3-31-2015

Precalculus Students' Achievement When Learning Functions: Influences of Opportunity to Learn and Technology from a University of Chicago School Mathematics Project Study

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by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Teaching and Learning
College of Education
University of South Florida

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Date of Approval:
March 31, 2015

Keywords: mathematics education, technology, graphing calculators, learning functions, teaching functions, UCSMP, path analysis, HLM

Dedication

I dedicate this dissertation to my parents, Jack and Dorothea Drittler. For fifty years they have given me unconditional love and support. They taught me I could do anything or be anything I wanted to be. They also taught me the value of education and the importance of family. I would not be who or where I am today without you. I love you both.

To my children, A.J., Melissa and Daniel Hauser, for all the nights you not only fixed yourselves dinner but fixed mine too so I could study or write. You gave me three reasons to never give up and to keep going even when things got tough. You helped me remember opportunities are not given: we make them through hard work, determination, and sometimes a little luck and divine intervention. I look forward to returning the favor and supporting you while you work to reach your goals and dreams. I love you three more than anything in this world and I thank God every day you are a part of my world.

Acknowledgments

There are no words to adequately express the gratitude I have for many people who have helped and supported me through my doctoral journey. First and foremost Dr. Denisse Thompson, I can't thank you enough for all your support, understanding, patience, encouragement, and when I needed it, scolding. I am eternally grateful for your guidance and your limitless patience. To Dr. Janet Richards, thank you for being there every time I needed you. You were always ready with a shoulder to cry on, an inspiring word to keep me going, or encouragement when I faltered. Thank you as well to Dr. Gladis Kersaint. The three of you show me what excellence looks like in everything you do, and inspire me on a daily basis. I would also like to thank Dr. Chen for his efforts and support.

There are numerous doctoral students I have met, worked with and studied with at USF without whom I would never have completed this adventure. I would especially like to thank Richard Osorio, Andrea Bennett, and Barbara Zorin. Your unwavering friendship, support and encouragement will always be appreciated.

I also want to acknowledge my family and friends who have encouraged and supported me. I especially want to thank my brother John for all the weekends he took my children out for some fun while I was working.

Last, but surely not least, I give thanks to God. You celebrated in my triumphs and carried me through the low spots.

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Abstract

The concept of function is one of the essential topics in the teaching and learning of secondary mathematics because of the central and unifying role it plays within secondary and college level mathematics. Organizations, such as the National Council of Teachers of Mathematics, suggest students should be able to make connections across multiple representations of mathematical functions by the time they complete high school. Despite the prominent role functions play in secondary mathematics curriculum, students continue to struggle with the complex notion of functions and especially have difficulty using the different representations that are inherent to functions (algebraic, graphical and tabular).

Technology is often considered an effective tool in raising student achievement, especially in learning functions where the different representations of a graphing calculator are analogous to the different representations of a function. Opportunity to learn is another important consideration when examining achievement and is generally considered one of, if not the most important, factor in student achievement. Opportunity to learn, or the measure of to what extent students have had an opportunity to learn or review a concept, is often measured with self-reports of content coverage.

This study examined the relationship between opportunity to learn, students' use of graphing calculators, and achievement within a curriculum that supports integrated use of technology and focuses on conceptual understanding of mathematical concepts. The research questions focused on what opportunities students had to learn functions from the enacted curriculum, what calculator strategies students used when solving function problems, how both opportunity to learn and calculator strategies influenced student achievement, and what

relationships exist between opportunity to learn, use of calculator strategies, and student achievement.

This study is an in-depth secondary analysis of a portion of data collected as part of the evaluation study of *Precalculus and Discrete Mathematics* (Third Edition, Field-Trial Version) developed by the University of Chicago School Mathematics Project. Participants in this study ($n = 271$) came from six schools, seven teachers, and 14 classes. Instruments in this study include two pretests (one with technology and one without) and three posttests (two with technology and one without) and a calculator usage survey for one posttest. In addition to five student assessments, teachers completed opportunity-to-learn surveys for the posttests and chapter evaluations forms on which they indicated the lessons taught and the homework problems assigned from the textbook. Some students ($n = 151$) had access to graphing calculators equipped with computer algebra systems (CAS) while others ($n = 120$) had access to graphing calculators.

Students had multiple opportunities to learn functions as measured by lessons taught, homework assigned, and posttest items teachers reported as having taught or reviewed the content necessary for students to correctly answer the items. Overall, students showed a positive increase in achievement between the pretests and posttests. In general, achievement was positively correlated to OTL Lessons, negatively correlated to OTL Homework, and had no correlation to OTL Posttests when controlling for prior knowledge. Results indicate students appear to be, for the most part, making wise choices about when and how to use graphing calculators to solve function items. Students prefer the graphical representation and are rarely using CAS features or tables, even when they are the best choices for solving a problem.

Results from hierarchical linear models (HLM) show use of strategies ($\beta = 0.96$), access to CAS ($\beta = 5.12$), and OTL lessons ($\beta = 0.75$) all had significant and positive impacts on

student achievement for one of the posttests, when controlling for prior knowledge. Results from path analyses also indicated use of strategies had a direct and positive effect (β =0 .14) on student achievement but showed access to CAS had a negative indirect effect (β = -0.64) on student achievement for the same posttest mitigated through OTL Lessons (β = 0.30).

The results of this study have implications for both researchers and mathematics educators who seek to understand ways in which teachers can increase students' understanding of functions and student achievement. The relationship between the use of technology and student achievement in relation to opportunity to learn is complex, but use of calculator strategies appears to have a positive effect on students' opportunity to learn functions and student achievement when used in a curriculum that focuses on conceptual understanding and integrates technology.

Chapter 1: Introduction

The concept of function is one of the most important topics in secondary school mathematics curriculum in the United States. Just as developing a sense of numbers is the goal of elementary mathematics curriculum, developing a sense of functions should be the goal of secondary curriculum (Eisenberg, 1992). In addition, the function concept is essential to the teaching and learning of mathematics because of the central and unifying role it plays within secondary mathematics (Dubinsky & Harel, 1992), and is one of the key differentiations between classical and modern mathematics (Kleiner, 1989). Kashefi, Ismail, and Yusof (2010) posit understanding functions is a prerequisite for learning many other mathematical concepts; without understanding functions, the learning of other concepts in secondary or undergraduate mathematics may become difficult, if not impossible.

Researchers are not the only ones who have advocated for the importance of functions in mathematics curriculum. The National Council of Teachers of Mathematics [NCTM] (1989, 2000) suggests students should be able to make connections across multiple representations of mathematical functions by the time they complete high school. In the more recent Common Core State Standards for Mathematics [CCSSM] (National Governors Association [NGA] Center for Best Practices and Council of Chief State School Officers [CCSSO], 2010), functions are one of seven topics students should master as part of the secondary mathematics curriculum.

Students struggle with the complex notion of functions (Akkoc & Tall, 2002; Akkoc & Tall, 2003; Akkoc & Tall, 2005; Duval, 2006; Sajka, 2003; Schoenfeld, Smith, & Arcavi, 1993; Schwarzenberger, 1980; Sfard, 1991; Sierpinska, 1992; Siti Aishah Sh, 2010; Tall, 1992; Tall, 1993; Thompson, 1994; Vergnaud, 1998), a trend that mathematicians, researchers, and policy makers perceive as concerning, given the importance they collectively place on understanding and applying functions in the secondary mathematics curriculum. In particular, students have difficulty with different representations that are intrinsic to the different facets of functions. Each representation (equations, graphs, tables, and words) offers information about particular aspects of the concept but does not describe it completely (Duval, 2006; Gagatsis & Shiakalli, 2004; Kaldrimidou & Ikonomou, 1998). Students often have a hard time working with functions due to the need to coordinate and translate among these multiple representations (Abdullah, 2010; Schoenfeld, Smith & Arcavi, 1993; Schwarz, Dreyfus, & Bruckheimer, 1990).

There are many factors that influence student achievement. Porter (2002) contends knowing the content of instruction is essential to researching factors that affect student achievement. Indeed, the National Research Council considered opportunity to learn (OTL), one aspect of content instruction, to be “the single most important predictor of student achievement” (National Research Council, 2001, p. 334).

Although the concept of OTL sounds simple, there are many interpretations of it (Flodin, 2002). For instance, one can measure OTL by examining how much emphasis a topic receives in written materials such as a textbook. One can alternatively measure OTL by the instructional time devoted to a particular topic, either in terms of teaching or the amount of time students are engaged in learning it. Herman and colleagues (Herman, Klein, & Abedi, 2000) operationalized four overlapping categories to measure OTL: curriculum content, instructional strategies, instructional resources, and general assessment preparation.

Whereas there is considerable research on the difficulties students face when learning functions, there is little research examining students’ opportunity to learn functions by examining criteria from the categories developed by Herman and colleagues (Herman, Klein, & Abedi, 2000). Thus, there is a need to investigate the relationship between achievement in solving function problems, and factors that affect students’ opportunity to learn functions.

Rationale

Data from large-scale assessments show U.S. students struggle when solving function problems (Center, 2004; Livingstone, 1986; Martin, Mullis, & Chrostowski, 2004). In recent years, efforts to raise student achievement on assessments have focused on legislation such as No Child Left Behind (NCLB, 2001), high-stakes testing, and distribution of educational resources (Schmidt & Burroughs, 2013). According to a recent study, “students [in mathematics classes] are exposed to widely varying content not only across states and school districts but within schools. Such inequities in content coverage deny students equal learning opportunities” (Schmidt & Burroughs, 2013, p. 1). This disparity in opportunity to learn was among the impetus behind the creation of the CCSSM (CCSSO, 2010). The development of these rigorous standards, which all states could choose to adopt, was one effort to close the achievement gap by giving all students the same opportunity to learn mathematics.

Although high-stakes testing, equity of resources, and common curriculum standards are important considerations in reducing the achievement gap, what happens in the classroom is equally as important. Many experts advocate the best way to raise student achievement is through a standards-based curriculum such as the CCSSM, which is a curriculum that emphasizes conceptual understanding, problem solving, thinking, reasoning, use of multiple representations, integrates use of technology and real-world applications, and deemphasizes memorization of rules and procedures (CCSSO, 2010; Marzano & Kendall, 1996; McLaughlin & Shepard, 1995; Senk & Thompson, 2003).

Researchers recognize the important role the curriculum plays in students’ opportunity to learn, and therefore, in their achievement (McDonnell, 1995; Tarr, Chávez, Reys, & Reys, 2006). The most central tool to any curricula is the textbook. In practice, the textbook is considered by some to have more impact on student learning than state or local standards (Tarr, Chávez, Reys,

& Reys, 2006). Perhaps this is why many educational professionals consider the textbook to be the most influential part of the curricula. Begle (1973) expressed it best:

> The textbook has a powerful influence on what students learn. If a mathematical topic is in the text, then students do learn it. If the topic is not in the text, then, on the average, students do not learn it….The evidence indicates that most student learning is directed by the text rather than the teacher. (p. 209)

Curricula that support conceptual understanding over procedural fluency frequently use multiple representations as recommended by NCTM (1989). Many espouse that complete understanding of function concepts only occurs when students are able to move seamlessly among the representations (i.e., students should be able to use each representation, translate between them, and know when to use each representation) (Gagatsis & Shiakalli, 2004; Herman, 2007; Hitt, 1998; Janvier, 1987; Kaput, 1985, 1987; National Council of Teachers of Mathematics [NCTM], 2000; Owens & Clements, 1998). In particular, Huntley and Davis (2008) refer to this ability as representational fluency, namely: “the ability to translate across different representations, to draw meaning about a mathematical entity from different representations, and to generalize across different representations” (p. 381). The four representations (i.e., symbolic, graphical, numerical [tabular], and verbal) form the well-known Rule of Four as shown in Figure 1. Huntley and colleagues (Huntley, Marcus, Kahan, & Miller, 2007) used the Rule of Four to define the strategies students use when solving function problems with technology by connecting the strategies (algebraic, graphical, tabular, arithmetic) to their different representations. In this document, I use the term strategies to refer to the use of a variety of multiple representations to solve function problems.

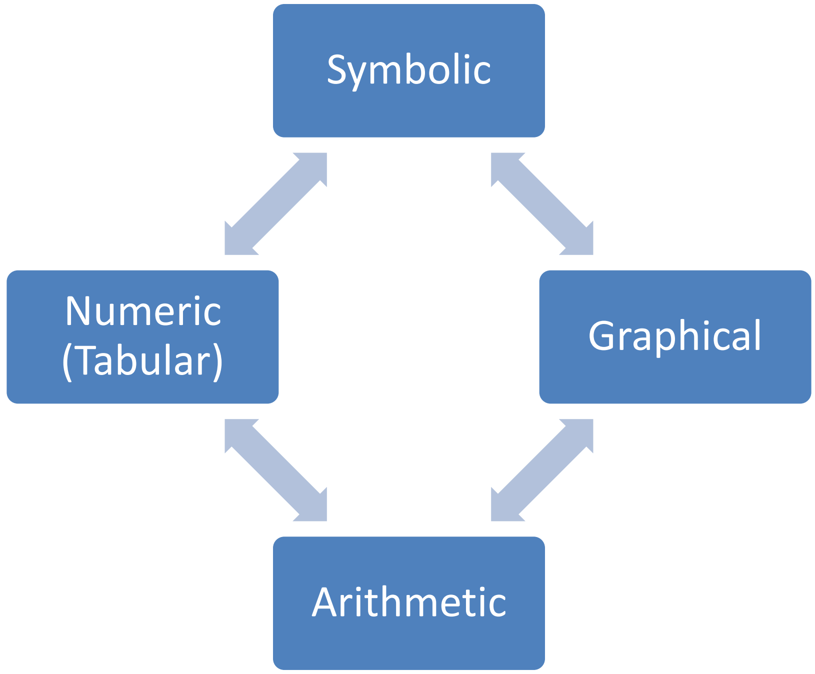


Figure 1. Rule of Four Representations of Functions.

Despite the large body of research, it is not clear whether mathematics educators have yet accomplished NCTM's goal of having all students "translate among tabular, symbolic, and graphical representations of functions" (NCTM, 1989, p 154). Research has shown students often prefer a symbolic strategy even when a different one would be more helpful; although students may attempt to use more than one strategy, they often regress to using the symbolic representation (Huntley & Davis, 2008; Senk & Thompson, 2006). Moreover, when students are taught to use all four representations, they rarely use a tabular strategy (Huntley & Davis, 2008; Huntley, Marcus, Kahan, & Miller, 2007).

One way to help students master different representations is through the use of graphing technology. Most graphical technologies provide a graphical representation of a function and can display at least two representations at the same time, such as an equation and a graph, or an ordered pair of numbers and a point on a graph. They can also display a table of values and help show the relationship among the ordered pairs, the graph, and the algebra (Leinhardt, Zaslavsky,

& Stein, 1990). Ruthven (1990) reported students who used graphing calculators had more strategies available to them and therefore attained higher achievement on tests than those who did not use graphing calculators. Harskamp, Suhre, and van Streun (1998, 2000) examined the effect graphing calculators had on the strategies students used when solving function problems. They reported students who used graphing calculators were three times more likely to use graphical strategies when solving problems than students who did not use a calculator. Additionally, Herman (2007) reported a positive correlation between achievement and the number of strategies students used.

Although the studies cited appear to suggest the use of graphing calculators can have a positive effect on student achievement, they provide no information about how students use the various strategies when they have access to a graphing calculator (henceforth referred to as *calculator strategies*) and, therefore, provide no explanation as to why students benefit from using the graphing calculator. For instance, Burrill et al. (2002) revealed weaknesses and gaps in the extant body of research on the use of graphing calculators. She and her colleagues called for more research into the kinds of mathematical problems for which students choose to use handheld graphing technology and how students use the calculator to solve these problems. Harskamp and associates (Harskamp, Suhre, & Van Streun, 2000) also advocated for more research focusing on how students' choice of strategies changes when they use a calculator, and how students move from using one strategy to using multiple strategies.

A few studies have investigated students' use of graphing calculator strategies when problem solving (Herman, 2007; Huntley & Davis, 2008; Huntley, Marcus, Kahan, & Miller, 2007; Senk & Thompson, 2006). Nevertheless, Herman (2007), Huntley and Davis (2008) as well as Leng (2010) have supported calls for additional research to examine solving strategies when students have a choice of using a calculator, particularly because research shows students often do not use calculators when that option is provided (Huntley and Davis, 2008; Huntley, Marcus, Kahan, & Miller, 2007). Others have also noted a scarcity of research on the use of calculators in advanced mathematics (i.e., Algebra II, trigonometry, precalculus, calculus, probability and statistics, and discrete mathematics) where there are more opportunities for productive graphing calculators (Crowe & Ma, 2010) and use of computer algebra systems.

Because students' use of problem solving strategies and students' opportunity to learn have been shown to play an important role in what mathematics students learn, it is a useful endeavor to investigate the relationship among achievement, strategies used when solving function problems, calculator usage, and students' opportunity to learn. It would also be useful to frame any investigation of function problem solving within the context of the CCSSM because these new standards are the impetus for the development of new curriculum materials. Results from this study could advance the current body of knowledge documenting students' difficulties solving function problems. The findings could also impact how function problems and strategies are presented in curricula materials and the classroom.

Research Questions

The purpose of this study is to examine students' use of strategies (i.e., function representations) when they solve function problems and the relationship, if any, among: strategies, opportunity to learn, use of technology, and student achievement. The following research questions guided this investigation:

1. What are students' opportunities to learn about functions in a precalculus course?
2. What calculator strategies do Precalculus students use when solving function problems? In particular, in what ways do students use these strategies when using a graphing calculator to solve function problems from both teachers' and students' perspectives?

3. How is Precalculus students' achievement in solving function problems related to their use of calculator strategies? In particular, what relationship, if any, exists among opportunity to learn, achievement and calculator strategies students use when solving function problems?
4. What effect does the use of technology, including calculator strategies, and opportunity to learn have on achievement when technology usage is reported from the students' perspective on a multiple choice assessment and from the teachers' perspective on a free response assessment?

Significance of the Study

There has been little research on problem solving strategies students use when solving function problems with or without calculators. Few studies have examined the relationship between OTL and achievement in relation to the strategies used in solving problems. Even fewer studies have examined these variables within the context of a curriculum based on NCTM (1989) recommendations, which includes standards-based instruction with multiple representations and technology integrated into the curriculum. Hence, this study addresses this gap in the literature.

The results of this study have the potential to inform teachers, researchers, and professional development facilitators on how students' use of strategies can influence achievement when solving function problems, and the degree to which students' opportunity to learn functions influences the relationship among problem solving strategies, technology, and achievement. In sum, results from studies such as this can help fill the documented gap in the extant body of research to investigate *in what contexts* technology influences, either positively or negatively, student achievement, and what relationships exist between achievement when using technology and opportunity to learn.

Definition of Terms

The following terms will be used frequently throughout this study. Whenever possible I use the commonly accepted mathematical definitions based on applicable mathematics education research. However, when there is disagreement among the research community for a particular term, I will provide an operational definition based on the needs of this study.

Function. In general terms, a function is defined as a correspondence between two variables, so that to any value of the independent variable (domain) it associates one and only one value of the dependent (range) variable (da Ponte, 1992). For purposes of this study I will use the characterization provided by the CCSSM 9-12 (NGA Center for Best Practices and CCSSO, 2010) which states:

> Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. (p. 67)

Function Standards. The CCSSM (CCSSO, 2010) specify four domains (learning outcomes) related to the teaching and learning of functions at the high school level. Those domains are: a) interpreting functions; b) building functions; c) linear, quadratic, and exponential models; and d) trigonometric functions.

Multiple representations. This is used to describe or symbolize mathematical concepts based on the Rule of Four. Multiple representations are used to understand, to explain, and to communicate different mathematical aspects of the same object as well as connections among different aspects. Multiple representations can include graphs and diagrams, tables and grids, formulas, symbols, words, physical and virtual manipulatives, pictures, and sounds. The four representations used in this study are algebraic, graphical, tabular, and verbal.

Algebraic representation. The representation of the relationship between an independent variable and a dependent variable using letters as variables, numbers, and mathematical operations (such as + or -). Examples of functions in algebraic representation would include $f(x) = 2x\text{-}4$, $y = 2x^3$, and $y = [x]$ (a step function with a domain of all real numbers).

Graphical representation. A representation of a function that uses points and/or curves in a Cartesian coordinate plane (or other appropriate visual coordinate systems for higher or other dimensions) to display some or all of the elements of the independent variable of the function with its corresponding dependent element. Some examples are shown in Figure 2.

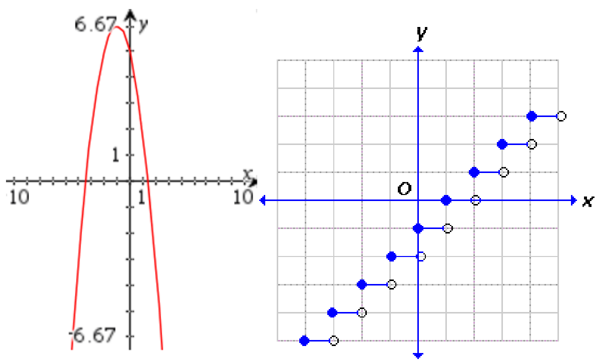


Figure 2. Graphical Representations of Functions.

Tabular representation. A representation of a function where some or all of the independent elements are listed in pairs with their corresponding dependent elements and displayed in a table. An example is shown in Table 1.

Table 1

Example of Tabular Representation of Function

x	*f*(*x*)
-1	1
0	0
2	4

Verbal representation. A representation of a function using words, and possibly numbers, to describe the relationship between the independent and dependent variables. An example of a verbal representation of a function would be "Mary earns $3.50 for every hour she works".

Strategies. For the purposes of this study I will use the strategy definitions Huntley and Davis (2008) proposed for solving problems both with and without calculators as shown in Table 2.

Table 2

Strategies Used to Solve Function Problems

With Calculator	Without Calculator
Algebraic: Use of CAS features to solve a problem.	*Algebraic:* Use of symbol manipulation
Graphical: Use of a graph to solve a problem.	*Graphical*: Use of a graph to solve a problem.
Tabular: Use of a table to solve a problem.	*Tabular*: Use of a table to solve a problem.
Arithmetic: Substituting in values (includes trial and error) and checking answers	*Arithmetic:* Substituting in values (includes trial and error) and checking answers
Unknown: Cannot determine how the calculator was used, if at all, to solve a problem.	*Other:* None of the above strategies were used

Note: CAS refers to graphing calculators equipped with computer algebra systems. Based on strategies developed by Huntley et al. (2007), Huntley and Davis (2008).

