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Decision Support Models for A Few Critical Problems in Transportation System Design and Operations

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Operations

by

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DEDICATION

I dedicate this dissertation to my family, my friends and all others who provide continuous support and encouragement throughout my Ph.D. work.

I would like to express my special appreciation to my wife, who continuously give me encouragement and support through the past five years. I also dedicate this work to my little daughter, who bring happiness to my life and make my life meaningful.

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ABSTRACT

Transportation system is one of the key functioning components of the modern society and plays an important role in the circulation of commodity and growth of economy. Transportation system is not only the major influencing factor of the efficiency of large-scale complex industrial logistics, but also closely related to everyone's daily life. The goals of an ideal transportation system are focused on improving mobility, accessibility, safety, enhancing the coordination of different transportation modals and reducing the impact on the environment, all these activities require sophisticated design and plan that consider different factors, balance tradeoffs and maintaining efficiency. Hence, the design and planning of transportation system are strongly considered to be the most critical problems in transportation research.

Transportation system planning and design is a sequential procedure which generally contains two levels: strategic and operational. This dissertation conducts extensive research covering both levels, on the strategic planning level, two network design problems are studied and on the operational level, routing and scheduling problems are analyzed. The main objective of this study is utilizing operations research techniques to generate and provide managerial decision supports in designing reliable and efficient transportation system. Specifically, three practical problems in transportation system design and operations are explored. First, we collaborate with a public transit company to study the bus scheduling problem for a bus fleet with multiples types of vehicles. By considering different cost characteristics, we develop integer program and exact algorithm to efficiently solve the problem. Next, we examine the network

design problem in emergency medical service and develop a novel two stage robust optimization framework to deal with uncertainty, then propose an approximate algorithm which is fast and efficient in solving practical instance. Finally, we investigate the major drawback of vehicle sharing program network design problem in previous research and provide a counterintuitive finding that could result in unrealistic solution. A new pessimistic model as well as a customized computational scheme are then introduced. We benchmark the performance of new model with existing model on several prototypical network structures. The results show that our proposed models and solution methods offer powerful decision support tools for decision makers to design, build and maintain efficient and reliable transportation systems.

CHAPTER 1: INTRODUCTION

Transportation system is a sophisticated, integrated and large-scale functional component of modern society. It plays a key role in improving circulation of commodity and mobility of population. In last decades, due to the rapid urbanization and population expansion in the world, many cities suffer from severe transportation problems, in the forms of air pollution, traffic congestion etc., These issues impede the development of most cities therefore policy makers struggle to find optimal and effective business solutions. With limited resources, efficient design and development of the transportation systems are greatly important in providing decision supports to city planner and policy maker.

The designing of efficient transportation systems typically involves two levels: strategic and operation. On the strategic level, policy maker must determine transportation network structure and configuration, while on operational level, decisions are made to identify optimal operational strategy, for example vehicle and crew scheduling, vehicle routing. Three specific research studies on both levels are presented in this dissertation as follow:

In chapter 2, we collaborate with a public transportation company in China to study the scheduling problem for a bus fleet with conventional diesel and alternative fuel vehicles. We propose a multi-depot multiple-vehicle-type bus scheduling model and consider various cost components of different types of buses. A time-spaced network approach is implemented and experiments are conducted on the real-world data.

In chapter 3, we study the reliable emergency medical service location problem and develop set of two-stage robust optimization models with/without consideration of relocation operations. Then, customized computational algorithms and approximation procedure are developed and implemented. A real-world case study is then discussed.

In chapter 4, we introduce a novel pessimistic optimization model for the vehicle sharing program network design. We develop a tight relaxation for level reduction and propose an exact solution approach. Numerical study is conducted on several prototypical networks and solutions are discussed and benchmarked with regular optimistic formulation.

The major research contributions of this dissertation are presented as follows.

1. Developed a multi-depot multiple-vehicle-type alternative fuel vehicle scheduling model that explicitly considering cost characteristics of different types of vehicles.

2. Integrated traffic congestion levels into alternative fuel vehicle scheduling model and conducted analysis on a real-world case to provide business solutions.

3. Developed two-stage robust optimization models to design emergency medical service systems and incorporated the impact of relocation operations on initial locations.

4. Developed customized *column-and-constraints generation* algorithm and approximation procedure for solving large-scale real world instance.

5. Proposed a novel pessimistic bi-level optimization model to determine the optimal network configuration of vehicle sharing program.

6. Developed an exact solution approach and benchmark results with regular optimistic formulation. Results demonstrate the capability of pessimistic formulation for providing reliable and robust VSP network design.

CHAPTER 2: SCHEDULING PROBLEM FOR BUS FLEET WITH ALTERNATIVE FUEL VEHICLES

Public transportation industry is the major contributor in achieving sustainable development goal. It provides an affordable and energy efficient travel alternative that could potentially change people's travel behavior, help reduce the ownership and utilization of private cars and reduce greenhouse gas (GHG) emission. Thus, air pollution in urban areas can then be minimized. With increasing pressure and strict government policy on environmental conservation, public transportation provider is making great effort on adopting alternative fuel vehicles using renewable energy. However, this transition requires significant financial support and increases the economic burden of public transportation providers. Therefore, they are seeking cost-effective ways to meet tight constraints.

This chapter focuses on implementing sustainable transportation system on the operational level. Specifically, a study of vehicle scheduling problem is conducted in collaboration with a public transportation company in China.

2.1 Introduction

In recent years, China has become the world's second largest producer of greenhouse gas (GHG). To reduce emission and protect the environment, Chinese Government starts to put great efforts on supporting the development of alternative fuel technology. Transportation becomes one of the major contributors to the air pollution problem and accounts for about 61% of the global oil consumption and 28% of the total energy consumption [1]. The public transit industry is considered to

have a significant impact on air pollution in urban cities in China. In general, conventional heavy diesel buses take a great portion in the fleet, therefore they are major source of producing toxic air pollutants and greenhouse gasses, especially under heavy traffic conditions in urban area. With increasing environmental pressures and financial supports from the government policy, public transit providers begin to replace old conventional diesel buses with alternative fuel buses (AFVs), such as compressed natural gas bus, plug-in electric bus, liquid natural gas bus, hybrid bus. Those alternative fuel buses typically have better fuel economy and low greenhouse gas emission, especially under heavy traffic conditions. Nevertheless, huge attainment and maintenance costs impede the adoption of AFVs in transit companies. With a fleet of conventional diesel buses and AFVs, public transit provider struggles to obtain cost effective solutions on their daily service schedules.

In traditional bus scheduling problem, a set of trips with starting and ending time is predetermined, the objective is to find a feasible schedule so that each bus takes a series of trips and the total costs can be minimized. A lot of attention has been paid to the bus scheduling problem, [2] proposed a single depot vehicle scheduling model solved by the two-phase approach. Later, the multi-depot case was proposed and studied by [3], [4], [5]. To the best of our knowledge, there is only one paper addresses the scheduling problem of the AFVs [6], however, the mixed integer model they developed is a single-depot, multiple-vehicle-type model. The cost characteristics of AFVs and the effect of traffic patterns on trips are not explicitly considered.

In this chapter, a multi-depot, multiple-vehicle-type vehicle scheduling model for a bus fleet with conventional diesel and alternative fuel vehicles is proposed. Moreover, we study different cost characteristics of different types of vehicles and integrate traffic patterns into the model. A time-space network approach is implemented and case study is conducted based on real-world data provided by a public transit provider in China.

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2.2 Problem Summary

2.2.1 The Development of Alternative Fuel Buses in China

The rapid growth of urbanization in China leads to the substantial increases in passenger and freight transportation demand. Furthermore, according to QY Wang's investigation [7], the total energy consumption per year by the transportation activities has quadrupled from 2000 to 2007. In regard to energy demand and GHG emission in China [8], public transit becomes the most rapidly growing industry. With the pressure from both global and national environmental consideration, great efforts have been put on energy-saving technologies and AFV applications by the Chinese government [9], [10]. In 2014, the State Council of China has released the implementation guide for alternative fuel vehicles and pointed out that among newly bought or updated vehicles in the city, the proportion of AFVs in the areas of public transit, taxi and city logistics should not be less than 30% [11]. By now, some AFV buses have already been applied in most of the major cities. The department of transportation in Beijing has announced that the government would invest more than 10 billion Chinese Yuan on promoting the application of alternative fuel vehicles, and would try to achieve zero emission by the year of 2019. The government of Shenzhen proposed a plan that 35,000 alternative fuel vehicles would be promoted for public use from the year of 2013 to 2015. The public transportation development plan of Zhengzhou projects a goal of increasing 3,000 AFVs in three years [12].

2.2.2 Emissions and Costs

More and more public transit providers in China begin to replace their old conventional diesel buses with AFVs not only because conventional diesel buses are a major source of producing GHG, but also are less energy efficiency. Table 2.1 shows the fuel consumption and GHG emission rate for each fuel type, where the LPG is liquefied

petroleum gas and CNG is compressed natural gas [13]. More importantly, the population in China is highly concentrated in city area, severe traffic congestion occurs in two peak periods every day. [14] shows that as traffic congestion increases, so do fuel consumption and GHG emissions. Table 2.2 gives the energy efficiency values based on multi-criteria analysis by [15]. In this table, higher value means better fuel efficiency. Diesel-powered buses have the lowest energy efficiency values comparing to alternative fuel vehicles. Therefore, with the progress of bus replacement project, conventional diesel buses should be gradually eliminated since their poor performance in fuel economy and are less environmental friendly. However, the new project requires tremendous capital and relatively longer periods to implement. Data in Table 2.3 published by Institute of Transportation (2000) indicates that the operational costs of AFVs are far less than diesel buses [15]. In fact, AFVs cost three times more on attainment and twice more on maintenance than the conventional diesel buses. At present, public transit provider is seeking solutions to optimally utilize both conventional diesel buses and alternative fuel vehicles on their daily service schedules.

2.3 Model and Methods

2.3.1 Vehicle Scheduling Problem

The bus scheduling problem in public transportation involves a set of timetabled trips with start/end stations, trip durations, lengths and depot numbers, given the capacity of depots, the objective is to find an optimal schedule so that total costs is minimized. The operational costs usually refer to the cost incurred by buses running on deadhead trips. Deadhead trips are the movement of an empty vehicle from the end station of a finished trip to the start station of an upcoming trip or from/to depot. In traditional bus scheduling problem, costs are calculated only based on the distances. Costs variation of different types of buses running on same trips are not

considered explicitly. In practice, attainment and maintenance costs should not be overlooked. As we mentioned, they vary considerably among vehicles. Additionally, due to special situation in China, the duration of trips with the same distances is highly influenced by the traffic patterns. Therefore, the various costs should be considered and modeled explicitly in AFV scheduling.

Vehicle scheduling problem is well recognized as complicated problem due to its numerous possibilities to assign vehicle to each trip, to schedule sequences of service trips for each bus, and to assign buses to depots. Multi-depot vehicle scheduling problem is well-known as the NP-hard problem [16]. With the increase of the number of trips and depots, the problem size can grow exponentially. Therefore, standard solvers cannot handle instances with thousands of trips.

2.3.2 Time-space Network

[17] first proposed a time-space network model for routing problems in airline schedule application. The same approach has not been applied in vehicle scheduling until recently. [18] applied the technique to the multi-depot vehicle scheduling in 2006. In Kliewer's approach, the arcs are aggregated and total number of arcs are highly reduced. Figure 2.1 shows the proposed time-space network. Arcs in this network can be categorized as follows:

1. Pull-in/pull-out arcs: Buses pull out from depots to serve scheduled trips, or pull in from finished trips to depots.

2. Service trip arcs: The timetabled or scheduled trips.

3. Waiting arcs: Buses waiting in a station before serving next trips.

4. Deadhead arcs: The movements of empty buses traveling from finished trips to next scheduled trips.

By definition, trips A and B are compatible if the ending time of trip A plus travel time between ending station of trip A and starting station of trip B is no later than the starting time of trip B. For example, in Figure 2.1 trips 1 and 3, 4, 5 are compatible and trips 3, 4, 5 are compatible, etc. By using latest-first matches, there is no need to model connections within one station explicitly. For example, connections between trip 1 and trip 4, trip 2 and trip 4 can be aggregated into the waiting arc in station 2, therefore there is no need to connect trip 2 and 4 since trip 1 and 2 have the same ending station. If the problem contains m depots and n trips, the number of total arcs in the time-space network is $O(nm)$ comparing to $O(n^2)$ in traditional connection-based network [18].

2.3.3 Costs Components

In AFV scheduling problem, the costs associated with each arc should be modeled explicitly. In this research, the total costs consist of the following components.

1. Operational cost

2. Maintenance cost

3. Waiting outside depot time cost

Operational cost is based on the fuel cost incurred by bus operations. For example, the fuel cost of pull out/pull in operations between depots and scheduled trips, the fuel cost of deadhead trips. Fixed costs are assigned to trips that are originated from any depots. Maintenance cost is defined by the cost of maintaining buses after servicing a series of trips and is depending on the accumulated trip lengths that buses have served. Waiting outside depot time cost is the cost of time on waiting. We further discovered that fuel efficiency level changes with respect to traffic pattern. For instance, buses have better fuel efficiency in trips under normal traffic

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conditions than those under heavy traffic conditions. Therefore, fuel economies vary according to specific traffic patterns.

2.3.4 Mathematical Formulation

We define T as the set of timetabled trips, D the set of depots, L the set of bus types. $i, j \in T \cup D$ represent the timetabled trip, $d \in D$ represents depot, $l \in L$ denotes bus type. Each trip has its corresponding start and end stations, denoted by sk_i, ek_i, start and end times, denoted by st_i, et_i, as well as trip distances td_i. We construct a vehicle scheduling network $G_D = (V_D, A_D)$ for each depot, with nodes $V_D = V_T \cup V_d$ and arcs $A_D = A_T \cup A_d$, where the set of nodes V_D denotes service trip nodes V_T and depot nodes V_d, A_d connect depot d and trips starting from it. Deadhead arcs are denoted by A_T.

We also define Q_d^l as the number of available buses in type l at depot d, K_d as the capacity of depot d and u_l as the fuel price of bus type l. For total costs, we let C_{ij}^{dl} represents the cost of buses traveling from end station of trip i to the start station of trip j plus operational cost for serving trip i. F is the fixed cost of operating a new bus. woc denotes the waiting time spent outside depot to serve next trip. h_{ij} be the distances from end station of trip i to the start station of trip j or distances between depots and trips. Let ρ be the traffic pattern indicator denotes the level of traffic congestion. FE_l be the fuel efficiency of bus type l. We also define m_l as the average maintenance cost per unit distance of bus type l. Therefore, C_{ij}^{dl} can be calculated as follows:

1. When i, j are both service trips and end station of i is different from start station of j,

$$C_{ij}^{dl} = \rho * u_l * (td_i + h_{ij})/FE_l + m_l(h_{ij} + td_i) + woc * (st_j - et_i) \tag{1}$$

2. Similarly, when i, j are trips and end station of i and start station of j are equal,

$$C_{ij}^{dl} = \rho * u_l * td_i/FE_l + m_l * td_i + woc * (st_j - et_i) \tag{2}$$

3. When i is depot and j is service trip,

$$C_{ij}^{dl} = F + \rho * u_l * h_{ij}/FE_l + m_l * h_{ij} \tag{3}$$

4. When i is trip and j is depot,

$$C_{ij}^{dl} = \rho * u_l * h_{ij}/FE_l + m_l * h_{ij} \tag{4}$$

Finally, we let x_{ij}^{dl} be binary variables, with $x_{ij}^{dl} = 1$ if bus type l from depot d serve trip j after finish trip i, and $x_{ij}^{dl} = 0$ otherwise. Then we propose the multi-depot, multiple vehicle type alternative fuel vehicle scheduling integer programming model (MDMVSP):

$$\min_{x} \sum_{l \in L} \sum_{d \in D} \sum_{i,j \in A_T \cup A_d} C_{ij}^{dl} x_{ij}^{dl} \tag{5}$$

$$s.t. \sum_{i:(i,j) \in A_T \cup A_d} x_{ij}^{dl} - \sum_{i:(j,i) \in A_T \cup A_d} x_{ij}^{dl} = 0, \qquad \forall j \in V_T \in V_D, d \in V_D, l \in L \tag{6}$$

$$\sum_{l \in L} \sum_{d \in D} \sum_{j \in V_T \cup V_D} x_{ij}^{dl} = 1, \qquad \forall i \in V_T \tag{7}$$

$$\sum_{l \in L} \sum_{j:(i,j) \in A_T} x_{ij}^{dl} \leq K_d, \qquad \forall d \in V_D, i \in V_D \tag{8}$$

$$\sum_{j:(i,j) \in A_d} x_{ij}^{dl} \leq Q_d^l, \qquad \forall i \in V_D, l \in L, d \in V_D \tag{9}$$

$$x_{ij}^{dl} = \{0,1\}, \qquad \forall (i,j) \in A_T \cup A_d, d \in D, l \in L \tag{10}$$

The objective function (5) in MDMVSP model seeks to minimize the sum of the total costs. Constraints (6) are the flow conservation constraints, representing that the out-flow and in-flow of each node should be equal, constraints (7) indicates that each trip should only be served by one bus of type l from depot d, (8) and (9) are the depot and vehicle type capacity constraints respectively.

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2.4 Numerical Study

In this study, we collaborate with a public transit company, Zhengzhou Bus Communication Company (ZZBC) in Zhengzhou, Henan, China. The real-world data is obtained from ZZBC's database, including bus routes, bus types, scheduled timetables and depot information. There are total of 17 Bus Rapid Transit (BRT) routes with two directions and they are grouped into 4 categories based on their service range. Four timetables were obtained and denoted by T1 to T4, with their corresponding number of trips and depots. Each timetable contains a combination of bus routes with different vehicle-type availability. There are 7 types of bus in total, we treat 18m Diesel and 12m Diesel as two distinct types since their fuel efficiency varies due to differences in length.

The detailed parameters can be seen in Table 2.5. Fuel economy and prices were obtained from ZZBC based on a report conducted by Grutter consulting company [19]. Maintenance cost was reported by [15] and converted to US dollar per kilometer based on estimated annual average distances of 55,000 kilometers. We assume that the maintenance costs for buses of the same type with different lengths are equal. Attainment costs were excluded since we are optimizing the schedule based on the existing buses and it does not require purchasing new buses. [20] reported that congestion can take great effects on fuel consumption, and the simulation shows that an approximate 50 percent increase in fuel consumption under heavily congested condition. To better capture the traffic congestion patterns, we set ρ to three levels $\rho = 1,2,3$ representing the congestion levels of light, medium, heavy, respectively. The cost of bus waiting outside depot for one-minute is set to be 1 and the fixed cost of operating a new bus to be $1,000. Each depot has a capacity of 100. ZZBC requires that the limit of idle time for buses waiting outside depot is 20 minutes.

The algorithm was implemented in MATLAB R2013a on Lenovo ThinkPad T420s with Intel Core i5-2520M CPU at 2.50GHz, 8GB of RAM and Windows 7 operating system (64). C++ and CPLEX 12.5 are used to solve integer programs. Table 2.6 presents the computational results for the four instances with $\rho = 1,2,3$ levels of traffic congestion and the number of bus in each type. Column AFV Utilization is the percentage of alternative fuel vehicles used among mixed fleet. Column Vehicle Reduction is the percentage of the total number of buses reduced comparing to the existing schedule. Column cost is the total cost required for each timetable under different congestion scenarios. Column CPU denotes the solution time.

The computational results and Figure 2.2 show that the multi-depot multiple vehicle type bus scheduling model can reduce the total fleet size significantly with an average of 18% comparing to current schedules. For example, the optimal schedule reduces total number of buses from 176 to 149 in T1, from 128 to 95 in T2, from 224 to 186 in T3, and from 179 to 154 in T4. Under each level of congestion, the total number of bus remains the same for all timetables which means the optimal fleet size is achieved. Figure 2.3 shows the AFV utilization rate under each congestion scenario for each timetable. We notice that, when the traffic congestion level increases from 1 to 3, the corresponding AFV utilization rates increase in T1 to T4. For example, when there is light traffic congestion where $\rho = 1$, the number of 18-meter diesel bus is 46 and the number of CNG bus is 39, when congestion level is medium, where $\rho = 2$, the number of 18-meter diesel decreases to 35 while the number of CNG bus increases to 50 which is the maximum availability of CNG bus. As congestion level increases to $\rho = 3$, which means heavy congestion. The number of 18-meter diesel buses is 29 and the number of 18-meter diesel-electric hybrid buses increases to 37. This is because when there is light traffic congestion, total fuel cost may be less than the total maintenance cost, the model prefers to choose buses

with low maintenance cost. However, when traffic level increases, fuel consumption increases for the same trips and fuel cost becomes the dominant cost. Hence higher utilization of alternative fuel vehicle is achieved due to their high fuel efficiency. Figure 2.4 to Figure 2.7 show the comparison of utilization of AFVs in normal and heavy congestion periods trips in timetable T1 to T4.

Furthermore, to study the traffic effects on bus schedules, we applied heavy congestion indicator only to those trips in heavy congestion periods, from 6:00am to 9:00am, and from 5:00pm to 8:00pm. We redo the experiment on T1 under normal and heavy congestion periods and acquire detailed schedule for each bus. Then we calculated AFV utilization only for those trips. The result is shown in Table 2.7. There are 350 trips in heavy congestion periods, 164 of them using AFVs under normal traffic condition. In contrast, if the congestion level is specifically considered and applied to those trips, the number of trips using AFVs increases to 234. The total AFV utilization in those trips increases from 46.9% to 66.9%. It shows that traffic congestion level takes great effect on the type of vehicles to be scheduled and should not be neglected by transit planner. Therefore, the utilization of AFVs should be high on heavy-congestion-periods-trips since fuel costs exceed maintenance costs and low on normal-traffic-trips since maintenance costs are the major costs.

