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Thinking Nature, "Pierre Maupertuis and the Charge of Error Against Fermat and Leibniz"

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Abstract

The purpose of this dissertation is to defend Pierre Fermat and Gottfried Wilhelm Leibniz against the charge of error made against them by Pierre Maupertuis that they errantly applied final causes to physics. This charge came in Maupertuis' 1744 speech to the Paris Academy of Sciences, later published in different versions, entitled *Accord Between Different Laws Which at First Seemed Incompatible.* It is in this speech that Maupertuis lays claim to one of the most important discoveries in the history of physics and science, The Principle of Least Action. From the date of this speech up until the end of the twentieth century, Maupertuis was credited with this discovery. Fermat discovered least time in optical physics, and Leibniz co-discovered infinitesimal calculus. When the credited discoverer of least action in physics accuses the discoverer of least time in optics and the co-discoverer of infinitesimal calculus of error before and audience of mathematicians, physicists and scientists, it is an event that calls out for investigation.

The idea of final causes in physics is the idea that bodies move for an end purpose. During the early modern period, this challenged the intellectual establishment of the day with the idea of thinking in nature. The question which fueled the research for this dissertation is why such a man would accuse two other prominent intellects with such an unprovable metaphysical assumption. The research for this project started with a study of the positions of all three of these men regarding final causes in physics. The second phase was to research the historical context in which *Accord Between Different Laws Which at First Seemed Incompatible* was

written and delivered. This context included the life of Maupertuis as member of the Paris Academy of Sciences and as later President of the Berlin Academy of Sciences. It also included the workings of three highly competitive, state funded, academies of science in Berlin, London and Paris.

Research showed that no formal positions were held on the subject of final causes by either Maupertuis or Fermat. Only Leibniz demonstrated an established and well thought out position on the subject. Research did reveal the story of an ambitious man in Maupertuis, who made it the fulfillment of his ambition to rise in the ranks of math and science within the academies and establish himself as an intellectual great in European culture. Consequently, the life and career of Maupertuis illustrates the sociological dimension of scientific achievement. *Accord Between Different Laws Which at First Seemed Incompatible* turned out to be a politically calculated speech delivered for the purpose of career advancement. In 1744, Maupertuis was being considered by the King of Prussia, Frederick the Great, and the leadership of the Berlin Academy of Science for the presidency of that institution. Maupertuis knew this. Therefore, the work must be interpreted in this context. Consequently, the charge of error against Fermat and Leibniz by Maupertuis must be interpreted likewise.

Discovery was made in contextual and Leibnizian research that Maupertuis was aware of Leibniz's idea of action from 1738 on, and knowingly claimed to have discovered a generalized notion of action in physics which was not his. It is the story of ambition clouding human judgment. The Leibnizians attacked Maupertuis on this matter, led by a member of the Berlin Academy named Samuel Konig. In the "Konig Affair", Konig accused Maupertuis of what is essentially plagiarism, and Maupertuis countered with charging Konig with forgery for claiming to have in his possession a letter from Leibniz to Jacob Hermann demonstrating Leibniz's knowledge of least action. Maupertuis buries Konig in legal proceedings, but loses his reputation in the process. During the final stages of the Konig affair, Maupertuis admits to Patrick d'Arcy, a member of the Paris academy, that he had used Leibniz's theory of action. Having lost his effective leadership as President of the Berlin Academy, Maupertuis spends the last years of his life in his native France without ever relinquishing his title and office.

When at first examining the charge of error, the immediate notion is that this is a cause and effect argument. It appears to be an argument about the order of metaphysics before physics. This turns out not to be the case. Maupertuis agrees with Fermat and Leibniz at every turn. The charge is all about career success.

Introduction

In this dissertation, I will analyze, and put into context, a statement made by the mathematician, philosopher, and then Paris Academy of Sciences member Pierre de Maupertuis (1698 – 1759),[1] that attorney and mathematician Pierre Fermat (1601 – 1665)[2] and mathematician and philosopher Gottfried Wilhelm Leibniz (1646 – 1716)[3] were in error regarding their application of final causes to physics. In his 1744 work entitled *Accord de differentes loix de la Nature qui avoient jusqu'ici paru incompatibles,* (*Accord Between Different Laws of Nature that at First Seemed Incompatible*) Maupertuis makes the following statement which serves as "focal passage" for this dissertation:

> I know the distaste that several mathematicians have for *final causes* applied to physics, a taste that I share up to some point. I admit it is risky to introduce such elements; their use is dangerous, as shown by the error made by Fermat and Leibniz in following them. Nevertheless, it is perhaps not the principle that is dangerous, but rather the hastiness in taking as a principle that which is merely *some consequences*.[4]

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[1] Beeson, David. *Studies on Voltaire and the Eighteenth Century, "Maupertuis: An Intellectual Biography"*. p. 7 – 13; Robert Audi does not include Maupertuis in his *Cambridge Dictionary of Philosophy.* This is probably because Maupertuis is considered a mathematician rather than a philosopher, if he is considered at all, and that is the problem. Maupertuis is obscure compared to Leibniz and Newton. On page 1 of his introduction, Beeson opens his work on Maupertuis with an explanation for this situation. "Although Maupertuis does not deserve the general obscurity into which he has sunk, the few rays of light that relieve it do little justice to his reputation. The texts that comment on his life and work fall into two categories: overblown tributes or unjust contempt, with the latter, thanks chiefly to Voltaire, predominating. The *Diatribe du docteur Akakia* paints a picture of Maupertuis as a bumptious fool whose incompetence was matched only by his arrogance. The accusation carries far more weight than the defense, because Voltaire's attack retains the ability to inspire laughter, while there is little to entertain a reader in Maupertuis' response. In any case, Maupertuis' position is fatally undermined, for those who know a little about him, by the bitter flavor of his treatment of Koenig, arraigned before a Berlin Academy of Sciences perverted for the purpose of a kangaroo court and inevitably returning a verdict of guilty of forgery on the flimsiest of evidence."

[2] Nahin, Paul. *When Least is Best. "How Mathematicians Discovered Many Clever Ways to make Things as Small (or as large) as Possible"* Princeton University Press: Princeton, N.J., 2004, p. 99

[3] Audi, Robert (Editor). *Cambridge Dictionary of Philosophy.* Second Edition Cambridge University Press: New York, N.Y., 1999, p. 491

[4] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même Année. Tirez des Registres de Cette Académie, p. 543; Richard Lamborn, Translator, in all cases where another translator is not specified (unless otherwise indicated). « Je connais la répugnance que

In this same work Maupertuis lays claim to the discovery of "The Principle of Least Action", a claim he is credited with broadly today.[5] Pierre Fermat is the discoverer of "The Principle of Least Time" in optical physics and Gottfried Leibniz is the co-discoverer of "Infinitesimal Calculus" and known for his promotion of *vis viva,* "living force", which today science recognizes as kinetic energy.[6]

In this dissertation, the following thesis will be advanced. Maupertuis' 1744 accusation of error is a politically calculated career move within the Paris Academy of Sciences.[7] His charge is not founded on any metaphysical disagreement, nor a disagreement over perceived sequence of occurrence between metaphysics and physics in terms of cause and effect. It is a charge pulled from the Paris memoires and inserted into *Accord*[8] for the purpose of gaining favor

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plusieurs Mathématiciens ont pour les causes finales appliquées à la physique, & l'approuve même jusqu'à un certain point; j'avoue que ce n'est pas sans péril qu'on les introduit: l'erreur ou sont tombez des hommes tels que Fermat & Leibniz en est suivant, ne prouve que trop combine leur usage est dangereux. On peut cependant dire que ce n'est pas le principe qui les a trompez, c'est la précipitation avec laquelle ils ont pris pour le principe ce qui n'en était que des conséquences. »

[5] Pierre Louis Maupertuis – Wikipedia, the free encyclopedia; en.wikipedia.org/wiki/Pierre_Louis_Maupertuis

[6] Leibniz discovered "living force" while working with the collisions laws of Huygens and Wren. In his chapter "Leibniz and the Vis Viva Contraversy" (Dascal, Marcelo, Editor, *The Practice of Reason, "Leibniz and His Contraversies",* John Benjamins: Philadelphia, Pa., 2010, p. 57), Idan Shimony says, "Leibniz was well acquainted with the works of Huygens, Wallis and Wren. He was thus fully familiar with the difference between the scalar 'quantity of motion' given by the product of mass and speed on the one hand, and the vector of mass times velocity, later called momentum on the other. Accordingly, his public arguments against Descartes were intended to show the advantage of living force or mv^2 over the erroneous Cartesian quantity of motion, and not to reject the valid quanity of momentum or *mv*."

Descartes discovered "momentum", and both the Cartesians and the Leibnizians made claims to the discovery of the real nature of force. In his essay "Leibniz versus the Cartesians on Motion and Force" (*Studia Leibnitiana,* 1975, Vol. 7, No.1, p. 135 – 136), Marshall Spector says, "Descartes and his followers, who considered quantity of motion to be the true measure of force, were 'really' speaking about momentum ; while Leibniz, who emphasized the importance of the square of the velocity of a body in measuring its force, was 'really' speaking about Kinetic Energy. Thus there was no real disagreement, but only a confusion, when each side called its concept by the same name - 'force', and went on to say that force is conserved in the universe."

[7] In the history of science and philosophy of science, there exist no scholarly works exclusively on the 1744 edition of *Accord Between Different Laws Which at First Seemed Incompatible* in either book or peer reviewed essay form. Likewise, there exist no scholarly works on Maupertuis' charge of error against Fermat and Leibniz in either book or peer reviewed essay form. Consequently, the commentary on this issue is scant, and appears in connection to discussions concerning the principle of least action and issues connected to the subsequent editions of his works published by Maupertuis between 1750 and 1756.

[8] This is how *Accord Between Different Laws Which at First Seemed Incompatible* will be referred to in the rest of this essay.

with factions within the Paris and Berlin Academies of Science. As the focal passage indicates, Maupertuis is not against the application of final causes to physics. He does so himself, and in a Leibnizian manner. It is just that he shares in the disgust with the misapplication of final causes in physics "up to a point". For political expediency, Maupertuis never defines where the "point" is, nor who the mathematicians are.

Τελικη αιτια is Greek for "final cause". In the early modern period, the term is written in Latin as "causa finales". It refers to the end(s) towards which action is performed. In terms of optical physics, it refers to the end purpose for which a ray of light, or light in general, moves. The Greek word for "end" is τελος, "telos". The study of "ends", teleology", is a word coined by the German philosopher Christian Wolff.[9]

The ancient philosophers Plato and Aristotle, and the writer of the Biblical book of Genesis, provide the theological and philosophical background to the early modern debate in theoretical physics of optics. Plato says in the *Timaeus,*

> The god wanted everything to be good and nothing to be bad so far as that was possible, and so he took over all that was visible – not at rest but in discordant and disorderly motion - and brought it from a state of disorder to one of order, because he believed that order was in every way better than disorder. Now it wasn't permitted (nor is it now) that one who is supremely good should do anything but what is best. Accordingly, the god reasoned and concluded that in the realm of things naturally visible no unintelligent being could as a whole be better than anything which does possess intelligence as a whole, and he further concluded that it is impossible for anything to come to possess intelligence apart from soul. Guided by this reasoning, he put intelligence in soul, and soul in body, and so he constructed the universe. He wanted to produce a piece of work that would be as excellent and supreme as its nature would allow. This, then, in keeping with our likely account, is how we must

[9] In his essay "Teleology and the Concepts of Causation" (*Philosophica*, 1990, Vol. 46, No. 2, p. 17), Ernst von Glasersfeld says "The term teleology was coined by the philosopher Christian Wolff in his Latin treatise of 1728 and he defined it as indicating the part of natural philosophy that explained the ends or purposes of things." See FN 1, p. 40, where Glasersfeld identifies the Wolffian work as *Philosophia rationalis sive logica,* and the definition of the term given "in #85 of his preliminary discourse". In his essay "Leibniz on Natural Teleology and the Laws of Optics" (*Philosophy and Phenomenological Research*, 2009, Vol. 78, No. 3, p. 505), Jeffrey McDonough says, "…a teleological explanation is an explanation that attempts to explain a behavior or event by appealing to an outcome or consequence of that behavior or event. 'Betty is going to law school in order to get rich;' 'The bear is swatting at the beehive in order to get honey;' 'The heat seeking missle swerved left in order to hit its target;' 'Stones fall in order to reach their natural resting place at the center of the earth' are thus all at least candidates for teleological explanations."

> say divine providence brought our world into being as a truly living thing, endowed with soul and intelligence.[10]

This is an example of intentional teleology where bodies in motion move because of the intentions of an outside agent purposively setting them into motion. They move for the purpose of goodness and that which is best, according to the wishes of the intelligent agent.

An example of non-intentional teleology where no outside agent causes motion is Aristotle. In *Physica*, he outlines a fourfold explanation of causation which influenced physics and metaphysics up to and through Maupertuis' day. The fourth cause, for Aristotle, is the final cause.

> Now that we have established these distinctions, we must proceed to consider causes, their character and number. Knowledge is the object of our inquiry, and men do not think they know a thing until they have grasped the 'why' of it (which is to grasp its primary cause). So clearly we too must do this as regards both coming to be and passing away and every kind of physical change, in order that, knowing their principles, we may try to refer to these principles each of our problems.
>
> In one sense, then, (1) that out of which a thing comes to be and which persists, is called 'cause', e.g. the bronze of the statue, the silver of the bowl, and the genera of which the bronze and the silver are species.
>
> In another sense (2) the form of the archetype, i.e. the statement of the essence, and its genera are called 'causes' (e.g. of the octave the relation of 2:1, and generally number), and the parts in the definition.
>
> Again (3) the primary source of the change or coming to rest; e.g. the man advice is a cause, the father is cause of the child, and generally what makes of what is made and what causes change of what is changed.
>
> Again (4) in the sense of end or 'that for the sake of which' a thing is done, e.g. health is the cause of walking about…The same is true of all the intermediate steps which are brought about through the action of something else as means towards the end, e.g., reduction of flesh, purging, drugs, or surgical instruments as means towards health.[11]

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[10] Cooper, John (Editor). *Plato, "Complete Works".* Hacket: Indianapolis, In., 1997, p. 1236; *Timaeus* 30a – b; In his book *Plato's Natural Philosophy, "A Study of the Timaeus-Critias"* (Cambridge University Press: New York, N.Y., 2004, p. 2) Thomas Johansen says, "Far from being value-free, cosmology for Plato is centered on the representations of goodness and beauty. He sees it as the central task of cosmology to articulate the way in which the cosmos manifests those values. Another word for this conception of cosmology is 'teleology'. For Plato goodness and beauty do not just happen to be found in the cosmos. They are there because the cosmos is so designed."

[11] McKeon, Richard (Editor). *The Basic Works of Aristotle.* Random House: New York, N.Y., 1941, p. 240 – 241, *Physicsa* II, 3, 194b20 – 35; These four causes must be understood within the context of *Physica*. In his essay "Philosophy of Science" (Barnes, Jonathon (Editor). *The Cambridge Comanion to Aristotle.* Cambridge Press: New York, N.Y., 1995, p. 118), R.J. Hankinson says, "The *Physics* is concerned with the world of change. In a distinction that was to form the backbone of the medieval world-picture which persisted at least until Galileo, Aristotle considered the universe to divide into two distinct parts. There was the domain of the heavenly bodies where perfection and eternal existence rule – nothing in that divine realm really alters. By contrast, in the region

To apply this to optics, Aristotle would say that light, like all other natural phenomena, is completely known only when science and physics can explain the "why?" of it. Why, in other words, does the light shine? Why does it move towards surfaces of differing medium thicknesses? When it arrives at the surface, why does it reflect and refract? The answer to these questions, Aristotle posits, yields a complete knowledge of light.

A third example of teleological explanation that heavily influenced every thinker in the early modern period was the words of the writer of Genesis concerning days one and four of creation. To describe day one he says, "In the beginning God created the heavens and the earth. Now the earth was formless and empty, darkness was over the surface of the deep, and the Spirit of God was hovering over the waters. And God said, 'Let there be light,' and there was light. God saw that the light was good, and he separated the light from the darkness. God called the light 'day' and darkness he called 'night'. And there was evening and there was morning, the first day." To describe day four he says,

> And God said, 'Let there be lights in the expanse of the sky to separate the day from the night, and let them serve as signs to mark seasons, and days and years, and let them be lights in the expanse of the sky to give light on the earth.' And it was so. God made two great lights – the greater light to govern the day and the lesser light to govern the night. He also made the stars. God set them in the expanse of the sky to give light on the earth, to govern the day and the night, and to separate light from darkness. And God saw that it was good. And there was evening and there was morning – the fourth day.[12]

below the orbit of the moon (hence 'sublunary'), change and decay are the norm." The common opinion among scholarship is that Aristotle's teleology is non-intentional. On page 127 Barnes notes that, although no atheist, Aristotle's God does not participate in change, nor is the creator of the natural order, nor the efficient cause of that order. He says, "Aristotle,...repudiates the divine Artificer. But neither does he rely on pure mechanical necessity to explain the world, rather he espouses a form of non-intentional teleology. Natural processes are goal directed, we cannot understand them other than in terms of the end-states to which they (other things being equal) tend." In her essay "Aristotle, Teleology, and Reduction" (*The Philosophical Review,* 1992, Vol. 101, No. 4, p. 791) Susan Suave Meyers argues that Aristotle's thesis of natural teleology where teeth are made for chewing and eyes are made for seeing is in line with necessity. This is in line with the position that there is no outside agent thinking in Aristotle's natural philosophy.

[12] *The Holy Bible*, New International Version, Zondervan Press: Grand Rapids, Mi., 1984; Genesis 1:1-4, 14 - 19

For the writer of Genesis, God creates the light and, like Aristotle's sculptor, stands back from his handy-work and proclaims it "good". This is not to say, like Plato, that God thought bringing already existing matter into order out of disorder was a "good" thing to do. Rather, as the Augustinian interpretation dictated in the early modern period, God created the light "ex nihilo", out of nothing.[13] The Christian emphasis in the early modern period, like Plato, is that God directs everything for the best according to what is good. This is also Leibnizian.

The influences of Greek philosophy, Christian theology, and the mechanical philosophy of the early modern scientific revolution, all converge to form foundation points of argumentation concerning light and its motion. It is the debate between these influences over the behavior of light in refraction specifically that serves as the point of reference for this dissertation. When Maupertuis accuses Fermat and Leibniz of error in their application of final causes to physics, he is referring to the issue of refraction in optics and the resulting argument about thinking in nature.

The birth of modern theoretical physics takes place in the early part of the seventeenth century. By the time of Maupertuis, the debate is well under way. Willibrord Snell (1580 – 1626)[14] galvanized this debate with his discovery of the law of refraction in 1621.[15] The

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[13] In his chapter "Time and Creation in Augustine" (Meconi, David Vincent and Eleonore Stump, Editors. *Cambridge Companion to Augustine.* Cambridge Press, July 2014, Online ISBN: 9781139178044), Lewis Ayres says, "Augustine's most extensive discussions of philosophical and theological cosmology are found in his commentaries on Genesis (*De Genesi ad litteram libri duodecim*), in the last three books of the *Confessions*, and in Books 11 and 12 of the *De civitate Dei.* The main lines of his view of creation are as follows. God created both the spiritual realm of angels and the visible world, including the incarnated souls, out of nothing, (*ex nihilo*), without any preexisting matter of other things outside of God (*Gn. adv. Man.* I.6.10; *conf.* 11.5.7, 12.7.7, 12.8.8; *Gn. litt.* I.14.28 – 15.29; *civ. Dei* II.21,24). It took place through God's omnipotence without toil, effort, and industry (*div. qu.* 78; *Gn. litt.* 9.17.32; *civ. Dei* 12.18)."

[14] Nahin, p. 102

[15] The actual credit is debated between Descartes and Snell. In his article "Refraction, Index of" (Rigden, John S. Editor. *MacMillan Encyclopedia of Physics*, Volume 3, Simon and Schuster: New York, N.Y., 1996, p. 1348) Charles Head says, "Most science historians credit Willebrord Snell with the discovery of the law that defines the index of refraction." Some argue that Thomas Harriot (1560 – 1621) knew the law before either Snell or Descartes. In his essay "An Early Experimental Determination of Snell's Law" (*American Journal of Physics*, 1951, 19, p.

geometry for Snell's law as applied to the early modern debate in refraction for purposes of this essay looks like this:

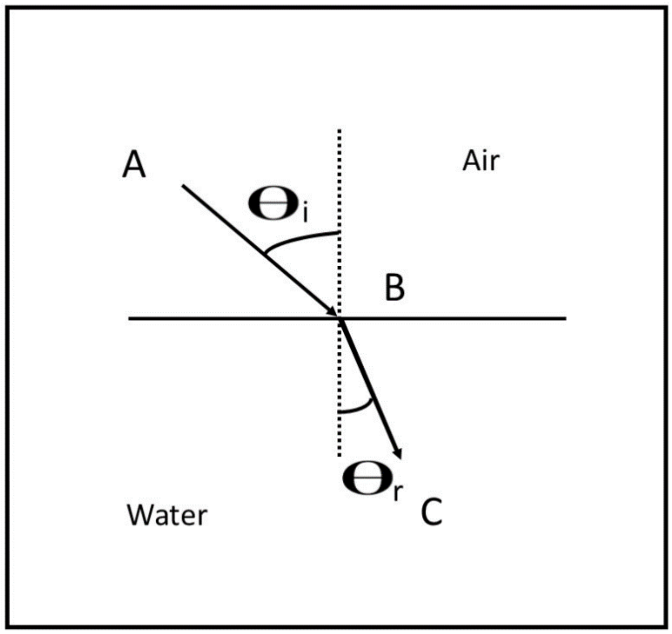

Figure #1. Snell's Law of Refraction as Depicted Geometrically; Upon entering the water at point *B*, the ray of light "refracts" and bends towards the normal. This is how light works as depicted on a flat plane surface.

The mathematical formula for Snell's Law is

$$\frac{\sin i}{\sin r} = \frac{Vi}{Vr} = \frac{w}{v} = \text{C}.$$

The *C* stands for "constant", and is a function of the nature of the two environments air and water. Snell says that if the medium of water is denser than the medium of air, and a ray of light

507), John Shirley says, "the law of sines was discovered independently in England by Thomas Harriot and his friends, Walter Warner and Sir Thomas Aylesbury."

travels from air into the water, then the constant is greater than one.[16] That is, sin(θi) > sin(θr) or, equivalently, $\theta i > \theta r$; i.e. upon entering the water the light beam bends towards the normal.[17]

This phenomenon is a philosophical, mathematical, and theological puzzle for the early moderns. All three scientific communities, the Royal Society of London, the Royal Academy of Science in Paris, and the Royal Prussian Academy in Berlin, were interested in solving this puzzle.[18] The problem, however, was that refraction came with an unquestionably metaphysical component which placed solving it somewhat outside of the laboratory. The paradigm for optics under which thinkers at that time labored was, therefore, unsettled.[19] The behavior of light in refraction was mathematically incompatible with the mathematics of reflection.

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[16] The ray of light travels with velocity. In his entry "Velocity" (Rigden, John, Editor. *Macmillan Encyclopedia of Physics.* Volume 4, Simon and Schuster: New York, N.Y., p. 1672) Justin Sanders says, "Velocity is the rate that the position of a body, relative to some frame of reference, changes with time. Velocity is a vector quantity, and it possesses both a direction and a magnitude called 'speed'. For example, an automobile may be traveling at a speed of 88 kilometers per hour (kph). If it is driving toward the northwest, then its velocity is 88 kph northwest. Another automobile may have the speed as the first, but it will have a different velocity if it travels in a different direction (88 kph south, for example) If a body travels in a straight line, and its initial position is x_0 at time t_0, and its position is x_1 at some later time $t_{1,}$ then the magnitude of the average velocity u is

$$u = \frac{x_{1-}x_0}{t_{1-}t_0} = \frac{\Delta x}{\Delta t}.$$

[17] See Nahin, p. 103, for a similar way to put this. For all drawings in this introduction, the vertical dotted line represents "normal". A vertical line is historically how "normal" is shown in explaining refraction on a flat plane, two dimensional, surface. In his essay "On Explanations in History" (*Philosophy of Science.* 1956, Vol. 23, No. 1, p. 17), Arthur Danto says, "In the statement 'The ray of light bends towards the normal in a certain medium because the index of density of the medium of origination is smaller than that of the medium of passage,' the notion of 'because' is somewhat gratuitous, for the phenomena happens to be covered by a known general law which merely requires that we furnish values for its variables in order to deduce a sentence describing the situation at hand. Generally in science, such laws, together with specifications of initial conditions, constitute the minimal equipment for a successful explanation." In his book *Geometrical and Physical Optics* (Longman: New York, N.Y., 1973, p. 5), R.S. Longhurst describes "normal" in wave reflection and refraction by saying, "In the case of reflection and refraction one again has the energy propagated along the wave normals. Consequently, if the rays are defined to be wave normal, one can say that the light behaves as a stream of energy traveling along the rays which are directed outwards from the source and obey the classical laws of reflection and refraction…." On p. 6 Longhurst says that "laws governing the behavior of the rays can be summarized in one fundamental law known as Fermat's principle."

[18] In his book *The Structure of Scientific Revolutions*. Fourth Edition, University of Chicago Press: Chicago, Il., 2012, p. 36), Thomas Kuhn defines scientific puzzles as "that special category of problems that can serve to test ingenuity or skill in solution."

[19] Kuhn, p. 10 and 11, says that scientific paradigms share two things in common; one, "There achievement was sufficiently unprecedented to attract an enduring group of adherents away from competing modes of scientific activity. Simultaneously, it was sufficiently open ended to leave all sorts of problems for the redefined group of

Of further debate in the early modern period was the question of the physical nature of light. Was the ray of light a corpuscular body or a wave function? If one thought of it as corpuscular, then one's paradigm tended to be Newton's *Optics.* If one thought of light as wave, then one's paradigm tended to be Huygen's *Traite de la Lumiere*. Newton said that light traveled faster in thicker mediums, and Huygens said that light did the opposite. These two competing seventeenth century opinions drew upon the ancient optical paradigm of Ptolemy. Concerning reflection, Ptolemy held that the angle of incidence was equal to the angle of reflection on a flat plane mirror surface. Ptolemy borrowed from Hero of Alexandria the idea that "the angle of incidence of the visual ray on the mirror was equal to the angle of reflection."[20] Ptolemy, however, failed to discover the law of refraction.[21] Though Snell discovered the law of refraction in 1621, there was no paradigm for optics due to the fact that the laws of motion as understood at that time, combined with the differing speeds of light in various mediums, did not allow for compatible harmony between reflection and refraction. So, the crisis brewed.

By the time of Maupertuis in 1744, the accepted opinion was that, if light was of a corpuscular nature, then it moved according to the laws of motion as established at that time. Accompanying this understanding was the common thinking in optics that light moved in propagation and reflection according to the shortest distance and least time, by virtue of a

practitioners to resolve." He lists Aristotle's *Physica*, Ptolemy's *Almagest*, Newton's *Principia* and *Optiks,* Franklin's *Electricity,* Lavoisier's *Chemistry* and Lyell's *Geology* as examples of paradigms.

[20] Darrigol, Olivier. *A History of Optics from Greek Antiquity to the Nineteenth Century.* Oxford University Press: New York, N.Y., 2012, p. 13

[21] In his essay "Ptolemy's Search for a Law of Refraction: A Case-Study in the Classical Methodology of 'Saving the Appearances' and its Limitations" (*Archive for History of Exact Sciences,* 1982, Vol. 26, No. 3, p. 221) Mark Smith says, "It is generally accepted that, after an auspicious beginning with Euclid's formulation of the ray-concept and an equally auspicious sequel, during which both optics proper and catoptrics were put on a firm scientific footing, the early development of mathematical optics was cut short by Ptolemy's failure to find the sine-law of refraction. This failure, which apparently stemmed the growth of mathematical optics for some fifteen centuries, has long puzzled historians of science. We know for a start that, in attempting to derive the law experimentally, Ptolemy was on the right track, and we also know that the experiment he devised could have yielded the right results. In short, the means of discovery available to Ptolemy, were essentially the same as those available to his seventeenth century successors."

uniform speed in a uniform environment. The problem with this was that the three scientific communities in Berlin, London, and Paris knew that light changes speeds in differing medium thicknesses. This, therefore, meant that if light moved faster in air than in water, it could not take the straight line route because this would take too much time. The straight line route, however, was what they understood to be natural law.

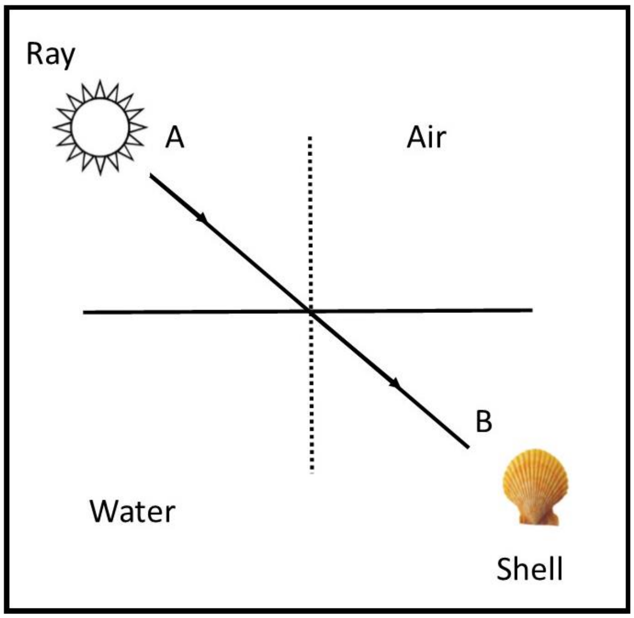

Figure #2 – Problem of Straight line route from air to water: This is not how it works! Light moves from point *A* to point *B* in the least time possible. This straight line takes more time than necessary due to the slowness of the route in the water.

The problem with refraction, and those refractive indexes which are being developed in the late seventeenth and early eighteenth centuries, was that light seemed to take a path that favored least time over shortest distance, as Snell and Fermat pointed out. This phenomenon, however, gave light the quality of intelligent decision making in that it seemed to be choosing a path of least time over shortest distance. This is best illustrated with the following life guard illustration.

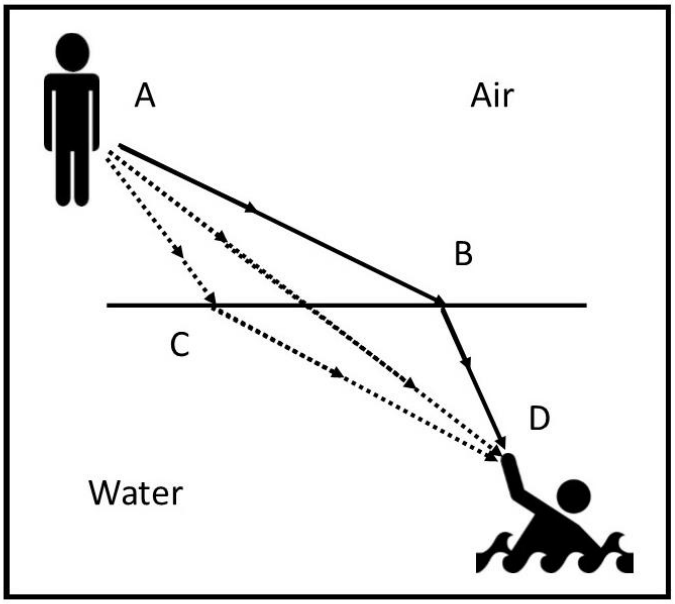

Figure #3 – In the act of saving the life of a drowning person, time is of the essence. Any given lifeguard chooses the route of least time in consideration of their ability to run faster over sand and through air, than swim through water. The lifeguard does not take path *AD* because the quicker route is path *ABD.* By running further downsand, the lifeguard knowingly eliminates the extra time it would take to swim through the extra water in the straight line route. Path *ACD* is not even considered because this route contains even more water than the straight line route. Therefore, it is the slowest route of all. The problem for the early modern thinkers was that the least time route, as they understood it from Snell and Fermat, was not the shortest distance, straight line, route. This finding severely challenged their thinking concerning natural law. Yet, the idea that light bent toward normal in refraction seemed persuasive to many. In the early modern period, two questions emerged from this phenomena: 1) Was human-like intelligence involved in the process? 2) For what purpose did light take the path of least time over shortest distance?

No one in the eighteenth century, however, wanted to admit that the nature of light contained a decision – making intelligence. Also, few wanted to admit that God was doing the thinking for light, directing its movements according to divine design. Additionally, few wanted to go the route of brute necessity without reason, but these three choices seemed to be their only options. The crisis in optical studies possessed an unsettling threefold philosophical, scientific, and theological dilemma. It is this crisis concerning the nature and activity of light that Maupertuis addresses in *Accord.*[22]

[22] Kuhn associates the early modern argument concerning the nature of light a crisis. He illustrates this on p. 86 along with Lavoisier and Priestley. "Often a new paradigm emerges, at least in embryo, before a crisis has

Maupertuis and the participants in the three academies had no firm answers to these questions. With no firm experiment in sight which would settle the question of the speed of light in various mediums, the "facts" boiled down to academic politics and the sociology of science.[23] The "discovery" of least action by Maupertuis is a microcosm of these political and sociological factors, with the three academies of science serving as laboratory incubators for such phenomena. This is how the science for the "discovery" of least action was done. It is within this context that Maupertuis charges Fermat and Leibniz of error.

developed far or been explicitly recognized….Thomas Young's first accounts of the wave theory of light appeared at a very early stage of a developing crisis in optics, one that would almost be unnoticeable except that, with no assistance from Young, it had grown to an international scientific scandal within a decade of the time he first wrote."

[23] The sociological approach to science is a controversial topic. Two philosophers of science who comment on the phenomenon are Thomas Kuhn and Steven Shapin. In *Structures,* Kuhn, p. 150, quotes Max Planck commenting on his own career in *Scientific Autobiography* who writes, "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because it's opponents eventually die, and a new generation grows up that is familiar with it." This is the case with the Cartesians and the speed of light in differing mediums. When one's career is made based on the "fact" that light moves quicker in water and glass than air, acceptance of the other side's "proof" is not possible for the sake of the career. Kuhn says on p. 151, "Though a generation is sometimes required to effect the change, scientific communities have again and again been converted to new paradigms. Furthermore, these conversions occur not despite the fact that scientists are human but because they are. Though some scientists, particularly the older and more experienced ones, may resist indefinitely, most of them can be reached in one way or another. Conversions will occur a few at a time until, after the last hold-outs have died, who whole profession will be practicing under a single, but now a different, paradigm."

Steven Shapin takes the legitimacy of the sociology of science for granted. In his book *The Scientific Revolution,* (University of Chicago Press: Chicago, Il., 1998, p. 9), he says, "For a long time , historian's debates over the propriety of a sociological and historically 'contextual' approach to science seemed to divide practitioners between those who draw attention to what were called 'intellectual factors' – ideas, concepts, methods, evidence – and those who stressed 'social factors' – forms of organization, political and economic influences on sciences, and social uses or consequences of science. That now seems to many historians, as it does to me, a rather silly demarcation,…If science is to be understood as historically situated and in its collective aspect (i.e. sociologically), then that understanding should encompass all aspects of science, its ideas and practices no less than its institutional forms and social uses. Anyone who wants to represent science sociologically cannot simply set aside the body of what the practitioners *knew* and how they went about obtaining that knowledge. Rather, the task for the sociologically minded historian is to display knowledge making and knowledge holding *as social processes.*" This approach is absolutely necessary for understanding Pierre Maupertuis, especially the political vantage point.

Chapter One: The Context

On April 15, 1744, forty-five year old Pierre Maupertuis stood before the Royal Paris Academy of Sciences and delivered a speech which contained his discovery of *The Principle of Least Action*. The speech was entitled *Accord Between Different Laws of Nature Which At First Seemed Incompatible.* Having established his world reputation in 1736 by flattening the earth at Lapland, Maupertuis needed another discovery, another scientific achievement, that would set him apart from the crowd. A French Newtonian who, along with Voltaire,[24] had promoted Newton's ideas in France, Maupertuis' goal was to convince Frederick the Great of Prussia to formally offer him the Presidency of the Berlin Academy of Science. As he stood before his audience of academic peers that day, Maupertuis delivered his address with the full knowledge that Frederick had been considering him for the job. He found his competitive edge that year in the idea of action formulated in such a way so as to reconcile seemingly irreconcilable laws of optics – propagation, reflection, and refraction.

Founded in 1666, the Royal Paris Academy of Sciences was a state-funded institution dedicated to the advancement of knowledge and understanding within mathematics and science under the leadership of Louis XV (1710 – 1774).[25] Taking the throne in 1715, Louis put the

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[24] In his work *The Newton Wars and the Beginning of the French Enlightenment* (University of Chicago Press: Chicago, Il., 2008, p. 245), J.B. Shank says, "Maupertuis announced his Newtonianism from the highest levels of the French scientific establishment. Voltaire, by contrast, proclaimed his from the public space of the Republic of Letters. However, since establishment French science was linked inextricably to the wider public culture of the Republic of Letters, and since the Republic of Letters, Maupertuis' prime arena of activity, was especially attached to these wider communities, Maupertuis' proclamations were public addresses as well. The academician in fact came to his Newtonianism through a negotiation between his public and academic identities, and his self-fashioning as the first French Newtonian resulted from an interplay between them."

[25] *Encyclopedia Britannica Online*; www.brittanica.com/EBchecked/.../Louis-XV; (accessed, 2014)

academy through a complete overhaul, which included changing the membership structure by instituting a three tiered promotion ladder of *adjoint, associe,* and *pensionnaire*.[26] Maupertuis, a self-promoting ladder climber, put his intellect and rhetorical skills to work and exploited the situation to his advantage. Beginning as an "adjoint geometre",[27] he set his sights on achieving *pensionnaire* status. Turning down the *vis viva* debate as a career promotion tool, he convinced the crown in 1735 to finance a trip to Lapland, Sweden where he measured for the curvature of the earth.[28] The project was a ladder climber's dream. Newton had previously measured for the curvature of the earth and had pronounced the planet to be flattened at the poles. Cassini after Newton had measured again, and had pronounced the earth to instead be egg shaped. Here were two completely opposite scientific opinions. Each opinion had its supporters within and without the Paris Academy. All Maupertuis had to do was re-measure, and become the deciding factor, which he did.[29] Returning from Sweden, he pronounced the globe to indeed be flattened at the poles, but even flatter than Newton had calculated. Maupertuis had saved the day for

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[26] Shank, p. 86

[27] In her work *The Man Who Flattened the Earth, "Maupertuis and the Sciences of the Enlightenment "* (Chicago Press: Chicago, Il., 2002, p. 47), Mary Terrall says, "Maupertuis, at the beginning of his career, had not committed himself to any particular physics or metaphysics, at least publically. He was looking to make a name for himself, and sharpening his mathematical skills was one route to his goal. In Paris, he had already learned all he could from Nicole and Saurin. As an associate member anxious to rise to one of the few pensioned positions, he needed to break new ground. He knew his own abilities well enough to realize that he would not be able to do this alone. In allying himself with Bernoulli, he was exploring possibilities for himself rather than adopting a cause, just as he had explored English mathematics and astronomy without taking up the Newtonian banner."

[28] Terrall, p. 100, says "The records of the Academy are silent about exactly when or how the decision to seek government approval for a second expedition was taken, or how the scientific team was put together. Talk of a new expedition was circulating publically as early as June 1735, following Maupertuis' speech to the public session, and well before it had been officialy sanctioned."

[29] Maupertuis flattened the earth flatter than Newton. This not only gave him the victory in the academy, but also set him as superior over Newton. In his book *Measure of the Earth* (Basic Books: New York, N.Y., 2011, p. 249) Larrie Ferreiro says, "When Maupertuis' data was analyzed and compared with the earlier survey of France, it showed that the earth's diameter through the poles was smaller than its equatorial diameter (the polar flattening) by 1 part in 178 (1/178). This was much flatter than what Newton had originally predicted (1/230) or what Clairaut's updated equations showed (between 1/573 and 1/230). Suspicion was immediately cast on the veracity of Maupertuis' observations, with critics the short length of his chain of triangles and his lack of a verifying baseline, as well as questioning whether he had used his 'English' instruments properly".

Newtonianism and the Paris Academy.[30] He had defeated Cassini.[31] In 1739, he was made a *pensionnaire.*[32]

Seated there before him that April day were the members of the Paris academy; mathematicians, scientists, and philosophers. They were heavily influenced by, if not outright partisans of, the thinking of the four most influential philosophers of the time, Rene Descartes, Baruch Spinoza, Gottfried Wilhelm Leibniz, and Isaac Newton. By the time Maupertuis stood before the Academy on that day, they were informally divided into highly competitive factions known as the Cartesians, the Leibnizians, and the Newtonians. Although highly influential in the academy, there was no faction called the "Spinozists", nor a group called the "Fermatians". The social and political relationships that existed between these groups and even between members within each group was extremely complex and interwoven.

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[30] Terrall, p. 219, says that the battle over the shape of the earth was a battle that started with Huygens and Newton, and carried over into the next century in Paris between the Newtonians and Cartesians.

[31] The opinion of some scholars is that Maupertuis' instruments were crude, and therefore, his findings were instantly called into question. Whether his calculations were purposefully exaggerated to make it appear that he was greater than Newton, or whether his instruments were actually unworthy, is a matter of debate. In her essay "Representing the Earth's Shape: The Polemics Surrounding Maupertuis' Expedition to Lapland," (*Isis.* 1992, Vol. 83, No. 2, p. 218 – 237), Mary Terrall says, "Historical accounts of quantification in the physical sciences in the Eighteenth Century have often been described as a straightforward series of steps in a process of maturation, as instruments and standards advanced in precision….In the case of the dispute over the shape of the earth, centered in Paris in the 1730's, the precision of measurements was a matter to be interpreted, attacked, defended, and represented. The whole messy business, undertaken by the participants to win consensus from their contemporaries, took place in the context of academic politics and the intellectual fashions of the salons and the court. All parties to the dispute claimed to be drawing on precision measurements; evaluating precision turned out to require the use of a range of intellectual, mathematical, instrumental, political, and textual resources. The alleged precision was then used to construct and defend rival scientific programs and practices."

[32] Terrall, p. 150 – 151, says, "Maurepas, minister of both the navy and the Academy, created a post especially for Maupertuis, charging him with the task of 'perfecting navigation'. Unlike the pension he had scorned earlier, this one, being unique as well as munificent, brought honor and distinction along with a substantial income. Maupertuis writes, 'It is a position (place) created expressly for me by M. de Maurepas….Everything was done in a most gracious way for me. It makes me very happy and quite comfortable'….'This position', he crowed to his old mentor Bernoulli, 'does me so much honor that nothing remains to be desired, other than to…merit the confidence which the minister has in me. The way in which it was done added greatly to its value. It makes all the professors of hydrography (at the new Ecole de la Marine) subordinate to me; I am charged with examining their theory (science) and practice."

Present in the room that day, and presiding over the occasion, was the secretary of the Paris Academy, Jean Jacques d'Ortous de Mairan (1768 – 1771). A Cartesian with Spinozistic leanings,[33] de Mairan had succeeded another Cartesian, "secretary for life" Bernard Le Bovier de Fontenelle (1657 – 1757).[34] de Mairan took over for Fontenelle in 1743, the year before Maupertuis wrote *Accord*. Under Fontenelle, the Paris Academy functioned as a scientific institution and an instrument of the state.[35] It functioned the same under de Mairan.

The Royal Paris Academy of Sciences was in competition with two other academies, The Royal Prussian Academy of Sciences and the Royal Society of London. These three organizations were competing against one another for the glory and prestige that went with cultural advancement in math, science, and philosophy. Members were expected to produce papers and books which put their respective academies at a competitive advantage.[36] The Royal

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[33] For the de Mairan correspondence with Malebranche that demonstrates de Mairan's Spinozistic leaning see *Malebranche's First and Last Critics, "Simon Foucher and Dortous de Mairan",* Translated by Richard Watson and Marjorie Grene (Watson, Richard and Charles Young, Editors. *The Journal of the History of Philosophy*, Monograph Series. Southern Illinois University Press: Carbondale, Il, 1995)

[34] According to *Encyclopedia Brittanica Online* (www.britannica.com/.../Bernard-Le-Bovier-sieu...Fontennele, accessed 2014), Fontenelle published *Entretiens sur la pluralite des moindes* (1686/88) which promoted the Copernican system and the Cartesian theory of vortices.

[35] See Hahn, Roger. *The Anatomy of a Scientific Institution, "The Paris Academy of Sciences, 1666 – 1803.* University of California Press: Berkeley, Ca., 1971, p. 3

[36] One example of this is the Berlin Academy, which held essay contests in an attempt to raise its stature in European intellectual opinion in the early modern period. In his essay "The Berlin Academy Under Frederick the Great" (*History of the Human Sciences,* 1989, Vol. 2, No. 2, p. 195 - 199), Hans Aarsleff says that in 1744 the Berlin Academy was divided into four classes, one of which was 'philosophie speculative'. This was unique among the three academies, and Frederick, fancying himself the philosopher king (see Terrall, p. 235), promoted essay contests. On page 42 of his book *Frederick the Great, A Historical Profile* (University of California Press: Berkeley, Ca., 1970), Gerhard Ritter says that Frederick held an intellectual round table at Sans Souci in which both Voltaire and Maupertuis participated. Of the king's relationship with these men, Ritter writes, "It was indeed a circle of companions rather than friends. Voltaire, whose mind he most admired, most deeply disappointed him as a human being, and rewarded his hospitality with cowardly abuse and slander from afar. That in the end Frederick conquered his disappointment and resumed their correspondence with repeated expressions of admiration for Voltaire's genius, shows how indifferent he really was to the personalities of his French associates. Basically, they were no more than a means of self-education. If that was his attitude towards Voltaire, it held even more true for the braggart Algarotti and the adventurer Lamettrie. Even that superficial *bel esprit,* the Marquis d'Argens, on whom he squandered his most beautiful letters, was nothing more to the king than a kind of intellectual sounding board. He felt more warmly toward the demanding, never satisfied invalid, Maupertuis, the President of the Royal Academy, whom he treated with a strange mixture of paternal, friendly care, and inner detachment."

The nature of the monad was one of the essay contest topics. Beeson, p. 189, says, "The topic as formulated reflected not only the fundamental hidden divisions within the Academy between the followers of

Society of London had commenced operations in 1660 and found its competitive niche, even its advantage, in its most famous and productive member, Isaac Newton.[37] The Berlin Academy was co-founded in 1700 by Frederick I of Brandenburg (1688 – 1740) and its first president, Gottfried Wilhelm Leibniz.[38] During Maupertuis' time it was under the rule of Frederick the Great (1712 – 1786).[39]

De Mairan and the "Cartesians" within the Paris Academy aligned themselves in varying degrees with Descartes and his mechanical philosophy founded upon the attributes of God. For Descartes, God's will was immutable. This was one of the principles that Descartes used in his tree of knowledge to argue for a metaphysical foundation to physics.[40] God was the creator of all things in terms of being the efficient cause, but final cause in physics, according to Descartes,

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Newton and the disciples of Wolff and Leibniz, it symbolized as well the ever-widening gap between the old physics which saw the hypotheses of science and metaphysics as closely inter-related, and the new science which made a complete distinction between them(…)Boasting as it did an eclectic membership which included such leading partisans from both camps as the Newtonians, Maupertuis, and Euler on the one hand, the Leibnizians, Wolff, and Forney on the other, the Berlin Academy afforded an almost ideal battleground for the heated debate." Mary Terrall says, "There is no evidence that Maupertuis tried to use the occasion to challenge Wolff's authority or that he pushed for one side of the question over the other." See *The Man Who Flattened the Earth,* p. 257 – 258

[37] The Royal Society History, The Royal Society, London; https://royalsociety.org/visit-us/london; In their introduction to *The Cambridge Companion to Newton,* (Cambridge Press: Cambridge Companions Online, 2015, p. 11 – 12) Bernard Cohen and George Smith say, "In late 1679, in an effort to reinvigorate the activities of the Royal Society, Robert Hooke wrote to Newton posing various research issues, with the goal of stimulating Newton to renew his active association with the Society. During the insuing exchange of letters, Hooke told Newton of his 'hypothesis' that curved or orbital motion could be analyzed by supposing two components: an intertial tangential motion and an accelerated motion directed towards a center of force. He also raised the question of the precise trajectory described by a body under an inverse-square force directed toward a central point in space. During the course of this brief correspondence, Newton discovered the relation between inverse-square centripetal forces Keplerian motion that comprises the initial stepping stone of the *Principia.*

[38] This was not an easy relationship, nor an easy transition for Leibniz from Hanover. In their book *Frederick I*: *The Man and His Times* (Columbia University Press: New York, N.Y., 1984,p. 74, 78, 102). Linda and Marsha Frey record that Frederick's government was dominated by Eberhard von Dankelmann, Frederick's former tutor, who viewed Leibniz as a Hanoverian spy, but kept the opinion to himself. Dankelmann controlled the money, but Leibniz had the confidence of the crown, especially Sophie Charlotte. In 1697, when Dankelmann no longer controlled the treasury, Leibniz was invited to Berlin.

[39] For a family chart of this royal household, see the "Family of Frederick I" chart one page after the index in *Frederick I*: *The Man and His Times*, by Linda and Marsha Frey

[40] Ariew, Roger. *Synthese.* 1992, Vol. 92, No. 1, "Descartes and the Tree of Knowledge", p. 103; In his essay Ariew takes pains to point out that Descartes is not talking about an order of discovery. In other words, it is not possible to construct an interpretation of Descartes from the tree that one's knowledge of final causes preceeds the laws of nature.

was beyond the ability of the human intellect to comprehend. Descartes states this position concerning final causes and its inappropriate application for mechanics in at least two places in his writings. In *Meditations* IV (1641) he says,

> For since I know that my nature is very weak and limited, whereas the nature of God is immense, incomprehensible, and infinite, I also know without more ado that he is capable of countless things whose causes are beyond my knowledge. And for this reason alone I consider the customary search for final causes to be utterly useless in physics; there is considerable rashness in thinking myself capable of investigating the <impenetrable> purposes of God.[41]

Here Descartes rejects the notion that he is capable of rationally grasping the "ends" by which bodies supposedly move in physics. He makes this overall point again in *Principles of Philosophy* I 28.

