<td>229°</td>
</tr>
<tr>
<td>vii → viii</td>
<td>( v_8 - v_7 = -0.85i + 1.1j )</td>
<td>1.39 mi</td>
<td>-52°</td>
<td>322°</td>
</tr>
<tr>
<td>viii → ix</td>
<td>( v_9 - v_8 = -0.9i - 0.7j )</td>
<td>1.14 mi</td>
<td>38°</td>
<td>232°</td>
</tr>
<tr>
<td>ix → x</td>
<td>( v_{10} - v_9 = -0.5i + 0.3j )</td>
<td>0.58 mi</td>
<td>-31°</td>
<td>301°</td>
</tr>
<tr>
<td>xi → xi</td>
<td>( v_{11} - v_{10} = -1i + 0.1j )</td>
<td>1.00 mi</td>
<td>-6°</td>
<td>276°</td>
</tr>
<tr>
<td>xi → xii</td>
<td>( v_{12} - v_{11} = -0.3i - 0.7j )</td>
<td>0.76 mi</td>
<td>67°</td>
<td>203°</td>
</tr>
</tbody>
</table>

d. What is the sinuosity of the River in from the Fletcher Bridge to the Sligh Ave bridge? The sinuosity is the total distance traveled divided by the straight line distance.

1.20. Principal meridian and baseline for the Florida PLSS grid. Where is (0,0) for the system of townships and ranges in our state?

The Sulphur Springs quad is largely in T29S, R19E, which means there are 28 rows of townships to the north and 18 columns of ranges to the west of the northwest corner of S6, T29S, R19E. (The location is
Nebraska Ave, a mile north of Fletcher Ave.). At 6 miles per row and 6 miles per column, the northwest corner of S6, T1S, R1E would be 168 mi to the north and 108 mi to the west, if all the section squares are truly 1 mile squares.

Assuming a 40,000-km circumference world and hence 111 km (69 mi) per degree of latitude, or 8.63 mi/map, the northwest corner of S6, T1S, R1E would be 168 mi to the north and 108 mi to the west, if all the section squares are truly 1 mile squares.

At the bridge, the elevation of the water is 20 ft. As it happens, the river’s edge is at the southern boundary of Section 22. Go north on 56th St, and you rise in elevation one 5-foot contour after another to at least Busch Blvd, where you are half-way across the section and at an elevation of 56 ft.

**1.21. Topographic features: Slopes.**

**a. Example: north from the River to Busch Blvd on 56th St. (S22, T28S, R19E).**

At the bridge, the elevation of the water is 20 ft. As it happens, the river’s edge is at the southern boundary of Section 22. Go north on 56th, and you rise in elevation one 5-foot contour after another to at least Busch Blvd, where you are half-way across the section and at an elevation of 56 ft.

i. With the information so far, what are the slope, grade and angle of your rise in elevation?

\[ \Delta z = 36 \text{ ft and } \Delta y = 0.5 \text{ mi}, \text{ where } \Delta z \text{ is the increase in elevation, and } \Delta y \text{ is the displacement north.} \]

Then rate of “ascent” is 72 ft/mi, or \( \frac{36 \text{ ft}}{0.5 \text{ mi}} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 0.0136 \). The latter quantity, which is \( \Delta z/\Delta y \), is dimensionless, and is equivalent to 1.36% and called the grade. To find the angle, use the arctangent: \( \arctan(\Delta z/\Delta y) = \arctan(0.0136) = 0.78^\circ \),

ii. The foregoing result is the overall average for the entire 0.5-mi “journey.” What are the slopes for the individual contour-to-contour 5-ft changes in elevation?

For this question, it is convenient to develop a table (spreadsheet) such as the following. For convenience (because the distances on the map are small), we use the JPG attachment and Paint, and we zoom in (to 200-400%) and use pixels as our length unit. The first data column lists the successive contours from the River to Busch Blvd. The second column, as read from the horizontal pixel ruler, is the number of pixels (on the map) from the left edge of the map to 56th Street. The third column, read from the vertical pixel ruler, is the number of pixels (on the
map) down from the top edge of the map to the contour of interest. The fourth column is the number of pixels that the location is south of the prior location on the map. The fifth column is the elevation (from column 1 which gives elevation in ft on the ground) converted to would-be pixels on the map.

Before going on, what is the conversion factor?

We measure the position of Busch Blvd on the map to be 2705 pixels below the top edge of the map, and the position of the river’s edge to be 2897 pixels below the top edge of the map. So, then a length of 192 px on the map scales to 0.5 mi on the ground. Then, for example, the 56-ft elevation at Busch and 56th converts to 4.07 px.

Now complete the table. Note that the \( \Delta y \) column and the three slope columns involve differences between rows, and in concept actually plot on the line between the row in which the number appears and the row below it. Thus the \( \Delta z / \Delta y \) column of the row for the 55-ft contour is calculated from:

\[
\frac{(4.00 \text{ px} - 3.64 \text{ px})}{(2786 \text{ px} - 2733 \text{ px})}
\]

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<th>Elev (px)</th>
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</table>

iii. Based on the succession of grades, how would you characterize the shape of the topographic cross-profile?

**convex up:** relatively steep as it rises from the river (5 and 6% grade) lessening as you go north (to a little less than 1%). Monotonic, i.e., no reversals.

b. Example: the southern boundary of Section 2,T28S, R19E, from the river to the east edge of the map. At the river, the elevation of the water is 20 ft. Go east along the southern boundary of Section 23, and you rise in elevation one 5-foot contour after another to the east edge of the map at an elevation of 65 ft.

i. With the information so far, what are the slope, grade and angle of your rise in elevation?

\[
\Delta z = 45 \text{ ft. For the } \Delta x, \text{ first measure the one-mile bar scale at the bottom and it reads } 384 \text{ px therefore the scale of the map is } \frac{384 \text{ px}}{1 \text{ mi}}. \text{ And then measure the distance from river to the east edge of the map and it reads 169 px, therefore } \Delta x = 169 \text{ px} \times \frac{384 \text{ px}}{1 \text{ mi}} = 0.44 \text{ mi.}, \text{ and } \Delta x \text{ is the displacement east. Then rate of “ascent” is } 102 \text{ ft/}1 \text{ mi, or } \left(\frac{102 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right) = 0.0194, \text{ or a grade of 1.94%. From the arctangent, the angle is: arctan}(\Delta z / \Delta y) = \text{arctan}(0.0194) = 1.11^\circ.
\]
ii. The foregoing result is the overall average for the entire transect. What are the slopes for the individual contour-to-contour 5-ft changes in elevation? It is convenient to develop a table like in the first example. The conversion factor for elevations is the same as calculated before: 1 pixel on the map corresponds to a length of 13.75 feet on the ground.

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</tbody>
</table>

iii. Based on the succesion of grades, how would you characterize the shape of the topographic cross-profile?

concave up: then lessens at top: very low angle as it rises from the river (<1% then 1.3%) for first 10 ft of rise., then increases from <2% to 4% for next 20 ft, and tapers off a little before cresting on the next map. Monotonic, no reversals.

c. Example: south to north transect from the river about 35 pixels to the east of the southwest corner of S22, T28S, R19E, on the south end, to Busch Blvd. At the river, the elevation of the water is 20 ft. Go north along the transect, and you rise in elevation one 5-foot contour after another till 80 ft, and then drop to 65 ft, and then rise again to Busch Blvd. at an elevation of 67 ft.

i. With the information so far, what are the slope, grade and angle of your rise in elevation?

\[ \Delta z = 47 \text{ ft} \]. For the \( \Delta y \), we already know that the scale of the map is \( \frac{304 \text{ px}}{1 \text{ mi}} \). And then measure the distance from river to the east edge of the map and it reads 223 px, therefore \[ \Delta y = 223 \text{ px} \times \frac{304 \text{ px}}{1 \text{ mi}} = 0.58 \text{ mi} \]. Then rate of “ascent” is 81 ft/mi, or \( \left( \frac{81 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{\text{mi}}{5280 \text{ ft}} \right) = 0.015 \), or a grate of 1.5%. From the arctangent, the angle is \( \arctan(0.0153) = 0.88^\circ \).