Calculator active. Calculator-active problems are those that would be difficult, if not impossible, to solve without the use of a calculator (Greenes & Rigol, 1992; Harvey, 1992).

Calculator inactive. Calculator-inactive problems are those for which there is no advantage (perhaps even a disadvantage) to using a calculator (Greenes & Rigol, 1992; Harvey, 1992).

Calculator neutral. Calculator-neutral problems are those that can be solved without a calculator, although a calculator might be useful (Greenes & Rigol, 1992; Harvey, 1992).

Opportunity to Learn (OTL). Opportunity to learn has been defined as "whether or not students have had an opportunity to study a particular topic or learn how to solve a particular type of problem" (Husen, 1967, p. 162). In this study, I am viewing opportunity to learn as the extent to which teachers have taught lessons and assigned homework, and the extent to which teachers taught or reviewed content on assessments.

Curriculum. The term is used to describe mathematical topics that comprise a specific course of study – the *what* of mathematics teaching and learning (Stein, Remillard, & Smith, 2007). Researchers have identified different types of curricula. This study examined the effects of the intended, implemented, assessed and achieved curricula.

Intended curriculum. What is articulated in local, state or national frameworks generally for a particular subject at a specified grade level is referred to as the intended curriculum (Tarr, Chávez, Reys, & Reys, 2006).

Implemented curriculum. The mathematics presented to students by the teacher is known as the implemented curriculum (Robitaille et al., 1993).

Assessed curriculum. The content that is assessed to determine achievement is described as the assessed curriculum (Porter, 2006).

Achieved curriculum. Students' observed performance (what they actually know) about a particular topic is referred to as the achieved curriculum.

Summary

In this chapter, I provided a background to the study and a brief account on the important role the concept of function plays in the teaching and learning of secondary mathematics. I also discussed the role opportunity to learn plays in achievement and the importance of understanding how curriculum content, instructional strategies, instructional resources, and general assessment preparation factor into the measurement of opportunity to learn and achievement. This was followed by the rationale for the study, the research questions, and the definition of key terms. Chapter 2 presents a review of literature that grounds this study. Chapter 3 contains the methods used to collect and analyze the data. Chapter 4 presents the results of the analyses. Chapter 5 provides a discussion of the results.

Chapter 2: Review of the Literature

This chapter consists of two main sections. The first section examines the difficulties students face when learning functions and the role multiple representations play in the learning of functions. The second section begins with a brief discussion of opportunity to learn and some of the different models used to explain and measure OTL, and continues with an examination of the research associated with curriculum content, instructional strategies, instructional resources and general assessment preparation in terms of how they impact students' OTL and achievement.

Functions in the Curriculum

For over two centuries mathematicians grappled with the concept of function and how it should be defined. It is, therefore, not surprising that students have difficulty identifying functions and making sense of their representations. In this section, I review the research on difficulties students encounter when learning about functions. Then, I examine the large body of research regarding multiple representations and the struggles students face when using different representations of functions.

Identifying functions. When students are introduced to a formal definition of a concept, they do not always use that concept definition when deciding whether a given mathematical object is an example or a non-example of the concept (Tall and Vinner, 1981; Vinner & Dreyfus, 1989). Students often fail to recognize what is and what is not a function. This can be true when students view functions as algebraic quantities, graphs, or are given a verbal description.

Students often mistakenly think functions can always be represented by a single formula. In their research on the concept or definition of function, Vinner and Dreyfus (1989) had 271 college students review the graph in Figure 3; some of those who identified the graph as a non-function justified their decision on their inability to represent the graph with a single equation or formula.

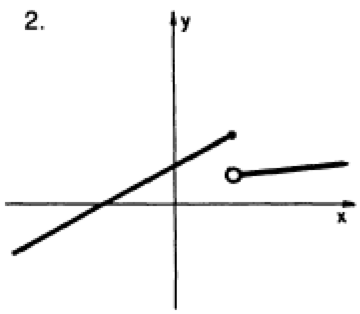


Figure 3. A Function Identification Problem. Adapted from "Images and Definitions for the Concept of Function," by S. Vinner, and T. Dreyfus, 1989, *Journal for Research in Mathematics Education*, 20(4), p. 359. Copyright 1989 by National Council of Teachers of Mathematics. Adapted with permission.

Cansiz, Küçük, & Isleyen (2011) obtained similar results in their study of functions with 61 9th, 10th and 11th grade students. They reported over 96% of the students failed to correctly identify a function when the description or graph could not be represented by a single formula.

Thompson (1994) concluded many students define functions as "two written expressions separated by an equal sign" (p. 25). For example Thompson (1994) noted a student who, when asked to prove $S_n = 1^2 + 2^2 + \cdots n^2$, provided the following as part of his proof: $f(x) = \frac{n(n+1)(2n+1)}{(n+6)}$. Thompson concluded the student found the explanation correct and sufficient because the latter was a general representation of the former. Students mistakenly believe the equality sign implies continuity in a function, which is not a requirement.

Students also struggle with continuity of functions. Many students believe all functions should be *nice* and should be one-to-one (Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Selden & Selden, 1992). Carlson (1998) reported students were likely to misclassify $f(x) = \begin{cases} 0, & x \le 0 \\ e^{-\frac{1}{2}x}, & x > 0 \end{cases}$ as two separate functions and would classify functions such as Dirichlet's example, $\begin{cases} 0, & x \text{ is irrational} \\ 1, & x \text{ is rational,} \end{cases}$ as not a function because it is not *well-behaved*. Misconceptions that arise from students' ill-conceived notion of one-to-one include difficulties with constant functions because they are not one-to-one (Dubinsky & Harel, 1992; Even, 1990; Markovits, Eylon, & Bruckheimer, 1986), and over reliance on the vertical line test (Wilson, 1994).

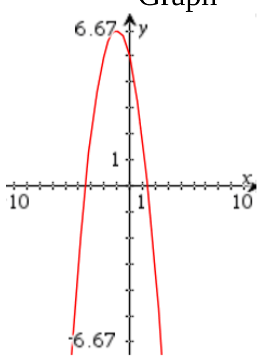
Sfard (1991) argued mathematical concepts, in general, and functions, in specific, are dual in nature. She posited the concept of function consists of both the object of the function, a set of ordered pairs, and a process or "a method of getting from one system to another" (Skemp, 1971, p. 246). Students often look at the object and the process as distinct, mutually exclusive entities instead of different lenses or representations by which to view the same whole. The propensity to look upon the object and the process separately has been a source of difficulties for students attempting to develop conceptual understanding of functions.

Multiple representations of functions. Although Sfard (1991) viewed the different aspects of a function as *different sides of the same coin*, it may be more appropriate to view the function as a multi-dimensional entity. A function has different representations and "each representation is of a different nature, has limited representation capabilities and describes different aspects of the object it represents" (Elia, Panaoura, Eracleous, & Gagatsis, 2007, p. 538). Each of the different representations of a function – graphical, tabular and algebraic – highlights a different aspect of the concept and no one representation can adequately describe the concept (Gagatsis & Shiakalli, 2004; Kaldrimidou & Ikonomou, 1998). For example, graphs display qualitative data including information regarding the shape and direction of the relationship between the variables (Ainsworth, Bibby, & Wood, 2002). Formulas, however, focus more on the procedural aspects of the concept (Kollöffel & de Jong, 2005), and tables highlight patterns and regularities (Ainsworth et al., 2002).

A function can have different representations: verbal, graphical, algebraic, and tabular (See Figure 4). Moving from one representation to another provides students opportunities to visualize the relationships and helps them obtain a better conceptual understanding, which strengthens their ability to solve problems (National Council of Teachers of Mathematics, 2000). Because different information can be obtained by viewing the function in different representations, it is critical students are able to move seamlessly between representations.

Verbal: Mary is standing on the roof of a building. She is 5 foot tall and throws a penny off the building. At one second the penny is 2 feet above the roof and at two seconds the penny is three feet below the roof.

Graph



Equation

$$f(x) = -x^2 - 2x + 5$$

Table

	x	B
◆		=f2('x)
2	-1	6
3	0	5
4	1	2
5	2	-3
6	3	-10

Figure 4. Different Representations of the Same Function Problem.

Yet, students at many levels face difficulties when attempting to move from one representation of a function to another (Abdullah, 2010; Artique, 1992; Gagatsis, 2004; Hitt, 1998). In one study of 195 college students, Gagastis and Shiakalli (2004) reported a significant correlation between translation ability and problem solving ability, and noted translation ability accounted for 53% of the regression equation ($p < .05$). They noted no significant relationship between the verbal and algebraic representations of the problem. Students did not recognize that the verbal and algebraic forms were two different views of the same concept; instead, they saw them as separate concepts. The researchers concluded success with the function concept requires students to master at least two different representations. In another study, Abdullah (2010) interviewed students who were working with function concepts. He noted students encountered difficulties going from the algebraic representation of a function to a graphical representation and did not make the connection that points on the graph corresponded to (x, $f(x)$) pairs in the equation. The results from these studies were consistent with earlier research (Eisenberg, 1992; Hitt, 1998).

Also, consistent with the work of Eisenberg (1992), more recent research has shown students who understand how to use the function concept in one representation often experience difficulties applying the same concept when using a different representation (Abdullah, 2010; Gagastis, 2004). In fact, students may not make the connection that an equation, table, and graph all communicate the same information in different forms, but instead view the different representations as separate entities. Student difficulties in using multiple representations go beyond their inability to switch representations. Research has also noted students tend to prefer some representations over others.

In a recent study of 44 pairs of high achieving high school students, Huntley and Davis (2008) reported students were most likely to use algebraic methods when solving function problems, even when a different representation would be more helpful. They also reported students preferred graphical solutions over using tables and rarely used tables. In another study of 38 students, Herman (2007) noted students were more likely to utilize symbolic manipulation when solving function problems. In solving six function problems, students, while they preferred algebraic solutions, did use graphical methods. However, none of the students used a table on any of the problems. Abdullah (2010), Herman (2007), and Huntley and Davis (2008) reported similar findings.

Multiple representations using calculators. Some educators believe that when students are able to move more frequently between representations as they solve problems, they become more aware of the connections between these representations and begin to see how information about functions is presented in different ways in different representations (Kaput, 1989). However, research and international assessments have shown students have not been successful in using multiple representations with functions (Kaput, 1987, 1989). Kaput argued dynamic technologies, such as graphing calculators, can be instrumental in helping students understand linked representations (1989). Others have argued it is difficult for students to move between different representations of functions without technology because using pencil and paper techniques to perform the necessary computations can be very tedious and prone to error (Harvey, Waits, and Demana, 1995). Graphing calculators can carry out computations quickly and accurately, allowing students to see multiple representations with ease. Harvey, Waits, and Demana (1995) used the example of a fifth-degree polynomial. Drawing this graph by hand would be a difficult process of finding roots and maxima and minima. But, by using a calculator, students can easily move between symbolic and graphical representations to describe the polynomial.

There is extensive research in mathematics education showing a correlation between translation ability and mathematical problem solving (Elia, Panaoura, Gagatsis, Gravvani, & Spyrou, 2006; Gagatsis & Shiakalli, 2004; Herman, 2007; Hitt, 1998; Kaput, 1987; NCTM, 2000). In one study of 197 university students studying functions, Gagatsis and Shiakalli (2004) found a significant, positive correlation between students' ability to translate between multiple representations and their problem solving ability when using graphing calculators. Although researchers agree modeling student success is complicated and not well understood, Gagatsis and Shiakalli (2004) suggested translation ability is clearly one important factor that cannot be overlooked. In another study of 193 high school students, Elia and colleagues (Elia, Panaoura, Gagatsis, Gravvani, & Spyrou, 2006) reported similar results linking problem solving success with functions to translation among different representations.

In a survey of 27 students, Hennessy, Fung, and Scanlon (2001) found when students had experience using the graphing calculator they were able to translate between representations frequently and coordinated information from different representations (usually the table and graph) to solve problems. Later research conducted by Herman (2007) found students were more likely to solve problems using more than one representation when they used graphing calculators. In her study of 38 students who had been taught to use the TI-83 graphing calculator, Herman found students were better at solving function problems using multiple representations after the end of her class. She also reported most students continued to prefer working with algebraic symbols rather than graphs or tables.

In their study of 44 pairs of high school students, Huntley and Davis (2008) reported use of graphing calculators and multiple representations helped students be more successful at problem solving. Students still overwhelmingly preferred symbolic manipulation, but use of multiple representations helped some students solve problems that were difficult to solve using algebra; in some cases, the calculator helped students recognize symbolic errors. They concluded, “students who learn about and become facile with multiple representations and strategies may become more reliant on themselves and less reliant on teachers for detecting and correcting their errors” (pp. 386-387).

Kastberg and Leatham (2005) reviewed the body of research on using graphing calculators in the teaching and learning of mathematics. They concluded that when the use of graphing calculators was embedded in the curriculum, students were able to integrate information obtained from multiple representations and were better problem solvers than their peers at interpreting mathematics problems in context. Dunham (2000), who has done extensive work compiling literature on graphing calculators, had reported similar results. She found that, in general, "students who use graphing calculators display better understanding of function and graph concepts, improved problem solving, and higher scores on achievement tests for algebra … skills" (p. 40).

Not all the research on multiple representations has been positive. Some studies (Gagatsis & Shiakalli, 2004; Moreno & Duran, 2004; Moreno & Mayer, 1999; Seufert, 2003; Yerushalmy, 1991) have provided evidence to suggest that, instead of supporting conceptual learning and problem solving, multiple representations can sometimes have the opposite effect, impeding the learning process or decreasing students' ability to solve problems. For example, Ainsworth et al. (2002) explain that students need to devote a significant amount of perceptual and attention resources to fully grasp the meaning behind the different representations. Some students become overwhelmed with such demands, which can hinder learning. In another example, Gagatsis and Shiakalli (2004) found students' ability to translate between different translations had a direct impact on problem solving success, especially when using the graphical representation of the function. Some studies have shown integrating multiple representations can be extremely demanding for the learner (Moreno & Duran, 2004; Moreno & Mayer, 1999; Seufert, 2003); others (Elia et al. 2006; Sierpinska 1992; Yerushalmy 1991) have suggested when students struggle with multiple representations their learning, problem solving abilities, and achievement are negatively impacted.

Strategies and multiple representations of functions. Most of the research on the use of graphing calculators in the teaching and learning of mathematics has focused on the areas of student achievement, attitude, or the teaching and learning of a particular mathematics topic such as functions (Burrill et al., 2002; Ellington, 2003). However, there has also been research examining how students use graphing calculators as a strategy to solve problems (Berry & Graham, 2005; Harskamp, Suhre, & van Streun, 2000; Ruthven, 1990).

Ruthven (1990) investigated the strategies students used as they answered six function questions by writing down the equation to a given graph. The sample consisted of 87 high school students of which 47 had access to graphing calculators and 40 did not. Ruthven identified three strategies the students used to obtain their solutions which he was able to link back to the graphing calculator usage: analytic construction, graphic trial, and numeric trial.

Students who used analytic construction utilized their existing knowledge of parent functions to build up the function. Those in the calculator group were able to quickly and effectively check their solutions. The second group used graphic trial. Students used their calculators to modify the equation of the graph until they found the correct answer. The third approach was numeric trial in which students made a symbolic guess, usually by examining the numeric pattern of a few points, and adjusted their conjecture by plotting points to see if the graphs matched.

Other researchers have also explored how students use graphing calculators as tools to solve problems. For instance, Harskamp and colleagues (2000) investigated the effect a graphing calculator had on students' solution strategies and their knowledge of functions. They examined written student solutions and coded the strategies the students used to solve function problems. They used the following strategy categories: heuristic; graphic; algorithmic, or analytic; and none. When students used their own strategies, including trial and error, the solution was coded as heuristic. Graphic was used to indicate students created a graph to solve a problem. When students relied on algebraic techniques, the strategy was coded as algorithmic. No solution was used when the problem was left blank.

Other researchers suggest relying on student solutions provides an incomplete picture of how students use calculators because students may not write everything down or may write down one solution but use another. In their research on the use of graphing calculators, Berry and colleagues (Berry and Graham, 2005; Berry, Graham, & Smith, 2006; Berry, Graham, & Smith, 2005) used key stroke capturing software to record every key students pressed while using the calculator. They found three main strategies.

One strategy students used was to enter the function into the calculator and then copy the graph on their paper. Students who used this approach did not make any changes to the display or use any additional strategies to verify the correctness of their graphs. In another group, students entered the function in a calculator but also made changes to the display, such as the size of the window, to get a better picture. Again, the graph may not have been correct and students did not verify their solution. The third strategy students utilized involved using the graphing calculator as a tool to check their work. These students already knew what the function looked like and merely used the calculator to verify their solution.

Still others have categorized students' solution strategies by the features students used on the graphing calculator. Huntley and Davis (2008) identified these strategies – algebraic (using the equation solver), graphical, and tabular – to show how students solved linear function problems. Graham and colleagues (2008) compared how students use calculators to how teachers expect them to be used and reported the most common strategies students used were algebraic (checking answers) and graphical. Other researchers (Herman, 2007; Huntley, Marcus, Kahan, & Miller, 2007; Senk & Thompson, 2006) have also used the algebraic, graphical and tabular categories in their research. All of these studies based their categories on the *Rule of Four*, which was adapted from the Rule of Three (see Hughes-Hallett, 1991) that began during the calculus reform movement. These strategies reflect the NCTM recommendation (2000) that students should be skilled in using each representation, and should be able to move seamlessly between the different representations, including knowing when to use a particular tool and when to utilize a different tool.

Multiple representations and the strategies to use them are crucial in the teaching and learning of the function concept. However, for students to learn they must be provided with opportunities to learn the concepts. Consequently, in the next section I review the extant body of research on opportunity to learn and how it affects student achievement.

Opportunity to Learn

The term *opportunity to learn* was first used in the early 1960s as a way to capture differences in achievement in international mathematics studies based on the extent to which the assessed content was taught (Boscardin, et al., 2005; Gau 1997; Husen, 1967; McDonnell, 1995). In the First International Mathematics Study (FIMS), OTL was defined as a measure of "whether or not students have had an opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (Husen, 1967, p. 162). Although this definition is simple and clear, Floden (2002) noted OTL has been interpreted in many ways since the term was first introduced. In one interpretation, Porter (2002) used three broad categories to describe OTL: educational inputs, processes, and outputs. Inputs refers to monetary resources, teacher quality (e.g., training), and student socio-economic status (SES). Processes include school and community characteristics such as quality of standards, type and quality of curriculum, and teacher quality. Finally, outputs refer to factors such as student achievement, participation and attitudes.

In another interpretation, Herman and colleagues (Herman, Klein, & Abedi, 2000) interviewed teachers and conducted student surveys to examine various aspects of OTL. They defined four overlapping categories that conceptualize OTL: curriculum content, instructional strategies, instructional resources, and general assessment preparation. These categories, discussed in the following section, are used to summarize the current literature on the impact of OTL on student achievement.

Curriculum content. One of the key factors affecting student achievement is the content of instruction (Porter, 2002), which focuses on how much students are exposed to specific subjects and topics. Different manifestations of curriculum content in relation to OTL have included content coverage, content exposure, and content emphasis (McDonnell, 1995; Porter, 2002; Wang, 1999). Content coverage, generally viewed as the most frequently studied aspect, has been measured in different ways, such as by teachers' self-reports, direct observations, and analysis of the lessons or content taught. Content exposure is generally measured using direct observation to document the amount of time a teacher spends covering specific content. Content emphasis considers how a content area is treated: as a major topic, a minor review, or not taught at all (Wang, 1999).

One of the most important influences on content curriculum, especially in the United States, is the textbook. In fact, many consider the textbook to be the most influential part of the curricula (Begle, 1973). Others have also documented the critical role the textbook plays in student learning; Yerushalmy and colleagues (1993) reported students generally explore problems as they are written and do no more than asked by the instructions. They found students were unwilling to alter or expand a problem unless they had been specifically instructed to do so, and concluded that the textbook greatly determines student activities. Schmalz (1990) observed that the mathematics textbook almost totally determined the day-to-day instruction in the classroom. He reported teachers often started on page 1 and continued through the text without skipping, or adding, anything. In their analysis of French, English and German classrooms, Haggarty and Pepin (2002) also found there were clear differences in students' OTL, not only between countries but within countries as well. They concluded students have different and varying OTL depending on the textbook used and how a teacher chose to use the given textbook. In a study of 39 middle grades teachers, Tarr and colleagues (Tarr, Chávez, Reys, & Reys, 2006) reported 34% of the teachers used the textbook on 90% of instructional days and 70% relied on the text at least 3 out of every 4 instructional days.

Instructional strategies. OTL factors associated with instructional strategies include whether or not students have been exposed to the kinds of teaching and instructional experiences that would prepare them for success (Herman, Klein, & Abedi, 2000). Research at the high school level shows achievement is higher when the instructional strategies are in agreement with the philosophy of the curriculum. For example, achievement using Core-Plus Mathematics curriculum is lower when taught using traditional instructional strategies and higher when taught using the instructional strategies outlined by NCTM to stress conceptual understanding and that are aligned with the philosophy of the curriculum developers (Schoen, Ziebarth, & Hirsch, 2010).

Instructional resources. OTL factors associated with instructional resources focus on whether there are appropriate resources to prepare students for success on assessments and standards. Criteria in this category include teacher preparation, level of education, amount of experience, type of experience, participation in in-service professional development, and attitudes (Herman, Klein, & Abedi, 2000). Both the textbook and use of technology are examples of this category because the resources to which teachers have access can facilitate or hinder a school's ability to provide a high-quality instructional program (Oakes, 1989). Technology, in general, and the calculator especially, is one instructional resource which can help students master the concept of function in all its dimensions. Both textbooks and use of technology, especially the calculator, have a direct impact on students' opportunity to learn mathematics.

Textbooks and their use in teaching mathematics. Textbooks are among the most widely used and trusted resources in most parts of the world (Beaton, et al., 1996), often defining the mathematics students have an opportunity to learn (Haggarty, & Pepin, 2002; Tornroos, 2005). In most cases students are not given the opportunity to learn material not present in the textbook (Porter, 1995; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Schmidt, 2002) and teachers are unlikely to present material that is not in the textbook (Reys et. al, 2003). Some have noted the textbook is such a critical resource for learning that the textbook often directs instruction instead of the teacher (Begle, 1973). For example, a national survey of 364 mathematics and science researchers reported the textbook assigned for a class is a major factor in the teacher's selection of content for a lesson (Weiss, Pasley, Smith, Banilower, & Heck, 2003).

Classroom uses of calculators. Textbooks are not the only classroom resource shown to have a positive impact on opportunity to learn. In their seminal research on how calculators are used in the classroom, Doerr and Zangor (2000) identified five categories of classroom usage of the graphing calculator. The graphing calculator was used as (a) a computational tool, (b) a transformational tool, (c) a data collection tool, (d) a visualization tool, and (e) a checking tool (Doerr & Zanger, 2000).