2.5 Conclusions

Emission reduction is the primary consideration in all alternative fuel vehicle scheduling models. However, public transit provider may focus more on total costs because maintenance cost could become a potential huge factor. This paper studies AFV scheduling problem and proposes a multi-depot multiple-vehicle-type scheduling model minimize total costs and considering both fuel costs and maintenance costs under different traffic congestion levels. A

time-spaced network approach is used to model the network. It is proven that this approach can significantly reduce the total number of arcs so that the model can be solved by standard solver under reasonable time. A real-world instance from China is tested and the computational results show that our model can significantly reduce fleet size comparing to previous schedule. Moreover, we also study the trade-offs between fuel costs and maintenance costs. As the result, the optimal schedule generated by the model can balance utilization of conventional diesel buses and AFVs. Results also suggest that AFVs should be highly utilized in peak hours due to their better fuel economy while conventional diesel buses should be highly utilized in off-peak hours because of their lower maintenance cost. It is demonstrated that our proposed model and algorithm provided a decision support tool for public transportation company to utilized their resources efficiently.
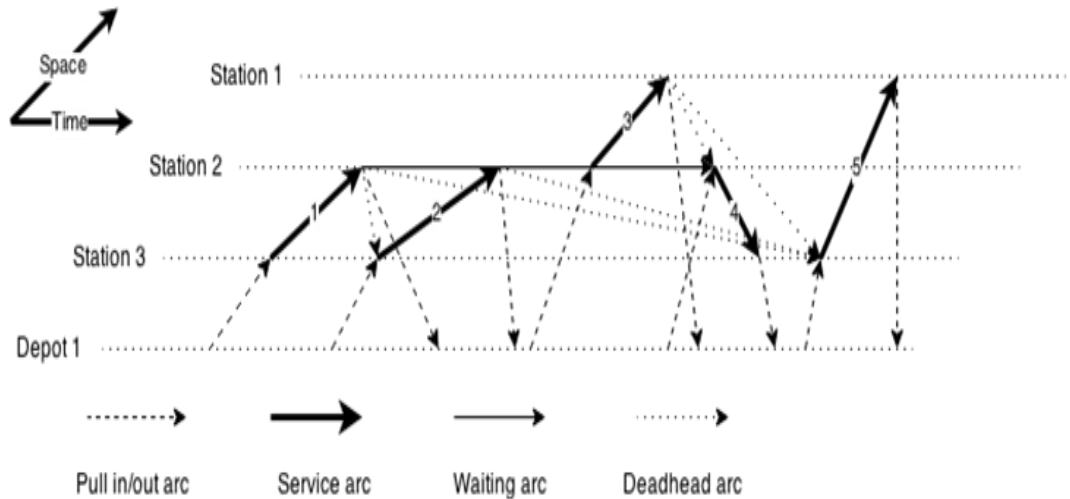
Figure 2.1: Time-space Network

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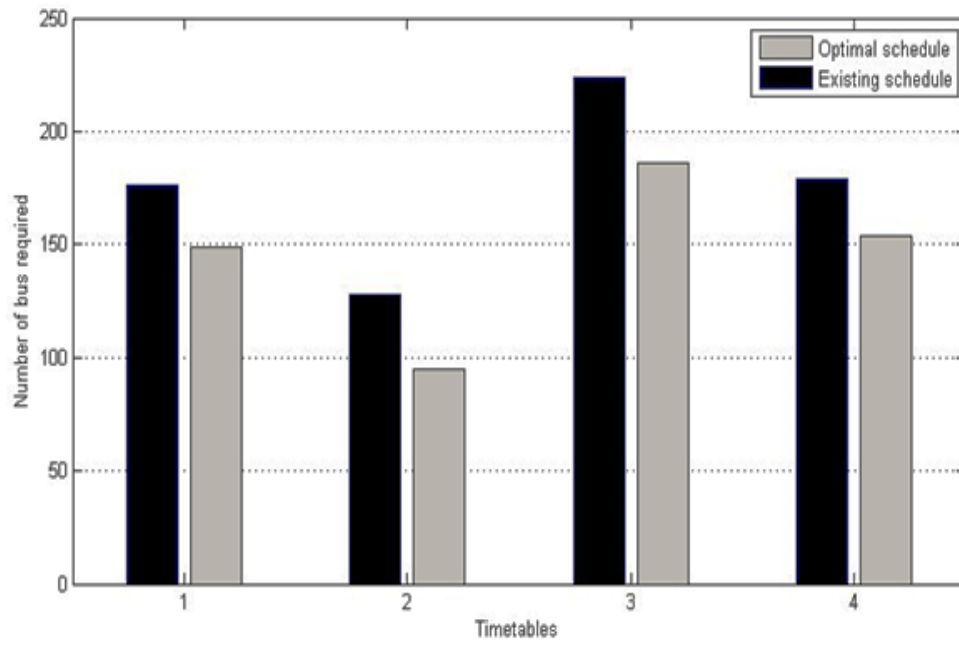
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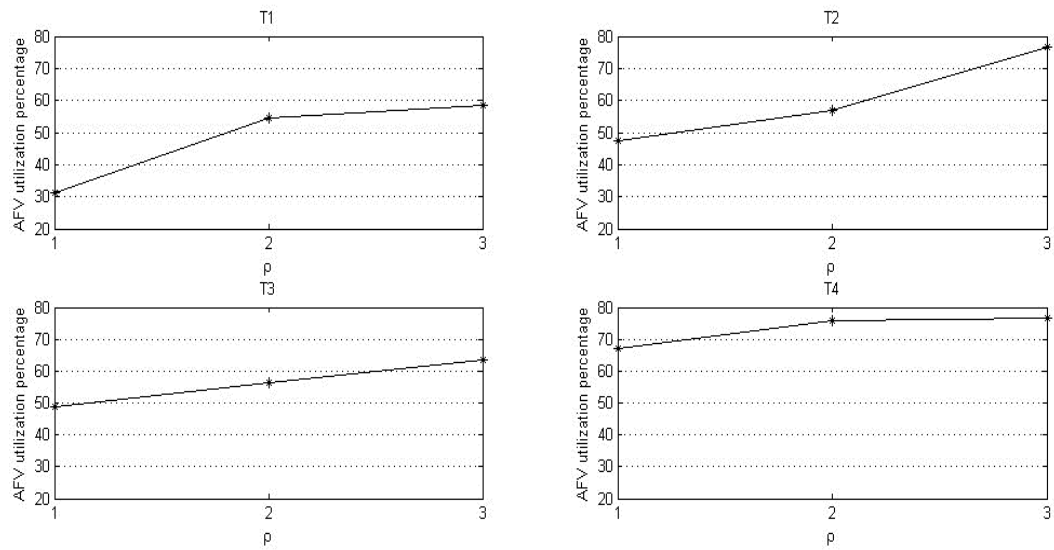
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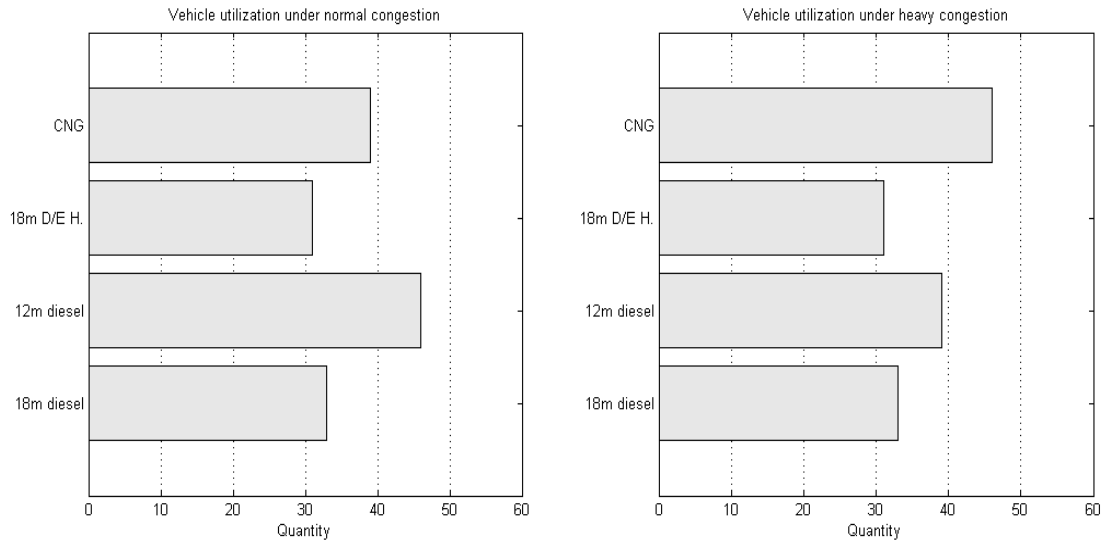
Figure 2.4: The Utilization of AFVs in Normal and Heavy Congestion Periods Trips (T1)
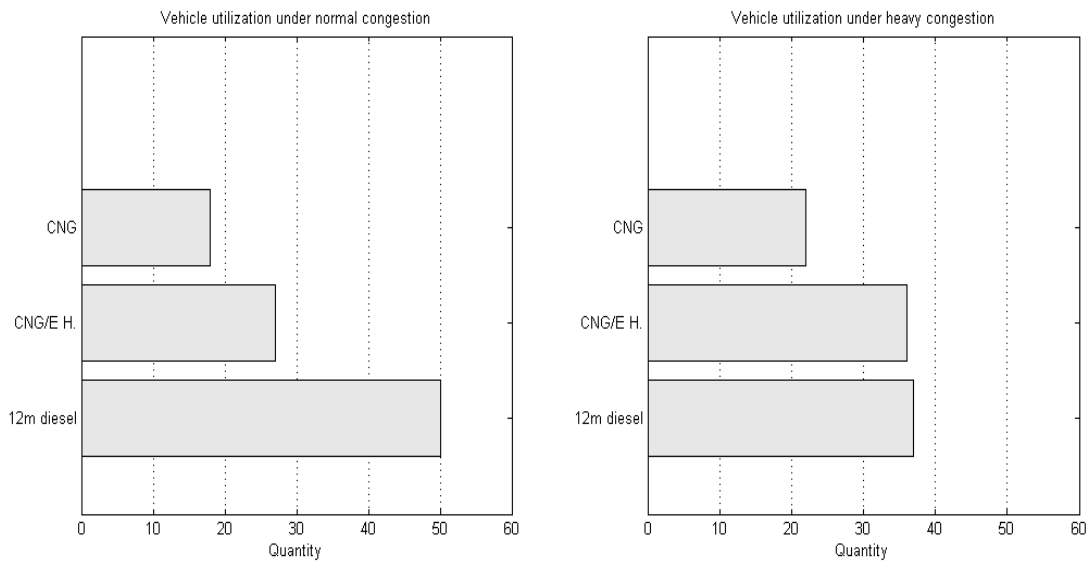
Figure 2.5: The Utilization of AFVs in Normal and Heavy Congestion Periods Trips (T2)
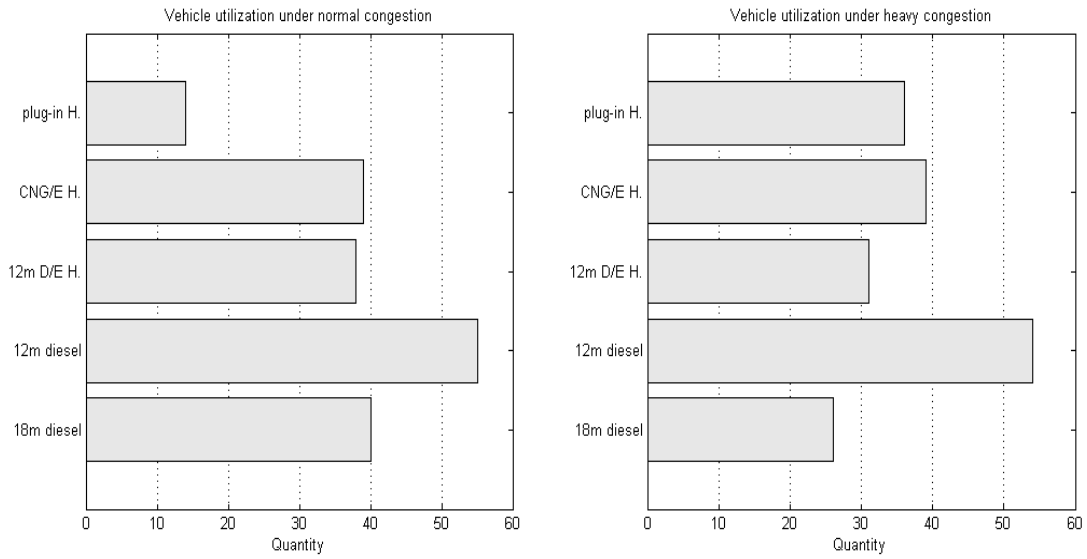
Figure 2.6: The Utilization of AFVs in Normal and Heavy Congestion Periods Trips (T3)
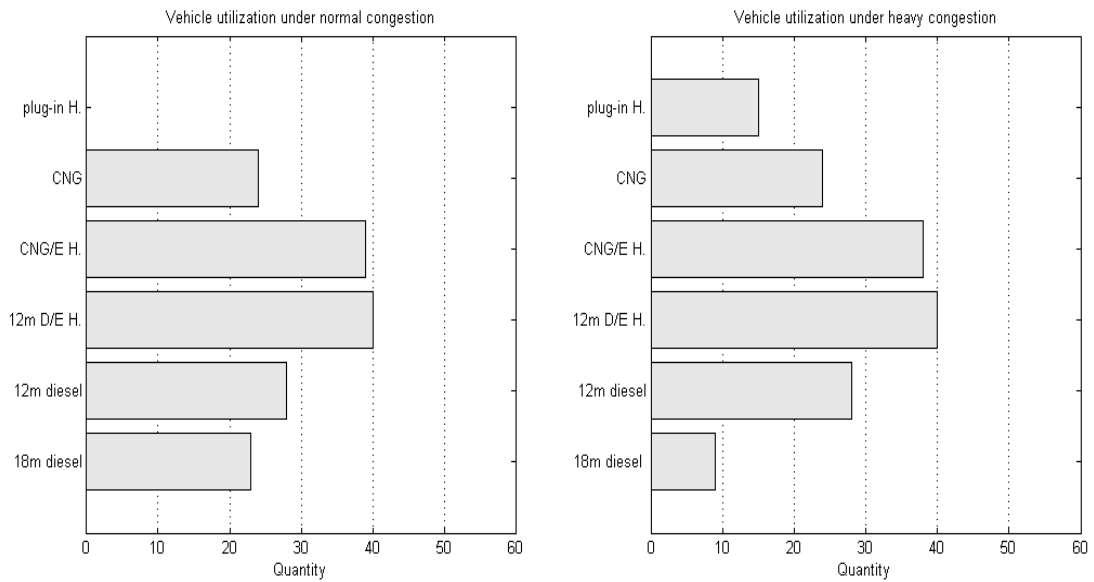
Figure 2.7: The Utilization of AFVs in Normal and Heavy Congestion Periods Trips (T4)

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Table 2.1: GHG Emission of Each Fuel Type

	Fuel Consumption / 100km	$GHG_{PTW} \left(\frac{gCO_2}{km} \right)$
Gasoline	55l	1191.78
Diesel	45l	1171.32
LPG	30kg	963.18
CNG	45kg	893.9
Electricity	150kWh	0

Table 2.2: Energy Efficiency Comparison between Different Fuel Types

Alternatives	Energy efficiency
Diesel	0.59
LPG	0.7
CNG	0.7
Electric with battery	0.79
Electric w dir.	0.79
Hybrid-diesel	0.63
Hybrid-CNG	0.73
Hybrid-LPG	0.73

Table 2.3: Cost of Diesel and Alternative-Fuel Buses (Unit: 1000 NT$)

Cost Items		Diesel	Pure Electric	Hybrid Electric	Natural Gas
Attainment cost	Purchase	90,000	300,000	360,000	300,000

Table 2.3: (Continued)

		10,000	40,000	40,000	120,000
	Recharge Equipment	**10,000**	**40,000**	**40,000**	**120,000**
	Total Cost	100,000	340,000	400,000	420,000
Operation Cost	Fuel Cost	12,000	1875	5880	9450
	Management Cost	2000	2000	2000	2000
	Total Cost	14,000	3875	7880	11,450
Maintenance Cost	Bus Maintenance Cost	11,400	18,495	22,200	7440
	Recharge Equipment Cost	0	0	0	2970
	Total Cost	11,400	18,495	22,200	10,410

Table 2.4: The Summary of Data

Timetable	Trips	Depots	Bus Type							
			18m D	12m D	12m D/E H.	18m D/E H	CNG/E H.	CNG	Plug-in	Total
T1	802	3	50	36	0	45	0	45	0	176
T2	596	2	0	54	0	0	36	38	0	128
T3	924	2	49	55	45	0	39	0	36	224
T4	659	3	30	28	40	0	39	24	18	179

Table 2.5: Parameters

Bus Type	Fuel Economy (100 km)		Fuel Price ($)	Maintenance cost ($/km)
	FE1	FE2		
18m Diesel	66.5 L	99.8 L	1.16 $/L	6.62 $/km
12m Diesel	40.0 L	60.0 L	1.16 $/L	6.62 $/km
12m D/E H.	29.5 L	44.3 L	1.16 $/L	15.89 $/km
18m D/E H.	43.9 L	65.9 L	1.16 $/L	15.89 $/km
CNG/E H.	39 m^3	58.5 m^3	0.52 $/m^3$	19.89 $/km
CNG	47.9 m^3	71.9 m^3	0.52 $/m^3$	9.04 $/km
Plug-in H.	100 kWh	150 kWh	0.08 $/kWh	26.75 $/km

D: Diesel E: Electric H: Hybrid CNG: Compressed Natural Gas

Table 2.6: Computational Results

Congestion Level	Timetable	18m D	12m D	12m D/E H.	18m D/E H.	CNG/E H.	CNG	Plug-in H.	Total	AFV Utilization (%)	Vehicle Reduction (%)	Cost	CPU(s)
$\rho = 1$	T1	46	33	0	31	0	39	0	149	31.3	15.3	193873	260.574
	T2	0	50	0	0	27	18	0	95	47.4	25.8	119006	73.957
	T3	40	55	38	0	39	0	14	186	48.9	17.0	241430	367.598
	T4	23	28	40	0	39	24	0	154	66.9	14.0	186472	252.257
$\rho = 2$	T1	35	33	0	31	0	50	0	149	54.4	15.3	211477	262.994
	T2	0	41	0	0	36	18	0	95	56.8	25.8	129052	75.506
	T3	26	55	38	0	39	0	28	186	56.5	17.0	252009	600.261
	T4	9	28	40	0	39	24	14	154	75.9	14.0	187629	265.377
$\rho = 3$	T1	29	33	0	37	0	50	0	149	58.4	15.3	226777	256.975
	T2	0	22	0	0	36	37	0	95	76.8	25.8	152832	77.93
	T3	26	42	43	0	39	0	36	186	63.4	17.0	260301	472.72
	T4	9	27	40	0	39	24	15	154	76.6	14.0	196897	264.975

Table 2.7: AFV Utilization of Trips in Peak Hours

Congestion Level (ρ)	18m Diesel	12m Diesel	18m D/E H.	Plug-in H.	Total Trips	Trips using AFVs	Utilization (%)
$\rho = 1$	33	46	31	39	350	164	46.9
$\rho = 3$	33	39	31	46	350	234	66.9

CHAPTER 3: AMBULANCE LOCATION AND RELOCATION THROUGH TWO STAGE ROBUST OPTIMIZATION

3.1 Introduction

Emergency Medical Services (EMS) is a necessary, complicated and highly organized system designed to provide medical care and transportation services under emergency which has a strongly restricted time frame. The process of intervention of ambulance to the scene in the incident typically includes four stages: (1) detection and reporting of the incident, (2) emergency call screening, (3) ambulance dispatching and (4) actual intervention by paramedics [21].

To provide this service to population, a fleet of ambulances need to be located over the region they serve. The measurement of the quality of service including response times, the types of care that EMS staffs are trained to provide, and the equipment to which they have access, etc., The most important attribute is response time which strongly correlated with patients' survival rate. Response time in EMS is the time interval from patients calling for service until being reached. Regulations based on the United States Emergency Medical Services Act of 1973 specified that 95% of calls should be reached within 10 minutes in urban areas, and 30 minutes in rural areas. [22]. A survey of the 200 largest cities in the United States by [23] indicated that over 75% of EMS providers use a target of 8:59 min or less as the common response time standard. Two types of covering constraints were proposed by [24], denoted by r_1 and r_2. In their notations, r_2 denotes the absolute covering constraints, it ensures all the demand must be covered by an ambulance within r_2 minutes. More strictly, the relative covering constraints is also

introduced. In that constraints, a proportion α of the total demand must be covered by ambulance

within r_1 minutes ($r_2 > r_1$). The absolute covering constraint coincides with the United States

EMS Act of 1973 while the relative covering constraint can be determined by EMS service

provider in order to achieve better service standard. Thus, the selection of the emergency medical

service locations is the major issue to the service provider in designing an efficient network. The

goal of emergency medical services is to provide better service coverage for the population and

quicker responses to the service calls, and further increase survival rate of the patients since time

is vital in emergency situations. In general, an emergency medical service call or demand is said

to be covered if they can be reached within a specific time. Hence it is important that ambulances

be at all time located to ensure an adequate coverage and quick response time. [25].

The ambulance location problem has been addressed by many authors previously. Some

focused on the strategic planning, including the system network design. Others emphasized on

the operational support and tactical decisions, for example, dynamically dispatch and relocation.

It is possible that the network components could lose functions and ambulances could

simultaneously become unavailable when disaster or crisis occur, for example, hurricanes in

Florida, tornado in Oklahoma, or even terrorist attack. In recent years, reliability of the service

system has addressed many attentions in terms of maintaining an efficient and effective

emergency medical service network. To hedge uncertainty, more authors study the stochastic

nature of the problem and proposed approaches using either stochastic programming paradigm or

queueing framework. [26], [27], [28], [29], [30], [31]. Those studies either incorporated

probabilistic constraints or used simplified assumptions by embedding the hypercube model or

queueing theory into the mathematical programming models. However, the exact probability

distribution is hard to be characterized by either accurate method or sufficient data in many

circumstances. In addition, most of the probabilistic models rely on simplified assumptions that will cause problems and become impractical in real life. Hence, probabilistic models could be improper and result in infeasible solutions. Two-stage robust optimization method is introduced to deal with this issue. In this method, the randomness is simply captured by an uncertainty set so that robust solutions can be found within the uncertainty set. Therefore, we can make second stage recourse decision after the uncertainty is identified. The two-stage robust optimization method has been widely applied in many areas [32], [33], [34], for example the facility location problems [35].

This study focuses on adopting two-stage robust optimization approach to study reliable emergency medical service location problem. There is no research study the impact of relocation operation on initial locations. While in the strategic design stage, the existing probabilistic models either don't consider relocations or only assign uncovered demand to other nearby available service sites. [26], [27], [36], [37], [38]. As [39] mentioned, the two-stage robust optimization modelling framework precisely represents the real decision making processes. Furthermore, this modelling framework enables us to consider relocation decisions when designing the network to address uncertainty issues. A customized solution algorithm, i.e., column-and-constraint generation method and approximation framework are implemented and tested on a real-world instance to demonstrate the performance of the algorithm and the network design modelling framework. The remainder of this chapter is organized as follows. Section 2 presents a review of relevant literature on probabilistic ambulance location models. In Section 3, two-stage robust optimization emergency medical service location models are proposed. In Section 4, a customized column-and-constraint generation method and approximation framework

is introduced and experiments are tested on a real-world instance. In Section 5, numerical results are presented and discussed. Conclusions and future research are presented in Section 6.

3.2 Literature Review

Most early studies of the ambulance location problem focused on static and deterministic location problems. The location set covering model (LSCM) introduced by [40] aims to minimize total number of ambulances needed to cover all demand points. This model is not realistic since most of the time all ambulances are not available due to service calls. The maximal covering location problem (MCLP) was proposed by [41] to counter some of the shortcomings of the LSCM. Instead of minimize total ambulances needed, MCLP maximizes the coverage of the demands. Several extensive static models have been developed by [42], [43], [44] and [24] to consider more aspects, such as different types of service vehicles and additional backup coverage. In practice, static models are not sufficient to capture the uncertainty of the operations of emergency services. Demand may become uncovered due to the unavailability of ambulances once vehicles are dispatched to calls. One way to address this issue is considering multiple demand coverages. [24] first introduce the double standard model (DSM) and solved by Tabu search algorithm. [45] modified the DSM to formulate a double-coverage objective function and single-coverage standard constraints, Tabu Search algorithm was used to find near optimal solution. Subsequently, [46] developed a dynamic DSM and the model was tested on the Island of Montreal. [47] introduced a multi-period covering model, considering the fact that coverage areas change throughout the day. Vehicle reposition are allowed to maintain a certain coverage standard. A metaheuristic method using variable neighborhood search was implemented to solve the problem. [48] modified the DSM to maximize the vehicle crash site coverage with a predefined number of ambulances dedicated to provide EMS services. A genetic

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algorithm was proposed to solve a real-world problem in the city of Thessaloniki, Greece. An extension of DSM developed by [49] considers multiple EMS vehicle types and variant coverage requirements for demand sites with different priority levels. The model was applied for allocating EMS vehicles in the Chicago urban area.

Another way to address the reliability matter are probabilistic models. One of the first probabilistic models for ambulance location problem is the maximum expected covering location problem (MEXCLP) studied by [26]. This model incorporates a busy fraction to represents the probability of unavailability of each ambulance. An extension of MEXCLP, called TIMEXCLP, was developed by [27], explicitly considered the variations in travel speed throughout the day. [36] proposed two other probabilistic models to formulate the maximum availability location problem (MALP I and MALP II). In MALP I, the busy fraction q is assumed to be the same for all sites while MALP II assigns different busy fraction to each site. Moreover, several articles proposed the estimation methods of the busy fraction. [28] developed the adjusted MEXCLP model (AMEXCLP) assumes that ambulances are not independently operated and the correction factor has been added to the objective function. Later, [29] proposed the queueing probabilistic location set covering problem (QPLSCP) to compute the minimum number of ambulances required to cover a demand point such that the probability of all of them being simultaneously busy does not exceed a given threshold. The extension of LSCM, called Rel-P, was developed by [30] incorporated a constraint that ensures the probability that a given demand call will not be satisfied does not exceed a certain value. A hypercube model was introduced by [50], which is considered to be capable of computing the busy fraction. Later studies have tried to incorporate the hypercube model in optimization frameworks for determining the location of emergence vehicles, such as [51], [52], [53], [54], [55] and [56]. The major drawback of the hypercube

model is that its computational complexity grows exponentially in terms of the number of applied ambulances. Recently, [57] introduced a new stochastic programming model incorporates probabilistic constraints within the traditional two-stage framework. [56] developed a new probabilistic coverage model based on MEXCLP and mixed with the hypercube queuing model. [58] developed two separate models for ambulance location and relocation with predefined crisis location, models are solved sequentially in three phases.

According to aforementioned models, they all involve very complicated mathematical programs. Standard solver CPLEX is usually not capable of dealing with large-scale problems, hence heuristic or exact solution approaches are developed. Those complicated programs require exact probabilistic information, which is not appropriate in practice. Robust optimization-based location models not rely on probability distribution, instead, it uses the uncertainty set to capture all the uncertainty scenarios and search for the robust locations in that uncertainty set [39]. Benders decomposition method is typically used to solve stochastic programming models due to its strong performance to generate cuts and reduce the feasible region. However, it can't handle large-scale practical problems. [59] demonstrates that *column-and-constraint generation* algorithm has greater performance in computation comparing to Benders method. This novel algorithm has been applied to facility location and power scheduling problems. Results show that the *column-and-constraint generation* algorithm can solve large-scale problem in very short time.