ii. The foregoing result is the overall average for the entire transect. What are the slopes for the individual contour-to-contour 5-ft changes in elevation? It is convenient to develop a table like in the first example. The conversion factor for elevations is the same as calculated before: 1 pixel on the map corresponds to a length of 13.75 feet on the ground.
iii. Based on the succession of grades, how would you characterize the shape of the topographic cross-profile

*convex up* from the river, then crests at a hill, goes through a swale and rises to Busch: quite steep as it rises from the river (10 and 20% grade) lessening northward as it crests (3 and 4% grade %). With 2 reversals: rises up to 80 ft, then drops to 65 ft, and then rises up to 67 ft at Busch (and then continues to another 80 ft ridge on the other side of Busch).

d. Example: Northeast across the NW1/4 S14, T28S, R19E, and continuing to the river, the transect is at the end of the map area. There are elevation markers where the transect starts (80 ft) and where it crosses out of S14 (i.e., at Fowler) (36 ft). The elevation of the river is still 20 ft. Proceed same as the first three parts, but here both $\Delta x$ and $\Delta y$ change along the transect (actually in equal amounts). It will be good to add a column, $\Delta s$, for the distance along the transect, from the Pythagorean theorem. (Note the closed depression contour.)

i. With the information so far, what are the slope, grade and angle of your rise in elevation?

$\Delta z = 60$ ft. For the $\Delta y$, we already know that the scale of the map is $\frac{384 \text{ px}}{1 \text{ mi}}$. And then measure the distance from river to the east edge of the map and it reads $253 \times 253$ px, therefore $\Delta y = \sqrt{(253 \text{ px})^2 + (253 \text{ px})^2} \text{ px} = 253 \frac{384 \text{ px}}{1 \text{ mi}} = 0.93 \text{ mi}$. Then rate of “ascent” is $65 \text{ ft/mi}$, or $\left(\frac{65 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right) = 0.0123$, or a grate of 1.23%. From the arctangent, the angle is $\arctan(0.0123) = 0.71^\circ$. 

---

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</table>
ii. The foregoing result is the overall average for the entire transect. What are the slopes for the individual contour-to-contour 5-ft changes in elevation? It is convenient to develop a table like in the first example. The conversion factor for elevations is the same as calculated before: 1 pixel on the map corresponds to a length of 13.75 feet on the ground.

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<th>Δs</th>
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iii. Based on the succession of grades, how would you characterize the shape of the topographic cross-profile

convex up from the river, then crests at a hill, goes through a swale and rises to the end of the transect quite steep as it rises from the river (10 and 20% grade) lessening northward as it crests (2% grade %). With 1 reversals: rises up to 35 ft, then drops to 35 ft at the swamp.

e. What two conclusions can you draw from these examples about the relation between the spacing of contours and the slope of hillsides?

- They are inversely related: as the slope steepens, the contours are spaced more closely together. For example, for a slope with 10% grade, the spacing between contours with a 5-foot contour interval is 50 ft on the ground and about a quarter of a tenth of an inch apart on a 1:24,000-scale map. For a slope with 1% grade, the space between contours with a 5-foot contour interval is 10x as large: 500 ft on the ground and a quarter inch on the map.

- As seen in cross-profile, convex slopes lessen in grade as they go uphill (i.e., the second derivative is negative), and concave slopes steepen as they go higher (second derivative is positive). Obviously a slope can’t increase indefinitely; commonly it will roll into a convex profile as it tops out.
Central Park Quadrangle

This section consists of word problems based on the Central Park, NY, Quadrangle to have a comparison with the ones for Sulphur Spring Quadrangle.

To download the map from Topoview, go back to the online shown in Figure B4, type in “Central Park, NY” in the search tool, scroll all the way down till you see the “Central Park, NY, 1995, 1999 ed.” Click and download the JPEG file (4 MB). Unzip the file and the map should look like Figure B4.

Figure B3. Central Park, NY 1995 (HTMC, 1999 ed.)

Scale and the size of the map area

2.1. Geographic coordinates. The map area is defined by the latitudes $28^\circ$ and $28\frac{1}{8}^\circ$ and longitudes $-82\frac{3}{8}^\circ$ and $-82\frac{1}{2}^\circ$. What are the coordinates of the corners of the quadrangle, in degrees-minutes format?

<table>
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<th>(40° 52.5′N / 73°52.5′W)</th>
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</thead>
<tbody>
<tr>
<td>(40° 45′N / 74°W)</td>
<td>(40° 45′N / 73°52.5′W)</td>
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</table>
2.2. Estimating quad size. Assume a spherical Earth with circumference 40,000 km,
a. What is the “height” dimension (i.e., NS) of the quadrangle on the ground?
   
   On the ground: \( NS \text{ edges} = \left( \frac{0.125^\circ}{90^\circ} \right) (10,000 \text{ km}) = 13.889 \text{ km} \)

b. What is the “width” (i.e., EW length) of the northern edge of the map area?
   
   On the ground: \( northern \text{ edge} = \left( \frac{0.125^\circ}{90^\circ} \right) (10,000 \text{ km} \cos (40.875^\circ)) = 10.502 \text{ km} \)

c. What is the “width” of the southern edge of the map area?
   
   \( southern \text{ edge} = \left( \frac{0.125^\circ}{90^\circ} \right) (10,000 \text{ km} \cos (40.75^\circ)) = 10.522 \text{ km} \)

d. How much shorter is the northern edge than the shorter edge.
   
   \( 20 \text{ m} \)

e. How much longer are the vertical edges than the horizontal edges.
   
   3.387 and 3.367 km

Location of the map area

2.3. Nearby quadrangles. Return to your live view of the Get Maps | topoView interface seeking the Central Park 24K quadrangle as shown in Figure 3.2. Scroll out to where the scale is large enough that the names of the quadrangles appear. What are the names of the surrounding eight quadrangles?

<table>
<thead>
<tr>
<th>Hackensack</th>
<th>Yonkers</th>
<th>Mount Vernon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weehawken</td>
<td>Central Park</td>
<td>Flushing</td>
</tr>
<tr>
<td>Jersey City</td>
<td>Brooklyn</td>
<td>Jamaica</td>
</tr>
</tbody>
</table>

2.4. Smaller scales. Stay with the Get Maps | topoView interface to explore what can be seen at other scales.

a. While the blue pointer is still marking the location of the Central Park, double-click on the 100K circle in the navigation column to reveal a grid of 100K maps. Describe the location of the Central Park 7.5-min quad in this grid of 100K maps.

The 100K maps are \( 1^\circ \times 30' \) quadrangles

<table>
<thead>
<tr>
<th>Middletown</th>
<th>Bridgeport</th>
<th>New Haven</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newark</td>
<td>\textbf{Long Island West}</td>
<td>Long Island East</td>
</tr>
<tr>
<td>Trenton</td>
<td>Long Branch</td>
<td></td>
</tr>
</tbody>
</table>

The Long Island West map area, which includes the Central Park map area, is bounded by latitudes 40.5° N and 41° N and longitudes 73° W and 74° W.
b. Now double-click on the 250 circle to reveal a grid of 250K maps. Describe the location of the Central Park 7.5-min quad in this grid of 250K maps.

The 250K maps are 1° × 2° quadrangles.

<table>
<thead>
<tr>
<th>Binghamton</th>
<th>Albany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scranton</td>
<td>Hartford</td>
</tr>
<tr>
<td>Newark</td>
<td>New York</td>
</tr>
</tbody>
</table>

The New York map area, which includes the Central Park map area, is bounded by latitudes 40° N and 41° N and longitudes 72° W and 74° W.

2.5. **Lengths via the digital map.** Recall in WP 4.2 we estimated the size of the map area by calculating how long the bordering lengths would be on a sphere 40,000 km in circumference. Our results were 13.889 km for the height (NS length) and 12.249 and 12.263 km for the upper (northern EW) and lower (southern EW) widths, respectively. Rather than estimate the lengths that way, we can determine them on the map itself by measuring the length of one of the bar scales and dividing by the scale.

a. What is the measured length of the 1-km bar scale?

