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Cybersecurity: Stochastic Analysis and Modelling of Vulnerabilities to Determine the Network Security and Attackers Behavior

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
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Dedication

This doctoral dissertation is dedicated to my father, my mother and my husband.

Acknowledgments

It was a wonderful opportunity that I was able to study at USF. I would like to pay my gratitude to everyone for helping me in numerous ways during my time of study at USF.

It is with my utmost respect and gratitude that I mention here, proper guidance, selfless gifts of knowledge, encouragement and motivation for independent research interests given by my advisor, the Distinguished University Professor Chris P. Tsokos. His considerate attention for students was exceptional. Through out my doctoral program, directions and advice inspired me in my research efforts.

I am thankful to Prof. Kandethody Ramachandran, Prof. Dan Shen, Prof. Lu Lu for their kind assistance and time in my dissertation research.

It is with great pleasure and gratitude that I remind all the faculty members in the Department of Mathematics and Statistics for their effort full teaching in the courses I have taken. I express my gratitude to the administration of the the Department of Mathematics and Statistics for all the helps and resources made available to me as a student.

I would also like to express my gratitude to the Florida Center of Cyber Security at USF (FC^2), for the Summer internship granted me in 2016.

It is with great pleasure and love that I mention the helpfulness of my husband,

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Many thanks to my fellow friends, Xing Wang, Jason, Muditha, Bashar and all other graduate students for their support and friendship in last four years.

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Abstract

Development of Cybersecurity processes and strategies should take two main approaches. One is to develop an efficient and effective set of methodologies to identify software vulnerabilities and patch them before being exploited. Second is to develop a set of methodologies to predict the behavior of attackers and execute defending techniques based on attacking behavior. Managing of Vulnerabilities and analyzing them is directly related to the first approach. Developing of methodologies and models to predict the behavior of attackers is related to the second approach. Both these approaches are inseparably interconnected. Our effort in this study mainly focuses on developing useful statistical models that can give us signals about the behavior of cyber attackers.

Analytically understanding of vulnerabilities in statistical point of view helps to develop a set of statistical models that works as a bridge between Cybersecurity and Abstract Statistical and Mathematical knowledge. Any such effort should begin with properly understanding the nature of Vulnerabilities in a computer network system. We start this study with analyzing "Vulnerability" based on inferences that can be taken from National Vulnerability Database (NVD).

In Cybersecurity context, we apply Markov approach to develop suitable predictive models to successfully estimate the minimum number of steps to compromise a security goal that an attacker would take using the concept of Expected Path Length (EPL).

We have further developed Non-Homogeneous Stochastic model by improving EPL estimates in to a time dependent variable. This approach analytically applied in a simple model of computer network with discovered vulnerabilities resulted in several useful observations exemplifying the applicability in real world computer systems. The methodology indicated a measure of the ”Risk” associated with the model network as a function of time indicating defending professionals on the threats they are facing and should anticipate to face.

Further more, using a similar approach taken in well known Google page rank algorithm, a new ranking algorithm of vulnerability ranks with respect to time for computer network system is also presented in this study.

With better IT resources analytical models and methodologies presented in this study can be developed into more generalized versions and apply in real world computer network environments.

1 Introduction

Present study is a series of Statistical and Mathematical applications that is proposed as set of decision making tools in the filed of Cybersecurity. This dissertation consists of five chapters where the first two chapters present the background of the research of interest and other chapters present with the methodologies, applications and research outcomes.

In this chapter, we discuss our research problems and their background, importance of Statistical Approaches in the area of Cybersecurity, current trends in the filed of Cybersecurity and basis of our strategies from Statistical perspective.

1.1 Research Problems and Their Background

Cyber-attacks are one of the most formidable security challenge faced by many governments and large scale companies. Cyber criminals are increasingly using sophisticated network and social engineering techniques to steal the crucial information which directly affects the operational effects of the Government or Companys objectives. According to the Secunia [1] report 2015 we can see how crucial the volume and magnitude of increasing Cybersecurity threats. Thus, in understanding the performance, availability and reliability of computer networks, security measuring techniques plays an important role in the subject area.

Quantitative measures are now commonly used to evaluate the security of computer network systems. These measures help administrators to make important decisions regarding their network security [3].

However, security of any computer system at last is about the weaknesses it has and capability of defending the system from attackers who would try to use those weaknesses to exploit the system. It is logical to compare, this situation to a war fare, where an army of enemies would tries to break in to a security station using its weaker points while defending forces are trying to track attackers, understand their own weaknesses and deploy solutions to those weaknesses so that the ability of the enemy forces to exploit those weaknesses are eliminated or minimized. This means that Cybersecurity require a great deal of analysis in two main aspects. The first one is the analysis of the weaknesses in computer systems. The second is the analysis of the behavior of attackers and defenders. Indeed the analysis of the behavior of the attacker is critical.

While we paid our attention to both the behavioral aspect analysis and system susceptibility analysis this dissertation focus more into the behavioral aspect of the analysis. In other words, we pay our specific attention to develop analytical models to analyze ”attackers behavior”.

There are indeed a lots of efforts taken by Information Technology specialists and related professionals in this respect, developing models to track attackers.

However, our mission here is to develop set of Statistical Models that can be used in analyzing or predicting the attackers behavior.

Subil Abrahim and Suku Nair [3] in 2014 introduced a stochastic model for cybersecurity quantification using absorbing Markov chain. In [3] they introduced a non homogeneous Markov model for security quantification. Kijsanayothin Phongphun [4] in her/his thesis "Network Security Modeling with Intelligent and Complexity Analysis" discussed the need for generating automated preventive network security systems and an approach to do it using "host centric attack graphs". This study used both quantitative and qualitative approaches in security model analysis named "exploit based analysis" and "preference based analysis" respectively. All three of these studies mentioned have used Markov Chain methods in their methodologies in Cybersecurity analysis.

In 2006 Stefan Frei [10] and others, in their study titled "Large-Scale Vulnerability Analysis" presented with several important analytical outcomes. In this study, an overall statistical analysis on the vulnerability data [2] uptodate was conducted using over 80,000 vulnerabilities recorded from 1995.

Yashwant K. Malaiya and Alhazmi [5] also have conducted a series of very useful and important studies analyzing the vulnerability data in several aspects. Malaiya talks about vulnerability life cycle and CVSS Metrics in an analytical view point of security risk and proposed a method for risk evaluation [8]. Malaiya discuss and analyze different states of vulnerabilities and proposed some models to evaluate risk associated with those states [6] & [7].

Hiroyuki Okamura [36] in 2013 proposed a quantitative security evaluation method using stochastic process for software systems using vulnerability data that the discovery dates, patch dates and exploit publish dates are available. Leyla Bilge and Tudor Dumitras in 2012 described a method to automatically identify ”zero-day” vulnerabilities [9].

Considering these efforts in the subject area of Cybersecurity it is our understanding that the need for better and improved analytical tools for vulnerability analysis and the likelihood to be exploited is at the highest importance. Therefore, we raise our research problems from the defenders point of view to seek answers. First, how to develop a set of successful statistical models to analyze the vulnerability status of a computer network system. Second, could an analysis on vulnerability and their likelihood to be exploited be observed and relate those observations in predicting the attackers success. To answer those problems, we first need to take better statistical and probabilistic approaches in analyzing the attacking process. Next few chapters as summarized below discuss these problems in details with the methodologies used and results obtained.

1.2 Vulnerability Data Base and Analysis of Vulnerability Types

In Chapter two, an overview about the nature of vulnerabilities and Common Vulnerability Scoring System (CVSS) [2],[14] is given. The methodology used in CVSS and metric system used to arrive the CVSS are discussed. The concept of Vulnerability Life Cycle and its stages are also introduced. Following this introduction and background is an analysis on various type of vulnerabilities. In this chapter, we take the entire vulnerability data base [2],[19] into consideration and rank number of vulnerabilities of 13 different types as given in the National Vulnerability Database. Resulting ranks of vulnerabilities important in terms of the frequency are presented graphically. Further, a suitable forecasting models are arrived based on the shape of the scatter plots where the number of different types of vulnerabilities are plotted with respect to time.

Main objective of chapter two is to have the basic analysis and gain the overall understanding about vulnerabilities and National Vulnerability Database which will make the platform for our deeper analysis in chapters to follow.

1.3 A Statistical Predictive Model for the Expected Path Length

In Chapter three, the concept of Expected Path Length (EPL)[3],[4] as an application in Cybersecurity filed is introduced. The main objective of the chapter is to develop a successful Statistical Model to estimate the Expected Path Length. Using a model computer network example, a methodology and an analysis on Attack Prediction Method is discussed. Steps that were taken using the Markov Chain Process [16] and the development of the Transition Probability Matrix are presented under Methodology. Concept of Attack graph and developing of Attack Graphs are also introduced. Finally, successful Statistical Models to predict the minimum number of steps (Expected Path Length) that an Attacker would take to compromise a security state presented. Developed Statistical Models are tested for the quality and accuracy.

1.4 NonHomogeneous Stochastic Model for Cyber Security Predictions

Objective of Chapter four is to develop and present a Non-Homogeneous Stochastic Model for better Cybersecurity predictions. In this chapter Cyber Security Analysis Method of Vulnerability Life Cycle is used as an index of Risk Factor[20],[21] associated with a particular vulnerability. But, the chapter further develop this analysis method and Risk Analysis to develop a time dependent analytical methodology. The new methodology proposed is applied in a model network system which allows to analyze the behavior of EPL as a function of time. This improvement is significant because the Risk is now calculated for a set of vulnerabilities in a network system and also observable with respect to time.

1.5 Nonhomogeneous Risk Rank Analysis Method for Security Network System

Chapter five in this dissertation presents a new Risk Rank Algorithm [13],[27] were the risk ranks of vulnerabilities exist in a computer network system can be obtained. Development of this new algorithm is based on well-known Google Page Rank Algorithm. New algorithm we propose in this chapter is significant since the Ranks of vulnerabilities of a computer network system can be observed with respect to time as well. Therefore, the results from this method can be used by network administrators and defending professionals to understand the priorities and allocate resources based on these priorities at the time of consideration.

2 Vulnerability Data Base and Analysis of Vulnerability Types

2.1 Introduction

In this chapter, an overview about the nature of vulnerabilities and Common Vulnerability Scoring System (CVSS)[14] is given. The methodology used in CVSS and metric system used to arrive the CVSS are discussed. The concept of Vulnerability Life Cycle and its stages are also introduced. Following this introduction and background is an analysis on various type of vulnerabilities. In this chapter, we take the entire vulnerability data base into consideration and rank number of vulnerabilities of 13 different types as given in the National Vulnerability Database [2]. Resulting ranks of vulnerabilities important in terms of the frequency are presented graphically. Further, a suitable forecasting models are arrived based on the shape of the scatter plots where the number of different types of vulnerabilities [19] are plotted with respect to time. .

However, it should be noticed that our study results in some new definitions also. Applying of Mathematical and Statistical approaches into the field of Cybersecurity is relatively new compared to many such other fields that use Mathematical and Statistical approaches. Therefore, in some cases we needed to define concepts our own. In all such cases, we tried our best to maintain the integrity of Mathematical, Statistical and Computer science practices.

2.2 Vulnerability

Properly understanding what is called ”vulnerability” [2] in the field of Cybersecurity is of the core in our study. Therefore, in this chapter we discuss all the basic aspects of vulnerability including vulnerability information, vulnerability data base and vulnerability life cycle [8].

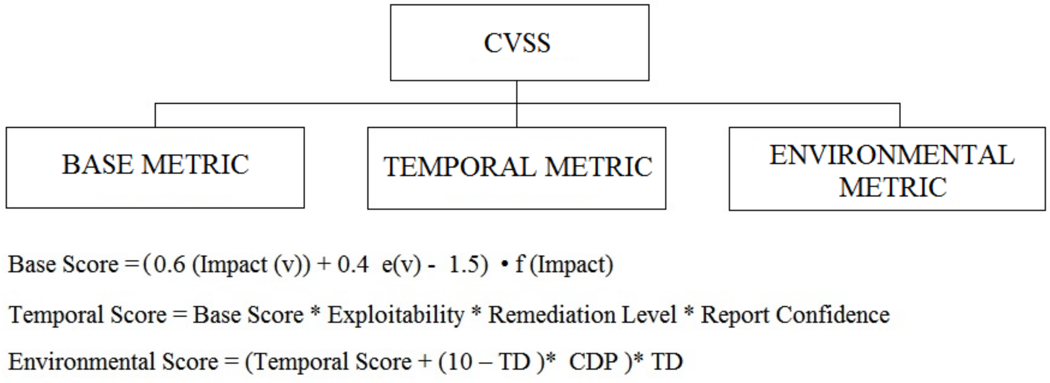
In computer security, a vulnerability is a weakness which allows an attacker to reduce a system's information assurance. Vulnerability is the intersection of three elements, which are, systems susceptibility to the flaw, attacker access to the flaw, and attacker capability to exploit the flaw.

To exploit a vulnerability, an attacker must have at least one applicable tool or technique that can connect to a system weakness. In this frame, vulnerability is also known as the attack surface.

The attack surface of a software environment is the sum of the different points (the ”attack vectors”) where an unauthorized user (the ”attacker”) can try to enter data to or extract data from an environment.

2.3 Common Vulnerability Scoring System (CVSS)

Common Vulnerability Scoring System (CVSS)[14] is a free and open industry standard for assessing the severity of computer system security vulnerabilities. It is under the custodianship of the Forum of Incident Response and Security Teams (FIRST). CVSS is composed of three metric groups, Base, Temporal, and Environmental, each consisting of a set of metrics. It attempts to establish a measure of how much concern a vulnerability warrants, compared to other vulnerabilities, so efforts can be prioritized. The scores are based on a series of measurements (called metrics) based on expert assessment. The scores range from 0 to 10 [2]. Vulnerabilities with a base score in the range 7.0-10.0 are High, those in the range 4.0-6.9 as Medium, and 0-3.9 as Low. Figure 2.1, 2.2 and 2.3 below give a schematic presentation of the Common Vulnerability Scoring System (CVSS) which is the basis of the metric calculation model and the temporal and environmental matrices calculation model, respectively. Calculation methodology for the CVSS is given in Appendix A.