In this research, the reliable emergency medical service location design problem is formulated into the two-stage robust optimization framework. We introduce models with/without consideration of relocation operations. This unique feature has not been studied in previous literatures. In addition, we propose customized computational scheme for solving those two-

stage robust optimization models. In the relocation model, relocation costs are also considered to best capture realistic situation. An approximation algorithm on top of the customized scheme is developed. Experiments are performed on a real-world instance.

3.3 Two-stage Robust EMS Location Models

In this section, a set of two-stage robust ambulance location models with/without the consideration of relocation are introduced and discussed. As we mentioned, in two-stage robust optimization framework, an uncertainty set is adopted to represent all the possible uncertainty scenarios. The worst-case scenarios are then identified and optimal solutions in those cases are evaluated. In our ambulance location study, we follow the idea in [39] that all ambulances are homogeneous and we consider all the possible situations with up to k ambulances simultaneously become unavailable. We then represent the uncertainty set as

$$\boldsymbol{B} = \{ \boldsymbol{z} \in \{0,1\}^{|W|} : \sum_{j \in W} z_j \leq k \} \tag{11}$$

where $z_j = 1$ indicates ambulance at location j is unavailable and $z_j = 0$ otherwise. Note that this compact mathematical form captures an exponential number of scenarios.

3.3.1 Two-stage Robust EMS Location Model without Relocation

In the following, we first propose our two-stage RO EMS location models without relocation. Let I represents the set of demand sites with d_i being the estimated demand volume, W represents the set of potential ambulance sites. p ambulances need to be placed among those sites to maximize the coverage. Let a_{ij}, b_{ij} be adjacent matrices where $a_{ij} = 1$ indicates demand i is covered by site j within r_2 time units and $b_{ij} = 1$ indicate site i is covered by site j within r_1 time units, and $a_{ij} = b_{ij} = 0$ otherwise. Parameter $\alpha \in [0,1]$ is introduced as a weight coefficient to represent our consideration of unavailability situations.

In our two-stage decision framework, we use y to denote the first stage decision variable with $y_j = 1$ indicates an ambulance is located at j, $y_j = 0$ otherwise. In addition, u_i is introduced to reflect its relative coverage, i.e., $u_i = 1$ if demand at i is covered by an ambulance within r_1 time units under normal situation, $u_i = 0$ otherwise. Noting that $r_1 < r_2$, it would be desired to have a higher portion of demand under the relative coverage criterion. Similar variable v_i is introduced in the second stage when ambulance unavailability occurs. We use p to denote the total number of ambulances. The two-stage robust ambulance location model RO_EMSL$_1$ without relocation is formulated as follows:

$$C(p, k) = \max_{y,u} \sum_{i \in I} d_i u_i + \min_{z \in B} \max_{v \in S(y,z)} \alpha \sum_{i \in I} d_i v_i \tag{12}$$

$$s.t. \sum_{j \in W} a_{ij} y_j \geq 1, \quad \forall i \in I \tag{13}$$

$$\sum_{j \in W} y_j = p, \tag{14}$$

$$u_i \leq \sum_{j \in W} b_{ij} y_j, \quad \forall i \in I \tag{15}$$

$$u_i \in \{0,1\}, \forall i \in I; y_j \in \{0,1\}, \forall j \in W \tag{16}$$

$$S(y,z) = \{ v_i \leq \sum_{j \in W} b_{ij} \delta_j, \quad \forall i \in I \tag{17}$$

$$\delta_j \leq y_j, \quad \forall j \in W \tag{18}$$

$$\delta_j \leq 1 - z_j, \quad \forall j \in W \tag{19}$$

$$\delta_j \geq 0, \forall j \in W; 0 \leq v_i \leq 1, \forall i \in I \} \tag{20}$$

The objective function in (12) is to maximize the total relative coverage within r_1 time units under normal situation and in the worst case scenarios included in uncertainty set B.

Constraint (13) imposes that for every demand site, at least 1 ambulances is located within its r_2 time-unit neighborhood. Constraint (14) defines the total number of ambulances equals p. Constraint (15) indicates that $u_i = 0$ if there is no ambulance located within r_1 time units of demand site i, and $u_i = 1$ otherwise. Constraint (17), (18) and (19) indicate that $v_i = 0$ if there is no ambulance available within r_1 time units of demand site i, $v_i = 1$ otherwise. Constraint (20) imposes variable restrictions. Remarks are presented as follows.

1. In this study, we focus on the non-trivial cases where $k \leq p - 1$. Otherwise, in the worst-case scenario, there is no ambulance available and the problem reduces to maximize the relative coverage within r_1 time units in the normal situation.

2. It can be easily proven that the optimal value $C(p, k)$ is non-decreasing in regard to p; and non-increasing in regard to k.

    In the following section, we present an extension of EMS location model with the consideration of relocation operations.

3.3.2 Two-stage Robust EMS Location Model with Relocation

In real life, it is not possible to reposition all ambulances when uncertainty situation occurs, hence the advantage of RO_EMSL$_1$ model is limited to study the importance of those candidate sites. The two-stage robust EMS location model with relocation is an extension of the RO_EMSL$_1$ model. As mentioned, when a sufficient coverage cannot be maintained due to ambulance unavailability, ambulance relocation operations are often implemented to employ remaining ones to achieve a better coverage. To the best of our knowledge, the impact of relocation operations on the initial locations in EMS location problem has been paid no attention. The possible reason is that in those probabilistic models, all of the stochastic scenarios have to be evaluated, hence the model complexity and solution time increase exponentially with respect to

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the number of unavailable ambulances. Therefore, it is computationally prohibitive for those

models to capture all the possibilities and be solved in a reasonable time.

In RO_EMSL$_2$, variable $P_{ij} = 1$ represents relocating ambulance from site j to i, $P_{ij} = 0$

otherwise. Variable $q_j = 1$ denotes the case where an ambulance stays at site j (after

relocations), $q_j = 0$ otherwise. β is the relocation cost coefficient which represents the system

decision maker's attitude towards relocation cost. Also, **D** is the distance matrix. The two-stage

robust EMS location model RO_EMSL$_2$ with relocation is shown as follows:

$$C^r(\boldsymbol{p}, \boldsymbol{k}) = \max_{\boldsymbol{y},\boldsymbol{u}} \sum_{i \in I} d_i u_i + \min_{\boldsymbol{z} \in B} \max_{(\boldsymbol{P},\boldsymbol{q},\boldsymbol{v}) \in S^r(\boldsymbol{y},\boldsymbol{z})} (\alpha \sum_{i \in I} d_i v_i - \beta \sum_{i \in W} \sum_{j \in W} D_{ij} P_{ij}) \tag{21}$$

$s.t. \ (13) - (16)$

$$S^r(\boldsymbol{y}, \boldsymbol{z}) = \{ v_i \le \sum_{j \in W} b_{ij} q_j, \quad \forall i \in I \tag{22}$$

$$\sum_{j \in W} P_{ij} \ge q_j, \quad \forall i \in W \tag{23}$$

$$P_{ij} \le 1 - z_j, \quad \forall i, j \in W \tag{24}$$

$$P_{ij} \le y_j, \quad \forall i, j \in W \tag{25}$$

$$\sum_i P_{ij} \le 1, \quad \forall j \in W \tag{26}$$

$$\sum_j P_{ij} \le 1, \quad \forall i \in W \tag{27}$$

$$P_{ij} \in \{0,1\}; \ 0 \le v_i \le 1, \forall i \in I; \ 0 \le q_j \le 1, \forall j \in W \} \tag{28}$$

The objective function in (21) maximize total relative coverage within r_1 time units under

normal situation and in the worst-case scenarios included in uncertainty set B, cost penalty based

on distances is considered. Constraint (22) defines upper bound of v_i, which depends on the

status of q_j for site j within r_1 time units. Constraint (23) indicates the relation between P_{ij} and q_i. Note that $q_i = 0$ if there is no ambulance moving to i. Constraints (24) and (25) ensure that relocation operation does not occur if no ambulance available at site j. And constraint (26) and (27) guarantee that the ambulance at j can only be relocated to one site and one site can only host one ambulance. Constraint (28) imposes variable restrictions. Remarks are presented as follows:

1. It also can be easily proven that the optimal value $C^r(p, k)$ is non-decreasing with respect to p; and non-increasing with respect to k.

2. Since binary variables are introduced in the second stage problem, the recourse problem is a mixed integer program (MIP), standard *column-and-constraint generation* method is not applicable. Hence, we seek to extend the *column-and-constraint generation* in an approximation framework to achieve a balance between the solution quality and computational speed.

3.4 Computation Algorithm for Two-stage Robust Optimization Models

In this section, we present the implementation of the computational algorithms for both RO_EMSL_1 and RO_EMSL_2 models. We first describe the *Column-and-constraint generation* method which is a two level procedure involving computing master problem (MP) and subproblem (SP). In master problem, we consider a small subset B and build an MIP model with recourse problems for each individual scenario of this subset. Its optimal value provides an upper bound. Then, a lower bound can be derived from the subproblem. Once upper bound and lower bound match, we can conclude that y^* provides optimal locations for ambulances.

3.4.1 Implementation of RO_EMSL_1 Model

Notice that, the third level problem of RO_EMSL_1 model is a linear program, by applying the strong duality, we could transform the third level problem by finding its dual problem to a

minimization problem. Together with the second level minimization problem, they can be reduced to a single level minimization problem. And we let m, w, t, γ, o be the corresponding dual variables of constraints (17) - (20). Then we have the following non-linear minimization formula of subproblem $SP1_{nl}$:

$$V_1(y^*, u^*) = min \sum_j w_i y_i + \sum_j t_j(1 - z_j) + \sum_i o_i \tag{29}$$

$$s.t. \quad m_i + o_i \geq \alpha d_i, \quad \forall i \tag{30}$$

$$w_j - \sum_i b_{ij} m_i + t_j - \gamma_j \geq 0, \quad \forall j \tag{31}$$

$$\sum_j z_j \leq k \tag{32}$$

$$w_i \geq 0, \forall i; t_i \geq 0, \forall i; o_i \geq 0, \forall i; z_j \in \{0,1\}, \forall j \tag{33}$$

The non-linear term are products of binary variables and continuous variables. Then we can linearize those constraints by replacing the non-linear terms with a set of new variables, i.e., $T_j = t_j z_j$, and using Big-M method. The linearized subproblem $SP1_l$ can be written as:

$$V_1(y^*, u^*) = min \sum_j w_i y_i + \sum_j t_j - \sum_j T_j + \sum_i o_i \tag{34}$$

$$s.t. \quad (30) - (32)$$

$$T_j \leq t_j, \quad \forall j \tag{35}$$

$$T_j \leq M z_j, \quad \forall j \tag{36}$$

$$T_j \geq t_j - M(1 - z_j), \quad \forall j \tag{37}$$

$$T_j \geq 0, \quad \forall j \tag{38}$$

$$w_i \geq 0, \forall i; t_i \geq 0, \forall i; o_i \geq 0, \forall i; z_j \in \{0,1\}, \forall j \tag{39}$$

Next, we present steps of C&CG algorithm where UB and LB are upper and lower bounds respectively, n is the iteration limit, ϵ represents the optimality tolerance.

Algorithm 1

Column-and-Constraint generation method for **RO_EMSL$_1$**

---

1. Set UB $= \infty$; LB $= -\infty$, and iteration counter n $= 0$.

2. Solve the following master problem (MP), derive an optimal solution $(\boldsymbol{y^n}, \boldsymbol{u^n}, \boldsymbol{\eta^n})$, UB is obtained and set to its optimal value $\sum_i d_i u_i^n + \eta^n$.

   MP: $max \sum_i d_i u_i + \eta$

   $s.t. (13) - (16)$

   $$\eta \le \alpha \sum_i d_i v_i^t, \quad \forall t = 1,2,\dots,n,$$

   $$v_i^t \le \sum_{j \in W} b_{ij} \delta_j^t, \quad \forall i, t = 1,2,\dots,n,$$

   $$\delta_j^t \le y_j, \quad \forall j \in W, \quad \forall t = 1,2,\dots,n,$$

   $$\delta_j^t \le 1 - z_j^t, \quad \forall j \in W, \quad \forall t = 1,2,\dots,n,$$

   $$\delta_j^t \ge 0, \forall j \in W, t = 1,2,\dots,n; 0 \le v_i^t \le 1, \quad \forall i, t$$

   $$= 1,2,\dots,n; \eta \ free$$

3. Solve SP1$_l$, derive $\boldsymbol{z^o}$ and update LB $=$ $\max\{LB, \sum_{i \in I} d_i u_i^n + \tilde{V}_1(y^n, u^n)\}$ accordingly.

4. If either one of the following occurs, (i) $(\text{UB} - \text{LB})/\text{LB} \le \epsilon$; (ii) $\mathbf{z}^o \in \{\mathbf{z}^1, \dots, \mathbf{z}^n\}$; (iii) $n \ge \bar{n}$, we terminate the algorithm and obtain solution $(\mathbf{y}^n, \boldsymbol{u}^n)$ with $(\text{UB} - \text{LB})/\text{LB}$ as its quality. Otherwise, we set $\mathbf{z}^{n+1} = \mathbf{z}^o$, $n = n + 1$, and go to Step 2.

3.4.2 Implementation of RO_EMSL$_2$ Model

For RO_EMSL$_2$ model, the subproblem is given by

$$SP2: V_2(y^*, u^*) = \min_{z \in B} \max_{(P,q,v) \in S^r(y^*,z)} \left(\alpha \sum_{i \in I} d_i v_i - \beta \sum_{i \in W} \sum_{j \in W} P_{ij} D_{ij} \right) \tag{40}$$

Due to integer restriction on variable P_{ij}, strong duality is not valid anymore. To address this challenge, we adopt the following procedure to approximately solve SP2.

The approximation procedure to compute SP2 is:

1. We consider LP relaxation of the recourse problem in computing SP2 to derive the optimal $\mathbf{z}^*$;

2. Compute the MIP recourse problem with respect to $(\boldsymbol{y}^*, \boldsymbol{u}^*, \mathbf{z}^*)$ to derive optimal relocation decisions $\boldsymbol{P}^*$;

3. Fixing $P_{ij} = P_{ij}^*$ for all i; j, we re-compute SP2 by using strong duality, and obtain the optimal value $\widetilde{V_2}(\boldsymbol{y}^*, \boldsymbol{u}^*)$ and $\mathbf{z}^o$.

Although the aforementioned procedure computes SP2 approximately, the value $\sum_{i \in I} d_i u_i^* + \check{V}_2(y^*, u^*)$ is a lower bound to the optimal value $C^r(p, k)$ of RO_EMSL$_2$. Again, if we note the upper and lower bounds match, optimal y^* is certainly derived. In other cases, the

gap between the upper and lower bounds can be used to evaluate the quality of a feasible

solution.

Let $m, t, e, \theta, g, l, h, o, s$ be the corresponding dual variables of constraints

$(22) - (28)$. Then we have the following non-linear minimization formula of subproblem $SP2_{nl}$:

$$V_2(y^*, u^*) = min \sum_i \sum_j e_{ij}(1 - z_j) + \sum_i \sum_j \theta_{ij} y_j + \sum_j g_j + \sum_i l_i + \sum_i h_i$$

$$+ \sum_j o_j + \sum_{ij} s_{ij}$$

(41)

$$s.t. f_i + o_i \geq \alpha d_i, \quad \forall i$$

(42)

$$-m_i + e_{ij} + \theta_{ij} + g_j + l_i + s_{ij} \geq -\beta D_{ij}, \quad \forall i, j$$

(43)

$$M_i - \sum_i b_{ij} f_i + o_j \geq 0, \quad \forall i, j$$

(44)

$$\sum_j z_j \leq k$$

(45)

$$m_i, f_i, l_i, h_i \geq 0, \forall i; e_{ij}, \theta_{ij}, s_{ij} \geq 0, \forall i, j; g_j, o_j \geq 0, \forall j; z_j \in \{0,1\}, \forall j$$

(46)

Then we replace the non-linear terms, i.e., $E_{ij} = e_{ij} z_j$ and using Big-M method. The

linearized subproblem $SP2_l$ can be written as:

$$V_2(y^*, u^*) = min \sum_i \sum_j e_{ij} - \sum_i \sum_j E_{ij} + \sum_i \sum_j \theta_{ij} y_j + \sum_j g_j + \sum_i l_i$$

$$+ \sum_i h_i + \sum_j o_j + \sum_{ij} s_{ij}$$

(47)

$$s.t. (42) - (46)$$

$$E_{ij} \leq e_{ij}, \quad \forall i, j$$

(48)

$$E_{ij} \leq M z_j, \quad \forall i, j$$

(49)

$$E_{ij} \geq e_{ij} - M(1 - z_j), \quad \forall i, j \tag{50}$$

$$E_{ij} \geq 0, \quad \forall i, j \tag{51}$$

Next, we present steps of our approximate algorithm for RO_EMSL$_2$.

<div align="center">Algorithm 2</div>

<div align="center">Column-and-Constraint generation method for RO_EMSL$_2$</div>

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1. Set UB $= \infty$; LB $= -\infty$, and iteration counter n $= 0$.

2. Solve the following master problem (MP) and obtain an optimal

   solution (y^n, u^n, η^n), UB is obtained and set to its optimal value

   $\sum_i d_i u_i^n + \eta^n$.

   MP: $\max \sum_i d_i u_i + \eta$

   $s.t. (13) - (16)$

   $$\eta \leq \alpha \sum_i d_i v_i^t - \beta \sum_i \sum_j D_{ij} P_{ij}^t, \quad \forall t = 1, 2, \ldots, n,$$

   $$v_i^t \leq \sum_j b_{ij} q_j^t, \quad \forall i, t = 1, 2, \ldots, n,$$

   $$\sum_j P_{ij}^t \geq q_i^t, \quad \forall i, t = 1, 2, \ldots, n,$$

   $$P_{ij}^t \leq 1 - z_j^t, \quad \forall i, j, t = 1, 2, \ldots, n,$$

   $$P_{ij}^t \leq y_j, \quad \forall i, j, t = 1, 2, \ldots, n,$$

   $$\sum_i P_{ij}^t \leq 1, \quad \forall i, j, y = 1, 2, \ldots, n$$

   $$\sum_j P_{ij}^t \leq 1, \quad \forall i, j, y = 1, 2, \ldots, n,$$

   $$P_{ij}^t \in \{0, 1\}, \quad \forall i, j, t = 1, 2, \ldots, n;$$

$$0 \le v_i^t \le 1, \quad \forall j, t = 1, 2, \dots, n,$$

$$0 \le q_j^t \le 1, \quad \forall j, t = 1, 2, \dots, n; \eta \; free$$

3. Solve SP2$_l$ by the approximation procedure, derive z^o and

   update $LB = max\{LB, \sum_{i \in I} d_i u_i^n + \widetilde{V}_1(y^n, u^n)\}$ accordingly.

4. If either one of the following occurs, (i) $(UB - LB)/LB \le \epsilon$; (ii)

   $z^o \in \{z^1, \dots, z^n\}$; (iii) $n \ge \bar{n}$, we terminate the algorithm and

   obtain solution (y^n, u^n) with $(UB - LB)\backslash LB$ as its quality.

   Otherwise, we set $z^{n+1} = z^o, n = n + 1$, and go to Step 2.

3.5 Numerical Study

In this section, the description on a real-world data is introduced first. Followed by experimental setup. Then, the results are illustrated and discussed to produce insights on those robust EMS location models.

3.5.1 Data Description and Computational Experiments

For our computational experiments, we used real-world data from the city of Tampa (Florida, USA). The demand site information, including population and coordinates are obtained using census tract data from [60]. There are total of 171 census tracts in Tampa CCD with population ranging between 0 and 9162. Demand are represented by the probability of calls that are proportional to the population. Two datasets with 30 and 50 potential sites were randomly generated. For simplicity, we used the Euclidean distance between any two sites. The maps are presented in Figure 3.1 and Figure 3.2.

In our study, r_1 is set to 10 minutes and r_2 is set to 12 minutes assuming a constant vehicle speed of 60 MPH. Also, we set $Gap = 0.05$ and time limit to 60 minutes. Models and

algorithms are implemented in C++ on a Dell OptiPlex 7020 desktop computer (Intel Core i7-4790 CPU, 3.60 GHz, 16 GB of RAM) in Windows 7 environment. We use a mixed integer programming solver, CPLEX 12.5 to solve the master problems and subproblems.

3.5.2 Computational Results of RO_EMSL$_1$ Model

Table 3.1 shows the computational results of RO_EMSL$_1$ model. In this table, column Time (s) is the computational time in seconds; column Iter represents the number of iterations; column Obj is the best optimal value ever found; column Gap (%) indicates the relative gap in percentage if it is larger than ϵ. From Table 3.1, we observe that all the instances can be solved within time limit and limited iterations. This indicates that the C&CG algorithm has a superior performance in solving reliable ambulance location problem. Table 3.2 shows the optimal locations for different scenarios.

3.5.3 Computational Results of RO_EMSL$_2$ Model

Table 3.3 shows the computational results of RO_EMSL$_2$ model. From the table, we observe that most of the instances can be solved within very short time and limited iterations. The computation complexity increases with respect to *p, k*.

Next, we demonstrate how changes of β values affect optimal solutions. In RO_EMSL$_2$ model, penalty is based on the Euclidean distances between sites, and is determined by both *p* and *k*. We randomly select one instance, i.e., $p = 12, k = 6$ and generate corresponding maps to show our model's capability of reflecting decision maker's attitude towards relocation costs. Table 3.4 shows the results, column Optimal solutions are the solutions for *Pij*. And we consider four levels of β which indicate the decision maker's attitude towards relocation cost from low to high. Low β indicates that decision maker does not concern about the relocation cost, and vice versa. From the table, we observe that the no. of ambulances need to be relocated decreases

when β value increases. A graphical display of ambulance locations and relocations are presented in Figure 3.3, Figure 3.4, Figure 3.5 with different β values. The hollow and solid hospital symbol representing 12 selected sites. In the worst-case scenarios, ambulances at solid ones are unavailable or busy, and hollow ones indicate ambulances are available. Black solid line with arrows indicates the relocation movements.

