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A Qualitative Content Analysis of Early Algebra Education iOS Apps for Primary Children

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by

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Dedication

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Abstract

Educational software applications (apps) on multi-touch, mobile devices provide a promising space to help learners work toward long-term educational goals, like *learning with understanding* (Bransford, Brown, & Cocking, 2000). Such goals are particularly relevant in supporting a learner's efforts to become more mathematically literate. Yet, a number of current apps do not appear to be living up to this potential. As such, this study drew upon the theoretical framework of Learning Science and the conceptual framework of TPACK theory (Mishra & Koehler, 2006) to define curricular characteristics that ideally support primary children's potential to learn early algebra concepts with understanding, through multi-touch, mobile, iOS mathematics education apps. Using qualitative content analysis these characteristics, then, were compared to the curricular characteristics of three authentic (i.e., real-world) apps in order to describe the general extent to which the two sets of characteristics aligned. This study found the authentic apps did not align with the majority of curricular characteristics that ideally support learning with understanding. Additionally, a number of qualitative findings emerged from the study that may be used to inform future app design. These ideas include themes related to the kinds of characteristics the authentic apps tended to align with or not, and suggested adaptations to a number of contemporary theories and models related to pedagogical content knowledge and its application toward the goal of learning with understanding. These findings have direct implications for the theory and practice of app design, and suggest revisions to the way in which the field of instructional design, historically, has been approached.

Chapter One:

Introduction

1.1 Background

Disciplinary literacy is progressively required of global-American citizens. Work place requirements are rooted in discipline-specific habits of mind. Decision making within a democratic society requires the effective comparing and weighing of contextualized concepts, and daily lives increasingly require the competent execution of disciplinary-rich ideas. True disciplinary literacy, however, relies upon successful learning, and the primary contributor of successful learning is *learning with understanding* (Bransford, Brown and Cocking, 2000, p. 8).

Formal schooling is generally positioned as a place that supports children's ability to learn with understanding. However, in formal school classrooms this underlying goal is often displaced by other factors, including "routine conditions of the classroom" (Kennedy, 2005, p. 2) and short-term goals driven by politics, economics, and public opinion (Darling-Hammond, 2010; Giroux, 2010). One of the more recent factors to influence public school education, in the U.S., has been the accountability movement and its consequent redefinition of "time on task". This recasting has more narrowly demarcated what working and learning look like in the formal school classroom- in most cases, to the exclusion of play and playful contexts. Arguably, this has had more impact on primary learners (defined loosely as six to eight-year-olds) than other learners. This is because primary learners largely prefer to learn and, often, learn best in playful contexts (see Hatch, 2002). Additionally, this loss of play in formal schooling has yielded at a time in which views toward this age group as unsophisticated and incapable learners are disappearing. While this latter change is positive and promising, the elimination of playful learning has resulted in a new set of constraints for primary learners. In other words, instead of the provision of learning experiences that are both playful and sophisticated, the two components have merely been swapped. As such, the educational goal of learning with understanding may be no more accessible to primary learners than it was in decades past, when perceptions of children's development limited the kinds of concepts and experiences to which they were exposed.

Several alternative learning spaces seem to offer the conceptual space to accommodate both playful and sophisticated learning. One of the most promising is educational software applications (apps) for multi-touch, mobile devices. Indeed, the specific induction of finger-sensitive, multi-touch, mobile computing into modern society has opened new landscapes of possibility for learning. Since the introduction of Apple Inc.'s iPhone to the mass market in 2007 (Apple, 2007), in addition to Apple's iPad and Samsung's Galaxy Tab in 2010 (Apple, 2010; Gideon, 2010), and many digital protégés since, multitudes of world citizens have gained access to a medium of inquiry boasting a unique combination of qualities. Seemingly, in one swoop, over the course of just a few years, the public was introduced to a new genre of tool that combines the many features of digital computing, such as hyperlink capabilities and faculties for multi-modal experiences, with the qualities of mobility and increased interactivity through touch. This distinctive combination of qualities, in addition to the tool's popularity and seemingly ubiquitous presence, has the potential to help learners achieve contemporary educational goals in ways other media cannot. As such, this type of device inspires grand visions of encouraging possibilities for transformed learning (Shuler, 2009).

While learners of all ages benefit from interactive educational experiences to deepen their understanding of a topic (Bransford, Brown, & Cocking, 2000), interactive learning through multi-touch, mobile technology seems particularly apt for children. This seeming suitability is partly supported by the fact that recent generations of American children have, for the most part, grown up with computer technologies as a prevalent part of their lives. Those children, many of whom are now adults, struggle to imagine daily existence without computer-mediated experiences, and digital media are perceived as a fundamental part of childhood- both in formal education and informal, out-of-school experiences (e.g., Palfrey & Gasser, 2008; Prensky, 2006; Tapscott, 1998). As such, a wide variety of interactive digital technologies, such as tablets and smart phones, seem to have secured a unique position in society as mutually educational and entertaining- qualities seized upon by many contemporary American adults when considering ways to both occupy children's time and prepare them for academics (see Nielsen, 2012). Additionally, this type of tool tends to be easy for children to use independently (Chiong & Shuler, 2010), often accepting imperfect touch-control from hands and fingers with emerging fine motor manipulation. Similarly, when clad in rubber casing and a screen protector, these mobile tools tend to be conveniently rugged and portable- generally withstanding sticky fingers and a certain level of turbulent handling. Given children's seemingly inherent fascination with such media (Shuler, 2009), as well as ever-evolving improvements in mobile capabilities, there is "enormous untapped educational potential for today's generation" (Shuler, 2009, p. 4).

Although the prospective of this tool to serve as a child's educational device is widely acknowledged, the extent to which it has a positive impact on a child's learning is vociferously debated and seems dependent on three primary forces- context of use, child, and virtual curriculum (sometimes called "content") [Guernsey, 2012]. The first, "context of use", conveys conditions of the physical environment in which the device is used. These contextual conditions include: the amount of screen time to which a child is exposed, the amounts and kinds of interactions between the child and present adults (e.g., *joint media engagement* [Takeuchi & Stevens, 2011]) or interactions between the child and peers during screen play (e.g., collaboration), the kinds and levels of external learning support available to a child during these activities (e.g., *scaffolding* [Wood, Bruner, & Ross, 1976]), the ways in which the technology is integrated into various environments, and the ways in which a child's participation can be assessed by others. As such, much research has focused on the ways in which multi-touch, mobile devices should be, or are being, integrated into early learning environments like a child's home and classroom (e.g., Cole & Stanton, 2003; Fails, 2007). For example, there has been long-standing debate over the amount of screen time to which children are exposed through their viewing of television, computer usage, and video game playing, and now, mobile devices are yet another way for children to increase their amount of screen time (Gutnick, Robb, Takeuchi, & Kotler, 2011). Yet, research has also shown not all screen time is created equally. There is a difference between active participation and passive viewing. This research reminds us, while it is important for parents and teachers to limit the amount of screen time a young learner consumes, it is equally important to consider the ratio of active to passive screen time in which the learner is engaged. As such, some bodies of research have focused on how teachers and parents can encourage a learner's use of mobile applications (apps) for creative collaboration (e.g., Fails, 2007). Overall, research in this area is promising in helping educators and parents understand not all contexts in which mobile devices are used, are identical. As such, there are particular contextual characteristics that potentially increase the quality of a young learner's experience.

Similarly, advocates, educators, and researchers have also considered how the force of "child" affects learning within these contexts. "Child" relates to the ways in which an individual child or a group of children utilizes and interacts with a device and any substantive curricular elements of its software. A child or group of children, for instance, may commandeer a digital device and use it to teach him or herself, without adult intervention. In fact, this is the premise behind minimally invasive education (MIE) initiatives, such as the "hole-in-the-wall experiments" (see Mitra & Rana, 1999) and *One Laptop Per Child* (Negroponte, 2005). This process can, not only, result in an educational experience, but one that is more profound than was originally thought possible (Mitra, Dangwal, Chatterjee, Jha, Bisht, & Kapur, 2005). Thus, research related to "child", provides important reminders that children construct their own meanings and exercise their power to create and redefine the educational process.

Hence, without discounting the enduring potential of the child to repurpose a device or app at any time, and while acknowledging the importance of contextual conditions under which the app is used, it is also important to consider the curricula of mobile, educational apps. In an age when the formal education system appears entangled in the ropes of accountability- in which the measurements themselves, seemingly, have become the focus of education- apps and other informal education systems provide an opportunity to refocus on critical learning goals, like learning with understanding. Accordingly, it is the curricula of these apps, with their potential to serve as an alternative learning space to support educational goals that may not be possible to pursue in current formal learning contexts, that constitutes the third force of "curriculum" as worthy of discussion.

In this study, "curriculum" refers to the whole of a learning program- the virtual learning environment, the content concepts to which learners are exposed, the embedded pedagogical practices, the way in which the program "interacts" with learners. Thus, an app's curriculum refers to a range of teaching and learning components that make up its educational substance. Specifically, this includes components such as learning goals (stated or implied), programmed scaffolding techniques, human and virtual relationships realized through the app, entry points into subject matter and conceptual pathways "through" content, physical design or layout of an app, learning "climate" within an app, specific activities or provocations, forms of representation related to topical ideas, key subject matter concepts, and kinds or levels of cognitive demands required by the app's activity. Hence, an educational app's curriculum not only refers to the disciplinary concepts it aims to portray, but also to the learning environment and pedagogical practices that are inherently a part of its design.

The curriculum of an educational app is important to consider for a number of reasons. Primarily among those reasons, curricular elements are a significant influential force in how learners conceptualize subject matter ideas (Guernsey, 2012; Shuler, 2012). In a traditional classroom, the knowledge and practices of a teacher affect her design and enactment of the learning curriculum, which in turn, shapes learners' conceptualizations. In an app-mediated learning environment, the knowledge and practices of the app designer(s) affect the way learners perceive concepts. This is because an app designer and her programming fulfill some of the primary roles of "teacher" through the design and enactment of the educational app curriculum. Thus, just as the curricular elements of a traditional classroom, as designed and enacted by the teacher, provide substantive framing for the ways in which learners perceive concepts (e.g., Ball, 1993), so do an app's curricular elements, as designed and enacted by the app designer(s).

1.2 Problem Statement

In many cases, curriculum, as defined here, is a key influence on the ways in which subject matter concepts are perceived. In this study, the curriculum of an educational app potentially becomes problematic when its characteristics do not align with those that support the educational goal of *learning with understanding* (Bransford, Brown, & Cocking, 2000, p. 8), as defined by the field of Learning Science. This is because Learning Science findings provide essential insight into fostering learning experiences that are, "cognitively active, deeply engaging, meaningful, and socially interactive within the context of a learning goal" (Hirsh-Pasek & Zosh et al., 2015, p.26). In turn, these characteristics support larger 21st century educational goals, such as learning with understanding. Hence, in broad terms, a misalignment between Learning Science findings and curricular components of an educational app implies learners have a slimmer chance of attaining important 21st century educational goals.

Despite the fact educational apps offer considerable potential to serve as an alternative space for primary learners to learn with understanding, an initial review of literature revealed discontent with the current curricular qualities of many educational apps, among a number of researchers. Regrettably, according to recent work by Hirsh-Pasek and Zosh[1] et al. (2015), first-generation apps- of which most current educational apps belong- primarily, do not align with key tenets of Learning Science. According to Hirsh-Pasek and Zosh et al., most educational apps promote high levels of activity, but fall short of being truly educational. Likewise, Shuler (2012b) notes, almost a third of gaming apps made some claim about being "educational" (p. 4).

While the affordances of multi-touch, mobile education apps have remained largely untapped, their potential is in no way diminished. Hence, second generation apps have the capability to support learning in ways their predecessors, to date, have not (Hirsh-Pasek & Zosh et al., 2015; Shuler, 2012b) As such, simply through designers making better use of the medium's inherent characteristics and features, emerging apps have real promise in helping learners realize 21st learning goals.

However, it is not for this reason alone that it is of growing importance how an educational apps' curricular substance aligns with key characteristics abstracted from Learning Science. In addition to broad 21st century educational goals like learning with understanding, learning goals in many content areas like mathematics are also in the process of shifting. This means, there are additional content-specific learning goals that, both, aim to support the larger educational goals and endeavor to reflect research findings specific to the particular discipline of focus. Accordingly, second generation apps have the potential to support both broad and specific educational goals that reflect new understandings of learning and new aspirations for learners. However, as noted by Hirsh-Pasek and Zosh et al. (2015), to accomplish this, app designers must rely on more than their "intuitive sense of how learning happens or what children will find enjoyable" (p. 4).

Another recently observed social phenomenon, known as the *pass-back effect* (Chiong & Shuler, 2010; Shuler, 2009) may also contribute to the need to examine the curricular characteristics of educational apps. The pass-back effect describes a trend in which parents habitually pass their own mobile devices to their children for use in cars, lines, restaurants, and in other places where children are typically required to wait. In most cases, parents are busy attending to other things during these times (e.g., driving, paying for groceries, ordering food, talking with other adults). Consequently, during pass-back, the number of interactions between child and adult tends to be limited. When interactions do occur, the adult may not be fully attentive to the child's play and learning (Chau, unpublished dissertation, 2014). Likewise, the nature of the collaboration is also apt to be shorter and less focused on the co-construction of knowledge. Furthermore, the presence of mobile devices seems to have contributed, not only, to the pass-back effect, but also to a dramatic increase in children's digital connectivity in general (Gutnik et al., 2011); and, frequency seems to be increasing (Chiong & Shuler, 2010). In other words, given the common context in which multi-touch, mobile apps are increasingly being used (i.e., frequently and often through pass-back) children are findings themselves interacting with an app's curricular substance in isolation more often than ever. Hence, modern habits such as "pass-back", and the contexts that surround them, may place additional emphasis on the interaction between learner and the educational substance of an app. Subsequently, the ways in which learners perceive subject matter concepts may be more heavily reliant upon an app's programmed curriculum.

Although the foremost dimensions of an app's curriculum depend upon one's view, from a broad pedagogical standpoint, they likely include, (a) The planned learning experiences, program of learning experiences, and prepared internal "environment" of an app that are often a part of its overall design and, (b) The enacted activities and interactions within the app that are a part of the program's execution. From an alternate perspective these dimensions, instead, might include, (a) What characteristics are ideally imparted within an outlined curriculum and, (b) How curricular characteristics are ideally imparted. With either choice, within each of these two sets of primary dimensions, there are specific elements such as learning goals, embedded pedagogical practices, and virtual relationships. As such, when further specified and defined, these elements can describe curricular characteristics that ideally support primary children's potential to learn concepts "with understanding", through app-mediated contexts.

As mentioned previously, Hirsh-Pasek and Zosh et al. (2015) began the important work of outlining general characteristics of an app's curriculum, based on the tenets of Learning Science. I posit, however, that expanding and tailoring these curricular characteristics, specifically to a particular content area, group of learners, and type of technology, seems fundamental to outlining traits that are of the most practical use to describe the nature of current apps, and to inform the creation of future apps. Thus, the purpose of this study was two-fold.

1.3 Purpose of the Study

Since educational apps and their subsequent curriculum provide a promising space for learners to work toward long-term educational goals, like *learning with understanding* (Bransford, Brown, & Cocking, 2000, p. 8), but currently do not appear to be living up to their potential, this study had two aims. The first was to define curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps. The second was to compare those "ideal" curricular characteristics to the curricular characteristics of three authentic apps, in order to describe alignment between the two sets of characteristics.

To accomplish this, I first utilized the *Technological Pedagogical Content Knowledge* (TPACK) framework (Koehler & Mishra, 2008; Mishra & Koehler, 2006) as a conceptual guide for identifying characteristics discussed in the professional literature, and tailored them to the specific educational circumstance I considered in this study. Thus, I used the TPACK framework to abstract characteristics of curricula related to technological pedagogical knowledge (TPK), technological content knowledge (TCK), pedagogical content knowledge (PCK), and technological pedagogical content knowledge (TPACK), from research in Learning Science related to that reflected the goal of learning with understanding.

Second, this study used qualitative content analysis to examine the ways in which, and the general extent to which, the curricular components of three current, multi-touch, mobile, iOS, mathematics education apps for primary children, compared with ideal characteristics of app-mediated, early algebra curricula.

As such, the formal research questions were:

1. What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?
2. To what extent do three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

To answer these questions, I created a coding frame that contains categories, definitions, indicators, and examples that demarcate characteristics of curricula related to app-mediated, early algebra learning with primary children. Then, I used these criteria to conduct a systematic, qualitative content analysis of current mathematics education apps for six to eight-year-olds, and described the findings. The intent was for these findings to, (a) Provide an initial sense of the current state of app-mediated, early algebra learning for primary children and, (b) Begin the conversation regarding the future design of early algebra education apps for primary children that might aim toward the goal of learning with understanding. In doing so, I joined a number of educational researchers who believe the curricular substance of apps require further analyses and consideration, so that this information, eventually, might inform the design of second generation education apps. Accordingly, my study built upon preceding scholarly work, by drawing upon related analyses (see Chau, unpublished dissertation, 2014; Hirsch-Pasek & Zosh et al., 2015; Shuler, 2012).

While my aim was similar to these previous studies in its broad intent, my study also differed in a number of specific ways. First, while Chau's study (unpublished dissertation, 2014) examined a range of educational apps for young children, my work specifically focused on educational apps that aspire to support young children's learning of early algebra concepts. This is because, (a) I have a specific interest in science, technology, engineering, and mathematics (STEM) education with primary learners and thus, (b) I endeavored to look more deeply at characteristics that potentially support learning in this area. Second, my study focused on a different age range than Chau's, whose work examines those apps designed for three to five-year-olds, while mine looked at apps for six to eight-year-olds. Third, my study proceeded from different theoretical and conceptual frameworks than Chau's, as I describe in detail below (see 1.4). Likewise, while Hirsh-Pasek's work abstracts four broad "pillars" (p. 3), or major characteristics, from research in Learning Science, as noted previously, I abstracted *specific* characteristics that relate to my particular focus. Contrastingly, Shuler's work examines components like pricing, branding, and usage trends of apps, while mine was concerned with curricular characteristics.

In comparison to these works, my study aimed to outline a *specific* set of characteristics by which to describe current apps and (eventually) inform future apps that are explicitly related to multi-touch, mobile app-mediated, early algebra learning with primary children. My intent on specificity emerged as a result of the conceptual framework I embraced. Thus, to better understand the aims of my study, it is important to learn about the conceptual framework I held in mind.

1.4 Conceptual & Theoretical Frameworks

1.4.1 Conceptual framework.

The primary goal of my study was to outline a set of observable indicators that describe "ideal" characteristics of curriculum, suggested by research findings in Learning Science, and explicitly related to app-mediated, early algebra learning with primary children. Through the process of qualitative content analysis, these indicators, then, were used as a means to compare the current authentic curricular characteristics of early algebra education iOS apps for primary children, against the "ideal" characteristics of quality for this learning situation. This was a very specific focus.

My interest in specificity was a product of the conceptual framework I embraced, *Technological Pedagogical Content Knowledge* (Mishra & Koehler, 2006). Technological Pedagogical Content Knowledge known as "TPCK", or the "TPACK" model, provides a narrative and visual illustration of keys types of knowledge that are important for designing and enacting effective, technology-enhanced educational situations. The TPACK model shows the intersection of three primary types of teacher knowledge- namely, technological knowledge, pedagogical knowledge, and content knowledge (Mishra & Koehler, 2006), although, in later models, additional types of knowledge were added- knowledge of context and learners (Koehler & Mishra, 2008).

As I interpret it, the TPACK model offers a broad visual analogy for the ideal outcome that should occur to a teacher's knowledge, in situations whereby that teacher designs and enacts technology-enhanced educational curricula. That is, specifically, her technological knowledge should intersect with her pedagogical content knowledge in transformative ways- the implied results of which have an impact on the ways she designs and enacts aspects of the technology-enhanced curricula. While the TPACK model has been used primarily to reference the design and enactment of technology integration in traditional learning settings, I posit the TPACK model could also effectively serve to illustrate the ideal outcome between types of teacher knowledge that are used to design and enact technology-*mediated* curricula. The implied results of which, similarly, have an impact on the ways a technology designer (fulfilling a number of teacher roles), designs and enacts aspects of a curricula. This is because in both technology-enhanced and technology-mediated learning situations "teacher roles" are executed in partially analogous ways.

When a teacher in a traditional setting is serving in the capacity of curriculum designer, the efficacy of a technology-enhanced learning environment or an activity she designs is dependent (at least in part) upon her consideration of how a particular technology can support or transform aspects of specific content, for a specific group of learners, in a specific context (Koehler & Mishra, 2008; Mishra & Koehler, 2006; also see Grandgenett, 2008). The implications she derives by considering these knowledge intersections, and the degree to which her design reflects those implications, are key in describing the educational quality of her design, as measured (in this study) in terms of a learner's potential to *learn with understanding* (Bransford, Brown & Cocking. 2000, p. 8). Likewise, when a teacher is enacting a technology-enhanced educational activity, or interacting with learners within a technology-enhanced environment, she is drawing upon the same basic types of knowledge and knowledge intersections, described above. Hence, during both design and enactment of traditional, technology-enhanced curricula, a teacher is relying upon her TPACK.

Similarly, in the design of educational apps and their associated virtual curricula, an app designer serves in the role of teacher as curriculum planner. The app designer is relying upon her knowledge of how a particular technology can support or transform aspects of specific content, for a specific group of learners, in a specific context. And, just as in a traditional setting, the implications she derives by considering this knowledge, and the degree to which her design reflects those implications, are key in describing the educational quality of her design. Additionally, the moment a learner participates in an educational app, the learning activities and virtual environment within the app are enacted. During this time, the programming and design of the app, as created by the app designer, serve in the role of "teacher as enactor". Thus, in app-mediated learning, an app designer, ultimately, fulfills two primary roles of the traditional teacher. As such, the TPACK model, as a conceptual framework that illustrates the ideal relational outcome between types of teacher knowledge used to design and enact technology-enhanced educational curricula, is apropos- regardless of whether a curriculum is Earth-bound or virtual, and regardless of whether the person fulfilling these roles is a traditional classroom teacher or an app designer.

As discussed previously, in a similar study, Hirsh-Pasek and Zosh et al. (2015) abstracted "pillars" of Learning Science (p. 3) that might be used to inform the curricular characteristics of apps. These researchers identified key characteristics of general pedagogical knowledge (PK) and considered how those characteristics might play out in app-mediated learning (TK), although not described using these (TPACK) terms. Although these researchers' aim was broader than mine, the integrative process they utilized, which might be described as PK+TK, is similar to the process I used in this study. With a more specific aim, I identified key characteristics of curriculum along the intersections of TCK, TPK, PCK, and TPACK, related to my situation of focus. Therefore, in acknowledgement of the "integrated approach" I take within this study, I describe my process as moving back and forth between key characteristics related to PK+TCK, , TK+PCK, PCK+TPK (see *Chapter 2*), and TPACK (see *Chapter 4*).

1.4.2 Theoretical framework.

While I envision the TPACK model as the specific conceptual framework that illustrated the ideal relational outcome between teacher knowledge types used to design and enact technology-enhanced (or technology-mediated) curricula, I embraced Learning Science as my overarching theoretical framework. Learning Science references a discipline that is, essentially, a

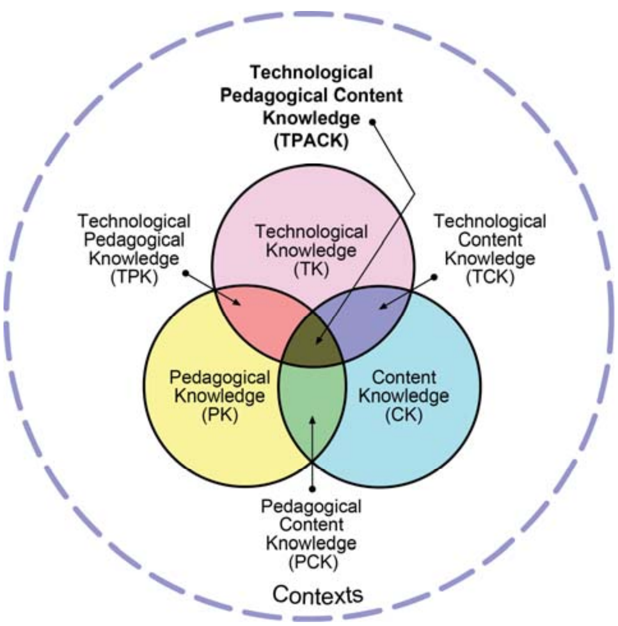

Figure 1. TPACK Image. Reproduced with permission. 2012 © tpack.org

compilation of findings from learning-related research in a variety of emerging fields, such as neuroscience, cognitive psychology and socio-cultural studies, as well as learning-related findings from long-established fields that have held true under modern day empirical scrutiny. In general, research in Learning Science suggests there are research-based approaches to acquainting learners with traditional subject matter concepts that make it possible for most people to construct a thorough understanding of these important concept, and thus, *learn with* *understanding* (Bransford, Brown, & Cocking, 2000, p. 8). As such, I used TPACK as a conceptual framework, seated within the theory of Learning Science, to provide an essential lens for describing curricular characteristics aimed at supporting primary children's potential to learn early algebra concepts with understanding, through multi-touch, mobile, iOS mathematics education apps. These specific characteristics are discussed in more detail in the Literature Review.

1.5 Significance of the Study

The findings from this study have multiple potential bearings on both practice and theory. As it relates to practice, one aim was that by examining the mathematics education apps for primary children, the otherwise latent TPACK knowledge, embedded within app-mediated curricula, eventually may be made visible to the everyday consumer (perhaps after refinement of the Coding Frame and its publication to a journal for consumers) . Likewise, my identification of characteristics of curricular components that support primary children's potential to learn early algebra concepts with understanding, through multi-touch, mobile, iOS mathematics education apps, provide descriptors that are specific, and thus relevant and useful, for the analyses of first-generation apps of this kind and the design of second-generation apps. Further, it remains my long-term vision that only after myself and other researchers outline the specifics for a wide and extensive range of content areas and learners (beyond the scope of this study), perhaps broader patterns can be realized and general rules can be abducted, in hopes of creating a broad set of curricular standards that support learning with understanding within this type of medium, across a range of content areas. Therefore, this study also marks an additional step toward responding to educational technology researchers' broad appeal to, "Create standards for products marketed as educational" (Shuler, 2007; 2012). As it relates to theory, this study contributes to the still-unfolding understanding of the TPACK framework. While all TPACK researchers may not share my interpretation of this framework's application, perhaps the Findings of this study contribute, in some way, to the larger discussion on interpreting and applying the TPACK model.

Of final note, qualitative content analysis (QCA) is a methodological approach that is used with regularity in European countries, such as Germany (Schreier, 2012), but is not yet widely used in the United States. This study not only increases the visibility of QCA within the U.S., and positions it as a viable and useful research method, it contributes to the development of the methodology itself. This is because, while QCA is most often applied to static texts, this study examines "temporal" texts. By "temporal", I mean texts that continue to unfold in real time. This compares with static texts that are produced and fixed. As such, I developed a tool within the course of this study that may help future QCA researchers analyze temporal texts more effectively (see *App Observation and Coding Frame*).

In summary, TPACK and its incorporated intersections are inherently embedded within an educational app a designer creates. These knowledge intersections are represented through the app's curriculum- the characteristics of which are informed by the designer's theoretical, conceptual, and philosophical frameworks. Even when incomplete, inaccurate, or devoid of high quality characteristics, this curriculum has the power to shape the way learners conceptualize ideas. Hence, particularly during a time when 75% of US children are spending an average of 43 minutes per day using mobile devices (Common Sense Media, 2013), nearly 40% of which claim to be educational (Shuler, 2012), it is important to continue examining their implicit curricular components. Unfortunately, the latent nature of embedded knowledge means the average consumer may find it difficult to identify the underlying guiding principles that inform an app-mediated curriculum; a situation Shuler (2012) describes as, "a long-standing issue in the educational toy and game industry and one (she) hopes can be tackled early in the evolution of the app market" (Shuler, 2012). Employing research findings from Learning Science, through the TPACK framework, provides a new means of systematically describing the alignment between current multi-touch, mobile, mathematics education iOS apps, and curricular characteristics that ideally support young children's capacity to learn early algebra concepts with understanding. Since apps are the largest growing activity related to device usage, describing this alignment is important (Judge, Floyd, & Jeffs, 2015). These aims continue to be essential, not only for understanding the general extent to which these current apps meet characteristics of curriculum that ideally support learning with understanding, but for imagining what curricular characteristics might look like when apps are being utilized to support this learning goal in transformative ways.

1.6 Operational Definitions

- *Early childhood education*- This discipline typically concerns the education of children, age's birth to eight-years-old.
- *Primary children*- Defined, here, as children ages six to eight years old. As such, the education of primary children is seen as a part of the broader field of early childhood education.
- *Curriculum*- Used, in this study, in a broad sense, to refer to the sum components found within a traditional early childhood curriculum; namely- the Learner, the Teacher's Role, the Learning Environment, "What" is Learned, and "How" it is Learned (with the exclusion, in this case, of the Family's Role).
- *App-mediated curriculum*- Used, in this study, to describe the sum components of a curriculum (described above), as they are expressed through the substance of an educational app.

- *Learning Science*- A contemporary discipline, which has emerged over the last 20+ years that is an amalgamation of learning-related findings in various disciplines, including neuroscience, cognitive psychology and socio-cultural studies, as well as learning-related findings from long-established fields that have held true under modern day empirical scrutiny.
- *Early Algebra*- A mathematical domain, typically introduced in primary and elementary classrooms, in which students learn about the relationships among quantities, use of symbols, modeling of phenomena, and the mathematical study of change (see NCTM, 2000). As discussed in Chapter 2, it is often treated as a domain of mathematics that is separate from other domains (e.g., number, arithmetic, measurement), but it can be (and should be) treated as an approach to domains, like arithmetic.

Chapter Two:

Literature Review

Despite the fact that educational apps offer considerable potential to serve as an alternative space for primary learners to, *learn with understanding* (Bransford, Brown, and Cocking, 2000, p.8), the affordances of multi-touch, mobile education apps have remained largely untapped (Hirsh-Pasek & Zosh et al., 2015; Shuler, 2012), as it relates to the support of similar goals. As such, outlining the particular curricular characteristics that support this aim is especially important. Hence, the aims of this study were to identify:

1. What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?
2. To what extent do three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

Accordingly, in this chapter I consider seminal and current research in Learning Science, surrounding the knowledge intersections outlined in the TPACK model. These intersections include the consideration of pedagogical knowledge (PK) as it relates to technological content knowledge (TCK) [PK+TCK], technological knowledge (TK) as it relates to pedagogical content knowledge (PCK) [TK+PCK], and Technological Pedagogical Content Knowledge (TPACK) as a uniquely situated knowledge set.

Therefore, in identifying studies for this Literature Review, I used five strategies to search for relevant literature (Galvan, 2009). First, I reviewed references from existing literature reviews related to research in teaching and learning mathematics, primary education, instructional design, learning with understanding, and characteristics of curricula. Second, I searched *Google Scholar, ERIC on EBSCO, Find It,* and *Education Full Text* databases for articles published in the last seventeen years (2000 and later), using the keywords “early childhood education”, “primary education”, “elementary education”, and “young children”, in conjunction with “math teaching”, “math learning”, “math curricula”, “mobile apps”, “apps”, “digital devices”, “technology”, and “curriculum”. Third, I reviewed reference lists of relevant articles (from the keyword search) and located specific articles in the University of South Florida library catalogs and virtual databases. Fourth, as themes emerged from my analysis of key works, I identified common aspects between literature works, and identified more literature related to those themes, such as “ meaning”, “knowledge construction”, “conceptual relations” and “understanding” (see Galvan, 2009, p. 87). As a result of this search, I have limited my analysis of studies from the last seventeen years to those focused on concept-centered learning in primary mathematics and early algebra. Additionally, classic or seminal studies are also considered in these sections. Finally, there are a number of sources, used here, to which I was exposed during my doctoral coursework. This literature, primarily those related to aesthetic and artistic inquiry, the Reggio Emilia “approach” to early childhood education, information and communication technology, and mathematics pedagogy, are also a part of my Review.

Each section of my literature analysis contains both background information gleaned from the literature review, as a well as an analysis of empirical studies and theoretical papers, published in peer-reviewed professional journals. However, this review does not attempt to cover discussion of play, technology usage, or mathematics learning on a broad scale.

2.1 A Changing World

In a world that is becoming progressively more technical, the number of people who need to practice mathematics with proficiency, as well as the number of people who need to deeply understand mathematical concepts, is rapidly climbing (NCTM, 2000; NRC, 2001). Mathematical literacy is increasingly required in the personal life of the average citizen- in the workplace, in decision-making within a modern democratic society, and within everyday existence (NCTM, 2000, p. 4).

In the workplace, mathematics is frequently intertwined with technology. It is present in workplace systems, techniques, and employees' cognitive competencies (Wedege, 2010, p. 89)- though it may be difficult to conceptually untangle from its production-centric context (Wake, 2014, p. 272). Further, the presence of mathematics in the workplace is increasing. Not only is the number of highly-technical jobs growing (e.g., cyber security and digital communication, medicine, engineering)- each of which requires more than basic arithmetic- but historically low-tech occupations (e.g., manual labor) continue to be infiltrated by algebraic, geometric, and statistical mathematics in the form of technology, accountability, and forecasting (Hodgen & Marks, 2013; Wedege, 2010). For example, as Hodgen and Marks (2013) note, "An increased focus on efficiency measures have resulted in mathematical application and understanding becoming an essential skill for all people in the workplace, even in relatively unskilled jobs [sic]" (p. 4). The need for mental mathematics, including estimation, approximation and proportional reasoning, graphical representations, data collection and interpretation, and geometric measurement are commonplace (Hodgen & Marks, 2013, p. 7). Applying mathematics in complex situations, such as statistical modeling and numeric data analysis, also is becoming more frequent, as it helps workers and companies avoid costly (or even deadly) mistakes.

In everyday life, mathematical concepts form the backbone of daily routine, as well as specific projects. While sometimes these concepts are hidden, as within the digital coding behind most contemporary household appliances and devices, other times they peek out through familiar activities, like cooking, shopping, assembling items, or planning logistics. On a wider scale within contemporary global-American society, a person's ability to make sense of information and news by applying discipline-specific knowledge and literacy strategies to construct understanding or respond critically supports essential democratic debate about vital technological, economic, and environmental issues (Yore, Pimm, & Tuan, 2007). Thus, understanding the ways mathematically literate people behave, within all of these contexts, is important.

Yet, the idea of literacy, in general, can be difficult to unpack. Within some disciplines, literacy seems tied to a person's familiarity with subject matter and his or her competent performance in discipline-based problem solving. Other times, it also seems to include hallmarks of acculturation, as might be seen in a person's ability to critique the credibility of subject-dependent methods or findings, ask relevant questions, propose empirically-based explanations, and use disciplinary ideas in everyday life (AAAS, 1993). According to Merriam-Webster (2015), literacy also includes a person's expression of "lucid(ity)" and "polish" (www. merriam-webster.com), which implies a certain practiced or, even, artful eloquence in the sharing of disciplinary ideas. Furthermore, Yore, Pimm, and Tuan (2007) note the importance of neither "overlook(ing) (n)or underemphasize(ing) the fundamental *literacy* component of (disciplinary) literacy" (p. 559) [emphasis added]. In other words, these researchers denote accentuating a person's ability to use certain information communication technology (ICT) strategies, discipline-specific language, and habits of mind to critically analyze information and render meaning. Hence, definitions of disciplinary literacy are vast and vague.

While defining literacy as it relates to each discipline is complex, disciplinary literacy- particularly in mathematics and science- is "embraced worldwide as a worthy education goal, even though there is no consensus (on its meaning)…"(McEneaney, 2003, p. 218). Scientific and mathematical literacy, in particular, seem to be cherished above other forms of disciplinary literacy. The reason for this is difficult to untangle, but in American education, its roots likely took hold as a result of Sputnik and frustration during the 1950s "space race" (see Asher, 2003, p. 199). Consequently, this embrace has resulted in considerable attention and analysis from educational researchers, over the last 50+ years.

To explicate the meaning of disciplinary literacy in terms of educational aims, numerous researchers have focused on the intellectual behaviors of experts within and across various fields (see Bransford, Franks, Vye, & Sherwood, 1989; deGroot, 1965; Glaser & Chi, 1988). Perhaps, this focus is due to the idea that experts are typically regarded as those persons "most literate" in their respective fields, and thus, can offer the highest benchmarks toward which learners might aspire. Or perhaps, attention is due to the wider recognition that thinking effectively about problems in various fields is key to developing solutions for thriving and surviving on local, national, and global levels. In any case, the cognitive and dispositional behaviors of experts have been a focus of educational researchers over the last two decades, and in the words of Bransford, Brown, and Cocking (2000), these experts "provide an important model of successful learning" (p.48).

In conclusion, the nature of the time period and society in which Americans currently live seems to require literate citizens (see Bruning, Schraw, Norby, 2011). Disciplinary literacy, within a specific subject like mathematics and across subjects as it relates to democratic citizenship, seems a lofty but essential goal. Contemporary citizens should be erudite in a range of key subjects, for securing safety and jobs, for preserving cultural heritage, and for individual decision-making (NCTM, 2000; NRC, 2001).

2.2 Successful Learning

While numerous educational benchmarks (e.g., autonomy, creativity) contribute to the overall characterization of successful learning, Bransford, Brown, and Cocking (2000) posit one of the primary benefactors is *learning with understanding* (p. 8). To modern ears this may sound like a basic stipulation of the formal education process, but deep understanding was not always the goal of education- in formal schooling or otherwise. At one time, learning to write one's name was the aim and definition of functional literacy (Resnick & Resnick, 1977). Likewise, prior to World War I, memorizing familiar passages from classical texts comprised language arts (Wolf, 1988) and executing the barebones of arithmetic computation satisfied goals in mathematics (National Research Council, 2001). Now, a certain degree of conceptual understanding seems an essential requirement for functioning effectively in a modern, global society.

Yet, in many contemporary public school classrooms, this key provision is often displaced in a teacher's scramble to meet federal, state, and district mandates, while simultaneously managing the clamber caused by "routine conditions of classroom life" (Kennedy, 2005, p. 2). Teachers experience time constraints, ever-changing reform agendas, contrasting personal and professional beliefs, constant interruptions, and increasing administrative requirements. These elements typically do not align with the curricular characteristics that ideally support a child's ability to learn with understanding. Learning with understanding takes time, emotive engagement, and a certain degree of sustained thought (Bransford, Brown, and Cocking, 2000; Kennedy, 2005). Moreover, this displacement, arguably, has more impact on learners in the early grades of elementary school. This is because, despite unfolding beliefs of young children's capabilities as sophisticated learners- releasing them from one set of constraints- the playful contexts, in which children often learn best and which were once a part of primary classrooms, have given way to increased top-down mandates related to measures of increased accountability. In short, contemporary mandates for increased accountability have collided with ever-present "routine conditions…" (Kennedy, 2005, p. 2). This combination has resulted, effectively, in a new set of constraints under which primary children must attempt to learn with understanding; a seeming shuffle of one set of flawed expectations for another.

2.2.1 Shifting expectations for primary learners.

2.2.1.1 Release from past cognitive constraints.

Over the last twenty to thirty years, research in Learning Science has led to revolutionary insights into the complicated act of educating people of all ages. Learning Science, as a discipline, is a compilation of findings from learning-related research in a variety of emerging fields, such as neuroscience, cognitive psychology and socio-cultural studies, as well as learning-related findings from long-established fields that have held true under modern day empirical scrutiny. Bransford, Brown, and Cocking (2000), perhaps most notably, delineate these ideas in their seminal work, *How People Learn*.

Among the most profound ideas to materialize from Learning Science are the realizations that, (a) Understanding, or *the construction of meaning* (Prawat, 1996), is created and re-created on the foundation of existing knowledge and understanding (Piaget, 1978; Vygotsky, 1978) and, (b) Learners are active constructors of knowledge from birth (Bruner, 1972; Carey & Gelman, 1991; Gardner, 1991; Gelman & Brown, 1986; Wellman & Gelman, 1992). Such ideas have challenged long-standing conceptions in the field of early childhood education, in particular, by providing deeper (and sometimes new) understanding of the ways learning, in general, takes place. For example, Learning Science theory supports the notion that, although children's cognitive capabilities are not unlimited (neither are anyone's), their thinking is decidedly more sophisticated than previously believed (Bruning, Schraw, Norby, 2011); a position supported by multiple, empirical research evidence from a broad variety of disciplines that had previously been excluded from educational theories, or were otherwise non-existent.

Two such (relatively) recent bits of evidence are that, (a) Children have an innate understanding of physical causality, biology, narrative and number from birth (Carey & Gelman, 1991) and, (b) Children have the capacity to think in sophisticated and abstract ways- although their conceptions may not always be accurate (Wellman, 1990). Another comparatively recent research outcome is that, while older learners tend to perform better on tests of memory (interpreted in the past as evidence of increased cognitive capacity with age), their success is likely attributed to their awareness of, and experience with, specific knowledge and strategies that help them make better use of their brain (Bransford, Brown, & Cocking, 2000, p. 96). In other words, the more experience one has had with the task of memorizing, the higher likelihood one has to become a better memorizer. This is because people tend to utilize cognitive strategies that make a task easier (by reducing *cognitive load*) [see van Merriënboer & Sweller, 2005] such as “chunking” information (see Miller, 1956) or “rehearsing” it (see Belmont & Butterfield, 1971), as they gain experience in *that* task. This means, as a person is cognitively engaged in a task or problem situation, their mind is busy, not only constructing and reconstructing ideas related to the solution or achievement of the task, but also becoming more efficient in and adept at the construction process itself.

Relatedly, neuroscience research findings show the quality of the learning experience within which one is exposed to a particular set of concepts, rather than the nature of the concepts themselves, is most significant in how the brain organizes the information (Bransford, Brown, & Cocking, 2000, p. 118). Specifically, quality of a learning experience is defined, in this case, in terms of relative cognitive complexity and emotional engagement. If the learner actively engages in a cognitive task (particularly one that is sophisticated), such as creating a physical model of a house, this leads to the formation of many neural synapses in the area of the brain associated with the content at hand. Thus, in this example, synapses form in, both, the “model building” area of the brain and the area of the brain storing information on residential structures. Additionally, synapses form between the two. The more synapses formed between these areas, and within each of these areas, the easier it is for the learner to understand related information. Thus, a certain emotive engagement must accompany cognitive engagement, and in fact, some note the cognitive cannot exist without the emotive. If the learner in this example, instead, listens to her teacher lecture on residential structures (a decidedly more passive and less complex cognitive-emotive task), fewer synapses are formed in the area of the brain storing information on residential structures, none in the “model building” section, and none between the two. More synapses make learning more effortless. In short, engagement in active, complex cognitive tasks within a particular concept area begets easier learning within the same area.

In yet another set of Learning Science findings, socio-cultural research shows the cultural and/or situational context(s) in which a person constructs the meaning of a particular idea, are of paramount influence to the ways that person perceives related concepts (Rogoff, 1999; 2003). As such, knowledge is seen as *situated* (Lave and Wenger, 1991). In part, this means a person's knowledge is tied to the specific cultural tools that were present during knowledge construction, such as materials, concepts, activities, and orientations provided by the informal curricular components (Rogoff, 2003). Thus, experience with these tools, both in and out of school, make it easier for learners to learn more easily. This speaks directly of young learners who share a different cultural context than formal schooling. These ideas also relate to the "growth mindset". The growth mindset is outlined by the recognition that intelligence in people of all ages is "elastic" instead of fixed (Dweck, 1989; Dweck & Legget, 1988), and that particular habits of mind are a matter of acculturation.

Thus, much of what was once presumed to be the age-related cognitive incapability (deficit) of a young learner is now viewed as the *differing* understanding of a learner who has had fewer experiences with, and often divergent from, the situations, activities, materials, tasks, concepts, language, orientations, and learning strategies valued in most formal schooling contexts and on standardized assessments (Delpit, 2006; Rogoff, 2003). A young learner, in most cases, possesses and utilizes the same sophisticated ability to make sense of the world and his or her place in it, which he or she has utilized since birth- regardless of formal schooling educators' interpretation of this ability.

This view of young learners is vastly different from that of the recent past. Perhaps this is because, just as the idea of early childhood education was emerging in its own right in the mid-twentieth century, Developmental Stage Theory (DST) was the primary conceptual lens through which learners were viewed- particularly in America. While DST is founded on the idea of constructivism, which has been a conceptual seedling since the beginning of Piaget's work in the 1920s, it also holds that children's information-processing capacities (e.g., short-term memory, metacognition) increase with age and general life experience. Thus, educators who embrace DST acknowledge children's active construction of knowledge, but expect children to consistently and equivalently improve across domains of learning as they mature. Hence, these educators believe that children's construction of meaning is not significantly influenced by specific instructional approaches, and see learning as synonymous with development, and development with learning (Bransford, Brown, & Cocking, 2000, pp. 95-96).

Since research in Learning Science shows most children execute sophisticated and abstract thinking from birth, and findings suggest the primary limitation of young children's (or most novices') learning is likely lack of experience with particular situations and/or school-aligned learning tools and contexts, chronological maturation seems a shaky premise upon which to predominantly base learning expectations and outcomes. Yet, components of DST are still enmeshed in the public's general conception of young learners today, as well as a number of educators and researchers. Its theoretical premise can be seen in the ways a number of educators measure children's progress and plan their learning experiences. Ideas such as " 'basics' first" or "children must learn in 'concrete' ways", still linger in classrooms, contemporary curriculum guides, and teacher education materials (e.g., Charlesworth & Lind, 2010)- even though "concreteness" has been clarified to mean one's *conceptual relationship* to an object (i.e., the intellectual distance between learner and object of learning), instead of the property of an object (Wilensky, 1990; also see Bers, 2008), and the idea of "(basic) skills first, concepts later" (Bruning, Schraw, & Norby, 2011, p. 314) has been uncovered as a faulty premise (see Carraher, Schliemann, Brizuela, & Earnest, 2006). Such lingering beliefs tend to restrict curricular aims (see Metz, 1995) and tend to define "child appropriate" learning experiences too narrowly, superficially, and universally.

In my experience, American educators and researchers, largely, have come to embrace a refined and expanded sense of young children's cognitive capabilities. However, there remains some ambiguity as to how to apply this new outlook toward the design of practical learning experiences for primary children. One example of uncertainty is whether the early childhood education process is usefully approached with or without predefined learning objectives. In this case, I use the term "learning objectives" to describe a list of declarative, procedural, and dispositional knowledge, in addition to performance skills, which a group of people (i.e., typically educational experts) has deemed essential for learners to know and do. Hypothetically, learning objectives reflect the fundamental knowledge, skills, and mindsets of those who are literate within a discipline. There is debate, however, because many learning objectives are currently organized according to universally-applied trajectories that outline what children should know about various disciplines, at various points in time- according to chronological maturation and/or grade level.

Putatively, learning objectives differ from developmental trajectories because they describe *what* learners should theoretically and ideally realize about a discipline, but not *how* they should get there. However, both predefined objectives and developmental trajectories tend to be framed by a declaration of *when* a learner should meet a benchmark. It is this sense of "when" that is troublesome, as such timelines tend to be applied in universal ways that hearken to DST. Thus, by one view, predefined learning objectives are acceptable when they are not strictly sequenced or organized according to the chronological age or grade level of the learner.

In this way, alone, they might serve in a capacity that supports the educational goal of learning with understanding. Additionally, since learning objectives often represent what might be described as “minimal requirements”- with the intent being that learning can go beyond the suggested items- ideally, they are used as a springboard for curriculum design (e.g., Common Core State Standards Initiative, 2009). In other words, the intention is that contemporary learning experiences will meet outlined learning objectives, but that these experiences are in no way bound by this goal, alone.

Alternatively, the “Reggio Emilia approach” in Italy, sees predefined learning objectives as restrictive to children’s learning. “Reggio” is widely known and respected as an exemplar of effective practice in early childhood education (Edwards, Gandini, & Foremann, 2012). While its philosophy did not grow out of Learning Science, many pedagogical practices of Reggio are comparable to those based in Learning Science, because both approaches proceed from the pedagogical standpoint of child competency and the epistemological stance that children have a right to literacy across disciplines. Both approaches also draw from some of the same theoretical foundations.

Reggio schools seem to excel at guiding children’s learning without predefined learning objectives. Its educational model functions with a small team of educators for each child, and the educational team is small enough to encourage shared agreement among educators. As such, collectively, these team members are able to hold in mind concepts that are important to know as it relates to various disciplines, while agreeing upon ways to engage a child in individually-driven learning that is emergent and adaptive (Gandini, 2012). As such, this approach also supports the educational goal of learning with understanding.

Thus, whether utilizing predefined learning objectives that have been sequestered from their chronological trajectories, or using emergent learning objectives that have reached shared consensus among team members, either practice might benefit primary children's learning with understanding. From my stance as an American educator, given the U.S.'s history of inequity in education (e.g., Kozol, 1991; Spring, 2010), I do not see the use of predefined learning objectives as a negative proposition (although, certainly not all American educators agree). However, even with my proposed acceptance of predefined learning objectives, I maintain it is important to avoid thinking of them in terms of developmental or chronological timelines. In divorcing proposed learning objectives from their developmental and/or chronological models, or by using emergent learning objectives as Reggio does, educators support young learners' release, at least in part, from a set of beliefs that have historically limited their learning.

2.2.1.2 Replacement constraints.

Despite a refined sense of children's learning capabilities, another set of constraints seems to have appeared to take its place. The elimination of playful contexts in formal schooling, through which most children prefer to learn, has cast a new set of shackles on children's propensity to learn with understanding- especially, in the early primary grades. Play is long-since acknowledged as essential for learners of all ages (if it is divorced from developmental models). Play is important for learning with understanding because, (a) Understanding requires cognitive perseverance, and a person is far more likely to persevere when motivated by playful engagement and, (b) Play creates support for emerging understandings (Bodrova & Leong, 2007, pp. 131-132).

Regardless of these findings, play is still seen as a waste of time by those who are unaware of, or otherwise dismiss, the science behind it (Bodrova & Leong, 2007, p. 129). Thus, despite efforts, over the last several decades, to raise awareness of the value of play (Edwards, 2013), those in charge of circumscribing measures of increased accountability in formal schooling have defined work, or “time on-task”, in ways that exclude play. In practice, this means primary learners (defined here as six to eight year olds) have far fewer opportunities for socializing, and freely moving about the classroom, and little to no opportunity to build structures and engage in dramatic play. This move has, not only, further delegitimized play as a powerful context for learning (Edwards, 2013)- particularly, in the eyes of the public- but also has greatly reduced primary learners’ inclination to learn with understanding.

Equally, superficial and time-consuming mandates take up much time in formal schooling contexts. Contemporary learning standards (e.g., Common Core State Standard Initiative, 2009; NCTM, 2001), most of which were designed to serve as sources for learning objectives that support disciplinary literacy, can be misused. While many standards display objectives organized according to grade level, school district-mandated curricular calendars (i.e., “Scope and Sequence” syllabi) exacerbate a focus on universal developmental trajectories by requiring young learners to demonstrate “mastery” of outlined learning objectives, along specific and rapid timelines. In my personal experience, some school districts go as far as requiring all teachers at a certain grade level, across the school district, to be on a particular page in a specific textbook, on a particular day and time. Furthermore, it is often compulsory for teachers to use district-adopted, pre-packaged curricula to help students reach learning benchmarks by the requisite date. This is problematic for a number of reasons, not the least of which is, adoption of these curricula (e.g., textbooks, software programs) by the school district is a highly politically driven process (Giroux, 2010). As such, the textbooks and software are merely interpretations of politics. In the words of Koehler and Mishra (2008), “…Greater emphasis should be placed on *teachers* as 'curriculum designer'" (p.3, emphasis added), instead of their need to rely upon imposed curricula. Thus, even the most motivated teachers, often, have lost their power and autonomy to design lessons for their own students; much less, detach proposed objectives from the sources or models from which they came, beforehand.

In brief, in contemporary US formal schooling (particularly in the public sector) short-term goals for learning tend to supersede long-term goals. Longstanding concerns (e.g., Public opinion, political reelection, perceived economic implications) and current preoccupations (e.g., Accountability measures) fixate on the short-term. In many ways, these short-term goals are in direct opposition to goals like learning with understanding, which is widely acknowledged to require time, patience, and a balance of persistence and flexibility (Bransford, Brown, & Cocking, 2000). Despite occasional pledges to the contrary, contemporary Americans still seem preoccupied with quick progress (Asher, 2003). As such, the end goal of disciplinary literacy, with its primary underpinning of learning with understanding, is easily lost to stopgap measures.

Unfortunately, formal schooling contexts may not shake loose from current accountability measures for some time, and they may never be free from excessive political, social, or economic influence. Consequently, the ways in which expectations for early learners are leveraged in the primary classroom (and may remain so for the near future) are troublesome to educators like myself who perceive young learners as capable of sophisticated thinking and interactions, but who also value learners' rights to the construction of deep understanding through playful means. Thus, while children stand to benefit from greater validation of their evolving status as serious scholars, short-term exigencies change the way education is enacted in formal schooling, and consequently, limit those benefits. Although this general phenomenon is not new, the most significant recent effect- the disappearance of play from the early gradeshinders children’s inclination to learn with understanding, despite higher expectations of their cognitive abilities. Consequently, young children may be no closer to learning with understanding in contemporary formal schooling contexts than they were previously.

In my analysis, incongruities between short-term and long-term educational goals in most public, primary classrooms (and other formal schooling spaces) play a more powerful role in inhibiting young children’s ability to learn with understanding than a failure to return to frameworks of “developmental appropriate(ness)” (Bredekamp and Copple, 1986/1996/2009) or developmental stage theory. Thus, providing less restrictive learning expectations, seated within playful contexts, seems most beneficial for the education of young children (Edwards, 2013). Finding ways for young learners to work toward disciplinary literacy while immersed in engaging experiences seems key to their learning with understanding. Further, utilizing platforms or learning spaces that can support these means and ways, without the constraint of (or with fewer constraints of) excessive political influence that seem inherent in formal schooling contexts, is also key. The first step, however, is better discerning what learning with understanding means within educational practice.

2.2.2 Learning with understanding.

Learning with understanding comprises a number of key characteristics that rely upon, (a) The differentiation between knowledge and understanding and, (b) The ability to unpack what is meant by the term “to understand”, which is loaded with varied connotations. According to Tomlinson and McTighe (2006), the major difference between knowing and understanding is that knowing is “binary”, whereas understanding is “more a matter of degree” (p.65). Hence, a person either knows something or they do not, but their understanding can be categorized as any value between extremely limited and exhaustive. Additionally, in many cases, a person’s degree of understanding is continuously evolving, which is not the case with knowing (unless one counts the acts of forgetting and remembering as evolutionary).

The term "to understand" implies a variety of meanings. Meanings range from a person's ability to *use* one's understanding, to an ability to *reflect on* one's own understanding, to an ability to *empathize* with another's view or "understand" a situation from a certain perspective (Tomlinson and McTighe, 2006). In fact, "to understand" is used in such diverse ways in the English language some researchers and educators suggest avoiding the term, altogether, when defining learning goals. Alternatively, one group of researchers, Wiggins and McTighe (1998; 2005), note that instead of sidestepping the phrase, its varied meanings can help to formulate major indicators of understanding, which can be used to evaluate the approximate extent of a learner's comprehension.

Wiggins and McTighe (2005) define these indicators of understanding in their book, *Understanding by Design* (UbD). This book outlines a theory by the same name, and in collaboration with Tomlinson's *Differentiated Instruction* (DI) theory (1999), has resulted in the *UbD/DI* approach to education. Consequently, Tomlinson and McTighe's (2006) book, *Integrating Differentiated Instruction + Understanding by Design* (2006) is a well-cited reference for educators seeking to design curricula in ways that help all learners work toward learning with understanding. UbD/DI, as an approach to curriculum planning, denotes ways an educator might infuse Learning Science principles into the contemporary curriculum, while also respecting the culture of each learner.

As it relates to learning with understanding, UbD/DI defines understanding along six facets that can be observed in learners. These facets include: apply, empathy, perspective, explain, interpret, and self-knowledge (Wiggins & McTighe, 1998; 2005). These indicators represent the idea that understanding is broader than a learner's comprehension of a concept's consensus-driven meaning. It involves other ways of understanding beyond cognitive comprehension, as well as other kinds of meaning beyond those that are highly consensus-driven. These ways include, but are not limited to, the six facets of understanding (listed above), and help to explicate the possible breadth of components to consider when aiming for helping a learner enlarge his or her understanding.

Another key definition of understanding comes from researchers, Bransford, Brown, and Cocking (2000). They define learning with understanding as, both, the degree to which meaning is constructed (Bransford & Stein, 1993) and one's ability to apply what has been learned in one circumstance, to new conditions and within new contexts (e.g., Byrnes, 1996; Morris, Bransford, and Franks, 1977). As such, these researchers posit learning with understanding is a complicated act that requires time for learners to build and refine meaning (Bransford, Brown, & Cocking, 2000, p.8). They note the complexity of such an aim must be acknowledged, before it has a chance of being realized.

While Bransford, Brown, and Cocking (2000) and UbD/DI (2006) both proceed from a Learning Science framework, their definitions of understanding vary somewhat from one another. In my mind, they both contribute to a working definition of learning with understanding, but their definitions required synthesis. Therefore, in order to reconcile these two definitions, I examined their similarities and differences, and abstracted a set of three characteristics that I believe captures the elements of both.

The first characteristic comes directly from Bransford, Brown, and Cocking (2000)- "meaning construction". The second characteristic, "externalizing meaning", was a synthesis of ideas found in both sources. It emerged as a result of my realization that applying meaning within new contexts and under new conditions (as expressed by both sets of researchers and sometimes called *transfer* [Byrnes, 1996])- whether through the creation of product outcomes or participation in specific actions- was essential in two ways. First, (a) It assists the learner with further construction of meaning and, (b) It provides opportunity for learners to come to realize that transfer is the ultimate practical aim of learning.

The third characteristic of understanding surfaced from my acknowledgment that it is important for a learner to make his or her meaning (which is personal and internal) “available” for application in external contexts, and specifically, to new contexts beyond those faced in initial meaning construction. In this way, meaning must be existing, accessible, useable, and/or useful for *transfer* (Byrnes, 1996). As such, there are certain key conditions that must be present to make transfer possible. Thus, the third characteristic, “neural organization”, describes the *conditions* of meaning construction and the ways in which meaning is cognitively organized in order to support its externalization.

In this project, I use all three elements to define understanding and to frame it in practical terms. I reason that to “possess” and express understanding, meaning must be both internally constructed and applied externally to new contexts. However, the application process is mediated by the use-ability and usefulness of that internal meaning, just as subsequent revisions to that meaning are mediated by the use-ability and usefulness of applications. Thus, given the interrelatedness of these characteristics (which are distinct from one another only in an academic sense), it is important to include all three elements of understanding when considering how to help primary learners learn in this way.

2.2.3 Purpose of learning experiences.

A learning experience refers to the educative activities, environment, and programming with which a learner engages. When an educator's goal is to help the learners in his or her charge to learn with understanding, there are two primary purposes for the educator to provide learning experiences. These are, (a) To support learners' construction of meaning and, (b) To provide learners externalization of meaning- either to support further meaning construction or to transfer meaning to new contexts of application. These are very different purposes than those of, (a) Satisfying content requirements and, (b) Evaluating knowledge and skills acquisition. Since this latter set of aims seems to be the primary driver in providing learning experiences in contemporary, American, primary classrooms, it is important to understand the difference in rationale behind the two sets.

2.2.3.1 Rationale.

While there is nothing wrong with an educator satisfying content requirements and developing a sense for a learner's current acquisition of knowledge and skills, doing so cannot be the primary motivator of curriculum design or the main descriptor in defining the extent of the learning program. Unfortunately, in many contemporary, American, primary classrooms, such a rationale seems embedded within the provision of learning experiences. In practice, the implications are such that, once the learner has demonstrated his or her understanding or has met a requirement, to the extent outlined by a specific mandate (see *Extent*, below), the learner's understanding of the concept is deemed complete. In the mind of the educator, the requirement has been satisfied, and the learner can "move on" to the next learning experience provided, to meet the next requirement. This approach is known as "content coverage" (Bransford, Brown, & Cocking, 2000, p. 20). While there are many potential reasons for this (e.g., limited time), particularly within formal schooling contexts, its effects are worrisome. As, too, are curriculum design rationales based on evaluation.