> When dealing with natural things we will, then, never derive any explanations from the purposes which God or nature may have had in view when creating them <and we shall utterly banish from our philosophy the search for final causes>. For we should not be so arrogant as to suppose that we could share in God's plans. We should, instead, consider him as the efficient cause of all things; and starting from the divine attributes which by God's will we have some knowledge of, we shall see, with the aid of our God-given natural light, what conclusions should be drawn concerning those effects which are apparent to our senses. At the same time we should remember, as noted earlier, that the natural light is to be trusted only to the extent that it is compatible with divine revelation.[42]

Descartes argued that the nature of body was extension, and with this as the foundation of his mechanical philosophy, commenced the overthrow of Aristotelian based, scholastic physics.[43] For Descartes, and the philosophy of the Cartesians which followed him, no end purpose was

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[41] Cottingham, John , Robert Stoothoff and Dugald Murdoch (Translators). *The Philosophical Writings of Descartes,* Vol. II, Cambridge University Press: New York, N.Y., 1984, p. 39; In their book *G.W. Leibniz, "Discourse on Metaphysics and Related Writings"* (Manchester University Press: New York, N.Y., 1988, p. 9), R.N.D. Martin and Stuart Brown say, "When Descartes and other modernists insist that final cause should not be invoked in natural sciences and that all explanations in physics should be in terms of efficient causes, they raised the bogies of mechanism, materialism, and determinism. The Cartesians, following Descartes himself, had claimed that animals were machines – physical systems whose workings were to be understood in non-purposive terms – and other seemed, explicitly or implicitly to extend the analogy in the obvious direction of human beings."
[42] Cottingham, Vol. 1, p. 202- 203
[43] In his chapter "Descartes Physics" (*Cambridge Companion to Descartes,* Cambridge Press: New York, N.Y., p. 292) Daniel Garber says, "Descartes natural philosophy begins with his conception of body. For Descartes, of course, extension is the essence of body or corporeal substance. Or, to use the technical terminology that Descartes adopted in the *Principles,* extension is the principle attribute of corporeal substance."

knowable in optical physics. They joined mechanists who had preceded them in the rejection of final causes in physics; namely Bacon, Galileo, and Hobbes among others.[44]

The direction of motion in bodies was a "determination". This was part of Descartes' calculation of the conservation of motion within the immutability of God.[45] Extension is characterized by modes of corporeal substance, a particular extension being manifest in shape, or state of rest or motion. These are ways in which attributes of substance manifest themselves, and these are first order modes. Second order modes qualify the mode of motion and that, for Descartes, is determination.[46] His idea of determination is seen in his demonstration of the third law of motion in optics with the tennis ball experiment. The motion of the tennis ball remains the same in reflection while its determination changes. Speed is conserved while direction is altered.[47] The Cartesian view of determination is the direction of a body according to the laws of motion.

This Cartesian influence was countered in the Paris Academy by the contrarian Leibnizian position. Gottfried Leibniz countered Descartes (and Spinoza) by stressing the significance of final causes in physics. In *Discourse on Metaphysics* he says,

> Since I do not like to judge people wrongly, I do not accuse our new philosophers who claim to banish final causes from physics. But I am nevertheless obliged to confess that the consequences of this opinion appear dangerous to me, especially if I combine it with the one I refuted at the beginning of this discourse, which seems to go so far as to eliminate final causes altogether, as if God proposed no end or good in acting or as if the good were not the object of his will. As for myself, I hold, on the contrary, that it is here we must seek the principle of all existences and laws of nature, because God always intends the best and most perfect.[48]

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[44] For Bacon on final causes, see Underhill, G.E. *Mind,* New Series, 1904, Vol. 13, No. 50, "The Use and Abuse of Final Causes", p. 221; For Galileo on final causes see Duchene, Steffen. *Journal of the History of Ideas.* 2006, Vol. 67, No. 3, "Galileo's Interventionist Notion of Cause", p. 444; For Hobbes on final causes, see Martinich, A.P. *Hobbes,* Routledge: New York, N.Y., 2005, p. 39 – 40

[45] Gaukroger, Stephen. *Descartes' System of Natural Philosophy.* Cambridge University Press: New York, N.Y., 2002, p. 115

[46] Gaukroger, p. 116; In FN 41, p. 116, Gaukroger refers the reader to 'Descartes to Clerselier, 17 November 1645.'

[47] Gaukroger, p. 122

[48] Ariew and Garber, p. 52; In his book *Radical Enlightenment, Philosophy and the Making of Modernity* (Oxford Scholarship Online, 2011, p. 35), Jonathon Israel argues that the new philosophy is Cartesianism.

During Maupertuis' time, the Leibnizians were strong in both the Paris and Berlin academies. Combined with the Leibnizians were the Wolffians, thinkers who adhered to the Leibnizian philosophy of Christian Wolff, which gave greater weight to apperception in corporeal bodies. When the corporeal ray of light traversed from point *A* in the air, and bent towards normal while traversing in the water towards point *B,* an apperceptive thinking process was going on in the ray beyond that of the natural tendency of Leibnizian conatus. This was beyond what Leibniz intended, and totally unacceptable to the Cartesians. The Paris academy had a flavor of this Wolffian idea, but it was much more pronounced in Berlin.

Another Leibnizian philosophy which was challenged in Paris by the Cartesians as well as by the Newtonians was the idea of *vis viva.*[49] Cartesians and Newtonians both tried to clarify their positions in response to this idea that nature was "living". [50] At the same time, however, both groups also positioned themselves away from Spinozist materialism.[51] If nature was living, then, did it think? This was the question for both physics and biology in the early modern period. John Locke (1632 – 1704), another influence on the academies, had no problem with the idea that nature thought.

> We have the ideas of matter and thinking, but possibly shall never be able to know, whether any mere material being thinks, or no; it being impossible for us, by the contemplation of our own ideas, without revelation, to discover, whether Omnipotency has not given to some systems of matter fitly disposed, a power to perceive and think, or else joined and fixed to matter so disposed, a thinking immaterial substance: it being, in respect of our notions, not much more remote from our comprehension to conceive, that God can, if he pleases, super add to matter a faculty of thinking; since we know not wherein thinking consists, nor to what sort of substances the Almighty has been pleased to give that power, which cannot be in any created being, but merely by the good pleasure and bounty of the Creator. For I see no contradiction in it, that the first eternal thinking being should, if he pleased, give to certain systems of created matter which is evidently in its own nature void of sense and thought: thought I think as I have proved.[52]

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[49] Shank, p. 426
[50] Shank, p. 426
[51] Shank, p. 426
[52] Woolhouse, Roger (Editor). *John Locke, "An Essay Concerning Human Understanding",* Penguin Group: New York, N.Y., p. 479 - 480

This was not Leibniz's position. The Leibnizian philosophy was about "conatus", the tendency in nature to action. This idea of thinking matter, however, never left Paris, or anywhere else in Europe in the early modern period. In his 1748 work *Man a Machine*, Julien La Mettrie gives an example of the eighteenth century counter to the notion of thinking nature. He says,

> Those metaphysicians who have attempted to prove that matter has a power of thinking, have not dishonoured their reason, because they have an advantage (for such indeed it is here) in expressing themselves with impropriety. In fact, to ask whether matter is capable of thinking without considering it otherwise than in itself, is asking whether matter is capable of pointing out the hours. Here tis obvious, we shall avoid the rock on which Mr. Locke unfortunately split. The Leibnizians with the monads, have raised an unintelligible hypothesis. They have rather spiritualized matter, than materialized the soul. How is it possible to define a being, whose nature is absolutely unknown to us? Descartes and all his followers, among whom the Mallebranchists are also reckoned, have committed the same mistake. They admit two distinct substances in man, as if they had really seen told them.[53]

The idea of thinking nature provided backdrop for Maupertuis in 1744. If nature did not think, then did God do the thinking for nature, or did it move by necessity without reason?

As Maupertuis stood to deliver *Accord*, the Paris Academy had been under the ever increasing influence of Newtonian thinking. Isaac Newton discovered the gravitational effect of planetary motion and believed that it was strong evidence for the existence of God. Newton's *Principia* presented Paris, Berlin, and the rest of Europe with a deterministic framework for

[53] Julien Offrey de (Translated From the French of the Marquiss d'Argens). London : printed for W. Owen, 1749, 5 - 6; In his essay "La Mettrie: The Robot and the Automaton" (*Journal of the History of Ideas*, 1970, Vol. 31, No. 4, p. 556), Blair Campbell says that La Mettrie was a biological scientist and philosophe in the eighteenth century. He says, "Although his major work, *L'Homme Machine*, made La Mettrie an object of loathing and outrage among most of his contemporaries, it earned him great respect from the philosophes; indeed Voltaire openly borrowed from him. But as a result of his later writings, La Mettrie was ostracized even by the philosophes. It is doubtful that a mere scientist would have provoked such scorn and ridicule as were directed against La Mettrie by cleric and philosophe alike." In his essay "The Philosophes and Religion: Intellectual Origins of the Dechristianization Movement in the French Revolution" (*Church History*, 1971,Vol. 40, No. 3 p. 273), Charles Gliozzo includes La Mettrie among those philosophes involved in the dechristianization movement in the French enlightenment/revolution. Gliozzo makes two points on page 273: 1) "In the Revolution, dechristianization took the following forms: aggressive anti-clericalism, prohibition of any Christian practice or worship either in public or private life, closing of the churches, the formation of a revolutionary calendar to replace the Christian one, and the establishment of new religious cults-the Cult of Reason and the Cult of Supreme Being. It is argued here that a direct influence can be traced from the philosophes to the dechristianizers of the Revolution.", 2) "Voltaire and Rousseau can be considered the spiritual mentors of the moderate dechristianizers with the former having the Civil Constitution of the Clergy as his memorial and the latter having a particularly strong influence on Robespierre. The materialist and atheist creeds of Jean Meslier, Jules La Mettrie, Claude Helvetius, and Paul d'Holbach inspired the more extreme policies of dechristianization."

mechanics such that, using the calculus, mathematicians and scientists could predict future physical events. Accompanying this deterministic framework was Newton's position in the scholium to *Principia* that physical reality was the product of God's intelligent design.

> The six primary planets are revolv'd about the sun, in circles concentric with the sun, and with motions directed towards the same parts and almost in the same plane. Ten moons are revolv'd about the earth, Jupiter and Saturn, in circles concentric with them, with the same direction of motion, and nearly in the planes of the orbits of those planets. But it is not to be conceived that mere mechanical causes could give birth to so many regular motions: since the comets range over all parts of the heavens, in very eccentric orbits. For by that kind of motion they pass easily through the orbs of the planets, and with great rapidity, and in their aphelions, where they move the slowest, and are detain'd the longest, they recede to the greatest distances from each other, and thence suffer the least disturbance from their mutual attractions. This most beautiful system of the sun, planets, and comets, could only proceed from the counsel and dominion of an intelligent and powerful being….This being governs all things, not as the soul of the world, but as Lord over all: And on account of his dominion he is wont to be called *Lord God* παντοκρατωρ, or *Universal Ruler.* [54]

Newtonian optics also weighed heavily on Paris. For Newton and the Newtonians, a ray of light was of a corpuscular nature that adhered to the laws of nature, which, of course were part of God's design. Some Newtonians promoted an idea that Newton himself toyed with; the idea that light rays moved according to gravitational attraction. Newton himself believed that refracted light obeyed Snell's law.[55]

Newton was friendly towards Huygens' wave theory of light. Though his objection was that it violated rectilinear propagation, Newton thought his corpuscular theory and wave theory

[54] The Newton Project Canada; Andrew Motte's Translation of the General Scholium to Isaac Newton's *Principia* (1729), p. 1-2; isaacnewton.ca/newtons-general-scholium

[55] In his chapter submission "Newton's Optics and Atomism" (*Cambridge Companion to Newton.* Cambridge University Press: New York, N.Y., p. 230) Alan Shapiro describes Newton's theory of light: "The essential point of his theory of light and color is that sunlight or white light is a mixture of rays differing in degree of refrangibility and color. He found that, at the same angle of incidence, rays of different color are refracted different amounts and that there is a constant correspondence between color and degree of refrangibility; that is, the red rays are always least refracted, the violet most and the intermediate colors intermediate amounts (he refers to Fig. 7.1). Rays of each color apart obey Snell's law of refraction, but with a different index of refraction for each."

could be compatible.[56] Consequently, the particle – wave debate in optics was argued in the Paris Academy during Maupertuis' time.

Starting with Descartes, science in Europe had moved into the field of theoretical physics regarding the nature of light. What was the true nature of light in the new mechanistic age, and how did it actually behave? As Maupertuis began to deliver his speech these groups were hotly debating whether light was a body or a wave. They were also debating the speed of light in different mediums. Specifically, did light move faster in a thinner medium versus a thicker medium, or visa versa. Most importantly though, how was refraction to be correctly calculated?

Christian Huygens had introduced the idea of light as a wave function in his 1690 work *Traite de la Lumiere.*[57] This laid the groundwork for the particle/wave duality debate which existed in 1744. The question was whether light was a body or a wave calculation. Scientists did not consider that it could be both. Therefore, either the corpuscular nature was correct, or the wave function was correct, but it could not be both.

Huygens bought the Fermatian idea that light traveled faster in thinner mediums and slower in thicker. The Cartesians, Leibnizians, and Newtonians, believed the opposite, that light moved quicker in thicker mediums and slower in thinner. It was this disagreement over the speed of light which formed the hypothesis backdrop for Maupertuis' 1744 *Accord.* This debate had a name. It was the Descartes – Fermat controversy.

This controversy began with Descartes' 1633 work on light *The World or Treatise on Light*,[58] and his 1637 work *Optics.* Descartes argued that light works according to mechanical

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[56] Shapiro, p. 232
[57] Bell, A.E. *Christian Huygens and the Development of Science in the Seventeenth Century* (Edward Arnold and Co.: London, U.K., 1947,P. 176)
[58] In their three volume work *The Philosophical Writings of Descartes* (Cambridge University Press: New York, N.Y., 1985, p. xi) Cottingham, Stoothoff and Murdoch note that, due to the mistreatment of Galileo, Descartes forsook plans to publish this work in 1633. It was later published posthumously.

laws undergirded by the immutable will of God. He argued that light moved quicker in thicker mediums than in thinner ones. He argued that the rule of sines was correct, and the relationship between the sines of the angles in refraction were inversely related to the speeds in order to produce a constant. Fermat disagreed, saying nothing about God, but agreeing with Snell's law. Descartes agreed with Snell, with the exception that Snell worked because light moved quicker in thinner mediums. Fermat argued that the inverted relationship between sines in refraction was correct, but in relation to the mediums instead of the velocity. Fermat also stressed that least time was the bottom line factor which lead to correct mathematical analysis of refraction. Thus, this argument in theoretical physics ensued and carried on into Maupertuis' day. The reason it was still debated was because no experiment had been performed which determined the speed of light in air compared to water or glass. That experiment was still one hundred and six years away.[59] In the meantime, and, especially in 1744 in the Paris, Berlin and London academic societies, it was all theory. No one really knew whether light moved quicker in air than in water, or visa versa. Nevertheless, battlelines were drawn around "fact". The two sides to this debate made up a section of Maupertuis' audience that day. It was posed as fact by one side that light moved quicker in air than in water. It was posed as fact by the other side that light moved quicker in water or glass than in air.

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[59] In his book *Pendelum, "Leon Foucault and the Triumph of Science"* (Atria Books: New York, N.Y., 2003, p. 87), Amir Aczell says that in 1850 Foucault solved for light being quicker in air than in water. "Foucault built a small steam engine and used it to drive a mirror at a lower speed of 800 revolutions per second. His experimental setup was only 4 meters long and consisted of a spinning mirror and a stationary one. Because light travels at a limited – albeit very high – speed, and one mirror spins fast, the reflected light does not arrive at its starting point, but rather is deflected somewhat. This relative deflection can be measured when air separates the two mirrors and also when a transparent tube of water is inserted between them. In April 1850, Foucault successfully completed his experiment, proving that light traveled slower in water than in air, as predicted by the wave theory of light."

The former was argued by the backers of Fermat and Huygens. The latter was argued by backers of Descartes, Leibniz, and Newton. No one, however, knew for sure since there was no scientific experiment to determine the truth of the matter. Therefore, the argument was not scientific. It was social. It was political within the academies, and it was personal. That was the science of it.

Within this context Maupertuis steps to the podium and delivers *Accord*. It is within *Accord* that he accuses Fermat and Leibniz of error regarding their application of final causes to physics. It is also within *Accord* that Maupertuis claims to have "discovered" the *Principle of Least Action*. This is an overreach on Maupertuis' part, the result of a speech designed to persuade politically and entice socially rather than convince scientifically.

A Leibnizian/Wolffian philosopher and member of the Berlin academy named Samuel Konig, caught the overreach and severely embarrassed Maupertuis in 1751 by publishing an essay crediting the discovery of least action to Leibniz. Maupertuis begins changing the wording of *Accord* in 1751 when the Konig affair strikes. Suddenly, Leibniz is no longer in error regarding his application of final causes to physics in *Accord*.

Samuel Konig was a mathematician who had been a journeyman academic, seeking teaching posts in German speaking Europe wherever he could find them. He spent the rest of his time as a tutor, translator and librarian. In 1749 he became a member of the Berlin Academy of Sciences.[60] The charge was made by Konig that Leibniz had communicated the principle of least action in a 1705 letter to Jacob Hermann. According to Konig, the letter contained the following quote:

> The action is not what you think, consideration of the times enters there; it is like the product of the mass by (the speed and by) the time, or the time by the living force. I remark that in the modifications of the movements, it ordinarily deviates a maximum, or a minimum. We can deduce several propositions

[60] Terrall, p. 295

> of great consequence useful to determine those curves which describe bodies attracted to one or several centers. I wanted to deal with these things among others in the second part of my dynamics, which I suppressed; the prejudiced bad reception, the first part having received disgusted me.[61]

Hence, in Konig's opinion, Maupertuis had plagiarized. The dispute which arose is known the Konig Affair of 1751/52.

Konig initially publicized his position in an essay in the *Acta Eruditorum.* Before publishing his finding, however, he sent the essay to Maupertuis for review and approval. Had Maupertuis reviewed the essay, he would have caught Konig's charge against him, and most likely, there would have never been a "Konig Affair". Maupertuis, however, did not review the essay, and instead sent it back to Konig as automatically approved. So, Konig went ahead and published the essay.

When Maupertuis read the published essay, he immediately saw that his claim of discovery was being challenged. To protect his reputation from a plagiarism charge, Maupertuis attacked Konig. It should have been a minor dispute, but Maupertuis turned it into a major issue that greatly divided and disturbed the entire Berlin Academy. He demanded that Konig produce Leibniz's letter to Hermann, which, of course, Konig could not. Konig claimed that it was in the possession of Henzi, who the French had branded as a traitor and subsequently had destroyed his possessions. If this was the case, the letter was lost forever. Over the absence of this letter, Maupertuis dragged the Academy into the dispute, demanding, and getting, a plenary session on April 13, 1752.

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[61] Beeson, p. 44 – 45; The French text found in Beeson reads as follows: L'Action n'est point ce que vous penses, la considération du tems y entre; elle est comme le produit de la masse par (la vitesse et par) le tems, ou du tems par la force vive. J'ai Remarqué que dans les modifications des mouvements elle déviant ordinairement un Maximum, ou un Minimum. On en peut déduire plusieurs propositions de grande conséquence elle pourrait server a déterminer les courbes que décrivent les corps attires a un ou plusieurs centres. Je voulais traiter de ces choses entr'autres dans la seconde partie de ma Dynamique, que j'ai supprimée; le mauvais accueil, que le préjuge a fait a la première, m'ayant dégoute.

Supporting Maupertuis among others in the academy was the King himself, and Leonhard Euler. Many in the academy supported Konig, who insisted on the authenticity of the letter to Hermann, and refused to back down even at the charge that he had forged the quote in question himself. Euler presided over the court that ruled against Konig and in favor of Maupertuis, sending Konig to personal and professional ruin. Parts of Euler's ruling pertain to the mathematics of the issue and some to the missing letter.

Euler opens with a statement in support of Maupertuis' discovery of least action: "Mr. *de Maupertuis*, President of the Royal Academy, has shown (by several very convincing arguments) that the action is always minimized, not only at equilibrium but also in the motions of bodies under external forces; this remarkable *principle of least action* expresses the most general law of Nature."[62] Then Euler declares Konig in error regarding his calculations of rest.

> Professor *König*, however, has tried in several ways to destroy this great discovery. First, he does not believe that it pertains to equilibrium problems, which he suggests corresponds not to the *minimum* of the action, but rather to where the action is *zero*; to support his case, he gives several examples in which minimization of the action results in zero action. However, this criticism is irrelevant, since it is recognized in the calculus of minima and maxima that a minimized quantity disappears entirely. Although König's "zero action principle" may apply in certain cases, it cannot be extended to all cases; for there are an infinite number of cases in which the action is minimized, but not zero. Hence, beyond any doubt, Nature's goal is to *minimize* the action, rather than set it to zero. If we consider the example of the "chain curve" (the catenary), where the total action is defined as the distance of the center of gravity of the chain to the center of the Earth, clearly this distance cannot be zero; rather, and most effectively, it should adopt the smallest possible value. It is indeed true that the force of gravity, given no resistance, would pull the chain to the center of the Earth, and that the only equilibrium position of the chain in that case would be at the center of the earth. However, since the chain is impeded at its points of suspension, the effect of gravity is limited to making the distance between the centers of gravity of the chain and the Earth as small as possible. Of course we agree with Mr. *König* that the action is indeed zero in those cases where it is true; however, this is not always possible, as we may see from the brachistochrone problem; rather, the action is to be minimized. It is as though Nature wishes to achieve some effect, and approaches it as closely as possible. Not only is the truth of this principle evidence, but also the reasoning upon which it is based, and to destroy entirely the objections of Mr. *König* who, far from striking a blow against this principle, has wonderfully helped to confirm it. It is a specious criticism of the principle of least action to note that there are certain cases in which the action disappears entirely, since the action can assuredly not become smaller than zero. Nevertheless, this objection might have some merit, if the action were reduced to zero in every case of equilibrium, as Mr. *König* seems to insinuate; but that cannot be proven, since there is an infinite number of cases in which the action is clearly not zero, but rather adopts the smallest possible value. In addition to the

[62] Wikisourse Translation; "Investigation of the Letter of Leibniz"; https://en.wikisource.org/wiki/Investigation_of_the_Letter_of_Leibniz_; (Wikisourse date: Feb. 2, 2013)

catenary problem, Mr. *D. Bernoulli* and Mr. *Euler* have shown that the elastic curves of every type and other figures adopted by flexible bodies (when the forces acting on them are in equilibrium) may be found by a method of maxima and minima applied to a formula that always contains the action, which is to be minimized and not set to zero.[63]

Euler goes on to admit that if the quote in Konig's essay is authentic, and the letter actually did exist, then indeed, Leibniz knew about the principle of least action before Maupertuis.

Euler presents Leibniz's correspondence with Bernoulli is as circumstantial evidence to show that this could not be the case. In the correspondence between Leibniz and Bernoulli, Leibniz never mentions the principle, and Euler reasons that such a monumental discovery would surely have been mentioned. Therefore, Euler argues, Leibniz did not know about the principle.

The charge in the affair was forgery on Konig's part, and plagiarism on Maupertuis' part. Euler and the king protected Maupertuis. The end of Euler's ruling reads as follows:

These matters are such that they may be reported. The quotation is intrinsically suspicious; and Mr. *König*, after learning that the original letter of Mr. *Leibniz* could not be found in the papers of *Henzi* (to whom he had referred), has not produced the original, nor has he been able to identify the place where it is preserved. Hence, it is assuredly obvious that his cause is bad, and that this quotation is a forgery, either to put Mr. *de Maupertuis* in the wrong, or to exaggerate (as if by pious fraud) the praises of the great *Leibniz*, who without question does not need such help. Having weighed these considerations appropriately, the Academy does not hesitate to declare this quotation fraudulent and to strip it of any credibility it might have ever possessed.

Approved by the assembled Academy on Thursday, 13 April 1752[64]

The final outcome of the "Konig Affair" by 1752 was the ruination of Maupertuis as president of the Berlin Academy of Sciences. Voltaire knew Konig personally and found this ruling incredible. On May 23, Voltaire says in a letter to Madame Denis:

This world is an huge Temple of Discord – our Berlin Academy is a chapel given over entirely to the protection of this Goddess. Maupertuis has just committed a little act of tyranny there which is unworthy of a philosopher. By his personal authority, he has denounced as a forger, in convocation, one of the Academy's members – Koenig, a great geometrician, librarian to the Princess of Orange, and Professor of Public Law at the Hague. This Konig is a man of merit, who is quite incapable of being a forger. I lived for nearly two years with him in the house of Madame du Chatelet….He is not a man to suffer

[63] Wikisourse Translation; "Investigation of the Letter of Leibniz"; https://en.wikisource.org/wiki/Investigation_of_the_Letter_of_Leibniz_; (Wikisourse date: Feb. 2, 2013)
[64] Wikisourse Translation; "Investigation of the Letter of Leibniz"; https://en.wikisource.org/wiki/Investigation_of_the_Letter_of_Leibniz; (Wikisourse date: Feb. 2, 2013)

such an affront.[65]

In late September, Konig fires back with *Appeal to the Publick*, "a formidable denunciation of Maupertuis, of his Academy, and of his procedure."[66] Voltaire accompanied Konig's work with his pamphlet "The Reply of An Academician of Berlin to an Academician of Paris", "a short but horribly concise account of the Konig affair."[67] This left the reputation of Maupertuis "in grave danger."[68] In his October 1 letter to Madame Denis, Voltaire says,

> I send you bravely Koenig's *Appeal to the Public.* You will read the history of the proceedings with interest. The work is extremely well composed: innocence and reason are victorious. Paris will think as Germany and Holland. Maupertuis is here looked on as a ridiculous tyrant….He has conducted himself in this affair as a man more skilled in intrigue than in geometry; he has secrectly irritated the king of Prussia against Koenig, and has cunningly availed himself of his authority to cause a search for the original letters of Leibniz in a place where he knew well they could not be found…."[69]

So incensed was Voltaire that he lampooned Maupertuis in the work *Diatribe du Docteur Akakia, me´decin du Pape.* . Although both Konig and Maupertuis expressed apologies to one another, it was too late to restore the reputations of either.

During the 1751 Konig affair, Maupertuis began to change *Accord* by removing Leibniz's name. Suddenly, Leibniz is no longer guilty of error regarding application of final causes to physics. Fermat is now the only guilty party. By 1756, Maupertuis published a second edition of *Accord* in volume four of his *Oeuvres de Maupertuis* with even more editorial changes regarding Fermat and Leibniz. Clearly, Maupertuis' position on Fermat and Leibniz changes. Yet, Maupertuis does not produce a new work to justify his new position. Instead, he drastically alters *Accord.* The thesis for this essay is that Maupertuis' charge of error is a politically calculated career move within the Paris Academy while keeping his eye on Berlin. This is in opposition to the possibility that Maupertuis' actually had an honest philosophical or scientific

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[65] Vulliamy, C.E. *Voltaire.* Kennicat Press: Port Washington, N.Y., 1970, p. 211 – 212
[66] Vulliamy, p. 212
[67] Vulliamy, p. 212
[68] Vulliamy, p. 213
[69] Vulliamy, p. 213

disagreement with the two over causation in physics. In order to determine this distinction, it is necessary to establish the positions of Maupertuis, Fermat, and Leibniz.

Chapter Two: Maupertuis' Position

Pierre de Maupertuis does not articulate a position concerning final cause. He does not define it, nor include a philosophical discussion of it in any section on causation. What he does articulate as being his is that from the attribute of God's perfection he can deduce the laws of rest and motion. From 1740 to 1746, he develops this position starting with the concept of rest in 1740, and developing least action from a specific application in optics in 1744 to a general application for all of physics in 1746. This development is worked out in three pieces during this period, *Law of the Body At Rest* (1740), *Accord Between Two Laws Which At First Seemed Incompatible* (1744 and 1756), and *The Laws of Movement and of Rest Deduced From A Metaphysical Principle* (1746).[70]

In *Law of the Body at Rest* he discusses the physics of equilibrium. The Newtonian position, and subsequently, the Maupertuian position, is that a ray of light is of a corpuscular nature. Consequently, therefore, a ray of light is in equilibrium if the system it is in does not move or moves "with constant speed in a fixed direction".[71] In *Accord Between Different Laws*

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[70] In 1750 Maupertuis published *Essay de Cosmology*. This work contains his comprehensive position on physics, biology, and cosmology. It is not included as a major part of this essay because it does not deal specifically with Fermat, Leibniz, and final causes, and because it repeats the positions of *du Repos, Accord,* and *de Mouvement and de Repos*. However, it should be noted that in his position on biology, Maupertuis does refer to a "living minima" endowed with desire, memory, and intelligence.

[71] In his article "Equilibrium" (Rigden, John, Editor. *Macmillan Encyclopedia of Physics,* Vol. 2, Simon and Schuster: New York, N.Y., p. 503 – 504), Michael Lieber says, "In mechanics, one is concerned with the motion of bodies. In this context, the state of equilibrium means that the system under consideration does not move or moves uniformly (i.e. with constant speed in a fixed direction). First, consider a single particle moving in one dimension, under the action of a force that can be derived from a potential energy function $V(x)$. At points where the derivative of $V(x)$ vanishes, the net force of the particle is zero. At such points, a particle at rest may remain at rest. At points where the derivative is nonzero, there will be a net force, and the particle must move, in accordance with Newton's second law, $F = ma$. Now equilibrium points, that is, points where the derivative of $V(x)$ is zero, may be further classified as points of stable or unstable equilibrium, depending on whether the second derivative of $V(x)$ is positive or negative. In the case that the second derivative is positive $V(x)$ has a local minimum. If the particle is displaced

Which at First Seemed Incompatible, Maupertuis discusses how *The Principle of Least Action* enables him to solve for both reflection and refraction in optics. In this case, a ray of light leaves its state of equilibrium and moves advantageously in reflection and in refraction through different mediums. In *The Laws of Movement and of Rest Deduced From A Metaphysical Principle,* he argues from metaphysics to physics, saying that from the attribute of God's perfection, namely power and wisdom, he can deduce the laws of both motion and equilibrium.[72] In part II he says,

> I could start from these laws, such as the mathematicians give and such as experience confirms, and seek the characteristics of wisdom and power of the supreme being. However, as those who have given them to us have relied on hypotheses which are not purely geometrical, and by their certainty seem not founded on rigorous demonstrations, I thought it more sure and useful to deduce the laws from the attributes of the omnipotent and wise supreme being. If what I find in this way follows the same effect we observe in the universe, is this not the strongest proof that this being exists and is the author of these laws?[73]

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slightly from this point, it will perform oscillatory motion when released, remaining always near the point of stable equilibrium. The force near the point of equilibrium always pushes the particle back toward that point (a restoring force). On the other hand, if the second derivative is negative, the function $V(x)$ has a relative maximum." In their book *Physics,* Third Edition (John Wiley and Sons: New York, N.Y., 1995, p. 110), John Cutnell and Kenneth Johnson say, "An object is in equilibrium when it has zero acceleration. According to Newton's second law, this definition implies that the net force acting on an object in equilibrium is zero. To put it another way, the forces acting on an object in equilibrium must balance. If the net force is zero, then in two dimensions the x component and the y component of the net force must each be zero. Thus, the equilibrium condition can be expressed by two equations: $\Sigma Fx = 0$ $\Sigma Fy = 0$."

[72] Motion in this case is in terms of least action. In his essay "Availing the Physics of Least Action" (*New Literary History,* 1995, Vol. 26, No. 2, p. 419), James Dunn says, "In physics, least action means, among other things, that natural events use the smallest measure of energy necessary for acting along a path adjusted by the pressures of reciprocating circumtances." Dunn, p. 421, credits Galileo with deriving least action from his first principle of inertia, and Newton with deriving least action from his first law of inertial motion.

This is Descartes in Maupertuis. In his essay "How God Causes Motion: Descartes, Divine Sustenance, and Occasionalism", (*The Journal of Philosophy,* 1987, Vol. 84, No. 10, p. 568), Daniel Garber says, "Descartes emphasizes in a number of places that 'we have no force through which we conserve ourselves,' and so for this we must turn to God, who 'continually reproduces us, as it were, conserves us' (Pr I 21). Descartes appeals to this doctrine of divine conservation in proving his laws of nature, both in *LeMonde* and in the *Principia Philosophiae*, arguing that God is the first and continuing cause of motion in the world, and that acting with constancy in preserving his material creation, he must necessarily sustain the world in such a way that certain general constraints on motion are satisfied; quantity of motion is thus conserved, as is motion along a straight path (Pr II 36 – 42)."

[73] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Chez Ambroise Haude, MDCCXLVI, p. 279, Richard Lamborn, Translator : J'aurais pu partir de ces loix, telles que les Mathématiciens les donnent, & telles que l'expérience les confirme; & y chercher les caractères de la sagesse & de la puissance de l'Etre suprême. Cependant, comme ceux qui nous les ont données, se sont appuies sur des hypothèses qui n'étaient pas purement géométriques, & que par la leur certitude ne parait pas fondée sur des démonstrations rigoureuses; j'ai cru plus sur & plus utile de déduire ces loix des attributs d'un Etre tout puissant & tout sage. Si celles que je trouve par cette voie, sont les mêmes sui sont en effet observes dans l'Univers, n'est-ce pas la preuve la plus forte que cet Etre existe, & qu'il est l'auteur de ces loix?

Here his knowledge of metaphysics leads him to correct physics. Maupertuis reasons from the power and wisdom of God to the laws of motion and rest.

Loi du Repos des Corps, "Law of the Body at Rest", is a paper on statics presented to the Paris Academy. This is the first of two extremization principles proposed by Maupertuis, the other being least action.[74] In this work, Maupertuis considers the appropriateness of a metaphysical application to physics.[75] He starts with the statics of bodies at rest and moves towards the dynamics of optics.[76] The principle he develops in this work easily converts to least action when generalized.[77] In this work, Maupertuis articulates his hierarchy of scientific law which provides structure for the positions he adopts and the arguments that he makes, for the rest of his career in Paris and Berlin. He says,

> If from the start the sciences are founded on certain simple and clear principles from which all the truths which are their object depend. They then have other principles, yet less simple in truth and often difficult to discover, but which, once being discovered, are of a very great usefulness. These ones are in some fashion what nature follows in certain circumstances, and we learn what they will do in similar occasions. By the evidence which the mind examines, the first principles have little need of demonstration. The last would not be knowable from general demonstration, because it is impossible to cover all the cases where they have occurred.[78]

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[74] Diaz, Penha Maria Cardoso. *Archive For History of Exact Sciences.* 1999, Vol. 54, No. 1, "Euler's 'Harmony' Between The Principle of 'Rest' and 'Least Action', p. 68

[75] Beeson, David. *Maupertuis, An Intellectual Biography.* The Voltaire Foundation: Oxford Press Oxford, England, 1992, p. 164

[76] "Statics is the branch of mechanics that deals with the forces on nonrotating rigid bodies at rest or in equilibrium. As such, it is distinguished from the other main branches of mechanics, called dynamics, which deals with the motions of bodies subject to external forces." See John Sharpe's entry "Statics" in *Macmillan Encyclopedia of Physics*, Vol. 4, p. 1526

[77] Beeson, p. 164

[78] *Histoire de l'académie royale des sciences.* Chez Hippolyte-Louis & Jacques Guérin, Année MDCCXL, avec les mémoires de mathématique & de physique pour la même année, p. 170; "Si les sciences sont fondées sur certains principes simples & clairs dès le premier aspect, d'où en dépendent toutes les vérités qui en sont l'objet, elles ont encore d'autres principes, moins simples a la virete, & souvent difficiles à découvrir, mais qui étant une fois découverts, sont d'une très grande utilité. Ceux-ci sont en quelque facon les lois que la nature suit dans certaines circonstances, & nous apprennent ce qu'elle fera en de semblables occasions. Les premiers principes n'ont guère besoin de démonstration, par l'évidence dont ils sont des que l'esprit les examine; les derniers ne sauraient avoir de démonstration générale, parce qu'il est impossible de parcourir tous les cas où ils ont lieu."

First in his hierarchy is what Maupertuis says is “from the start”. This is metaphysical law, or “principle”. They need no scientific demonstration because they cannot be known this way. Second are intermediate principles which can be known through demonstration. From them you can predict phenomena, but they are not universal in the sense that they are fundamental to all applications as law.[79] These intermediate principles require scientific proof. The third are principles of observable phenomena which do need demonstration but whose occurrences of which are too numerable to count.

For Maupertuis, Leibniz’s living force, “vis viva” is an example of the second variety. He argues that Leibniz’s discovery of living force is capable of predicting concrete phenomena, but is not valid as a fundamental, self-evident universal law. Maupertuis puts it this way concerning the nature of intermediate principles like *vis viva* and statics:

> Such is, for example, the well known and useful principle in ordinary statics, that in assemblies of bodies has a common center of gravity which descends to the lowest that is possible. Such is one of the conservation of living forces. No one ever gives a rigorous general demonstration of these principles. No one, however, accustomed to judging within the sciences, and who is experienced in the force of induction, will doubt their truth. When he has seen on a thousand occasions nature

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Beeson, p. 165, says about this statement, “A hierarchy of scientific law can therefore be drawn up, between fundamental metaphysical law, which needs no demonstration, and the observed phenomenon. An intermediate level covers principles which can provide guidance in predicting concrete phenonema, but which do not have the universal validity of fundamental law or the same self-evidence. They therefore require proof; however, since they are not general principles, they have no general demonstration; they can only be shown to apply in particular cases.”

In his essay “On the Nature of Scientific Law” (*The Scientific Monthly*, 1952, Vol. 74, No. 5, pp. 247ff), Laurence Lafleur comments on the defining of scientific law throughout history. Concerning the early modern period, he says that scientific law was influenced by a rediscovery of the Greek idea, which was the concept that the universe had a plan or pattern. Its pattern meant rationality, and its plan meant the work of a supernatural thinking supreme being. Either one could be scientific law. Lafleur says that in the early part of the modern period, roughly Maupertuis’ time, natural laws were understood to be simple. “For example, many laws state that one quantity varies directly or inversely as another, or as the square or other integral power of another. Another favorite form of law is one that asserts that events happen in such a way that a given quantity is maximized or minimized, and the most common form of all asserts the equality of two values, as the conservation of mass, energy, or angular momentum. If laws must be simple, it is possible to estimate the probability of scientific hypothesis in advance of experimentation.” Lafleur argues that this is the method of Bacon, Descartes, and Mill. He also says that the simplicity of scientific law meant its self-evidence. For a scientific law to be accurate meant that it was possible “to discover real principles that describe precisely how objects behave.” It is evident from Mr. Lafleur’s clarification here that Maupertuis is not a unique thinker, but, rather, someone who follows the intellectual pattern of his day. The simpler the better.

act in a certain manner, no man of good sense believes that on the thousand and one time it will follow other laws.[80]

Applied to *vis viva*, this means that, though it is not a universal law, it works every time it is tested. No one with scientific experience and expertise would doubt that after a thousand tests, it would act differently.

Maupertuis argues that, though they are dependable, intermediate principles are not *a priori* verifiable. He says, "They seem to belong to a superior science."[81] Maupertuis thinks that these intermediate principles are built up by induction from empirical observation.[82] He greatly prefers to deduce nature's workings from *a priori* truth, but he believes that neither he nor anyone else can intellectually handle the "necessary deductions." Therefore, all minds must reason "in terms of intermediate principles."[83] He starts with a ray of light existing at point *A*. The ray moves to point *B* and reflects. Whether refraction is involved does not matter. He makes the point that if one is going to calculate the motion as "living force", the calculation must be done by a combination of physics and metaphysics. This is in keeping with his belief that *vis viva* belongs to a higher science'."[84]

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[80] *Histoire de l'académie royale des sciences.* Année MDCCXL, avec les mémoires de mathématique & de physique pour la même année, p. 170; Tel est, par exemple, le principe si connu & si utile dans la Statique ordinaire; que dans tous les assemblages de corps, leur commun centre de gravité descend le plus bas qu'il est possible. Tel est celui de *la conservation des forces vives.* Jamais on n'a donné de démonstration générale à la rigueur de ces principes; mais jamais personne, accoutume à juger dans les sciences, & qui connaitra la force de l'induction, ne doutera de leur vérité. Quand on aura vu que dans mille occasions la Nature agit d'une certaine manière, il n'y a point d'homme de bon sens qui croie que dans la mille-unième elle suivra d'autres lois.

[81] *Histoire de l'académie royale des sciences.* Année MDCCXL , avec les mémoires de mathématique & de physique pour la même année, p. 170; Quant aux démonstrations *a priori* de ces fortes de principes, il ne parait pas que la Physique les puisse donner; elles semblent appartenir à quelque science supérieure.

[82] Beeson, p. 165

[83] Beeson, p. 166

[84] Jourdain, Philip. *The Monist.* 1912, Vol. 22, No. 3, "Maupertuis and the Principle of Least Action", p. 416 – 417; Dascal, p. 92, FN 6, shows that Leibniz told Papin in his April 14, 1698 letter that he achieved conservation of motive action *a priori* :

> "Prior demonstration hoc est: 1 – Actio absolvens duas leucas duabus horis est duplum
> This is the first demonstration: 1 – Action loosening two/both leagues two hours is to double
> actionis absolventis unam leucam una hora. 2 – Actio absolvens unam leucam una
> action absolving single league one hour. 2 – Action absolving one league one

Maupertuis finds *vis viva* appropriate within its class, and as correctly applied. The problem with living force, as he sees it, is that it does not help him solve for both elastic and inelastic collisions.[85] The position of Maupertuis, as he writes it in the 1746 work, is that living force is only applicable to inelastic collisions, and, consequently, is not universal. This betrays either a knowledge of mature Leibnizian writings after 1686, which fill the contents of the 1744 *Accord*, or he is overly influenced by the Leibnizians and Wolffians who stressed monadic reality as ultimate. This view, at least as it was understood by many in the academies, was that hard bodies did not actually exist. Leibniz himself does not embrace monadic structure until the late 1690s.[86] In 1746 Maupertuis couples *vis viva* with Descartes' momentum as both being equally intermediate in nature. It is the difference, however, between living force and least action which will create the challenge for Maupertuis in Berlin.

In *Loi du Repos* Maupertuis begins with statics and works towards optics. Statics is calculated at an extreme value. "The extreme will be a maximum in the case of unstable equilibrium, and a minimum in the case of stable equilibrium."[87] Maupertuis' "force du repos" is potential energy, and in *Loi du Repos*, Maupertuis moves towards Leibniz's architectonic

hora est duplum actionis absolventis unam leucam duabas horis.
hour to double
Posterior demonstration ita habet
Later demonstration thus has
1 – Spatium est in ratione composite temporum et velocitatum
1 – Space to in reason composed of time and velocity
2 – Actio est in ratione composite spatiorum et velocitatum
2 – Action is in reason composed of space and velocity
3 – Ergo action est in ratione composita ex simplice temporum et duplicate velocitatum."
3 – Therefore action is in reason composed of single time and double velocity. »

[85] Beeson, p. 84; Beeson also speculates, p. 84 – 85, that Bernoulli so alienated Maupertuis personally concerning the proof of, that Maupertuis walked away from full support of Leibnizianism because of it.

[86] In his essay, "Leibniz and the Fardella Memo" (*Studia Leibnitiana,* 2009, Vol. 41, No. 1, p. 67), Shane Suarte refers to Garber's position that 'from the late 1670s to the late 1690s, Leibniz had not yet come upon the monadological metaphysics that will characterize his later years, and that what one finds in this period instead is metaphysics grounded in corporeal substance, extended unities of matter and form."

[87] Beeson, p. 167

reasoning. He denies Leibniz's *vis viva* as pure mechanical law, regarding it rather as intermediate reasoning for lack of intellectual capacity to arrive at the actual law. However, in *Loi du Repos,* Leibnizian architectonic reasoning is employed towards "an extremum principle".[88] This law of rest is "proven" by Maupertuis by induction, which by his own standards makes his law of rest an intermediate principle.

Maupertuis accepts the Newtonian idea that light is of a corpuscular nature, and, because it is a body, it must adhere to the laws of motion. The problem for Maupertuis, and for everyone else in the early modern period, is that the laws of optics are incompatible according to the known laws of nature at that time. Maupertuis addresses this problem in 1744 with his first work on optics, *Accord Between Different Laws Which at First Seemed Incompatible*.

This work can be roughly divided into three sections; a section introducing the problem, a section discussing the various approaches taken to the problem, and Maupertuis' solution to the problem of least action. He opens the work with the "problem" of the mathematical challenge before his audience. Their fundamental truths were being challenged by "reasonings of geometry" and "the calculus of algebra". The contradiction presented by mathematics, Maupertuis declares, is overwhelming, "accablant".[89] Simply put, the mathematics for reflection and refraction did not harmonize for the Paris Academy members, and mathematicians and thinkers in general! The ancients, Maupertuis says, knew about both reflection and refraction, but could not harmonize them, and, he notes that "since the renewal of the sciences, and ever since their origins, we have discovered no fact more beautiful than the laws which follow light, whether moving in a uniform medium, whether upon encountering opaque bodies being reflected

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[88] Beeson, p. 167; Beeson says that in producing his own principle, Maupertuis employed a "wholly Leibnizian methodology." "In rejecting Leibniz's physics, Maurpertuis was more Leibnizian than Leibniz in his metaphysics.
[89] Present particle of accabler - overwhelming

by the surface, or whether translucent bodies oblige it to change course when traversing them. These laws are the foundations of all the science of light and colors."[90]

Maupertuis says that he thinks it best to get right to the point rather than focus on the vastness of this subject matter. Snell, he says, discovered how refraction worked. Descartes, he says, "undertook to explain" Snell, and he says that Fermat, attacked Descartes' explanation. Maupertuis says that ever since then, many great geometers have worked on the relationship between refraction and another law of nature that must be obeyed, but no one has been able to harmonize them.[91]

So as to assist in clarification of the problem, Maupertuis lists the laws of optics as he assumes are understood by everyone in that day:

> The first is that in a uniform environment, it moves in a straight line.
>
> The second is that when light encounters a body it cannot penetrate, it is reflected, and the angle of reflection is equal to the angle of incidence. That is to say that after its reflection it makes with the surface of the body an angle equal to which it had in the encounter.
>
> The third is that when light passes in its translucent environment into another, its path**,** after its encounter with the new medium, is at an angle from the one it had at first, and the sine of refraction will always be three quarters the sine of its new incidence.[92]

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[90] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 417: Depuis le renouvellement des Sciences, depuis même leur premier origine, on n'a fait aucune découverte plus belle que celle des loix que suit la lumière, soit qu'elle se meuve dans un milieu uniforme, soit que rencontrant des corps opaques elle soit réfléchie par leur surface, soit que des corps diaphanes l'obligent de changer son cours en les traversant. Ces loix sont les fondemens de toute la science de la lumière & des couleurs.

"since the renewal of the sciences" is the best way to translate Depuis le renouvellement des Sciences. Wikisourse translates this "since the Renaissance", which is incorrect. Renaissance is a term used by Dictionnaire de l'Académie française, 4th Edition (1762). It means "rebirth, renewal". So, the word does exist for Maupertuis, but he does not use it here.

[91] The phrase is *cette loi avec une autre que la Nature doit suivre encore plus inviolablement* which should be translated "this law with another which nature must obey even more inviolably." Wikisourse translates as "although no one has yet found a way of harmonizing the law of refraction with more fundamental laws that Nature must obey." The word fundamental is not in the text, and loi is singular. Maupertuis is talking about one particular law of nature.

[92] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 418:
La première est que *dans un milieu uniforme elle se meut en ligne droite.*

This is Maupertuis' mental image of Snell's law in that the ray of light bends towards towards the normal. Maupertuis says that laws one and two work according to mechanical laws of bodies and the law of reflection. Law number three, however, is a different matter. It represents an intellectual puzzle. If the shortest distance between two points is a straight line, and such a line is representative of least time between those points, then the refractive bend, if it is the least time, is a longer distance, which is the shortest distance in terms of least time. This is so, because if the ray was to move from air to water in a straight line towards the shell, then it would necessarily take longer than necessary. Herein lay the early modern crisis in optics.

Maupertuis reduces all of history's great thinkers who have ever dealt with this matter to three classes and then proceeds to consider a select few involved in the debate. The first class consists of those who want "to deduce refraction from the most simple and ordinary mechanical principles."[93] These thinkers, he says, are Descartes and his followers, the Cartesians. Maupertuis puts them in this class because of the tennis ball experiment. "They consider the movement of light like that of a ball, that meeting a surface that cedes nothing, reflects to the side from which it had just come, or which, in encountering one which cedes, continues to advance by changing only the direction of its route."[94] Descartes and his followers, he argues,

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La seconde que *lorsque la lumière rencontre un corps qu'elle ne peut pénétrer, elle est réfléchie, & l'angle de sa réflexion est égal à l'angle de son incidence,* c'est-à-dire qu'après sa réflexion elle fait avec la surface du corps un angle égal à celui sous lequel elle l'avait rencontre.

La troisième est que *lorsque la lumière passé d'un milieu diaphane dans un autre, sa route, après la rencontre du nouveau milieu, fait un angle avec celle qu'elle tenait dans le premier, & le sinus de l'angle d'incidence.* Si, pa exemple, un rayon de lumière passant de l'air dans l'eau s'est brise de manière que le sinus de l'angle de sa réfraction soit les trois quarts du sinus de son angle d'incidence, sous quelqu'autre obliquité qu'il rencontre la surface de l'eau, le sinus de sa réfraction sera toujours les trois quarts du sinus de sa nouvelle incidence.

[93] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 419: déduire la réfraction que des principes les plus simples & les plus ordinaires de la mécanique.

[94] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 419: ils ont considère le mouvement de la lumière comme celui d'une balle qui rencontrant une surface qui ne lui cède aucunement, rejaillirait vers le cote

are imperfect in their approach to refraction, but still deserve credit for having tried to derive it from the basic mechanical laws.