8.75 cm on our map image.

b. What is the apparent RF scale (meaning the *scale corresponding to our measured length of the bar scale on the map image*)?

The apparent RF scale is \( \frac{8.75 \text{ cm}}{1 \text{ km}} \left( \frac{1 \text{ km}}{100,000 \text{ cm}} \right) = 1:11.429 \) (or 1 cm : 0.1143 km)

c. What are the “height” and “widths” measured on the map?

*Height* = 120.53 cm (47.45 in.)

*Short width* = 91.46 cm (36.01 in.)

*Long width* = 91.60 cm (36.06 in.)

d. What are the corresponding lengths on the ground?

NS edges: \( \frac{120.53 \text{ cm}}{1 \text{ cm}/0.1143 \text{ km}} = 13.775 \text{ km} \) (vs. 13.889 km estimate)

EW length at the northern edges: \( \frac{91.46 \text{ cm}}{1 \text{ cm}/0.1143 \text{ km}} = 10.453 \text{ km} \) (vs. 10.502 km estimate)

EW length at the southern edges: \( \frac{91.60 \text{ cm}}{1 \text{ cm}/0.1143 \text{ km}} = 10.469 \text{ km} \) (vs. 10.522 km estimate)

e. How do the estimated values compare with the actual values?

NS edges: estimate is longer than the actual by 114 m (or 0.8%)

EW edges: estimates are longer than actual values by 49 m (0.5%) and 53 m (0.5%)

2.6. **Size of the map area at nominative scale.** Imagine you wish to cover a wall with a large-scale map of New York City by making a mosaic of 7.5-min. quadrangle maps
a. What are the length of the edges of the Central Park map area on an actual paper map where the RF scale really is 1:24,000, i.e., the nominative scale?

This is an exercise in scaling. How does the image change when you enlarge or reduce the scale by a given percentage? Here, we know the lengths on the map at our apparent RF scale. The nominative scale is 52.37% smaller; i.e., \( \frac{1}{24,000} \div \frac{1}{11,430} = 0.4763 \). Therefore lengths “on the wall” would be smaller than on the map with the apparent RF, namely:

- **NS edge**: \((120.53 \text{ cm})(0.4763) = 57.4 \text{ cm} = 22.6 \text{ in.}\)
- **Upper EW edge**: \((91.46 \text{ cm})(0.4763) = 43.56 \text{ cm} = 17.15 \text{ in.}\)
- **Lower EW edge**: \((91.60 \text{ cm})(0.4763) = 43.63 \text{ cm} = 17.18 \text{ in.}\)

b. How may such maps could you tile together to fit on a wall 10-ft high?

Round down to integer value: \( \frac{10 \text{ ft}(\frac{\text{ft}}{12 \text{ in}})}{22.6 \text{ in/\text{map}}} = 5.3 \text{ maps} = 5 \text{ whole maps} \)

c. How much horizontal length of the wall would your map mosaic take?

Computationally: **Wall width** = \((5\text{ maps})(\frac{18 \text{ in}}{\text{map}})(\frac{\text{ft}}{12 \text{ in}}) = 7.5 \text{ ft} \)

d. Describe the geographic coverage of the mosaic. Assume the Central Park quad is at the center of the mosaic?

Going back to topoView, the mosaic would consist of the following 25 quads:

<table>
<thead>
<tr>
<th>Ramsey</th>
<th>Park Ridge</th>
<th>Nyack</th>
<th>White Plains</th>
<th>Glenville</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paterson</td>
<td>Hackensack</td>
<td>Yonkers</td>
<td>Mount Vernon</td>
<td>Mamaroneck</td>
</tr>
<tr>
<td>Orange</td>
<td>Weehawken</td>
<td>Central Park</td>
<td>Flushing</td>
<td>Sea Cliff</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>Jersey City</td>
<td>Brooklyn</td>
<td>Jamaica</td>
<td>Lynbrook</td>
</tr>
<tr>
<td>Arthur Kill</td>
<td>The Narrows</td>
<td>Coney Island</td>
<td>Far Rockaway</td>
<td>Lawrence</td>
</tr>
</tbody>
</table>

The mosaic would be bounded by latitudes 40°30′N and 41°7′30″N and by longitudes 73°37′30″W and 74°15′W. It includes the whole New York City, and downtown New Jersey. On the southeast, it would barely reach the Atlantic Ocean in the Far Rockaway quad and Lawrence quad. It the northeast, it would cover small area of Connecticut State in the Glenville quad.
The UTM Coordinate System

UTM coordinate is a coordinate system derived from Transverse Mercator projection system. Its unit is meter, and it is very convenient in calculation distance, angle, and area by knowing the coordinate of the key point. We will show its applications in the following section:

Generally speaking, the UTM system divides the Earth into 60 zones, each of them is 6° of longitude in width. Zone 1 covers longitude 180° to 174° W; zone numbering increases eastward to zone 60, which covers longitude 174°E to 180°. The naming of UTM zones of North and South America is shown as a figure below:

![Figure B4. UTM zones for North and South America (from Snyder and Voxland 1989)](image)

2.7. Identifying UTM zones.

a. In the EW (longitude) direction, which of the UTM zones are U.S. covering?
   From Zone 10 to Zone 10

b. In the NS (latitude) direction, which of the UTM zones are U.S. covering?
   From Zone T to Zone U.

c. Which zone is New York City in?
   Zone 18T

2.8. Reading UTM grid lines. Open the Central Park 1:24,000 sheet again. For the purpose of measurement, use the on downloaded from topoView. Magnify the scale to where you can
see detail and scroll to the northern edge of the map area. Notice the black numbers and the vertical and horizontal black lines halfway between them.

a. First look at the northwest corner of the map, what is the coordinate of the crossing of the north-most grid line and west-most grid line (south of interstate highway I-95)?
   \[ \text{x-coordinate is: 585,000 m} \]
   \[ \text{y-coordinate is: 4,525,000 m} \]

b. What is the interval of the grid?
   \[ 1000 \text{ m, and that is 1 km} \]

c. Zoom in at the southeast corner of the map, what is the coordinate of the crossing of the south-most grid line and east-most grid line (at the crossing of Northern Blvd and 81st St near a fire station)?
   \[ \text{x-coordinate is: 594,000 m} \]
   \[ \text{y-coordinate is: 4,512,000 m} \]

d. Are the coordinates increasing or decreasing on the eastward and southward direction?
   \[ \text{x-coordinate increases eastward, while y-coordinate decreases southward.} \]

e. How many UTM grid lines are there in the map?
   \[ \text{There are 14 horizontal grid lines and 10 vertical grid lines.} \]

f. What is the dimension of the area covered by the 4 boundary grid lines? Compare it with the dimension of the map area you calculated and measured in Word Problem 4-2 and 4-5
   \[ \text{It is a rectangle that has 9 km in width and 13 km in height. It is slightly smaller than the quadrangle area calculated and measured in Word Problem 4-2 and 4-5.} \]

2.9. Reading UTM coordinates. It is straightforward to read the UTM coordinates at the crossing of grid lines. In order to get the coordinates of any points on the map, you will need to take measurements. We will take steps to measure the coordinate the west-most point of Central Park (Columbus Circle)

a. Identify the “block” of UTM grid line that includes the Columbus Circle. What are the boundary grid lines?
   \[ \text{Northern boundary is: 4,514,000 m N} \]
   \[ \text{Southern boundary is: 4,513,000 m N} \]
   \[ \text{Western boundary is: 585,000 m E} \]
   \[ \text{Eastern boundary is: 586,000 m E} \]

b. Measure the distance from Columbus Circle to the southern boundary of “block”. What is the distance in real world based on the scale calculated in Word Problem 4-5?
   \[ \text{The measured distance is 4.44, therefore the distance in real world is} \]
   \[ \frac{4.44 \text{ cm}}{1 \text{ cm/11.430 cm}} = \]
   \[ 50,800 \text{ cm} = 508 \text{ m} \]
c. Measure the distance from Columbus Circle to the eastern boundary of “block”. What is the distance in real world based on the scale calculated in Word Problem 4-5?