Base Score =(0.6 (Impact (v)) + 0.4 e(v) - 1.5) • f (Impact)

Temporal Score = Base Score * Exploitability * Remediation Level * Report Confidence

Environmental Score = (Temporal Score + (10 – TD)* CDP)* TD

Figure 2.1: Common Vulnerability Scoring System

2.3.1 Base Metric

Base Metric [2] is derive from two sub metrics,Exploitability metric and Impact metric. Calculation methodology is given in Appendix B.

$$BaseScore = (0.6 * Impact + 0.4 * e(v) - 1.5) * f(Impact),$$

$$e(v) = 20 * AV * AC * AU, Impact(v) = 10.41(1 - (1 - C)(1 - I)(1 - A)),$$

$$f(Impact) = \begin{cases} 0, & \text{if } Impact(v) = 0 \\ 1.176, & \text{Otherwise} \end{cases}$$

Access Vector (AV)

This measures whether a vulnerability is exploitable locally or remotely. Local: The vulnerability is only exploitable locally Remote: The vulnerability is exploitable

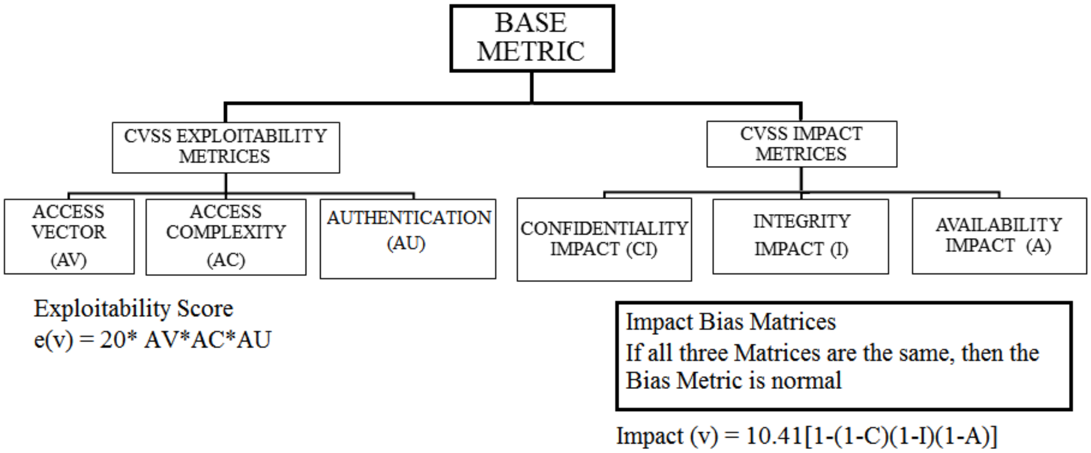


Figure 2.2: Common Vulnerability Scoring System- Base Metric Calculation Model

remotely (and possibly locally as well)

Access Complexity (AC)

This measures the complexity of attack required to exploit the vulnerability once an attacker has access to the target system. High: Specialized access conditions exist such as specific window of time (a race condition), specific circumstance (non-default configurations) or victim interaction such as tainted e-mail attachment. Low: Specialized access conditions or extenuating circumstances do not exist. In other words, it is always exploitable. This is the most common case

Authentication (AW)

This measures whether or not an attacker needs to be authenticated to the target system in order to exploit the vulnerability. Required: Authentication is required to access and exploit the vulnerability. Not Required: Authentication is not required to access or exploit the vulnerability.

Confidentiality Impact (C)

Confidentiality Impact measures the impact on Confidentiality of a successful exploit of the vulnerability on the target system. None: No impact on confidentiality. Partial: There is consider able informational disclosure. Complete: A total compromise of critical system information.

Integrity Impact (I)

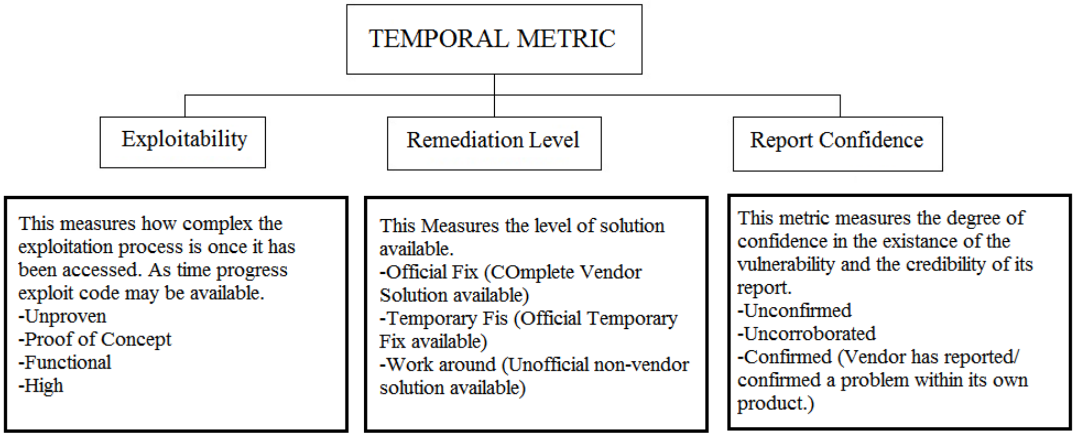
Integrity Impact measures the impact on Integrity of a successful exploit of the vulnerability on the target system. None: No impact on integrity. Partial: Considerable breach in integrity. Complete: A total compromise of system integrity.

Availability Impact (A)

Availability Impact measures the impact on Availability of a successful exploit of the vulnerability on the target system. None: No impact on availability Partial: Considerable lag in or in eruptions in resource availability Complete: Total shutdown of the affected resource

2.3.2 Temporal Metric

The Temporal metrics [2] measure the current state of exploit techniques or code availability, the existence of any patches or workarounds, or the confidence that one has in the description of a vulnerability.



This measures how complex the exploitation process is once it has been accessed. As time progress exploit code may be available.
-Unproven
-Proof of Concept
-Functional
-High

This Measures the level of solution available.
-Official Fix (COmplete Vendor Solution available)
-Temporary Fis (Official Temporary Fix available)
-Work around (Unofficial non-vendor solution available)

This metric measures the degree of confidence in the existance of the vulnerability and the credibility of its report.
-Unconfirmed
-Uncorroborated
-Confirmed (Vendor has reported/ confirmed a problem within its own product.)

Figure 2.3: Common Vulnerability Scoring System- Temporal Metric Calculation Model

2.3.3 Environmental Metric

These metrics enable the analyst to customize the CVSS score depending on the importance of the affected IT asset to a users organization, measured in terms of complementary/alternative security controls in place, Confidentiality, Integrity, and Availability. The metrics are the modified equivalent of base metrics and are assigned metrics value based on the component placement in organization infrastructure.

Collateral Damage Potential (CDP)

This metric measures the potential for loss of life or physical assets through damage or theft of property or equipment. The metric may also measure economic loss of productivity or revenue. The possible values for this metric are listed in Appendix A. Naturally, the greater the damage potential, the higher the vulnerability score.

Target Distribution (TD)

This metric measures the proportion of vulnerable systems. It is meant as an environment-specific indicator in order to approximate the percentage of systems that could be affected by the vulnerability. The possible values for this metric are listed in Appendix A. The greater the proportion of vulnerable systems, the higher the score.

2.4 Vulnerabilities Life Cycle

The Life Cycle of a Vulnerability [10] can be introduced with different stages that a vulnerability passes through. We shall discuss specific stages that are commonly identified in a given situation. Commonly identified stages are involved with the events such as the Birth (Pre-discovery Stage), Discovery, Disclosure, Availability for Patching and Availability for Exploiting.

2.4.1 Birth (Pre-Discovery)

The birth of a vulnerability occurs at the development of a software, mostly due to a weakness or a mistake in coding of the software. At this stage the vulnerability is not yet discovered or exploited. In a well-developed software package where its reliability has been identified, one can identify the probability of the birth of the problem.

2.4.2 Discovery

Vulnerability is said to be discovered once someone identifies the flaw in the software. It is possible that the vulnerability is discovered by the system developers themselves, skilled legitimate users or by the attackers also. If the vulnerability is discovered internally or by white hackers, (who are making breaking attempts on a system to identify the flaws and vulnerabilities with good intentions of helping them to be patched so that the system security is strengthened) it will be notified to be fixed as soon as possible. But, if a black hacker [40] discovers a vulnerability it is possible that he or she will try to exploit it, or sell in the black market or distribute it among hackers to be exploited.

It should be noted here that while vulnerabilities could actually exist prior to the discovery, until it is discovered, it is not a potential security risk. ”Time of the discovery” is the earliest time that a vulnerability is identified. In a vulnerability life cycle the ”time of discovery” is an important and critical event. Exact discovery time might not be published or disclosed to the public due to the other risks that could be associated with a vulnerability. However, in general after the ”disclosure” of a vulnerability, public may know the time of discovery subject to security risk review.

We would like to mention here that in developing our statistical model, we consider only pre-exploit discovery. There are rare chances that a discovery of a vulnerability could occur after it is actually exploited. As an example, an attacker could run an exploit attempt aiming to exploit a particular vulnerability. But, the exploit actually breaks into the system through another unidentified or undiscovered vulnerability instead of expected vulnerability at that time. Such rare occurrences are not taken into account in our our present study.

2.4.3 Disclosure

Once a vulnerability is discovered, it is subject to be disclosed. Disclosure could take place in different ways based on the system design, authentication and who discovered it. However, ”disclosure” in widely accepted form in the information security means the event that a particular vulnerability is made known to public through relevant and appropriate channels. Definition for the disclosure of vulnerability is however presented differently by different individuals.

In general, public disclosure of a vulnerability is based on several principles. The availability of access to the vulnerability information for the public is one such important principle. Another such important principle is validity of information. Validity of information principle is to ensure the users ability to use those information, assess the risk and take security measures. Also, the independence of information channels is also considered to be important to avoid any bias and interferences from organizational bodies including the vendor.

2.4.4 Scripting (Exploiting) and Exploit Availability

A Vulnerability enters to the stage of exploit availability [22],[23] from the earliest time that an exploit program of code is available. Once the exploits are available even low skilled crackers (or in other words a black hat hacker) could be capable of exploiting the vulnerability. As we mentioned earlier, there are some occurrences that the exploit could happen even before the vulnerability is discovered. However in the present study we consider the modeling of Vulnerability Life Cycles with exploit availability occurs only after the discovery.

2.4.5 Patch Availability and Death: (Patched)

Patch is a software solution that the vendor or developer release to provide necessary protection from possible exploits of the vulnerability. Patch will act against possible exploit codes or attacking attempts for a vulnerability and protect the system and ensure the integrity. The vulnerability dies when one applies a security patch to all the vulnerable systems.

When a White Hat Researcher discovers a vulnerability, the next transition is likely to be the internal disclosure leading to patch development. On the other hand, if a Black Hat Hacker discovers a vulnerability, the next transition could be an exploit or internal disclosure to his underground community. Some active black hats might develop scripts that exploit the vulnerability.

2.5 Categorizing and Ranking Different Types of Vulnerabilities

There are 13 types of vulnerabilities [19] identified and recorded in the National Vulnerability database. These different types are defined and identified based on the nature of the vulnerability in its execution and effect. In this section we look into these vulnerability types and try to identify them based on their vulnerability level. We also rank these different types of vulnerabilities based on the number of discovered vulnerabilities recorded in the National Vulnerability Database. All 75705 recorded vulnerabilities [19],[15] in the National Vulnerability Database from 1999 to 2016 are considered here.

To observe the Ranks and then to see if we can develop suitable models to predict the number of vulnerabilities in yearly basis for vulnerabilities that are crucial in numbers, we needed to obtain number of discovered and reported vulnerabilities in each level Low, Medium and High. However, CVE details website provides us with only the total number of vulnerabilities in each year for different categories with respect to 10 different levels of vulnerability scores from 0 to 10. To obtain the information we needed, it was required to manually calculate numbers in each category in each year data and then add them up to get the sum of number of vulnerabilities for different levels. To obtain the information needed for our preliminary analysis on the data base, there were no other easier information source available. Therefore, it was necessary to go through this manual process. Following schematic network presents the steps we used in the calculation process.

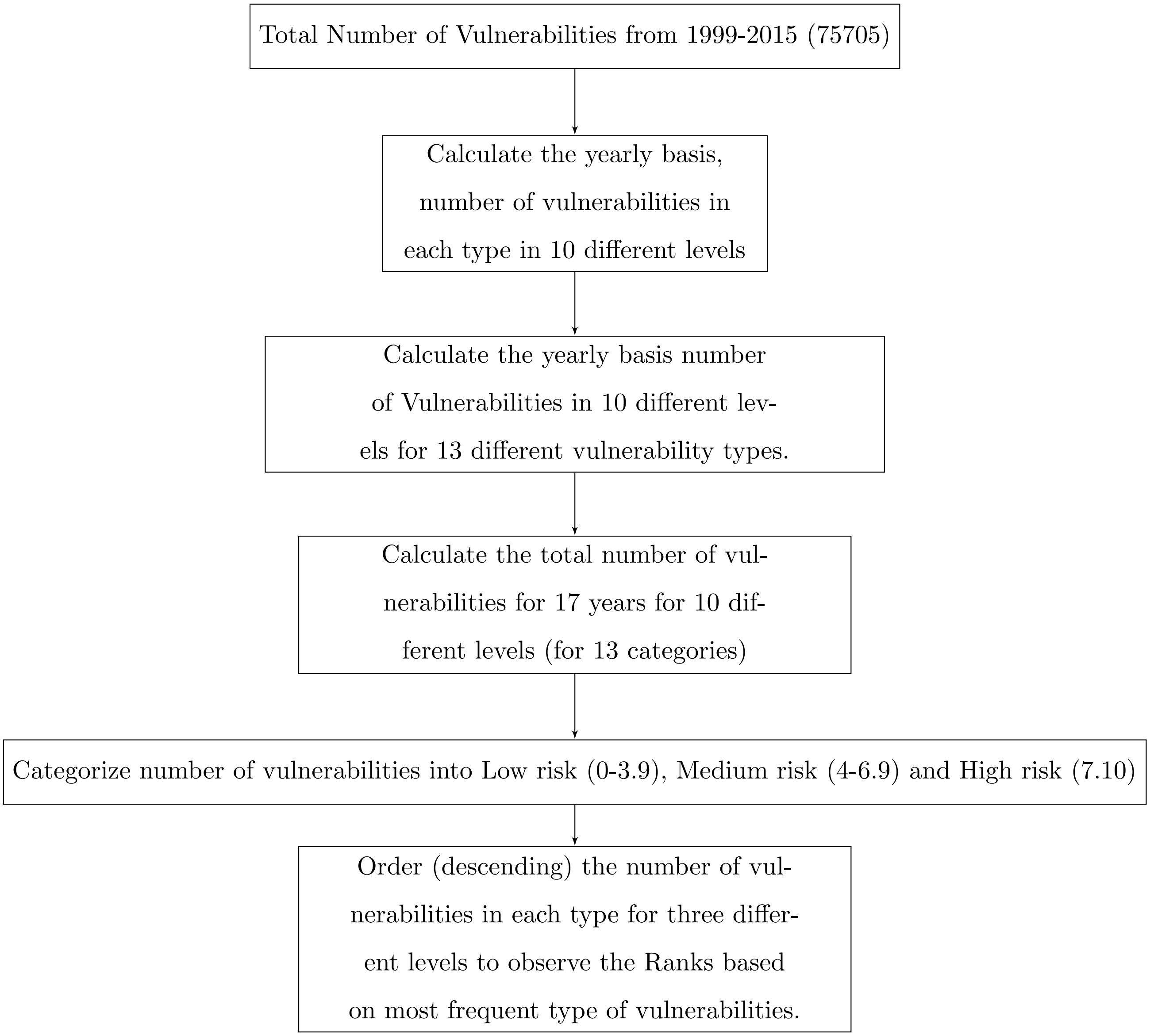

Total Number of Vulnerabilities from 1999-2015 (75705)
Calculate the yearly basis, number of vulnerabilities in each type in 10 different levels
Calculate the yearly basis number of Vulnerabilities in 10 different levels for 13 different vulnerability types.
Calculate the total number of vulnerabilities for 17 years for 10 different levels (for 13 categories)
Categorize number of vulnerabilities into Low risk (0-3.9), Medium risk (4-6.9) and High risk (7.10)
Order (descending) the number of vulnerabilities in each type for three different levels to observe the Ranks based on most frequent type of vulnerabilities.

Figure 2.4: Key Steps of the Rank Vulnerability Types.