The second class belongs to Newton and the Newtonians who believe in the theory of attraction. The ball of light moves towards the object of reflection because of a natural tendency to do so. Maupertuis credits his Lapland associate Clairaut for producing a memoire outlining attraction over Descartes' view. He does not, however, carry this particular argument any further in *Accord.* It is simply the second way to unsuccessfully harmonize optical physics.

The third class consists of those thinkers who approach the problem metaphysically. Maupertuis puts Fermat in this class. He says, "Fermat had first felt the lack of explanation of Descartes. He apparently despaired to deduce the phenomena of refraction from a ball which would thrust against obstacles in resistant mediums. But he also did not have recourse to atmospheres around bodies, nor to attraction, though one knows that the latter principle was neither unknown to him nor disagreeable. He had sought the explanation in an entirely different and purely metaphysical principle."[95]

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d'où elle venait; ou qui, en rencontrant une qui lui cède, continuerait d'avancer, en changeant seulement la direction de sa route.

[95] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles,* p. 420: Fermat avait senti le premier le défaut de l'explication de Descartes, il avait aussi désespère apparemment de déduire les phénomènes de la réfraction de ceux d'une balle qui serait poussée contre des obstacles ou dans des milieu résistants; mais il n'avait eu recours ni à des atmosphères autour des corps, ni à l'attraction, quoiqu'on sache que ce dernier principe ne lui était ni inconnu ni désagréable; il avait cherché l'explication de ces phénomènes dans un principe tout diffèrent & purement métaphysique.

In *A History of the Theories of Aether and Electricity From the Age of Descartes to the Close of the Nineteenth Century,* p. 102 – 103, E. T. Whitaker says, "according to Descartes the velocity of light is greatest in dense media, while according to Fermat the propogation is swiftest in free aether. The arguments of the corpuscular theory convinced Maupertuis that in this particular point Descartes was in the right; but nevertheless he wished to maintain for science the beautiful method by which Fermat had derived his result. This he now proposed to do by modifying Fermat's principle so as to make it agree with the corpuscular theory; instead of assuming that light follows the quickest path, he supposed that 'the path described is that by which the quantity of action is the least'; and this action he defined to be proportional to the sum of the spaces described, each multiplied by the velocity with which it is traversed. Thus, instead of Fermat's expression

$$\int dt \quad \text{or} \quad \int \frac{ds}{v}$$

Maupertuis outlines the metaphysical principle and Fermat's employment of it which led to the lawyer's resolution of the problem. Maupertuis frames it in terms of what everyone already knew about propagation and reflection: "All the world knows that when light or some other body goes from one point to another through a straight line, it goes by the shortest route and by the most prompt time."[96] He presents the ideas of "shortest route" and "most prompt time" as being inseparably wed together. The same combination principle worked with reflection as well. The angles of incidence and reflection are equal because the path taken is both shortest in distance and least in time. He concludes from this that there is a metaphysical principle at work in optics, and he terms it "that nature, in the production of its effects, always acts through the most simple means."[97]

(where *t* denotes time, *v* velocity, and *ds* an end element of the path) Maupertuis introduced

$$\int v\, ds$$

as the quantity which is to assume its minimum value when the path of integration is the actual path of the light. Since Maupertuis' *v*, which denotes the velocity according to the corpuscular theory, is proportional to the reciprocal of Fermat's *v*, which denotes the velocity according to the wave-theory, the two expressions are really equivalent, and lead to the same law of refraction. Maupertuis' memoir is, however, of great interest from the point of view of dynamics; for his suggestion was subsequently developed by himself and by Euler and Lagrange into a general principle which covers the whole range of nature, so far as Nature is a dynamical system."

Terrall, p. 177, says, "Maupertuis set out to vindicate Newton's physics with a Leibnizian metaphysical argument, although he couched it in terms of an attack on 'the edifice that Fermat built'. Time is only minimized if light slows down when it moves to a denser medium, so siding with Newton on this point meant amending the principle of least time. Calling on the Leibnizian principle of sufficient reason, Maupertuis argued that there would be no reason for light to choose time rather than distance as the minimum quantity in the case of refraction. Rectilinear propogation and reflection minimize both time and path length; why should refraction be different? 'What preference could light have for time over distance? This language of preference and choice recalled Maupertuis' speculations, back in 1732, about God's reasons for choosing an inverse-square law of gravity over all other possibilities. In that case, he had based his argument on symmetry and consistency, a different kind of efficiency. Here, God prefers a world functioning economically, where all changes or motions cost the least 'expenditure'. In both cases, the metaphysical language stood out starkly from the other papers in the Academy's *Memoires.*"

[96] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles,* p. 420: Tout le monde scait que lorsque la lumière ou quelque autre corps va d'un point à un autre par une ligne droite, ils vont par le chemin & par le temps le plus court.

[97] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles,* p. 421: que *la nature dans la production de ses effets agit toujours par les moyens les plus simples.*

Maupertuis next applies this metaphysical principle of the wedded relationship between shortest distance and least time to refraction and specifically how Fermat employed it to "resolve" the problem. What he says next is not his position, but only an explanation of Fermat's application.

> To apply this principle to refraction, consider two mediums penetrable by light separated by a plane of their common surface. Suppose that the point from where a ray of light must leave is either one of these environments, and that one must arrive, or the other, but the line which joins these points is not perpendicular to the surface of the environment. We ask again by which cause it should happen, that the light is to move in each environment with different speeds. It is clear that the line which joins the two points will always be the shortest path going from one to another, but it will not be that of the shortest time. This time depends on the different speeds of light in the different environments. If it is necessary that the ray employ the least time possible, then at the encounter with the common surface, it breaks in a manner in which the greater part of its route takes place in the environment where it moves more quickly, and the lesser in the environment where it moves more slowly.
>
> This is what light seems to do when it passes from air into water. The ray breaks in the manner that the greatest part of its route is in the air, and the lesser in water. So if, as it was quite reasonable to suppose, light moves more quickly in the more rare environment than in the densest. If it moves more quickly in air than in water, it must follow a route to arrive on point as promptly or it leaves and goes to the point it must reach.[98]

The problem for Maupertuis in this case is that this destroys the inseparable metaphysical relationship between space and time in propagation and reflection because the quickest time is no longer the "straight line" shortest distance. That is, of course, if one considers only a straight line as qualifying as shortest distance. Yet, the reasoning seems to go, if one believes that the

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[98] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles,* p. 420 – 421; The French text reads as follows:

Pour appliquer ce principe a la réfraction, considérons deux milieu pénétrables a la lumière, séparez par un plan qui soit leur surface commune: supposons que le point d'où un rayon de lumière doit partir, soit dans un de ces milieux, & que celui où il doit arriver, soit dans l'autre, mais que la ligne qui joint ces points, ne soit par perpendiculaire à la surface des milieu: posons encore par quelque cause que cela arrive, que la lumière se mauve dans chaque milieu avec différentes vitesses, il est clair que la ligne droite qui joint les deux points, sera toujours celle du plus court chemin pour aller de l'un a l'autre, mais elle ne sera pas celle du temps le plus court, ce temps dépendent des différentes vitesses que la lumière a dans les différents milieux, il faut si le rayon doit employer lippe moins de temps qu'il est possible, qu'à la rencontre de la surface commune il se brise de manière que la plus grande partie de sa route se fasse dans le milieu où il se meut le plus vite, & la moindre dans le milieux ou il se meut le plus lentement.

C'est ce que parait faire la lumière lorsqu'elle passé de l'air dans l'eau, le rayon se brise de manière que la plus grande partie se sa route se trouve dans l'air, & la moindre dans l'eau. Si donc, comme il était assez raisonnable de la supposer, la lumière se mouvoir plus vite dans les milieu plus rares que dans les plus denses, qu'elle doit suivre pour arriver le plus promptement du point d'où elle part au point où elle doit parvenir.

metaphysical principle of the wedded relationship between shortest distance and briefest time holds for refraction, then the longer path must necessarily also be the shortest distance. This means that refraction is a consideration of time over space, which is how Maupertuis says that Fermat finally "resolved" the matter.

> It was by this principle that Fermat resolved the problem, by this principle so verisimilar that light, in its propagation and in its reflection, will always go by the shortest time possible. It also follows the same law also in refraction. And he did not hesitate to believe that light moves with more ease and faster in the more rare medium than in those in which for the same space. It finds a greater quantity of matter: In effect, could anyone believe at first sight that light traverses easier and faster in crystal and water than in air and the void? We also saw several of the most famous mathematicians embrace the opinion of Fermat. Leibniz is the one who argued it the most, which he has given to this problem by name and by a more elegant analysis: His was so charming of a metaphysical principle, and to rediscover here his final causes to which one knows to what extent he was so attached to. He regarded as an indubitable fact that light moves more quickly in air than in water or glass.[99]

Maupertuis says that Fermat "resolved" refraction.[100] He did so with the (verisimilar) principle of least time.[101] He did so by applying least time to propagation, reflection, and refraction in priority fashion. Maupertuis says that is how Fermat "resolved" the

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[99] *Histoire de l'Académie Royal des Sciences.* Année MDCCXLIV, Avec les Mémoires de Mathématique & de Physique, pour la même Année. *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles* p. 422; The French text reads as follows: Ce fut par ce principe que Fermat résolut le problème, par ce principe si vraisemblable, que la lumière qui dans sa propagation & dans sa réflexion va toujours par le temps le plus court qu'il est possible, suivit encore cette même loi dans sa réfraction; & il n'hésita pas à croire que la lumière ne se meuve avec plus de facilite & plus vite dans les milieu les plus rares que dans ceux ou, pour un même espace, elle trouvait une plus grande quantité de matière: en effet, pourvoit- on croire au premier aspect que la lumière traverserait plus facilement & plus vite le crystal & l'eau que l'air & l'vuide?
Aussi vit-on plusieurs des plus célèbres Mathématiciens embrasser le sentiment de Fermat; Leibniz est celui qui l'a le plus fait valoir; & par son nom & par une analyse plus élégante qu'il a donnée de ce problème; il fut si charme du principe métaphysique, & de retrouver ici ces causes finales auxquelles on sait combien il était attache, qu'il regarda comme un fait indubitable que la lumière se mouvoir plus vite dans l'air que dans l'eau ou dans le verre.

100 The instrumental phrase is *Ce fut par ce principe que Fermat résolut le problème,…* Dictionnaire de l'Académie française, 4th Edition (1762) defines *resolut* as "Resolved, EU, participates This is a resolved item. Company resolved. Addressed…It is also an adjective and means, Determined, bold…It also takes substantively. This is a big solved." The issue here is that Maupertuis is not describing *le problem*, so the adjectival sense cannot be used. Maupertuis has Fermat doing to something to *le problem.* Fermat is either resolving the problem or addressing the problem. Since the 1762 dictionary gives the primary meaning as "resolved", we went with this word over "addressed". This cannot, however, be translated outright "solved", because that is the word *resoudre*, which is not the word used here. Wikisourse translates this, "The path-of-least-time principle of Fermat seems reasonable" is incorrect. The word *raisonnable* is not used.

[101] The hyphenated term *vrai-sembable* is used, which is best translated "verisimilar/verisimilitude"; Dictionnaire de l'Académie française, 4th Edition (1762) defines *vrai* as "true, which is conforming to the truth" and defines *sembable* as "similar…which is of the same nature". Another way to possibly translate it is "truly likely".

problem of the relationship between the three, and he refers to this as truth. He notes that several of the most famous mathematicians embraced this resolution, including Leibniz.

Now enters the Descartes – Fermat debate in *Accord*. Maupertuis says that Descartes believed, contrary to Fermat, that light moves quicker in thicker mediums.[102] He says that this supposition is Descartes' deductive faultline. He points out that this Cartesian supposition is posed as fact in his day. If this is the case, he notes, "the whole edifice that Fermat and Leibniz had built is destroyed".[103] Maupertuis thinks that the position of Fermat and Leibniz is built upon the idea that light moves faster in thinner mediums than in thicker, e.g. faster in air than in water. He finds this to be problematic and explains: "Light, when traversing different mediums, will go neither by the shortest path or the quickest time. The ray which passes from air into water making the greatest part in the air, arrives later than if it had made the lesser."[104] They think that light travels faster in water, therefore, if the greater part of travel is in the air, it will take more time. This is problematic and he refers to de Mairan's memoir on the issue: "We can see in the memoir that de Mairan gave on reflection and refraction, the history of the dispute between Fermat and Descartes, and the embarrassment and helplessness where it has been so far to harmonize the law of refraction with the metaphysical principle."[105]

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[102] In his book *A History of Mechanics* (www.Doverpublications.com, p. 374) Rene Dugas says, "When Maupertuis wrote, it was generally agreed that light moved more quickly in denser media, in the manner specified by the Newtonian law of the proportionality of the indices of refraction to the velocities of propagation."

[103] *Histoire de l'Academie Royal des Sciences.* Annee MDCCXLIV, Avec les Memoires de Mathematique & de Physique, pour la meme Annee. *Accord de differentes loix de la nature quijusqu'ici paru incompatibles*, P. 422: tout l'edifice que Fermat & Leibniz avoient bati, est detruit. This is not part of an if-then clause. That would require *ce* to have the definition of "if", we which it does not have. The conditional particle *si*, which is "if", is not used.

[104] *Histoire de l'Académie Royal des Sciences.* Année MDCCXLIV, Avec les Mémoires de Mathématique & de Physique, pour la même Année. *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles*, P. 422 – 423: la lumière, lorsqu'elle traverse différents milieux, ne va ni par le chemin le plus court, ni par celui du temps le plus prompt; le rayon qui passé de l'air dans l'eau faisant la plus grand partie de sa route dans l'air, arrive plus tard que s'il n'y faisait que la moindre.

[105] *Histoire de l'Académie Royal des Sciences.* Année MDCCXLIV, Avec les Mémoires de Mathématique & de Physique, pour la même Année. *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles*, p. 423: On peut voir dans le Mémoire que M. de Mairan a donné sur la Réflexion & la Réfraction, L'historien de la dispute

Having outlined the problem in optics concerning the speed of light in differing mediums and its effect on distance traveled versus time taken, Maupertuis moves to present his solution – least action. He says,

> While meditating deeply on this matter, I think that light, when passing from one medium into another, already abandoning the shortest path, which is the straight line, also does not follow that of the most prompt time. In effect, what preference ought it to have for time over space? Light cannot go just by the shortest distance, and by that of the most prompt time. Why should it go with one of these rather than the other? It does not follow either. It takes a path that has a real advantage. The path it takes is the one in which the amount is the least action.[106]

He gives away his thinking when he says that in going from air into water, light abandons the shortest path. He does not think that what he has earlier declared to resemble truth can at the same time contain the shortest path. That would require time to predominate over distance, and evidently Maupertuis cannot abide this. Perhaps he thinks that this means that light "chooses" a path of least time over shortest distance. The words for such an interpretation of thinking nature are not in the text.[107] So he abandons both, which means that he rejects the metaphysical notion of time over distance, and embraces a different metaphysics – a notion of action over against the wedded relationship of space and time.

In the first half of *Accord*, Maupertuis appears to promote Fermat, but then he rejects the idea of a consideration of time over distance. His definition of action has no time element. It also contains the metaphysical notion that, although he does not word it as such, light thinks in

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entre Fermat & Descartes, & l'embarras & l'impuissance ou l'on a été jusqu'ici pour accorder la loi de la réfraction avec le principe métaphysique.

[106] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles*, p. 423 : Translator: En méditant profondément sur cette matière, j'ai pensé que la lumière, lorsqu'elle passé d'un milieu dans un autre, abandonnant déjà le chemin le plus court, qui est celui de la ligne droite, pourvoit bien aussi ne pas suivre celui du temps le plus prompt: en effet, quelle préférence devrait-il y avoir ici du temps sur l'espace? la lumière ne pouvant plus aller tout-à-la-fois par le chemin le plus court, & par celui du temps le plus prompt, pourquoi iroit-elle plutôt par un de ces chemins que par l'autre? Aussi ne suit-elle aucun des deux, elle prend une route qui a un avantage plus réel: *le chemin qu'elle tient est celui par lequel la quantité* pourquoi irait-elle plutôt par un de ces *d'action est la moins.*

[107] Wikisource reads: *After meditating deeply on this topic, it occurred to me that light, upon passing from one medium to another, has to make a choice,…*The words for "has to make a choice" are nowhere to be found in the memoire. Maupertuis does not say that light has to make a choice, "choix".

terms of choosing an advantageous path. He describes this advantageous path according to his explanation for action: "When a body is carried from one point to another, this requires a certain action. This action depends on the speed the body has and the space it traverses, but it is not the speed or the space taken separately."[108]

Maupertuis moves from a calculation of light in terms of a combination of space and time to a calculation of speed and space. He says, "The quantity of action is all the more greater when the speed of the body is greater, and when the path it traverses is longer. It is proportional to the sum of the space each multiplied by the speed in which the body travels."[109]

Maupertuis assures his listeners in the next few statements that this is the "household economy" of nature; its budget.[110] It is a theory of action without a force of cause and without the element of time. He includes the nature of light in this concept of action: "That's it! That is this quantity of action that is here the true expense of Nature, and it is this household economy that is more than possible in the movement of the light."[111] He proceeds to describe it geometrically and mathematically:

> There are two different environments separated by a common surface represented by the line C D such as the speed of light in the environment which is on top, meaning = V, and the speed of light which is beneath, meaning = W. There is a ray of light A R that leaves from a given point A and arrives at a given point B.

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[108] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles*, P. 423: Lorsqu'un corps est porte d'un point à un autre, il faut pour cela une certaine action, cette action dépend de la vitesse qu'a le corps & de l'espace qu'il parcourt, mais elle n'est ni la vitesse ni l'espace pris séparément.

[109] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles*, P. 423: La quantité d'action est d'autant plus grande que la vitesse du corps est plus grande, & que la chemin qu'il parcourt est plus long, elle est proportionnelle à la somme des espaces multiplies chacun par la vitesse avec laquelle le corps les parcourt.

[110] The word used is *menage,* which is the word for "household" and carries the idea of management.

[111] Histoire de l'académie Royale des Sciences. Année M.DCC.LIV, *Accord de différentes loix de la nature qui jusqu'ici paru incompatibles*, P. 423: "C'est cela, c'est cette quantité d'action qui est ici la vraie dépense de la Nature, & ce qu'elle ménage le plus qu'il est possible dans le mouvement de la lumière."

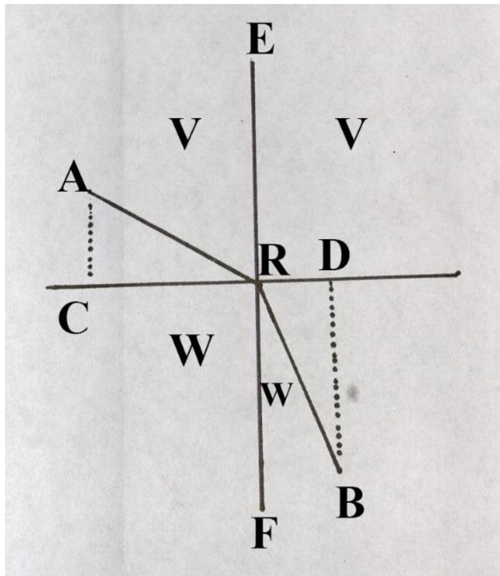

Figure #4 Likeness of the geometrical drawing of Pierre Maupertuis in *Accord Between Different Laws Which At First Seemed Incompatible,* 1744 Paris Academy Memoires

To find the point R at which the light is bent, I seek a point that minimizes the action, i.e. V · AR + W · RB should be minimized.

Now

$$V \cdot \sqrt{AC2 + CR2} + W \ \cdot \sqrt{BD2 + CD2 - 2CD\ x\ CR + CR2} = \text{min.}$$

Therefore *AC*, *BD & CD* being constant, I have

$$\frac{V \ \cdot \ CRdCR}{\sqrt{AC2 + CR2}} - \frac{w \ \cdot \ (CD - CR) \ \cdot \ dCR}{\sqrt{BD2 + DR2}} = 0$$

or

$$\frac{V \ \cdot \ CR}{AR} = \frac{W \ \cdot \ DR}{BR} \cdot \frac{CR}{AR} : \frac{DR}{BR} :: W : V$$

that is to say, the ratio of the sine of incidence to the sine of refraction is inverse of the speed that light moves in each environment.[112]

[112]*Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tires des Registers de cette Academie, P. 424

Here Maupertuis is invoking Snell's law, and Fermat's application of it. Alex Montwill and Ann Breslin explain this in their book *Let There Be Light, "The Story of Light From Atoms to Galaxies"* (World Scientific

Publishing Co.: ISBN, 9781848167605, p. 42 – 43). They use the lifeguard illustration whereby a lifeguard will calculate a path to the distressed swimmer which cuts a path that takes the least time based on a knowledge that he or she can run faster in air than swim in water. The mathematical calculation, however, is inversed:

$$\frac{\sin i}{\sin r} = \frac{\text{speed in water}}{\text{speed on land}} = \frac{n2}{n1} = n$$

In his entry "A Geometric Proof of the Equivalence of Fermat's Principle and Snell's Law" (*The American Mathematical Monthly*, 1964, Vol. 71, No. 5, p. 543 – 544), Daniel Pedoe says, "Let l be the line separating the media, and let $P_1Q^*P_2$ be the actual path of the ray according to Snell's L, where the sine $\theta_1/\ \theta_2 = v_1v_2$. We wish to show that for any other point Q on the line

$$P_1Q/v_1 + P_2Q/v_2 > P_1Q^*/v_1 + P_2Q^*/v_2,$$

so that the time taken for traversing the actual path is a minimum. Draw the circle through the points P_1, Q^* and P_2, and let the perpendicular to l through Q^* interect this circle again at the point A. Then we note that $AP_1 = 2R \sin \theta_1$, and $AP_2 = 2R \sin \theta_2$, where R is the radius of the circle $P_1Q^*P_2$. Therefore $AP_1/AP_2 = v_1v_2$; that is, $AP_1 = k/v_2$ and $AP_2 = k/v_1$, where k is a constant.

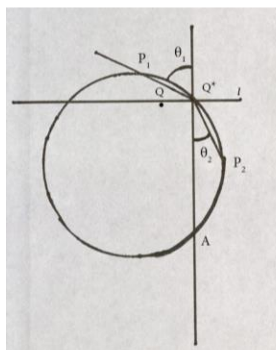

Figure #5 Likeness of Pedoe's figure in this article

Applying the theorem of Ptolemy to the four concyclic points P_1, Q^*,P_2 and A, we have the equality

$$P_1P_2 \cdot AQ^* = P_1Q^* \cdot AP_2 + P_2Q^* \cdot AP_1,$$

Whereas if Q is any point $\neq Q^*$ on the line l, the extension of the Ptolemy theorem which arises naturally by inverting the triangle inequality (1) gives the inequality

$$P_1P_2 \cdot AQ < P_1Q \cdot AP_2 + PQ_2 \cdot AP_1.$$

If we substitute for AP_1 and AP_2, we obtain the equality

$$k(P_1Q^*/v_1 + P_2Q^*/v_2) = P_1P_2 \cdot AQ^*,$$

and the inequality

$$k(P_1Q^*/v_1 + P_2Q/v_2) > P_1P_2 \cdot AQ.$$

Hence the Fermat principle of minimum time is established for a ray which satisfies Snell's law, since

$AQ > AQ^*$."

After this demonstration, Maupertuis says that refraction is now brought into the understanding that nature, in the production of all of its effect, is as simple as possible. He says, "From this principle it follows that when light passes from one medium into another, the sine of the angle of refraction is to the sine of the angle of incidence in inverse ratio of the speeds light has in each medium."[113] This is Snell's law as understood by Fermat.

It is at this location of *Accord* that Maupertuis begins his accusation of error against Fermat and Leibniz. He says that the amount of action in nature in the movement of light is always the smallest possible, and in propagation and reflection, the speed of light remains the same. The smallest quantity of action, he notes, gives at the same time the shortest path and most prompt time. "However," he says, "this shortest and fastest path is a sequence of the smallest quantity of action, and it is this sequence that Fermat and Leibniz had taken to be a principle."[114] The principle he is referring to here is the idea that, for Fermat and Leibniz, no matter the length of the distance due to the bend in refraction, the path is always the shortest. Maupertuis argues that this concept cannot be a principle. He finds this distasteful and says, "I know the distaste that several mathematicians have for *final causes* applied to physics, a taste that I share up to some point. I admit it is risky to introduce such elements; their use is dangerous, as shown by the error made by Fermat and Leibniz in following them. Nevertheless, it is perhaps not the principle that is dangerous, but rather the hastiness in taking as a principle that which is merely a *consequence*."

[113] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 424: De ce principe suit que *lorsque la lumière passé d'un milieu dans un autre, le sinus de son angle de réfraction est au sinus de son angle d'incidence en raison inverse des vitesses qu'a lumière dans chaque milieu.*

[114] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 425 : mais ce chemin le plus court & le plûtôt parcouru n'est qu'une suite de la plus petite quantité d'action, & c'est cette suite que Fermat & Leibnitz avoient prise pour le principe.

Maupertuis' change in vocabulary causes some interpretive difficulty. He ends the focal passage for this essay with the word *consequence*, which carries the idea of cause and effect. Something is the consequence of something else. However, what he claims that Fermat and Leibniz mistook for a principle was actually, in his opinion, the *sequence* of something, which carries no causal meaning. Something necessarily follows something else. In this case, shortest distance necessarily follows from least action. In changing from *sequence* to *consequence*, Maupertuis appears to be trying to argue a fine point of difference. However, *Accord* is a speech, and the change could be rhetorical technique which just so happens to unfortunately produce a misunderstanding in interpretation. It is difficult to discern. If he means final causes per se, then he means that the error was in their application of final causes in order to understand the idea of shortest distance, which, Maupertuis thinks, should be the other way around.

He follows the focal passage with his customized intelligent design argument, which is a mix of Christian theism, Spinozist and Leibnizian philosophy, and his fall-back Cartesian position that the human mind is not capable of comprehending the end purposes of God's design. He begins by saying,

> We cannot doubt that all things are settled by a supreme being who impressed on matter forces which denote his power, has destined them to execute effects which mark their wisdom; And the harmony of the two attributes is so perfect, that probably all effects of nature themselves could be deduced from each taken separately.[115]

The idea that the laws of nature can be deduced adequately from either efficient or final causes is Leibnizian, and straight out of *Specimen Dynamicum* where Leibniz says, "Next I arrived at a true estimation of forces and at exactly the same one, moreover, by widely different ways. One

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[115] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, P. 425 : On ne peut pas douter que toutes choses ne soient réglées par un Etre suprême qui, pendant qu'il a imprime a la matière des forces qui dénotent sa puissance, l'a destinée à exécuter des effets qui marquent se sagesse; & l'harmonie de ces deux attributs est si parfaite, que sans doute tous les effets de la nature se pourraient déduire de chacun pris séparément.

was *a priori*, based on the most simple consideration of space, time, and action; this I shall explain elsewhere. The other was *a posteriori*, by calculating force by the effect it produces in expending itself."[116] Maupertuis follows this with what he thinks is a Leibnizian intelligent design maxim, but which is half Spinozist: "A blind and necessary mechanics follows the designs of the most enlightened and free intelligence." The idea of mechanics being "blind" and "necessary" is Spinozist.[117] The idea that mechanics follows the designs of an intelligence is Leibniz. Leibniz, however, does not think that mechanics is blind and necessary. Maupertuis falls back to his Cartesian fail safe position that the human mind is not capable of grasping the ends of God. He finishes his customized design argument by saying, "And if our minds were large enough, it would also see the causes of physical effects, either in calculating the properties of bodies, or in seeking what was most suitable to execute them."[118] With this, all the bases have been covered. Every political group within the Paris Academy has been recognized and affirmed.

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[116] Loemker, p. 442

[117] In their Introduction to their edited work *The Cambridge Companion to Spinoza's Ethics* (Cambridge University Press: New York, N.Y., 2009, p. 2) Olli Koistinen and Valtteri Vilianen say, "The most important theses in the *Ethics*, Part I, are the following three: (i) God necessarily exists; (ii) God is the only true substance; (iii) everything follows from God by geometrical necessity." There is no teleological meaning for Spinoza. Whereas Leibniz, via Plato, argue that nature moves for the good, Spinoza says that this cannot be. It should be noted that Maupertuis is not consistent here.

In his essay "Substance, Causation and Free Will in Spinoza and Leibniz" (arche, 2008, Vol. 2, No. 1, p. 11 - 12), Ross Wolfe says, "…The Ethics lays forth the provocative assertion that God lacks free will. Spinoza contends that the order and relation of God's modal (i.e. existential) emanations follow with necessity from the logic of His essence, in much the same way that the sum of the interior angles in a triangle (180o) proceeds from his own immanent logic. Constrained by geometric necessity, the creative activity of God could no more said to be free willed than this angular relation is freely willed by the triangle."

In Maupertuis' 1746 work, it is mathematics that is blind and necessary. See *Histoire de l'Académie Royale des Sciences et des Belles Lettres,* Annee MDCCXLVI, p. 279

[118] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 425: & si notre esprit était assez vaste, il verrait également les causes des effets Physiques, soit en calculant les propriétés des corps, soit en recherchant ce qu'il y avait de plus convenable à leur faire exécuter.

He next reaffirms his commitment to the negation of the idea that the extended distance in refraction is an appropriate quantity in nature: "The first of these means is the most within our reach, but it does not lead us very far. Sometimes the second leads us astray because we do not quite know what is the purpose of nature, we can be mistaken on the quantity that we must regard as its expense in the production of its effects."[119] In other words, Maupertuis is saying, understanding time over space in refraction is beyond our comprehension. It requires an understanding that, in the production of its effects in refraction, light moves "for the sake of", or, "for the end purpose of" least time and not for space. This violates what he knows to be the law of optics number two and is a metaphysical leap beyond mechanics.

Maupertuis ends with a compliment of the ancient geniuses who knew about the issues debated in optics, but before he does, he makes a final appeal to the members in the Paris academy and his readership at large. It is a Leibnizian appeal to merge the study of efficient causes with the study of final causes: "To join the extended security in our research, it must employ the one and the other of these means. Calculate the movement of bodies, but consult also the designs of the intelligence which makes them move."[120]

Maupertuis' 1746 work, *The Laws of Motion and Rest Deduced From a Metaphysical Principle* is about the metaphysical application to physics and evidence of theistic design in nature.[121] Maupertuis organizes this work into three sections. Section one is entitled

[119] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 426: Le premier de ces moyens est le plus à notre portée, mais il ne nous mené pas fort loin. Le second quelquefois nous égare, parce que nous ne connaissons point assez quel est le but de la nature, & que nous pouvons nous méprendre sur la *quantité* que nous devons regarder comme sa *dépense* dans la production de ses effets.

[120] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 426: Pour joindre l'étendue à la sureté dans nos recherches, il faut employer l'un& l'autre de ces moyens. Calculons les mouvements des corps, mais consultons aussi les desseins de l'intelligence qui les fait mouvoir.

[121] In his essay "Euler's "Harmony" Between the Principles of "Rest" and "Least Action": The Conceptual Making of Analytical Mechanics" (*Archive For History of Exact Sciences,* 1999, Vol. 54, No. 1, p. 68), Penha Maria

"Examination of the Proofs of the Existence of God Drawn from the Marvels of Nature". Section two is entitled "To Look For the Existence of God in the General Laws of Nature". Section three is entitled. "The Research of the Laws of Movement and Rest".

Before beginning part one, Maupertuis compliments Leonhard Euler on his book which provides a metaphysical application to physics entitled "Method Found for the Pleasing Properties of Maximum and Minimum Curved Lines, Or Solution to the Problem of Isoperimetrics I Felt Accepted". Maupertuis notes that "In the supplement thereto had been added this illustrious demonstration in the trajectories that describe bodies by central forces, the speed multiple through (multiplied by) the element of the curve, the fact of which is always a minimum."[122]

Cardoso Diaz says that Maupertuis' two laws are metaphysical in two senses. One, they point to the existence of the supreme being, and, two, "they point to the way in which nature acts." In his book *Science and the Enlightenment* (Cambridge University Press: New York, N.Y., 1995, p. 30), Thomas Hankins says, "Whereas *vis viva* was a measure of God's desire to conserve his creation, 'action' was a measure of his efficiency. God, as a perfect being, was expected to act always by the most economical means, and therefore the 'action' in any motion in the universe should be a minimum. The Principle of Least Action stated just that – that in any motion the action consumed (measured by the product of the mass, the velocity, and the distance) would be minimum. The concepts of both *vis viva* and action had philosophical and theological roots in the idea of the economy and simplicity of nature."

[122] *Histoire de l'Académie Royale des Sciences et des Belles Lettres,* Annee MDCCXLVI, p. 267 : Dans le supplement qui y avoit ete ajoute, cet illustre Geometre demontre; Que dans les trajectoires, que des corps decrivent par des forces centrales, la vitesse multiple par l'element de la courbe, fait toujours un minimum ; In his essay entitled «The Origins of Euler's Variational Calculus » (*Archive in History for Exact Sciences.* 1994, Vol. 47, No. 2, p. 104 – 105), Craig Fraser says, "Euler's theory is based on his derivation of the variation equations for these three classes of problems. I. We are given a curve in the plane joining the points *a* and *z* (Figure 1) The curve represents geometrically the analytical relation between the abscissa *x* and the ordinate *y*. Let *M,N,O* be three points of the interval *AZ* infinitely close together. We set $AM = x$, $AN = x'$, $AO = x''$, and $Mm = y$, $Nn = y'$, $Oo = y''$. The differential coefficient or derivative *p* is defined by the relation $dy = pdx$. Euler represented the relations

$$P = \frac{v' - v}{Dx}$$

$$P = \frac{v'' - v'}{Dx}$$

which give the values of *p* at *x* and *x'* in terms of *dx* and the differences of the ordinates *y, y'* and *y''*.

Suppose now that *Z* is some expression composed of *x, y* and *p*….The definite integral $\int Zdx$ corresponding to the interval from *A* to *Z* is

In section one and two of this work, Maupertuis discusses how the existence of God is seen by many to be "deduced" from the animal kingdom and known through the laws of nature. "For all time people have applied themselves to the contemplation of the universe," he argues, "and have found marks of the wisdom and power of a being who governs. The more the study of physics progresses, the more these proofs multiply. Some *proofs* strike confusedly at the character of divinity which one finds in all moments in nature, while others, by an evil zeal to talk religion, have given too much sight to some proofs when they are not *actual proofs*."[123]
(Italics mine)

Isaac Newton, Maupertuis argues, surpasses everyone in proofs from nature for the existence of an all-powerful, all-wise, and supreme being because of his thoughts concerning planetary motion. However, he says, all such great arguments from all the great thinkers who postulated that nature and physics demonstrates the existence of God are all at best probabilities.

$$(2)\ \int Zdx(A \text{ to } M) + Zdx + Z'dx + \ldots,$$

where Z, Z',…are the values of Z at x,y,p; x',y',p';…Suppose the given curve joining a to z is such that (2) is a maximum or minimum. Increase the ordinate y' by the infinitely small quantity nv, obtaining in this way a comparson curve *amvoz*. The change in the value of the integral calculated along the given and comparison curves must by hypothesis be zero. The only part of the integral that is affected by varying y' is $Zdx + Z'dx$. Euler wrote

$$(3)\ \begin{aligned} dZ &= Mdx + Ndy + Pdp \\ dZ' &= M'dx + N'dy' + P'dp'. \end{aligned}$$

He proceeded to interpret the differentials in (3) as the infinitesimal changes in Z, Z', x,y,y', p, p' that result when y' is increased by nv. Evidently dx=0, dy = 0 and $dy' = nv$. From (1) we see that dp and dp' equal nv/dx and $-nv/dx$ respectively. Hence (3) becomes

$$dz = P\cdot nv/dx \quad :$$

[123]Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin*, Chez Ambroise Haute, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 269; De tout tems ceux qui se sont appliqués à la contemplation de l'Univers, y ont trouvé des marques de la sagesse & de la puissance de Celui qui le gouverne. Plus l'étude de la Physique a fait de progrès, plus ces preuves se sont multipliées. Les uns frappés confusément des caractères de Divinité qu'on trouve à tous moments dans la Nature ; les autres par un zèle mal à propos religieux, ont donné à quelques preuves plus de force qu'elles n'en dévoient avoir ; & quelque fois ont pris pour des preuves, ce qui n'en était pas.

He says, "If one conceives as Newton, that all celestial bodies attracted to the sun move in a void, it is true that it was not likely that chance caused them to move as they move. However, there remains some probability. One cannot say that this uniformity was necessarily the result of a choice."[124] He calculates the chance probability of planetary orbits at $17^5 – 1$; "that is to say, 1419856 to 1."[125]

In the section of *Optiks* which influenced Maupertuis, Newton says,

> All these things being consider'd, it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, movable particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear of break in pieces; no ordinary Power being able to divide what God himself made on in the first Creation. While the Particles continue entire, they may compose Bodies of one and the same Nature and Texture in all Ages: But should they wear away, or break in pieces, the Nature of Things depending on them, would be changed. Water and Earth, composed of old worn Particles and Fragments of Particles, would not be of the same Nature and Texture now, with water and Earth composed of entire Particles in the Beginning. And therefore, that Nature may be lasting, the Changes of corporeal Things are to be placed only in the various Separations and new Associations and Motions of these permanent particles; compound bodies being apt to break, not in the midst of solid Particles, but where those Particles are laid together, and only touch in a few Points.[126]

Newton's views are *a posteriori* and by his own admission, his results are probable.[127] Newton says in the preface to the third edition of *Principia* and Book III "Rules of Philosophizing" that the mechanical practice of geometry is based on experience and so is everything we know about bodies.[128]

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[124] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 271: Si l'on conçoit comme Newton, que tous les corps célestes attires vers le Soleil, se meuvent dans le vide, il est vrai qu'il n'était guerres probable que le Hazard les eut fait mouvoir comme ils se meuvent. Il y restait cependant quelque probabilité ; & des-lors on ne peut pas dire que cette uniformité soit l'effet nécessaire d'un choix.

[125] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 270 – 271

[126] Newton, Isaac. *Optiks* in Dover Histories, Biographies and Classics of Medicine. Dover Publications: New York, N.Y., 1952, P. 400

[127] Stein, Howard. *Cambridge Companion to Newton,* p. 261

[128] Stein, p. 262

The Newtonian argument from the design of the animal kingdom seems strong in Maupertuis' eyes. In language classically anti-Spinozist and pro-Leibnizian he says,

> The argument drawn from the suitability of different parts of animals with their needs seems stronger. Their feet are they not made for walking, their wings to fly, their eyes to see, their mouths to eat, others to reproduce their kind? Does not all this mark an intelligence and a purpose that led to their construction? This argument struck Newton as it struck the ancients: and it is in vain that the greatest enemy of providence responds that use has not been the goal. It was the result of the construction of parts of animals: that chance shaped the eyes, ears, language, we use to see, to hear, and to talk.[129]

Unfortunately, Maupertuis writes, two extremes exist in regards to the metaphysical application to the laws of nature and the construction of the animal kingdom. There are those who find no traits of intelligent design anywhere and would ban all final causes, and there are those who find signs of it everywhere. Those who do not see it anywhere believe in a blind mechanism.[130] He concludes part one by saying,

> It is true that our view being as bounded as it is, cannot in any way require it to continue far enough around the order and the linking of things. If it could, without a doubt it probably would be as struck by the wisdom of patterns, as well as intelligence in the execution. But in our helplessness, so let us not in any way confuse these different attributes. Because, what an infinite intelligence necessarily supposes to be wisdom; a bounded intelligence might fail in the power of wisdom: and there is as much worth that the universe had its origin in a blind fate, than that it was the work of such intelligence.[131]

Though persuasive, and though Newtonian, this is not quite good enough for Maupertuis. The truth of the origin of the universe from this perspective is a toss-up.

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[129] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 272: L'Argument tire de la convenance des differentes parties des Animaux avec leurs befoins, paroit plus solide. Leurs pieds ne sont-ils pas faits pour marcher, leurs ailes pour voler, leurs yeux pour voir, leur bouche pour manger, d'autres parties pouir reproduire leurs semblables ? Tout cela ne marque-t-il pas une intelligence & une dessein qi ont préside à leur construction? Cet argument avait frappé les anciens comme il a frappé Newton : & c'est en vain que le plus grand ennemi dela providence y répond, que l'usage n'a point été le but, qu'il a été la suite de la construction des parties des animaux: que le Hazard aidant forme les yeux, les oreilles, la langue, on s'en est servi pour voir, pour entendre, pour parler.

[130] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 272

[131] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 277: Il est vrai que notre vue étant aussi borne qu'elle l'est, on ne peut pas exiger, quelle poursuive assez loin l'ordre & la enchainement les choses. Si elle le pouvoit, sans doute qu'elle serait autant frappe de la sagesse des motifs, que de l'intelligence dans l'exécution. Mais dans cette impuissance ou nous sommes, ne confondons pas ces différents attributs. Car, quoi qu'une intelligence infinie suppose nécessairement la sagesse; une intelligence borne pourrait en manquer, & il vaudrait autant que l'universel dut son origine a un Destin aveugle, que s'il était l'ouvrage d'une telle intelligence.

In part two Maupertuis offers an intelligent design argument based on the attributes of God, namely, the perfection. This is a design argument that actually persuades him. He says that the laws of physics in nature are proof for a "supreme intelligence." His position is that the attributes of God are the foundation for the laws of motion. The best way to observe this, he argues, is in the simple laws of mathematics:

> So, let us not stop at the mere speculation of the most marvelous objects; the organization of animals, the multitude and the small parts of insects, the immensity of the celestial bodies, their distances and their revolutions, are more likely to surprise our minds than to illuminate it. The supreme being is everywhere; However, he is not also equally visible by all. We shall see better in the most simple objects: seeking it in the first laws he imposed upon nature; in these universal rules, according to which the movement is conserved, distributed, or destroyed; and not in phenomena the laws of which are just too complicated sequences.[132]

The idea is to deduce the laws of nature from the attributes of the supreme being rather than induce them through experience. Simply put, only a perfect supreme being would run a world in the most economical of ways – least action.

Here he considers an objection against his own argument. "However, might we say that although the rule of motion and of rest have not hitherto been demonstrated by hypothesis and experience, they are perhaps necessary sequences of the nature of bodies. And there was nothing arbitrary in their establishment. You attribute to providence that which is a necessary effect?"[133] So, he replies to the objection: "If it is true that the laws of motion and of rest are indispensable

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[132] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 278 – 279: Ne nous arrêtons donc pas à la simple spéculation des objets les plus merveilleux ; l'organisation des animaux, la multitude & la petitesse des parties des insectes, l'immensité des corps célestes, leurs distances & leurs révolutions, ont plus propres a étonner notre esprit qu'a l'éclairer. L'être suprême est par tout ; mail il n'est pas par tout également visible. Nous le verrons mieux dans les objets les plus simples : cherchons-le dans les premières loi qu'il a imposées à la nature ; dans ces règles universelles, selon lesquelles le mouvement se conserve, se distribue, ou se détruit ; & non pas dans des phénomènes qui ne sont que des suites trop compliques de ces loix.

[133] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 279: Mais, pourrait-on dire, quoique les règle du Mouvement & du Repos n'aient été jusqu'ici démontrés que par des hypothèses & des expériences, elles sont peut-être des suites nécessaires de la nature des Corps : & n'y aidant rien eu d'arbitraire dans leur établissement, vous attribuez à une Providence ce qui n'est l'effet que de la Nécessité ?

sequences of the nature of bodies, it proves even more the perfection of the supreme being. It is that all things are so ordered, that a blind and necessary mathematics executes that which the free and clear intelligence prescribes."[134]

Maupertuis points out how mentally awesome and difficult it is to contemplate the nature of bodies in motion. He says that what is observed is that some bodies are at rest, and some are in motion. Therefore, motion is "not an essential property of matter."[135] What causes a body to go into motion apart from another body causing it to do so, Maupertuis conceded, is so far unknown to him.[136] What he does know as a Newtonian is that bodies are indifferent to motion and rest. They remain in their state until something causes them to change. The question regards the nature of the force that causes the change. Is it living, or is it dead? He does not answer this question, but, rather, discusses the Cartesian – Leibnizian dispute over the conservation of motion. This is so as to set up his argument that the laws of nature are deduced from the perfection of God.

Mathematicians, Maupertuis argues, all agree about the most complicated cases, but do not agree about the simple cases. The situation concerning the relationship between elastic and inelastic collision is so bad that some deny the reality of hard bodies altogether. He says, "They claim that bodies taken as such are only elastic bodies whose bending stiffness makes their parts

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[134] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 279: S'il est vrai que les loix du Mouvement & du Repos soient des suites indispensables de la nature des Corps ; cela même prouve encore plus la perfection de l'Etre suprême : C'est que toutes choses soient tellement ordonnées, qu'une Mathématique aveugle & nécessaire exécute ce que l'Intelligence la plus éclairée & la plus libre prescrirait.

[135] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 281

[136] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Annee MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 281

and their recovery imperceptible."[137] Descartes, Maupertuis says, believed in hard bodies, but calculated the conservation of the quantity of movement to be the same, which is incorrect.[138] Other philosophers came along and calculated another conservation law called "living force", which is "the product of each mass by the square of its speed." The problem here, he says, is that *vis viva* does not work with elastic body collisions. Therefore, those philosophers claimed that there were no other bodies in nature but elastic bodies.

Maupertuis rules that because conservation of motion is true only in certain cases, neither conservation law can be considered "a universal principle, or a general result of the laws of motion."[139] They are intermediate. He expresses surprise that these men were able to make their discoveries due to their faulty methodology of working backwards from experience rather than forwards from deduction.

Maupertuis ends section two by announcing his discovery of least action and the solutions in physics it has enabled him to garner.

> After so many great men who have worked on this matter, I would almost dare to say that I have discovered the universal principle on which all these laws are based, which also extends to the hard and the elastic body; from which depends the movement and rest of all corporeal substances.
>
> This is the principle of the least quantity of action: a principle so wise, so worthy of the supreme being, and to which nature seems constantly attached. She observes not only in all changes, but in its permanence it tends also to observe it. Movement is distributed so that the amount of action that involves the change as it happens, is as small as possible. In rest, bodies which are held in equilibrium must be located so as if they happened to some small movement, the quantity of action would be the least.
>
> The laws of motion and rest are deduced from this principle, being precisely the same that are observed in nature: we admire its application in all phenomena. The movement of animals, the vegetation of plants, the revolution of stars, are only sequences: and the spectacle of the universe deviating much larger, far more beautiful, far more worthy of its author, when we know that a few laws, most wisely established, suffices to all these movements. That is when we can have a fair idea

[137] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 283
[138] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 283
[139] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 285

of the wisdom and power of the supreme being, and not when discerned by some small part which we know neither the construction nor the use, nor the connection it has with others. What satisfaction for the human mind in contemplating these laws which are the principles of the movement and rest of all bodies of the universe, to find the expected life of the one who governs it![140]

With this statement, Maupertuis finishes Part II.

Part III contains his mathematics of rest and motion. It is here that Maupertuis calculates using force and time. He employs his theory of action to solve for the three problems of elastic and inelastic collisions, and equilibrium. He opens by saying, "Whether bodies are at rest or in movement, they have a certain force that works to maintain them in their current state of motion; this force, denoted inertia, is proportional to the amount of matter they contain."[141] Maupertuis applies his general principle to all three problems: "*When any change happens in nature, the amount of action necessary for this change is as small as possible*." As in *Accord,* he follows his announcement of least action with his definition of action. In this work, however, he changes his definition to say the following: "The quantity of action is the product of the mass of the bodies

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[140] Histoire de l'Académie Royale des Sciences et des Belles Lettres, Année MDCCXLVI, *A Berlin,* Chez Ambroise Haude, Libraire de la Cour & de l'Académie Royale MDCCXLVIII, p. 286 – 287: Après tant de grands hommes qui ont travaillé sur cette matière, je n'ose presque dire que j'ai découvert le principe universel, sur lequel toutes ces loix sont fondées ; qui s'étend également aux Corps durs & aux Corps élastiques ; d'où dépend le Mouvement & le Repos de toutes les substances corporelles.

C'est le principe de la *moindre quantité d'action* : principe si sage, si digne de l'Etre suprême, & auquel la Nature paroi si constamment attachée ; qu'elle l'observe non seulement dans tous ses changements, mais que dans sa permanence, elle tend encore à l'observer. *Dans le Choc des Corps, le Mouvement se distribue de manière que la quantité d'action, que suppose le changement arrivé, est la plus petite qu'il soit possible. Dans le Repos, les Corps qui se tiennent en équilibre, doivent être tellement situés, que s'il leur arrivait quelque petit Mouvement, la quantité d'action serait la moindre.*

Les loix du Mouvement & du Repos déduites de ce principe, se trouvant précisément les mêmes qui sont observées dans la Nature : nous pouvons en admirer l'application dans tous les Phénomènes. Le mouvement des Animaux, la végétation des Plantes, la révolution des Astres, n'en sont que les suites : & le spectacle de l'Univers devient bien plus grand, bien plus beau, bien plus digne de son Auteur, lors - qu'on sait qu'un petit nombre de loix, le plus sagement établies, suffisent à tous ces mouvements. C'est alors qu'on peut avoir une juste idée de la puissance & de la sagesse de l'Etre suprême ; & non pas lors qu'on en juge par quelque petite partie, dont nous ne connaissons ni la construction, ni l'usage, ni la connexion qu'elle a avec les autres. Quelle satisfaction pour l'esprit humain, en contemplant ces loix, qui sont le principe du Mouvement & du Repos de tous les Corps de l'Univers, d'y trouver la preuve de l'existence de Celui qui le gouverne !