One effect of these two common motivators is that the "program of learning experiences" that results typically lacks the characteristics essential to supporting learning with understanding. For example, minimal time is typically allotted for engagement with a single concept, which in turn, limits depth and breadth of the learner's meaning construction. Additionally, contextualization is positioned as superfluous and long-term encoding is not the focus. Instead, when the primary motives behind providing learning experiences are to support learners in the construction and externalization of meaning across various contexts and under various conditions, the effect is to preserve opportunities to include the characteristics of a learning experience (and program of learning experiences), essential to learning with understanding. Therefore, the difference between these two motives directly affects the nature of the learning experiences to which learners are exposed.

When a learning experience is focused on meeting a requirement, understanding is framed as knowledge acquisition, and the nature of knowledge acquisition centers on obtaining a set of facts and skills (Bruning, Schraw, & Norby, 2011). Contrastingly, learning with understanding inspires more than a "one and done" mentality, or the "one shot" approach described by Bransford, Brown, and Cocking (2000, pp.65-66). Understanding is extensive and multi-faceted, and while it is an ongoing lifelong process, even within the confines of the formal education process, it includes far more than a simple procurement of facts and skills. The same is true of learning experiences designed from the perspective of evaluation. Since evaluation is focused upon an outsider's (i.e., usually an educator's) assignment of value to the learner's cognitive status, the primary concern is for the learning experience to encompass easily measureable actions and outcomes, and result in an abundance of documentation or evidence to justify the appointed value. Learning experiences that are easily measureable, or that result in the amounts and kinds of documentation or justifiable evidence expected within contemporary evaluation-based models, however, do not always effectively support the goal of learning with understanding. Learning with understanding relies upon characteristics, such as elaborating on concepts by exploring and explaining how ideas may be affected by various conditions. In short, these two rationales mark the difference in attitude between, (a) Trying to meet the minimum, instead of aiming for all that is comfortably possible for the learner and, (b) Making the act of evaluation easily accessible, convenient, and documentable for the educator instead of making the acts of meaning construction and transfer accessible to the learner. As such, the choice between these two rationales can affect the extent to which learners may expand their understanding.

2.2.3.2 Extent and boundary.

One challenge of learning with understanding is that, because it is an ongoing process, no single set of learning experiences- no matter how extensive- can provide complete satisfaction of this goal. In fact, possibly, learning with understanding can never be truly satisfied, and may be marked by a lifelong commitment to learning. As such, when designing learning experiences, curriculum designers need to provide, both, a boundary for the program of learning experiences (i.e., they cannot supply infinite learning experiences), and the extent of fulfillment as it relates to each learning objective (e.g., represent key algebraic symbols *through three distinct media*).

While there is no clear marker for either, McTighe and Wiggins (2004) suggest that certain actions and outcomes, whether internal or external in nature, might be hallmarks of breadth of understanding (e.g., empathizing with others, relating to various perspectives, self-knowledge, interpretation). Hence, as it relates to the program of experiences, various suggestions for types of actions, outcomes, conditions, contexts, and phases related to breadth and depth of meaning construction and transfer (beyond those suggested by McTighe and Wiggins, 2004) might delineate a broad sense of boundary. Likewise, predefined learning objectives (e.g., As found in formal learning standards or within research publications) can add to this sense of boundary by providing a list of disciplinary concepts for primary learners. Liberating these objectives and concepts from rigid timelines allows the list of possible concepts, with which young learners might engage, to become more expansive.

As it relates to the fulfillment of each learning objective, “mastery” has been a term recently used to describe this extent. However, questions have arisen related to, (a) Whether “mastery” is a fitting term to use and, (b) Whether satisfaction of a few verbs outlined by learning objectives (e.g., Name three plant parts) would be enough to qualify as mastery, in any case. Research suggests a negative answer to both. Since, the sources from which predefined learning objectives are commonly derived (e.g., Formal learning standards) do not always allude to the specific extent of fulfillment for each objective or concept, it is important to compare individual learning objectives and concepts with key characteristics related to depth and breadth of meaning construction. In this way, extent of “satisfaction” for each concept informs, in part, the boundary of the program. Together, these aspects provide a sense of what it means for a learner to satisfy all that might be comfortably possible for him or her to understand about a concept, at a given point in time.

While the rationale and extent of learning experiences may be simply realigned with the goal of learning with understanding, the nature of the learning experience, itself, can be more difficult to calibrate to this aim. This is because, while learning can be supported through such experiences, “No one can give (understanding) to anyone else” (Carpenter, Fennema, Franke, Levi, & Empson, 1999). It is, therefore, important for educators to design a program of learning experiences that aims for learners’ meaning construction.

2.2.4 Constructing meaning.

2.2.4.1 What is meaning?

“Meaning” is a multi-faceted term that refers to a person’s cognitive grasp of a concept’s substance- its essence, its significance, its conditions of applicability, and in some cases, its connotation and nuance (Merriam-Webster, 2017). When a learner constructs a concept’s meaning, she builds mental representations of that concept in long-term memory- in various forms, such as words, numbers, images, action clips (e.g., “mental movies”), sensations (e.g., body memory), sounds, smells, tastes, and emotions (e.g., “linguistic” imprints like, humor or spiritual awe)- all of which refer to and symbolize a particular notion. By mental representations, I refer to a learner’s schema or “Mental frameworks (people) use to organize knowledge” (Bruning, Schraw, & Norby, 2011, p. 6). These representations may include, but are not limited to, a concept’s definitions, synonyms and analogies, its nature as captured in supporting principles or theories, the emotion it evokes, its detailed substance, the individual features, functions, and behaviors that make it unique, and the features, functions, and behaviors it shares with other concepts.

These mental representations might be in the forms of many modes and “languages”. By mode, I mean a type of sensory arrangement (e.g., visual, auditory, tactile). In this case, by “language” I refer to genres of symbolic representation, “including the expressive, communicative, symbolic, cognitive, ethical, metaphorical, logical, imaginative, and relational” (Reggio Children, 2010, p. 4)- the emotional imprints of which, often linger. Potentially, this will result in a learner "knowing more about" a concept, by knowing more features that define it. Additionally, these features will become connected to other concepts to form a kind of web of understanding, with meaning as chunks of mental representations connected to one another. As such, these "Abstract representations become a part of larger, related events or schemata" (Bransford, Brown, & Cocking, 2000, p. 65). This becomes important, particularly when it comes time to transfer meaning to new contexts.

2.2.4.2 What is construction?

Constructing something generally refers to the action of "making". In this case, the thing being made is of a conceptual nature- as in a learner's meaning-making or construction of meaning. Such construction connotes an active-minded endeavor, in which the learner is doing the work.

To begin the construction process, a learner's related prior knowledge must be awakened. Sometimes, a learner sees an object or hears a word that sparks a memory or sensation within their schema and, subsequently, this automatically awakens their existing knowledge. Other times, awakening requires more effort or an explicit cue to activate prior knowledge. Once prior knowledge is activated, there is an opportunity for conceptual change to occur. Broadly speaking, conceptual change refers to "the major reorganization in memory of the conceptual framework for a domain of knowledge" (Bruning, Schraw, & Norby, 2011, p. 361). This process entails comparing and contrasting new information with existing information. This results in either conceptual reinforcement or conceptual change.

Conceptual change sounds straightforward. However, conceptual change is not always a smooth progression, nor is it a passive process for the learner. This is because at the heart of conceptual change is the act of conversion. A learner must be convinced to adopt a new point of view or refine an existing conception. In order for this to occur, several factors need to be met. First, the learner must doubt or question his or her existing knowledge. Second, the new or revised concept must carry weight. Third, the learner must be convinced the new or revised conception offers more value than his or her existing belief (Bruning, Schraw, & Norby, 2011).

Perhaps most importantly in the construction process is that a learner has the chance to cognitively "wrestle" with a problem or build knowledge for him or herself (with coaching or other forms of scaffolding available as needed), and with limited direct instruction beforehand.

2.2.5 Depth and breadth of construction.

"Depth" and "breadth" of meaning construction are analogical descriptors. When I use these terms I refer to the expansion and refinement of a learner's understanding, in ways that move beyond the confinement of the traditionally accepted "ladder model" (Tomlinson & McTighe, 2006, p. 119). The ladder model represents the faulty view that learners who have not mastered "basic" facts cannot apply or understand more abstract ideas [sic]. In my view, it also serves as an unsound metaphor for grade level progression. As a more fitting metaphor, McTighe and Wiggins (2004) suggest the analogy of a "web", in which various indicators of understanding are honored equally and are described through the pictorial representation of connected breadth. Using the web as a model for the learning process communicates the idea there are plenty of opportunities for learners to extend understanding, with is no rung to "climb" to the next level, but also no ceiling to keep a learner down . Instead, engagement with concepts, as they relate to each indicator of understanding, can be discussed with reference to another metaphor- the learning cycle.

2.2.5.1 The learning cycle.

According to one general understanding, the process of education can be pictured as a continuous cycle of learning and assessment. As such, the learning cycle is a design model that describes a learning "sequence" based on a series of experiential phases, through which the learner can "move" (Bredekamp & Rosegrant, 1992; Bybee, 1997; 2002; National Academy of Sciences, 1998). However, this process has been described in various ways, according to different disciplines. In the early childhood field, it is described in terms of four phases (Bredekamp & Rosegrant, 1992); namely, *awareness*, *exploration*, *inquiry*, and *utilization*- with each phase of learning intended to help a child investigate an idea with progressively greater depth. In yet another definition, the learning cycle is described in terms of inquiry-based learning, which emerged within the field of science education- though its components may be relevant to a wide variety of fields. This model is known as the *5Es* (Bybee, 1997/ 2002; National Academy of Sciences, 1998), and its five phases include: *engage*, *explore*, *explain*, *elaborate*, and *evaluate*. Since both models proceed from the viewpoint of constructivism, the general premise is that when a curriculum planner explicitly plans for learners to spend time in each phase of the learning cycle, they are more likely to construct meaning for him or herself, with depth and focus.

Thus, the learning cycle represents a theoretical structure around which curriculum planners can design learning experiences that help learners move through multiple types of learning experiences over time, for the purpose of revisiting concepts multiple times and from multiple conceptual angles (e.g., The "web" of various indicators of understanding). In general, the learning cycle is seen as a process through which a teacher can help a learner move his or her thinking through phases that may help the child construct knowledge. In this way, one might qualify it as the framework upon which a series of learning experiences are molded. Among other things, this provides sustained focus and opportunities for the learner to investigate numerous examples of the same concept at work. Thus, the “phases” within the cycle are not developmental stages. Each phase represents an allotted conceptual space in which learners are provided with opportunities to construct meaning. Thus, when educators plan learning experiences according to each phase of the learning cycle, they provide learners with the conceptual space and time to construct meaning, while avoiding *cognitive overload* (see van Merriënboer & Sweller, 2005). While cognitive overload is discussed in more detail, below (see *Cognitive-emotive balance*), briefly this negative effect is said to occur when a combination of high-level demands are taking place. This series of learning experiences, designed according to the suggested phases of the cycle, offer the learner multiple opportunities for the construction and refinement of concepts.

Both, the early childhood (EC) learning cycle and 5Es learning cycle overlap in many ways. However, there are some differences, and the phases from each model are not an exact comparison with one another. There are elements of Exploration (EC cycle), for instance, that share descriptive qualities with Engage and Explore (5Es cycle). Likewise, there are elements of Inquiry (EC cycle) that share descriptive qualities with Explore and Explain (5Es). Additionally, there is no equivalency with Evaluate within the Early Childhood model. Another difference is that the descriptors within the early childhood learning cycle are fairly broad, while the 5Es model is more specific. Because of these differences, I have merged the characteristics of the two into one table (see Table 1.).

In general, the phases and descriptions I accept tend to draw more heavily from the 5Es model. Perhaps this is because the 5Es model also suits my goal of divorcing curricular characteristics from platforms like developmental stage theory, the subtle aspects of which are represented in the early childhood (EC) model. However, there are modifications I have made to the merged learning cycle, and of those changes there is one primary component I have adopted from the EC model. That is, the absence of a phase related to summative evaluation.

I have made other changes to the merged learning cycle, as well. First, I have placed less emphasis on "Awareness", given that children are naturally aware of their surroundings and attempt to make sense of them from birth onward.

Table 1. Merged learning cycles.

Phase of Learning Cycle	Characteristics
Engage/ Awareness.	Initiates learning task; Makes connections between prior and current experiences; Cognitive engages learner in concepts of focus; Introductions to new objects, people, and events; Provoke interest through a problem or question for learner.
Explore/ Exploration.	Opportunities for learner to test ideas against new conditions, contexts, and others'; Promotes exploration of environment and manipulation of materials; Extend play through scaffolding; Respect learner's rule systems and ways of thinking, and allow for constructive error.
Explain/ Inquiry.	Opportunities for learner explanations; Introduction of formal language and disciplinary-based terms; Help learner refine understanding via answering more focused questions; Help learner make conceptual connections.
Elaborate/ Utilize.	Opportunities for learner to extend or apply concepts under new conditions and within new contexts; Chances for learner to deepen and broaden meaning; Real world application.

Table 1. (Continued)

Evaluate.	I have renamed this last phase, "Cumulative Project" (see my translation, below).

The table above represents a combination of, both, Bredekamp and Rosegrant (1992), "Cycle of learning and teaching". *From Reaching Potentials: Appropriate Curriculum and Assessment for Young Children, Vol. 1*, p. 33 and Bybee (2002); National Academy of Sciences (1998), *The 5E Model of Science Instruction*, according to my interpretation of how the two might be merged for this project.

Thus, I reasoned Awareness does not require its own phase, distinct from the Engagement phase. Second, in addition to removing "Evaluate" as a descriptor for the fifth phase, I renamed the fifth phase "Cumulative Project" (the reasoning behind which is explained in more detail, below).

Third, I added "reflection and assessment" to each phase of the revised cycle to remind educators these acts should happen throughout the learning process. Fourth, I expanded the "Elaborate" phase to provide "room for" the various facets of understanding with which the learner will invariably engage (as mentioned briefly in the "web" analogy, above and as discussed in detail, below). The expansion of the Elaborate phase also helps to emphasize how extensive this phase is, when it accommodates characteristics of learning with understanding, such as breadth of meaning construction. Fifth, I added several sets of arrows and an important note to the revised learning cycle. The arrows refer to the need for learners to revisit the phases of "Explore" and "Explain" when new conditions, contexts, and facets are introduced in the Elaborate phase. These arrows represent the acts of revisiting and revising both meaning and outcomes. This helps learners to deepen and broaden meaning construction by offering opportunities to do and redo meaning across multiple facets of understanding.

Likewise, individual learners may "move through" the cycle at different rates. Some learners may quickly arrive at the "Elaborate" phase, while others may linger in the "Explore" or

“Explain” phases for a while. Each learner’s movement may also change according to various concepts. Additionally, the note explains the learning cycle need not always be applied sequentially. While there needs to be general coherence in the way concepts are introduced, the learning cycle need not be applied rigidly, and educators may choose to place learners in the heart of the “Explain” phase before formally visiting the “Engage” and “Explore” phases (these additional changes can be seen in Figure 15).

While helping a learner move through the learning cycle is one way to assist him or her in increasing the degree to which meaning is constructed, this outcome is not automatic. Nor is it the only way to expand and refine meaning construction. However, in this study the revised learning cycle I adapted represents a combination of the early childhood learning cycle and the 5Es learning cycle, and it serves as a primary informant in defining curricular characteristics that support learning with understanding.

2.2.5.2 The "Six Facets of Understanding".

Wiggins and McTighe’s (1998/ 2005) *Six Facets of Understanding* also serve as one set of several that inform the way breadth of meaning construction is outlined within this study. In general, designing learning experiences that account for these facets help learners expand their breadth of meaning, and consequently, their extent of true understanding. Since understanding is described by a number of researchers as multi-faceted, it is important to envision each facet of understanding as indispensable to the whole of the construction process. This is because each time the learner must accommodate or construct a different facet of understanding through engagement in a learning experience, the potential for him or her to construct more meaning and more connections between various meanings, is compounded. Hence, the accommodation of various facets of understanding (as well as other indicators) within the “Elaborate” phase of the learning cycle is not an optional luxury when it comes to learning with understanding.

As such, it is important to consider the qualities of each facet of understanding, and how each might relate to the whole of a learning program. Wiggins and McTighe’s (1998/ 2005) *Six Facets of Understanding* include: explain, interpret, perspective, empathy, self-knowledge, and apply (p. 67). It is important to note that each facet may be realized in a number of ways- namely, external outcomes, internal outcomes, or learner actions. Below, I examine each Facet, in turn.

2.2.5.2.1 Explain.

The fact that, according to Wiggins and McTighe (1998/ 2005), “explain(ing)” represents one of six facets of understanding, and one of only two major acts of understanding (i.e., applying and explaining), shows that its weight is substantial. Wiggins and McTighe describe “explain” as, the act of communicating principles and generalizations, providing orderly and justified accounts of data, facts, and phenomena, providing illustrations and examples, and making perceptive connections (p. 67). The general implication of explaining is that, in some part or combination, the significance, essence, conditions of applicability, and nuance of a concept (i.e., The substance of its more consensus-driven meaning), as well as less consensus-driven meanings (some of a highly personal nature), will be explained to others and/or to one’s self through learning experiences that encompass external outcomes (e.g., work products), actions (e.g. translating), or a combination. For example, the task of building a home for the class guinea pig might serve as a visual representation that can be used to explain the essence of the pet’s needs.

2.2.5.2.2 Interpret.

Wiggins and McTighe (1998/ 2005) also suggest “interpret(ing)” as an additional facet of understanding. Wiggins and McTighe describe interpretation as, “offering fitting translations, telling meaningful stories, making concepts accessible and personal to one’s self through analogies, models, images and anecdotes, and providing a personal or historical component to ideas or events” (p. 67). In practice, learners might construct new mental models of translations, analogies, models, and images, and/or they might share these meanings through external outcomes and actions. Interpretation is involved in a range of actions, such as solving, designing, building, or collecting. However, its focus rests in verbs, like translate, interpret, reframe, redesign, and decipher. Using the example above, the task of building a home for the class guinea pig would shift to something like, “using the same measurements for area, redesign a home for the guinea pig with features that will make the house more enjoyable for the guinea pig”. While the emphasis here is on the learner’s personal interpretation of “enjoyable features” (e.g., slides, a maze, a treat dispenser), the task of carrying out the original assignment (with accuracy) remains an underlying requirement. If both tasks were to be provided by the teacher (separately, across multiple lessons), learners would have the opportunity to increase his or her depth of understanding related to mathematical concepts (among others), by “doing meaning” in more than one way. This is because, often, when the act of redesign or translation occurs, the original product with its conceptual underpinnings requires renegotiation (Forman, 1994). Original constructions of meaning have the opportunity to be dismissed, reaffirmed, or modified, as learners attempt to fulfill similar requirements across varied circumstances. This description also illustrates the value of trafficking across the “web” in order to engage with various facets of understanding (and other elements of breadth of construction).

2.2.5.2.3 Perspective.

Similar to interpretation, Wiggins and McTighe (1998/ 2005) suggest "hav(ing) perspective" is another facet of understanding (p. 67). They define this facet as perceiving the "big picture", as well as, "filtering what one sees and hears through a critical lens" (p. 67). As this facet fits within the construction of meaning through learning experiences, verbs such as judge, prioritize, assume, imagine, and evaluate affect the processes and outcomes of a task. Thus, a learner might be said to have perspective if he or she was able to account for the viewpoint of another or consider multiple physical or conceptual angles, during the process of his or her designing, building, collecting, measuring, modeling or calculating and/or the process of his or her translating, redesigning, or deciphering. Likewise, a learner might be said to have perspective through an external outcome, like as product that captures the viewpoint of another, or acknowledges multiple physical or conceptual angles. Referring to the established example, the redesign of the guinea pig's cardboard home might disregard features a human would find enjoyable (e.g., a pool), and instead, use the given area to provide features a small rodent might enjoy (e.g., separate spaces for feeding and sleeping; ramps, levers, and buttons the pet can reach and operate). Cultivating perspective is reliant upon one's ability to, both literally and metaphorically, vary one's perspective. Yet, despite this variance in view, gaining perspective does not always result in empathy toward the other view-holder.

2.2.5.2.4 Empathy.

In many ways, empathy is tied closely to perspective. While varying one's perspective does not always result in feeling empathy, feeling empathy (genuinely) is reliant upon one's ability to vary his or her perspective. Wiggins and McTighe (1998/ 2005) describe having "empathy" as "find(ing) value in what others might find odd, alien, or implausible" (p.67). As it relates to learning experiences, actions such as relating, regarding, embodying, and perceiving may be of focus. Cultivating empathy is typically perceived to be an internal outcome, but it may also be expressed in external outcomes. Using the standing example of the class guinea pig, a learner might display empathy by designing the "enjoyable features" of the cardboard home in a way that reflects a balance of fun, health, and safety, out of concern for the pet's wellbeing. Or, a learner might revise his or her design multiple times, after testing whether the pet seems comfortable navigating the space.

2.2.5.2.5 Self-knowledge.

Wiggins and McTighe (1998/ 2005) describe "hav(ing) self-knowledge" as have self-awareness of one's own personal prejudices, habits, and styles that shape understanding (p.67). Having self-knowledge is typically regarded as an internal outcome. As such, arguably, self-knowledge is most effectively externalized by communicating about it, although certain actions can also express self-knowledge. Actions such as reflect (aloud), be aware of, and realize spring to mind. As it relates to the guinea pig project, self-knowledge may be expressed by a learner's reflection on a particular phase of the creative process that was troublesome to enact, and consideration of how he or she overcame the challenge.

2.2.5.2.6 Apply.

The name of this facet is somewhat confusing, because of diverse definitions of the word "apply". In this case, Wiggins and McTighe (1998/ 2005) define "appl(ication)" as using and adapting what is known in various, authentic contexts (p. 67). Further, Wiggins and McTighe (1998/ 2005) describe this facet as "do(ing) the subject" (p. 67). Often, this means direct performance or a specific use of a concept is implied. For instance, a task in which learners must design and build a cardboard home for the class guinea pig that meets certain parameters of area, based on learners' knowledge of multiplication arrays (a second grade learning standard [Common Core State Standards Initiative, 2012]). In this standing example, while various groups of learners would certainly offer different interpretations of a guinea pig home that meets the established requirements, in this version of the task, the emphasis is on the application (i.e., the accuracy of the solution and fulfillment of the challenge).

These six facets of understanding inform descriptors related to the breadth of meaning construction. However, while these facets provide some suggestions, when considering other definitions of understanding in the literature, more elements emerge. Within this study, these elements include, (a) Ideas related to mathematics and early algebra understanding (see The five mathematical facets, under 2.4 Early Algebra), (b) Ideas related to trafficking and translating across various modes, "languages", media, and contexts (see Varied media, modes,"languages", and contexts, below) and, (c) Ideas related to various conditions of applicability related to a concept (see Conditions of applicability, under Aspects of meaning, below).

Even given these additional examples, it is not easy to determine how many facets should be included in outlining breadth, or what range of facets constitutes "enough" diversity for a learner to expand meaning sufficiently, during a particular point in time. Tomlinson and McTighe (2006) suggest the boundary of sufficient provision is defined by the quantity of learning experiences that will likely result in a "preponderance of evidence", across several facets of understanding (p.63). While the notion of "evidence" is based on an evaluative model of curriculum design, Tomlinson and McTighe's reasoning provides some general guidance for non-evaluative curriculum design, as well. Thus, in the case of this study, I define the boundaries of facet provision in terms of, the quantity of learning experiences that will likely result in a preponderance of "opportunities" across many facets of understanding (as determined by all the sources above).

2.2.6 Co-construction of meaning.

One of the primary elements that set humans apart from other Great Apes is our propensity to share our knowledge with others, or teach others. Unsurprisingly, then, the nature of meaning construction is, in part, social. This is not to say meaning is not constructed within the individual (i.e., meaning is personal) or that construction cannot occur without others. However, collaboration with others can significantly assist learners with the construction process. While a learner's creation of (personal) meaning is the ultimate goal of meaning construction, collaborative settings exponentially assist with this individual meaning construction (Bodrova & Leong, 2007). This is because others' stories and ideas can become a part of a learner's personal schema, and others' ideas can awaken an idea not previously activated within the learner's mind. The collaborative construction process also can frame ideas in terms of a unique analogy offered by a group member, which can help the learner form new views. As such, the act of co-construction involves negotiating meanings, exchanging concepts, responding to others and provoking thought. Therefore, while not all learners prefer to work in groups, all should have the option to collaborate. However, brainstorming and ideas and working as an effective group member can be tricky at first. Explicit guidance in this area, especially for novice learners, is important. Perhaps this is why co-construction is helpful in the creation of knowledge. This distributes cognitive burden and introduces relationships more quickly.

2.2.7 Aspects of meaning.

2.2.7.1 Essence.

One aspect of meaning is a concept's essence. Constructing the essence of a concept depends upon a learner's formation of mental representations, and their storage in long-term memory. Essence is the "who", "what", "when", "where", "why", and "how" of a concept and details related to the concept, like operational definitions, features, behaviors, and functions that define it, synonyms, antonyms, and underlying principles. In short, the nature of a concept's essence is explanatory.

Some of the details that comprise a concept's essence are specific and some are more general. As such, learners typically move between the acts of constructing a concept's essence and abstracting the general principles underlying the concept. This is because the acts of construction and abstraction reinforce one another. The process of abstraction means to remove embedded meaning from its context or source, in order to construct a summary of a concept's important features. "Complete" construction of essence rarely precedes some degree of abstraction, and vice-versa. Many times, abstraction occurs when considering multiple cases. For example, a learner might summarize the essence of a concept in a couple phrases- each with a supporting detail (e.g., a fact, an example, a definition). Collectively, these phrases might represent the big idea of the concept. Likely, the learner is comparing each of the concept's essential phrases with one another to determine ways they relate, and abstracting a theme, big idea, or generality before returning to the act of constructing additional essential details. This process is continuously recurring.

2.2.7.2 Significance.

Another aspect of meaning is a concept's significance. Just as with the process of constructing a concept's essence, constructing significance depends upon a learner's formation of mental representations, and their storage in long-term memory. While the nature of essence tends to be explanatory, the nature of significance is comparative. A concept's significance refers to how its essence relates to one's self, the natural world, the human world, and other ideas.

Constructing significance means coming to realize how and why a concept relates to which aspects of life. Thus, in order to truly comprehend a concept's significance, the learner must know a bit about it. This involves reflecting on the essence of a concept and comparing it to the elements above. As it relates to neural organization, engaging in the process of reflection and comparison often leads to enlarging one's perspective, and thus, theoretically results in the learner building more interrelations among conceptual chunks. As such, a concept's significance may serve as a "big idea" (see Wiggins & McTighe, 2004) under which its essential meaning is stored in long-term memory.

2.2.7.3 Conditions of applicability.

Another aspect of meaning is a concept's "conditions of applicability" (Bransford, Brown, & Cocking, 2000, p. 43). By conditions of applicability I refer to when, where, and why to apply a concept in life, as well as to various contexts in which the concept might be used. While it is key for learners to grasp the essence and significance of a concept, comprehending the conditions under which the concept is applicable, is imperative to its ultimate usefulness. Therefore, it is important to acknowledge that conditionalization has implications for the kinds of meaning construction that takes place. By this, I mean the construction of condition-action pairs. Condition-action pairs are frequently thought of in terms of "If/Then…" statements.

Additionally, conditionalization has implications for the organization of this meaning and the role this organization will play in supporting later information retrieval (see *Neural Organization*). For example, a concept's conditions of applicability can be a kind of "big idea", under which details of meaning (e.g., the essence and/or significance) related to the concept might be "filed". Alternatively, the concept, itself, might be filed with various conditions of applicability (i.e., condition-action pairs) nested within it. Alternatively, some conditions might be connected to a specific problem type, which in turn, calls for the application of a particular concept and its related condition-action pairs. In short, the precise neural organization is less important than the presence of "condition-action pairs" (Bransford, Brown, & Cocking, 2000, p. 43) and the number of connections between ideas (see *Connections*, below). This is because there are multiple conditions under which a person might apply a learned concept, and just as many potential connections between ideas.

While, it is likely impossible to identify all conditions and connections, a learner's awareness of key conditions and circumstances is important. Transfer is highly related to understanding a concept's conditions of applicability, and one cannot find a solution to a problem without understanding the concepts that potentially might be part of the solution. As such, information needs to be *conditionalized* upon the potential contexts, in which it may be useful (see Simon, 1980; Glaser, 1992) to become truly "meaning-full".

Additionally, aspects of a culture- be it a sub-culture, a national culture, or a professional culture- provide different conditions under which a certain meaning is useful. As it relates to disciplinary literacy, this is certainly an aspect to consider and may inform additional facets of breadth of construction (see *The Six Facets of Understanding,* above).

2.2.7.4 Connotations and nuance.

In some cases, knowing the connotations or nuances of a concept is also an essential aspect of meaning. Distinctions related to special rules and exceptions to those rules, multiple meanings of wording, and inferences and implications associated with the concept can be an important aspect of meaning. Traditionally, these were topics with which young learners were thought to struggle. However, contemporary research findings suggest otherwise.

2.2.7.5 Personal situation.

The kinds of meanings described above denote aspects of understanding that are more consensus-driven. To an extent, this means the learner adopts meaning that is, in part, shared by others. However, all meaning is cognitively *situated* (Lave and Wenger, 1991) within the context of an individual's schema, and it is this fact that defines the act of true meaning construction. Hence, through the act of personal situation, a learner comes to "own" an idea for him or herself. By "owning" an idea, I refer to the notion that a learner has linked current concepts of focus to his or her own *prior knowledge* (Bruning, Schraw, & Norby, 2011).

Without such links, the learner may attempt to make sense of or memorize a new concept, but he or she will not fully grasp its substance without an analogical connection to previously known ideas. In other words, people see new ideas in terms of existing ideas, by establishing a cognitive relationship between the two ideas in the form of an analogy (Bransford, Brown, & Cocking, 2000). Thus, for the sake of education (in its most lasting sense), the only kind of meaning worth pursuing is personal meaning. This implies that the essence, significance, conditions of applicability, and the nuance of a concept (i.e., its substance [among other kinds of meaning]) must be framed in terms of the learner's existing understanding. In part, this process may happen involuntarily. However, if no related prior knowledge exists (there is typically some existing concept in the mind of the learner that can serve as an analogy- even if such a concept diverts from historically honored perspectives), or if a learner connects broad meaning to unrelated prior knowledge by creating an incomparable analogy, conceptions can be lost or misconceptions can be formed, respectively.

Fortunately, some sort of existing analogy (even if tenuous) usually exists. However, one challenge rests in the teacher's ability (or lack thereof) to recognize opportunities for analogy that are diverse in content (see Delpit, 2006). Yet, even beyond recognizing diverse cues of analogical potential, this very personal process can be tricky for an educator to "manage". On one hand, the creation of non-traditional analogies can be a hallmark of individual creativity- especially, if such correlations are justifiably comparable. On the other hand, part of an educator's aim is typically to assist a learner in making some specific and commonly held metaphorical connections; connections that eventually may prove to be essential for literacy within a discipline. As such, educators must be aware of the existing knowledge each learner potentially possesses about a concept, be aware of the connections each learner makes during the construction process, and the learning objective must include but not be limited to specific elements of concepts and particular connections between concepts that are key to the discipline (see *Neural Organization,* below). Accordingly, there are specific kinds of personally situated meaning to consider.

Table 2. Kinds of personal connection and "situated" meaning.

Kinds of Personal Connection & Situation
Specific kinds of big ideas or key themes: - Algebraic themes; - A class of mathematical problems;

Table 2. (Continued)

- A mathematical principle, rule, or operational property; - "Essential questions" (Wiggins & McTighe, 1998/2005); - an observation about the nature of mathematics.
Specific kinds of connections between concepts: - Details with various big ideas; big ideas with various details; - Ideas (in terms of key features) to other ideas within a disciplinary domain; - Ideas (in terms of key features) to other ideas across disciplinary domains, across the curriculum, and across individual learning experiences; - Concepts with skills; skills with concepts - Big ideas with other big ideas; - Details with other related details; e.g., How the structure of a concept is linked to its function or behavior.
Specific kinds of relationships between learner and concept: - relationship descriptors; label those types of relationships. By labeling relationships between concepts, this helps learners recognize patterns of useful information and help determine solutions to problems or pathways to solutions. e.g., "examples of", "features of", "possible solutions for", "analogies for"; - consensus-driven meanings with more personal meanings; - new concepts with existing concepts; -out-of-app experiences with in-app experiences; - conceptual proximity to an idea.
Specific reflections on meanings- in- the-moment: - current degree and summary of understanding; - current self-performance; - current "reading" of a learning situation.

2.2.8 External outcomes and actions.

Externalizing meaning through the creation of certain kinds of outcomes and products is important. This is because the act of translating internal outcomes (i.e., certain realizations and meanings) into an externalized form, (a) Conveys meaning to others, (b) Provides the learner with opportunities to cognitively "wrestle" with the challenge of translation (and hence further shapes the learner's own meaning) and, (c) Provides the learner with experience in applying ideas under new conditions (it generally acquaints him or her with the act of transfer).

Likewise, the externalization of certain processes, or the "undergoing" of particular actions is also important. These actions provide learners with similar benefits as above- particularly (a) and (c), but also provide another potential advantage. Depending upon its nature, the action may decrease the conceptual distance between the learner and the object or concept of focus (i.e., increase "concreteness"). Actions, such as representation, translation, and observation and/or perception tend to increase a learner's proximity to the concept of focus. Thus, specific tasks, like modeling and drawing, can reinforce internal meaning-making in ways some other actions cannot (e.g., completing a multiple choice worksheet). Perhaps unsurprisingly, then, not all actions are as beneficial in supporting the goal of learning with understanding.

Actions also provide conditions under which learners can "practice" transfer. The second major aspect of learning with understanding relates to a learner's ability to apply what has been learned in one circumstance, to new conditions and within new contexts (i.e., transfer) [e.g., Byrnes, 1996; Morris, Bransford, and Franks, 1977]. While there is no guarantee transfer of a specific act (required at a later time) will have been practiced in formal education (in fact, the chances are unlikely), opportunities for the learner to experience *the purpose and general nature of* transfer help to familiarize him or her with this goal.

The nature of transfer is very specific and there is no way to provide learners with practice in all potential situation types (an educator's ability to do so would eliminate the phenomenon of transfer). Real life is ill defined and successful transfer is marked by a person's ability to accommodate an infinite variety of unknowns. None-the-less, "practicing" transfer by engaging with concepts across multiple contexts, under multiple conditions, and toward multiple ends, not only helps the learner understand that transfer is the ultimate aim of meaning construction, but familiarizes him or her with the kinds of dispositions and general procedures that might be required when attempting to apply a concept under a new condition. Thus, as it relates to providing learners with these conditions, aspects of breadth of meaning (e.g., explain, interpret, empathize, problem solve) can provide guidance.

2.2.9 Neural organization.

What makes transfer possible is the process of *encoding* (Miller, 1956). Encoding is the process by which information is transferred from one's short-term memory to one's long-term memory (Bruning, Schraw, & Norby, 2011, p. 363) and it is at the heart of the construction process. Without meaning construction and the encoding of that meaning, there would be nothing to transfer. There are several key components that are part of the encoding process. These include, storage of concepts and organization of concepts- with particular attention paid to connections between and among concepts, and mental representation, in various forms such as words, numbers, images, action clips (e.g., "mental movies"), sensations (e.g., body memory), sounds, smells, tastes, and emotions- all of which refer to and symbolize a particular notion. These representations may include, but are not limited to, a concept's definitions, messages, synonyms and analogies, its nature as captured in supporting principles or theories, its detailed substance, the individual features, functions, and behaviors that make it unique, and the features, functions, and behaviors it shares with other concepts. This serves as a repertoire of multi-modal, multi-linguistic, and multi-media representations related to one concept.

Teachers can assist learners in creating a mental "toolbox" of big ideas, and their related principles and concepts that might be applicable to certain situations. (Multiple chunks with some of the same concepts in each equate to flexible mental representations of concepts.) This also involves assisting learners in "reading" a situation, in order to recognize patterns of useful information or configurations and their implications, analyzing a situation by formulating reasoned interpretations or solutions, and identifying what makes a concept a "high-quality possibility" in a given situation. Additionally, it can be helpful to learners when teachers organize information into "chunks" of related information. This makes these ideas easier to encode in organized conceptual structures (i.e. schemata).

2.2.9.1 Storage.

Awakening prior knowledge is not the same thing as awakening long-term storage. It is not enough to form mental representation(s) of a concept- even if they are detailed and plentiful- if those representations are not lasting. As such, encoding, which is the process of storing these representations in long-term memory, is an implicit and essential part of the construction process. Triggering the response to store ideas, involves several elements. In part, this involves both a cognitive and social-emotional "availability" on the part of the learner. For example, *during* an education experience a learner might create images, sounds, words, mental movies, and many other mental representations of the idea of gravity. In that moment, she may appear to have constructed the essence of gravity- it's definition, analogies, and underlying theories. Yet, if neither content nor related learning experiences provide an affective stir, she will not encode this essence, long-term. This is because the brain is most likely to encode representations, long-term, when some part of the learning experience is aesthetic or emotive. As such, memorizing is not the same as encoding, as the former may or may not engage the social-emotional domain. For example, an opportunity to investigate instances in which one might apply a concept, within the context of a simulation, is much more likely to trigger the social-emotional domain than memorizing a list of instances in which others think one should apply a concept. In contrast, storing mental representations in mind, long-term always requires emotional or affective engagement, and this is most likely to occur through a learner's direct experience with, observation of, and reflection upon an idea.

Knowing what to store, then, is dependent upon cognitive attentiveness and direction of attention. Knowing what to store also involves the act of abstracting the general features of a concept, over many instances. While having more conceptual chunks in memory and more relations or features defining each chunk, is important for expanding understanding, possessing an abundance of information is moot if the concepts are poorly organized. Thus, while affective engagement might trigger long-term storage, and attention, direction, and abstraction might help the learner decide what to store, the brain must decide where and how to store this meaning- lest relevant information will never be found or useful.

2.2.9.2 Organization.

Deciding where to store meaning and how to arrange it is not always a conscious process. Often the brain decides where and how to organize concepts subconsciously, and thus, a learner may not even be cognizant of this element. Organization, however, does inform the efficiency with which a learner retrieves the information. As discussed previously (see *Conditions of Applicability*, above), information needs to be *conditionalized* upon the potential contexts, in which it may be useful (Simon, 1980; Glaser, 1992) to become truly “meaning-full”.

This amounts to a kind of cognitive filing. It is hoped this filing results in clustering ideas into related units, governed by underlying concepts and principles. Hence, usually smaller details (e.g., facts, examples, quotes, images) are "filed under" bigger ideas. Typically, these big ideas are the essential and significant features of a concept, but they can take a variety of forms. Examples of big ideas may be a concept's conditions of applicability or dispositional attitudes and mindsets that can help a person complete an act of application or see it as useful to life.

2.2.9.3 Connections.

The word "connection" can be ambiguous and diversely used. In this context, it typically refers to conceptual connections between ideas. Even then, such conceptual connections can refer to multiple acts and aspects of the learning process. Connecting smaller concepts to larger ideas, in order to answer life's big questions about the world and one's place in it, connotes one type of connection. Forming interrelations among chunks by actively seeking and creating connections between ideas is another. Awakening prior knowledge to connect new information with the existing, through analogical thinking, in order to create personally derived representations *and* meanings that are commonly shared, is another.

Sometimes an educator must make explicit connections between concepts that highlight relationships between ideas. Although this is often required as an inherent part of engaging learners in learning experiences, conceptual connections are often not made explicit. In other words, learners may be writing as they record scientific observations in a lab notebook, but they may not perceive the parallel between writing in language arts and writing in other subjects, such as science. By drawing learners' attention to the comparison, learners come to see writing as an applicable and relevant endeavor. Other times, learners inherently perceive features or patterns across examples that can lead to the abstraction of rules, conjectures, or big ideas. Hence, the construction and abstraction process often leads to the learner building his or her own explicit connections.

2.2.9.4 Recall, retrieval, interpretation, analysis, and selection.

The processes of recalling and retrieving information, for the purpose of applying the information within new contexts, have traditionally been described in terms of efficiency and automaticity. Automaticity is one way to engage in efficient recall and retrieval, and thus, one way to reduce cognitive load. For example, instantly knowing 5 x 7 = 35 is much more efficient than needing to recalculate the answer anew each time, and thus, reduces the load on one's working memory. Bransford, Brown, and Cocking (2000) denote, a learner's familiarity with acts of applying meaning during the learning process may *remind the learner of the need* to reference her mental "toolbox" of related concepts when she is faced with applications in the real world (p. 43). In other words, although a learner's broad familiarity with various acts of application does not typically assist him or her in retrieving a specific chunk of information associated with a particular application task, this familiarity can remind the learner that all problems of application can be approached calmly and systematically by referring to the cognitive toolbox she has built related to that task. This equates to a kind of dispositional automaticity.

Yet, despite ease of recall through automaticity, learning with understanding requires more than just the simple recollection of facts. Thus, another way to reduce cognitive load is described by Bransford, Brown, and Cocking (2000), who define this process in terms of fluency and accessibility. They denote fluent recall and accessible retrieval are reliant upon organized mental "toolboxes of meaning" (p. 43). As such, instead of memorizing facts, this process involves interpreting or "reading" a situation. Reading a situation means the learner can perceive patterns of useful information or configurations of data within context. It also implies a learner's ability to segment the perceptual field, develop sensitivity to potential patterns, and recognize problem types (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Once a situation has been interpreted, further analysis is required. This means attempting to understand the situation, in order to assess the implications of identified patterns of information. This is the point at which fluent recall and accessible retrieval come into play. Further, if the situation is a problem to be solved, analysis involves identifying a possible relevant and "high quality" solution. This involves knowing when certain laws may be useful in solving problems. Hence, "practicing" transfer helps learners weigh the relevancy of a concept to a situation. Potentially, it is also a reminder to the learner that he or she should try to understand the problem and reference what she knows about both the associated content and processes, instead of simply looking for surface features or plugging numbers into formulas (Bransford, Brown, & Cocking, p. 41). According to Bransford, Brown, and Cocking (2000), these processes help a learner find the most workable solution path (p. 38), when faced with a problem in which multiple concepts must be applied to form the solution, or when a number of ideas represent possible solutions and a learner must select the most fitting.

2.2.9.5 Reflection.

Practice applying concepts under new contexts and conditions (i.e., transfer) provides an opportunity for the learner to practice reflecting on one's own meaning construction and whether or not one has a sufficient amount of relevant information to make transfer possible. Processes of application raise questions of whether one has enough of the right kind of meaning to solve a problem or to practice an act, like sharing information with others. Likewise, acts of application also allow the learner to reflect upon and construct knowledge related to his or herself as a learner. This form of reflection is known as *metacognition* (Brown, 1975; Flavell, 1973). Metacognition is the act of being mindful of how one thinks. Ultimately, the end goal of metacognition is to develop self-knowledge and develop the ability to teach one's self. It implies developing a vocabulary related to one's self as a learner, reflecting on one's own growth, and setting goals. This involves reflection on the acts themselves, reflection one's ability to enact the act, and reflection on what one can do to improve his or her ability to enact the act.

2.2.10 Summary of successful learning.

In summary, successful learning is a tricky act, requiring the consideration of: shifting expectations, learning with understanding, the purpose of learning experiences, constructing meaning and its various aspects, depth and breadth of the construction process, and neural organization. It is more than just temporarily holding in mind superficial information. In the words of Bransford, Brown and Cocking (2000) it, "…means having more conceptual chunks in memory, more relations or features defining each chunk, more interrelations among the chunks, and efficient methods for retrieving the chunks" (p. 38; also see Chi, Feltovich, & Glaser, 1981). Nor is the construction process merely a matter of receiving information and remembering it. While "meaning" implies a connection between concept and self, and a connection between new and existing knowledge, "making" connotes creation- an active-minded endeavor. As such, meaning must be made for one's self, and it is understood that "No one can *give* knowledge (i.e., understanding) to anyone else" (Carpenter, Fennema, Franke, Levi, & Empson, 1999, pp. 99-100) [emphasis added]. Learning with understanding is a complex process that requires the learner to construct meaning, become familiar with certain applications of meaning, and do these things in ways that make reciprocal transfer possible. Bransford, Brown, and Cocking (2000) denote how the use-ability and usefulness of, both, meaning and application are augmented by the learner's construction of specific types of meaning, and are improved when certain traits of the construction process are present. Additionally, use-ability and usefulness are enhanced by a learner's participation in specific learning experiences related to communication and application, and by his or her development of specific skills, knowledge, and habits of mind that lubricate the reciprocal transfer process between meaning-making and application. *Transfer* (Byrnes, 1996, p. 74) between meaning and the acts of sharing and doing (and vice-versa), are less assured without the use-ability and usefulness of meaning as it relates to these experiences, and the use-ability and usefulness of sharing and doing as it relates to meaning-making. Thus, while each element of understanding is interrelated, explicit focus during the formal learning process, on each of these areas, is needed.

One thing is sure, deeply constructing meaning takes time (Bransford, Brown, & Cocking, 2000, p. 58) and requires use of conceptual spaces to engage with, explore, explain, and elaborate on ideas. Remaining faithful to such principles seems most probable when applied to curricular contexts in which short-term goals are less likely to influence, and long-term goals like learning with understanding, when properly understood as, (a) The construction of meaning, (b) The application of meaning and, (c) Characteristics that aid in transfer, are supported. When an educator creates learning experiences in which these ideas are a natural part of the environment and activities, these characteristics can *support* an individual's meaning-making.

2.3 The Curriculum

2.3.1 What is "curriculum"?

The term "curriculum" can mean many things, to many people. In my experience, people often view curriculum as "what should be learned"- with the "what" sometimes packaged as a unit of educational content and materials, available for purchase and implementation by educators and parents. Other times, people define curriculum in slightly more comprehensive terms, including "how" content is learned- primarily, pedagogical methods enacted by the educator and portrayed through the learning environment. Yet, other times, people envision curriculum, not as a set of components, but as *hidden* (Jackson, 1968) [or, sometimes, apparent] ideologies embedded within the curricular components (e.g., Martin, 1994). In its effect on learners, curriculum is likely all these things and more.

Traditionally, within the realm of early childhood education, there are several categories around which a curriculum is theoretically organized. In simplified terms, these categories include: the Learner, the Teacher's Role, the Family's Role, the Learning Environment, "What" is Learned, and "How" it is Learned (Bredekemp & Copple, 2009; Dodge, Colker, & Heroman, 2002). As such, curriculum may be defined in a more comprehensive way than in other educational fields- at least in terms of the range of distinct curricular components. Perhaps this is because, in America, during inception of the "early childhood curriculum", in the mid twentieth century, Developmental Stage Theory (DST) was the primary conceptual lens through which curriculum was developed, defined, and viewed (Brainerd, 2003). The teacher, who was perceived as one of knowledgeable authority, and the young learner, who was considered less cognitively developed (Bransford, Brown, & Cocking, 2000), were each thought to carry out distinct roles. As such, the teacher was responsible for most of the work associated with those roles.

Despite the fact that, in recent years, principles from the discipline of Learning Science have reshaped the way many early childhood educators view conceptual aspects of particular components, the curricular elements themselves appear largely unchanged- at least in the traditional classroom context. However, when one considers non-traditional curricular contexts (e.g., after school programs, museum exhibitions, app-mediated learning) components of the traditional early childhood curriculum, such as the Teacher's Role, Family's Role, the Learner, and the Learning Environment, as well as boundaries between those components, begin to crumble.

2.3.2 Non-traditional curricular contexts.

Non-traditional curricular contexts have always blurred the lines between distinct curricular components. However, at it relates to this project, this distortion is only useful if it helps to provide greater freedom from the constraints of formal schooling. Finding ways to work toward disciplinary literacy through platforms that can support learning with understanding, without the constraints of excessive political influence (and other powers that generate short-term goals) inherent in formal schooling contexts, is a must. This is because, despite one set of constraints on young learners fading, it has been replaced with narrow definitions of time on task (Bransford, Brown, & Cocking, 2000, pp.58-59). To this end, I propose within this project, the option of utilizing a curricular space for primary learners, separate from the confines of formal schooling. If those in charge of designing the curriculum within alternative spaces proceed from this orientation, separate contexts like museums, out-of-school programs, and digital media may offer a platform that provides the conceptual space to learn with understanding.

A number of platforms potentially meet this guideline. Museums, zoos, aquariums, nature preserves, historical centers, landmarks, and some parks, art centers, community centers, and libraries offer learning environments with exhibits, camps, tours, discussions and collaborative exchanges, interactive experiences, and access to educators and reference materials. When executed well, these places provide a positive learning experience, and in my experience, many provide playful contexts for learning. However, these spaces are limited in their capacity to reach a wide scope of primary learners. Historically, those learners that do attend are a demographically narrow group (primarily Euro-normative middle class), and sustained learning is typically limited to a day's visit.

Extracurricular and summer programs that are academic in nature offer another potential platform. One challenge, however, is that these programs vary considerably. Before and after-school programs and summer camps are sponsored by a wide variety of organizations, and as such, their aims also vary considerably- from the provision of basic daycare services to exposing learners to advanced robotics. Yet again, even with lofty goals, limited time and resources restrict potential.

Despite the fact that these platforms tend to provide playful contexts (and other positive qualities), such contexts are not accessible to a broad range and number of learners. Hence, beyond playful contexts, what is needed is mass accessibility. Fortuitously, educational apps provide this hypothetical accessibility to many learners (Hirsh-Pasek and Zosh et al., 2015). World wide, 33% of families have access to a functioning touch-based, mobile computing device (Shuler, 2012). In Western society, the numbers are as high as 75% (Common Sense Media, 2011) of the population that have mobile devices.

Hence, almost by default, multi-touch, mobile apps have secured a unique position in society as mutually educational and entertaining. They also seem especially fitting for young children because of certain physical characteristics of, and qualities found within, this type of device (e.g., Bers, 2008; Clements & Sarama, 2007). The app platform offers at least some freedom from the politics and economics that tend to weigh down the efforts of many formal school primary classrooms. Programmed curricular attitudes are another way the medium adds inherent value to the broad learning experience. Apps are even-handed in their treatment of *all* learners, and dignity and respect will be present, if programmed accordingly. Thus, educational apps have the capacity to serve in a unique position.

Yet, hypothetical capacity is not enough. Educational apps must provide curricula with characteristics that support learning with understanding, to justify their use as a platform for supplemental education. Unfortunately, a number of studies have shown first generation educational apps- many of which are still on the market, have fallen disconcertingly short of their potential, and in some cases of being considered "educational" at all- at least according to the primary tenets of Learning Science (see Hirsh-Pasek and Zosh et al., 2015; Shuler, 2012). In short, the inherent value of multi-touch, mobile educational apps are present, but the specific educational experiences, themselves, are lacking. In the words of Hirsh-Pasek and Zosh et al., (2015), "Apps present a significant opportunity for out-of-school, informal learning when designed in educationally appropriate ways…(but), only a handful of apps are designed with an eye toward how children actually learn" (pp. 4-5).

This lack raises questions about the characteristics of an educational app's curriculum that, potentially, make it supportive of helping primary children *learn with understanding* (Bransford, Brown, & Cocking, 2000, p.8). As such, it is essential to consider how curriculum designers can support learners in meeting the specific goals associated with learning with understanding.

2.3.3 Responsibilities of educational program design.

App-mediated learning demands significant responsibility of its programmatic designers. While there is no teacher standing in front of a classroom or acting as primary director of action, there is much "teaching"- in a redefined sense- made visible through programming decisions. The goal of helping learners learn with understanding is inherently present, and the platform is primed to support that goal. However, just as in other educative settings, a teacher's practices and her own understanding provide substantive framing for the ways in which her learners perceive concepts- from her use of particular metaphors (Lakoff & Johnson, 1980/ 2003), to her beliefs and attitudes about learners and the discipline at hand (Kagan, 1992). Thus, with the "education" label, comes great responsibility.

Even in educational situations that are less teacher-centric, a teacher's knowledge, attitudes, and professional practices affect the ways she documents learners' understanding and guides their thinking, the ways she prepares activities, provocations, and the physical environment, the ways she frames topics, the kinds of feedback she gives, and the ways she analyzes learning situations. Hence, by designing an educational app, an app designer necessarily agrees to fulfill a number of teaching responsibilities.

In the case of app-mediated curricula, this is not to say trained educators should be the only ones to engage in educational app program design. In fact, a convincing argument is easily made that designers who are not formally teacher-trained bring satiating pedagogical perspective, due to their freedom from the acculturation that typically accompanies the quintessential educator. Nor can one discount the inherent "learning potential" of apps that are designed for other purposes, such as entertainment. Yet, when a designer classifies an app as "educational", with her action comes particular consumer expectations; implicit prospects for a unique learning experience that, largely, has been unsubstantiated at the time of this writing (Hirsh-Pasek & Zosh et al., 2015; Ito, 2009; Shuler, 2012). In brief, these expectations reflect the notion that programs that are deemed educational reflect a certain degree of pedagogical *intentionality* (Epstein, 2006).

In traditional learning contexts, intentionality implies that an educator's approach to teaching and curriculum planning is thoughtful and deliberately aimed at providing high quality learning, based on her understanding of principles behind the art and science of teaching. For instance, a teacher is intentional when she selects a metaphorical entry point for introducing a concept that she knows might prevent learner misconception. As such, intentionality implies a teacher's curriculum design is solicitous, purposeful, and thorough as she attempts to anticipate how a particular practice, action, approach, interaction, or activity might influence the learner, based on personal knowledge of the learners in her charge and her understanding of contemporary research. Consequently, when teaching is approached with intentionality, there is a better chance learners will realize the goal of the program (Bredekamp & Copple, 2009; Epstein, 2006).

In the case of this project, the stated goal is to help learners learn with understanding, as they move toward increased disciplinary literacy. If the app-teacher-designer is going to aim for this goal, then certain considerations need to be undertaken. These include ideas such as, (a) "Learning with understanding" trumps evaluation (hence, assessment is "seamless" [Abell & Volkmann, 2006], and separate "assessment activities" designed for the sake of evaluation are moot), (b) "Learning with understanding" is supported by the provision of learning experiences, programming, and environmental considerations that are created with intentionality and, (c) Dignity is a given part of a learner's quest for understanding. As such, in this case, intentionality can be realized through the extent app programming aligns with the rationale, extent and boundary, and unique models that support learning with understanding. Hence, while it is understood that "No one can *give* knowledge (i.e., understanding) to anyone else" (Carpenter, Fennema, Franke, Levi, & Empson, 1999, pp. 99-100, emphasis added), it is important to note, others can *support* an individual's meaning-making and quest to learn with understanding.

Accordingly, the general attributes of supporting learners' learning with understanding can be linked to specific app-teacher-designer roles and the ways in which those roles are carried out.

2.3.4 App designer roles.

One role of the app designer is to provide opportunities for learners to experience specific actions and realize particular outcomes (whether internal or external) that support his or her effort to learn with understanding. These aims are enacted through the planning and provision of varied-structured learning experiences, and programming and environmental characteristics. With an eye toward this goal, a curriculum designer should plan learning experiences that provide opportunities for learners to construct meaning, over various days and over multiple types of activities, at each phase of the learning cycle, and provide opportunities to externalize meaning to reinforce meaning construction and increase familiarity with the idea of transfer (Byrnes, 1996, p. 74).

Another app designer role relates to assessment. While there are two primary purposes for an educator to provide learning experiences- namely, (a) To support learners' construction of meaning and, (b) To provide learners with opportunities to practice transfer- the provision of such experiences provides a useful effect. That is, the experiences and their outcomes (e.g., physical products, carrying-out actions) provide opportunities for teachers to assess the educational status of the learners in their care. By "assess", I mean to check a person's understanding. Assessment is important because it plays a dynamic part in the instruction process (Tomlinson & McTighe, 2006, p. 20). Ultimately, this allows characteristics of learning experiences to be personalized, or at the very least differentiated, to meet the learners' needs. It also means that while the learning objectives, themselves, are not generated emergently (i.e., developed in-the-moment), pathways can be largely individual, flexible, and learner selected.

However, above all, it is an adherence to the purpose behind curriculum design that demands the nature of assessment be “seamless”. “Seamless assessment is inseparable from regular instruction” (Abell & Volkmann, 2006, p.1). Likely, learners will already be solving problems and interpreting others’ views within the course of expanding meaning. In order to inform instruction, educators can analyze a learner’s actions and outcomes that are already occurring within the context of the learning experience. It is simply a matter of utilizing existing learning experiences for an additional purpose. Thus, opportunities for expanding meaning can also be used to assess a learner’s extent of understanding, when the assessment process is “seamlessly” approached. That is, assessment can occur seamlessly as learners externalize meaning for the sake of meaning construction and expansion of understanding. When assessment is not approached seamlessly or when assessment is undertaken for the purpose of evaluation, often, educators create and use distinct tests, quizzes, drills, and other activities that are distinct from the purpose of supporting a learner’s capacity to learn with understanding. While it would be difficult to program an app to perceive and “read” a child’s understanding as he or she expresses it through his or her virtual participation, assessment can be approached within the context of activities in which the learner is already engaged. The results of informally assessed activities are not shared with parents or the learner, other than in terms of the concept or skill the learner is currently developing. Thus, as it relates to app programming, the idea of seamless assessment is no less applicable.

2.3.5 In consideration of broad characteristics.

There are a number of broad characteristics the app designer should include within the learning experiences, environment, and program he or she provides to support a learner’s effort to learn with understanding. Since these characteristics, hypothetically, may affect a range of experiential, environmental, and programmatic provisions, they have been listed in a separate section, below.

2.3.5.1 Decontextualized and contextualized tasks.

A decontextualized task is an activity in which the problem to solve is the task, itself. The purpose of providing a learner with a decontextualized task, generally, is to allow the learner to intentionally focus on one concept, skill, process, act, medium, or types of knowledge (e.g., declarative, procedural or dispositional), at a time. Such an approach can be beneficial in assisting learners with meaning construction or refinement, by helping them avoid cognitive overload. Yet, decontextualized activities are not inherently constructivist in nature. An educator must intentionally design a learning experience to include the conceptual space for a learner to construct meaning, rather than simply “receive” another’s meaning. Thus, although worksheets are a kind of decontextualized activity, they generally do not support constructivist learning, and thus do not qualify as the type of decontextualized activity that supports learning with understanding.

A contextualized task is an activity in which the problem to solve is part of a larger task or context. The purpose of providing a learner with a contextualized task is to allow him or her to intentionally experience the whole context, in which specific concepts, skills, processes, acts, media, or types of knowledge are embedded within a physical and/or conceptual setting. This is beneficial because life is nearly always contextualized, and as such, learners have opportunity to learn the habits of mind essential to contextualized learning and application of ideas (e.g., perceiving patterns of useful information embedded in context). Yet, contextualized activities are not inherently constructivist in nature, either. As with decontextualized activities, an educator must intentionally design a learning experience to include the conceptual space for a learner to construct meaning. Hence, the provision of drills within "play" or playful settings qualifies as neither, (a) A constructivist approach (instead, it merely asks learners to "memorize or adopt" another's meaning in a passive sense) nor, (b) Is it what is meant by contextualization. This is particularly true within early childhood education. "Child-friendly" characters and settings, many of which draw upon clichés, are sometimes substituted for weighty contexts (see *Authenticity*, below). However, providing contextualized activities is not always straightforward.

There is a continuum of contextualized activity (Clough, 2006). At one end, is the highest form of context: "real life contribution". This is defined not only by participation in the modes of inquiry and practices of a field, but *also* by contributions toward the advancement of disciplinary knowledge. Some say this is the only "true" context- particularly as it relates the fields of science and mathematics education. At the other end of the spectrum, is lack of context (i.e., decontextualization). As discussed above, this is characterized by tasks such as, the completion of drills and the execution of raw calculations, and through isolated tasks of a more constructivist nature, like "black box" activities (see Lederman & Abd-El-Khalick, 1998). In between these two extremes are what I have come to think of as "scaled contexts". On the highly contextualized end, after "real life contribution", is "real life practice"- this is signified by participation in the modes of inquiry and practices of a field, without contributions toward the advancement of disciplinary knowledge. This is followed by "immersive environments"- such as simulations and virtual laboratories, in which learners can submerge him or herself in virtual reproductions of contexts and conditions found in the non-virtual world. In my mind, this is the point at which contextualization, for purposes of education, begins. This degree of contextualization is followed by "playful games"- typically, game-based drills or other decontextualized activities, couched in terms of play or games. While these kinds of "contexts" are often fun and may provide a greater aesthetic stir than typical drills, the degree of genuine contextualization they offer is fairly superficial. This is because, with each degree of decreased contextualization, tasks become less reflective of those enacted within real life disciplines. Hence, I consider neither “playful games”, nor the last degree- “decontextualization”, to provide educational context in a way that supports learning with understanding. While some degree of “fun contest” occurs within disciplines and everyday practices, the weight of such a context does not seem to strongly affect one’s ability to negotiate either a disciplinary field or everyday life. Likewise, while decontextualized activities, like raw skill or simple knowledge-execution, are a part of real life practice and disciplinary literacy, these tasks need not masquerade as anything other than what they are- one piece in helping learners construct and refine their understanding.