Computational tool. The first category of calculator usage is as a computational tool. Doerr and Zangor (2000) defined computation tool usage as "evaluating numerical expressions and estimating and rounding" (p. 151). A study conducted by Hollar and Norwood (1999) examined 46 students who used the graphing calculator in their study of functions. They found the students who used the graphing calculator had higher levels of procedural skill and conceptual understanding than those students who did not use graphing calculators. Equally as important, they reported there was no significant difference between the treatment and control groups in computational skills. In other words, students who used the calculator as a computational tool did not suffer from a decrease in computational skills. Similarly, in a 2003 meta-analysis of the effect sizes of 15 studies, Ellington found no significant difference in computational skills between students who used calculators and those who did not.

Transformational tool. The second category of calculator usage is as a transformational tool. Using the calculator as a transformational tool was defined by Doerr and Zangor (2000) as using a graphing calculator to transform tedious computational tasks into interpretative tasks. One example given by Doerr and Zangor (2000) was a problem in which students examined the rate of change for a given function. Doerr and Zangor (2000) reported the students were able to find a rate function and generate a table of values for a rotating Ferris wheel. Then the students were able to determine the rate of change (Doerr & Zangor, 2000). In another study of 179 grade 11 students, Elia and colleagues (Elia, Panaoura, Eracleous, & Gagatsis, 2007) found, even when students were unable to solve function problems, they were more likely to understand the function concepts when they used the graphing calculator as a transformation tool to switch between the algebraic and graphical representations.

Data collection and analysis tool. The third category of calculator usage is as a data collection and analysis tool. Doerr and Zangor (2000) defined the third category as "gathering data, controlling phenomena, and finding patterns" (p. 154). Elia and colleagues (2006) assert graphing calculators are critical to helping students see and create patterns and functions from data. Several researchers have espoused the use of graphing calculators with CAS as a tool to generate patterns from data, assisting students in obtaining conceptual understanding (Drijvers, 2004; Lagrange, 2005a; Lagrange, 2005b; Stacey, Kendal, & Pierce, 2002).

Visualizing tool. The fourth category of calculator usage is as a visualizing tool. Doerr and Zangor (2000) defined the fourth category as "finding symbolic functions, displaying data, interpreting data, and solving equations" (p. 151). There is ample evidence from research to support the use of graphing calculators as a visualization tool. Burrill and colleagues (Burrill et al., 2002) reported students who used the graphing calculator had a better understanding of functions, solving equations, and interpreting graphs than those students who did not use the graphing calculator. They concluded that students who use graphing calculators to produce quick and accurate graphs become better problem solvers. Results from Burrill and colleagues (2002) mirror earlier findings by Slavit (1994) who posited "the graphing calculator provided the instructor a means of quickly changing symbolic function parameters in order to better discuss global functional properties of a given function class" (p. 11-12). In his seminal work, Ruthven (1990) hypothesized regular use of graphing calculators exposed students to the relationships between symbols and graphs, making it easier for students to recognize salient features of graphs and connect them to their symbolization.

Checking tool. The fifth category of calculator usage is as a checking tool. Doerr and Zangor (2000) defined the fifth category as "confirming conjectures and understanding multiple symbolic forms" (p. 151). Research has shown graphing calculators are commonly used as tools for checking or verifying work done by hand (Doerr & Zangor, 2000; Harskamp, Suhre, & Van Streun, 2000; Hennessy, Fung, & Scanlon, 2001; McCulloch, 2005; McCulloch, Kenney, & Keene, 2012). Many researchers believe students use a graphing calculator to check answers more often than any other use. Ruthven (1990) reported one third of the students used a graphing calculator to determine if an equation matched a given function.

General assessment preparation and OTL. The final category of variables used by Herman et al. (Herman, Klein, & Abedi, 2000) focused on general measures of preparation for the assessment. These variables sought to capture indicators of general preparation for the assessment by teachers and students. For example, teachers were asked how many class periods they spent directly preparing their students for the assessment. Although many use curriculum content OTL to include assessment items, others measure OTL on assessment items directly, generally using interviews or questionnaires to ascertain if teachers had taught the material necessary for students to answer specific assessment questions.

Herman and colleagues (2000) reported a positive correlation between OTL and achievement when considering how much time teachers spent in preparing students for assessments. Other methods for measuring the achieved curriculum include work by Cooley and Leinhardt (1980) who asked teachers to estimate the percentage of their students who had been taught the minimum material necessary to pass each item on a standardized achievement. In a related study, Leinhardt, Zigmond, and Cooley (1981) asked teachers to indicate whether each student or sample of students had been taught the information required to answer specific test items. Extending the concept of measuring OTL by examining assessment questions, Thompson and Senk (2001) adjusted achievement scores to account for students' OTL on assessment questions. They found the average scores rose when OTL was controlled, which suggests a positive correlation between OTL and achievement at the assessment level.

Summary

Researchers have illustrated the important role curricula materials; instructional strategies; instructional resources, including technology; and assessment preparation play in understanding how the research reviewed in this chapter provides the foundation for this study. Opportunity to learn impacts student achievement. Researchers have documented the important role technology can play in students' understanding of functions and the importance of learning the different representations of functions, as well as helping students obtain representational fluency and ease when shifting between representations. It is also clear from looking at the research that the ways in which teachers use the textbook play a critical role in students' opportunity to learn mathematics in K-12 education, and ultimately on their achievement on mathematics assessments. Each of these areas of research has influenced not only my understanding of the phenomenon under study in this investigation, but also the methods and interpretations used to conduct this investigation.

Chapter 3: Methods

This study analyzed students' opportunities to learn and their use of calculator strategies when solving function problems. The description of the study is divided into six sections. In the first section I provide an overview of the overall research design. In the second section I provide the research questions. The third section contains a description of the University of Chicago School Mathematics Project (UCSMP) and the sample to provide context for the current study. In the fourth section I discuss the data collection procedures. The fifth section contains a discussion of the analyses, and in the final section I discuss the reliability of the statistical methods.

Research Design

This study utilized quantitative methods in a correlational study that used secondary data analysis to determine to what extent opportunity to learn and the use of technology affect student achievement when learning functions. Opportunity to learn was analyzed by investigating the actual lessons and homework teachers assigned and teachers' reported coverage of content assessed by the items on the three posttests. Use of technology was examined using students' reported use of technology and strategies on one posttest and the codes teachers used to describe student solutions on the problem solving test. Specifically the study addressed the following research questions:

1. What are students' opportunities to learn about functions in a precalculus course?
2. What calculator strategies do Precalculus students use when solving function problems? In particular, in what ways do students use these strategies when using a graphing calculator to solve function problems from both teachers' and students' perspectives?

3. How is Precalculus students' achievement in solving function problems related to their use of calculator strategies? In particular, what relationship, if any, exists among opportunity to learn, achievement and calculator strategies students use when solving function problems?
4. What effect does the use of technology, including calculator strategies, and opportunity to learn have on achievement when technology usage is reported from the students' perspective on a multiple choice assessment and from the teachers' perspective on a free response assessment?

Context of the study

This study represented an in-depth secondary analysis of student achievement data collected as part of the field trial evaluation of Precalculus and Discrete Mathematics ([PDM] Field-Trial Version, Third Edition) developed by UCSMP (See Thompson and Senk in preparation). The following sections provide background on the UCSMP study which provides context for the current study. Following the overview is a brief description of the sample used for the field study.

Overview of The University of Chicago School Mathematics Project. UCSMP was established in 1983 in an effort to improve K-12 mathematics education and reflected collaboration between two departments at the University of Chicago (mathematics and education). Zalman Usiskin and Sharon Senk co-directed the secondary component, which designed a mathematics curriculum for students in grades 7-12. These materials were developed as one implementation of the recommendations in many documents throughout the 1980s that culminated in the content and process standards in the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards* (1989). Three editions of the curriculum were developed between 1983 and 2010. The main goals in the development of the

First and Second Editions of the UCSMP Secondary materials were: "(a) to upgrade students' achievement in mathematics; (b) to update the mathematics curriculum in terms of content; and (c) to increase the number of students who take mathematics beyond algebra and geometry" (Thompson, Senk, & Yu, 2012, p. 5). The Second Edition materials were developed and tested between 1992 and 1998 and the materials for the Third Edition were developed between 2005 and 2010. The development of the Third Edition materials built upon the goals implemented in the First and Second Editions but included, among other things, more use of technology and the use of graphing calculators with computer algebraic systems (CAS) capability.

This study focuses on the curriculum for *Precalculus and Discrete Mathematics* (PDM), which is:

> designed to prepare students for rigorous mathematical study in college. Precalculus topics include polynomial and rational functions, a study of advanced properties of functions, including limits, and the underpinnings of the derivative and integral. Polar coordinates and complex numbers are also topics of study. Discrete mathematics topics include work with recursion, permutations and combinations, and logic. Mathematical thinking, with particular attention to proof, is a unifying theme of the course. Computer algebra systems are assumed throughout the course (Thompson et al., p. 7).

The curriculum in PDM is comprised of seven major concept areas: proof, functions and their properties, discrete mathematics, trigonometry, foundations of calculus, polar and complex numbers, and polynomials and their operations.

The research questions for the field-study had two main areas of focus: how the teachers used the materials and what students learned when taught using the materials. The main research questions for the evaluation study (D. R., Thompson, personal communication, October 29, 2013) were:

1) How do teachers implement their respective curriculum materials?
2) What support, if any, do teachers need when using the UCSMP *Precalculus and Discrete Mathematics* (Third Edition, Field-Trial Version) curriculum materials?
3) How does the achievement of students in classes using UCSMP *Precalculus and Discrete Mathematics* (Third Edition, Field-Trial Version) compare to that of students using the Second Edition curriculum already in place at the school, when applicable?
4) How do students' achievement and understanding of key content topics change over the course of the school year?
5) How do students use technology relevant to their curriculum?

Sample for the field evaluation study. Schools were recruited using UCSMP and NCTM publications. The director of evaluation, in consultation with other UCSMP personnel, selected participating schools based on a broad range of educational conditions in the United States in terms of curriculum and demographics such as location, size, type of community, and socioeconomic status. This led to six schools being selected to participate in the field study. Among the six schools, seven teachers taught 14 classes of PDM. It was not feasible to use random selection of schools or teachers for participation in the field study.

As part of the Third Edition field study, for comparison purposes, two teachers at two schools used the Second Edition of the textbook. There are minor differences in the textbooks between the two editions but the major difference between the implementation of the two editions was the use of technology. Teachers using Third Edition materials were provided with enough CAS capable graphing calculators to assign one to each student for home and classroom use. Students using the Second Edition used graphing calculators, potentially without CAS, and may or may not have had access to graphing calculators at home.

Data Collection

My study utilized existing data collected as part of the larger evaluation study conducted during the 2007-2008 school year that examined student achievement and opportunity to learn from the enacted curriculum using the Third Edition ($n \approx 253$) of UCSMP *Precalculus and Discrete Mathematics* (PDM) compared to students using the Second Edition ($n \approx 47$). Data were collected from the following sources: Teacher Initial Questionnaire, Teacher End-of-Year Questionnaire, Teacher Interviews, Classroom Visits, Chapter Evaluation Forms (Third Edition) or Textbook Chapter Coverage Forms (Second Edition), Teacher Opportunity-to-Learn (OTL) Form, 2 pretests, 3 posttests, end of year student survey, and student posttest calculator usage. Only some of the data were used in my study as delineated in the following sections.

Student Instruments. Students completed two pretests, three posttests, a calculator usage form, and a student survey. Although the student instruments contained items that assessed a variety of skills and knowledge, only items that assessed students' knowledge of functions are utilized in this study. This section details each of the relevant student assessments.

Pretests. Students completed two pretests designed to assess prerequisite knowledge of concepts taught in PDM. The pretests were also used to determine if classes were comparable (i.e., Third Edition and Second Edition in the same school) when appropriate, in terms of prerequisite knowledge, and to provide a baseline score to measure growth over the course of the year.

Pretest Form One (see Appendix A) consisted of 35 multiple-choice questions. Items were developed to provide information about seven major sub-topic areas: proof (3 items); functions and their properties (16 items); discrete mathematics (3 items); trigonometry (6 items);

foundations of calculus (2 items); polynomials (1 item); and basic algebra (4 items). Pretest Form Two (see Appendix B) contained 25 questions. Students were permitted to use a calculator only Pretest 2. Items on pretest 2 were developed to provide information about eight major sub-topic areas: proof (1 item); functions and their properties (10 items); discrete mathematics (1 item); trigonometry (4 items); foundations of calculus (4 items); polar and complex numbers (1 item); polynomials (2 item); and basic algebra (2 items).

All assessments were designed to be completed in no more than 40 minutes. Data include responses to each question in raw form (the actual multiple-choice letter selected), gender, grade level, and name. UCSMP project staff scored the items indicating correct (1) or incorrect (0) for each question and assigned unique IDs to all students.

Posttests. The items on the posttests assessed knowledge in the areas of basic algebra, functions, trigonometric functions, discrete mathematics, and exponential and logarithmic functions. Posttest Form One (see Appendix C) consisted of 30 multiple-choice questions. Items were developed to provide information about eight major sub-topic areas: proof (7 items); functions and their properties (15 items); discrete mathematics (1 item); trigonometry (1 item); foundations of calculus (2 items); polar and complex numbers (1 item); polynomials (1 item); and basic algebra (2 items). There were 16 items repeated from pretest 1. Of these sixteen items, three assessed basic algebraic knowledge; eight assessed knowledge of functions; three tested knowledge of discrete mathematics; and one assessed knowledge on trigonometric functions.

Posttest Form Two (see Appendix D) contained 22 multiple-choice questions on which students were permitted to use a calculator. Items were developed to provide information about six major sub-topic areas: functions and their properties (7 items); discrete mathematics (1 item); trigonometry (4 items); foundations of calculus (6 items); polar and complex numbers (2 items);

and polynomials (2 items). There were 13 items repeated from the pretest. Two of the repeated items tested basic algebraic knowledge; seven assessed function knowledge; three tested knowledge of trigonometric functions; and one assessed knowledge of exponential functions.

Posttest calculator usage. After students completed posttest two, they were asked to complete a survey regarding their usage of a calculator on that posttest. The students were asked to identify the type of calculator (i.e., it can graph or it can simplify equations using CAS) and for each question to identify how they used the calculator in solving the problem. Options included a) did not use the calculator, b) used only for arithmetic, c) used graphing features, d) used CAS features, and 5) other. Data were entered into EXCEL spreadsheets for each student, including the type of calculator used, and the manner in which the calculator was used for each question.

Problem solving posttest. In addition to the multiple-choice tests, students also took an open-ended problem solving test (see Appendix E). This posttest contained five items designed to measure students' abilities to solve multi-step problems, do proofs, and explain their thinking of the core concepts in precalculus and discrete mathematics. The items were chosen because each was solvable using several strategies, including numeric, symbolic, and graphical methods, and each required students to explain their reasoning. As part of this exam, students indicated if they used a calculator, and if so, what type they used: a) it cannot graph equations, b) it can graph equations, or c) it can simplify algebraic equations (CAS). Students were directed to show all of their work, including how the calculator was used to solve the problem. Problems were scored using a 0-4 rubric; if the scoring team was able to ascertain the type of solution (e.g., numeric, graphic or symbolic) it was also recorded.

Student survey. At the end of the school year the students were asked to complete a survey with 18 questions (see Appendix F). Ten of the questions inquired about how technology was used in the teaching and learning of mathematics, including: a) the type of calculator used in class and at home, b) how often the calculator was used in class and at home, c) for what purpose the calculator was used in class and at home, d) how often students used CAS in class and at home, and e) how helpful the calculator was. There was a desire to collect student names on this survey to enable the researchers to connect the data with individual student achievement results. Due to privacy concerns and issues related to parental permission, student names or IDs were not collected. Instead, the researchers included two questions to help identify responses from students who were likely to have been in class during the entire year. Students were asked if they were in the class at the beginning of the year and at the time of the first report card. Students who answered *yes* to both questions were included in the final data set because it is likely they completed all the pretests and posttests. Results from this survey are reported only at the class level.

Teacher Instruments. Teachers were asked to complete beginning and end of year questionnaires. In addition they completed chapter evaluation forms for each chapter they taught, which included information about students' opportunity to learn lessons and homework assigned from each lesson. Teachers also completed opportunity to learn questionnaires for each posttest. Each of the instruments is described in this section.

Beginning of the year teacher questionnaire. This questionnaire was administered to all teachers participating in the study at the beginning of the school year. The initial questionnaire was used to collect teacher demographics and baseline data about the classroom (i.e., block or traditional scheduling) and anticipated instructional approaches. The survey included 34 items that addressed the following factors: teachers' beliefs about what is important in the teaching and learning of mathematics, the importance of specific pedagogical practices, and their experience with technology and calculator features. Two open ended questions enabled teachers to provide additional information and describe what they expected to be their greatest challenge for the upcoming school year. For the present study, only the demographic information (Questions 1-4), questions pertaining to the importance of using calculators (6n, 6o), and the intent to use technology (7l and 8) will be used as shown in Table 3.

Table 3

Items Used from Beginning of the Year Teacher Questionnaire

Number	Item
1	[List your] Education/Degrees
2	List your teaching Certifications
3	[List your] teaching experience Number of years teaching prior to this year Number of years teaching mathematics prior to this year Number of years teaching at present school prior to this year
4	Please check one of the following: ____ UCSMP Third Edition Teacher ____ UCSMP Second Edition Teacher
6n	Help students learn to use a graphing calculator as a tool for learning mathematics Of little importance Somewhat important Quite important Of highest importance
6o	[How important is it to] Help students learn to use a symbolic manipulator as a tool for learning mathematics Of little importance Somewhat important Quite important Of highest importance
7l	[How often do you plan to] Ask students to use multiple representations (e.g., numerical, graphical, geometric, etc.) _____almost every day _____ 2-3 times per week _____ 2-3 times a month _____ less than once a month _____ almost never
8	[Describe your experience using] a. graphing features Never used Seldom used Use frequently b. table features Never used Seldom used Use frequently c. statistics features Never used Seldom used Use frequently d. equation modeling features Never used Seldom used Use frequently e. symbolic algebra features (e.g., computer algebra systems). Never used Seldom used Use frequently

Table 4

Items Used From End of the Year Teacher Survey

Number	Item
1	What book did your students use in the classes in this study? ___ UCSMP Third Edition *Precalculus and Discrete Mathematics* ___ UCSMP Second Edition *Precalculus and Discrete Mathematics*
6	What calculator technology was available for use by the majority of students **during this mathematics class**? _____ calculators not available _____ a class set of scientific calculators _____ student-owned scientific calculators _____ class set of graphing calculators without computer algebra system capability _____ student-owned graphing calculators without computer algebra system capability _____ class set of graphing calculators with computer algebra system capability _____ student-owned graphing calculators with computer algebra system capability _____ the loaner calculators provided by UCSMP _____ other (Please specify. ____)
7	About how often did students use calculator technology **during this mathematics class**? _____almost every day _____ 2-3 times per week _____ 2-3 times a month _____ less than once a month _____ almost never
8	For what did your students use calculator technology **in this mathematics class**? (Check all that apply.) _____checking answers _____doing computations _____solving problems _____graphing equations _____ working with a spreadsheet _____making tables _____analyzing data _____finding equations to model data _____ simplifying algebraic equations _____other features of CAS _____ other (Please specify.____)
9	If you had students use the computer algebra system capability on this calculator, if applicable, about how often did your students use the calculator for this purpose in your mathematics class? _____almost every day _____ 2-3 times per week _____ 2-3 times a month _____ less than once a month _____ almost never
10	How helpful was this calculator for students learning mathematics **in this mathematics class**? _____very helpful _____somewhat helpful _____not very helpful
17n	[[How important is it to] Help students learn to use a graphing calculator as a tool for learning mathematics Of little importance Somewhat important Quite important Of highest importance
17o	[[How important is it to] Help students learn to use a symbolic manipulator as a tool for learning mathematics Of little importance Somewhat important Quite important Of highest importance
18	[About how often did you] Ask students to use multiple representations (e.g., numerical, graphical, geometric, etc.) Almost never Sometimes Often Almost all

End of the year teacher questionnaire. This questionnaire was administered to all teachers participating in the study at the end of the school year. This questionnaire was used to collect teacher data about pedagogical and instructional approaches used during the year. The questionnaire included 23 items that addressed the following factors: the time spent on different aspects of mathematics instruction, the nature of instructional activities, particular instructional practices, and the use of calculators. Two open ended questions enabled teachers to provide additional information and describe the greatest challenge they faced during the school year. For this study only the edition of textbook used (Question 1), questions pertaining to the use of calculators (Questions 6-10), and importance of specific calculator features (Questions 17n, 17o and 18) were used as shown in Table 4.

Chapter evaluation/ chapter coverage form. At the end of each chapter of *Precalculus and Discrete Mathematics*, teachers who used the Third Edition completed a chapter evaluation form (see Appendix G) on which they indicated the lessons taught, the questions assigned, and provided ratings for each lesson and question set on a scale from 1 to 5 (1= Disastrous; scrap entirely, 5= Excellent; leave as is). Teachers who used the Second Edition completed a modified chapter coverage form on which they indicated lessons taught and questions assigned, but did not provide ratings for lessons or questions.

For each chapter, teachers who used the Third Edition also were queried regarding their use of Teacher Notes, Chapter Test, supplementary materials, and calculator or computer technology. Most questions were consistent from chapter to chapter. However, some questions were specific to a given chapter to determine views on a given approach or technique or to request comments regarding changes made from the Second Edition to the Third Edition. The curriculum developers used the information from these forms to make any necessary changes to the materials before they were released for commercial publication.

The chapter evaluation form regularly included 12 free response questions, addressing the following areas: sections covered and homework problems assigned, evaluation of materials (e.g., What comments do you have on the sequence, level of difficulty, or other specific aspects of the content of this chapter?), How technology was used in teaching (e.g., What comments or suggestions do you have about the way calculator technology is incorporated into this chapter?). One open-ended question was also included to provide teachers the opportunity to provide any additional information they deemed relevant. The lesson coverage data were used to calculate students' opportunity to learn functions. The data from the free response questions pertaining to the use of technology may be used as context when reporting the results of the analyses.

Teacher Opportunity-to-Learn (OTL) Posttest Form. Teachers completed an OTL form for each of the three posttests. The intent was to determine if the teacher taught or reviewed the material necessary for students to answer each assessment question. This is important and different from simply asking teachers if they taught the section. For example, a teacher might acknowledge teaching the quadratic equation but might indicate she did not teach students enough to answer a word problem involving projectile motion.