[141] *Histoire de l'Académie Royale des Sciences et des Belles Lettres* MDCCXLVI, p. 287: Les corps soit en repos, soit en mouvement, on une certain force pour persister dans l'état ou ils sont: cette force appartenant à toutes les parties de la Matière , est toujours proportionnelle à la quantité de Matière que ces Corps contiennent, & s'appelle leur inertie.

by their speed and the space through which they travel. When a body is transported from one place to another the action is all the greater as the mass is greater, the speed is faster, the space through which it is transported is longer."[142]

In the first two problems, Maupertuis not only employs the concepts of force and time to solve for both elastic and inelastic collisions, but he also uses *vis viva*. The following are problems one (inelastic bodies) and two (elastic bodies) respectively:

Problem One: Solving for Inelastic Bodies

> Given two inelastic bodies whose masses are *A* and *B* which move in the same direction, with speeds *a* and *b*: but, let *A* move quicker than *B*, so that it overtakes *B* and impacts with it. Let the common speed of both bodies after impact = $x < a$ & $> b$. The change that occurs in the universe consists in body *A*, which moves with speed *a*, and which in certain time travels a space = *a*, it now moves only at speed *x*, and travels that of a space = *x*: body *B* that moves with speed *b*, and traverses with space = *b*, moves with speed *x* and traverses a space = *x*.
>
> The change is the same if it happens that, if that body *A* moves with speed *a*, and traverses space = *a* (per unit time), then if it were being transported backwards by an moving with a speed $a – x$, by a space = $a – x$ (per unit time): and that body *B* moves with the speed *b*, and traverses space = *b*, then if it were being transported forwards by an immaterial plane, that was moving with speed $x – b$, by a space = $x – b$(per unit time). Now, these bodies *A* and *B* move with their own speeds on these mobile planes, will be the same whether they will be in rest, or moving relative to the planes: the quantity of action produced in nature will be $A(a – x)^2$, and $B(x – b)^2$; of which the sum will be the smallest possible. It is therefore
>
> $$Aaa – 2Aax + Axx + Bxx – 2Bbx + Bbb = \text{minimum.}$$
>
> Or
>
> $$- \quad Aadx + 2Axdx + 2Bxdx – 2Bxdx = 0$$
>
> Or where one draws for the common speed
>
> $$X = \frac{Aa+Bb}{A+B}$$
>
> The same reasoning is easily applied to this case, where these bodies move one towards another where it well suffices to consider *b* as negative by relationship to *a:* and the common speed will be

[142] *Lorsqu'il arrive quelque changement dans la nature, la Quantité d'Action,* nécessaire pour ce changement, est la plus petite qu'il soit possible. La Quantité d'Action est le produit de la Masse des Corps, par leur vitesse & par l'espace qu'ils parcourent. Lors qu'in Corps est transporté d'un lieu dans un autre, l'Action est d'autant plus grande, que la Masse est plus grosse; que la vitesse est plus rapide; que l'espace, par lequel il est transporté, est plus long.

$$X = \frac{Aa - Bb}{A+B}$$

If one of these bodies was at rest before impact, b = 0, and the common speed is

$$X = \frac{Aa}{A+B}$$

Problem Two: Solving for Elastic bodies

Given are two elastic bodies whose masses are *A* and *B*, which move to the same side with speed *a* and *b*. But *A* is faster than *B*. It overtakes *B* and collides with it. The sum or the difference of the speeds is the same as before the collision.

The change that happens in the universe consists of that which is body *A*, that moves in a certain time with speed *a*, and traverses a space = *a*. Body *B* moves with speed *b*, and traverses a space = *b* (per unit time).

The change is the same that would be happening if, while body *A* was moving with the speed = *a* and traversing a space = *a*, was transported backwards by an invisible massless plane moving at speed *B – b* and by a space = *B – b* (per unit time). Now, whether the bodies *A* and *B* move with their own speed on moving planes orthey are at rest, their movement is the same: the quantity of action produced in nature will be $A(a-a)^2$ & $B(b-B)^2$, the sum of which is as small as possible. It is therefore

$$Aaa - 2Aaa + Aaa + Bbb - 2BbB + Bbb = Minimum.$$

Or

$$-2Aada + 2Ada + 2BBdB - 2Bbdb = 0.$$

Now for elastic bodies, the speed after the collision should be the same speed as after. Therefore, we have

$$B - a = a - b, \text{ ou } B = a + a - b, \text{ \& } dB = da,$$

that was substituted in the previous equation to give speeds

$$a = Aa - Ba + 2Bb/A + B \quad \& \quad B = 2Aa - Aa + Bb/A + B$$

If these bodies move towards one another, it is easy to apply the same reasoning or just consider *b* as negative in relationship to *a*, and the speed will be

$$a = Aa - Ba\ 2Bb/A + B \ \ \& \ \ 2Aa + Ab - Bb/A + B.$$

If one of the bodies is an unwavering obstacle, considering this obstacle body *B* of infinite mass, the speed will be $a = -a$: that is to say, the body *A* rebounds with the same speed it had before hitting the obstacle. If the sum of the forces is taken, it will be seen that after the impact, it is the same as before. That is to say that

$$Aaa + Bbb = Aaa + Bbb.$$

The sum of the forces is conserved after the impact, but this conservation occurs only for the elastic body and not for the hard bodies. The general principle, which extends to one and the other, is that the quantity of action, necessary to cause a change in the nature is as small as possible. This principle is so universal and so fruitful, that one can derive the law of rest, or equilibrium from it. It is evident that there is no more difference between hard bodies and elastic.

In 1751, the same year as the Konig Affair, Maupertuis changes his position in *Accord.* The evidence is an unpublished manuscript of *Essai de Cosmologie* followed by an altered edition of *Accord.* Maupertuis further alters *Accord* in the 1756 *Oeuvres de Maupertuis,* volume four. The 1744 *Accord* and the 1756 *Accord* are two different works in grammar, structure and meaning. Between the two, there are twenty grammatical category changes that occur, ranging from everything from spelling and punctuation changes to wording and verb conjugation changes.[143]

For purposes of this dissertation, the following changes are mentioned to clarify the change in Maupertuis' position over time. In the 1744 memoire, the structure is a presentation of the two sides of the Descartes – Fermat debate, with the "Now" dividing the two positions. Fermat is credited with "resolving" for refraction, and Leibniz is credited with being the mathematician who praised Fermat the loudest among all those mathematicians who "embraced the opinion of Fermat." Maupertuis accuses both men of error regarding the speed of light in air versus water and glass, and he accuses them both of error regarding their application of final causes to physics. According to Maupertuis, they both took for a principle that which was merely the consequences of a principle. Maupertuis says that the debate over refraction between Fermat and Descartes can be seen in the 1723 memoire put out by de Mairan.

In the unpublished 1751 manuscript of *Accord*, the "Now" remains, but Leibniz's name is removed and only Fermat's edifice is destroyed.[144] This text is still a presentation of two sides of

[143] A few examples of these are 1) the 1744 memoire, p.417, reads "scait render a son….", and the 1756 edition reads "sait render a son…."; 2) the 1744 memoire, p. 421, reads "…l'air dans l'eau,…", and the 1756 edition reads "…l'air dans l'eau:"; 3) the 1744 memoire, p. 422, says, ""C'est cependant tout le contraire, Descartes avoit avance le premier,…", whereas the 1756 edition reads ""C'est cependant ce qui arrive. Descartes avoit avance le premier se meut le plus vite dans les milieu les plus denses:…". These three examples are but a small sample of the numerous changes made.

144 Essai de Cosmologie - Maupertuis, Pierre Louis Moreau ¬de ...
books.google.com/books/about/Essai_de_Cosmologie.html?id=MMMAAAAAcAAJ, p. 227

the argument, with Maupertuis' "least action" as the solution to the problem. Leibniz's name is also removed in the paragraph where several mathematicians embrace the sentiment of Fermat. In the paragraph before the focal passage, Leibniz's name is removed from error regarding shortest distance being the principle, and his name is removed from error regarding the application of final causes to physics. Leibniz's name is completely removed from this unpublished text.

In 1756, Maupertuis changes his position even further. The "Now" is removed and *Accord* goes from being a presentation of two sides of an argument to an embracing of one side over the other. The fact is now posed that light moves faster in water and glass than in air, and the edifice of Fermat is destroyed. It appears that Maupertuis now assumes this position as his own. Leibniz's name is removed in all the locations mentioned in the previous paragraph. However, the entire paragraph which mentions mathematicians embracing the sentiment of Fermat, Leibniz included, is removed. Now the only mathematicians mentioned are those who have a repugnance for final causes applied to physics.

The 1756 edition contains some additions which add to the change in position. Leibniz's name is added in a comment not found in either the 1744 memoire of the 1751 draft. The 1744 reads:

> C'est cependant tout le contraire, Descartes avait avancé le premier, que la lumière se meut le plus vite dans les milieux les plus denses, & quoique l'explication de la réfraction qu'il en avait déduite, sut insuffisante, son défaut ne venait point de la supposition qu'il faisait. Tous les systèmes qui donnent quel qu'explication plausible des phénomènes de la réfraction, supposent le paradoxe, ou le confirment.[145]
>
> This is, however, all contrary. Descartes had first advanced the explanation of refraction that he had deduced, that light moves more quickly in the more dense environment. It was insufficient. Its default was that it was just not the assumption he was making. All systems give some plausible explanation for the phenomena of refraction, supposing the paradox, or the confirmation.

[145] *Histoire de l'Academie Royale des Sciences.* Annee MDCCXLIV. Avec les Memoires de Mathematique & de Physique, pour la meme annee. Tirez des Registres de Cette Academie, p. 422

The 1756 edition reads:

> C’est cependant ce qui arrive. Descartes avait avancé le premier que la lumière se meut les plus vite dans les milieux les plus denses: & quoique l’explication de la réfraction, qu’il en avait déduite, sut insuffisante, son défaut ne venait point de la supposition qu’il faisait. Tous les systèmes qui donnent quelque explication plausible des phénomènes de la réfraction, supposent le paradoxe, ou le confirment. Leibniz voulut concilier le sentiment de Descartes avec les causes finales: mais ce ne fut que par des suppositions insoutenables, & qui ne quarraient plus avec les autres phénomènes de la nature.
>
> This is, however what happens. Descartes had been the first to advance that light moves more quickly in more dense environments: and though the explanation of refraction which he had deduced was insufficient, his default had just been the point of his supposition. All systems give some plausible explanation for the phenomena of refraction, supposing the paradox or the confirmation. Leibniz wanted to conciliate the sentiment of Descartes with final causes: but this was not due to unsustainable suppositions which did not square with the other phenomena of nature.[146]

This change is not in the 1751 draft. In 1756 Maupertuis says that Leibniz desired to unite the “opinion of Descartes with final causes”, but does he mean by “sentiment” Descartes’ paradox itself or the idea that light moves quicker in water and glass than in air? Either way, Leibniz is not able to do this because of his “unsustainable suppositions which did not square with the other phenomena of nature.” Maupertuis footnotes this line on page 15 of the 1756 edition, saying that this addition is taken from “v. la Remarque de Mr. Euler a la fin de ce memoire.”

At the end of the 1756 edition, Maupertuis adds an appendix not found in either the 1744 or 1751 works which discuss a disclaimer regarding a “misunderstanding” on the part of both he and de Mairan concerning Leibniz’s position in 1682 as well as Euler’s commentary on the superiority of Maupertuis’ discovery to Leibniz’s vis viva. Maupertuis would have us believe that both he and de Mairan mistook Leibniz’s position on refraction to be that of Fermat’s. The disclaimer reads as follows:

> When we read the preceding memoire in the Royal Academy of Sciences in Paris, we know that which Leibniz had as fact on this matter by what de Mairan said in his memoire on the reflection of

[146] *Oeuvres de Mr. de Maupertuis.* Nouvelle Edition corrigee & augmente. Tome Quatrieme. Chez Jean-Marie Bruyset, Libraire, grande rue Merciere, au Soliel. MDCCLVI, “Accord de Differentes Loix de la Nature qui Avoient Jusqu’ici Paru Incompatibles”, p. 15

> bodies. We had confused, as he, this sentiment of Leibniz with that of Fermat: here is this developed sentiment; drawn from the memoire of M. Euler, volume seven of the memoires of the Berlin Royal Academy of Sciences.[147]

The part of Euler's commentary which Maupertuis presents, and by its inclusion endorses, is Euler's interpretation of Leibniz's 1682 essay found in Euler's work "Principle of Least Action", sections X and XI. In sections one and two, Euler says that, although it is true that the ancients posed that nature does nothing in vain, Maupertuis' claim to discovery is legitimate in that he is the first to claim the universal application of minimization in physics. The essence of what Euler says concerning Leibniz's 1682 position is summarized by the following.

In 1682 Leibniz argued for a return of final causes which Descartes had banished from his physics, and to reintroduce Descartes' deduction concerning the collision of bodies. Leibniz set out to reverse the position of Descartes, but in fact, demonstrates a Cartesian position with one exception. Whereas Descartes believed that a ray of light moves with greater speed in a more dense medium because the resistance was less, Leibniz believed that a ray of light moves quicker in the more dense medium where the resistance was greater. Euler finds Leibniz to be in paradox because Leibniz seemed to agree with Fermat who said that light moved more quickly in air. Euler says that Leibniz has a Cartesian position despite the fact that Leibniz claims that his easiest path is a minimum according to his calculus of maximum and minimum.

The problem with Leibniz's easiest path, according to Euler, is that Leibniz claims a universality for it. Euler says that it only works in a few cases, and the claim of universality is unacceptable. Leibniz never applied it to any other case, nor taught it as such. As it turns out, the easiest path can only successfully be applied to optics. If one applies Leibniz's idea of an

[147] *Œuvres de Mr. de Maupertuis.* Nouvelle Edition corrigée & augmente. Tome Quatrième. Chez Jean Marie Bruyset, Libraire, grande rue Mercière, au Soliel. MDCCLVI, "Accord de Différentes Loi de la Nature qui Avoient Jusqu'ici Paru Incompatibles, p. 23

increase in speed with greater resistance, one has a proportionality between resistance and speed. In this case the calculation of the difficulty is reduced to the described route multiplied by the speed. If this is taken as a minimum, Euler argues, then it accords with Maupertuis calculation for action being the product of the length of the space traveled by the speed. If one looks at this as a minimum, then it would appear that Leibniz had least action in mind. The problem here, according to Euler, is that nature gives us plenty of examples where resistance minimizes speed. Consequently, Leibniz's easiest path, in this respect, is not universally applicable. In fact, Euler argues, examples where the speed of a body increases where the resistance increases are quite rare.[148] Euler finishes section XII with the statement: "Leibniz never held the principle of least action, far from it. On the contrary, he had an entirely opposite principle, the use of which, except in a single case, was never applicable or led to error."[149]

It is clear from an examination of the two *Accords* that Maupertuis changes his position concerning Leibniz's error regarding final causes being applied to physics. His addition of the de Mairan and Euler evidence indicates Maupertuis uses Euler's commentary to shift the charge against Leibniz away from the consequences of applying final causes to physics and towards the causal effect of resistance on the speed of light in differing mediums. As regards Fermat and the charge of error, the charge remains throughout all publications of *Accord*. In order to better assess his position regarding the focal passage in this essay, it necessary to give summaries of the positions of both Fermat and Leibniz regarding final causes applied to physics.

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[148] Leonhard Euler, "On the Principle of Least Action", from Euler Archives, http://eulerarchive.maa.org/docs/translations/E198tr.pdf

[149] Euler, p. 8, http://eulerarchive.maa.org/docs/translations/E198tr.pdf

Chapter Three: Fermat's Position

Pierre Fermat never uses the term "final cause". Nor does he give a theological interpretation of motion in terms of a reference to God. Fermat appears to be all mathematics. His stated position that comes the closest is in *Oeuvres de Fermat* where he says, "Our demonstration rests on a single postulate: that nature operates by the easiest and most expedite ways and means."[150] This statement is metaphysical. Fermat is saying that the action taken by nature is according to what is easiest and most expedite. He does not use the word action. Had he done so, he would not have been the first. Leibniz mentions it in a 1669 work. Additionally, Christian Wolf says in a 1726 work that action consists of "mass, velocity and space".[151]

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150 Osler, Margaret. *The Monist.* 1996, Vol. 79, No. 3, "From Immanent Natures to Nature as Artifice: The Reinterpretation of Final Causes in Seventeenth-Century Natural Philosophy", p. 392; In her essay "Whose Ends? 'Teleology in Early Modern Natural Philosophy' " (*Osiris, 2nd Series,* 2001, Vol. 16 p. 157), Osler says that this statement is Fermat's concept of immanent finality. Ms. Osler appears to fall into the thinking of many since the Descartes' overthrow of Aristotelian science of motion which categorizes anything metaphysical under the heading of final causes.

151 In his essay "The Principle of Action" (*Facta Universitatis, Series* "Mechanics, Automatic Control and Robotics", 1999, Vol. 2, No. 9, p. 843 – 844) V. A. Vujicic says that this information is found in an essay written by L.S. Polak (in Russian) in *Variational Principles of Mechanics.* He also says, "The concept of *action* can be found in the work given by Leibnitz (1669) as *action formalis*, whose dimension is product of mass, velocity and way (Mathematische Schriften, Harausg. Von Gerhard, t. II, 1; t. III, 1860). Christian Wolff (1726) wrote: action consist of mass, velocity and space. In "Accord de differentes lois de la Nature qui avaient jusqu'ici paru incompatibles", Pierre Maupertius (1744-1746) was the first who write about the *principle of least action*. Two years later (1748) in "Reflexions sur quelques lois generales de la Nature qui s'observent dans les effects des forces quelconques", Euler found the functional form of the action as

$$\int Tdt,$$

where is T kinetic energy and t is time, and gave a definition of principle of least action. Analytical form of this principle

$$\delta(M \square \int udt) = \delta(M \int \square u^2 dt) = 0 \ ; \ \delta \sum Mi \square \int uidsi = 0$$

(1) where u is velocity, m mass of body, was given by Lagrange (1760)."

In his 1637 letter to Mersenne in which he responds to Descartes' *Dioptrics* and specifically the tennis ball analogy,[152] Fermat expresses doubt that geometry can adequately explain reflection and refraction of light.

> First of all, I doubt, with reason it seems, whether the inclination towards motion must follow the laws of motion proper, since there is as great a difference between one and the other, as between the ability to act and the action itself. Otherwise on this subject, it seems to me that there is a particular incongruity, as the motion of a ball is more or less violent to the same degree that it is pushed by different forces, whereas light penetrates transparent bodies instantly and appears not to be affected by succession. But geometry is not at all suited to further explain matters of physics.[153]

His clear inference is that physics requires a metaphysical application in order to be adequately explained. He simply never explicitly says so.

As regards Fermat's thinking concerning any application of final cause in physics, he is to be understood in reference to his reaction to Descartes' tennis ball experiment in *Dioptrics* and Clerselier's arguments in defense of Descartes. Descartes' published views on final causation in *Meditations* and *Principles of Philosophy* emerge several years after Fermat's correspondence with Descartes through Mersenne. Therefore Fermat cannot be interpreted via Descartes' abolishment of "ends" in physics. He should be interpreted via his own mathematical development which ultimately leads to *The Principle of Least Time.* Fermat calls it his "principle of natural economy."[154] With his principle, he is able to account for the curvature of

[152] In his essay, "Leibniz's Two Realms Revisited" (*Nous,* 2008, Vol. 42, No. 4, p. 676) Jeffrey McDonough says, "The proof of the law of reflection published in Descartes's *Dioptrics* in 1637 relies on three crucial assumptions. First, the behavior of a ray of light can be modeled on the behavior of material bodies, and thus can be treated as being governed by the laws of motion. Second, the motion of a ray of light can be decomposed into orthogonal motions, which can in turn be considered independently of one another, and subsequently recomposed to yield the overall motion of the body under consideration. Third, the direction of the downward component of a ray of light is reversed by a reflective surface, but is not otherwise altered, i.e. a ray of light loses none of its "speed" in rebounding off of a reflective surface."

153 Ross, Jason, (Translator). *Light*: *A History, "Fermat's Complete Correspondence on Light"*; science.larouchepsc.com/fermat, p. 20; Ross notes that Fermat's reference to light penetrating bodies instantly is a reference to Descartes, who Fermat believed thought that light propagated instantaneously.

[154] Basdevant, Jean-Louis. *Variational Principles in Physics.* Springer: New York, N.Y., 2000, p. 9

light, something that the Snell-Descartes law cannot do.[155] His personal development mathematically is revealed in *Method of Finding Maxima and Minima,[156] Synthesis For Refraction,[157]* and in his correspondence with de La Chambre.[158]

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[155] Basdevant, Jean-Louis. *Variational Principles in Physics.* Springer: New York, N.Y., 2007,P. 22; In his article "Analytic Geometry, the Discovery of Fermat and Descartes" (*The Mathematics Teacher,* 1944, Vol. 37, No. 3), Carl Boyer notes on page 104, "it had long been known that, for a given curve, certain distance relationships are determined which may be interpreted as equations of the curve with respect to coordinate systems. However, there appears to have been no connection before their time of the converse, the fact that in general an arbitrary *given equation* involving *two unknown* quantities can be regarded as determining *per se* with respect to a coordinate system a plane curve. This latter recognition, together with its fabrication into a formalized algorithmic procedure, constituted the decisive contribution of Fermat and Descartes. This is the sense in which these men may be regarded as the founders of analytic geometry, much as Newton and Leibniz are generally regarded as the inventors of the calculus through precise formulation of the mutually inverse nature of area and tangent problems in terms of a definitely regularized operational procedure."

[156] Boyer, p. 388 – 389, says, "It is possible that Fermat was in possession of his analytic geometry as early as 1629, for about this time he made two significant discoveries that are closely related to his work on loci. The more important of these was described a few years later in a treatise, again unpublished in his lifetime, entitled *Method of Finding Maxima and Minima.*"

[157] Sabra, p. 149, says, "This work was written after the '*Analysis ad Refractions'*, and was sent to La Chambre in February 1662." Sabra, p. 150 – 152, says, "In the *Synthesis*, Fermat seeks to show with his sin calculation that "the actual path of light in refraction is that for which the time of the movement is a minimum, while being consistent with the supposition…that the velocity is greater in rarer media." See Sabra, p. 150. He draws La Chambre the following diagram and provides a proof.

Fig. 109.

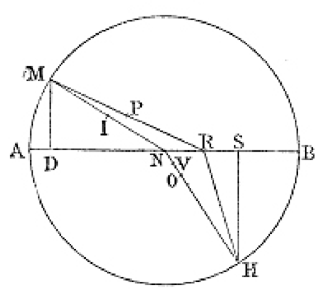

Figure #6 Fermat to de La Chambre Demonstrating Minimum Time

The proof from the diagram demonstrated by Fermat is the following: The diameter *ANB*…seperates the rare medium of incidence above from the sense medium below; *MN* is an incident ray that is refracted into *NH*; *MD* and *HS* are perpendiculars. *R* is any point (other than N) of the diameter *AB* – taken, for example, on the radius *NB.* It is assumed that

$$(1)\ DN = \frac{\sin i}{\sin r} = \frac{v_i}{v_r} = \frac{MN}{NI} = \frac{MR}{RP} = n > 1$$

(where *I* and *r* are the angles respectively made by *MN* and *NH* with the normal to *AB* at *N*: and v_i, v_r, the velocities of incidence and of refraction). It is to be proved that the time along *MNH* is less than the time along *MRH.*

Since

$$\frac{time\ along\ MN}{time\ along\ NH} = \frac{MN}{NH}\ x\ \frac{v_r}{v_i} = \frac{MN}{NH}\ x\ \frac{NI}{MN} = \frac{NI}{NH}$$

and

$$\frac{time\ along\ MR}{time\ along\ RH} = \frac{MR}{RH}\ x\ \frac{v_r}{v_i} = \frac{MR}{RH}\ x\ \frac{RP}{MR} = \frac{RP}{RH}$$

therefore

$$\frac{time\ along\ MNH}{time\ along\ MRH} = \frac{NI + NH}{RP + RH}$$

It is now proved that $RP + RH > NI + NH$. Let

(2) $\frac{MN}{DN} = \frac{RN}{NO}$ and $\frac{DN}{NS} = \frac{NO}{NV}$

S*ince,* by construction, *DN* < *MN* and *NS* < *DN*, we have:

(3) *NO* < *RN* and *NV* < *NO*.

But, $MR^2 = MN^2 + RN^2 + 2DN.RN$.

Therefore, by

(2), $MR^{2} = MN^2 + RN^2 + 2MN.NO$.

From which it follows, by

(3), that $MR^2 > MN^2 + NO^2 + 2MN.NO = (MN + NO)^2$.

Therefore, (4) *MR* >*MN* +NO. But, from (1) and (2),

$$\frac{DN}{NS} = \frac{MN}{NI} = \frac{NO}{NV} = \frac{MN + NO}{NI + NV} = \frac{MR}{RP}.$$

Therefore by (4), *RP* >*NI* + *NV*. It now remains to be proved that *RH* > *HV*, For then it follows that

RP + *RH* > *NI* + *NV* + *HV*, *RP* + *RH* > *NI* + *NH*.

In the triangle *NHR*, (5) $RH^2 = NH^2 + NR^2 - 2NS.NR$. From (2) *MN*(- *NH*). *NO* = *DN.NR NS.NO* = *DN.NV*. Therefore,

(6) $\frac{NH}{NS} = \frac{NR}{NV}$.

From (5) and (6), $RH^2 = NH^2 + NR^2 - 2NH.NV$ But since, by (3), *NR* > *NV*, $NR^2 > NV^2$, It follows that $RH^2 > HN^2 + NV^2 - NH.NV$, And we finally have: *RH* > *HN* – *NV*, *RH* >*HV* which was to be proved."

[158] Marin Cureau de la Chambre (1594 – 1669) is a French physician who authored *A Physical Discourse Touching the Nature and Effects of the Courageous Passions* (Printed by Thomas Newcomb and sold by Thomas Bassett:1658) and *The Art How To Know Men* (Thomas Bassett: 1670), as well as several other works on the passions. His work on light, which enters into his correspondence with Fermat, is his 1657 publication entitled *La Lumiere a Monseignevr L'Eminentissime Cardinal Mazarin*; See Sabra, p. 138, FN #6

In sections one and two of *Dioptrics*, Descartes spells out his understanding of light, and its action in reflection and refraction. Concerning what light is he says, "I would have to think that light is nothing other, in bodies that we call luminous, than a certain movement, or a very quick and strong action which moves towards our eyes through the medium of the air and other transparent bodies in the same fashion as the movement or the resistance of bodies encountered by this blind person pass to his hand by the intermediary of the stick."[159] Here Descartes thinks that time and distance, however slight, are involved with the movement of light. That would subject light to his laws of motion in physics. In his next statement, however, he refers to the idea of light as an "instantaneous propagation". How certain Descartes is of this idea is a debated topic in Cartesian studies.[160] He says, "This example will prevent you from thinking it strange that light can extend its rays in an instant from the sun to us; for you know that the action by which one end of the stick is moved must pass in an instant to the other, and that light must pass

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Claude Clerselier (1614 – 1684) is the editor and translator of several of Descartes' works. In her article entitled "Figuring the Dead Descartes: Claude Clerselier's *Homme de Rene´ Descartes* (1664)" (*Representations,* 2003, Vol. 83, No. 1, p. 42), Rebecca Wilken says, "Clerselier's publications of Descartes's textual estate earned him recognition as the premier Cartesian of the day from fellow Cartesians and anti-Cartesians alike, who unanimously emphasized his role as a guardian, producer, and solicitor of books. Florent Schuyl, the editor of *Renatus Des Cartes de homine*, lauded him in 1662 as ''that guardian and faithful curator of the posthu- mous works of Mr. Descartes''; Pierre Bayle (1647–1706) described him as ''the ornament and pillar of Cartesianism''; Baillet called him ''the second author of Cartesianism''; and the Jesuit Louis le Valois, alias de la Ville (1639–1700), viewed him as the very ''soul'' of the Cartesian ''sect,'' singling out his quarrelsome habit of engaging ''those with the most wit [*esprit*] and talent to continually further its reach with new books.''

159 Ross, p. 5; In his essay "Cartesian Optics and the Mastery of Nature" (*Isis,* 1997, Vol. 88, No. 1, p. 48), Neil Ribe points out that Descartes uses three analogies for practical purposes only. He quotes Descartes from part one, "Now because my only reason for speaking of light here is to explain how its rays enter the eye, and how they are diverted by the different bodies they encounter. I need not attempt to say what is its true nature. It will, I think, suffice to use two or three comparisons that facilitate the conception of light which seems to me most suitable for explaining all those of its properties that experience acquaints us with, and then to deduce all the others that cannot be noticed so easily."

160 In his essay "Descartes on Refraction: Scientific versus Rhetorical Method" (*Isis*, 1984, Vol. 75, No. 3, p. 481 – 482), Bruce Eastwood refers to Sabra, *Theories,* pp. 105 – 135, esp. pp. 110 – 113, and Buchdahl, *Metaphysics,* pp. 141 – 147 on this discussion. He says, "The idea of instantaneous propagation (or transmission) is generally taken as one of the hallmarks of Descartes's physics. Yet that emphasis seems to be more the critics' than Descartes's. The idea emerges in his letters and Le monde as well as the Dioptrics, but in the Dioptrics the treatment of refraction transcends the idea. For Descartes, the assumption that transmission is instantaneous is the simpler assumption in an account that could have assumed a finite velocity for light and still have been worded in a similar way. But although Descartes's mode of presentation eliminates time from the refraction of light, most modern authors insist on reintegrating time into the formulation, thereby obscuring the formulation's original purpose."

in the same way between the earth and the heavens, even though there would be more distance."[161] For Descartes, the nature of light is a sense phenomenon which adheres somehow to the mechanical laws of physics, and he knows that there is a working relationship between reflection and refraction, but he does not understand how.

In *Dioptrics* Descartes describes the movement of light with three analogies, the blind man's walking stick, the wine vat with holes in it, and the tennis ball.[162] The walking stick is a sense perception analogy. Fermat takes issue with neither it nor the porous wine vat. He does take issue with the tennis ball analogy. Descartes shows a tennis player serving a tennis ball onto a hard surface for reflection and into water for refraction.[163]

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[161] Ross, p. 5; In his 1638 letter to Morin, Descartes speaks of light as being both a movement and an instant propagation. He seems to think it is both without being contradictory. See Jonathon Bennett's "Selected Correspondence of Descartes", www.earlymoderntexts.com – Jonathon Bennett translator

[162] In his book *The Optics of Rays, Wavefronts, and Caustics* (Academic Press: New York, N.Y., 1972, p. 4) O.N. Stavroudis says, "Descartes' physical reasoning – and his explanations – occupied two levels of sophistication. The higher level was pure physics with perhaps a touch of applied theology to make things reasonable. Here facts were jostled into position to reveal ultimate truth. The goal was a theory of the universe on the grand scale. The lower level was where details were taken care of….Descartes knew that rays existed (there was never any argument about these), that these rays were associated with light's propagation, which took place at an infinite speed. He was committed to a mechanical universe – events were described in terms of moving particles, collisions, pressures, and the like. He deftly created a hydrostatic analogue for the propagation of light that is mechanistic and at the same time consistent with his notion of infinite velocity."

In his chapter "Thomas Hobbes" (Nadler, Stephen. *A Companion to Early Modern Philosophy.* Blackwell: Oxford, U.K., 2002, p. 324 – 325) Tom Sorell says that both Descartes and Hobbes put out a mechanical explanation for the transmission of light. They differed, however, over refraction. "In a treatise (*Tractatus Opticus I* (c 1640)) that originated in correspondence with Descartes over the *Dioptrics,* Hobbes proposed that an ordinary ray of light has length and breadth, and sections of the ray form parallelograms. When light is transmitted, the path of the ray is like that of a rolling cylinder. When light passes from air to a denser medium different points along the breadth of the ray can move at different speeds….For Descartes, the behavior of light was comparible to the behavior of a ball striking different sorts of surfaces."

[163] Concerning Descartes' concept of light in connection with the tennis player, Stavroudis, p. 4, says, "Light is a pressure applied to the transparent medium by a luminous object. Rays are in the direction of the force. The pressure is applied throughout the medium at exactly the same time. The hydraulic fluid, a substance that he called the *second element,* which was inelastic and incompressible, would be the kind of medium that could transmit a pressure instantaneously (Sabra, 1967, p. 51 *et seq.*). This was the serious physics at the higher level of sophistication."

Concerning Descartes' use of a tennis ball, Stavroudis, p. 4, says, "For an explanation of reflection and refraction Descartes descended to his second level (Sabra, 1967, Chapter 3). Here the mechanical analogue is more obvious and more concrete. He harks back to a rather old idea, due originally to Hero of Alexandria, who explained reflection as a process of balls bouncing off of a plane surface. For both a reflecting beam of light and a bouncing ball, the angle of incidence always equals the angle of reflection or of rebound. Hero observed that the path of a ray between two fixed points that included a reflection off of a mirror was such that the path length was a minimum. This *minimum-path* principle was true, according to Hero, because nature does nothing in vain."

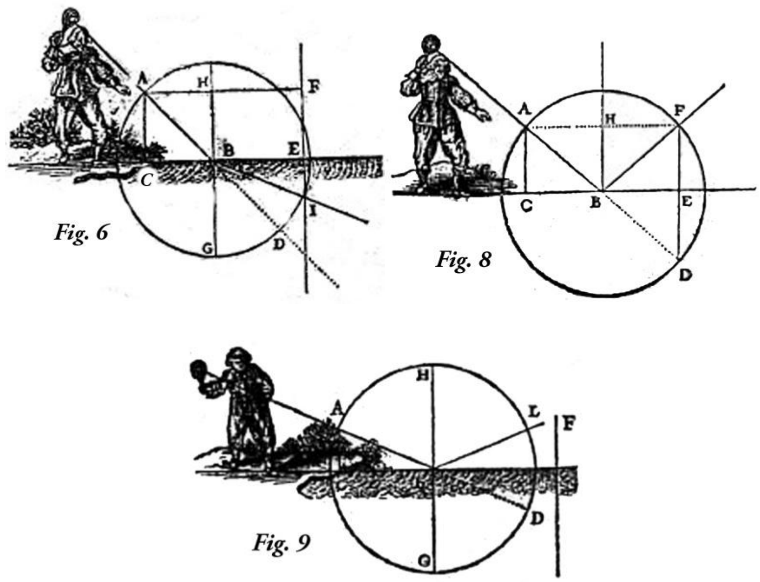

Figure #7 Descartes Drawings of the Tennis Ball experiment in *Dioptrics*; From *Light*: *A History, The Complete Correspondence of Pierre Fermat,* Jason Ross, Translator, p. 9, 10 and 12

These drawings have the following factors in common. The tennis ball is served by a thinking agent using an instrument, the tennis racquet.[164] The force and direction behind the action of the tennis ball is caused by the thinking agent. Descartes, however, calculates force as *mv*, an equation supposedly free of any metaphysical considerations such as final causes.[165] The

[164] Descartes' drawings bring up the question of his commitment to a mechanical universe. More pointedly, given these drawings, what is his real thinking regarding the relation of physics to metaphysics. In his essay "Descartes on the Immutabality of the Divine Will" (*Religious Studies*, 2003, Vol. 39, No. 1, p.) David Cunning argues that the metaphysics of Descartes in terms of God's immutability is all about necessity. He gets this idea from Descartes' "reported" conversation with Burman who quotes Descartes as saying, "Concerning ethics and religion…the opinion has prevailed that God can be altered, because of the prayers of mankind; for no one would have prayed to God if he knew, or had convinced himself, that God was unalterable….From the metaphysical point of view, however, it is quite unintelligible that God should be anything but completely unalterable. It is irrelevant that the decrees could have been separated from God; indeed, this should not really be asserted. For although God is completely indifferent with respect to all things, he necessarily made the decrees he did, since he necessarily will what was best, even thought it was of his own will that he did what was best."

[165] This calculation lays the groundwork for the vis viva controversy fueled by Leibniz. See Iltis, Carolyn. *Isis.* 1971, Vol. 62, No. 1, "Leibniz and the Vis Viva Contraversy", p. 21; In his essay "Force, Determination, and Impact" (Gaukroger, Steven, John Andrew Schuster and John Sutton, Editors. *Descartes' Natural Philosophy.* Routledge: New York, N.Y., 2000, p. 83), Peter McLaughlin says, "Descartes' concept of force or motion, as developed in various writings and applied to impact in the *Principia*, is most easily grasped if we first look at its extension. The concept of force was supposed to cover at least the following phenomena: 1. the ability of a body in motion to *act* upon other bodies, 2. the work necessary *to raise a load*, or the work performed by lowering the center

velocity of the tennis ball coming off of the racquet is determined by the thinking agent.[166] The velocity in reflection and refraction, however, is determined by the medium. The thinking agent determines the direction of the tennis ball off of the racquet, but the direction in reflection and refraction is determined by the ground and water.

In his attempt to explain the physics of optics using this analogy, Descartes neglects to include the thinking process of the tennis player in his mathematical calculations. Why he does this is an interpretation call in Cartesian studies, because the earliest date that Descartes publically or privately "abolishes" final causes, or "ends", from physics is *Meditations.* It can be argued that at the time he writes *Dioptrics,* he abolished in his mind the Aristotelian notion that the ball itself moves towards the object of reflection because it thinks that it should, or because it loves the object. The question is whether or not at the time of *Dioptrics* he has decided that the

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of gravity of a complex machine, and 3. the causal action (of God or whatever) that is conserved in the world system."

[166] This is teleological. It is metaphysics before physics, final causes before efficient causes. The tennis player forces the ball into motion by the raquet for the purpose of reflection and refraction. In his book *Descartes' Imagination, "Proportion, Images, and the Activity of Thinking"* (University of California Press: Berkeley, Ca., 1996, p. 4) Dennis Sepper associates the tennis player with Descartes imagination and his attempt to illustrate mechanical problems in physics.

In his essay "Descartes Argument From Design" (*The Journal of Philosophy.* 2008, Vol. CV, No. 7), Daniel Dennett says that Descartes possessed a not explicitly laid out design argument. On page 337 he refers to Descartes reply to Caterus in the "First Set of Replies": "Thus if someone possesses in their intellect the idea of a machine of a highly intrinsic design, it is perfectly fair to ask what is the cause of this idea. And it will not be an adequate reply to say that the idea is not anything outside the intellect and hence that it cannot be caused but can merely be conceived. For the precise question being raised is what is the cause of its being conceived. Nor will it suffice to say that the intellect itself is the cause of the idea, in so far as it is the cause of its own operations; for what is at issue is not this, but *the cause of the objective intricacy which is in the idea* (my emphasis)…and what applies to the objective intricacy belonging to this idea also applies to the objective reality belonging to the idea of God. Now admittedly there could be various causes of the intricacy contained in the idea of the machine. Perhaps the cause was a real machine of this design which was seen on some previous occasion, thus producing an idea resembling the original. Or the cause might be an extensive knowledge of mechanics in the intellect of the person concerned, or perhaps a very subtle which enabled him to *invent the idea* (my emphasis) without any previous knowledge. But notice that all the intricacy which is to be found merely objectively in the idea must necessarily be found, either formally or eminently, in its cause, whatever this turns out to be. And the same must apply to the objective reality in the idea of God (AT VII,104)." Dennett interprets Descartes as meaning that one can have a set of blueprints for a machine in hand and be the author of the prints, or one copied the blueprints from the author. He says, p. 337, that Descartes alludes to this idea of machine in the introductory synopsis of *Meditations*. In his book *The Making of Buddhist Modernism* (Oxford University Press: Oxford, U.K., 2008, p.79) David McMahon associates Descartes meaning of a mechanistic universe to be a watchmaker's analogy like Newton.

"ends" towards which the tennis player serves the ball are completely irrelevant in mathematically calculating the reflection and refraction of the tennis ball.

Descartes does admit that the direction of the ball is "determined" by the player. This determination, however, is always in a straight line.[167] In his explanation of reflection, he says that a tennis ball is impelled to move from point *A* to the ground, point *B*, where it reflects off of the ground. He says, "The power that causes the movement of the ball to continue, whatever it may be, is different from that which determines it to go towards one direction rather than another, as it is easy to know from the fact that its movement depends on the force by which it was pushed by the racket, and that this force could just as easily have moved it towards any other direction instead of B."[168]

Descartes says that the movement of the tennis ball has a factor of quantity of motion and of time. Regarding quantity of motion he says,

> Furthermore, it must be noted that the determination to move in one direction can, just as movement, and in general any sort of quantity, be divided into all the parts of which we imagine it is composed, and that we can easily imagine that the motion of the ball which moves from *A* towards *B*, is composed of two others, one causing it to descend from the line *AF* towards the line *CE*, and the other at the same time causing it to go from the left *AC* to the right *FE*, such that these two combined direct it towards *B* along the straight line *AB*. And then, it is easy to understand that the encounter with the ground can only prevent one of these two determinations, and not in any way the other: for it must prevent that which causes the ball to descend from AF towards CE, because it occupies all the space below CE; but why would it hinder the other which causes it to advance towards the right, considering that it is in no way opposed to this direction?[169]

[167] Ross, p. 11; In *PP* #39 Descartes assures us that is impossible for the determination to ever be curved: When a stone is rotated in a sling, whirling around in a circle, at any given instant in its journey the stone is inclined to leave the circle and move along its tangent—·so that (for instance) at the bottom-most point of the circle it is inclined to shoot off straight ahead, parallel with the ground·. The suggestion that it is inclined at each instant to move in a circle is an impossible story: it involves the thought that the stone will be inclined to go on moving in a circle, but at any given instant the fact that it has been moving in a circle is a fact purely about the past; it's absurd to think that that past circular motion is somehow still with the stone, still in the stone, at this instant. And we know from experience that at the instant the stone is released from the sling, it shoots off in a straight line. So there we have it: any body that is moving in a circle constantly tends to move away from the centre of the circle that it is following. See Jonathon Bennett Translation, p. 34

[168] Ross, p. 11

[169] Ross, p. 11

The question of direction in descent leaves him puzzled. He believes that the quantity of motion is conserved, and the mind/body relationship is separate from the physics of motion. As soon as the tennis ball strikes the ground or water, force is removed from his calculations.[170]

By calculating force as purely *mv,* Descartes considers only "purely directional changes in the vector of motion".[171] He introduces determination as more than direction. It "has a quantity and can be divided into parts, but it is also not an independent magnitude since its value is always fixed by the body's quantity of motion, of which it is only a mode."[172] In the deductive core of his physics, located in *Principles of Philosophy,* Book II, Descartes defines the minimal identity conditions in terms of two concepts: "$\sum$ m = constant and $\sum$ *mv* = constant (where m is the quantity of matter and v the speed). The material universe is considered to be the same over time as long as neither its size nor the amount of action or motion going on within it undergoes change."[173] Foundational for this is the immutable God who creates all things including the conservation law. In the tennis ball analogy "determination is used to analyze the trajectory of a tennis ball colliding with the court surface." Descartes' calculation using determination is "(vertical and horizontal) according to the parallelogram rule (vector addition)." When he derives the inverse sign law of refraction "determinations are added vectorially and motions

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[170] Leibniz writes in *Monadology* #80, "Descartes recognized that souls cannot impart a force to bodies because there is always the same quantity of force in matter. However, he thought that the soul could change the direction of bodies. But that is because the law of nature, which also affirms the conservation of the same total direction in matter, was not known at that time. If he had known it, he would have hit upon my system of pre-established harmony." See Ariew and Watkins, p. 292; Leibniz, of course, thinks that Descartes is mistaken about conservation of force. See McLaughlin, Peter. *The Philosophical Review.* 1993, Vol. 102, No. 2, "Descartes on Mind-Body Interaction and the Conservation of Motion", p. 159 McLaughlin says on page 159 that Descartes' calculation of force as *mv* is "wrong".
[171] McLaughlin, p. 159
[172] McLaughlin, p. 160; In FN #11 on page 160,. McLaughlin says "Thus, in spite of what one often reads, it is not the case that "a change can occur in one without any change in the other" (Hatfield, "Force," 118); on the contrary, any change in the quantity of motion entails a change in the (quantity) of determination, though the directional aspect of determination can change without any change in the quantity of motion."
[173] McLaughlin, p. 160, AT 8:61-62; CSM 1:240.

arithmetically. The problems which result from this lead to his discussions with Fermat, Hobbes and Bourdin.[174]

Regarding time, he says, "Therefore to truly find in which direction the ball must rebound, let us describe a circle with center *B* passing through point *A*, and let us say that in as much time as it will take to move from *A* to *B*, it will undoubtedly return from *B* to some point in the circumference of the circle, since all the points which are equally distant from *B*, such as *A*, are found on the circumference, and assuming that the movement of the ball is equally fast."[175] From here he moves to the correct notion of reflection, even though the ball will actually show down upon impact in incidence. He says, "And thus you easily see how reflection occurs, to wit: along an angle always equal to that which is named the angle of incidence; just as if a ray coming from point A falls upon point B on the surface of a flat mirror CBE, is reflected towards F, such that the angle of reflection FBE is neither larger nor smaller than the angle of incidence ABC."[176]

Descartes' tennis ball experiment for refraction using cloth is heuristic, and does not

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[174] McLaughlin, p. 162; In his essay "Physico-Mathematics and the search for causes in Descartes' optics – 1619 – 1637" (*Synthese*, 2012, Vol. 185, No. 3, p. 484), John Schuster says that the laws of reflection and refraction in Descartes' tennis ball experiment should be understood in the light of *Le Monde*. Here Descartes "was concerned with the nature and 'mechanical' properties of microscopic corpuscles and a causal discourse, consisting of a theory of motion and impact, explicated through key concepts of the "force of motion" and directionally exerted "tendencies to motion" or "determinations"." In *Le Monde* the properties of light are: "(1) that it extends circularly in all directions around those bodies one calls luminous; (2) to any distance whatever; (3) instantaneously; (4) and ordinarily in straight lines, which should be taken as rays of light; (5) and that several of these rays coming from different points can collect together at the same point; (6), or, coming from the same point, can put out towards different points; (7) or, coming from different points and going to different points, can pass through the same point without impeding one another; (8)and that they can sometimes impede one another, namely when they're of very unequal force, that of some rays being far greater than that of others; (9) and, finally, that they can be diverted by reflection; (10) or by refraction; (11) and that their force can be increased, (12) or diminished by the different dispositions or qualities of the matter that receives them." See Gaukroger, Steven (Editor). *Descartes, "The World and Other Writings",* Cambridge Press: New York, N.Y., 2004, p. 62 – 63

[175] Ross, p. 11 - 12

[176] Ross, p. 12

consider the true nature of light.[177] Determinism is in play as his ball moves through flimsy cloth which changes the ball's direction, but the movement differs from the determination.[178] The quantity of motion must now be divided and considered separately.[179] As Ross notes in his translation, "So for Descartes, if f_i and f_r were the "forces" of light of incidence and refraction, then the condition that the horizontal determinations were to be the same, would be expressed as $f_i \sin(i) = f_r \sin(r)$, giving $fi/fr= \sin(r)/\sin(i)$. Isn't that convenient?"[180]

Descartes serves his tennis ball into water. Descartes understands that in refraction through a thicker medium, light moves towards the center. In order to do this, however, he needs

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[177] Ribe, p. 43 – 44; Ribe says on page 44, "The practical intent of Descartes's treatment of light is revealed by the explicitly instrumentalist character of his derivation of the law of refraction, which avoids any consideration of light's "true nature." In particular, I suggest that Descartes used a heuristic "tennis ball" model for the derivation precisely because that model was mathematizable, as the natural philosophical model presented in The World was not." Buchdahl says that Descartes does not understand, and realizes that he actually does not understand, the true nature of light. He compares Descartes' *Regulae* 8 and 9 with *Dioptrics* 1 and 2 and says, "Descartes is…perfectly aware that he has no direct evidence for, and hence cannot be certain of, the actual nature of the fundamental physical process involved. He expressly remarks that it may be that when we are trying to develop the theory *deductively*, we are in fact 'unable to determine straightforwardly the nature of the action of light.' In that case we should run over the various possibilities which earlier inspection of the variety of natural powers has already suggested….Descartes…suddenly switches to the remarkable position that a knowledge of one of the other alternatives may 'help us understand it (sc. the action of light), at least by analogy.' The intended analogy must be…that of a body in local motion…, moving with finite velocity. We see, then, that at this point…the deductive line is snapped and original restrictions are relaxed; the reasoning thereafter proceeds in accordance with a series of models. The models of the *Dioptrics* are in fact on Descartes' own admission employed with the utmost abandon and disregard for mutual consistency in their actual physical action, though founded on the basic paradigm of the mechanical laws of matter and motion. We might describe all this as a procedural tightening of the qualitative hypothetical situation, which was initially introduced to begin the movement from the data to the colligating formula." See Eastwood, Bruce. *Isis.* 1984. Vol. 75, No. 3, "Descartes on Refraction: Scientific versus Rhetorical Method", P. 484; Buchdahl, *Metaphysics,* p. 141 – 142

[178] Ross, p. 13

[179] Ross, p. 13

[180] Ross, p. 14; In his essay "Cartesian Optics and the Mastery of Nature" (*Isis*, 1997, Vol. 88, No. 1, Abstract p. 42), Niel Ribe says, "Descartes's *Dioptrics* is more than a mere technical treatise on optics; it is an essay in the "practical philosophy" that he claimed could render us "masters and possessors of nature." Descartes's practical intent is indicated first by the instrumentalist character of his derivation of the sine law of refraction, which is based on a heuristic and readily mathematizable model that requires no consideration of light's "true nature." In his essay "Descartes' Theory of Light and Refraction: A Discourse on Method" (*Transactions of the American Philosophical Society*. 1987, Vol. 77, No. 3, p, 13), Mark Smith says, "Judged by its internal logical structure alone, Descartes' 'proof' of the sine-law in the *Dioptrique* is unquestionably a failure because it is glaringly inconsistent at certain points with the theoretical principles upon which it is supposedly built."

another racket to force the tennis ball that way at the surface of the water. Otherwise, it would behave the way it does through cloth and move away from center. He says,

> But let us make yet another assumption here, and let us consider that the ball, having first been impelled from A towards B (fig. 10), is impelled anew, once at point B, by the racket CBE, which increases the force of its movement by, for example, a third, such that it afterwards traverses the same distance in two moments of time that it earlier traversed in three, which will have the same effect as if it encountered at point B a body of such a nature that it passed through the surface CBE a third more easily than through air. And it obviously follows from what I have already demonstrated that, if we describe the circle AD as before, and the lines AC, HB, FE such that there is a third less distance between FE and HB than between HB and AC, the point I, where the line FE and the circumference AD intersect, will designate the location towards which this ball, being at point B, must be deflected.[181]

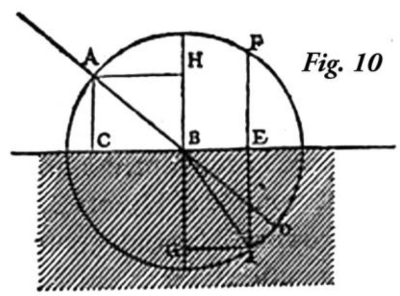

Figure #8 Descartes' *Dioptrics* drawing where the tennis racket strikes the ball anew at line CBE; From *Light*: *A History, The Complete Correspondence of Pierre Fermat,* Jason Ross, Translator, p. 15

Here, the "force" of another tennis racquet redirects the motion of the tennis ball. It does not appear in the text that Descartes considers this to be a "foreign cause".[182] Is a thinking agent, i.e. a tennis player, holding this *CBE* racquet as well? He does not indicate in *Dioptrics.*

The demarcation point between Descartes and Fermat, and the point at which Fermat is interpreted metaphysically, is understanding the tennis ball to act like light in refraction. Descartes says towards the end of section II of *Dioptrics,*

> Finally, inasmuch as the action of light follows in this respect the same laws as the movement of this

[181] Ross, p. 15
[182] See Ross FN 7, p. 1

> ball, it must be said that, when its rays pass obliquely from one transparent body into another, which receives them more or less easily than the first, they are deflected in such a way that they always find themselves less inclined to the surface of these bodies on the side which receives them more easily; and this is in the same proportion as they are more easily received on one side than on the other side.[183]

As is shown in his 1637 letter to Descartes through Mersenne, Fermat's response to this is that, no, light does not behave like a tennis ball. He disagrees on metaphysical grounds, finding geometry to fall short of an adequate explanation for the nature of light.