2.5.1 Rank and Distribution of Vulnerability Types in Low level (CVSS score 0-3.9)

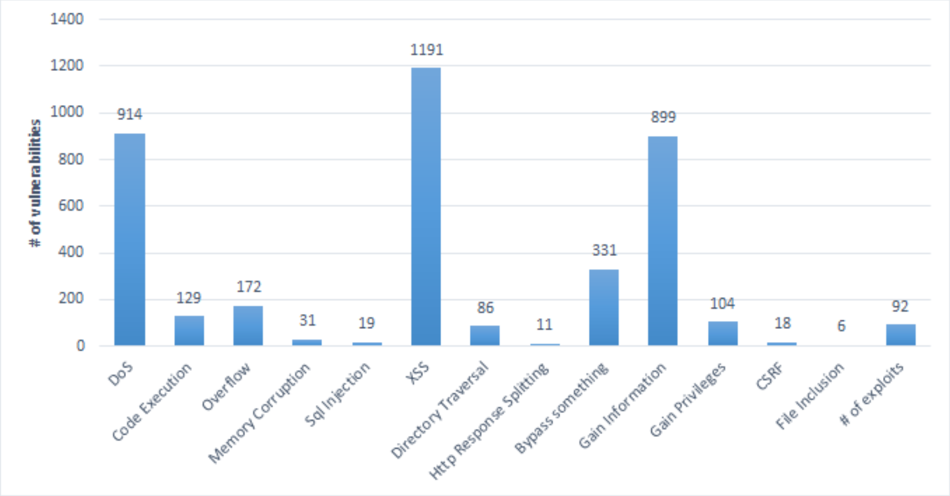


Figure 2.5: Distribution of vulnerability Types in Low Category

The most common type of vulnerability that is in lower level category is XSS (Cross-site scripting Vulnerabilities). According to the number of vulnerabilities in each type, the second and third in the lower level are DOS (Denial of service) and Gain Information respectively. Table below illustrates the rank of vulnerability types, based on number of discovered vulnerabilities in the lower level category.

2.5.2 Rank and Distribution of Vulnerability Types in Medium Level (CVSS score 4-6.9)

In the Medium level, DOS (Denial of service) vulnerabilities have the highest frequency in the database. Types XSS and code executions are the other most frequent vulnerabilities. Table 2.2 below illustrates ranks based on the number of different

Table 2.1: Rank of Vulnerability Types in Low Category

Vulnerability Type	Rank	Vulnerability Type	Rank
XSS	1	Directory Traversal	8
DOS	2	Memory Corruption	9
Gain Infor	3	Sql Injection	10
Bypass	4	CSRF	11
overflow	5	Http Response Splitting	12
Code Execution	6	File Inclusion	13
Gain Privileges	7	-	-

types of vulnerabilities in the medium level.

Table 2.2: Rank of Vulnerability Types in Medium Category

Vulnerability Type	Rank	Vulnerability Type	Rank
DOS	1	Gain Privileges	8
XSS	2	CSRF	9
Code Execution	3	Sql Injection	10
Gain Infor	4	Memory Corruption	11
overflow	5	File Inclusion	12
Bypass something	6	Http Response Splitting	13
Directory Traversal	7	-	-

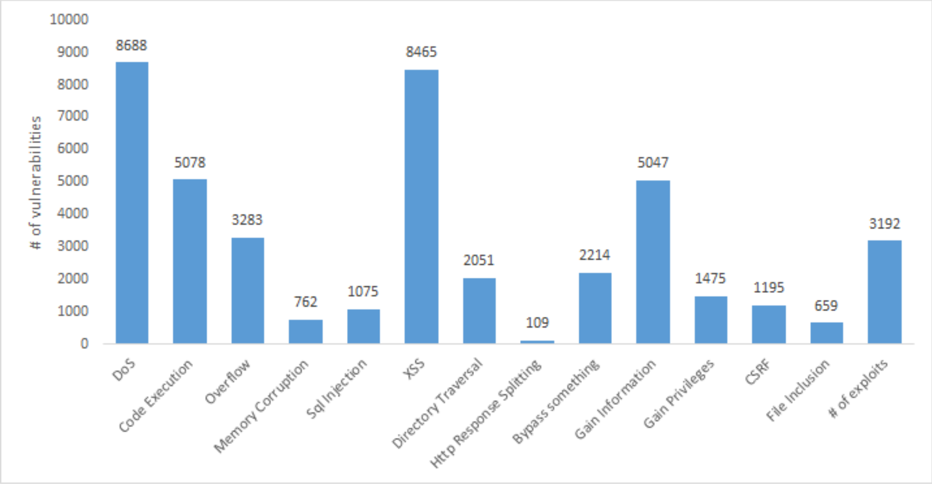


Figure 2.6: Distribution of vulnerability Types in Medium Category

2.5.3 Rank and Distribution of Vulnerability Types in High Level (CVSS score 7-10)

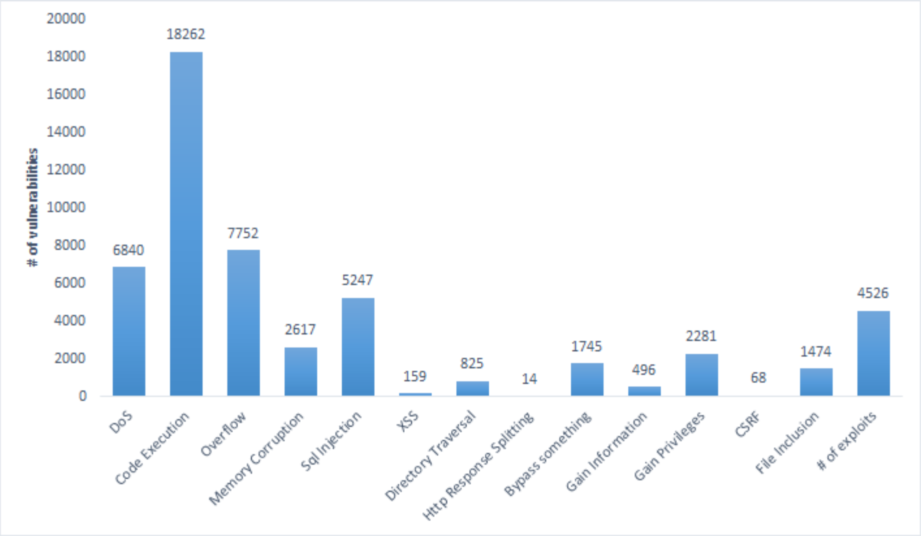


Figure 2.7: Distribution of vulnerability Types in High Category

For the high level CVSS score category Code Executions dominates in numbers. The second most common type is Over flow. Type DOS is the third most frequent type as shown in Table 2.3 below.

From these three tables, we can easily observe that the importance in terms of the frequency in 3 different levels of vulnerabilities are different. However, it is clear that some categories of vulnerabilities are most common than the others in each level. As examples, types Code execution, Overflow and DOS are more frequent in all three categories. Therefore, in this section, it would be important that we look into these major types (in terms of numbers discovered) of vulnerabilities and see if there is a noticeable pattern for discovering them. We would start with a very basic regression models. Such analysis, even though very simple will certainly give us a main picture of number of vulnerabilities in each type to be expected in the future.

Table 2.3: Rank of Vulnerability Types in High Category

Vulnerability Type	Rank	Vulnerability Type	Rank
Code Execution	1	File Inclusion	8
overflow	2	Directory Traversal	9
DOS	3	Gain Infor	10
Sql Injection	4	XSS	11
Memory Corruption	5	CSRF	12
Gain Privileges	6	Http Response Splitting	13
Bypass something	7	-	-

2.5.4 Behavior of DOS Vulnerability Type with Respect to Time

Scatter plot in the Figure 2.8 illustrates the behavior of number of vulnerabilities in type DoS over time. It looks reasonable to assume a linear relation as a function of time. Therefore we propose a linear regression model to predict the behavior of this vulnerability type. The predicted model is given below,

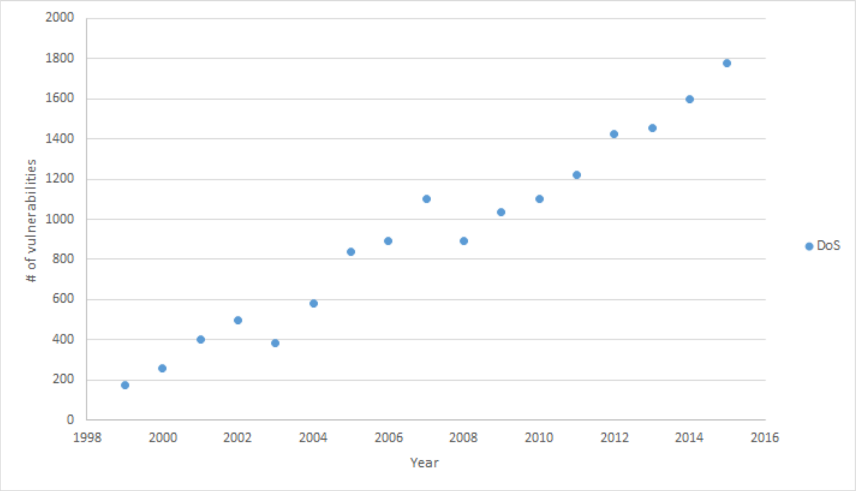


Figure 2.8: Distribution of DOS vulnerability types with time

$$Y(t) = 94.292t - 188323.85. \tag{2.5.1}$$

For this model, the Coefficient of Determination, R^2 is obtained to be 0.96488 and R^2_{adj} is obtained to be 0.96254. Both these results indicates the model accuracy to be higher as they are closer to 1.

2.5.5 Behavior of Memory Corruption Vulnerability Type with Respect to Time

Scatter plot in the Figure 2.9 illustrates the behavior of number of vulnerabilities in type Memory Corruption over time. It looks reasonable to assume a linear relation as a function of time. Therefore we propose a linear regression model to predict the behavior of this vulnerability type. The predicted model is given below,

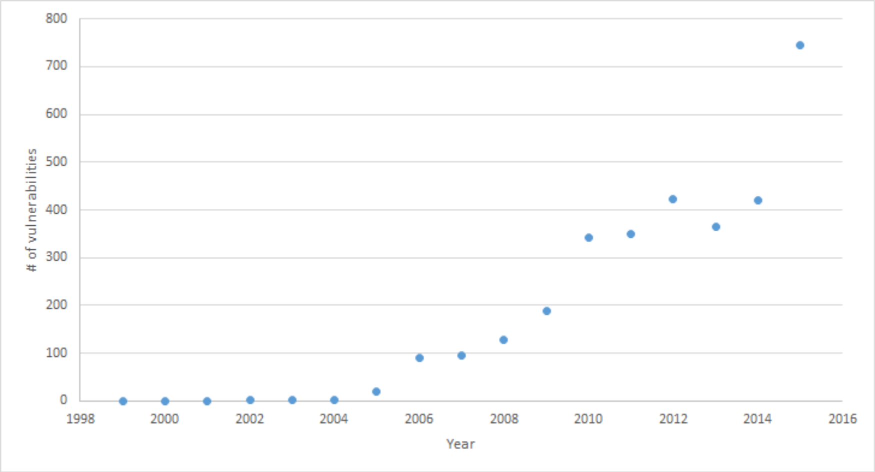


Figure 2.9: Distribution of Memory Corruption vulnerability types with time

$$Y(t) = 52.9945t - 106221.73. \tag{2.5.2}$$

For this model, the Coefficient of Determination, R^2 is obtained to be 0.8835 and R^2_{adj} is obtained to be 0.8729. Since Memory Corruption has rapid increased after 2005 we proposed exponential model to predict the behavior of this vulnerability type and the predicted model is given below,

$$Y(t) = 1.381E^{-05} * e^{(t)} + 29.586t - 110.4284. \tag{2.5.3}$$

For exponential model, the Coefficient of Determination, R^2 is obtained to be 0.9221 and R^2_{adj} is obtained to be 0.9109. Both these results indicates the model accuracy to be higher as they are closer to 1. Since R^2_{adj} for second model is much higher than linear regression model, we proposed exponential model to predict the behavior of Memory Corruption.

2.6 Contribution

In this chapter, we discussed the vulnerability data base in several aspects including the computational methodology used in the calculation of "Common Vulnerability Scoring System (CVSS)[39]. We observed thirteen different vulnerability types in the vulnerability database for over 75000 different software vulnerabilities. In this chapter, we conducted the basic analysis of number of vulnerabilities for the use of following chapters.

To further understand different types of vulnerabilities and their role in the cyber world we observed the data available in the vulnerability data base with respect to different types of vulnerabilities and rank them based on the number of vulnerabilities. Once the number of different types of vulnerabilities are calculated for three different levels of vulnerabilities recorded in each year from 1996 to 2015, we ranked the vulnerability types based on number of vulnerabilities discovered or recorded in each of the three levels. This analysis is important since different types of vulnerabilities have different effects on the security and solutions to be implemented are also different based on these different types.

Finally, taking the ranks based on number of vulnerabilities into consideration, we observed the vulnerability type DOS constitute a major role in all three different levels based on the number of vulnerabilities. Therefore, to have a general predictive model for DOS we have developed a Linear Regression Model with higher degree of the Coefficient of Determination (R^2).

We also developed a non linear predictive model where we observed an exponential type of behavior for the vulnerability type "Memory Corruption" with very good prediction accuracy.

3 A Statistical Predictive Model for the Expected Path Length

3.1 Introduction

Cyber-attacks are the most formidable security challenge faced by most governments and large scale companies. Cyber criminals are increasingly using sophisticated network and social engineering techniques to steal the crucial information which directly affects the operational effects of the Government or Companys objectives. According to the Secunia report 2015 [1] one can see how crucial the volume and magnitude of increasing cybersecurity threaten. Thus, in understanding the performance, availability and reliability of computer networks, measuring techniques plays an important role in the subject area.

Quantitative measures are now commonly used to evaluate the security of computer network systems [35],[36]. These measures help administrators to make important decisions regarding their network security.

In the present study we have first proposed a stochastic model for security evaluation [17] based on vulnerability exploitability scores and attack path behavior. Here, we consider small case scenarios which include three vulnerabilities (high, medium and small) as a base model to understand the behavior of network topology.

We structure the attack graph which includes all possibilities that the attacker reach the goal state and used probabilistic analysis to measure the security of the network [30]. In addition we propose a statistical model that is driven by the mentioned vulnerabilities along with the significant interactions that is highly accurate. This statistical model will allow us to estimate the Expected Path Length and Minimum number of steps to reach the target with probability one. Having these important estimates we can take counter steps and acquire relevant resources to protect the security system from the attacker. In addition, utilizing this model we have identified the significant interaction of the key attributable variables. Also we can rank the attributable variables (vulnerabilities) to identify the percentage of contribution to the response (Expected path length and Minimum number of steps to reach the target) and furthermore one can perform surface response analysis to identify the acceptable values that will minimize the Expected Path Length among others.

3.2 Background and Related Methodologies

The background of the inherent problems with the Markov property [16],[44] that we are going to use in our new model are discussed in the following subsections.