The measured distance is 4.44, therefore the distance in real world is

\[
\frac{0.51 \text{ cm}}{1 \text{ cm/11,430 cm}} = 5,800 \text{ cm} = 58 \text{ m}
\]

d. Based on your measurements and calculation, what is the UTM coordinates of Columbus Circle?

The Easting coordinate is: 586,000 − 58 = 585,942.
The Northing coordinate is: 4,513,000 + 508 = 4,513,508

2.10. Calculate distance and angle with UTM coordinate. It is very convenient to calculate distance and angle with UTM coordinate once you know the coordinates of two points.

a. Measure and calculate the UTM coordinate of the south-most point of Central Park

The measured distance to the south boundary is 2.82, therefore the distance in real world is

\[
\frac{0.91 \text{ cm}}{1 \text{ cm/11,430 cm}} = 10,400 \text{ cm} = 104 \text{ m}
\]
The measured distance to the east boundary is 2.82, therefore the distance in real world is

\[
\frac{2.82 \text{ cm}}{1 \text{ cm/11,430 cm}} = 32,200 \text{ cm} = 322 \text{ m}
\]
The Easting coordinate is: 587,000 − 322 = 586,678.
The Northing coordinate is: 4,513,000 + 104 = 4,513,104

b. What is length of the southwest boundary of Central Park (calculate using UTM coordinates)?

The length of the southwest boundary of Central Park equals to the distance between Columbus Circle and William Tecumseh Sherman Monument. To calculate the distance, first construct a vector from William Tecumseh Sherman Monument pointing to the Columbus Circle, the vector \( \mathbf{v} = (585,942 − 586,678) \hat{i} + (451,3508 − 451,3104) \hat{j} = -736 \hat{i} + 404 \hat{j} \)

The distance between the two points equals to the length of the vector:

\[
\sqrt{x^2 + y^2} = \sqrt{(-736 \text{ m})^2 + (404 \text{ m})^2} = 840 \text{ m}
\]

c. What is the angle between the southwest boundary of Central Park with respect to the UTM grid?

The angle is: \( \tan^{-1} \frac{x}{y} = \tan^{-1} \frac{404 \text{ m}}{-736 \text{ m}} = -29^\circ + 180^\circ = 151^\circ \)
The azimuthal is: 90° − (−29°) = 119°
The bearing is: S(90° + (−29°))E = S61°E

2.11. Application of UTM coordinates: museums around Central Park. Museums are one of the most popular attractions in New York City. There are five museum in the map area. Their names and locations are listed below in the table. Measure and calculate their UTM coordinates:
<table>
<thead>
<tr>
<th>Name</th>
<th>Location</th>
<th>UTM coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Museum of Natural History</td>
<td>81st St-Central Park West</td>
<td>586757E, 4515060N</td>
</tr>
<tr>
<td>The Metropolitan Museum of Art</td>
<td>82nd St-5th Ave</td>
<td>587564E, 4514740N</td>
</tr>
<tr>
<td>Asia Society and Museum</td>
<td>70th St-Park Ave</td>
<td>587381E, 4513717N</td>
</tr>
<tr>
<td>Museum of Arts and Design</td>
<td>59th St-Columbus Circle</td>
<td>585942E, 4513508N</td>
</tr>
<tr>
<td>American Folk Art Museum</td>
<td>Broadway-65th Street</td>
<td>585897E, 4514051N</td>
</tr>
</tbody>
</table>

a. Which is the northmost one of these five museums? Which is the southmost one? What is the distance between these two museum (km, mi)?

The northmost museum of the five is American Museum of Natural History and the southmost one is Museum of Arts and Design. To calculate the distance, first construct a vector from Museum of Arts and Design to American Museum of Natural History: \( \mathbf{v} = (586757 - 585942)i + (4515060 - 4513508)j = 815i + 1552j \)

The distance between the two points equals to the length of the vector: \( \sqrt{x^2 + y^2} = \sqrt{(815 \text{ m})^2 + (1552 \text{ m})^2} = 1753 \text{ m} = 1.753 \text{ km} = 1.089 \text{ mi} \)

b. What is the angle between Broadway and Central Park West?

The angle between Broadway and Central Park West is the angle formed by American Folk Art Museum-Museum of Arts and Design-American Museum of Natural History. Take Museum of Arts and Design as a “base station”; the vector from it to American Folk Art Museum is \( \mathbf{v}_a = (585897 - 585942, 4514051 - 4513508) = (-45, 543) \); the vector from it to American Museum of Natural History is \( \mathbf{v}_b = (586757 - 585942, 4515060 - 4513508) = (815, 1552) \).

Therefore, the angle between these two vectors is: \( \cos^{-1}\frac{\mathbf{v}_a \cdot \mathbf{v}_b}{|\mathbf{v}_a||\mathbf{v}_b|} = \frac{(-45) \times 815 + 543 \times 1552}{545 \times 1753} = 32.5^\circ \)

c. What is the perimeter of the polygon (pentagon) defined by the five museums? (km, mi)

First calculate the length of the five sides separately with the Pythagorean theorem used in 5.4 and 5.5a, and then calculate the Perimeter by adding up the length of the five sides.

To calculate the length of the five sides:

From Museum of Arts and Design to American Folk Art Museum
\[ \sqrt{(585897 \text{ m} - 585942 \text{ m})^2 + (4514051 \text{ m} - 4513508 \text{ m})^2} = 545 \text{ m} \]

From American Folk Art Museum to American Museum of Natural History
\[ \sqrt{(586757 \text{ m} - 585897 \text{ m})^2 + (4515060 \text{ m} - 4514051 \text{ m})^2} = 1326 \text{ m} \]

From American Museum of Natural History to the Metropolitan Museum of Art
\[ \sqrt{(587564 \text{ m} - 586757 \text{ m})^2 + (4514740 \text{ m} - 4515060 \text{ m})^2} = 868 \text{ m} \]

From the Metropolitan Museum of Art to Asia Society and Design
\[ \sqrt{(587381 \text{ m} - 587564 \text{ m})^2 + (4513717 \text{ m} - 4514740 \text{ m})^2} = 1039 \text{ m} \]

From Asia Society and Museum to Museum of Arts and Design
\[ \sqrt{(585942 \text{ m} - 587381 \text{ m})^2 + (4513508 \text{ m} - 4513717 \text{ m})^2} = 1454 \text{ m} \]
The Perimeter is 1228 m + 1326 m + 868 m + 1039 m + 933 m = 5202 m = 5.202 km = 3.232 mi

d. What is the area of the polygon (km², mi²)?

Area can be calculated as the same way using vectors in the Part 4 of Appendix A. In this case, let the Museum of Art and Design be the “base station”:
Vector from Museum of Art and Design to American Folk Art Museum is
\[ \mathbf{v}_1 = (585897 - 585942)i + (4514951 - 4513508)j = -45i + 1443j \]
Vector from Museum of Art and Design to American Museum of Natrual History is
\[ \mathbf{v}_2 = (586757 - 585942)i + (4515060 - 4513508)j = 815i + 1552j \]
Vector from Museum of Art and Design to the Metropolitan Museum of Art is
\[ \mathbf{v}_3 = (587564 - 585942)i + (4514740 - 4513508)j = 1622i + 1232j \]
Vector from Museum of Art and Design to Asia Society and Museum is
\[ \mathbf{v}_4 = (587381 - 585942)i + (4513717 - 4513508)j = 1439i + 209j \]
Area of the triangle equals to half of the magnitude of the cross-product of the vectors, thus:
Area of the triangle formed by Museum of Art and Design, American Folk Art Museum, and American Museum of Natrual History is:
\[ A_1 = \frac{1}{2} \left| \mathbf{v}_1 \times \mathbf{v}_2 \right| = \frac{1}{2} \left| (-45 \text{ m}) \times (1552 \text{ m}) - (543 \text{ m}) \times (815 \text{ m}) \right| = 256193 \text{ m}^2 \]
Area of the triangle formed by Museum of Art and Design, American Museum of Natrual History, and the Metropolitan Museum of Art is:
\[ A_1 = \frac{1}{2} \left| \mathbf{v}_2 \times \mathbf{v}_3 \right| = \frac{1}{2} \left| (815 \text{ m}) \times (1232 \text{ m}) - (1552 \text{ m}) \times (1622 \text{ m}) \right| = 756326 \text{ m}^2 \]
Area of the triangle formed by Museum of Art and Design, the Metropolitan Museum of Art, and Asia Society and Museum is:
\[ A_1 = \frac{1}{2} \left| \mathbf{v}_3 \times \mathbf{v}_4 \right| = \frac{1}{2} \left| (1622 \text{ m}) \times (209 \text{ m}) - (1232 \text{ m}) \times (1439 \text{ m}) \right| = 716925 \text{ m}^2 \]
The area of the polygon is 256193 m² + 756326 m² + 716925 m² = 1729750 m² = 1.730 km² = 0.668 mi²

e. What is the distance from American Folk Art Museum to Central Park West?