In the 1637 letter, Fermat refers to Descartes' position on light: "This is nearly his argument: Light is nothing other than the inclination of luminous bodies to move; yet this inclination towards movement must probably follow the same laws as movement itself; and thus, we can establish the effects of light based on the knowledge we can derive from motion."[184] Fermat doubts Descartes' position for several reasons. One, light does not behave like a tennis ball. It is not "pushed by different forces", and "appears not to be affected by succession" like a tennis ball.[185] Two, Descartes' demonstration is not legitimate proof. He argues this lack of proof in points five and six of the letter:

> For example, in the following figure (fig. 54), in which AF is no longer parallel to CB, and where the angle CAF is obtuse, why may we not imagine that the determination of the moving ball from A towards B is composed of two others, one of which moves from line AF towards line CE, and the other advances towards AF?

[183] Ross, p. 16
[184] Ross, p. 20
[185] Ross, p. 20

Fig. 54.

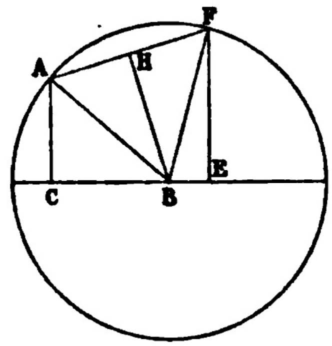

Figure #9 Fermat's Letter to Mersenne 1637; From *Light*: *A History, The Complete Correspondence of Pierre Fermat,* Jason Ross, Translator, p. 21

> For it is true to say that to the extent the ball falls on line AB, it moves towards AF, and that this advance must be measured by perpendiculars drawn from various points which can be taken between A and B along the line AF. And this must be understood when AF makes an acute angle with AB; otherwise, if it were right or obtuse, the ball would not advance towards AF, as it is easy to understand. Assuming this, by the same reasoning as the author, we would conclude that the shiny body CE prevents only the first movement, not being opposed except in direction; such that, giving no hindrance to the second, the perpendicular BH being drawn, and HF made equal to HA, it follows that the ball must reflect towards point F, and thus angle FBE will be greater than ABC.
>
> Thus it is clear that of all the infinite divisions of the determination of motion, the author has only chosen the one which serves to reach his conclusion; therefore he has accommodated his means to his end, and we know as little as we did before. Certainly, it seems that an imaginary division, which could be varied in an infinite number of ways, could never be the cause of a real effect. We can, by a similar reasoning, refute the proof of these foundations of *Dioptrics*, since they are established by a similar train of reasoning.
>
> 6. There you have my thoughts on these new propositions, from which he draws out such beautiful consequences when he treats the shape that lenses must have, that I would wish that the foundations upon which they are established were better proved than they are now; but I fear that they are as lacking of truth as they are lacking of proof.[186]

Three, Descartes' understanding of determination is incorrect.[187] He uses Descartes' Fig. 56 for

[186] Ross, p. 21 – 22

[187] Sabra, p. 153 – 154, says, "Fermat's geometry is metaphysical. He agrees that a ray of light moves in the same direction as long as it is in the same medium. He also agrees with Descartes that upon entering a different medium, a ray of light alters course. This is what Descartes shows.[187] Descartes, however, does not consider the metaphysical notion of light's movement brought about by the change in time in refraction. The difference between Descartes and Fermat is an interpretation of "force" and "determinism". Fermat ignores Descartes' view of movement through different media, and supports a metaphysical assumption that light slows down going from thinner to thicker mediums. For this Clerselier accuses him of forcing "geometry to adapt itself to false opinions. Clerselier attempts to back Descartes example using canvas in *Dioptrics* with his ballistic model inserted in his May 1662 letter to Fermat.

Concerning Clerselier's model, Sabra, p. 134 – 135, says, "Clerselier's model, in spite of its elaborations, only succeeds in making more manifest the difficulties which it was proposed to remove. It in fact fails to satisfy the second of two examples he uses against Descartes idea of determination. He says, "if we imagine that the ball has been directed from point H towards point B, and that it then falls perpendicularly upon the cloth CBE, it is clear that it will traverse it along line BG, and thus its motive force will be weakened and its movement retarded without a change in direction, since it continues its motion along the same line HBG."[188]

Fig. 56.

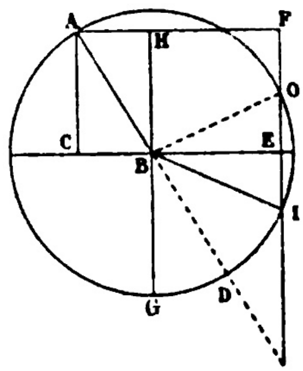

Figure #10 Fermat to Mersenne, 1637; From *Light*: *A History, "Fermat's Complete Correspondence on Light",* Jason Ross, Translator, p. 28

In this diagram Fermat explains the incorrect argument that determinism is something "which remains constant when the ball continues to move in the same line even though its speed has changed. That is, determination is identical with direction."[189] Fermat disagrees with this.

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the main condition which it was required to fulfil: namely, the conservation of the horizontal speed. For although the condition is preserved at the moment of impact, if the further resistance of the medium subsequently changes the actual speed without altering the direction (as is supposed by Clerselier), both the perpendicular and the horizontal components of the motion will be diminished in the same proportion.   Moreover, the model fails to explain, even in a general way, how the velocity can be increased by refraction. To suppose that the ball *A* yields more easily to the impinging ball does not mean that the latter will thereby acquire new momentum. And to say, as Clerselier did, that the incident sphere can be supposed to be 'dragged' into the refracting medium would be to introduce something like an attractive force which would seem completely out of place in this mechanical picture.   (It is interesting to note, however, that Clerselier was led to introduce an idea which Newton later used in his explanation of refraction.) Cleselier's model thus raises more problems than it solves, and it shows that no solution of the fundamental physical difficulties was forthcoming along the lines suggested by him and earlier by Descartes."

[188] Ross, p. 28

[189] Sabra, p. 118

Descartes says that when the ball goes through the canvas perpendicularly, its changes speeds.[190]

Fermat's argument runs as follows:

> what does that mean, if the distinction is precisely between the determination and 'the moving force or speed of the motion? If the distinction is to be maintained, 'one cannot say that, since the movement of the ball has been weakened, the determination which makes it go from above downwards has changed'. Again, if the determination is to be distinguished from the speed, how could Descartes deduce from his assumption about the conversation (after refraction) of the horizontal 'determination' that the speed of the ball will be the same in that direction as before? Understood in this manner, it is obvious that Descartes' argument would lead to absurdities.[191]

After doing battle with Descartes and the Cartesians, Fermat discovers, much to his chagrin, that his sine law of refraction is identical with Descartes'! He therefore makes peace with the Cartesians. The Cartesian publisher Clerselier, however, recognizes the superiority of Fermat's argument, and responds in a letter to Fermat conceding to Fermat's combination of geometry and metaphysics. Clerselier outlines in eight points the Cartesian position, just so that Fermat and the world would not forget what the master thought.

In a May 6,' 1662 letter, Clerselier concedes to Fermat's position on refraction, saying that for those who could not understand the mathematics, Fermat explained it metaphysically, and for those who could not understand it metaphysically, he explained in geometrically.[192] Although he does not envy his discovery, Clerselier promises Fermat "to publish everywhere and confess openly that I have seen nothing as ingenious nor as well conceived as the demonstration that you have brought forward."[193]

In point number one, Clerselier says that Fermat's principle is a moral principle, not a physical principle. Consequently, he argues, it is not, and cannot be, "the cause of any effect of nature." The Cartesian complaint is that Fermat's principle insinuates that light decides

[190] Sabra, p. 118
[191] Sabra, p. 118
[192] Ross, p. 111
[193] Ross, p. 111

something during the refraction process. Two, the publisher says that it puts it in an "unresolved state." This runs counter to the Cartesian natural philosophy that the laws of nature operated according to the immutable nature of God, which is unchanging. Cleserlier asked the question that Maupertuis asks in *Accord* in a slightly different way.[194] Maupertuis asks why it is that light should favor time over space, or visa versa. Clerselier puts it another way:

> Because, I ask you, if it is true that nature must always act by the shortest and simplest pathways, and since the straight line is undoubtedly both the shortest and the simplest of all, then when a ray of light has to travel from a point in a rare medium to a point in a dense medium, is it not the case that nature must hesitate? For if you wish her to act by the principle of following a straight line immediately after the break, then isn't your path the shortest in time, while the straight line is shorter and simpler in measure? Who will decide, then, and who will pronounce himself on this matter?[195]

Third, Fermat seems to be giving time causative power, and the Cartesians do not believe that

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[194] Sabra, p. 156 - 158, says, "Supposing, however, that the principle of economy was generally true, there was still no *a priori* reason why time, rather any other quantity, should be the minimum. As it happened, however, Huygens later showed in his *Traite de la Lumiere* (1690) that the principle of least time was deducible from his own law if secondary waves. Fermat's principle thus received a physical foundation by being shown to follow from wave considerations. Before the appearance of Huygen's *Traite*, Newton had already published in *Principia* a dynamical explanation of refraction which required the velocity of light to be greater in denser media – in contradiction to Huygens' theory and Fermat's principle. In the eighteenth century Newton's explanation was accepted as part of a corpuscular theory of light which had completely overshadowed Huygens' theory; and Fermat's principle was regarded as a detached metaphysical principle to be examined on its own merits, or, rather, demerits. Final causes had not then completely disappeared from physical considerations; but the adherents of Newton's theory could point to an arbitrary character of Fermat's principle: if light could not at once travel in the shortest time by the shortest path, why should it choose one of these courses rather than the other?- that was Maupertuis' argument in 1744. Instead of the shortness of either space or time, he proposed that the path really followed by light is that for which the quantity of action is minimum. Thus Maupertuis accepted the principle of economy, but he offered a new interpretation. By action he understood a quantity proportional to the sum of the distances covered, each multiplied by the velocity with which it is traversed; and he showed that this principle of least action (as called it) leads to the Newtonian law of refraction giving the sines of incidence and of refraction in the inverse ratio of the velocities.

Maupertuis' procedure was no less metaphysical than Fermat's. In a sense it was even more arbitrary. For whereas Fermat could have at least argued that time was a natural and basic concept whose intimate relation to motion should recommend it as the minimum quantity, Maupertuis' 'action' was an artificially constructed quantity introduced to fit the already formulated Newtonian law. Similarly to what had happened with Fermat's principle, however, Euler showed (in 1744) that the principle of least action was applicable in several important cases of particle dynamics, as, for example, when a body moves under the action of a central force. And later, LaGrange extended Euler's applications of Maupertuis' principle (as a law of extremum) which he regarded, not as a metaphysical explanation, but as a 'simple and general result of the laws of mechanics'.

Thus, as far as optical refraction is concerned, the incompatibility of Fermat's principle with that of Maupertuis' became a conflict, not between two interpretations of a metaphysical view of nature, but between the wave and the dynamical (or corpuscular) theories of light in which these two principles were respectively verified. Until 1850 there had been no experiment to decide between the wave and the corpuscular theories of refraction. When, in that year, Foucault performed his famous experiment, he found that the velocity of light was greater in air than in water – in agreement with Huygens' explanation and against the Newtonian view.

[195] Ross, p. 112

time determines any motion.[196] Time, the Cartesians believe, is not involved in efficient causation of body to body impact, and the aspects of force and determination associated with such. Fourth, the Cartesians believe that a straight line is the only line that can be determined, whereas Fermat thinks that the bent line in refraction is determined. Fifth, Fermat believes that bodies move easier through thinner mediums than through thicker, and the Cartesians believe that not only do bodies behave opposite, they avoid the thicker medium where possible. Cleserlier claims experimental knowledge! He says,

> This is what experiment confirms: when a ball is impelled obliquely from air into water, far from continuing its movement along a straight line, and farther still from diving in still more to approach the perpendicular, it actually moves away from it as much as it can, approaching the surface [instead of the perpendicular]. Surely you have recognized the force of this objection, although you call it weak, but you would be incapable of resolving it without the principle of M. Descartes, which completely destroys your own [principle]. For if, with your very principle, the ball must move away from the perpendicular, why would light move towards it? And if the ball does not follow your principle, as indeed it does not, why would light follow it? Does this not rather show, by these examples, that nature does not follow your principle?[197]

Sixth, Cartesians believe that light has no "intention", therefore the quickest path cannot be determined by it. They believe that Fermat gives intentionality to light. Seventh, Clerselier repeats himself regarding time not being a factor is light determining its path. Eight, *Dioptrics* seemed to win because Descartes was making a proof "that light moves more easily through dense bodies than through rare ones" whereas Fermat was assuming "that light moves more easily through rare bodies than through dense ones."[198]

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[196] Ross, p. 112
[197] Ross, p. 115
[198] Ross, p. 116; If one reads beyond the eight points in Clerselier's letter, one finds where Clerselier trys to help Descartes' frail canvas model with a substitute ballistic model for thoughts concerning horizontal speed; see Ross, p. 124; Concerning this failed Clerselier model, Sabra says on page 134 – 135, "Clerselier's model, in spite of its elaborations, only succeeds in making nmore manifest the difficulties which it was proposed to remove. It in fact fails to satisfy the main condition which it was required to fulfil: namely, the conservation of the horizontal speed. For although the condition is preserved at the moment of impact, if the further resistance of the medium subsequently changes the actual speed without altering the direction (as is supposed by Clerselier), both the perpendicular and the horizontal components of the motion will be diminished in the same proportion. Moreover, the model fails to explain, even in a general way, how the velocity can be increased by refraction. To suppose that the ball *A* yields more easily to the impinging ball does not mean that the latter will thereby acquire new momentum. And to say, as Clerselier did, that the incident sphere can be supposed to be 'dragged' into the refracting medium

Clerselier ends this letter with hopes that Fermat should win over Descartes: "And likewise, since this geometric proof was the most difficult to find and to fathom, I would very much like you to win over him, and from this moment I sign and subscribe to an eternal peace with you, and I no longer wish to raise this question of the ineffectiveness of your principle or of the difference between your principle and his, when they lead to the same thing and teach us the same truth."[199] Fermat, knowing that he has won Clerselier over, responds humbly on May 21, 1662 by saying,

> Sir,
>
> I received your two letters of the sixth and the thirteenth of May at the same time. They did me more honor than I could reasonably have expected, and far from being shocked by your Latin words, I am convinced that in the assumption of your thoughts on the subject of the demonstration of M. Descartes, there has been nothing more truthful anywhere else in your letters.
>
> For if this demonstration is made according to the rules of certain and infallible demonstrations, than nothing is more true than to say that those who are not convinced by it do not understand it. The essential quality of a demonstration is to force belief, such that those who do not feel this force, do not feel the demonstration itself, which is to say that they do not understand it.
>
> Therefore, sir, I attribute this mollification that the Gentlemen of your assembly have inspired In you, only to an excess of courtesy and civility, and I give you my very humble thanks.
>
> For the main question, it seems to me that I have often told both you and M. de La Chambre that I neither claim nor have ever claimed to be in the secret confidence of nature. She has obscure and hidden ways that I have never tried to penetrate. I have only offered her a small help with geometry on the subject of refraction, if she were ever to have need of it. But since you assure me, sir, that she can manage her affairs without this and she is happy enough with the movement that M. Descartes has dictated to her, I give up to you with no ill-feelings my supposed conquest of physics, and it is enough for me that you leave me in possession of my geometry problem, completely pure, and *in abstracto*, by means of which one can find the path of a moving body which passes through two different media and which tries to complete its movement as soon as it can."[200]

The geometry which so impressed Clerselier was developed by Fermat in those years in which he strived to prove his principle to de La Chambre. In 1657, de La Chambre had

would be to introduce something like an attractive force which would seem completely out of place in this mechanical picture. (It is interesting to note, however, that Clerselier was led to introduce an idea which Newton later used in his explanation of refraction.) Cleselier's model thus raises more problems than it solves, and it shows that no solution of the fundamental physical difficulties was forthcoming along the lines suggested by him and earlier by Descartes."

[199] Ross, p. 118

[200] Ross, p. 128

presented Fermat with a problem which took Fermat over four years to solve,[201] and prepared him for his correspondence with Clerselier. The answer involved time in refraction and was metaphysical. The mathematics for least time in optics of refraction did not work for Fermat until he assumed least time over shortest distance.[202] Then the mathematical proof presented itself, and he overcame Descartes' law of refraction.[203]

de La Chambre sent Fermat Book II, Chapter VI ("On the Reflection of Light"), of his work on light *La Lumiere a Monseignevr Cardinal Mazarin.* In Article 4, de La Chambre acknowledges the truth in optics that the angle of incidence is equal to the angle of reflection. He then acknowledges that if lines are drawn past the line of incidence, the angles made on both sides of this line are equal, and that because nature always does that which is shortest. Then equal opposite angles are a consequence of the shortest distance taken by rays of light in incidence and reflection.

---

[201] Sabra, p. 140

[202] Sabra, p. 136, says, "To discover the true law of refraction, Fermat appealed to his own method of maxima and minima which he had invented about eight years before the beginning of the controversy with Descartes in 1637. In order to being his method to bear on the problem under investigation, however, he relied on the metaphysical principle that nature performed its actions in the simplest and most economical ways. His law of least time was for him a justifiable specification of this principle. Thus, in the hands of Fermat, geometry and metaphysics joined forces in an attempt to defeat Descartes and his followers by going beyond mere opposition to their arguments and actually producing the true law of refraction." On page 155 Sabra says, "It is very doubtful that he would have embarked on the difficult task of applying his method of maxima and minima to refraction, had he not been convinced that the general principle of economy, which inspired this application, was a true law of nature. Indeed, he was not convinced of the truth of the refraction law itself, even after it had been confirmed by experiments, until he had derived it from his principle of least time."

[203] Sabra, p. 143ff, records the complete mathematical proof.

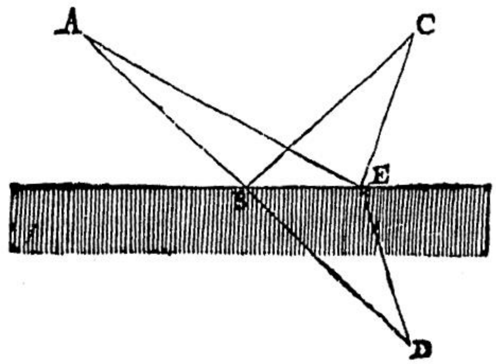

Figure #11 Found in *Light* by Marin Cureau de la Chambre, 1657; From *Light*: *A History, "Fermat's Complete Correspondence on Light",* Jason Ross, Translator p. 44

de La Chambre writes concerning this illustration,

> By extending AB to D, making an extension of the same length as the reflected line BC, such that ABD be equal to ABC; and also extending line AE to D, extension ED having length EC, such that AED be equal to AEC, it is evident that ABD is shorter than AED, because together they make a triangle with ABD as its base and AE, ED the two legs; and in any such triangle two legs are longer than the third. We see therefore by all this reasoning that the equality of angles in reflection is made along the shortest lines,1 and that this is not something particular to Light, since nature observes the same order in all the movements that she causes.[204]

Reflection off of plane surfaces, or convex surfaces, is not the problem which vexes thinkers according to de La Chambre. It is what happens in concave surfaces. Angles can be equal with unequal line lengths! de La Chambre illustrates with the following drawing:

[204] Ross, p. 44; Ross notes that the sentence "We see therefore by all this reasoning that the equality of angles in reflection is made along the shortest lines" is essentially the demonstration made by Heron of Alexandria in the first century. In his essay "Aristotelian References to the Law of Reflection" (*Isis*, 1946, Vol. 36, No. 2, p. 94 - 95) Carl Boyer argues that the Aristotelian school had knowledge of the equality of angles in the case of plane mirrors. Boyer credits Euclid with stating the formula itself: "The earliest explicit statement of the law appears to be found in the Optics ascribed to EUCLID, composed probably near the end of ARISTOTLE'S life. Here (Prop. XIX) in finding the height AB of a tree, a mirror is so placed on the ground at C that the top of the tree B is visible to the eye E of an observer standing on the ground at D in the plane of ABC. Knowing the lengths AC, CD, and DE, the height AB is found from the proportion AB:AC=ED:CD'."

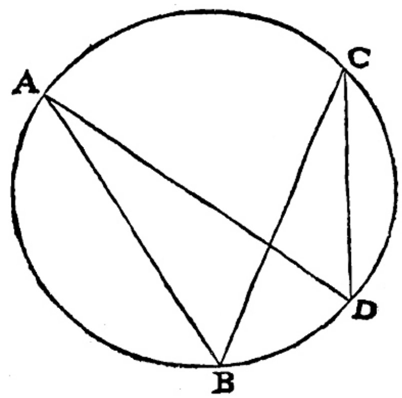

Figure #12 Found in *Light* by Marin Cureau de la Chambre, 1657; From *Light*: *A History, "Fermat's Complete Correspondence on Light",* Jason Ross, Translator p. 45

de La Chambre uses this drawing to talk about the opposition to the natural certainty of shortest distance and equal angles. He makes two points saying,

> But there may be opposition to this: first, because there are reflections which are made in concave mirrors, where the angles are equal, although the lines are longer than if they were unequal. For lines AB, BC which form equal angles are longer than AD, DC which make unequal angles, as can be demonstrated with Geometry. Therefore it is not a general rule that the equality of angles comes from the shortest lines.
>
> Second, if nature makes its movements by the shortest lines, it would be necessary that they be made in refraction as well. However, the lines which contain the angles of incidence and refraction, are longer than the line drawn from one of their extremities to the other. For lines AB, BC made by refraction, are longer than AD, DC, since AB, BC form the two legs of a triangle, whose base is AD, DC. And two legs are longer than the third side considered by itself.[205]

de la Chambre concludes these two points with a laymen's finding of maxima and minima. "Finally, if all the rays have determined extensions, as we have shown, whether they go straight or be reflected, they have always the same length, and consequently the lines which compose them can be neither shorter nor longer for any other motion that they undergo."[206]

de La Chambre next asks the question which many others were asking in the Early Modern period. "How shall we respond then to the preceding objections, which show that light does not always move by the shortest lines?"[207] Fermat sends de La Chambre a "thank you note"

[205] Ross, p. 45
[206] Ross, p. 46
[207] Ross, p. 49

for the letter and refers to his figure #87 on page 315 and connects his principle with the figure.

Fig. 87.

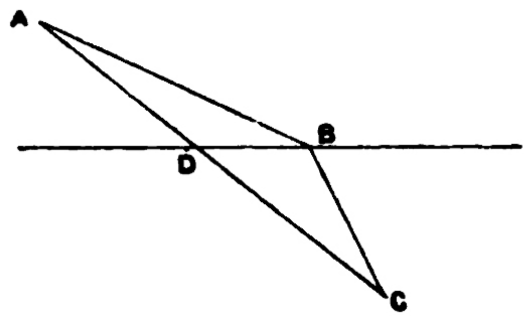

Figure #13 Fermat to de La Chambre, 1657; From *Light*: *A History, "Fermat's Complete Correspondence on Light",* Jason Ross, Translator, p. 53

Fermat suggests trying a different idea to solve the problem posed by de La Chambre. Instead of shortest path, Fermat turns to easiest course.[208] This would aid in understanding light as it passed from one medium to another. The question is, where along DB is the path a minimum.

Fermat calculates as follows:

> If we suppose that point B has been found, and that nature acts always along the shortest and easiest path, the resistance along CB, added to the resistance along BA, will contain the sum of the two resistances, and this sum, in compliance with the principle, must be the least of all those sums which could be created at any point along line DB. So, in this case, these two combined resistances are represented, as we have proven: either by CB plus half BA, or by the same CB plus twice BA.[209]

By employing this thinking, Fermat says that the problem reduces to geometry:

> Given two points C and A and the line DB, to find a point on line DB for which, if you extend lines CB and BA, the sum of CB with half BA is the least of all similarly taken sums, or rather that the sum

[208] Fermat makes this determination without a certainty of the speed of light. In 1676 Ole Roemer makes the first determination of the speed of light. On page four of their book *Patterns of Light*, "*Chasing the Spectrum from Aristotle to LEDs* (Springer: New York, N.Y., 2008), Steven Beeson and James Mayer say, "Ole Roemer, a Danish Astronomer, wanted to use the light of Jupiter's moons as they disappeared and reappeared behind the giant planet to determine how far the Earth had spun in a given time (and thus measure one's change in longitude in the same time). Instead, he ended up measuring the speed of light as it traveled across the solar system, and he used only a telescope and a clock. He observed the moons of Jupiter as they emerged from the far side of the gaseous planet, measuring their periods, or the time it takes to make one revolution. He noticed that if the earth was on one side of its orbit, traveling toward Jupiter, the period of a certain moon would be eleven minutes faster than average. And if the earth was on the other side of its orbit (six months later), traveling away from Jupiter, the period of the Jovian moon was eleven minutes slower. Roemer knew that, if light traveled infinitely fast, there should be no discrepancy between the sightings of the moon on either side of the earth's orbit. Parting from popular opinion, Roemer was the first to show that light had a finite (but truly great) speed."

[209] Ross, p. 54

of CB and double BA be the least of all similarly taken sums; and the point B which will be found by the construction of this problem will be the point where the refraction will take place.[210]

This statement serves as Fermat's first statement of his principle of refraction.[211]

On January 1, 1662, Fermat sent de La Chambre his geometrical demonstration of least time in optical physics. This is in response to line length and shortest distance problems in reflection off of concave surfaces and refraction through differing mediums. Fermat says,

> Let there be, in the separate figure (fig. 100), the circle ACBG, with diameter AOB, center O, and another diameter GOC. From the points G and C let there be drawn perpendiculars GH, CD onto the first diameter. Let us suppose that the first diameter AOB separates the two different media, of which the lower of the two, AGB, is denser, and the upper, ACB, rarer. Let it be rarer to such a degree that, for example, the passage through the rarer medium be easier than passage through the denser by a ratio of two to one.

Fig. 100.

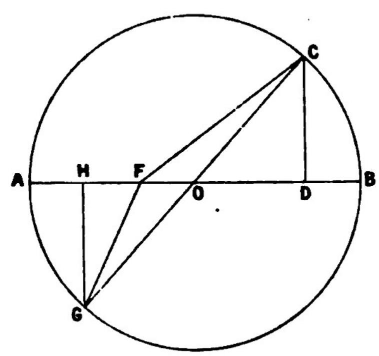

Figure #14 Fermat to de La Chambre, 1662; From *Light*: *A History, "Fermat's Complete Correspondence on Light",* Jason Ross, Translator, p. 106

> From this assumption, it follows that the time expended by the body in motion, or by light, to go from C to O is less than that taken to go from O to G, and that the time of movement from C to O, made in the rarer medium, is only half the time of the movement from O to G. Consequently, the measure of the entire movement of the two lines CO and OG can be represented by the sum of half of CO and the entirety of OG. Similarly, if you take another point, such as F, the time of motion along the two lines CF and FG can be represented by the sum of half of CF with the entirety of FG. Now, assume that the radius CO be 10, and consequently that the total diameter COG will be 20, that line HO be 8, and OD also be 8, and finally that line OF be only 1. I say that in this case the movement along line COG will be made in a longer time than the motion along the two sides of the triangle CF, FG. For if we prove that half CO plus OG is greater than half CF plus FG, the conclusion will be clear, since these two sums are the precise measures of the times

[210] Ross, p. 54 – 55
[211] Ross, p. 54, FN #4

of these two movements. Now, the sum of half CO with OG is exactly 15, and it is obvious by the construction that the line CF is equal to the square root of 117 and that FG is equal to the square root of 85. But half of the first root added to the second, is less than 59/4 , and 59/4 is less than 15. Thus, the sum of half CF with FG is less than the sum of half CO with OG, and therefore the movement along the lines CF, FG is made more quickly and in less time than the movement along the base COG.
4. I arrived at this point without much difficulty, but it was necessary to take the study further, and because, to satisfy my principle, it is not sufficient to have found a point such as F, through which the natural movement is made more quickly, easily, and in less time than along COG: but rather it is still necessary to find the point which allows the journey to be made in less time than any other point. For this, it was necessary to have recourse to my method *of maxima and minima*, which tackles this sort of question with fair success.[212]

Fermat had possessed an understanding of maxima and minima before *Dioptrics.* In his pre-1638 work *Methodus ad Disquirendam Maximam et Minimam*, Fermat explains how he calculates the point which allows the journey in less time than any other point. He starts by expressing " the maximum or minimum quantity in terms of *a* by means of terms of any degree." He says,

We will then substitute *a+e* for the primitive unknown *a*, and express the maximum or minimum quantity in terms containing *a* and *e* to any degree. We will *ad-equate,* to speak like Diophantus, the two expressions of the maximum and minimum quantity, and we will remove from them the terms common to both sides. Having done this, it will be found that on both sides, all the terms will involve *e* or a power of *e*. We will divide all the terms by *e*, or by a higher power of *e*, such that on at least one of the sides, *e* will disappear entirely. We will then eliminate all the terms where *e* (or one of its powers) still exists, and we will consider the others equal, or if nothing remains on one of the sides, we will equate the added terms with the subtracted terms, which comes to be the same. Solving this last equation will give the value of *a*, which will lead to the maximum or the minimum, in the original expression. [213]

Fermat gives an example of what he means.

Divide the line AC (fig. 91) at E, such that AE×EC be a maximum.

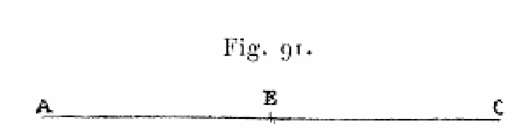

Figure #15 *Method for the Study of Maxima and Minima* by Pierre Fermat; Translated from *Oeuvres de Fermat,* Vol. 3, by Jason Ross, first page

Let us take AC = b; let a be one of the segments, and let the other be b – a, and the product whose maximum we have to find is: ba–a2. Now let a+e be the first segment of b, the second b–a–e, and

[212] Ross, p. 106 – 107
[213] Ross, English translation of *Method for the Study of Maxima and Minima* in *Oeuvres de Fermat* Vol. 3, pp. 121 – 156, science.larouchepac.com/fermat; Light: A History,

> the product of the two segments will be: ba–a2+be–2ae–e2. It must be co-equal to the preceding: ba–a2; Removing the common terms: be ~ 2ae + e2; Dividing all the terms: b ~ 2a + e; Remove e: b = 2a. To solve the problem, therefore, the half of b must be taken. It is impossible to give a more general method.[214]

In *Maxima et Minima* Fermat purposefully chooses against shortest distance:

> Our demonstration rests solely upon the postulate that nature operates by the easiest and most convenient means and pathways. For we believe it must be stated this way, and not in the ordinary way, which says that nature always operates by the shortest lines. Indeed, in addition to speculating on the natural movements of heavenly bodies, Galileo measured their relationships in time as well as in space, similarly, we will not consider the shortest spaces or lines, but rather those pathways which can be most easily traveled through with the greatest of ease, in the most accommodating way, and in the least time.[215]

Fermat gave a copy of his method to Descartes through Mersenne "who received it around the 10th of January, 1638".[216] Descartes rejected his findings in an attempt to defend *Geometrie.* Fermat's "variational principle" is an application of final causes to physics.[217] "The *Principle of Least Time* relates the length and orientation of a light ray to the time required for light to propagate along the ray path. All the facts of ray optics are a consequence of this principle."[218]

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[214] Ross, English translation of *Method for the Study of Maxima and Minima* in *Oeuvres de Fermat* Vol. 3, pp. 121 – 156, science.larouchepac.com/fermat; Light: A History,
[215] Section IX "Synthesis for Refractions"
[216] Ross, English translation of *Method for the Study of Maxima and Minima* in *Oeuvres de Fermat* Vol. 3, pp. 121 – 156, science.larouchepac.com/fermat; Light: A History, FN #1
[217] Lemons, Don. *Perfect Form, "Variational Principles, Methods, and Applications in Elementary Physics."*. Princeton University Press: Princeton, N.J., 1997, p. x
[218] Lemons, p. 4 says that Fermat's principle can easily derived from Snell's law.

Chapter Four: Leibniz's Position

Pierre Maupertuis accuses Gottfried Wilhelm Leibniz of being erroneous in his application of final causes to physics. Leibniz applies final causes to physics in order to reconcile the positions of Fermat and Descartes.[219] Additionally, he applies final causes to physics because he believes in the rightness of applying metaphysics to physics. His metaphysics is an application "from the material to the formal".[220] His application, finally, is in the spirit of reconciling the "mechanist" Aristotle with the moderns.[221] In *Two Sects of Naturalists,*

> "Leibniz explicitly divides his contemporaries into two groups of philosophers distinguished by their attitudes toward materialism but united in their rejection of final causes. In opposing both groups, he counsels a return to "the sect of Socrates and Plato" as being "so much more suitable for piety," and quotes approvingly, and at length, the famous passage from the *Phaedo* in which, as Leibniz describes it, Socrates "maintains that final causes are the principles in physics and that we must seek them in order to account for things."[222]

Leibniz's application of final causes to physics is located doctrinally in two places, *Specimen Dynamicum* (1695) and *Tentamen Anagogicum* (1696). These statements are properly understood within the works *How the Soul Acts in the Body* (1677/78), *Unicum Opticae, Catoptricae et Dioptricae Principium* (*Acta Eruditorum,* Leipzig, 1682), *Elementa Physicae,*

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[219] Reconciling the good things about everyone was at the heart of Leibniz the man and thinker. To Redmond he writes, "Besides always taking care to direct my study toward edification, I have tried to uncover and unite the truth buried and scattered under the opinions of all the different philosophical sects, and I believe I have added something of my own which takes a few steps forward." See Loemker, p. 654

[220] In his January 10, 1714 letter to Redmond, Leibniz says that his conversations with Huygens in Paris and his inquiry into the ultimate causes of mechanism caused him to turn back to metaphysics. See Loemker, p. 654; In his essay "Leibniz on Form and Matter" (*Early Science and Medicine* , 1997, Vol. 2, No. 3, p. 328), Daniel Garber specifies this turning point being when Leibniz discovers *vis viva* in 1678.

[221] See Leibniz's 1714 letter to Redmond.

[222] McDonough, *Midwest Studies in Philosophy.* 2011, Vol. 35, "The Heyday of Teleology in Early Modern Philosophy", p. 194

Discourse on Metaphysics (1686), *Specimen Dynamicum* (1695), *New System* (1695), *Tentamen Anagogicum* (1696) and *Monadology* (1714). In *Specimen Dynamicum* Leibniz states his "Doctrine of Interpenetration of Causation":

> In general we must hold that everything in the world can be explained in two ways: through the *kingdom of power,* that is through efficient causes, and through the *kingdom of wisdom,* that is through final causes, through God governing bodies for his glory, like an architect, governing them as machines that follow the laws of *size* or mathematics, governing them, indeed for the use of souls, and through God governing for his glory souls capable of wisdom, governing them as his fellow citizens, members with him of a certain society, governing them like a prince, indeed like a father, through *laws of goodness* or *moral laws.*[223]

In *Tentamen Anagogicum* Leibniz states his "Most Determined Path" principle:

> What is more, our thinking sometimes furnishes us with considerations revealing the value of final causes, not merely in increasing our admiration for the supreme author, but also in making discoveries among his works. Someday I shall show this in a special case in which I shall propose as a general principle of optics that a ray of light moves from one place to another by the path which is found to and which holds that in the absence of a minimum it is necessary to hold to the *most determined*, which can be the simplest, even when it is a maximum.[224]

In regards to optics and the movement of a ray of light, bodies move according to physical laws and the will of God towards the good. Leibniz believes that rational agents act towards ends which they believe to be good. Will, Leibniz believes, "consists in the inclination to do something in proportion to the good it contains."[225] Where this concerns God, Leibniz says that God has an antecedent will and a consequent will. They both work toward the best possible outcome. In *Theodicy*, Leibniz says

> This will is called antecedent when it is detached, and considers each good separately in the capacity of a good. In this sense it may be said that God tends to all good, as good, . . . Now this consequent will, final and decisive, results from the conflict of all the antecedent wills, of those which tend towards good . . .

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[223] Ariew, Roger and Daniel Garber. *G.W. Leibniz, "Philosophical Essays".* Hackett: Indianapolis, In., 1989, p. 125; In his essay "Leibniz's Two Realms Revisited, p. 692, Jeffrey McDonough makes three points regarding this doctrine: One, Leibniz's doctrine can be traced to his desire to reconcile Descartes and Fermat. Two, "that that doctrine can be found to rest on surprisingly sober foundations provided that Leibnizian laws are viewed as abstractions from the intrinsic, well-founded, derivative forces attributable to bodies", and three, "a proper understanding of Leibniz's position with respect to explanatory over determination within the domain of nature might provide insights into his understanding of monadic teleology and the supposed pre-established harmony between mind and body."
[224] Loemker, p. 479
[225] McDonough, *Heyday,* p. 194

> and from the concurrence of all these particular wills comes the total will. Thence it follows that God wills antecedently the good and consequently the best. (T 22–23)[226]

That which is best in the case of optics and the movement of a ray of light is found in the most determined path, a geometrical calculation which is based on a metaphysical foundation. This is an attempt at reconciliation between Descartes and Fermat, efficient and final causes, Aristotle and the moderns.

In a short note dated by the Akademie editors as between 1677 – 1678, Leibniz says,

> For the general laws of mechanics are decrees of divine will, and the special laws of mechanics in each body (which follows from the general laws) are decrees of its soul or form, striving towards its own good or perfection. Therefore God is that mind which leads everything to general perfection. The soul however is that sentient force which in everything tends to special perfection. Souls have arisen, however, while God impresses on all a conatus to special perfection so that from this conflict there should arise the greatest possible perfection. Everything in the whole of nature can be demonstrated both through final causes and through efficient causes. Nature does nothing in vain, nature acts through the briefest (*brevissimas*) paths provided that they are regular paths. Hence the briefest paths are to be sought not in reflecting surfaces themselves but along tangents. But this in passing. Souls do not act on bodies outside of order. Nor God in nature, even if things appear to happen outside of order; from the beginning, things have been so arranged that the general order is involved in some way in extraordinary appearance.[227]

Leibniz demonstrates this position geometrically in his 1682 *Acta Eruditorum* article "*Unicum Opticae, Catoptricae et Dioptricae Principium.* Leibniz opens his position with a statement which is metaphysically similar to Fermat: "The following can be set down as the first principle common to these sciences, from which the direction of any ray of light is determined geometrically: *Light radiating from a point reaches an illuminated point by the easiest path, which is to be determined first with respect to planar surfaces, but is accommodated to concave*

[226] McDonough, *Heyday*, p. 194, McDonough cites G. W. Leibniz. *Essais de Theodicée*. Cited by section number as in GP 6:21–471. English translation in G. W. Leibniz. *Theodicy*. Trans. E. M. Huggard. New Haven: Yale University Press, 1952. Reprint, La Salle, IL: Open Court, 1985 (p. 201)
[227] McDonough translation from Home Page; McDonough notes, "This short note by Leibniz has been dated by the Akademie Editors to the beginning of 1677 to the beginning of 1678. The Latin text appears as *Anima Quomodo Agat in Corpus*, 257 in G.W. Leibniz, *Sämtliche Schriften und Briefe* (Berlin: Akademie Verlag 1923-) volume VI.iv.B.

and convex surfaces by considering their tangent planes."[228] He illustrates his position with this drawing:

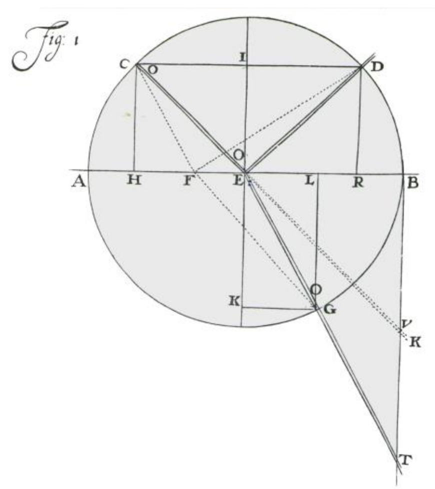

Figure #16 Leibniz's geometrical drawing of the easiest path taken by a ray of light for the purpose of beauty; From Lamborn, Richard. "Commentary on Gottfried Wilhelm Leibniz's Discourse on Metaphysics #19," (Master's Thesis, University of South Florida, 2012) p. 65

In this drawing line segment CE is teleological! It operates for an end purpose according to final causes! Leibniz argues that "If the lines *m* and *n* represent resistance with respect to light – the former of air, the latter of water – the difficulty of the path from C to E will be as the rectangle formed by CE and m; from E to G as the rectangle formed by EG and *n.* Therefore so that the difficulty of the path CEG is the least of all, the sum of the rectangles CE by m and EG by *n* should be the least possible, or less than CF by m and FG by n – where F is taken to be any point whatsoever except E. E is sought."[229] He then affirms the adequacy of his calculations which have final cause as their base:

> We have therefore reduced all the laws of rays confirmed by experience to pure geometry and calculation by applying one principle, taken from final causes if your consider the matter correctly: Indeed a ray setting out from C neither considers how it could most easily reach point E or D or G, nor is it directed through itself to these, but the Creator of things created light so that from its nature that most beautiful event might arise. And so those who reject final causes in physics

[228] McDonough Translation; McDonough notes "The Latin text of this essay was first published by Leibniz in the *Acta Eruditorum*, June 1682. Reprinted in reprinted in L. Dutens (ed.), *Opera Omnia* (Geneva: Fratres De Tournes, 1768), volume III, pages 145-150.
[229] Lamborn, p. 65

with Descartes err greatly – not to speak more harshly – since even besides the admiration of divine wisdom, they would also supply to us the most beautiful principle for discovering some properties of those things whose interior nature is still not so clearly known to us that we would be able to use proximate efficient causes and explain the machines which the Creator employed in order to produce those effects and in order to obtain his ends."[230]

Regarding the issue of the speed of light in thinner and thicker mediums and the Fermat – Descartes over refraction, Leibniz says that light moves faster in water and glass than in air, but he does not agree with Descartes calculations. To calculate for refraction, he says,

> In *Dioptrics*, the complimenting sines, EH and EL of the angles of incidence CEA and of refraction GEB always preserve the same ratio, which is reciprocal to the resistance of the media. Let IE be air, EK water, or glass, or some other medium denser than air, C a radiating point in the air, G an illuminated point beneath the water; the question is, by which path does the light shine from the former to the later, or what is the point E on the surface of the water AB such that the ray emitted from C is to be refracted sending it to G? This (point) E should be taken such that the path is the easiest of all. Now, in different media, the difficulties of the path are in the ratio composed of the lengths of the paths and the resistances of the media. If the lines m and n represent resistance with respect to the light – the former of air, the latter of water – then the difficulty of the path from C to E will be as the rectangle formed by CE and m; and from E to G as the rectangle formed by EG and n. Therefore, so that the difficulty of the path CEG should be the least of all, the sum of the rectangles CE by m and EG by n ought to be the least possible, or less than CF by m and FB by n – where F is taken to be any other point except E. We are seeking E. Since the points C and G and also the straight line AB are given by supposition, the straight lines perpendicular to the plane – we will call CH, 'c'; and GL, 'g'; and HL itself 'h' are therefore given as well. Also what is sought, EH, we will call 'y', so EL will be h-y, and CE will be$\sqrt{(c2 + y2)}$ which we will call 'p', and EG will be $\sqrt{g2 + y2 - 2hy + h2)}$, which we will call 'q'. Therefore m $\sqrt{c2 + y2} + n\sqrt{(g2 + y2 - 2by + b2)}$ – or mq + pq – should be the least of all quantities similarly expressed and y is sought so that it will be the least. From my method of maximum and minimum, which above all notations known thus far shortens miraculously the calculation, right away, at first glance – almost without any calculation it is clear that mq times y will be equal to np times h – y, or that np will be to mq as y will be to h – y, or that the rectangle CE by n will be to the rectangle EG by m, as EH will be to EL. Therefore, having postulated that CE and EG are equal, the resistance of the water with respect to light will be to the resistance of the air with respect to light as EH (the sine of the complementing angle of incidence CEA in the air) is to EL (the sine of the complimenting angle of refraction GEB in water), or (seu) the complimenting sines will be in a reciprocal ratio to the resistance of the medium – which is what was claimed. And so, if EL in one example or experiment should be discovered to be 2/3 of EH, it will be (2/3 of EH) in all other cases wherever C and G are taken to be in air and glass respectively. If E is in air and G beneath water, EL will be approximately ¾ of EH."[231]

From this calculation Leibniz concludes that he has reduced optics to mathematics and geometry based on the principle of final causes.

[230] Lamborn, p. 66

[231] McDonough, Jeffrey. "Jeffrey McDonough's Homepage – Harvard University; Papers, Translations, "Unicum Opticae, Catopticae, et Dioptricae Principium"; www.people.fas.harvard.edu/~jkmcdon/

He states that this is the method most likely used by the ancients and also by Snell and Fermat. He also says that the Cartesian theorem can be seen in "the ratio of the sines of the compliments of the angles of refraction and incidence". There is one problem, however, as Leibniz notes. The Cartesian idea of the nature of the resistance is "different from, indeed contrary to" his.[232] Leibniz recognizes this as problematic and responds by saying,

> Moreover, because Descartes wanted to demonstrate his theorem using his own tools, he fell into great difficulties: indeed, since he saw that the ray CE having entered into the water from the air, and being refracted there into EG and so towards the perpendicular EK, is therefore rendered more similar to that ray whose action (actio) is stronger, namely, the perpendicular, he suspected that it met with less resistance in water or glass than in air. And yet, in supposing the contrary, which is more consistent with reason, the same conclusion is reached applying our principle of the easiest way. From which Fermat rightly concluded that Descartes had not given the true reason for his own theorem.[233]

In this text Leibniz says that Descartes' position only needs amended. He does not throw it out. Neither does he completely disagree with Fermat. As regards the optics of refraction in the Fermat – Descartes debate, Leibniz does not take sides. He does, however, state his position that the formulas for optics are known through application of final causes. That is his affirmed position.

Leibniz's metaphysical application to physics is tied to his scientific method.[234] In *Elementa Physicae,* an unpublished and unfinished work from his early Hanover years, Leibniz reveals his scientific method, which includes a statement about light as well as "incorporeal

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[232] Translation from McDonough's "Homepage"

[233] Translation from McDonough's "Homepage"

[234] In his essay "Leibniz's Philosophy of Science" (*Studia Leibnitiana*, 1976, Vol. 8, No. 1, p. 2-3), L.J. Russell says that Leibniz was influenced by Bacon, Descartes, and Huygens. Russell notes that concerning scientific method "Bacon had stressed the need for going to nature to discover the laws of nature, and he had outlined the processes of inductive examination of facts. Like his contemporaries, Leibniz agreed with this. There were of course on the continent a great many scientists who thought that an empirical examination of facts is not sufficient to establish general laws of nature, and who agreed with Descartes that the basic principles needed for an understanding of the world have to be established in the same way as mathematics is established, namely by an examination of concepts, and of the relations which can be intuited to hold between concepts. Leibniz agreed with this as well." He says about Huygen's influence on Leibniz, "Probably the most important influence in bringing about a change in Leibniz's outlook is that of Huygens, in Paris. Leibniz was in Paris from 1672 to 1676. It was Huygens who showed him how little he knew of modern mathematics and guided his studies. Huygens gave him a copy of his own *Horologium Oscillatorium*, and initiated him into modern physics. From Huygens he took certain general ways of looking at the problems of motion, and it was through Huygens that he saw the importance of the conservation of vis viva."

matters" and "force and power".[235] Concerning "incorporeal matters" he says,

> Certain things take place in a body which cannot be explained from the necessity of matter alone. Such are the laws of motion, which depend on the metaphysical principle of the equality of cause and effect. Therefore, we must deal here with the soul and show that all things are animated. Without soul or form of any kind, body would have no being, because no part of it can be designated which does not in turn consist of more parts. Thus nothing could be designated in a body which could be called 'this thing', or a unity. On the nature of soul or form; that there is a kind of perception and appetite which are the passions and actions of the soul. Any why; because souls result from God's knowledge of things, or they are imitations of the ideas. All souls are indestructible, but those especially are immortal which are citizens in the commonwealth of the universe, or those to whom God is not merely author but also king, for to them he is connected by an entirely special basis, and they are therefore called minds. These minds never forget themselves. They alone think of God and have distinct conceptions of things. It is improper to try to ascribe perception to man alone. Since all bodies are able to have some perception according to the measure of their perfection, they will have it, for whatever can happen without detriment to other things will in fact happen, because everything occurs in the perfect way. Here too can be explained the nature of joy and grief, which if merely the perception of one's own success or perception. Thus, when a striving (*conatus*) is satisfied, the result is success; when it meets resistance, there arises grief. There are as many mirrors of the universe as there are minds, for every mind perceives the whole universe, but confusedly.[236]

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[235] In his essay "Boyle and Leibniz" (*Journal of the History of Ideas*, 1955, Vol. 16, No. 1, p. 25 – 26) Leroy Loemker says that *Elementa Physicae* was intended, along with *Elementa de Corpore* to be a section of *Catholic Demonstrations*."