3.2.1 Markov Chain and Transition Probability

A discrete type stochastic process $X = \{X_N, N \geq 0\}$ is called a Markov chain if for any sequence $\{X_0, X_1, ..., X_N\}$ of states, the next state depends only on the current state and not on the sequence of events that preceded it, which is called the Markov property. Mathematically we can write this as follows.

$$P(X_N = j|X_0 = i_0, X_1 = i_1, ..., X_{N-2} = i_{N-2}, X_{N-1} = i) = P(X_N = j|X_{N-1} = i). \tag{3.2.1}$$

We will also make the assumption that the transition probabilities $P(X_N = j|X_0 = i_0, X_1 = i_1, ..., X_{N-2} = i_{N-2}, X_{N-1} = i)$ do not depend on time. This is called time homogeneity. The transition probabilities $(P_{i,j})$ for Markov chain can be defined as follows.

$$(P_{i,j}) = P(X_N = j|X_{N-1} = i). \tag{3.2.2}$$

The transition matrix P of the Markov chain is the NxN matrix whose (i, j) entry $P_{i,j}$ satisfied the following properties.

$$0 \leq P_{i,j} \leq 1, 1 \leq i, j \leq N, \tag{3.2.3}$$

$$\sum_{j=1}^{N} P_{i,j} = 1, 1 \leq i, j \leq N. \tag{3.2.4}$$

Any matrix satisfying the above two equations is the transition matrix for a Markov chain. To simulate a Markov chain, we need its stochastic matrix P and an initial probability distribution π_0.

3.2.2 Transient States

Let P be the transition matrix [16],[4] for Markov chain X_n. A state i is called transient state if with probability 1 the chain visits i only a finite number of times. Let Q be the sub matrix of P which includes only the rows and columns for the transient states. The transition matrix for an absorbing Markov chain [16] has the following canonical form.

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}. \tag{3.2.5}$$

Here P is the transition matrix, Q is the matrix of transient states, R is the matrix of absorbing states and I is the identity matrix.

The matrix P represents the transition probability matrix of the absorbing Markov chain. In an absorbing Markov chain the probability that the chain will be absorbed is always 1. Hence, we have,

$$Q^n \to \infty, n \to \infty.$$

Thus, is it implies that all the eigenvalues of Q have absolute values strictly less than 1. Hence, I-Q is an invertible matrix and there is no problem in defining the matrix

$$M = (I - Q)^{-1} = I + Q + Q^2 + Q^3 + ...$$

This matrix is called the fundamental matrix of P. Let i be a transient state and consider Y_i, the total number of visits to i. Then we can show that the expected number of visits to i starting at j is given by M_{ij}, the (i, j) entry of the matrix M. Therefore, if we want to compute the expected number of steps until the chain enters a recurrent class, assuming starting at state j, we need only sum M_{ij} over all transient states i.

3.3 Cybersecurity Analysis Method

The core component of this method is the attack graph [11],[12],[13]. When we draw an attack graph for a cybersecurity system it has several nodes which represent the vulnerabilities that the system has and the attackers state [11]. We consider that it is possible to go to a goal state starting from any other state in the attack graph. Also an attack graph has at least one absorbing state or goal state. Therefore we will model the attack graph as an absorbing Markov chain.

Absorbing state or goal state is the security node which is exploited by the attacker. When the attacker has reached this goal state, attack path [18] is completed. Thus, the entire attack graph consists of these type of attack paths.

Given the CVSS score for each of the vulnerabilities in the attack Graph [43], we can estimate the transition probabilities of the absorbing Markov chain by normalizing the CVSS scores over all the edges starting from the attackers source state.We define,

p_{ij} = probability that an attacker is currently in state j and exploits a vulnerability in state i.

n = number of outgoing edges from state i in the attack model.

v_j = CVSS score for the vulnerability in state j.

Then formally we can define the transition probability below,

$$P_{ij} = \frac{v_j}{\sum_{k=1}^{n} v_k}. \tag{3.3.6}$$

By using these transition probabilities we can derive the absorbing transition probability matrix P, which follows the properties defined under Markov chain probability method.

3.3.1 Attack Prediction

Under the Attack prediction, we consider two methods to predict the attackers behavior.

Multi Step Attack Prediction

The absorbing transition probability matrix shows the presence of each edge in a network attack graph [29]. This matrix shows every possible single-step attack. In other words, the absorbing transition probability matrix shows attacker reachability within one attack step. We can navigate the absorbing transition probability matrix by iteratively matching rows and columns to follow multiple attack steps, and also raise the absorbing transition probability matrix to higher powers, which shows multi-step attacker reachability at a glance.

For a square $(n \times n)$ adjacency matrix P and a positive integer k, then P_k is P raised to the power k: Since P is an absorbing transition probability matrix with time, this matrix goes to some stationary matrix Π, where the rows of this matrix are identical. That is,

$$\lim_{k \to \infty} P^K = \Pi. \tag{3.3.7}$$

At the goal state column of this matrix Π has ones, so we can find the minimum number of steps that the attacker should try to reach to the goal state with probability 1.

Prediction of Expected Path Length (EPL)

The Expected Path Length (EPL)[3],[26] measures the expected number of steps the attacker will take starting from the initial state to reach the goal state (the attackers objective). As we discussed earlier P has the following canonical form.

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}. \tag{3.3.8}$$

Here, P is the transition matrix, Q is the matrix of transient states, R is the matrix of absorbing states and I is the identity matrix. The matrix P represents the transition probability matrix of the absorbing Markov chain. In an absorbing Markov chain the probability that the chain will be absorbed is always 1. Thus, we have

$$Q^n \to \infty, n \to \infty.$$

Thus, is it implies that all the eigenvalues of Q have absolute values strictly less than 1. Hence, I-Q is an invertible matrix and there is no problem in defining the matrix

$$M = (I - Q)^{-1} = I + Q + Q^2 + Q^3 + ...$$

Using this fundamental matrix M of the absorbing Markov chain we can compute the expected total number of steps to reach the goal state until absorption.

Taking the summation of first row elements of matrix M gives the expected total number of steps to reach the goal state until absorption and the probability value relates to the goal state gives the expected number of visits to that state before absorption.

3.3.2 Illustration: The Attacker

To illustrate the proposed approach model that we discussed in section 3.3.1, we considered a Network Topology given in Figure 3.1, below. The network consists of two service hosts IP 1, IP 2 and an attackers workstation, Attacker connecting to each of the servers via a central router.

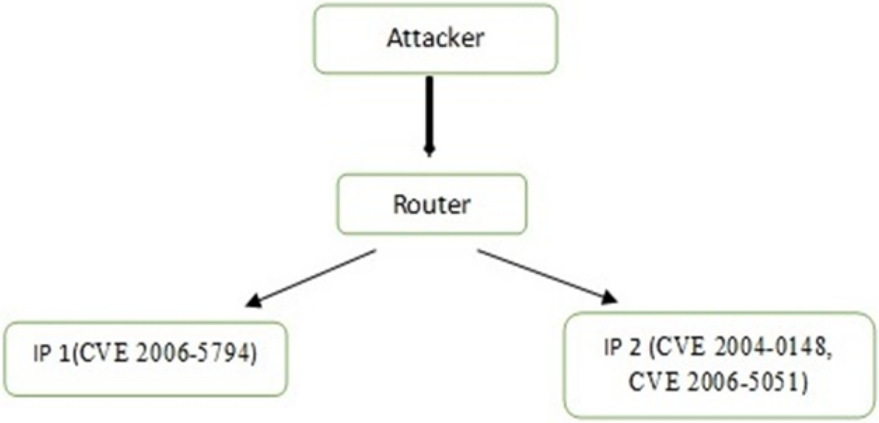


Figure 3.1: Network Topology

In the server IP 1 the vulnerability is labeled as CVE 2006-5794 and lets consider this as V_1. In the server IP 2 there are two recognized vulnerabilities, which are labeled CVE 2004-0148 and CVE 2006-5051. Lets consider this as V_2 and V_3, respectively. We proceed to use the CVSS score of the above vulnerabilities. And the exploitability score ($e(v)$ in Figure 3.1) of each vulnerabilities as given in Table 3.1.

Table 3.1: Vulnerability Scores

Vulnerability	Exploitability score
V_1(CVE 2006-5794)	6
V_2(CVE 2006-5051)	5
V_3 (CVE 2004-0148)	1

Host Centric Attack graph

The host centric attack graph [25],[34] is shown by Figure 3.2, below. Here, we consider that the attacker can reach the goal state only by exploiting V_2 vulnerability. The graph shows all the possible paths that the attacker can follow to reach the goal state.

Note that IP1,1 state represents V_1 vulnerability and IP2,1 and IP2,2 states represent V_2 and V_3 vulnerabilities, respectively. Also, the notation 10 represents the maximum vulnerability score and this provides attacker the maximum chance to exploit this state. Attacker can reach each state by exploiting the relevant Vulnerability.

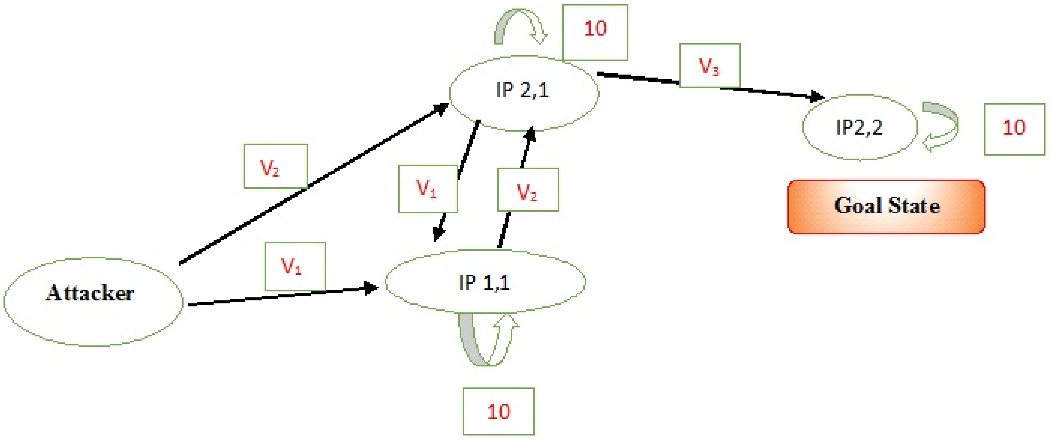


Figure 3.2: Host Centric Attack Graph

3.3.3 Adjacency Matrix for the Attack Graph

Let s_1, s_2, s_3, s_4, represent the attack states for Attacker, (IP1,1), (IP2,1) and (IP2,2), respectively. To find the weighted value of exploiting each vulnerability from one state to another state, we divide the vulnerability score by summation of all out going vulnerability values from that state. For our attack graph the weighted value of exploiting each vulnerability is given below.

1st row probabilities:

Weighted value of exploiting V_1 from s_1 to s_2 is $V_1/(V_1+V_2)$

Weighted value of exploiting V_2 from s_1 to s_3 is $V_2/(V_1+V_2)$

2nd row probabilities:

Weighted value of exploiting V_2 from s_2 to s_3 is $V_2/(10+V_2)$

3rd row probabilities:

Weighted value of exploiting V_1 from s_3 to s_2 is $V_1/(V_1+V_3+10)$

Weighted value of exploiting V_3 from s_3 to s_4 is $V_3/(V_1+V_3+10)$

4th row probabilities:

Weighted value of exploiting V_3 from s_4 to s_4 is 1.

For the Host Centric Attack graph we can have the Adjacency Matrix as follows.

$$A = \begin{bmatrix} \frac{0}{(V_1+V_2)} & \frac{V_1}{(V_1+V_2)} & \frac{V_2}{(V_1+V_2)} & \frac{0}{(V_1+V_2)} \\ \frac{0}{(10+V_2)} & \frac{10}{(10+V_2)} & \frac{V_2}{(10+V_2)} & \frac{0}{(10+V_2)} \\ \frac{0}{(V_1+V_3+10)} & \frac{V_1}{(V_1+V_3+10)} & \frac{10}{(V_1+V_3+10)} & \frac{V_3}{(V_1+V_3+10)} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.3.9}$$

Utilizing the information given in Table 3.1, the matrix A is given by

$$A = \begin{bmatrix} 0 & 0.5455 & 0.4545 & 0 \\ 0 & 0.6667 & 0.3333 & 0 \\ 0 & 0.3529 & 0.5882 & 0.0588 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (3.3.10)$$

Here, 0.5455 is the probability that attacker exploit V_1 vulnerability in first step, from s_1 to s_2. We can explain 0.0588 as the probability that once in state IP2,1 can exploit V_3 vulnerability and reach to IP2,2 in first attempt. Similarly each probability represent the chance to exploit relevant vulnerability from one state in the first attempt.

We want to use this matrix to answer the important question in cyber security analysis. We want to find the minimum number of steps to reach the goal state (final destination) with probability one and the expected path length metric.

3.4 Finding Stationary Distribution and Minimum Number of Steps

By using the above matrix A, we can find the probabilities with two, three and several attempt by the attacker to reach the goal state using A^2, A^3, A^4,..., A^p matrices. From these matrices we can find all possible probabilities from one state to another that the attacker can reach by two steps A^2, three steps A^3 and four steps A^4 and up to p steps A^p respectively. We continuous this process until we reach the absorbing matrix and that p value gives the minimum number of steps that the attacker is required to reach the goal state with probability one.

We proceed by changing the CVSS score and calculate for each combination of V_1, V_2 and V_3 the minimum number of steps that the attacker will reach the goal state with probability one. These calculations are given in Table 3.4.

For example, it will take minimum 68 steps with vulnerability configuration of $V_1 = 10$, $V_2 = 9$, $V_3 = 8$ for the attacker to reach the final goal with probability one. The largest number of steps for the attacker to achieve his goal is 844 steps by using the vulnerabilities, $V_1 = 10$, $V_2 = 2$ and $V_3 = 1$, with probability one.

3.5 Expected Path Length (EPL) Analysis

As described under Section 3.3.1 we measure the expected number of steps the attacker will take starting from the initial state to compromise the security goal. In Table 3.5, we present the calculations of the Expected Path Length of the attacker for various combinations of the vulnerabilities V_1, V_2 and V_3.

For example, it will take 8.25 EPL with vulnerability configuration of $V_1 = 10$, $V_2 = 9$, $V_3 = 8$ for the attacker to compromise the security goal. The largest Expected Path Length of the attacker is 72.8 using $V_1 = 8$, $V_2 = 2$ and $V_3 = 1$.

3.6 Development of the Statistical Models

The primary objective here is to utilize the information that we have calculated to develop a statistical model [32],[33], [46] to predict the minimum number of steps to reach the stationary matrix and EPL of the attacker. We used the application software package R [41] for required calculations in developing these models.

3.6.1 Developing a Statistical Model to Predict the Minimum Number of Steps

By using the information in Table 3.4, we developed a statistical model that estimates the minimum number of steps the attacker takes to reach the goal state with probability one.

Table 3.2: Parametric Model: R^2 and adjusted R^2 values

Model	R^2	AjustedR^2
$Y1 = 344.167 + 35.284V_1 - 34.115V_2 - 67.803V_3$	0.7244	0.7173
$Y2 = 446.865 + 67.645V_1 - 81.662V_2 - 149.982V_3$ $-1.24V_1V_2 - 13.7V_1V_3 + 29.354V_2V_3$	0.8835	0.8773
$Y3 = 689.84 + 51.177V_1 - 138.815V_2 - 328.093V_3$ $-0.3626V_1V_2 + 9.29V_1V_3 + 39.114V_2V_3$ $-0.084V_1^2 + 8.479V_2^2 + 17.96V_3^2 3.47V_1V_2V_3$	0.9428	0.9376

The quality of the model is measured by R^2 and adjusted R^2 values as defined below. The first model in Table 3.2 does not include interactions of the three Vulnerabilities, V_1, V_2 and V_3 and R^2 and R^2_{adj} reflect its quality of 0.7244 and 0.7173. The second model shows that there is a significant binary interaction of the each factors and the statistical model shows a significant improvement with R^2 and R^2_{adj} of 0.8835 and 0.8773 respectively. However, the best statistical model is obtained when we consider in addition to individual contributions of V_1, V_2 and V_3, two way and three way significant interactions. Thus, from the above table the third model with R^2 and R^2_{adj} of 0.9428 and 0.9376 respectively attest to the fact that this statistical model is excellent in estimating the minimum number of steps that an attacker will need to achieve his goal.