Decontextualized and contextualized learning experiences are both important, but each kind of provision contains common pitfalls. Within contextualized experiences, educators frequently assume learners have abstracted key ideas embedded in the contexts, without educators’ explicit reference to these ideas. Thus, educators must make key ideas explicit to learners by leading them to reflect, summarize, or otherwise focus on key ideas in context. If not, chances are ideas will remain embedded in context and important ideas will be lost to learners (Clough, 2006). As Tomlinson and McTighe (2006) put it, using the analogy of baseball, the coach (educators) must help the players (learners) to identify the concepts and skills, in this case batting, running, fielding, that make up the game.

Far more common in contemporary American education, is the presence of decontextualized learning experiences. The concern with this is two-fold. First, context isn’t a bonus. When spending most of one’s time concentrating on a part without knowing the larger whole. Second, not only are decontextualized experiences used too frequently, but in my experience these activities are very rarely constructivist in nature. Historically, the ways a teacher's role has been defined, as giver of knowledge, makes it easy to forget that meaning, in fact, is personally constructed. The habit of primarily relying upon direct instruction and the assignment of worksheets to complete is rampant and unfortunate. While these techniques are not always negative, for the most part, the nature of these tasks fails to provide learners with the conceptual space required for meaning construction. Additionally, mistakes have been made in assuming so-called cognitive "process skills" (e.g., observing, measuring) devoid of content and/or regardless of context, are helpful for learners to know and practice. In truth, procedural knowledge devoid of context and content is meaningless, and isolated process skills do not reflect the way professionals or everyday people behave in real life (Bybee, 2002; Clough, 2006). Thus, educators must help learners "keep their eye on the game", even when focused on developing specific skills or concepts (Tomlinson & McTighe, 2006, p. 120).

It is, therefore, important for educators to design a program of learning experiences that moves between decontextualized and contextualized tasks (Clough, 2006). To break the common habit of providing excessive decontextualized tasks, some educators prefer to begin with ideas in context, and resort to increasingly decontextualized tasks as necessary for scaffolding. Helping learners to reflect on contexts, in order to identify embedded concepts, aligns with a constructivist approach. However, there is no "right way". Beginning with a decontextualized task, if the task is constructivist in nature, can be equally helpful to learners' refinement of meaning. As such, some educators prefer to design tasks other than worksheets on which learners can focus their attention. Additionally, contextualization, or lack thereof, can be a part of cognitive support whereby learners are able to construct and refine meaning within degrees of contexts that are most helpful in scaffolding their expanded understanding.

2.3.5.2 Authenticity.

As it relates to learning, one component of authenticity is that it draws from real-life situations that occurred in the past, are occurring at present, or which represent a possible future occurrence. Another essential aspect of authenticity is that the modes of inquiry employed by learners in a learning activity reflect the modes of inquiry one might use in the real world. Examples of such modes include: interviewing, capturing a memoir, concept mapping, observation, physical sorting or manipulation of tactile objects, making a 2D or 3D model, creating a data table or another graphic organizer, teaching or sharing with others, summarizing through writing and/ or drawing, field work, literature review, scientific research, experiencing a simulation, exploring a concept or medium. As such, it is easy to qualify "real" contexts, acts, products, and processes as those acts found in particular professions or which are otherwise required in life (in fact, this may be part of the inspiration for using experts as educational models- they theoretically represent authenticity).

"Doing (a) subject" (Tomlinson & McTighe, 2006) in an authentic way implies less emphasis on skill and drill. In fact, it implies a certain ratio of application in comparison to raw meaning construction. Hence, another characteristic of authenticity is that it mimics learning in real life, which allows us to seek help from others, and engage in first-hand experiential learning and reflection. Relatedly, an additional element of authenticity is that learners see learning experiences as genuinely useful to life. In order for a learning experience to be useful to life, learners must come to see its connectivity to big ideas, the learners' lives, the natural world, the human world, and other ideas. Thus, authenticity can also be judged based on how well an activity helps a learner work toward answering provocative *essential questions* (McTighe & Wiggins, 2004, pp.89-90). Essential questions are questions that can be used to frame and "uncover" content. In one example, the larger questions of, "How and why do people use patterns? Are there patterns in the natural world, and if so, what is their purpose?" can be answered in part by an exploration task. In this example, there are patterns in the natural world that people have observed and identified, and by learning to perceive these patterns, people can better understand how the world is organized.

Besides cultural-professional literacy, and besides better cognitive connectivity, there is another reason to provide authentic tasks in which learners might engage. That is, there is a better chance of a learner becoming cognitively-affectively engaged in content that is seated within authentic, real world tasks.

2.3.5.3 Cognitive-affective engagement.

By "cognitive-affective engagement" I mean that a learner's attention and interest is piqued and sustained by his or her current learning (See Dewey on "aesthetic experience"). This engagement can be sparked by any number of catalysts or triggers in the learning experience- the content, contexts or conditions of learning, or by the learning process, itself. Activities, challenges, and tasks (assigned by a teacher or self-assigned) that require active exertion (void of excessive frustration) across a combination of learning domains (i.e., cognitive, physical, social-emotional, linguistic, creative), usually inspire cognitive-affective engagement within a learner. It may be triggered by the content itself, or the process of conceptual change (i.e., reconciling new information with existing information), the act of sharing meaning with others, or the act of applying meaning within context(s). It can also be triggered by the act of reflecting upon whether the conceptual meaning one currently has is use-able or useful "enough" to share with others or apply to a certain situation (i.e., through metacognition). In short, any element of a learning experience may provide cognitive-affective engagement for a learner. While many aspects of learning *can be* cognitively-affectively engaging, some aspects *are* engaging, typically.

This is because such activities tend to draw upon common human motivators (e.g., Wagner, 1996). Although the factors that determine whether or not a specific learning aspect is cognitively-affectively engaging are individually based, in general, such engagement is based on common human motivators (e.g., pride, freedom, hope, curiosity, entertainment, disequilibrium, outrage, joy, anticipation, awe or wonder, belonging). As such, there is a much more significant chance a learner will encode conceptual representations long-term if she partakes in learning experiences that draw upon the mainsprings of human emotions that motivate. This means, from a Learning Science standpoint, theoretically she will hold more conceptual chunks in memory.

2.3.5.4 Scaffolding.

The act of *scaffolding* has been defined in numerous ways, since its inception in 1976 by Wood, Bruner, and Ross. According to ERIC's thesaurus (2001), scaffolding is, "temporary support or assistance, provided by a teacher, peer, parent, or computer, that permits a learner to perform a complex task or process that he or she would be unable to do alone -the technique builds knowledge/skills until learners can stand on their own, similar to scaffolding on a building" (http://eric.ed.gov/?ti=Scaffolding+(Teaching+Technique). A teacher can also use scaffolding as a technique to help lead to conceptual change, if a child has a misconception about a particular concept. In this way, it is a means of assisting a learner while he or she constructs new knowledge or revises existing knowledge. Scaffolding can take many forms. Berk and Winsler (1995) note, effective scaffolding utilizes various strategies, is responsive to each child's needs, is flexible in the moment, and can differ significantly across cultures (for the latter, also see Rogoff, 1990). One of the most comprehensive lists of scaffolding techniques I've found is in Notari-Syverson, O'Connor, and Vadasy (1998), in which the authors describe 24 scaffolding strategies (in italics), grouped into six main categories (in quotation marks). Ranging from low teacher support to high teacher support, these include:

> The use of "open-ended questions" to help a learner *describe* events or objects, *predict* and *plan* future events, alternatives, hypotheses, or generalize to new situations, provide *explanations*, and *relate (a new situation) to* his or her *(previous) experience*; *providing feedback* through *encouragements*, *evaluations*, *thinking aloud*, *clarification requests*, *interpretation of meaning*, and *information talk*; providing *cognitive structuring* by making underlying *rules and logical relationships* explicit, assisting the learner with *sequencing*, and stating *contradictions*; *holding (an idea) in memory* for a child by *restating goals* or objectives, or making *summaries and reminders*; assisting with *task regulation* by *matching (a learner's) interest to experience*, by *making (a situation) more concrete* [sic], by *rearranging elements*, or by *reducing alternatives*; and, lastly, by *instructing* through *modeling* or demonstrating an idea, *orienting* by suggesting specific tools or strategies, through *direct questioning*, making *elicitations* or direct requests, and through *co-participation* (pp. 29-33).

Perceiving learner cues is essential to identifying instances in which scaffolding may be needed. One way a teacher can recognize frustration is when a learner actively seeks help. Sometimes learners need help with just one aspect of a task. By considering all domains- physical, cognitive, social-emotional, creative, linguistic, etc. helps to isolate the aspect of the goal with which learners are struggling (e.g., word problems), without resorting to strategies related to "instructing". This is not to say strategies related to instructing, as listed above, are wrong or harmful. They simply make lower cognitive demands on a learner (Notari-Syverson, O'Connor, & Vadasy, 1998), and historically, American teachers have tended toward these techniques. While the goal is never for a child to become frustrated with a task to the point in which he or she becomes unreceptive to an idea or learning activity (see Vygotsky, 1930/1978), at times, the process of knowledge construction may involve a degree of "mental wrestling". This means, in large part, higher cognitive demands are made on the learner. By, first, implementing scaffolding techniques that offer less teacher support, such as open-ended questioning, providing feedback, or cognitive structuring, this allows learners to "wrestle" (cognitively) with ideas before direct instruction strategies are utilized. This opportunity means learners have a greater chance of constructing knowledge for themselves, without the teacher merely telling or otherwise leading him or her directly to "the answer".

If a teacher is not present to either "read" a child's frustration or misconception, or respond by enacting scaffolding techniques in person, there are ways to make use of specific characteristics of the app medium to support "in-the-moment" assessment and scaffolding (see *Chapter 4*). For instance, Universal Design for Learning (UDL) is a framework for creating flexible instructional methods, goals, assessments, materials, and learning environments that can adjust to individual learning differences. Learners may not use the given provisions, but including UDL elements is a way to provide a figurative safety net for undiagnosed problems.

2.3.5.5 Cognitive-emotive "balance".

Cognitive-emotive "balance" is different from cognitive-affective engagement. By cognitive-emotive balance I mean cognitive and emotional functioning as an interrelated affect. The emotive aspect of this sense of balance is often not included, but when neglected can lead to prolonged frustration in the learner. A situation will fail to become engaging if cognitive overload occurs. While cognitive overload typically leads to frustration, misconception may, as well (perhaps unknowingly to the learner). Additionally, when learners are not accustomed to high-level demands or active cognitive effort, frustration can occur. Prolonged frustration, as a result of any of these causes, can lead learners to mentally and emotionally “check out”. Thus managing this sense of balance is important.

Feeling success or doing something well is important for cognitive-emotive balance. This is the *Zone of Proximal Development* (ZPD) Vygotsky discussed in 1978. The right amount of cognitive effort is typically motivating. “Bite-sized chunks” (Spiro, Feltovich, Jackson, & Couley, 1991) are key. It is important to remember not too much essence, significance, conditions of app, or nuance at once- or the sum of them. A situation will fail to become or remain aesthetic in a positive way if cognitive overload occurs. Not only does cognitive overload lead to frustration, but misconceptions can lead to frustration as well. For this reason, it is important to note that a learner’s exposure to many forms of representation at once, does not necessarily contribute to better learning. Instead, bombardment of representational forms can result in cognitive or sensory overload- a common complaint of modern learning. Thus, what is required is the building of multiple forms of representation, through various modes (e.g., visual, auditory, kinesthetic) and through various media (e.g., analytical writing, sculpting, dance), over time.

Another goal is for a learner to sense frustration in herself and notice it in others. Knowing specific indicators of frustration (or common misconceptions), say, related to abstracting big ideas over mathematical cases or examples in order to form conjectures, is important in order to anticipate the need for additional help. We are allowed to seek help from others in life. In learning, there is a fine line between overwhelm and motivation, but maintain that cognitive-emotive balance can be tricky.

All this said, it is better for teachers to err on the side of assuming a learner is capable of much, then add scaffolding as needed, rather than beginning with "basics". Scaffolding in the form of guidance from others or environment can put learners back in their zone of proximal development. Preventing or ending frustration early can maintain an experience's aesthetic nature.

2.3.5.6 Movement.

By movement, I mean the way in which a learner progresses through his or her construction of key concepts. As it relates to education, this refers to "moving through" content concepts in particular ways.

2.3.5.7 Coherence.

It is important to provide coherence across concepts in a way that is cognizant of prerequisite understanding, without requiring all learners to move unnecessarily through every activity on a predefined linear trajectory. Meets a learner's needs, but not through a linear trajectory, except in cases where sequencing is required for coherence of a concept. It also relates to the learning cycle, in that there must be space to move through each phase of construction. This also prevents cognitive overload.

2.3.5.8 Repetition.

Likewise, the old adage of "one and done", is never applicable to learning. Educators must encourage learners to repeatedly revisit concepts in new ways or with increasing depth.

2.3.5.9 Spiraling.

Spiraling is not quite the same as repetition. Spiraling refers to the cumulative- building of ideas related to the same concept. Thus, spiraling is not just a review of concepts. The purpose of spiraling is to revisit an activity or idea with differing focuses, in "bite-sized" chunks (see

Spiro, Feltovich, Jackson, & Couley 1991). This provides opportunities for learners to reorganize concepts around big ideas. These big ideas may be in the form of broad declarative knowledge that helps to answer essential life questions, or principles used to solve a problem. By visiting multiple representations related to a single concept, over time, will help learners create flexible views in an organized and balanced way.

2.3.5.10 Time.

As it relates to education, time refers to the allotted chronology for meaning construction within a given period. This characteristic is important because cognitive representations need time to mentally integrate. If representations do not have time to mentally integrate, there is likelihood a learner will not encode meaning. Likewise, adequate time also relates to the learning cycle and provides learners with "space to move" through each phase in the ways described above- with repetition, with spiraling, with coherence, and with elaboration.

2.4 Content: Early Algebra

A conceptual focus of this project is to consider curricular characteristics that will help learners learn with understanding, as they move toward the goal of disciplinary literacy. Mathematical literacy is important because it is increasingly required in the personal life of the average citizen- in the workplace, in decision-making within a modern democratic society, and within everyday existence (NCTM, 2000, p. 4). Yet, defining exactly what learners should strive for, as they make their way toward this goal is not always easy.

2.4.1 Mathematical literacy.

2.4.1.1 Literacy versus proficiency in mathematics.

Proficiency, as described by the National Research Council (NRC) in their work, *Adding it Up: Helping Children Learn Mathematics* (2001), is defined as five interconnected *strands* (p.

116) that are necessary aspects for anyone aiming to learn mathematics with efficacy. These strands, identified as *conceptual understanding*, *procedural fluency*, *strategic competence*, *adaptive reasoning*, and *productive disposition*, are seen as " interwoven and interdependent" (p.116). They are composed of a set of beliefs, skills, knowledge, and abilities that allow a learner to overcome mathematical challenges in everyday life and within the formal schooling curriculum. In this seminal work, the National Research Council (2001) proposed the idea of mathematical proficiency as the specific goal of pre-collegiate, formal, American mathematics education.

The first strand of proficiency is conceptual understanding. As defined by the NRC (2000), conceptual understanding is seen as, "an integrated and functional grasp of mathematical ideas" (p. 118). This idea is at the root of learning mathematics "with understanding" (noted previously in this paper), as expressed by Learning Science researchers to be a primary goal of contemporary education (see Bransford, Brown, & Cocking, 2000). A large defining factor of conceptual understanding is a person's ability to represent situations that are mathematical in nature in various ways, and understand how those varied representations might be useful in different contexts or for different purposes (NRC, 2001, p. 119). As such, it is vital to comprehend how various representations connect to one another. In large part, this understanding of connection is constructed through the act of representing, itself. If learners are guided to represent one idea in a number of ways, over time, the act of moving fluently between representations goes a long way toward helping learners construct a deeper conceptual understanding of the concept. In part, this is because learners come to see the ways superficially related situations and representations are profoundly related (NRC, 2001, p. 120). Thus, "trafficking" (Forman, 1994) between representational forms, is essential to building conceptual understanding, and conceptual understanding is a key aspect of learning mathematics with understanding.

The second strand of proficiency is procedural fluency. This involves understanding "knowledge of procedures… when and how to use them appropriately" (NRC, 2001, p. 121). This means learners should display a level of accurate and efficient performance in computing basic calculations of whole numbers, without their need to refer to charts, tables or other visual aids. However, this does not mean learners should simply drill and memorize sums, difference, products, and quotients. In fact, procedural fluency is closely linked with conceptual understanding, in that, through learners' multiple and varied experiences representing and reasoning about various concepts, they tend to become very familiar with number combinations and computations (Carpenter et al., 1999). This said, while a number of researchers recommend that students initially develop their own algorithms to solve mathematical problem situations (e.g., Carpenter et al., 1999), the NRC (2001) notes there are some common algorithms that are seen in American society as "important concepts in their own right" (p. 121). Thus, there are some common algorithms learners should eventually come to understand. This said, the NRC also makes a point that by studying *all* algorithms as "general procedures" (p. 121) [emphasis added] learners come to understand that a procedure that has been carefully developed, can be a potent tool for carrying out customary tasks.

The third strand of proficiency is strategic competence. Strategic competence, in the words of the NRC (2001) is "similar to what has been called problem solving and problem formulation" (p. 124). When learners encounter mathematical problem situations outside of the classroom, part of the challenge is in determining precisely what the problem is. Thus, learners need practice in formulating a problem, before they can use mathematics to solve it. Once they are able to describe the problem at hand, learners must represent the problem mathematically in some way (e.g., graphically, verbally, symbolically, numerically), and to do this they must construct a mental image of its key components. For example, if learners are trying to solve a problem related to getting from home to school, they might draw a scaled map of the neighborhood, and possible routes they could take. This type of activity helps a learner construct an understanding of the key components involved in a problem situation (NRC, 2001, p. 124). Understanding the situation is essential to building authentic strategic competence, as opposed to looking superficially at "key" words (which are more often than not, misleading) [Bruning, Schraw, & Norby, 2011] or "number grabbing" (NRC, 2001, p. 124).

The fourth strand of proficiency is adaptive reasoning. The NRC (2001) describes adaptive reasoning as "the capacity to think logically about the relationships among concepts and situations" (p. 129). There are a number of metaphors that have been used to describe this reasoning- from "the glue that holds everything together", to an essential "navigation" tool that steers learning (p. 129). While formal proofs and other forms of deductive reasoning are a part of adaptive reasoning, they are only one aspect. Particularly, at the primary level, adaptive reasoning is better conceived of as informal justification and explanation, and intuitive and inductive reasoning based on perceived patterns and analogies (p. 129). This complements the other strands by helping learners determine the legitimacy of potential strategies and solution paths.

The fifth and final strand of proficiency is developing a productive disposition. This refers to learners' belief that mathematics is "useful and worthwhile… that steady effort in mathematics pays off, and that… (one's self) is an effective learner and doer of mathematics" p. 131). Unsurprisingly, a mathematics teacher plays a key role in helping learners build these positive attitudes. As such, this strand is discussed in more detail in a section following (see The app designer teaches early algebra).

The strands of mathematical proficiency, as outlined above, are important in helping learners become adept at the practice of mathematics. However, proficiency is only one aspect of mathematical literacy. Thus, it is important to examine other components and definitions of mathematical literacy.

2.4.1.2 Definitions of literacy.

As noted previously, the idea of disciplinary literacy in general can be difficult to unpack, and mathematical literacy is no exception. Even within the discipline there are differing views. Yore, Pimm, and Tuan (2007) denote that mathematical literacy entails one's ability to use certain information communication technology (ICT) strategies, discipline-specific language, and habits of mind to critically analyze information and render meaning. Jablonka (2015) discusses the evolution of the term "mathematical literacy" as it relates to an increasingly disparate set of definitions and expressions, from "critical literacy in mathematics" (Gutstein, 2006) to "mathemacy" (Alrø & Skovsmose, 2002), and from "reformist critical mathematics" (Brantlinger, 2013) to "criticalmathematical numeracy" (Frankenstein, 2010). Jablonka also uses the term interchangeably with "numeracy".

One thing is certain. Most contemporary definitions of mathematical literacy include more requirements than a person's understanding of a few arithmetical concepts and basic operations. In fact, "understanding" may not be an accurate way to describe the goal, at all. "Engag(ing)" with mathematics is likely a better descriptor (Bass & Ball in Carpenter, Franke, & Levi, 2003, p. vii). In this sense, literacy might be framed as a process or "state" of interaction, instead of a destination. Being able to engage with mathematics equates to far more than knowledge recall, or even competency in calculation. Engagement involves representing numeric ideas, investigating them, and generating new ideas based on one's perception of established patterns and mathematical reasoning (Carpenter, Franke, & Levi, 2003). Engaging with mathematics, likely also involves being literate in the discourses of the discipline (Jablonka, 2015.). This characteristic is supplemental to knowing the content of the discipline.

Thus, as mathematical literacy relates to primary learners, its full realization is certainly not immediate. Yet, particular habits of mind like engaging with mathematics and utilizing some terms of formal discourse may help to establish young learners on a path toward this goal.

2.4.2 The primary mathematics learner.

It has already been established (see *Release from past cognitive constraints*.) that children bring much informal knowledge about number with them to formal learning situations. In short, children arrive capable of, and ready to, build upon their broad mathematical understanding. However, children's representations of their mathematical understanding, in general, often appear different than the way adults perceive mathematical representations should look (Carpenter et al., 1999). For example, young learners often use natural language for expressing their understanding of mathematical relations, even though, by adult standards, it is far more ungainly than symbol systems like arithmetical-algebraic notation (Carraher, & Schliemann, 2007). Research findings in Learning Science have shown adults take many symbolic representations in mathematics for granted (Carpenter, et al., 1999; Carpenter, Franke, & Levi, 2003; NCTM, 2000). Since particular forms of representation have long since been a part of school mathematics, and since these representations have often been taught "as if ends unto themselves", many adults have lost sight of mathematics as more than the manipulation of particular symbols (NCTM, 2000). Additionally, adults often define "complex" mathematics in terms of a person’s use of symbols and algorithms. Thus, if a person is manipulating exotic-looking symbols and formulas, they are seen to be engaging in complex mathematics. Similarly, adults’ perceptions of complex mathematics, also, may be linked to outdated understandings of abstractness and concreteness. As discussed previously, concreteness refers to the status of a person’s relationship to an object, not to the object itself. However, for many years, the idea of concreteness was referred to (and is often still referred to) in terms of concrete objects, such as physical manipulatives. Likewise, children’s use of these concrete manipulatives were often seen a “basic” act. Thus, particularly in early mathematics, concepts that can be supported by physical manipulations, like addition and subtraction, and their associated “facts” (i.e., “basic facts”) are positioned as enduring ideas that all children must learn, in order to form a solid conceptual foundation, *before* proceeding to more “complex” work- such as algebra. Therefore, despite the fact children are capable of engaging in complex reasoning, often, they are not given an opportunity to do so (Bass & Ball in Carpenter, Franke, & Levi, 2003, p. v). While many beginning mathematics learners use direct modeling (sometimes with physical manipulatives like objects and fingers) to represent the amounts and actions in a problem (Carpenter et al., 1999), learners are actually engaging in complex ways with ideas.

In their seminal work, *Children’s Mathematics: Cognitively Guided Instruction*, Carpenter, Fennema, Franke, Levi, & Empson (1999) describe their findings of young children’s mathematical capabilities, based on 20 years of research (previous to 1999). After studying the ways in which children solve mathematical problem situations naturally (i.e., without being required to use adult-directed methods for solving), Carpenter et al. realized young children are very capable of solving complex mathematical situations, by drawing upon their own reasoning and techniques. In the words of one teacher, Kerri Burkey, who worked with the research team, “In the past I thought children didn’t understand subtraction with regrouping, when what they didn’t understand was how to use the process I was insisting they use” (Carpenter et al., 1999, p. xiii).

Carpenter et al.’s realizations began a movement in early childhood mathematics, which is known today as Cognitively Guided Instruction (CGI). It is not a formal mathematics program or boxed curriculum, but rather an approach to teaching mathematics that makes use of children’s natural inclination to solve mathematical problem situations through mental reasoning, direct representation and modeling, and development of their own algorithms and conjectures. With adult guidance, children construct mathematical understandings by building on what they already know, and by developing the skills to solve problems, through the act of solving them (Carpenter, 1999).

One of the most profound ideas Cognitively Guided Instruction (CGI) reflects is the understanding that computational operations as found in addition and subtraction are habitual for most adults, and thus, from an adult’s point of view seem to be only procedural (Carpenter et al., 1999). As such, adults’ near-automatic performance hides the deeply conceptual nature of the problem at hand (Bruning, R. H., Schraw. G. J., Norby, M. M., 2011). Studying children’s errors (and approaches) has contributed to better understanding of the nature of arithmetic problem solving (Bruning, R. H., Schraw. G. J., Norby, M. M., 2011). As such, children do not naturally approach all the problems adults might see as, say, subtraction problems, through pencil and paper algorithms related to subtraction. Instead, they may add up parts of an amount to determine a subtracted whole amount; sometimes with the aid of manipulatives, sometimes not. Children need to construct their understanding of mathematical ideas by building on what they already know, and using techniques that draw on their own capable reasoning. CGI has legitimized this natural approach.

2.4.3 Why early algebra?

2.4.3.1 Practical reasons.

In this study, my focus is on primary children's learning of early algebra ideas. Early algebra (EA) has been defined in many ways, and is sometimes used interchangeably with terms like "algebrafied" arithmetic, algebraic reasoning, algebraic thinking, and occasionally, pre-algebra. In this study, early algebra refers to learning expectations that are delineated by NCTM (2000) as algebraic in nature and fitting for Pre-kindergarten to Grade 2. These expectations are comprised of specific objectives for this general age group, within each of the four broad standards recommended for all pre-collegiate grade levels. These broad standards include, enabling learners to: "understand patterns, relations, and functions"; "represent and analyze mathematical situations and structures using algebraic symbols"; "use mathematical models to represent and understand quantitative relationships"; and, "analyze change in various contexts" (p. 395).

As discussed previously (see Chapter 1), my concentration on early algebra (EA) is driven, partially, by a need to focus on a specific content area- given the conceptual framework I embrace within this study (*TPACK* theory [Mishra & Koehler, 2006]). A researcher cannot examine how technological knowledge, pedagogical knowledge, and content knowledge intersect, without identifying "the content" in the latter. However, more importantly, I have selected EA because it is of significant personal and professional interest, and because it is a topic of considerable weight for the mathematics education community. Algebra, typically seen as a subject for select high school students, has been described as the "gatekeeper" to higher education (Carpenter, Franke, & Levi, 2003, p. 6). In fact, in my own schooling experience, it almost became a barricade for me. This is not unusual. Learners often face algebraic stumbling blocks, such as not comprehending letters as a generalized variable or failing to understand concepts of equivalency. Yet, many of these trip-ups can be traced back to false divisions established between algebra and other mathematics topics, like arithmetic and data analysis- sometimes as early as preschool. As such, many mathematics educators like Carraher & Schliemann (2007) believe concepts like equivalence, which underlay much of primary arithmetic, "should be treated early on in ways consistent with their usage in more advanced mathematics" (e.g., algebra) [p.671]. Moreover, a number of researchers believe arithmetic should be seen as a part of algebra and consequently arithmetical concepts approached with this vision in mind (Carpenter, Franke, & Levi, 2003; Carraher, & Schliemann, 2007). They believe children are not only capable of learning arithmetic concepts in this way, and that doing so is vital to a child's future understanding of advanced algebra concepts, but that it is also essential to his or her *current* deep understanding of arithmetic itself.

2.4.3.2 Theoretical reasons.

Algebra is not a separate mathematical domain, in my mind. It is a facet of all mathematical domains. Algebraic ideas are simply the underlying principles and relationships between amounts. Envision key early algebra (EA) ideas as doorways into various domains of mathematics- key ideas, like: equality, conjecture, pattern, problem types/principles and how they relate to arithmetic, geometry, and measurement and data. Utilizing EA ideas as entryways into all mathematical domains is an essential analogical framework from which to proceed. In fact, the key EA ideas, themselves, seem to align with essential characteristics of "learning with understanding". In some ways this is confusing because EA "content" and the approach to learning the EA content are very similar. This is simply because algebra is often treated as a distinct content subject, when in fact, it is a kind of framework through which other mathematical content can be viewed and analyzed. Because of these reasons, however, some see early algebra as illegitimate; not a "real" mathematical domain.

2.4.3.3 Early algebra as legitimate and essential.

However, despite this acknowledgement by numerous researchers and research councils (see Kaput, 1995; NCTM, 2000; NRC, 2001), the idea that EA might be described as algebra at all, is still contestable for some (see Balacheff, 2001). Instead, these researchers and mathematicians argue that seeing algebraic ideas in arithmetic (and other classically distinct domains of mathematics) is "reading" too much into it. Yet, in my mind (and presumably in the minds of other EA proponents), this is akin to suggesting a person is "reading" history into geography, if that person identifies events that may have led to patterns of global migration over time. As such, my view of early algebra aligns with advocates of EA who emphasize that current primary mathematics content is not completely and mutually distinctive from algebra (Carraher & Schliemann, 2007, p. 671; also see Bass, 1998; Carraher, Schliemann, Brizuela, 2000). Hence, a part of teaching early algebra is coming to understand where algebraic ideas are evident within the broader discipline of mathematics. Perceiving the algebraic meaning embedded in arithmetical operations, geometric and numeric patterns, mathematical representations, and situations involving change, is an essential leap toward reaching this understanding.

This is not to say every concept, idea, or method from arithmetic (or other domains) is patently algebraic (Carraher & Schliemann, 2007, p.). In fact, most algebraic ideas are latent in the existing curriculum (Bass, 1998). Likewise, some aspects of primary mathematics like rote counting and reading or writing numerals are a matter of becoming familiar with standard conventions. Even counting on from a given number is partly non-algebraic, in that, success in this endeavor is partially due to a person's familiarity with conventional number sequence. However, when "young children come to understand the relationship between numerals and quantities, and when they connect this to counting to answer how many objects, they are arguably beginning to splash about in the pre-algebraic pool" (Carraher & Schliemann, 2007). For example, when a child comes to understand a numeral represents a quantity; when she perceives the numeral "2" or the word "two" as representative of "twoness", and when she begins to understand the word "three", or numeral "3", is representative of one more than twoness, the child is beginning to understand the relationships between quantities- the nature of which is potentially algebraic.

Despite the potential algebraic nature of many mathematical concepts within the primary curriculum, adults often still conceive of algebra as the manipulation of particular symbols (NCTM, 2000, p. 37). As such, algebraic notation and its associated procedures are sometimes seen as the practice of "true" algebra. However, a number of symbol systems play a role in algebraic reasoning (Carraher & Schliemann, 2007). Hence, although algebraic notation, historically, has been privileged over other systems of representation, there are currently four symbolic systems considered fundamental in EA. According to Carraher and Schliemann (2007), those systems include arithmetical-algebraic notation, tables, graphs, and natural language; they are accepted because mathematicians can use them to represent functions. Within the context of elementary mathematics, a function can be defined as "a rule that assigns each element from a domain, to a unique element in the co-domain" (Carraher & Schliemann, 2007, p. 688). As such, the extent to which the four symbol systems, mentioned above, represent functions and capture functional reasoning, is the chief means for determining whether these systems are propitious to algebraic reasoning.

In spite of debate, studying arithmetic and other mathematical topics through the entry point of early algebra, not only, is legitimate, but essential to helping young learners learn all current mathematical concepts with understanding. It is also fundamental to providing a solid foundation for later algebra learning, and subsequently, preventing the gateway to higher education from slamming shut.

2.4.3.4 Algebra in the context of traditional mathematics domains.

Arithmetic, as defined by Carraher and Schliemann (2007), is "the science of numbers, quantities, and magnitudes" (p.669). In part, researchers recommendation is based on the fact that the study of number and arithmetic are a large part of the primary and elementary mathematics curriculum. Additionally, however, as Carraher, Schliemann, and Brizuela (2000) posit, arithmetic inherently has an "algebraic character". For example, the characteristics of number that make arithmetical calculations feasible, are the same characteristics that make simplifying expressions and solving equations possible. The same operational properties that are at work in one domain, are at work in the other. Hence, in the words of Carpenter, Franke, and Levi (2003), "The artificial separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics in the early grades and makes it more difficult for them to learn algebra in later grades" (p. 1). Yet, because arithmetic and algebra are often separated, characteristics of number and operations (like those described above) often go unnoticed by learners during their execution of arithmetic, and their knowledge becomes over-contextualized. Thus, when learners (later) attempt algebra they, not only, fail to recognize familiar concepts, but their perceptions of arithmetic as a series of rote calculations can actually impede algebra learning (Carpenter, Franke, & Levi, 2003, p. 2). Many learners end up needing to re-learn arithmetic as well.

Beyond arithmetic, algebra is connected to other areas of mathematics, as well- including geometry and data analysis- and is seen as a key component and unifying element of the collective school mathematics curriculum (http://standards.nctm.org/document/chapter3/alg.htm). In fact, the Rand Mathematics Study Panel (2003), suggested algebra as the primary topic “for focused and coordinated research and development” because of its axial role in investigating most areas of mathematics, in addition to exploring engineering and science ideas (p. 47). Even at the primary level, one can see these links between algebraic ideas and ideas from other domains. For instance, the act of analyzing repeating and growing patterns (if patterns are geometric in nature), essentially, is exploring the algebraic nature of geometry. Recognizing a constant rate of change between two sets of data is the perception of a functional relationship in the context of data analysis. Hence, if algebraic ideas are already potentially present in primary mathematics concepts, it is simply a matter of making those ideas explicit.

2.4.4 Learning early algebra with understanding.

Learning mathematical concepts, with understanding, is a key part of the larger goal of becoming mathematically literate. However, as second grade teacher Ann Badeu posits in an interview with Carpenter el al. (1999), “It is only when you build from within that you truly understand something…(otherwise) it is only rote, and that’s not really understanding” (p. xiii). In general, learning with understanding requires a number of considerations (e.g., constructing knowledge, avoiding over-contextualized thinking by examining the specific and the general, representing across contexts, breadth, depth, transfer) [as discussed previously]. As it happens, these ideas are also applicable to learning early algebra with understanding. In their seminal work, *Principles and Standards for School Mathematics,* NCTM (2000) outlines a number of standards that are applicable across mathematical domains. These standards share remarkable similarity with the elements that help to define learning with understanding. Specifically, NCTM identifies that prekindergarten through grade 12 learners should be able to: problem solve, reason mathematically and use methods of proof, communicate about mathematics, recognize and use connections between mathematical ideas, and use and create representations of mathematical ideas (pp. 52-71). Each of these standards is discussed below, in brief, as many of these ideas have been mentioned previously, within the context of strands of proficiency.

Learning specific concepts, with understanding, is a key part of the larger goal of becoming mathematically literate. In general, learning with understanding requires a number of considerations (e.g., constructing knowledge, avoiding over-contextualized thinking by examining the specific and the general, representing across contexts, breadth, depth, transfer) [as discussed previously]. As it happens, these ideas are also applicable to learning early algebra with understanding. In their seminal work, *Principles and Standards for School Mathematics,* NCTM (2000) outlines a number of standards that are applicable across mathematical domains. These standards share remarkable similarity with the elements that help to define learning with understanding. Specifically, NCTM identifies that prekindergarten through grade 12 learners should be able to: problem solve, reason mathematically and use methods of proof, communicate about mathematics, recognize and use connections between mathematical ideas, and use and create representations of mathematical ideas (pp. 52-71). Each of these standards is discussed below, in brief, as many of these ideas have been mentioned previously, within the context of strands of proficiency.

2.4.5 Facets of mathematical understanding.

Just as there are facets of general understanding, as suggested by Wiggins and McTighe (1998/ 2005) [e.g., explain, apply, interpret], there are mathematical recommendations made by the NCTM (2000) that are similar in effect. While the NCTM does not phrase these ideas in terms of "facets of understanding", the way in which the NCTM describes these ideas is similar to the way Wiggins and McTighe (1998/ 2005) frame their general "facets of understanding". As such, I have come to think of the following mathematical recommendations as additional "facets of understanding" that are directly related to mathematics.

2.4.5.1 Problem solving.

Problem solving is not equivalent to solving word problems, but instead references the act of investigating and solving mathematical problem situations or completing mathematical tasks in which the "solution method is not known in advance" (NCTM, 2000, p. 52). In other words, word problems are just one type of mathematical problem learners should solve. It is also important that learners are helped to think in a systematic way about potential solution paths and to record their thinking in an organized way (p. 53).

2.4.5.2 Reasoning and proof.

There are particular types of reasoning that back up forms of proof. These include, but are not limited to, algebraic reasoning, geometric reasoning, proportional reasoning, probabilistic reasoning, and statistical reasoning. Proofs, themselves, can take multiple forms, which, at the primary level, tend to be more informal. These include, but are not limited to, narrative argument, two-column proof, visual argument, and proof by cases. The conventional form of proof, logical deduction, tends to be enacted by children beyond the primary level. This is satisfactory. However, teachers can also help young learners work toward the conventional form by introducing it as such, and helping them to use truths that already have been established (i.e., which a learner has already constructed and discussed), to reason deductively. As such, the NCTM (2001) notes, “conjecture is a major pathway to discovery” (p. 57; also see Carpenter, Franke, & Levi, 2003).

2.4.5.3 Communication.

Clarifying one’s ideas about mathematics is, in part, accomplished through sharing ideas with others (NCTM, 2000, p. 60). Additionally, learning to communicate in a way that is clear and convincing to others is also important. As such, learners need to work on mathematical tasks that are worthy of social discussion (p. 60). Furthermore, learners will only feel comfortable expressing their ideas, within a community of learners with whom they feel psychologically safe. It is the teacher’s responsibility to create such a community.

2.4.5.4 Connections.

Blanton & Kaput (2000) found, in their research, there is a constant shifting back and forth between a specific number or set of numbers, and a more general class of numbers. Seeing connections between mathematical topics and the interrelatedness of ideas, is essential to deep understanding of concepts. As discussed previously, however, the mathematics curriculum generally tends to be a collection of isolated topics. Yet, there are good arguments for treating mathematical (and scientific) concepts- particularly those that are more abstract- as relational (Carraher & Schliemann, 2007). Part of developing this perception is through the act of teachers making ideas explicit (NCTM, 2000, p. 64). Teachers play a major role in drawing learners’ attention to the same concepts across seemingly different contexts. In general, effectively using mathematics requires the acquisition of networks of mental representations (Bruning, R. H., Schraw. G. J., Norby, M. M., 2011).

2.4.5.5 Representation.

Learners should represent their ideas in ways that make sense to them, even if not conventional at first. Conventional representations are important to learn, but should be explicitly defined as such, and there should not be a premature rush to introduce them (Carpenter et al., 1999). The term "representation" can refer to either (or both) product or process (NCTM, 2000, p. 67). It also refers to both internal (in one's mind) and externally observable products. New forms of representation that have arisen from digital technology create a need for even greater attention to kinds and forms of representation (p. 67). Since different forms of representation illuminate different components of a mathematical relationship or concept, it is important for learners to gain considerable experience with many forms of representation- including conventional and student-derived forms (p. 69).

2.4.6 Content: "What" to learn.

In any contemporary learning situation, it is no secret that what children "need to know" is a moving target. Mathematics education is no exception. Some people feel "basic" facts are the priority. Others feel children should be introduced to more concepts sooner- from basic to complex- in an attempt to squeeze in all they will purportedly need to know for the 21st century. Yet, others feel it is impossible to predict what concepts today's learners will need to know in the future, and consequently, propose focusing on broader processes like, problem solving, communicating, creativity, and reasoning. In fact, however, research findings in Learning Science show learners need to learn, both, concepts and processes- but not as they have historically been conceptualized (Bransford, Brown, & Cocking, 2000; NCTM, 2000).

Despite the fact it is impossible to predict exactly what concepts learners will need to know in the future, there are key ideas that, as far as is known, children will continue to need to know in the near future. Thus, in deciding on what concepts are applicable to most primary children, I refer to recommendations outlined by NCTM (2000), a highly respected work rooted in Learning Science principles. As such, in the following section, I have included a number of suggestions for what primary learners should know and do as it relates to learning early algebra, as suggested by NCTM (2000). These ideas center on key subject matter concepts, recommended by the source noted above.

The recommendations below represent key components from the *Algebra Standard for Grades Pre-K-2*, outlined by NCTM (2000, p. 90). They are phrased as educational objectives for learners. Thus, they should be read as, "Learners will be able to…". The authors describe eight specific learning objectives (in italics), grouped into four main categories (in quotation marks). These include:

> *Sort(ing), classify(ing), and order(ing) object(s) by size, number, and other properties*; *Recogniz(ing), describ(ing), and extend(ing) patterns, such as sequences of sounds and shapes or simple numeric patterns and translat(ing) from one representation to another*; and *Analyz(ing) how both repeating and growing patterns are generated*, within the category of "Understand(ing) patterns, relations, and functions". *Illustrat(ing) general principles and properties of operations, such as Commutativity, using specific numbers*; and *Us(ing) concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations,* within the category of "Represent(ing) and analyz(ing) mathematical situations and structures using algebraic symbols" *Model(ing) situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols*, within the category of "Us(ing) mathematical models to represent and understand quantitative relationships". Lastly, *Describ(ing) qualitative*

change, such as a student's growing taller; and *Describ(ing) quantitative change, such as a student's growing two inches in one year*, within the category of "Analyz(ing) change in various settings" (NCTM, 2000, p. 90).

2.4.7 The app designer "teaches" early algebra.

As Bass and Ball note, teachers need even more than "a perspective on mathematics, a view of children as capable, … (and) a rich set of resources for bridging young students with mathematics" (Bass & Ball, 2003, p. vii). The major aspects of "how" learners might learn with understanding can be difficult to tease apart. This is because, in practice, individual learning activities, educational program, learning environment, and medium of learning blend together to create an amalgamation of events, spaces, experiences, and interactions that serve as a collective vehicle for perception, comprehension, and competence. As such, "Teaching mathematics well is a complex endeavor, and there are no easy recipes" (NCTM, 2000, p.17). Accordingly, the approaches suggested below are not meant to indicate there is only one right way of teaching. They do suggest, however, there are particular aspects to consider in the design and enactment of high quality, early algebra learning activities, programs, and environments. Yet, there are infinite ways these aspects might be realized. As such, in accordance with the range of recommendations provided by mathematics education researchers that I considered for the generation of my Coding Frame, I have provided only examples of ideas for how teachers or app programming can support learners' construction of mathematical understanding.

Bass & Ball (in Carpenter, Franke, & Levi, 2003) recommend offering weighty problem situations that are not overly burdensome to learners, and helping learners maneuver those problems to a useful end. They also recommend listening for the mathematics in learners' talk, and offering tasks that "lead learners to generate new questions and ideas" and "pull learners into encounters with challenging, generative, and fascinating kinds of math work" (p.vii). Carpenter, Franke, & Levi (2003) suggest, "Engag(ing) students in articulating conjectures about properties they think are true and provid(ing) them with the opportunity and means to express these conjectures clearly and accurately using words and symbols" (p.47). They also suggest, initially, giving learners the opportunity to "use basic properties of arithmetic without explicitly identifying the properties they are using" (p.3). The NCTM (2000) recommends a wide range of teaching considerations, including the ideas of making problem solving strategies explicit by first asking learners to solve problems using intuitive strategies, then helping learners compare strategies across a group of learners to make a list of strategies used (p. 54), providing opportunities for learners to read, write, speak, listen, and represent mathematical ideas with the group or community to help them determine if they are understood and adequately convincing (p.60), and "Fram(ing) representations as tools to support understanding and apply(ing) mathematics to problem situations- instead of introducing representations as an end unto themselves" (p.15).

As previously stated, the examples above only represent a few suggestions for teaching, made by a few scholarly sources. However, it is interesting to note that, even these limited examples, may not yet be present in many non-virtual, contemporary classrooms. As such, this may be another way app-mediated learning can bring value to primary children's learning of algebraic ideas.

2.5 The Multi-Touch, Mobile iOS App

Apps offer particular value as a promising educational context in which educational goals like learning with understanding might be realized. While there are other non-traditional contexts in the U.S. that also provide increased freedom from the short-term goals and conditions that seem to plague contemporary formal schooling, these settings have less potential than apps in terms of inherent affordances and ubiquitous accessibility. In general, apps offer some unique educational benefits. Apps can provide access to (potentially) high-quality curricula for many learners, greater freedom from traditional schooling politics, even-handed treatment of learners, and can limit children's tendency to separate learning from out-of-school contexts. Apps are not bound by typical time constraints, and the nature of digital programming can provide the conceptual space for learners to construct meaning with depth and breadth, as well as other characteristics that help learners realize long-term educational goals. Thus, theoretically, app designers have greater freedom in their role of helping learners learn with understanding than is currently available in either traditional formal schooling contexts or other alternative educational contexts. Yet, according to recent studies, current first generation educational apps are not presently enacting characteristics that qualify them as truly "educational".

2.5.1 The current state of educational apps.

Shuler (2012) and Ito (2009) have each conducted analyses of children's educational technologies, and have found, in general, adult consumers' expectations for these devices are extremely lofty. Thus, some consumer expectations seem founded upon inflated prophecies that will never be feasible (Ito, 2009). After all, educational apps are no "magic bullet" to the sum of pervasive challenges in education (Shuler, 2012, p. 13). Yet, both researchers also mention, when a product bears the educational descriptor, it is realistic to expect it to align with certain precepts of contemporary pedagogy.

A number of researchers have begun the process of outlining such precepts. In their 2015 analysis of Learning Science principles, Hirsh-Pasek and Zosh with colleagues', abstracted four characteristics of high quality pedagogy from the Learning Science literature, which they posit can help to determine an app's "pedigree" (p.24). Those characteristics are: active or "minds on" learning (requiring mental effort), engagement in the learning process (requiring cognitive and emotional attentiveness), meaningful learning (that is relevant and purposeful to the learner), and social interaction (that supports the focus of learning). Hirsh-Pasek and Zosh et al. posit that, if an app has recognizable learning goals *and* satisfies the other characteristics of high quality pedagogy, listed above, it is likely to promote "deep learning" (p. 25), and thus might be considered educational. If an app falls short on learning goals and/or the other Learning Science-inspired characteristics, it is likely to, (a) Be primarily entertaining (many characteristics, few learning goals), (b) Provide shallow learning (many learning goals, few characteristics) or, (c) Offer neither entertainment nor educational value (few characteristics, few learning goals).

In addition to the proposed standards, above, Hirsh-Pasek and Zosh et al. (2015) delineate a number of common hazards app designers might face in their endeavor to design an educational app. As the researchers note, to date, many app designers have succumbed to a variety of design traps that have made first-generation educational apps less than educational. Among those pitfalls are, what Hirsh-Pasek and Zosh et al. call, *the fire-alarm syndrome* (e.g., Do all the whirligigs and sound effects increase engagement, or cause distraction?), *the too-many-choices trap* (self-explanatory), *the masquerading "educational" app* (the rote memorization of letters and numbers is not sufficiently educational), *the empty calories* (fun and engaging, not much educational content), and *the attention-deficit design* (constant changes and visual switching).

Hirsh-Pasek and Zosh et al.'s (2015) proposed standards and observations provide a number of essential considerations for, both, assessing current apps and designing second generation educational apps. Yet, more description is required as it relates to specific educational apps and their content of focus.

2.5.2 The app medium.

Since apps have such potential, of which they seem to be falling short, it is important for app designers to make better use of apps' strengths. Like any medium, apps have strengths and weaknesses- affordances and biases. Despite some similarities between traditional learning spaces and those of the virtual world, the parameters of design are quite different within each. For obvious reasons, a virtual space behaves differently than a space that is situated within the confines of Earthly physics. Virtual learning spaces, like those found within educational apps, can provide opportunities to explore and manipulate things in ways that are not physically possible in the Earth-bound world. Learners can investigate inaccessible terrain, experiment with materials, tools, substances and forces that are dangerous or costly on Earth. Additionally, the medium of computer programming allows virtual learning spaces to mimic almost any Earthly environment, set of principles, or physical properties. Thus, without negating the value of thoughtfully designed, Earth-bound learning spaces, a virtual space offers a unique and valuable laboratory, if its features are utilized.

2.5.2.1 Structure.

As it relates to apps in particular, the architectural, aesthetic, and organizational features of a virtual learning space share some similarities with a traditional classroom. In both virtual and non-virtual contexts, orderliness and clarity as expressed through clear directions and organized systems help a child effectively navigate a space with autonomy. A space that is well equipped offers inviting provocations, abundant and engaging activities and resources, and a culture of creativity and investigation. A learning environment with a balanced cognitive-emotive space provides enough stimulation to be interesting, while preventing sensory overload. A space that invites seamless integration includes images and characters that reflect people like those in the learner's out-of-app life, incorporates Universal Design features, and provides optional learning supports a learner can choose to utilize without disrupt to the activity or content of focus. A learning environment that protects personal boundaries by enforcing rules of respectful interaction, confidentiality and privacy, offers safety and trustworthiness. Just as in an Earth-bound learning environment, relevant materials, labels posted, signs and directions in multiple modes and media (e.g., visual/audio, words/pictures) are supportive of the primary learner.

As it relates to structures and routines established for learning, such as daily routines, scheduling, and pacing, these elements tend to be objectified differently in the virtual learning space, or are otherwise inapplicable. Since app-mediated learning is not dependent upon linear sequencing, or bound by predetermined time restrictions, the structural features of a multi-touch, mobile app appear to play a larger role in dictating the arrangement and flow of activities than any traditional, temporal considerations. In app-mediated tasks, the learner controls the order in which they participate in virtual activities, as well as how long they participate (at least to a degree). As such, as long as the app provides learners with opportunities to complete small tasks or achieve small goals throughout the storyline, pause an activity at various points in action, save work that is in progress, and repeat tasks that were completed unsuccessfully without beginning anew, even potential hurdles such as a particular child's short attention span appear largely irrelevant. Indeed, if an app designer utilizes the inherent structural characteristics of multi-touch, mobile apps, storyline and learning goals will drive the composition of the program, instead of time constraints. Thus, to this end, it is important to recognize the structural characteristics of multi-touch, mobile apps and the potential affordances they offer (see Table 5).

In learning contexts where the teacher is not present to enact a lesson, structure plays a significant intermediary role in the arrangement of content ideas and their communication to learners (Forman, 1994). While “physical” traffic patterns of an app affect a learner’s ability to easily navigate the site, conceptual traffic patterns (e.g., learning trajectories, entry points into content concepts, interdisciplinary links between concepts)- as theoretically afforded by the structural characteristics of multi-touch, mobile apps- directly affect a child’s ability to learn content with understanding.

2.5.2.2 Social climate.

Another major component of a traditional early childhood learning environment is the social climate (Dodge, Colker, Heroman, 2002, p. 102). As it relates to the social climate of a virtual learning environment, there are two major components it shares with its traditional counterpart; those are, relationships and the communication of dignity.

2.5.2.2.1 Relationships.

Virtual environments cannot, of course, support the same kinds of relationships as traditional learning environments. While this is a potential limitation to app-mediated learning, app designers can make an intentional effort to reduce this constraint by thoughtfully considering the kinds of relationships that *can* be supported. For instance, a quasi-relationship (of the social-emotional kind) can be established between learner and app-teacher, between learner and online peers, and between learner and the app characters with whom he “interacts”. By “quasi-relationship” I mean that, in these types of interactions, the same sorts of connections are not typically possible, as they are between two animals- particularly, between two humans- who interact with regularity. A relationship, by definition, is reciprocal. In this case, social-emotional reciprocity is not possible, because a computer program cannot synthesize emotions. The computer-teacher and app characters, then, only appear to reflect emotion. This said, there are many long-standing examples of cases in which children form attachments to both robots with whom they interact regularly, and fictional characters (e.g., Kanda, Sato, Saiwaki, & Ishiguro, 2004). Without diving too deeply into the complex psychology of social-emotional relationships, however, it is perhaps enough to note, at the very least in this situation, quasi-relationships are possible between learner, app-teacher, and fictional "friends". Likewise, it is also possible to establish relationships between the learner and online peers (with safety in mind). While these relationships are between two humans, interactions are often temporary, brief, and "distant". As such, social-emotional connections are likely fairly superficial.

Another kind of relationship that can be supported through thoughtful programming is of a traditional type between humans. This is, perhaps, the most important kind of relationship. Collaborations between learners and others established for purposes of inquiry or problem solving, such as virtual interviewing or joint research between two groups of learners or between learners and professionals in a field, are potentially both cognitively and social-emotionally fulfilling. However, apps not only offer opportunities for learners to connect with others at a distance, with whom they otherwise would likely never connect, but opportunities for learners to connect to those around them in unique ways (e.g., Interviewing family members about numeric patterns they notice at work or school).

Regardless of whether the connections between learner and "other" are of a kind that might be qualified as a full and mutual relationship, or a quasi-relationship, the nature of the interactions are of equal importance. For example, there are a number of authentic purposes for interaction within a virtual learning environment. Purposes such as “getting to know” the learner by inquiring into interests, out-of-school practices, preferred ways of learning and communicating, and current understandings of content skills and concepts, all offer opportunities to differentiate, or even personalize instruction, as well as express respect, promote a sense of individual worth, and communicate a feeling of care. Other genuine purposes for interaction include both learner initiated and app-teacher initiated aims, such as providing specific feedback or prompts to the learner, serving as a sounding board for a learner to express his or her feelings and hear them reflected back, communicating expectations and rules that a learner agrees to follow, and collaborating with others on activities (e.g., connections to the larger community).

2.5.2.2.2 Dignity.

Views of the learner are embedded within all aspects of a curriculum, and the app curriculum is no exception. As such, treating the learner with dignity is a must. Dignity is defined here, as being held in esteem because of one’s worth. It can be expressed in a multitude of ways, such as through an app designer’s use of an authentic historical figure in an app (e.g., Benjamin Franklin is just as potentially kid-friendly as a loveable teddy bear). Similarly, accepting and even prompting multiple solutions for solving problems, representing ideas, or executing tasks, as well as expressing polite manners and avoiding sarcasm and “baby talk” (see Dodge, Colker, & Herroman, 2002), all communicate respect for the learner. Providing specific guidance on how to befriend and interact with online peers, or structuring tasks in a way that encourage learners to interact with familiar humans in their lives, shows support of collaboration and other healthy social relationships. Conveying the notion that it is acceptable to test ideas and make mistakes, and that there are boundaries that prevent others from treating you poorly or invading your private space, impart a risk-free and safe space.

While many of these outlooks seem self-evident, a few require further discussion- particularly as it relates to how they potentially might translate in an app. For example, when providing “appealing and engaging” activities, app designers must be leery of how this translates. Sometimes with children’s content, there seems to be a lingering perception of a young learner’s need for simplicity and “happy” content. Yet, designing virtual characters that are substantive in nature, and not random manifestations that are included simply because they are perceived of as being “kid-friendly”, are often more engaging for learners. This is not to say app designers should avoid “happy” content; simply, that there is a line between genuine good cheer and fluff. The same is true as it relates to exposing children to violence, gore, and frightening content. Like most people, children want to feel safe. Violence, gore, and frightening content can be emotionally unsettling and scary for many people. However, there is a difference between exposing learners to pugnacious content, particularly without educational cause, and introducing topics that have authentic grounding in real life. There are ways in which cognitively sophisticated and emotionally complex subject matter (e.g., war), that are important for learners to consider, can be introduced without causing fear or frustration.

Another example of how dignity potentially might translate in an app, is through the provision of certain types of learning experiences. Tomlinson and McTighe (2006) note, learners need opportunities to provide input and make meaningful contributions to the work at hand (p. 46). Tomlinson and McTighe also note, learners require “respectful work… (with a) focus on what matters most” (p. 162). These ideas are fitting in both virtual and non-virtual contexts.

Additionally, as it relates to previously discussed ideas within this study, communicating dignity within an app also means “respecting” children’s innate, informal, and diverse understandings. By this, I mean the app appears to reflect an underlying belief in, and appreciation of, children's capacity to make sense of the world and their place in it, through terms that are relevant to the learner (e.g., A learner's metaphors and frameworks for meaning are treated seriously- "A function is like when my sister cries. The more my sister cries, the more attention my mommy gives her. The less my sister cries, the less attention my mommy gives her.") Likewise, it also means the app's programming avoids old stereotypes tied to children, like "basics first", "concrete experience only", use of developmental stage theory, and use of universal trajectories. Instead, the app "communicates" high expectations for content and higher-level thinking. (e.g., Learners guided to "think" in complex ways. Tasks are designed to foster complex and creative thinking.)

2.5.3 Features and characteristics of an app.

In traditional curricular contexts, educational media are often invisible (Mishra & Koehler, 2006). Yet. the implication in many early childhood settings is that learning will occur through the manipulation of "realia" that comply with principles of Earthly physics. With apps, this implication is no longer valid. In fact, almost anything is possible. As such, the digital, virtual, structural, and social features of an app must be explicitly discussed, as well as the physical features of the multi-touch, mobile device, on which the app runs. While the list below is in no way extensive, it provides examples of features of multi-touch, mobile apps and the affordances these features allow.

Digital artifacts are often multi-modal. In terms of hardware, this means a device has the capacity to switch between or, more often, combine modes of communicating. A multi-touch, mobile tablet, for instance, allows users to receive and read content in visual, auditory, temporal-motion, and kinesthetic forms. It also has the capacity to allow users to respond to content in

Table 3. Multi-touch mobile app features and affordances.

Nature of Multi-touch, Mobile Apps	-mimicry of Earth World; -creation of virtual worlds; -layering of virtual worlds on Earth world; -multi-modal, multi-media, multi-linguistic; -portable; -finger-sensitive.
Digital Tools	
Physical	-add-ons/ plug ins for external equipment (e.g., screen readers, microscopes); -internal equipment (e.g., camera, flashlight, voice recognition, word prediction, multi-touch recognition).
Collaboration & Communication	-email; -voice & videophones; -internet access to blogs, wikis, social and professional networking sites.
Structural	-non-linear structure (vertical layers, web-like linkages).

these same ways. In other words, the device can "receive" various types of content added by the user (e.g., images, sound bytes, video files, word documents). However, what ultimately makes the artifact useful to the everyday user is the software. Software applications (apps) or websites allow users to upload photos, record sounds, make movies, and manipulate the virtual world through touch and voice control. Designers of software have to consider how a learner may utilize these physical characteristics, which can be enabled through the designer's programming.

By drawing on the features and characteristics of the app-medium, an app designer and her programming can design and enact educational activities, programs, and environments that satisfy certain curricular characteristics. However, it is also important to note, even if an app fulfills these ideas in ways that are unique to the medium, app-mediation does not necessarily add value to learning, in ways that transform how content is understood.

2.5.4 TPACK.

One goal of this study is to outline curricular characteristics that describe how the features and affordances of apps can be used to support learning early algebra concepts with understanding. This is a very specific aim. My interest in specificity is a product of the conceptual framework I embrace, *Technological Pedagogical Content Knowledge* (Mishra & Koehler, 2006). Technological Pedagogical Content Knowledge known as “TPCK”, or the “TPACK” model, provides a narrative and visual illustration of keys types of knowledge that are important for designing and enacting effective, technology-enhanced educational situations. The TPACK model shows the intersection of three primary types of teacher knowledge- namely, technological knowledge, pedagogical knowledge, and content knowledge (Mishra & Koehler, 2006), although, in later models, additional types of knowledge were added- knowledge of context and learners (Koehler & Mishra, 2008).