3.6 Conclusions

In this study, we present two-stage robust optimization models to determine locations of emergency medical service with/without consideration of relocation operations. We implemented the standard *column-and-constraint generation* method for RO_EMSL$_1$ model. Then, in order to solve the challenging model RO_EMSL$_2$ with a mixed integer recourse problem, we customized and implemented an approximation extension to the standard *column-and-constraint generation* algorithm as the solution method. The experiments are conducted to a practical-scale instance with real world data, we note our approximation method demonstrates strong capability in producing either optimal solutions or solutions with very small optimality gaps.
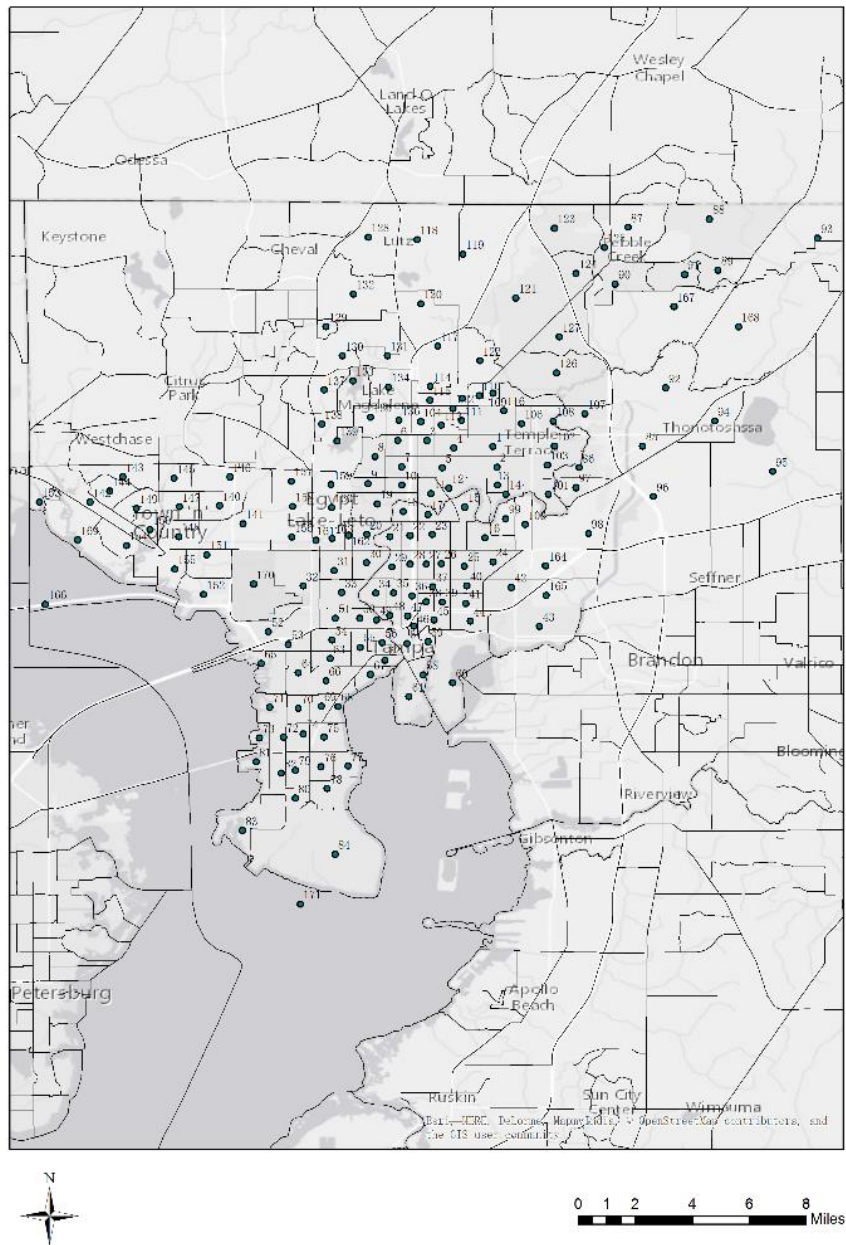
Figure 3.1: Tampa CCD Demand Map

Population
POP
- .000000 - 1444.000000
- 1444.000001 - 3165.000000
- 3165.000001 - 4520.000000
- 4520.000001 - 6140.000000
- 6140.000001 - 9162.000000
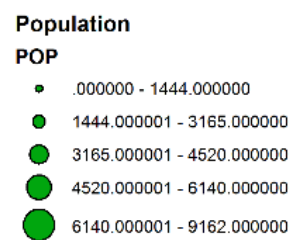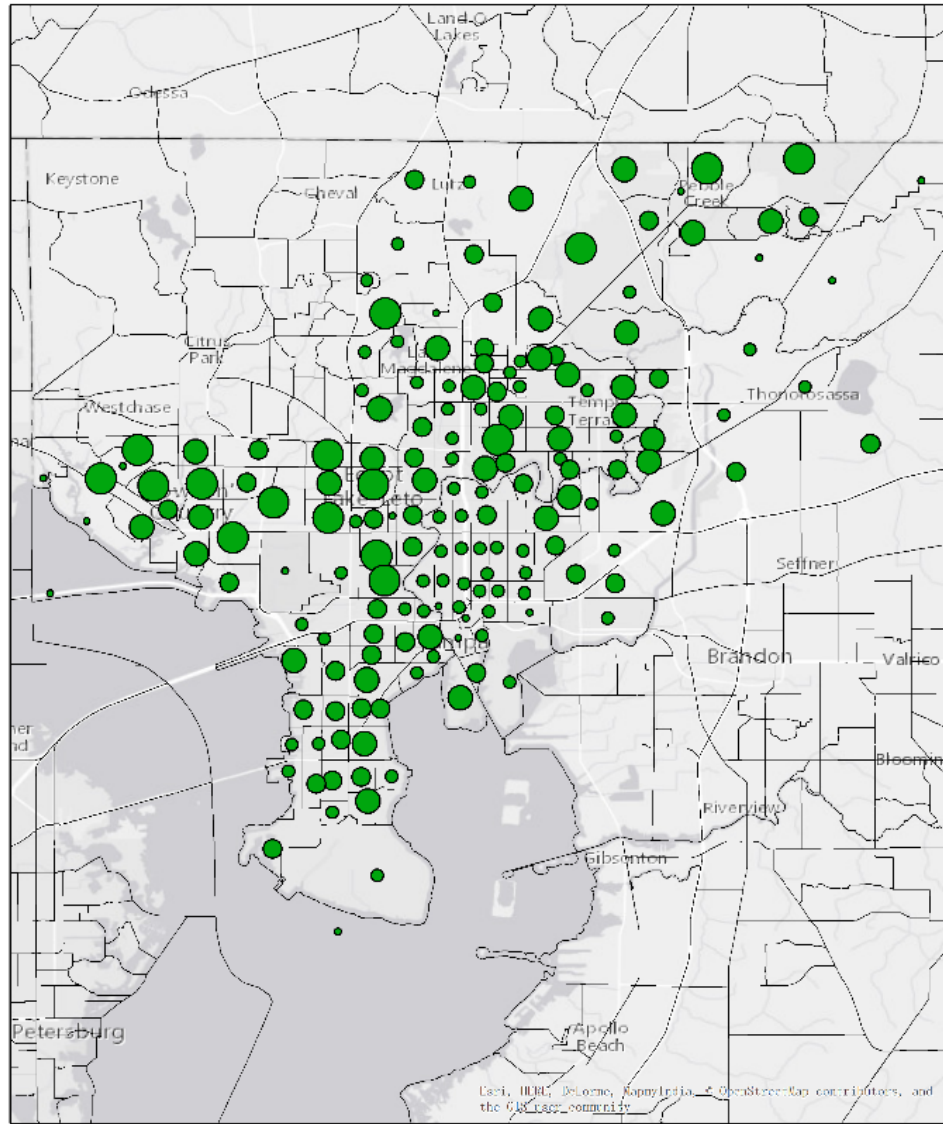
Figure 3.2: Tampa CCD Demand Map with Population
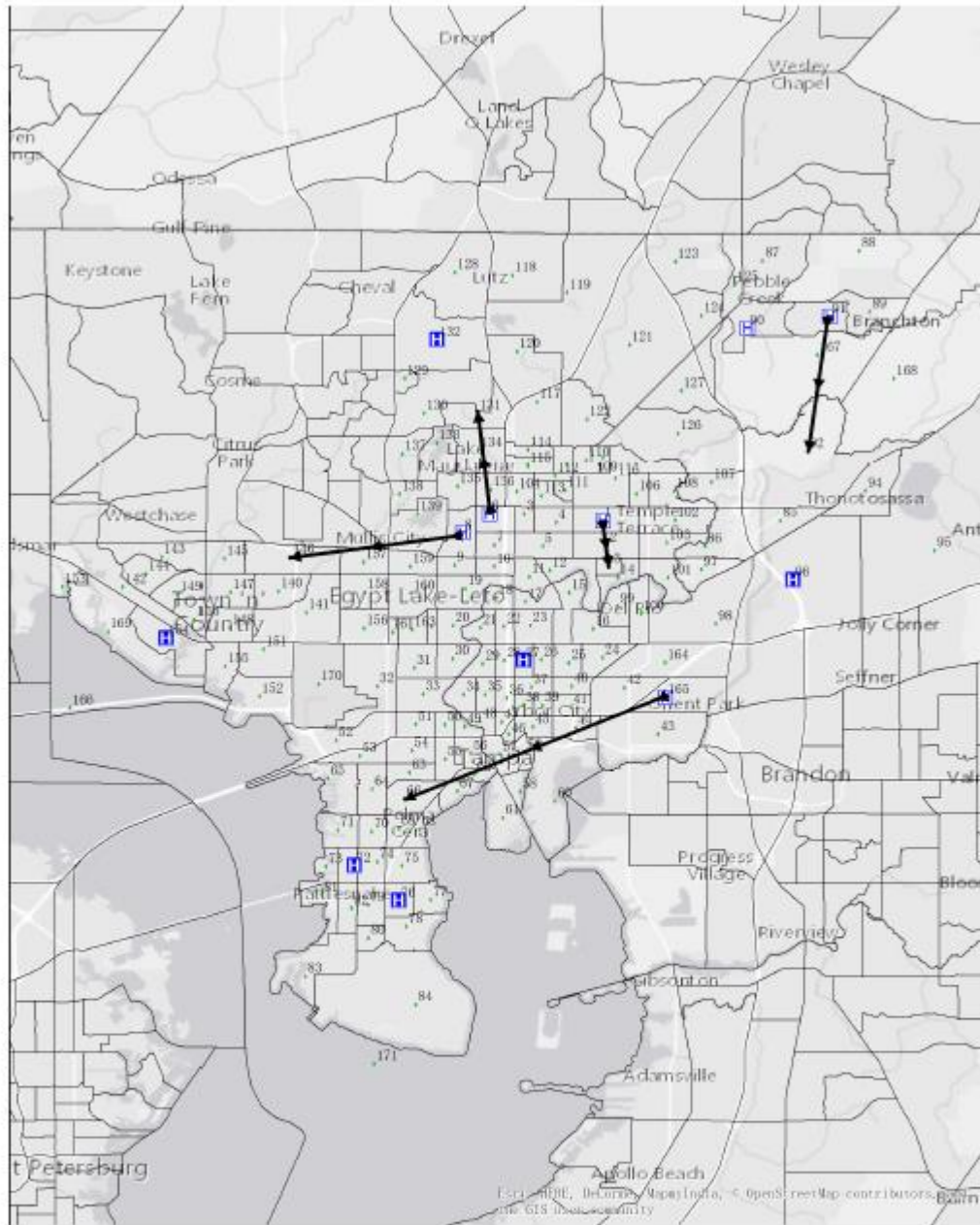
Figure 3.3: Relocation Strategy for β = 3000

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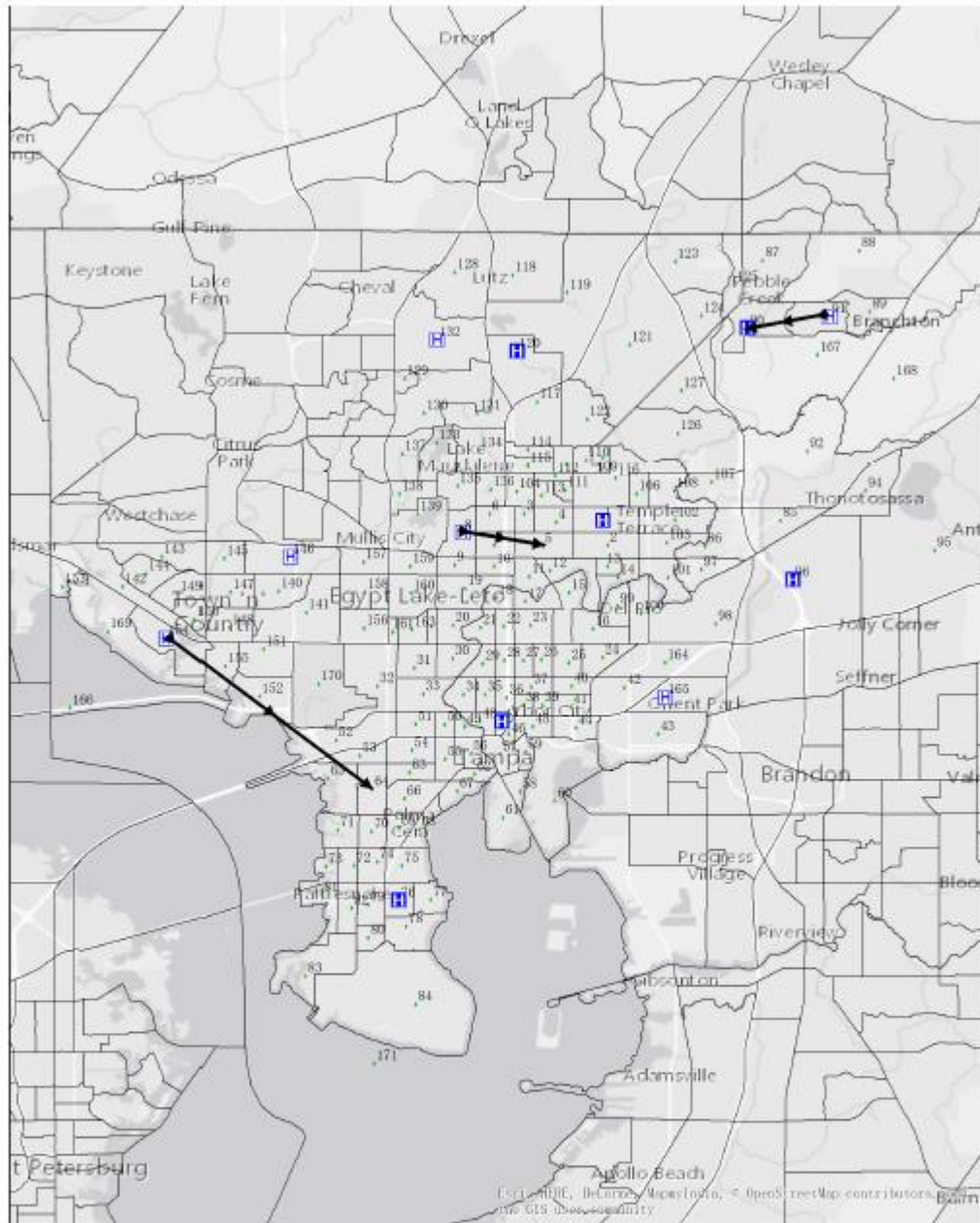
Figure 3.4: Relocation Strategy for β = 6000
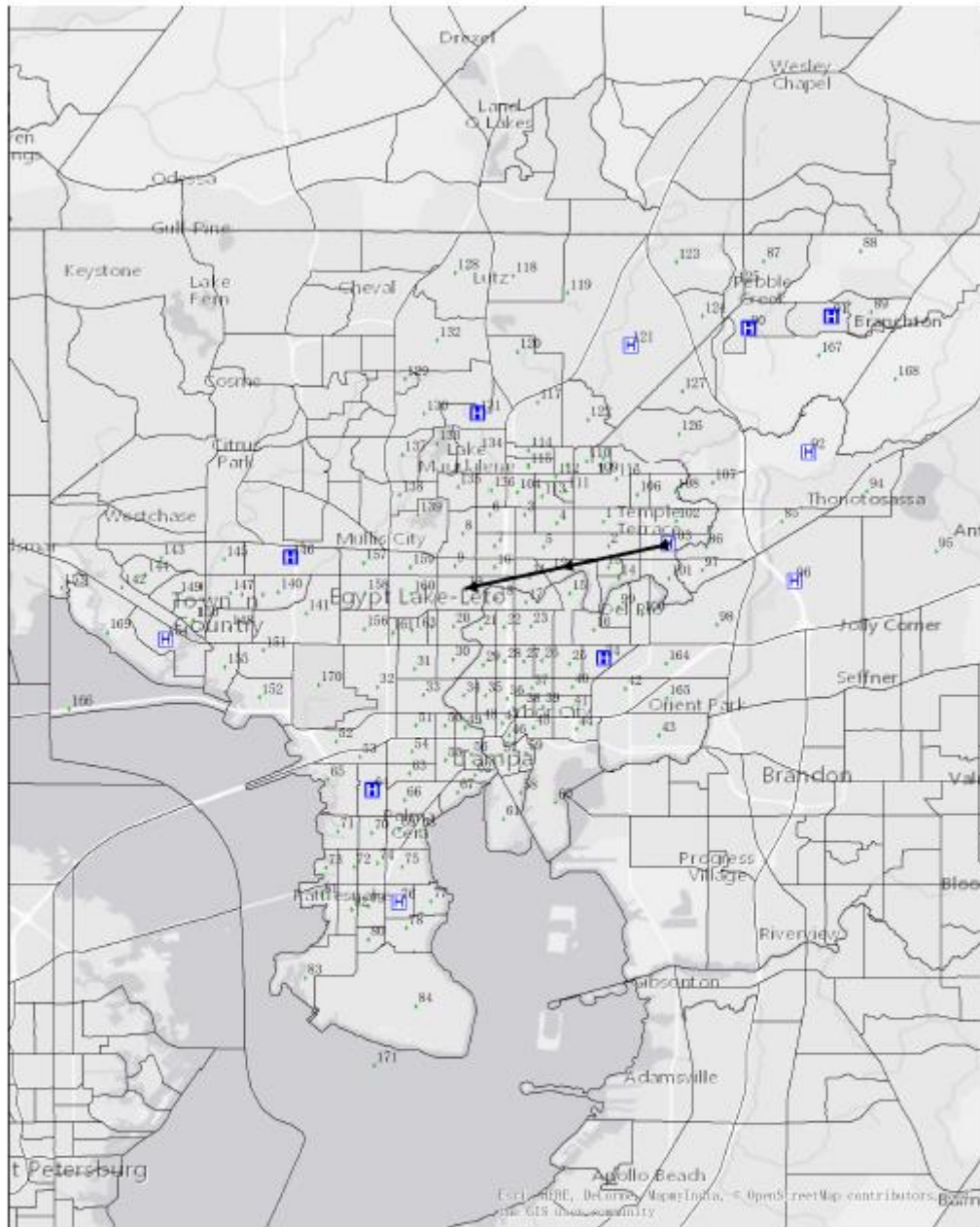
Figure 3.5: Relocation Strategy for β = 9000

Table 3.1: Computational Results of RO_EMSL_1 Model

$|W|$	p	k	Time (s)	Iter	Obj	Gap (%)
30	10	2	1.888	7	1190610	
		4	2.955	7	1158640	
		6	5.922	11	1123420	
		8	3.399	8	1088490	
	12	2	0.465	3	1088490	
		4	1.953	7	1181420	
		6	4.904	9	1151420	
		8	6.212	11	1112780	
50	10	2	2.282	9	1191140	
		4	39.605	17	1157670	
		6	840.079	35	1119820	
		8	847.714	40	1089440	
	12	2	1.081	5	1207040	
		4	354.995	36	1165480	
		6	T	46	1120760	6.9
		8	T	57	1080510	9.2

Table 3.2: Optimal Locations of RO_EMSL_1 Model

$|W|$	p	k	y^*
30	10	2	2 19 68 76 90 92 99 120 149 167
		4	2 23 63 76 90 92 120 146 149 167
		6	9 40 68 76 90 92 106 120 149 167
		8	30 63 76 85 90 99 134 140 149 167
	12	2	23 47 52 72 76 90 92 106 120 146 149 167
		4	23 63 76 90 92 99 120 126 131 140 149 167
		6	8 49 72 76 92 99 106 131 140 146 149 167
		8	2 40 63 72 76 92 110 120 140 146 149 167
50	10	2	5 40 72 76 90 91 92 132 151 154
		4	14 58 76 90 91 96 120 135 151 154
		6	11 66 80 91 96 99 105 120 151 154
		8	8 27 32 72 80 91 92 103 120 149
	12	2	32 47 49 80 90 91 96 108 132 135 146
		4	8 24 47 72 80 90 91 92 105 132 151 154
		6	10 32 47 72 76 90 91 92 103 120 149 151
		8	6 32 51 76 80 90 91 92 99 107 132 154

Table 3.3: Computational Results of RO_EMSL_2 Model

$|W|$	p	k	Time (s)	Iter	Obj	Gap (%)
30	10	2	0.049	3	1190820	
		4	8.184	6	1160610	
		6	83.413	11	1126910	
		8	259.63	15	1094680	
	12	2	0.596	2	1211460	
		4	2.288	4	1188950	
		6	9.814	6	1157500	
		8	207.23	14	1123960	
50	10	2	0.708	2	1194020	
		4	127.063	7	1186730	
		6	2784.76	14	1158480	
		8	1229.72	11	1146710	
	12	2	0.634	2	1215140	
		4	2.817	4	1196870	
		6	272.072	8	1181330	
		8	T	23	1160160	5.6

Table 3.4: Optimal Locations of RO_EMSL_2 Model

$|W|$	p	k	y^*
30	10	2	40 72 76 90 92 126 131 146 149 167
		4	23 68 76 92 99 120 134 146 149 167
		6	40 52 76 85 90 110 120 146 149 167
		8	9 22 47 72 76 85 106 120 149 167
	12	2	52 68 76 90 92 99 106 110 131 140 149 167
		4	8 40 68 76 90 92 99 110 120 146 149 167
		6	2 31 68 76 90 92 99 120 131 146 149 167
		8	2 31 47 68 76 90 92 131 134 146 149 167
50	10	2	2 6 8 13 47 76 91 92 120 149
		4	5 24 64 76 91 92 121 131 146 154
		6	24 66 76 91 96 107 120 131 154 162
		8	10 38 64 76 91 92 107 121 132 154
	12	2	5 6 8 10 11 31 40 80 91 92 120 149
		4	6 8 27 47 72 76 90 91 96 120 132 149
		6	8 13 27 66 76 90 91 92 121 131 149 154
		8	38 66 80 90 91 96 108 120 121 146 154 162

Table 3.5: Solution of Instance with p=12, k=6

p	k	β	No. of Ambulances Need Relocation	From	To	From	To
12	6	3000	5	13	1	66	165
				90	90	92	91
				131	6	146	8
		6000	3	5	8	64	154
				90	90	131	132
				146	146	165	165
		9000	1	19	103	76	76
				92	92	96	96
				121	121	154	154

CHAPTER 4: PESSIMISTIC OPTIMIZATION FOR VEHICLE SHARING PROGRAM NETWORK DESIGN

4.1 Introduction

Vehicle sharing programs (VSPs) have gained its popularity rapidly in recent years. In 2010, there were an estimated 100 bicycle sharing programs in 125 cities worldwide with more than 139,300 bicycles, and 45 were being planned in 22 nations [61]. Car sharing programs exist in 600 cities, in 18 nations and on 4 continents [62]. The main factors for the growth of vehicle sharing are the increasing congestion levels in urban cities; growing costs of personal vehicle ownership; people's mind change towards the ownership of cars. Unlike private vehicle, VSP gives user the opportunity to utilize vehicle on demand. User can check out vehicles (bicycles, cars) close to their origin and drop them off at VSP stations near their destinations. The cost of utilizing such system is usually determined by the time of vehicle usage and mileage driven. Some programs implemented an incentive mechanism that encourages user return vehicles to designated VSP stations. The principle of shared-use vehicles is simple: users take advantage of VSP to reduce their travel time and improve travel utility without worrying about the costs and responsibilities of ownership. [63].

The VSP is an economic and sustainable urban transportation solution. VSP can either be functioned as the individual modal which is independent of public transits, or as the extension of the public transit network which provides the "last-mile" connections to the existing transit services. For instance, for shorter trips, shared vehicles can be the sole transit modal from origin

to destination, while for longer trips, they can serve as the intermodal connect to existing transit services so that users can take it as part of their travel itinerary. Therefore, the system network design need to be carefully considered by the VSP operators. At system planning stage, VSP operators determine the locations of the VSP stations, the number of slots and initial number of shared-vehicles in each station. To exploit the potential of intermodalism, an optimal VSP network design should be strongly collaborated with the existing transportation networks [64]. The VSP network design problem can be formulated as bi-level equilibrium network design model. The model involves two stakeholders, VSP provider and user. On the upper level, VSP provider (leader) determines the location of VSP stations, capacity and initial number of vehicles in each station, while on the lower level, user (follower) react to the configuration by selecting routes travel from origin to destination. VSP provider then adjusts the configuration in order to achieve maximum utilization of the system. This leader-follower framework is essentially a Stackelberg equilibrium, since the non-cooperative nature of the two stakeholders.

In this chapter, we study the pessimistic bi-level VSP network design problem which receives no attention previously. We propose a pessimistic bi-level VSP network design model and customized computational scheme. The model is tested on several prototypical transit network instances. Experimental results show the significant differences between optimal solutions in optimistic and pessimistic formulations. The remainder of this chapter is organized as follows: in section 2, we review the existing literature on VSP network design; the pessimistic VSP network design formulation and computational scheme are proposed in section 3; numerical studies are conducted on various instances and results are presented in section 4; in section 5, conclusions are made and future works are discussed.

4.2 Literature Review

The literature on VSP system design is growing in past years. Early studies were focused on determining the feasibility of such novel systems [65], [66], potential of environment consequences and effects on mobility behavior [67], [68], [69], [70].

From the operational perspective, some researchers study the operational characteristics of vehicle sharing program. [71] developed a stochastic chance constrained mixed-integer program to manage the fleet for vehicle sharing operations. The model incorporates a probabilistic level of service constraint to ensure the redistribution plan must satisfy a proportion of all demand scenarios. enumeration based technique and cone generation technique were proposed for solving large systems. [72] proposed a model to estimate the capacity of bicycle system and examine the viability of re-distribution of bicycles in the network to support more flows. [73] developed a time-stepping simulation model to assess the efficiency of the different relocation methods. More recent works seek optimal strategy for the fleet balancing and vehicles repositioning. [74] proposed a variable neighborhood search with an embedded variable neighborhood descent that exploits a series of neighborhood structures to generate candidate routes for vehicles to visit unbalanced rental stations. [75] developed a mathematical programming based 3-step heuristic to determine the routes for repositioning operations. The proposed heuristic reduces the size of the routing problem by clustering together sets of points that are likely to be visited consecutively. Routes among clusters are optimized and solution is used to search for a good feasible route in the original network. [76] introduced a new planning approach addressing both routing and assignment aspects of vehicle repositioning. Vehicle routing model with sub tour elimination constraint and flow assignment model are developed. [77] developed a proportional network flow based linear program to study optimal locations of

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bike sharing docks and optimal bike allocations at each dock. Results show that the system performance of the stochastic system could be approximated using deterministic linear programing model.