Not only an individual could do the predictions but also he could perform surface response analysis. For example using the third model $Y3$ one could maximize the number of steps for an attacker to reach his goal state by taking defending steps to ensure to maintain their systems with the minimum vulnerability scores. Similarly one could estimates confidence intervals for vulnerability scores so that the necessary steps can be taken to maintain the expected security level.

3.6.2 Developing a Statistical Model to Predict the Minimum Number of Steps

By using Table 3.5 results we developed a model to find the Expected Path Length [46] that the attacker will take starting from the initial state to reach the security goal. To utilize the quality of the model we use R^2 concept and by comparing the values in Table 3.3, the third model gives the highest $R^2 = 0.943$ and $R^2_{adj} = 0.9378$ value. Therefore we can conclude that the third model gives the best prediction of EPL. Not only an individual could predict the EPL but also he could perform surface response analysis. For example using the third model "$Y3$" one could max-

Table 3.3: Parametric model (EPL): R^2 and R^2_{adj} values

Model	R^2	R^2_{adj}
$Y1 = 35.975 + 3.622V_1 3.497V_2 6.845V_3$	0.7253	0.7181
$Y2 = 46.301 + 6.904V_1 - 8.28V_2 15.178V_3$		
$-0.128V_1V_2 - 1.384V_1V_3 + 2.97V_2V_3$	0.8839	0.8778
$Y3 = 70.62 + 5.338V_1 14.108V_2 33.144V_3$		
$-0.041V_1V_2 + 0.942V_1V_3 + +3.943V_2V_3$		
$-0.015V_1^2 + 0.864V_2^2 + 1.814V_3^2 0.35V_1V_2V_3$	0.943	0.9378

imize the EPL for an attacker to reach his goal state by taking defending steps to ensure to maintain their systems with the minimum vulnerability scores. Similarly one could estimates confidence intervals for vulnerability scores so that the necessary steps can be taken to maintain the expected security level.

We proceed by changing the CVSS score and calculate for each combination of V_1, V_2 and V_3 the minimum number of steps that the attacker will reach the goal state with probability one. These calculations are given in Table 3.4.

For example, it will take minimum 68 steps with vulnerability configuration of $V_1 = 10$, $V_2 = 9$, $V_3 = 8$ for the attacker to reach the final goal with probability one. The largest number of steps for the attacker to achieve his goal is 844 steps by using the vulnerabilities, $V_1 = 10$, $V_2 = 2$ and $V_3 = 1$, with probability one.

In Table 3.5, we present the calculations of the Expected Path Length of the attacker for various combinations of the vulnerabilities V_1, V_2 and V_3.

For example, it will take 8.25 EPL with vulnerability configuration of $V_1 = 10$, $V_2 = 9$, $V_3 = 8$ for the attacker to compromise the security goal. The largest Expected Path Length of the attacker is 72.8 using $V_1 = 8$, $V_2 = 2$ and $V_3 = 1$.

3.7 Contribution

We have developed a very accurate statistical model that can be utilized to predict the minimum steps to reach the goal state and predict the expected path length. This developed model can be used to identify the interaction among the vulnerabilities and individual variables that drive the EPL.

We ranked the attributable variables and their contribution in estimating the subject length. By using these rankings security administrators can have a better knowledge about priorities. This will help them to take the necessary actions regarding their security system.

Here we develop a model for three vulnerabilities and we can expand this model to any Large Network System. Thus, the proposed methods will assist in making appropriate security decisions in advance.

Table 3.4: Number of Steps for Absorbing Matrix

Steps	V_1	V_2	V_3	Steps	V_1	V_2	V_3	Steps	V_1	V_2	V_3	Steps	V_1	V_2	V_3
68	**10**	**9**	**8**	407	9	8	1	92	7	6	5	75	10	9	7
87	9	7	6	109	7	6	4	85	10	9	6	100	9	7	5
138	7	6	3	99	10	9	5	121	9	7	4	197	7	6	2
119	10	9	4	154	9	7	3	374	7	6	1	153	10	9	3
222	9	7	2	118	7	5	4	221	10	9	2	424	9	7	1
149	7	5	3	424	10	9	1	107	9	6	5	212	7	5	2
78	107	8	7	128	9	6	4	400	7	5	1	88	10	8	6
163	9	6	3	165	7	4	3	102	10	8	5	234	9	6	2
233	7	4	2	124	10	8	4	447	9	6	1	439	7	4	1
159	10	8	3	138	9	5	4	269	7	3	2	229	10	8	2
176	9	5	3	504	7	3	1	439	10	8	1	252	9	5	2
634	7	2	1	93	10	7	6	480	9	5	1	107	6	5	4
107	10	7	5	195	9	4	3	135	6	5	3	129	10	7	4
279	9	4	2	191	6	5	2	166	10	7	3	529	9	4	1
359	6	5	1	239	10	7	2	323	9	3	2	149	6	4	3
458	10	7	1	610	9	3	1	210	6	4	2	114	10	6	5
774	9	2	1	393	6	4	1	137	10	6	4	81	8	7	6
242	6	3	2	176	10	6	3	93	8	7	5	450	6	3	1
253	10	6	2	112	8	7	4	564	6	2	1	484	10	6	1
143	8	7	3	134	5	4	3	148	10	5	4	205	8	7	2
187	5	4	2	190	10	5	3	390	8	7	1	348	5	4	1
272	10	5	2	99	8	6	5	215	5	3	2	520	10	5	1
119	8	6	4	396	5	3	1	211	10	4	3	151	8	6	3
493	5	2	1	301	10	4	2	216	8	6	2	187	4	3	2
574	10	4	1	411	8	6	1	342	4	3	1	350	10	3	2
128	8	5	4	423	4	2	1	664	10	3	1	163	8	5	3
351	3	2	1	**844**	**10**	**2**	**1**	232	8	5	2	74	9	8	7
440	8	5	1	83	9	8	6	180	8	4	3	96	9	8	5
256	8	4	2	115	9	8	4	484	8	4	1	148	9	8	3
296	8	3	2	212	9	8	2	557	8	3	1	-	-	-	-

Table 3.5: Expected Path Length for several Vulnerabilities.

EPL	V_1	V_2	V_3	EPL	V_1	V_2	V_3	EPL	V_1	V_2	V_3	EPL	V_1	V_2	V_3
8.25	**10**	**9**	**8**	34.25	9	3	2	8.98	10	9	7	63.25	9	3	1
9.96	10	9	6	79.91	9	2	1	11.33	10	9	5	9.53	8	7	6
13.39	10	9	4	10.78	8	7	5	16.81	10	9	3	12.65	8	7	4
23.67	10	9	2	15.77	8	7	3	44.22	10	9	1	22.01	8	7	2
9.32	10	8	7	40.72	8	7	1	10.33	10	8	6	11.39	8	6	5
11.75	10	8	5	13.36	8	6	4	13.87	10	8	4	16.64	8	6	3
17.42	10	8	3	23.19	8	6	2	24.5	10	8	2	42.86	8	6	1
45.75	10	8	1	14.35	8	5	4	10.81	10	7	6	17.85	8	5	3
12.29	10	7	5	24.85	8	5	2	14.5	10	7	4	45.85	8	5	1
18.19	10	7	3	19.67	8	4	3	25.57	10	7	2	27.33	8	4	2
47.71	10	7	1	50.33	8	4	1	13	10	6	5	31.48	8	3	2
15.33	10	6	4	57.82	8	3	1	19.22	10	6	3	**72.8**	**8**	**2**	**1**
27	10	6	2	10.57	7	6	5	50.33	10	6	1	12.35	7	6	4
16.5	10	5	4	15.32	7	6	3	20.67	10	5	3	21.27	7	6	2
29	10	5	2	39.1	7	6	1	54	10	5	1	13.25	7	5	4
22.83	10	4	3	16.42	7	5	3	32	10	4	2	22.75	7	5	2
59.5	10	4	1	41.75	7	5	1	37	10	3	2	18.06	7	4	3
68.67	10	3	1	24.98	7	4	2	87	10	2	1	45.73	7	4	1
8.798	9	8	7	28.7	7	3	2	9.73	9	8	6	52.37	7	3	1
11.04	9	8	5	65.67	7	2	1	13	9	8	4	12.14	6	5	4
16.27	9	8	3	14.97	6	5	3	22.82	9	8	2	20.64	6	5	2
42.44	9	8	1	37.64	6	5	1	10.18	9	7	6	16.43	6	4	3
11.54	9	7	5	22.6	6	4	2	13.58	9	7	4	41.1	6	4	1
16.99	9	7	3	25.89	6	3	2	23.79	9	7	2	46.89	6	3	1
44.22	9	7	1	58.5	6	2	1	12.2	9	6	5	14.78	5	4	3
14.35	9	6	4	20.19	5	4	2	17.93	9	6	3	36.44	5	4	1
25.1	9	6	2	23.04	5	3	2	46.6	9	6	1	41.38	5	3	1
15.43	9	5	4	51.29	5	2	1	19.26	9	5	3	20.14	4	3	2
26.93	9	5	2	35.81	4	3	1	49.93	9	5	1	44	4	2	1
21.26	9	4	3	36.6	3	2	1	29.67	9	4	2	54.92	9	4	1

4 NonHomogeneous Stochastic Model for Cyber Security Predictions

4.1 Introduction

In this study, we continue our research efforts in integrating Mathematical and Statistical theories into better understanding the complex behavior of computer network systems in the perspective of Cybersecurity. Thus, we propose a new method to estimate the EPL as a function of time t. The EPL is a major factor in determining the risk level of a given computer system and lesser the EPL, the network system is more vulnerable and probable to be exploited.

In our recent studies [20],[21] and [46] we introduced several stochastic models to better understand the behavior of vulnerabilities, network systems with respect to cybersecurity. Initially, we introduced a stochastic model that can estimate the Expected Path Length of a system with any three vulnerabilities and two machines. Then, we introduced a new approach of estimating the probability of a given vulnerability being exploited at a time t, using Markovian approach with respect to the Vulnerability life cycle. We have further introduced a set of three stochastic time dependent models for each categories of vulnerabilities with Low, Medium and High exploitability scores that can estimate the probability of a given vulnerability getting exploited without going through the Markovian process each time. Additionally, the concept of Risk Factor that we introduced and its analytical formulation allowed us to present a more sophisticated way of estimating the risk associated with a specific vulnerability of a computer network system. In the present study, we introduce a Non Homogeneous Stochastic Model that allows the computer system administrators to predict the time, that the system is most vulnerable for an attack in terms of the EPL. This estimate is based on the assumption that a system is more susceptible to be exploited when the EPL is at a minimum at a particular time t. In developing this model we have used a network system of two IPs with three vulnerabilities as a base model.

With the introduction of this new approach we will be re-defending the capability to estimate the probability of getting exploited as a function of time for a computer network system with given set of vulnerabilities. Even though we have already developed a successful statistical model to find the EPL of a possible attack, it is more important to estimate the EPL as a function of time. Thus, for a system with a given set of vulnerabilities, estimating of most probable exploit times can be modelled on the logical assumption that a system is more susceptible to be exploited at a time where the Expected Path Length (number of steps that an attacker needs to pass before achieving the goal state) is at its minimum.

4.2 Background and Related Methodologies

4.2.1 Cybersecurity Analysis Method

The core component of this method is the attack graph [11]. When we draw an attack graph for a cybersecurity system it has several nodes which represent the vulnerabilities that the system has and the attackers state [12]. We consider that it is possible to go to a goal state starting from any other state in the attack graph. Also an attack graph has at least one absorbing state or goal state. Therefore we will model the attack graph as an absorbing Markov chain [12].

Absorbing state or goal state is the security node which is exploited by the attacker. When the attacker has reached this goal state, attack path is completed. Thus, the entire attack graph consists of these type of attack paths.

Given the CVSS score for each of the vulnerabilities in the attack Graph, we can estimate the transition probabilities of the absorbing Markov chain by normalizing the CVSS scores over all the edges starting from the attackers source state.We define,

p_{ij} = probability that an attacker is currently in state j and exploits a vulnerability in state i.

n = number of outgoing edges from state i in the attack model.

v_j = CVSS score for the vulnerability in state j.

Then formally we can define the transition probability below,

$$P_{ij} = \frac{v_j}{\sum_{k=1}^{n} v_k}. \tag{4.2.1}$$

By using these transition probabilities we can derive the absorbing transition probability matrix P, which follows the properties defined under Markov chain probability method.

4.2.2 Risk Factor Model

p_{ij} , the transition probabilities for each state in an attack graph, that is, it represents the risk of a particular state (with a given vulnerability) of being exploited. Therefore, it is logical to consider it the same as a risk variable. In our previous studies we have introduced a more convenient and powerful tool named Risk Factor [21] that can estimate the risk associated with a particular state of a given vulnerability.

It is important that, when we consider a given vulnerability its exploitability factor should vary with time. But the exploitability factor calculated under the CVSS score is discrete and is not suitable for inclusion in a non-homogenous model. However, our Risk Factor model is based on the Vulnerability Life Cycle and it is time dependent. This allows us to develop a non-homogeneous model which is our objective in this study. Therefore, in this study we will extend the Transition Probability Matrix Model, replacing vulnerability with the CVSS score, v by its Risk Factor r.

The probability of an exploitation for a given vulnerability can be obtained using the three stochastic models given in Table 4.1 below. These time dependent stochastic models were developed in our previous study [20],[21], and we used the general classification of vulnerability risks based on the CVSS score identified as Low, Medium and High. Details of the process and methodology in developing the subject models along with their validation accuracy are given in our previous study [21].

Table 4.1: Model Equations of Risk Factors for three different categories of vulnerabilities.

Category	Model Equation	R^2	R^2_{adj}
Low (0-4)	$Y(t) = 0.1354410.308532(1/t) - 0.002030 \ln(\ln t)$	0.9576	0.9566
Medium (4-7)	$Y(t) = 0.169518 - 0.356821(1/t) - 0.007011 \ln(\ln t)$	0.962	0.961
High (7-10)	$Y(t) = 0.1917010.383521(1/t) - 0.00358 \ln(\ln t)$	0.9588	0.9577

In each of the equations, t is the age of vulnerability and is calculated by taking the difference between the dates that the vulnerability was first discovered and the attacking attempt started. Thus, for a given vulnerability and time we can obtain the probability of being exploited. We can now define the transition probability as follows.

$$P_{ij} = \frac{R(v_j(t))}{\sum_{k=1}^{n} R(v_k(t))}, \tag{4.2.2}$$

where

$R(v_j(t))$ = Risk Factor of a given vulnerability in state j at time t,

$e(v_j)$ = Exploitability sub score that is related to the CVSS score for the given vulnerability in state j,

$$R(v_j(t)) = Y(t) * e(v_j). \tag{4.2.3}$$

Above equation gives the analytic form of the risk factor as a function of $Y(t)$ and $e(v_j)$ where Y(t) is the exploitability probability factor as a function of time and $e(v_j)$ is the exploitability score taken from the CVSS.

4.3 Attack Prediction

Under the Attack Prediction, we consider two methods to predict the attackers behavior.