As I interpret it, the TPACK model offers a broad visual analogy for the ideal outcome that should occur to a teacher’s knowledge, in situations whereby that teacher designs and enacts technology-enhanced educational curricula. That is, specifically, her technological knowledge should intersect with her pedagogical content knowledge in transformative ways- the implied results of which have an impact on the ways she designs and enacts aspects of the technology-enhanced curricula. While the TPACK model has been used primarily to reference the design and enactment of technology integration in traditional learning settings, I posit the TPACK model can also effectively serve to illustrate the ideal outcome between types of teacher knowledge that are used to design and enact technology-*mediated* curricula. The implied results of which, similarly, have an impact on the ways a technology designer (serving in the role of teacher), designs and enacts aspects of the curricula. This is because in both technology-enhanced and technology-mediated learning situations “teacher roles” are executed in analogous ways.

When a teacher in a traditional setting is serving in the capacity of curriculum designer, the efficacy of a technology-enhanced learning environment or activity she designs, is dependent (at least in part) upon her consideration of how a particular technology can support or transform aspects of specific content, for a specific group of learners, in a specific context (Koehler & Mishra, 2008; Mishra & Koehler, 2006; also see Grandgenett, 2008). The implications she derives by considering these knowledge intersections, and the degree to which her design reflects those implications, are key in describing the educational quality of her design. Likewise, when a teacher is enacting a technology-enhanced educational activity, or interacting with learners within a technology-enhanced environment, she is drawing upon the same basic types of knowledge and knowledge intersections, described above. Hence, during both design and enactment of traditional, technology-enhanced curricula, a teacher is relying upon her TPACK. Similarly, in the design of educational apps and their associated virtual curriculum, an app designer serves in the role of “teacher as curriculum planner”. The app designer is relying upon her knowledge of how a particular technology can support or transform aspects of specific content, for a specific group of learners, in a specific context. And, just as in a traditional setting, the implications she derives by considering this knowledge, and the degree to which her design reflects those implications, are key in describing the educational quality of her design. Additionally, the moment a learner participates in an educational app, the learning activities and virtual environment within the app are enacted. During this time, the programming and design of the app, as created by the app designer, serve in the role of "teacher as enactor". Thus, in app-mediated learning, an app designer, ultimately, fulfills both primary roles of the traditional teacher. As such, the TPACK model, as a theoretical framework that illustrates the ideal relational outcome between types of teacher knowledge used to design and enact technology-enhanced educational curricula, is apropos- regardless of whether a curriculum is Earth-bound or virtual.

While, as I see it, Mishra and Koehler's TPACK model (2006) illustrates the ideal relational outcome between teacher knowledge types used to design and enact technology-enhanced curricula, there remains some debate about the nature of the knowledge-relationships this model suggests. Like Mishra and Koehler (2006), most TPACK researchers posit Technological Pedagogical Content Knowledge must reflect an authentic merger of knowledge types. In practice, however, TPACK researchers tend to take two approaches toward realizing this knowledge merger (Angeli & Valanides, 2015, vi). Researchers who align with the first approach visualize TPACK as the outcome of a knowledge merger that preludes a reference to the specific situation of focus (Angeli & Valanides, 2015; see Angeli & Valanides, 2005). In other words, TPACK is conceptualized as the merger of all three knowledge types, whose interconnected parts are impossible to untangle and isolate. In practice, this means only those research findings that are seated explicitly within the TPACK intersection are genuinely relevant to the technology-enhanced, learning situation of focus. For example, as it relates to this study, only those research findings related specifically to app-mediated, early algebra learning with primary children (mutually) are seen as genuinely relevant to this study. This suggests temporarily disregarding any previous research on, say, ways primary children effectively learn early algebra concepts in other settings.

Comparatively, researchers who align with the second approach visualize TPACK as an outcome that occurs during knowledge merger within a specific situation (see Mishra & Kohler, 2006). In practice, this means researchers may look at a relationship between two knowledge types, such as PCK, and subsequently consider how the third knowledge type (i.e., TK) may change what is known about the original knowledge intersection, and thus transform PCK into TPACK. For instance, one aspect of TCK knowledge is that professionals use particular kinds of technology within a content area. Thus, as it relates to this study, when a teacher knows professional mathematicians use calculators in certain scenarios, this reflects a part of her technological knowledge of a content area (i.e., one aspect of her TCK). Further, when she determines how best to share this information with her learners, she is consulting her PK as it relates to this aspect of her TCK. As such, one aspect of the teacher's TPACK, in this situation, is knowledge about how her learners might meaningfully construct an understanding of professional mathematicians' use of calculators in certain scenarios. This particular aspect of TPACK may or may not surface in research focused specifically upon app-mediated, early algebra learning with primary children. Thus, differences in these TPACK approaches have important implications for research and practice.

While research that is explicitly situated at the intersection of TPACK potentially promises the greatest *accuracy* in informing the design and enactment of its corresponding, technology-enhanced learning situation, there are a number of prospective challenges I see in embracing this approach (i.e., the first approach- sometimes called the "transformative" approach), in this study. First, I question whether disregarding research related to the other three knowledge intersections (i.e., TCK, TPK, PCK) is helpful in informing the design and enactment of second-generation education apps. For example, I embrace the idea that previous research on ways primary children effectively learn early algebra concepts in non-app-mediated settings, can provide a starting point for reflecting upon ways an educational app might support this aim, and perhaps even surpass approaches in non-app-mediated settings. Outlining examples of possibilities can help curriculum designers realize what is creatively feasible (see Koehler & Mishra, 2008), while maintaining empirically grounded roots.

Second, with either approach, the issue remains of identifying the latent aspects of TPACK that should be considered- or at least, as many as are possible to identify. Admittedly, as several researchers who embrace the first approach point out, identifying instances of TPACK's subcomponents is "difficult and… (not necessarily) methodologically plausible" (Angeli & Valanides, 2015, p. vi). Thus, they argue for viewing TPACK holistically, instead of as integrative intersections. However, viewing TPACK holistically does not negate the need to examine its multiple layers. For example, even if a researcher were to draw upon a study related specifically to app-mediated, early algebra learning with primary children, that study would still have a particular focus. For instance, it may examine how an apps' programming can effectively scaffold primary children's learning of early algebra concepts. While this hypothetical study is particularly relevant to my area of interest, and although the examination of a series of highly-situated studies, such as these, would certainly help to accurately inform my identification of curricular characteristics, I would have no way of ensuring I have exhaustively gathered study findings related to the major aspects of TPACK. While Angeli & Valanides (2015) have attempted to outline major aspects of TPACK that may be applied universally to any tech-enhanced learning situation, in my mind, this undermines TPACK's intent of specificity. In other words, just because there is existing research on aspects of the specific situation of focus, does not mean the researchers of these existing studies have considered the major aspects of TPACK, as it relates to the situation. Hence, as I see it, with either approach, a theoretical framework is required to outline the major aspects of TPACK related to the situation of focus. While embracing a theoretical framework does not guarantee a researcher will be able to identify the major aspects of TPACK related to the situation of focus, its principles seem to provide a structure for reflection.

Since both conceptualizations of TPACK theory seem to require the additional support of a conceptual framework, I have chosen to embrace the second approach within this study. This is because, as I see it, the second approach (i.e., the "integrative" approach, attributed to Mishra & Koehler, 2006), enables me to consider research related to the four major knowledge intersections within the TPACK model; three of which (TPK, TCK, & PCK), I feel, contribute to my consideration of, not just "what is", but "what could be". Subsequently, the quality of imagination is an essential element of visualizing how elements of a particular curriculum might play out (Koehler & Mishra, 2008), before they have been objectified.

As discussed previously, in a similar study, Hirsh-Pasek and Zosh et al. (2015) abstracted "pillars" of Learning Science (p. 3) that might be used to inform the educational quality of apps. These researchers identified key characteristics of general pedagogical knowledge (PK) and considered how those characteristics might play out in app-mediated learning (TK). Although their aim was broader than mine, the integrative process they utilized, which might be described as PK+TK, is similar to the process I used within this study. With a more specific aim, I identified key characteristics of knowledge along the intersections of TCK, TPK, PCK, and TPACK, related to my situation of focus. Therefore, in acknowledgement of the "integrated approach" I took within this study (see Angeli & Valanides, 2015), I describe my process as moving back and forth between key characteristics related to PK+TCK, CK+TPK, TK+PCK (see *Chapter 2*), and TPACK (see *Chapter 4*).

2.6 Implications for future study.

Educational apps offer great potential to provide a space to support learning with understanding. Currently, however, many apps are not living up to their "educational" title. Thus, if apps are to serve in this position well, app designers need to know what curricular characteristics ideally support this educational goal. Development of a detailed coding frame that outlines these characteristics may be the first step toward a taxonomy that might, later, assist in providing guidelines to app designers. Likewise, by applying the coding frame to a handful of authentic apps, designers might better understand the extent to which these apps are currently aligning with ideal characteristics.

Chapter Three:

Methods

3.1 Research Design

3.1.1 Purpose of study.

The first purpose of this study was to outline characteristics of curricula that ideally support *learning with understanding* (Bransford, Brown, & Cocking, 2000, p.8), as it relates to primary children's app-mediated learning of early algebra concepts. The second purpose of this study was to use qualitative content analysis to describe the ways in which, and the general extent to which, the curricular components of three current iOS, mathematics education apps for primary children, compare with the "ideal" characteristics of curricula, I previously outlined. To this end, my research questions are as follows:

1. What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?
2. To what extent do three, multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

To answer these questions, I adopted a qualitative approach to this research. In general, the use of a qualitative approach in study enables a person to interpret covert material by allowing for the exploration of personal and social meaning. Specifically, I aimed to analyze certain "textual" meanings of multi-touch, mobile iOS software applications (apps). Hence, to accomplish the kind of descriptive analysis I was seeking, on the specific content of my research interest, I utilized a method of research called Qualitative Content Analysis (QCA).

QCA is a systematic method for "describing the meaning of qualitative material" (Scherier, 2012, p. 1). It is an established empirical method of study, calling for the creation of a coding frame that contains categories, definitions, examples, and indicators, and later, the application of these descriptors to the material of focus. This type of approach enables a researcher to focus on the contextual particulars of a situation, and study a phenomenon in depth (Schreier, 2012, p. 28), and often, across multiple descriptive facets. In this study, because of the latent nature of the meanings within the multi-modal texts (verbal, visual, conceptual), and my desire to focus on the contextual particulars of the texts, qualitative content analysis (QCA) offered a fitting methodological choice.

Qualitative content analysis (QCA), differs from other qualitative methodologies, in that, it requires a researcher to assign *every* relevant unit of coding taken from the material to at least one subcategory of the coding frame during analysis (Schreier, 2012) [see Exhaustiveness]. This diverges slightly from other qualitative approaches that ask the researcher to abstract themes from data, but not necessarily to divide all relevant data into units of coding and classify each unit. Additionally, QCA differs in its aim to reduce data instead of offering expanded views (Schreier, 2012, p. 7). While this characteristic is also found in forms of reductive coding (Schreier, 2012, p. 38), other qualitative approaches tend to use coding as a conceptual device or otherwise aim to "open up" expanded meanings of texts (Schreier, 2012, p. 39; also see Coffey & Atkinson, 1996; Saldaña, 2009). Thus, in QCA, the description of analytical findings is typically expressed in a way that is less narrative than other forms of qualitative research, and which provides qualitative description, at least partially, through the extent to which the text material satisfies categories of the coding frame. In other words, in QCA, the detailed description that tends to be a cherished part of qualitative research is typically found within the categories, definitions, examples, and indicators of the coding frame, instead of within verbose narrative passages.

In some ways, QCA differs from other types of qualitative analysis, such as in its use of traditional quantitative criteria for assessing the quality of the coding frame (e.g., validity, reliability), but as a method it shares most similarities with qualitative approaches. First, as in all forms of qualitative study, QCA researchers do not attempt to infer that patterns interpreted from the phenomenon under study, apply to a larger population or other phenomena. Second, the aims of QCA do not center on frequency counts of highly discernable content, as found in quantitative approaches to content analysis. Instead, QCA researchers seek to "systematically describe the meaning of qualitative material… by classifying (its) parts… as instances of the categories of a coding frame" (Schreier, 2012, p. 8).

Although, as in most qualitative research, the findings of a QCA are never presented as the only interpretation of meaning, they are meant to represent the conclusions of those who share the perspective of the researcher (Schreier, 2012, p. 34). As such, a primary objective of a systematic analysis like QCA is to help a researcher not only exceed his or her own current understanding (Scherier, 2012, p. 6), but also to represent the viewpoint of peers who share the researcher's perspective. This is achieved in QCA by the researcher's adherence to a step-by-step process that leads him or her to generate well-defined categories and definitions that are typically both concept-driven and data-driven, and to conduct a data analysis using these categories. Additionally, in both the category generation phase and the data analysis phase, the researcher consults like-minded peers to check for reliability. Likewise, to ensure peers share the perspective of the researcher, as well as to safeguard readers' authentic vicarious experience of the case, the researcher's thinking and beliefs are made as transparent as possible.

3.1.2 A brief history of QCA.

QCA is not used very widely (yet) in the US, but is frequently employed as a research method in Europe- particularly in Germany. Yet, even within the European nursing profession (long time qualitative researchers) there still remains debate over related terminology (i.e., "thematic analysis" versus "content analysis") [see Vaismoradi, Turunen, & Bonden, 2013]. Perhaps this is because, historically, in many parts of the world, content analysis has been synonymous with frequency counts and the identification of concepts that are easily discernable. In contrast, thematic analysis has focused on the study of ideas embedded within material that tend to be latent; the "conceptual". For this reason, some qualitative researchers may label QCA as a form of thematic analysis, or even concept analysis. Without entering into debate over terminology, I have chosen to utilize QCA in this study because many researchers consider it to be a unique method in its own right (see Groeben & Rustemeyer, 1994; Hsie & Shannon, 2005; Hussy, Scherier, & Echterhoff, 2009; Mayring, 2010; Rustemeyer, 1992; Scherier, 2012). While concept analysts tends to utilize techniques such as cluster analysis to interpret meanings behind concept maps (see Kane & Trochim, 2007), QCA researchers use categories to classify concepts into descriptive groupings and interpret meanings from patterns that emerge from the classification process. Similarly, while various forms of thematic coding use categories in a way that's similar to QCA- namely, to reduce and describe data- its methodological steps seem less defined than those found in QCA.

In summary, while QCA retains a few qualities that can be linked to its historical roots in quantitative content analysis, its methodological approach and its aims are distinctly qualitative in nature. Additionally, its unique combination of attributes is valuable in circumstances like those of this study, whereby a sizeable group of multi-modal "texts", like apps, need to be analyzed for concepts that are primarily covert. As such, by creating a descriptive coding frame and "reading" three primary, mathematics education apps as texts in order to interpret a specific part of their hidden meanings, this study has resulted in a number of practical and theoretical implications for the field.

3.2 Significance of the Study

The practical consequences of this study, potentially, are two-fold. First, these findings provide a snapshot or preliminary sense of the absence or presence of specified curricular characteristics across three current mathematics apps for primary children, as a result of comparing the characteristics of these apps with "ideal" curricular characteristics of "learning with understanding", abstracted from decades of research in Learning Science. Second, by sharing these descriptions with the larger research community, these ideas can begin to inform, or at least extend the conversation regarding, characteristics essential to the design of future apps. Specifically, I hope descriptive details of the coding frame will provide inspiration for tangible ways in which global-American citizens might be able to recapture the educational goal of learning with understanding, with a context that is hypothetically less influenced by politics and large-scale economics than the current, typical, public primary classroom.

In terms of theoretical consequences, this study potentially contributes to the field in multiple ways. First, this study illustrates an example of applying the TPACK model (Mishra & Koehler, 2006) to generate specific, research-based characteristics of a technology-mediated (instead of technology-infused) curriculum. While some TPACK researchers (e.g., Angeli & Valanides, 2015) may argue this coding frame, as an outcome of Phase I of this project, stops slightly short of representing a complete or authentic application of the TPACK model, it may contribute to larger discussion on interpreting and applying the still-emerging TPACK framework.

3.3 Data Corpus

In QCA, there are two distinct phases in the data analysis process. The first involves determining the dimensions you aim to examine through your study, and their subsequent subcategories and category definitions- which include a category name, description, example, and set of decision rules (Boyatzis, 1998; Rustemeyer, 1992; Schreier, 2012). In other words, Phase I involves creating the coding frame. Phase II involves applying the coding frame to the material, by classifying instances of the material into categories, and observing classification patterns. As such, in this study, there were two distinct types of material from which to collect and analyze data- professional literature and apps.

3.3.1 Boundaries of the material.

In general, my research interests in science, technology, engineering, and mathematics (STEM) education, as it relates to young children focused my attention to this particular research topic and group of apps. As such, and because of my own experience as a parent, teacher educator, researcher, doctoral student, and former preschool and primary classroom teacher, I recognized that mathematics apps for young children were worthy of investigation. My familiarity with the historical changes occurring in US mathematics education (in particular with early algebra education), and the evolution of early childhood pedagogical beliefs and learning goals, as well as my awareness of the educational position of apps within the context of contemporary society, provided me with an initial sense for the potential depth of meaning within the material- both in terms of professional literature and apps. Specifically, however, the boundaries of the material, focused upon within this study, were circumscribed according to more explicit guidelines. Additionally, in this case, I intentionally excluded the curricular component of "Family" from this study.

3.3.2 Specific criteria for selecting literature.

As its primary focus, this study aimed to describe curricular characteristics that ideally support primary children's potential to learn early algebra concepts with understanding, through multi-touch, mobile, iOS mathematics education apps, by generating a coding frame that captured these characteristics. Since the number of potential curricular characteristics, was enormous, I established specific boundaries for selecting concept-driven material. The TPACK model (Mishra and Koehler, 2006) [see Chapter Two], and its subsequent conceptual intersections, was the chief guide in helping me determine the boundaries for selecting concept-driven material.

Once boundaries were established, I found I needed to further delineate the relevant aspects of the literature from the irrelevant, in order for the material I selected to align with the ideals of my research focus- primarily, supporting the educational goal of learning with understanding. As such, beyond the TPACK model (Mishra and Koehler, 2006; 2008) serving as a broad theoretical framework for binding the totality of my material, the tenets of Learning Science in which the educational goal of learning with understanding is based, served as a means for isolating relevant pieces of material within the broad boundary. In this way, tenets surrounding the educational goal of learning with understanding served as the conceptual framework for selecting material, particularly as it related to Pedagogical Knowledge (PK) and it's corresponding knowledge intersections (i.e., PCK; TPK). Likewise, early algebra education served as an additional criterion for selecting material related to Content Knowledge/

Pedagogical Content Knowledge (CK/ PCK), and it's corresponding knowledge intersections (PCK + PK-related to Learning Science; TCK). As the final criterion for selecting material, app features/traits and app-based education served as the last informant. This classifier informed the selection of materials related to Technological Knowledge/ Technological Pedagogical Knowledge (TK/ TPK), and its corresponding knowledge intersections (TPK + PK-related to Learning Science; TCK). Hence, in summary, the following criteria guided my selection of literature.

1. Learning Science tenets surrounding the educational goal of *learning with understanding*- As a chief premise of this research's rationale, I sought literature related to this ideal, which was first defined by Bransford, Brown, & Cocking (2000, p.8), as a primary goal of education. This ideal served as the primary conceptual framework through which I filtered potential curricular characteristics;
2. … As they relate to primary children's education- In addition to my focus on Learning Science principles related to learning with understanding, I was particularly concerned with the way these tenets circumscribe the education of primary children (approximate ages of 6-8 years old). Hence, "primary children's education" served as an additional conceptual lens through which potential curricular characteristics were vetted;
3. Early algebra (EA) education and concepts- given my research focus in STEM (Science, Technology, Engineering, and Mathematics) education and primary children, the topic of EA was identified as a topic of particular professional interest. Since much learning of EA concepts occurs with primary children, this criterion was pre-satisfied. Hence, I sought research related to early algebra

education and early algebra content concepts, particularly at their overlap with curricular characteristics that might support learning with understanding, and app-based or tech-mediated learning;

4. App features/traits and app-based education- In addition to qualifying as an additional topic of particular professional interest within my research focus, apps served as the medium of the material I examined within this study. Thus, I sought research related to " multi-touch, mobile iOS apps", app-mediated learning with primary children, and app features/ traits that might support learning with understanding. I also considered app learning at its intersect with early algebra and mathematics education.

Additionally, I needed to consider potential curricular characteristics related to the data-driven portion my coding frame development. Since the potential number was also enormous, I established specific boundaries for selecting the data-driven material. In large part, these boundaries were influenced by the secondary focus of my study, because the nature of QCA demands that a researcher adapt one's coding frame to the specific material of one's study. Provided that the secondary focus of my study aimed to compare the curricular characteristics of a handful of real apps with the ideal curricular characteristics outlined in my Coding Frame, the details of this latter aim provided guidelines for binding the material for the data-driven portion my coding frame development. Since my study focused upon curricular characteristics that ideally support primary children's potential to learn early algebra concepts with understanding, through multi-touch, mobile, iOS mathematics education apps, "multi-touch, mobile, mathematics education, iOS apps for primary children" describe the broad boundaries of my data-driven material. Specific criteria I followed for the selection of these apps within the secondary aim of this study are listed below.

3.3.3 Specific criteria for selecting apps.

As mentioned above, part of the aim of this study's secondary focus, was to apply the coding frame to a handful of real apps, in order to compare the ideal curricular characteristics with those of the real apps. Additionally, I also aimed to describe the general extent to which the two sets of characteristics aligned. Fortunately, the same set of criteria for selecting apps supported both aims. Since the size of Apple's app marketplace is vast, with over 1.5 million apps available (as of May 2017)- 150,000 of which are presented as children's educational apps- I applied a specific procedure for selecting the material for the application phase of this study.

Further, my decision to select three apps was based on a number of factors. First, after a review of comparable studies, the number of apps reviewed by each researcher or research team was variable, depending on the nature of the research. Some studies showed that researchers reviewed between 50 to 100 apps at a time, while others examined far fewer (e.g., Chau, 2014; Handal, El-Khoury, Campbell, & Cavanagh, 2013; Shuler, 2012; Watlington, 2011). This variance in numbers seemed to depend upon the position of the study along the qualitative/ quantitative research continuum.

Second, a Qualitative Content Analysis (QCA) involves the generation of a formal coding frame, as compared with other qualitative studies that generally do not. Developing a formal coding frame is a challenge; generating a coding frame of "high complexity" (see Scherier, p. 67), as my coding frame is, was a sizeable undertaking. As this relates to other QCA studies, the dimensions and hierarchical levels of my coding frame are much more extensive than the number found in comparable studies.

Third, and perhaps most importantly, I was only able to identify four apps that fit within the parameters of my selection criteria, and one did not download correctly (it seemed to have some bugs). Thus, three was the total number of apps available for my analysis, without expanding the parameters of my search.

Given these factors, limiting the number of apps to three seemed fitting. Application of the Coding Frame across three apps provided enough material by which to “pilot test” the frame and make procedural adjustments, provide some data-driven concepts to shape the content of the coding frame, and also provided enough material to create a snapshot of the extent to which some mathematics education apps for this age group compare with 95 ideal curricular characteristics.

Thus, I selected the apps, for the application phase of this study, by identifying apps that satisfied the following criteria. It should be noted, these measures draw heavily upon sampling criteria in Chau’s study (unpublished dissertation, 2014, p. 76), though they differ in some respects.

1. iOS apps labeled “educational” by their creators- A “Category” search within Apple’s App Store offers “Education” as one of the options. I selected this category because I sought apps that were labeled by their creators as educational;
2. iOS apps within the “Elementary” collection- I sought apps targeted for primary learners; the approximate age range of six to eight years old;
3. iOS apps with mathematical content- Not all elementary education apps contain mathematical content. Since this study focused on early algebra concepts, only those apps that contained mathematical content were included. It should be noted, I looked across two “domains” (as presented in the App Store) of mathematical

apps for this age group. This is because early algebra concepts are often embedded across mathematical domains. I ended up selecting apps from both of these domains (i.e., the "Number System" and "Beyond Drill- Strategy"). Other mathematical domains included: "Drill & Practice", "Shapes & Spatial Reasoning", "Measurement & Data", and "Beyond Drill- Brain Busters". After a search through all of these categories, the only other apps that appeared as though they may have contained algebraic content, were two apps found within the "…Brain Busters" category;

4. $4.99 and under- I decided on $4.99 as the upper limit of my price point, because I sought apps that were free or "affordable". This is, of course, a relative term. However, the prices of many apps seemed to increase sharply, after the $4.99 price point. The average price of a meal at McDonald's is currently $5.00, so I chose this as my measure of affordability;

5. Target age range of six to eight years old- Apple's App Store allows app designers to choose from three main age groups for categorizing children's apps- five years old and younger, ages six to eight years old, and ages nine to eleven years old. Since this study focused on apps intended for primary children, I included apps that fall into the second age group (i.e., six to eight year olds);

6. Other parameters- No apps from textbook publishers; No apps for an entire year's worth of mathematics, along a grade level (e.g., second grade math);

7. Full Version- Many apps offer a free trial version of an app, as well as a full version available at cost. Since the free trial version of an app often contains only a fraction of content, and can have incomplete functionality, I downloaded and

purchased (if required) full versions of the apps. This assisted in allowing me to fully evaluate the material;

8. Available on May 1, 2017- Allowing that the inventory of the App Store grows and changes daily, I collected all data on the same day. In order to avoid confusion, I did not include apps in this study that were unavailable on May 1, 2017;

9. English content on the US App Store- At the time of this study, Apple's App Store was offered in 126 countries and regions, as a virtual entity. Likewise, each regional marketplace offered a different selection of content. Due to this study's focus on early algebra education in the United States, only material from the US App Store was included. Additionally, as the primary researcher, since my principal language proficiency is in English, I included only those apps written and presented in English;

10. For Apple iPhones- I selected those iOS apps that were created specifically for Apple iPhones, for two reasons. First, the ubiquity of mobile phones is well noted. iPhones are more prevalent than other mobile iOS devices, such as iPads and iPod Touch(es). Second, apps for the iPhone can also be "mirrored" on iPads, but not vice-versa (i.e., an iPhone app can be projected onto the screen of an iPad for play, but an iPad app cannot be projected onto the screen of an iPhone).

Thus, my material included: Apple iPhone mathematics education apps, related to the "number system" and mathematical "strategies beyond drills", the full versions of which were presented in English and found in the US App Store on May 1, 2017, aimed at children ages six to eight years old.

3.4 The Coding Frame

3.4.1 Structure of the coding frame.

In QCA, coding frames typically include specific key elements. Those elements include dimensions (main categories), subcategories, and category definitions. Categories, both main and sub, are particularly important since they are the “filter through which (a researcher) views (her) data” (Schreier, 2012, p. 90). As such, several considerations must be taken into account during category generation.

First, dimensions and their subcategories can be developed in a way that is either data-driven (based on the material), concept-driven (based on previous studies, theory, and logic), or both. As discussed previously, my study was both concept-driven and data-driven, although the extent of the data-driven aspects were limited to the material of the three identified apps.

Second, in QCA, it is important for the researcher to consult with others who can notify her of material she inadvertently overlooked (p. 94), as well as, those who can scrutinize the categories of the coding frame, itself. In my study, I called upon volunteers to assist me in the ways mentioned above and, later, to aid in the process of classifying data from the app, within categories of the frame.

After a researcher has outlined her coding frame by deciding what the dimensions and categories look like, she must define the rules for coding the data, by creating category definitions and other details of the coding frame (Schreier, 2012, p. 94). These details include naming the categories and subcategories, and detailing what is meant by a specified category name, by either furnishing indicators of the category, by describing characteristics of the category, or both. Additionally, the researcher provides examples that illustrate the subcategories, and decision rules if there is conceptual overlap between categories. In Chapter

Four, there is a snapshot detailing the structure of my finished coding frame (Version 2) [the Coding Frame in its entirety is available in *Appendix A*.]

3.4.2 Evaluating the coding frame.

Criteria for evaluating the quality of qualitative study components can be diverse. Schreier (2012), on whose description of QCA I most heavily rely, suggests a range of criteria for evaluating the overall study design, as well as the coding frame generated by the primary researcher. Since, the coding frame is the “heart” of QCA (Schreier, 2012, p. 58), much attention is given to its quality. As such, Schreier suggests the following criteria for evaluating one’s coding frame. I have tried to meet each of these criteria within this study.

3.4.2.1 Reliability.

According to Schreier (2012), reliability plays two major roles in QCA (p. 35). Since double coding is often used as a technique for achieving reliability in QCA, the first role relates to consistency of coding between researchers (or research consultants) and between time periods. Double coding is a technique whereby the researcher either completely codes or classifies her material over two distinct time periods, or utilizes other like-minded consultants to classify her material, in the aim of achieving consistent results. Sometimes QCA researchers utilize both approaches, as I did in this study.

While consistency scores (e.g., coefficients of agreement) are sometimes reported in the final research report, if coherent interpretations are not achieved over different time periods or between coders in a QCA study, typically researchers use this information to adjust the coding frame or analytical interpretations of the study, instead (Scherier, 2012, p. 167). Since, inconsistency is a sign the categories of one’s coding frame are not defined with enough clarity, revising one’s coding frame or analytical interpretations, in order to achieve greater articulacy, results in immediate resuscitation of the study's quality (p. 168-169). However, even when one's goal is to resolve any disagreements, reliability is more difficult to attain when researchers work with material filled with latent meaning (Scherier, 2012, p.16). Consequently, in addition to utilizing a double-coding technique, QCA researchers also aim to increase the reliability of their study by exercising transparency and systematicity (Schreier, 2012, p. 35). To achieve these aspects, researchers communicate to their readers how they arrive at their interpretations with as much clarity and forthcoming as possible (p. 34). Likewise, they follow the same sequence of steps each time they code material and require consulting coders to do the same.

Within this study, consistency was a challenge, initially, in two small cases. As suggested by QCA researchers, I spoke to the coder with whom I disagreed and it became obvious that, (a) In one case, I had not defined a category clearly enough and we had interpreted its meaning in two different ways and, (b) We disagreed on the extent to which the app fulfilled a characteristic defined in the frame. In both cases, discussion easily remedied the differences in interpretation, and the coding frame and results of the coding were revised, accordingly. Additionally, I have aimed for both transparency and systematicity by utilizing the techniques, suggested above.

3.4.2.2 Validity.

There are two types of validity to consider when designing a qualitative study. The first relates to the overall quality of the study and design, and whether the design and methods are laid out adequately and represent an effective way to go about answering the research questions. The second type of validity is specific to QCA. As it relates to the QCA coding frame, validity refers to how well a researcher's categories represent the concepts in her research questions (Scherier, 2012, p. 7). According to Scherier, a coding frame that has been tailored to the material has greater validity than one that has not. Thus, in this study, while my categories were initially generated based on findings from the professional literature (i.e., concept-driven categories), they were also customized to reflect the material in the study (i.e., data-driven categories).

However, despite the fact my coding frame is both concept-driven and data-driven, it is only intended to describe the specific data I analyzed (Schreier, 2012, p. 17). Thus, in describing specific curricular characteristics, and the subsequent extent of their presence (or lack thereof) in this group of apps, I do not extend my analysis to describe other groups of apps, learning effects that may result from interacting with the app, or conditions under which these apps were designed and produced. Therefore, this study is a systematic, descriptive inventory of the curricular characteristics of three apps, as they compare with "ideal" curricular characteristics suggested by the professional literature. It's purpose is to potentially inform constituencies (i.e., consumers and app designers) how scholars and teacher educators in the field of early childhood mathematics education might classify the curricular characteristics of these apps, as they relate to supporting primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, mathematics education, iOS apps.

3.4.2.3 Exhaustiveness and mutual exclusivity.

Exhaustiveness is another indicator of a high-quality coding frame (Holsti, 1969; Rustemeyer, 1992; Schreier, 2012). A coding frame is said to be exhaustive if the researcher can assign *every* relevant unit of coding, taken from the material, to at least one subcategory in the coding frame (Schreier, 2012, p. 76). Since all researchers invariably bring personal biases to their work, systematically assigning each applicable unit of coding to a subcategory helps to ensure the researcher transcends biases (many of which she may not be aware of). As such, exhaustiveness is closely related to validity (Schreier, p. 77). The need to achieve exhaustiveness is also the main reason why each hierarchical category in a coding frame typically has a miscellaneous category. A miscellaneous category serves as a place for pieces of information that are unique, and is an important tool in securing exhaustiveness.

In this study, I achieved exhaustiveness by asking coders (including myself) to take observational notes of the app's characteristics (as well as several screen shots) during his or her "tour" of the app's program and virtual environment, as well as during their participation in the app activities. Coders could, then, classify these observed characteristics by matching them to categories of the coding frame. Any characteristics "left over", which did not fit within a category of the frame, were placed within a "miscellaneous" category. Additionally, the process was reversed, whereby coders looked for each categorical descriptor of the coding frame within the material of the app, and identified those that were missing from the material. As results will show, in this study, plenty of descriptors from the coding frame were missing from the material (an important finding), but the only descriptors missing from the coding frame can be described as characteristics of gaming theory design, and were intentionally excluded from the study (and, thus, irrelevant).

Similarly, categories within a coding frame are considered mutually exclusive if a unit of coding can be assigned to *only one* category, within a given dimension (Schreier, p. 75). If the categories are not mutually exclusive, this may indicate categories are too broad or vague and it may be difficult for a researcher to judge where to place a unit of coding, as it may fit under multiple categories. Thus, mutual exclusivity is another mark of quality, as it safeguards against researchers accounting for data more than once or classifying data in ways that differ from one another. For this reason, mutual exclusivity is closely linked to reliability. In this study, I achieved mutual exclusivity, in part, by engaging in many rounds of editing the coding frame. I combined categories of overlap and removed redundant categories. I also outlined a number of “decision rules”, which helped to further differentiate between certain categories, in order for coders to more easily determine where to place a unit of coding.

Additionally, mutual exclusivity is a benchmark of structural completeness. Not only does a structurally complete coding frame require the inclusion of all possible subcategories within a given set (Scherier, 2012, p. 93), those categories must be mutually exclusive of one another. In this study, this latter aspect of mutual exclusivity was initially a challenge. This is because, the TPACK framework (see Mishra & Koehler, 2006), through which I generated categories of the coding frame, focuses on the conceptual intersections of technological, pedagogical, content knowledge. As such, the categories that resulted from applying this framework were not always exclusively bound from one another (see Chapter 4 for further discussion). Ultimately, I achieved mutual exclusivity by designing “layered” categories, the individual characteristics of which are independent of one another, despite their relational proximity. I also achieved mutual exclusivity by assigning units of coding to only one category within a particular dimension. Additionally, to ensure structural completeness, I drew upon logic (in the everyday sense) to ensure all possible subcategories within a given set were included in my frame. For example, since one of my subcategories is “Meets”, it follows I also need the subcategory “Does not meet”.

3.4.2.4 Saturation.

In qualitative research, a single dimension of interest is considered to be of adequate size to study because the focus of such research is on understanding the particulars of the specific situation; its main objective is not generalization. In this sense, *saturation* (Strauss & Corbin, 1998, p. 136) is reached in a QCA because it results in a deep exploration and description of the particulars surrounding at least one dimension of interest. While my study satisfies this definition (i.e., I explore and describe two dimensions of interest), an additional meaning of saturation requires further discussion, as it relates to this study.

Saturation also refers to the amount of data it takes for a researcher to reach a satisfactory understanding of phenomena occurring across or within his or her identified dimensions of interest. In QCA, saturation is generally said to have occurred when the researcher stops adapting the coding frame because viewing additional material no longer produces insights that result in new or revised categories (Schreier, 2012, p. 91). As this relates to my study, saturation was easier to recognize within Version 1 of my coding frame, which included concept-driven curricular characteristics based on *TPACK* dimensions (Mishra and Koehler, 2006). In Version 2 of my coding frame, which includes data-driven curricular characteristics from three apps, the question became whether data from three apps led to enough saturation to assure all aspects of the identified dimensions were well represented. This is an important consideration because it supports the comparability of diverse material. As it relates to this project, the answer to this question is both "yes" and "no".

Since this study represents the first phase of a potentially larger long-term project, the inclusion of curricular characteristics from three apps is enough, at this time. As such, an important caveat is worth mentioning. That is, the coding frame used within, and resulting from Phase I of, this study is not yet ready to be applied to other educational apps, including those that are similar to the apps analyzed in this study, without its undergoing further research. Schreier notes, within QCA, the more diverse a researcher's material, the better the odds she will need to view *all* material before the coding frame is complete (p. 91). Given the rate at which iOS apps are being developed, in addition to the sheer volume of existing apps, it may never be possible to view all material before a "complete" coding frame is realized. However, due to the diversity of material found in mathematics education iOS apps for children, I anticipate future researchers will need to view more material and adjust this coding frame accordingly, before it reaches a more satisfactory point of saturation.

Further, in addition to providing a criterion for evaluating the quality of a coding frame, saturation is also defined a third way. It is sometimes used in QCA to refer to the idea that a researcher must use each subcategory of his or her coding frame at least once, with no subcategory remaining unused during the data collection process (Schreier, 2012, p.77). However, this criterion is not applicable in all situations. When utilizing a coding frame that is highly concept-driven, lack of saturation (i.e., The presence of unused categories within the frame) can serve as a tool for analysis. Indeed, non-saturated coding frames can be particularly valuable in revealing the absence of concepts in material (Rustemeyer, 1992; Scherier, 2012). In other words, empty categories may show gaps between theory and practice. Consequently, as it relates to my study, saturation in this latter sense was not applicable as a measure of quality. Rather, non-saturation of my coding frame was used as a filter for revealing a pattern of characteristics that appear to be missing from current educational mathematics apps for this age group (see Chapter 4).

3.4.2.5 Unidimensionality.

Yet another indicator of quality is unidimensionality. This signifies that the main categories or dimensions through which the researcher views a study cannot be enmeshed (Scherier, 2012, p. 75). In other words, the researcher cannot create categories that are mutually inclusive of one another, or attempt to portray how two or more dimensions relate to one another. The researcher can study relationships between dimensions during a successive stage of data processing, or by using software to check for co-occurrences of a phenomenon (Scheier, p. 75).

Thus, during the main data analysis phase (i.e., Generation of the frame), two categories cannot be conceptually interwoven.

In this study, although I faced the challenge of converging conceptual intersections, related to technological knowledge, pedagogical knowledge, and concept knowledge (i.e., TPACK) in my generation of coding frame categories, I was able to achieve unidimensionality by creating separate categories within the coding frame for each of my two dimensions of interest (i.e., "How" particular characteristics are ideally imparted within a curriculum, and "What" particular characteristics are ideally imparted within a curriculum). Further, across both dimensions, I analyzed every subcategory to ensure each only attempted to capture either the "How" *or* the "What" of curricular characteristics (the ways in which), and not both simultaneously. As a result, I have more subcategories within my coding frame than typical QCA researchers, because of my need to isolate the highly related dimensions of each of my characteristics.

3.5 Data Collection

In QCA, the term "text" is often used as a broad term to mean all types of qualitative material. Hence, QCA is considered to be an effective approach for analyzing texts whose meaning is less discernable or uniformly agreed upon (Scherier, 2012, pp. 2-3), or in which there is an abundance of rich material with many conceptual layers. However, in order to analyze the meaning within these conceptual layers, the researcher first has to collect and organize the embedded data. Therefore, in QCA, data collection can involve a number of steps.

In Phase I of this study, data collection involved three parts. First, it involved turning raw material from the determined *unit of analysis* (Scherier, 2012, p.) into relevant data or *units of coding* (Boyatzis, 1998, p.; Krippendorff, 2004, p.; Scherier, 2012, p. 131). In this case, my primary units of analysis were, (a) The body of documents found in the professional literature and, (b) The body of apps, as multi-modal texts. While, at times, a single unit of analysis can contain several units of coding- the nature of which can vary depending upon the subcategory considered by the researcher at the time (Scherier, 2012, p. 132)- the unit of coding did not vary within this study. The same unit was examined within each of two dimensions of my Coding Frame. Within both dimensions, the unit of coding was "curricular characteristics" embedded within the literature and the apps, respectively. Identifying instances of this unit of coding (i.e., Instances of curricular characteristics) within the literature and apps was accomplished by utilizing *contextual units* (Scherier, 2012, p.), found within the literature and apps.

Second, I organized these curricular characteristics into the distinct structure of the Coding Frame. This resulted in Version 1 of the Coding Frame (see 3.3 *The Coding Frame*, above, for the specific procedure of coding frame development). Third, I "pilot tested" the Frame by applying it to each app. The purpose of this was to adapt the frame to reflect any ideal curricular characteristics that may have been present in the apps, but not within the literature (the data-driven content). I also drew upon the assistance of volunteers- namely, other early childhood education researchers and mathematics education researchers, whose educational philosophies primarily match my own (i.e., A team of individual "frame generators"), in order to assist me in identifying any curricular characteristics I may have missed (both from the literature and from the apps). They also assisted me in editing the structure of the Coding Frame. Finally, I made changes to the content and structure of the Frame, as recommended, and evaluated its quality (see *Evaluating the Coding Frame*, below).

By utilizing volunteers to assist me generating and organizing categories of the coding frame, and through my own multiple iterations of Frame editing (as commenced over an extended time period), increased the validity of the Frame categories and descriptors, through triangulation. Triangulation is the use of multiple forms of data collection, data sources, theories, and analysts in order to corroborate evidence for the validity of qualitative research findings (Dedrick, personal communication). Since the Coding Frame contents represented the "findings" from Phase I of this study, triangulation was an effective means for increasing validity.

In Phase II of this study, data collection involved three main parts- download, data collection, and "segment(ation)" (Scherier, 2012, p.). First, since the nature of apps and their subsequent curricular components can be altered at any moment [by their designer(s)], it was important to "preserve" this raw material by downloading all apps at a single point in time (for all coders). Next, there were two sets of curricular characteristics I needed to segment and collect. Since the primary unit of analysis is based on the kinds of categories generated by the researcher for the coding frame, my study aimed to compare the "ideal" curricular characteristics with those of real apps. Hence, I provided space for all four of these processes by creating an *App Observation and Classification Form*, on which all data could be collected and segmented.

As such, one space on the Form is for "Observational Notes". In this space, coders can record their observations of the app during play and participation. Specifically, coders were guided to: (a) Tour the environment and programmatic features of the app for approximately 20 minutes, and play the major individual learning activities within the app and, (b) Look for curricular characteristics of the app and record these observations in bulleted form within the "Observational Notes" section of the *App Observation and Classification Form*. After this, coders were asked to classify their observations according to the categories of the Coding Frame, which also were provided on the form. Any observations that were "leftover" were assigned to a

"Miscellaneous" category. In other words, the space for "Observational Notes" secured a place for coders to collect and segment data of the real app and classify it under the categories of the Coding Frame.

The second space on the form is called, "Guiding Questions". I created these questions based directly on the categories of the Coding Frame. They outline the "ideal" curricular characteristics, as informed by the concepts of the Literature and data of the three apps, and essentially translate the content of the Coding Frame, into a form that is more "coder-friendly". Using these two aspects of the Form, in tandem with one another, assisted me (and other coders) in, (a) Determining relevant and irrelevant material in the app, (b) Segmenting the relevant data from the app, into units of coding and, (c) Collecting and classifying the data from the app, according to the categories of the Coding Frame.

Since the modes of the app-texts are auditory, visual, kinesthetic, conceptual, and temporal (i.e., multi-modal), utilizing a template (i.e., The *App Observation and Classification Form*) that asked the coding team to gather relevant data through four collection techniques (participant observation, screen shots, a written description of key elements, and answering conceptual questions) increased the validity of the data through triangulation. As well, using multiple collection "instruments" (i.e., Individual members of the "coding team"), and multiple collection time periods (i.e., Two distinct times, 14 days apart), also increased the validity of the data collected.

Thus, to collect data for this study, coders and I abided by the following procedures:

1. I downloaded the selected apps onto one iOS mobile device. (Each coder downloaded these apps on the same day as one another, on their respective devices.);

2. I participated in each learning experience offered by the app, and explored the program and virtual environment within each app, to develop a broad sense of the material;
3. I applied Version 1 of the coding frame (i.e., The concept-driven frame, based solely on literature) to determine the extent to which this frame reflected all relevant material across the three apps. This helped me identify *all* relevant material from each app, and ensure it became part of the structure and substance of the coding frame. I, then, made adjustments to the coding frame, so it reflected additional data-driven material, not originally included in Version 1 of the frame. This resulted in Version 2 of the coding frame. In this way, this process helped me “overcome the shortcomings of (my own) everyday understanding” (Scherier, 2012, p. 5) by generating a coding frame that was both data-driven and concept-driven (Schreier, 2012, p. 33);
4. I also amended my Literature Review to include these additional data-driven ideas;
5. I created the *App Observation and Classification Form* as a data collection tool; This form reflects the content from Version 2 of the Coding Frame, but organizes it in a format that makes it easier for coders to locate relevant data within the app, and classify the data under the subcategory he or she determines most fitting;
6. I utilized the *App Observation and Classification Form* for each of the three identified apps. This form accomplishes multiple functions. First, it offers a uniform format for observing, participating in, and describing the app. This section of the form asks the coder to, (a) Tour the app and participate in the

learning experiences of the app for approximately 20 minutes, in order to become familiar with its curricular characteristics, (b) Capture digital screen shots during tour and participation and, (c) Take "observational notes" to describe the features of the app and its curricular characteristics, such as characters that are utilized, activity objective, and a summary of the activity. Second, this form offers "Decision Rules" (from the coding frame) in the form of "Guiding Questions" which, when answered by the coders, helps him or her determine the subcategory under which the data from the app should be classified. Third, the form offers a place for the coder to mark his or her decision about where data should be classified. This allowed me to catalogue my unit of coding (i.e., Curricular characteristics of the app) as occurrences of the categories of my coding frame (Scherier, 2012, p. 1);

7. I completed the *App Observation and Classification Form* twice for each app, with a 14 day separation between the first and second application, as recommended as a minimum by Scherier (2012). After data was collected and classified, I analyzed it.

3.6 Data Analysis

Broadly, Qualitative Content Analysis (QCA) is a systematic method for "describing the meaning of qualitative material" (Scherier, 2012, p. 1). Specifically, QCA also refers to a specific method of data analysis within a study. Thus, as explained previously, QCA consists of two distinct, but related, phases of research (i.e., Coding frame generation and coding frame application), which I call Phase I and Phase II of this study, respectively.

During Phase I, I drew upon an approach to data collection and initial analysis within the professional literature called “themeing the data” (Saldaña, 2009, p.139). Themeing the data is a kind of foundational coding in which the researcher, first, identifies the theme “under investigation” (Kvale, 1996, p. 88) before analysis of the text begins. This theme serves as a kind of filter through which data is later analyzed. In this case, the theme of *learning with understanding* (Bransford, Brown, & Cocking, 2000, p. 8) emerged during this initial phase. To further analyze this theme within the literature, I then utilized *axial coding* (Strauss & Corbin, 1998, p. 124). Axial coding is described by Charmaz (2006, p. 60) as an approach to data analysis that “relates categories to subcategories [and] specifies the properties and dimensions of a category”. The iterative process used in axial coding involves fluently moving between data analysis and category generation. Accordingly, the results of this coding eventually formed the category details of my Coding Frame (i.e., Categories, definitions, examples, indicators, and decision rules). Generation of the frame, itself, marked the end of this two-cycle phase. Hence, I abided by the following procedures to analyze data during this phase:

1. Within the identified professional literature, I first used an approach called “themeing the data” (see Saldaña, 2009, p.139);
2. The theme of “learning with understanding” emerged as my concept of focus;
3. I then used *axial coding* (Strauss & Corbin, 1998, p. 124) to identify characteristics of the theme, “learning with understanding” (across the domains and intersections of TPACK theory [Mishra & Koehler, 2006]), in order to identify the dimensions, subcategories, and properties of my coding frame (see Charmaz, 2006, p. 60-62);

4. I created the *Coding Frame* and *App Observation and Classification Form*, both of which reflect the specific characteristics that emerged during the axial coding process.

During Phase II of the study, I compared the results from the *App Observation and Classification Form* for each app with, (a) Those of other volunteer "coders" and, (b) My subsequently completed forms (created 14 days later), in order to reach consensus. Then, I created a table summarizing the results of the *App Observation and Classification Forms* applied to the three apps (see *Table B4*). Later, I analyzed the completed Forms and *Results Summary Table*, in order to identify patterns and themes that emerged within and across the data, pertaining to the three apps. Hence, I abided by the following procedures to analyze data during this phase:

1. I compared my completed *App Observation and Classification Forms* with, (a) Those of other "coding team" members and, (b) My subsequently completed forms (created 14 days later), in order to reach consensus;
2. I created a table summarizing the results of the *App Observation and Classification Forms* applied to the three apps (see *Figure A1*);
3. I analyzed the summary table, as well as descriptive details of the Forms, in order to identify patterns and themes that emerged within and across the data, pertaining to the three apps;
4. I presented these results in Chapter Four of this study.

3.7 Ethical Considerations

No human participants were involved in this study, and thus IRB approval was not required. However, I did receive official confirmation that such was the case, as it related to this study (see *Appendix C* for official confirmation). Additionally, since all apps analyzed within this study were publicly available in Apple's App Store, I did not use pseudonyms for the apps in this analysis.

3.8 Assumptions, Limitations, & Delimitations

This study was intentionally focused on the curricular characteristics that ideally support primary children's potential to learn early algebra concepts with understanding, through multi-touch, mobile, iOS mathematics education apps, and the extent to which existing apps met these characteristics. While the curricular characteristics of the Coding Frame appear to be couched in terms of merit, use of the term "ideal" does not mean these characteristics are superior to other suggestions. Instead, it merely marks these characteristics as theoretically representative of the professional literature and apps. Nor, does the Coding Frame attempt to serve as a measurement tool for evaluating the learning potential of an app. The development of this Coding Frame, which considers the interplay between technological knowledge, pedagogical knowledge, and content knowledge for the formation of its categories, provides only a beginning step toward a classification system that, eventually, may help others identify examples of ideal curricular characteristics that relate to learning early algebra with understanding, as they are outlined by the conceptual framework in this study. A QCA coding frame is not a true taxonomy- it stops, perhaps, a bit short of even being applicable to other math education apps related to algebra for this age group- especially in this case, with the data-driven portion being limited to three apps analyzed.

Likewise, participation and observation of the app from a researcher/coder's point of view differs from a child's. Some indicators are relative (I did not feel rushed), and the coder's point of view is no "guarantee" of a learner's point of view. Additionally, the results of my search for authentic apps was limited by the parameters I used within the Apple App Store, and the ways in which app designers classified their own apps. As such, searching by means of another method would likely yield different apps. Additionally, I did not assume the designers of the authentic apps intended to help learners learn with understanding. Instead, this was meant to be an exercise in examining what could be, according to these curricular characteristics, if that was the designer's goal.

Chapter Four:

Results

4.1 Overview

4.1.1 Problem.

Literacy is increasingly required of people in the world, and true literacy relies upon successful learning- the primary contributor of which is *learning with understanding* (Bransford, Brown, & Cocking, 2000, p. 8). Formal schooling is generally positioned as a place that supports children's ability to learn with understanding- especially in contemporary public schools (in the U.S.). However, in public school classrooms this underlying goal is often displaced by other factors, including "routine conditions of the classroom" (Kennedy, 2005, p. 2) and short-term goals driven by politics, economics, and public opinion. One of the more recent factors to influence public school education, in the U.S., has been the accountability movement and its consequent redefinition of time on task. This movement and redefinition has more narrowly demarcated what working and learning look like in the formal school classroom- in most cases, to the exclusion of play and playful contexts. Arguably, this has had more impact on primary learners (defined loosely as six to eight-year-olds) than other learners- the former of whom primarily prefer to learn and, often, learn best in playful contexts. This loss of play in formal schooling has given way at a time in which views toward this age group of learners as unsophisticated and incapable are disappearing. While this latter change is positive and promising, the elimination of playful learning has resulted in a new set of constraints for primary learners. In other words, instead of learning that is both playful and sophisticated, the two components have merely been swapped. As such, the educational goal of learning with understanding may be no more accessible to primary learners than it was in decades past, when perceptions of children's unsophistication as learners limited the kinds of concepts and experiences to which they were exposed. Additionally, there is concern it might not be possible, in the near future, for formal schooling to accommodate learning that is both playful and sophisticated. For this reason, consideration of alternative learning spaces that are (hypothetically) freer from shifts in politics, economics, and public opinion is a worthwhile aim.

While several alternative learning spaces seem to satisfy this description, one of the most promising is educational software applications (apps) and the multi-touch, mobile devices on which they often are found. This is because these apps and devices are ubiquitous, place power in the hands of non-educators, and are hypothetically freer from hyper-politics and economics. However, despite this potential, review of the professional literature reveals discontent among a number of researchers with the current curricular qualities of many educational apps. Hence, although apps provide a promising alternative learning space, they do not currently appear to be living up to their potential. Accordingly, this study aimed to, (a) Outline curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps and, (b) Compare those "ideal" curricular characteristics to the curricular characteristics of three multi-touch, mobile, mathematics education, iOS apps, in order to describe the general extent to which the two sets of characteristics aligned. Accordingly, this study was guided by the following research questions:

1. What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?
2. To what extent do three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

4.1.2 Material of focus.

As such, I used these three knowledge types as a guide for my review of literature when generating the Coding Frame. In the area of Pedagogical Knowledge (PK), I focused upon ideas based in Learning Science, with particular focus on the idea of learning with understanding. I also focused on ideas related to the education of primary children, which encompasses ideas from both early childhood education and aspects of elementary education. In the area of Content Knowledge (CK), I focused on ideas related to early algebra. Since, early algebra as a content area exists primarily within the scope of teaching young children algebraic ideas, this area qualified as Pedagogical Content Knowledge (PCK). In the area of Technological Knowledge (TK), I focused on ideas related to educational apps for multi-touch, mobile devices. Specifically, I focused on educational apps for young children, so this area qualified as Technological Pedagogical Knowledge (TPK).

4.1.3 Literature of focus.

In addition to my initial analysis of the professional literature, discussed in Chapter Two (and briefly summarized, above, see 4.1 *Overview*), I also focused on literature during Phase I of my study. In summary, the following criteria guided my selection of literature.

1. Learning Science tenets surrounding the educational goal of *learning with understanding*- As a chief premise of this research's rationale, I sought literature related to this ideal, which was first defined by Bransford, Brown, & Cocking (2000, p. 8), as a primary goal of education. This ideal served as the primary conceptual framework through which I filtered potential curricular characteristics;
2. As they relate to primary children's education- In addition to my focus on Learning Science principles related to learning with understanding, I was particularly concerned with the way these tenets circumscribe the education of primary children (approximate ages of six to eight years old). Hence, "primary children's education" served as an additional conceptual lens through which potential curricular characteristics were vetted;
3. Early Algebra (EA) education and concepts- Given my research focus in STEM (Science, Technology, Engineering, and Mathematics) education and primary children, the topic of EA was identified as a topic of particular professional interest. Since much learning of EA concepts occurs with primary children, this criterion was pre-satisfied. Hence, I sought research related to early algebra education and early algebra content concepts, particularly at their overlap with curricular characteristics that might support learning with understanding, and app-based or tech-mediated learning;
4. App features/traits and app-based education- In addition to qualifying as an additional topic of particular professional interest within my research focus, apps served as the medium of the material I examined within this study. Thus, I sought research related to " multi-touch, mobile iOS apps", app-mediated learning with

primary children, and app features/ traits that might support learning with understanding. I also considered app learning at its intersect with early algebra and mathematics education.

4.1.4 Apps of focus.

Within the secondary part of this study, I selected three apps by which I compared their curricular characteristics with those of ideal curricular characteristics from the Coding Frame. The apps I selected satisfied the following criteria:

1. iOS apps labeled “educational” by their creators- A “Category” search within Apple’s App Store offers “Education” as one of the options. I selected this category because I sought apps that were labeled by their creators as educational;
2. iOS apps within the “Elementary” collection- I sought apps targeted for primary learners; the approximate age range of six to eight years old;
3. iOS apps with mathematical content- Not all elementary education apps contain mathematical content. Since this study focused on early algebra concepts, only those apps that contained mathematical content were included. It should be noted, I looked across two “domains” (as presented in the App Store) of mathematical apps for this age group. This is because early algebra concepts are often embedded across mathematical domains. I ended up selecting apps from both of these domains (i.e., The “Number System” and “Beyond Drill- Strategy”). Other mathematical domains included: “Drill & Practice”, “Shapes & Spatial Reasoning”, “Measurement & Data”, and “Beyond Drill- Brain Busters”. After a search through all of these categories, the only other apps that appeared as though

they may have contained algebraic content, were two apps found within the "…Brain Busters" category;

4. $4.99 and under- I decided on $4.99 as the upper limit of my price point, because I sought apps that were free or "affordable". This is, of course, a relative term. However, the prices of many apps seemed to increase sharply, after the $4.99 price point. The average price of a meal at McDonald's is currently $5.00, so I chose this as my measure of affordability;
5. Target age range of six to eight years old- Apple's App Store allows app designers to choose from three main age groups for categorizing children's apps- five years old and younger, ages six to eight years old, and ages nine to eleven years old. Since this study focused on apps intended for primary children, I included apps that fall into the second age group (i.e., Six to eight year olds);
6. Other parameters- No apps from textbook publishers; No apps for an entire year's worth of mathematics, along a grade level (e.g., Second grade math);
7. Full Version- Many apps offer a free trial version of an app, as well as a full version available at cost. Since the free trial version of an app often contains only a fraction of content, and can have incomplete functionality, I downloaded and purchased (if required) full versions of the apps. This assisted in allowing me to fully evaluate the material;
8. Available on May 1, 2017- Allowing that the inventory of the App Store grows and changes daily, I collected all data on the same day. In order to avoid confusion, I did not include apps in this study that were unavailable on May 1, 2017;

9. English content on the US App Store- At the time of this study, Apple's App Store was offered in 126 countries and regions, as a virtual entity. Likewise, each regional marketplace offered a different selection of content. Due to this study's focus on early algebra education in the United States, only material from the US App Store was included. Additionally, as the primary researcher, since my principal language proficiency is in English, I included only those apps written and presented in English;

10. For Apple iPhones- I selected those iOS apps that were created specifically for Apple iPhones, for two reasons. First, the ubiquity of mobile phones is well noted. iPhones are more prevalent than other mobile iOS devices, such as iPads and iPod Touch(es). Second, apps for the iPhone can also be "mirrored" on iPads, but not vice-versa (i.e., An iPhone app can be projected onto the screen of an iPad for play, but an iPad app cannot be projected onto the screen of an iPhone).

Thus, my app material included: Apple iPhone mathematics education apps, related to the "number system" and mathematical "strategies beyond drills", the full versions of which were presented in English and found in the US App Store on May 1, 2017, aimed at children ages six to eight years old.

4.2 Methodology

To answer the research questions, and thus, respond to the aims of this study, I adopted a qualitative approach. In general, the use of a qualitative approach to research enables a person to interpret covert material by allowing for the exploration of personal and social meaning (name, date). Specifically, I aimed to analyze certain "textual" meanings of multi-touch, mobile iOS apps. Hence, to accomplish the kind of descriptive analysis I was seeking, related to the specific content of my research interest, I utilized a method of research called Qualitative Content Analysis (QCA).

QCA is a systematic method for "describing the meaning of qualitative material" (Scherier, 2012, p. 1). It is an established empirical method of study, calling for the creation of a coding frame that contains categories, definitions, examples, and indicators, and later, the application of these descriptors to the material of focus. This type of approach enables a researcher to focus on the contextual particulars of a situation, and study a phenomenon in depth (Schreier, 2012, p. 28), and often, across multiple descriptive facets. In this study, because of the latent nature of the meanings within the multi-modal texts (auditory, visual, kinesthetic, conceptual, and temporal), and my desire to focus on the contextual particulars of the texts, qualitative content analysis (QCA) offered a fitting methodological choice.

4.2.1 Phase I.

As its primary focus, this study aimed to capture curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps. This was accomplished via the multi-step process of coding frame-generation. Accordingly, I analyzed professional literature (as well as three apps) to create a coding frame that was largely concept-driven (and secondarily data-driven), which could be used to describe "ideal" curricular characteristics. This occurred as a result of analyzing and synthesizing ideas (through "themeing the data" [Saldaña, 2009] and axial coding [Strauss & Corbin, 1998, p. 124]) from the related professional literature and apps. Specifically, I moved back and forth between the Frame and the units of context within the literature, and later, between the Frame and units of context within the apps. As such, the categories and descriptors of my coding frame are concept-driven and data-driven.