While on the strategy design level, the VSP network design problem received very limited attention. [78] presents an analytical hierarchy process (AHP) for car sharing stations selection. The process ranks the stations by weights obtained from various criteria and experts' on-hand experience, then select the stations whose total weights above a certain limit. [79] presented an optimization approach to determine depot location, capacity considering relocation of vehicles. The objective is to maximize daily profits considering revenues and operation costs. Trip requests are known, three formulations with different types of service schemes are proposed. [80] developed a comprehensive methodology for implementation, including the estimation of potential demand and determining of optimal locations. [81] proposed a model trading off the interests of both users and operators. However, the decisions of operators and users are formed into a single objective function. This approach can be viewed as the system-optimal utilization of the vehicle sharing resources. [82] developed a mixed-integer program maximize the net revenue. User preferences are considered and estimated at zone level.

In practice, the objectives of users and VSP operators are often conflicting with each other. Since VSP operators' objective is to maximize the utilization of the system to gain profit, however, user utilize the system only if it can reduce their travel disutility. To capture the different objectives of two stakeholders, bi-level programming model was proposed by [64]. At upper level, VSP operators (leader) first determine the optimal configuration of the system (supply), users (follower) make corresponding response to the configuration by constructing routes from origin to destination (demand). VSP operators then make adjustment to optimize

system utilization. They also proposed an exact solution approach to solve this problem. However, the bi-level programming model is optimistic formulation. Users' decision is based on assumption that they are cooperative and always select routes in favor of the VSP operators' interests. In practice, users are selfish and often behave non-cooperatively by taking solution against VSP operators' interest. The optimistic formulation is no longer sufficient to provide an optimal solution, therefore pessimistic formulation (PBL) should be considered [83].

No previous work has studied the pessimistic aspect of the bi-level VSP network design problem. This study filled the gap in the literature and contributions are provided in the following. A novel pessimistic formulation of the bi-level VSP network design model and tight relaxation for level reduction are proposed. A customized computational scheme is then developed. The model is tested on several synthetic networks, comparisons of the solution to the optimistic formulation are presented. Experimental results show by adopting pessimistic formulation, the resulting optimal network configuration is significantly different from the one from optimistic formulation. In fact, solutions obtained from pessimistic bi-level optimization are more reliable and have the promising potential of identifying practical and robust VSP network configurations.

4.3 Model and Evaluation

In this section, we first describe the optimistic formulation of the Bi-Level VSP network design model (OBL-VSP), then we evaluate the reliability of the OBL-VSP and present a pessimistic reformulation (PBL-VSP).

4.3.1 The Bi-level VSP Network Design Model

The vehicle-sharing network can be represented by a graph $G(V, A)$, where the set of nodes V including transit stations, candidate VSP stations and demand origins, destinations, the

set of arcs A consist of transit links, sharing links, transfer links and walk links. Arcs connect

nodes in the graph G. A subset of arcs A denoted by A_F includes the frequency based arcs. Each

arc in A is characterized by a pair (c_{ij}, g_{ij}). Where c_{ij}, g_{ij} denote the travel time (costs) and

waiting time of transit service respectively [84]. User travel through arcs (i, j) with waiting

include a waiting time at node i leading up to the frequency based arcs. And waiting time is

determined by the frequency of the transit services f_{ij} at terminal node j. The vehicle sharing

sub-network is denoted by $G_s(V_s, A_s)$ where V_s are the candidate VSP stations and A_s are the

shared vehicle links. For each arc in A_s, (r_{ij}, c_{ij}) pair is used to denote the revenue to the VSP

operators and costs to the users.

At the upper level, vehicle sharing network configuration is denoted as x by a 3-tuple

(x, y, z). x_i is binary decision variable determining whether to build VSP station at location i,

$i \in V_s$. y_i represents the capacity of VSP station i, z_i are the number of vehicles at i. c_s, c_p, c_v are

setup costs, capacity expansion costs and unit vehicle costs respectively. The total cost denoted

by a scalar C. At the lower level, flow decision variables v_{ijk} denotes the flows from node i to j

of origin-destination pair k. w_{ik} represents the total waiting time in OD pair k at node i. In

addition, the checkout replacement ratio is denoted by a, where $a = 1$ means that capacity

satisfies checkouts and returns and $a > 1$ otherwise.

The objective function for upper level is denoted by

$$F(v) = \sum_k \sum_{i,j \in A_s} r_{ij} v_{ijk} \tag{52}$$

Objective function for lower level is denoted by

$$f(v, w) = \sum_k \left(\sum_{(i,j) \in A} c_{ij} v_{ijk} + \sum_{i \in V} w_{ik} \right) \tag{53}$$

Then the bi-level VSP network design model OBL-VSP can be written as:

$$\max_{v \in S(x,y,z)} F(v) \tag{54}$$

$$s.t. \quad \sum_{i \in V_s} c_s x_i + c_p y_i + c_v z_i \leq C \tag{55}$$

$$M x_i \geq y_i, \quad \forall i \in V_s \tag{56}$$

$$z_i \leq y_i, \quad \forall i \in V_s \tag{57}$$

$$y_i \leq y^{ub}, \quad \forall i \in V_s \tag{58}$$

$$x_i \in \{0,1\}, \quad \forall i \in V_s; y_i, z_i \in \mathbb{Z}^n, \quad \forall i \in V_s \tag{59}$$

$$S(x,y,z) = \underset{v,w}{argmin}\{ f(v,w): \tag{60}$$

$$\sum_{j:(i,j) \in A} v_{ijk} - \sum_{j:(j,i) \in A} v_{jik} = D_{ik}, \quad \forall i \in V, k \in K \tag{61}$$

$$v_{ijk} \leq f_{ij} w_{ik}, \quad \forall (i,j) \in A_F, k \in K \tag{62}$$

$$M x_i \geq \sum_k v_{ijk}, \quad \forall (i,j) \in A_s \tag{63}$$

$$M x_j \geq \sum_k v_{ijk}, \quad \forall (i,j) \in A_s \tag{64}$$

$$\sum_k \sum_{j:(i,j) \in A_s} v_{ijk} \leq z_i, \quad \forall i \in V_s \tag{65}$$

$$\sum_k \sum_{j:(j,i) \in A_s} v_{jik} \leq a(y_i - z_i), \quad \forall i \in V_s \tag{66}$$

$$w_{ik} \geq 0, \quad \forall i \in V, k \in K; v_{ijk} \geq 0, \quad \forall (i,j) \in A, k \in K\} \tag{67}$$

In this model, upper level program denotes VSP operator's decision and determines the optimal VSP configuration (x,y,z) to maximize the revenue of shared-flows (54), subject to costs and capacity constraints (55) – (59). The lower level problem represents user's decision

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and seeks to minimize total costs for a given upper level VSP network configuration. The lower level problem involves a set of flow constraints (61) – (67) to determine the optimal flows for user travel through the network from origin to destination.

In general, bi-level mixed-integer program typically has a non-convex feasible region which results in difficulties in computation. This type of problems is recognized as NP-hard [85]. To address this challenge, various computational algorithms have been developed [86], [87]. When lower-level problem is convex, we can adopt Karush-Kuhn-Tucker (KKT) conditions and replace the lower level problem by its KKT conditions. Therefore, the bi-level program becomes a single level mathematical program with equilibrium constraints (MPEC) and can be readily solved by existing MIP solvers.

For convenience, we represent the OBL-VSP using concise notation as follows:

$$\max_{x \in X} \quad F(\boldsymbol{v})$$

$$s.t. \quad G(\boldsymbol{x}, \boldsymbol{v}) \leq 0$$

$$\min_{\boldsymbol{v} \in S} f(\boldsymbol{v}) \tag{68}$$

$$s.t. \ g(\boldsymbol{x}, \boldsymbol{v}) \leq 0$$

where $\boldsymbol{x} = (x, y, z)$ and $\boldsymbol{v} = (v, w)$ are the operator and user decision vectors respectively. $G(\boldsymbol{x}, \boldsymbol{v})$ and $g(\boldsymbol{x}, \boldsymbol{v})$ are the upper and lower level constraints sets.

4.3.2 Reliability of OBL-VSP Solution

Although OBL-VSP provides an effective approach for modeling the conflicting objectives of VSP operators and users. However, it may be inefficient from user's perspective since user may not always follow operator's decision. Hence solutions obtained from OBL-VSP can be viewed as leader-optimal solutions. In fact, two critical factors have been neglected, which may significantly affect the performance of OBL-VSP in practice:

1. For a given optimal configuration $\boldsymbol{x}^*$, the lower-level solution may not be unique and optimal to the user.

2. Given the non-unique lower-level solution, the user may not always select routes that maximize revenue for the VSP operator.

Figure 4.1 illustrates the impact of those two factors using a small network instance. The network has 4 VSP candidate station nodes (A, B, C, D) and one OD pair (O, D) with a flow of 10 units. The numbers for each shared-link and walk link represent the flow revenue and cost respectively. Given the upper-level configuration B, D, figure on the left shows the optimal lower-level solution to the upper-level objective, the resulting revenue and cost are both equal to 30 units. However, if user decides routes without considering revenue, the resulting solution to the lower-level is shown on the right figure. In this situation, the corresponding revenue reduces to 0 unit and cost remains at 30 units. Notice that, for two different flow decisions, the costs remain the same while the revenues show a huge difference.

We then formulate a problem to evaluate the impact of those two factors on OBL-VSP. We first solve OBL-VSP (68) and derive optimal configuration $\boldsymbol{x}^*$, then for given $\boldsymbol{x}^*$ we solve lower-level program and obtain optimal value l^*. After that, we solve the following formulation and get optimal value $\underline{u}^*$. The comparison results are shown in section 4.4.

$$min\, F(\boldsymbol{x}^*, \boldsymbol{v})$$

$$\textit{s.t.} \quad g(\boldsymbol{x}^*, \boldsymbol{v}) \leq 0 \tag{69}$$

$$f(\boldsymbol{x}^*, \boldsymbol{v}) \leq l^*$$

where $\boldsymbol{x}^* = (\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*), \boldsymbol{v} = (\boldsymbol{v}, \boldsymbol{w})$. Apparently, for a given $\boldsymbol{x}^*$, (69) computes the performance in pessimistic environment. As showed in the illustrative example, user may behave non-cooperative and their decisions may not be in favor of operator's interests. Hence, OBL-VSP

may not work in practice. To study this challenge, we introduce the pessimistic formulation in the next section.

4.3.3 Pessimistic Bi-Level VSP Network Design Model

In the last subsection, we observe that regular OBL-VSP assume that users are always cooperative by generating solutions to maximize VSP operator's objective. However, users are in fact selfish. When making the decision, they tend to overlook VSP operator's interest and select solutions that only minimize their travel costs. In many practical scenarios, those solutions are against VSP operator's interest. Therefore, we should consider the pessimistic bi-level formulation for modeling and computing. We propose such pessimistic formulation PBL-VSP in the following:

$$\min_{x_i, y_i, z_i} \max_{(v_{ijk}, w_{ik}) \in S(x_i, y_i, z_i)} -F(\boldsymbol{v}) \tag{70}$$

$$s.t. \quad \sum_{i \in V_s} c_s x_i + c_p y_i + c_v z_i \leq C \tag{71}$$

$$M x_i \geq y_i, \quad \forall i \in V_s \tag{72}$$

$$z_i \leq y_i, \quad \forall i \in V_s \tag{73}$$

$$y_i \leq y^{ub}, \quad \forall i \in V_s \tag{74}$$

$$x_i \in \{0,1\}, \quad \forall i \in V_s; \quad y_i, z_i \in \mathbb{Z}^n, \quad \forall i \in V_s \tag{75}$$

$$(v_{ijk}, w_{ik}) \in S(x_i, y_i, z_i) = \underset{v,w}{argmin}\{ f(\boldsymbol{v}, \boldsymbol{w}): \tag{76}$$

$$\sum_{j:(i,j) \in A} v_{ijk} - \sum_{j:(j,i) \in A} v_{jik} = D_{ik}, \quad \forall i \in V, k \in K \tag{77}$$

$$v_{ijk} \leq f_{ij} w_{ik}, \quad \forall (i,j) \in A_F, k \in K \tag{78}$$

$$M x_i \geq \sum_k v_{ijk}, \quad \forall (i,j) \in A_s \tag{79}$$

$$Mx_j \geq \sum_k v_{ijk}, \quad \forall (i,j) \in A_s \tag{80}$$

$$\sum_k \sum_{j:(i,j)\in A_s} v_{ijk} \leq z_i, \quad \forall i \in V_s \tag{81}$$

$$\sum_k \sum_{j:(j,i)\in A_s} v_{jik} \leq a(y_i - z_i), \quad \forall i \in V_s \tag{82}$$

$$w_{ik} \geq 0, \quad \forall i \in V, k \in K; v_{ijk} \geq 0, \quad \forall (i,j) \in A, k \in K\} \tag{83}$$

Bi-level problem is typically very hard to solve, even the lower level problem is linear program, it is NP-hard. The PBL-VSP is a three-level problem so that it is much more difficult than the OBL-VSP in computation. However, a new computational scheme developed by [83] is practically useful in solving pessimistic bi-level problems. By introducing a set of variables and constraints that replicate those in the lower-level OBL-VSP, we obtain a tight relaxation RPBL-VSP for PBL-VSP in the following:

$$\min_{x_i, y_i, z_i, \bar{v}_{ijk}, \bar{w}_{ik}} -F(v) \tag{84}$$

$$s.t. \quad \sum_{i\in V_s} c_s x_i + c_p y_i + c_v z_i \leq C \tag{85}$$

$$Mx_i \geq y_i, \quad \forall i \in V_s \tag{86}$$

$$z_i \leq y_i, \quad \forall i \in V_s \tag{87}$$

$$y_i \leq y^{ub}, \quad \forall i \in V_s \tag{88}$$

$$x_i \in \{0,1\}, \quad \forall i \in V_s; y_i, z_i \in \mathbb{Z}^n, \quad \forall i \in V_s \tag{89}$$

$$\sum_{j:(i,j)\in A} \bar{v}_{ijk} - \sum_{j:(j,i)\in A} \bar{v}_{jik} = D_{ik}, \quad \forall i \in V, k \in K \tag{90}$$

$$\bar{v}_{ijk} \leq f_{ij}\bar{w}_{ik}, \quad \forall (i,j) \in A_F, k \in K \tag{91}$$

$$Mx_i \geq \sum_k \bar{v}_{ijk}, \quad \forall (i,j) \in A_s \tag{92}$$

$$Mx_j \geq \sum_k \bar{v}_{ijk}, \quad \forall (i,j) \in A_s \tag{93}$$

$$\sum_k \sum_{j:(i,j) \in A_s} \bar{v}_{ijk} \leq z_i, \quad \forall i \in V_s \tag{94}$$

$$\sum_k \sum_{j:(j,i) \in A_s} \bar{v}_{jik} \leq a(y_i - z_i), \quad \forall i \in V_s \tag{95}$$

$$\bar{w}_{ik} \geq 0, \quad \forall i \in V, k \in K; \bar{v}_{ijk} \geq 0, \quad \forall (i,j) \in A, k \in K \tag{96}$$

$$\left(v_{ijk}, w_{ik} \right) \in argmin\{ F(\boldsymbol{v}): \tag{97}$$

$$\sum_{j:(i,j) \in A} v_{ijk} - \sum_{j:(j,i) \in A} v_{jik} = D_{ik}, \quad \forall i \in V, k \in K \tag{98}$$

$$v_{ijk} \leq f_{ij} w_{ik}, \quad \forall (i,j) \in A_F, k \in K \tag{99}$$

$$Mx_i \geq \sum_k v_{ijk}, \quad \forall (i,j) \in A_s \tag{100}$$

$$Mx_j \geq \sum_k v_{ijk}, \quad \forall (i,j) \in A_s \tag{101}$$

$$\sum_k \sum_{j:(i,j) \in A_s} v_{ijk} \leq z_i, \quad \forall i \in V_s \tag{102}$$

$$\sum_k \sum_{j:(j,i) \in A_s} v_{jik} \leq a(y_i - z_i), \quad \forall i \in V_s \tag{103}$$

$$f(\boldsymbol{v}, \boldsymbol{w}) \leq f(\bar{\boldsymbol{v}}, \bar{\boldsymbol{w}}) \tag{104}$$

$$w_{ik} \geq 0, \quad \forall i \in V, k \in K; v_{ijk} \geq 0, \quad \forall (i,j) \in A, k \in K\} \tag{105}$$

According to [83], constraint (104) ensures that v_{ijk}, w_{ik} are an optimal solution of the

lower-level. Notice that, the lower-level problem of RPBL-VSP is still a convex program and we

can apply KKT reformulation to convert the bi-level program into a single-level mixed integer programming problem.

4.3.4 Karush-Kuhn-Tucker Reformulation for RPBL-VSP

As we mentioned, the lower-level problem of RPBL-VSP is a program with linear objective and constraints, Karush-Kuhn-Tucker conditions for such program are both necessary and sufficient. Therefore, lower-level problem can be replaced by its KKT conditions and RPBL-VSP can be reduced to a single level mixed integer programming model which can be readily solved using standard solver.

First we let $\alpha_{ik}, \beta_{ijk}, \gamma_{ij}, \delta_{ij}, \xi_i, \eta_i, \omega, \lambda_{ik}, \mu_{ijk}$ be the corresponding dual variables for constraints (98) – (105). Then the stationarity conditions are:

$$D + \alpha_{ik} - \alpha_{jk} - \mu_{ijk} + c_{ij}\omega + B + C = 0, \quad \forall (i,j) \in A, k \in K \tag{106}$$

$$-\lambda_{ik} - A' + \omega = 0, \quad \forall i \in V, k \in K \tag{107}$$

$$A' = \begin{cases} \sum_j \beta_{ijk} f_{ij}, & if\ (i,j) \in A_F \\ 0, & o.w \end{cases}$$

$$B = \begin{cases} \beta_{ijk}, & if\ (i,j) \in A_F \\ 0, & o.w \end{cases}$$

$$C = \begin{cases} \gamma_{ij} + \delta_{ij} + \xi_i + \eta_j, & if\ (i,j) \in A_s \\ 0, & o.w \end{cases}$$

$$D = \begin{cases} r_{ij}, & if\ (i,j) \in A_s \\ 0, & o.w \end{cases}$$

followed by the complementarity conditions:

$$\beta_{ijk}(f_{ij}w_{ik} - v_{ijk}) = 0, \quad \forall (i,j) \in A_F, k \in K \tag{108}$$

$$\gamma_{ij}\left(Mx_i - \sum_k v_{ijk}\right) = 0, \quad \forall (i,j) \in A_s \tag{109}$$

$$\delta_{ij}\left(Mx_j - \sum_k v_{ijk}\right) = 0, \quad \forall (i,j) \in A_s \tag{110}$$

$$\xi_i\left(z_i - \sum_k \sum_{j:(i,j)\in A_s} v_{ijk}\right) = 0, \quad \forall i \in V_s \tag{111}$$

$$\eta_i\left(a(y_i - z_i) - \sum_k \sum_{j:(j,i)\in A_s} v_{jik}\right) = 0, \quad \forall i \in V_s \tag{112}$$

$$\omega\left(\sum_k\left(\sum_{(i,j)\in A} c_{ij}\bar{v}_{ijk} + \sum_{i\in V}\bar{w}_{ik}\right) - \sum_k\left(\sum_i w_{ik} - \sum_{(i,j)\in A} c_{ij}v_{ijk}\right)\right) = 0, \tag{113}$$

$$\lambda_{ik}w_{ik} = 0, \quad \forall i \in V, k \in K \tag{114}$$

$$\mu_{ijk}v_{ijk} = 0, \quad \forall (i,j) \in A, k \in K \tag{115}$$

In order to deal with nonlinear constraints in complementarity conditions, we use a set of binary variables $u_{ijk}, e_{ij}, g_{ij}, h_i, l_i, m$ for constraints (108)-(115). Then, using Big-M method, those nonlinear constraints can be transformed to a set of linear constraints in the following ways:

$$\beta_{ijk} \leq Mu_{ijk}, \quad \forall (i,j) \in A_F, k \in K \tag{116}$$

$$f_{ij}w_{ik} - v_{ijk} \leq M(1 - u_{ijk}), \quad \forall (i,j) \in A_F, k \in K \tag{117}$$

$$u_{ijk} \in \{0,1\} \tag{118}$$

$$\gamma_{ij} \leq Me_{ij}, \quad \forall (i,j) \in A_s \tag{119}$$

$$Mx_i - \sum_k v_{ijk} \leq M(1 - e_{ij}), \quad \forall (i,j) \in A_s \tag{120}$$

$$e_{ij} \in \{0,1\} \tag{121}$$

$$\delta_{ij} \leq Mg_{ij}, \quad \forall (i,j) \in A_s \tag{122}$$

$$Mx_j - \sum_k v_{ijk} \leq M(1 - g_{ij}), \quad \forall (i,j) \in A_s \tag{123}$$

$$g_{ij} \in \{0,1\} \tag{124}$$

$$\xi_i \leq Mh_i, \quad \forall i \in V_s \tag{125}$$

$$z_i - \sum_k \sum_{j:(i,j)\in A_s} v_{ijk} \leq M(1 - h_i), \quad \forall i \in V_s \tag{126}$$

$$h_i \in \{0,1\} \tag{127}$$

$$\eta_i \leq Ml_i, \quad \forall i \in V_s \tag{128}$$

$$a(y_i - z_i) - \sum_k \sum_{j:(j,i)\in A_s} v_{jik} \leq M(1 - l_i), \quad \forall i \in V_s \tag{129}$$

$$l_i \in \{0,1\} \tag{130}$$

$$\omega \leq Mm \tag{131}$$

$$\left(\sum_k \left(\sum_{(i,j)\in A} c_{ij} \bar{v}_{ijk} + \sum_{i\in V} \bar{w}_{ik} \right) - \sum_k \left(\sum_i w_{ik} - \sum_{(i,j)\in A} c_{ij} v_{ijk} \right) \right) \leq M(1 - m) \tag{132}$$

$$m \in \{0,1\} \tag{133}$$

$$\lambda_{ik} \leq Mp_{ik}, \quad \forall i \in V, k \in K \tag{134}$$

$$w_{ik} \leq M(1 - p_{ik}), \quad \forall i \in V, k \in K \tag{135}$$

$$p_{ik} \in \{0,1\} \tag{136}$$

$$\mu_{ijk} \leq Mq_{ijk}, \quad \forall (i,j) \in A, k \in K \tag{137}$$

$$v_{ijk} \leq M(1 - q_{ijk}), \quad \forall (i,j) \in A, k \in K \tag{138}$$

$$q_{ijk} \in \{0,1\} \tag{139}$$

In addition, the dual feasibility constraints are:

$$\beta_{ijk} \geq 0, \quad \forall (i,j) \in A_F, k \in K \tag{140}$$

$$\gamma_{ij}, \delta_{ij} \geq 0, \quad \forall(i,j) \in A_s \tag{141}$$

$$\xi_i, \eta_i \geq 0, \quad \forall i \in V_s \tag{142}$$

$$\lambda_{ik} \geq 0, \quad \forall i \in V, k \in K \tag{143}$$

$$\mu_{ijk} \geq 0, \quad \forall(i,j) \in A, k \in K \tag{144}$$

$$\omega \geq 0 \tag{145}$$

Finally, the mathematical program with equilibrium constraints (MPEC) is defined by objective function (84), upper level constraints (85)-(96), lower level KKT primal constraints (98)-(105), KKT stationarity constraints (106), (107), linearized complementarity constraints (116)-(139), and dual feasibility constraints (140)-(145).

4.3.5 *Relaxation-and-Correction* **Computational Scheme**

Next, we adopt the *Relaxation-and-Correction* computational scheme introduced by [83] to solve this problem. In practice, by computing RPBL-VSP, we may not obtain feasible solutions to PBL. This issue can be easily addressed by the *Relaxation-and-Correction* computational scheme shown in the following.

Algorithm 3

Relaxation-and-Correction computational scheme

1. **Relaxation:** Compute the tight relaxation RPBL-VSP and derive an optimal solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*, \overline{\mathbf{v}}^*, \overline{\mathbf{w}}^*, \mathbf{v}^*, \mathbf{w}^*)$.