[236] Loemker, Leroy, p. 278 – 279; The Latin word "conatus" has two primary meanings; (1) an effort, exertion, struggle endeavor, (2) an impulse, inclination, tendency; see *Harper's Latin Dictionary*, the Lewis and Short revision, American Book Company: New York, N.Y. The is what Leibniz means by "strive", associating it with the Latin "conatus"? In his essay "Leibniz on Conatus, Causation, and Freedom" (*Pacific Philosophical Quarterely*, 2004, Vol. 85, No. 4, p. 365), Laurence Carlin argues that Leibniz is a causal determinist, and, consequently, a compatibilist**.** On page 369 he cites the letter to de Volder where Leibniz says, "it is obvious that primitive forces can be nothing but the internal tendencies of simple substances, tendencies by means of which they pass from perception to perception." Carlin interprets this as Leibniz meaning "that it is force that is the proper efficient cause of perceptual states, as opposed to previous perceptual states themselves, which presumably (qua static representative states) are causally inefficient." Carlin refers to Leibniz's 1702 – 1704 work for his meaning of conatus. In FN 14 (p. 378) he notes that conatus and final cause are interchangeable. He quotes Leibniz's table of definitions: "The end is that, an appetite for which is a sufficient cause of *conatus* in the agent." About the early modern definition of *conatus* and Leibniz's understanding of it, he says, "In the context of the early modern period, *conatus* is typically understood as an instantaneous "endeavor," or "striving," or"tendency," or "effort" toward change. Leibniz seems to understand it this way too. According to Leibniz, "*conatus* is a state from which is born another state, unless something impedes it" (C 474; cf. C 481; Grua 513, 538). The idea seems to be this: a thing *x* exhibits *conatus* at time *t* if and only if *x* will do some activity **A** unless impeded from doing **A** at time *t*. What *x* will do – that is, what the relevant **A** is – is, of course, a function of what state *x* is in at time *t*. In some passages, Leibniz claims that *conatus* is "halfway" between being a mere faculty endowed with the potential for activity, and action itself (G IV, 469 (L 433)). It would thus seem that *conatus* is not itself an activity (such as motion), but a *force* which incites activity." Scholarship does not present us much in the way of essays on what Leibniz means by conatus. Woolhouse says that Leibniz's concept of the essence of matter is to be interpreted as a rejection of Descartes. In his essay "Leibniz's Collision Rules for Inertialess Bodies Indifferent to Motion" (*History of Philosophy Quarterly*, 2000, Vol. 17, No. 2, p. 143), R. L. Woolhouse says, "A recurring theme in Leibniz's philosophy from the early 1680s is that Descartes' account of the essence of matter, as extension, is unsatisfactory. Such a notion of matter, Leibniz began to argue around 1680, is not substantial enough to fill in the basic outlines of the matter-in-motion picture of the world drawn by the new mechanical philosophy of the seventeenth century. The world cannot, he argued, just be a world of movements of matter whose essence consists in no more than extension. Matter would not behave in the ways we know it does if that is there were to it."

Concerning force, Leibniz says that it is to be "estimated from the quantity of its effect". He also says that "the power of the effect and of the cause are equal to each other." That way physical reality does not succumb to mechanical perpetual motion or to a loss of "continuous (perpetuous) physical motion."[237] Concerning "refraction in the transition from one fluid to another" he says, "All things seem in fact to be fluid but merely variously folded into each another without a break in the continuity."[238]

Next Leibniz talks about scientific discovery and causation through *a priori* and *a posteriori* reasoning. He says,

> Just as there is a two-fold way of reasoning from experiments, one leading to the application, the other to the cause, so also there is a two-fold way of discovering causes, the one *a priori*, the other *a posteriori*, and each of these may be either certain or conjectural. The *a priori* method is certain if we can demonstrate from the known nature of God that structure of the world which is in agreement with the divine reasons and from this structure, can finally arrive at the principles of sensible things. This method is of all the most excellent and hence does not seem to be entirely impossible. For our mind is endowed with the concept of perfection, and we know that God works in the most perfect way. I admit, however, that, though this way is not hopeless, it is certainly difficult and that not everyone should undertake it. Besides, it is perhaps too long to be covered by men. For sensible effects are too greatly compounded to be readily reduced to their first causes. Yet superior geniuses should enter upon this way, even without the hope of arriving at particulars by means of it, in order that we may have true concepts of the universe, the greatness of God, and the nature of the soul, through which the mind can be most perfected, for this is the most important end of contemplation. Yet we believe that the absolute use of this method is conserved for a better life.[239]

A priori reasoning, he argues, "proceeds from hypotheses, assuming certain causes, perhaps, without proof, and showing that the things which now happen would follow from these assumptions."[240] Such hypotheses lead men to great truths, but not all, and consequently, "it is also useful to apply less perfect hypotheses as substitutes for truth until a better one occurs, that is, one which explains the same phenomena more happily or more phenomena with equal felicity." He says that there is no danger in this provided that "we carefully distinguish the certain from the probable."[241]

[237] Loemker, p. 279
[238] Loemker, p. 279
[239] Loemker, p. 283
[240] Loemker, p. 283
[241] Loemker, p. 283

"The hypothetical method *a posteriori*, which proceeds from experiments," he argues, "rests for the most part upon analogies."[242] Leibniz points out the method for discovering design. If everything else works by least action, why not light? Leibniz does not speak of light here, but of other analogous findings. He says,

> For instance, seeing that many terrestrial phenomena agree with magnetic phenomena, some men teach that the earth is a great magnet, that the structure of the earth corresponds to this, and that heavy bodies are drawn to the earth as as a magnet draws iron. Others explain everything by fermentation, even the ebb and flow of the tides. Still others, seeing that lye fights against acids reduce all corporeal conflicts to those of acid and alkali. We must guard against the abuse of analogies. Yet they can be of exceedingly great use in making inductions and in setting up aphorisms from inductions by means of which we can also make predictions about matters of which we as yet have little experience. This too is useful in investigating the causes of things, for it is always easier to discover the cause of a phenomenon which several things have in common.[243]

Experiment, Leibniz writes, "resolves the phenomenon into its attributes and seeks the causes and effects of each attribute." He points out that if the cause of all the attributes of a phenomenon is discovered, then, of course, the cause of the phenomenon is discovered.[244] What if the effects, but not the causes, of a phenomena are discoverable? Leibniz says that the method is the same; "the effects of the separate attributes will have to be examined."[245] He identifies three kinds of attributes of phenomena; composite, confused, and distinct. Experimentation yields the causes and effects of each attribute. For this he gives his method:

> Composite attributes are made up of more simple attributes. Simple attributes "include heat, firmness, duration; composites are such as fusibility, which consists of a body ceasing to be firm when heated."[246] Simple attributes are simple by reason of our senses. Such endure, or are themselves; i.e. they cannot be reduced. Sensible attributes can be divided into confused and distinct. "Confused attributes", Leibniz says, "are those which are indeed composite in themselves…but are simple to the senses and whose definition therefore cannot be explained."[247] "Distinct attributes," he says, "are either simple to the intellect itself or understood in themselves as 'to be', 'to endure'; or they can explain through a definition, that is, they can be recognized by us through certain signs, as for example, roundness or the equidistance of all points from one, and gravity or a striving toward the center of the earth."[248]

[242] Loemker, p. 284
[243] Loemker, p. 284
[244] Loemker, p. 284
[245] Loemker, p. 285
[246] Loemker, p. 285
[247] Loemker, p. 285
[248] Loemker, p. 285

Leibniz says that light is a confused attribute.[249] To investigate the cause of any confused attribute, "we must relate them to other attributes as well as to the subjects which contain them."[250] To bring together an attribute with a subject requires "the bringing together of the attribute with an aggregate of others which concur in the same subject."[251] Consequently, "a confused attribute can be related either to other confused attributes or to distinct attributes."[252] In this scientific process, Leibniz says that "there sometimes occurs another kind of resolution of confused attributes which I call experimental." These are distinguished from intellectual resolution.[253] In order to investigate the cause of light, "the way in which it is produced or increased," its contrary, "the way in which it is destroyed or diminished", and its effects, "we do this by bringing it together with an aggregate of many other confused or distinct attributes taken together."[254] Mathematics assists in this process to make determinations of Catoptrics and Dioptrics.

> The application of mathematics to physical sciences consists in such consideration of the distinct attributes which accompany confused ones. Once we have learned that the angles of incidence and reflection of a ray of light are equal and that these angles are taken with respect to the perpendicular striking a plane tangent to the surface at the point of incidence, then we can easily establish the science of catoptrics. Similarly few experiments about refraction are needed to set up the foundations of Dioptrics.[255]

After this brief description of scientific method, Leibniz turns to a lengthy statement about efficient and final causes, a portion of which is repeated by Maupertuis. Leibniz says,

> I know too that there are excellent and most learned men who cannot abide having all bodily phenomena explained mechanically. For they think that this injures religion, and they believe that if it were accepted, the world mechanism would need neither God nor any other incorporeal substance. This they rightly regard as absurd and dangerous. Hence some of them make use of an immediate intervention of God everywhere, while others introduce intelligences or angels as moving forces here and there. Some set up a kind of world soul or a hylarchic

[249] Loemker, p. 287
[250] Loemker, p. 287
[251] Loemker, p. 287
[252] Loemker, p. 287
[253] Loemker, p. 287
[254] Loemker, p. 287
[255] Loemker, p. 287

> principle,[256] through whose operations heavy bodies are made to strive toward the earth and other things happen which are needed to conserve the world system. But all these things are insufficient to explain things, for whether we introduce God or an angel or a soul or whatever corporeal operative substance, the cause and the mode of operating can always be explained in the truth which we have about the things themselves. But the way in which a body operates cannot be explained distinctly unless we explain what its parts contribute. This cannot be understood, however, unless we understand their relation to each other, and to the whole in a mechanical sense, that is, their figure and position, the change of this position or motion, their magnitude, their pores, and other things of this mechanical kind, for these always vary the operation. I admit that these outstanding men have unimpeachable grounds for shrinking back from the philosophy of certain recent thinkers, because many philosophers today resort to efficient and material causes only, completely neglecting formal and final causes. But those who are wise know that every effect has a final as well as an efficient cause – final because everything that happens is done by a perceiving being, efficient because everything that happens naturally in a body takes place through the corporeal organ and according to the laws of bodies.[257]

Leibniz says that there is an unfortunate and unnecessary war going on between science and religion in which both sides have overstepped their bounds. This is because neither side properly understands the necessary application of final causes to physics. Leibniz says, "This is what I think. Everything is by nature to be understood clearly and distinctly and could be manifested to our understanding by God if he willed to do so. And the operation of a body cannot be understood adequately unless we know what its parts contribute; hence we cannot hope for the explanation of any corporeal phenomenon without taking up the arrangement of its parts."[258] He finishes this work with a statement about final causes: "For the action of the soul is determined by the state of the organ of the soul and its object, and the operation of God by the conditions of the individual things, and this not by the necessity of matter but by the impulsion of the final cause or the good."[259]

In 1686 Leibniz wrote a treatise against Descartes and Spinoza entitled *Discourse on Metaphysics.* In this work he mentions final causes. First, however, he lays a divine foundation for his doctrine. In DM #1 he finishes with, "Whence it follows that God possessing supreme and infinite wisdom, acts in the most perfect manner, not only metaphysically, but also morally

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[256] This is most notably Henry More and his work *An Antidote Against Atheism.*
[257] Loemker, p. 288
[258] Loemker, p. 289
[259] Loemker, p. 289

speaking, and that, with respect to ourselves, we can say that the more enlightened and informed about God's works, the more we will be disposed to find them excellent and in complete conformity with what we might have desired."[260] Geometrically speaking, the perfect God draws the most perfect and best of all lines. In DM #6 he says, "Thus, one can say, in whatever manner God might have created the world, it would always have been regular and in accordance with a certain general order. But God has chosen the most perfect world, that is, the one which is at the same time the simplest in hypotheses and the richest in phenomena, as might be a line in geometry whose construction is easy and whose properties and effects are extremely remarkable and widespread."[261]

In DM #10 Leibniz pins down Descartes and the mathematicians mentioned by

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[260] Ariew and Garber, p. 35; This differs from Descartes, whose foundation, of course is God's immutability. Leibniz says that God is perfect, and his works come from his perfection. Descartes says the same thing to Elizabeth! In his essay "God as Total Efficient Cause" (*History of Philosophy Quarterly,* 1995, Vol. 12, No. 2, p. 196), Kenneth Clatterbaugh says, "to Elizabeth he writes: God ". . . would not be supremely perfect if anything could happen without coming entirely from Him" and he continues "God is the universal cause of everything in such a way as to be also the total cause of everything; and so nothing can happen without His will" (CSM III. 272). " Clatterbaugh says that Descartes is a concurrentist in causal notions. On page 199 he says, "It is most accurate to read Descartes as advocating concurrentism, which using the language of Saint Thomas Aquinas is the view that "the same effect is ascribed to a natural cause and to God, not as though part were affected by God and part the natural agent; but the whole effect proceeds from each." Descartes, then, defends a position in which it is true both that God is the total efficient cause of everything and that created substances have a real efficient causal role in each and every effect after the initial creation. Descartes' efforts in this direction are greatly aided by a conception of cause that treats *any proposition that occurs as a premise in a scientific explanation* as a cause." On page 199 he says that Descartes believes that God is the efficient cause of physical laws, and those laws cause everything to sort themselves out. In her essay "Formal Causation and the Explanation of Intentionality in Descartes" (*The Monist*, 1996, Vol. 79, No. 3, p. 379), Amy Schmitter says, "Indeed when "form" is reconceived as a mode such as shape, size, or configuration of parts, Descartes has no problem with admitting them into his metaphysics or physics: "essential forms explained in our fashion, on the other hand, give manifest and mathematical reasons for natural actions, as can be seen with regard to the form of common salt in my Meteorology" (AT 506, CSMK 209)"

[261] Ariew and Garber, p. 39; In his essay "Perfection and Happiness in the Best Possible World" (Jolley, Nicholas, Editor. *Cambridge Companion to Leibniz,* Cambridge Press: New York, N.Y., 2006, Online ISBN: 9781139000277, p. 383 – 384), David Blumenfeld says, "Leibniz identifies perfection with harmony, which he defines in classical fashion as "unity within variety." There are many terminological variations on this theme: harmony is said to be "agreement in . . . variety," "similarity in variety or diversity balanced by identity," and order (regularity, uniformity, etc.) within plurality (Gr 12, 267; G W 171-72: AG 233-34; A VI.iii 116, G V1 616; L 648). But these variations are evidently intended to signify the same thing, namely, a certain order that unifies diversity. The specific kind of order that determines harmony, however, is the one expressed by the variety/simplicity criterion." In his essay "Leibniz on Divine Concurrence" (*The Philosophical Review,* 2004, Vol. 113, No.2, p. 203-204), Sukjae Lee says that Leibniz is a divine concurrentist who agrees with Malebranche that "conservation is but continuous creation". Both God and creature are involved in the occurrence of effects.

Maupertuis (perhaps Descartes is one Maupertuis has in mind) with his words on forms. With his banishment of final causes from physics, Descartes, who clings to the formal causation of mathematics, falls into the very trap he accuses the scholastics of doing.[262] He and they both attempt to describe how the world works with only the form, and none of the efficiency and finality of the sculpture. The mathematical formula is substantial form which provides information on the property of matter, but does not cause anything. It therefore explains nothing. Leibniz says,

> It seems that the ancients, as well as many able men accustomed to deep meditation who have taught theology and philosophy some centuries ago (some of whom are respected for their saintliness) have had some knowledge of what we have just said; this is why they introduced and maintained the substantial forms which are so decried today. But they are not so distant from the truth, nor so ridiculous as the common lot of our new philosophers imagine.
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> I agree that the consideration of these forms serves no purpose in the details of physics and must not be used to explain particular phenomena. That is where the scholastics failed, as did the physicians of the past who followed their example, believing that they could account for the properties of bodies by talking about forms and qualities without taking the trouble to examine their manner of operation. It is as if we were content to say that a clock has a quality of clockness derived from its form without considering in what all this consists; that would be sufficient for the person who buys the clock, provided he turns over its care to another.[263]
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> But this misunderstanding and misuse of forms must not cause us to reject something whose principles or elevate our minds sufficiently well to the knowledge of incorporeal natures and the wonders of God. However, just as a geometer does not need to burden his mind with the famous labyrinth of the composition of the continuum, there is no need for any moral philosopher and even less need for a jurist or statesman to trouble himself with the great difficulties involved in reconciling free will and God's providence, since the geometer can achieve all his demonstrations and the statesman can complete all his deliberations without entering into these discussions, discussions which remain necessary and important in philosophy and theology. In the same way, a physicist can explain some experiments, at times using previous simpler experiments and at times using geometric and mechanical demonstrations without needing general considerations from another sphere. And if he uses God's concourse or else a soul, animating force (*archee*), or something else of this nature, he is raving just as much as the person who, in the course of an important practical deliberation, enters into a lofty discussion concerning the nature of destiny and the nature of our freedom. In fact, people often commit this fault without thinking when they encumber their minds with the consideration of fatalism and sometimes are even diverted from a good resolution or a necessary duty in this way.[264]

[262] In their essay "Descartes on Causation" (*The Review of Metaphysics*, 1997, Vol. 50, No. 4, p. 841 – 842), Daniel Flage and Clarence Bonnen say that Descartes' idea that God essence is his existence is declared by Descartes, in response to Arnauld and Caterus, to be of formal, and not efficient, causes. Flage and Bonnen argue that, upon examination, "Cartesian natural laws are ontologiocally and epistemologically indistinguishable from eternal truths: they constitute the form of the world."
[263] Ariew and Garber, p. 42
[264] Ariew and Garber, p. 42 – 43

In DM#11 Leibniz says that the scholastics demonstrate the inadequacy of geometry in explaining optical physics.[265] In DM 12 he says that "the nature of body does not consist merely in extension, that is, in size, shape, and motion, but that we must necessarily recognize in body something related to souls,…"[266] In DM #13 he says that these souls move without an inclination towards necessity.[267]

In DM #17 Leibniz gives an example from physics how explanation requires a metaphysical application. God, Leibniz say, "always conserves the same force but not the same quantity of motion."[268] This, he admits, is in direct conflict with the Cartesian understanding that God always conserves the same quantity of motion in the world. The difference here is in the distance light travels in refraction. Leibniz, however, uses a different example. He says, "Descartes and many other able mathematicians have believed that the quantity of motion, that is, the speed multiplied by the size of the moving body, coincides exactly with the moving force, or, so to speak geometrically, that the forces are proportional to the product of the speeds and (sizes of) bodies."[269] Leibniz makes three observations; one, "the same force is always conserved in the universe, two, "there is no perpetual mechanical motion, because then the force of a machine, which is always diminished somewhat by friction and which must sooner or later come to an end, would restore itself, and consequently would increase by itself without any new external impulsion," three, "the force of a body is diminished only in proportion to the force it imparts to some bodies contiguous to it or to its own parts, insofar as they have separate

[265] Ariew and Garber, p. 43
[266] Ariew and Garbar, p. 44; In their essay "Leibniz on Body, Matter and Extension" (*Proceedings of the Aristoltelian Society, Supplemental Volumes*, 2004, Vol. 78, p. 23), Daniel Garber and Jean Baptiste Rauzy say that DM #12 is "a critique of the Cartesian conception of body." The Cartesian view of body is extension alone. Leibniz's argument is that body consists of substantial form in addition to extension.
[267] Ariew and Garber, p. 44
[268] Ariew and Garber, p. 49
[269] Ariew and Garber, p. 49

motion."[270]

Leibniz says that the Cartesians believe that what can be said about force can also be said about motion. To show the difference Leibniz resorts to the physics of falling bodies.

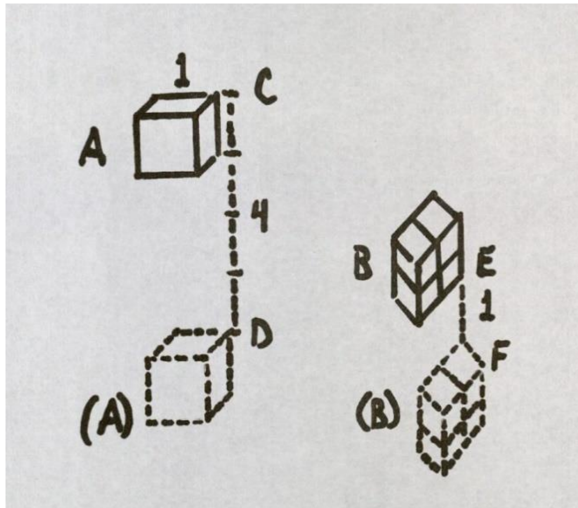

Figure #17 Located in *Discourse on Metaphysics* by Leibniz; Found in Ariew and Garber, *G.W. Leibniz, "Philosophical Essays,* p. 50

Leibniz says,

> Let us now see whether the quantity of motion is also the same in each. But here we will be surprised to find a very great difference. For Galileo demonstrated that the speed acquired by the fall *CD* is twice the speed acquired by the fall *EF*, even though one height is four times the other. I assume also that as much force is required to elevate *A*, a body of one pound to *CD*, a height of four fathoms, as to elevate *B*, a body of four pounds, to *EF*, a height of one fathom. All this is admitted by our new philosophers. It is therefore evident that, having fallen from height *CD*, body *A* required exactly as much force as body *B*, which fell from height *EF*; for since body (*B*) reached *F* and acquired the force to rise to *E* (by the first assumption), it has the force to carry a body of four pounds, that is, itself, to *EF*, the height of one fathom; similarly, since body (*A*) reached *D* and acquired the force to rise to *C*, it has the force to carry a body of one pound, that is, itself, to *CD*, a height of four fathoms. Therefore (by the second assumption), the force of these two bodies is equal.
>
> Let us therefore multiply body *A*, proportional to 1, with its speed, proportional to 2; the product or quantity of motion will be proportional to 2. On the other hand, let us multiply body *B*, proportional to 4, by its speed, proportional to 1; the product or quantity of motion will be proportional to 4. Therefore the quantity of motion of body (*A*) at point *D* is half of the quantity of motion of body (*B*) at point *F*; yet their forces are equal. Hence, there is a great difference between quantity of motion and force – which is what needed to be proved.[271]

From this example Leibniz concludes, "Thus we see that force must be calculated from the

[270] Ariew and Garber, p. 49 – 50
[271] Ariew and Garber, p. 50

quantity of the effect it can produce."[272]

In DM#18 Leibniz says that it is this distinction between force and quantity of motion that causes the thinker to resort to metaphysical considerations. Force, he says is something apart from size, shape, and motion. Therefore, "one can judge that not everything conceived in body consists solely in extension and in its modifications, as our moderns have persuaded themselves."[273] For this reason, Leibniz argues, some "beings or forms" they banished must be reinstated. Even though all particular phenomena of nature can be explained mathematically or mechanically, "nevertheless the general principles of corporeal nature and of mechanics itself are more metaphysical than geometrical, and belong to some indivisible forms or natures as the causes of appearances, rather than to corporeal mass or extension."[274] In DM #19 he says that examples like the one in DM #17 show the usefulness of final causes in physics.

Leibniz says that it is the "end" or "good in acting" that God has proposed which provides the place where thinkers must seek "the principle of all existences and laws of nature". This is because "God always intends the best and most perfect."[275] Good effects and perfection in nature are the product of God's proposal. There is nothing by chance, and Leibniz says error in applying final causes in physics need not be feared "provided we limit ourselves to affirmations and avoid negative propositions that limit God's designs."[276]

This statement must be interpreted in light of his earlier statement in #19 where he says, *"I am quite willing to admit that we are subject to deception when we wish to determine God's ends or counsels. But this is only when we try to limit them to some particular design, believing that he had only one thing in view, when instead he regards everything at the same time."*[277]

[272] Ariew and Garber, p. 50
[273] Ariew and Garber, p. 51
[274] Ariew and Garber, p. 51 - 52
[275] Ariew and Garber, p. 52
[276] Ariew and Garber, p. 52
[277] Ariew and Garber, p. 52

This is a specific reference to Descartes statements in *Meditations* and *Principles of Philosophy* where Descartes says that the "ends" of God are beyond his intellectual grasp.[278] God regarding "everything at the same time" is a declaration by Leibniz of God's omniscience, and eliminates the notion that bodies move towards an end in any Aristotelian or scholastic notion.

Necessity and chance are problems for Leibniz. He says,

> When one seriously holds these opinions ascribing everything to the necessity of matter or to some chance (even though both must appear ridiculous to those who understand what we have explained above), it is difficult to recognize an intelligent author of nature. For the effect must correspond to its cause; indeed, the effect is best recognized through a knowledge of the cause. Moreover, it is unreasonable to introduce a supreme intelligence as orderer of things and then, instead of using his wisdom, use only the properties of matter to explain the phenomena.[279]

Leibniz, like Newton his contemporary, believes that physical reality is an intelligent design. Mechanical rules that depend on geometry without metaphysics, Leibniz argues in DM #21, produce an entirely different phenomena.[280]

In DM #22, entitled *Reconciliation of Two Ways of Explaining Things, by Final Causes and by Efficient Causes, in Order to Satisfy Both Those Who Explain Nature Mechanically and*

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[278] Cottingham, John, Robert Stoothoff and Dugald Murdoch (Editors). *The Philosophical Writings of Descartes.* Vol. 1, page 202, and Vol. 2, page 39, Cambridge Press: New York, N.Y., 1984; In her essay "Descartes and Charleton on Nature and God" (*Journal of the History of Ideas*, 1979, Vol. 40. No. 3, p. 445), Margaret Osler says that positions on matter and motion, like the one held by Descartes, are based on epistemological positions. She argues that seventeenth century epistemological positions emerged from certain theological traditions already prevalent. The theological tradition which focuses on God's will leads to empiricism, and the tradition which emphasizes God's intellect leads to an *a priori* approach. The first says that properties of matter are knowable through observation and the second emphasizes rational deduction. Osler says that Descartes represents the latter, which explains his aversion to knowledge of God's "ends". In her essay "Whose Ends? Teleology in Early Modern Natural Philosophy" (*Osiris,* 2001, Second Series, Vol. 16, p. 155), Osler says that the question facing Descartes and other mechanists was whether the principle of activity in matter was internal or external. Strictly against the scholastics, Descartes says it is external. Therefore there can be no final causes in matter. Leibniz agrees with this, but final cause belongs to God who directs matter by his wisdom as well as his mechanical laws. This is affirmed by Osler on page 155.

[279] Ariew and Garber, p. 53

[280] Ariew and Garber, p. 53; Concerning this Leibnizian position, Garber says in his essay "Leibniz on Form and Matter" (*Early Science and Medicine,* 1997, Vol. 2, No.3, p. 333) "for Leibniz, the basic constituents of the world are simple substances, what he comes to call "monads" sometime in the late 1690s. These simple substances are non-extended, non-compound substances, understood on analogy with Cartesian souls. They possess what Leibniz calls perception and apperception, perceptions being the momentary states of simple substances and apperception being the faculty which carries the substance from one perceptual state to the next. But in addition, in this period, Leibniz also recognized corporeal substances. These complex substances, apparently extended and made up of other smaller substances, form the subject matter of his mechanist physics from the early 1680s to roughly the middle of the first decade of the next century. Matter and form enter into Leibniz's account of both."

Those Who Have Recourse to Incorporeal Natures, Leibniz alludes to his doctrine and his most determined path. Leibniz says that we can praise the skill of the great artisan by the way that efficient causes and final causes work together. When it comes to refraction in optics, Leibniz says, final causes is much easier and produces results quicker. He says,

> I find that the way of efficient causes, which is in fact deeper and in some sense more immediate and *a priori*, is, on the other hand, quite difficult when one comes to details, and I believe that, for the most part, our philosophers are still far from it. But the way of final causes is easier, and is not infrequently of use in divining important and useful truths which one would be a long time in seeking by the other, more physical way; anatomy can provide significant examples of this. I also believe that Snell, who first discovered the rules of refraction, would have waited a long time before discovering them if he first had to find out how light is formed. But he apparently followed the method which the ancients used for catoptrics, which is in fact that of final cause. For, by seeking the easiest way to lead a ray from a given point to another point given by reflection on a given plane (assuming that this is nature's design), they discovered the equality of angles of incidence and angles of reflection, as can be seen in a little treatise by Heliodorus of Larissa, and elsewhere. That is what, I believe, Snell and Fermat after him (though without knowing anything about Snell) have most ingeniously applied to refraction. For when, in the same media, rays observe the same proportion between sines (which is proportional to the resistance of the media), this happens to be the easiest or, at least, the most determinate way to pass from a given point in a medium to a given point in another.[281] And the demonstration Descartes attempted to give of this same theorem by way of efficient causes is not nearly as good. At least there is room for suspicion that he would never have found the law in this way, if he had learned nothing in Holland of Snell's discovery.[282]

In his *Specimen Dynamicum,* Leibniz elaborates on his "new dynamics" with the notions of *conatus* and *living force.* There is in nature, Leibniz argues, a force implanted by the author of nature which is prior to extension.[283] This force is a striving or effort, a "conatus seu nisus". It is a full striving unless impeded by "a contrary striving". This striving is present everywhere in matter, is sometimes known by the senses, and is to be understood on rational grounds. It is

[281] In his essay "Leibniz on Natural Teleology and the Law of Optics" (*Philosophy and Phenomenological Research*, 2009, Vol. 78, No. 3, p. 507) Jeffrey McDonough says, "The law of refraction – first published by Descartes – states that the ratio between the angle at which a ray of light strikes a refractive surface and the sine of the angle at which it is refracted is a constant determined by the mediums involved."

[282] Ariew and Garber, p. 54 - 55

[283] Garber notes that Leibniz believes that not only extension but also force, both active and passive, is phenomenal. See *Leibniz: Body, Substance, Monad,* p. 362: Garber quotes Leibniz's June 1704 letter to de Volder, "Indeed, considering the matter carefully, it should be said that there is nothing in things except simple substances and in them perception and appetite. Moreover, matter and motion are not so much substances or things as the phenomena of perceivers, the reality of which is located in the harmony of perceivers with themselves (at different times) and with other perceivers."

due to the miraculous of God, and produced by him in every body. It is the nature of body. A body acts, and "extension means only the continuation or diffusion of an already presupposed acting and resisting substance.[284]

Motion is that momentary state in which force strives towards change. Active force, or power, is of two kinds, primitive and derivative. Primitive refers to a suffering or a resisting. It is to the derivative force that action is calculated. Leibniz says, "Here, therefore, we understand by derivative force, or the force by which bodies actually act and are acted upon by each other, only that force which is connected with motion (local motion, that is) and which in turn tends to produce further local motion."[285]

Conatus is the direction of velocity.[286] Impetus is "the product of the mass (*molis*) of the body by its velocity."[287] Force is of two kinds, dead and living. Living force is conatus. After the pendulum illustration, Leibniz says,

> Hence, force is also of two kinds: the one elementary, which I also call dead force, because motion does not yet exist in it but only a solicitation to motion, such as that of the ball in the tube, or a stone in a sling even while it is still being held by the string; the other is ordinary force combined with actual motion, which I call living force (vis viva). An example of dead force is centrifugal force, and likewise the force of gravity, or centripetal force; also the force with which a stretched elastic body begins to restore itself. But in impact, whether this arises from a heavy body which has been falling for some time, or from some similar cause, the force is living and arises from an infinite number of continuous impressions of dead force. This is what Galileo meant when in an enigmatic way, he called the force of impact infinite as compared with the simple impulsion of gravity.[288]

Leibniz says that living force is understood in two ways, total and partial. Partial force is either

out the difference between dead and living force:

> As we pointed out long ago, this happens for a special reason, namely, that when for example, different heavy bodies fall, the descent itself or the quantities of space passed through in the descent are,

[284] Loemker, p. 435
[285] Loemker, p. 437
[286] Loemker, p. 437
[287] Loemker, p. 437
[288] Loemker, p. 438

> at the very beginning of motion while they remain infinitely small or elementary, proportional to the velocities or to the conatuses of descent. But when some progress has been made and living force has developed, the acquired velocities are no longer proportional to the spaces already passed through in the descent but only their elements.[289]

The mechanics behind what he means is found in *New System* (1695) and *Tentamen Anagogicum* (1696). In *New System* he says,

> But later, after trying to explore the principles of mechanics itself in order to account for the laws of nature which we learn from experience, I perceived that the sole consideration of *extended mass* was not enough but that it was necessary, in addition, to use the concept of *force,* which is fully intelligible, although it falls within the sphere of metaphysics. It seemed to me also that though the opinion of those who transform or degrade beasts into pure machines seems possible, it goes beyond appearances and is even contrary to the order of things.[290]

The principles of physical reality, Leibniz says, are not to be found in matter alone. A revival of the substantial forms, apart from their abuses, is necessary. All matter is of *primitive forces*, or in Aristotelian terms *first entelechies.* They contain "not only the *actuality* or the *completion* of possibility but an original *activity* as well.[291]

They think, but not like humans think.[292] They have minds, and God governs them. Rejecting atomism as understood by the new mechanists, Leibniz says that the true single entity of which nature is made up is the *atom of substance*. "It is only atoms of substance, that is to say, real unities that are absolutely destitute of parts, which are the sources of action and the absolute first principles out of which things are compounded, and as it were, the ultimate elements in the analysis of substance. One could call them *metaphysical points*. They have

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[289] Loemker, p. 439; On the same page Leibniz goes on to mention Galileo and Descartes: "Yet we have already shown, and will show more fully, that the force must be calculated in terms of these spaces themselves. Though he used another name, and indeed, another concept, Galileo began the treatment of living force and was the first to explain how arises from the acceleration of heavy falling bodies. Descartes rightly distinguished between velocity and direction and also saw that in the collision of bodies that state results which least change the prior conditions. But he did not rightly estimate this minimum change, since he changes either the direction alone or the velocity alone, while the whole change must be determined by the joint effect of both together. He failed to see how this was possible, because two such heterogeneous things did not seem to him to be capable of compariskon or of simultaneous treatment – he being concerned with modalities rather than with realities in this connection; not to speak of his other errors in his teaching on this problem."

[290] Loemker, Leroy, *Gottfried Wilhelm Leibniz, "Philosophical Papers and Letters".* D. Reidel: Boston, Ma., p. 454

[291] Loemker, p. 454

[292] Loemker, p. 454

something vital, and a kind of perception and *mathematical points* are the points of view from which they express the universe."[293]

The Divine Wisdom brings things about "in conformity with the particular concept of the subject in question."[294] Leibniz says, "God has originally created the soul, and every other real unity, in such a way that everything in it must arise from its own nature by a perfect *spontaneity* with regard to itself, yet by a perfect *conformity* to things without."[295] The perfection of the works of God is that souls, or substances, by nature move and change. This is a necessity, and "they would have no force of action without it."[296]

Leibniz says that, though some think that we are free only in appearance, the truth is that "we are determined only in appearance and that in metaphysical strickness we are in a state of perfect independence as concerns the influence of all the other created beings."[297] The motion of intelligent souls fits nicely into mechanics. He ends *New System* with

> For it can be said that in the collision of bodies each suffers only from its own elasticity, caused by the motion that is already within it. As for absolute motion, nothing can determine it mathematically, since everything ends in relations. The result is always a perfect equivalence in hypothesis, as in astronomy, so that no matter how many bodies one takes, one may arbitrarily assign rest or some degree of velocity to any one of them we wish, without possibly being refuted by the phenomena of straight, circular, or composite motion.[298]

In a 1696 letter to Beauval, Leibniz clarified *New System*. The harmony of two substances are like "two clocks or watches that are in perfect agreement." This can happen in three ways; one, they are naturally influenced, as Huygens argued from his two pendulum experiment. Two, they are constructed by a skilled craftsman, who continually adjusts them in order to keep them in agreement. This is the Newtonian/Clark position which he argued against.[299] Third, they are

[293] Loemker, p. 456
[294] Loemker, p. 457
[295] Loemker, p. 456
[296] Loemker, p. 458
[297] Loemker, p. 458
[298] Loemker, p. 459
[299] In his first letter to Clarke, November 1715, Leibniz says, "Sir Isaac Newton and his followers also have a very odd opinion concerning the work of God. According to them, God Almighty needs to wind up his watch from time

constructed with such skill and precision in the first place "that they can be assured of their subsequent agreement."[300] This third position is Leibniz's idea of the body/soul relationship and the indestructible reality harmoniously run by God according to his wisdom and mechanical laws.

Leibniz insists to the Cartesians, and anyone else so concerned in his time, that final causes be applied to mechanics in order to properly understand nature. In his 1696 work *Tentamen Anagogicum*, he uses optics to make his point. Of the infinite number of paths a ray of light can possible take, it always takes the one that is best and most perfect. This is the *most determined* path.

Leibniz's aim in optics was to find a position which reconciles the Cartesian and the Fermatian approaches to optics.[301] In *Metaphysical Definitions and Reflections* (1678 – 1680), Leibniz proposes an efficient causal explanation for the first two assumptions in Descartes' reflection and makes a new argument for the third.

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to time, otherwise it would cease to move. He had not, it seems, sufficient foresight to make it a perpetual motion. No, the machine of God's making is so imperfect, according to these gentleman, that he is obliged to clean it now and then by an extraordinary concourse and even mend it, as a clockmaker mends his work, who must subsequently be so much the more unskillful a workman as he is often obliged to mend his work and to set it right. According to my opinion, the same force and vigor remains always in the world and only passes from one part of matter to another agreeably to the laws of nature and the beautiful pre-established order. And I hold that when God works miracles, he does not do it in order to supply the wants of nature, but those of grace. Whoever thinks otherwise must needs have a very mean notion of the wisdom and power of God." See Ariew and Garber, p. 320 – 321; Ariew and Garber note at the bottom of page 320 that Leibniz is referring to Newton's *Optiks*, p. 402 "in which Newton intimates that there are regularities in planetary motions caused by the (gravitational) attraction of the planets with one another, and that these irregularities will increase 'till this system wants a reformation."

[300] Loemker, p. 459 – 460

[301] McDonough, Jeffrey, *Philosophy and Phenomenological Research.* 2009, vol. 78, No. 3, "Leibniz on Natural Teleology and the Law of Optics", p. 511;

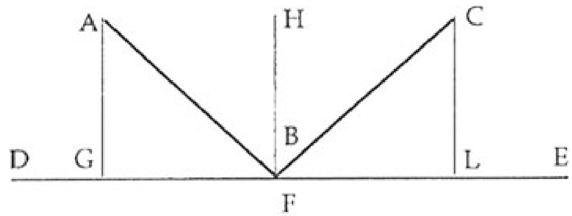

Figure #18 Found in Jeffrey McDonough's article "Leibniz's Two Realms Revisited", p. 676

> Letting DBE represent reflecting surface, AB an incident ray of light, and GB and HB its decomposed orthogonal components, Leibniz asks us to consider the following diagram (where BE represents a straight line, and F represents a point some distance below B) Leibniz argues that upon striking the reflecting surface, the ray of light causes the surface DBE to deform to a curve DFE so that when the surface is restored it imparts a motion BH to the ray of light. He boasts that in providing this basis for Descartes's third assumption, "we now have the reason for that which is elsewhere assumed without reason" (LC 1404/DC 255). He goes on to argue in Cartesian fashion that the horizontal component GB is unaffected by the impact with the reflective surface, and thus that the composed motions GB=BL and BH=CL result in the reflected ray being as BC, and the angle of incidence ABG equaling the angle of reflection CBL.[302]

Leibniz says next that the same results can be garnered using Fermat's employment of final causes.

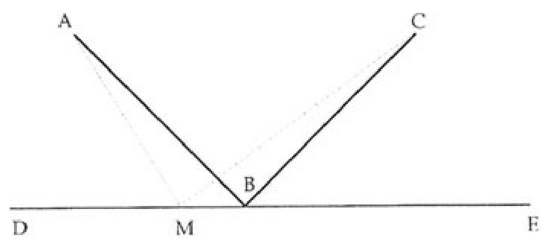

Figure #19 Found in Jeffrey McDonough's article "Leibniz's Two Realms Revisited", p. 677

> Letting DE be a reflecting surface, A a source of light (e.g. a flashlight) and C a light sink (e.g. an eye), Leibniz sets up the problem by asking which path AMC or ABC represents the actual path of the ray of light (where M could be any point on the surface DE)…. Leibniz suggests that the problem may be solved by appealing to the principle that "nature, proposing some end to itself, chooses the optimal means," and thus that—given the uniformity of the medium—"there ought to be a reason only for a long or short journey" (DC 1405/LC 255). Since there is no unique longest path, Leibniz maintains that the actual path must be the shortest, i.e. it must be ABC from which it follows that the angle of incidence ABD must be equal to the angle of reflection CBE. Leibniz concludes that, at least for a standard class of cases, the path of a ray of light can be determined by calculating the shortest reflected route from its source to its sink.[303]

This is an example of Leibniz striving to reconcile Cartesian optics and Fermatian optics,

[302] McDonough, "Leibniz's Two Realms Revisited", p. 67
[303] McDonough, "Leibniz's Two Realms Revisited", p. 676 – 677

efficient and final causes, and mechanism and grace. "In attempting to improve upon Descartes' broadly mechanical derivations, Leibniz charactistically emphasizes considerations of continuity and elasticity. In order to improve upon Fermat's patently non-mechanical approach, Leibniz attempts to address the two technical objections raised by Cartesians to the great mathematician's derivations."[304] The *Most Determined Path* is Leibniz's way of improving on Fermat. He argues that his MDP supercedes Fermat's idea that "the greatest ease be obtained in relation to planes".[305] It is the idea that light from among the infinite number of paths that a ray of light can take from source to sink, "a ray of light will travel along the path which is unique with respect to ease; where 'ease' is understood as the quantity obtained by multiplying the distance of the path by the resistance of the mediums."[306]

Leibniz combines his MDP with his infinitesimal calculus to offer new derivations for the laws of reflection and refraction. He begins this process in *TA* by showing maxima and minima. "Leibniz begins by first illustrating the use of his calculus to find local maxima and minima, asking his readers to consider ''a curve AB, concave or convex, and an axis ST to which the

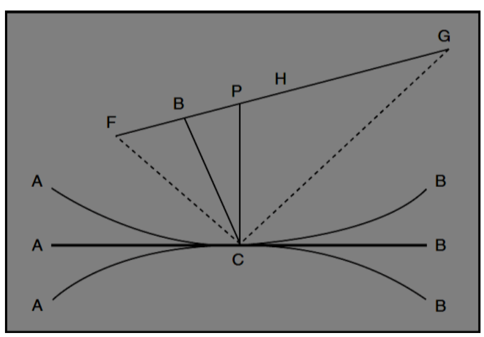

Figure #20 Found in Lamborn Master's Thesis, p. 67 and in Jeffrey McDonough's article, "Leibniz on Natural Teleology and the Law of Optics", p. 51

304 McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 511
305 McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 511
306 McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 512

ordinates of the curve are referred: His point is that *C* is the only point on *AB* "where the derivative of the line with respect to *ST* equals zero."[307] Consequently, given line *AB*, one can find *C* "by taking the derivative of the equation and equaling it to zero."[308] With this technique, Leibniz says that his derivations of the laws of reflection and refraction are greatly simplified.[309]

Next in *TA* Leibniz employs his MDP along with his calculus to derive the law of reflection. Leibniz wants his readers to consider a ray of light traveling between F and G in relation to a mirror ACB, which can be either flat plane, concave, or convex.

> Tacitly assuming that the medium through which the light travels is homogenous and isotropic, Leibniz reduces the problem of finding the unique path with respect to distance times resistance to the problem of finding the point C such that the path FCG is unique with respect to its length He then (i) constructs an equation for the length of the path from F to G via some point C on ACB, (ii) uses the technique previously trigonometry to show that for such a path the angle of incidence FCA must be equal to the angle of reflection GCB. illustrated to find the value of the equation of the path such that the value is unique or ''stationary,'' and then (iii) uses elementary trigonometry to show that for such a path the angle of incidence FCA must be equal to the angle of reflection GCB.[310]

Leibniz employs the same strategy for the law of refraction. He gives us a refracted surface of *ACB.* It can be either plane surface, concave, or convex. The ray of light starts at point *G* and moves to point *F*. The point of refraction is *C.*

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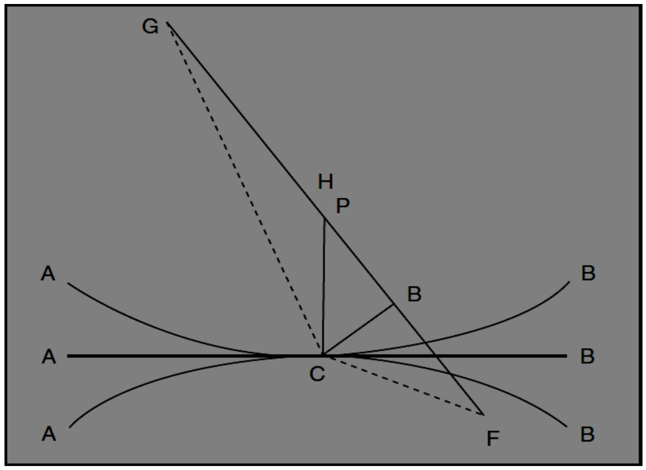
[307] McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 512
[308] McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 513
[309] McDonough notes that it took Fermat over four years to figure the same thing out. (FN 14, p. 513)
[310] McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 513 – 514; "More precisely, Leibniz lets, HF = HG = a, HB = x, CB = y, CB ⊥ FG, CP ⊥ ACB. Since CBP is a right triangle, dy = PB ⁄ CB. Substituting y for CB, dy = PB ⁄ y. Multiplying through by y and taking the distance from B to P to be negative, -ydy=BP. Now since CBF is also a right triangle, CF = $\sqrt{((CB)^2 + (BF)^2)}$. Substituting y for CB yields CF = $\sqrt{(y^2 + (BF)^2)}$. But BF = a –x, so by substitution, CF = $\sqrt{(y^2 + (a - x)^2)} = \sqrt{(y^2 — 2ax + a^2 + x^2)}$. Similar considerations show that CG = $\sqrt{(y^2 + (a+x)^2)} = \sqrt{(y^2 + 2ax + a^2 + x^2)}$. In order to find the path FCG = CF + CG which is unique with respect to length, Leibniz differentiates and sets the resulting equation equal to zero: d • CF + d • CG = d • √(y2—2ax+ a2+x2) + d • $\sqrt{(y^2 + 2ax + a^2 + x^2)}$ = (ydy + xdx—adx) ⁄ CF + (ydy + xdx+ adx) ⁄ CG = 0. Rearranging terms yields: CF ⁄ CG = (a—x—ydy)dx ⁄ (a + x +ydy)dx. Substituting a-x for BF, and a+x for GB in turn yields: CF ⁄ CG = BF+BP⁄GB—BP = PF ⁄ PG. Trigonometry is now sufficient to show that if CF ⁄ CG= PF⁄ PG, then CP bisects FCG, and that the angle of incidence is therefore equal to the angle of reflection.



Figure# 21 Found in Lamborn Master's Thesis, p. 68 and Jeffrey McDonough's article, "Leibniz on Natural Teleology and the Law of Optics", p. 514

> Here Leibniz once again (i) constructs an equation for the path of the ray of light—this time taking into account the different resistances of the two mediums, (ii) applies his calculus to find the path that is unique with respect to ease (i.e. length times resistance), and (iii) uses trigonometry to show (a) that the ratio of the sine of incidence to the sine of refraction is inversely proportional to the ratio of incident velocity to the refractive velocity, and (b) that the ratio between the sine of the angle at which a ray of light strikes a refractive surface and the sine of the angle at which the ray is refracted is a constant determined by the mediums involved.[311]

With his final drawing and mathematical calculation for the MDP, Leibniz provides proof for the proportionality of sines and enters the element of time.

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311 McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 514 – 515; "Again, in greater detail, Leibniz lets HF = HG = a, HB = x, CB = y, $CB \perp FG$, $CP \perp ACB$. As before, FCG = CF + CG = $\sqrt{(y^2+ (a- x)^2)}$ + $\sqrt{(y^2+(a+x)^2)}$ = $\sqrt{(y^2 - 2ax + a^2+x^2)}$ + $\sqrt{(y^2+2ax + a^2+x^2)}$. Taking the resistance ofthe upper medium to the lower medium to be as f to g, then the measure of the ease of the path FCG = f • CF + g • CG = f • $\sqrt{(y^2 - 2ax + a^2+x^2)}$ + g • $\sqrt{(y^2+2ax + a^2+x^2)}$. In order to find the path FCG = CF + CG which is unique with respect to its ease (i.e. distance times length), Leibniz again differentiates and sets the resulting equation equal to zero: f(d • CF) + g(d • CG) = f(d • p(y2—2ax + a2+x2)) + g(d • p(y2+2ax + a2+x2)) = f • (ydy + xdx—adx)⁄CF + g • (ydy + xdx + adx)⁄CG = 0. Calculating as above, yields: CF⁄CG = (f • PF)⁄(g • PG). Trigonometry is now sufficient to complete the proof."