4.2.2 Phase II.

As its secondary focus, this study aimed to apply the Coding Frame to a handful of authentic apps in order to compare the "ideal" curricular characteristics with those curricular characteristics of the authentic apps, and describe the general extent to which the two sets of characteristics aligned. Accordingly, after the Coding Frame was developed and evaluated, I applied it to the relevant data within each app. To this end, I focused on three mathematics education, iOS iPhone apps for primary learners, available through Apple's App Store on May 1, 2017. Later, I compared the curricular characteristics of the "ideal" app to those within the authentic app, and vice-versa. The general extent to which the two sets of curricular characteristics aligned, could then be compared and analyzed.

4.2.3 Components of data collection and general research procedure.

The components, immediately below, represent processes, products, and tools used to collect data for the researcher to analyze, in order to answer the outlined research questions. Below, is the general research procedure I followed for this study, after I selected the materials (i.e., The literature and apps).

Table 4. Components of data collection.

Research Question	Data Collection	Instruments
1. What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?	-Observation and classification of curricular components from literature and three apps -Tool generation: *Coding Frame* and *App Observation and Classification Form* -Journal writing	-Reflective journal -*Coding Frame* -"Observational Notes" on *App Observation and Classification Form* -"Guiding Questions" on *App Observation and Classification Form*

Table 4. (Continued)

		-Miscellaneous Category on *App Observation and Classification*
2. To what extent do three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?	-Download of three apps, according to criteria for selection -Completion and collection of all *App Observation and Classification Forms* - Summarize apps' inventory of curricular characteristics	-iPhone - Three separate *Single Inventor(ies)* summarizing each app -Cross-*App Inventory* summarizing all three apps

4.3 Results

4.3.1 Phase I: "Ideal" curricular characteristics.

The *Coding Frame* and its complimentary tool, the *App Observation and Classification Form*, represent most of the analytical results of Phase 1 of this study (although, the specifics surrounding the categories and descriptors of the Frame and Form are discussed in more detail in the Literature Review) [see *Chapter 2*]. In addition to the Frame, Form, and contents of the Literature Review, I also discuss themes that emerged during the generation of the Coding Frame (see Table A1). These themes are based on a reflective journal I kept during the Phase I process. Within this journal, I recorded questions, conceptual "knots", and theoretical epiphanies related to the content of the Frame, during the cyclical process of literature analysis and Coding Frame-generation. The process of journal writing provided me with a means through which I could attempt to reconcile discrepant thoughts and identify patterns and conceptual connections, as they emerged over the process. Consequently, sharing the themes that emerged from these musings represents an opportunity to be as transparent as possible about the meanings I negotiated during the Literature Analysis and Frame-generation process.

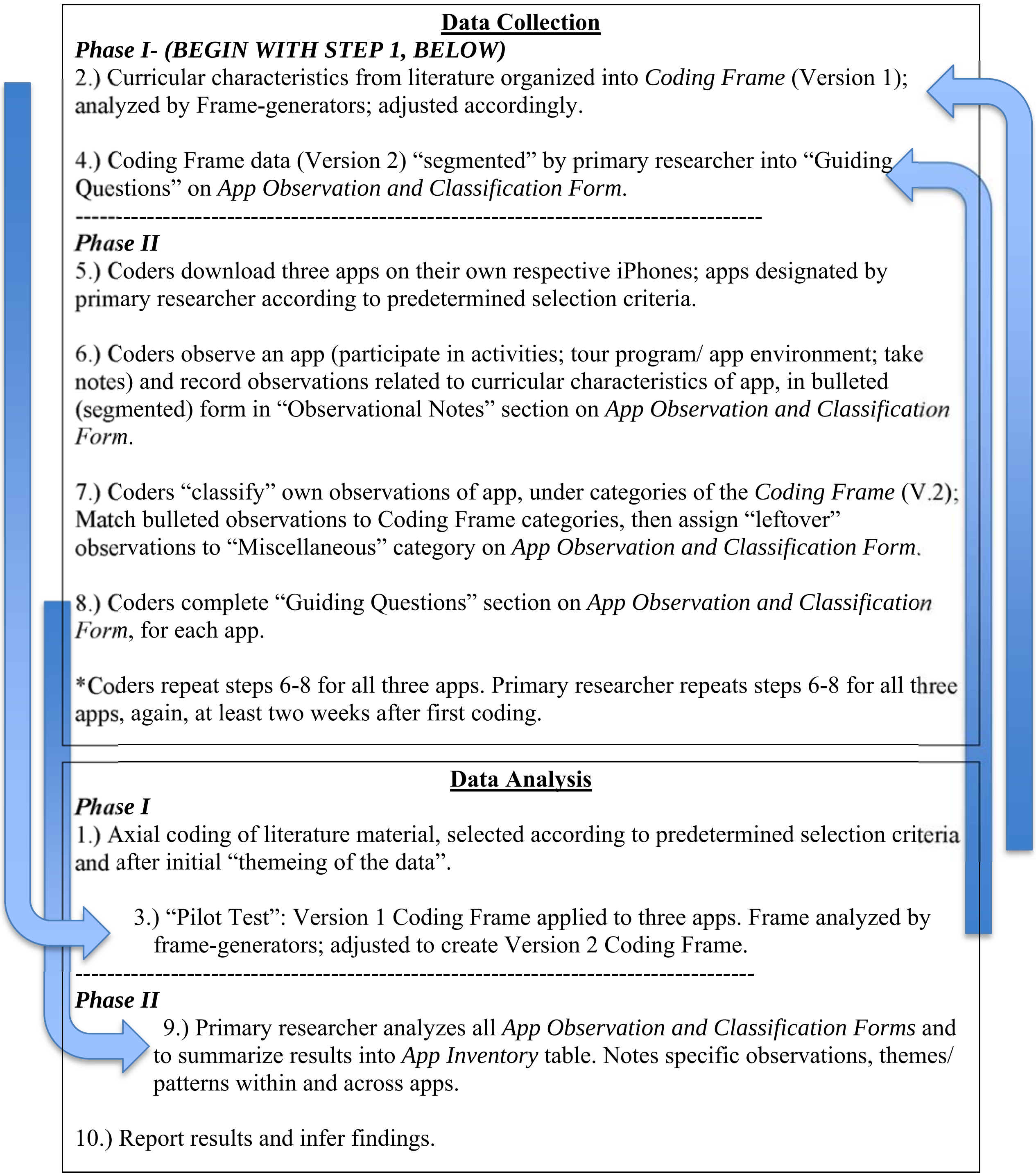

Figure 2. General research procedure.

Therefore, included below are, (a) A snapshot of one section of the *Coding Frame Model* (i.e., major categories and subcategories of the Coding Frame through all three tiers), (b) A snapshot of one section of the *Coding Frame*, (c) A snapshot of one section of the *App Observation and Classification Form*, (d) A brief narrative summary of the Coding Frame's structure and, (e) A brief narrative summary of the Coding Frame's content. The *Coding Frame Model*, the *Coding Frame*, and the *App Observation and Classification Form*, can be found in their entireties, within Appendix A.

First-tier subcategory
Second-tier subcategory
Third-tier subcategory

"What"	
A. Specific Learner Actions	
Explain and describe	
Meets	*Does Not Meet*
Choose from and use	
Meets	*Does Not Meet*
Create solutions and products	
Meets	*Does Not Meet*
Move through design cycle	
Meets	*Does Not Meet*
Weigh and evaluate	
Meets	*Does Not Meet*
Abstract	
Meets	*Does Not Meet*
Move between specific and general	
Meets	*Does Not Meet*
Represent	
Meets	*Does Not Meet*
Translate	
Meets	*Does Not Meet*
Explore	
Meets	*Does Not Meet*

Figure 3. Snapshot of Coding Frame model.

4.3.1.1 Narrative summary of the coding frame structure.

The two dimensions or main categories of my Coding Frame, included "What…" and "How…". These two descriptors essentially asked, "What particular characteristics are ideally imparted within a curriculum that aims to support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?" and, "How are particular characteristics ideally imparted, within a curriculum that aims to support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?", respectively. The "answers" to these questions are reflected in the subcategories, definitions, examples, and indicators of the Coding Frame.

The dimension of "What" included three first-tier subcategories, 1. Specific learner actions (Category A), 2. Specific learner outcomes (Category B) and, 3. Specific early algebra concepts (Category C). The dimension of "How", included two first-tier subcategories, 1. Through experiences, program, and environment (Category D) and, 2. App features (Category E). As such, there are two dimensions of the Frame, with five first-tier subcategories altogether. Each first-tier subcategory is identified by a letter (A- E), as noted in parentheses above.

Within the five first-tier subcategories there are 95 second-tier subcategories. One of the 95 subcategories is a "Miscellaneous" category, and the remaining 94 subcategories are listed in the Coding Frame model (see *Figure 2*). Each second-tier subcategory is identified by a number and letter that corresponds with the first-tier subcategory to which it belongs [e.g., The second-tier subcategory "Algebraic symbols" is identified by its demarcation "C6.)". This is because this example is the sixth second-tier subcategory under the first-tier subcategory "C.)", which is "Specific early algebra concepts".] Accordingly, each of the 94 second-tier subcategories, which outline the specific curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps, are easily identified by a letter and number. It should be noted the numbering begins anew under each letter.

There are two third-tier subcategories for each of the 94 second-tier subcategories. Those are "Meets" and "Does Not Meet". This amounts to 188 second-tier subcategories, in total.

Accordingly, coders classified each curricular characteristic, from each of the three authentic apps they observed, under one of these 188 second-tier subcategories. Alternatively, if it was more applicable, they classified the curricular characteristics of the authentic app under the "Miscellaneous" category of the App Observation and Classification Form, instead. Coders also had the opportunity to write descriptive comments concerning each of the 188 subcategories as they related to the authentic apps. This included, but was not limited to, "Partially Meets" or "Unsure".

Additionally, there was a separate subcategory of descriptors I created, called "Tech-Plus". These represent a set of fourth-tier subcategories that relate to 43 of the 94 second-tier subcategories. Tech-Plus is an extra descriptive "layer", designed to describe whether the app designer appears to have utilized the affordances of multi-touch, mobile app technologies or media in a way that changes a subcategory's descriptor (e.g., "Explain and describe meaning in one's own words") "for the better", as compared with the descriptor's hypothetical enactment in the non-virtual world. "For the better" refers to the idea that the learner's ability to enact the descriptor, or engage with the substance of the descriptor, has been enhanced by the app technology. (For a more in-depth discussion of this subcategory tier, see *Themes in content generation*, below). It is also important to note, the examples provided in the Tech-Plus tier of the Coding Frame are only examples. Just as with the examples outlined in the other subcategories of the Coding Frame, the Tech-Plus descriptors may be met in ways other than those outlined.

Likewise, observing the three apps also changed the structure of the Frame. First, applying the Frame to the apps helped to reduce and streamline the category descriptors. There were originally 107 second-tier subcategories that were ultimately were reduced to 95. Other coders and myself realized a number of the original subcategories were redundant, despite these categories having emerged from different aspects of the literature. Second, the categories were completely reorganized on two separate occasions, based on feedback from other coders and myself. This reorganization was also a result of moving between the App Observation and Classification Form (the direct coding tool) and the Coding Frame. These effects amounted to further reduction and reordering of first-tier and second-tier subcategories. It also led me to dependently link some second-tier subcategories to other second-tier subcategories. For example, A5.b.) "App provides learner with opportunities to move from the general back to the specific" was directly linked to A5.a. "App provides learner with opportunities to abstract (general ideas from the specific)". In this way, only if a coder determined A5.a. was met, did he or she need to consider A5.b.

4.3.1.2 Narrative summary of the coding frame content.

Primarily, professional literature circumscribed by the outlined criteria shaped my Coding Frame. As such, one goal during Coding Frame generation was that the reader of this final report might return to the Lit. Review section (i.e., Chapter Two) and see these ideas embedded within the discussion there. In some cases, however, the Frame is not an exact match with content in the Literature Review. First, some categories with the Literature Review proved to be either redundant or required further explication within the Frame. For example, while the general notion of "application" is discussed within Chapter Two as an essential component of meaning construction, subcategories A2. "Choose from and use…" and A3. "Create solutions and products…" explicate this idea further by differentiating between these two kinds of application. Second, some ideas discussed in Chapter Two were adapted slightly, in order to meet the educational goal outlined within this study- “learning with understanding” (Bransford, Brown, and Cocking, 2000, p. 8) [adaptations are discussed in Chapter Five].

Likewise, observing the three identified apps also changed the content of the frame. The potential source of these content changes was two-fold. First, in a data-driven qualitative content analysis (QCA), or in a study that is at least partially data-driven- like this study, characteristics may have emerged that were not accounted for within the original Coding Frame. Accordingly, I provided space for this possibility through the creation of a “Miscellaneous” category on the App Observation and Classification Form. However, I was surprised by the modesty of the content changes that emerged from this source. For the most part, the content placed within the Miscellaneous category of the App Observation and Classification Form reflected curricular characteristics that might be described as elements of gaming theory (see Van Eck, 2010). For example, there were opportunities for the learner to change the hair color of his or her avatar, and the learner accumulated “points” throughout the app program. Since these were characteristics I intentionally left out of my Coding Frame, this data was considered irrelevant, and did not lead to changes in the content of the Frame. The other source- the overall application process of the App Observation and Classification Form across the three apps- led to some minor content changes. Primarily, these changes involved revisions to a number of examples, descriptions, and indicators within the Coding Frame, itself. For example, subcategory D1. originally placed more emphasis on providing a learner with the opportunity to revise his or her work over multiple stages. After applying the Form to the apps, however, I realized that couching this idea in terms of “reset” or “adaptation” was a more fitting description.

After the Coding Frame was generated and organized, part of its detail was translated into an App Observation and Classification Form. This form was created because of a practical need that emerged during an initial pilot-testing phase of this study. Specifically, the nearly sixty-page Coding Frame, with its abundant detail, made its application on authentic apps too cumbersome. As such, the App Observation and Classification Form represents an abbreviated and slightly different format from the original Coding Frame. My aim was to include enough detail so the Form could be used on its own to classify the curricular characteristics of the real apps, but not so much detail that it might make the Form unwieldy to use. (Future studies may serve to refine the Form and enact this aim with greater effectiveness.) After creation of the App Observation and Classification Form, the Coding Frame filled the role of a detailed reference guide for the coder, should the details of the Form require further clarification in the coder's mind. This also helped the coder move through the temporal space of the text. Both the Frame and the Form have served as invaluable tools in this study. (Below, are snapshots of parts of the Coding Frame and App Observation and Classification Form, see Figures 3 and 4, respectively.)

4.3.2 Phase II: Comparison between "authentic" and "ideal" curricular characteristics.

The summary tables: (a) Three *App Inventory of Curricular Characteristics- Single App* forms and, (b) One *App Inventory of Curricular Characteristics- Across Apps* represent the analytical results for Phase II of this study. I also included a brief narrative summary of each. Below, is a snapshot of part of the App Inventory of Curricular Characteristics- Single App, for one app. All three apps were summarized, individually. As well, the results for the three individual apps were combined into one summary table. Thus, below is also a snapshot of part of the App Inventory of Curricular Characteristics- Across Apps. All four Inventories, in their entirety, can be found in Appendix B.

			characteristic in the virtual world.
Movement through content.	**App "moves through" algebraic concepts in certain ways, and for particular purposes. One theme throughout is, there is no rush for content coverage.**		
D1. *Reset* activity, fix/adapt work, or try again.	App provides learner with opportunities to reset activity, fix/adapt work, or try again. Allows learner to demonstrate through another similar mathematics-based activity or fix the current one. e.g., An activity allows learner to manually "erase" his virtual work by using his fingertip to erase similarly to how a rubber eraser might be used in the non-virtual world.		*Tech-Plus: Uses app tech. to help learners *reset/ try again, fix work* better than in the non-virtual world. e.g., Makes use of digital programming to provide ease of reset/ ease of correction (instant reset button), and/or allows learner to isolate certain "layers" to erase. This latter characteristic, in particular, improves upon the physical limitations of the non-virtual world.
*D2. *More than one experience for a single phase* of learning cycle.	App provides more than one learning experience for a single phase of the learning cycle. e.g., Learners can describe the essence of a concept in their own words in a "sound lab" and explain the concept's essence by making a virtual poster- both activities of which could be part of the "Explain" phase of the learning		Tech-Plus: N/A Ultimately, this category is about learner accessibility to increased learning time and dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is

Figure 4. Snapshot of Coding Frame.

Guiding Questions:

Coder Directions: Please refer to the "Coding Frame" if you have questions about definitions, require further examples, or seek more detail in regard to indicators or decision rules. The specific characteristic, for which you are looking, is represented by the italicized words in each question. The example provided may or may not match the specific way the characteristic is enacted within the app. If a characteristic is not met, the Tech-Plus box [in green] does not require consideration. If a characteristic is met, please consider the descriptor in the Tech-Plus box [in green], perhaps during a subsequent stage in coding.

A1.) App provides learner with opportunities to explain and describe an algebraic concept in his or her own words? (e.g., An activity/ tool, such as a virtual "sound lab" asks the learner to describe the pattern she made and explain the way in which it repeats [unit of repeat]).

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

*Uses app tech. to help learners *explain and describe*, better? (e.g., uses voice recognition/ word prediction/ recording as a dictation tool to capture descriptions or explanations by the learner; playback to self-assess or to share and compare descriptions with others.)

☐ Check box.

Figure 5. Snapshot of "Guiding Questions" on App Observation and Classification Form.

4.3.2.1 Narrative summary of individual app inventory.

4.3.2.1.1 DragonBox Algebra 5+.

DragonBox 5+ most often met curricular characteristics in category D, “Programmatic, Experiential and Environmental Provisions”. This means DragonBox 5+ aligned with “ideal” curricular characteristics most often in this category, or at least in certain aspects of this category. The aspects of this category in which this app’s characteristics aligned most often were “Movement through content”, “Orderliness and clarity”, and a “Balance of cognitive-affective” space. For instance, as an example of “Movement through content”, DragonBox Algebra 5+ introduced several new conceptual conditions during the learning cycle, in addition to the original condition under which the algebraic concept was first introduced.

The aspect of category D with which this app’s characteristics tended to most often misalign was “Kinds of Contexts”. For example, the app did not provide contextualized learning or make embedded algebraic ideas explicit. Additionally, DragonBox 5+ did not align with “ideal” curricular characteristics most often in the category “Specific Learner Actions”, “Externalized outcomes” within “Specific Learner Outcomes”, and what might be described as the aspect related to differentiated instruction within “Trust, safety, and respect”. Also, of note, DragonBox 5+ did not meet any of the Tech-Plus descriptors, outlined on the App Observation and Classification Form. This means the app designers did not appear to utilize the affordances of the app medium or technology in a way that changed the descriptor (to which it is associated) for the “better”, as compared with the descriptor’s hypothetical enactment in the real world. For instance, if the technology of the app, or the medium of the digital programming within the app, was seemingly utilized by the app designer(s) in a way that benefitted the learner’s ability to explain and describe meaning in his or her own words (see A1. in the Coding Frame), the app would have met the qualification of "Tech-Plus" associated with this descriptor. As it was, DragonBox 5+ met none of the 43 Tech-Plus descriptors of the Coding Frame.

Figure 6. "DragonBox 5+" screenshot of main activity response.

Figure 7. "DragonBox 5+" screenshot of main activity.

4.3.2.1.2 Math Motion: Zoom.

Similarly, Math Motion: Zoom met curricular characteristics in category D, "Programmatic, Experiential and Environmental Provisions". This means Math Motion: Zoom aligned with "ideal" curricular characteristics most often in this category, or at least in certain aspects of this category. The aspects of this category with which this app's characteristics aligned most often were the same as DragonBox 5+- namely, "Movement through content", "Orderliness and clarity", and a "Balance of cognitive-affective" space. The aspect of this category with which this app's characteristics tended to most often misalign was "Kinds of Contexts". Additionally, Math Motion: Zoom did not align with "ideal" curricular characteristics most often in this category "Specific Learner Actions" and "Specific Learner Outcomes", and what might be described as the aspect of differentiated instruction within "Trust, safety, and respect". However, this app met more characteristics in more areas than either of the other two apps. This was particularly true of "Specific Early Algebra Concepts". Of note, Math Motion: Zoom also met the Tech-Plus descriptor associated with helping learners move between the general and specific ideas of a concept. This means the designers appeared to leverage the technology of the app medium to help learners in ways that might be considered better than if they would have attempted this action in the non-virtual world. Specifically, Math Motion: Zoom utilizes a “zooming” motion to increase the learner’s proximity to algebraic concepts and help the learners move between the specific and the general (and vice-versa).

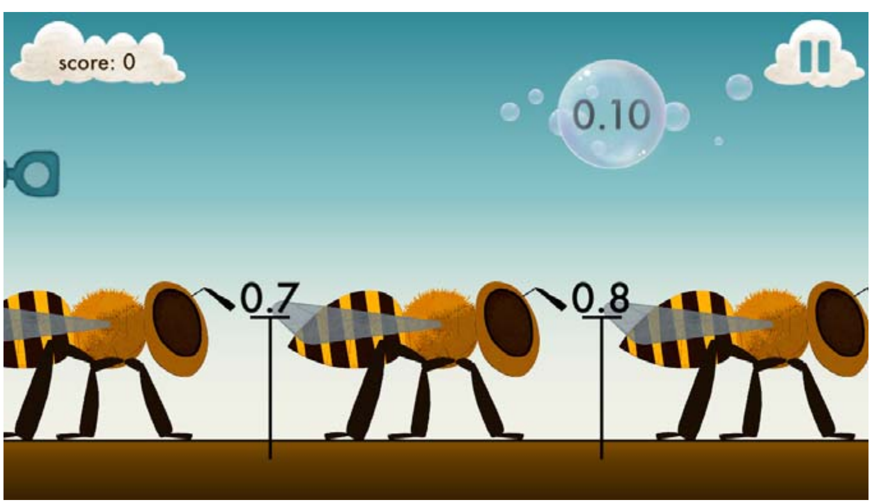

Figure 8. “Math Motion: Zoom” activity.

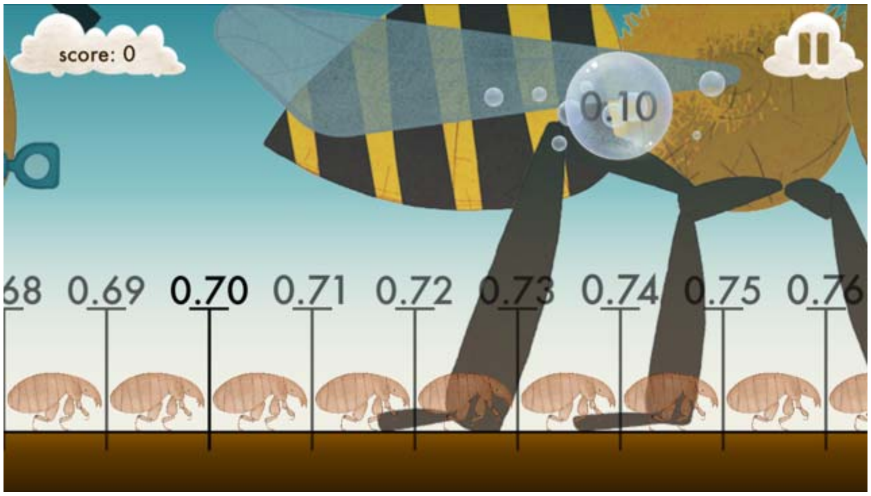

Figure 9. “Math Motion: Zoom” activity- “zoomed in”.

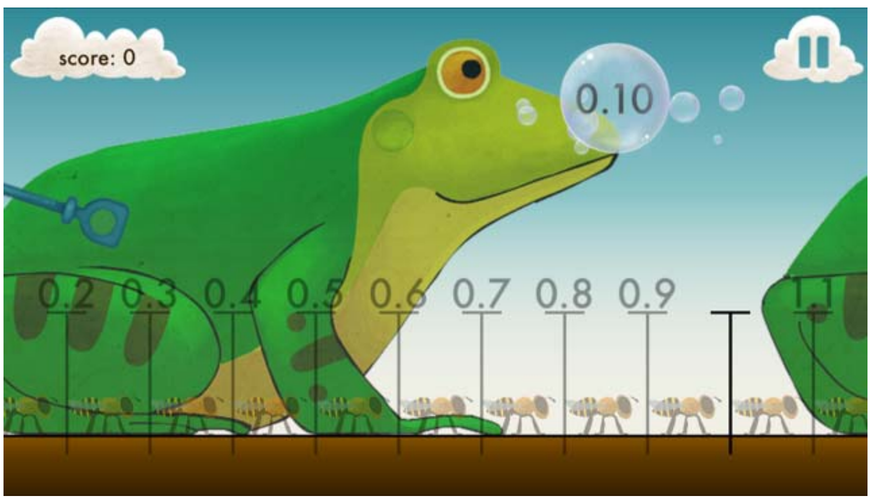

Figure 10. “Math Motion: Zoom” activity- “zoomed out”.

4.3.2.1.3 Slice Fractions.

As with the other two apps, Slice Fractions met curricular characteristics in category D, "Programmatic, Experiential and Environmental Provisions", most often. This means Slice Fractions aligned with "ideal" curricular characteristics most often in this category, or at least in certain aspects of this category. Again, the aspects of this category with which this app's characteristics aligned most often were "Movement through content", "Orderliness and clarity", and a "Balance of cognitive-affective" space. The aspect of this category with which this app's characteristics tended to most often misalign was "Kinds of Contexts", and what might be described as the aspect of differentiated instruction within "Trust, safety, and respect". Additionally, Slice Fractions did not align with "ideal" curricular characteristics most often in the categories of "Specific Learner Actions" and Specific Learner Outcomes". Of note, Slice Fractions also did not meet any of the Tech-Plus descriptors.

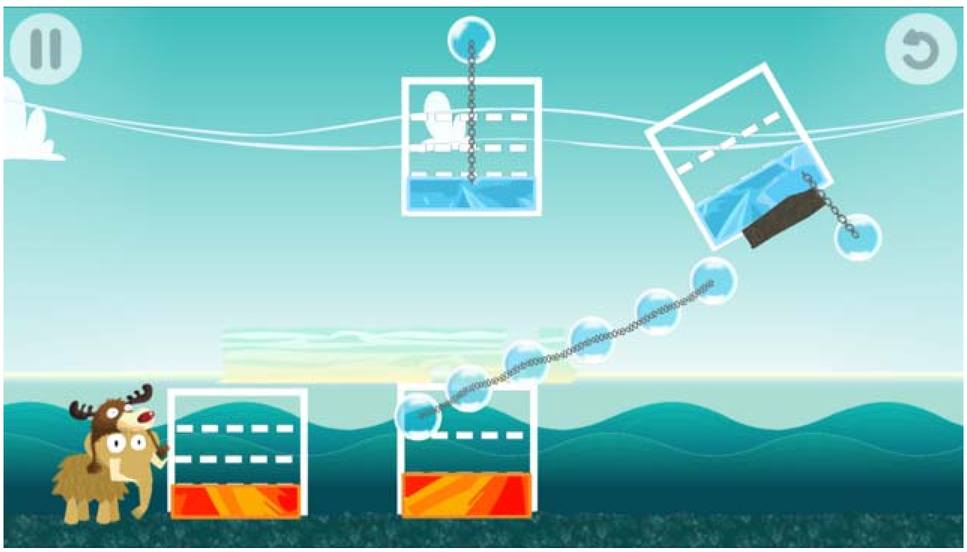

Figure 11. “Slice Fractions” main activity.

4.3.2.2 Narrative summary across app inventory.

All three apps aligned with one another in most areas. This means, often, all three apps either did “Meet” or “Did Not Meet” the same descriptors. The areas in which the apps differed most from one another were in the areas of “Kinds of meaning” under “Specific Learner Outcomes” and “Specific Early Algebra Concepts”. As it relates to the latter, these differences likely reflect the variable content choices made by the individual designers, regarding which specific algebraic ideas to include. As it relates to the former, this is discussed in more detail in Chapter Five.

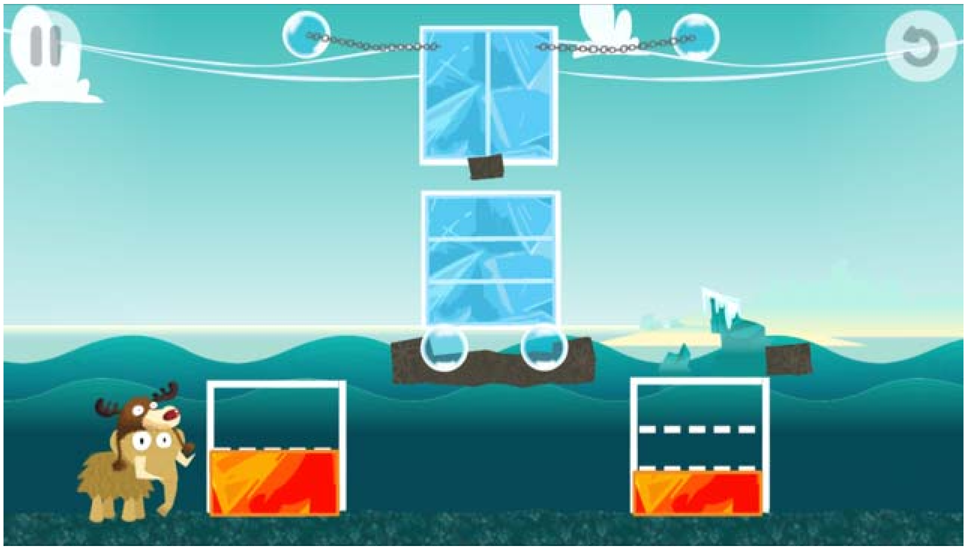
Figure 12. “Slice Fractions”- another level of main activity.

Perhaps, what is most interesting is noting the alignment or misalignment between the curricular characteristics of the three authentic apps and the ideal characteristics outlined in the Coding Frame. This was characterized in terms of “Meets” or “Does Not Meet”, respectively. In the majority of cases, the three apps did not meet the characteristics outlined by the Coding Frame. In several instances, however, all three apps aligned with the “ideal” curricular characteristics of the Coding Frame. These included the “Specific Learner Action” of “Choosing from and using…”, supporting the relationship between learner and concept through increased “Concreteness…”, and providing learners with “New conditions…”, “Constructivist activities…”, and concepts that are “Divorced from timelines…”. Likewise, all three apps seemed to meet multiple descriptors associated with the programmatic and environmental layout of the app, such as “Clarity and organization”.

App: DragonBox Algebra 5+

	Coding Frame Category	Meets	Does Not Meet	Tech-Plus	Comments
	Specific Learner Actions				
A1.	Explain and describe		X		
A2.	Choose from and use	X			
A3.a.	Create solutions and products		X		
A3.b.	Move through design cycle		X		
A4.	Weigh and evaluate	X			
A5.a.	Abstract		X		
A5.b.	Move between specific and general	X			
A6.	Represent		X		
A7.	Translate		X		
A8.	Explore		X		
A9.	Collaborate w/ local		X		
A10.	Collaborate w/ distance		X		
A11.	Collaborate in specific ways		X		
A12.	Justify		X		
A13.a.	Listen and respond		X		
A13.b.	Cultivate empathy		X		
	Specific Learner Outcomes				
	Internalized Outcomes				

Figure 13. Snapshot of part of the App Inventory of Curricular Characteristics- Single App.

4.4 Findings

4.4.1 Phase I.

4.4.1.1 Question 1.

What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?

The curricular characteristics I identified as ideally supporting primary children's potential to learn early algebra concepts "with understanding" through multi-touch, mobile, iOS mathematics education apps, are both broad and specific in nature. By this, I mean these characteristics represent a broad range of descriptors from across most components of a traditional primary/ early childhood curriculum. Yet, the characteristics are also specific in that, their descriptors detail specific ways of supporting the educational goal of “learning with understanding”. One of the things I noticed while generating the descriptors of the Coding Frame across such a broad range of literature was the repetition of certain themes. In particular, were patterns of specific kinds of actions that seem to mark the difference between a learner’s ability to truly “own” a concept for him or herself and a learner simply learning about a concept. These ideas included an emphasis on the learner representing a concept and translating between representations, the learner attempting to “read” or perceive important ideas in the world, and the idea of a learner revising his or her ideas. (See Chapter Five for further discussion of these themes.)

In contrast, even though the parameters of my literature search centered on pedagogical knowledge and early algebra content knowledge related to the theoretical framework of Learning Science and early childhood/ primary pedagogy, I also found several inconsistencies across and between these fields that demanded reconciliation before they could be included in the Coding Frame. As such, numerous concepts required slight adaptation in order to remain loyal to the goal of supporting learning with understanding.

Even among concepts rooted in Learning Science (as is learning with understanding), it seems some ideas have been hi-jacked by the aim of making them more palatable to the contemporary classroom teacher. Accordingly, since contemporary formal schooling is highly concerned with accountability at the moment, some Learning Science principles have been adjusted or applied to evaluation-based and accountability-based models. Thus, in some cases, I needed to divorce particular ideas that support the goal of learning with understanding from their theoretical or conceptual frameworks, or from the models upon which they were resting. Some examples of this separation of Learning Science principles from the models upon which they have been grafted are: seamless assessment (Abell & Volkmann, 2006) from an evaluation-based model, the "Six Facets of Understanding" (Wiggins & McTighe, 1998; 2005) also from an evaluation-based model (as well as other elements from the UbD/DI framework, like "Essential Questions" [Wiggins & McTighe, 1998; 2005]), the idea of play from developmental stage theory, and new outlooks on children as sophisticated learners from narrow definitions of accountability and time on task. Additionally, I aimed to release contemporary learning approaches, in general, from non-constructivist models.

Summary Across Apps

	Coding Frame Category	Meets	Does Not Meet	Tech-Plus	Comments
	Specific Learner Actions				
A1.	Explain and describe		# O ~		
A2.	Choose from and use	# O ~			
A3.a.	Create solutions and products		# O ~		
A3.b.	Move through design cycle		# O ~		
A4.	Weigh and evaluate	# ~	O		
A5.a.	Abstract		# O ~		
A5.b.	Move between specific and general	# ~	O		
A6.	Represent		# O ~		
A7.	Translate		# O ~		
A8.	Explore		# O ~		
A9.	Collaborate w/ local		# O ~		

KEY:	
DragonBox Algebra 5+	**#**
Math Motion: Zoom	**~**
Slice Fractions	**O**

Figure 14. Snapshot of part of the App Inventory of Curricular Characteristics- Across Apps.

In other cases, the ideas, theories or models, themselves, required modifying. This result also grew out of the need to adapt various ideas from the literature to fit the aims of my theoretical framework, learning with understanding. As suggested by Wiggins & McTighe (1998; 2005), I rejected the ladder as the model of learning progression, in favor of their web model (p.119). However, when considering the relation between the web model and the 5Es learning cycle (Bybee, 1997; 2002; National Academy of Sciences, 1998), this required me to modify the 5Es learning cycle, somewhat considerably. Further, when considering the learning cycle in relation to spiraling and coherence, the learning cycle model required further adaptation. The result is that this adaptation now represents how a breadth of facets (i.e., Wiggins & McTighe's "web") might be accommodated within the cycle, in order to explicitly account for the aim of learning with understanding. The adaptation also reflects adjustments related to primary education and releases the original 5Es model from its evaluation-based model. (A model of my adaptation to the 5Es learning cycle is below. See Figure 14.)

I also aimed to follow in the footsteps of Mishra and Koehler (2006) by divorcing the idea of app-mediated learning from its tech-centric model. Specifically, I aimed to realign app-mediated learning with a child-centric or concept-centric model. Hence, the curricular characteristics I identified needed to reflect technology's influence without being based upon a tech-centric model. While the TPACK theory (Mishra & Koehler, 2006) expresses the need to relinquish tech-centric views, its framework has typically been applied to tech-integrated educational settings, instead of tech-mediated settings. Therefore, in the following paragraph, I describe the way in which I approached the generation of curricular characteristics in a tech-mediated setting.

In my experience, in tech-mediated contexts like app design, a designer's starting point is often with Technological Knowledge (TK). Then, the implications of this TK are applied to Pedagogical Content Knowledge (PCK) in order to inform curriculum design (TK ➔ PCK). Instead, I aimed to do the opposite (PCK 🡸 TK). While the latter graphic may appear to represent the same idea as the former, given the direction of the arrows, in my mind it does not.

Engage

- App program initiates learning task and helps learner make connections between prior and current experiences;

- App program engages learner in concepts of focus.
  - May introduce learner to new objects, people, and events;
  - May provoke learner interest through a problem or question for learner.

Learner reflects on experience; self-assesses.

App program assesses.

Explore

- Explore environment (virtual and/or non-virtual world) and manipulate materials.

- Test ideas within one condition and/or context.

- Compare own ideas against others' ideas.

Learner reflects on experience; self-assesses.

App program assesses.

Explain

- Learner attempts to explain ideas;

- App program introduces formal language and disciplinary-based terms;

- App program helps learner refine understanding and make conceptual connections via offering more focused questions.

Learner reflects on experience; self-assesses.

App program assesses.

Elaborate

- Test and revise idea under a new condition (Conditions informed by app program's mathematical content knowledge related to early algebra).
  - Retest concept with change in aspect of the case;
  - Reconcile concept given a discrepancy in case;
  - Solve new problems related to case;
  - Revise concept to apply to all cases in set.

- Test and revise idea within a new context (Contexts informed by app program's pedagogical content knowledge related to "facets of understanding"- both general and mathematical).
  - For a different audience;
  - To represent another view;
  - To frame within another physical, disciplinary, or subject matter context.
  - To represent another way/ translate idea into different medium or mode.

Learner reflects on experience; self-assesses.

App program assesses.

Cumulative Project

- Can be a summative project, completed at the end of this learning cycle;

Or…

- Can be a formative project, whereby final revisions to on-going project are made at the end of this learning cycle.

Learner reflects on experience; self-assesses.

App program assesses.

Figure 15. Modifications to the Early Childhood/ 5Es merged learning cycle. (For original "Merged learning cycle", see Table 1). Note: In Figure above, app program can begin with any of the top three phases.

The first begins with Technological Knowledge (TK) and associated technological capabilities (of the device and medium) and asks what designers can do with these capabilities that relates to educating people on a particular subject. The other model begins with Pedagogical Content Knowledge and its associated principles and asks how technology can support these principles. Application of the TPACK model has been debated for some time (see Angeli &Valanides, 2015), particularly as it relates to the use of arrows to symbolize influential force and starting points. However, I view the TPACK model (Mishra and Koehler, 2006) as representative of a "finished product", instead of a process. So, instead of joining in the critique of the model as a representation that has fallen short, I aimed to add to the continued conversation on how TPACK theory might be applied to additional technology-enhanced contexts. One way of contributing to the conversation was to consider how this particular application might be represented through diagrams that compliment the existing model. These simple models are expressed above, and in Figure 16, below.

(TK → PCK)
Traditional approach to tech-mediated curriculum development.

(PCK ← TK)
Revised approach to tech-mediated curriculum development.

Figure 16. Traditional versus revised approaches to tech-mediated curriculum development.

The curricular characteristics I identified also needed to reflect aspects of a restructured early childhood curriculum. Thus, the characteristics I generated attempted to maintain the focus of learning on the child and concept, instead of on the teacher. This goal was realized more fully when I eliminated the sixth first-tier subcategory I initially included, the “App-Teacher’s Roles”. I realized this category of characteristics need not be included (i.e., some of the characteristics could be subsumed by other categories) if the app-teacher-designer was truly serving in a less teacher-centric role (see Chapter Five).

There were also more significant challenges in creating characteristics that reflect a restructured curriculum. Since the “App-Teacher-Designer” works via pre-programmed design decisions and “responses”, this approach does not always support ideal characteristics that support learning with understanding. In one sense, preprogramming provides continuity and freedom from potential human errs, such as prejudice. From another perspective, however, it makes adaptive teaching much more difficult. Adaptive teaching is a technique used by educators to fine-tune his or her teaching to an individual’s learning needs. As such, enacting these characteristics through preprogrammed responses is difficult in the best case, and impossible in others. Consequently, despite multiple exclusions and adjustments to models and adaptations to theory, several conceptual knots remain for me. Adaptive teaching and its related concepts are one set of issues that linger.

4.4.1.2 Conceptual “knots” in content generation.

Within this study, certain themes emerged during the generation of content for the Coding Frame. These themes are based on a reflective journal I kept during the Phase I process. Within this journal, I recorded questions, conceptual “knots”, and theoretical epiphanies related to the content of the Frame, which emerged during the cyclical process of literature analysis and

Coding Frame-generation. It should be noted these conceptual knots may not present a problem within others' thinking. Likewise, I may resolve these ideas in my own mind, at a future point. However, in the name of transparency, I share those ideas about which, upon ending this project, I do not yet have a sense of peace or full resolve.

The first relates to the limitations of the app medium- especially its mostly-asynchronous environment, in which responsive teaching becomes a challenge. This was of particular frustration in this study as it relates to assessment. Since I aimed to remove the role of assessment from its evaluation-based model, and in my attempt to remain loyal to idea of learning with understanding and my commitment to primary learners, the idea of *seamless assessment* emerged as a fitting paradigm. As previously discussed, seamless assessment is "inseparable from instruction" (Abell and Volkmann, 2006), and aligns with a similar approach of naturalistic observation, sometimes used in early childhood education settings. By definition, this type of assessment is enacted to inform future instruction for the learner, and not as an evaluative measure. Accordingly, this means learning experiences, actions, and outcomes designed for the learner should remain focused on helping the young learner learn with understanding, and should not be commandeered by the need for activities and outcomes that might be more easy to measure and evaluate.

Initially, I was quite content with the idea of focusing on seamless assessment and abolishing evaluation and evaluation-based thinking. However, while seamless assessment seemed compatible with the PCK aspect (i.e., Early algebra learning with young children), I ran into challenges when attempting to determine how TPK could be used to support a non-evaluative, seamless assessment model. This is because many of the ways in which a non-evaluative, seamless assessment model is effectively implemented in the primary classroom are not as well suited to the characteristics of app-mediated learning. Observations, individual interviews, and conversations with learners are important aspects of assessing young children's learning, without relying upon evaluative activities (see Edwards, Gandini, & Forman, 2012). Yet, these very techniques are not easily supported by the app medium. As such, I aimed to provide some examples of more untraditional approaches to assessment within this category of the coding frame, in order to satisfy the goal of learning with understanding with primary children, within the confines of the app's strengths and capabilities. As a result, I suggested several unique approaches to assessment, including documenting and considering a learner's past participation in the real world, documenting and considering his or her interests, considering his or her history of participation in experiences within the app, and considering self-assessment results. These suggestions serve as a collective means for the app-teacher to assess current learning, and use those results to inform future instruction.

On one hand, these approaches utilize the strengths of an app's features while working around the limitations of those features. They also provide learners with metacognitive tools (in the case of self-assessment) and help learners develop habits of mind related to their interests, previous experiences and strengths as a learner. On the other hand, however, I remain bothered by the idea that characteristics related to responsive teaching (e.g., Personal conversations and meetings about the learner's work), which seem to work well in supporting young learners' learning with understanding, remain mostly absent from my Coding Frame because of their perceived dysfunction in the app medium.

Another primary point of contention was how to organize the technology component of this study. In particular, there was great challenge in using the TPACK model to guide the literature review and subsequent category generation for the Coding Frame. Despite the fact this model illustrates my conceptual view toward the integration of technology within a curriculum, and I cannot envision proceeding from another viewpoint, the specific methodology of qualitative content analysis (QCA), at times, appeared at odds with the underlying premise of the TPACK model. The TPACK model illustrates the theory that educators need to consider the conceptual intersections of three kinds of knowledge in order to effectively design and enact technology-enhanced educational curricula. Thus, an underlying idea behind the theory is that, by examining ideas within these kinds of knowledge and within their corresponding intersections, the use of technology in educational settings will be employed in a less superficial, technology-centric way. However, working within the confines of the Coding Frame was difficult, because of the need for each category of a QCA coding frame to be mutually exclusive of one another. As such, for the technology-related aspect of this study, I decided to create a "layer" to account for the possibility that the app designer may have seized upon the technology and medium of the app to leverage learning related to each characteristic. Yet, I am not completely comfortable with this approach, because I wonder if the unintentional effect was that I treated technology too superficially – as a sort of "bonus" (see Angeli & Valanides, 2015). Likewise, brainstorming examples of ways in which the app medium and technology might make learning "better" was exhilarating. However, I wonder if the examples I listed are, (a) The most fitting and, (b) Too restrictive in supporting coders' ability to envision multiple ways the app designers' may have utilized app technology to leverage learning. As such, I see the "Tech-Plus" layer I created as a starting point in a longer conversation. Additionally, I see the eventual need to define types of potential Tech-Plus benefits in terms of the hypothetical levels of advantage they offer. For example, if an app provides the learner with a wide variety of representational tools (wider than he or she would otherwise have access to in the non-virtual world), this may rank differently than an app that prompts the learner to document a specific example of an algebraic concept in his or her personal world. While the former represents unrestricted opportunity that the learner may or may not seize upon, the latter represents a conditionalized challenge he or she is prompted to fulfill. This is not to say open-ended opportunity is less beneficial than structured activity. Instead, this example simply raises further questions about how Tech-Plus benefits might be framed and redefined.

A final issue, related to the challenge above, involves the way I attempted to isolate and define the curricular characteristics throughout the Coding Frame. As discussed previously, the intersections of the TPACK knowledge types- Technological Knowledge, Pedagogical Knowledge, and Content Knowledge- tend to generate curricular characteristics that are multi-faceted. Yet, given the nature of qualitative content analysis (QCA), this meant I had to dissect some of the TPACK-driven ideas in order to isolate the individual aspects of a multi-faceted characteristic and classify them as either “What” or “How”, in order to assure mutual exclusivity. For instance, I could not create a subcategory that outlined both “what” and “how” a learner should participate in a learning experience (e.g., Use algebraic notation in open-ended and highly structured contexts). In the example provided, I was required to separate this idea into two characteristics. As such, I remain uncertain of whether I accomplished the need for mutual exclusivity without losing the benefit of examining the intersections suggested by the TPACK model, in the first place. Thus, as with the issues above, I see my Coding Frame, with its categories, definition, indicators and examples as a springboard for continued conversation and evolution.

4.4.2 Phase II: Question 2.

To what extent do three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

When considering the extent that three multi-touch, mobile, mathematics education, iOS apps reflected curricular characteristics that ideally support primary children's potential to learn early algebra concepts with understanding certain themes emerged. The first is related to the kinds of characteristics all three apps tended to "Meet". In looking across the five first-tier subcategories (i.e., A- E in *Coding Frame*), I noticed that many of these curricular characteristics could be described as minimal or standard requirements for any educational program. Characteristics such as, enabling learners to "choose from and use" materials, tools, and ideas to apply within a given context (see A2.), the provision of "decontextualized" learning experiences (see D11.a.), and the endowment of "clear directions" (see D16.), all satisfy these standard requirements. In part, the meeting of these types of requirements is positive, because even minimal characteristics have not always been enacted in educational apps. However, learning with understanding requires the support of many of the Coding Frame characteristics, including those that are less likely to be considered minimal or standard.

Another type of characteristics all three apps tended to "Meet", however, marked what might be called "step-up" pedagogy. By this, I mean this group of characteristics represent pedagogical decisions on the part of the app designer(s) that might be classified as more positively unexpected than the standard requirements anticipated in most educational settings (see paragraph above). Although fewer in number, these characteristics included the creation of "constructivist" learning experiences (see D10.), the provision of "new conditions" under which the learner must apply meaning (see D.4.a.), and the fact that many content concepts seemed divorced from strict developmental and chronological timelines (see D8.). Therefore, it is important to recognize where these apps are aligning with the "ideal", particularly as it relates to the enactment of more unexpected characteristics.

There were also some subcategories that were "partially met", which may indicate that the app designer was headed in a promising direction, but may not have extended a characteristic's application far enough. While there is no category on the App Observation and Classification Form entitled "Partially Meets", these characteristics were considered "partially met" because of descriptive comments written by the coder. An example of this was that, while all three apps offered activities that were constructivist in nature, the app programs each failed to offer ways that might make embedded ideas explicit or to help learners connect in-app learning with formal educational subject matter. For instance, in DragonBox 5+, while the context of a two-sided "play mat" served as a fitting analogy for the process of balancing algebraic equations, this idea was never made explicit (or was never made explicit to the extent required to meet related definitions).

The second theme related to the extent the three apps reflected curricular characteristics that ideally support primary children's potential to learn with understanding, is the kind of characteristics all three apps did not meet. In looking across the five first-tier subcategories (i.e., A- E in *Coding Frame*), I noticed that one set of these curricular characteristics could be described as "higher level" cognitive actions and outcomes. While I hesitate to frame actions and outcomes in terms of "level", given the controversy that surrounds the frequent misuse of Benjamin Bloom's (1957) taxonomy as a model for curricular design (see Tomlinson and McTighe, 2006), it may be a fitting use of Bloom's model here. Bloom's taxonomy (1957) was designed for the purpose of cognitively classifying assessment items on college exams. Thus, while the categories of my Coding Frame are not items on a college exam, they do represent cognitive activities that are intended to be (seamlessly) assessed. As such, it is interesting to consider where these actions and outcomes fall within the levels of Bloom's taxonomy. In this case, all those characteristics that might be described as "higher level" (i.e., levels 4-6 on Bloom's taxonomy) were not met by all three apps. The one characteristic of the group that was met by all three apps (i.e., A2. "Choose and use…") is considered, at best, a level 3.

Another set of curricular characteristics that none of the apps met related to the kinds of contexts in which the subject matter concepts were situated. Again, this mimics the characteristics one might expect to find in the majority of educational situations, in that the only category that was met was each app's provision of a decontextualized learning experience.

4.5 Summary

In summary, this study occasioned a number of findings related to the results of each of its two phases of research. In Phase I, I focused on describing curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps. I, then, outlined these characteristics, inspired by both the professional literature and three authentic apps, within a detailed Coding Frame.

Accordingly, certain themes emerged across the literature as I generated categories of the Frame. Of particular note, was the repetition of certain motifs that seemed to focus on certain learner actions that might promote learning with understanding. These included opportunities for the learner to represent concepts in a physical form, translate between representational forms, "read" and perceive patterns and concepts within the world, and revise his or her work.

In addition to repeated themes, there were also numerous concepts that required slight adaptation in order to remain loyal to the goal of this study- supporting learning with understanding. These included liberating certain Learning Science concepts from teacher-centric curricular models, non-constructivist models, developmental models, evaluation-based models, and other accountability models. Similarly, in my creation of the specific categories and descriptors of the Coding Frame I also embraced the "web" model (Wiggins & McTighe, 1998; 2005) of learning "progression", modified the 5Es learning cycle model in a number of ways (see Figure 15, and aimed to follow in the footsteps of Mishra and Koehler (2006; 2008) by divorcing app-mediated learning from its tech-centric model. As such, I created two very simple figures in an attempt to show the difference between my perception of a traditional approach to app-mediated curriculum design and the revised approach I attempted within this study. The latter model begins with Pedagogical Content Knowledge and its associated principles and asks how technology can support these principles, instead of the opposite.

Likewise, while the characteristics I outlined attempted to maintain the focus of learning on the child and concept (i.e., learning early algebra with understanding), instead of on the teacher, limitations of the app medium caused some frustration. Specifically, since the app-teacher-designer is primarily visible via pre-programmed design decisions and "responses", this situation does not always provide ideal characteristics that support learning with understanding.

Relatedly, while I was able to overcome several conceptual knots (primarily through the process of adapting ideas based on inconsistent models) [as described above], some disquiet remains. As it relates to limitations of the app medium, some frustration with means of assessment lingers. Many of the ways in which a non-evaluative, seamless assessment model is effectively implemented in the primary classroom (and which also align with responsive teaching) are not as well suited to the characteristics of app-mediated learning. While, I suggested several unique approaches to assessment within this medium, I am not completely satisfied with these outcomes.

Additionally, working within the confines of the Coding Frame was difficult as it related to the technological aspect of the study. Since qualitative content analysis (QCA) demands mutually exclusive categories, I could not combine characteristics of technology with other curricular characteristics. As such, I created an extra layer for some of the categories, which I called “Tech-Plus”. This layering system provides a reasonable starting point for future discussion and research. Likewise, my use of the TPACK model generated curricular characteristics that were multi-faceted. Yet, given the nature of QCA, this meant I had to dissect some of the TPACK-driven ideas in order to isolate the individual aspects of the characteristics before placing them within categories of the Frame. While the results appear to be successful, after dissecting these characteristics I am not completely comfortable that the results maintained the benefit offered by the TPACK model, in the first place

In Phase II, I applied the Coding Frame (in the format of the App Observation and Classification Form) to three authentic apps, in order to compare their curricular characteristics with those “ideal” curricular characteristics outlined in the Coding Frame. I then sought to describe the general extent to which the two sets of characteristics aligned. Consequently, themes emerged based on certain types of characteristics all three apps either met or did not meet (as compared with the ideal characteristics of the Form and Frame). I described the major type of curricular characteristic the three apps met as “standard requirements for any educational program”. This included ideas, such as provision of clear directions, coherence of content, and confidentiality and privacy (when applicable). I also described another type of curricular characteristic the three apps met, though in a considerably smaller amount, as “step-up” pedagogy. By this, I meant the three apps met some curricular characteristics that were more positively unexpected than the standard requirements anticipated in most educational settings. Examples of this were provisions of constructivist-based learning experiences and algebraic concepts divorced from strict timelines.

Additionally, there were a few curricular characteristics that were “partially met” by all three apps. For example, each of the three apps offered activities that were constructivist in nature, but each failed to offer ways that might make ideas embedded within the activity explicit. This potentially would result in a learner not knowing if the meanings he or she was constructing were those intended by the app-teacher-designer. Likewise, none of the apps aligned with the majority of learner actions and outcomes suggested in the Coding Frame.

Chapter Five:

Discussion

5.1 Overview

5.1.1 Focus of study.

This study aimed to describe curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps. These curricular characteristics were outlined within a Coding Frame that included a hierarchy of categories, definitions, examples, indicators, and decision rules used to describe these characteristics. Subsequently, this study also sought to apply the Coding Frame to a handful of authentic apps in order to compare the "ideal" curricular characteristics with those curricular characteristics of the authentic apps, and describe the general extent to which the two sets of characteristics aligned. Accordingly, this study was guided by the following research questions:

1. What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?
2. To what extent do the three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

5.1.2 Methodology.

To answer the research questions, and thus, respond to the aims of this study, I adopted a qualitative approach. Specifically, I utilized a method of research called Qualitative Content Analysis (QCA). QCA is a systematic method for "describing the meaning of qualitative material" (Scherier, 2012, p. 1). It is an established empirical method of study, calling for the creation of a coding frame that contains categories, definitions, examples, and indicators, and later, the application of these descriptors to the material of focus. This type of approach enables a researcher to focus on the contextual particulars of a situation, and study a phenomenon in depth (Schreier, 2012, p. 28), and often, across multiple descriptive facets. This was evidenced by the result of Phase I of this study- generation of the Coding Frame.

Based on the professional literature, my Coding Frame contained two, primary dimensions, under which fell five first-tier subcategories. In turn, 94 second-tier subcategories fell collectively within the five first-tier subcategories. Thus, these 94 second-tier subcategories provided multiple descriptive facets across which to study curricular characteristics of the three apps, in depth. Additionally, during generation of the Coding Frame, certain themes emerged. After creating the Coding Frame and after applying it to the three authentic apps, Phase II of this study provided an opportunity to analyze the results of the Coding Frame application.

5.2 Discussion of Findings

5.2.1 Question 1.

What curricular characteristics ideally support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?

5.2.1.1 Findings.

The parameters of my literature search centered on pedagogical knowledge and early algebra content knowledge related to the theoretical framework of Learning Science and early childhood/ primary pedagogy. Despite the seemingly cohesive nature of this literature, I found several inconsistencies across and between these ideas that demanded reconciliation before their implications could be considered for category-generation. As such, in my examination of ideas across the professional literature related to curriculum, I found numerous concepts required adaptation, in order to remain loyal to the goal of supporting learning with understanding. These included the liberation of certain Learning Science- inspired ideas from teacher-centric curriculum models, non-constructivist models, developmental models, evaluation-based models, and other accountability-friendly models. Similarly, in my creation of the specific categories and descriptors of the Coding Frame I also embraced the "web" model (Wiggins and McTighe, 1998; 2005) as a new metaphor for a learner's "progression" through content, and I modified the 5Es learning cycle model in ways that provide more time and conceptual space for the depth and breadth of meaning construction required of learning with understanding (see Figure 15). I also followed in the footsteps of Mishra and Koehler (2006) by attempting to divorce app-mediated learning from its tech-centric model.

5.2.1.2 Discussion: The need for curricular reimagining.

While I never set out to dissect theories or adapt existing models, I always envisioned apps as an alternative space to reclaim educational goals. Since alternative spaces, like educational apps, are not yet tied to a persistent theoretical or pedagogical mindset, they mark an opportunity to salvage displaced educational goals. As such, they provide permission for researchers and educators to dissect theories from dysfunctional models, or vice-versa. They offer a place to put into practice educational goals like learning with understanding, or to employ ideas drawn from inspirational practices around the world. Thus, it is important to utilize apps and other alternative spaces in ways that remain loyal to one's educational purpose.

Remaining loyal to one's educational purpose may also include a complete revision of curricular structures. It is not a new idea that the structure of a curriculum might need to be redefined in its entirety, given the aim of meeting a new educational goal. After World War II, the villagers of Villa Cella in Reggio Emilia, Italy needed to rebuild their bomb-ravaged nursery schools, and they decided to do so one physical and pedagogical brick at a time (Malaguzzi, 1998). Village parents and citizens asked themselves what kind of school they aimed to rebuild, and the response was unilateral- a different kind of school; one they would build with their own hands and one that would educate their children in a different way than previously (Malaguzzi, 1998, p.58). After experiencing the horrors of war, the villagers' had new insight into human rights; beliefs that extended to the education of their children. As they saw it, each child had the legitimate right to develop his or her intelligence, and prepare himself, through formal education, for the success that was undoubtedly a part of his or her future as a citizen of humanity. Accordingly, above all, they felt children should be taken seriously and their knowledge and ideas respected (Malaguzzi, 1998, p.58). With keen perceptiveness, these villagers realized their revised view of education could not be built upon previous educational structures that expressed indifference toward children, advocated the advancement of "prepackaged knowledge", and placed unbalanced emphasis on authority (Malaguzzi, 1989, p. 50). Thus, began the long process of constructing one of the most revered approaches to early childhood education in the world today (Edwards, Gandini, & Forman, 1998)- what is casually and affectionately referred to as “the Reggio approach”; the philosophy of which is formally expressed in *Hundred Languages* theory (Malaguzzi, 1987).

Nearly seventy years later, it is difficult to summarize the Reggio approach. In brief and oversimplified terms, it is an approach that protects and advances specific rights for young learners- the right to sustained and engaging inquiry of his or her own choosing; the right to competently use a wide range of representational tools that help him or her understand and communicate his or her understanding about the world; the right to revisit his or her work again and again to improve upon it; the right to be an active co-constructor of understanding, with others who can help him or her discover new meanings and relations (Edwards, Gandini, Forman, 1998, p. 464). While these epistemological and pedagogical underpinnings make many of its characteristics remarkably inspirational, a specific aspect of the Reggio approach is worth mentioning as it relates to the findings of this study.

Reggio’s story of development might serve as a meaningful analogy and conjectural model for the kind of work that must be done in rethinking app-mediated learning. Ultimately, Reggio’s story is one of continuous pedagogical refinement (Rinaldi, 2006). With the Reggio approach, the driving need to reimagine curriculum was sparked by the strong desire to fulfill, what was seen as, the educational rights of humanity (Edwards, Gandini, & Forman, 1998; Malaguzzi, 1998; Rinaldi, 2006). With app-mediated learning, the motives are, perhaps, different. While the act of uniting the potential of educational technology with humanity, also, could be conceived of as a way to fulfill educational rights, there are perfunctory needs as well. As previously stated, over 80,000 apps are categorized as learning- or education-based (Apple, 2015)- 10% of which are aimed at young children under eight-years-old. Given that 58% of US parents profess to have downloaded apps for their children (Common Sense Media, 2013), the increasing entanglement of non-traditional learning contexts, like app-mediated experiences, with the aim of education has created a need for reassessing the traditional components of a curriculum, themselves.