4.3.1 Multi Step Attack Prediction

The absorbing transition probability matrix shows the presence of each edge in a network attack graph. This matrix shows every possible single-step attack. In other words, the absorbing transition probability matrix shows attacker reachability within one attack step. We can navigate the absorbing transition probability matrix by iteratively matching rows and columns to follow multiple attack steps, and also raise the absorbing transition probability matrix to higher powers, which shows multi-step attacker reachability at a glance.

For a square $(n \times n)$ adjacency matrix P and a positive integer k, then P_k is P raised to the power k: Since P is an absorbing transition probability matrix with time, this matrix goes to some stationary matrix Π, where the rows of this matrix are identical. That is,

$$\lim_{k\to\infty} P^K = \Pi. \tag{4.3.4}$$

At the goal state column of this matrix Π has ones, so we can find the minimum number of steps that the attacker should try to reach to the goal state with probability 1.

4.3.2 Prediction of Expected Path Length (EPL)

The Expected Path Length (EPL) measures the expected number of steps the attacker will take starting from the initial state to reach the goal state (the attackers objective). As we discussed earlier P has the following canonical form.

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}. \tag{4.3.5}$$

Here, P is the transition matrix, Q is the matrix of transient states, R is the matrix of absorbing states and I is the identity matrix. The matrix P represents the transition probability matrix of the absorbing Markov chain. In an absorbing Markov chain the probability that the chain will be absorbed is always 1. Thus, we have

$$Q^n \to \infty, n \to \infty.$$

Thus, is it implies that all the eigenvalues of Q have absolute values strictly less than 1. Hence, I-Q is an invertible matrix and there is no problem in defining the matrix

$$M = (I - Q)^{-1} = I + Q + Q^2 + Q^3 + ...$$

Using this fundamental matrix M of the absorbing Markov chain we can compute the expected total number of steps to reach the goal state until absorption.

Taking the summation of the first row elements of matrix M gives us the expected total number of steps to reach the goal state which is defined as the Expected Path Length. Given below is an application that illustrate a computer network system of our proposed analytic process to estimate the EPL of a hacker.

4.4 Attack Graph and Attack Risk Evaluation

In this section we present an example illustrating the application of the usefulness of our method. We combine the application of methodology with an attack graph relevant to a typical network exemplified with three different recorded vulnerabilities.

4.4.1 Application: The Attacker

To illustrate the proposed analytical approach model that we have developed as discussed above, we considered a Network Topology [4],[37], given by Figure 4.1, below.

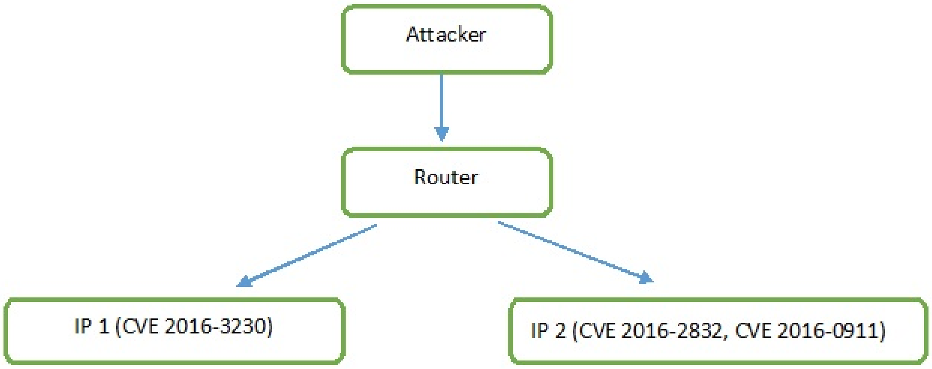


Figure 4.1: Network Topology

The computer network consists of two service hosts IP 1, IP 2 and an attackers workstation, Attacker connecting to each of the servers via a central router.

In the server IP 1 the vulnerability is labeled as CVE 2016-3230 and shall denote as V_1. In the server IP 2 there are two recognized vulnerabilities, which are labeled CVE 2016-2832 and CVE 2016-0911. Lets denote them as V_2 and V_3, respectively.

We proceed to use the CVSS score of the above vulnerabilities in our analysis. The exploitability score (e (v) in Figure 4.1) of each vulnerabilities as given in Table 4.2, below.

Published date is in general considered as the date that a vulnerability is made known to the public. CVSS score is the score given to the vulnerability based on exploitability factors by the Forum of Incident Response and Security Teams, (FIRST). Calculation of this score is established and updated time to time and the relevant details are available in the CVE detail and other relevant official websites.

Calculating Risk factor

For example, lets consider the vulnerability V1 (CVE 2016-3230). The CVSS score has given the exploitability score for this vulnerability as 8. Taking the difference between the published date (June 15th) and the attack date (June 24th), the age of this vulnerability is calculated as 9 days. Since this is a vulnerability of the category High, we can now use our model given in the Table 4.1 and calculate the Risk Factor as follows.

$$R(v_j(t)) = Y(t) \times e(v_j),$$

$$R(v_1(t)) = [0.1917010.383521(1/t) - 0.00358 \ln(\ln(t))] \times 8,$$

and

$$R(v_1(9)) = 1.702.$$

Similarly, Risk factors for two other vulnerabilities are also calculated and presented in the Table 4.2 below.

Table 4.2: Vulnerability Scores.

Vulnerability	Published date	CVSS score	$e(v_j)$	(t_j)	$R(v_j(t))$
V_1(CVE 2016-3230)	6/15/2016	9 (High)	8	9	1.702
V_2(CVE 2016-2832)	6/13/2016	4.3 (medium)	2.8	11	0.3667
V_3(CVE 2016-0911)	6/19/2016	1.9 (Low)	3.4	5	0.2474

4.4.2 Host Centric Attack graph

The host centric attack graph is shown by Figure 4.2, below. Here, we consider that the attacker can reach the goal state only by exploiting V_3 vulnerability. The graph shows all the possible paths that the attacker can follow to reach the goal state.

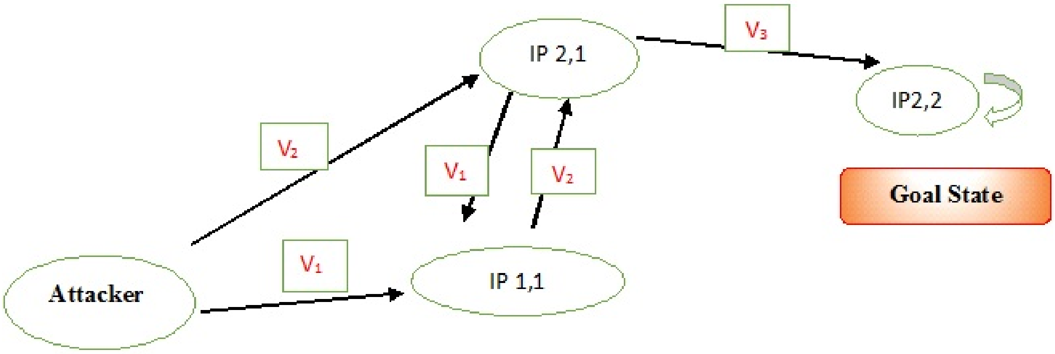


Figure 4.2: Host Centric Attack Graph

Note that IP1,1 state represents V_1 vulnerability and IP2,1 and IP2,2 states represent vulnerabilities V_2 and V_3 respectively. Attacker can reach each state by exploiting the relevant Vulnerability.

4.5 Adjacency Matrix for the Attack Graph

Let s_1, s_2, s_3, s_4, represent the attack states for Attacker, (IP1,1), (IP2,1) and (IP2,2), respectively. To find the weighted value of exploiting each vulnerability from one state to another state, we divide the vulnerability score by summation of all out going vulnerability values from that state. For our attack graph the weighted value of exploiting each vulnerability is given below.

1^{st} row probabilities:

Weighted value of exploiting V_1 from s_1 to s_2 is $R_1/(R_1 + R_2)$

Weighted value of exploiting V_2 from s_1 to s_3 is $R_2/(R_1 + R_2)$

2^{nd} row probabilities:

Weighted value of exploiting V_2 from s_2 to s_3 is $R_2/(R_2)$

3^{rd} row probabilities:

Weighted value of exploiting V_1 from s_3 to s_2 is $R_1/(R_1 + R_3)$

Weighted value of exploiting V_3 from s_3 to s_4 is $R_3/(R_1 + R_3)$

4^{th} row probabilities:

Weighted value of exploiting V_3 from s_4 to s_4 is 1.

For the Host Centric Attack graph we can have the Adjacency Matrix as follows.

$$A = \begin{bmatrix} \frac{0}{(R_1+R_2)} & \frac{R_1}{(R_1+R_2)} & \frac{R_2}{(R_1+R_2)} & \frac{0}{(R_1+R_2)} \\ \frac{0}{(R_2)} & \frac{0}{(R_2)} & \frac{R_2}{(R_2)} & \frac{0}{(R_2)} \\ \frac{0}{(R_1+R_3)} & \frac{R_1}{(R_1+R_3)} & \frac{10}{(R_1+R_3)} & \frac{R_3}{(R_1+R_3)} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{4.5.6}$$

Applying the information given in Table 4.2, the matrix A can be obtained as follows.

$$A = \begin{bmatrix} 0 & 0.7614 & 0.2386 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.8255 & 0 & 0.1745 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{4.5.7}$$

Here, 0.7614 is the probability that attacker exploits vulnerability V_1 in the first step, the step from s_1 to s_2. Similarly, we can explain 0.1745 as the probability that attacker exploits the vulnerability V_3 in the step s_2 to s_3 in his first attempt. Similarly, each probability represents the likelihood to exploit relevant vulnerability from one state to another state in the first attempt.

We can use this matrix to answer several important questions in cyber security analysis. First, using the Adjacency Matrix we expect to find the Expected Path Length. Then, we can analyze the behavior of Expected Path Length over the time. To calculate the EPL over the time we follow the steps given below.

Step 01:

Calculate the Risk Factor of each vulnerability on the date of the first attack assumed (June 24th in our example). That is, calculate the age of each vulnerability by taking the difference between the published date and the June 24th. And, substitute this value of t in relevant model equation given in the Table 4.1.

Step 02:

Using those Risk Factors, develop the transition matrix A and calculate the EPL.

Step 03:

Repeat the same process for all the following dates that we need to calculate the Expected Path Length.

4.5.1 Expected Path length

The Table 4.3 below show us the EPL for this computer system for 100 days starting from 24th of June.

From the Table 4.3 above, we can identify that the number of days a hacker will take to reach his goal of exploitability for the given computer network system we have structured.

For example, lets consider 20^{th} day. Under step 1, we calculate Risk factors for V_1, V_2 and V_3. For 20th day age of three vulnerabilities V_1, V_2 and V_3 are, $t_1 = 9 + 20$, $t_2 = 11 + 20$ and $t_3 = 5 + 20$ respectively. Then, by substituting these ages in the respective model equation from the Table 4.1 and multiplying the answers by respective exploitability score, we calculate three risk factors.

V_1 is a vulnerability of category High. Therefore we use the 3rd model equation from Table 4.3 and obtain the Risk factor.

Substituting $t_1 = 29$ to the Risk factor model we obtain, $R_1 = 1.393$. Similarly for V_2 and V_3 we obtain following Risk factors calculated using relevant model equations. For V_2 substitute $t_2 = 31$ and obtained $R_2 = 0.4182$ and similarly for V_3, $t_3 = 25$ and $R_3 = 0.4105$.

Table 4.3: Expected Path Length relative to number of days after first attack

Days	EPL	Days	EPL	Days	EPL	Days	EPL
1	12.2205398	26	9.5175367	51	9.4453151	76	9.4239414
2	11.3052188	27	9.5116554	52	9.4440137	77	9.4234008
3	10.7998722	28	9.5062477	53	9.4427673	78	9.4228753
4	10.4850373	29	9.5012611	54	9.4415727	79	9.4223643
5	10.2729754	30	9.4966504	55	9.4404267	80	9.4218672
6	10.1220591	31	9.4923761	56	9.4393265	81	9.4213834
7	10.0101501	32	9.4884044	57	9.4382695	82	9.4209123
8	9.9244658	33	9.4847053	58	9.4372532	83	9.4204536
9	9.8571518	34	9.4812528	59	9.4362752	84	9.4200067
10	9.8031388	35	9.478024	60	9.4353336	85	9.4195711
11	9.7590231	36	9.4749985	61	9.4344263	86	9.4191465
12	9.7224429	37	9.4721584	62	9.4335516	87	9.4187324
13	9.6917134	38	9.4694878	63	9.4327076	88	9.4183284
14	9.6656039	39	9.4669724	64	9.4318929	89	9.4179342
15	9.643197	40	9.4645994	65	9.431106	90	9.4175493
16	9.6237964	41	9.4623576	66	9.4303454	91	9.4171736
17	9.606865	42	9.4602366	67	9.4296099	92	9.4168066
18	9.5919829	43	9.4582272	68	9.4288983	93	9.416448
19	9.5788176	44	9.4563211	69	9.4282094	94	9.4160975
20	**9.5671025**	45	9.4545107	70	9.4275421	95	9.415755
21	9.5566222	46	9.4527892	71	9.4268956	96	9.41542
22	9.5472004	47	9.4511503	72	9.4262687	97	9.4150924
23	9.5386921	48	9.4495884	73	9.4256606	98	9.4147719
24	9.5309766	49	9.4480984	74	9.4250706	99	9.4144583
25	9.5239531	50	9.4466754	75	9.4244978	100	9.4141513

Once we have calculated the Risk Factors for all the vulnerabilities in the network system, the second step is to develop the Transition Matrix A as given in the Equation 4.5.7.

Step 3 is to calculate the EPL. Applying the methodology we explained in the section 4.3 we can calculate the EPL using the transition matrix A by obtaining the matrix M. The sum of the first row of matrix M is the EPL of this computer network system at the 20th day (from June 24th) from the first attack attempt assumed. We have obtained, EPL=9.567 for 20th days after the first attack created as given in the

Table 4.3.

Behavior of Expected Path Length over the time

Figure 4.3 below illustrates the results shown in the Table 4.3, graphically.

By examining the distribution of Expected Path Length of the attacker over 100 days, it will take fewer steps for an attacker to compromise the security goal as the age of vulnerabilities increases. Security practitioners in a typical organization can establish a threshold score for the system and the security teams can planned in advance and mark the critical points to establish a strategy to defend the security of the computer system and introduce relevant patches before such critical stages approach.

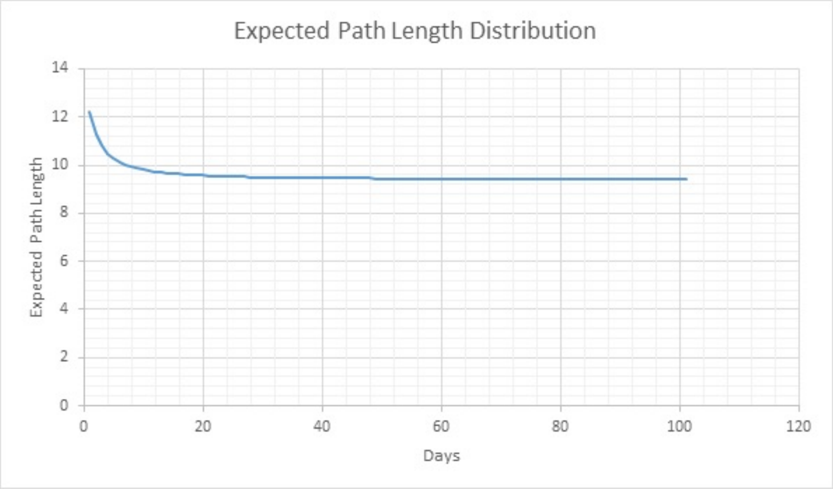


Figure 4.3: Behavior of Expected Path Length over the time

In this system, it is clear that the threshold score of the EPL is approximately 9.5 steps and the defending professionals can conclude that the system in their network is relatively safe from exploits only for the next 21 days as EPL score is above the threshold value.