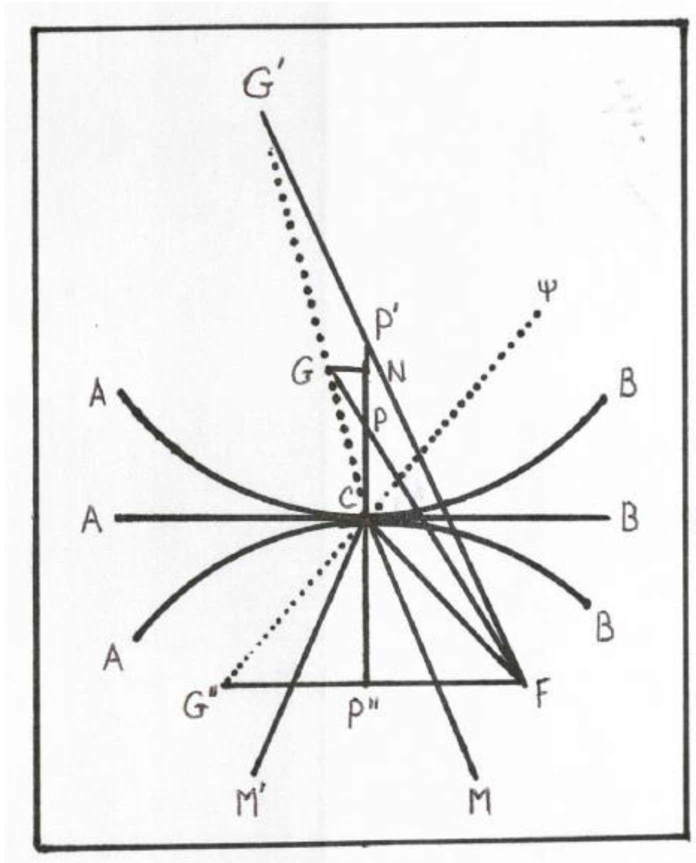

Figure #22, Found in Loemker, Leroy. *G.W. Leibniz, "Philosophical Papers and Letters",* p. 482

He says,

> Now it is easy to derive from this theorem the proportionality of sines. For let the ray *FC* strike the refractive surface *ACB* at *C*, and let the refracted ray, *CG*, be taken taken equal to the incident ray *FC*. Draw *FG* cutting the straight line *CP* perpendicular to the surface at *P*. From points *F* and *G* draw normal *FL* and *GN* to *CP* (Fig. 35). Now, since *CG* and *CF* are taken equal, it follows by the equation of the preceding paragraph that *PF* is to *PG* as *g* to *f*. Then because of the similar triangles *PLF* and *PNG* the sine *FL* will be to the sine
>
> *GN* as *g* to *f*, that is, reciprocally as the resistances of the media. And the sines of the angles of refraction will be proportional to the sines of the angles of incidence. This makes us see, finally, that the rule of the unique path, or the path most determined in length of time, applies generally to the direct and the broken ray, whether reflected or refracted, whether by plane or by curved surfaces, whether convex or concave, without distinguishing in the process whether the time is the longest or the shortest, though it is in fact the shortest when that which should provide the rule is taken into consideration, that is, the tangent planes; nature being governed, as it is, by sovereign wisdom, shows the general design throughout of controlling curves by straight lines or planes tangent to them, as if the curves were composed of these, although this is not strictly true.[312]

In this last drawing, the ray of light bends towards the normal. It does so in the shortest time.

[312] Loemker, p. 482 - 483

This, according to Leibniz, is the architectonic nature of nature.

With his *Most Determined Path*, Leibniz continues his effort to reconcile Descartes and Fermat. There are no special assumptions needed to apply the MDP to special cases of reflection. The MDP finds the path of a ray of light unique in respect to some quantity rather than minimal. "As a result, the possibility that a ray of light might maximize distance, speed, or ease does not present even a prima facie difficulty for his treatment, and he thus has no need to appeal to planes tangent to reflecting surfaces in order to handle non-standard cases of reflection."[313] His "ease" of quantity allows him to side with Descartes concerning light traveling faster through thicker mediums.[314] It also keeps him in line with the current thought concerning the corpuscular behavior of light. In his focus on reconciling the technical matters between the two sides, Leibniz is able to turn everyone's attention to the metaphysical aspects of light's behavior. He says, "This principle of nature, that it acts in the most determinate ways which we may use, is purely architectonic in fact, yet it never fails to be observed."[315] There is, in Leibniz's opinion a fundamental difference between architectonic and geometric determinations. "Geometric demonstrations introduce an absolute necessity, the contrary of which implies a contradiction, but architectonic determinations introduce only a necessity of choice whose contrary means imperfection – a little like the saying I jurisprudence: *Quae contra bonos mores sunt, ea nec facere nos posse credendum est.*"[316]

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[313] McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 515
[314] Terrill, p. 177, says, "Maupertuis situated his idea relative to the work of his predecessors: Fermat, Descartes, Leibniz and Newton. Fermat, assuming that the speed of light decreases with the density of the medium, had said that the sine law minimized time; for Leibniz (who assumed that light moves faster in denser media) the 'law' minimized 'resistance'. Newton also argued that velocity increases in direct proportion to density, but analyzed light as a steam of material particles attracted by the media through which they travel. They all agreed on the mathematical description of refraction, whereby the sines of the angles of incidence and refraction stand in a fixed proportion to each other.
[315] Loemker, p. 484
[316] Loemker, p. 484

Leibniz's *MDP* is a teleological law, an illustration of final cause as applied to physics. It aids in showing how prior events can be linked to subsequent events "by appealing to consequences of those subsequent events."[317] All of reality is like this. As Leibniz says in his opening in *TA* with : "I have shown on several occasions that the final analysis of the laws of nature lead us to the most sublime principles of order and perfection, which indicate that the universe is the effect of a universal intelligent power."[318]

[317] McDonough, "Leibniz on Natural Teleology and the Law of Optics", p. 518
[318] Loemker, p. 477

Chapter Five: Critical Analysis

In defense of Fermat and Leibniz, the thesis for this essay is that the charge against them is motivated by career building political gamesmanship on Maupertuis' part. Two other options are that the charge is philosophically or scientifically based. There are three reasons why the charge cannot be philosophically or scientifically based and five reasons why it is a politically motivated career move.

In accordance with the argument that there are three reasons against a philosophical or scientific basis, the following points are made. One, Maupertuis does not articulate a position on causation in general or final causation in particular. In the entire body of Maupertuian academic literature, there is nowhere a section on causation per se. The charge is therefore baseless from this standpoint.

Two, Pierre Fermat articulates no position on final causes either. Therefore, there is no causation claim made by Fermat that Maupertuis can point to and successfully argue against. Fermat's formal works are mathematical, and his correspondence is free from substantive discussion on causation. The closest he comes is in his work *Maxima et Minima* were he says, "Demonstratio nostra unico nititur postulato: *naturam operari per modos et vias faciliores et expeditiores".* As Osler translates, "Our demonstration works on a single postulate: *that nature operates by the easiest and most expedite ways and means.*"[319] This statement, however, is not an

[319] See page 69

explanation of why nature operates as it does. It is a description of how, and explanations of how nature works are not final cause arguments. Should Fermat have said that nature operates for the purpose of achieving "the easiest and most expedite ways and means", then Maupertuis would have had a position to argue against. Fermat does not word it that way, however. Therefore, from this standpoint, Maupertuis has no case against Fermat.

Three, Leibniz does articulate a position concerning final causation and its relation to physics, and Maupertuis agrees with him in both *Accord* and *Les Loix du Mouvement et Repos*. First of all, Maupertuis agrees with Leibniz in terms of the significance of the ancients and their influence. In the final section of *Accord* where he calls the academy to action and embrace his position by saying "We cannot doubt that all things are settled by a supreme being who impressed to matter forces which denote his power, has destined them to execute effects which mark his wisdom," he says,

> It seems a fact that the ancient philosophers possessed the first essays of this kind of mathematics. They had sought metaphysical property relationships in numbers and bodies, and when they said that God's occupation was geometry, they had probably understood it only of that science which compares the work of his power with the views of his wisdom. Too few geometers for the company they formed, what they have left us is little founded or intelligible. The artful perfection which has been acquired from them, makes us better able to reach success, and perhaps is more than compensation for the advantage that these grand geniuses had on us.[320]

Leibniz embraced the ancients in this regard as well. Early on in his career Leibniz turned away from total mechanism and embraced the final causes of the ancients as an equal in the search for

[320] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 426: Il semble que les anciens philosophes aient fait les premiers essais de cette espèce de mathématique; ils ont cherché des rapports métaphysiques dans les propriétés des nombres & des corps; & quand ils ont dit que l'occupation de Dieu était la géométrie, ils ne l'ont entendu sans doute que de cette science qui compare les ouvrages de sa puissance avec les vues de sa sagesse. Trop peu géomètres pour l'Enterprise qu'ils formaient, ce qu'ils nous ont laissé est peu fonde, ou n'est pas intelligible. La perfection qu'a acquise l'art depuis eux, nous met mieux à portée de réussir, & fait peut-être plus que la compensation de l'avantage que ces grands génies avoient sur nous.

philosophical wisdom.[321] An example of this is in *Discourse on Metaphysics* #19 and #20 where Leibniz urges an embracing of the utility of final causes and a rejection of over indulgence in materialistic philosophy which was a theme in *Phaedo*.[322]

Secondly, Maupertuis employs a customized version of Leibniz's intelligent design argument, complete with the theme of harmony and interpenetration of causation.

> We cannot doubt that all things are settled by a supreme being who impressed unto matter forces which denote his power, has destined them to execute effects which mark his wisdom; and the harmony of the two attributes is so perfect, that probably all effects of nature themselves could be deduced from each taken separately. A blind and necessary mechanics follows the designs of the most enlightened and free intelligence. And if our minds were large enough, it would also see the causes of physical effects, either in calculating the properties of bodies, or in seeking what was most suitable to execute them.[323]

Here Maupertuis says that a harmony exists between the wisdom of God and the mechanistic workings of nature.[324] Harmony is a concept used by Leibniz, but with meanings not strictly referring to the relationship between mechanics and God's wisdom. In *Elementa Verae Pietatis*,

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[321] In the introduction to the *Labyrinth and the Continuum* page xviii and xix, translator Richard Arthur says, "According to his later testimony Leibniz first began to doubt the scholastic philosophy he had learned in school as early as 1961 when, at the tender age of fifteen, he was in his first year at university of Leipzig. "…by the time Leibniz left university in the late 1660s he was firmly committed to finding a rapprochement between Aristotle and the moderns. Like his teachers Jacob Thomas and Erhard Weigel, he maintained that an Aristotelian account of substance was not only compatible with the new mechanistic physics of Descartes, Galileo, Hobbes, and Gassendi, but essential for its proper function." (See Richard Lamborn, "A Commentary on Gottfried Wilhelm Leibniz's Discourse on Metaphysics #19", p. 14)

[322] Ariew and Garber, p. 52 - 53

[323] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 425 – 426: On ne peut pas douter que toutes choses ne soient réglées par un Etre suprême qui, pendant qu'il a imprime a la matière des forces qui dénotent sa puissance, l'a destinée à exécuter des effets qui marquent se sagesse; & l'harmonie de ces deux attributs est si parfaite, que sans doute tous les effets de la nature se pourraient déduire de chacun pris séparément. Une mécanique aveugle & nécessaire suit les desseins de l'intelligence la plus éclaire & la plus libre, & si notre esprit était assez vaste, il verrait également les causes des effets Physiques, soit en calculant les propriétés des corps, soit en recherchant ce qu'il y avait de plus convenable à leur faire exécuter.

[324] In his essay, "On the Very Concept of Harmony in Leibniz" (*The Review of Metaphysics.* 2000, Vol. 54, No. 1, p. 124, Laurence Carlin acknowledges that Leibniz does indeed see the relationship between mechanism and divine wisdom as a feature of harmony, but he sees it as problematic. "…there are numerous passages where Leibniz speaks of a thing or an object being harmonious. The idea here seems to be that if the concept of the relevant thing is resolved into its component concepts in such a way that a range of properties may be inferred from it, the object exhibits a certain level of harmony. But, more often, Leibniz claims that harmony is exhibited by a collection of entities which do not, in any obvious sense, constitute a thing or an object. For example, Leibniz envisioned the totality of efficient causes as harmonizing with the totality of final causes."

Sive De Amore Dei Super Omnia (1679?), he says, "Harmony is unity in variety….Harmony is when many things are reduced to some unity. For where there is no variety, there is no harmony. Conversely, where variety is without order, without proportion, there is no harmony. Hence, it is evident that the greater the variety and the unity in variety, this variety is harmonious to a higher degree."[325] In *A New System of Nature* (1695) Leibniz promotes the hypothesis that the relationship between body and soul in the world can be understood as a divinely inspired harmony. It can be argued that Leibniz would agree that this can extend to body on body contact in mechanics.

The idea that the universal harmony of bodily interaction can be explained by efficient causes alone without final causes and final causes without efficient causes is Leibniz's idea in *Specimen Dynamicum.*

> These two kingdoms everywhere interpenetrate each other without confusing or disturbing their laws, so that the greatest obtains in the kingdom of power at the same time as the best in the kingdom of wisdom. But we had promised to establish here the general rules of effective forces, rules which we can then use in explaining particular efficient causes. Next, I arrived at the true way of measuring forces, indeed, I arrived at the very same measure but in widely different ways, the one *a priori*, from a very simple consideration of space, time, and action…the other, *a posteriori*, namely, through measuring the force by effect it produces in consuming itself.[326]

The idea that mechanics is blind and necessary, and that it functions according to the design of a free and clear intellect is Leibnizian in its teleological order, but Spinozist in terms of mechanics being blind and necessary. Maupertuis is Leibnizian in his promoting of the idea that mechanics follows from metaphysics. Metaphysics is the principle, and mechanics is the consequence. Leibniz indicates this to be the way reality works in *Unicum, Opticae, Catoptricae & Dioptrice Principium* when he writes, "We have therefore reduced that all the laws of rays proven from

[325] Carlin, Laurence, *The Review of Metaphysics.* 2000, Vol. 54, No. 1, "On the Very Concept of Harmony in Leibniz", p. 101
[326] Ariew and Garber, p. 126 – 127

experience to have been confirmed by pure geometry and calculus are from final causes, a single restricted principle."[327]

Maupertuis' mix of these Leibnizian themes with each other, along with an inclusion of Spinozist mechanics, is intended to appear Leibnizian while at the same time appealing to de Mairan's Spinozist leanings. It can even be argued that the last line of Maupertuis' statement is from Descartes. The entire point made by Descartes regarding the application of final causes applied to mechanics is that he could not get his mind around the end purposes which God might intend in such a mix. The inability of the human mind to comprehend the relationship between a priori deduction and a posteriori methodology, and therefore to make the jump from intermediate principles to universal principles, is a philosophy that Maupertuis carried with him from *du Repos* forward. This statement in *Accord* should be interpreted as Maupertuis finding agreement with Cartesians and Leibnizians, as well as those influenced by Spinoza, for the purpose of persuading them to affirm his idea of action. He wants them to know that he agrees that nature works according to the Leibnizian interaction between efficient and final causes, that mechanics is blind and necessary, that such a mechanics operates according to the design of God in harmony, that mechanics can be explained by final causes apart from efficient causes, and visa versa, and that it is only our lack of mental ability that keeps us from seeing the true nature of cause and effect.

Thirdly, Maupertuis thinks, as did Leibniz, that the wonders of physical reality point to the existence of God. He says in the opening of his 1746 work, ""Whether we abide withdrawn in ourselves, or we come out to go through the marvels of the universe, we find many proofs of

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[327] Jeffrey McDonough's Homepage – Harvard University, www.people.fas.harvard.edu/~jkmcdon/; Translations, "A Unitary Principle of Optics, Catoptrics, and Dioptrics"

the existence of an all powerful and all wise being."[328] Leibniz says in *DM* #19, "Anyone who sees the admirable structure of animals will find himself forced to recognize the wisdom of the author of things."[329]

Fourthly, Maupertuis employs the idea of the principle of perfection, as did Leibniz.

> I could have departed from these laws, such as the mathematicians give, and such as experience confirms, and seek the characteristics of wisdom and power of the supreme being. However, as those from which we have data have relied on hypothesis which were not purely geometrical, and that by their certainty seem not founded on rigorous demonstrations, I thought about it being more useful to deduce the laws from the attributes of the omnipotent and wise supreme being. If what I find in this way follows the same effect we observe in the universe, is it not the strongest expectation that this being exists, and that he is the author of these laws? But, one might say, although the rule of motion and rest have not hitherto been demonstrated by hypothesis and experiments, they are the necessary consequences of the nature of the body: and there being nothing arbitrary in their establishment, do you assign a providence which is the necessary effect? Although the laws of motion and of rest are essential consequences of the nature of body; it proves even more the perfection of the supreme being: This is that all things are so ordained, a blind and necessary mathematics executes what the most enlightened intelligence prescribes.[330]

Leibniz says in *Tentamen Anagogicum*, "I have shown on several occasions that the final analysis of the laws of nature leads us to the most sublime principles of order and perfection, which indicate that the universe is the effect of a universal intelligent power."[331]

It appears from these themes employed in *Accord* and *Les Loix du Mouvement et du Repos* that Maupertuis is a Leibnizian himself, but this is not necessarily the case. It is more likely that he is familiar with *Specimen Dynamicum* and a few other well circulated

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[328] *Histoire de l'Académie Royale des Sciences et des Belles Lettres,* 1746, p. 265

[329] Ariew and Garber, p. 52

[330] *Histoire de l'Académie Royale des Sciences et des Belles Lettres,* Année MDCCXLVI, p. 279: J'aurais pu partir de ces loix, telles que les Mathématiciens les donnent, & telles que l'expérience les confirme; & y chercher les caractères de la sagesse & de la puissance de l'Etre suprême. Cependant, comme ceux qui nous les ont données, se sont appuies sur des hypothèses qui n'étaient pas purement géométriques, & que par la leur certitude ne parait pas fondée sur des démonstrations rigoureuses; j'ai cru plus sur & plus utile de déduire ces loix des attributs d'un Etre tout puissant & tout sage. Si celles que je trouve par cette voie, sont les mêmes sui sont en effet observes dans l'Univers, n'est-ce pas la prévue la plus forte que cet Etre existe, & qu'il est l'auteur de ces loix? Mais, pourrait-on dire, quoique les règle du Mouvement & du repos n'aient été jusqu'ici démontres que par des hypothèses & des expériences, elles sont peut-être des suites nécessaires de la nature des corps: & n'y aidant rien eu d'arbitraire dans leur établissement, vous attribuez a une Providence ce qui n'est l'effet que de la Nécessite? S'il est vrai que les loix du Mouvement & du Repos soient des suites indispensables de la nature des Corps; cela même prouve encore plus la perfectionne l'Etre suprême: C'est que toutes choses soient tellement ordonnées, qu'une Mathématique aveugle & nécessaire exécute ce que l'intelligence la plus éclairée & la plus libre prescrirait.

[331] Loemker, p. 477

Leibnizian works, and employs his familiarity strategically to achieve his goal. What his employment of these themes do demonstrate is that Maupertuis does not accuse Leibniz of error because of a philosophical or scientific disagreement regarding the legitimacy and applicability of final causes. The only criticism by Maupertuis of Leibniz to be found in the Maupertuis corpus is the nature of the monad and the proper view of *vis viva.* In 1740, Maupertuis says that *vis viva* is not a universal principle because it does not solve for both elastic and inelastic collisions. Then, in 1746 he employs *vis viva* to solve for both elastic and inelastic collisions.

There are five reasons why the charge of error should be interpreted as a politically motivated career move on Maupertuis' part. One, he is vague concerning the wording in the focal passage of the identity of the mathematicians and the phrase "up to a point". Two, the structure of the 1744 *Accord* is so that an appeal can be made to all political constituencies within the Paris Academy. Three is the fact that de Mairan is a Cartesian, and Maupertuis is aware that he is being considered for the Berlin Presidency. Four is the Konig affair with the admission to d'Arcy, and five are all of the editorial changes made by Maupertuis in subsequent editions of *Accord.*

Maupertuis identifies two groups of mathematicians in *Accord.* First, he mentions the group who embrace the sentiment of Fermat, Leibniz being the loudest, and second, he identifies the group who finds the application of final causes to physics to be disgusting. The political nature of *Accord* is revealed in the fact that Maupertuis does not identify any individual by name, except, of course, Leibniz. Furthermore, he does not clarify "the point" at which he no longer shares the latter group's disgust with the application of final causes to physics. This is so as to be acceptable enough to members of both groups without being found so unacceptable as to be ostracized. Knowing that he is being considered for the Berlin Academy presidency, his primary goal is not to be philosophically or mathematically groundbreaking. His primary goal is to be politically acceptable to Frederick based on enough shared intellectual common ground between himself and Frederick's constituent groups in the king's academy.

Based on the contents of *Accord*, the two groups of mathematicians are most likely the Cartesians and the Leibnizians. The Cartesians are led by the new academy secretary de Mairan, and the Leibnizians are presently under the influence of the German mathematician/philosopher Christian Wolff. Cartesian mechanical philosophy is completely opposed to any application of final causes to physics. Leibnizian mechanical philosophy, on the other hand, emerges from out of an understanding of nature based on final cause. By recognizing both positions in a generic sense in *Accord*, Maupertuis successfully plays to the center without alienating either group. This explains the complimentary tone by which he mentions both groups.

The second reason the charge of error should be interpreted as a political move is the structure and content of *Accord* itself. *Accord Between Different Laws of Nature Which at First Seemed Incompatible* is a speech designed to win votes from all parties. It appeals to Newtonians, Leibnizians, and Cartesians. It has the elements of a persuasive speech in terms of an introduction, the establishment of a problem, the need for a solution to the problem, the satisfaction of the problem by presentation of the solution, and an appeal to the audience to adopt the solution.[332] Maupertuis presents the problem as the inability of mathematicians down through history to solve for the incompatibility between reflection and refraction. This incompatibility, he argues, is a puzzle that must be solved for the scientific community at large to move forward, and he presents his new paradigm of "least action" as the solution to the puzzle.

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[332] See Monroe, Alan H. and Douglas Ehninger. *Principles of Speech Communication.* Foresman: Glenview, Il. , 1969; For an online presentation of the Monroe outline, see "Persuasive Speech Outline – Monroe's Motivated Sequence, www.mindtools.com>CommunicationSkills"

His appeal for acceptance is a connection between science and religion and a common recognition of the wisdom of the ancients who conceived that the language of God as geometry.[333]

In his introduction, Maupertuis recognizes the general consensus among early modern mathematicians and scientists that the mathematics of refraction produced contradictory results from what they all understood to be the laws of nature. He says, “We should not require that the different means we have to increase our knowledge lead us to the same truths, but it would be overwhelming to see that the propositions that we give as fundamental truth were to be found contradicted by reasonings of geometry, or by the calculations of algebra.”[334] Maupertuis follows this opening line with an emotional appeal to the intellect. The mathematical calculations for refraction contradict the known laws of nature, and the consequence, as Maupertuis knew many of them were thinking there that April day, was thinking nature. In refraction, a decision was being made choosing time over distance in the path through different mediums!

Having gotten their attention with the problem, Maupertuis turns to the recent attempts made in history to solve the problem, by presenting the three most well-known approaches to refraction at that time – the approaches of Descartes, Newton, and Fermat. He declares Descartes’ approach to be insufficient, but praiseworthy. Finding Descartes’ approach insufficient but praiseworthy is a Leibnizian approach to Descartes on light, which pleases the Leibnizians.[335] This does not necessarily make de Mairan and the Cartesians happy, but

[333] See Plutarch, *Convivialium disputationum, liber 8.2.*
[334] *Histoire de l’Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 417: On ne doit pas exiger que les différents moyens que nous avons pour augmenter nos connaissances, nous conduisent aux mêmes vérités, mais il serait accablant de voir que des propositions que la Philosophie nous donne comme des vérités fondamentales, se trouvassent démenties par les raisonnements de la Géométrie, ou par les calculs de l’Algèbre.
[335] Leibniz praises Descartes’ optics in *Unicum Opticae, Catoptricae, et Dioptricae Principium*; “…it seems that the way in which Descartes explains the reflection as well as the refraction of light by an imitation of the motion of

Maupertuis is going to placate them later on in the accusation of error. He mentions Newton's lament and the application of gravitational attraction, but does not promote Newtonian attraction. He compliments Clairaut's memoire promoting Newtonian attraction over against Descartes.[336] This is an honorable mention for his Lapland partner, Clairaut, another Paris Academy Cartesian in the audience. It also pleases the Newtonians.

Then he turns the audience's attention to the approach of Fermat and the theoretical physics debate occurring in the academy at that time. He drives his audience straight into the debate over the speed of light in differing mediums, the experimental proof and epistemological certainty of which had not yet been established. At the time of this delivery of *Accord*, neither the Newtonians, Leibnizians nor Cartesians knew for certain whether light moved faster in thinner or thicker mediums. Everyone, including Maupertuis, was guessing. The perplexity of the argument was that if a ray of light was corpuscular, and water was more dense than air, then when it struck the water it should behave like a tennis ball and break away from normal. This was intuitively convincing. However, since this made so much sense to everyone, then it also made sense to them that the tennis ball, once served by the force of the racket, moved much faster in air than in water. In fact, it stopped in the water. They found out that the same was true with bullets fired from a gun. Water slowed the bullet down and brought it to a halt. Such was the unavoidable reasoning behind hard bodies moving from air to water. Yet, precisely because air was less dense than water, Snell's calculation showed the constant being greater than one, and the ray of light moving towards perpendicular. If Snell was correct, then why did Descartes commit to his tennis ball experiment as well as his thinking that light traveled faster in denser

other bodies is worthy of his genius and not to be rejected but only emended." See McDonough's Homepage translation.

336 See Alexis Clairaut's 1739 Paris Academy memoire "Sur les explications Newtonienne and Cartesienne de la reflection de la lumiere"

mediums, while at the same time embracing Snell's law. That was paradoxical. If not outright contradictory. How could a ray of light be corpuscular and yet behave contrary according to the mathematics of refraction?

Notably absent in *Accord* is any mentions of Huygens, who said that light was a wave function which moved faster in air than in water.[337] The reason for this omission is probably because the argument concerning the corpuscle – wave debate was not as heated as the argument over the speed of light in differing mediums. Newton was accepting of Huygens' wave function idea, so the fight was simply not there.

Maupertuis' focuses the problem of incompatibility on Fermat's "resolving" of refraction by embracing least time as the solution over against shortest distance. This is a perceived

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[337] Sabra, p. 216 – 218, describes Huygens on refraction as follows: "In the figure (Fig. VIII. 3) let *AC* be a plane wave-front which obliquely strikes the separating surface *AB* at *A*. *AC* is perpendicular to the direction of incidence *DA*. Let the distance *CB* (parallel to *DA*) be equal to $v_i t$, where v_i is the velocity of the medium above and *t* the time required for *C* to arrive at *B*. In the same time, the secondary (See Sabra drawing) wave generated at *A* will have traveled in the medium of refraction a distance equal to $v_r t$, where v_r is the velocity in this medium.

Supposing, then, that

$$\frac{v_i}{v_r} = n,$$

a constant *greater* or *less* than unity, the circumference *SNR*, having *A* as center and a radius equal to

$$\frac{1}{n}\, CB,$$

will give the position of the secondary wave at the time when the part of the incident wave-front at *C* arrives at *B*. The other arcs in the figure are similarly drawn (with points *K* as centres and radii that are equal to the distances *LB* each divided by *n*) by considering the wave-fronts *KL* in order. All these arcs will have as a common tangent the straight line *BN* which is the same as the tangent from *B* to the arc *SNR*. *BN* therefore gives the position which the wave-front has reached successfully by taking the positions *LKO* in order. And the direction of propagation after refraction is represented by the perpendicular *AN*.

From this the sine relation is readily obtained. For since the angle of indicence *EAD* is equal to the angle *CAB*, and the angle of refraction *FAN* is equal to *ABN*, it follows that

$$\frac{\sin i}{\sin r} = \frac{CB}{AB} x \frac{AB}{AN} = \frac{CB}{AN} = \underline{v_i t}/v_r t = v_i v_r = n.$$

This law implies that when the angle of refraction is smaller than the corresponding angle of incidence, the velocity must have been diminished by refraction. And since light in passing from a rare into a dense medium is deflected towards the normal, it must be concluded that the velocity of light is greater in rarer media. Huygens' law is the same as that deducted by Fermat (from the least-time principle) and maintained by Pardies and Ango."

employment of final cause on Fermat's part by the Cartesians, and Maupertuis splits the politics of the academy down the middle between those mathematicians who embrace Fermat's solution and those who do not. Of those who embrace Fermat, he says that Leibniz embraced it the most, and he plays to the Leibnizians in this regard with a compliment of Leibniz's "elegant and charming" metaphysical principle. This principle, of course, is most likely the "Interpenetration of Causation" doctrine in which Leibniz argues that nature can be equally understood mechanically (efficient cause) and teleologically (final cause, or, the wisdom of God).

Maupertuis knows, however, that the Cartesians will have none of that, so he plays to their sentiments by declaring that Fermat and Leibniz had gotten their principles turned around and had erroneously applied final causes to physics. Maupertuis plays politics with this charge when he claims to agree with these mathematicians who also charge Fermat and Leibniz with error, but will not name them. He artfully does not name those mathematicians who agree with Fermat either, except for Leibniz. Like a politician "moving to the middle" in a campaign for office, Maupertuis says that he agrees with those mathematicians who side against Fermat and Leibniz, "up to a point", but does not say exactly where that point is. This leaves the question open between the Leibnizians and Cartesians as to exactly whose side Maupertuis is on. This also leaves the question open for King Frederick. Neither de Mairan nor the King can be "pointedly" offended.

The "point" cannot be the question of the speed of light in different mediums, because that is not a teleological question. Maupertuis is being purposefully ambiguous here. If interpreters of Maupertuis must go beyond this ambiguity, they should consider that it is either the specific issue of employing time over space and giving light the characteristic of thinking nature, or it is the general Cartesian argument that final causes have no place in the mechanics of physics at all. At the end of *Accord* Maupertuis admonishes the audience with the Leibnizian belief that science and philosophy should pursue understanding by employing both efficient and final causes. This steers his audience away from an understanding that Maupertuis actually embraces the Cartesian argument. Another possible understanding of where the "point" is is in terms of the application of least time in refraction, because he abandons the time element in his pronouncement of least action.

Perhaps the most brilliant stroke of political genius in *Accord* is Maupertuis' description of action. Having established the problem and the failed attempts down through history to solve it, Maupertuis' announces his solution to the problem, i.e., his solving of the puzzle. Light does not prefer time over space, or visa versa, he argues. Rather, light takes the path of minimum quantity – least action. The action of a ray of light, he says, is understood as distance multiplied by speed. This is the Cartesian understanding of the nature of body as extension. At the same time, however, he says that the path taken by the ray of light in this action is the path of greatest "advantage". This is Fermat in *Maxima and Minima*, and it is especially de Mairan's understanding of Fermat. He has skillfully appealed to both sides of the debate! This is mechanism combined with thinking nature! Maupertuis gives the Cartesians in his audience, as well as the Leibnizians who support Fermat, something they both like. This has the same effect on the two groups in Berlin.

The problem with Maupertuis' account of action is that it is meaningless. Maupertuis' wording for least action contains the elements of distance, and speed, but with no time element or force of cause.[338] He leaves his audience without certainty whether he is talking about the

[338] Jourdain, p. 420, comments on Maupertuis' adding of time in 1746. He says, "In the third part, Maupertuis states the general principle that 'when some change occurs in nature, the quantity of action necessary for this change is the smallest possible,' and adds: '*The quantity of action* is the product of the mass of the bodies by their velocity and by the space which they describe. When a body is transported from one place to another, the action is greater in

Newtonian mechanics of accelerating mass, *ma*, Cartesian momentum, *mv* or Leibnizian living force (kinetic energy), mv^2. He inappropriately multiplies space by speed (velocity) without time, which does not work.[339] Distance *x* velocity is meaningless. This is, of course, assuming

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proportion as the mass is greater, as the velocity is greater, and as the path by which it is transported is longer.' From this principle, Maupertuis deduces the laws of of impact of hard (or inelastic) and elastic bodies, and of the lever. When treating of impact of hard (inelastic) bodies of masses A and B, which move with the velocities *a* and *b* respectively in a straight line and in the same sense, Maupertuis considers the spaces (*a* and *b*) *described in a certain time* (the unit of time), so that *m.v.s* becomes mv^2, as Mach notices, and so he points out Maupertuis' inconsistency."

[339] This is picked up on by both Mach and Jourdain. In his work *The Science of Mechanics, "A Critical and Historical Account of It's Development"* (Open Court Publishing Company: Chicago, Il., 1919, p. 364), Mach says, "Maupertuis enunciated, in 1747, a principle which he called '*le principe de le moindre quantite d'action',* the Principle of Least Action. He declared this principle to be one which eminently accorded with the wisdom of the creator. He took as the measure of the 'action', the product of the mass, the velocity, and the space described, or *mvs.* Why, it must be confessed, is not clear. By mass and velocity, definite quantities may be understood; not so, however, by space. If, however, unit of time is be meant the distinction of space and velocity in the examples treated by Maupertuis is, to say the least, perculiar. It appears that Maupertuis reached this obscure expression by an unclear mingling of his ideas of *vis viva* and the principle of virtual velocities. Its distinctiveness will be more saliently displayed by the details.

2. Let us see how Maupertuis applies his principle. If *M*, *m* be two inelastic masses, *C* and *c* their velocities before impact, and *u* their common velocity after impact, Maupertuis requires, (putting here velocities for spaces,) that the action expended in the change of the velocities in impact shall be a minimum. Hence, $M(C-u)^2 + m(c-u)^2$ is a minimum; that is,

$$M(c-u) + m(c-u) = 0; \text{ or } u = \frac{MC+mc}{M+m}.$$

For the impact of elastic masses, retaining the same designations, only substituting *V* and *v* for the two velocities after impact, the expression $M(c-v)^2 + m(c-v)^2$ is a minimum; that is to say,

$$M(C-V)dV + m(c-v)dv = 0 \ldots\ldots\ldots\ldots\ldots..(1)$$

In consideration of the fact that the velocity of approach before impact is equal to the velocity of recession after impact, we have

$$C - c = -(V-v) \text{ or } C + V - (c+v) = 0\ldots..(2)$$

and

$$dV - dv = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

The combination of equations (1), (2), and (3) readily gives the expressions for *V* and *v*. These two cases may, as we will see, be viewed as processes in which the least change of *vis viva* by reaction takes place, that is, in which the *least – counter* work is done."

On page 367, Mach comments on Maupertuis drawing that Maupertuis has no theory of action. Mach says, "Following the precedent of Fermat and Leibniz, Maupertuis also treats by his method the *motion of light.* Here again, however, he employs the notion 'least action in a totally different sense.

that Maupertuis is thinking in terms of light as a hard body, which he probably is. Therefore, the ray of light starts out at point *A* and travels to point *B*. This implies a distance, not an instantaneous propagation. Two different mediums might necessitate the inversion in relation to the differing velocities to find a constant, but after that, distance and velocity of any inelastic body without time is void of meaningful content. For example, a car traveling at sixty miles per hour over three hundred miles does not mean 18,000 anything! In order for Maupertuis' action to have meaning, he has to account for time taken and the force which drives the body to move. He does neither, therefore, his action has neither actual movement nor meaning.

Maupertuis also leaves his audience uncertain about his notion of cause and effect. Without spelling out the nature of force causing the ray of light to move, how was one supposed to know that the effect on the amount of action was least? He simply says so by virtue of the inverted relationship to the speed of light in the two differing mediums. His reference is to Snell's law as understood by Fermat: "All the phenomena of refraction agree now with the

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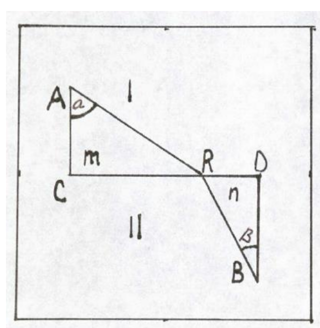

Figure #23 Mach's geometrical drawing of refractive optics in argument against Maupertuis, p. 367

The expression which for the case of refraction shall be a minimum, is *m.AR + n.RB*, where *AR* and *RB* denote the paths described by the light in the first and second media respectfully, and *m* and *n* the corresponding velocities. True, we really do obtain here, if *R* be determined in conformity with the minimum condition, the result sin*a*/sin*b* = *n/m = const.* But before, the 'action' consisted in the *change* of the expressions mass *x* velocity *x* distance; now, however, it is constituted of the *sum* of these expressions. Before, the spaces described in units of time were considered; in the present case, the *total* spaces traversed are taken. Should not *m.AR – n.RB* or (*m – n*)(*AR - RB*) be taken as a minimum, and if not, why not? But if we accept Maupertuis' conception, the reciprocal values of the velocities of light are obtained, and not the actual values. It will thus be seen that Maupertuis really had no principle, properly speaking, but only a vague formula, which was forced to do duty as the expression of different familiar phenomena nor really brought under one conception."

grand principle that nature, in the production of its effect, is always by the most simple ways. This principle follows that when light passes from one environment into another, the sine of its angle of refraction is the sine of its angle of incidence in inverse ratio from the speed of light in each environment."[340]

Why would Maupertuis offer such a nebulous understanding of action as a new paradigm? The reason is because he knows that if he introduces mass and force into the description of action, then he will be dragged into arguments about conservation which would lead to least action of momentum and/or living force arguments in the minds of his audience. Such a disagreement within his audience would be counterproductive to his goal in *Accord* of being acceptable to both sides. Maupertuis had studied both Leibnizian calculus and *vis viva* in the years preceding his appointment to the membership to the Paris Academy. The omission is purposeful. Maupertuis is not trying to be philosophically or scientifically correct. He is trying to be politically acceptable, especially to Frederick, who is looking for someone has demonstrated that he can work with warring factions within the Berlin academy. In his 1744 description of action and the path taken by a ray of light, Maupertuis said everything to everyone, and nothing to anyone.

The third reason that the charge of error is political is that the charge and the terminology that Maupertuis uses are all found in one of de Mairan's four memoires on optics. Maupertuis merely parrots de Mairan. Jean – Jacques de Ortous Mairan (1678 – 1771) is a Cartesian who was appointed as secretary of the Paris Academy in 1743 to succeed another Cartesian,

[340] *Histoire de l'Académie Royale des Sciences.* Année MDCCXLIV. Avec les Mémoires de Mathématique & de Physique, pour la même année. Tirez des Registres de Cette Académie, p. 424: Tous les phénomènes de la réfraction s'accordent maintenant avec le grand principe, que *la Nature dans la production de ses effets agit toujours par les voies les plus simples.* De ce principe suit que *lorsque la lumière passé d'un milieu dans un autre, le sinus de son angle de réfraction est au sinus de son angle d'incidence en raison inverse des vitesses qu'a lumière dans chaque milieu.*

Fontenelle. De Mairan makes his charge of error in his 1723 memoire entited "Suite des researches physic-mathematiques sur la reflexion des corps".[341]

All four of de Mairan's works on optics can be found in the Paris Academy memoires. They are, in order by date, "Reserches Physico-Mathematiques sur la Reflexion des Corps" (1722), "Suite des Reserches Physico-Mathematiques sur la Reflexion des Corps" (1723), "Sur la Propagation du son dans les differens tons qui le modifient" (1737), and "Troisieme partie des Reserches Physico-Mathematiques sur la Reflexion des Corps" (1738)." The common theme running through these four works is the early modern term for the mixing of physics and mathematics, "physico – mathematics". This is the scientific revolution term which began with Descartes and was well known and used by the time of Maupertuis.

The Cartesian influence on the early modern understanding for this mixture is articulated by Archibald Pitcairn in his 1718 work *The Philosophical and Mathematical Elements of Physick, In Two Books, The First Containing the Theory*: *The Second Continuing the Practise*:

> I do advise indeed all diligently to consider the principles of the Cartesian philosophy, and to compare them with those of Democritus, so far as geometry will conduct them, especially that part of it whereby are demonstrated the laws of motion; and what they shall find most conformable to those laws, may be reserved as of great service in the true theory of Physick; and therefore, as a qualification for the study of medicine, I rather recommend an acquaintance with mathematics than with the philosophy which is so much in esteem.[342]

Descartes was involved in physico – mathematics early in his career. In his essay "Physico – Mathematics and the Search for Causes in Descartes' Optics – 1619-1637", John Schuster says that by this term Descartes and Beekman meant in a broad sense, "a questioning of the Scholastic

[341] De Mairan succeeded Fontanelle as associate secretary of the Paris Academy in 1743 (see Westfall, Richard. "Mairan , Jean-Jacques d'Ortous de", *The Galileo Project,* Rice University). In his book *Science and Immortality, "The Eloges of the Paris Academy of Sciences" (*1699 – 1791) (University of California Press: Berkeley, Ca., 1980, pp. 35 – 40), Charles Paul refers to de Mairan as a "maverick Cartesian" who championed Descartes' vortex theory and refused the notion of non-mechanical action at a distance. Paul, p. 36, says, "In 1722, 1723, 1738 and 1740 Mairan published four long memoires,…on 'Reserches Physico-Mathematiques sur la reflexion des corps" in which he argued that diffraction was only a species of refraction, and refraction, a species of reflection."

[342] Pitcairn, Archibald. *The Philosophical and Mathematical Elements of Physick, In Two Books, The First Containing the* Theory: *The Second Contining the Practise,* London, 1718, p. 7

Aristotelian view of the mixed mathematical sciences as subordinate to natural philosophy, non-explanatory, and merely instrumental." More pointedly, Schuster says that the core of physico – mathematics was "the belief that solid geometrical results in the mixed mathematical sciences offered windows into the realm of natural philosophical causation in the sense that one could read natural philosophical causes out of geometrical representations of such mixed mathematical results, and hence, in a way, 'see the causes'."[343] This is what Descartes thought.

Physico-mathematics grew in popularity within the mechanistic philosophy of the early modern period, and by the time of Maupertuis, found one of its greatest proponents in d'Alembert. In his essay "The Evolution of the term 'Mixed-Mathematics' ", Gary Brown says,

> For d'alembert rational mechanics was mathematics, at least 'mixed mathematics'. Mechanics was an extension of geometry and calculus to the equally ideal science of motion. All ideas originated in experience, and our most basic sensory impressions of geometrical extension and temporal succession provided the primitive materials with which rational mechanics began. In his *Traite de Dynamique* of 1743 he introduced the concept of body as impenetrable extension moving in space. Such perfectly hard bodies interacted by contact according to laws that were a priori derivable from reason. No experimental verification was given and the theory was applied to a wide range of dynamical systems. 'Motive causes' such as force and mass were thrown out.[344]

It is under the influence of this concept that de Mairan submits his four works to the Paris Academy. The other major theme running through de Mairan's four memoires is Newton's work on color and the refrangibility differences according to color.

D'Alembert omits force and mass as causes in his dynamical system. This is a possible reason why Maupertuis omits them in his definition of action in *Accord*. However, seeing that Maupertuis includes them in 1746, he either rejected d'Alembert's position sometime in 1745, or he omitted them in 1744 because he knew that to include them would force him to include *vis viva* in his definition of action. The latter is the position of this essay. In 1744, Maupertuis

[343] Schuster, p. 468
[344] Brown, Gary, *Journal of the History of Ideas.*1991, Vol. 51, No. 2, "The Evolution of the Term Mixed-Mathematics," p. 97

defines action as a Cartesian body by extension idea with distance *x* speed. He knows that this is all the more he needs to please de Mairan seated in the audience.

Although he mentions Descartes and Fermat in other parts of the four works, the only place where de Mairan mentions Descartes, Fermat, and Leibniz together is in his 1723 work "Suite des Reserches Physico-Mathematiques sur la Reflexion des Corps". In this work, de Mairan has a section on refraction in which he refers to the Descartes – Fermat debate and Leibniz's 1682 Acta Eruditorum submission "A Unitary Principle of Optics, Catoptrics, and Dioptrics". It is to this section of de Mairan's 1723 work that Maupertuis refers in *Accord.*
In section LXII, remark 12, de Mairan refers to corollaries, LX Corol. #26 and LXIV Corol. #27 which refer to the fundamental argument in the Descartes – Fermat debate as he saw it, the bend of light through differing mediums in refraction. The debate centered around the reaction of light as it passed from the less dense medium to the more dense medium. De Mairan believes that Descartes has evidence on his side. Rays of light move with more ease in more dense mediums like water or glass. Rays of light which move from less dense mediums to more dense mediums bend away from perpendicular if the difficulty of movement is greater, while rays of light which move into denser mediums with greater ease bend towards the perpendicular – both of which are contrary to one another. Due to the refractive line being longer than the straight reflective line, the time in refraction is longer. The shortest path must necessarily be in 'the most difficult medium' where the ray moves slower. The shortest path, therefore, is in air, and the longer path is in the more dense medium. De Mairan says that the path of refraction is actually contrary to all this. The longer, more sharply slanted path is in the air and away from perpendicular, and the shorter path slants towards the perpendicular in water or glass. These points are in line with the Cartesian paradox.

De Mairan offers the rest of corollary 27, and all of corollaries 28 and 29, as mathematical "proof" of his position. Then, in LXVI. Corol. 30, he says, "From which it follows that *in the case of the same quantities, and of the same relationship of forces before and after impact, the time of reflection is equal to that of refraction, and that in general, in both reflection and refraction, the path that suites the mobile is not that of the shortest time."* In LXVI. Corol. 31, de Mairan says, *"It follows again that in general, the path of reflection, no more than that of refraction, is not the shortest length."*

After these positions, de Mairan enters into the Descartes – Fermat debate, and Leibniz's position in it. This is the section from which Maupertuis drew the words and concepts he used to phrase the focal passage. In LXVIII, remark 14, he says that Fermat argued against Descartes concerning the path of light in passing from a less dense and less resisting medium into a denser and more resisting medium. Fermat argued that, in this situation, light approaches the perpendicular. de Mairan says of Fermat's entrenched, geometric argument, "In effect, if we receive his principle and suppose that light employs a time to pass from one medium to another, it will necessarily confess the consequence, that light passes by the path of most short duration." He equates Fermat's position with a harmonious relationship between mechanism and "views customarily attributed to the theory of final causes."[345] That is, light moves according the most simplest path. This act of simplicity, de Mairan asserts, is final cause.

De Mairan says that Leibniz, the "grand protector" of final causes, agreed with Fermat and cites Leibniz's 1682 work on optics as reference point. He says, however, that in the case of light moving towards the perpendicular in the more resistant medium in which it moves more slowly, Leibniz agreed with Fermat only out of a sense of obligation. He says that Leibniz

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[345] *Histoire de l'academie royale des sciences*, Avec les Memoires de Mathematique & de Physique, pour la meme annee. Tirez des Registres de Cette Academie, Rue S. Jacques : Paris, Annee M.DCCXXIII, p. 380 – 381

employed his calculus of maxima and minima, "conjointly with the differential calculus" to get this done. The real problem, de Mairan notes, was that in the process of agreeing with Fermat in order to promote final causes, Leibniz attempts to discredit Descartes' mechanistic philosophy. He complains that, not only did Leibniz call Descartes erroneous for abandoning final causes in this work, Leibniz pronounced Descartes in error in other of his works as well. In this accusation of error against Descartes, de Mairan argues, Leibniz is errant. Leibniz's idea, that from God's end purpose of a beautiful world one can discover the geometry of optics, is erroneous.

In the face of Leibniz's errant methodology in application of final causes to physics, de Mairan says that the disciples of Descartes are appropriate in their conduct against Leibniz in this matter. Nothing, de Mairan says, is more reckless in physics than Leibniz's methodology and conviction concerning final causes. What needs to be preferred, he argues, is the position that "nature always acts by the shortest route". In his application of final causes, Leibniz has found in nature something that his design attributes, and this is an errant methodology for physics.[346] Maupertuis pulls this position out of de Mairan's work and employs it as his own in *Accord.*   In de Mairan's piece, are found the ideas of, error, recklessness (hastiness in *Accord*), and final cause inappropriately applied to optical physics.

De Mairan understands the Cartesian objection to Fermat in terms of light choosing the path of least time. Fermat, however, never says that this is final cause in terms of a choice being made by light. Fermat simply responds to Clerselier that, upon being charged with giving light the quality of thinking, he would be glad to keep his geometry. It just so happens that the mathematics worked for Fermat only after working from the assumption of least time. Historically, therefore, Fermat can be shown to have finally succeeded because of this

[346] *Histoire de l'Academie Royale des Sciences*. Avec les Memoires de Mathematique & de Physique, pour la meme annee. Tirez des Registres de Cette Academie, Rue S. Jacques : Paris, Annee MDCCXXIII, p. 380 - 384

metaphysical assumption. In this regard, de Mairan understands Fermat. From the perspective of the "ends" towards which light supposedly moved, de Mairan does not correctly interpret Fermat.

In his 1682 essay, Leibniz comments on the drawing inserted in the page before the article, saying that line CE was both straight and teleological.[347] He says, "The complementing sines, EH and EL of the angles of incidence CEA and of refraction GEB always preserve the same ratio, which is reciprocal to the resistance of the media."[348] This is in keeping with Snell's law. Of all the possible paths CE could be, it happens to be the easiest path. Leibniz says that the geometry for this calculation is from out of final causes. Of this phenomenon and his calculation of it, he says this in relation to Descartes "error".

> We have therefore reduced all the laws of rays confirmed by experience to pure geometry and calculation by applying a single principle, taken from final causes if you consider the matter correctly: for a ray setting out from C neither considers how it could most easily reach point E or D or G, nor is it directed to itself to these, but the Creator of things created light so that from its nature the most beautiful result would arise. Hence those who reject final causes in physics do err greatly – not to speak more harshly – since even besides the admiration of divine wisdom, they would also supply to us the most beautiful principle for discovering properties of those things whose interior nature is still not so clearly known to us that we would be able to use proximate efficient causes and explain the machines which the Creator employed in order to produce those effects and in order to obtain his ends.[349]

Leibniz, who in 1682 thinks that light moves quicker through mediums of greater resistance, says that both Snell and Descartes are right.

> Since that ratio would always be the same – keeping the same mediums – the ratio therefore between the true ray ET and the apparent (ray) EV, will always be the same which was Snell's theorem. In the same way, the ratio of the sines of the compliments of the angles of refraction and incidence, EL and EH, which for us is reciprocal to the resistance, will always be the same: that is indeed the Cartesian theorem though Descartes' view of the resistance of the media was different from, and contrary to, ours.[350]

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[347] Translation from McDonough's Homepage
[348] Translation from McDonough's Homepage
[349] Translation from McDonough's Home Page
[350] Translation from McDonough's Home page; In his translation of Leibniz's work, FN 3, McDonough says, "This would appear to be Leibniz's understanding of the situation: Descartes had maintained that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is *inversely* proportional to the ratio of the velocity of the ray of incidence to the velocity of the ray of refraction, *directly* proportional to the ratio of the resistance of the medium of incidence to the resistance of the medium of refraction, and constant. In algebraic terms:

Leibniz finishes his 1682 work with an explanation of light in refraction which sheds light on the issue in question:

> In order to explain refraction, it should be noted that a medium more resistant to light (yet still more opaque) seems to be that which impedes the diffusion of light more, or its distribution through more parts of the medium, and one can say that is less illuminable for indeed it is the nature of light to try to diffuse itself. Conversely, the more that light will affect equally the parts of the medium it illuminates, or where it will communicate its own force to more insensible parts of the illuminated place, the medium will be more illuminable and less resistant to light. Hence where the affected particles of the illuminated medium are solid and small, or less interspersed with some other heterogeneous material not affected by light, to that extent the medium will be said to be more illuminated.[351]

de Mairan leaves this thinking alone. It is not Leibniz's position on the speed of light through mediums of varying resistances that bothers de Mairan. It is the issue of teleology.