The OTL form is based on forms used in international studies (Schmidt, Wolfe, & Kifer, 1992). For each item on the posttests, teachers were asked to answer the following: During this school year, did you teach or review the mathematics needed for your students to answer this item correctly? The inclusion of this question allows achievement to be analyzed by holding OTL constant or by using it as a covariate.

Data Analysis

Various approaches were used during the analysis of the data to identify which, if any, of the data variables discussed have a significant effect on student achievement when learning functions. Although identification of major relationships between variables is important, the main goal of the analysis was to develop a model which uses technology and OTL measures along with other significant data relationships to predict achievement on function problems. Because achievement on all three posttests is a continuous dependent variable linear regression, multiple regression, path analysis and hierarchal linear modeling (HLM) were used to identify relationships. The following sections contain a description of the sample used in analysis, the creation of opportunity to learn variables, and the methods I used to answer each of the research questions.

Data sample for analysis. Prior to being provided to me, the raw data from the field trial, provided in the form of EXCEL workbooks, were cleaned and blinded by the UCSMP director of evaluation to remove any teacher or student names. The student data had two components: assessments and survey data. For the assessments, only the data from students who completed all pretests and posttests were used. For the student surveys only data from students who were present at the beginning of the year and at the end of the first marking period were used. Student IDs were used to link pretest and posttest data and student demographic information such as gender and grade level.

OTL variables. Three operational variables relating to opportunity to learn were created and used in this study: OTL function lesson coverage, OTL function homework coverage, and OTL function posttest coverage. The focus of this study was students' OTL functions. To properly connect with the UCSMP data it was necessary to remove the data from textbook sections, homework problems and assessment questions that did not relate to functions. Each of these three variables is described below.

OTL function lesson coverage. This is based on the percentage of sections in the textbook whose main focus is functions. I examined the textbook's table of contents to determine which lessons pertained to functions, verified my results with the UCSMP Director of

Evaluation, and resolved any discrepancies by consensus. For each teacher, OTL function lesson coverage was calculated using the number of function lessons taught by the teacher as the numerator and the total number of function lessons in the textbook (either Second or Third Edition) as the denominator.

OTL function homework coverage. Most of the teachers in the field study used the Third Edition of the textbook. The questions for each section come under one of four categories: *Covering the ideas*, *Applying the mathematics*, *Review*, and *Exploration*. The Second Edition has four categories as well with the only difference being the first category is entitled *Covering the reading*. In general, the expected homework assignment is all of the problems with the exception of the exploration, so all textbook problems within the first three types were included in the analysis.

The OTL for homework is based on the percentage of function homework assigned by each teacher, but based only on the function lessons they taught. The numerator is the number of function homework problems assigned from the textbook by the teacher, and the denominator is the total number of homework problems in the function lessons taught, as defined in the function lesson coverage variable.

As stated previously, only homework problems addressing the function concept were used to determine the OTL function homework coverage variable, using homework problems from the function lessons previously identified in the OTL lesson coverage variable. I worked together with the UCSMP Director of Evaluation to identify the homework problems that pertain to one of the four function domains as outlined by the Common Core State Standards for Mathematics [CCSSM] (CCSSO, 2010): interpreting functions; building functions; linear, quadratic and exponential models; and trigonometric functions as summarized in Table 5. Any discrepancies between the two reviewers were resolved by consensus.

Table 5

CCSSM Domains for High School Functions

Code		Identifier	Learning Outcome
1		F-IF	**Interpreting functions**
	1a		Understand the concept of a function and use function notation
	1b		Interpret functions that arise in applications in terms of the context
	1c		Analyze functions using different representations
2		F-BF	**Building Functions**
	2a		Build a function that models a relationship between two quantities
	2b		Build new functions from existing functions
3		F-LE	**Linear, Quadratic, and Exponential Models**
	3a		Construct and compare linear and exponential models and solve problems
	3b		Interpret expressions for functions in terms of the situation they model
4		F-TF	**Trigonometric Functions**
	4a		Extend the domain of trigonometric functions using the unit circle
	4b		Model periodic phenomena with trigonometric functions
	4c		Prove and apply trigonometric identities

Note: Information from CCSSM content standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, pp. 69-71)

The original intent was to not only identify problems in the textbook as function or non-function items, but also to classify each item by its corresponding function domain within the CCSSM (2010). Too many function items did not fall within the definition of the four domains. For example, functions relating to calculus, many exponential functions, polynomial functions and many trigonometric functions are contained in other domains within the CCSSM (2010), such as algebra or geometry, or not included in the CCSSM at all, such as in the case of calculus functions. My choices were to either remove items that did not fall within the four domains, include the other domains, or not classify them by domain. The first option resulted in a small sample of items that was not representative of the items contained in the textbook. There was too much overlap between the different domains to meaningfully include the other domains in the analysis. Therefore I made the decision to only classify items as being function items using the definition of function contained in the CCSSM (2010).

OTL function posttest coverage. This variable represented the percentage of function problems on the posttests for which each teacher reported having taught or reviewed the material necessary for their students to successfully answer the question. The numerator is the number of function problems on the posttest for which each teacher reported teaching or reviewing the material and the denominator is the total number of problems relating to functions on the posttests. The contents of the posttests are identical for students using the Second and Third Editions of the textbooks. I used the same procedures to identify the posttest function problems that I used to identify the homework function problems, and again verified my results with the UCSMP Director of Evaluation.

Methods to answer question one. To answer question one, *what are students' opportunities to learn about functions in a precalculus course*, I used descriptive statistics using the OTL variables. I report the OTL function lesson coverage, the OTL function homework coverage, and the OTL posttest coverage. Frequencies, means and standard deviations were calculated and are reported for each OTL variable measured.

Methods to answer question two. To answer question two, *what strategies do Precalculus students use when solving function problems?*, I first used descriptive statistics to report how many students used each strategy and to show the distribution of strategies by use of technology for technology neutral and technology inactive items. Next I examined students' use of technology and strategies for the function items on the two posttests in which students were permitted to use technology. Then I examined strategies at the class level and across curricula levels. Finally I examined how students used technology to solve function problems on the posttests.

Methods to answer question three. Question three is, *how is Precalculus students' achievement in solving function problems related to their use of strategies*? First descriptive statistics were used to examine students' prior knowledge as measured on the pretests. Then descriptive statistics were used to examine student achievement on the posttests. Next chi-squared goodness of fit tests were performed on each function item to determine if there was a difference in student achievement between students who used no strategy compared to those who used any strategy, grouped by students' access to CAS capable calculators. Technology neutral items, items that can be solved without a calculator (although a calculator might be useful), were compared to technology inactive items, items for which there is no advantage (perhaps even a disadvantage) to using a calculator. On problems in which the overall achievement was significant, additional chi-squared goodness of fit tests were performed to determine which strategies, if any, resulted in a difference of achievement when compared to students who did not use a strategy. For achievement on the problem solving test, the strategies teachers coded the students as possibly using to solve the items were analyzed using chi-squared goodness of fit tests to determine if there was a difference in achievement between students who appeared to have used a strategy as compared to those students who did not appear to have used a strategy. The next step was to examine the achievement on each of the three posttests by performing multiple linear regressions and using pretest achievement scores to control for prior knowledge. The independent variables that were compared to achievement include calculator type, number of strategies used, and OTL measures. All of the independent variables are categorical with the exception of the OTL variables which are continuous. For each posttest three sets of multiple regression analyses were conducted using technology measures only, OTL measures only, and both technology and OTL measures.

Methods to answer question four. Path analysis was used to answer question four, *what effect does the use of technology, including strategies, and opportunity to learn have on achievement when technology usage is reported from the students' perspective on a multiple choice assessment and from the teachers' perspective on a free response assessment*. Multiple regression analysis was conducted comparing achievement to technology and OTL measures. However, regression has one critical weakness compared to path analysis: we are not be able to see the interrelationships of achievement, technology and OTL variables concurrently. Path analysis offers a way to examine the interrelated relationship within the variables, and allows one to see the path, and the path coefficients, while holding other variables constant. According to Hair and colleagues (2006), path analysis has three distinguishing characteristics:

> (1) an estimation of multiple and interrelated dependence relationships, (2) an ability to represent unobserved concepts in these relationships and correct for measurement error in the estimation process, and (3) defining a model to explain the entire set of relationships. (p. 706)

Path analysis can also provide insight into the strength and types of relationships (e.g., mediating and moderating) and can identify relationships in which a variable may be independent in one situation and dependent in another. Path analysis uses exogenous variables (Ex) to indicate variables that have both indirect and direct effects on the endogenous (EN) variables (dependent). This study did not use unobserved, or latent, variables for analysis. However, based on the observed variables, this study attempted to test the fit of the hypothesized path analysis of manifest variables. The variables used in the three-tiered model are illustrated in Figure 5.

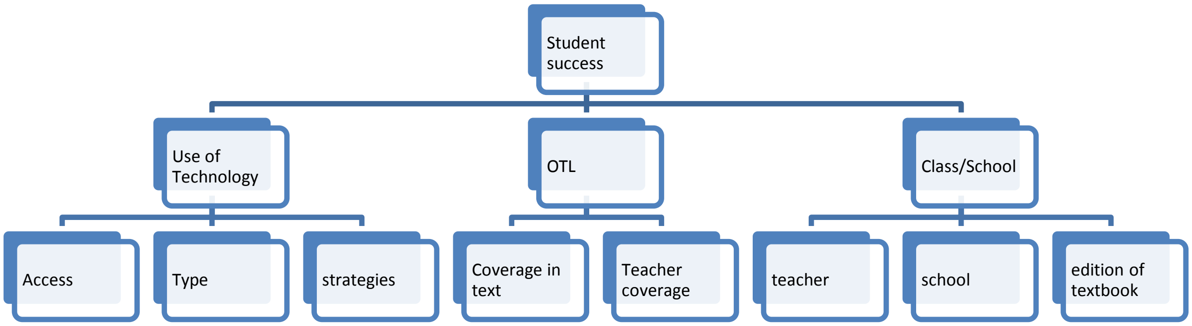


Figure 5. A Framework of Graphing Calculators in Studying Functions for Modeling.

The framework includes three categories of variables: use of technology, opportunity to learn, and school/class. Some of the variables relating to technology, such as strategy chosen and type of calculator used on the posttests, can be assigned to a student level because there is data linking posttest data to individual students through the use of a student ID. Other variables, such as how often technology was used at home and other data obtained from the student survey, were used at the class level because there is no way to link that data back to an individual student. Percent averages were calculated for class variables used in calculations.

The model in Figure 6 was the initial model created and tested using Path Analysis. Results from the multiple regression tests and prediction models were used to refine the path analysis model, and results from the path analysis were used to refine the prediction models. To determine the parameter estimates for the variable coefficients, I used the following procedures: model specification, model identification, evaluating the model fit, making modifications to the model, and presenting the final model (Hoyle, 1995; Schumacker & Lomax, 2010). Model specification is the creation of a baseline model based on the relationships between the variables. For this study, information from the literature review and existing theories was used to create an initial model for analysis. The purpose of the initial, or hypothesized, model is to study the underlying relationship that might exist among the variables (Hoyle, 1995; Schumacker & Lomax, 2010).

After the hypothesized model is created, the next step is to examine the model identification. According to Hoyle (1995), "identification concerns the correspondence between the information to be estimated-the free parameters-and the information from which it is to be estimated-the observed variance and covariance" (p. 4). There are three levels of identification:

1) A model is under-identified (or not identified) when there is not enough data in the covariance matrix to uniquely identify one or more of the parameters.
2) A model is just-identified when there is just enough data in the covariance matrix to uniquely identify all the parameters.
3) A model is over-identified when there is more than enough information in the covariance matrix resulting in multiple solutions for at least one of the parameters (Schumacker & Lomax, 2010).

If a model is under-identified the results cannot be considered reliable. For this study, degrees of freedom were used to establish model identification: a model is considered identified if the degree of freedom is 1 and over-identified if the degree of freedom is greater than 1 (0 or negative implies under-identified) (Schumacker & Lomax, 2010).

After analysis was performed on the hypothesized model, I examined the fit of the model. Hatcher (2007) noted a model with ideal fit has the following characteristics:

- The *p* value associated with the model chi-square test should exceed .05; the closer to 1.00, the better.
- The comparative fit indices should exceed 0.9; the closer to 1.00, the better.
- The multiple squared correlation value for each endogenous variable should be relatively large compared to what typically is obtained in research with these variables.

- The absolute value of the *t* statistic for each path coefficient should exceed 1.96, and the standardized path coefficients should be nontrivial in magnitude (i.e., absolute values should exceed .05) (p. 197).

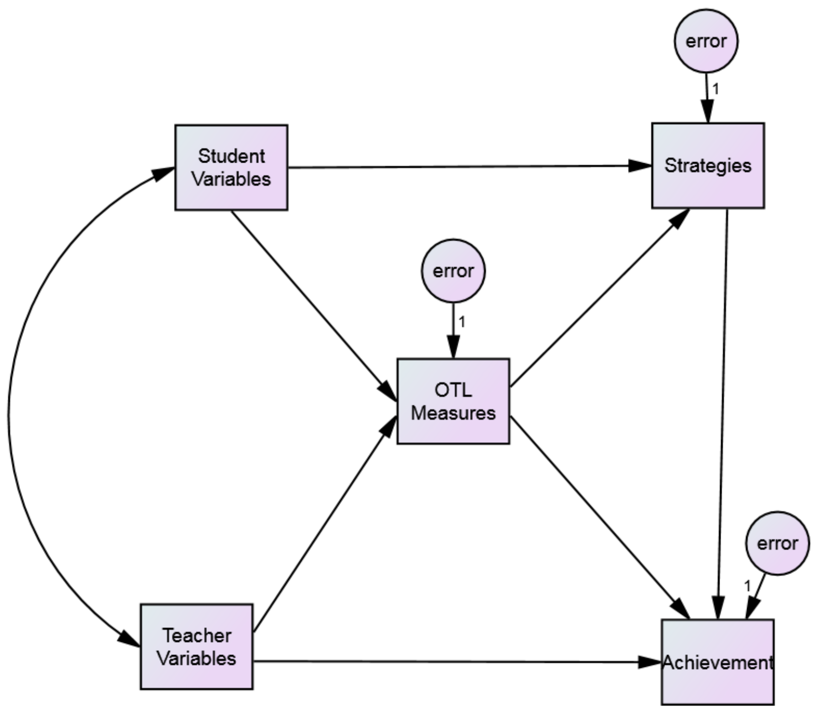


Figure 6. Hypothesized Path Analysis for Achievement with Technology and OTL Measures.

Revisions were made to the model following Hatcher's (2007) recommendations to remove any non-significant paths, and add any new paths as recommended by the modification indices in SPSS, but only if the new path had practical or theoretical value based on the literature review. This procedure was followed until the final model met the ideal characteristics of model fit. All of the final models provided in this study meet the characteristics of an ideal model fit.

Reliability of Statistical Methods

Table 6 shows the number of items on each test, the number of function items, and the Cronbach's Alpha for each. Although the assessments were designed to have overlap of coverage between the three posttests, there was little overlap of coverage within each test. In measurement, an assessment designed to intentionally have little overlap in coverage is viewed as a formative measurement model in which there is no assumption that items correlate to each other by design (Edwards, 2011), thus resulting in a somewhat low alpha level.

Table 6

Internal Consistency Reliability for Achievement on Posttests 1, 2 and Problem Solving Test

	Number of Items		Cronbach's Alpha	
	Entire Test	Function Items only	Entire Test	Function Items only
Posttest 1	30	16	.72	.66
Posttest 2	22	16	.57	.56
Problem Solving Test	5	3	.43	.26

Strategies to increase reliability include thoroughly documenting the research process, including listing all statistical tests, showing the variables used and including copies of all code. In this study all statistical calculations, including path analyses, were performed using SPSS version 21 and HLM for Windows version 7.01, which are available through the university.

For each test all assumptions that need to be satisfied were documented. Descriptive statistics such as skew and kurtosis were used to verify the assumptions have been met before conducting the analysis. See Table 7 for the independent and dependent variables for each research question and the list of tests that were conducted.

Table 7

Methodologies Used to Answer Research Questions

Research Question	Type of test	Independent Variable(s)	Dependent Variable
Opportunity to learn (1)	Descriptive statistics		
Strategies (2)	Descriptive statistics		
Achievement (3)	Chi-squared GOF	• Strategy • Class • Access to technology • Calculator type	Accuracy
	Multiple Regression	• Access to technology • Class • OTL Lessons • OTL posttest • OTL Homework • Achievement on pretest 2	Score on posttest
Achievement (4)	Path Analysis	• Access to technology • Class • OTL Lessons • OTL posttest • OTL Homework • Achievement on pretests	

Summary

Several different analysis tools were employed to analyze the relationships between technology strategies and student achievement on function items against multiple teacher and student variables. Data obtained from the field study of the Third Edition of the PDM curriculum, as well as data variables obtained from the textbooks, pretests, and posttests, were used to derive models that can be used to answer the research questions. The procedures used to answer each research question were described in detail and included the initial indication of the assignment of independent and dependent variables. Table 7 summarizes the methodologies employed in this study.

Chapter 4: Results

In this chapter I present the results of the analyses that were conducted on data obtained from the evaluation of the UCSMP *Precalculus and Discrete Mathematics* (Third Edition, Field-Trial Version) curriculum on students' opportunities to learn and their use of technology when solving function problems. I analyzed three data sets using SPSS and HLM statistical software. The data sets pertained to: a) results of pretests, posttests and the use of technology, from the students' perspective, on posttest 2; b) results and use of technology, as inferred from solution approaches and coded by teachers on the Problem Solving test; and c) opportunity to learn data, specifically lesson coverage and percent of questions assigned from function lessons. The analyses of the three data sets are presented separately and then combined together for a final analysis. As part of this study I employed descriptive analyses, inferential statistics, correlation analyses, and path analysis. First, basic descriptive statistics for students' opportunity to learn functions are presented. After that, the descriptive statistics regarding what technology was used and how it was used when students solve function problems are discussed. Following that, the results of both descriptive and inferential statistics, performed on achievement data, are presented to examine the impact OTL and technology have on student achievement for posttests 1, 2 and the problem solving test. Finally, the correlation and path analysis are presented to examine the relationships among student achievement and OTL, student achievement and use of technology, and student achievement with both OTL and use of technology.

Students' Opportunity to Learn when Solving Function Problems

In this study teachers reported the lessons they covered, the homework they assigned, and whether they taught or reviewed the content needed to answer the questions on the posttests. The following section details students' opportunities to learn functions.

Lesson coverage. Figures 7 and 8 report the number of lessons teachers taught from which the percentage of function lessons taught was calculated. Only function lessons are shown in the table and the lessons each teacher taught are indicated by shading. Sections that did not contain function material in the lessons were excluded from the analysis.

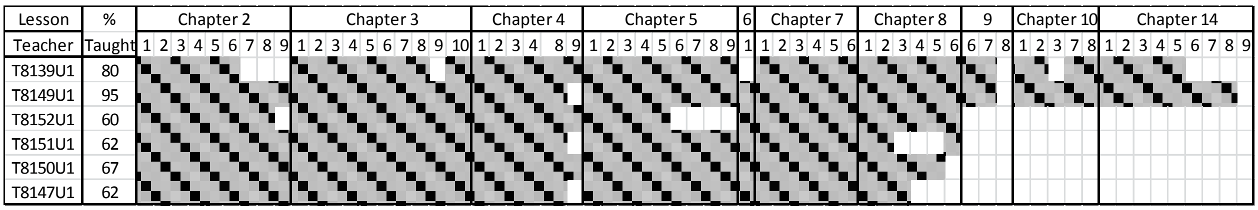


Figure 7. Pattern of Lesson Coverage and Percent of Function Lessons Taught in the UCSMP *Precalculus and Discrete Mathematics* (Third Edition). Shading indicates the lesson was taught.

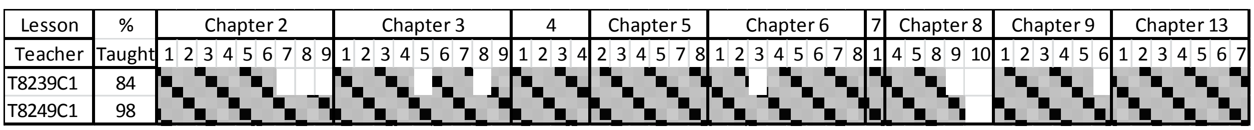


Figure 8. Pattern of Lesson Coverage and Percent of Function Lessons Taught in the UCSMP *Precalculus and Discrete Mathematics* (Second Edition). Shading indicates the lesson was taught.

Because teachers could have covered vastly different sections resulting in similar percentages, the actual patterns of lesson coverage are illustrated in Figure 7 (3rd Edition) and Figure 8 (2nd Edition) using a display similar to those by Tarr, Chávez, Reys, and Reys (2006) in their curriculum study. In the Third Edition there were 55 lessons related to the topic of functions. No teacher taught all of the lessons but they all taught at least 60% of the function lessons as shown in Figure 7. Teachers using the Third Edition taught generally between sixty and eighty percent of the total function lessons in the textbook. In the Second Edition, there were also 55 function lessons and teachers taught at least 84% of function lessons in the text as indicated in Figure 8.

Table 8

Percent of Function Homework Problems Assigned From Function Lessons Taught in Precalculus and Discrete Mathematics (Second and Third Editions)

Teacher	Maximum Number of Possible Function Problems in Function Lessons Taught	Percentage of Homework Problems Assigned From Function Lessons Taught
Third Edition		
T8139U1	777	52
T8147U1	957	94
T8149U1	689	61
T8150U1	718	94
T8151U1	742	87
T8152U1	712	79
Total	*M* = 765.83 *SD* = 98.31	*M* = 77.83 *SD* = 17.66
Second Edition		
T8239C1	705	59
T8249C1	835	55
Total	*M* = 770 *SD* = 91.92	*M* = 57.0 *SD* = 2.83

Note: There were 55 function lessons available in both the 2nd and 3rd Editions.
In the 3rd Edition, there were a total of 1042 function problems available in those lessons for teachers to assign.
In the 2nd Edition, there were a total of 926 function problems available in those lessons for teachers to assign.

Questions assigned for homework. Teachers also reported which homework problems they assigned for students to complete. Table 8 shows the percentage of function homework problems teachers assigned to students from the function lessons they taught. Percentages ranged from 52% to 94% with a high degree of variability; in general, Third Edition teachers reported assigning more problems from the text than did Second Edition teachers. A low number of problems assigned for practice could impact students' opportunity to learn, therefore, their achievement. In some cases, teachers assigned practice problems from other sources, therefore there was no way to determine if students completed the assigned problems, regardless of the source.