Responsible design of app-mediated learning requires redefinitions of roles, without changes to overall responsibilities. Mishra & Koehler's (2006) theory of *Technological Pedagogical Content Knowledge* (TPACK) provides guidance here- although, perhaps not in these words. As TPACK theory denotes, a Teacher's Role- in particular, her knowledge and teaching practices- must change to account for alterations that occur when new *technological knowledge* intersects with existing *pedagogical content knowledge* (Mishra & Koehler, 2006). Although the extent and nature of these changes remain highly debated (see Angeli & Valanides, 2015), most TPACK researchers seem to agree, at the heart of this framework lies the epistemological belief that when technology (and its surrounding knowledge) collides with the goal of learning, it is impossible to proceed with a "business as usual" mindset, while retaining educational integrity. Hence, TPACK provides a poignant framework for reconsidering teacher knowledge and practices related to planning and enacting app-mediated curricula. I propose the reciprocal is also true. When the goal of learning collides with technology, it is impossible to proceed as usual.

With all this said, while apps seemingly are positioned within this study as an alternative educational space, they may be better described as a complementary educational space. Educational apps are not a replacement for human-led education; as exemplified through my frustration with the limitations of the medium to fit harmoniously with adaptive teaching techniques. Instead, apps offer a complement to it. As such, it is essential to note that, while apps can fill a less tech-centric role, they are not a replacement for a human teacher.

5.2.2 Question 2.

To what extent do three multi-touch, mobile, mathematics education, iOS apps reflect curricular characteristics that ideally support primary children's potential to learn early algebra concepts "with understanding"?

5.2.2.1 Findings.

Particular themes emerged based on certain types of characteristics that all three apps either met or did not meet (as compared with the "ideal" characteristics of the Form and Frame). I described the major type of curricular characteristic the three apps met as "standard requirements for any educational program". This included ideas, such as enabling learners to "choose from and use" provided components to apply within a given context (see A2.), the provision of "decontextualized" learning experiences (see D11.a.), and the endowment of "clear directions" (see D16.). I also described another type of curricular characteristic the three apps met, though in a considerably smaller amount, as "step-up" pedagogy. By this, I meant the three apps met some curricular characteristics that were more positively unexpected as compared with the standard requirements anticipated in most educational settings. Examples of this were constructivist-based activities and algebraic ideas divorced from strict developmental timelines.

5.2.2.2 Discussion: Expanding theoretical influence in instructional design.

While, the meeting of standard educational requirements is positive (given these minimal requirements have not always been enacted in educational apps- see Hirsh-Pasek and Zosh et al., 2015). I posit most educational goals that are of consequence require the support of curricular characteristics that are more sophisticated than the standard provided. As it happens, however, educational app designers may not be aware of the difference between various pedagogical characteristics. Further, this may be due to the field in which they work.

As it is now, one of the most prominent theories to inform the field of instructional design is gaming theory. While gaming theory has many valuable qualities when it comes to informing the creation of entertaining apps, the theory proceeds from a highly tech-centric viewpoint. For example, in gaming theory incentives are not only considered important to motivate the learner-player, but are also often tied to the storyline, or otherwise help to advance the game in some way (see Van Eck, 2010). Typically, these incentives take the form of a virtual reward for the player, such as earning points or advancing the player to the next level. Thus, from a gaming standpoint these incentives are a foundational aspect of design. From an educational standpoint, however, there are varied perspectives on the idea of a learner earning incentives beyond self-satisfaction. Participating in app activities that are "appealing and engaging", for instance, may be motivation enough for the learner-player from a pedagogical standpoint.

As such, the discipline of instructional design may need to expand its reference point. Indeed, instructional design expert Van Eck (2010) notes there are not yet "meaningful models" related to digital game design in its support of key characteristics of cognition (xvii). He posits the instructional design field must, "look to theory across disciplines" to determine relevant ideas for the field, and use those theories to design digital games (xvii). He points to the field of Learning Science in particular for the acquisition of such theories.

Given the pattern exhibited within this study, and given my personal experience working with a team of instructional game designers (although admittedly very limited), I wonder if the discipline of instructional design might benefit from consideration of more theories that are not primarily tech-centric. Pedagogical knowledge that grows out of non-virtual spaces (e.g., Principles of Learning Science) may offer a more promising beginning point than more tech-centric theories. Adaptation of "non-virtual" pedagogical knowledge can be shaped by considering its intersection with content knowledge and technological knowledge, as suggested by TPACK theory (see Mishra & Koehler, 2006; 2008). By expanding the sphere of theoretical influence in the field, educational app designers may become more aware of curricular characteristics that are more sophisticated than the standard fallback.

5.3 Implications

5.3.1 Implications for theory.

Arguably, the most significant implications for theory were my adaptations of two conceptual models. One related to adaptation of the 5Es learning cycle model to accommodate the breadth of meaning construction through a longer elaboration phase (see Figure 15). I also added arrows to the model to represent the importance of revisiting the "explore" and "explain" phases when a new condition or context is imposed within the learning experience. Additionally, I added a note about the flexible beginning point of the learning cycle, I added reflection and assessment to each phase of the cycle, and I deleted the "evaluate" phase in favor of a cumulative project. In my mind, this adapted model better represents characteristics that support learning with understanding. I also created a simple set of models showing my interpretation of the difference between a designer's traditional starting point in app design (i.e., as springing from Technological Knowledge [TK]) and my aim in this study (see Figure 16). The implications of the traditional approach start with TK and apply it to Pedagogical Content Knowledge (PCK) in order to inform curriculum design (TK ➔ PCK). Instead, I aimed to do the opposite (PCK 🡐 TK). While the latter graphic may appear to represent the same idea as the former, given the direction of the arrows, in my mind it does not. This is because the latter model begins with

Pedagogical Content Knowledge and its associated principles and asks how technology can support these principles, instead of the opposite.

5.3.2 Implications for practice.

Given the latter model above, one implication for practice is that publishing findings from studies such as this, which proceed from a Pedagogical Content Knowledge perspective instead of a Technological Knowledge perspective, in peer-reviewed journals for the field of Instructional Design, may help to introduce additional perspective related to educational app design.

Further, even though the implications are not immediate, my hope is that someday the Coding Frame can be streamlined into a kind of checklist for the development of 2nd generation apps, aimed at helping learners learn with understanding. One challenge lays in streamlining the frame without sacrificing the necessary detail or over simplifying the list. Similarly, sharing this otherwise latent knowledge with the everyday consumer (eventually) will help to make curricular characteristics embedded within app-mediated curricula more visible, and raise public awareness. Likewise, the continuing education of app designers and/or the importance of teaming with someone who understands pedagogy and educational aspects, are also important.

5.3.3 Limitations.

While I have tried to be as comprehensive as possible in abstracting “ideal” characteristics of curricula from the literature and apps, given the nature of coding frame generation, there are likely characteristics I have missed, or which other researchers would have worded differently. As such, I see this as a beginning step toward the identification of characteristics of educational apps that support children’s learning of early algebra with understanding, which others surely will build upon.

Likewise, the nature of any study that results in new theory or models, or adapts existing theories or models, is provisional. Perhaps to some extent, wooly ideas cannot be avoided if one is to get beneath the surface of basic educational rhetoric. As such, my definition of various terms, and my provision of various examples and indicators are tentative, and subsequent studies will help to refine these ideas further.

Lastly, the data collection and analytical tools utilized in this study, such as the *App Observation and Classification Form* (as well as the coder) are limited in their scope. As such, they are unable to capture every curricular characteristic of an app through observation, documentation, and classification. Again, future related research will help to refine these ideas further.

5.4 Recommendations for Further Research

As it relates to future research, one recommendation is to revisit Phase I of this study. By focusing on each curricular component again, in greater depth, this will help to enhance the Coding Frame and App Observation and Classification Form. Additionally, due to the nature of the concepts of focus within this study (i.e., early algebra education; educational apps) I never examined technological knowledge (TK) and content knowledge (CK) as isolated components. Nor did I examine the TPACK model-recommended TCK intersection, because I never examined apps (without education) or pure algebra (without the early education component). As such, it may be worthwhile to consider the intersection of “raw” algebra concepts (or the professional practice of algebraic ideas) and digital technology (or the multi-touch, mobile app medium). This intersection may suggest curricular characteristics that were not already considered in the current Coding Frame.

Similarly, another idea for future research also relates to generation of the Coding Frame. I chose a more comprehensive view of curriculum by considering five of six traditional components of an early childhood curriculum (the *Teacher's Role*, the *Learner*, the *Environment*, *What to Learn*, and *How to Learn* it), because my aim was to examine how an app might be affected by most curricular elements. Too often, it seems, one specific element is seen as the "magic bullet" for curricular reform. With a view toward avoiding the "magic bullet" approach, I aimed to examine a "whole-curriculum" (or nearly "whole"). However, since I chose this comprehensive definition, my inclusion of all components resulted in a rather broad view of ideal curricular characteristics. In truth, any one of the curricular components could have been a research project in and of itself. As such, likely it will be beneficial to examine each of these components again, in even greater detail, in order to refine the curricular characteristics I have attempted to capture in this study. Hence, additional phases of research that examine each component again, in greater depth, will likely be valuable.

A final recommendation relates to extending Phase II of this study; that is, to examine more apps of the same kind, or a different set of apps, such as the most popular mathematics education apps for primary children. These same apps might also be used to refine the Coding Frame by adding more data-driven characteristics, or adding data-driven influence on existing characteristics.

5.5 Conclusions

Since apps are the largest growing activity related to device usage (Judge, Floyd, & Jeffs, 2015), considering curricular characteristics that might be used to support young learners, on such devices, is important. Further, since there is acknowledgement that first generation apps have fallen short of their educational potential (Hirsh-Pasek and Zosh et al., 2015; Shuler, 2012), the TPACK model encourages educators to revisit technologies or media in order to become more comfortable with them. Over time, those who revisit may find increasingly sophisticated ways to leverage the medium's affordances, instead of being tempted to simply move on to new technologies. Beyond utilizing an app's affordances with greater sophistication, the educational app curricular structure, itself, may need redefinition. Likewise, the discipline that utilizes the structure may require revision before better use can be made of educational apps. Overall, educational apps provide tremendous educational potential. Those who are willing and able to reimagine how these tools may be better utilized for children's learning, may offer the next generation of learners access to true understanding.

References

American Association for the Advancement of Science (1993). *Benchmarks for science literacy*. New York, NY: Oxford University Press.

Abell, S. K. & Volkmann, M. J. (2006). *Seamless assessment in science: A guide for elementary and middle school teachers*. Portsmouth, NH: Heinemann.

Angeli, C. & Valanides, N. (2005). Pre-service teachers as ICT designers: An instructional design model based on an expanded view of pedagogical content knowledge. *Journal of Computer Assisted Learning*, *21*(4), 292-302.

Angeli, C. & Valanides, N. (Eds.). (2015). *Technological pedagogical content knowledge: Exploring, developing, and assessing TPCK*. New York, NY: Springer.

Apple Computers. (2007). *Apple reinvents the phone with iPhone*. Retrieved on August 15, 2015 at http://www.apple.com/pr/library/2007/01/09Apple-Reinvents-the-Phone-with-iPhone.html

Apple Computers. (2010). *Apple launches iPad*. Retrieved on August 15, 2015 at http://www.apple.com/pr/library/2010/01/27Apple-Launches-iPad.html

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection and critique*. Dordrecht, The Netherlands: Kluwer.

Asher, J. W. (2003). The rise to prominence: Educational psychology 1920-1960. In B. J. Zimmerman & D. H. Schunk (Eds.), *Educational Psychology: A century of contributions* (pp. 189-205). Mahwah, NJ: Lawrence Erlbaum.

Balacheff, N. (2001). Symbolic arithmetic vs. algebra: The core of a didactical dilemma. In R. Sutherland, T. Rojano, A. Bell, & R. C. Lins (Eds.), *Perspectives on School Algebra* (pp. 249-260). Dordrecht, The Netherlands: Kluwer.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal*, *93*, 373-397.

Bass, H. (1998). Algebra with integrity and reality. *The nature and role of algebra in the K-14 curriculum: Proceedings of a National Symposium* (pp. 9-15). Washington, DC: National Academy Press.

Bass, H. & Ball, D. L. (2003). Foreword. In T. Carpenter, M. Franke, & L. Levi, *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Belmont, J. M. & Butterfield, E. C. (1971). Learning strategies as determinants of memory deficiencies. *Cognitive Psychology*, *2*, 411-420.

Berk, L. E. & Winsler, A. (1995). *Scaffolding children's learning: Vygotsky and early childhood education*. Washington, DC: National Association for the Education of Young Children.

Bers, M. U. (2008). Engineers and storytellers: Using robotic manipulatives to develop technological fluency in early childhood. In O. N. Saracho & B. Spodek (Eds.), *Contemporary perspectives on science and technology in early childhood education*. Charlotte, NC: Information Age Publishing.

Blanton, M. & Kaput, J. (2000). Generalizing and progressively formalizing in a third grade mathematics classroom: Conversations about odd and even numbers. In M. Fernández (Ed.), *Proceedings of the 20th Annual Meeting of the Psychology of Mathematics Education, North American Chapter* (p. 115). Columbus, OH: ERIC Clearinghouse.

Bloom, B. S. (1957). The 1955 normative study of the Tests of General Educational Development. *School Review, 64*(3), 110-124.

Bodrova, E. & Leong, D. J. (2007). Tools of the mind: The Vygotskian approach to early childhood education (2nd ed.). Upper Saddle River, NJ: Pearson Education.

Boyatzis, R. E. (1998). *Transforming qualitative information. Thematic analysis and code development*. Thousand Oaks, CA: Sage.

Bransford, J. D., Brown, A. L. & Cocking, R. R. (Eds.) (2000). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academies Press.

Bransford, J. D. & Stein, B. S. (1993). *The ideal problem solver*. New York, NY: Freeman.

Bransford, Franks, Vye, & Sherwood, (1989). New approaches to instruction: Because wisdom can't be told. In S. Vosniadou & A. Ortony (Eds.), *Similarity and Analogical Reasoning*. Cambridge, UK: Cambridge University Press.

Brantlinger, A. (2013). Critical mathematics discourse in a high school classroom: examining patterns of student engagement and resistance. *Educational Studies in Mathematics*. doi:10.1007/s10649-013-9506-2

Bredekamp, S. & Copple, C. (1986/1996/2009). *Developmentally appropriate practice in early childhood programs serving children birth through age eight*. Washington, D. C.: National Association for the Education of Young Children. Retrieved from: http://www.naeyc.org

Bredekamp, S. & Rosegrant, T. (Eds.). (1992). *Reaching potentials: Appropriate curriculum and assessment for young children* (Vol. 1). Washington, DC: National Association for the Education of Young Children.

Brainerd, C. J. (2003). Jean Piaget, learning research, and American education. In B. J. Zimmerman & D. Schunk (Eds.), *Educational psychology: A century of contributions*. New York, NY: Routledge.

Brown, A.L. (1975). The development of memory: Knowing, knowing about knowing, and knowing how to know. In H.W. Reese (Ed.), *Advances in Child Development and Behavior, 10*. New York, NY.

Bruner, J. S. (1972). Toward a sense of community. Review of Gardner et al. (1971), "Children teach children". *Saturday Review*, *55*, 62-63.

Bruning, R. H., Schraw. G. J., Norby, M. M. (2011). *Cognitive psychology and instruction* (5th ed.). Boston, MA: Pearson Education, Inc.

Bybee, R. (1997). *Achieving scientific literacy: From purposes to practices*. Portsmouth, NH: Heinemann.

Bybee, R. (2002). *Learning science and the science of learning*. Arlington, VA: NSTA Press.

Byrnes, J. P. (1996). *Cognitive development and learning in instructional contexts*. Boston, MA: Allyn and Bacon.

Carey, S. & Gelman, R. (1991). *The epigenesis of mind: Essays on biology and cognition*. Hillsdale, NJ: Erlbaum.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T.P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Carraher, D. W. & Schliemann, A. D. (2007). Early algebra and early algebraic reasoning. In F. K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 669-705). Charlotte, NC: Information Age Publishing.

Carraher, D. W., Schliemann, A. D., & Brizuela, B. (2000). *Early algebra, early arithmetic: Treating operations as functions*. Plenary address at the 22nd Meeting of the Psychology of Mathematics Education, North American Chapter, Tucson, AZ.

Carraher, D., Schliemann, A., Brizuela, B., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, *37*, 87-115.

Charlesworth, R., & Lind, K. K. (2013). *Math and science for young children*. 7th ed. Belmont, CA: Wadsworth.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Thousand Oaks, CA: Sage.

Chau, C. L. (2014). *Positive technological development for young children in the context of children's mobile apps*. Unpublished manuscript, Child Development, Tufts University, Medford, Massachusetts.

Chi, M.T.H., Feltovich, P.J. & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science,5*, 121-152.

Chiong, C. & Shuler, C. (2010). *Learning: Is there an app for that?* New York, NY: The Joan Ganz Cooney Center at Sesame Workshop.

Clements, D. H., & Sarama, J. (2007). Early Childhood Mathematics. In Frank J. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 461-555). National Research Council. Charlotte, N.C.: Information Age Publishing.

Clough, M. (2006). Learner's responses to the demands of conceptual change: Considerations for effective nature of science instruction. *Science Education*, 15, 463-494. doi: 10.1007/s11191-005-4846-7

Coffey, A. & Atkinson, P. (1996). *Making sense of qualitative data*. Thousand Oaks, CA: Sage.

Cole, H. & Stanton, D. (2003). Designing mobile technologies to support co-present collaboration. *Personal and Ubiquitous Computing, 7*(6), 365-371.

Common Core State Standards Initiative. (2009). The National Governors Association (NGA) and the Council Chief State School Officers (CCSSO). Washington, D.C.

Common Sense Media. (2013). *Zero to eight: Children's media use in America*. San Francisco, CA: Victoria Rideout.

Darling-Hammond, L. (2010). *The flat world and education: How America's commitment to equity will determine our future*. New York, NY: Teacher's College, Columbia University.

deGroot, A. D. (1965). *Thought and choice in chess*. The Hague, The Netherlands: Mouton.

Delpit, L. (2006). *Other people's children: Cultural conflict in the classroom*. New York, NY: The New Press.

Dewey, J. (1934). *Art as experience*. New York, NY: Capricorn Books.

Dodge, D. T., Colker, L. J., & Heroman, C. (2002). *The creative curriculum for preschool* (4th ed.). Washington, DC: Teaching Strategies.

Dweck, C. S. (1989). Motivation. In A. Lesgold and R. Glaser (Eds.), *Foundation for a psychology of education* (pp. 87-136). Hillsdale, NJ: Erlbaum.

Dweck, C. & Legget, E. (1988). A social-cognitive approach to motivation and personality. *Psychological Review*, *95*, 256-273.

Edwards, C. P. (2002). Three approaches from Europe: Waldorf, Montessori and Reggio Emilia. *Early Childhood Research and Practice*, *4*(1).

Edwards, C., Gandini, L. & Forman, G. (Eds.). (1998). *The hundred languages of children: The Reggio Emilia approach- Advanced reflections* (2nd ed.). Westport, CT: Ablex Publishing.

Edwards, S. (2013). Digital play in the early years: A contextual response to the problem of integrating technologies and play-based pedagogies in the early childhood curriculum, *European Early Childhood Education Research Journal, 21*(2), 199-212, DOI: 10.1080/1350293X.2013.789190

Epstein, A. S. (2006). *The intentional teacher: Choosing the best strategies for young children's learning*. Washington, D.C.: National Association for the Education of Young Children.

Fails, J.A. (2007). *Mobile collaboration for young children*. Proceedings of the 6th International Conference on Interaction Design and Children, (pp. 181-184). June 6-8, 2007. Aalborg, Denmark.

Flavell, J. H. (1973). Metacognitive aspects of problem solving. In L. B. Resnick (ed.), *The Nature of Intelligence*. Hillsdale, NJ: Erlbaum.

Forman, G. (1994, May). *Different media, different languages*. Paper presented at the Study Seminar on the Experience of Municipal Infant-Toddler Centers and Preprimary Schools of Reggio Emilia, Reggio Emilia, Italy.

Paper retrieved from: EDRS, ERIC Document Reproduction Services.

Frankenstein, M. (2010). Developing critical mathematical numeracy through real real-life word problems. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference*, (pp. 248–257). Berlin: Freie Universität Berlin.

Galvan, J. (2009). *Writing Literature Reviews: A Guide for Students of the Social and Behavioral Sciences* (4th ed.). Glendale, CA: Pyrczak Publishing.

Gandini, L. (1998). Educational and caring spaces. In C. Edwards, L. Gandini, & G. Forman, *The hundred languages of children: The Reggio Emilia approach- Advanced reflections* (2nd ed.). Westport, CT: Ablex Publishing.

Gandini, L. (2012). Connecting through caring and learning spaces. In C. Edwards, L. Gandini, & G. Forman, *The hundred languages of children: The Reggio Emilia experience in transformation* (3rd ed.). Santa Barbara, CA: Praeger.

Gardner, H. (1991). *The unschooled mind: How children think and how schools should teach*. New York, NY: Basic Books.

Gelman R. & Brown, A. L. (1986). Changing views of cognitive competence in the young. In N. Smelser and D. Gerstein (Eds.), *Discoveries and trends in behavioral and social sciences*. Commission on Behavioral and Social Sciences and Education, National Research Council. Washington, D.C.: National Academy Press.

Giroux, H. A. (2010). In defense of public school teachers in a time of crisis. *In Policy Futures in Education, 8*(6), 709-714.

Glaser, R. (1992). Expert knowledge and processes of thinking. In D.F. Halpern (Ed.), *Enhancing Thinking Skills in the Sciences and Mathematics* (pp. 63-75). Hillsdale, NJ: Erlbaum.

Glaser, R. & Chi, M. T. H. (1988). Overview. In M.T.H. Chi, R. Glaser, & M. J. Farr (Eds).,*The Nature of Expertise* (pp. xv- xxvii). Hillsdale, NJ: Erlbaum.

Grandgenett, N. F. (2008). Perhaps a matter of imagination: TPCK in mathematics education. In AACTE Committee on Innovation and Technology (Ed.), *Handbook of technological pedagogical content knowledge (TPCK) for educators*. New York, NY: Routledge.

Groeben, N. & Rustemeyer, R. (1994). On the integration of quantitative and qualitative methodological paradigms (based on the example of content analysis). In I. Borg & P. Mohler (Eds.), *Trends and perspectives in empirical social research* (pp. 306-326). Berlin, Germany: De Gruyter.

Guernsey, L. (2012). *Screen time: How screen time affects children from birth to age five*. New York, NY: Basic Books.

Gutnik, A. L., Robb, M., Takeuchi, L. & Kotler, J. (2011). *Always connected: Young children's media use on the rise.* New York, NY: The Joan Ganz Cooney Center at Sesame Workshop.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York, NY: Routledge.

Handal, B.,El-Khoury, J., Campbell, C., & Cavanagh, M. (2013). A framework for categorizing mobile applications in mathematics education. *Education Conference Papers. Paper 70.*

Hatch, J. A. (2002). Accountability shovedown: Resisting the standards movement in early childhood education. *Phi Delta Kappan, 2*, 457-462.

Hirsh-Pasek, K., Zosh, J. M., Golinkoff, R. M., Gray, J. H., Robb, M. B., & Kaufman, J. (2015). Putting Education in “Educational” Apps Lessons From the Science of Learning. *Psychological Science in the Public Interest*, *16*(1), 3-34.

Hodgen, J. & Marks, R. (2013). *The employment equation: Why our young people need more maths for today’s jobs*. The Sutton Trust, London, UK: King’s College.

Holsti, O. (1969). *Content analysis for the social sciences and humanities*. Don Mills: Addison-Wesley Publishing.

Hsie, H. F. & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, *15*(9), 1277-1288.

Hussy, W., Scherier, M., & Echterhoff, G. (2009). Research methods in the social and behavioral sciences. For bachelor’s students. Heidleberg: Springer.

Ito, M. (2009). *Engineering play: A cultural history of children’s software.* Cambridge, MA: MIT Press.

Jablonka, E. (2015). The evolvement of numeracy and mathematical literacy curricula and the construction of hierarchies of numerate or mathematically literate subjects. *ZDM Mathematics Education 47*, 599. doi:10.1007/s11858-015-0691-6

Jackson, P.W. (1968). *Life in classrooms*. New York, NY: Holt, Reinhart, & Winston.

Judge, S., Floyd, K. & Jeffs, T. (2015). Using mobile media devices and apps to promote young children’s learning. *Young Children and Families in the Information Age, 10*, 117-131.

Kagan, D. M. (1992). Implications of research on teacher belief. *Educational Psychologist*, *27*, 65-90.

Kanda, Sato, Saiwaki, & Ishiguro (2004). *Friendly social robot that understands human's friendly relationships*. Proceedings of the 2004 International Conference on Intelligent Robots and Systems, September 28- October 2, 2004, Sendal, Japan.

Kane, M. & Trochim, W. M. K. (2007). *Concept mapping for planning and evaluation*. Thousand Oaks, CA: Sage.

Kaput, J. (1995, April). *Transforming algebra from an engine of inequity to an engine of mathematical power by 'algebraifying' the K-12 curriculum*. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Boston, MA.

Kennedy, M. M. (2005). *Inside teaching: How classroom life undermines reform*. Cambridge, MA: Harvard University Press.

Koehler, M. & Mishra, P. (2008). Introducing TPACK. In AACTE Committee on Innovation and Technology (Ed.), *Handbook of technological pedagogical content knowledge (TPCK) for educators*. New York, NY: Routledge.

Kozol, J. (1991). *Savage inequalities: Children in America's schools*. New York, NY: Broadway Paperbacks.

Krippendorff, K. (2004). *Content analysis: An introduction to its methodology*. Thousand Oaks, CA: Sage.

Kvale, S. (1996). *Interviews: An introduction to qualitative research interviewing*. Thousand Oaks, CA: Sage.

Lakoff, G. & Johnson, M. (1980/ 2003). *Metaphors we live by*. Chicago: University of Chicago Press.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York, NY: Cambridge University Press.

Lederman, N. & Abd-El-Khalick, F. (1998). Avoiding de-natured science: Activities that promote understandings of the nature of science. In W. F. McComas (Ed.), The Nature of Science in Science Education (pp. 83-126). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Malaguzzi, L. (1998). History, ideas, and basic philosophy: An interview with Lella Gandini. In C. Edwards, L. Gandini, & G. Forman, *The hundred languages of children: The Reggio Emilia approach- Advanced reflections* (2nd ed.). Westport, CT: Ablex Publishing.

Martin, J. R. (1994). *Changing the educational landscape: Philosophy, women, and curriculum.* New York, NY: Taylor & Francis.

Mayring, P. (2010). *Qualitative content analysis: Basics and techniques* (11th ed.). Weinheim, Germany: Beltz.

McEneaney, E. (2003). The worldwide cachet of scientific literacy. Comparative Education Review 47(2), 217-237.

McTighe, J. & Wiggins, G. (2004). *The Understanding by Design professional development workbook*. Alexandria, VA: Association for Supervision and Curriculum Development.

Metz, K.E. (1995). Reassessment of developmental constraints on children's science instruction. *Review of Educational Research*, 65(2), 93-127. Retrieved from: http://www.jstor.org

Miller, G. A. (1956). The magical number seven, plus-or-minus two: Some limits on our capacity for processing information. *Psychological Review*, *63*, 81-97.

Mitra, S. & Rana, V. (1999). *Children and the internet: An experiment with minimally invasive education in India.* Retrieved on September 17, 2015 at http://www.iicd.org/search/show-entry.ap?entryid=3778

Mitra, S. Dangwal, R. Chatterjee, S. Jha, S., Bisht, R., & Kapur, P. (2005). Acquisition of computing literacy on shared public computers: Children and the hole-in-the-wall. *Australasian Journal of Educational Technology*, *21*(3), 407-426.

Mishra, P. & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

Morris, C. D., Bransford, J. D., & Franks, J. J. (1977). Levels of processing versus transfer appropriate processing. *Journal of Verbal Learning and Verbal Behavior*, *16*, 519-533.

National Academy of Sciences. (1998) Working group on teaching evolution. *Teaching About Evolution and the Nature of Science*. Washington, D.C.: National Academy Press.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Negroponte, N. (2005). *One Laptop Per Child*.
Retrieved on September 29, 2015 at http://one.laptop.org/about/people/negroponte

Nielsen. (2012). *American families see tablets as playmate, teacher and babysitter*.
Retrieved from: http:/Nielsen.com

Notari-Syverson, A., O'Connor, R. E., & Vadasy, P. F. (1998). *Ladders to literacy: A preschool activity book*. Baltimore, MD: Brookes Publishing.

Palfrey, J. & Gasser, U. (2008). *Born digital: Understanding the first generation of digital natives*. New York, NY: Basic Books.

Piaget, J. (1978). *Success and understanding*. Cambridge, MA: Harvard University Press.

Prawat, R. S. (1996). Constructivisms, modern and post-modern. *Educational Psychologist*, *31*, 215-225.

Prensky, M. (2006). *Don't bother me mom- I'm learning*. St. Paul, MN: Paragon House.

RAND Mathematics Study Panel. (2003). Mathematical proficiency for all students: Toward a strategic development program in mathematics education. Santa Monica, CA: Author.

Reggio Children. (2010). *The infant-toddler centers and preschools of Reggio Emilia: Historical notes and general information*. Reggio Emilia, Italy: Municipal Infant-Toddler Centers and Preschools of Reggio Emilia.

Resnick, D. P. & Resnick, L. B. (1977). The nature of literacy: An historical exploration. *Harvard Educational Review*, *47*, 370-385.

Rinaldi, C. (2006). *In dialogue with Reggio Emilia: Listening, researching and learning.* New York, NY: Routledge.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context.* Oxford, U.K.: Oxford University Press.

Rogoff, B. (2003). *The cultural nature of human development*. New York, NY: Oxford University Press.

Rustemeyer, R. (1992). *Content analysis step-by-step*. Münster, Germany: Aschendorff.

Saldaña, J. (2009). *The coding manual for qualitative researchers*. London, U.K.: Sage.

Schreier, M. (2012). *Qualitative content analysis in practice*. London, UK: Sage. Shuler, C. (2012). *Where in the world is Carmen Sandiego? The edutainment era: Debunking myths and sharing lessons learned*. New York, NY: Joan Ganz Cooney Center at Sesame Workshop.

Shuler, C. (2012). *iLearnII: An addendum: An analysis of the education category of the iTunes app store*. New York, NY: The Joan Ganz Cooney Center at Sesame Workshop.

Shuler, C. (2009). *Pockets of potential: Using mobile technologies to promote children's learning*. New York, NY: The Joan Ganz Cooney Center at Sesame Workshop.

Shuler, C. (2007). *D is for digital: An analysis of the children's interactive media environment with a focus on mass marketed products that promote learning*. New York, NY: The Joan Ganz Cooney Center at Sesame Workshop.

Simon (1980). Problem solving and education. In D.T. Tuma and R, Reifs (Eds.), *Problem Solving and Education: Issues in Teaching and Research*. Hillsdale, NJ: Erlbaum.

Spring, J. (2010). *Deculturalization and the Struggle for Equality* (6th Edition). McGraw Hill. New York, New York.

Spiro, R.J., Feltovich, P.L., Jackson, M.J., & Coulson, R.L. (1991). Cognitive flexibility, constructivism, and hypertext: Random access instruction for advanced knowledge acquisition in ill-structured domains. *Educational Technology, 31*(5), 24-33.

Strauss, A. & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Strong-Wilson, T. & Ellis, J. (2007). Children and place: Reggio Emilia's environment as third teacher. *Theory Into Practice*, *46*(1), 40-47. DOI: 10.1080/00405840709336547

Takeuchi, L. & Stevens, R. (2011). *The new coviewing: Designing for learning through joint media engagement.* New York, NY: Joan Ganz Cooney Center at Sesame Workshop.

Tapscott, D. (1998). *Growing up digital: The rise of the net generation*. New York, NY: McGraw-Hill.

Tomlinson, C. (1999). The differentiated classroom: Responding to the needs of all learners. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C. A. & McTighe, J. (2006). *Integrating differentiated instruction and understanding by design: Connecting content and kids*. Alexandria, VA: Association for Supervision and Curriculum Development.

TPACK Image. (2012). Retrieved from http://tpack.org

Van Eck, R. (2010). *Gaming and cognition: Theories and practices from the learning sciences*. Hershey, PA: IGI Global.

van Merriënboër, J. J. G. & Sweller, J. (2005). Cognitive load theory and complex learning: Recent developments and future directions. *Educational Psychology Review, 17*, 147-177.

Vaismoradi, M., Turunen, H. & Bonden, T. (2013). Content analysis and thematic analysis: Implications for conducting a qualitative descriptive study. *Nursing and Health Sciences*, *15*, 398-405. doi: 10.1111/nhs.12048

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. (Originally published 1930, New York, NY: Oxford University Press.)

Wake, G. (2014). Making sense of and with mathematics: The interface between academic mathematics and mathematics in practice. *Educational Studies in Mathematics, 86*, 271-290.

Watlington, D. (2011). Using iPod Touch and iPad educational apps in the classroom. In M. Koehler & P. Mishra (Eds.), Proceedings of Society for Informational Technology & Teacher Education International Conference 2011 (pp. 3112-3114). Chesapeake, VA: AACE.

Wedege, T. (2010). People's mathematics in working life: Why is it invisible? *ALM International Journal, 5*(1), 89-97.

Wellman, H. M. (1990). *The child's theory of mind*. Cambridge, MA: MIT Press.

Wellman, H. M. & Gelman, S. A. (1992). Cognitive development: Foundational theories of core domains. *Annual Review of Psychology*, *43*, 337-375.

Wiggins, G. & McTighe, J. (2005). *Understanding by design* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

Wilensky, U. (1990). Abstract meditations on the concrete and concrete implications for mathematics education. In I. Harel & S. Papert (Eds.), *Contructionism*. Norwood, NJ: Ablex.

Wolf, D. P. (1988). Becoming literate. *Academic Connections: The College Board 1*(4).

Wood, D. Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17*: 89-100.

Yore, L. D., Pimm, D., & Tuan, H. (2007). The literacy component of mathematical and scientific literacy. *International Journal of Science and Mathematics Education, 5*, 559-589.

Appendix A:

Coding Frame

<u>What:</u> These descriptors answer the hypothetical question, "What particular characteristics are ideally imparted within a curriculum that aims to support primary children's potential to learn early algebra concepts 'with understanding', through multi-touch, mobile, iOS mathematics education apps?"

"What" Categories	Description / Examples	Indicators	Decision Rules
A.) Specific learner actions.	These descriptors refer to the actions that learners, hypothetically, engage in during provided learning opportunities. They do not refer to the outcomes of those actions. (e.g., the act of adapting is different than its outcome- an adaptation). Outcomes are listed within a separate section of this coding frame. "Learning experiences" are defined as the structured and unstructured activities, the overall program, and the environment within the app. This term is used interchangeably with "opportunities". *These "learner actions" cannot be treated as isolated process skills. Each must be linked to an Early Algebra (EA) concept, within the app. They can occur either within open-ended settings and/or within structured learning experiences.	Questions provided are examples of the types of questions an app may "essentially" ask the learner to answer, through the actions or outcomes of their work and play within the app or as prompted by the app. These questions do not need to be explicitly asked.	*"Meet(ing)" the main descriptor is separate from meeting the "Tech-Plus" descriptor. *"Tech-Plus" is an extra descriptive layer, designed to describe whether the app designer appears to have utilized the affordances of multi-touch, mobile app technologies/ media in a way that changes the descriptor "for the better", as compared with the descriptor's hypothetical enactment in the non-virtual world. "For the better" refers to the idea that the learner's ability to enact the descriptor or engage with the substance of the descriptor has been enhanced, specifically, by the app technology. The examples

			provided are only examples. Tech-Plus may be met in other ways.
A1. *Explain* and *describe* meaning in one's own words.	App provides learner with opportunities to explain and describe an algebraic concept in his or her own words. e.g., An activity/ tool, such as a virtual "sound lab" asks the learner to describe the pattern she made and explain the way in which it repeats (unit of repeat).	The app essentially asks learner to answer questions, such as: "What do I see?", "What happened?", "What is this about?", "Why do I think that happened?", "How can I describe and explain what it is, and how it works?" App possibly allows or encourages: - Learners to label/name an idea (sometimes a short phrase or word helps to explain/ describe), but the app must also require more than just labeling or naming an algebraic concept. - Acceptable for app to allow learner to explain in personal and informal ways (without use of conventional disciplinary vocabulary and language.)	*Tech-Plus: Uses app tech. to help learners *explain and describe* better than in the non-virtual world. e.g., An activity/ tool, such as a virtual "sound lab" uses voice recognition/ word prediction/ recording as a dictation tool to capture descriptions or explanations by the learner; playback to self-assess or to share and compare descriptions/ explanations with other learners/ family. This allows for easy sharing beyond the immediate circle of those present during learning, provides a different medium through which learners can translate ideas, and/or provides playback for learners to hear and reflect upon his or her own words.
A2. *Choose from and use* concepts to apply within provided contexts.	App provides learner with opportunities to "choose from and use" mathematical strategies, ideas, and algebraic concepts (from a bank/menu of options), and apply them to provided contexts. e.g., An environmental provocation within the app prompts the learner to build or extend patterns with virtual sounds, images, taps/touches provided.	The app essentially asks learners to answer questions, such as: "How can I use my understanding of this concept, in this situation?"	*Tech-Plus: Uses app tech. to help learners *choose and use* concepts better than in the non-virtual world. e.g., Learner is able to access, choose from, and use a wider variety of virtual "tools" and multi-modal manipulatives. This wider variety supports a learner's

			construction of flexible conceptual representations.
A3. *Create solutions or products* to solve problem situations.	a.) App provides learner with opportunities to create solutions or products to solve mathematical problem situations. e.g., A task within the app prompts the learner to create a virtual machine out of available digital "parts" that serves as an "input/output" device that demonstrates algebraic change. b.) App provides learner with opportunities to move through the phases of a formal problem solving process/ engineering-design process for purpose of: proposing ideas, testing ideas, evaluating test results, making revisions and adapting ideas, and repeating the process until a working solution is created. These phases must be made explicit to the learner. e.g., A "checklist" within an activity specifically guides the learner to "move through" the formal phases of a design/ problem solving process (made explicit to him or her), in order to link the learner's personal experience of product/ solution-design to the larger procedural process of: proposing ideas, testing ideas, evaluating test results, making revisions and adapting ideas, and repeating the process until a working solution is created.	The app essentially asks learner to answer questions, such as: a.) "How can you solve this problem?" b.) "How can you change this, in order to…?" App possibly allows or encourages: - There is more than one representation of, both, the general engineering-design process and the general problem solving process. There is no such thing as "the" official process for either. These processes may be represented in different forms, with varying indicators of "movement through the phases" and varying title and descriptions of each phase. The point is that the learner's individual experience of product or solution-design are linked to a larger procedural process of some kind, that might be used as a general guideline in other problem situations.	*<u>Tech-Plus:</u> Uses app tech. to help learners *create solutions or products* to solve problem situations better than in the non-virtual world. Uses app tech. to help learners *use a formal problem-solving/design process* better than in the non-virtual world. e.g., Learner is able to better relate to the design/ problem solving process by using touch to track his or her virtual path through the phases of the process on app, and/or take photos of their project at different stages of design to reflect each phase of the process. This tracking/ documenting increases a learner's conceptual proximity to the problem solving/ design process and, thereby, supports his or her construction of procedural knowledge related to problem solving/ design. e.g., The creation made by the learner can be animated, or otherwise utilize motion to see "effects" of the design on algebraic ideas.

A4. *Weigh and evaluate* interpretations and executions of meaning.	App provides learner with opportunities to weigh, judge, and/or evaluate one's own and others' interpretations, and executions of algebraic meaning. e.g., A virtual "gallery" provides the learner with a chance to compare and evaluate several of peers' interpretations of, explanations of, or solutions for algebraic problems or concepts, and judges them in mathematical terms such as, "a fitting representation of equality"; "the best solution for this mathematical problem"; "the easiest 'solution path' in this context".	The app essentially asks learners to answer: "Did the idea work?", "Does it look right?", "What makes this concept (the most) appropriate or fitting, in this situation?"	*<u>Tech- Plus:</u> Uses app tech. to help learners *weigh, judge, and/or evaluate* products/ ideas better than in the non-virtual world. e.g., <u>Link to</u> <u>online</u> virtual "display boards"/ a "gallery" within the app allows the learner to post/ share his or her own work to a wider audience, so that others might respond. Online gallery also encourages the learner to respond to others' work. Additionally, multi-modal guidelines for evaluation are provided in "kid-friendly" terms to assist him or her in weighing the various ideas, and the app helps the learner consider each guideline in reference to his or her evaluation of others' work. This personalized assistance with the application of evaluation criteria may not be available in the non-virtual classroom. Neither might feedback from a large audience.
A5. *Abstract* the general *over many specific instances*, over time.	a.) App provides learner with opportunities to abstract "big ideas" related to early algebra after observing many specific instances of those ideas, over time. To "abstract" means to remove embedded	The app essentially asks learners to answer: "What do all these ideas have in common?"	To "Meet": App must not "rush" the learner's observation of specific instances (of a general idea). There must be plenty of

	meaning from its context or source, in order to construct a summary of the idea's important features. e.g., An activity prompts the learner to notice and verbalize that all of these examples have something in common. They are all problems where you have to join two groups together to figure out the sum. b.) App provides learner with opportunities to move from the general back to the specific. The learner is provided with opportunities to identify specific examples, based on the "big idea" of focus; opportunities to illustrate the general with the specific and/or move between the two perspectives, as it relates to one concept. e.g., A "scavenger hunt" provides context for the learner to notice and verbalize that a situation is another example of a joining problem.		varying instances to observe and plenty of time in which to do it. *<u>Tech-Plus:</u> a.) Uses app tech. to help learners *abstract general* ideas better than in the non-virtual world. e.g., Makes use of digital programming to provide access to many examples, cases, representations for learners to observe, over time. The range of examples, and the extended time in which to observe those examples, may not be available in the non-virtual classroom. b.) Uses app tech. to help learners *illustrate the general/ move between the general and specific* better? e.g., Makes use of digital programming traits to "layer" the general "over" the specific and the specific "within" the general. Use of motion, hyperlinks, visual "layers" to fluidly move between the general and the specific with ease.
A6. *Represent* ideas through models.	App provides learner with opportunities to represent ideas through the creation of direct physical/ visual models of	The app essentially asks learners to answer: "How can I represent this idea, through______?", "What do I see or know about this idea? <u>And,</u>	*<u>Tech-Plus:</u> Uses app tech. to help learners *represent* ideas *through models*

	mathematical problem situations, algebraic concepts, and/or mathematical systems. e.g., A task asks the learner to draw or illustrate functional change through a "comic strip" or a series of images that represents the specific change. Like, the learner's daily routine in three hour increments: 7 o'clock- breakfast, three hours later at 10 o'clock- art class, three hours later at 1 o'clock- nap time, three hours later at 4 o'clock- snack and homework, three hours later at 7 o'clock- bath time.	how can I describe what I see or know, through this medium?"	better than in the non-virtual world. e.g., App provides access to different representational tools, media, and modes for modeling algebraic concepts than what, otherwise, might be available in the non-virtual classroom. For instance, using photographs and video- which are beyond those media typically valued in formal school. By some definitions, this increases the learner's disciplinary literacy, in addition to supporting the learner in his or her construction of flexible conceptual representations of ideas.
A7. *Translate* ideas across representational forms.	App provides learner with opportunities to translate algebraic ideas through the creation of multiple products and models, across different modes, media, and "languages". e.g., A task asks the learner to translate the information from his or her comic strip into the format of a functional table.	The app essentially asks learners to answer: "Do different conditions call for different representational forms?", "How does the same idea appear differently and similarly through various modes, media, and "languages"?	To "Meet": App must provide at least one opportunity for translation within a different medium, mode, and/ or "language", beyond the original representation (described in the category, above). *<u>Tech- Plus:</u> Uses app tech. to help learners *translate* ideas across modes, media, "languages" better than in the non-virtual world. e.g., A virtual "photo lab" prompts the learner to translate

			equivalency, as originally captured through algebraic symbols (4 + 1 = 3 + 2), into photographs of examples of equivalency found in the learner's real world. App provides a "virtual atelier", with access to more representational tools, media, and modes for translating algebraic concepts than what, otherwise, might be available in the non-virtual classroom. In some cases, the fluidity with which apps can switch between modes and media supports the ease of a learner's trafficking across said modes and media. In some ways this is beneficial, in that learners can visualize various representations with efficiency, rather than awaiting extensive translation.
A8. *Explore* the real world environment.	App encourages learner to explore his or her real world environment for a particular mathematical/algebraic purpose. e.g., A task prompts learner to identify an algebraic idea in the world around them. Such as, a scavenger hunt for particular algebraic symbols.	The app essentially asks learners to answer: "Is there an example of this idea in the world around me?"	*<u>Tech-Plus:</u> Uses app tech. to help learners *explore the real world environment* for a particular mathematical/ algebraic purpose better than in the non-virtual world. e.g., A task prompts learner to use the mobile device to identify and document particular algebraic symbols with the device's camera. The portability of mobile

			devices allows the learner to explore a wider variety of environments than what, otherwise, might be available in the non-virtual classroom. Learners can find "embedded" ideas in a variety of contexts as they relate to his or her out-of-school environments. This helps to bridge "in-school" learning with "out-of-school" learning, and helps learners construct the significance of algebraic ideas as they relate to the larger world.
*A9. *Collaborate* with others in the "immediate" real world.	App provides learner with opportunities to collaborate with others in his or her "immediate" or local real world, toward a particular mathematical/algebraic purpose. e.g., A "family challenge" asks learners to work with his or her parent, grandparent, or caregiver to classify objects in the app, in three different ways.	The app essentially asks learners to answer: "What would people around me say about this idea?"	To "Meet": Learner must see/ know the identity of the real world person(s) with whom they are collaborating. <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with opportunities to collaborate with immediate others about algebraic ideas in the non-virtual world (at

			least within the context of formal schooling).
A10. *Collaborate* with others "at a distance", in/via the virtual world	App provides learner with opportunities to collaborate with others "at a distance" in or via the virtual world, toward a particular mathematical/algebraic purpose. e.g., A game asks the learner to "join in" and "help virtual friends" (could be simulated characters) to extend a pattern with a long unit of repeat, over a long (virtual) distance.	The app essentially asks learners to answer: "What do different people around the world say about this idea?" App possibly provides or encourages: -Collaboration based on interest groups.	To "Meet": Learner may or may not know the identity of the person(s) with whom they are collaborating, other than "screen names/ identities". Either is acceptable, but those with whom learner collaborates must be "at a distance" from one's immediate local circle. *Tech-Plus: Uses app tech. to help learners *collaborate with others "at a distance"* better than in the non-virtual world. e.g., App program utilizes communication technology to help learners "Facetime" with a class in South Africa in order to work together on a joint algebra project. Such communication technologies allow learners to communicate with others at a distance, and/or with whom they might not otherwise have access. Or, in a way (like face-to-face) with people they might not otherwise interact with in this way.

*A11. *Collaborate* in specific ways.	App provides learner with opportunities to collaborate/work with others, in at least one of these three ways: with turns to lead the group; within two different sized groupings (e.g., partners, triads, small groups, large groups); synchronously, in real-time, toward a particular mathematical/algebraic purpose. e.g., A task prompts the learner to work with a virtual partner (an actual learner, not a simulated character) to order objects from tallest to shortest.		To "Meet": App must prompt the learner to contribute to work/ discussion in some way. <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this opportunity (to collaborate in these ways) in the non-virtual world (at least within the context of formal schooling).
*A12. *Use evidence to justify* reasoning.	App provides learner with opportunities to use corroborating evidence to justify their mathematical reasoning. In this case, "corroborating evidence" refers to physical and conceptual indications that validate an algebraic idea in some way. These indications might be in the form of examples- a learner's experiential observations across cases, visual evidence like models or drawings, or the use of logical reasoning as expressed through mathematical rules and properties). Justification refers to the range of arguments used by the learner to show that	The app essentially asks learners to answer: "How do I know this conjecture is always true for all numbers?"	<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with opportunities to justify their

	a conjecture is true (Carpenter, Franke, & Levi, 2003, p. 85). e.g., App prompts learner to justify his or her reasoning by choosing from a "bank" of options- create a drawing, model an idea, use words or mathematical symbols to express a mathematical rule or property.		reasoning in the non-virtual world (at least within the context of formal schooling).
A13. Actively *listen* and *respond* to situations and others' communications.	a.) App provides learner with opportunities to listen and respond to situations and others' communications. Active listening is a communication technique whereby learners make a conscious effort to understanding what others are really saying. As it relates to educational practice, it includes certain characteristics. These include: focusing on the person speaking, showing interest through body language, listening without interruption or judgment, paraphrasing what was said, and asking questions to discover more about the person's ideas. e.g., App activity provides "space" for synchronous listening to others' interpretations of algebraic ideas. Learners respond by paraphrasing what was said, and asking questions to discover more about the person's ideas. *b.) Cultivate and/or express empathy toward others. e.g., App activity invites learner to relate to others' personal experiences with algebraic	The app essentially asks learners to answer: "How can I show I am interested in what the person is saying?", "What did the person say?", "Do I understand everything that was said, or do I have questions?" App possibly provides or encourages: - Respectful and empathic interactions, "professional" regard through accountable talk, celebrating others' victories, supporting others' efforts and variances. - The elicitation of all students to participate.	*<u>Tech-Plus:</u> Uses app tech. to help learners *"actively listen" and respond* better than in the non-virtual world. e.g., App program utilizes communication technology to assist learners in actively listening to/ synchronously viewing others' interpretations of algebraic ideas, to which they may not otherwise have had access- like, listening to global interpretations.

	ideas by sharing his or her own personal experiences with algebraic ideas.		
Category	**Description / Examples**	**Indicators**	**Decision Rules**
B.) Specific learner outcomes.	These descriptors refer to the outcomes that learners, hypothetically, realize as a result of/ within the course of provided app learning experiences. This study does not attempt to measure or identify whether the learner has realized the outcome. Instead, it attempts to identify if the app provides learning opportunities that hypothetically support these outcomes, through specific indicators. Some "outcomes" are types of meaning constructions that take place within the mind of the learner. Other outcomes are external outcomes that might be observable to outsiders. "Learning experiences" are defined as the structured and unstructured activities, the overall program, and the environment within the app. This term is used interchangeably with "opportunities".	*Questions provided are examples of the types of questions an app may "essentially" ask the learner to answer, through the actions or outcomes of their work and play within the app or as prompted by the app. These questions do not need to be explicitly asked.	
Internalized outcomes.	Internalized outcomes are constructions of meaning within the learner. They are the internal outcomes of learning experiences, and/or interactions with the app program/ environment.	The coder is NOT assessing if these constructions/internalized outcomes are present in the learner, but whether the app has indicators of opportunities for the learner to construct these meanings.	Learners must be allowed to construct these meanings for these indicators to be met. To "construct" means to "build" meaning for one's self. Direct instruction, alone, does not qualify as construction. However, when a learner constructs a concept, the <u>concept must be made explicit to the learner</u>, in some way. This might be achieved through guided reflection by learner on learning experiences and summary of meanings constructed during the course of the learning experiences. A direct statement or summary by the app is acceptable, if the learner has

			first been provided with opportunities to negotiate meaning for him self or her self.
Kinds of meaning:	All meaning is personal. However, there is a continuum of "kinds of meaning" that denotes a range from more consensus-driven meanings to more personally situated meanings. Consensus-driven meanings are agreed upon by wide social and professional circles (shared meaning), jointly negotiated meanings are agreed upon by a smaller group of people, and highly-personal meanings are not reliant upon consensus with others. The kinds of meanings below reflect a range of meaning types.		
B1. *Essence* of a concept.	App provides learner with opportunities to construct the fundamental nature and defining features of an algebraic concept- its "what", "where", "why", and "how". Especially "how" and "why" an algebraic concept functions or behaves in a certain way, and how this relates to its features. Remembering the properties of a concept is not the same as understanding why a concept has those properties, and how those properties relate to its details; come to know how structure relates to function. e.g., A set of activities that allow the learner to construct the essence of the equal sign. An example of the essence of the equal sign: The equal sign is a symbol used in mathematics. An equal sign represents an equivalent relationship between two numbers or sets of numbers. It does NOT	The app essentially asks learners to answer: "What's important about this concept that makes it unique?", "What behaviors did the features exhibit?", "How would I define or describe this idea?", "How do the features of this concept affect its function?" App possibly provides or encourages: - The chance to think in terms of a metaphor like a balance scale (or see saw), whereby the equal sign is symbolized by the scale and the two numbers (or sets of numbers) are represented by each side of the scale. When the same amount (of the same object) is on each side of the scale, the sides are equal and the scale is balanced. Learners must be able to manipulate the	To "Meet": App must provide learners with an opportunity to construct more than just the big ideas of a concept. Essence involves some level of detail. Additionally, if the app does not address all the elements of a concept's fundamental nature, at some point across the "program of apps", or does not address common misconceptions related to the concept's nature, this category cannot be met. *<u>Tech-Plus:</u> Uses app tech. to help learners construct the *essence of a concept* better than in the non-virtual world. e.g., Utilization of multi-modal representations of the concept will likely help learners construct

	symbolize the output of a numeric operation (i.e., the meaning of the equal sign in 4 + 3 = 7 is NOT the same as stating 4 + 3 ➔ 7). The equal sign is a symbol, invented by a human. Many people agreed, a long time ago, that this symbol should mean an equivalent relationship between two or more ideas or objects.	scale by placing objects on and off each side to observe the results. The equal sign also must be explicitly linked to the metaphor of the balance scale.	a more "complete" sense of essence. Like, through observing and analyzing growing patterns in visual, spatial, auditory, and temporal forms.
B2. *Significance* of a concept.	App provides learner with opportunities to construct how an algebraic concept's essence (see above) relates to one's self, the natural world, the human world, and other ideas. Significance, often, is determined through increased conceptual proximity between the concept and the learner, and through the learner's comparing, contrasting, judging, and observing multiple perspectives on how an algebraic concept relates the natural world, the human world, and other ideas. e.g., A set of activities that allow the learner to construct one aspect of the significance of the equal sign, by observing its global and local relevancy. Observe how the equal sign and other symbols of mathematical equality and/or inequality are used in a variety of individual, community-based, domestic, global, and professional/disciplinary cultures.	The app essentially asks learners to answer: "How might_____ person think about this idea?" App possibly provides or encourages: - A chance for learners to invent their own symbols to represent various mathematical relations (between amounts). - Real world contexts in which the concept might be enacted. For instance, the equal sign is used in the creation of a computer chip program that runs a household appliance, in an internet search engine, or in air traffic control software.	To "Meet": Three of the four components of the concept's relationship- to self, to natural world, to human world, and to other ideas- must be satisfied for this category to be met. The example for this category illustrates only one of those components- a concept's relationship to the human world. *<u>Tech-Plus:</u> Uses app tech. to help learners construct the *significance of a concept* better than in the non-virtual world. e.g., Uses motion to move between concept and aspects of its significance.

	An example of one aspect of the significance of the equal sign: An equal sign, which represents an equivalent relationship, helps answer (in part) the Essential Question: How do people communicate their ideas in mathematics? (Answer: Often, through invented symbols, like: =, <, >, ≠, ≈.)		
B3. *Conditions of applicability* of a concept.	App provides learner with opportunities to construct when, where, and why (given conditions in which) to apply an algebraic concept. Conditions are different than contexts. In this case, a condition refers to a specific situation (with certain needs) in which application is required. For example, a condition is a need to represent a relationship between two numbers or sets of numbers, using mathematic symbols. In contrast, a context is a setting. This category is primarily concerned with conditions. At least one condition, often, is presented to the learner during the "Explore" or "Explain" phase of the learning cycle. Additional conditions are often presented during the "Elaboration" phase of the learning cycle. e.g., A set of activities that allow the learner to construct a condition of applicability of the equal sign.	The app essentially asks learners to answer: "Under what conditions is this concept applicable?" App possibly provides or encourages: - The presence of condition-action pairs, or "If/Then" statements or situations. - Provides immediate feedback through a learning experience in which the learner can manipulate virtual conditions and watch results. Like trying to use an equal sign in various virtual simulations, in which two numbers or sets of numbers are sometimes equal and sometimes unequal, and receiving feedback for "misuse" of the sign in the wrong instance. - Provides feedback to learners as to the *degree* learners know when, where, and how conditions apply (e.g., App says, "It looks like you	To "Meet": The app must signal (verbally, visually, etc.) how a specific idea is relevant for transfer to other conditions (e.g., App says, "When you added blocks to the balance scale to make both sides equivalent, the equal sign stayed in place. I wonder what you can do to keep the equal sign in place, if there are no more blocks to add and the sides are not equivalent." *Tech-Plus: Uses app tech. to help learners construct the *conditions of applicability of a concept* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to virtual "rooms" in which various conditions are presented.