It is clear that, any vulnerability that exists creates a threat to the computer system and the risk of probable exploitation will increase over the time of its existence without being patched. In other words, for a particular network system a higher Expected Path Length for an attacker to reach a goal state represents more difficulty for the hacker and would be reasonable to assume that the attacker has to face many defending measure with a higher expected path length compared a lower Expected Path Length. Now using the probabilistic models that we have developed in our previous studies, using the Vulnerability Life Cycle approach enables us to develop a time dependent stochastic models so that we could extend their application to develop a relevant and well defined process of monitoring the behavior of threats. Thus, our proposed analytic process illustrates its capability of estimating a Risk Index as a function of attacking time for a given computer system with known vulnerabilities.

4.6 Contributions

In the present study, we have developed a nonhomogeneous stochastic model for predicting the Expected Path Length (EPL) of a computer network system with a given set of vulnerabilities at time t.

Knowing EPL as a function of time is extremely important in developing defending strategies. Such strategy will reduce the likelihood of the computer network system being hacked. As we observe the behavior of the EPL over the time, it is possible to identify the time ranges where EPL reached a minimum. Low EPL implies higher chance for hacker to be successful. In other words, a computer network system is more vulnerable to be exploited on the days where the EPL is at lowest. On such time t, vulnerabilities and the system hence are more susceptible to be hacked. The same scenario from an attackers point of view can be explained. That is, on the days where EPL is at its lowest, likelihood of making a successful attack attempt is higher. Therefore, an attacker (hacker), who identifies the set of vulnerabilities in a given computer system would put more attempt on exploiting the system on such date where the EPL is at its lowest. This means, we can use this method as a prediction method of attacking (hacking) time.

By knowing this time for any computer network system, security engineers or IT architectures can take the necessary actions in advance to protect their computer system. Finally, we have developed over methodology based on a typical computer network system that exists in a real world situation with given vulnerabilities that identifies the EPL and actual time that the subject computer system could be exploited. Thus, industry can apply developed methodology in their own computer network system with given (known) vulnerabilities to predict the EPL and most probable time of being exploited.

5 Nonhomogeneous Risk Rank Analysis Method for Security Network System

5.1 Introduction

This chapter describes our approach to rank a set of vulnerabilities based on the risk, called ”nonhomogeneous risk rank analysis” that identifies the rank of vulnerabilities for each attack state in a network security model. We study and use the approach taken in wellknown Google Page Rank Algorithm. Here section 5.2 presents the method of Google page rank algorithm [24],[27],[28]. Section 5.3 presents our proposed Risk Rank algorithm, which provides the core of the ranking analysis approach along with an illustration in Section 5.4. Section 5.5 discusses the generality and theoretical implications of the approach.

A Network systems could have numerous vulnerabilities. We understand the process of generating vulnerabilities is highly stochastic and outcomes are hard to predict. Similarly the behavior of attacks and attackers also have higher level unpredictability. When considering a particular system based on the discovered vulnerabilities the analysis must consider the dynamic nature of the effect of vulnerabilities over time. As we observed in the previous chapters, effect of the vulnerabilities vary with the time over their life cycle. Therefore, for a particular system, the most threatening vulnerability at time t_1 might not be the same at time t_2. Hence, it would be very useful to have analytical models to observe the behavior of the rank of vulnerabilities based on the magnitude of the threat with respect to time for a given network system.

Such ranking distribution over time would empower the defenders by giving the priority directions to attend on fixing vulnerabilities. In this chapter we attempt to address this need.

5.2 Google Page Rank Algorithm

This section provides a background for our quantitative analysis of risk rank algorithm method. Ranking web pages is an important function of an internet search engine [27],[4]. Google Page Rank Algorithm is one of the most accurate and efficient page ranking methods in use. Methodology behind this algorithm will be briefly discussed below.

Output of this algorithm gives a probability distribution which is used to represent the likelihood that a person randomly clicking on links will arrive at any particular page. Using this method we can rank the likelihood of clicking on any web link. This can be calculated for any number of web links. In this algorithm, the sum of the page rank values of all the considered web links is equal to be one and it is assumed that the probability of selecting a web page initially is equal for any available option.

Google page rank algorithm simulates the clicking behavior of a web link in two ways. First is to visit a web link via an incoming link to the current web page and second way is to pick a web page randomly. Google page rank theory holds that an any surfer who is randomly clicking on web links will eventually stop clicking. At any of these stages, a damping factor d is the probability that the web surfer will continue surfing. Many researches have tested various damping factors but in generally it is assumed that the damping factor will be set around 0.85.

Let $p_t(v)$ be the probability of visiting web page v at time t and V be a set of all web pages under consideration. Here $out(v)$ represents the set of web pages in V with an outgoing link from v, and $in(v)$ represents the set of incoming link to v. The page rank computation can be viewed as a Markov process whose states are pages and the links between pages represent state transitions. This computation is given in the Equation (5.2.1) below.

$$P_{t+1}(v) = (1-d) * \sum_{\forall u \in V} \frac{P_t(u)}{|V|} + d * \sum_{\forall u \in in(v)} \frac{P_t(u)}{|out(u)|}. \tag{5.2.1}$$

Let, $|V|$ be the number of pages considered. Surfer will stop clicking on any link with probability $1-d$. Since there are $|V|$ number of pages and probability of visiting v from any page is equally likely, the probability for each case is equal to $\frac{1}{|V|}$.

Here $d * \sum_{\forall u \in in(v)} \frac{P_t(u)}{|out(u)|}$ represents the case when the surfer continues clicking links with probability d and goes to page v at time $t+1$ from page u that has an incoming link to v.

Initially at $t = 0$, each page has the same ranking value probability which is equal to $\frac{1}{|V|}$. Then iterations are executed over time until the stability is achieved. Once the probability distribution for each page becomes stable, considering high to low probabilities ranks are assigned.

5.3 Risk Rank Algorithm

By developing the concept applied in Google Page Rank Algorithm here we introduce a ranking method for risk [31] of vulnerabilities in a network system.

To estimate the probabilities in Risk Rank Algorithm Markov model techniques can be applied similarly as in Google Page Rank Algorithm. However there is a difference between web surfing behavior and Cyber security attacking behavior. A web surfing user can randomly select a web page but in cyberattacks an attacker doesn't have the same freedom. In web surfing user can arrive at any web page in one single step by using it's URL. But attacker has many restrictions. In computer network system an attacker doesn't have the access to all vulnerabilities in the network system. To achieve attacker's target state he must exploits several vulnerabilities in a particular order and enters in to the target system.

In the attacking process an attacker has two options. He can either continue or quit from his current path. If it is too difficult for him to achieve his goal state he can quit on the current path and try an alternative path by starting over from one of the set of initial states. Base on these assumptions here we propose our model to calculate the probability distribution of a given security attack model.

To obtain the risk rank we used the risk factor $R(v(t))$ of each vulnerability at each state and calculated normalized risk factor matrix $A(V, R)$ for the attack network system by using $\psi(u, v)$ transition probabilities from state u to v. Thus, we can calculate transition probabilities using the equation $\psi(u,v) = \frac{R(v(t))}{\sum_{\forall w \in out(u)} R(w(t))}$

Let $P_k(v)$ be the probability of exploiting state v at time k and V be a set of all states under consideration. Here $out(v)$ represents the set of states in V with an outgoing link from v, and $in(v)$ represents the set of incoming link to v. The Risk rank computation can be viewed as a Markov process whose state are vulnerabilities and the links between vulnerabilities represent state transitions. This computation is given in the Equation (5.3.2) below.

$$P_{k+1}(v) = \begin{cases} \frac{(1-d)}{|I|} + d * \sum_{\forall u \in in(v)} P_k(u).\psi(u,v), & \text{if } v \text{ is an initial state} \\ d * \sum_{\forall u \in in(v)} P_k(u).\psi(u,v), & \text{if } v \text{ is not an initial state} \end{cases} \tag{5.3.2}$$

Let $|I|$ be the number of initial states and attacker will stop his current path with probability $1-d$. Since there are $|V|$ number of states and probability of exploiting v from any other state is equally likely, the probability for each case is $\frac{1}{|V|}$.

Here in equation 5.3.2,

$$d * \sum_{\forall u \in in(v)} P_k(u).\psi(u,v)$$

represents the case when the attacker continues his current path with probability d and attack to state v at time $t+1$ from state u that has an incoming link to vulnerability v.

Initially at $t = 0$ each state has the same ranking value which is equal to $\frac{1}{|V|}$. Then computing iterations over time once the stability achieved. Once the probability distribution for each state become stable, assigning rank to each vulnerability.

Algorithm 1 Risk Rank Algorithm

```
procedure Risk Rank Algorithm(Input)
    Determine the input values.
    $A(V,R)$= Probability transition matrix, $V$ - Set of states in the attack graph, $R(v(t))$-Risk factor of vulnerability $v$ at time $t$, $\psi(u,v)$- transition probability from state $u$ to $v$, $I$ - Set of initial states. $out(v)$ - Set of all states out going from state $v$, $|V|$- Number of states.
    begin each $v \in V$
    set $P_0(v) = \frac{1}{|V|}$
    for $\forall u \in out(v)$ do
    $\psi(u,v) = \frac{R(v(t))}{\sum_{\forall w \in out(u)} R(w(t))}$
    end
    for $k = k+1$
    $P_{k+1}(v) = d \sum_{\forall u \in in(v)} P_k(u).\psi(u,v)$
    if $v \in I$ then
    $P_{k+1}(v) = P_{k+1}(v) + \frac{(1-d)}{|I|}$
    end if
    end for
    break if $P_{k+1}(v) = P_k(v)$, $\forall v \in V$
end procedure
```

This procedure is illustrated by the following schematic network.

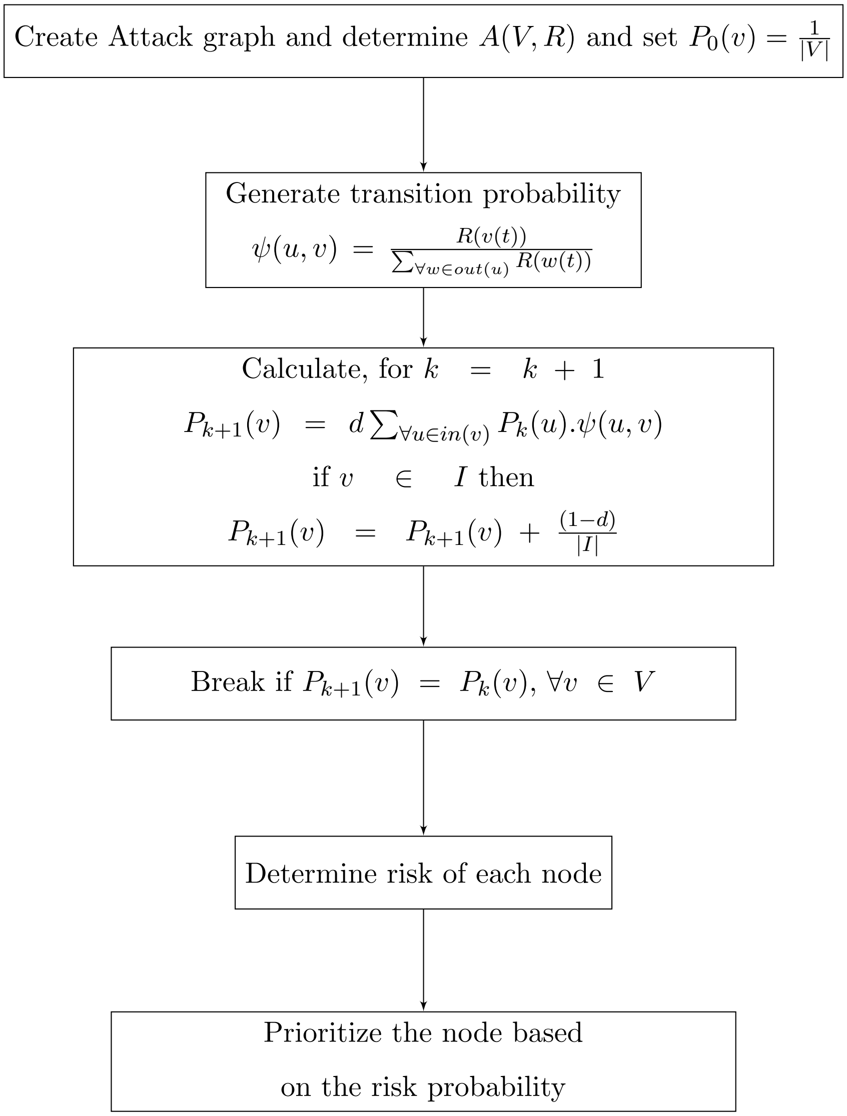

Create Attack graph and determine $A(V, R)$ and set $P_0(v) = \frac{1}{|V|}$
Generate transition probability
$\psi(u, v) = \frac{R(v(t))}{\sum_{\forall w \in out(u)} R(w(t))}$
Calculate, for $k = k + 1$
$P_{k+1}(v) = d \sum_{\forall u \in in(v)} P_k(u).\psi(u, v)$
if $v \in I$ then
$P_{k+1}(v) = P_{k+1}(v) + \frac{(1-d)}{|I|}$
Break if $P_{k+1}(v) = P_k(v), \forall v \in V$
Determine risk of each node
Prioritize the node based
on the risk probability

Figure 5.1: Key Steps of the Risk Rank Algorithm.

5.4 Illustration of Applying of the Risk Rank Algorithm

In this section we use the same attack graph we discussed in Chapter 4 and apply the Risk rank algorithm to this attack graph. Then we calculate the risk rank of each vulnerability in the network system [21].

To illustrate the proposed analytical approach model that we have developed as discussed above, we considered a Network Topology, given by Figure 5.2, below.

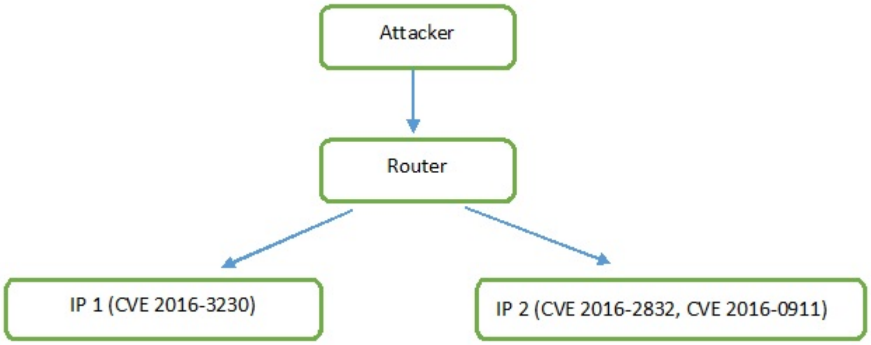


Figure 5.2: Network Topology

The computer network consists of two service hosts IP 1, IP 2 and an attackers workstation. Attacker is connecting to each of the servers via a central router. In the server IP 1 the vulnerability is labeled as CVE 2016-3230 and shall denote as V_1. In the server IP 2 there are two recognized vulnerabilities, which are labeled as CVE 2016-2832 and CVE 2016-0911. Lets denote them as V_2 and V_3, respectively.