Table 9

Percent Opportunity-to-Learn on Function Post Assessment Items as Reported by UCSMP Precalculus and Discrete Mathematics Teachers

Teacher	OTL Posttest1	OTL Posttest 2	OTL Problem Solving Test
Third Edition			
T8139U1	100	81	67
T8147U1	100	94	100
T8149U1	100	100	100
T8150U1	100	100	100
T8151U1	100	100	100
T8152U1	100	88	100
Total	*M* = 100.0 *SD* = 0.0	*M* = 93.83 *SD* = 7.91	*M* = 94.50 *SD* = 13.47
Second Edition			
T8239C1	100	94	67
T8249C1	100	100	100
Total	*M* = 100.0 *SD* = 0.0	*M* = 97.0 *SD* = 4.24	*M* = 83.5 *SD* = 23.33

Note: Posttests 1 and 2 each contained 16 function items. The problem solving test contained three function items. n = 6 for 3rd Edition teachers; n = 2 for 2nd Edition teachers.

Preparation for assessments. Students for both Second and Third Editions completed two multiple-choice and one free response post assessments to measure achievement. Teachers were asked if they taught or reviewed the material necessary to answer each question on each posttest. Table 9 reports the percentage of function assessment problems for which students had the opportunity to learn or review the content.

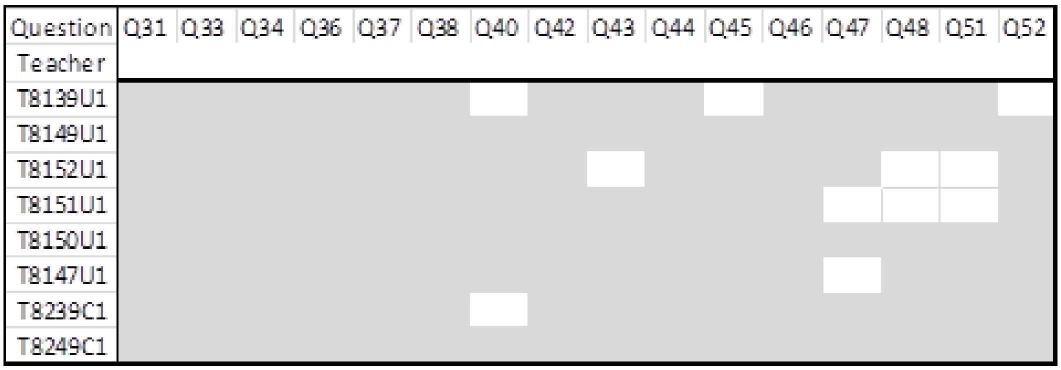


Figure 9. Opportunity-to-Learn on Multiple Choice Posttest 2 as Reported by Second and Third Edition Teachers of *Precalculus and Discrete Mathematics*. Shading indicates function whose content was reported as taught or reviewed.

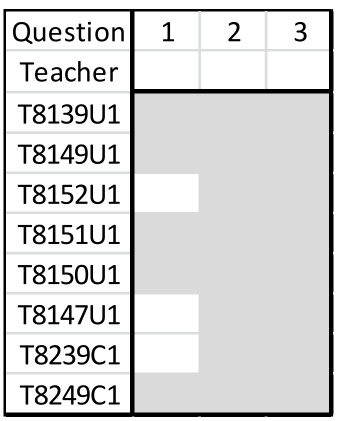


Figure 10. Opportunity-to-Learn on the Problem Solving Test as Reported by Second and Third Edition Teachers of *Precalculus and Discrete Mathematics*. Shading indicates function whose content was reported as taught or reviewed.

Every teacher reported covering or reviewing the content for all items on posttest 1. Again, because percentages can be similar while coverage is vastly different, Figures 9 and 10 show the patterns of coverage for function problems for posttest 2 and the problem solving test. Coverages for posttest 2 range from 81% to 100% with an average of over 90%. The Second Edition teachers, on average, covered more of the problems. Coverage of function items on the problem solving test was generally higher than on posttest 2, with only three teachers covering less than 100% of the assessed function items.

Table 10

Correlation of OTL variables

OTL Variable	Lessons	Homework	Posttest 2	PSU
Lessons	—	-.53**	.79**	.025
Homework		—	.00	.67**
Posttest 2			—	.32**
Problem Solving Test				—

Note. OTL = Opportunity to learn; PSU = problem solving test. All OTL variables are measured as percentages and ranges from 0 to 100%. These variables are collected at the class level from $n = 8$ teachers.
** $p < .01$.

Correlation of OTL variables. The correlation for each pair of OTL variables is shown in Table 10. There was a significant and positive relationship between OTL Lessons and OTL posttest 2 (r = .79), but no relationship between OTL Lessons and the PSU (r = .025). This means that generally the teachers who taught more lessons also covered more of the assessed function items. OTL Homework was significant and positively related to the PSU (r = .67), indicating teachers who assigned more homework generally covered more of the items assessed on the PSU. The relationship between OTL posttest 2 and PSU was significant and moderately positive (r = .32), meaning in general teachers who covered more of the assessed items on posttest 2 covered more of the assigned items on the PSU. There was also a negative correlation between OTL Lessons and OTL Homework (r = -.53), meaning, on average, the more lessons teachers taught the fewer homework problems they assigned from the text.

Students' Use of Technology When Solving Function Problems

This section presents the results regarding what technology students used to solve function items and how students used the technology for both posttest 2 and the problem solving test. Recall students were not permitted to use calculators on posttest 1. Descriptive statistics are used to analyze the results of the supplemental questions students answered about their use of technology on posttest 2 and the data received from teachers' coding of student responses for the problem solving test.

What technology students had access to for solving function problems. All classes using the Third Edition curricula materials were provided access to CAS (computer algebra system) capable calculators as part of the field trial, although teachers may not have loaned them out for continual access. Second Edition classes generally had access to non-CAS graphing calculators. It is unknown how many students used an assigned calculator and how many used their own calculator either in class or at home. Table 11 shows the number of students who reported having access to a CAS calculator grouped by both class and curriculum. Table 12 reports the number of students who reported using each type of calculator by class and curriculum. Of the 271 students who completed posttest 2 and the problem solving test, 54% indicated taking the exams using a calculator equipped with CAS. In the remaining eight classes, students reported using either graphing calculators or CAS calculators.

Table 11

Number of Students Who Had Access to CAS Capable Graphing Calculators by Class

	3rd Edition												2nd Edition			
Class No.	410	411	414	415	416	418	419	420	421	422	423	Total	412	413	417	Total
No CAS	3	3	5	19	12	0	0	4	4	6	4	60	22	19	19	60
Had CAS	13	17	12	0	1	18	11	15	20	19	23	149	1	1	0	2

Note. CAS refers to graphing calculators equipped with computer algebra systems. Number of students using CAS capable calculators is n = 151 .

Table 12

Number of Students Reporting Type of Calculators Used on PDM Assessments by Class and Curriculum

Type of Calculator	3rd Edition Classes 410	411	414	415	416	418	419	420	421	422	423	Total	2nd Edition 412	413	417	Total	Total
TI Non CAS [a]	1	0	5	19	12	0	0	4	4	6	4	55	22	19	19	60	115
TI-Nspire	2	3	0	0	0	0	0	0	0	0	0	5	0	0	0	0	5
TI-Nspire CAS	13	17	0	0	0	0	0	0	0	0	0	30	0	1	0	1	31
Casio CAS	0	0	0	0	0	0	0	15	20	0	0	35	0	0	0	0	35
TI-89	0	0	12	0	1	18	11	0	0	19	23	84	1	0	0	1	85
Total	16	20	17	19	13	18	11	19	24	25	27	209	23	20	19	62	271

Note. CAS refers to graphing calculators equipped with computer algebra systems. Number of students using CAS $n = 151$. Number of students using graphing calculators without CAS is $n = 120$.
[a]TI Non CAS graphing calculators include TI-NSpire,TI-84/TI-82/83/85/86. CAS capable graphing calculators include Casio, TI-NSpire CAS and TI-89 family.

How students used technology to solve function problems on posttest 2. Table 13 reports the strategies students reported using when solving the 16 function problems (out of 22 problems) on posttest 2. When taking posttest 2, almost all of the students (n = 255) reported using no calculator for at least one question. When students did use a calculator, the most utilized strategy reported was the graphing feature. The distribution of strategy use between students who had access to CAS and those who did not was similar. On the seven calculator neutral items, students, on average, used a calculator strategy 3.6 times. On the nine calculator inactive items, students, on average, did not use a calculator on eight of the items. Students who had access to CAS reported using CAS, on average, 0.60 times on the seven neutral items.

Table 13

Students' Use of Technology When Solving Function Items on Posttest Two

Strategy	Students who reported using strategy on at least one problem		Mean			
			CAS		No CAS	
	CAS	No CAS	Neutral Items (7)	Inactive Items (9)	Neutral Items (7)	Inactive Items (9)
Did not use	142	113	3.56	7.77	3.58	8.1
Used only to do arithmetic	96	82	.93	.41	1.23	0.2
Used graphing features	131	107	1.51	.26	1.55	0.09
Used CAS features	61	N/A	.60	.18	N/A	N/A
Used some other feature	26	18	.11	.03	0.13	0.04

Note: CAS refers to calculators equipped with computer algebra systems Number of students who reported using CAS n = 151. Number of students who reported using non CAS graphing calculators n = 120. One student (using CAS) did not record the use of any strategies. That record was removed for all analyses involving strategies. Seven students (4 with CAS and 3 without) reported not using a calculator on any item. Four students (CAS) reported using a strategy on every item. The mean (average number of times each strategy was reported used) is the total number of times a strategy was reported being used divided by the total number of students who reported using that strategy.

Table 14 displays the number of students who reported using calculator strategies per class as well as the mean and standard deviation. Almost all students used at least one calculator strategy on posttest 2. The mean number of strategies ranged from a low of 3.6 (the mean number of strategies used per student in class 412) to 7.5 (the mean number of strategies used per student in class 419).

Table 14

Use of Strategies on Function Items for Posttest 2 Per Class

	3rd Edition Classes											2nd Edition		
	410 n=16	411 n=20	414 n=17	415 n=19	416 n=13	418 n=18	419 n=11	420 n=19	421 n=24	422 n=25	423 n=27	412 n=23	413 n=20	417 n=19
Number of Students using strategies	16	20	17	19	13	17	11	19	23	25	27	23	19	19
M	4.1	4.2	5.4	5.0	4.8	6.6	7.5	4.8	3.6	4.6	5.5	4.2	4.5	4.5
SD	2.0	2.0	2.3	2.3	1.6	4.4	5.2	2.0	2.5	4.3	3.7	3.2	1.9	1.9

Note. Mean is the total number of times students reported using a strategy to solve a problem divided by the number of students who reported using any strategy.

Table 15

Number of Students Who Reported Using a Calculator Strategy on Calculator Neutral Posttest 2 Function Items.

	Access to CAS	% students who reported Calculator Strategy					
		None	Arithmetic	Graph	CAS	Other	Any[a]
31*. Given the function *h* defined by $h(x) = \frac{(2x+4)(x-1)}{(x+2)}$. What is the behavior of the function near *x* = -2?	Yes (n = 146)	45*	3	40	5	4	52*
	No (n = 113)	58*	0	35	N/A	2	37*
37. Suppose $f(x) = x^{1/2}$. What is the set of all values of *x* for which $f(x)$ is a real number?	Yes (n = 146)	46	30	19	3	2	54
	No (n = 112)	43	35	20	N/A	1	56
38. Evaluate $\lim_{x \to 10} \frac{x^2 - 100}{2x^2 - 23x + 30}$.	Yes (n = 145)	28	16	23	32	1	72
	No (n = 112)	38	26	29	N/A	5	60
45*. Which of the following is (are) true for all values of θ for which the functions are defined? I. sin(-θ) = -sin θ II. cos(-θ) = -cos θ III. tan(-θ) = -tan θ	Yes (n = 147)	55	27	8	6	3	44
	No (n = 112)	46*	43	8	N/A	3	53*
46**. Which of the following could be an equation for the graph at the right? [graph of polar function shown]	Yes (n = 147)	29**	3	63**	6	0	72**
	No (n = 114)	25**	7	68**	N/A	0	75**
48. The line in the figure at right is the graph of $y = f(x)$. What is the value of $\int_{-2}^{3} f(x)dx$?	Yes (n = 143)	77	8	5	10	0	23
	No (n = 111)	78	19	1	N/A	0	20
52**. Which equation is graphed at the right? [graph of sine function shown]	Yes (n = 142)	29	5	59**	6	1	71**
	No (n = 110)	20	6	71**	N/A	3	80**

Note: CAS refers to graphing calculators equipped with computer algebra systems. Posttest 2 contained 16 function items, 9 of which are considered technology inactive and 7 of which are considered technology neutral. The *n* varies by problem because not all students reported using a strategy on all items.

[a]Any refers to the use of Arithmetic, Graph, CAS, or other strategies and is compared to the use of no strategy (none). *N* represents the number of students who reported using a strategy for each item. Rows add up to more than 100% due to rounding and the fact that some students reported using more than one strategy.

* $p < .05$. ** $p < .01$.

Table 15 shows the strategies students used on the seven calculator neutral items on posttest 2. *Calculator neutral* refers to items on which students could have used a calculator to solve the item, but could have also solved it without the calculator. In four of the seven calculator neutral items (37, 38, 46, and 52), students used a calculator strategy more often than not and the difference in use, when tested with a chi-square test, showed significantly more students chose to use a strategy than not. In three of the items (37, 45, and 48), students reported using arithmetic most often. It is unknown if students substituted values in an attempt to prove the equations true or used some other strategy involving arithmetic. In three other items (38, 46, and 52), students reported using a graph to solve the items most often. In two of the items (38 and 45) students could have solved the items using CAS features. Thirty-two percent of students used CAS to solve the item using limits (38) but only six percent of students who had access to CAS attempted to prove the trigonometric identities (45) using CAS features. More students tried to prove the trigonometric identities (45) using arithmetic, but only one of the equations could be disproved with a counterexample. Few students attempted to use a graph on item 45, which would have been a feasible strategy. It is unknown how or why students chose or used the strategies they did.

Table 16 shows the strategies students used on the nine calculator inactive problems for posttest 2. *Calculator inactive* refers to items for which there is no advantage (perhaps even a disadvantage) to using a calculator. For all the calculator inactive items, the majority of students did not use a calculator. In eight of the nine items (all but 33), at least 10% of students attempted to use a calculator strategy to solve the item. In two of the items arithmetic was the most commonly used strategy. Students rarely chose to use other strategies when they were solving the calculator inactive items.

Table 16

Number of Students Who Reported Using a Calculator Strategy on Calculator Inactive Posttest 2 Function Items.

Item	Access to CAS	% Students Who Reported Using Strategy					
		None	Arithmetic	Graph	CAS	Other	Any[a]
33. For a function g, the derivative at 2 equals -1, that is $g'(2) = -1$. Which of the following describes the meaning of $g'(2)$?	Yes (n = 145)	90	3	3	1	0	7
	No (n = 112)	93	1	0	N/A	0	1
34. Refer to the graph of function f at right. On which of the following intervals is f increasing?	Yes (n = 146)	93	2	2	3	0	7
	No (n = 113)	97	1	1	N/A	1	3
36. A function h is graphed at right. As $x \to +\infty$, what is true about $h(x)$?	Yes (n = 142)	88	3	2	2	0	7
	No (n = 113)	88	1	5	N/A	1	7
40. Charlie got a car loan for $30,000. Each month, interest of 1/2% is added and then he makes a $600 car payment. If A_n describes the amount he owes for the car at the beginning of month n and $A_1 = 30,000$, which equation is true?	Yes (n = 146)	81	15	1	1	1	18
	No (n = 113)	90	10	0	N/A	0	10
42. Use the graph of the function $f(x) = ax^3 + bx^2 + cx + d$ shown at right. How many real solutions are there to the equation $f(x) = ax^3 + bx^2 + cx + d = -2$?	Yes (n = 145)	90	4	3	3	1	11
	No (n = 113)	96	3	0	N/A	1	4
43. What is the value of $g(1)$? [using the graph]	Yes (n = 146)	91	1	5	2	0	8
	No (n = 113)	96	0	3	N/A	1	4
44. What is the value of $f(g(1))$? [using the graph]	Yes (n = 145)	89	4	4	2	1	11
	No (n = 111)	98	1	0	N/A	1	2
47. A woman is standing on a cliff 200 feet above the water. Through a set of high-powered binoculars, she sees a boat on the water off in the distance. If θ represents the angle of depression, which of the following gives a formula for determining the angle of depression in terms of the distance d of the boat from the bottom of the cliff?	Yes (n = 144)	90	8	2	1	0	11
	No (n = 112)	96	3	1	N/A	0	4
51. Which of the following is the derivative of function f at x?	Yes (n = 145)	89	3	4	4	0	11
	No (n = 112)	96	3	1	N/A	0	4

Note: CAS refers to graphing calculators equipped with computer algebra systems. The *n* varies by problem because not all students reported using a strategy on all items. Any refers to the use of Arithmetic, Graph, CAS, or other strategies and is compared to the use of no strategy (none). *N* represents the number of students who reported using a strategy for each item. Rows add up to more than 100% due to rounding and the fact that some students reported using more than one strategy.

How students used technology to solve function problems on the problem solving test. For the problem solving test, teachers scoring the responses recorded the strategy they believed the student used when solving the problem, based on reading the student's solution, but had no information about whether a student actually used a calculator to solve the item. Table 17 reports the strategies teachers coded students as using on the three function problems (out of 5 problems with 11 subparts) on the problem solving test. Teachers reported most students (n = 178) used arithmetic as a problem solving strategy at least once, and 238 students reported using a graph to solve at least one item.

Table 17

Number of Times Teachers Code Indicated Use of Technology in Solutions to Function Items on Problem Solving Test

Strategy	Students who used strategy on at least one problem	Number of times used	Mean
CAS	48	50	1.04
Graph	191	390	2.04
Arithmetic	108	120	1.11
Substitution	239	523	2.19
Table	24	25	1.04
Other	216	519	2.04

Note. N=271. CAS refers to graphing calculators equipped with computer algebra systems. Substitution includes the use of algebra. Other includes none and not reported. Mean is a weighted mean with the summation of number of times a strategy is reported in numerator and number of students coded as using the strategy in denominator.

All of the items on the problem solving test are calculator active or neutral. A CAS capable calculator could have been used to solve items 1, 2c, 2d and 3, but students only appeared to use CAS to solve item 1. Table 18 reports the strategies teachers coded for students' solutions to the six function items which were all classified as calculator active or neutral items. Unlike posttest 2 where students were more likely not to use a calculator, on the problem solving test students used a calculator strategy far more often than not. Arithmetic was the most often utilized strategy followed by graphing and then other. It is impossible to know, however, if students used a calculator to perform the strategy.

Table 18

Students' Strategies on Problem Solving Function Items as Coded by Teachers

Item	Access to CAS	% coded as using					
		Symbolic (Algebra)	Arithmetic	Graph	Table	CAS	Other
1**. Solve the following system. $\begin{cases} y = x^2 - 3x + 3 \\ y = 2^x \end{cases}$	Yes (n = 145)	0	28	24	1	25	19
	No (n = 112)	0	29	38	1	N/A	18
2a**. A ball is thrown so that its height (in meters) after t seconds is given by $h(t) = -4.9t^2 + 18t + 15$. After how many seconds does the ball reach its maximum height?	Yes (n = 145)	0	5	53	9	0	29
	No (n = 119)	0	31	56	6	N/A	31
2b**. A ball is thrown so that its height (in meters) after t seconds is given by $h(t) = -4.9t^2 + 18t + 15$. What is the maximum height reached by the ball?	Yes	17	0	53	8	0	38
	No	29	0	39	3	N/A	27
2c**. A ball is thrown so that its height (in meters) after t seconds is given by $h(t) = -4.9t^2 + 18t + 15$. Find the instantaneous velocity of the ball 3.4 seconds after it is thrown. Include units	Yes	51	28	0	0	0	13
	No	58	23	0	0	N/A	14
2d**. A ball is thrown so that its height (in meters) after t seconds is given by $h(t) = -4.9t^2 + 18t + 15$. Find the acceleration of the ball 3.4 seconds after it is thrown. Include units.	Yes	38	7	1	0	0	36
	No	41	8	0	0	N/A	43
3**. Are the functions f and g with $f(x) = 3x + 2$ and $g(x) = \frac{x+2}{3}$ inverses of each other?	Yes	19	34	19	9	1	12
	No	17	47	18	1	N/A	18

Note: CAS refers to graphing calculators equipped with computer algebra systems. Unless otherwise stated n = 151 students reported using CAS and n = 120 students reported not using CAS. The n varies by problem because not all students reported using a strategy on all items.

Student Achievement on Function Items

This section presents the results of the analysis of student achievement for posttests 1 and 2 and the problem solving test. Descriptive statistics were used to examine students' prior knowledge before taking *Precalculus and Discrete Mathematics*. Next I analyzed student achievement on assessment items based on the use of technology and students' access to CAS capable calculators.

Table 19

Mean Percentage and Standard Deviation for Student Achievement on Pretests by Class

Class	*n*	Pretest 1		Pretest 2	
		M	*SD*	*M*	*SD*
Third Edition					
410	16	58.79	12.60	42.19	17.00
411	20	54.13	15.48	44.06	11.91
414	17	63.43	16.91	46.69	20.15
415	19	57.44	18.02	49.01	14.77
416	13	62.54	10.13	49.52	11.26
418	18	43.96	15.27	33.33	12.13
419	11	33.60	14.43	23.86	14.74
420	19	52.17	14.35	37.50	12.33
421	24	52.72	18.01	38.54	15.05
422	25	56.17	15.47	38.25	15.13
423	27	51.69	16.42	41.44	14.41
Total	209	53.69	16.75	40.67	15.54
Second Edition					
412	23	55.39	12.03	47.28	12.90
413	20	56.52	15.45	41.88	15.85
417	19	60.64	16.62	48.68	11.71
Total	62	57.36	16.33	45.97	13.68

Note: Pretest 1 mean is the percentage score each student received on pretest 1 for only the 23 function items and ranges from 0 to 100. Pretest 2 mean is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100.

Students' prior knowledge of functions. To compare student achievement, it is important to first determine a baseline of students' knowledge of functions. At the beginning of the school year, students completed two pretests. Table 19 shows the means and standard deviations for pretest 1 and pretest 2 by class. For more details on comparisons between class, curriculum and achievement on pretests see Appendix H for pretest 1 and Appendix I for pretest 2. Figure 11 shows the achievement scores for students by class on the function items for pretest 1 (23 items) taken without the use of any technology and pretest 2 (16 items) on which students were permitted to use technology.