Area of the triangle formed by Museum of Art and Design, American Folk Art Museum, and American Museum of Natrual History is:
\[ A_1 = \frac{1}{2} \left| \mathbf{v}_1 \times \mathbf{v}_2 \right| = \frac{1}{2} \left| (-45 \text{ m}) \times (1552 \text{ m}) - (543 \text{ m}) \times (815 \text{ m}) \right| = 256193 \text{ m}^2 \]
The distance between Museum of Art and Design and American Museum of Natrual History is
\[ \sqrt{(586757 \text{ m} - 585942 \text{ m})^2 + (4515060 \text{ m} - 4513508 \text{ m})^2} = 1753 \text{ m} \]
The distance from American Folk Art Museum to Central Park West is: 256193 m² × 2 ÷ 1753 m = 292 m

2.12. Coordinate transfering. If you set a new coordinate system where the Columbus Circle is the origin point, x-axis is the southwest boundary of Central Park, y-axis is the northeast boundary of Central Park. What are the coordinates of the five museums under this coordinate system?

First move the origin point to the Columbus Circle, but let the axis stay parallel to UTM grid, and then rotate the coordinate system to fit with the boundary of Central Park.

The new coordinate of the five museums after moving the origin point to Columbus Circle are:
American Museum of Natural History:
\[ \mathbf{v}_1 = (586757 - 585942)i + (4515060 - 4513508)j = 815i + 1552j \]
The Metropolitan Museum of Art:
\[ \mathbf{v}_2 = (587564 - 585942)i + (4514740 - 4513508)j = 1622i + 1232j \]

Asia Society and Museum:
\[ \mathbf{v}_3 = (587381 - 585942)i + (4513717 - 4513508)j = 1439i + 209j \]

Museum of Arts and Design:
\[ \mathbf{v}_4 = (585942 - 585942)i + (4513508 - 4513508)j = 0i + 0j \]

American Folk Art Museum:
\[ \mathbf{v}_5 = (585897 - 585942)i + (4514951 - 4513508)j = -45i + 1443j \]

Then rotate the coordinate system by \( \tan^{-1} \frac{815}{1552} \approx 27.7^\circ \) clockwise. The new coordinate can be calculated through the matrix production as follows:

American Museum of Natural History: \[ \mathbf{v}_1' = [815 \ 1552][\begin{array}{cc} \cos 27.7^\circ & \sin 27.7^\circ \\ -\sin 27.7^\circ & \cos 27.7^\circ \end{array}] = [0 \ 1753] \]

The Metropolitan Museum of Art: \[ \mathbf{v}_2' = [1622 \ 1232][\begin{array}{cc} \cos 27.7^\circ & \sin 27.7^\circ \\ -\sin 27.7^\circ & \cos 27.7^\circ \end{array}] = [863 \ 1845] \]

Asia Society and Museum: \[ \mathbf{v}_3' = [1439 \ 209][\begin{array}{cc} \cos 27.7^\circ & \sin 27.7^\circ \\ -\sin 27.7^\circ & \cos 27.7^\circ \end{array}] = [1177 \ 854] \]

Museum of Arts and Design: \[ \mathbf{v}_4' = [0 \ 0][\begin{array}{cc} \cos 27.7^\circ & \sin 27.7^\circ \\ -\sin 27.7^\circ & \cos 27.7^\circ \end{array}] = [0 \ 0] \]

American Folk Art Museum: \[ \mathbf{v}_5' = [-45 \ 543][\begin{array}{cc} \cos 27.7^\circ & \sin 27.7^\circ \\ -\sin 27.7^\circ & \cos 27.7^\circ \end{array}] = [-292 \ 460] \]
APPENDIX C:

WORD PROBLEM EXAMPLES (BASED ON WORLD MAP)

1.1. What is map projection used for? How can they be classified?

Map projection is used to systematically transform the latitudes and longitudes of location from the surface of sphere Earth into locations on a flat plane of map.

Based on the types of projection surface, projection system can be classified as cylindrical projection, conic projection, and plane projection. Based on the properties of model that they preserve, projection system can be classified as azimuthal projection that preserve directions, conformal projection that preserve shape locally, equal-area projection that preserve area, equidistant projection that preserve distance, gnomonic projection, that preserve shortest route.

![World Map produced with Plate Carree Projection (From Snyder and Voxland 1989)](image)

**Figure C1.** World Map produced with Plate Carree Projection (From Snyder and Voxland 1989)

1.2. Plate Carree Projection is a projection system for World Map, in which 1 degree of latitude and longitude equal to the same distance on map. Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians? How are the poles represented.

There are 13 parallels and 25 meridians on the map. The interval of parallels is $180^\circ \div (13 - 1) = 15^\circ$.

The Interval of meridians is $360^\circ \div (25 - 1) = 15^\circ$.

The two poles are both represented as parallels as long as equator? Similarly, the parallels at 60o N and 60o S are each twice as long as they would be on a map with no distortion (i.e., a globe).
1.3. Variable scale is commonly used in Plate Carree Projection map. We measure the equator on the Mercator Projection map shown above and it is 6.25 inches

![Diagram of Plate Carree Projection with scale comparison]

**Figure C2.** Variable Scale Example

a. What is the scale along the parallel at the equator?
   The scale along the parallel at equator is: 6.25 in : 40000 km = 1 : 252,000,000.

b. What is the scale along the meridians?
   The scale along the meridian is the same as the scale along the parallel at equator, which is 1 : 252,000,000.

c. What is the scale along the parallel at the 45\textdegree parallel?
   The scale along the parallel at 45\textdegree parallel is: 6.25 in : (40000 km \times \cos 45\textdegree) = 1 : 178,000,000.

d. What is the scale along the parallel at the 75\textdegree parallel?
   The scale along the parallel at 75\textdegree parallel is: 6.25 in : (40000 km \times \cos 75\textdegree) = 1 : 65,000,000.

1.4. What can you infer about the shapes of geographic features on a Plate Carree Projection Map?

1 degree of latitude and longitude equal to the same distance on map, while in fact 1 degree of longitude get smaller with the increase of latitude. Therefore the high latitude area have a much small scale along the parallel than that on the meridian. As a result, the shape of geographic features get “stretched” on the parallel in the high latitude area.
1.5. Equirectangular Projection is similar with Plate Carree Projection. However, the scale along the meridian is now equal to that along two standard parallel (in the example above, 30° N and E). Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 13 parallels and 25 meridians on the map. The interval of parallels is $180° \div (13 - 1) = 15°$.
The Interval of meridians is $360° \div (25 - 1) = 15°$.

1.6. Variable scale is commonly used in Plate Carree Projection map. We measure the equator on the Mercator Projection map shown above and it is 6.25 inches

a. What is is the scale along the parallel at the equator?
The scale along the parallel at equator is: $6.25 \text{ in:} 40000 \text{ km} = 1:252,000,000$.

b. What is is the scale along the parallel at the 30th parallel?
The scale along the parallel at 30th parallel is: $6.25 \text{ in:} (40000 \text{ km} \times \cos 30°) = 1:218,000,000$.

c. What is is the scale along the meridians?
The scale along the meridian is the same as the scale along the parallel at equator, which is $1:218,000,000$.

d. What is is the scale along the parallel at the 45th parallel?
The scale along the parallel at 45th parallel is: $6.25 \text{ in:} (40000 \text{ km} \times \cos 45°) = 1:178,000,000$.

e. What is is the scale along the parallel at the 75th parallel?
The scale along the parallel at 75th parallel is: $6.25 \text{ in:} (40000 \text{ km} \times \cos 75°) = 1:65,000,000$. 
1.7. Compare with a Plate Carree Projection Map, what can you infer about the shapes of geographic features on Equirectangular Projection Map?