If

$$\sum_k \left(\sum_{(i,j) \in A} c_{ij} \overline{v}_{ijk}^* + \sum_{i \in V} \overline{w}_{ik}^* \right) > \theta(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$$

$$\theta(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*) = \min \sum_k \left(\sum_{(i,j) \in A} c_{ij} v_{ijk} + \sum_{i \in V} w_{ik} \right):$$

$$st. \sum_{j:(i,j)\in A} v_{ijk} - \sum_{j:(j,i)\in A} v_{jik} = D_{ik}, \quad \forall i \in V, k \in K$$

$$v_{ijk} \le f_{ij}w_{ik}, \quad \forall (i,j) \in A_F, k \in K$$

$$Mx_i^* \ge \sum_k v_{ijk}, \quad \forall (i,j) \in A_s$$

$$Mx_j^* \ge \sum_k v_{ijk}, \quad \forall (i,j) \in A_s$$

$$\sum_k \sum_{j:(i,j)\in A_s} v_{ijk} \le z_i^*, \quad \forall i \in V_s$$

$$\sum_k \sum_{j:(j,i)\in A_s} v_{jik} \le a(y_i^* - z_i^*), \quad \forall i \in V_s$$

$$w_{ik} \ge 0, \quad \forall i \in V, k \in K$$

$$v_{ijk} \ge 0, \quad \forall (i,j) \in A, k \in K$$

Perform the correction step. Otherwise, we obtain $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*, \mathbf{v}^*, \mathbf{w}^*)$ as an optimal solution to PBL-VSP.

2. **Correction:** compute the following program

$$min \sum_k \sum_{(i,j)\in A_s} r_{ij}v_{ijk}$$

$$st. \sum_{j:(i,j)\in A} v_{ijk} - \sum_{j:(j,i)\in A} v_{jik} = D_{ik}, \quad \forall i \in V, k \in K$$

$$v_{ijk} \le f_{ij}w_{ik}, \quad \forall (i,j) \in A_F, k \in K$$

$$Mx_i^* \ge \sum_k v_{ijk}, \quad \forall (i,j) \in A_s$$

$$Mx_j^* \ge \sum_k v_{ijk}, \quad \forall (i,j) \in A_s$$

$$\sum_k \sum_{j:(i,j)\in A_s} v_{ijk} \leq z_i^*, \quad \forall i \in V_s$$

$$\sum_k \sum_{j:(j,i)\in A_s} v_{jik} \leq a(y_i^* - z_i^*), \quad \forall i \in V_s$$

$$w_{ik} \geq 0, \quad \forall i \in V, k \in K$$

$$v_{ijk} \geq 0, \quad \forall (i,j) \in A, k \in K$$

$$\sum_k \left(\sum_{(i,j)\in A} c_{ij} v_{ijk} + \sum_{i\in V} w_{ik} \right) \leq \theta(x^*, y^*, z^*)$$

Derive an optimal solution (v', w'). Optimal solution (x^*, y^*, z^*, v', w') is obtained.

4.4 Numerical Experiment

In this section, the key objective is to examine the efficiency of pessimistic formulation and solution method, study the effect of input parameters and different network characteristics, compare the performance and differences between pessimistic and optimistic formulations in regard of the solutions and provide management insights in designing a reliable VSP network.

First, we demonstrate the results using the illustrative example to gain initial insights on the solutions. Then, we conduct experiments on various instances with different network structures followed by discussion.

Figure 4.2 shows the results obtained by solving OBL-VSP and PBL-VSP. In optimistic solution, VSP stations B and D are built and corresponding user's flow is O-B-D. The resulting revenue and cost are both 30 units. Nevertheless, in pessimistic solution, VSP stations A and D are built with route decision O-A-D, the revenue and cost associated with this design are 20 units and 10 units. If VSP operator build stations based on OBL-VSP solution, user's non-cooperative behavior will lead to 0 unit revenue to the VSP operator.

In the experiments, three types of network structures are considered, shown in Figure 4.3, Figure 4.4, Figure 4.5, with circles represent transit stations, squares denote candidate VSP stations and the black lines represent transit services. Figure 4.3 shows the corridor network (COR), in this network, transit services run along a corridor with shared-vehicles providing access to transit stations. Figure 4.4 shows the grid network (GRID) which mimic the transit services in the downtown area, shared-vehicles provide additional intermodal path choices to the users. Figure 4.5 denotes the random network (RND), where the nodes and arcs are randomly generated. For each network structure, we generated several instances denoted by the network type (e.g. COR) and the number of node (e.g. COR23). Table 4.1 shows the summary of each instance. We observe that, for the same instance, the numbers of integer variables in PBL-VSP and OBL-VSP are nearly identical, while the number of continuous variables in PBL-VSP are larger since a set of new variables and constraints that replicate the lower level program were added.

Nodes are categorized by transit stations, VSP candidate stations, and origins/destinations. Links are classified into three types, walk links, shared-vehicle links, and transit links. For walk links, we assume a constant walking speed of 5 miles/hour. And 30 miles/hour for shared-vehicle links and transit links. Parameter Budget is varied from $0 - 100$ with the increment of 20 for each instance. The parameters of cost components c_s, c_p, c_v are set to 4,2,1 respectively. Solution time limit set to one hour. The models and computational scheme are implemented in C++, solved by CPLEX 12.5 solver. Table 4.2 shows the computational results. The upper and lower level objective values of PBL-VSP and OBL-VSP as well as solution time in seconds are also reported. For instances that solution time exceeds limit, we report the MIP gap (%). The values in column OBL-P denotes that OBL performance in

pessimistic environment as we discussed in section 4.3.2. From the experiments, we notice that solution time increases with respect to the number of nodes and budget. Due to the combinatorial nature of this problem, networks with more than 40 nodes and budget greater than 100 require huge amount of time (>10 hours) for CPLEX to find solution with relatively large gaps. To benchmark the optimal network configurations, we omit those instances and only include instances that can be solved to optimality within time limit.

Figure 4.6 and Figure 4.8 present the VSP operator's revenue and user's cost as a function of budget in PBL-VSP. Figure 4.7 and Figure 4.9 show the same information in OBL-VSP. It can be seen from those figures that VSP operator's revenue is slightly higher in OBL-VSP than in PBL-VSP among tested instances, while lower level user cost in PBL-VSP is lower than that in OBL-VSP. To better understand the results, we plot upper level objective values in PBL-VSP, OBL-VSP and OBL-P, lower level objective values in PBL-VSP and OBL-VSP with respect to budget for each instance, see Figure 4.10 -Figure 4.18. Results show that OBL-VSP has highest upper and lower level optimal values since users make corresponding route decisions to maximize VSP operator's revenue while sacrificing their travel utility if the cooperative assumption holds. OBL-P solutions are the least favorable solutions to VSP operators if users behave non-cooperative and seek solutions only to minimize their costs regardless of VSP operator's interest. For example, in instance COR32, when C=80, PBL-VSP solution yields 36 units of revenue comparing to 58 units in OBL-VSP solution. However, the corresponding costs for users in PBL-VSP equal to 519.667 which is lower than 526.667 in OBL-VSP. In non-cooperative situation, OBL-VSP solution only results in 8 units of revenue. This result shows that if users behave non-cooperatively, VSP operator can only gain 8 units of revenue from the optimal OBL-VSP configuration. Comparing to the expected revenue of 58 units, the result is

less desirable and therefore we can conclude that OBL-VSP cannot provide optimal network configuration in practice.

Furthermore, the networks derived from PBL-VSP and OBL-VSP for instance RND 34 are presented in Figure 4.19 - Figure 4.29 with budget C varies from 0-100. The detailed optimal capacities and number of vehicles in each station are shown in Table 4.3. When budget C is in the range of 0-100, the budget constraint is binding, hence only a subset of the VSP candidate stations will be selected to build. It is obvious that the network configuration obtained from PBL-VSP and OBL-VSP are different with the increase of budget. When C=100, we observe that for a same origin-destination pair, user select route segments with lower costs in PBL-VSP than that in OBL-VSP, as the result, lower level optimal value $f(v)$ in PBL-VSP is 785 which is less than 853 in OBL-VSP. On the contrary, route segments with higher revenue are chosen in OBL-VSP, the resulting upper level optimal value $F(v)$ is 54 in OBL-VSP and 30 in PBL-VSP. The results show that in OBL-VSP, user make decisions to incorporate with VSP operator's interest. The network configuration is optimal to VSP operator, however, it is suboptimal to user and lead to larger travel costs. By computing OBL-P, we obtain the upper level objective value $F(v)$ =21, which represents the OBL-VSP configuration in pessimistic environment if the non-cooperative assumption holds. Since VSP operator cannot control over the user's behavior, solution obtained from OBL-VSP could give arbitrarily low revenue if cooperative assumption is not effective. Clearly, PBL-VSP will derive robust and reliable network design when dealing with the users' irrationality.

4.5 Conclusion and Future Work

In this study, we examine the reliability issue raised from the optimistic bi-level equilibrium design of the vehicle sharing program (OBL-VSP). The equilibrium solution is

inefficient since two stakeholders do not cooperate and could get arbitrarily low revenue. To address this issue, a pessimistic formulation (PBL-VSP) is proposed. A tight relaxation is developed for level reduction and a *Relaxation-and-Correction* computational scheme is implemented for solving the problem. We conduct experiments on several random prototypical networks with different structures to show the capability of the model and benchmark results with optimistic formulation. Results show PBL-VSP is more efficient in designing a reliable and robust VSP network.

The future work of this study could be extended to adopt more realistic origin-destination estimation method and incorporate congestion effects on the networks to obtain additional insights. Furthermore, other solution algorithms and decomposition methods could be explored to deal with large-scale instances in practice.
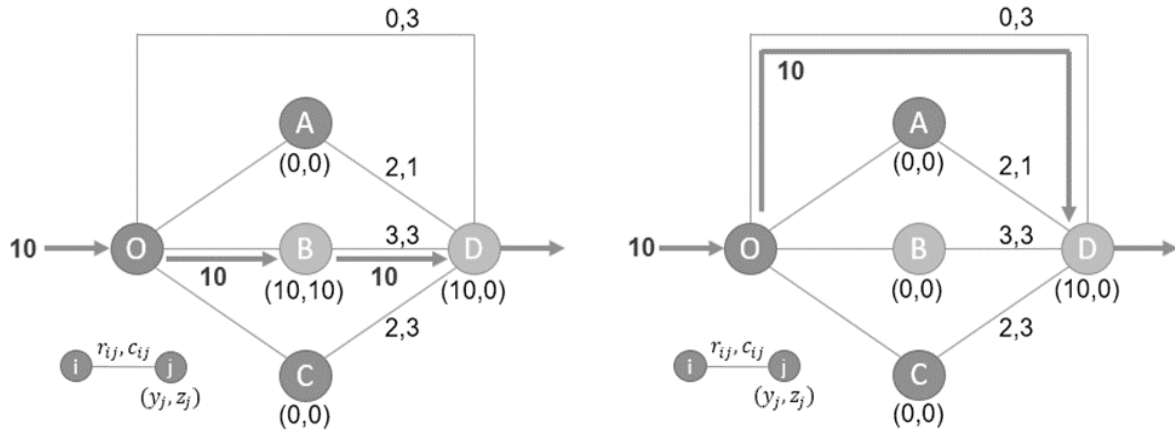
Figure 4.1: Illustrative Example
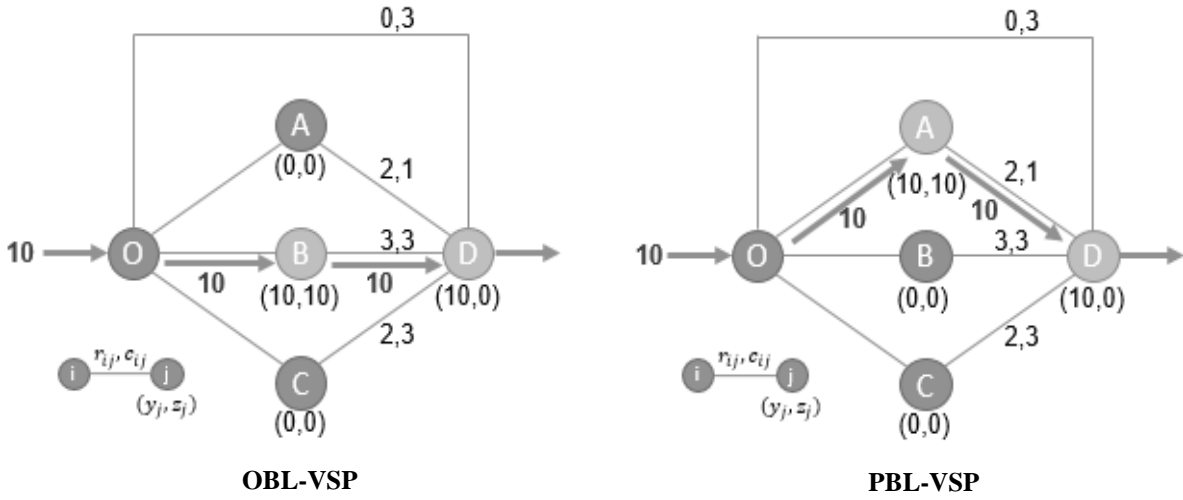
OBL-VSP   **PBL-VSP**

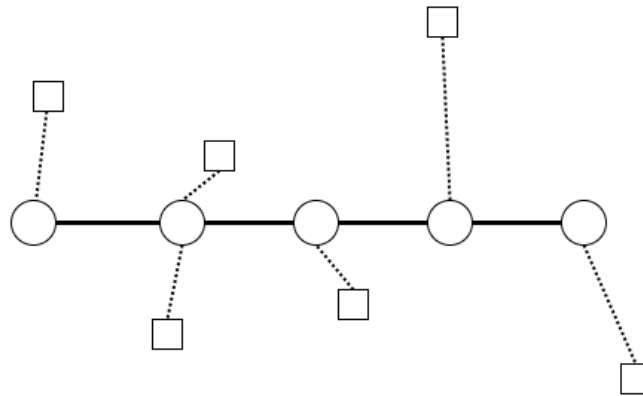Figure 4.2: Result for Illustrative Example

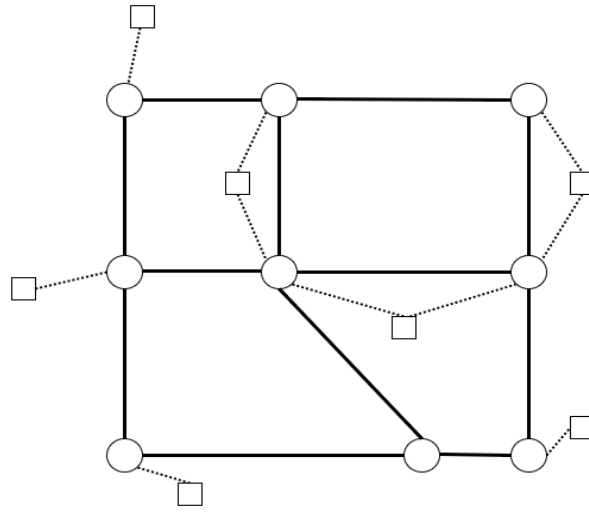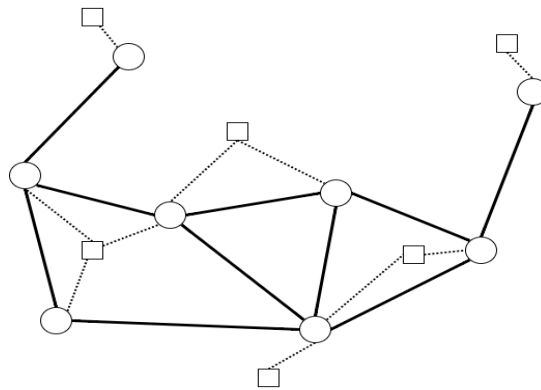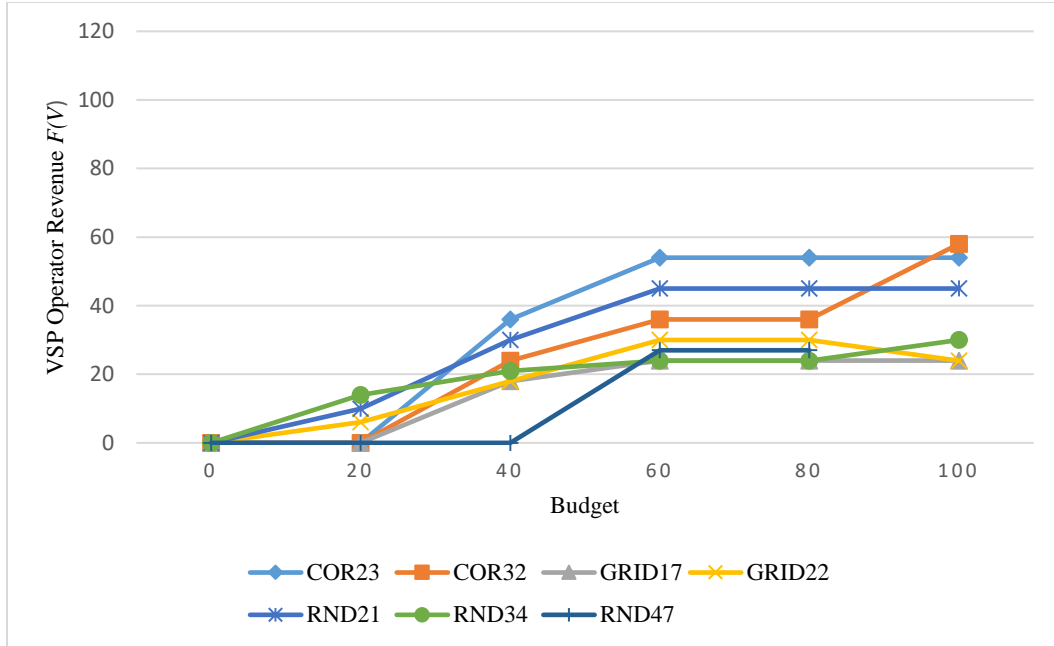Figure 4.3: Corridor

Figure 4.4: Grid

Figure 4.5: Random

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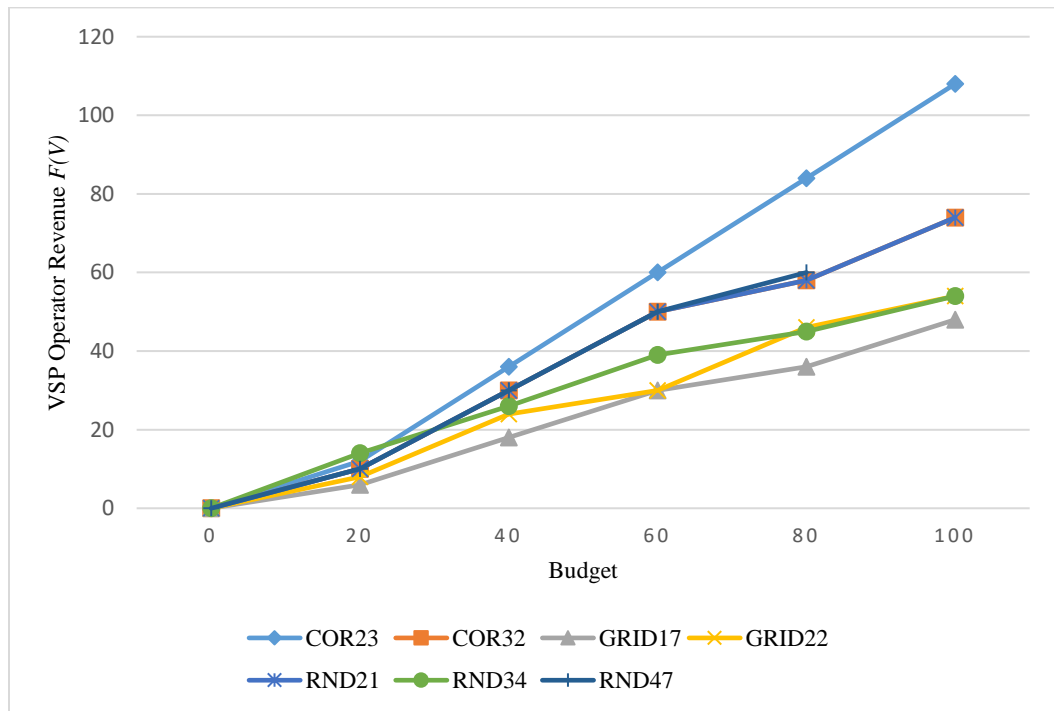Figure 4.6: VSP Operator Revenue *F(v)* of PBL-VSP
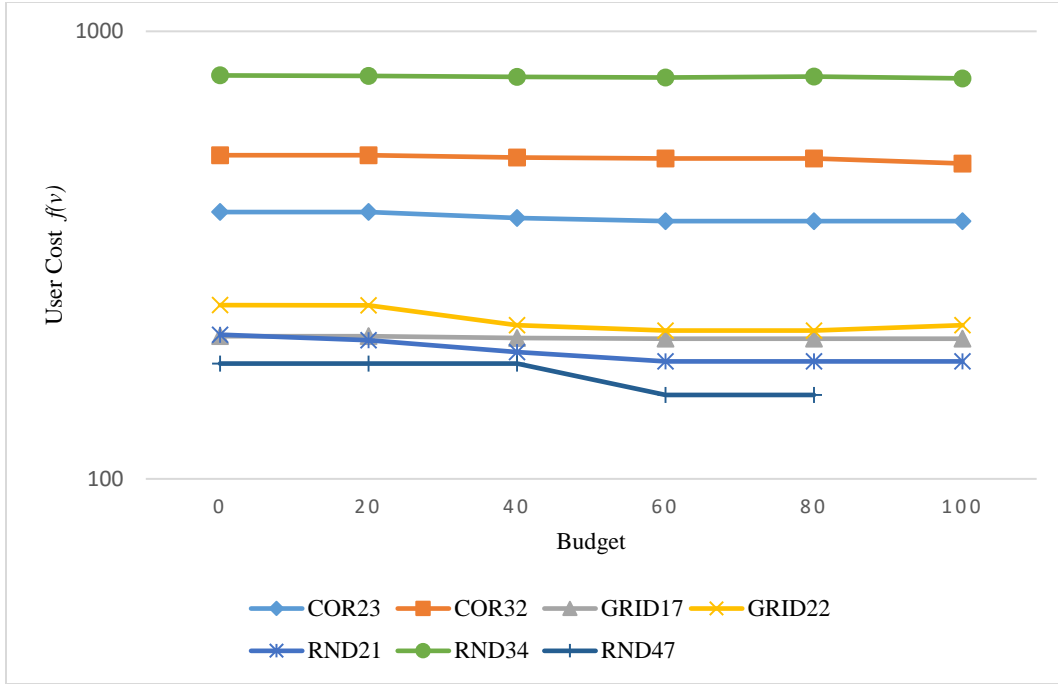