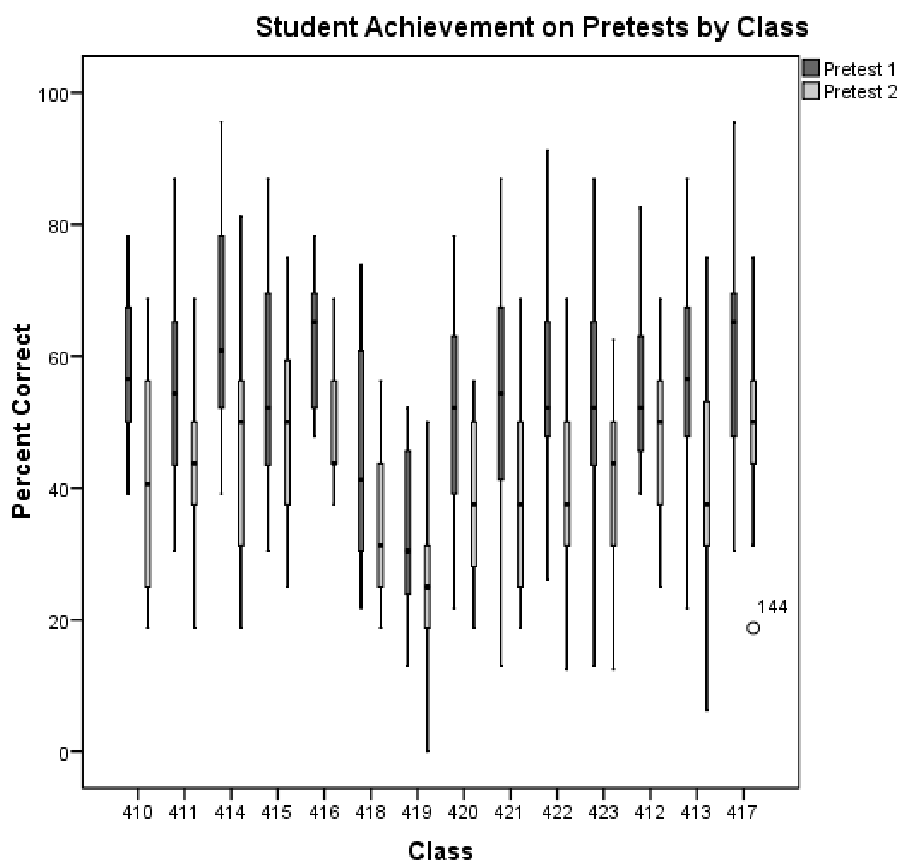


Figure 11. Box plots of Percent of Function Items Correct by Class for Students Using PDM (2nd and 3rd Ed). Classes 412, 413 and 417, located on the far right, all used 2nd Edition curriculum materials.

Prior to conducting an ANOVA to compare the achievement scores on the pretests by classes, I found no evidence of lack of normality in the skewness (pretest 1 = -.01, pretest 2 = -.06) and kurtosis (pretest 1 = -.24 , pretest 2 = -.37) values, and no violation of the homogeneity of variation from the Levene's test (pretest 1 Levene's statistic = .84, $p = .62$; pretest 2 Levine's statistic = 1.7, $p = .06$). Results of the ANOVA for pretest 1 indicated a significant difference in achievement between the different classes (pretest 1 $F(13, 257) = 3.46$, $p < .001$). Results of the ANOVA for pretest 2 also indicated a significant difference, $F(13, 257) = 3.56$, $p < .001$, in achievement scores between classes. I conducted Scheffe's post-hoc tests to determine which achievement scores at the class level were significantly different from the average. The achievement on pretest 1 for the students in class 419 ($n = 11$) was lower when compared to all other classes in general, but the difference in achievement scores was most significant between students in class 419 ($\Delta M = -25.66$, $p < .001$) and students in class 416 ($n = 13$). There was, however, no interaction when examining achievement scores controlling for class and curricula. I also used Scheffe's test to perform post-hoc comparisons on the achievement scores from pretest 2. Scheffe's test showed no significant difference for the achievement in any class when compared to the achievement in other classes. However, a Tukey test indicated a significant difference between the achievement scores in class 419 and several other classes as shown in Table 20. The significant difference in student achievement on the pretest scores indicates a need to control for prior knowledge when examining achievement on posttests.

Table 20

Difference in Achievement Scores for Pretest 2 Function Items for Classes Significantly Higher Than Class 419.

Class	ΔM	Class	ΔM
Third Edition		Second Edition	
411	-20.20*	412	-23.42***
414	-22.83***	417	-24.82***
415	-25.15***		
416	-25.66***		
423	-17.57*		

Note: ΔM is difference in mean scores.
* $p < .05$. *** $p < .001$.

Pretests and the use of technology. Figure 12 reports achievement scores on the function items on pretest 2 grouped by whether students had access to a CAS capable calculator (See Appendix I). On pretest 2, there is a significant percentage difference, $F(1, 269) = 17.37$, $p < 0.01$, overall in achievement between students with and without CAS, with students who reported having access to CAS ($n = 151$, $M = 38.53$, $SD = 14.90$) scoring lower than students who reported not having access to CAS ($N = 120$, $M = 46.09$, $SD = 14.74$), even when controlling for prior knowledge by using achievement scores on pretest 2 as a covariate. There was, however, no difference, $t(115) = -0.12$, $p = 0.91$, when comparing percent achievement for students who reported having access to CAS and who used the 2nd Edition ($n = 60$, $M = 37.50$, $SD = 17.68$) to the achievement of students who reported having access to CAS and used the Third Edition ($n = 60$, $M = 38.55$, $SD = 14.93$); or when comparing achievement of students who reported not having access to CAS ($t(1) = 0.09$, $p = 0.94$) and who used the Second Edition ($n = 149$, $M = 38.59$, $SD = 14.93$) or the 3rd Edition ($n = 2$, $M = 37.50$, $SD = 17.68$) . There was also no interaction between class, curricula, and reported access to CAS.

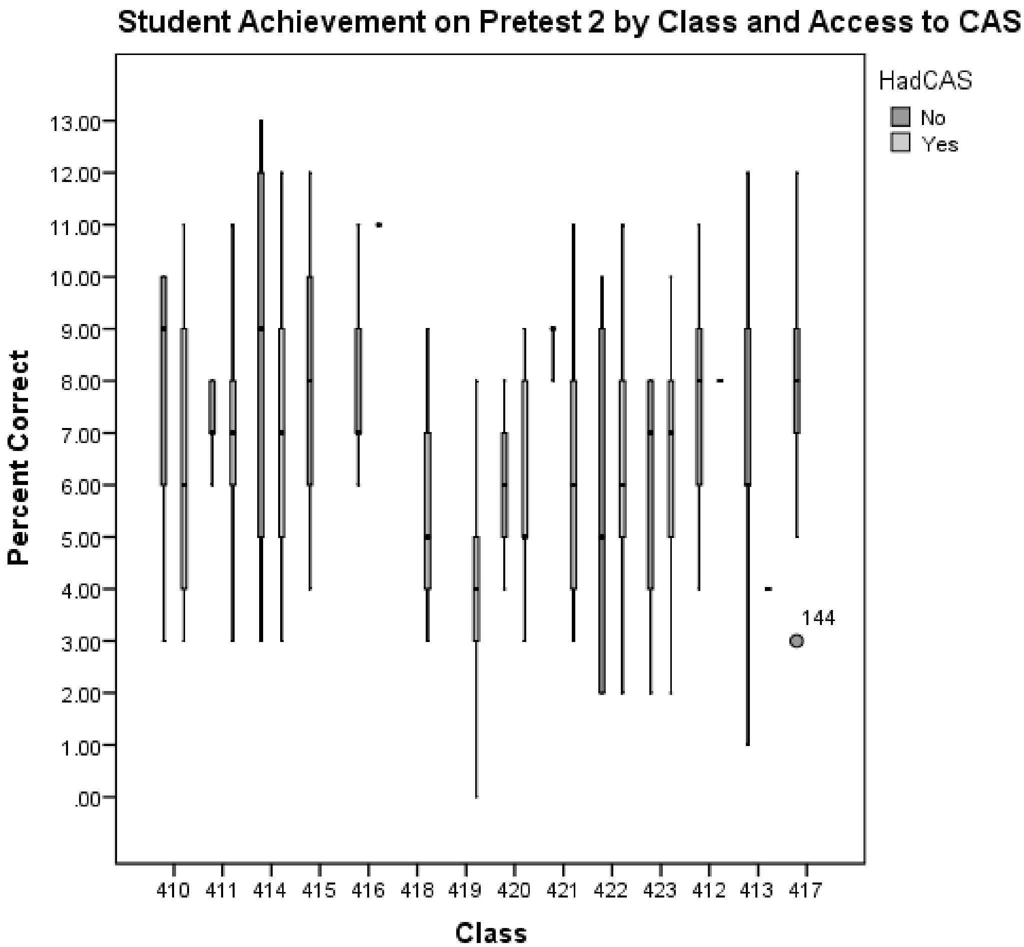


Figure 12. Percent Achievement on Pretest 2 Function Items by Class and Grouped by Access to CAS. Classes 412, 413 and 417, located on the far right, all used 2nd Edition curriculum materials.

Student achievement on posttests. At the end of the year, students completed three assessments which included items to assess their knowledge of functions. Achievement scores for students taking posttest 1 and posttest 2 are shown in Figure 13. In six of the classes the median score on the posttest 1 was higher than posttest 2. There was also substantially more variation on the scores for the problem solving test than for posttest 2. I performed an ANOVA test to look for differences in achievement scores on the posttest by class. Prior to conducting the ANOVA, I found no deviations from normality in skewness (posttest 1 =0.11, posttest 2 =

0.02, and PSU = -0.34) and kurtosis (posttest 1 = -0.40, posttest 2 = -0.48, and PSU = -0.57) values and no evidence of violation of homogeneity of variance from the Levene's test (posttest 1 Levene's statistic =1.10, $p = 0.37$; posttest 2 Levene's statistic = 0.81, $p = 0.66$; PSU Levene's statistic = 1.2, $p = 0.26$). Results of the one-way ANOVA indicate significant differences in posttest 1, $F(13, 257) = 12.35$, $p < 0.001$, between classes. Conducting Scheffe's post-hoc tests showed the achievement differences for posttest 1 in all classes were significantly different when compared to at least one other class. However, there was also a significant difference on posttest 1 achievement scores between the different curricula ($t(269) = -2.08$, $p < 0.05$) with achievement of students using Second Edition materials being higher. Results of the one-way ANOVA indicate significant differences in posttest 2, $F(13, 257) = 7.64$, $p < 0.01$, between classes. Conducting a Scheffe's post-hoc test showed the achievement differences for posttest 2, in all classes except 410, 412, and 420, were significantly different when compared to at least one other class. However, there was no significant difference on posttest 2 achievement scores between the different curricula, $t(269) = -1.22$, $p = 0.15$.

Students' use of technology on posttests. There was a significant difference, $F(1, 269) = 33.69$, $p < 0.001$, in the percent achievement scores of function items for posttest 1 between students who reported not having access to CAS ($M = 63.54$, $SD = 19.11$) and students who reported having access to CAS ($M = 51.32$, $SD = 15.54$), with students who did not have access to CAS during instruction scoring higher even though students were not permitted to use technology on posttest 1. There was also a significant difference, $F(1, 269) = 16.09$, $p < 0.001$, in the percent achievement scores of function items for posttest 2 between students who reported not having access to CAS ($M = 62.86$, $SD = 17.31$) and students who reported having access to CAS ($M = 54.84$, $SD = 14.79$), with students who did not have access to CAS scoring higher. However, because there was a significant difference in the achievement of the groups on pretest

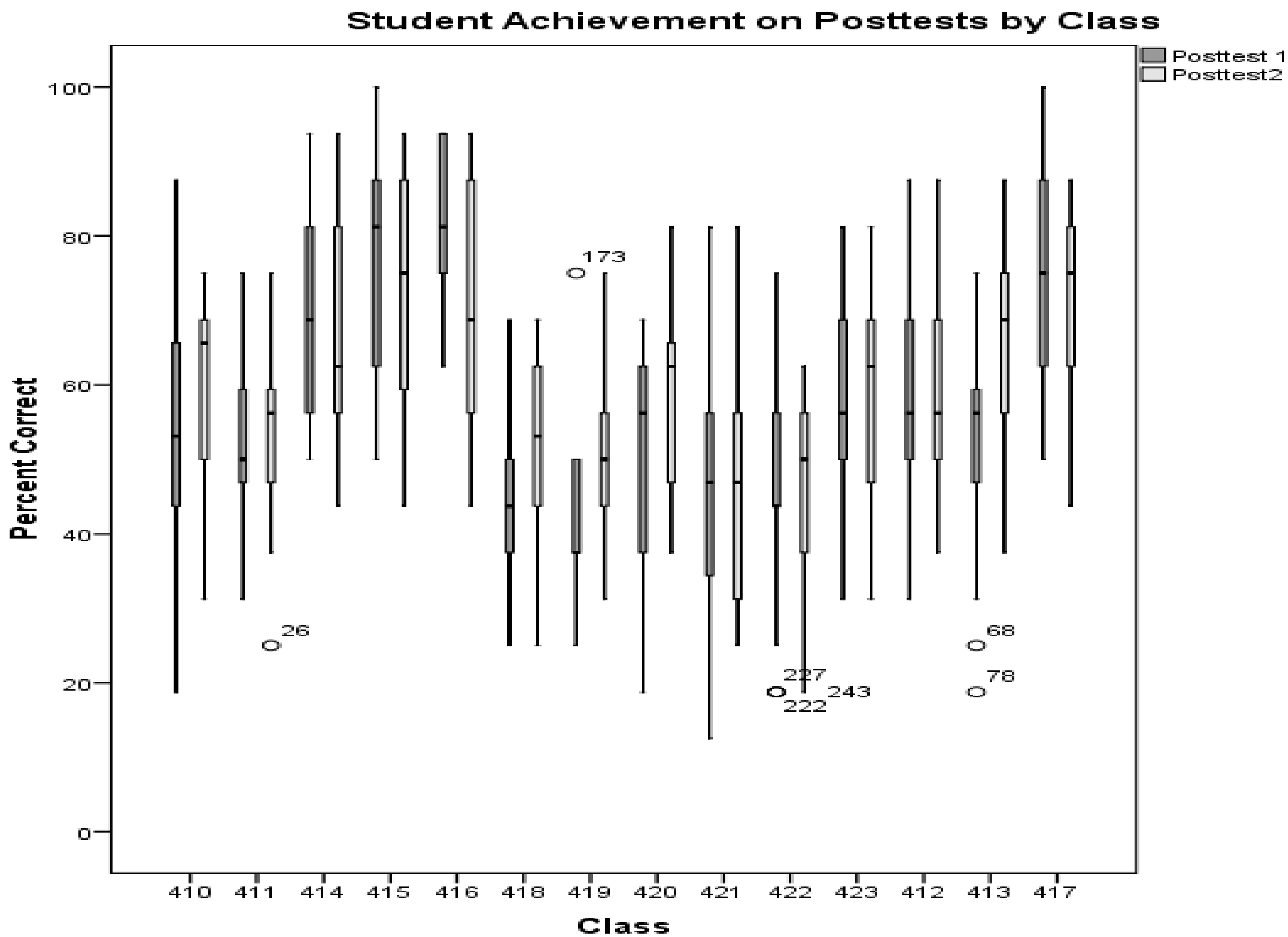


Figure 13. Box Plots of Percent of Function Items Correct on posttest 1 and 2 by Class for Students Using PDM (2nd and 3rd Ed). Classes 412, 413 and 417, located on far right, all used 2nd Edition curriculum materials. Outliers are indicated by record number in SPSS.

scores, it was important to statistically control for those differences by including pretest scores as a covariate in subsequent analyses of achievement.

Achievement and the Use of Technology on Posttest Function Items

In this section, I examine achievement on a per item basis for posttests on which students were permitted to use technology, namely posttest 2 and the PSU, delineated by technology strategy and access to CAS calculators. First I examine achievement from the perspective of how having access to technology influenced students' answers to function items. Then I examine achievement from the perspective of how having access to CAS influenced students' answers to function items.

Achievement and the use of technology on posttest 2. In order to compare the different strategies students reported using on posttest 2 to their achievement in terms of accuracy, I performed chi-square tests on the frequency counts for strategies used to solve the function items on a per item basis. Prior to conducting the chi-square tests, I had already checked the data for normality and homogeneity of variance. I also confirmed each strategy was independent of each other, meaning there was no overlap in strategies. When conducting the chi-squared tests on achievement, I used an experiment wise $\alpha = .05$ and then adjusted it to account for the five comparison tests resulting in a test-wise alpha of .01. Therefore, tests in which achievement differences were significant at $\alpha/5 = .01$ or lower are reported as significant. Post-hoc chi-square tests were performed comparing individual strategies to no strategy and then comparing the use of *any strategy* to the use of *no strategy*.

Of the seven items which were calculator neutral, achievement on only two items (31 and 48) showed no significant achievement differences between students who reported using a calculator and those who did not (see Table 21). In all other cases, students who reported using a calculator answered the item correctly more often than students who did not report using a calculator. Frequency counts from Table 22 show most students did not report using a calculator

Table 21

Number of Students Indicating Use of Calculator Features on Technology Neutral Items on Posttest 2 and Percent of Those Obtaining Correct Solution

	Item	Access to CAS	Number of Students Reporting Strategies and Percentage of Students Obtaining Correct Solution											
			None		Arith		Graph		CAS		Other		Any[a]	
			n	%	*n*	%	*n*	%	*n*	%	*n*	%	*n*	%
31.	Given the function *h* defined by $h(x) = \frac{(2x+4)(x-1)}{(x+2)}$.	Yes	68	51	5	40	60	30	7	29	6	67	78	33
	What is the behavior of the function near *x* = -2?	No	69	2	0	0	42	48	N/A	N/A	2	48	44	50
37.	Suppose $f(x) = x^{1/2}$. What is the set of all values of *x* for which $f(x)$ is a real number?	Yes**	67	30	44	34	28	82	4	50	3	67	79	53
		No*	49	39	40	43	23	74	N/A	N/A	1	100	64	55
38.	Evaluate $\lim_{x \to 10} \frac{x^2 - 100}{2x^2 - 23x + 30}$.	Yes **	40	5	23	17	34	24	49	76**	1	0	107	46**
		No	43	19	29	24	32	19	N/A	N/A	6	33	67	22
45.	Which of the following is (are) true for all values of θ for which the functions are defined?	Yes **	81	33	40	80	12	58	10	60	5	60	67	72**
	I. $\sin(-\theta) = -\sin\theta$ II. $\cos(-\theta) = -\cos\theta$ III. $\tan(-\theta) = -\tan\theta$	No**	51	49	48	83	9	78	N/A	N/A	3	67	60	82**
46.	Which of the following could be an equation for the graph at the right? [graph of polar function shown]	Yes **	42	48	4	75	92	85**	10	80	0	0	106	84
		No**	28	57	8	88	77	86**	N/A	N/A	1	0	86	86
48.	The line in the figure at right is the graph of	Yes	111	32	11	36	7	14	15	27	0	0	33	27
	$y = f(x)$. What is the value of $\int_{-2}^{3} f(x)dx$?	No	87	68	21	82	1	0	N/A	N/A	0	0	22	50
52.	Which equation is graphed at the right? [graph of sine function shown]	Yes **	41	34	7	29	84	70**	9	67	1	100	101	67**
		No**	22	27	7	100	78	78**	N/A	N/A	3	67	88	80**

Note: [a]*Any* refers to the use of Arithmetic, Graph, CAS, or other strategies and is compared to the use of no strategy (none). Rows add up to more than 100% due to rounding or if students reported using more than one strategy. For each item 5 post-hoc tests were conducted comparing use of no strategy to arithmetic, graph, CAS, other or any for students who had access to CAS then repeated for those who did not have access to CAS capable calculators.
* $p < .05$. ** $p < .01$

Table 22

Number of Students Indicating Use of Calculator Features on Technology Inactive Items on Posttest 2 and Percent of Those Obtaining Correct Solution

Item	Access to CAS	Number of Students Reporting Strategies and Percentage of Students Obtaining Correct Solution											
		None		Arith		Graph		CAS		Other		Any[a]	
		N	%	N	%	N	%	N	%	N	%	N	%
33. For a function g, the derivative at 2 equals -1, that is $g'(2) = -1$. Which of the following describes the meaning of $g'(2)$?	Yes	136	58	4	25	4	0	1	0	0	0	9	11
	No	111	56	1	100	0	--	N/A	N/A	0	0	1	100
34. Refer to the graph of function f at right. On which of the following intervals is f increasing?	Yes	136	82	3	83	3	100	4	100	0	100	10	100
	No	110	94	1	100	1	0	N/A	N/A	1	100	3	67
36. A function h is graphed at right. As $x \rightarrow +\infty$, what is true about $h(x)$?	Yes	133	74	4	75	3	67	4	75	0	0	11	73
	No	105	79	1	100	5	80	N/A	N/A	1	100	7	67
40. Charlie got a car loan for $30,000. Each month, interest of 1/2% is added and then he makes a $600 car payment. If A_n describes the amount he owes for the car at the beginning of month n and A_1 = 30,000, which equation is true?	Yes	220	20	33	23	2	50	2	50	2	0	39	26
	No	102	36	11	27	0	0	N/A	N/A	0	0	11	27
42. Use the graph of the function $f(x) = ax^3 + bx^2 + cx + d$ shown at right. How many real solutions are there to the equation $f(x) = ax^3 + bx^2 + cx + d = -2$?	Yes	130	74	6	67	4	25	4	75	1	0	15	53
	No	109	78	3	0	0	0	N/A	N/A	1	100	4	25
43. What is the value of g(1)? [using the graph]	Yes	133	93	2	50	8	88	3	100	0	0	13	85
	No	109	96	0	0	3	100	N/A	N/A	1	100	4	100
44. What is the value of $f(g(1))$? [using the graph]	Yes	129	69	6	67	6	67	3	67	1	100	16	69
	No	109	83	1	100	0	0	N/A	N/A	1	100	1	100
47. A woman is standing on a cliff 200 feet above the water. If θ represents the angle of depression, which of the following gives a formula for determining the angle of depression in terms of the distance d of the boat from the bottom of the cliff?	Yes	129	39	11	18	3	33	1	0	0	0	15	20
	No	108	32	3	67	1	0	N/A	N/A	0	0	4	50
51. Which of the following is the derivative of function f at x?	Yes	129	50	4	50	6	17	6	17	0	0	16	25
	No	108	52	3	33	1	100	N/A	N/A	0	0	4	50

Note: [a] Any refers to the use of Arithmetic, Graph, CAS, or other strategies and is compared to the use of no strategy (none). Rows may add up to more than 100% due to rounding or if students reported using more than one strategy. For each item 5 post-hoc tests were conducted comparing use of no strategy to arithmetic, graph, CAS, other or any for students who had access to CAS then repeated for those who did not have access to CAS capable calculators.
* $p < .05$. ** $p < .01$

on calculator inactive function items. Post-hoc chi-square or Fisher's exact tests conducted on the calculator inactive items failed to demonstrate any significant differences in achievement between students who did and did not report using the calculator.