The shape of geographic features that are above the upper standard parallel or below the lower standard parallel get “stretched” on the parallel direction, while the geographic features in between the two standard parallels get “pressed” on the parallel direction.

1.8. Mercator Projection is a commonly-seen projection system for World Map. Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 13 parallels and 25 meridians on the map. The interval of parallels is \(180^\circ \div (13 - 1) = 15^\circ\).

The Interval of meridians is \(360^\circ \div (25 - 1) = 15^\circ\).
1.9. Variable scale can also be applied on Mercator Projection map. We measure the equator on the Mercator Projection map shown above and it is 6.25 inches

f. What is the scale along the parallel at the equator?

The scale along the parallel at equator is: 6.25 in: 40,000 km = 1: 252,000,000.

g. What is the scale along the parallel at the 45th parallel?

The scale along the parallel at 45th parallel is: 6.25 in: (40000 km × cos 45°) = 1: 178,000,000.

h. What is the scale along the parallel at the 75th parallel?

The scale along the parallel at 75th parallel is:

\[
6.25\text{ in: } (40000\text{ km} \times \cos 75°) = 1: 65,000,000.
\]

1.10. Compared with Plate Carree Projection, the scale of each point keep consistency in every direction in Mercator Projection. What is the distance from equator to the 45th parallel in a Mercator Projection Map of which the scale at equator is 1:252,000,000?

Take a very small angle along the meridian as \(d\theta\), the arc is cover on the earth surface is \(R \, d\theta\). The scale along that point is \(\frac{1:252,000,000}{\cos \theta} = 1: 252,000,000 \sec \theta\). Therefore the distance from equator to 45th parallel on the map is:

\[
\int_{0}^{45°} R \sec \theta \cdot (1:252,000,000) \, d\theta = 6370 \text{ km} \times \frac{1}{252,000,000} \times (\ln|\sec 45° + \tan 45°| - \ln|\sec 0° + \tan 0°|) = 2.228 \times 10^{-5} \text{ km} = 0.877 \text{ in}
\]

1.11. Because Mercator Projection preserve the scale on all direction in the certain point, the rhumb lines are picted as straight line on the map. Word Problem 5.10 give the equation for the bearing of rhumb line, derive that equation.

The tangential value of the bearing of rhumb line equals to the ratio of horizontal (parallel) distance over vertical (meridian) difference. The horizontal difference is:

\[H = \frac{(\phi_1 - \phi_2)}{180°} \times R.\]

The vertical difference is:

\[V = (\ln|\sec \theta_1 + \tan \theta_1| - \ln|\sec \theta_2 + \tan \theta_2|) \times R.\]

Therefore the bearing is:

\[
\tan^{-1} \left(\frac{H}{V}\right) = \tan^{-1} \left(\frac{(\phi_1 - \phi_2)}{180° \ln|\sec \theta_1 + \tan \theta_1| - \ln|\sec \theta_2 + \tan \theta_2|}\right)
\]

1.12. Word Problem 5.11 give the equation for the true distance of the rhumb line, derive that equation?

For a very small angle of the meridian, the length along the meridian is \(\frac{\pi}{180°} R \, d\theta\). The length of the rhumb line crossing the small part of meridian is \(\frac{\pi}{180°} R \, d\theta \div \cos \alpha\). The True distance is:

\[
\int_{\theta_1}^{\theta_2} \frac{\pi}{180°} R \, d\theta \div \cos \alpha = \left(\frac{\theta_1 - \theta_2}{180°}\times \pi R\right) \div \cos \alpha
\]

Figure C5. World Map produced with Lambert Cylindrical Equal-Area Projection (From Snyder and Voxland 1989)
1.13. Again, read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 13 parallels and 25 meridians on the map. The interval of parallels is \(180° \div (13 - 1) = 15°\).

The Interval of meridians is \(360° \div (25 - 1) = 15°\).

1.14. Variable scale can also be applied on Lambert Cylindrical Equal-Area Projection map. We measure the equator on the Mercator Projection map shown above and it is 6.25 inches.

a. What is the scale along the parallel at the equator?

The scale along the parallel at equator is: \(6.25\text{ in} : 40000\text{ km} = 1:252,000,000\).

b. What is the scale along the parallel at the 45th parallel?

The scale along the parallel at 45th parallel is: \(6.25\text{ in} : (40000\text{ km} \times \cos 45°) = 1:178,000,000\).

c. What is the scale along the parallel at the 75th parallel?

The scale along the parallel at 75th parallel is: \(6.25\text{ in} : (40000\text{ km} \times \cos 75°) = 1:65,000,000\).

1.15. As indicated from its name, Lambert Cylindrical Equal-Area is an equal-area projection system. What would be the scale along the meridian of \(\theta\) degree, if the scale along the equator is 1:252,000,000.

Image we have a small rectangle at the meridian of 0 degree. Because the scale along the parallel at this point is \((1:252,000,000) \div \cos \theta\). In order to have the area displayed correctly, the scale need to be is \((1:252,000,000) \times \cos \theta\) along the meridian at this point.

1.16. What is the distance from equator to the 45th parallel in a Mercator Projection Map of which the scale at equator is 1:252,000,000?

Take a very small angel along the meridian as \(d\theta\), the arc is cover on the earth surface is \(R d\theta\), the scale along that point is \(1:252,000,000\cos \theta\). Therefore the distance from equator to 45th parallel on the map is:

\[
\int_0^{45°} R \cos \theta \cdot (1:252,000,000) \ d\theta = 6370 \text{ km} \times \frac{1}{252,000,000} \times (\sin 45° - \sin 0°) = 1.788 \times 10^{-5} \text{ km} = 0.704 \text{ in}
\]

Figure C6. World Map produced with Equidistance Conic Projection, central meridian 90°W, standard parallel 20°N and 60°N (From Snyder and Voxland 1989)
1.17. Conic projection system is produced based on a cone placed on one of the polar of Earth like a “hat”. Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 13 parallels and 25 meridians on the map. The interval of parallels is \( 180° \div (13 - 1) = 15° \).
The interval of meridians is \( 360° \div (25 - 1) = 15° \).

1.18. Notice that the distances between parallels are constant in Equidistance Conic Projection. Therefore the scale along the meridian is constant. We measure the distance from 90°N to 90°S on this map and it is 2.81 in. What is the scale along the meridian?

The scale along the meridians is: \( 2.81 \text{ in}: 20000 \text{ km} = 1:280,000,000 \).

1.19. Notice that the map area is in a fan-shape, and there is some blank in the center. Knowing that standard parallels are 20°N and 60°N (that means the scale along these to parallels are constant in all direction), solve for the radius of the blank area \( (r) \) and the angle of the fan shaped sector \( (\theta) \)?

Length of the meridian is 2.81 in on this map, therefore the length of every 10° along the meridian is \( 2.81 \text{ in} \div 18 = 0.156 \text{ in} \).

At the 60°N parallel: \( ((0.156 \times 3 + r) \times \theta) : (40000 \times \cos 60°) = 1:280,000,000. \)

At the 20°N parallel: \( ((0.156 \times 7 + r) \times \theta) : (40000 \times \cos 20°) = 1:280,000,000. \)

Solve for \( r \) and \( \theta \) by the two function above and get: \( r=0.240 \text{ in}, \text{ and } \theta=3.97=228° \).

1.20. Based on your calculation in Word Problem A6.18, what is the scale along the parallel at the latitude of \( \varphi \) (\( \varphi \) is positive degree in the northern hemisphere and negative degree in the southern hemisphere)?