From de Mairan's 1723 memoire it is evident that Maupertuis' charge of error against Fermat and Leibniz came directly out of the Cartesian's piece. None of it is original with Maupertuis. He simply uses de Mairan to serve his own interests in the writing and presention of

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$$\frac{\sin i}{\sin r} = \frac{Vr}{Vi} = \frac{m}{n} = C$$

Snell, and later Fermat, had maintained that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is *directly* proportional to the ratio of the velocity of the ray of incidence to the velocity of the ray of refraction, *inversely* proportional to ratio of the resistance of the mediums of resistance to the resistance of the medium of refraction, and constant. In algebraic terms:

$$\frac{\sin i}{\sin r} = \frac{Vi}{Vr} = \frac{n}{m} = \mathrm{C}$$

Leibniz, who goes on to argue that light travels faster in more resistant (non-opaque) mediums, maintains that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is *inversely* proportional to the ratio of the velocity of the ray of incidence to the velocity of the ray of refraction, *inversely* proportional to the ratio of the resistance of the mediums of incidence to the resistance of the medium of refraction, and constant. In algebraic terms:

$$\frac{\sin i}{\sin r} = \frac{Vr}{Vi} = \frac{n}{m} = \mathrm{C}$$

He is thus able to maintain that Snell was *correct* with respect to the ratio of the velocity of the incedent ray to the refractive ray, but *incorrect* with respect to the ratio of the resistance of the incident medium to the refractive medium, and conversely, that Descartes was *correct* with respect to the resistance of the incident medium to the refractive medium, but *incorrect* with respect to the ratio of the velocity of the incident ray to the refractive ray. Contemporary optics agrees with Snell (and Fermat) on both counts."

[351] Translation from McDonough's Homepage

Accord to the Paris Academy. Why does he pick a piece from de Mairan? He does so because de Mairan had been appointed secretary of the Paris Academy in 1743, and Maupertuis wants to solidify his relationship with the Cartesian while pursuing the Berlin appointment.

In 1738, Voltaire had recommended Maupertuis to Frederick the Great for the presidency. Maupertuis was not even a member of the Paris Academy at that time. This happened in 1743, the same year that de Mairan was named secretary in Paris replacing Fontenelle. Maupertuis writes *Accord* with the full knowledge that Frederick is interested in his taking the Berlin job, which happens February 1, 1746.

Having adequately pitted the two sides against one another around the issue of the incompatibility of the laws of reflection and refraction in optics, Maupertuis presents himself as the great reconciler. It is his "discovery" of the principle of least action. Why, he asks, should light take the path of shortest distance, or the path of least time? It does neither. Rather, it takes the path of greatest advantage, least action. The problem with his discovery is that he used Leibniz's concept of action – knowingly!

The evidence that Maupertuis knowingly used Leibniz's theory of action is that he admits it to Count Patrick d'Arcy (1725 – 1779)[352] in the Paris Academy. In the 1752 Berlin Academy memoires, Maupertuis submitted a response to d'Arcy's 1749 submission to the Paris Academy memoires accusing Maupertuis of using Leibniz's concept of action. Maupertuis says, "In the public assembly of the Royal Academy of Sciences of Paris in 1744, I proposed the Principle of Least Action, by making application to the rules of optics, catoptrics, and dioptrics. My memoire was ruled on, I dare say, with some applause, and inserted into the annual academy

[352] In her entry "A Dynamic Irishman in Paris: Patrick d'Arcy, 1725 – 1779" (*History Ireland*, 2010, Vol. 18, No. 2, pp. 22 – 23), Mary Stratton Ryan says that d'Arcy became a member of the Paris Academy in 1749 and the director in 1775. He is best known for his discovery of the principle of angular momentum. He was mathematically trained by Claude Clairaut. This is the connection to Maupertuis.

volume. Shortly after, with which, I had the honor of being called by the King to Berlin to administrate his academy."[353] In this response to d'Arcy, Maupertuis reveals his strategy, and notes that it is a strategy that worked! Shortly after publishing the piece in the Paris memoires, King Frederick of Prussia appointed him to the presidency of the Berlin Academy. That was his goal all along, and his methodology in *Accord* worked.

In this 1752 memoire, Maupertuis states his understanding of d'Arcy's three fold complaint against him in his 1749 Paris memoire. d'Arcy accuses him of three things; 1) of being wrong about action being the product of mass *x* space *x* speed, 2) of being wrong in saying that the quantity of action necessary to produce change in nature is the least amount possible, and 3) of having an inadequate demonstration of lever action. It is in Maupertuis' response to complaint #1 that he makes his admission.

> About the first point, if I had to justify the name of action whose I myself flee serving, I should think I have good strong reasons to allege: but to settle shortly with M. d'Arcy, I can say that this is not my affair. Leibniz, and those who have followed have called thusly the product of bodies by the space and by the speed. I adopted an established definition, against which no argument was developed, and which I had no reason to change. This is what would suffice to answer.[354]

In response to this admission, the following points need to be made. One, this admission is made in the same year as the Konig ruling by Euler, 1752. So, while in Berlin Maupertuis is having one man ruined over a missing letter showing Leibniz to have least action, in Paris he is admitting to an academic peer that he used Leibniz's theory of action.[355] Two, in this admission, Maupertuis clearly disavows any association concerning Leibniz and least action. He says here that he flees serving such a connection. Three, it is not his doing. He disavows personal responsibility, claiming to only be using what everyone else was using, which was a theory of

[353] Histoire de l'Académie Royale des Sciences et des Belles Lettres, MDCCLII, A Berlin : chez Haute et Spener, libraires de la Cour & de l'Académie Royale p. 293
[354] *Histoire de l'Académie Royale des Sciences et des Belles Lettres,* MDCCLII p. 295
[355] Pierre Maupertuis was not present at the Konig trial.

action used by Leibniz and others after him. Four, here Maupertuis contradicts his claim of discovery in 1744. If his claim is to have discovered least action, and by his own admission, he used the “established definition” of Leibniz for action, then Leibniz’s theory of action must have been least in quantity as well. Therefore, Maupertuis must had known, that in this sense, he did not discover the principle of least action, but rather Leibniz. Five, also in this 1752 admission to d’Arcy, Maupertuis mentions the multiple attacks on him concerning this bordering on slander and libel. As Jourdain says in his essay, Maupertuis declares that the whole pack of Leibnizians descended upon him. Clearly, from 1747 on, Maupertuis’ claim to the discovery was under assault, and he found himself in a battle which, if he mishandled it, could spoil his leadership as president of the Berlin Academy. Mishandle it is exactly what he did in the form of the Konig affair. Consequently, between the 1751 publication by Konig in *Acta Eruditorum* and his 1752 admission to d’Arcy, the political pressure within the Berlin academy forced his admission.

If mass *x* space *x* speed is action is Leibnizian, and if Maupertuis’ claim is that his discovery of least action is universal in that it is generally applicable in areas of physics, then Leibniz’s theory of action is universally applicable as well. Maupertuis knows this, and takes steps to suppress this truth. That is why Konig and his charge against Maupertuis had to be completely discredited. Furthermore, Maupertuis’ use of Leibnizian action for his own career benefit is circumstantial evidence pointing to the strength of the argument for this essay that Maupertuis charged Leibniz with error regarding final causes in physics for the same reason, a political strategy to enhance his career while a member of the Paris academy of 1744. Leibniz, as well as Fermat for that matter, are merely tools for employment of the strategy.

As Maupertuis used Leibniz and Fermat for personal career gain, he also manipulated them for career preservation in Berlin. The editing of subsequent editions of *Accord* is evidence for this. There exists a copy of a work by Maupertuis combining *Essay de Cosmologie* and *Accord*. It is dated 1751, but the publisher is not named.[356] It contains the earliest known changes to *Accord* to date. On page 208 (page one of Accord) there is a footnote inserted by Maupertuis at the bottom of the page which reads as follows: “This memoire was read in the public assembly of the Royal Academy of Sciences of France, April 15, 1744, and was inserted into the collection of 1744.” It is the position of this essay that, emphatically no, it was not. The 1744 edition of *Accord* reads markedly different.

In 1751, Maupertuis removes Leibniz from the charge of error regarding the application of final causes to physics. This removal remains through all subsequent editions of *Accord.* The 1744 edition of the focal passage reads:

> Je connais la répugnance que plusieurs Mathématiciens ont pour les *causes finales* appliquées à la physique, & l’approuve même jusqu’à un certain point; j’avoue que ce n’est pas sans péril qu’on les introduit: l’erreur ou sont tombez des hommes tels que Fermat & Leibniz en est suivant, ne prouve que trop combine leur usage est dangereux. On peut cependant dire que ce n’est pas le principe qui les a trompez, c’est la précipitation avec laquelle ils ont pris pour le principe ce qui n’en était que des consequences.

The 1751 and 1756 editions read :

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Now, a man who articulated a position concerning how light moved in least time and never attached this to any explanation of end purposes, is given credit for error in a faulty application

[356] Essai de Cosmologie – Google Books; https://books.google.com/books/about/Essai_de_Cosmologie.htlm. A preserved copy of this edition is also available on Maupertuis, Pierre Louis Moreau de – Essai de Cosmologie, echo.mpiwg-berlin.mpg.de/MPIWG:D59WXSP9

of final causes to physics. The name of Fermat remains as charged in 1751 and through all subsequent editions of *Accord.*

In comparing the 1744 *Accord* with the 1756 *Accord*, twenty categories of editorial changes are made by Maupertuis. These changes range from issues like comma usage and word changes, to name omissions and paragraph deletions. The examples of name omissions and paragraph deletion are most telling concerning the political motivation and career preserving strategy employed by Maupertuis as he attempted to salvage his legacy and reputation during and after the Konig affair.[357]

The "Aussi-vit on" paragraph is the best example of this. In 1744 it is situated between the "Ce fut par" paragraph and the "C'est cependant tout" paragraph. Here is how they read together.

> Ce fut par ce principe que Fermat résolut le problème, par ce principe si vraisemblable, que la lumière qui dans sa propagation & dans sa réflexion va toujours par le temps le plus court qu'il est possible, suivit encore cette même loi dans sa réfraction; & il n'hésita pas à croire que la lumière ne se meuve avec plus de facilité & plus vite dans les milieux les plus rares que dans ceux où, pour un même espace, elle trouvait une plus grande quantité de matière: en effet, pourvoit-on croire au premier aspect que la lumière traverserait plus facilement & plus vite le crystal & l'eau que l'air & le vuide?
>
> Aussi vit-on plusieurs des plus célèbres Mathématiciens embrasser le sentiment de Fermat; Leibnitz est celui qui l'a le plus fait valoir, & par son nom & par une analyse plus élégante qu'il a donnée de ce problème: il fut si charmé du principe métaphysique, & de retrouver ici ses *causes finales* auxquelles on sçait combien il était attaché, qu'il regarda comme un fait indubitable que la lumière se mouvoir plus vîte dans l'air que dans l'eau ou dans le verre.
>
> C'est cependant tout le contraire, Descartes avait avancé le premier, que la lumière se meut le plus vîte dans les milieux les plus denses, & quoique l'explication de la réfraction qu'il en avait déduite, fût insuffisante, son défaut ne venait point de la supposition qu'il faisait. Tous les systèmes qui donnent quel qu'explication plausible des phénomènes de la réfraction, supposent le paradoxe, ou le conferment.

> It was by this principle that Fermat resolved the problem, by this principle so truly likely, that light in its propagation and in its reflection will always go by the shortest time possible, yet follows this even more in refraction. And he did not hesitate to believe that light moves with more ease in the more rare environment than in those for the same space it finds a great quantity of matter : In effect, could anyone believe the first aspect that light travels more easily and more quickly in crystal and water than air and the void ?
>
> We saw several of the most famous mathematicians embrace the sentiment of Fermat. Leibniz

[357] In her essay "Maupertuis and the Principle of Least Action", p. 16, Susanne Bachelard says that Maupertuis removed Leibniz's name from subsequent editions of *Accord* because of the Konig affair.

is that one who argued it most, by name and by a most elegant analysis which he has given to this problem. And here to find its final causes whose combinations it is known he was attached. He regarded as an undubitable fact that light moves more quickly in air than in water or in glass.

This is, however, all contrary. Descartes had first advanced the explanation of refraction that he had deduced, that light moves more quickly in the more dense environment, and though the explanation of refraction he had deduced was insufficient, his fault was not seeing the supposition he was making. All systems give some plausible explanation for the phenomena of refraction, supposing the paradox, or the confirmation.

In 1751, Leibniz's name is removed from the middle paragraph, but the paragraph itself remains in the body of the text. This is indicative of a panic move by Maupertuis under pressure. It is in haste as are all the removals of Leibniz's name in *Accord.* Here is why. Now, Fermat is credited with giving name and analysis to a most charming principle that he never articulated. Once Maupertuis realizes this mistake he removes the entire paragraph. The 1756 edition reads:

Ce fut par ce principe que Fermat résolut le problème, par ce principe si vraisemblable que la lumière qui dans sa propagation & dans sa réflexion va toujours par le temps le plus court qu'il est possible, suivoit encore cette même loi dans sa réfraction; suivoit encore cette même loi dans sa réfraction; & il n'hésita pas à croire que la lumière ne se meuve avec plus de facilité & plus vîte dans les milieux les plus rares, que dans ceux où, pour un même espace, elle trouvait une plus grande quantité de matière: en effet, pourvoit-on croire au premier aspect que la lumière traverserait plus facilement & plus vîte le crystal & l'eau que l'air & le vuide?

C'est cependant ce qui arrive. Descartes avait avancé le premier, que la lumière se meut le plus vîte dans les milieux les plus denses, & quoique l'explication de la réfraction, qu'il en avait déduite, fût insuffisante, son défaut ne venait point de la supposition qu'il faisait. Tous les systèmes qui donnent quelque explication plausible des phénomènes de la réfraction, supposent le paradoxe, ou le confirment. Leibniz voulut concilier le sentiment de Descartes avec les causes finales: mais ce ne fut que par des suppositions insoutenables, & qui ne quarraient plus avec les autres phénomènes de la nature.

It was by this principle that Fermat resolved the problem, by this principle so truly likely, that light in its propagation and in its reflection will always go by the shortest time possible, yet follows this even more in refraction. And he did not hesitate to believe that light moves with more ease in the more rare environment than in those for the same space it finds a great quantity of matter : In effect, could anyone believe the first aspect that light travels more easily and more quickly in crystal and water than air and the void ?

This is, however, what happens. Descartes had first advanced the explanation of refraction that he had deduced, that light moves more quickly in the more dense environment, and though the explanation of refraction he had deduced was insufficient, his fault was not seeing the supposition he was making. All systems give some plausible explanation for the phenomena of refraction, supposing the paradox, or the confirmation. Leibniz wished to reconcile the senses of Descartes with final causes, but it was only by unsustainable assumptions which no longer squared with other phenomena of nature.

Maupertuis has no felt convictions about the issues of which he is writing. He is doing what is expedient. Having removed Leibniz from error regarding final cause, Maupertuis now adds

Leibniz's name and refers the reader to the appendix where he changes the issue to Leibniz's mistaken physics of resistance.

At the beginning of the appendix, Maupertuis says that he was mistaken about Leibniz's agreement with Fermat only because de Mairan was mistaken. This is evidence that Maupertuis never read *Unicum Opticae.* He relied on de Mairan's interpretation. Instead of offering a correction of de Mairan's 1723 memoire, however, Maupertuis extracts the sections from Euler's commentary on least action where Euler criticizes Leibniz's physics of resistance in mediums. In this work, Euler asserts that Leibniz holds a position in resistance that is unmeasurable, and therefore, unacceptable as a scientific test. He also says that Leibniz's idea of resistance is non-applicable due to the common sense notion that all other hard bodies in nature slow down when entering thicker mediums. Therefore, if Leibniz's easiest path is applicable at all, it is applicable only in optics. Euler makes another assertion at the beginning of this work which reveals Maupertuis' real reason for inserting it in the appendix after the de Mairan criticism. However, Maupertuis leaves this part of Euler's commentary out of the appendix.

Euler says in the beginning of this commentary that everyone is aware that Maupertuis was not the original discoverer of least action. He says that Konig was correct in asserting that many people knew of it before Maupertuis. Euler says that it is not least action per se that Maupertuis discovered, but the general applicability of least action to all physics.[358] The general applicability of least action was what Maupertuis claimed to have discovered in the 1744 *Accord*, yet illustrated within a specific application in optics. Maupertuis says in the 1744 *Accord*, "The true principle once discovered, I deduce all the laws that follow light, whether in its propagation, in its reflection, and in its refraction."[359] This is the real reason for the appendix. In 1756

[358] Euler, Leonhard. On the Principle of Least Action. eulerarchive.maa.org/docs/translations/E198tr.pdf
[359] *Histoire de l'Academie Royale des Sciences.* Annee MDCCXLIV, P. 425

Maupertuis is still struggling for legitimacy in his claim for the discovery of least action. His employment of Euler in the appendix is most unfair to Leibniz. Leibniz never claimed a general application of action in physics based on his easiest path in optics.

In all three editions of *Accord* considered in this critical analysis, the wording of the focal passage remains the same. Therefore, he considers final causes to be a principle which is dangerous if misapplied. He seems to be using the word principle in the universal sense, but, as usual, Maupertuis is vague. In his works where he mentions final cause, he does not refer to it as only for particular applications. Rather, he employs it in the Leibnizian sense, as universally connected with blind and necessary mechanics, or mathematics.

Of the two charged with error, Fermat and Leibniz, only Leibniz employed the term final cause in the Aristotelian sense; "that for the sake of which", or, the end purpose for which an action is performed. He says in *Unicum Opticae* that light takes the easiest path for the sake of beauty. Here Leibniz uses the term in the correct sense. Taking the easiest path is how light moves. Why it takes the easiest path is for the production of beauty. This is the difference between Fermat and Leibniz. Whereas Leibniz answers the how and the why, Fermat only answers the how when he tells Descartes that nature works according to the most expedite ways and means. Once the ray of light moves for the simplest, or most expedite ways and means, however, Fermat does not indicate its end purpose in doing so. When he tells le Chambre that light takes the path of least time Fermat is explaining how light works. "The principle of Physics is that nature performs its movements by the most simplest paths."[360] He does not say for what purpose light takes the path of least time. He speaks of physics, not metaphysics.

[360] Ross, p. 52

Maupertuis does not remove Fermat's name from subsequent issues of *Accord* because there is no political faction in Berlin that can be defined as Fermatians and no Konig-like person to trumpet the cause of Fermat against him. It is not because he actually thought or cared that Fermat was guilty of error in his application of final causes. They both speak in terms of thinking nature. Fermat says that light takes a path of least time instead of shortest distance which infers a thinking choice being made. This is what Clerselier charged him with. Maupertuis says that light takes the path which gives it the greatest advantage. This too implies thinking nature. It can be argued that, if alive, Clerselier would have charged Maupertuis with thinking nature as well. If thinking nature is final cause, then they both commit this error, and Maupertuis is as hypocritical with Fermat as he is with Leibniz. However, thinking nature is not final cause. Both men describe how light works, not why. What Maupertuis is not saying is that light takes the path that it does for the sake of greatest advantage, and Fermat is not saying that light takes the path it does for the sake of least time. Maupertuis fundamentally understands Fermat, and uses him for career advancement along with Leibniz. Had the Konig affair never occurred with the Leibnizians descending on him as a pack, the 1756 *Accord* would have read like the 1744 *Accord.*

Conclusion

The charge made by Pierre Maupertuis in 1744 that Pierre Fermat and Gottfried Wilhelm Leibniz were in error for their faulty application of final causes to physics is a politically calculated career building statement on the part of Maupertuis. The charge of error is not a charge based on philosophical or scientific convictions on his part. The internal evidence of the 1744 edition of *Accord Between Different Laws Which at First Seemed Incompatible* and the external evidence from outside sources, as well as the historical context, all bear this out.

The body of Maupertuis' work in philosophy and science involves itself in the issues being contested in his day. He presents no formal thought out position on causation in general, or final cause in particular. Maupertuis, therefore, does not charge Fermat and Leibniz from a formal position based any systematic approach to causation. The jurist Pierre Fermat presents the world with no philosophical position on any subject, much less causation. His presentation to the world is mathematical; therefore, Maupertuis has no formal Fermatian argument on causation from which to hang the charge. The fact is that they both use the same metaphysical term to describe light in refraction, "simplest". Fermat's reference to metaphysics occurs within his correspondence with Descartes, le Chambre, and Clerselier. Fermat's response to Clerselier's remark that his optics gives light the quality of thinking should be interpreted as Fermat's tacit admission that purposiveness is involved in light's path of least time. From this standpoint, Maupertuis has room for attack, but he uses the same idea of purposiveness in *Accord* with the idea of advantage. Of the three men, Leibniz presents the world with the clearest overt argument on causation in general, and final cause in particular. As it regards the issue of optics, Leibniz presents the *Doctrine of the Interpenetration of Causation*. A ray of light moves according to two sets of laws, mechanical and divine wisdom. In the 1744 *Accord,* Maupertuis not only does not attack this doctrine, but instead compliments it, and attempts to assert it has his own with the statement “a blind and necessary mechanics follows the designs of a free and clear intellect”. It is evident that Maupertuis does not attack Fermat and Leibniz in *Accord* for reasons of actual disagreement concerning the issue of causation.

When one first glances at the charge, the obvious problematic question arises as to exactly how anyone can be erroneous in believing, or not believing for that matter, that a ray of light travels from point *A* to point *B* for the sake of something. Proving such an assertion is problematic. This was Descartes’ point about final causes. The ends which God has in mind, he says, are beyond his mind’s ability to grasp. This happened to be Maupertuis’ own point about knowledge of intermediate principles lodged in experiment in *du Repos*. Anything beyond that, he argued, was beyond the mind’s grasp.

The three alternatives presented to early modern thinkers concerning the cause behind the movement of light through mediums was that the ray of light itself thought about which path to take, or God directed its path for divine purposes, or that it moved by necessity without reason. None of these alternatives was satisfactory for everyone in this time. The idea that light moved the way it did was because it thought it should harkened back to the scholasticism which Descartes and other mechanists had taken great pains to break away from in their quest for a mechanistic universe. The idea of God’s involvement for many was unsatisfactory because, first of all, God could not be proven to exist, and, secondly, it seemed certain to many in this time that Christian theology in general, and the Bible in particular, were too wedded to Aristotelian and scholastic philosophy to any longer be counted as credible. Plato's idea of a grand designer was equally irreconcilable with the new science of mechanism.

The doing of science in the seventeenth century moved from the cathedrals of theology and philosophy to state-sponsored academies of math and science. In so doing, the third of these alternatives took center stage. The causal factors behind the movement of light fell to the arena of mechanistic philosophy. Leading this effort was Descartes, who exclaimed that final causes should be banished from physics. The new mechanistic philosophy brought with it not only a new way of doing science intellectually, it brought with it a new sociology for the doing of science.

These new academic institutions found themselves prone to the same sociological factors as had the Christian institutions. In these institutions, men of great intellect and ambition found a new arena in which to exercise their quest for significance. Whereas in the days before Descartes and Galileo, such significance was achieved in religious priesthood and theology chairs, now quests for personal significance could be achieved in secular academies outside of religion. This reflected the change that began to take place after Copernicus, Descartes and Galileo. Math and science now said that men dwelt in a heliocentric universe instead of a geocentric universe as posed by Aristotle and the Bible. So, as Aristotelian and Biblical cosmology lost favor, the cosmology produced by the math and science of mechanism in state funded academies of math, philosophy and science gained favor.

This early modern "scientific revolution" caused an intellectual battle over causation. Thinkers like Leibniz tried to balance the new philosophy of mechanism with the philosophy of final causes – the belief that natural motion could be explained in terms of end purposes. Thinkers like Spinoza argued to the other extreme, saying that there were no end purposes in nature, and that such thinking was wishful human imagination. In between were thinkers who attempted to balance the two without getting themselves unfairly branded as either a religious zealot or an atheist.

This early modern "scientific revolution" saw the overthrow of many an existing scientific paradigm, starting with the overthrow of the geocentric view of the universe to the heliocentric view. What did not change in this particular paradigm shift was the knowledge that light propagated from the sun to the earth. Consequently, the paradigm of the laws of the propagation, reflection, and refraction as understood from ancient times still stood. This understanding, however, produced a crisis in the early modern period over the reconciliation of these laws with each other according to the new laws of mechanical motion. The bend of light in refraction contradicted the new Newtonian laws of motion in physics and the Cartesian understanding of the immutability of the will of God.

The battle lines in this crisis became drawn between factions which emerged in the seventeenth century along with the state run science academies which housed them. The latter half of the seventeenth century and the first half of the eighteenth saw the rise of the Newtonians and their dominance in the Royal Society of London, the rise of the Cartesians and their dominance in the Royal Academy of Science in Paris, and the rise of the Leibnizians and their dominance in the Royal Prussian Academy in Berlin. A particular academic and sociological phenomenon occurred within each of these academies during this period where each of the three institutions became infected with members of the other two rival factions. Paris had Leibnizians and Newtonians. London had Cartesians and Leibnizians, and Berlin had Cartesians and Newtonians. Consequently, highly competitive debates naturally occurred within their midst such as the Leibniz – Clark and Descartes – Fermat arguments over the relationship of space and time and the behavior of light.

Pierre Maupertuis should be interpreted as a man of great personal ambition and intellectual ability who chose as his life's work academic success within this context instead of the military career planned out by his father. He should not be interpreted as a man seeking primarily to achieve philosophical and/or scientific truth as his life's work. Pierre Maupertuis wanted to run things. The arguments within the academies provided him with the tool necessary to rise in the ranks. The argument among scholarship today that Maupertuis was a Newtonian who promoted Newton's ideas in the Paris academy is half accurate. Although he promoted Newton's ideas in Paris, he was not a Newtonian by conviction. Pierre Maupertuis was a Maupertuian, who used the beliefs of Descartes, Leibniz, and Newton to his advantage as he rose through the ranks of French and German intellectual societies. He played the positions of the three sects against one another. His two greatest achievements, the calculation of the shape of the earth and the principle of least action, were both by-products of a strategy to take the understanding of two groups which were opposed to one another on an issue, and solve their crisis. In the case of the shape of the earth, Maupertuis solves in favor of the Newtonians which puts him in greater favor in London and Paris. By flattening the earth slightly more than Newton, he set himself apart as an intellect even greater than Newton, which gained him pensioner status in Paris. As a ranking member in Paris, he realized that de Marian's appointment the year before removed any opportunity for the top position in Paris. The Berlin position, however, was up for grabs.

To climb this ladder and win the Berlin appointment, Maupertuis chose the crisis in optics and the Descartes-Fermat debate in this area as his vehicle of advancement. It is within this understanding that *Accord Between Different Laws Which at First Seemed Incompatible*, as well as the focal passage for essay, should be interpreted. The announcement to the world of the discovery of the foundation for all of physics, least action, came in the form of a persuasive speech designed for the purpose of career advancement. Such was the sociology for the doing of science. One of the greatest physics "discoveries" in world history occurred within the context of career building.

The contents of *Accord*, down to the idea of action itself, belong to others beside Maupertuis. The form of *Accord* is that of a persuasive speech. The idea was not necessarily to be original or groundbreaking, but to persuade decisions makers in the Paris, and especially in the Berlin acadamies. He borrowed the general concept of action from Leibniz and Wolff. His intelligent design position was a unique combination of Spinoza and Leibniz, and his specific description of action was from Descartes idea of the nature of body being that of extension. By skillfully weaving these and other concepts together, Maupertuis spoke to the felt need of all three groups in his audience for a solution to the puzzle in optics concerning the harmonization of the laws of reflection and refraction. As they listened to the speech, all three groups in the Paris academy heard their positions being affirmed. In Berlin, Frederick the Great read *Accord* and perceived it to present a solution to one of the most complicated mathematical and scientific dilemmas of their day. It displayed abilities to manage the battlelines within warring groups existing within his own academy. The intended purpose for which Maupertuis constructed *Accord Between Different Laws Which at First Seemed Incompatible* worked. The Paris academy received it positively, and the next year, Maupertuis was the president of the Berlin Academy with full control of pensions.

Maupertuis surrounded the charge of error against Fermat and Leibniz with so much pro-Leibniz material, and posed the focal passage so ambiguously, that the Leibnizians and backers of Fermat in the Fermat – Descartes debate could hardly object. No one could tell exactly where the point was that Maupertuis ceased objecting to the application of final causes and approved their use. It is the position of this essay that this ambiguity is politically purposeful on the part of Maupertuis. He has no conviction in the matter. This is born out by the fact that shortly after charging the men with error, he will state that "a blind and necessary mechanics follows the designs of a free and clear intellect." In other words, Leibniz is correct. Metaphysics precedes physics. Optics in propagation, reflection and refraction worked mechanically according the designs of God. The principle is final cause. The consequence is the mechanics. Yet, Maupertuis says in the focal passage, that the mistake made by Fermat and Leibniz in this matter was that they mistook for a principle that which was the consequence of the principle. The reason Maupertuis words it as such is because that is the place in de Mairan's 1723 memoire that the Cartesian charged Leibniz with being "exceedingly errant".

The focal passage for this essay is simply one more borrowing by Maupertuis used to fill out his argument for least action. In order to fill out the Cartesian side of the argument concerning the crisis in optics, Maupertuis kills three birds with one stone. He extracts terms and usages from de Mairan's memoire and employs them on the Cartesian side, and, in so doing, pleases not only de Mairan, but Fontenelle and every other Cartesian in the audience. On top of that, Maupertuis extracts de Mairan's argument from Clerselier's correspondence with Fermat that in refraction, Fermat believed that light possessed thinking capabilities. The way de Mairan puts it is that, according to Fermat, light seeks the path of a calculated advantage. Skillfully, Maupertuis turns that de Mairan interpretation of Fermat to his own advantage and says that in refraction, light takes the path that gives it it's greatest advantage. With that, the Fermat backers in the audience look smugly at the Cartesians, knowing that Maupertuis was actually their man all along.

As an opportunist, Maupertuis correctly uses de Mairan's 1723 memoire. Although only Leibniz articulated a position on final cause within a more general scheme in causation, both Fermat and Leibniz employed final cause thinking in the calculating of their optics. Fermat said, "Our demonstration rests on a singular postulate: that nature operates by the easiest and most expedite ways and means," and Leibniz said, "We have therefore reduced all the laws of rays confirmed by experience to pure geometry and calculation by applying one principle, taken from final causes if you consider the matter correctly." De Mairan's charge of error is in terms of the inappropriate application of final causes to physics. Maupertuis' charge of error in 1744 is not in terms of the speed of light or of density and resistance factors. In the 1744 *Accord*, the speed of light factor is couched in terms of what is argued over between two sides, and the issues of density and resistance are not even mentioned. Maupertuis does not accuse Fermat and Leibniz of error in incorrect positions concerning the speed of light, density or resistance factors.

Maupertuis removes Leibniz's name from subsequent issues of *Accord* to assuage the political heat when the Leibnizians in the Berlin Academy descended on him after his publication of 1746 work *Les Loix du Mouvement et Repos Deduites d'un Principe Metaphysique*. The reason for this is because of his employment of the factors of force and mass in his definition of action, and because of his employment of time and Leibniz's mv^2 in his solving for both elastic and inelastic collisions. His reference to the 1744 *Accord* in his admission to d'Arcy is evidence that he knew he was using Leibniz's idea for action the whole time. He left the elements of force, mass, and time out of his idea of action in 1744 so as to not upset the political balance struck in *Accord*. He inserted them confidently in 1746 because he had achieved his goal of the Berlin presidency, and, as monarch over academy member pensions, assumed that the Leibnizians would not challenge his claim to discovery of least action. Furthermore, the Leibnizians were dominant in the Berlin academy, so getting Leibniz wrong in Berlin was a whole different consequential matter to getting Leibniz wrong in Paris where the Cartesians predominated.

The problem for Maupertuis was that his assumption of Leibnizian silence and submission to him as absolute authority was an error in judgment. Samuel Konig submitted an essay to him declaring Gottfried Leibniz to be the actual discoverer of the principle of least action. Maupertuis did not think enough of Konig's opinion to read the essay, so Konig published his opinion. Maupertuis' so severely mishandled the "Konig Affair" that his reputation was ruined, and he finished out his presidency in disgrace. His removal of Leibniz's name from *Accord* began in the same years as this scandal, 1751.

Maupertuis did not remove Fermat's name from the charge of error because there was no face-saving, political benefit, for him. There was no Fermatian faction in Berlin. The principle of least time in optical physics was still all theory, as the world was still one hundred and six years away from the experimental proof.

There are two approaches to interpreting *Accord Between Different Laws Which at First Seemed Incompatible*. One is to assume that Maupertuis did what every author and thinker does, and that is put their most accurate and true positions in either systematic works or the last edition of a series of works. Two is to look at an authors works from the beginning and interpret the subsequent works from the beginning. Maupertuis must be approached from the latter. This is because of the editorial changes made to *Accord* between 1744 and 1756. The argument as well as the form radically change. Maupertuis knew that most interpreters would judge him by the first technique and assume that his real position was what he posted in subsequent editions. So he pens a footnote at the bottom of page one in the 1751 edition which assures the reader that what they are looking at is what he read to the Paris Academy on April 15, 1744. It stretches the bounds of credulity to think that he thought he was referring to the same work in structure and meaning. In addition to the footnote, he inserted an appendix and put the blame for his misinterpretation of Leibniz totally on de Mairan. Then he includes his own hand-picked selection from Leonhard Euler's commentary on the principle of least action and leads his readers to believe that the actual problem lies in Leibniz's interpretation of Descartes' paradox. Now his charge of error against Leibniz is that of the mechanics of optics and light's behavior in traveling through mediums of differing densities and resistances.

Even though there are two ways to approach *Accord* and the charge of error, and even though this essay stresses that the most accurate interpretation of Maupertuis is to start from the 1744 work and look forward, it cannot be "posed as fact" that the 1744 work is what Maupertuis actually thinks. Consequently, it cannot be insisted upon that he actually thought that Fermat and Leibniz were actually in error regarding their application of final causes to physics. The nature of the political context within the academies of science at that time, the changing structure and meaning of *Accord*, and Maupertuis' career ambitions all point to an interpretation that he charged the two men with error in 1744 in order make political gains and upward career movement in Paris and Berlin.

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Appendix I:

July 6, 1751 Letter From Maupertuis to Johann Bernoulli II

Maupertuis wrote one hundred and thirty five letters to Johann Bernoulli II from 1743 to 1756. This correspondence, existing untranslated in the University of Basel, sheds light on Maupertuis' thinking during this period, specifically concerning the Konig Affair. This correspondence is now in the possession of the author of this dissertation, as well as the Philosophy Department Chair of the University of South Florida. A translation of this correspondence should be undertaken so as to reveal a clearer, more detailed, picture of Maupertuis as a member of the Paris Academy of Sciences and as President of the Berlin Academy of Sciences.

On page 179 - 181 of this appendix is the July 6, 1751 original hand written letter from Maupertuis to Bernoulli II detailing Maupertuis' disdain for Samuel Konig, and his strategizing to prove Konig's claim of the Leibniz letter to Hermann as fraudulent. The entire letter covers some matters that are unrelated to Konig and Leibniz. Here is a translation of the section which does pertain.

> From Potsdam 6 July 1751…Here is yet another piece of business concerning your fellow Swiss. Konig whom I have received here as I would my brother: hardly has he been back in Holland that he published in the *Acta Eruditorum* an article in which he gives himself the most advantageous airs against me, and takes up some old problems of your father that are not relevant to anything, and throws some smoke on the rest. He criticizes what I published in our memoires and in my essay *De Cosmologie*: not hoping however that could make the whole world believe that I do not say anything of worth, he ends by citing a fragment of a letter from Leibniz to Hermann by which he claims for Leibniz the principle of the least quantity of action, and by which also Leibniz would have discovered before Euler this beautiful product of the trajectories described by central forces in which the element of the curve multiplied by the speed of the body that describes it, that is, the action, always achieves a minimum.
>
> Since I do not think that Leibniz had resolved the problem of Euler; since I believe that if he had the principle of the least quantity of action he would have made the same application of it that I did; finally

since I knew Konig somewhat, I wrote him to ask him to point out to me this letter of Leibniz he cited without date, and to say where it can be found. After a silence of five weeks and a second letter sent to him by the diplomatic secretary responsible for the King's business at the Hague, can you guess what he answered me? That the copy of the letter whose fragment he cites, was communicated to him and several others by an M. Henry, who had his head cut off at Berhe. It has been two years since he had wanted to published a collection of letters of Leibniz. He sends me the entire copy of the one in which I am not unhappy to find my principle, given that all the discoveries since the death of Leibniz can be found in it. Mr. Leibniz, e.g., had seen *a priori* the polypes of M. Trembley. My suspicion is no longer just a suspicion for me, nor do I believe that it would hardly be so all equitable people: But since I want to bury Konig into the mud as much as he deserves, I want to gather all the possible evidence of his fraud. To that end, I began by writing to our Ambassador requesting to procure for us a collated copy of Leibniz's letters which were not burned as seditious papers at the death of Henri, and must be retrievable somewhere: and I hope that if they have never existed Mr. de Baulmy will have a copy made for us.

But you, my dear friend, can again better serve me in this than anyone else: by helping me with your advice, and indicating to me and providing me with the surest means for convicting the falsifier.

No doubt you know in which hands hands the papers of M. Hermann will have passed; perhaps they are still at Basel: and in this case I would make the same demand I made to our Ambassador, not to send me originals, because they must remain with a third part, but a copy of all the collected letters of Leibniz collated and and initialed by a notary. You can also put Mr. the Ambassador on this path. Finally, dear friend, use for me on this occasion everything your love for justice and your wisdom could inspire you, without sparing no care or expense. Because the preceding of K. if it is such, is so vile that I want to shine the greatest light on it. Keep the thing secret until we are in position to remove all his escape routes.

Here is the actual 1751 handwritten letter from Maupertuis to Bernoulli II.

De Potzdam 6 juillet 51 152

je recois votre lettre du 23 juin cher amy: j'avois deja fait pour Mr Passavant tout ce que vous demandés qu'il faste, et sans doute je n'osé vous le dire parceque cela le condamne. dans le temps meme ou je le fis exclurre de l'Academie je luy dis que cela n'etoit et ne seroit qu'un moyen de l'eprouver; qu'il n'avoit qu'à employer le temps qu'il passeroit auprès du jeune Thulmeier à faire connoitre sa capacité et sa volonté, desquelles nous n'avions encor aucune preuve. qu'après cela, l'education de son eleve finie, je ne m'ecarterois point de le faire rentrer dans l'Academie. cela dit depuis 2 ans, il n'a pu produire une ligne; l'autre jour il m'allegua que sa vue s'étoit affoiblie en veillant la petite Thulmeier pendant sa petite verole, comme s'il falloit voir bien clair pour faire des ouvrages de Metaphisique. je commence à ne pouvoir presque douter que ce ne soit un mauvais sujet quoyqu'il me promette encor des merveilles.

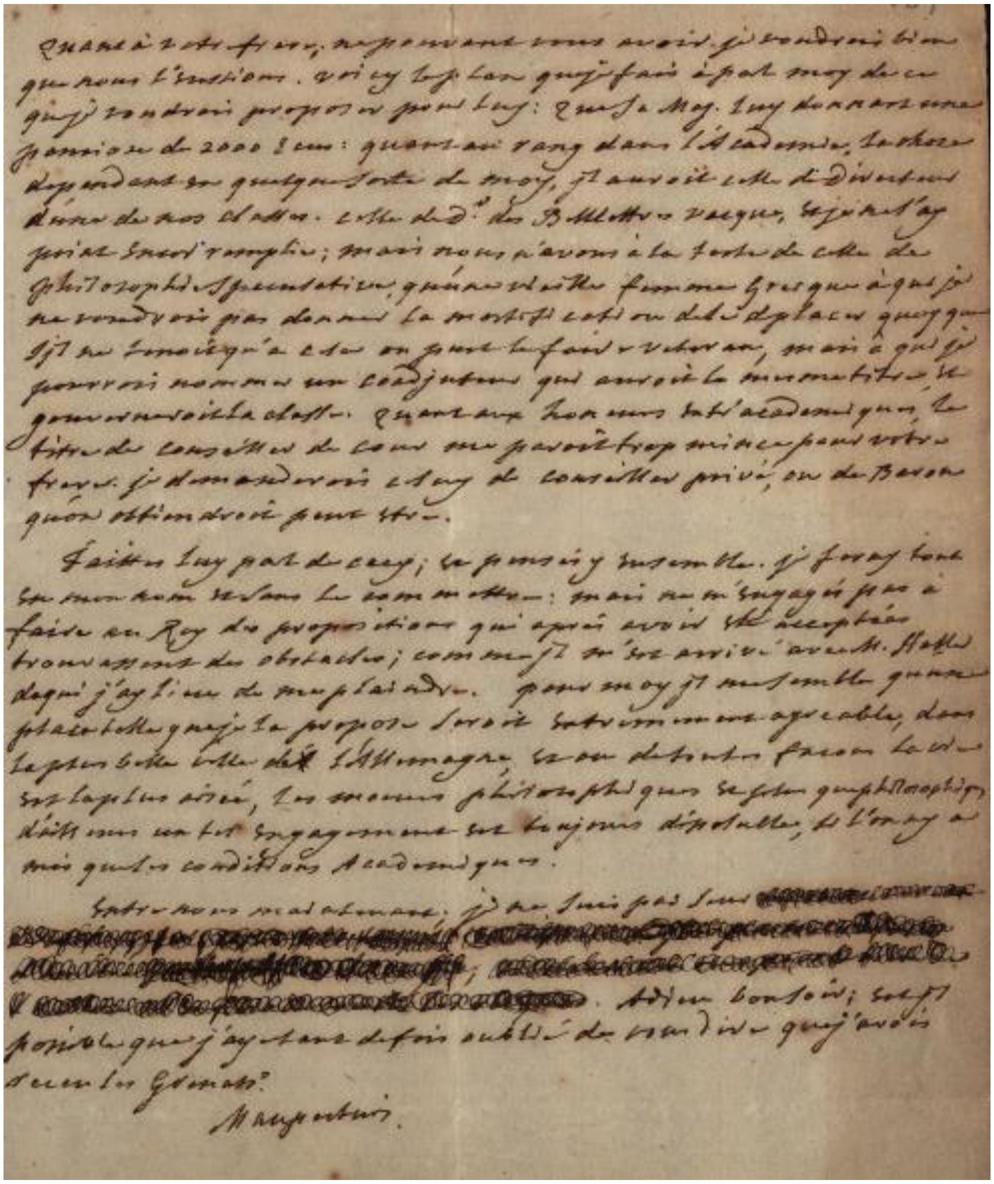
Voicy une autre affaire encor de vos Suisses. König que j'ay receu icy comme j'aurois fait mon frere: à peine a-t-il été de retour en Hollande qu'il a fait paroitre dans les Acta Erud. une piece où il se donne les airs avantageux avec moy, et en jettant quelques vieux problemes de votre pere qui ne font rien à la chose, en jettant des themes sur le reste critique ce que j'ay donné dans nos memoires et dans mon Essay de Cosmologie: n'esperant pourtant pas faire croire à tout le monde que je n'ai dit rien qui vaille il finit par citer un fragment de lettre de Leybnitz à Herman par lequel il revendiqueroit pour Leybnitz le principe de la moindre quantité d'Action, et par lequel aussy Leybnitz auroit avant Euler decouvert cette belle proprieté des Trajectoires decrittes par des forces centrales dans lesquelles l'element de la courbe multiplié par la vitesse du corps qui la decrit, c'est à dire l'Action, fait toujours un Minimum.

Comme je ne crois point que Leybnitz ait ny eust resolu le probleme d'Euler: comme je crois que s'il eust eu le principe de la moindre quantité d'Action il en auroit fait la meme application que moy: enfin comme je connoissois un peu Konig, je luy ecrivis pour le prier de m'indiquer cette lettre de Leybnitz qui etoit sans date, et sans dire où elle se trouvoit. Après un silence de 5 semaines et une 2e lettre à luy rendue par le secretaire de legation chargé des affaires du Roy à la Haye, devineriés vous bien ce qu'il m'a repondu?

Que la copie de la lettre dont il a cité le fragment luy avoit
été communiquée avec plusieurs autres par un M. Henzy
qui eut la teste coupée a Berne il y a 2 ans lorsqu'il devoit
faire imprimer un recueil de lettres de Leybnitz. il m'envoye
la copie entiere de celle cy ou je ne dois pas etre faché de trouver
mon principe, puisque toutes les decouvertes faittes depuis la mort
de Leybnitz s'y trouvent; M. Leybnitz par ex. avoit vu a priori
les polypes de M. Trembley. Mon soubçon n'est presque plus
un soubçon pour moy, et je crois ne le seroit guerres pour
tous les gens equitables: Mais comme je veux enfoncer Konig
dans la boue autant qu'il le merite, je veux rassembler toutes
les preuves possibles de son imposture. pour cela j'ay commencé
par ecrire à notre Ambassadeur pour le prier de nous procurer
une copie collationée des lettres de Leybnitz qui à la mort de Henzy
n'auront pas eté brulées comme papiers seditieux, et doivent
se retrouver quelque part: et j'espere qu'elles n'ont jamais
existé. M. de Paulmy vous en fera avoir la copie.

Mais vous, mon cher amy, pouvés encor mieux que persone
me servir en cecy: en m'aidant de vos conseils ~~[illegible]~~
~~[illegible]~~, en m'indiquant et me procurant
les moyens les plus seurs pour convaincre le faussaire.

vous scaurés sans doute en quelles mains seront passés les
papiers de M. Herman; peut etre mesme sont ils encor à Basle:
et dans ce cas je vous ferois la mesme demande qu'à notre
Ambassadeur; non de m'envoyer les originaux, car il faut qu'ils
restent en main tierce, mais une copie de toutes les lettres de
Leybnitz collationnée et paraffée par un notaire. vous pourriés
peut etre aussy mettre M. l'Ambassadeur sur la voye. Enfin
cher amy employés pour moy dans cette occasion tout ce que
votre amour pour la justice et tout ce que votre sagacité vous
pourront inspirer, sans epargner ny soins ny depenses; car
le procedé de K. s'il est tel, est si infame que je le veux mettre
dans le plus grand jour. Tenés la chose secrette jusqu'à ce que
nous soyons en etat de luy oter toutes les ~~[illegible]~~ echapatoires.



Quant à votre frere, ne pouvant vous avoir, je voudrois bien que nous l'eussions. voicy le plan que je fais à part moy de ce que je voudrois proposer pour luy: Que le Roy luy donnat une pension de 2000 écus: qu'il eut rang dans l'Academie; la chose dependant en quelque sorte de moy, il auroit celle de Directeur d'une de nos classes. celle de d. des Belles lettres vacque, et je ne l'ay point encore remplie; mais nous n'avons à la teste de celle de Philosophie speculative, qu'une vieille femme Grecque à qui je ne voudrois pas donner la mortification de deplacer quoyqu'il ne [illegible] on [illegible]; mais à qui je pourrois nommer un coadjuteur qui auroit le [illegible], et gouverneroit la classe. Quant aux honneurs hors l'academie, le titre de conseiller de cour me paroit trop mince pour votre frere. je demanderois celuy de conseiller privé, ou de Baron qu'on obtiendroit peut être.

Faites luy part de cecy; sa pensée y ressemble. je feray tout [illegible]; mais ne m'engagez pas à faire au Roy des propositions qui apres avoir été acceptées trouveroient des obstacles; comme il est arrivé avec M. Haller dequoi j'ay lieu de me plaindre. pour moy il me semble qu'une place telle que je la propose seroit extremement agreable, dans la plus belle ville de l'Allemagne, [illegible] la plus aisée, les moeurs philosophiques [illegible] toujours [illegible] quelques conditions Academiques.

Entre nous mal [illegible]; je ne suis pas sans [illegible]. Adieu bonsoir; est-il possible que j'ay tant de fois oublié de vous dire que j'avois reçu les Genevois?

Maupertuis.

Specifically, this letter reveals the confidence Maupertuis' placed in Johann Bernoulli II regarding Samuel Konig's article in *Acta Eruditorum.* Maupertuis is clearly upset about what he perceives to be Konig's unwarranted and even fraudulent assessment of Maupertuis' claim to least action. It also reveals his thinking that he has responded to the issues in academy memoires and *Essay de Cosmologie.* A study concerning what Maupertuis means here should be undertaken. In order to fill out the research, however, a translation should be performed on the complete correspondence between these two men from 1743 – 1752. It is evident that

Maupertuis confided in Bernoulli II, asking him to keep quiet about what Maupertuis was up to as he encircled Konig so as to defeat him. This material has the potential to shed light on the Konig Affair and Maupertuis' thoughts on least action and his claim to its discovery.

Appendix II:

Email Granting Permission to Use Figure #5

Dear Ms. Ruedi,

I am working with a researcher at my university who would like to use the figure:

Figure on pg 543. Pedoe, Daniel (1964). "A geometric Proof of the Equivalence of Fermat's Principle and Snell's Law." American Mathematical Monthly. 71(5): 543-544.

In his dissertation. Can you give us instructions on how we can obtain permission for this use?

Sincerely,
LeEtta

LeEtta Schmidt
Resource Sharing and Copyright Librarian
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Tampa, FL 33620

lmschmidt@usf.edu

Email Granting Permission

From: Beverly Ruedi [mailto:bruedi@maa.org]
Sent: Friday, September 18, 2015 10:33 AM
To: Schmidt, Leetta <lmschmidt@usf.edu>
Subject: Re: permission to use figure from AMM

It is fine to use that figure. Please ask him to be sure to give a full citation.

Thanks,

Bev

Beverly Joy Ruedi
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