Figure 4.7: VSP Operator Revenue *F(v)* of OBL-VSP

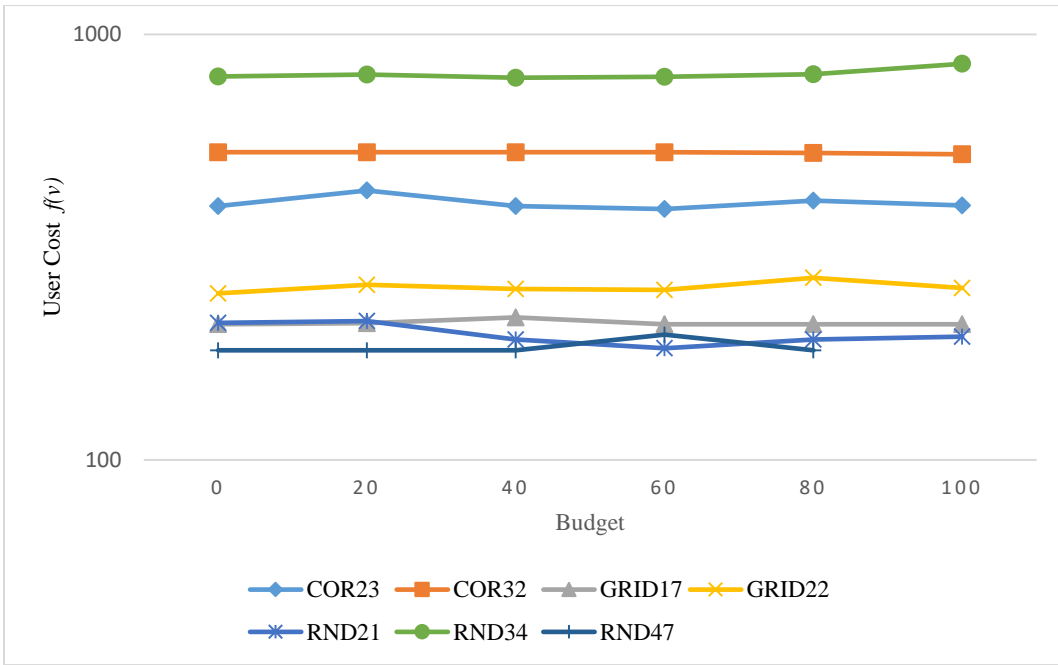Figure 4.8: User Cost $f(v)$ of PBL-VSP

Figure 4.9: User Cost $f(v)$ of OBL-VSP
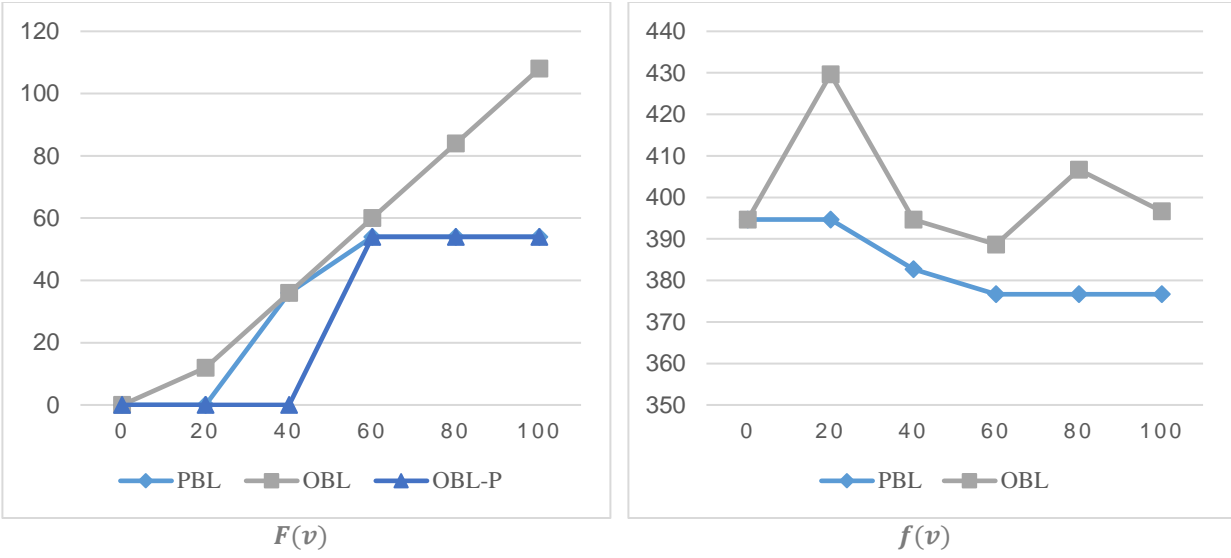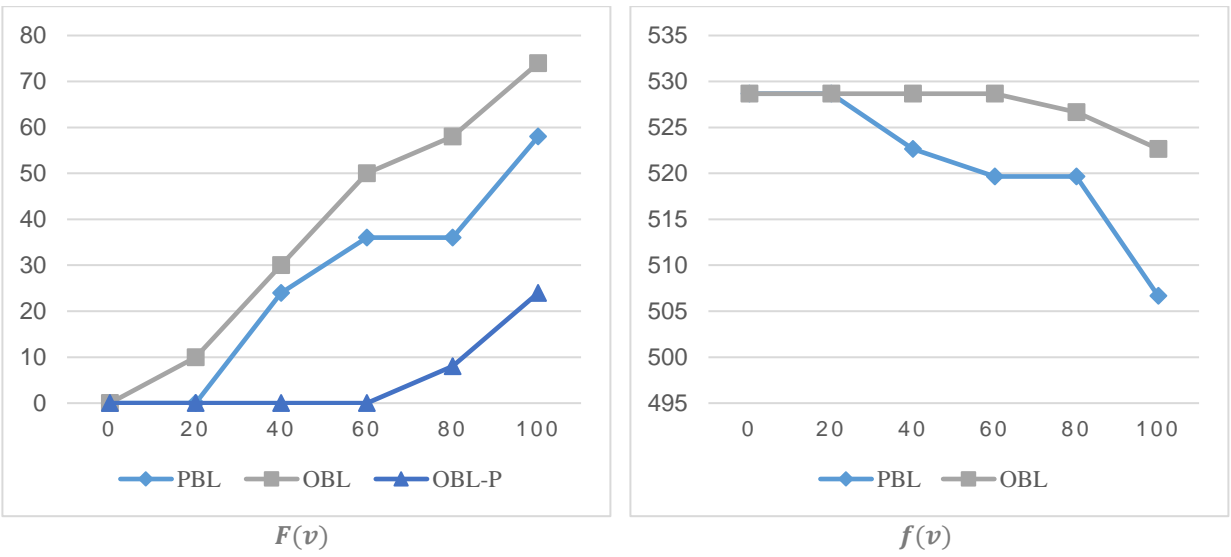
Figure 4.10: COR 23
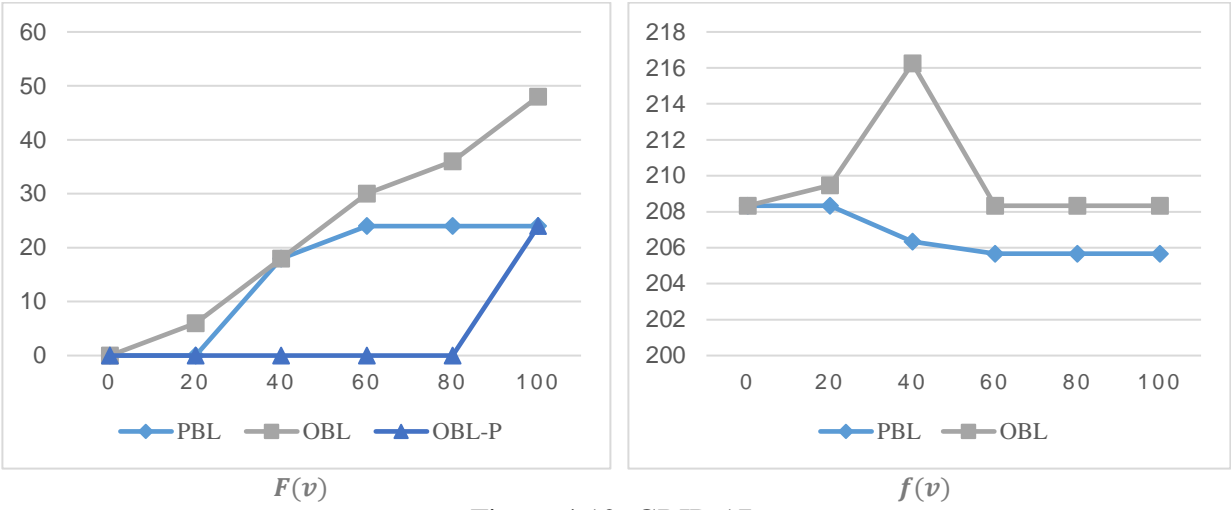

Figure 4.11: COR 32

Figure 4.12: GRID 17
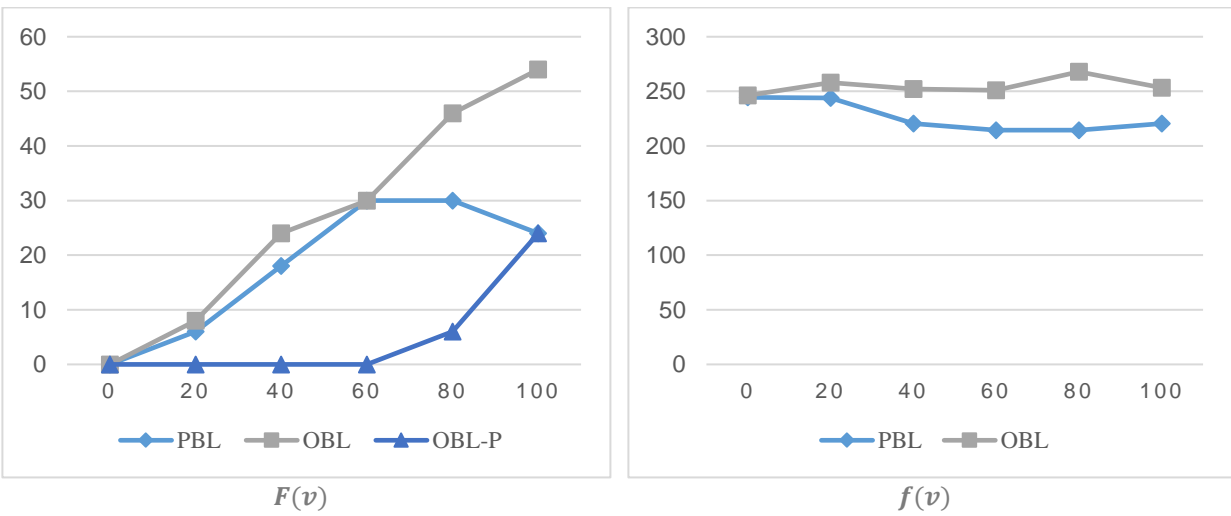
Figure 4.13: GRID 22

$F(v)$ $f(v)$

Figure 4.14: RND 21

$F(v)$ $f(v)$

Figure 4.15: RND 34

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$F(v)$

$f(v)$
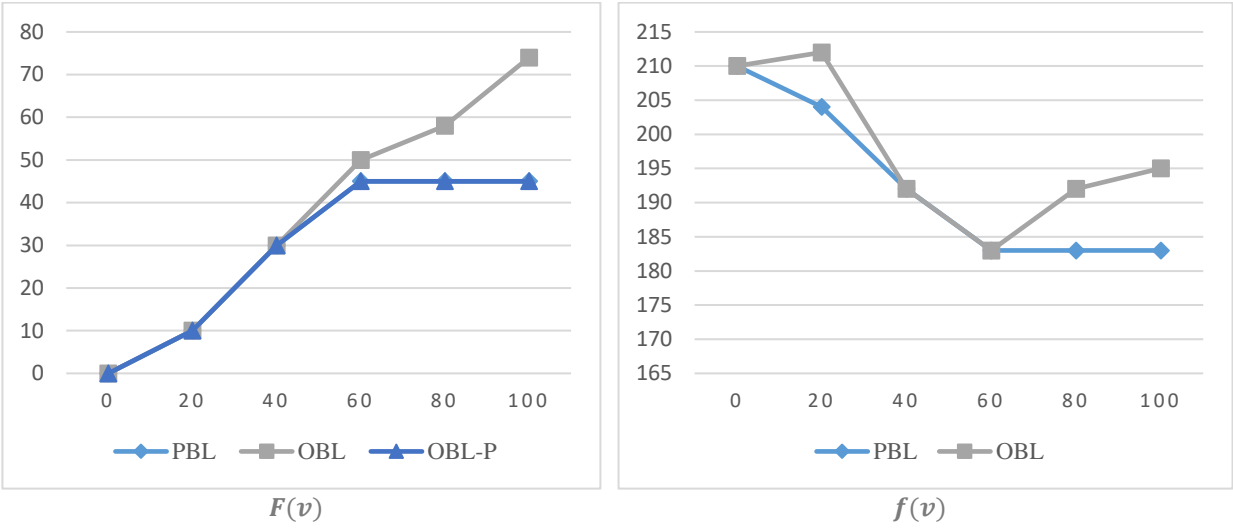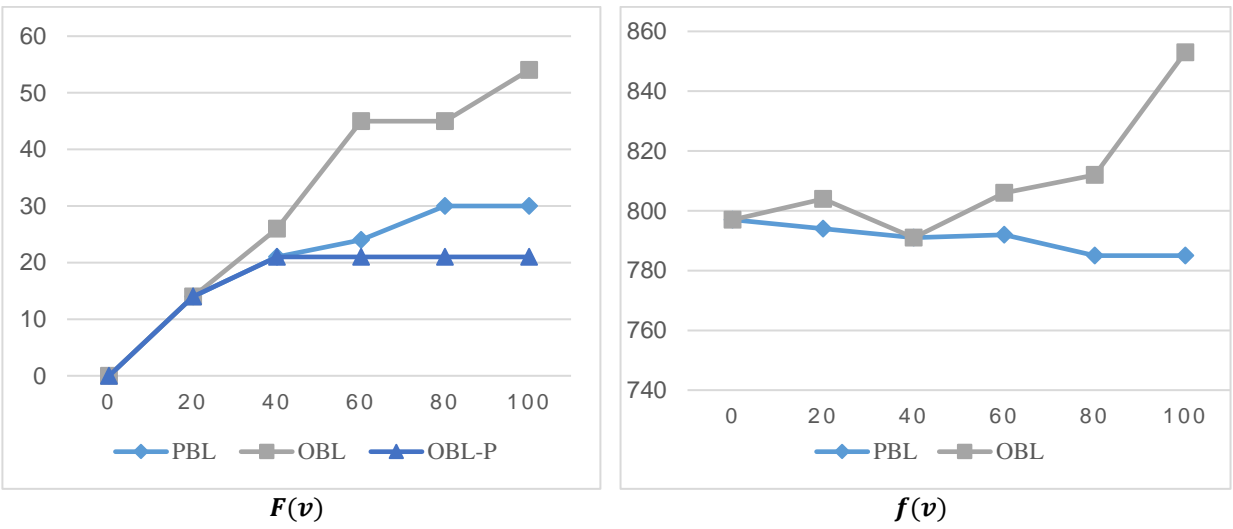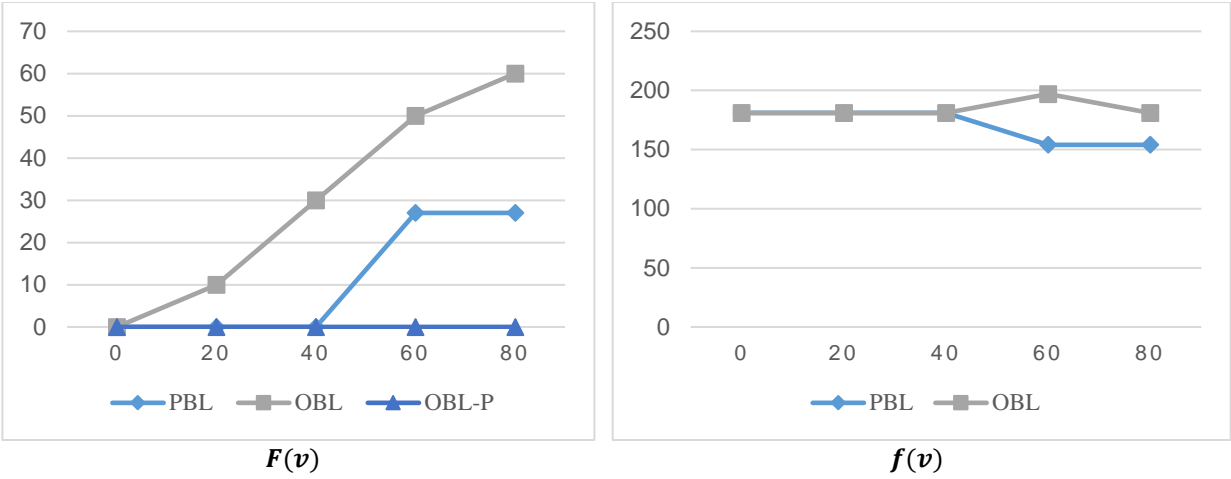
Figure 4.16: RND 47
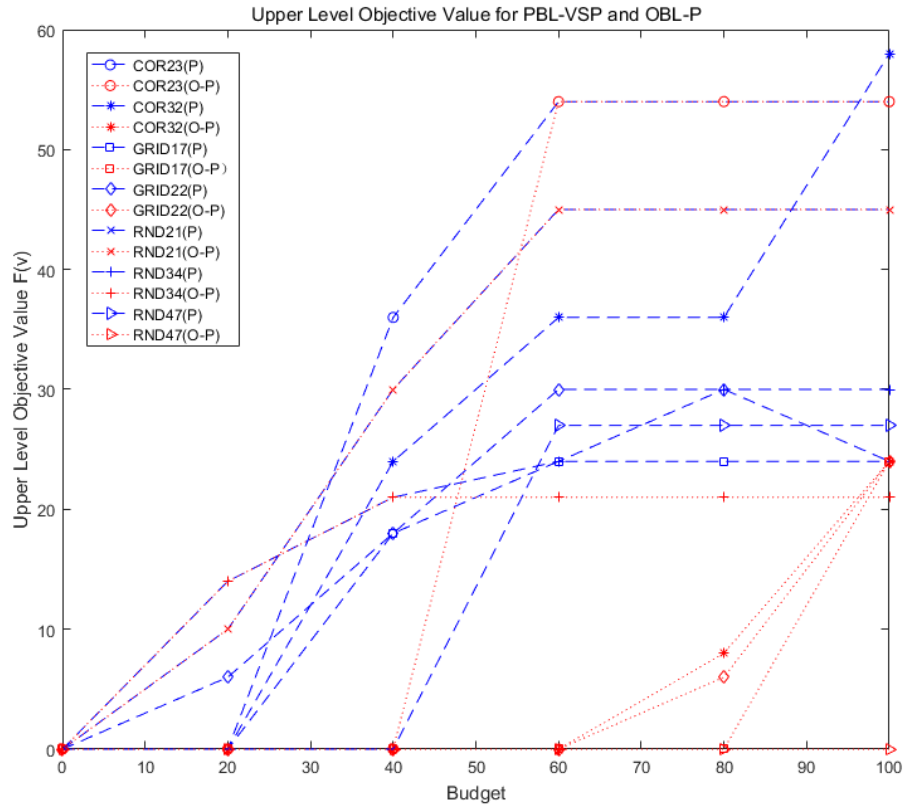
Figure 4.17: Upper Level Objective Values of PBL-VSP and OBL-P
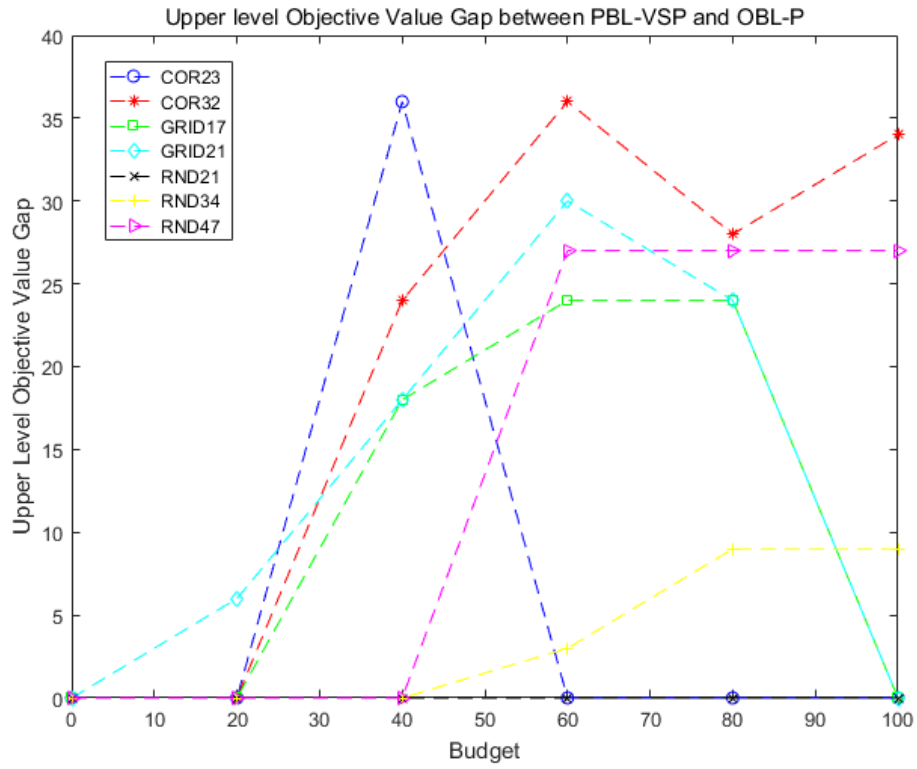
Figure 4.18: Upper Level Objective Value Gaps between PBL-VSP and OBL-P