The scale is: \( ((2.81 \text{ in} \div 180) \times (90 - \varphi) + 0.24 \text{ in}) \times 3.97 : (40000 \times \cos \varphi) \)

**Figure C7.** World Map produced with Albert Equal-area Conic Projection, central meridian 90°W, standard parallel 20°N and 60°N (From Snyder and Voxland 1989)
1.21. Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 13 parallels and 24 meridians on the map. The interval of parallels is $180^\circ \div (13 - 1) = 15^\circ$.

The interval of meridians is $360^\circ \div 24 = 15^\circ$.

1.22. Notice that the distances between parallels are not constant anymore. Given that the angle of the fan shaped sector ($\theta$) is $(\sin \alpha + \sin \beta) \times 180^\circ$, and the radius of the parallel ($\phi$) is $2R \sqrt{\frac{\cos^2 \alpha + (\sin \alpha + \sin \beta)(\sin \alpha - \sin \varphi)}{\sin \alpha + \sin \beta}}$, where $\alpha$ and $\beta$ are the standard parallel, and $R$ is the Earth radius of the map.

a. The standard parallel of the map above is 20°N and 60°N, what is the angle of the fan shaped sector of the map?

The angle is $(\sin 20^\circ + \sin 60^\circ) \times 180^\circ = 217.45^\circ$.

b. The Earth radius under the map scale is 1 in, what is the radius of the blank area in the center of the sector?

The radius of the blank area equals to the radius at the 90°N. Therefore the radius is $2 \times \sqrt{\frac{\cos^2 20^\circ + (\sin 20^\circ + \sin 60^\circ)(\sin 20^\circ - \sin 90^\circ)}{\sin 20^\circ + \sin 60^\circ}} \times 1 \text{ in} = 0.49 \text{ in}$

c. The Earth radius under the map scale is 1 in, what is the length of meridians?

The length of the meridians equals to the difference between the radius at the 90°N and 90°S. Therefore the radius is $2 \times \left(\sqrt{\frac{\cos^2 20^\circ + (\sin 20^\circ + \sin 60^\circ)(\sin 20^\circ - \sin 90^\circ)}{\sin 20^\circ + \sin 60^\circ}} - \sqrt{\frac{\cos^2 20^\circ + (\sin 20^\circ + \sin 60^\circ)(\sin 20^\circ - \sin (-90^\circ))}{\sin 20^\circ + \sin 60^\circ}}\right) \times 1 \text{ in} = 2.13 \text{ in}$

1.23. Based on the equation given in Word Problem 6.22, what is the scale along the parallel at the latitude of $\varphi$ ($\varphi$ is positive degree in the northern hemisphere and negative degree in the southern hemisphere)?

The radius of the parallel at the latitude of $\varphi$ is: $2R \sqrt{\frac{\cos^2 \alpha + (\sin \alpha + \sin \beta)(\sin \alpha - \sin \varphi)}{\sin \alpha + \sin \beta}}$, therefore the length of the Meridian is $2\pi (\sin \alpha + \sin \beta) R \sqrt{\frac{\cos^2 \alpha + (\sin \alpha + \sin \beta)(\sin \alpha - \sin \varphi)}{\sin \alpha + \sin \beta}}$. Therefore the scale along the meridian is: $(\sin \alpha + \sin \beta) \frac{R \sqrt{\cos^2 \alpha + (\sin \alpha + \sin \beta)(\sin \alpha - \sin \varphi)}}{R_E (\sin \alpha + \sin \beta)}$, where $R_E$ is the radius of the Earth.
1.24. Azimuthal projection system is produced based on a plane placed on certain point of Earth. Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 13 parallels and 24 meridians on the map. The interval of parallels is $180^\circ \div (13 - 1) = 15^\circ$.

The interval of meridians is $360^\circ \div 24 = 15^\circ$.

1.25. Notice that the distances between parallels are constant in Azimuthal Equidistance Projection. Therefore the scale along the meridian is constant. We measure the distance from $90^\circ$N to $90^\circ$S on this map and it is 2.37 in. What is the scale along the meridian?

The scale along the meridian is: 2.37 in: 20000 km = 1:332,000,000.

1.26. What is the scale along the parallel at the latitude of $\varphi$ ($\varphi$ is positive degree in the northern hemisphere and negative degree in the southern hemisphere)?

The scale is: \[
\left( \frac{90^\circ - \varphi}{90^\circ} \times 10000 \text{ km} \times (1:332,000,000) \right) \times 2\pi \times (40000 \text{ km} \times \cos \varphi) = (1 - \frac{\varphi}{90^\circ} \frac{\pi}{\cos \varphi}) (1:332,000,000)
\]

1.27. Given the latitude and longitude of a point on Earth ($\varphi, \lambda$). What’s its spherical coordinate in the map above? What about a Cartesian coordinate of which the x-axis points to the $0^\circ$ meridian and y-axis points to the $90^\circ$E meridian (like the map showing above)?

In spherical coordinate, the radium ($r$) is: $20000 \text{ km} \times \frac{90^\circ - \varphi}{180^\circ} \times 2\pi$, where $s$ is the scale along the meridian.

The angle ($\theta$) from is $90^\circ - \lambda$. 

\[ \text{Figure C8. World Map produced with Polar Azimuthal Equidistance Projection, central meridian 90°W (From Snyder and Voxland 1989).} \]
In Cartesian coordinate, the coordinate is: 

\[(r \cos \theta, r \sin \theta) = (20000 \text{ km} \times \frac{90-\varphi}{180} \times s \times 
\cos(90 - \lambda), 20000 \text{ km} \times \frac{90-\varphi}{180} \times s \times \sin(90 - \lambda))\]

**Figure C9.** World Map produced with Lambert Azimuthal Equal-Area Projection, central meridian 90°W, range 90° (From Snyder and Voxland 1989).

1.28. Unlike the Azimuthal Equidistance Projection, Lambert Azimuthal Equil-Area Projection has a range in the latitude displayed. Read the map above, how many parallels and meridians are there? What is interval of parallels and meridians?

There are 9 parallels and 36 meridians on the map. The interval of parallels is 90° ÷ 9 = 10°. The Interval of meridians is 360° ÷ 36 = 10°

1.29. Lambert Azimuthal Equil-Area Projection is produced based on the geometry shown below. Read the graph below and derive the sphere radius at the parallel at the latitude of \(\varphi\) on the map?

**Figure C10.** How Lambert Azimuthal Equil-Area Projection Works
The radius is: 

\[ 2R \sin \frac{90° - \varphi}{2} = 2R \sqrt{\frac{1 - \sin \varphi}{2}}, \]  

where \( R \) is the radius of Earth under the map scale.

1.30. What is the scale along the parallel at the latitude of \( \varphi \) based on the radius you calculated in World Problem?

The scale is: 

\[ 2R \sqrt{\frac{1 - \sin \varphi}{2}} \times 2\pi : (2R \times \cos \varphi) = 2 \sqrt{\frac{1 - \sin \varphi}{2}} \sec \varphi, \]  

where \( s \) is the scale of radius of Earth.

1.31. Given the latitude and longitude of a point on Earth \( (\varphi, \lambda) \). What’s its spherical coordinate in the map above? What about a Cartesian coordinate of which the x-axis points to the 0° meridian and y-axis points to the 90°E meridian (like the map showing above)?

In spherical coordinate, the radius \( (r) \) is: 

\[ 2R \sqrt{\frac{1 - \sin \varphi}{2}} \]. The angle \( (\theta) \) from the is 90° - \( \lambda \).

In Cartesian coordinate, the coordinate is: 

\[ (r \cos \theta, r \sin \theta) = \left( 2R \sqrt{\frac{1 - \sin \varphi}{2}} \times \cos(90° - \lambda), 2R \sqrt{\frac{1 - \sin \varphi}{2}} \times \sin(90° - \lambda) \right) = (2R \sin \varphi \sqrt{\frac{1 - \sin \varphi}{2}}, 2R \cos \varphi \sqrt{\frac{1 - \sin \varphi}{2}}) \]

1.32. Notice that we include equal-area maps in all three type of maps, the the gradicule intervals are all 15×15 degree. Calculate the area of the following “blocks” bounded by meridians and parallels (radius of the Earth under map projection is 1 in):

a. The “block” that includes the Tampa. Did your answers for the three different type of maps confirm?