	An example of a condition of applicability of the equal sign: If an equal sign is used in a number sentence, then a person has to assure both sides of the equation are equivalent, for the statement to be true. (*If both sides are not equivalent, the person must either make them so- the process for which has its own set of rules- or use a sign that accurately expresses the relationship between both sides of the number sentence, such as: <, >, ≠.)	know most of the times <u>when</u> a person should use the equal sign, but you don't yet know <u>how</u> to use it. Where does it belong in a number sentence?")	
B4. *Nuances/ connotations* of a concept.	App provides learner with opportunities to construct important rules, exceptions to rules, multiple meanings of, inferences, nuances, and implications of an algebraic concept's meaning, as needed for understanding the concept at an upper elementary level. e.g., A set of activities that allow the learner to construct a nuance of the equal signs' meaning. An example of a nuance of the equal sign is its varied meaning. The word equality has different meanings in different contexts. In mathematics, it signifies numeric equivalency or equal amounts. In social studies, it can mean something different (i.e., equity/justice versus equality).	The app essentially asks learners to answer: "Are there exceptions to this rule?", "Are there different meanings for this word?", "Is this idea used differently in other places?" App possibly provides: -A frequent indicator of nuance is varied word meanings within various contexts, situations, or disciplines.	*<u>Tech-Plus:</u> Uses app tech. to help learners construct the *nuance of a concept* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to tidbits of knowledge related to nuances of, inferences of, and exceptions to the concept.
*B5. *Realizations*	App provides learner with opportunities to construct awareness about his or herself as a mathematics learner; to develop	The app essentially asks learners to answer: "How do you feel when faced with this kind of problem?",	*<u>Tech-Plus:</u> N/A

about self as learner.	vocabulary related to self as learner, reflect on one's own degree of growth and set goals, and reflect on one's own strengths and weaknesses. e.g., A virtual "mirror" pops-up and the learner looks at his or her reflection while answering questions the "magic mirror" asks.	"What was your first reaction to this task?" (I panicked/ wanted to cry or scream/ froze and could not move forward, tried to breathe and look for clues, felt confident and proceeded to work out solution). "What am I good at?", "What can I do better?" App possibly provides: -May draw attention to a learner's strengths and encourage extending those strengths; facilitate ways to remediate or compensate a learner's weaknesses.	Ultimately, this category is about learner accessibility to this dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with opportunities to engage in self-reflection and to construct realizations of him or herself as a learner in the non-virtual world (at least within the context of formal schooling).
*B6. *Kinds of big ideas or key themes.*	App provides learner with opportunities to construct <u>at least one</u> specific kind of "big idea" related to early algebra and mathematics. "Big ideas" are key themes and enduring concepts in disciplinary subjects. Big ideas are usually not related to the surface features of mathematical practice. These include, but are not limited to: a.) Algebraic themes (e.g., "equivalency", "change"); b.) A class of mathematical problems (e.g., "joining" problems); c.) A mathematical principle, rule, or operational property (e.g., the associative property of addition);	The app essentially asks learners to answer: "What are all these ideas about?", "What is the theme here?"	To "Meet": This category is slightly different than "essence". Essence <u>may include</u> some big ideas about the concept of focus. However, this category notes the app must touch upon at least one idea from a list of specific kinds of big ideas related to early algebra. The app <u>must make the big idea(s) explicit</u> to the learner to meet this category. <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this concept. Thus, if the app meets the characteristics at far left, this

	d.) An "Essential Question" related to mathematics and/or its practice (e.g., How do people use mathematical symbols to describe the world around them?); e.) An observation about the nature of mathematics practice (e.g., a mode of inquiry in mathematics).		implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with opportunities to engage in self-reflection and to construct these kinds of big ideas in the non-virtual world (at least within the context of formal schooling).
B7. *Kinds of relations between concepts.*	App provides learner with opportunities to make <u>at least two</u> types of connections/ relations between mathematical concepts. This includes, but is not limited to: a.) Details with a big idea; a big idea with details (e.g., "People describe groups of objects in different ways. One way people describe a group of objects is by describing how part of the group compares with all of the group. For example, four of the six balloons are round.") b.) Ideas (in terms of key features) to other ideas <u>within</u> a disciplinary domain (e.g., Growing patterns and functions are both kinds of change, even though their units of change are different); c.) Ideas (in terms of key features) to other ideas <u>across</u> disciplinary domains, across the curriculum, and across individual learning experiences (e.g., In what other disciplines do people measure change?);	The app essentially asks learners to answer: "In what ways are these ideas related to one another?" "How are these ideas different from one another?", "How are these ideas the same as one another?", "How might I describe the way these ideas are related to one another?"	To "Meet": App <u>must make the relationship between ideas explicit</u>. *<u>Tech-Plus:</u> Uses app tech. to help learners construct *specific relations between concepts* better than in the non-virtual world. e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom in or out, isolate parts, observe cause and effect of input/ output. These features may make algebraic relations more explicit by allowing the learner to control perspective, negotiate the visual and conceptual space between ideas, and simulate effects as they relate to learner-controlled changes.

	d.) Concepts with skills; skills with concepts (e.g., “The clearer I write, the easier it will be for others to read my symbolic notation.”); e.) Big ideas with other big ideas (e.g., People describe groups of objects in different ways. People, also, describe patterns in different ways.); f.) Details with other related details; (e.g., “One way people describe a group of objects is by describing how part of the group compares with all of the group. Another way people describe a group of objects is by describing the total amount.”). Also, relationships must be described or labeled- preferably by learner before app. By labeling relationships between concepts, this helps the learner recognize patterns of useful information, “file” the information in a particular way in his or her mind, and helps make it easier for the learner to determine solutions to problems or pathways to solutions, in the future. e.g., “<u>An example of</u> equivalency”, “ <u>a feature of</u> a repeating pattern”, “<u>a possible solution for</u> this kind of problem”, “<u>an analogy for</u> classifying objects. ”		

B8. *Kinds of relationships between learner and concept.*	App provides learner with opportunities to form <u>at least three</u> kinds of relationships between him self or herself [including (d.) and the algebraic concept of focus]. This includes, but is not limited to: a.) Consensus-driven meanings with more personal meanings (e.g., Learner pictures a dumbbell when envisioning the equal sign and its meaning of equivalency. This is because the bar in the middle of the weight is horizontal like the lines in the equal sign symbol, and the learner's big brother often adds various weights, in equivalent amounts, to each side of his dumbbell- like five two-pound weights on one side and two five-pound weights on the other.); b.) New concepts with existing concepts (e.g., Learner has already noticed that some patterns repeat. Now, she understands that other patterns "grow", instead of repeat.); c.) Out-of-app experiences with in-app experiences (e.g., Learner has noticed the equal sign used at the grocery store. Now, he knows what it means, based on app learning experience.); e.g., App provides a "space" for learners to share their personal experiences/ stories with others, as they relate to algebraic concepts they are learning. "I've seen the equal sign at the grocery store!" And…	The app essentially asks learners to answer: "Has this ever happened to me before?", "What actions spark greater personal relation to this concept- increased conceptual proximity?/ How can I understand this idea better?", "What's it like?" "Have I seen something like this before?", "What does this mean to me and my life?"	*<u>Tech-Plus:</u> Uses app tech. to help learners construct *specific relationships between learner and concepts* better than in the non-virtual world. e.g., Dictation tools and "audio galleries" may assist learners in sharing their personal connections to algebraic ideas. e.g., As it relates to the last characteristic- increasing one's conceptual proximity to an idea- app utilizes temporal/ interactive features of digital programming and device, like motion to zoom in or out and observe aspects of the concept more closely. Or, use of multi-touch manipulation can also decrease the conceptual distance between learner and idea.

	d.) Increasing one's conceptual proximity to an idea (e.g., Learner better understands how growing patterns "work" because she has used virtual manipulatives to make her own growing pattern). This last idea is sometimes described as "concreteness". It involves increasing the conceptual distance between the learner and the object/concept. It involves the learner familiarizing him or herself with the concept or object, in a way that makes the idea more comprehensible. Usually during "Engagement", "Exploration", and "Explanation" phases of the learning cycle, a person needs to be at a greater conceptual proximity to a concept (though this is not always the case). Increased conceptual proximity occurs through description (direct modeling, drawing, or verbally describing "what is" according to the learner). To some degree, this is a way of describing one's relative relationship to the object or concept.		
*B9. *Kinds of meanings-in-the-moment.*	App provides learner with opportunities to reflect on <u>at least two</u> kinds of current understanding or meaning constructed during or directly following a learning experience. Usually these opportunities occur in the form of attempting to answer questions or prompts. (Learner responses are not evaluated for accuracy.)	App possibly provides or encourages: - Asynchronous feedback. - Synchronous live chat availability to assist with answering the questions.	To "Meet": There is some controversy surrounding whether or not a teacher should state "ahead of time" what he or she hopes the learner will learn from an experience. Some say that explicitly noting this directs a learner's attention to the ideas of focus within the experience. Others say explicitly mentioning

	Topics and questions/prompts include, but are not limited to: a.) Current summary of understanding; specific meanings constructed (e.g., "What does this mean?", "I can explain this idea." "What do I know so far?", "Are there any aspects of the concepts that are confusing to me?") b.) Current degree to which he or she understands a concept; the level with which the learner perceives he or she understands a concept (e.g., "How well do I understand so far?"- Rank understanding by provided categories. "Have my ideas changed? If so, in what ways?") c.) Current self-performance; self-evaluation of his or her own performance or participation within an experience; personal reaction to the experience (e.g., "What was this activity like for me?", "What did I need to do?", "Were there certain steps involved?", "Did certain attitudes help me accomplish this?", "Is my solution/ idea a high-quality possibility in the given circumstance?"); d.) Current "reading" of a learning experience or situation, as it relates to one or more of the following: 1.) Perceived purpose of the learning experience (e.g., "Why is this important to do?", "What is the purpose of this activity?"), 2.) How this experience compares with others (e.g., the degree to which experiences share		specific points and purposes of the experience beforehand limits the depth and breadth of learning, and the learner's potential for creative response. The emphasis here is on reflection upon the learning experience (after or during), not on predefinition before the experience. All wording can be easily understood by a six to eight year old. Tech-Plus: N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners typically are not provided with this circumstance (individual guidance on-demand and in-the-moment reflection on experiences) in the non-virtual world (at least within the context of formal schooling). These preprogrammed questions help the learner analyze a situation in-the-moment and reflect upon the

	cognitive elements- “How does this tie in with what I learned before?”) and, 3.) What is needed or required in a current situation or experience (e.g., “What is happening here?”, “What needs to happen? What do I require to make that happen?”, “What is needed in this situation?”). e.g., Provides a multi-modal survey upon which the learner selects a response. There are no “right” answers.		subsequent meanings they derive during and from these experiences.
Kinds of supporting knowledge:	Supporting knowledge is information that is helpful in (and sometimes essential to) comprehending a situation, problem, or concept. This kind of knowledge does not need to be shared with the learner through direct instruction. Learners can construct this knowledge in the same way they construct meaning related to algebraic ideas. However, it is acceptable to offer this knowledge in the form of direct instruction because the primary focus of learning is on the construction of algebraic/ mathematical concepts. The one exception to this rule is “specific procedural knowledge”. Learners must be allowed to construct this type of knowledge for themselves. In other cases, ideally, learners are provided with opportunities to explore these ideas/skills before the information is directly given.		
B10.*General procedural knowledge* related to content at hand.	App provides learner with opportunities to learn about general procedural knowledge related to the practice of mathematics and early algebra. Procedural knowledge is information about how to carryout an act. In its “broad” form it refers to knowledge	The app essentially asks learners to answer: ““How might a ‘good mathematician’ approach this problem situation?”, “What do I have and what do I need?”	To “Meet”: Procedural knowledge cannot be treated as generic process skills- devoid of content or context (e.g., how to observe, how to measure). Instead, knowledge must be

	about how to engage in mathematical practice, or how to carryout general acts related to applying mathematical knowledge. e.g., App provides general information on how learners can look for repeating patterns in the world around them.	App possibly provides or encourages: - Broader information related to: how to approach situations of transfer (new mathematical conditions and contexts of application).	related to early algebra ideas (see example at far left). *<u>Tech-Plus:</u> Uses app tech. to help learners learn *general procedural knowledge* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of general procedural knowledge. This "layer" of knowledge can be selected/deselected by learner.
B11. *Specific procedural knowledge related to content at hand.*	App provides learner with opportunities to construct specific procedures related to the solving of mathematical/ algebraic problems and problem situations. Specific procedural knowledge is information about how to execute a particular mathematical action, like adding two-digit numbers. Too often, procedural knowledge is shared with learners in the form of well known algorithms and introduced through direct instruction as "the way" to solve a specific mathematical problem. Learner procedural approaches need not match: adult approaches, traditional approaches, or the most efficient approach	The app essentially asks learners to answer: "How can I approach this problem situation?", "How can I solve this problem?", What do I need to do first, next, last?"	To "Meet": Learner must be allowed to construct specific procedural knowledge for him or herself. No teacher-derived algorithms are presented before a learner has had multiple opportunities to generate specific procedural approaches to an algebraic problem. Even though learners are provided with opportunities to construct this knowledge, supports must be in place to help the learner make these procedures explicit to him or herself (e.g., to reflect upon and summarize the procedures one used).

	to solving a mathematical problem situation. e.g., An example of specific procedural knowledge: One way to determine a sum when joining amounts with tens and ones is to, first, combined amounts, then trade in sets of ten "ones" (units) for a ten (rod), and then count the tens (rods) and ones (units) to see how many are altogether.		*<u>Tech-Plus:</u> Uses app tech. to help learners construct *specific procedural knowledge* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of specific procedural knowledge (i.e., a common algorithm) that "unlocks" after the learner has created some of his or her own specific procedures. This "layer" of knowledge also can be deselected by learner.
B12. *Dispositional knowledge related to content at hand.*	App provides learner with opportunities to learn about dispositional knowledge related to the practice of mathematics and early algebra. Dispositional knowledge is useful mindsets, attitudes, and/or habits of mind that are helpful in the practice of a discipline. e.g., An example of dispositional knowledge: "Mathematicians make calculation mistakes sometimes, but they don't become frustrated. Mistakes are easy to fix. Instead, mathematicians double-check their work to try to catch as many mistakes as possible."	The app essentially asks learners to answer: "What mindsets or habits of mind are important when analyzing the conditions of a situation or when analyzing whether a concept fits those conditions?" App possibly provides or encourages: -Observe professionals within a discipline and reflect on their behavior as it relates to adopted mindsets (e.g., Observe experts in the field. What qualities do they express? Persistence? Creativity?).	*<u>Tech-Plus:</u> Uses app tech. to help learners learn *dispositional knowledge* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of disciplinary knowledge. This "layer" of knowledge can be selected/deselected by learner.

		-Humor, perhaps, is used to share these tidbits of disciplinary knowledge. "Grandpa Joe says, 'Mathematicians make mistakes…'." … Is the message more likely to get through, if it doesn't take itself too seriously?	
B13. *Formal disciplinary language, conventional symbols*, and/or *supporting information.*	App provides learner with opportunities to learn about formal language, conventional symbols, or background information and/or pre-requisite knowledge related to the algebraic content at hand. The nature of the formal language and supporting knowledge provided by the app, aims to provide just enough support to make concepts easier for the learner to comprehend, describe, and/or explain. e.g., When learning about "the same amount in each group", the word "equivalent" is introduced, at some point.	The app essentially asks the app-teacher to answer: "What disciplinary vocabulary are important for learners to know?"	To "Meet": Formal language and conventional symbols are introduced only <u>after</u> providing learners with time to explore these ideas on their own, <u>and after</u> providing learners with opportunities to use informal/ invented language. *<u>Tech-Plus:</u> Uses app tech. to help learners learn *formal disciplinary language, conventional symbols, and supporting knowledge* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of supporting/ background information. A "layer" of disciplinary language/ vocabulary can be selected/deselected by learner.

B14. *Technical/ media knowledge related to content at hand.*	App provides learner with opportunities to learn practical technical knowledge and/or other media knowledge, in order to help him or her utilize these skills and ideas in the construction of algebraic ideas. e.g., App provides "How To"s related to: skills acquisition, dispositional/ procedural knowledge related to those skills, and/or practical technology help. "How to use a virtual paintbrush or work a calculator". e.g., App prompts, "Swipe here.", "Drag, like this."	The app essentially asks learners to answer: "Do I know how to use this tool, in order to ________, with this algebraic idea?", "How do I ____ within the app?", "Which tools, modes, media, and/or "languages" are important for communicating this algebraic idea?" App possibly provides or encourages: - Opportunities to gain help through: repetition of an experience, more information about a concept, or observe the demonstration or modeling of a skill. - Specific tasks for learners to become familiar with the properties of, uses of, and dispositional and procedural knowledge required of work with certain tools, media, and modes. - Opportunities to weigh the merits of, or become familiar with the biases of, a range of representational media.	To "Meet": Knowledge is available on how to use tools, media, modes, contexts, or how to approach a specific kind of learning experience, <u>as related to the content of the app</u>. *<u>Tech-Plus:</u> Uses app tech. to help learners learn about or seek *practical technical or media knowledge* better than in the non-virtual world. e.g., App utilizes features of digital programming- embedded "how to" videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of information on technological or media features and procedures.
Externalizations of meaning.	Externalizations are usually enacted in the form of a work product.		
B15. *A cumulative*	App provides learner with opportunities to enact a cumulative project or solve a	The app essentially asks learners to answer: "How can I use what I	*<u>Tech-Plus:</u>

project or problem to solve.	problem situation. "Cumulative" means- increasing in quantity, degree or force. In this case, it refers to refinement and accumulation of meaning along the learning cycle. (The cumulative project replaces the previous fifth "E" in the traditional 5Es learning cycle, *Evaluate*) [Bybee, 2002; National Academy of Sciences, 1998]. Project can be, either: - Summative: project is completed at the end of the learning cycle. (In this case, other activities or mini-projects form the contexts for learning along the rest of the learning cycle.) Thus, a summative project is an opportunity to summarize cumulative meaning near the end of study. or -Formative: the same project is the primary context for learning along all/most phases of the learning cycle. Thus, a formative project is an opportunity to "accumulate" meaning along the way, and make "final revisions" to the project (which was probably previously revised various times along the way). A formative approach may call for repeated adaptation of one project along the way, or may call for a series of related stages of a project. Cumulative project (either summative or formative) must meet one of three requirements: a.) a real world goal- (e.g., conduct plausible research, solve a "real" problem), b.) provide a meaningful role for	know or have been learning to ___?" App possibly provides or encourages: - A public endeavor or social outcome that affects or benefits another/others.	Uses app tech. to help learners enact *a cumulative project or solve a complex problem* better than in the non-virtual world. e.g., This depends upon the nature of the project. Learners might create a digital slideshow to share their observations and photographs of repeating patterns they documented at the playground. The possibilities are nearly endless.

	the learner (e.g., debates an issue, leads a group of learners, presents an idea) or, c.)a real world (actual or simulated) audience (e.g., public citizens, other learners).		
*B16. *Conjectures/ proposals.*	App provides learner with opportunities to make conjectures or proposals related to mathematical ideas. Conjectures are statements that seem true, but do not have sufficient evidence for proof. These conjectures can be in the form of hypotheses, principles, rules, theories, laws, or properties. Conjectures may arise as a result of abstracting ideas from their contexts/ over many observed cases, but the outcome of a conjecture/ proposal is different than the act of abstracting. Conjectures need not be expressed in conventional mathematical language (although the wording used needs to be precise), nor be expressed in a certain mode, media, or "language" (e.g., written symbols); There should be increasingly-formal and generalized wording of rules, principles, and properties, with support from app-teacher and slow integration of formal language. Conjectures can be shared in a number of forms (e.g., verbalized words; visual/ physical models). However, making a conjecture is different than "brainstorming" ideas. Conjectures are	The app essentially asks learners to answer: "What is the rule?", "Does this property apply in all cases?", "What's a way to state this idea more broadly or more clearly so everyone can understand?" App possibly provides or encourages: - Alternative conjectures or more than one proposal. Or, asks for edited proposals under different conditions.	To "Meet": Immediate feedback from the app is essential (e.g., "That rule works in this instance. Does it work in every instance?" <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this opportunity (to make conjectures) in the non-virtual world (at least within the context of formal schooling).

	restricted by conditions (e.g., a conjecture that represents: <u>all</u> cases/ a set of cases, <u>the most</u> likely outcome, <u>the most</u> "workable" solution path or solution, given certain constraints of time, contexts, or other conditions.) e.g., True/ False number sentences are provided for children to evaluate. After categorizing a series of true and false statements, the learner is asked to explain or identify "the rule" of the true statements.		
B17. *Formalized plans.*	App provides learner with opportunities to plan for projects in formalized ways Plan work products and anticipate needs for product execution. e.g., An activity asks the learner to make "blue prints" for solving a mathematical problem situation. "Draw a picture of what you will do first, next, last." e.g., A challenge asks the learner to "order supplies" they anticipate needing for a virtual project.	The app essentially asks learners to answer: "What do you need to do, to get _____ result?", ""What do you need to do first?", "What do you think might happen if….?", "What might go wrong?", "What do you need to bring?"	*<u>Tech-Plus:</u> Uses app tech. to help learners make *formalized plans* better than in the non-virtual world. e.g., Utilizes digital programming to provide learner with "guiding templates" for planning. Accessibility to a wide variety of visual organizers, embedded with template-specific scaffolding to assist the learner in its completion.
Category	**Description / Examples**	**Indicators**	**Decision Rules**
C.) Specific early algebra concepts.	Early algebra is refers to learning expectations that are delineated by NCTM (2000) as algebraic in nature, enabling learners to: "understand patterns, relations, and functions"; "represent and analyze mathematical situations and structures using algebraic symbols"; "use mathematical models to represent and		Learners must be allowed to construct these concepts for themselves. An essential aspect of construction, however, is time for learners to reflect upon and summarize meaning, throughout the learning process.

	understand quantitative relationships"; and, "analyze change in various contexts" (p. 395). This category encompasses early algebra concepts, as defined by "consensus-driven topics", beyond the timelines typically associated with an age group or grade level.		
*C1. *Rules/ principles/ properties of whole numbers and operations.*	App provides learner with opportunities to construct meaning related to rules, principles, and properties of whole numbers and operations. This is different than making a conjecture. While a learner might make a conjecture about an algebraic rule, principle, or property, this category relates to the provision of learning experiences that help the learner construct understanding of particular rules, principles, and properties of mathematics. Operational properties: Identity, Inverse, Commutative, Associative, Distributive. e.g., An activity is designed to help the learner notice that, when she adds the same numbers in a different order, she gets the same sum.		Tech-Plus: N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with the opportunity to construct mathematical rules, principles, and properties for his or herself in the non-virtual world (at least within the context of formal schooling).
C2. *Mathematical classifications.*	App provides learner with opportunities to classify, sort, and order objects, sets of objects, amounts, problems, and changes. Concepts include, but are not limited to: a.) Problem types and classes of problems (e.g., Join, Separate, Compare, Part-Part-Whole, Equalize) [Carpenter, Fennema, Franke, Levi, Empson, 1999].		*Tech-Plus: Uses app tech. to help learners *classify, sort, and order objects and amounts* better than in the non-virtual world. e.g., App utilizes multi-touch technology of device to allow learners to virtually manipulate

	b.) <u>Sort</u> and <u>order</u> objects by physical properties related to mathematics (e.g., size, shape, amount, shared features or functions).		objects and ideas, by sorting, ordering, and grouping.
C3. *Patterns- geometric and conceptual.*	App provides learner with opportunities to construct meaning related to the nature of mathematical patterns in sequences, organized configurations, and/or randomly distributed in the larger world. Patterns may be geometric, numeric, or conceptual in nature. Concepts include, but are not limited to: a.) Growing patterns (bigger or smaller) and their units of growth (e.g., +1, +2, +3); b.) Repeating patterns and their units of repeat (with increasing size and complexity of unit of repeat) (e.g., ABABAB, ABCAABCA); c.) Patterns in numeric data (e.g., The relationship between the 7, 17, and 27 on a Hundreds Chart is the same as the relationship between the 29, 39, and 49). e.g., Activity asks learner to describe, extend, repeat patterns of shape, sound, and number, and to explore different patterns to determine how they are generated.		To "Meet": The content of the app is not focused on visual patterns. While learners may recognize visual/ superficial patterns, at first (e.g., there are two zeros in each of the following numbers- 001, 100), the app's primary focus should be on the content at far left. *<u>Tech-Plus:</u> Uses app tech. to help learners construct meaning related to *the nature of mathematical patterns* better than in the non-virtual world. e.g., App provides simulations of physical contexts and subject-matter, for learners to <u>observe and manipulate</u> patterns of which he or she <u>might not otherwise have access</u> in the non-virtual world. This may include patterns of fish or reptile scales, the mirrored symmetrical patterns of enlarged snowflakes, or various types of structural or visual patterns in nature (spirals in plant and animal body configurations,

			waves on ocean floor, cracks on moon or desert surface). This also exposes learners to other types of patterns beyond simple geometric or numeric patterns.
C4. *Mathematical changes.*	App provides learner with opportunities to construct meaning related to mathematical changes. Concepts include, but are not limited to: - Quantitative and qualitative changes (e.g., Joe grew 2” this year; Joe grew taller this year); - Functional changes (e.g., +3, +3, +3- skip-counting by 3s).		*<u>Tech-Plus:</u> Uses app tech. to help learners construct meaning related to *mathematical changes* better than in the non-virtual world. e.g., App utilizes temporal/ interactive features of digital programming and device to isolate parts of a mathematical system and/or observe the effect of changes made by the learner. Like, the metaphor of an “input/output machine” within a virtual lab would allow learners to see the effects on output, as they make changes to both input and the unit of change.
C5. *Algebraic relations.*	App provides learner with opportunities to construct meaning related to algebraic relations of quantitative amounts. Concepts include, but are not limited to: a.) Relations between amounts, in terms of equality (e.g., =, <, >); b.) Relations between amounts, in terms of descriptive part to whole ratios (e.g., 3 out of 5 apples are red);		*<u>Tech-Plus:</u> Uses app tech. to help learners construct meaning related to *algebraic relations* better than in the non-virtual world. e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom in or out, isolate parts, observe

	c.) Relations between amounts, in terms of function (e.g., ratio of “input” to “output”); d.) Relation between addition and subtraction; e.) Relation between multiplication and division; f.) Relation between addition and multiplication; g.) Relation between subtraction and division.		cause and effect of input/ output. These features may make algebraic relations more explicit by allowing the learner to control perspective, negotiate the visual and conceptual space between ideas, and simulate effects as they relate to learner-controlled changes.
C6. *Algebraic symbols.*	App provides learner with opportunities to create or invent symbolic notation. Including through: writing, pictures, invented symbols. e.g., Activity asks learner to create symbols that mean two amounts are equal, greater than, less than, and unequal (A □ means two amounts are the same as one another). App provides learner with opportunities to gradually learn conventional algebraic symbols, <u>after</u> learners have developed their own. e.g., +, -, x, n	App possibly provides or encourages: - Opportunities to <u>use</u> invented symbols to describe real mathematical situations.	*<u>Tech-Plus:</u> Uses app tech. to help learners construct meaning related to *algebraic symbols* better than in the non-virtual world. e.g., App utilizes features of digital programming, like digital “layers” to compare different symbol systems with one another.

<u>How:</u> These descriptors answer the hypothetical question, "How are particular characteristics ideally imparted, within a curriculum that aims to support primary children's potential to learn early algebra concepts "with understanding", through multi-touch, mobile, iOS mathematics education apps?"			
"How" Categories	**Description**	**Examples/ Indicators**	**Decision Rules**
D.) Through programmatic, experiential, and environmental provisions.	Programmatic, experiential, and environmental provisions refer to the various potential learning experiences, and programmatic and environmental characteristics within the app. The "learning cycle" is an inquiry-based or experiential-based instructional design model that describes a learning "sequence", based on a series of experiential phases through which the learner can move. Several models exist that describe this general cycle (Bybee, 2002; National Academy of Sciences, 1998; Bredekamp & Rosegrant, 1992). The model to which I refer is a version of the 5Es learning cycle (Bybee, 2002; National Academy of Science, 1998) that I adapted for this study (see Ch. 2 Literature Review). My adapted cycle includes the five phases, in the order of: Engage, Explore, Explain, Elaborate, and Cumulative Project, and the entire cycle need not be "completed" before a phase is revisited.		Some categories related to the virtual environment of the app, showcase instances in which the meeting of the particular category inherently implies the meeting of Tech-Plus qualifications. This is because the meeting of said category implies the inclusion of preprogrammed pedagogical knowledge and its consequent enactment, which would likely not otherwise be enacted in a non-virtual formal schooling environment. Thus, in some cases, Tech-Plus is not applicable, and the category's description implies the app-designer's leverage of technology in an inherently value-added way. In these cases, an asterisk is added in front of the category number to differentiate these types of categories. Additionally, within other categories, the use of app technology does not necessarily support a better outcome than what might be found in the non-virtual world. Instead, the coder should observe whether or not the app utilizes affordances of the medium and device to <u>enhance the enactment</u> of the outlined characteristic in the <u>virtual</u> world.

Movement through content.	**App "moves through" algebraic concepts in certain ways, and for particular purposes. One theme throughout is, there is no rush for content coverage.**		
D1. *Reset* activity, fix/adapt work, or try again.	App provides learner with opportunities to reset activity, fix/adapt work, or try again. Allows learner to demonstrate through another similar mathematics-based activity or fix the current one. e.g., An activity allows learner to manually "erase" his virtual work by using his fingertip to erase similarly to how a rubber eraser might be used in the non-virtual world.		*<u>Tech-Plus:</u> Uses app tech. to help learners *reset/ try again, fix work* better than in the non-virtual world. e.g., Makes use of digital programming to provide ease of reset/ ease of correction (instant reset button), and/or allows learner to isolate certain "layers" to erase. This latter characteristic, in particular, improves upon the physical limitations of the non-virtual world.
*D2. *More than one experience for a single phase* of learning cycle.	App provides more than one learning experience for a single phase of the learning cycle. e.g., Learners can describe the essence of a concept in their own words in a "sound lab" <u>and</u> explain the concept's essence by making a virtual poster- both activities of which could be part of the "Explain" phase of the learning cycle.		<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to increased learning time and dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this amount of time/ dedicated conceptual space in the non-virtual world (at least within the context of formal schooling).

D3. *Exploration of* ideas in the virtual environment.	App provides learner with opportunities to explore the app environment, manipulate materials, test his or her ideas against a particular condition and/or context, compare his or her ideas against others', and explore the "simulated real world"- (e.g., virtual labs; simulations) to figure out properties and components of algebraic ideas. This represents the "Explore" phase of the learning cycle. e.g., Environmental provocations elicit curiosity and prompt learner investigation of algebraic ideas and their effects in various simulated environments, like a virtual "room" in which learners can lay tile flooring in various repeating patterns.	App possibly provides: - mathematical stimuli that promote learners' natural curiosity and inquiry. - ways to interact with those stimuli to discover more about the way they work.	To "Meet": App must offer enough "stimuli"/ "ways to interact with stimuli" to sustain learner interest for more than five minutes. *<u>Tech-Plus:</u> Uses app tech. to help learners *explore* ideas and their effect in various simulated environments better than in the non-virtual world. e.g., App provides <u>simulations of contexts, conditions, and subject-matter</u> that might not otherwise be possible <u>for learners to experience and explore</u> in the non-virtual world. For example, an underwater setting could allow the learner to observe and manipulate the patterns of scales. Virtual magnifiers can enlarge snowflakes so the learner can observe and manipulate their mirrored symmetrical patterns. Learners can observe and manipulate patterns within various types of plants, animals, and habitats for structural or visual patterns (spirals in plant and animal body configurations, waves on ocean floor, cracks on moon or desert surface).

*D4. *At least one new conceptual condition*, during learning cycle.	a.) App provides at least one new conceptual condition during the learning cycle, in addition to the original condition under which the algebraic concept was first introduced. This new condition is usually presented during the "Elaborate" phase of the learning cycle. This includes, but is not limited to: a.) A change in an aspect of the case (e.g., "What if there were no red or blue blocks? Could you create a pattern with yellow and green blocks, instead?"), b.) An introduction of well-chosen contrasting cases or presentation of an anomaly/ discrepancy (e.g., "People call this ____ a pattern. They also call this ___ a pattern. How can that be?"), c.) A proposal of problems during learning (e.g., "What if an arrangement of blocks used the same red square, over and over again, but varying spacing between the blocks that repeats?" i.e., X, big space, X, little space, X, big space, X, little space. "Would this be a pattern?") or, d.) The requirement that a concept apply to <u>all</u> cases in a given set (e.g., "What if an arrangement of blocks used the same red square, over and over again, <u>and</u> the same spacing between blocks?" i.e., X, little space, X, little space, X, little space. "Would this still be called a pattern?"). b.) App offers learner opportunity to "Explore" and "Explain" again, in light of the introduction of new conditions. Learner	The app essentially prompts/ asks learner: "Imagine needing to design________." Would it need to _________ (act a certain way)? Why or why not?", "What if…..?", "Does this idea still hold true, given this new condition?"	<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to increased learning time and dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this amount of time/ dedicated conceptual space/ particular circumstance in the non-virtual world (at least within the context of formal schooling).

	explanations need not be in the form of verbal or written language; could be adaptation of a picture or model. e.g., An activity encourages the learner to explore existing ideas given the introduction of a discrepant case.		
*D5. *At least one new contextual restraint on action/outcome*.	App provides at least one new restraint on learner action or outcome, in addition to the original context or set of restraints under which the action or outcome was first framed. This new restraint is usually presented during the "Elaborate" phase of the learning cycle. This includes, but is not limited to, asking learner to change an action or outcome: a.) For a different audience (e.g., "This time, change your 'input/output machine' to make it easier for the <u>shoemaker</u> to use. She needs to make pairs of shoes, not individual buttons."), b.) To represent another point of view, another solution, explanation, description, or application (e.g., "Use the microphone again to record another way someone might describe part of the apples" [Three of five are red. Two of five are green. Instead of, one of five is bumpy. Four of five are smooth.].) or, c.) To frame within another physical/ disciplinary/ subject matter context (e.g., "This time, make a pattern with three different textured seashells, on the beach". Instead of, making a pattern with two different sized bubbles, in a carwash.).	The app essentially prompts/ asks learner: "What might happen if I showed or shared this idea another way, or described the idea differently?"	<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to increased learning time and dedicated conceptual space. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. In this case, the ability to provide learners with a range of diverse contexts, in particular, improves upon the physical limitations of the non-virtual world.

*D6. *One condition, no more than five times consecutively.*	App presents a single algebraic concept, under one condition and/or context, no more than five times consecutively. (This helps with conceptual flexibility and avoids over-contextualization of a concept.) e.g., After initial introduction of a condition, app presents concept within random conditions/ contexts, or with changes to the conditions and contexts, intermittently.		<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance in the non-virtual world (at least within the context of formal schooling).
*D7. *Moves with coherence.*	App "moves" learners through increasingly complex ideas related to a single concept, in a way that respects the degree to which the more-complex ideas share characteristics with previous-known ideas. New concepts are approached with respect to a possible need for "supporting knowledge" that learners may require to comprehend or enact meaning. However, caution must be used against superficial sequencing of concepts, according to surface features or according to outdated perceptions of "basics first". e.g., A learner needs to understand the idea of equivalency (i.e., the same amount in each group) in conjunction with		To "Meet": App must organize concepts according to the principles that can be used to define them, not according to surface features or outdated perceptions of "basics first". <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed condition. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is

	determining the symbolism behind the equal sign.		inherently "better" than in the non-virtual world. This is because, while learners are generally provided with coherent curricular movement in the non-virtual world (at least within the context of formal schooling), the nature of coherence for this group of learners is not always free from rigid and superficial sequencing of mathematical concepts.
*D8. *Content concepts are divorced from timelines*.	App provides algebraic concepts divorced from strict timelines and developmental timetables. Learning trajectories are individual/ flexible, and there is a high ceiling on "complexity" of material, despite targeted age group of 6-8 year olds. e.g., A wide range of algebraic concepts are available (through the learning experiences provided by the app) to learners of any age. This may include content recommended by NCTM for grades 3- 5.		<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because, learners are rarely provided with this circumstance in the non-virtual world (at least within the context of formal schooling).
**D9. Differentiated learning routes* based on *various*	App provides individual/ differentiated learning routes based on various seamless assessment results. A "learning route" refers to an individual learner's trajectory of learning.	Possibly, the app-teacher encourages: - Learner to fill a virtual "portfolio" with app "products" and completes a checklist of	To "Meet": Assessment must occur seamlessly, within the context of learning experiences, actions, and outcomes that would otherwise be provided for

seamless assessment results.	Seamless assessment is defined as an act in which an educator gauges the nature and quality of a learner's understanding, in a way that is "indistinguishable from classroom instruction" (Abell & Volkmann, 2006, p.1). This implies the instruments, through which assessment might occur, are the same as those that would otherwise be used within the course of instruction. This does not include quizzes or tests designed to measure and evaluate learning. Instead, it includes analysis of learner actions, experiences, and outcomes that would otherwise be used to support learner's construction of meaning and the goal of "learning with understanding". Assessing a learner's current understanding of an algebraic concept, especially in qualitative/ descriptive ways (that are more informative in shaping instruction), is different than evaluation. Evaluation is making a judgment about the amount, number, or value of something (e.g., letter grades on assignments or within subjects, ranking and comparing learners with one another or according to degree of accomplishment). The purpose of evaluation is different than the purpose of assessment, as described in this study. In this study, the purpose of assessment is to help describe a learner's current understanding of a concept, in order to influence future instruction. As such,	various learning experiences, actions, and outcomes in which he or she has participated/ included in the portfolio.	meaning construction, or that support the goal of "learning with understanding". Due to the nature of such experiences, actions, and outcomes, assessment of a learner's understanding may be more difficult to carryout. This is because, unlike with evaluation-centric models, "measureable" learner outcomes and actions are not the primary focus of learning experiences, actions, and outcomes. Thus, the app may need to rely more heavily upon learner self-assessment and a wide variety of assessment techniques (e.g., observation, interview/ individual meetings, concept mapping, conceptual cartoons and drawings). Additionally, because such techniques, often, are less compatible with the app-medium, there may be fewer formal ways of assessing the learner's current understanding than what might be found in an evaluation-based model. Descriptive rubrics do not necessarily need to be formalized and/or shared. However, the programming needs to serve the same function as a descriptive rubric would- isolating and

	descriptive rubrics help to isolate and identify specific aspects of a concept, with which the learner might struggle. Assessing a learner's actions and outcomes based on these rubrics can help determine the specific aspect of the concept, on which the learner needs to work next. This directly influences a learner's individual learning route. e.g., App utilizes those learning experiences, outcomes, and actions already provided to support "learning with understanding", that are possible to assess through the app-medium (e.g., self-assessment reporting, some "close-ended" activities, virtual portfolios with corresponding checklists) to vary an individual learner's trajectory of learning.		identifying specific aspects of a concept with which the learner struggles, in order to determine an individual learning route that includes focus on that aspect. <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance in the non-virtual world (at least within the context of formal schooling)- either in terms of highly individualized learning routes or assessment models that are non-evaluative in nature.
*D10. High ratio of *constructivist learning experiences.*	App provides a high ratio of constructivist learning experiences across the total app "program". This means the app primarily provides mathematical learning experiences that are constructivist in nature. In this case, "constructivist" refers to a philosophical approach to education in	Reflective questions: "How can I help learners 'come to understand' this idea or concept in a way that allows him or her to grasp its premise for him or herself?"	To "Meet": The app must provide a way to make the concepts of focus explicit to the learner (e.g., through reflection and summary during and after the constructivist experience). <u>Tech-Plus:</u> N/A

	which learners are encouraged to mentally construct understanding by reflecting upon learning experiences and building a mental model that captures his or her sense of the world. It is based on the premise that no one, simply, can transfer meaning to anyone else. Instead, meaning must be "built" by the learner through a process of personally situating knowledge within his or her own mental model. An critical aspect of meaning construction, however, is time for learners to reflect upon and summarize meaning, throughout the learning process. In this way, algebraic concepts are made explicit to the learner. Important concepts being made explicit to the learner is another essential aspect of meaning construction. e.g., A constructivist learning experience might be a virtual lab in which the learner can manipulate each part of a change ratio and observe the effects. Like, adjusting the level of growth (1, 2, 3 inches) and the provision of time (1, 2, 3 years), and observing the visual and numeric change in a character's change in growth. Standard units of measurement need not be used (i.e., The character might grow 3 "paperclips" taller while in kindergarten).		Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance (a high ratio of constructivist learning experiences) in the non-virtual world (at least within the context of formal schooling).
Kinds of contexts.	**Content is presented within particular contexts and contexts have particular characteristics.**		

D11. *Decontextualized experiences* and *explicit connection to larger* context.	a.) App provides at least one decontextualized activity; one learning experience in which the problem to solve is the task, itself. e.g., An activity allows the learner to focus on one aspect of one operational property. The learner is tasked with rearranging two groups of objects and joining them together to determine the total. App guides learner to try various configurations- change amounts of individual objects in each group, change the order of the individual objects in each group, change the position, arrangement, or order of the two groups. Does the sum stay the same or change? Continuum of kinds of contextualized/decontextualized activities, includes: - Real life contribution - Real life practice - Immersive environments - Playful games - Decontextualized activity b.) Decontextualized activity is explicitly connected to the larger context. e.g., After the learner participates in the activity above, the next activity is one in which a character must add groups of an object (gold coins) to find out how much he has, within the context of an immersive environment. Some sort of explicit link		To "Meet": The activity is toward the decontextualized end of the context continuum. e.g., A playful game or decontextualized activity. *<u>Tech-Plus:</u> Uses app tech. to provide learners with *decontextualized activities explicitly connected to the larger context* better than in the non-virtual world. e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom out from individual activities to their larger contexts. These features can increase a learner's conceptual proximity to the idea of a relationship between a task and its larger application, more than features used in the non-virtual world, like words.

	between the two activities is made, beyond sequential order (words like, "Now that you've rearranged and joined amounts of gold coins to determine how many you have, you can use your gold coins to buy things.").		
D12. *Contextualized experiences* and *embedded concepts* made *explicit.*	a.) App provides at least one contextualized activity; one learning experience in which the problem to solve is part of a larger task or context. "Scaled contexts" are acceptable. These are contexts that are as close to authentic as possible, usually without contributing to the advancement of the discipline, itself. e.g., A virtual context in which the learner uses gold coins to buy things at a store. She must continuously combine amounts to determine how much she has to spend. This is because even as coins are spent, the learner's character earns more coins. The amount leftover (after spending some coins) must be combined with new income. b.) Concepts embedded within contexts are made explicit. e.g., As learner participates in activity above, the app points out concepts like, "Does it matter in what order you add up the coins in your two palms?"		"To "Meet": The activity is toward the contextualized end of context continuum. e.g., An immersive environment, real life practice, or real life contribution. *<u>Tech-Plus:</u> Uses app tech. to provide learners with *contextualized activities the embedded concepts of which are made explicit* better than in the non-virtual world. e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom in from larger contexts in order to temporarily focus on a specific concept. These features can increase a learner's conceptual proximity to the idea of a relationship between a larger application and the embedded concepts within, more than features used in the non-virtual world, like words.

D13. *Higher ratio of contextualized* learning experiences than decontextualized.	App provides a higher ratio of activities and learning experiences that are contextualized in nature as compared with those that are decontextualized in nature. e.g., More learning experiences are immersive environments, opportunities for real life practice, or opportunities for real life contribution than are games or decontextualized activities.		*<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance (a high ratio of contextualized learning experiences) in the non-virtual world (at least within the context of formal schooling).
D14. Balance of *varied-structured* learning experiences and contexts, and some open-ended learner response.	a.) App provides a balance of varied-structured learning experiences and contexts, on a continuum from open-ended mathematical activities to highly structured mathematical activities. e.g., Virtual "sand boxes", labs, and simulations available for exploring relevant materials, media, tools, contexts as it relates to algebraic content. Highly structured activities are also available. b.) Within activities toward the more "open-ended" end of the continuum, the app accepts/ provides space for the learner to input creative, open-ended ideas, or	App possibly provides or encourages: - Creative responses to mathematical content.	*<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance (a balance of varied-structured learning experiences) in the non-

	submit multiple answers/ solutions/ proposals.		virtual world (at least within the context of formal schooling). Many learning experiences in formal schooling are highly structured.
Orderliness and clarity.	**Content is orderly and clear. Form follows function.**		
D15. *Clear layout and ease of navigation through app.*	App provides ease of navigation through virtual curriculum. App provides clear spatial layout; learner can likely picture a clear map of the app space in his or her mind; learner can likely visualize how various parts of the app (e.g., individual activities) fit within the whole of the app. Learners are unlikely to get lost within the app- including through hyperlink "rabbit holes". e.g., Navigating the app is easy; buttons and commands are "user-friendly"; movements are easy for learner to execute- swipes, grasps, trace, tap, slide, etc. A "map" or reference to layout is easily accessible.		*Tech-Plus: Uses app tech. to enhance *ease of navigation through curriculum* in the app's virtual world. e.g., A "map" of the space is easily accessible (e.g., on the homepage). e.g., App provides multi-modal support for traversing the site and making navigational choices.
D16. *Clear directions.*	App provides clear directions for interacting with content and navigating the virtual environment. e.g., Directions are easy to understand for six to eight-year-old.		*Tech-Plus: Uses app tech. to enhance *clear directions* within the app's virtual world. e.g., Directions are available in multiple modes. "Multiple modes" means a message is communicated through more than

			one form of representation- typically drawing on more than one physical sense (e.g., sight, hearing, touch). Directions are visual (written words and pictures) *and* verbal (spoken words).
D17. *Universal Design features standard and controllable.*	App includes Universal Design for Learning (UDL) features across the app program. UDL is a framework for creating flexible instructional methods, goals, assessments, materials, and learning environments that can adjust to individual learning differences. At least five UDL features must be available. A primary benefit is that, by the inclusion of UDL characteristics in an app, modifications and assistance are available all the time- even for instances when unanticipated help is required. This results in a "safety net" for learning. While some learners may not use the offered features, others (many of whom are unpredictable) can be helped with undiagnosed or unpredictable problems (e.g., Just as in the non-virtual world of UD architecture, where wheelchair ramps are used for many additional reasons beyond wheelchair usage- strollers, hand trucks, walkers). e.g., Common UDL features include, but are not limited to: various degrees of text complexity and wording, flexible time frames, ELL support/ language translation,	App possibly provides or encourages: -A feeling of, "I can do many things on my own here." - Asynchronous guidance (e.g., visual cues, arrangements, graphic organizers and labels to show hierarchies, bold print, colors, self checklists, highlights, anchor charts/ reference tools.) -Seemingly-synchronous guidance (e.g., prompts from app program or character "suggestions" help learner reorient or question learner based on his or her interactions within app.) - Synchronous guidance (e.g., live chat, peer coaching through criteria-guided checklists, live mini-sessions, tutorials, or workshops.)	"To "Meet": UDL features are always available- at every level across program at all times, but can be turned on/off individually and easily. Additionally, at least five UDL features must be available. *<u>Tech-Plus:</u> Uses app tech. to provide learners with *UDL features* better than in the non-virtual world. e.g., App utilizes features of digital programming- pop-ups, embedded videos, roll-over definitions, physical perspective changes, programmed reminders about potential trouble spots. e.g., Learners can deselect/ select UDL features, like "layers" in Google Earth.

	voice command, narration, vocab. development/ definitions. These modifications to existing learning experiences result in enough adaptation to support the learner, without changes to desired outcomes.		
Balanced cognitive-affective space.	**Cognitive-affective refers to the act of thinking or reasoning as inherently influenced by or related to one's emotions. These two aspects are inexorably intertwined. Content is arranged in the space, within activities, and across the program in ways that help learners maintain a cognitive-affective balance.**		
D18. Helps learner *activate prior knowledge*.	App helps learners activate his or her prior knowledge related to algebraic content at hand, and helps learners make connections between these ideas and present learning. Prior knowledge refers to the understanding a learner already possesses, before a new learning experience is undertaken- particularly as they each relate to a single concept. e.g., App mentions a concept that a learner may have observed in her previous everyday experience. "Have you ever noticed that numbers are used to describe many different ideas?"	Reflective questions: "What big ideas have learners already formed?", "What are learners' present hypotheses and conjectures?", "Are there existing connections learners have made between ideas? Are they accurate?", "If not, what misconceptions might learners have and how can they be changed?" (Unless a specific change is not currently required. App may be "content" with the idea of "truth-for-now", knowing that over the course of the app program, a learner's understanding will evolve to align with more consensus-driven meaning.	*<u>Tech-Plus:</u> Uses app tech. to help learners *activate their prior knowledge and connect this to present learning* better than in the non-virtual world. e.g., Utilizing data from the learner's "life experience" survey may help to activate knowledge embedded within a certain life context. ("Have you ever noticed that numbers are used to describe many different ideas? "How are numbers used <u>at the zoo</u>?"/ "How are numbers used <u>in a game</u>?")
D19. *Initiates and maintains a*	App provides learning experiences and an environment that initiates and maintains	App possibly provides:	"To "Meet": Playful and engaging experiences and

learner's cognitive and affective engagement.	learner's cognitive-affective engagement in algebraic concepts of focus, by drawing upon natural human motivators (e.g., Draws upon common human motivators like, authorship/ autonomy, play, personal relevancy, and curiosity). e.g., Introduces new objects, events, or people in a way that speaks to learner's natural curiosity. e.g., Introduces algebraic content of focus in problem situations or contexts.	- Beauty and wonder- The virtual environment has a feeling of aesthetic satisfaction and interest. - Algebraic ideas framed with a sense of fascination and intrigue for mathematics' capabilities.	contexts can be "fun". However, "fun", as a superficially defined and overused stimulus, should not be the primary motivational driver. Authentic provocations should drive engagement. *<u>Tech-Plus:</u> Uses app tech. to *initiate and maintain a learner's cognitive-affective engagement* better than in the non-virtual world. e.g., The interactivity of the device, <u>if utilized by the app</u>, tends to offer nearly instant and long-lasting engagement with content.
*D20. A *balance of sensory stimulation and peace.*	App provides a balance of sensory stimulation and peace. Algebraic content and the contexts in which they are presented are interesting, but not over-stimulating. App avoids sensory overload. Sensory overload is defined as, excess noise/ sound effects, increasing volume, quick visual movements, fast video cuts, an over-abundance of bright or primary colors, and requisitions for the learner's constant touch or movement. There are cognitively and physically active learning activities, in addition to passive tasks (e.g., viewing a video clip being more passive than drawing). However, even "active" learning is not achieved	App possibly provides or encourages: - Places for the participant to go that are peaceful or stimulating, depending upon his or her needs in the moment. - A feeling of, "This is a good place to be."	"To "Meet": Most learning experiences/ environmental provocations require active cognitive effort. A few passive cognitive activities are permitted. App must avoid sensory overload, even within "spaces" that are intended to contain increased sensory output. <u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if

	through sensory overload. The primary emphasis of activities is on active cognitive activity, with some physical movement. Some quiet and passive activities are also encouraged. e.g., Learners can switch or move between virtual "rooms", some of which have greater sensory output, and some of which have less. Most may require active cognitive activity. e.g., The app relies upon active cognitive activity for stimulation, but the entire virtual environment is peaceful.		the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance (a balance of sensory stimulation and peace) in the non-virtual world (at least within the context of formal schooling).
*D21. Provides *plenty of time.*	App imposes no time limits within learning experiences or across the app program, unless there is a valid educational reason for a concept to be embedded within a timed activity (e.g., Competition with self or others is very important to learning this concept <u>with understanding</u>?) e.g., Digital programming allows for the control of time provisions for learning. Either there is no time limit for accomplishing a task/activity or exploring a simulated environment, or the learner can easily control the imposition of time limits.	App possibly provides or encourages: - A feeling of, "I did not feel rushed".	<u>Tech-Plus:</u> N/A Ultimately, this category is about learner accessibility to this preprogrammed learning condition/ circumstance. Thus, if the app meets the characteristics at far left, this implies the leverage of technology in a way that is inherently "better" than in the non-virtual world. This is because learners are rarely provided with this circumstance (plenty of time for learning) in the non-virtual world (at least within the context of formal schooling).

D22. *Scaffolds* learning from *least to most teacher-help*.	App-teacher scaffolds mathematical learning by moving along a continuum, from more cognitive demand on the learner to less cognitive demand on the learner (i.e., least to most teacher-help). Scaffolding is "a changing quality of support in which the (app-teacher) adjusts the assistance she provides to fit the (learner's understanding)" [CCCRT, 2005]. e.g., "Open-ended questioning" is provided before "direct instruction", along a continuum of scaffolding techniques. Continuum of kinds of scaffolding from least to most teacher help: - Open-ended questioning (e.g., questions related to descriptions, predictions and planning, explanations, and relations to one's own experience); - Providing Feedback (e.g., encouragements, evaluations, think aloud, requests for clarification, interpretation of meaning, acknowledgements and information talk); - Cognitive Structuring (e.g., rules and logical relationships, sequencing, contradictions); - Holding in Memory (e.g., restating goals, summaries and reminders);	Reflective questions: "How can I guide the learner in attending to a specific facet of this work, based on his or her individual needs?", "What are possible points of frustration or misconception related to the ideas learners are forming?"	*<u>Tech-Plus:</u> Uses app tech. to provide learners with a way to *scaffold learning from least to most teacher-help* better than in the non-virtual world. e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom, pop-ups, hyperlinks, and rollovers. These features may make it easier to, a.) Provide individual scaffolding on demand- due to constant availability of such features and, b.) Offer preprogrammed pedagogical knowledge of the scaffolding continuum and consequent steps in moving along the scale from least to most teacher assistance. Each of these aspects helps to keep learning as learner-centric as possible, while hypothetically avoiding learner frustration and premature exit from learning.

	- Task Regulation (e.g., matching interests and experience, making more concrete, rearranging elements, reducing alternatives); - Instructing (e.g., modeling, orienting, direct questioning, elicitations, coparticipation). [Notari-Syverson, O'Connor, Vadasy, 1998, p. 30].		
D23. *Presents ideas through a range* of materials, processes, tools, modes, media, and "languages".	a.) App presents key ideas through a range of materials, processes, tools, modes, media, and "languages", so as to increase conceptual accessibility of algebraic concepts and extend learners' multi-media literacy. At least three diverse materials, tools, modes, media, or "languages" need to be used to represent algebraic ideas. One of the three must be a material, tool, mode, medium, or "language" not traditionally valued in the practice of mathematics or algebra, and/or within formal schooling (e.g., collage, shadow play, music). e.g., A task to invent symbols that represent certain mathematical relationships between two sets of numbers, using symbolic "language", in a visual mode, through the medium of paint and paper. b.) Presents diverse (at least two) visual/spatial/conceptual perspectives. e.g., Two views on a repeating pattern are available- 1st person and 3rd person.		*Tech-Plus: Uses app tech. to *present algebraic ideas through a range of materials, processes, tools, modes, media, and "languages"* better than in the non-virtual world. In addition to the possibility of virtual equivalencies of traditional media (virtual painting, building, sculpting, making collages, etc.) within the app (which are not necessarily "better" than in the non-virtual world), there is the possibility of utilizing tools, modes, and media that are unique to digital programming and its corresponding device(s) of use, such as temporal capabilities like motion and sequenced animation (e.g., stop motion animation). Alternatively, the digital programming and device may provide a unique value that would

			not otherwise be available in the non-virtual world, such as convenience (e.g., shadow play may be enacted more easily in the virtual world because the light source and projection screen may be easier to control and set-up), or certain affordances, like easy trafficking between changes in visual perspective (e.g., flip between forms of symbolic notation in other cultures- present and ancient; similar to Google Translate).
Trustworthy, safe, and respectful.			
D24. *Space for reporting negative behaviors.*	App promotes only positive behaviors from participants. App posts reminders that only positive interactions are tolerated. Bullying behaviors and other negative behaviors are not tolerated. Reporting of negative behaviors is encouraged. Some functions in place to prevent such behaviors from occurring. e.g., You may "like" somebody's work they choose to share with other learners, but not dislike it or add comments. e.g., App provides a way to report negative behaviors or misuse of app, if applicable.		"To "Meet": Expected behaviors do not have to be stated, but in collaborative and social contexts, negative behaviors are discouraged through program design and/ or reminders to interact positively. There must be a way to report negative behaviors. <u>Tech-Plus:</u> N/A This is a minimal requirement for any learning space.

D25. *Confidentiality and privacy secure.*	App provides confidentiality and protects learners' personal boundaries. No sharing of personal information; others cannot delete or copy work. Players choose their level of communication with others. e.g., App utilizes <u>most recent and highest-level</u> digital security measures to protect the learner's identity and personal security.		<u>Tech-Plus:</u> N/A This is a minimal requirement for any learning space.
D26. *Diverse means and potential to reflect home lives of multiple learners.*	App presents mathematical ideas from diverse personal perspectives, so the app has potential to reflect home lives of multiple learners. This includes, but is not limited to: a.) Characters with whom a range of participants can identify (e.g., a range of races and ethnicities, sexual orientations, religions, disabilities, and genders are represented by positive images and avoid stereotypes) and, b.) Considers various perspectives, points of view, frames of reference, modes of communication, senses of identity, and cognitive styles without generalizing to a culture, gender, or type of learner (e.g., these include the use of idioms, stories, events, heroes, public figures, metaphors, illustrations, songs, materials, and examples).	App possibly provides: -A feeling of, "I belong here". - There are ways to create an avatar or individualize physical attributes and social behaviors of a character.	"To "Meet": Anamorphic characters (e.g., personified animals, creatures, objects) are acceptable, if through their actions and words they suggest diverse views and orientations in a way that avoids stereotypes. *<u>Tech-Plus:</u> Uses app tech. to *present mathematical ideas from diverse personal perspectives* better than in the non-virtual world. e.g., App offers video clips of diverse individuals and groups of people using mathematics/ algebraic ideas in their everyday lives- basketball players, tailor/ seamstress, Inuit kayak-maker, alpaca farmer, etc.
D27. Provides learners with way to *denote*	a.) App provides a survey (or other way) for learner to denote his or her personal interests. Survey results can be changed or updated at any time, by learner.	App possibly provides: -Choose-your-own-adventure activity map.	*<u>Tech-Plus:</u> Uses app tech. to *denote some personal interests* and utilizes those interests to *influence*

some *personal interests*, and utilizes interests *to influence instruction*.	e.g., "I like: singing, dressing-up, building with blocks, playing in the mud or sand, taking care of my pet, play basketball". b.) App-teacher utilizes learner interests to frame/ influence instruction. e.g., There are several different interest-based activities for the same mathematical concept. Like, a virtual game of shooting baskets from a basketball three-point line (functional change exemplified in change in score). Or, a virtual karaoke singing contest		*instruction* better than in the non-virtual world. e.g., Digital programming can capture a learner's interests and use those interests to individualize instruction. Hypothetically, this makes algebraic concepts more relatable and engaging to the learner.
D28. Provides learners with way to *denote* some *life experiences* and utilizes experiences *to influence instruction*.	a.) App provides a survey (or other way) for learner to denote some of his or her past life experiences. Survey results can be changed or updated at any time, by learner. e.g., I have been to: a zoo, the beach or shore, a park, a shopping mall; I have: played basketball, baked/cooked, painted a picture. b.) App-teacher utilizes learner past experiences to frame/ influence instruction. e.g., Metaphors for same content are varied, depending upon a learner's past experiences. Like, "the animals in a zoo are sorted according to their shared features". Or, "you could sort the seashells on the beach according to their shared features".		*Tech-Plus: Uses app tech. to provide learners with a way to *denote some life experiences* and utilizes those experiences to *influence instruction* better than in the non-virtual world. e.g., Digital programming can capture a learner's life experience profile, and use the descriptors from that profile to individualize instruction for the learner, with far more consistency and ease than might otherwise occur in the classroom.

D29. *Dignity* of young learner.	App treats learner with dignity; promotes dignity of the young learner. Dignity is defined here, as being held in esteem because of one's worth. This includes, but is not limited to: a.) Respect for children's innate, informal, and diverse understandings. App appears to reflect an underlying belief in, and appreciation of, children's capacity to make sense of the world and their place in it (e.g., A learner's metaphors and frameworks for meaning are treated seriously- "A function is like when my sister cries. The more my sister cries, the more attention my mommy gives her. The less my sister cries, the less attention my mommy gives her."). b.) Avoids old stereotypes tied to children, like "basics first", "concrete experience only", use of developmental stage theory, and use of universal trajectories. Instead, there are high expectations for content and higher-level thinking. (e.g., Learners guided to "think" in complex ways. Tasks are designed to foster complex and creative thinking.) c.) Chances to provide input (e.g., Public project) and make meaningful contributions to the work at hand. Respectful work with a focus on what matters. (e.g., An activity is "worthy of the learner's time and effort").		*<u>Tech-Plus:</u> Uses app tech. to provide learners with a way to *promote the dignity of the young learner* better than in the non-virtual world. e.g., App links to a site where the young learner's work can be shared and showcased; Virtual exhibitions of algebraic ideas, as presented in the words of children.

Category	**Description / Examples**	**Indicators**	**Decision Rules**
E.) Through specific app features.	The primary medium of the app- digital programming and its potential uses, are not accounted for in the features below. Instead, examples of these potential features and affordances are provided through examples of the "Tech-Plus" criteria, listed in the far right column beside each descriptor, above. Below, are two features requisite to the devices of focus within this study. As such, the descriptors below only attempt to account for whether the app has made use of the devices' inherent features.		
E1. *Mobility utilized.*	App provides learner with specific opportunity to use mobility/ portability of device to interact with his or her larger world. e.g., App asks learner to go outside their home and photograph examples of objects in his or her yard/ neighborhood that are ordered by size.		*Tech-Plus: N/A
E2. *Multi-touch utilized.*	App provides learner with specific opportunity to use multi-touch function of the device to interact with the content and curriculum of the app. e.g., Swiping, touching, pinching, and tracing the screen, and/or tilting the device in order to aid learner perception of an algebraic concept.		*Tech-Plus: N/A

Appendix B:

App Observation and Classification Form.

App Observation and Classification Form

Coder I.D.: ________________

App Information:
App Name: ____
Target Age: ________________________________
Price: _____________________________________
Version: ___________________________________
Seller: ____
Date of Release: ____________________________
In-app purchases? ____________________________
Seller's App Description:

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How would I modify the seller's description? If no description is provided by the seller, please provide a short description of the app. If you would not make any modifications to the seller's description, note "No modifications".

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____
____
____

Screen Shots of:
Home Page/ Landing Pad- ☐ taken ☐ attached to this form

One Activity- ☐ taken ☐ attached to this form

*Additional screen shots are always an option to include.

Guiding Questions:

Coder Directions: Please refer to the "Coding Frame" if you have questions about definitions, require further examples, or seek more detail in regard to indicators or decision rules. The specific characteristic, for which you are looking, is represented by the italicized words in each question. The example provided may or may not match the specific way the characteristic is enacted within the app. If a characteristic is not met, the Tech-Plus box [in green] does not require consideration. If a characteristic is met, please consider the descriptor in the Tech-Plus box [in green], perhaps during a subsequent stage in coding.

A1.) App provides learner with opportunities to explain and describe an algebraic concept in his or her own words? (e.g., An activity/ tool, such as a virtual "sound lab" asks the learner to describe the pattern she made and explain the way in which it repeats [unit of repeat]).

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

*Uses app tech. to help learners *explain and describe*, better? (e.g., uses voice recognition/ word prediction/ recording as a dictation tool to capture descriptions or explanations by the learner; playback to self-assess or to share and compare descriptions with others.)

☐ Check box.

Comments:

A2.) App provides learner with opportunities to "choose from and use" mathematical strategies, ideas, and algebraic concepts (from a bank/menu of options), and apply them to provided contexts? (e.g., An environmental provocation within the app prompts the learner to build or extend patterns with virtual sounds, images, taps/touches provided.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

*Uses app tech. to help learners *choose and use* concepts, better? (e.g., Learner is able to access, choose from, and use a wider variety of virtual "tools" and multi-modal manipulatives.)