$C=0$
$F(\boldsymbol{v}) = 0$
$f(\boldsymbol{v})=797$
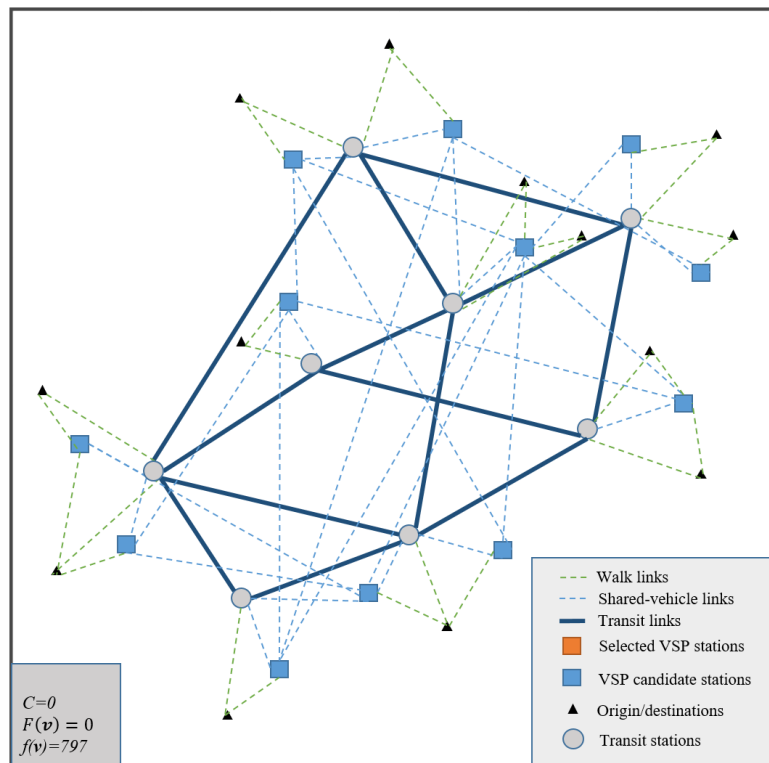
Figure 4.19: PBL-VSP and OBL-VSP Optimal VSP Stations for RND 34, C=0
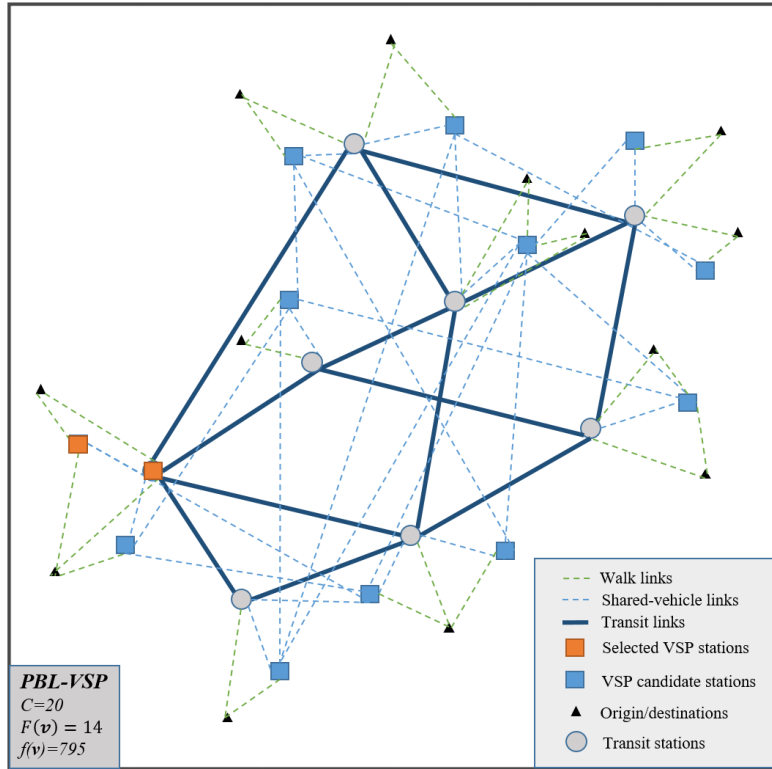
Figure 4.20: PBL-VSP Optimal VSP Stations for RND 34, C=20
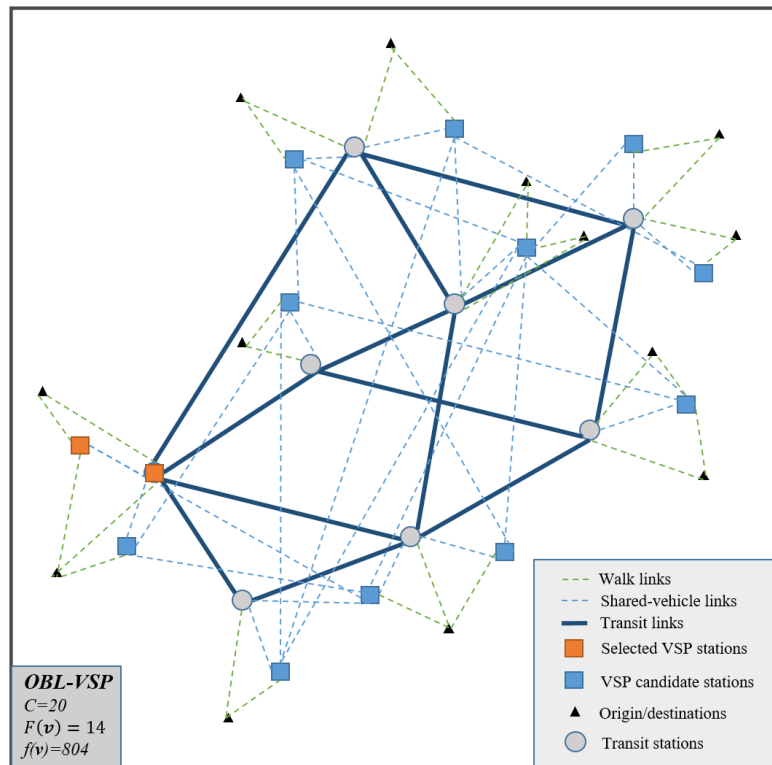
Figure 4.21: OBL-VSP Optimal VSP Stations for RND 34, C=20
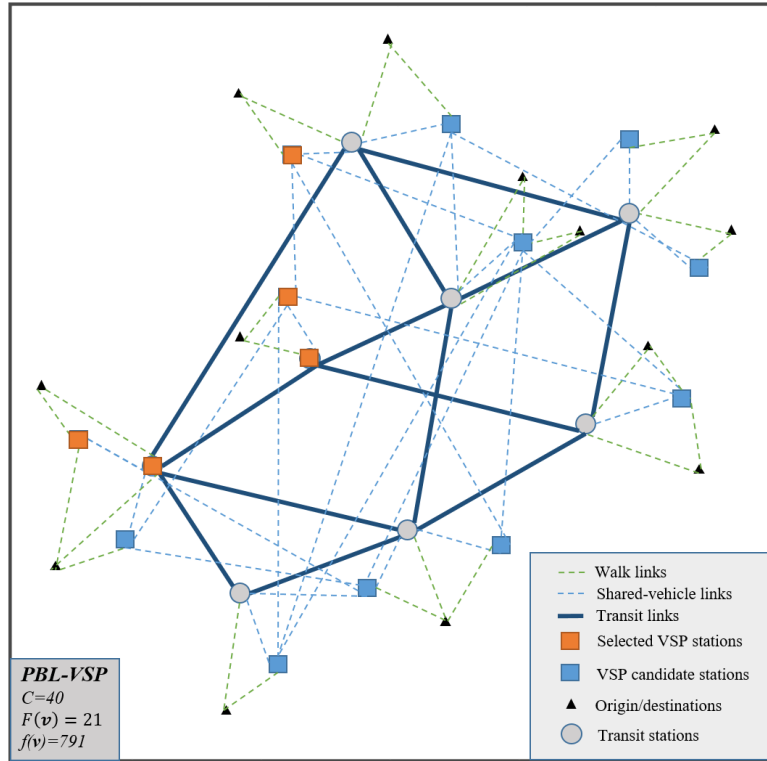
Figure 4.22: PBL-VSP Optimal VSP Stations for RND 34, C=40
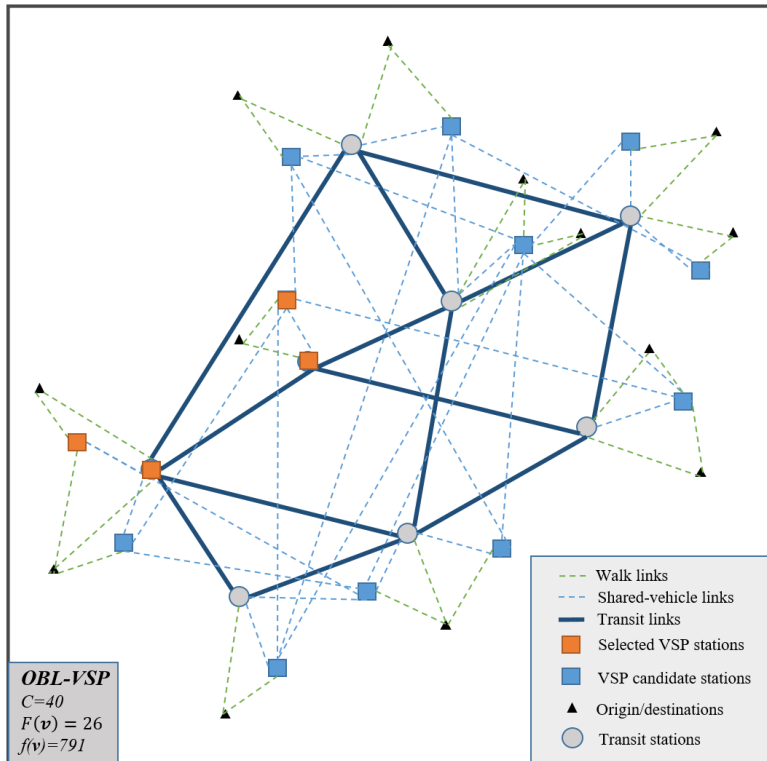
Figure 4.23: OBL-VSP Optimal VSP Stations for RND 34, C=40
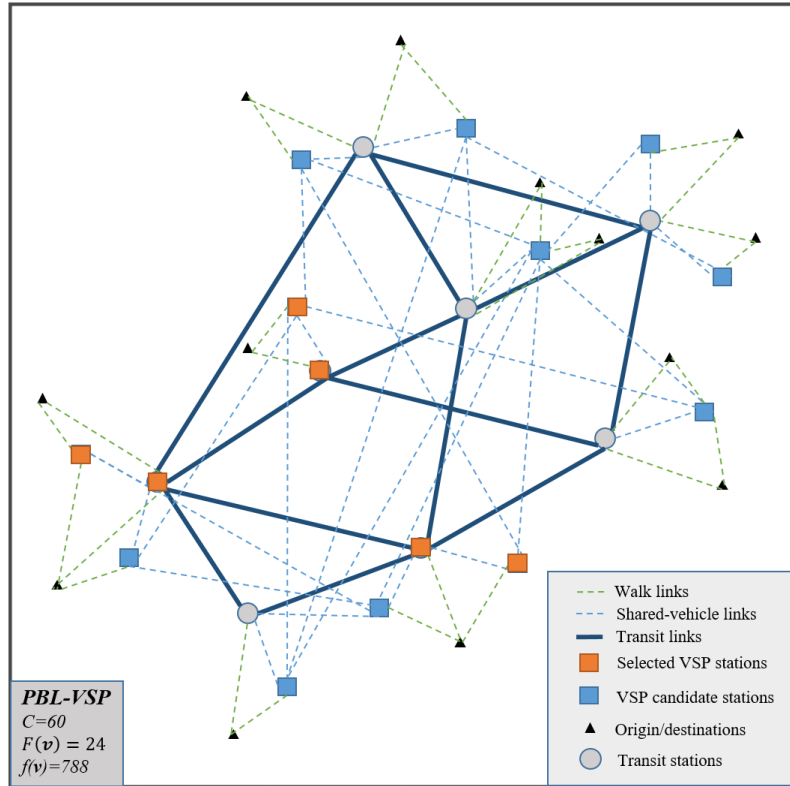
Figure 4.24: PBL-VSP Optimal VSP Stations for RND 34, C=60
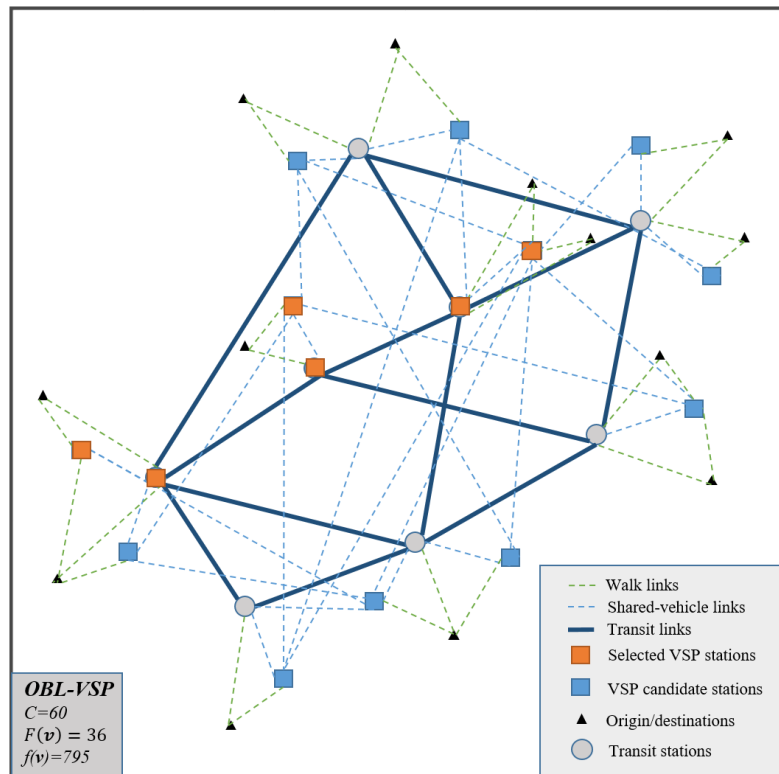
Figure 4.25: OBL-VSP Optimal VSP Stations for RND 34, C=60
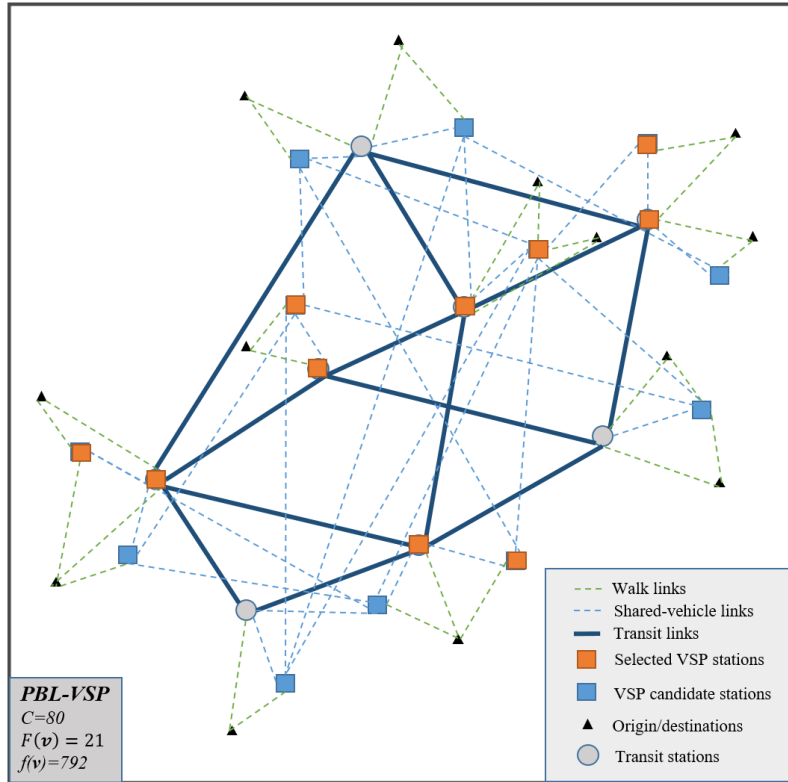
Figure 4.26: PBL-VSP Optimal VSP Stations for RND 34, C=80
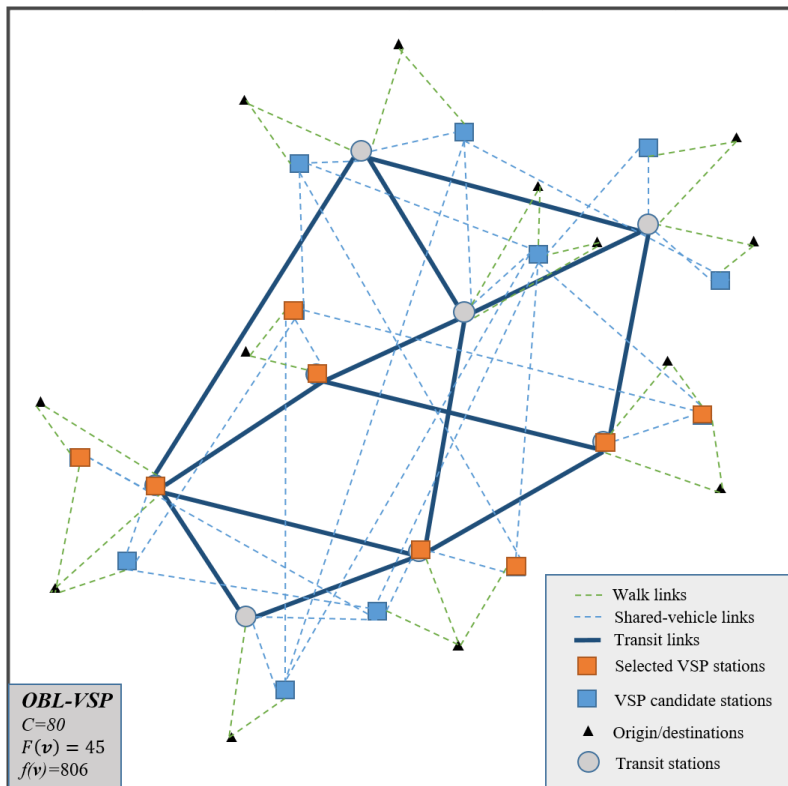
Figure 4.27: OBL-VSP Optimal VSP Stations for RND 34, C=80
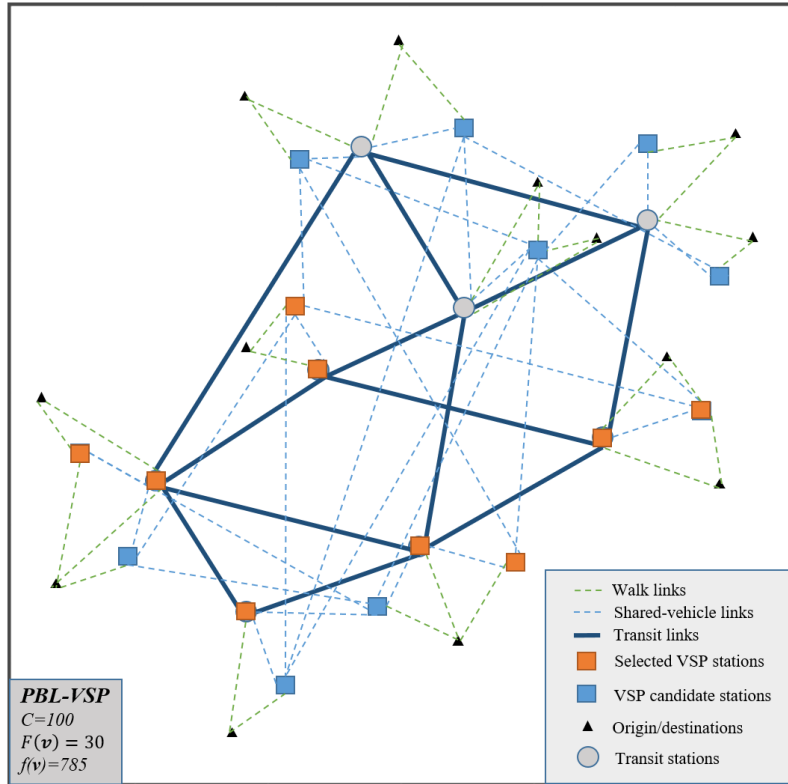
Figure 4.28: PBL-VSP Optimal VSP Stations for RND 34, C=100
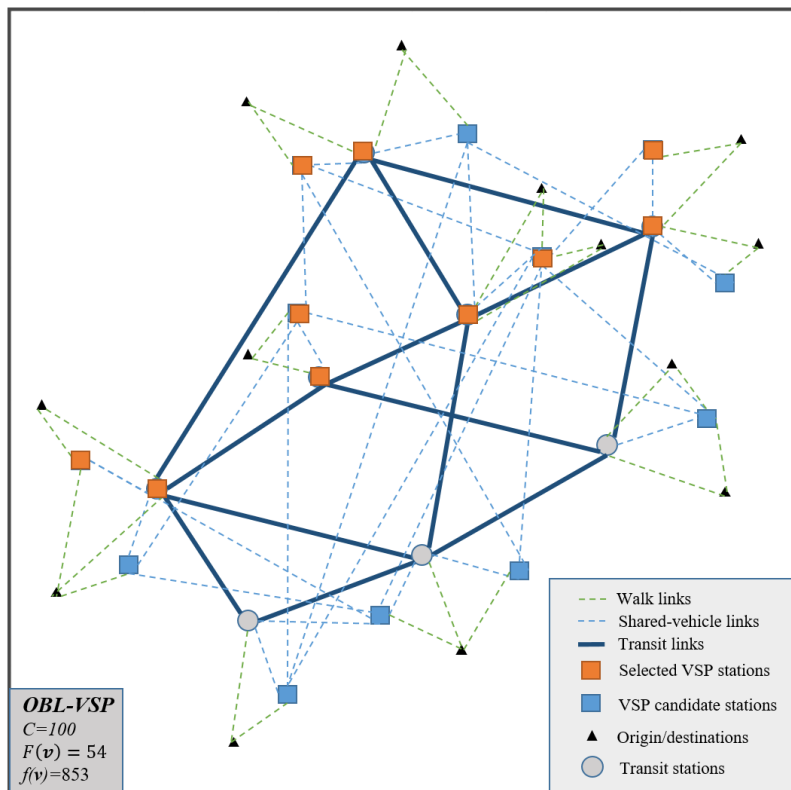
Figure 4.29: OBL-VSP Optimal VSP Stations for RND 34, C=100

Table 4.1: Summary of Network Instances

| Network	Instance	Nodes	Links	Transit Services	VSP sites	PBL		OBL	
						Int.vars	Cont.vars	Int.vars	Cont.vars
Corridor	COR23	23	74	2	7	497	1237	496	940
	COR32	32	102	4	11	684	1850	683	1314
GRID	GRID17	17	56	3	11	331	786	330	567
	GRID21	21	69	3	8	452	1129	451	889
RND	RND21	21	72	3	7	577	1284	576	912
	RND34	34	118	5	12	1509	2761	1508	1704
	RND47	47	164	6	19	1558	3049	1557	1783

Table 4.2: Computational Results

Instance	Budget	PBL			OBL			OBL-P
		$F(v)$	$f(v)$	Time	$F(v)$	$f(v)$	Time	$F(v)$
COR23	0	0	394.667	0.454	0	394	0.189	0
	20	0	394.667	2.47	12	429	0.377	0
	40	36	382.667	1.874	36	394	0.418	0
	60	54	376.667	74.798	60	388	50.594	54
	80	54	376.667	53.347	84	406	20.457	54
	100	54	376.667	40.272	108	396	30.73	54
COR32	0	0	528.667	0.671	0	528	0.205	0
	20	0	528.667	1.165	10	528	0.323	0
	40	24	522.667	71.575	30	528	50.323	0
	60	36	519.667	13.289	50	528	10.286	0
	80	36	519.667	87.97	58	526	60.632	8

	100	58	506.667	98.97	74	522.667	54.33	24
GRID17	0	0	208.333	0.256	0	208.333	0.203	0
	20	0	208.333	0.527	6	209.476	0.405	0
	40	18	206.333	2.549	18	216.250	0.393	0
	60	24	205.667	50.57	30	208.333	10.286	0
	80	24	205.667	81.117	36	208.333	62.661	0
	100	24	205.667	189.921	48	208.333	110.736	24
GRID22	0	0	244.5	0.15	0	246.283	0.254	0
	20	6	244	3.887	8	257.923	0.636	0
	40	18	220.5	44.237	24	252.067	23.385	0
	60	30	214.5	38.226	30	250.885	26.385	0
	80	30	214.5	183.152	46	267.924	89.357	6
	100	24	220.5	263.094	54	253.4	235.281	24
RND21	0	0	210	0.411	0	210	0.086	0
	20	10	204	0.509	10	212	0.218	10
	40	30	192	3.999	30	192	0.211	30
	60	45	183	40.502	50	183	30.641	45
	80	45	183	62.141	58	192	50.331	45
	100	45	183	149.804	74	195	120.954	45
RND34	0	0	797	1.083	0	797	0.219	0
	20	14	794	2.579	14	804	0.413	14
	40	21	791	24.309	26	791	20.649	21
	60	24	792	614.946	45	806	411.181	21
	80	30	785	1143.061	45	812	920.467	21

	100	30	785	1262.922	54	853	830.48	21
RND47	0	0	181	0.363	0	181	0.201	0
	20	0	181	114.645	10	181	80.455	0
	40	0	181	508.2	30	181	565.39	0
	60	27	154	1211.45	50	197	1086.4	0
	80	27	154	1372.35	60	181	1120.4	0
	100	27	145	3607.74 (16.74%)	8	181	3245.8	0

Table 4.3: PBL-VSP and OBL-VSP Optimal Configuration for RND 34

Budget	Configuration	PBL-VSP	OBL-VSP
C=20	$x_i = 1$	5 14	5 14
	y_i	2 2	2 2
	z_i	2 0	1 1
C=40	$x_i = 1$	3 5 11 14 17	3 5 11 14
	y_i	1 3 1 3 0	1 3 2 3
	z_i	1 0 0 3 0	1 0 0 3
C=60	$x_i = 1$	3 5 7 11 14 18	2 3 5 10 11 14
	y_i	3 3 1 3 3 1	1 3 3 1 3 3
	z_i	0 0 1 3 3 0	1 3 0 0 0 3
C=80	$x_i = 1$	2 3 4 5 10 11 12 14	3 5 7 8 11 14 18 20
	y_i	3 3 0 3 3 3 1 3	3 3 1 3 3 3 1 2
	z_i	1 3 0 1 2 0 1 2	1 0 1 0 2 3 0 2
C=100	$x_i = 1$	1 3 5 7 8 11 14 18 20	2 3 4 5 6 10 11 13 14 17
	y_i	1 3 3 3 3 3 3 3 3	3 3 2 3 1 3 3 2 3 1
	z_i	1 0 3 3 0 3 0 0 3	0 3 0 0 0 3 0 2 3 1

CHAPTER 5: CONCLUSION

The studies presented in this dissertation have advanced the knowledge and techniques in transportation system design and operations including vehicle scheduling, emergency medical service network design and vehicle sharing program network design. Three critical transportation problems are studied and discussed.

In chapter 2, we studied the scheduling problem for a bus fleet with alternative fuel vehicles. In collaboration with a public transportation company, we successfully addressed a practical problem in real life. We specifically study the cost characteristics of different types of vehicles and consider the traffic congestion effects on fuel economy in the model. It is demonstrated that the schedule generated by the model could significantly reduce total fleet size required to finish timetables and balance the utilization of conventional diesel buses and alternative fuel vehicles. Results show that the model provides a useful tool for transit provider optimally allocating resources in their business operations.

In chapter 3, we studied emergency medical service network design problem to determine the robust ambulance locations. Our study illustrated that traditional stochastic and probabilistic models fail to capture all possible uncertain scenarios when the problem size is large. To address this issue, we proposed two-stage robust optimization models. This special modeling technique incorporates a compact uncertainty set and enables the decision maker to fully take advantage of the revealed information to make less conservative decisions. The proposed model is the first model to consider the impact of relocation operations on the initial locations. Furthermore,

classical algorithms have limited capability of handling two-stage robust optimization model with mixed integer recourse problem, an approximate column-and-constraint generation algorithm was developed and implemented. We conducted numerical experiments on a practical-scale instance with real world data. Results show our method has strong capability of producing either optimal solution or solutions with very small optimality gaps.

In chapter 4, a novel research topic studies critical limitations that recently have been found in traditional bi-level optimization problems. We demonstrated that the regular optimistic bi-level programming model unable to identify the uncertainty introduced by the lower level model from the non-cooperative and non-unique issues. This counterintuitive finding totally overthrows our understanding of bi-level programming. We further developed a pessimistic optimization model that can derive solutions which are robust in any uncertain situations. A bi-level relaxation and relaxation-and-correction computational scheme are proposed for finding solutions that are both feasible and optimal to the pessimistic optimization model. Our experiments are conducted on three widely adopted prototypical transportation networks with several instances. Solutions are compared with that from optimistic formulation. Results demonstrated that the pessimistic optimization model has great potential in designing a robust and reliable vehicle sharing program network.

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