We proceed to use the CVSS score of the above vulnerabilities in our analysis. The exploitability score ($e(v)$ in Figure 5.2) and Risk Factor $R(v_j(t))$ of each vulnerabilities as given in Table 5.1, below.

Table 5.1: Vulnerability Scores.

Vulnerability	Published date	CVSS score	$e(v_j)$	(t_j)	$R(v_j(t))$
V_1(CVE 2016-3230)	6/15/2016	9 (High)	8	9	1.702
V_2(CVE 2016-2832)	6/13/2016	4.3 (medium)	2.8	11	0.3667
V_3(CVE 2016-0911)	6/19/2016	1.9 (Low)	3.4	5	0.2474

For example we can calculate the Risk Factor of V_1 as follows.

$$R(v_j(t)) = Y(t) \times e(v_j),$$

$$R(v_1(t)) = [0.1917010.383521(1/t) - 0.00358\ln(\ln(t))] \times 8,$$

and

$$R(v_1(9)) = 1.702.$$

Although our proposed algorithm can be applied to any form of network system, for simplicity we will use our host centric attack graph model introduced in chapter 4 to illustrate the process.

The host centric attack graph is shown by Figure 5.3, below. Here, we consider that the attacker can reach the goal state only by exploiting V_3 vulnerability. The graph shows all the possible paths that is available for the attacker to reach the goal state.

Note that IP1,1 state represents vulnerability V_1 and states IP2,1 and IP2,2 represent vulnerabilities V_2 and V_3 respectively. Attacker can reach each state by exploiting the relevant Vulnerability.

In this methodology for the Host Centric Attack graph we can have the Ad-

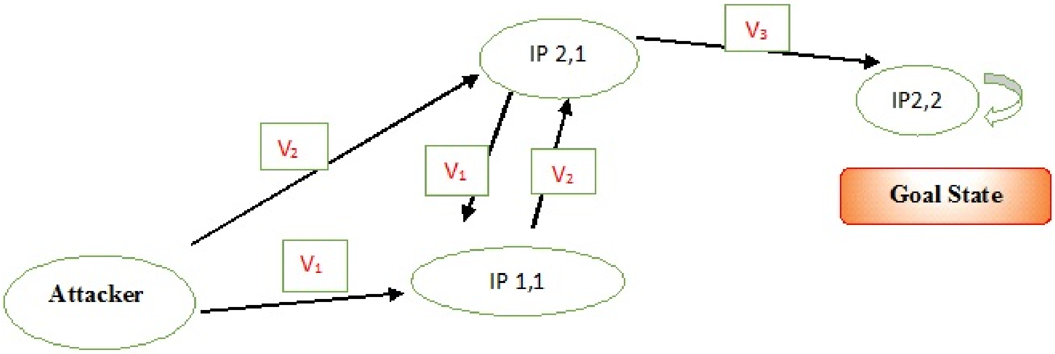


Figure 5.3: Host Centric Attack Graph

jacency Matrix as follows. Applying the information given in Table 5.1, the matrix $A(V, R)$ can be obtained as follows. Where we can find the transition probabilities from one state to another state.

$$A = \begin{bmatrix} 0 & 0.7614 & 0.2386 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.8255 & 0 & 0.1745 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5.4.3}$$

Applying this normalized risk matrix into Algorithm 1, we can obtain steady state probabilities for each state in the network which represent risk of being exploited. Results we obtained for each state are shown in the Table 5.2.

Table 5.2 results are in the order of being exploited by attacker at time t. Order of vulnerabilities based on the rank we obtained is s_1, s_2, s_3, s_0. This result

Table 5.2: Ranking results in attack states

States	Rank probability	Rank
s_0	0.15	4
s_1	0.293669	1
s_2	0.279731	2
s_3	0.2766	3

suggest that s_1 has the highest likelihood of being attacked. This means at time t, s_1 is the most vulnerable state. However according to Table 5.1 risk factor values for vulnerability v_1 is 1.702 which is higher than the risk factor values of vulnerabilities v_2 and v_3. Therefore it is reasonable to assume that reaching state s_1 from initial state s_0 (attacker's state) by exploiting v_1 vulnerability is easier than reaching states s_2 and s_3. Therefore, the risk rank of the state s_1 is higher than other states.

5.4.1 Behavior of Risk Ranks Over Time

In this section we extend our methodology to obtain the risk ranks of each attack state over time. Since our risk factor is a function of time, with the age of vulnerabilities the transition probability matrix with respect to the attack graph also varies. In our attack graph we consider dates according to the Table 5.1 and therefore after each day, transition probabilities in the matrix vary.

Table 5.3 illustrates risk ranks obtained for the next 10 days using the new algorithm we proposed. As these results indicate "risk ranks" for vulnerabilities varies

Table 5.3: Ranking results for each state with time

Time	S_0	S_1	S_2	S_3	rank state by highest risk
1	0.15	0.293669	0.279731	0.2766	S1, S2, S3, S0
2	0.15	0.2926	0.2723	0.2851	S1, S3, S2, S0
3	0.15	0.2799	0.2678	0.3023	S3, S1, S2, S0
4	0.15	0.2766	0.2648	0.3086	S3, S1, S2, S0
5	**0.15**	**0.2742**	**0.2628**	**0.313**	**S3, S1, S2, S0**
6	0.15	0.2725	0.2612	0.3163	S3, S1, S2, S0
7	0.15	0.2712	0.2601	0.3187	S3, S1, S2, S0
8	0.15	0.2702	0.2592	0.3206	S3, S1, S2, S0
9	0.15	0.2694	0.2585	0.3221	S3, S1, S2, S0
10	0.15	0.2688	0.2579	0.3233	S3, S1, S2, S0

over time. For example at time $t = 5$ risk probabilities are 0.15, 0.2742, 0.2628 and 0.313 for each state s_0, s_1, s_2 and s_3 respectively. As the Table 5.3 exemplifies with initial ranks state one (vulnerability, V_1) was most risky or vulnerable. But, after two days state s_3 (Vulnerability, V_3) becomes the most vulnerable, hence the most risky state and continue to be so afterwards. It should be noted that ”State 0” is not a vulnerability but represents the attacker. Therefore it is at the last of the order of ranks always. It is interesting to see that s_3 (Vulnerability, V_3) initially was at the least risk level so in the last position of the risk levels among vulnerabilities, and then just after one day becomes more risky and reach the second in the rank and after two dates become the dominating risk factor in this particular computer network model.

So, application of this algorithm in more a generalized real life network model would give us with the similar observations with respect to time. According to this model example, network administrators and defending resources must be allocated to resolve s_3 (Vulnerability, V_3) at priority.

5.5 Contributions

In this chapter a new Ranking Algorithm was introduced to rank the vulnerabilities in a particular computer network system. The methodology of well-known Google Page Rank Algorithm was used and we further developed it to fit a computer network environment. General assumptions used in Google Page Rank Algorithm with respect to the probability of selecting a particular web link were changed according to the probability distributions we obtained by normalized vulnerability scores in subject computer network system. Ranks were obtained for each vulnerability based on the likelihood of those vulnerabilities getting exploited.

We have further developed the algorithm so that the Distribution of Ranks of Vulnerabilities in the subject computer network system is given as a function of time. That is, using our new algorithm, a user (a network system administrator or a researcher) would be able to observe the behavior of the ranks of vulnerabilities with respect to time. This new methodology will greatly help relevant parties to make better decisions to protect network systems because at a particular time t, the algorithm will indicate which vulnerabilities are most vulnerable and needed immediate attention or priority.

6 Future Research

Our main objective in subject research area is to develop successful Statistical Models and Analytical tools to be effectively applied in the area of Cybersecurity. With respect to our analytical approaches on vulnerability data we have, two main research focuses are to apply Power Law process [38] and Non-Homogeneous Poison Process. Having identified the intensity function of the power-law process on existing vulnerabilities of a particular computer network system or a software, we will be able to analytically obtain a measure of successfulness of a network modification or a defending attempt such as network re-design, patching vulnerabilities, disclosing vulnerabilities, replacing or updating software etc. If we can acquire relevant data, such an analytical model with a highest degree of accuracy will be able to indicate whether the modification on the computer network system of a software resulted in decreasing, increasing or made no significant change of the likelihood of being exploited, increased or remained the same.

It is also possible to analytically model the hacking times using Non-Homogeneous Poison Process. If we can obtain relevant data with respect to times and time intervals for hacking attempts related to vulnerabilities, we would be able to make new approaches in developing models to further predict the behavior of attackers.

Further, from a subjective and analytical point of view, a vulnerability is a resource or an opportunity for the attacker. But, for defenders the software is the resource and the vulnerability is a threat that question the survival of the software. If we track the vulnerabilities and the exploitation data and simulate hacking approaches it is possible to execute a survival analysis on the relevant software. We would be able to define and estimate parameters indicating the behavior of the survival functions for different categories of vulnerabilities, software and computer network systems,

In this dissertation, we presented several important analytical tools and statistical models with respect to vulnerability analysis [42],[45]. In the future, we would develop these models further and apply software resources to improve the models to fit and apply in Large-Scale real world computer network systems with any number of computers and network nodes with variety of discovered vulnerabilities.

Another important research objective we have for the near future is to analyze the CVSS model. As it was discussed in the previous chapters, we have CVS scores given for each vulnerability in the CVE detail website [19] and presented in the National Vulnerability Database [2]. Calculation of these scores are based on several factors represented by several metrics and equations. However, at this point we do not know any methodology to measure the accuracy or the degree of confidence in these methodologies and calculations. We also do not know the relevant assumptions which these calculations are based on. Therefore, we expect to coordinate with the administration of the National Vulnerability Data base and other relevant government organizations to test and improve these scoring system further.

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Appendices

Appendix A - Common Vulnerability Scoring System (CVSS)Version 2.0 Calculations

Scoring equations and algorithms for the base, temporal and environmental metric groups are described below. Further discussion of the origin and testing of these equations is available at www.first.org/cvss.

The **Base equation** is the foundation of CVSS scoring. The base equation is:

```
BaseScore = (((0.6*Impact)+(0.4*Exploitability)1.5)*f(Impact))

Impact = 10.41*(1-(1-ConfImpact)*(1-IntegImpact)*(1-AvailImpact))

Exploitability = 20* AccessVector*AccessComplexity*Authentication


f(impact)= 0 if Impact=0, 1.176 otherwise


AccessVector (AV) = case AccessVector of

requires local access: 0.395

adjacent network accessible: 0.646

network accessible: 1.0


AccessComplexity (AC) = case AccessComplexity of

high: 0.35

medium: 0.61

low: 0.71
```

```
Authentication (AU) = case Authentication of
requires multiple instances of authentication: 0.45
requires single instance of authentication: 0.56
requires no authentication: 0.704

ConfImpact (C)= case ConfidentialityImpact of
none: 0.0
partial: 0.275
complete: 0.660

IntegImpact (I) = case IntegrityImpact of
none: 0.0
partial: 0.275
complete: 0.660

AvailImpact (A)= case AvailabilityImpact of
none: 0.0
partial: 0.275
complete: 0.660
```

Temporal Equation

If employed, the temporal equation will combine the temporal metrics with the base score to produce a temporal score ranging from 0 to 10. Further, the temporal score will produce a temporal score no higher than the base score, and no less than 0.33 lower than the base score. The temporal equation is:.

```
TemporalScore = (BaseScore*Exploitability*RemediationLevel*ReportConfidence)

Exploitability = case Exploitability of
unproven: 0.85
proof-of-concept: 0.9
functional: 0.95
high: 1.00
not defined: 1.00

RemediationLevel = case RemediationLevel of
official-fix: 0.87
temporary-fix: 0.90
workaround: 0.95
unavailable: 1.00
not defined: 1.00
```

```
ReportConfidence = case ReportConfidence of

unconfirmed: 0.90

uncorroborated: 0.95

confirmed: 1.00

not defined: 1.00
```

Environmental Equation If employed, the environmental equation will combine the environmental metrics with the temporal score to produce an environmental score ranging from 0 to 10. Further, this equation will produce a score no higher than the temporal score. The environmental equation is:

```
EnvironmentalScore =(AdjustedTemporal+(10-AdjustedTemporal) *CDP) *TD)

AdjustedTemporal = TemporalScore recomputed with the
BaseScores Impact sub- equation replaced with the AdjustedImpact equation

AdjustedImpact = min(10,10.41*(1-(1-ConfImpact*ConfReq)*(1-IntegImpact*IntegReq)
*(1-AvailImpact*AvailReq)))

CollateralDamagePotential (CDP) = case CollateralDamagePotential of
none: 0
low: 0.1
low-medium: 0.3
medium-high: 0.4
high: 0.5
not defined: 0

TargetDistribution (TD)= case TargetDistribution of
```

```
none: 0
low: 0.25
medium: 0.75
high: 1.00
not defined: 1.00

ConfReq = case ConfReq of
low: 0.5
medium: 1.0
high: 1.51
not defined: 1.0

IntegReq = case IntegReq of
low:0.5
medium: 1.0
high: 1.51
not defined: 1.0

AvailReq = case AvailReq of
low:0.5
medium: 1.0
high: 1.51
```

```
not defined: 1.0
```

Appendix B

Example-CVE-2002-0392

Below, we provide steps of how CVSS is used for three different metric.

```
----------------------------------------------------

BASE METRIC   EVALUATION SCORE

----------------------------------------------------

Access Vector [Network] (1.00)

Access Complexity [Low] (0.71)

Authentication [None] (0.704)

Confidentiality Impact [None] (0.00)

Integrity Impact [None] (0.00)

Availability Impact [Complete] (0.66)



Impact = 10.41*(1-(1)*(1)*(0.34)) == 6.9

Exploitability = 20*0.71*0.704*1 == 10.0 f(Impact) = 1.176

BaseScore = (0.6*6.9 + 0.4*10.0  1.5)*1.176 =7.8


----------------------------------------------------

TEMPORAL METRIC   EVALUATION SCORE

----------------------------------------------------

Exploitability [Functional]     (0.95)
```

```
Remediation Level [Official-Fix]   (0.87)

Report Confidence [Confirmed]      (1.00)


TEMPORAL SCORE=round(7.8 * 0.95 * 0.87 * 1.00) == (6.4)


----------------------------------------------------

ENVIRONMENTAL METRIC   EVALUATION SCORE

----------------------------------------------------

Collateral Damage Potential  [None - High]  {0 - 0.5}

Target Distribution  [None - High]  {0 - 1.0}

Confidentiality Req.        [Medium] (1.0)

Integrity Req. [Medium] (1.0)

Availability Req. [High] (1.51)


AdjustedImpact = min(10,10.41*(1-(1-0*1)*(1-0*1)*(1-0.66*1.51))= (10.0)

AdjustedBase =((0.6*10)+(0.4*10.0)1.5)*1.176 = (10.0)

AdjustedTemporal = (10*0.95*0.87*1.0) = (8.3)

EnvScore = round((8.3+(10-8.3)*{0-0.5})*{0-1})= (0.00 - 9.2)
```

Appendix C

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Journal: Journal of Information Security (JIS)

Paper ID: 7800364

Paper Title:Cybersecurity: A Statistical Predictive Model for the Expected Path Length(7800364)

Author(s): Pubudu Kaluarachchilage, Chris Tsokos, Sasith Rajasooriya

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