Tables 21 and 22 indicate there are some significant differences in achievement based on students' reported use of calculators. I wanted to further explore any role the use of CAS may have had on achievement. Therefore, I reanalyzed the problems on posttest 2 comparing achievement on the technology neutral items and students' access to CAS capable graphing calculators. Table 23 reports the results for the CAS neutral items. Differences in achievement on two of the CAS neutral items were significant in favor of students who did not have access to CAS capable calculators (item 45 at $p < .05$ and item 48 at $p < .01$).

Table 23

Percentage of Students Obtaining Correct Solution on CAS Neutral Items on Posttest 2 by Access to CAS Calculator

Item	Access to CAS calculator	
	No	Yes
	% correct	% correct
31. Given the function *h* defined by $h(x) = \frac{(2x+4)(x-1)}{(x+2)}$. What is the behavior of the function near $x = -2$?	48	43
38. Evaluate $\lim_{x \to 10} \frac{x^2 - 100}{2x^2 - 23x + 30}$	23	33
45*. Which of the following is (are) true for all values of θ for which the functions are defined? I. $\sin(-\theta) = -\sin\theta$ II. $\cos(-\theta) = -\cos\theta$ III. $\tan(-\theta) = -\tan\theta$	66	51
48**. The line in the figure at right is the graph of $y = f(x)$. What is the value of $\int_{-2}^{3} f(x)dx$?	63	30

Note: CAS refers to graphing calculators equipped with computer algebra systems. Students who had access to CAS $n = 150$. Students who did not have access to CAS $n = 120$.
* $p < .0$,. ** $p < .01$.

Achievement and the use of technology on problem solving test. In order to compare the different strategies teachers coded students as appearing to use on the problem solving test to their achievement in terms of accuracy, I performed chi-square tests on the frequency counts for the strategies students reported using on the function items on a per item basis. Prior to conducting the chi-square tests, I had already checked the data for normality and homogeneity of variance. When conducting the chi-squared tests on achievement, I used an experiment wise α = .05 and then adjusted it to account for the five comparison tests, resulting in a test-wise alpha of .01. Therefore, tests in which achievement differences were significant at $\alpha/5 = .01$ or lower are reported as significant using **.

Post-hoc chi-square tests were performed comparing individual strategies to no strategy and comparing the use of any strategy to the use of no strategy. When examining achievement on the problem solving test, (max score for function items = 11), there was not a significant difference ($t(269) = 0.85$, $p = .40$) in the achievement scores between students who had access to CAS ($M = 6.92$, $SD = 2.88$) and those who did not ($M = 6.61$, $SD = 3.04$).

All six items on the problem solving test were calculator active or neutral and CAS neutral. Differences in achievement were significant for all six items as reported in Table 24. Item number 1 is calculator active and indicates more students used CAS to solve this item than any other item. Students who reported using CAS strategies to solve the item were also more likely to obtain a correct solution than students who did not use CAS. Students who did not have access to CAS calculators reported using a graph most often to solve the item with 80% of those students obtaining the correct solution.

Table 24

Number of Students Teachers Reported Using Specific Strategies and Percent of Students Obtaining Correct Solution for Technology Neutral and Active Items on Problem Solving Test

		Strategy											
		Arithmetic		Algebra		Graph		Table		CAS		Other	
Item	Access to CAS	*n*	%	*n*	%	n	%	*n*	%	*n*	%	*n*	%
1**. Solve the following system. $\begin{cases} y = x^2 - 3x + 3 \\ y = 2^x \end{cases}$	Yes % Partially Correct[b]	0	0	0	0	13	28	0	0	15	32	19	40
	Yes % Correct[b]	0	0	0	0	19	48	0	0	19	58	2	5
	No % Partially Correct[b]	1	5	0	0	4	18	0	0	N/A	N/A	17	77
	No % Correct[b]	2	5	0	0	31	80	0	0	N/A	N/A	6	15
2a**. A ball is thrown so that its height (in meters) after *t* seconds is given by $h(t) = -4.9t^2 + 18t + 15$.After how many seconds does the ball reach its maximum height?	Yes	6	5	0	0	78	61	12	9	0	0	33	26
	No	5	4	0	0	66	58	6	5	N/A	N/A	36	32
2b**. What is the maximum height reached by the ball?	Yes	0	0	25	18	63	46	0	0	0	0	50	36
	No	0	0	33	29	47	41	0	0	N/A	N/A	34	30
2c** Find the instantaneous velocity of the ball 3.4 seconds after it is thrown. Include units.	Yes % Partially Correct[a]	33	79	5	12	0	0	0	0	0	0	3	7
	Yes % Correct	47	96	1	2	0	0	0	0	0	0	1	2
	No % Partially Correct	29	94	1	3	0	0	0	0	N/A	N/A	1	3
	No % Correct	46	94	0	0	0	0	0	0	N/A	N/A	2	4
2d** Find the acceleration of the ball 3.4 seconds after it is thrown. Include units.	Yes % Partially Correct	24	65	0	0	0	0	0	0	0	0	10	27
	Yes % Correct	32	63	0	0	0	0	0	0	0	0	19	37
	No % Partially Correct	20	57	0	0	0	0	0	0	N/A	N/A	15	43
	No % Correct	32	63	0	0	0	0	0	0	N/A	N/A	19	37
3**. Are the functions *f* and *g* with $f(x) = 3x + 2$ and $g(x) = \frac{x+2}{3}$ inverses?	Yes % Partially Correct	32	78	0	0	8	20	2	5	1	2	0	0
	Yes % Correct	47	80	0	0	11	19	1	2	0	0	1	2
	No % Partially Correct	37	82	0	0	8	18	0	0	N/A	N/A	0	0
	No % Correct	37	82	0	0	7	16	0	0	N/A	N/A	1	2

Note: CAS refers to graphing calculators equipped with computer algebra systems. Otherwise noted partially correct items are scored 0 for incorrect, 1 for partially correct and 2 for correct.
[a]Indicates scoring rubric for this problem was scored on 0-4 instead of a 0-2 scale. A score of 0 is considered incorrect, 1-2 is partially correct and 3-4 is essentially correct. Rows may add up to more than 100% due to rounding or if teachers coded a student as using more than one strategy.
* $p < .05$. ** $p < .01$.

Table 25 reports student achievement on function items from the problem solving test grouped by students' access to CAS calculators. Differences in achievement on only one problem were significant (Problem 2a) in favor of students who used CAS. In general, even if the results are not significantly different, those who had access to CAS scored at least as well as those who did not.

Table 25

Percent of Students Indicating Use of CAS on Problem Solving Test and Obtaining Correct Solution on CAS Neutral Items

		Had access to CAS calculator	
		No n = 120	Yes n = 150
1. Solve the following system. $\begin{cases} y = x^2 - 3x + 3 \\ y = 2^x \end{cases}$	% Partially Correct[a]	19	31
	% Correct	32	27
2a*. A ball is thrown so that its height (in meters) after t seconds is given by $h(t) = -4.9t^2 + 18t + 15$. After how many seconds does the ball reach its maximum height?	% Correct	94	86
2b. What is the maximum height reached by the ball?	% Correct	95	92
2c. Find the instantaneous velocity of the ball 3.4 seconds after it is thrown. Include units.	% Partially Correct	26	27
	% Correct	41	33
2d. Find the acceleration of the ball 3.4 seconds after it is thrown. Include units.	% Partially Correct	29	25
	% Correct	43	34
3. Are the functions f and g with $f(x) = 3x + 2$ and $g(x) = \frac{x+2}{3}$ inverses of each other?	% Partially Correct[a]	53	44
	% Correct	23	29

Note: CAS refers to graphing calculators equipped with computer algebra systems.
[a] Indicates scoring rubric for this problem was scored on 0-4 instead of correct/incorrect. A score of 0 is considered incorrect, 1-2 is partially correct and 3-4 is essentially correct.
* $p < .05$, ** $p < .01$.

The Relationships Between Achievement, OTL Measures, and the Use of Technology

In this section I examine what relationships exist between achievement, OTL measures, and the use of technology. Because independent variables can have confounding effects when examined together, I first examine the effects of only the OTL variables on achievement for each of the three posttests. Then I consider the effects of only technology variables on achievement for the three posttests. Finally I examine achievement in relation to both OTL and the use of technology.

Achievement on posttest 1 and OTL. A regression analysis was conducted to evaluate how well OTL measures predicted achievement on posttest 1. Before conducting the regression, an analysis of standard residuals was conducted, which showed that the data contained no outliers (Std. Residual Min = -2.79, Std. Residual Max = 2.9). Tests to determine if the data met the assumption of collinearity indicated that multicollinearity was not a concern (pretest 1, Tolerance = 0.95, *VIF* = 1.05; OTL Homework, Tolerance = 0.64 *VIF* = 1.57; OTL Lessons, Tolerance = 0.55, *VIF* = 1.84). The data met the assumption of independent errors (Durbin-Watson value = 1.80). The histogram of standardized residuals indicated that the data contained approximately normally distributed errors, as did the normal P-P plot of standardized residuals, which showed points that were not completely on the line, but close. Finally, to test the assumption of independence, I calculated an intra-class correlation coefficient. For posttest 1, (ICC = 0.35), the high ICC value indicates a severe violation of the independence assumption suggesting data should be analyzed at the class level, which was more in line with the overall design of the initial study, instead of the student level. When I examined the data at the class level, I found all of the above assumptions violated due to the small sample size (n = 14). Therefore, I used both linear regression and HLM to create the prediction models. Because the results were comparable only the regression results are presented for this model.

A regression analysis was performed with pretest 1, used to control for differences in prior knowledge, OTL Homework and OTL Lessons used as predictors, and percent achievement on posttest 1 as the criterion variable. Analysis of achievement for posttest 1 (N = 270; M =

56.73; *SD* = 18.22) showed the linear combination of OTL measures was significantly related to achievement, $F(3, 267) = 81.05$, $p < .0001$, $R^2 = .48$, $R^2_{adjusted} = 0.47$, indicating approximately 48% of the variance of achievement for posttest 1 in the sample can be accounted for by the linear combination of OTL measures when controlling for prior knowledge. Achievement on pretest 1 scores had the most impact on the regression model, meaning for every one percent higher students scored on pretest 1, on average, they scored 0.56 percentage points higher on posttest 1 after controlling for prior knowledge and OTL variables. OTL Lessons also had a significant impact on achievement ($\beta = 0.42$, $p < .01$), meaning for every additional lesson a teacher taught, student achievement increased, on average, 0.42 percentage points. Table 26 shows the descriptive statistics, the standardized (β) coefficients, the standard errors and the correlations for each variable.

Table 26

OTL Measures as a Set of Predictors for Achievement on Posttest 1 Function Items

Variable	Correlations				β	Std Error
	OTL Lessons	OTL Homework	Pretest 1	Posttest1		
Pretest 1				.56**	.52**	.05
OTL Homework			-.22**	-.31**	.02	.07
OTL Lessons		-.53**	.11*	.47**	.42**	.09
				Intercept = 32.01		
Mean	78.10	72.07	54.53	56.73		
SD	10.78	14.70	16.33	16.33	R^2=.48	

Note: *N* = 271. Pretest 1 is the percentage score each student received on the 23 function items and ranges from 0 to 100. Posttest 1 is the percentage score each student received on the 16 function items and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTL HW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100.
* $p < .05$, ** $p < .01$.

Achievement on posttest 2 and OTL. A regression analysis was conducted to evaluate how well OTL measures predicted achievement on posttest 2. Before conducting the regression, an analysis of standard residuals was carried out, which showed that the data contained no outliers (Std. Residual Min = -2.38, Std. Residual Max = 2.7). Tests to determine if the data met the assumption of collinearity indicated that multicollinearity was a concern (pretest 2, Tolerance = .90, *VIF* = 1.10; OTL Homework, Tolerance = 0.26 *VIF* = 3.78; OTL Lessons, Tolerance = 0.10, *VIF* = 10.22; OTL posttest 2, Tolerance = 1.36, *VIF* = 7.36) for only the variable OTL (Function) Lessons which was found to be highly and significantly correlated to OTL posttest 2 (r = 0.79), and was therefore removed from the predictors. The data met the assumption of independent errors (Durbin-Watson value = 1.72). The histogram of standardized residuals indicated that the data contained approximately normally distributed errors, as did the normal P-P plot of standardized residuals, which showed points that were not completely on the line, but close. Finally, to test the assumption of independence I calculated an intra-class correlation coefficient. For posttest 2, (ICC = 0.26), the high ICC value indicates a severe violation of the independence assumption, suggesting data should be analyzed at the class level instead of the student level. Therefore, I used both linear regression and HLM to create the prediction models. Because the results were comparable, only the regression results are presented for this model.

A regression analysis was performed with pretest 2, used to control for differences in prior knowledge, OTL Homework and OTL posttest 2 used as predictors, and percent achievement on posttest 2 as the criterion variable. Analysis of achievement for posttest 2 (N = 270; M = 58.39; SD = 16.42) showed the linear combination of OTL measures was significantly related to achievement, $F(3, 267) = 30.63$, $p < .01$, $R^2 = .26$, $R^2_{\text{adjusted}} = .25$, indicating approximately 26% of the variance of achievement for posttest 2 in the sample can be accounted for by the linear combination of OTL measures when controlling for prior knowledge as shown in Table 27. OTL Homework had the most impact on the regression model, meaning for every one percent more homework assigned a student scored, on average, 0.58 percentage points higher on posttest 2 after controlling for prior knowledge and OTL variables. OTL Posttest 2 had a significant but negative impact on student achievement (β = -0.184, $p < .01$). The descriptive statistics, the standardized (β) coefficients, the standard errors and the correlations for each variable are also reported in Table 27.

Table 27

OTL Measures as a Set of Predictors for Achievement on Posttest 2 Function Items

Variable	Correlations				β	Std Error
	OTL Posttest 2	OTL Homework	Pretest 2	Posttest 2		
Pretest 2				.415**	.363**	.059
OTL Homework			-.280**	-.287**	.583**	.136
OTL Posttest 2		.001	.023	.237**	-.184**	.219
				Intercept = 58.39		
Mean	91.35	72.07	41.88	58.38		
SD	6.55	14.70	15.28	16.42	R^2=.26	

Note: *N* = 271. Pretest 2 is the percentage score each student received on the 16 function items and ranges from 0 to 100. Posttest 2 is the percentage score each student received on the 16 function items and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTL HW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100.
* $p < .05$. ** $p < .01$.

Achievement on problem solving test and OTL measures. A multiple regression analysis was conducted to evaluate how well OTL measures predicted achievement on the problem solving test. Pretest 2 was chosen to control for prior knowledge because students were permitted to use technology on both pretest 2 and the problem solving test. Before conducting the regression, an analysis of standard residuals was carried out, which showed that the data contained no outliers (Std. Residual Min = -2.80, Std. Residual Max = 2.2). Tests to determine if the data met the assumption of collinearity indicated that multicollinearity was not a concern (pretest 2, Tolerance = 0.91, *VIF* = 1.11; OTL Homework, Tolerance = 0.24 *VIF* = 4.1; OTL

Lessons, Tolerance = 0.46, *VIF* = 2.16; OTL PSU, Tolerance = 0.35, *VIF* = 2.87). The data met the assumption of independent errors (Durbin-Watson value = 1.42). The histogram of standardized residuals indicated that the data contained approximately normally distributed errors, as did the normal P-P plot of standardized residuals, which showed points that were not completely on the line but close. Finally, to test the assumption of independence I calculated an intra-class correlation coefficient. For the problem solving test (ICC = 0.4) the high ICC value indicates a severe violation of the independence assumption suggesting data should be analyzed at the class level instead of the student level. Therefore I used both linear regression and HLM to create the prediction models. Because the results were comparable, only the regression results are presented for this model.

A regression analysis was performed with pretest 2 used to control for differences in prior knowledge, OTL Homework, OTL Lessons, and OTL PSU used as predictors, and achievement on the problem solving test as the criterion variable. Analysis of achievement for the problem solving test (N = 270; M = 61.69; SD = 24.55) showed that while the linear model was significant overall, $F(4, 266) = 6.54$, $p < .01$, $R^2 = .09$, $R^2_{adjusted} = 0.08$, none of the OTL variables were significant after controlling for prior knowledge. Only prior achievement as measured by pretest 2 had any impact on the achievement scores for the PSU. The low value of R^2 also indicates less than 10% of the variance of achievement for the problem solving test in the sample can be accounted for by the linear combination of OTL measures. Table 28 shows the descriptive statistics, the standardized (β) coefficients, the standard errors and the correlations for each variable.

Table 28

OTL as a Set of Predictors for Achievement on Problem Solving Test Function Items

Variable	Correlations					β	Std Error
	OTL Lessons	OTL PSU	OTL HW	Pretest 2	PSU		
Pretest 2			.283**	.	289**	.454**	.099
OTL Homework			.067	-.280**	-.121*	.113	.197
OTL PSU			-.102	-.092	-.081	-.165	.160
OTL Lessons		.025	.078	.206**	.098	.177	.196
						Intercept = 61.69	
Mean	78.09	90.28	72.07	41.88	61.69		
SD	10.78	15.18	14.70	15.28	24.55	R^2=.09	

Note: N = 271. PSU is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTL HW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. Pretest 2 is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100.
* $p < .05$; ** $p < .01$.

Achievement on posttest 1 and access to technology. I also conducted a regression analysis to evaluate how well reported use of technology predicted achievement on posttest 1 even when students were not permitted to use technology on the assessment. Before conducting the regression, an analysis of standard residuals was carried out, which showed that the data contained no outliers (Std. Residual Min = -2.54, Std. Residual Max = 2.62). Tests to determine if the data met the assumption of collinearity indicated that multicollinearity was not a concern (pretest 1, Tolerance = 0.98, *VIF* = 1.2 HadCas, Tolerance = 0.98 *VIF* = 1.02). The data met the assumption of independent errors (Durbin-Watson value = 1.55). Finally, the histogram of standardized residuals indicated that the data contained approximately normally distributed errors, as did the normal P-P plot of standardized residuals, which showed points that were not completely on the line, but close. Therefore, I used both linear regression and HLM to create the prediction models. Because the results were comparable, only the regression results are presented for this model.

A regression analysis was performed using pretest 1 to control for differences in prior knowledge (when students did not have access to technology), and HadCas (when students did have access to technology during instruction) as predictors, with achievement on posttest 1 as the criterion variable as shown in Table 29.   Analysis of achievement for posttest 1 (N = 270; M = 56.73; SD = 18.22) showed the linear combination of the use of technology was significantly related to achievement, $F(3, 264) = 81.51$, $p < 0.001$, R^2 =0.38, $R^2_{adjusted}$ = 0.38, indicating approximately 38% of the variance of achievement for posttest 1 in the sample can be accounted for by the technology measure when controlling for prior knowledge. Achievement on pretest 1 scores had the most impact on the regression model, meaning for every one percent higher students scored on pretest 1, on average, they scored 0.52 percentage points higher on posttest 1after controlling for prior knowledge. Access to CAS was also significant (β = -0.266, $p < .01$) and students who had access to CAS, on average, scored 0.27 percent points lower. Table 29 reports the descriptive statistics, the standardized (β) coefficients, the standard errors and the correlations for each variable.

Table 29

Access to Technology as a Predictor For Achievement on Posttest 1 Function Items

Variable	Correlations			β	Std Error
	HadCas	Pretest 1	Posttest 1		
Pretest 1			.558**	.524**	.054
HadCas		-.129*	-.334**	-.266**	1.78
				Intercept = 30.29	
Mean	.56	54.53	56.73		
SD	.50	16.33	16.33	R^2=.38	

Note: N = 271. Pretest 1 is the percentage score each student received on the 23 function items and ranges from 0 to 100. Posttest 1 is the percentage score each student received on the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes.
* $p < .05$, ** $p < .01$

Achievement on posttest 2 and the use of technology. Analysis on posttest 1 was an indication of how students performed without technology; posttest 2 offers insight in how student achievement relates directly to the use of technology. Before conducting a regression analysis on posttest 2 achievement with technology variables, an analysis of standard residuals was carried out, which showed that the data contained no outliers (Std. Residual Min = -2.40, Std. Residual Max = 2.35). Tests to determine if the data met the assumption of collinearity indicated that multicollinearity was not a concern (pretest 2, Tolerance = 0.92, *VIF* = 1.09; HadCas, Tolerance = 0.93 *VIF* = 1.07; UseofStrategies, Tolerance = 0. 96, *VIF* = 1.04). The data met the assumption of independent errors (Durbin-Watson value = 1.89). Finally, the histogram of standardized residuals indicated that the data contained approximately normally distributed errors, as did the normal P-P plot of standardized residuals, which showed points that were not completely on the line but close. I used both linear regression and HLM to create the prediction models. Because the results were comparable, only the regression results are presented for this model.

A regression analysis was performed using pretest 2 used to control for differences in prior knowledge, HadCas and DidUseStrategies used as predictors, and achievement on posttest 2 as the criterion variable. Analysis of achievement for posttest 2 with two levels of nesting (*N1* = 267; *N2* = 3; *M* = 58.39; *SD* = 16.42) showed the linear combination of technology measures was significantly related to achievement, $F(3, 264) = 7.66$, $p < 0.01$, $R^2 = 0.23$, $R^2_{adjusted} = 0.22$, indicating approximately 23% of the variance of achievement for posttest 2 in the sample can be accounted for by the linear combination of technology measures when controlling for prior knowledge. This model shows that each time a student used a calculator strategy to solve a problem, the student's score on posttest 2 went up 1.06 percentage points, on average. However, the use of CAS was not significant in this model. Table 30 shows the descriptive statistics, the standardized (β) coefficients, the standard errors and the correlations for each variable.