☐ Check box.

Comments:

A3.a.) App provides learner with opportunities to create solutions or products to solve mathematical problem situations? (e.g., A task within the app prompts the learner to create a virtual machine out of available digital "parts" that serves as an "input/output" device that demonstrates algebraic change.)

NO
↓
___ Does Not Meet

YES
↓
___ Meets

*Uses app tech. to help learners *create solutions, ideas, and products*, better?
(e.g., The creation made by the learner can be animated, or otherwise utilize motion to see "effects" of the design on algebraic ideas.)
☐ Check box.

A3.b.) If yes, app provides learner with opportunities to move through the phases of a formal problem solving process/ engineering-design process for purpose of: proposing ideas, testing ideas, evaluating test results, making revisions and adapting ideas, and repeating the process until a working solution is created. These phases must be made explicit to the learner? (e.g., A "checklist" within an activity specifically guides the learner to "move through" the formal phases of a design/ problem solving process [made explicit to him or her], in order to link the learner's personal experience of product/ solution-design to the larger procedural process.)

NO
↓
___ Does Not Meet

YES
↓
___ Meets

*Uses app tech. to help learners *use engineering-design process*, better?
(e.g., Learner is able to better relate to the design/ problem solving process by using touch to track his or her virtual path through the phases of the process on app, and/or take photos of their project at different stages of design to reflect each phase of the process.)
☐ Check box.

Comments:

A4.) App provides learner with opportunities to weigh, judge, and/or evaluate one's own and others' interpretations, and executions of algebraic meaning? (e.g., A virtual "gallery" provides the learner with a chance to compare and evaluate several of peers' interpretations of, explanations of, or solutions for algebraic problems or concepts, and judges them in mathematical terms such as, "a fitting representation of equality"; "the best solution for this mathematical problem"; "the easiest 'solution path' in this context".)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

*Uses app tech. to help learners *weigh, judge, and/or evaluate* products/ ideas, better? (e.g., Link to online virtual "display boards"/ a "gallery" within the app allows the learner to post/ share his or her own work to a wider audience, so that others might respond; multi-modal guidelines for evaluation.)

☐ Check box.

A5.a.) App provides learner with opportunities to abstract "big ideas" related to early algebra after observing many specific instances of those ideas, over time? (e.g., An activity prompts the learner to notice and verbalize that all of these examples have something in common. They are all problems where you have to join two groups together to figure out the sum.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": App must not "rush" the learner's observation of specific instances (of a general idea). There must be plenty of varying instances to observe and plenty of time in which to do it.

*Uses app tech. to help learners *weigh and evaluate* products/ ideas, better?
(e.g., Virtual display boards allow learners to post/ share own work and respond to others'.)

☐ Check box.

A5.b.) App provides learner with opportunities to move from the general back to the specific. The learner is provided with opportunities to identify specific examples, based on the "big idea" of focus; opportunities to illustrate the general with the specific and/or move between the two perspectives, as it relates to one concept? (e.g., A "scavenger hunt" provides context for the learner to notice and verbalize that a situation is another example of a joining problem.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

*Uses app tech. to help learners *illustrate the general/ move between the general and specific* better?
(e.g., Makes use of digital programming traits to "layer" the general "over" the specific and the specific "within" the general. Use of motion, hyperlinks, visual "layers" to fluidly move between the general and the specific with ease.)

☐ Check box.

Comments:

A6.) App provides learner with opportunities to represent ideas through the creation of direct physical/ visual models of mathematical problem situations, algebraic concepts, and/or mathematical systems? (e.g., A task asks the learner to draw or illustrate functional change through a "comic strip" or a series of images that represents the specific change. Like, the learner's daily routine in three hour increments.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

*Uses app tech. to help learners *represent* ideas *through models* better than in the non-virtual world? (e.g., App provides access to different representational tools, media, and modes for modeling algebraic concepts than what, otherwise, might be available in the non-virtual classroom. For instance, using photographs and video.)

☐ Check box.

Comments:

A7.) App provides learner with opportunities to translate algebraic ideas through the creation of multiple products and models, across different modes, media, and "languages"? (e.g., A task asks the learner to translate the information from his or her comic strip into the format of a functional table.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

To "Meet": App must provide at least one opportunity for translation within a different medium, mode, and/ or "language", beyond the original representation.

*Uses app tech. to help learners *translate* ideas across modes, media, "languages" better than in the non-virtual world? (Process, itself need not be made explicit.) (e.g., A virtual "photo lab" prompts the learner to translate equivalency, as originally captured through algebraic symbols ($4 + 1 = 3 + 2$), into photographs of examples of equivalency found in the learner's real world.)

☐ Check box.

Comments:

A8.) App encourages learner to explore his or her real world environment for a particular mathematical/algebraic purpose? (e.g., A task prompts learner to identify an algebraic idea in the world around them. Such as, a scavenger hunt for particular algebraic symbols.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

*Uses app tech. to help learners *explore the real world environment* for a particular mathematical/ algebraic purpose better than in the non-virtual world? (e.g., A task prompts learner to use the mobile device to identify and document particular algebraic symbols with the device's camera.)

☐ Check box.

Comments:

---

*A9.) App provides learner with opportunities to collaborate with others in his or her "immediate" or local real world, toward a particular mathematical/algebraic purpose? (e.g., A "family challenge" asks learners to work with his or her parent, grandparent, or caregiver to classify objects in the app, in three different ways.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": Learner must see/ know the identity of the real world person(s) with whom they are collaborating.

Comments:

A10.) App provides learner with opportunities to collaborate with others "at a distance" in or via the virtual world, toward a particular mathematical/algebraic purpose? (e.g., A game asks the learner to "join in" and "help virtual friends" [could be simulated characters] to extend a pattern with a long unit of repeat, over a long [virtual] distance.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": Those with whom learner collaborates must be "at a distance" from one's immediate local circle.

*Uses app tech. to help learners *collaborate with others "at a distance"* better than in the non-virtual world?
(e.g., App program utilizes communication technology to help learners "Facetime" with a class in South Africa in order to work together on a joint algebra project.)

☐ Check box.

Comments:

---

*A11.) App provides learner with opportunities to collaborate/work with others, in at least one of these three ways: with turns to lead the group; within two different sized groupings (e.g., partners, triads, small groups, large groups); synchronously, in real-time, toward a particular mathematical/ algebraic purpose? (e.g., A task prompts the learner to work with a virtual partner (an actual learner, not a simulated character) to order objects from tallest to shortest.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": App must prompt the learner to contribute to work/ discussion in some way.

Comments:

---

*A12.) App provides learner with opportunities to use corroborating evidence to justify their mathematical reasoning? (e.g., App prompts learner to justify his or her reasoning by choosing from a "bank" of options- create a drawing, model an idea, use words or mathematical symbols to express a mathematical rule or property.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

A13.a.) App provides learner with opportunities to listen and respond to situations and others' communications? (e.g., App activity provides "space" for synchronous listening to others' interpretations of algebraic ideas. Learners respond by paraphrasing what was said, and asking questions to discover more about the person's ideas.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

*Uses app tech. to help learners *"actively listen" and respond* better than in the non-virtual world? (e.g., App program utilizes communication technology to assist learners in actively listening to/ synchronously viewing others' interpretations of algebraic ideas.)
☐ Check box.

*A13.b.) Cultivate and/or express empathy toward others.
(e.g., App activity invites learner to relate to others' personal experiences with algebraic ideas by sharing his or her own personal experiences with algebraic ideas.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

B1.) App provides learner with opportunities to construct the fundamental nature and defining features of an algebraic concept- especially "how" and "why" an algebraic concept functions or behaves in a certain way, and how this relates to its features?

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

To "Meet": App must provide learners with an opportunity to construct more than just the big ideas of a concept. Essence involves some level of detail.

*Uses app tech. to help learners construct the *essence of a concept* better than in the non-virtual world? (e.g., Utilization of multi-modal representations of the concept will likely help learners construct a more "complete" sense of essence. Like, through observing and analyzing growing patterns in visual, spatial, auditory, and temporal forms.)

☐ Check box.

Comments:

B2.) App provides learner with opportunities to construct how an algebraic concept's essence (see above) relates to one's self, the natural world, the human world, and other ideas?

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

* Uses app tech. to help learners construct the *significance of a concept* better than in the non-virtual world. (e.g., Uses motion to move between concept and aspects of its significance.)

☐ Check box.

To "Meet": Three of the four components of the concept's relationship- to self, to natural world, to human world, and to other ideas- must be satisfied for this category to be met.

Comments:

B3.) App provides learner with opportunities to construct when, where, and why (given conditions in which) to apply an algebraic concept? (e.g., A set of activities that allow the learner to construct a condition of applicability of the equal sign.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

To "Meet": The app must signal (verbally, visually, etc.) how a specific idea is relevant for transfer to other conditions

*Uses app tech. to help learners construct the *conditions of applicability of a concept* better than in the non-virtual world?
(e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to virtual "rooms" in which various conditions are presented.)

☐ Check box.

Comments:

B4.) App provides learner with opportunities to construct important rules, exceptions to rules, multiple meanings of, inferences, nuances, and implications of an algebraic concept's meaning, as needed for understanding the concept at an upper elementary level? (e.g., A set of activities that allow the learner to construct a nuance of the equal signs' meaning.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

*Uses app tech. to help learners construct the *nuance of a concept* better than in the non-virtual world? (e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to tidbits of knowledge related to nuances.)

☐ Check box.

Comments:

---

*B5.) App provides learner with opportunities to construct awareness about his or herself as a mathematics learner? (e.g., A virtual "mirror" pops-up and the learner looks at his or her reflection while answering questions the "magic mirror" asks.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

Comments:

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*B6.) App provides learner with opportunities to *construct specific kinds of big ideas*, and makes these ideas explicit to learners?

These include, but are not limited to:

----

a.) Algebraic themes (e.g., "equivalency", "change");

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

b.) A class of mathematical problems (e.g., "joining" problems);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

c.) A mathematical principle, rule, or operational property (e.g., the associative property of addition);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

d.) An "Essential Question" related to mathematics and/or its practice (e.g., How do people use mathematical symbols to describe the world around them?);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

e.) An observation about the nature of mathematics practice (e.g., a mode of inquiry in mathematics)?

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

B7.) App provides learner with opportunities to make <u>at least two</u> types of connections/ relations between mathematical concepts?

*Uses app tech. to help learners construct *specific relations between concepts* better than in the non-virtual world? (e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom in or out, isolate parts, observe cause and effect of input/ output.)

☐ Check box.

To "Meet": App must make the relationship between ideas explicit. Also, relationships must be described or labeled- preferably by learner before app.

These include, but are not limited to:

----

a.) Details with a big idea; a big idea with details (e.g., "People describe groups of objects in different ways. One way people describe a group of objects is by describing how part of the group compares with all of the group. For example, four of the six balloons are round.");

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

b.) Ideas (in terms of key features) to other ideas <u>within</u> a disciplinary domain (e.g., Growing patterns and functions are both kinds of change, even though their units of change are different.);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

c.) Ideas (in terms of key features) to other ideas <u>across</u> disciplinary domains, across the curriculum, and across individual learning experiences (e.g., In what other disciplines do people measure change?);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

d.) Concepts with skills; skills with concepts (e.g., "The clearer I write, the easier it will be for others to read my symbolic notation.");

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

e.) Big ideas with other big ideas (e.g., People describe groups of objects in different ways. People, also, describe patterns in different ways.);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

f.) Details with other related details (e.g., "One way people describe a group of objects is by describing how part of the group compares with all of the group. Another way people describe a group of objects is by describing the total amount.")?

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

B8.) App provides learner with opportunities to form <u>at least three</u> kinds of relationships between him self or herself [including (d.) and the algebraic concept of focus]?

> * Uses app tech. to help learners construct *specific relationships between learner and concepts* better than in the non-virtual world? (e.g., Dictation tools and "audio galleries" may assist learners in sharing their personal connections to algebraic ideas.)
>
> ☐ Check box.

These include, but are not limited to:

----

a.) Consensus-driven meanings with more personal meanings (e.g., Learner pictures a see-saw when envisioning the equal sign and its meaning of equivalency.);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

b.) New concepts with existing concepts (e.g., Learner has already noticed that some patterns repeat. Now, she understands that other patterns "grow", instead of repeat.);

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

c.) Out-of-app experiences with in-app experiences (e.g., Learner has noticed the equal sign used at the grocery store. Now, he knows what it means, based on app learning experience.);

NO ↓ YES ↓

___Does Not Meet ___ Meets

AND

----

d.) Increasing one's conceptual proximity to an idea (e.g., Learner better understands how growing patterns "work" because she has used virtual manipulatives to make her own growing pattern)?

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

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*B9.) App provides learner with opportunities to reflect on <u>at least two</u> kinds of current understanding or meaning constructed during or directly following a learning experience?

To "Meet": The emphasis here is on <u>reflection</u> upon the learning experience (after or during), not on predefinition before the experience.

This includes, but is not limited to:

----

a.) Current summary of understanding; specific meanings constructed;

NO ↓ YES ↓

___Does Not Meet ___ Meets

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b.) Current degree to which he or she understands a concept;

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

c.) Current self-performance; self-evaluation of his or her own performance or participation within an experience;

NO ↓ YES ↓

___Does Not Meet ___ Meets

----

d.) Current "reading" of a learning experience or situation, as it relates to one or more of the following: 1.) Perceived purpose of the learning experience, 2.) How this experience compares with others, 3.) What is needed or required in a current situation or experience?

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

B10.) App provides learner with opportunities to learn about general procedural knowledge related to the practice of mathematics and early algebra? (e.g., App provides general information on how learners can look for repeating patterns in the world around them.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

To "Meet": Procedural knowledge cannot be treated as generic process skills- devoid of content or context (e.g., how to observe, how to measure). Instead, knowledge must be related to early algebra ideas.

* Uses app tech. to help learners construct *specific procedural knowledge* better than in the non-virtual world? (e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of specific procedural knowledge (i.e., a common algorithm) that "unlocks" after the learner has created some of his or her own specific procedures.)

☐ Check box.

Comments:

B11.) App provides learner with opportunities to construct specific procedures related to the solving of mathematical/ algebraic problems and problem situations? (e.g., An example of specific procedural knowledge: One way to determine a sum when joining amounts with tens and ones is to, first, combined amounts, then trade in sets of ten "ones" (units) for a ten (rod), and then count the tens (rods) and ones (units) to see how many are altogether.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meet

To "Meet": Learner must be allowed to construct specific procedural knowledge for him or herself. No teacher-derived algorithms are presented before a learner has had multiple opportunities to generate specific procedural approaches to an algebraic problem.

* Uses app tech. to help learners construct *specific procedural knowledge* better than in the non-virtual world? (e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of specific procedural knowledge (i.e., a common algorithm) that "unlocks" after the learner has created some of his or her own specific procedures.)

☐ Check box.

Comments:

B12.) App provides learner with opportunities to learn about dispositional knowledge related to the practice of mathematics and early algebra? (e.g., An example of dispositional knowledge: “Mathematicians make calculation mistakes sometimes, but they don’t become frustrated. Mistakes are easy to fix. Instead, mathematicians double-check their work to try to catch as many mistakes as possible.”)

NO ↓

___Does Not Meet

YES ↓

___ Meets

Comments:

* Uses app tech. to help learners learn *dispositional knowledge* better than in the non-virtual world? (e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of disciplinary knowledge. This “layer” of knowledge can be selected/deselected by learner.)

☐ Check box.

B13.) App provides learner with opportunities to learn about formal language, conventional symbols, or background information and/or pre-requisite knowledge related to the algebraic content at hand?

NO ↓

___Does Not Meet

YES ↓

___ Meets

To “Meet”: Formal language and conventional symbols are introduced only after providing learners with time to explore these ideas on their own, and after providing learners with opportunities to use informal/ invented language.

Comments:

* Uses app tech. to help learners learn *formal disciplinary language, conventional symbols, and supporting knowledge* better than in the non-virtual world? (e.g., App utilizes features of digital programming- embedded videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of supporting/ background information. A “layer” of disciplinary language/ vocabulary can be selected/deselected by learner.)

☐ Check box.

B14.) App provides learner with opportunities to learn practical technical knowledge and/or other media knowledge, in order to help him or her utilize these skills and ideas in the construction of algebraic ideas? (e.g., "Swipe here." "Drag, like this." App provides "How To"s: How to use a virtual paintbrush or work a calculator.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

To "Meet": Knowledge is available on how to use tools, media, modes, contexts, or how to approach a specific kind of learning experience, as related to the content of the app.

* Uses app tech. to help learners learn about or seek *practical technical or media knowledge* better than in the non-virtual world? (e.g., App utilizes features of digital programming- embedded "how to" videos, pop-ups, roll-overs, and hyperlinks that lead to additional tidbits of information on technological or media features and procedures.)

☐ Check box.

Comments:

B15.) App provides learner with opportunities to enact a cumulative project or solve a problem situation? (e.g., Project can be, either: Summative: project is completed at the end of the learning cycle, or Formative: the same project is the primary context for learning along all/most phases of the learning cycle.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

*Uses app tech. to help learners enact *a cumulative project or solve a complex problem* better than in the non-virtual world? (e.g., This depends upon the nature of the project. Learners might create a digital slideshow to share their observations and photographs of repeating patterns they documented at the playground. The possibilities are nearly endless.)

☐ Check box.

Comments:

B16.) App provides learner with opportunities to make conjectures or proposals related to mathematical ideas? (e.g., A series of true and false statements, the learner is asked to explain or identify "the rule" of the true statements.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

To "Meet": Immediate feedback from the app is essential (e.g., "That rule works in this instance. Does it work in every instance?")

Comments:

B17.) App provides learner with opportunities to plan for projects in formalized ways
Plan work products and anticipate needs for product execution? (e.g., An activity asks the learner to make "blue prints" for solving a mathematical problem situation. "Draw a picture of what you will do first, next, last.")

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

* Uses app tech. to help learners make *formalized plans* better than in the non-virtual world? (e.g., Utilizes digital programming to provide learner with "guiding templates" for planning.)

☐ Check box.

Comments:

____

*C1.) App provides learner with opportunities to construct meaning related to rules, principles, and properties of whole numbers and operations? (e.g., An activity is designed to help the learner notice that, when she adds the same numbers in a different order, she gets the same sum.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

____

C2.) App provides learner with opportunities to classify, sort, and order objects, sets of objects, amounts, problems, and changes? Concepts include, but are not limited to: a.) Problem types and classes of problems (e.g., Join, Separate, Compare, Part-Part-Whole, Equalize) [Carpenter, Fennema, Franke, Levi, Empson, 1999]; b.) <u>Sort</u> and <u>order</u> objects by physical properties related to mathematics (e.g., size, shape, amount, shared features or functions).

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

*Uses app tech. to help learners *classify, sort, and order objects and amounts* better than in the non-virtual world? (e.g., App utilizes multi-touch technology of device to allow learners to virtually manipulate objects and ideas, by sorting, ordering, and grouping.)

☐ Check box.

Comments:

____

C3.) App provides learner with opportunities to construct meaning related to the nature of mathematical patterns in sequences, organized configurations, and/or randomly distributed in the larger world? (e.g., growing patterns, repeating patterns, patterns in numeric data.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": The content of the app is not focused on design patterns (e.g., stripes/ polka-dots/ flowers.)

Comments:

*Uses app tech. to help learners construct meaning related to *the nature of mathematical patterns* better than in the non-virtual world? (e.g., Simulations of physical contexts and subject-matter, for learners to observe and manipulate patterns of which he or she might not otherwise have access.)

☐ Check box.

C4.) App provides learner with opportunities to construct meaning related to mathematical changes?

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

*Uses app tech. to help learners construct meaning related to *mathematical changes* better than in the non-virtual world? (e.g., App utilizes temporal/ interactive features of digital programming and device to isolate parts of a mathematical system and/or observe the effect of changes made by the learner.)

☐ Check box.

C5.) App provides learner with opportunities to construct meaning related to algebraic relations of quantitative amounts? (e.g., Relations between amounts, in terms of equality [e.g., =, <, >].)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

*Uses app tech. to help learners construct meaning related to *algebraic relations* better than in the non-virtual world? (e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom in or out, isolate parts, observe cause and effect of input/ output.)

☐ Check box.

C6.) App provides learner with opportunities to create or invent symbolic notation. Including through: writing, pictures, invented symbols?

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

*Uses app tech. to help learners construct meaning related to *algebraic symbols* better than in the non-virtual world? (e.g., App utilizes features of digital programming, like digital "layers" to compare different symbol systems with one another.)

☐ Check box.

D1.) App provides learner with opportunities to reset activity, fix/adapt work, or try again?
(e.g., An activity allows learner to manually "erase" his virtual work by using his fingertip to erase similarly to how a rubber eraser might be used in the non-virtual world.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

*Uses app tech. to help learners *reset/ try again, fix work* better than in the non-virtual world?
(e.g., Makes use of digital programming to provide ease of reset/ ease of correction (instant reset button), and/or allows learner to isolate certain "layers" to erase.)

☐ Check box.

---

*D2.) App provides more than one learning experience for a single phase of the learning cycle?
(e.g., Learners can describe the essence of a concept in their own words in a "sound lab" <u>and</u> explain the concept's essence by making a virtual poster- both activities of which could be part of the "Explain" phase of the learning cycle.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

D3.) App provides learner with opportunities to explore the app environment, manipulate materials, test his or her ideas against a particular condition and/or context, compare his or her ideas against others', and explore the "simulated real world"? (e.g., virtual labs; simulations)

NO ↓ YES ↓

___Does Not Meet ___ Meets

* Uses app tech. to help learners *explore* ideas and their effect in various simulated environments better than in the non-virtual world? (e.g., App provides simulations of contexts, conditions, and subject-matter that might not otherwise be possible for learners to experience and explore.)

☐ Check box.

Comments:

*D4.a.) App provides at least one new conceptual condition during the learning cycle, in addition to the original condition under which the algebraic concept was first introduced? (e.g., A change in an aspect of the case- "What if there were no red or blue blocks? Could you create a pattern with yellow and green blocks, instead?")

NO ↓ YES ↓

___Does Not Meet ___ Meets

D4.b.) App offers learner opportunity to "Explore" and "Explain" again, in light of the introduction of new conditions?

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

*D5.) App provides at least one new restraint on learner action or outcome, in addition to the original context or set of restraints under which the action or outcome was first framed? (e.g., For a different audience- "This time, change your 'input/output machine' to make it easier for the shoemaker to use. She needs to make pairs of shoes, not individual buttons.")

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

*D6.) App presents a single algebraic concept, under one condition and/or context, no more than five times consecutively? (e.g., After initial introduction of a condition, app presents concept within random conditions/ contexts, or with changes to the conditions and contexts, intermittently.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

*D7.) App "moves" learners through increasingly complex ideas related to a single concept, in a way that respects the degree to which the more-complex ideas share characteristics with previous-known ideas? (e.g., A learner needs to understand the idea of equivalency [i.e., the same amount in each group] in conjunction with determining the symbolism behind the equal sign.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": App must organize concepts according to the principles that can be used to define them, not according to surface features or outdated perceptions of "basics first".

Comments:

____

*D8.) App provides algebraic concepts divorced from strict timelines and developmental timetables? (e.g., A wide range of algebraic concepts are available [through the learning experiences provided by the app] to learners of any age.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

*D9.) App provides individual/ differentiated learning routes based on various seamless assessment results?
(e.g., App utilizes those learning experiences, outcomes, and actions already provided to support "learning with understanding", that are possible to assess through the app-medium (e.g., self-assessment reporting, some "close-ended" activities, virtual portfolios with corresponding checklists) to vary an individual learner's trajectory of learning.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": Assessment must occur seamlessly, within the context of learning experiences, actions, and outcomes that would otherwise be provided for meaning construction.

Comments:

---

*D10.) App provides a high ratio of constructivist learning experiences across the total app "program"?
(e.g., A constructivist learning experience might be a virtual lab in which the learner can manipulate each part of a change ratio and observe the effects.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": The app must provide a way to make the concepts of focus explicit to the learner (e.g., through reflection and summary during and after the constructivist experience).

Comments:

D11.a.) App provides at least one decontextualized activity; one learning experience in which the problem to solve is the task, itself? (e.g., An activity allows the learner to focus on one aspect of one operational property.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

To "Meet": The activity is toward the decontextualized end of the context continuum.

* Uses app tech. to provide learners with *decontextualized activities explicitly connected to the larger context* better than in the non-virtual world. (e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom out from individual activities to their larger contexts.)

☐ Check box.

D11.b.) Decontextualized activity is explicitly connected to the larger context?

NO ↓

___Does Not Meet

YES ↓

___ Meets

Comments:

D12.a.) App provides at least one contextualized activity; one learning experience in which the problem to solve is part of a larger task or context? (e.g., A virtual context in which the learner uses gold coins to buy things at a store. She must continuously combine amounts to determine how much she has to spend.)

NO ↓

___Does Not Meet

YES ↓

___ Meets

To "Meet": The activity is toward the contextualized end of context continuum. (e.g., An immersive environment, real life practice, or real life contribution.)

*Uses app tech. to provide learners with *contextualized activities the embedded concepts of which are made explicit* better than in the non-virtual world?
(e.g., App utilizes temporal/ interactive features of digital programming and device, like motion or digital layers to zoom in from larger contexts in order to temporarily focus on a specific concept.)

☐ Check box.

D12.b.) Concepts embedded within contexts are made explicit.

NO ↓

___Does Not Meet

YES ↓

___ Meets

Comments:

D13.) App provides a higher ratio of activities and learning experiences that are contextualized in nature as compared with those that are decontextualized in nature? (e.g., More learning experiences are immersive environments, opportunities for real life practice, or opportunities for real life contribution than are games or decontextualized activities.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

*D14.a.) App provides a balance of varied-structured learning experiences and contexts, on a continuum from open-ended mathematical activities to highly structured mathematical activities? (e.g., Virtual "sand boxes", labs, and simulations available for exploring relevant materials, media, tools, contexts as it relates to algebraic content. Highly structured activities are also available.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

D14.b.) Within activities toward the more "open-ended" end of the continuum, the app accepts/ provides space for the learner to input creative, open-ended ideas, or submit multiple answers/ solutions/ proposals?

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

D15.) App provides ease of navigation through virtual curriculum. App provides clear spatial layout?

NO

↓

___Does Not Meet

YES

↓

___ Meets

*Uses app tech. to provide learners with *contextualized activities the embedded concepts of which are made explicit* better than in the non-virtual world? (e.g., Uses app tech. to enhance *ease of navigation through curriculum* in the app's virtual world. [e.g., App provides multi-modal support for traversing the site and making navigational choices.])

☐ Check box.

Comments:

D16.) App provides clear directions for interacting with content and navigating the virtual environment? (e.g., Directions are easy to understand for six to eight-year-old.)

NO
↓

___Does Not Meet

YES
↓

___ Meets

* Uses app tech. to enhance *clear directions* within the app's virtual world? (e.g., Directions are available in multiple modes. "Multiple modes" means a message is communicated through more than one form of representation.)

☐ Check box.

Comments:

D17.) App includes Universal Design for Learning (UDL) features across the app program? (e.g., Common UDL features include, but are not limited to: various degrees of text complexity and wording, flexible time frames, ELL support/ language translation, voice command, narration, vocab. development/ definitions.)

NO
↓

___Does Not Meet

YES
↓

___ Meets

* Uses app tech. to provide learners with *UDL features* better than in the non-virtual world. (e.g., App utilizes features of digital programming- pop-ups, embedded videos, roll-over definitions, physical perspective changes, programmed reminders about potential trouble spots.)

☐ Check box.

"To "Meet": UDL features are always available- at every level across program at all times, but can be turned on/off individually and easily. Additionally, at least five UDL features must be available.

Comments:

D18.) App helps learners activate his or her prior knowledge related to algebraic content at hand, and helps learners make connections between these ideas and present learning? (e.g., App mentions a concept that a learner may have observed in her previous everyday experience.)

NO
↓

___Does Not Meet

YES
↓

___ Meets

*Uses app tech. to help learners *activate their prior knowledge and connect this to present learning* better than in the non-virtual world. (e.g., Utilizing data from the learner's "life experience" survey may help to activate knowledge embedded within a certain life context.)

☐ Check box.

Comments:

D19.) App provides learning experiences and an environment that initiates and maintains learner's cognitive-affective engagement in algebraic concepts of focus, by drawing upon natural human motivators? (e.g., Introduces new objects, events, or people in a way that speaks to learner's natural curiosity.)

NO
↓

___Does Not Meet

YES
↓

___ Meets

* Uses app tech. to *initiate and maintain a learner's cognitive-affective engagement* better than in the non-virtual world? (e.g., The interactivity of the device, <u>if utilized by the app</u>, tends to offer nearly instant and long-lasting engagement with content.)

☐ Check box.

"To "Meet": Playful and engaging experiences and contexts can be "fun". However, "fun", as a superficially defined and overused stimulus, should not be the primary motivational driver. Authentic provocations should drive engagement.

Comments:

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*D20.) App provides a balance of sensory stimulation and peace. Algebraic content and the contexts in which they are presented are interesting, but not over-stimulating? (e.g., Learners can switch or move between virtual "rooms", some of which have greater sensory output, and some of which have less. Most may require active cognitive activity.)

NO
↓

___Does Not Meet

YES
↓

___ Meets

"To "Meet": Most learning experiences/ environmental provocations require active cognitive effort. A few passive cognitive activities are permitted. App must avoid sensory overload, even within "spaces" that are intended to contain increased sensory output.

Comments:

---

*D21.) App imposes no time limits within learning experiences or across the app program, unless there is a valid educational reason for a concept to be embedded within a timed activity? (e.g., Digital programming allows for the control of time provisions for learning. Either there is no time limit for accomplishing a task/activity or exploring a simulated environment, or the learner can easily control the imposition of time limits.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

D22.) App-teacher scaffolds mathematical learning by moving along a continuum, from more cognitive demand on the learner to less cognitive demand on the learner (i.e., least to most teacher-help)? (e.g., "Open-ended questioning" is provided before "direct instruction", along a continuum of scaffolding techniques.)

NO ↓ YES ↓

___Does Not Meet __ Meets

* Uses app tech. to *initiate and maintain a learner's cognitive-affective engagement* better than in the non-virtual world? (e.g., The interactivity of the device, <u>if utilized by the app</u>, tends to offer nearly instant and long-lasting engagement with content.)

☐ Check box.

Comments:

D23.a.) App presents key ideas through a range of materials, processes, tools, modes, media, and "languages", so as to increase conceptual accessibility of algebraic concepts and extend learners' multi-media literacy?

NO ↓ YES ↓

___Does Not Meet __ Meets

* Uses app tech. to *initiate and maintain a learner's cognitive-affective engagement* better than in the non-virtual world? (e.g., The interactivity of the device, <u>if utilized by the app</u>, tends to offer nearly instant and long-lasting engagement with content.)

☐ Check box.

D23.b.) Presents diverse (at least two)visual/spatial/conceptual perspectives. (e.g., Two views on a repeating pattern are available- 1^{st} person and 3^{rd} person.)

NO ↓ YES ↓

___Does Not Meet __ Meets

Comments:

---

*D24.) App promotes only positive behaviors from participants. App posts reminders that only positive interactions are tolerated? (e.g., App provides a way to report negative behaviors or misuse of app.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

*D25.) App provides confidentiality and protects learners' personal boundaries? (e.g., App utilizes <u>most recent and highest-level</u> digital security measures to protect the learner's identity and personal security.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

Comments:

____

D26.) App presents mathematical ideas from diverse personal perspectives, so the app has potential to reflect home lives of multiple learners? (e.g., Considers various perspectives, points of view, frames of reference, modes of communication, senses of identity, and cognitive styles without generalizing to a culture, gender, or type of learner.)

NO ↓ YES ↓

___Does Not Meet ___ Meets

*Uses app tech. to *present mathematical ideas from diverse personal perspectives* better than in the non-virtual world? (e.g., App offers video clips of diverse individuals and groups of people using mathematics/ algebraic ideas in their everyday lives- basketball players, tailor/ seamstress, Inuit kayak-maker, alpaca farmer.)

☐ Check box.

"To "Meet": Anamorphic characters (e.g., personified animals, creatures, objects) are acceptable, if through their actions and words they suggest diverse views and orientations in a way that avoids stereotypes.

Comments:

____

D27.a.) App provides a survey (or other way) for learner to denote his or her personal interests. Survey results can be changed or updated at any time, by learner? (e.g., I like: singing, dressing-up, building with blocks, playing in the mud or sand, taking care of my pet, play basketball".)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

> *Uses app tech. to *present mathematical ideas from diverse personal perspectives* better than in the non-virtual world? (e.g., App offers video clips of diverse individuals and groups of people using mathematics/ algebraic ideas in their everyday lives- basketball players, tailor/ seamstress, Inuit kayak-maker, alpaca farmer.)
>
> ☐ Check box.

D27.b.) App-teacher utilizes learner interests to frame/ influence instruction? (e.g., There are several different interest-based activities for the same mathematical concept.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

____

D28.a.) App provides a survey (or other way) for learner to denote some of his or her past life experiences. Survey results can be changed or updated at any time, by learner? (e.g., I have been to: a zoo, the beach or shore, a park, a shopping mall; I have: played basketball, baked/cooked, painted a picture.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

> *Uses app tech. to provide learners with a way to *denote some life experiences* and utilizes those experiences to *influence instruction* better than in the non-virtual world? (e.g., Digital programming can capture a learner's life experience profile, and use the descriptors from that profile to individualize instruction for the learner.)
>
> ☐ Check box.

D28.b.) App-teacher utilizes learner past experiences to frame/ influence instruction? (e.g., Metaphors for same content are varied, depending upon a learner's past experiences.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

Comments:

____

D29.) App treats learner with dignity; promotes dignity of the young learner? (e.g., Respect for children's innate, informal, and diverse understandings; Avoids old stereotypes tied to children; Chances to provide input and make meaningful contributions.)

NO ↓ ___Does Not Meet

YES ↓ ___ Meets

* Uses app tech. to provide learners with a way to *promote the dignity of the young learner* better than in the non-virtual world? (e.g., App links to a site where the young learner's work can be shared and showcased.)

☐ Check box.

Comments:

____

Miscellaneous (If a characteristic or concept is present in the app, and fits nowhere else, please place it here):

Observational Notes (Please use a "bulleted" format to record your observations as they relate to curricular characteristics present within the app)**:**

____

____

____

____

*Coder: After you have recorded your observations, in the space provided above, classify each of the curricular characteristics you observed by matching them to one of the categories of the form, above. If there are observations you noted that cannot be matched to a category of the form, please write those observations in the "Miscellaneous" box, above.

- Did the app utilize the mobility/ portability feature of the mobile device? ☐ yes ☐ no

- Did the app utilize the multi-touch features of the device? ☐ yes ☐ no

Appendix C:

DragonBox 5+ Inventory

	Coding Frame Category	Meets	Does Not Meet	Tech-Plus	Comments
	Specific Learner Actions				
A1.	Explain and describe		X		
A2.	Choose from and use	X			
A3.a.	Create solutions and products		X		
A3.b.	Move through design cycle		X		
A4.	Weigh and evaluate	X			
A5.a.	Abstract		X		
A5.b.	Move between specific and general	X			
A6.	Represent		X		
A7.	Translate		X		
A8.	Explore		X		
A9.	Collaborate w/ local		X		
A10.	Collaborate w/ distance		X		
A11.	Collaborate in specific ways		X		
A12.	Justify		X		
A13.a.	Listen and respond		X		
A13.b.	Cultivate empathy		X		
	Specific Learner Outcomes				
	Internalized Outcomes				
	Kinds of meaning:				
B1.	Essence		X		
B2.	Significance		X		

B3.	Conditions of applicability		X		
B4.	Nuance		X		
B5.	Self as learner		X		
	Big Ideas-				
B6.a.	Algebraic themes	X			
B6.b.	Class of mathematical problems				
B6.c.	Mathematical principle of rule		X		
B6.d.	Essential mathematics questions		X		
B6.e.	Nature of mathematics		X		
	Relations between concepts-				
B7.a.	Details with big ideas		X		
B7.b.	Within disciplinary ideas		X		
B7.c.	Cross-disciplinary ideas		X		
B7.d.	Concepts with skills		X		
B7.e.	Big ideas with other big ideas		X		
B7.f.	Details with other details		X		
	Relationship between learner and concept-				
B8.a.	Consensus-driven with personal		X		
B8.b.	New concepts with existing		X		
B8.c.	Out-of-app with in-app		X		
B8.d.	Increased concreteness	X			
	Meanings-in-the-moment-				
B9.a.	Current summary of understanding		X		
B9.b.	Current degree of understanding		X		
B9.c.	Current self-performance		X		
B9.d.	Read current situation	X			Through feedback.
	Kinds of supporting knowledge:				
B10.	General procedural		X		
B11.	Specific procedural		X		
B12.	Dispositional		X		
B13.	Formal/ conventional language	X			
B14.	Tech. and media		X		
	Externalized Outcomes				
B15.	Cumulative project		X		

B16.	Conjectures		X		
B17.	Formal plans		X		
	Specific Early Algebra Concepts				
C1.	Rules, properties, principles		X		
C2.	Classifications		X		
C3.	Patterns		X		
C4.	Changes		X		
C5.	Relations	X			
C6.	Symbols		X		
	Program, Experience, Environment				
	Movement Through Content				
D1.	Reset/ adapt	X			
D2.	Multiple experiences for one phase		X		
D3.	Exploration of virtual environment		X		
D4.a.	New condition	X			
D4.b.	Chance to explore and explain again		X		
D5.	New contextual restraint		X		
D6.	One condition, five time restriction	X			
D7.	Coherence	X			
D8.	Divorced from timelines	X			
D9.	Differentiated learning routes		X		
D10.	Constructivist	X			
	Kinds of Contexts				
D11.a.	Decontextualized	X			
D11.b.	Connects to larger context		X		
D12.a.	Contextualized		X		
D12.b.	Embedded ideas made explicit		X		
D13.	Higher ratio of contextualized		X		
D14.a.	Varied-structured		X		
D14.b.	Accepts open-ended input		X		
	Orderliness and Clarity				
D15.	Clear layout	X			
D16.	Clear directions	X			
D17.	Universal Design features		X		

	Balance of Cognitive-Affective				
D18.	Activates prior knowledge	X			
D19.	Initiates and maintains engagement	X			
D20.	Sensory and peace	X			
D21.	Time	X			
D22.	Scaffold		X		Not in the way outlined.
D23.a.	Range of materials		X		
D23.b.	Diverse perspectives		X		
	Trust, Safety, Respect				
D24.	Reporting negative behaviors		X		
D25.	Confidentiality and privacy	X			N/A
D26.	Diverse reflections of home life		X		
D27.a.	Interest survey		X		
D27.b.	Interest utilized		X		
D28.a.	Life experiences survey		X		
D27.b.	Life experiences utilized		X		
	Dignity-				
D29.a.	Respect for children’s capacity	X			
D29.b.	Avoids child stereotypes	X			
D29.c.	Meaningful work		X		
	App Features				
E1.	Mobility/ portability		X		
E2.	Multi-touch	X			

Appendix D:

Math Motion: Zoom Inventory

	Coding Frame Category	Meets	Does Not Meet	Tech-Plus	Comments
	Specific Learner Actions				
A1.	Explain and describe		X		
A2.	Choose from and use	X			
A3.a.	Create solutions and products		X		
A3.b.	Move through design cycle		X		
A4.	Weigh and evaluate	X			
A5.a.	Abstract		X		
A5.b.	Move between specific and general	X			
A6.	Represent		X		
A7.	Translate		X		
A8.	Explore		X		
A9.	Collaborate w/ local		X		
A10.	Collaborate w/ distance		X		
A11.	Collaborate in specific ways		X		
A12.	Justify		X		
A13.a.	Listen and respond		X		
A13.b.	Cultivate empathy		X		
	Specific Learner Outcomes				
	Internalized Outcomes				
	Kinds of meaning:				
B1.	Essence	X			
B2.	Significance		X		

B3.	Conditions of applicability		X		
B4.	Nuance		X		
B5.	Self as learner		X		
	Big Ideas-				
B6.a.	Algebraic themes	X			
B6.b.	Class of mathematical problems				
B6.c.	Mathematical principle of rule		X		
B6.d.	Essential mathematics questions		X		
B6.e.	Nature of mathematics		X		
	Relations between concepts-				
B7.a.	Details with big ideas		X		Not in the way outlined.
B7.b.	Within disciplinary ideas		X		
B7.c.	Cross-disciplinary ideas		X		
B7.d.	Concepts with skills		X		
B7.e.	Big ideas with other big ideas		X		
B7.f.	Details with other details		X		
	Relationship between learner and concept-				
B8.a.	Consensus-driven with personal		X		
B8.b.	New concepts with existing		X		
B8.c.	Out-of-app with in-app		X		
B8.d.	Increased concreteness	X			Great example of this!
	Meanings-in-the-moment-				
B9.a.	Current summary of understanding		X		
B9.b.	Current degree of understanding		X		
B9.c.	Current self-performance		X		
B9.d.	Read current situation	X			Through feedback.
	Kinds of supporting knowledge:				
B10.	General procedural		X		
B11.	Specific procedural		X		
B12.	Dispositional		X		
B13.	Formal/ conventional language		X		
B14.	Tech. and media	X			
	Externalized Outcomes				
B15.	Cumulative project		X		

B16.	Conjectures		X		
B17.	Formal plans		X		
	Specific Early Algebra Concepts				
C1.	Rules, properties, principles	X			
C2.	Classifications	X			
C3.	Patterns	X			
C4.	Changes	X			
C5.	Relations		X		
C6.	Symbols		X		
	Program, Experience, Environment				
	Movement Through Content				
D1.	Reset/ adapt	X			
D2.	Multiple experiences for one phase		X		
D3.	Exploration of virtual environment		X		
D4.a.	New condition	X			
D4.b.	Chance to explore and explain again		X		
D5.	New contextual restraint		X		
D6.	One condition, five time restriction		X		
D7.	Coherence	X			
D8.	Divorced from timelines	X			
D9.	Differentiated learning routes		X		
D10.	Constructivist	X			
	Kinds of Contexts				
D11.a.	Decontextualized	X			
D11.b.	Connects to larger context		X		
D12.a.	Contextualized		X		
D12.b.	Embedded ideas made explicit		X		
D13.	Higher ratio of contextualized		X		
D14.a.	Varied-structured		X		
D14.b.	Accepts open-ended input		X		
	Orderliness and Clarity				
D15.	Clear layout	X			
D16.	Clear directions	X			
D17.	Universal Design features		X		

	Balance of Cognitive-Affective				
D18.	Activates prior knowledge		X		
D19.	Initiates and maintains engagement		X		
D20.	Sensory and peace	X			
D21.	Time	X			
D22.	Scaffold		X		Not in the way outlined.
D23.a.	Range of materials		X		
D23.b.	Diverse perspectives	X			
	Trust, Safety, Respect				
D24.	Reporting negative behaviors		X		
D25.	Confidentiality and privacy	X			N/A
D26.	Diverse reflections of home life		X		
D27.a.	Interest survey		X		
D27.b.	Interest utilized		X		
D28.a.	Life experiences survey		X		
D27.b.	Life experiences utilized		X		
	Dignity-				
D29.a.	Respect for children's capacity	X			
D29.b.	Avoids child stereotypes	X			
D29.c.	Meaningful work		X		
	App Features				
E1.	Mobility/ portability		X		
E2.	Multi-touch	X			

Appendix E:

Slice Fractions Inventory

	Coding Frame Category	Meets	Does Not Meet	Tech-Plus	Comments
	Specific Learner Actions				
A1.	Explain and describe		X		
A2.	Choose from and use	X			
A3.a.	Create solutions and products		X		
A3.b.	Move through design cycle		X		
A4.	Weigh and evaluate		X		
A5.a.	Abstract		X		
A5.b.	Move between specific and general		X		
A6.	Represent		X		
A7.	Translate		X		
A8.	Explore		X		
A9.	Collaborate w/ local		X		
A10.	Collaborate w/ distance		X		
A11.	Collaborate in specific ways		X		
A12.	Justify		X		
A13.a.	Listen and respond		X		
A13.b.	Cultivate empathy		X		
	Specific Learner Outcomes				
	Internalized Outcomes				
	Kinds of meaning:				
B1.	Essence		X		
B2.	Significance		X		

B3.	Conditions of applicability	X			Not explicit; many conditions.
B4.	Nuance		X		
B5.	Self as learner		X		
	Big Ideas-				
B6.a.	Algebraic themes		X		
B6.b.	Class of mathematical problems		X		
B6.c.	Mathematical principle of rule	X			
B6.d.	Essential mathematics questions		X		
B6.e.	Nature of mathematics		X		
	Relations between concepts-				
B7.a.	Details with big ideas		X		
B7.b.	Within disciplinary ideas		X		
B7.c.	Cross-disciplinary ideas		X		
B7.d.	Concepts with skills	X			A strength of app.
B7.e.	Big ideas with other big ideas		X		
B7.f.	Details with other details		X		
	Relationship between learner and concept-				
B8.a.	Consensus-driven with personal		X		
B8.b.	New concepts with existing		X		
B8.c.	Out-of-app with in-app		X		
B8.d.	Increased concreteness	X			
	Meanings-in-the-moment-				
B9.a.	Current summary of understanding		X		
B9.b.	Current degree of understanding		X		
B9.c.	Current self-performance		X		
B9.d.	Read current situation	X			Through feedback.
	Kinds of supporting knowledge:				
B10.	General procedural		X		
B11.	Specific procedural		X		
B12.	Dispositional		X		
B13.	Formal/ conventional language		X		
B14.	Tech. and media		X		
	Externalized Outcomes				
B15.	Cumulative project		X		

B16.	Conjectures		X		
B17.	Formal plans		X		
	Specific Early Algebra Concepts				
C1.	Rules, properties, principles	X			
C2.	Classifications		X		
C3.	Patterns		X		
C4.	Changes		X		
C5.	Relations		X		
C6.	Symbols		X		
	Program, Experience, Environment				
	Movement Through Content				
D1.	Reset/ adapt	X			
D2.	Multiple experiences for one phase		X		
D3.	Exploration of virtual environment		X		
D4.a.	New condition	X			
D4.b.	Chance to explore and explain again		X		
D5.	New contextual restraint		X		
D6.	One condition, five time restriction	X			
D7.	Coherence	X			
D8.	Divorced from timelines	X			
D9.	Differentiated learning routes		X		
D10.	Constructivist	X			
	Kinds of Contexts				
D11.a.	Decontextualized	X			
D11.b.	Connects to larger context		X		
D12.a.	Contextualized		X		
D12.b.	Embedded ideas made explicit		X		
D13.	Higher ratio of contextualized		X		
D14.a.	Varied-structured		X		
D14.b.	Accepts open-ended input		X		
	Orderliness and Clarity				
D15.	Clear layout	X			
D16.	Clear directions	X			
D17.	Universal Design features		X		

	Balance of Cognitive-Affective				
D18.	Activates prior knowledge		X		
D19.	Initiates and maintains engagement	X			
D20.	Sensory and peace	X			
D21.	Time	X			
D22.	Scaffold		X		Not in the way outlined.
D23.a.	Range of materials		X		
D23.b.	Diverse perspectives		X		
	Trust, Safety, Respect				
D24.	Reporting negative behaviors		X		
D25.	Confidentiality and privacy	X			N/A
D26.	Diverse reflections of home life		X		
D27.a.	Interest survey		X		
D27.b.	Interest utilized		X		
D28.a.	Life experiences survey		X		
D27.b.	Life experiences utilized		X		
	Dignity-				
D29.a.	Respect for children's capacity	X			
D29.b.	Avoids child stereotypes	X			
D29.c.	Meaningful work		X		
	App Features				
E1.	Mobility/ portability		X		
E2.	Multi-touch	X			

Appendix F:

Cross App Inventory Summary

	Coding Frame Category	Meets	Does Not Meet	Tech-Plus	Comments
	Specific Learner Actions				
A1.	Explain and describe		# O ~		
A2.	Choose from and use	# O ~			
A3.a.	Create solutions and products		# O ~		
A3.b.	Move through design cycle		# O ~		
A4.	Weigh and evaluate	# ~	O		
A5.a.	Abstract		# O ~		
A5.b.	Move between specific and general	# ~	O		
A6.	Represent		# O ~		
A7.	Translate		# O ~		
A8.	Explore		# O ~		
A9.	Collaborate w/ local		# O ~		
A10.	Collaborate w/ distance		# O ~		
A11.	Collaborate in specific ways		# O ~		
A12.	Justify		# O ~		
A13.a.	Listen and respond		# O ~		
A13.b.	Cultivate empathy		# O ~		
	Specific Learner Outcomes				
	Internalized Outcomes				
	Kinds of meaning:				
B1.	Essence	~	# O		
B2.	Significance		# O ~		

B3.	Conditions of applicability	O	# ~		
B4.	Nuance		# O ~		
B5.	Self as learner		# O ~		
	Big Ideas-				
B6.a.	Algebraic themes	# ~	O		
B6.b.	Class of mathematical problems		# O ~		
B6.c.	Mathematical principle of rule	O	# ~		
B6.d.	Essential mathematics questions		# O ~		
B6.e.	Nature of mathematics		# O ~		
	Relations between concepts-				
B7.a.	Details with big ideas		# O ~		
B7.b.	Within disciplinary ideas		# O ~		
B7.c.	Cross-disciplinary ideas		# O ~		
B7.d.	Concepts with skills	O	# ~		
B7.e.	Big ideas with other big ideas		# O ~		
B7.f.	Details with other details		# O ~		
	Relationship between learner and concept-				
B8.a.	Consensus-driven with personal		# O ~		
B8.b.	New concepts with existing		# O ~		
B8.c.	Out-of-app with in-app		# O ~		
B8.d.	Increased concreteness	# O ~			
	Meanings-in-the-moment-				
B9.a.	Current summary of understanding		# O ~		
B9.b.	Current degree of understanding		# O ~		
B9.c.	Current self-performance		# O ~		
B9.d.	Read current situation	# O ~			
	Kinds of supporting knowledge:				
B10.	General procedural		# O ~		
B11.	Specific procedural		# O ~		
B12.	Dispositional		# O ~		
B13.	Formal/ conventional language	#	O ~		
B14.	Tech. and media	~	# O		
	Externalized Outcomes				
B15.	Cumulative project		# O ~		

B16.	Conjectures		# O ~		
B17.	Formal plans		# O ~		
	Specific Early Algebra Concepts				
C1.	Rules, properties, principles	O ~	#		
C2.	Classifications	~	# O		
C3.	Patterns	~	# O		
C4.	Changes	~	# O		
C5.	Relations	#	O ~		
C6.	Symbols		# O ~		
	Program, Experience, Environment				
	Movement Through Content				
D1.	Reset/ adapt	# O ~			
D2.	Multiple experiences for one phase		# O ~		
D3.	Exploration of virtual environment		# O ~		
D4.a.	New condition	# O ~			
D4.b.	Chance to explore and explain again		# O ~		
D5.	New contextual restraint		# O ~		
D6.	One condition, five time restriction	# O	~		
D7.	Coherence	# O ~			
D8.	Divorced from timelines	# O ~			
D9.	Differentiated learning routes		# O ~		
D10.	Constructivist	# O ~			
	Kinds of Contexts				
D11.a.	Decontextualized	# O ~			
D11.b.	Connects to larger context		# O ~		
D12.a.	Contextualized		# O ~		
D12.b.	Embedded ideas made explicit		# O ~		
D13.	Higher ratio of contextualized		# O ~		
D14.a.	Varied-structured		# O ~		
D14.b.	Accepts open-ended input		# O ~		
	Orderliness and Clarity				
D15.	Clear layout	# O ~			
D16.	Clear directions	# O ~			
D17.	Universal Design features		# O ~		

	Balance of Cognitive-Affective				
D18.	Activates prior knowledge	#	O ~		
D19.	Initiates and maintains engagement	# O	~		
D20.	Sensory and peace	# O	~		
D21.	Time	# O	~		
D22.	Scaffold		# O ~		
D23.a.	Range of materials		# O ~		
D23.b.	Diverse perspectives	~	# O		
	Trust, Safety, Respect				
D24.	Reporting negative behaviors	#	O ~		
D25.	Confidentiality and privacy	# O ~			N/A
D26.	Diverse reflections of home life		# O ~		
D27.a.	Interest survey		# O ~		
D27.b.	Interest utilized		# O ~		
D28.a.	Life experiences survey		# O ~		
D27.b.	Life experiences utilized		# O ~		
	Dignity-				
D29.a.	Respect for children's capacity	# O ~			
D29.b.	Avoids child stereotypes	# O ~			
D29.c.	Meaningful work		# O ~		
	App Features				
E1.	Mobility/ portability		# O ~		
E2.	Multi-touch	# O ~			

KEY:	
DragonBox Algebra 5+	**#**
Math Motion: Zoom	**~**
Slice Fractions	**O**

Appendix G:

Coding Frame Model

"What"	
A. Specific Learner Actions	
Explain and describe	
Meets	*Does Not Meet*
Choose from and use	
Meets	*Does Not Meet*
Create solutions and products	
Meets	*Does Not Meet*
Move through design cycle	
Meets	*Does Not Meet*
Weigh and evaluate	
Meets	*Does Not Meet*
Abstract	
Meets	*Does Not Meet*
Move between specific and general	
Meets	*Does Not Meet*
Represent	
Meets	*Does Not Meet*
Translate	
Meets	*Does Not Meet*
Explore	
Meets	*Does Not Meet*
Collaborate w/ local	
Meets	*Does Not Meet*
Collaborate w/ distance	
Meets	*Does Not Meet*
Collaborate in specific ways	
Meets	*Does Not Meet*
Justify	
Meets	*Does Not Meet*
Listen and respond	
Meets	*Does Not Meet*
Cultivate empathy	
Meets	*Does Not Meet*
B. Specific Learner Outcomes	
Internalized Outcomes	
Kinds of meaning:	

Essence	
Meets	*Does Not Meet*
Significance	
Meets	*Does Not Meet*
Conditions of applicability	
Meets	*Does Not Meet*
Nuance	
Meets	*Does Not Meet*
Self as learner	
Meets	*Does Not Meet*
Big Ideas-	
Algebraic themes	
Meets	*Does Not Meet*
Class of mathematical problems	
Meets	*Does Not Meet*
Mathematical principle of rule	
Meets	*Does Not Meet*
Essential mathematics questions	
Meets	*Does Not Meet*
Nature of mathematics	
Meets	*Does Not Meet*
Relations between concepts-	
Details with big ideas	
Meets	*Does Not Meet*
Within disciplinary ideas	
Meets	*Does Not Meet*
Cross-disciplinary ideas	
Meets	*Does Not Meet*
Concepts with skills	
Meets	*Does Not Meet*
Big ideas with other big ideas	
Meets	*Does Not Meet*
Details with other details	
Meets	*Does Not Meet*
Relationship between learner and concept-	
Consensus-driven with personal	
Meets	*Does Not Meet*
New concepts with existing	
Meets	*Does Not Meet*
Out-of-app with in-app	
Meets	*Does Not Meet*
Increased concreteness	
Meets	*Does Not Meet*
Meanings-in-the-moment-	
Current summary of understanding	
Meets	*Does Not Meet*
Current degree of understanding	

Meets	*Does Not Meet*
Current self-performance	
Meets	*Does Not Meet*
Read current situation	
Meets	*Does Not Meet*
Kinds of supporting knowledge:	
General procedural	
Meets	*Does Not Meet*
Specific procedural	
Meets	*Does Not Meet*
Dispositional	
Meets	*Does Not Meet*
Formal/ conventional language	
Meets	*Does Not Meet*
Tech. and media	
Meets	*Does Not Meet*
Externalized Outcomes	
Cumulative project	
Meets	*Does Not Meet*
Conjectures	
Meets	*Does Not Meet*
Formal plans	
Meets	*Does Not Meet*
C. Specific Early Algebra Concepts	
Rules, properties, principles	
Meets	*Does Not Meet*
Classifications	
Meets	*Does Not Meet*
Patterns	
Meets	*Does Not Meet*
Changes	
Meets	*Does Not Meet*
Relations	
Meets	*Does Not Meet*
Symbols	
Meets	*Does Not Meet*
"How"	
D. Program, Experience, & Environment	
Movement Through Content	
Reset/ adapt	
Meets	*Does Not Meet*
Multiple experiences for one phase	
Meets	*Does Not Meet*
Exploration of virtual environment	
Meets	*Does Not Meet*
New condition	
Meets	*Does Not Meet*

Chance to explore and explain again	
Meets	*Does Not Meet*
New contextual restraint	
Meets	*Does Not Meet*
One condition, five time restriction	
Meets	*Does Not Meet*
Coherence	
Meets	*Does Not Meet*
Divorced from timelines	
Meets	*Does Not Meet*
Differentiated learning routes	
Meets	*Does Not Meet*
Constructivist	
Meets	*Does Not Meet*
Kinds of Contexts	
Decontextualized	
Meets	*Does Not Meet*
Connects to larger context	
Meets	*Does Not Meet*
Contextualized	
Meets	*Does Not Meet*
Embedded ideas made explicit	
Meets	*Does Not Meet*
Higher ratio of contextualized	
Meets	*Does Not Meet*
Varied-structured	
Meets	*Does Not Meet*
Accepts open-ended input	
Meets	*Does Not Meet*
Orderliness and Clarity	
Clear layout	
Meets	*Does Not Meet*
Clear directions	
Meets	*Does Not Meet*
Universal Design features	
Meets	*Does Not Meet*
Balance of Cognitive-Affective	
Activates prior knowledge	
Meets	*Does Not Meet*
Initiates and maintains engagement	
Meets	*Does Not Meet*
Sensory and peace	
Meets	*Does Not Meet*
Time	
Meets	*Does Not Meet*
Scaffold	
Meets	*Does Not Meet*

Range of materials	
Meets	*Does Not Meet*
Diverse perspectives	
Meets	*Does Not Meet*
Trust, Safety, Respect	
Reporting negative behaviors	
Meets	*Does Not Meet*
Confidentiality and privacy	
Meets	*Does Not Meet*
Diverse reflections of home life	
Meets	*Does Not Meet*
Interest survey	
Meets	*Does Not Meet*
Interest utilized	
Meets	*Does Not Meet*
Life experiences survey	
Meets	*Does Not Meet*
Life experiences utilized	
Meets	*Does Not Meet*
Dignity:	
Respect for children's capacity	
Meets	*Does Not Meet*
Avoids child stereotypes	
Meets	*Does Not Meet*
Meaningful work	
Meets	*Does Not Meet*
E. App Features	
Mobility/ portability	
Meets	*Does Not Meet*
Multi-touch	
Meets	*Does Not Meet*
Miscellaneous	
Relevant	Irrelevant

Appendix H:

TPACK Image Permission

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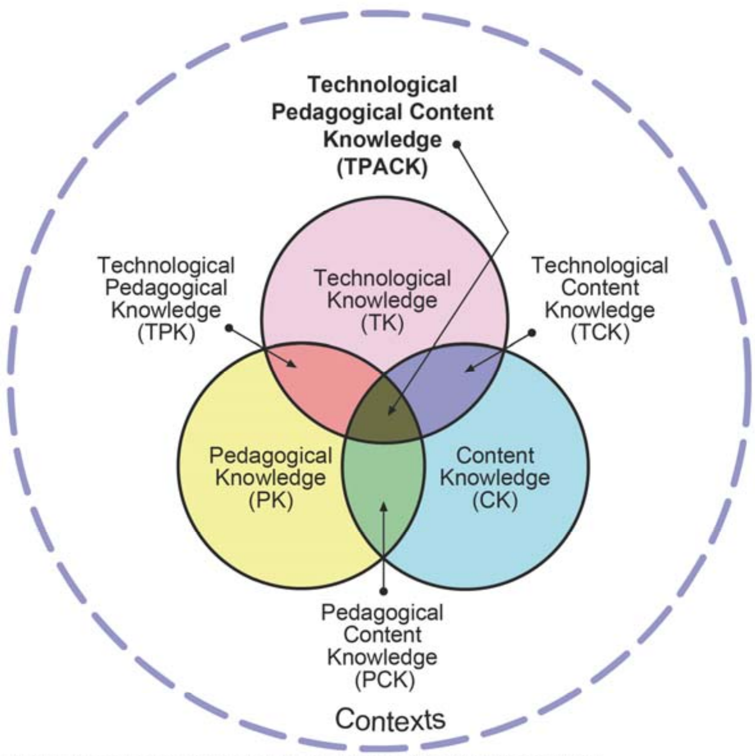

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