The “block” is bounded by 15th and 30th parallel, and 90th and 75th meridian.

In the Lambert Equal-Area Projection, the width is \( \frac{15°}{180°} \pi R \), the height is \( (\sin 30° - \sin 15°)R \), the area is \( \frac{15°}{180°} \pi R \times (\sin 30° - \sin 15°)R = 0.063 \text{ in}^2 \)

In the Albert Equal-Area Conic Projection, the “block” is the difference of the fan sector at the range of 15th and 30th parallel. The area is 

\[ \frac{15°}{360°} \times \frac{217.45°}{360°} \times \pi \left( (2 \times \frac{\cos^2 20° + (\sin 20° + \sin 60°)(\sin 20° - \sin 15°)}{\sin 20° + \sin 60°}) \times 1 \text{ in}^2 \right) = 0.063 \text{ in}^2 \]

In the Lambert Azimuthal Equal-Area Projection, the “block” is the difference of the fan sector at the range of 15th and 30th parallel. The area is 

\[ \frac{15°}{360°} \times \pi \left( (2R \sqrt{\frac{1 - \sin 15°}{2}}) - (2R \sqrt{\frac{1 - \sin 30°}{2}}) \right) = 0.063 \text{ in}^2 \]

The area calculated based on three different type of projections confirm with each other.

b. The “block” that includes UK. Did your answers for the three different type of maps confirm?

The “block” is bounded by 45th and 60th parallel, and 0th and 15th meridian.

In the Lambert Equal-Area Projection, the width is \( \frac{15°}{180°} \pi R \), the height is \( (\sin 60° - \sin 45°)R \), the area is 

\[ \frac{15°}{180°} \pi R \times (\sin 60° - \sin 45°)R = 0.042 \text{ in}^2 \]

In the Albert Equal-Area Conic Projection, the “block” is the difference of the fan sector at the range of 45th and 60th parallel. The area is 

\[ \frac{15°}{360°} \times \frac{217.45°}{360°} \times \pi \left( (2 \times \frac{\cos^2 20° + (\sin 20° + \sin 60°)(\sin 20° - \sin 15°)}{\sin 20° + \sin 60°}) \times 1 \text{ in}^2 \right) = 0.042 \text{ in}^2 \]

In the Lambert Azimuthal Equal-Area Projection, the “block” is the difference of the fan sector at the range of 45th and 60th parallel. The area is 

\[ \frac{15°}{360°} \times \pi \left( (2R \sqrt{\frac{1 - \sin 45°}{2}})^2 - (2R \sqrt{\frac{1 - \sin 60°}{2}})^2 \right) = 0.042 \text{ in}^2 \]

The area calculated based on three different type of projections confirm with each other.

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c. What is the true area of the two previous blocks on the Earth?

Based on the equation derived in 1.17, the area of the block that include Tampa is:

\[
A = \frac{(90° - 75°)}{180°} \times \pi \times (\sin 30° - \sin 15°) \times (6380 \text{ km})^2 = 2.57 \times 10^6 \text{km}^2
\]

The area of the block that include UK is:

\[
A = \frac{(15° - 0°)}{180°} \times \pi \times (\sin 60° - \sin 45°) \times (6380 \text{ km})^2 = 1.69 \times 10^6 \text{km}^2
\]

d. Calculate the “scale of area” based on the two blocks. Did your answers confirm?

Based on the block that include Tampa, the scale is: 0.063 in²: 2.57 \times 10^6 \text{km}^2 = 1: 6.32 \times 10^{16}.

Based on the block that include Tampa, the scale is: 0.042 in²: 1.69 \times 10^6 \text{km}^2 = 1: 6.23 \times 10^{16}.

The answers confirm. That means there is a constant “scale of area” among single equal-area map, which indicate that the area is preserved in this type of projection system.

1.33. Notice that the area of blocks is very different on the Mercator Projection map:

a. What is the size of “block” that includes the Tampa in the mercator Projection map (radius of the Earth under map projection is 1 in)?

The “block” is bounded by 15th and 30th parallel, and 90th and 75th meridian.

In the Mercator Projection, the width is \(\frac{15°}{180°} \pi R\), the height is \((\ln|\sec 30° + \tan 30°| - \ln|\sec 15° + \tan 15°|)R\), the area is \(\frac{15°}{180°} \pi R \times (\ln|\sec 30° + \tan 30°| - \ln|\sec 15° + \tan 15°|)R = 0.074 \text{ in}^2\)

b. The “block” that includes UK. Did your answers for the three different type of maps confirm?

The “block” is bounded by 45th and 60th parallel, and 0th and 15th meridian.

In the Mercator Projection, the width is \(\frac{15°}{180°} \pi R\), the height is \((\ln|\sec 60° + \tan 60°| - \ln|\sec 45° + \tan 45°|)R\), the area is \(\frac{15°}{180°} \pi R \times (\ln|\sec 60° + \tan 60°| - \ln|\sec 45° + \tan 45°|)R = 0.114 \text{ in}^2\)

c. Compare the area of these blocks in the Mercator Projection and those in equal-area projection system. What do you find?

In the Mercator Projection, the blocks in the higher latitude have larger area, the blocks in the lower latitude have smaller area.

Oppositely, in the equal-area projection, the blocks in the higher latitude have smaller area, the blocks in the lower latitude have larger area.
APPENDIX D:

WORD PROBLEM EXAMPLES (BASED ON LAND USE MAPS)

Figure D1. Land-use Map of Downtown Tallahassee (data source: Tallahassee County Planning Department)

1.1. What is the dimension of the map?
   Using the bar scale, the length is 5.09 mile, and the width is 3.67 mile

1.2. On the PDF as shown on your computer monitor with a zoom setting at 100%, the bar scale from 0 to 1.0 mile measures 1.5 inch with the ruler built into the software's display. What is the scale (RF) of the map with respect to the bar scale shown on the map.
   The scale (RF) is 1.5 inch:1 mile = 1.5 inch : 63360 inches = 1:42240.
1.3. What is the length of the bar scale from 0 to 1.0 mile on your monitor when the zoom setting is at 200%. What is the scale (RF) of the map as seen on your monitor with this setting?

When the zoom setting is 200%, the scale is twice as large as that in 100%, thus the scale (RF) is 1:21120.

1.4. Where are the government facilities locate in downtown Tallahassee?

In the south of downtown Tallahassee.

1.5. The average size of a city block in Tallahassee is 500 X 500 foot, count the numbers of Government Operation in south downtown tallahassee and estimate the area?

The average area of a city block in Tallahassee is 500 X 500=250000 square foot=0.0090 square mile. There are 37 city blocks classified as Government Operation in south downtown Tallahassee, the total area should be close to 0.33 square mile.

1.6. If you plan to build a shopping center within downtown Tallahassee, which would occupy at least 0.5 square mile of area. And meet the following requirement:

a. Must be build on “Open Space Undesignated” or “Open Space Common Areas”

b. Must be within 1.5 to 2.5 mile from the center of the city.

![Figure D2. Solution to Word Problem 1.6](image)

Draw 2 circles, of which the center locate in the center of the city, and the radius of 1.5 mile and 2.5 mile respectively. The “Open Space Common Areas” A and B shown in the figure meet all the requirements.
2.1. What is the land use type that covers the most area of Australia?

Open shrublands

2.2. What is the land use type that covers the most area along the equator? Why?

Evergreen Broadleaf Forest. The equator area is located in the Intertropical Convergence Zone, which brings abundant heat and rainfall for the growth of evergreen broadleaf plants.

2.3. Where on the Earth is the Barren/Sparsely Vegetated area mostly located? Why?

Barren/Sparsely Vegetated area is mostly located in the north Africa. North Africa is under the influence of subtropical high pressure, which means very limited rainfall and high temperature. This limited the growth of plants.

2.4. Compare the land cover map with climate. Do you see some correlations between climate type and land cover type? List some of pairing.

Evergreen Broadleaf Forest – Tropical Forest Climate.
Barren/Sparsely Vegetated area and Open Shrublands – Arid Desert Climate
Snow and Ice – Polar Climate
APPENDIX E:

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June 24, 2019

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Best wishes,

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