Table 30

Use of Technology as a Set of Predictors for Achievement on Posttest 2 Function Items

Variable	Correlations				β	Std Error
	DidUseStategy	HadCas	Pretest 2	Posttest 2		
Pretest 2				.415**	.380**	.057
HadCas			-.246**	-.243**	3.15	2.37
DidUseStategy		.032	-.012	.148*	1.06*	.48
					Intercept = 58.94	
Mean	3.70	.56	41.88	58.38		
SD	1.79	.50	15.28	16.42	R^2=.23	

Note: *N* = 270. Pretest 2 is the percentage score each student received on the 16 function items and ranges from 0 to 100. Posttest 2 is the percentage score each student received on the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the seven calculator neutral items on posttest 2 function items and ranges from 0 to 7
* $p < .05$; ** $p < .01$

Achievement on problem solving test and the use of technology. A multiple regression analysis was conducted to evaluate how well use of technology measures predicted achievement on the problem solving test. Pretest 2 was used to control for prior knowledge because technology was permitted on both pretest 2 and the problem solving test. Before conducting the regression, an analysis of standard residuals was carried out, which showed that the data contained no outliers (Std. Residual Min = -2.80, Std. Residual Max = 2.1). Tests to determine if the data met the assumption of collinearity indicated that multicollinearity was not a concern (pretest 2, Tolerance = 0.92, *VIF* = 1.09; HadCas, Tolerance = 0.93 *VIF* = 1.07; UseofStrategies, Tolerance = 0.96, *VIF* = 1.04). The data met the assumption of independent errors (Durbin-Watson value = 1.44). Finally, the histogram of standardized residuals indicated that the data contained approximately normally distributed errors, as did the normal P-P plot of standardized residuals, which showed points that were not completely on the line, but close. I used both linear regression and HLM to create the prediction models. The results of the HLM model are reported here because there was a difference in the results of the regression and HLM models, and the HLM model is more appropriate given the intra-class correlation. The results of the regression analyses performed using SPSS are found in Appendix J. A regression analysis was performed using HLM with pretest 2 used to control for differences in prior knowledge, and HadCas used as predictors, with achievement on the problem solving test as the criterion variable. Analysis of achievement for the problem solving test with two levels of nesting (*N1* = 267; *N2* = 3; *M* = 61.69; *SD* = 24.55) showed the model was significant overall but only achievement on the pretest was a significant predictor of achievement on the problem solving test (β = 0.44, p < 0.01). Table 31 reports the descriptive statistics, the standardized (β) coefficients, the standard errors and the correlations for each variable.

Table 31

Access to Technology as a Predictor For Achievement on Problem Solving Test Function Items

Variable	Correlations			β	Std Error
	HadCas	Pretest 2	PSU		
Pretest 2		.	.289**	.44**	.09
HadCas		-.246**	-.085	5.74	3.73
				Intercept = 61.86	
Mean	.56	41.88	61.69		
SD	.50	15.28	24.55	R^2=.12	

Note: PSU is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes.
** *p* < .01

Achievement on posttest 1 with OTL measures and the use of technology. A final regression analysis was performed to evaluate how well both OTL measures and access to technology during instruction predicted achievement on posttest 1. I used both linear regression and HLM to create the prediction models. HLM was used for this model because there was a difference in the results from linear regression and the HLM model is more appropriate given the intra-class correlation. The results of the regression analyses performed using SPSS are found in Appendix J. As the assumptions had been previously verified, I used HLM to conduct a regression analysis with pretest 1 used to control for differences in prior knowledge, HadCas, OTL Lesson, and OTL Homework used as predictors, and achievement on posttest 1 as the criterion variable. Analysis of achievement for posttest 1 (*N*1 = 253; *N* 2 = 13; *M* = 56.73; *SD* = 18.22**)** showed the linear model was significant overall but only achievement on pretest 1 and OTL Lessons had significant effect on achievement for posttest 1 ($\beta = 0.71$, $p < 0.001$) meaning, on average, student achievement increased 0.71 percentage points for every additional lesson a teacher taught. Table 32 shows the descriptive statistics, the standardized (β) coefficients, the standard errors and the *t*-ratios for each variable.

Table 32

Use of Technology and OTL Measures as a Set of Predictors for Achievement on Posttest 1 Function Items

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
INTRCPT $\beta 0$	56.70	1.71	33.07	13	<0.001
Pre1FcnScore	0.54	0.05	11.13	253	<0.001
HadCAS	-1.22	2.32	-.527	253	0.599
OTLHW	0.03	0.13	.221	253	0.825
OTL Lessons	0.71	0.19	3.77	253	<0.001

The final model is given by $POST1FCNSCORE_{ij} = \gamma_{00} + \gamma_{10}*PRE1FCNSCORE_{ij} + \gamma_{20}*HADCAS_{ij} + \gamma_{30}*OTLLESSON_{ij} + \gamma_{40}*OTLHW_{ij} + u_{0j} + r_{ij}$

Note: *N* = 271. All variables are grand mean centered. Post1FcnScore is the percentage score each student received on posttest 1 for only the 23 function items and ranges from 0 to 100. Pre1FcnScore is the percentage score each student received on pretest 1 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned only for the function lessons he/she taught and ranges from 0 to 100.

Achievement on posttest 2 with OTL measures and the use of technology. A final regression analysis was performed to evaluate how well both OTL measures and the use of technology measures predicted achievement on posttest 2. I used both linear regression and HLM to create the prediction models. HLM was used for this model because there was a difference in the results from linear regression and the HLM model is more appropriate given the intra-class correlation. The results of the regression analyses performed using SPSS are found in Appendix J. As the assumptions had been previously verified, I used HLM to conduct a regression analysis with pretest 2 used to control for differences in prior knowledge, HadCas, DidUseStrategies, OTL Lesson, and OTL posttest 2 used as predictors, and achievement on posttest 2 as the criterion variable. Analysis of achievement for posttest 2 (N1 = 270; N2 = 14; *M* = 58.39; *SD* = 16.42**)** showed the linear model was significant overall with all variables except OTL posttest 2 having a significant effect on achievement as reported in Table 33. In this model, students who had access to CAS scored, on average, 5.1 percentage points higher ($\beta = 5.12$, $p < .05$) and student achievement went up 1 percentage point every time a student used a strategy on function problems ($\beta = .96$, $p < .05$). OTL Lessons ($\beta = .75$, $p < .01$) also had a positive impact

Table 33

Use of Technology and OTL Measures as a Set of Predictors for Achievement on Posttest 2 Function Items

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
INTRCPT $\beta 0$	58.45	1.55	37.78	13	<0.001
Pre2FcnScore	0.37	0.058	6.42	252	<0.001
DidUseStrategy	0.96	0.480	2.00	252	0.047
HadCAS	5.12	2.47	2.08	252	0.039
OTLPosttest 2	-0.29	0.37	-0.78	252	0.439
OTL Lesson	0.75	0.25	3.02	252	0.003

The final model is given by $POST2FCNSCORE = \beta 0 + \gamma_{10}*PRE2FCNSCORE_{ij} + \gamma_{20}*DIDUSESTRATEGY + \gamma_{30}*HADCAS_{ij} + \gamma_{40}*OTLPOST2_{ij} + \gamma_{50}*OTLLESSON_{ij} + u_{0j} + r_{ij}$

Note: *N* = 270. All variables are grand mean centered. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTL Post2 is the percentage of the 16 function problems on posttest 2 for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the 7 calculator neutral items on posttest 2 function items and ranges from 0 to 7.

on student achievement. Table 33 also reports the descriptive statistics, the standardized (β) coefficients, the standard errors and the *t*-ratios for each variable.

Achievement on PSU with OTL measures and the use of technology. A final regression analysis was performed to evaluate how well both OTL measures and the use of technology measures predicted achievement on the problem solving test. As the assumptions had been previously verified, I used both linear regression and HLM to create a prediction model with pretest 2 used to control for differences in prior knowledge, HadCas, DidUseStrategies, OTL Lesson, and OTL PSU used as predictors, and achievement on PSU as the criterion variable. Because the results were comparable, only the regression results are presented in Table 34. Analysis of achievement on the problem solving test (N1 = 270; N2 = 14; *M* = 61.69; *SD* =

Table 34

Use of Technology and OTL Measures as a Set of Predictors for Achievement on Problem Solving Test Function Items

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
INTRCPT $\beta 0$	61.62	3.02	20.39	13	<0.001
Pre2FcnScore	0.43	0.09	4.61	253	<0.001
HadCAS	7.73	4.14	1.86	253	0.063
OTLPSU	-0.13	0.20	-0.67	253	0.507
OTL Lessons	0.33	0.30	1.07	253	0.286

The final model is given by $ACHIEVEMENT_{ij} = \beta 0 + \gamma_{10}*Pre2FcnScore + \gamma_{30}*HADCAS_{ij} + \gamma_{40}*OTLPSU_{ij} + \gamma_{50}*OTLLESSON_{ij} + u_{0j} + r_{ij}$

Note: *N* = 271. All variables are grand mean centered. Achievement (PSU) is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100.

24.55**)** showed that while the model was significant overall, $F(3, 267) = 22.18$, $p < 0.01$, R^2 =0.09, $R^2_{\text{adjusted}} = 0.08$, none of the OTL or technology measures was a significant predictor of achievement on the PSU. The low value for R^2 indicates less than 10% of the variation in achievement is explained by the model. Table 34 also shows the descriptive statistics, the standardized (β) coefficients, the standard errors and the *t*-ratios for each variable.

Effects of OTL and Technology on Achievement

This section contains the results of the path analyses performed on the achievement data for function items on posttest 2 and the problem solving test. I created different models for posttest 2 and the problem solving test because posttest 2 analyzed technology from the perspective of the student in the form of self-reported data on technology features used to solve each item while the problem solving test analyzed the use of technology from the perspective of a teacher who scored student responses and provided a code for approach used, including the use of technology after the fact. Models for achievement on both posttest 2 and the problem solving test were created using the same OTL measures and technology measures used in the regression analyses.

Path and correlational analysis for use of CAS with OTL. First, the variables were analyzed for normality. The descriptive statistics for the variables used in the path analysis of posttest 2 and the problem solving test are shown in Table 35. None of the variables needed transformation for normality. Next the correlation between the variables was computed and the results for posttest 2 are shown in Table 36. Having access to CAS calculators was significantly correlated ($p < 0.01$) to all other model variables on posttest 2. The correlation between CAS and posttest 2 score, OTL posttest 2 and OTL lessons was negative, meaning the students who had CAS had fewer opportunities to learn the material and, in general, scored lower on posttest 2. However, these results should be interpreted with caution. The results from the HLM models showed the relationship between HadCAS and achievement on posttest 2 was positive. The negative results here are likely due to suppression, meaning HadCAS has direct and indirect effects on posttest 2

Table 35

Mean, Min, Max, Standard Deviation, Skewness and Kurtosis for Variables Used in Achievement Path Analyses

	Min	Max	M	SD	Skewness		Kurtosis	
					Statistic	SE	Statistic	SE
Pre2FcnScore	0.0	81.25	41.89	15.28	-.06	.19	-.37	.30
Post2FcnScore	18.75	93.75	58.39	16.42	.02	.15	-.48	.30
PSUFcnPcnt	0.0	100.00	61.69	24.55	-.34	.15	-.57	.30
HadCAS	0.0	1.0	.56	.50	-.23	.15	-2.0	.30
OTLPost2	66.67	100.0	91.35	6.55	.12	.15	-1.28	.30
OTLPSU	66.67	100.0	90.28	15.18	-.92	.15	-1.16	.30
OTLLessons	67.27	98.18	78.09	10.78	.96	.15	-.45	.30
OTLHW	44.48	73.29	58.53	8.53	-.03	.15	-.81	.30
DidUseStrategy	0.0	7.00	3.70	1.79	-.29	.15	-.54	.30

Note: N = 270. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTL Post2 is the percentage of the 16 function problems on posttest 2 for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the 7 calculator neutral items on posttest 2 function items and ranges from 0 to 7.

Table 36

Correlations Between Variables Used in Path Analysis for Posttest 2

Subscale	Post2Fcn Score	OTL Post2	OTL Lessons	OTL HW	HadCAS	Pre2Fcn Score	DidUse Strategy
Post2FcnScore	—	.24**	.39**	-.29	-.24**	.42**	.15*
OTLPost2		—	.79**	0.00	-.43**	.02	.11
OTLLessons			—	-.53	-.64**	.21**	.14
OTL HW				—	.25**	-.28**	.00
HadCAS					—	-.25**	.032
Pre2FcnScore							-.10
DidUseStrategy							—

Note: N = 270. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes. OTLPost2 is the percentage of the 16 function problems on posttest 2 for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the 7 calculator neutral items on posttest 2 function items and ranges from 0 to 7.
* $p < .05$, ** $p < .01$.

and one of those paths is negative even though the entire relationship is overall a positive one. Negative results could also occur when variables are analyzed separately, but the relationship can change in models in which statistical controls have been applied. The correlation between CAS and homework was low but positive, meaning students with CAS were assigned more function problems than were students who did not have access to CAS. The scores for the pretest were also significantly correlated to all model variables with the exception of OTL on posttest 2. Correlation between pretest scores and posttest 2 scores was positive and moderately strong, indicating students who did well on the pretest generally did well on posttest 2. The pretest scores were also positively correlated, but not as strongly, with OTL Lessons and OTL Homework and negatively correlated with the use of CAS.

The correlations for the variables used in the problem solving test model are displayed in Table 37. On the problem solving test, CAS was not significant compared to achievement.

Table 37

Correlations Between Variables Used in Path Analysis for Problem Solving Test

Subscale	PSUScore	OTLPSU	OTLLessons	OTL HW	Had CAS	Pre2Score
PSUscore_fcn	—	-.08	.10	-.12*	-.09	.29**
OTLPSU		—	.03	.67**	.20**	-.09
OTLLessons			—	-.53**	-.64**	.21**
OTLHW				—	.53**	-.28**
HadCAS					—	-.25**
Pre2FcnScore						—

Note: $N = 271$. PSUScore_fcn is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100.
* $p < .0$, ** $p < .01$.

However, homework was negatively correlated and significant when compared to achievement. Pretest 2 was used to control for prior achievement because it was correlated to achievement on the PSU. Pretest scores were significantly correlated in a positive direction for lesson OTL but negatively correlated to use of CAS and OTL HW. The negative correlations are likely due to suppression or no use of statistical control. The path analysis provides another picture of the direct and indirect effects.

Use of CAS with achievement and OTL posttest 2: Path analysis. The initial path model run to test the theoretical model revealed a significant model chi-square value, $\chi 2$ (8) = 33.60, $p < .001$ and acceptable values for Normed Fit Index (NFI), Non Normed Fit Index (NNFI) and Comparative Fit Index (CFI). The chi-square results indicate the null hypothesis, that the model is a good fit, should be rejected. A low *p*-value associated with the chi-square test suggests the model provides a poor fit to the data and model modifications are necessary, implying that certain paths should be added or some paths might need to be dropped to increase the model fit. The chi-square test can be influenced by factors other than the validity of the theoretical model, such as sample size, departures from multivariate normality, and the complexity of the model. Because external factors can have an adverse effect on the chi-square test, some researchers suggest the ratio of the chi-square value to its degrees of freedom should be less than 2.00 (Hatcher, 1996). For the path model of Figure 14, the ratio of the chi-square value to the degrees of freedom is: 33.60 / 8 = 4.1. Because this ratio is greater than 2.00, it provides further evidence that the model has a poor fit to the data.

Other fit indices have been proposed as alternatives or supplements to the chi-square test. The normed fit index (NFI; Bentler & Bonett, 1980) and the comparative fit index (CFI; Bentler, 1989) can also be used to determine the fitness of a proposed model. Both indices NFI and CFI generally range in size from 0 to 1 (although the NFI may assume values less than 0 or greater than 1 in some rare instances). With both the NFI and CFI, values over 0.90 are said to be indicative of an acceptable fit. RMSEA is also a common fit index. The RMSEA value should be close to zero for a good fit. Even though the other fit indices are acceptable, the chi-square test suggests that the theoretical model inadequately accounts for the input covariance matrix reflecting relationships among the variables in the model. Therefore my next step was to identify modifications that would improve the model's fit.

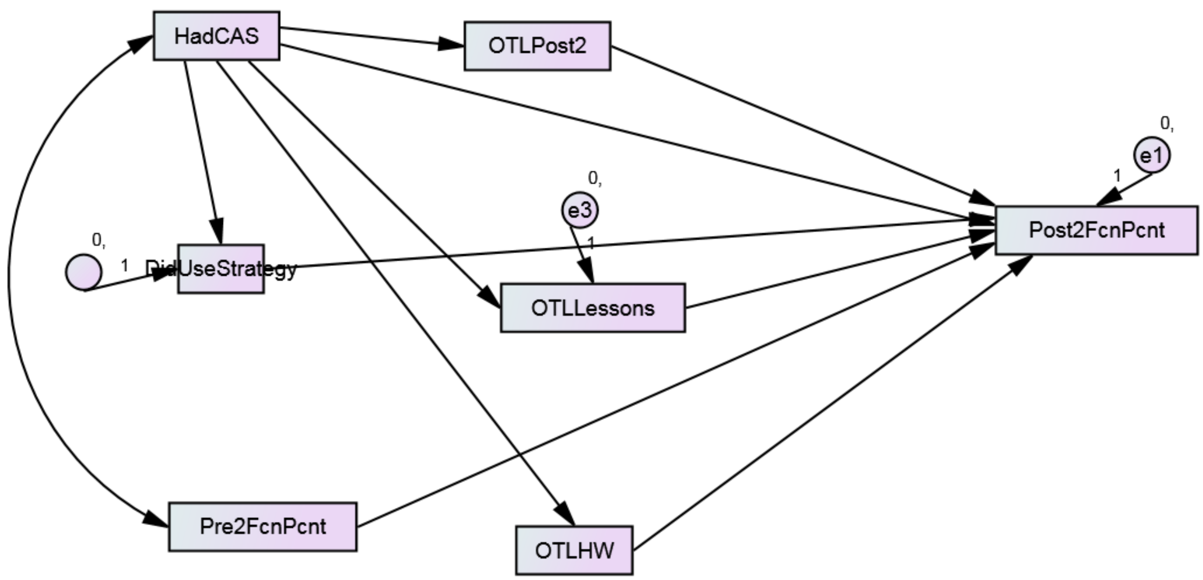


Figure 14. Technology and OTL Measures with posttest 2: Initial Model. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the seven calculator neutral function items on posttest 2 and ranges from 0 to 7. Post2FcnPcnt is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. Pre2FcnPcnt is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100.

As Hatcher (2007) recommended, the path coefficients were reviewed to identify the non-significant paths. The direct path coefficients that were not significant are shown in Appendix J. Next, I examined the normalized residual matrix to determine if any paths contained an absolute value of 2.58 or greater in the intersection of the following paths, which would indicate a specification problem with the theoretical model. None of the residuals were greater than 2.58 (Hatcher, 2007). After reviewing all model modification suggestions from the initial analysis, the non-significant paths were removed one at a time. Table 38 shows the non-significant paths that were removed from the model. After making the above modifications, I ran a new analysis on the revised model. The final model is shown in *Figure 15*. The model and fit paramaters for both the original and final models are reported in Table 39. A chi-squared difference test between the original and the final models (33.60 - 1.92 = 31.68) was significant: $\chi 2$ (6) = 18.55, $p < 0.05$ which indicates the final model is significantly better than the initial model. See Table 40 for the standardized parameter estimates and the R^2 values for the endogenous variables.

Table 38

Non Significant Path Coefficients for Posttest 2 Achievement

		Path	Estimate	*S.E.*	*C.R.*	*p*
HadCAS	→	Post2FcnScore	2.15	2.3	.95	.34
OTLPost2	→	Post2FcnScore	-.19	.21	-.90	.37
HadCAS	→	DidUseStrategy	.12	.22	.53	.60
OTLHW	→	Post2FcnScore	-.04	.07	-.63	.53

Note: N = 270. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTL Post2 is the percentage of the 16 function problems on posttest 2 for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 10. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the seven calculator neutral function items on posttest 2 and ranges from 0 to 7. OTL HW is the percentage of function problems an individual teacher assigned and is based only for the function lessons he/she taught and ranges from 0 to 100.
[a] C.R. is the critical ratio which is obtained by dividing the covariance estimate by its standard error.

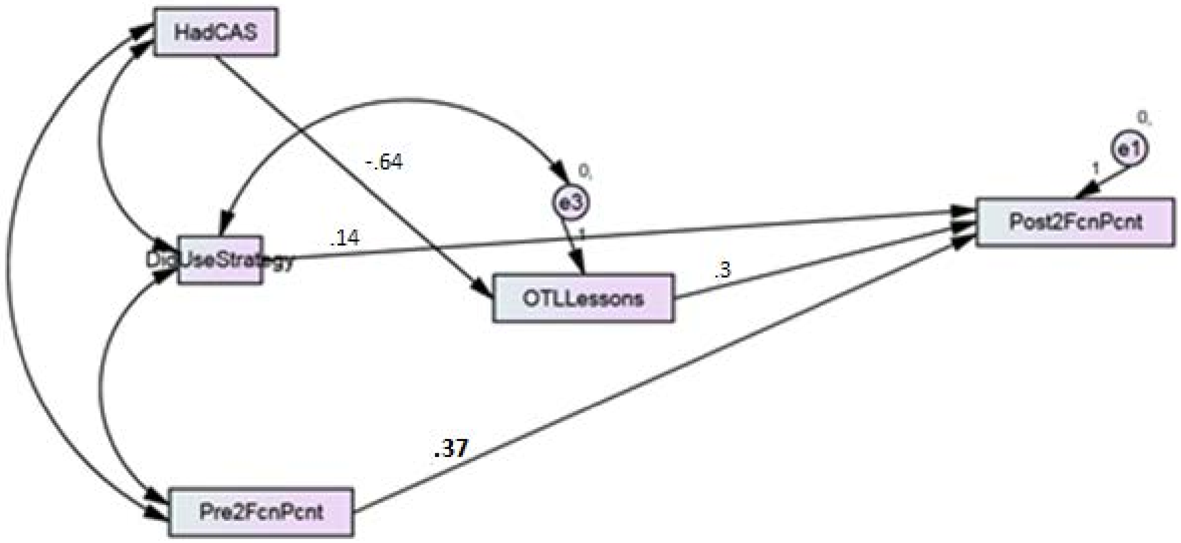


Figure 15. Final Model for OTL Posttest 2. Technology and OTL Measures with posttest 2: Initial Model. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the 7 calculator neutral items on posttest 2 function items and ranges from 0 to 7. Post2FcnPcnt is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. Pre2FcnPcnt is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100.

Table 39

Use of Technology and Opportunity to Learn on Achievement for Posttest 2: Comparison of Initial and Final models

Model	χ^2	*d.f.*	*p*	NFI	CFI	RMSEA
Initial model	33.60	8	0.00	0.96	0.97	0.11
Final model	1.92	2	0.38	0.99	1.00	0.00

Note: *N* = 270. NFI is normed fit index (NFI; Bentler & Bonett, 1980). CFI is comparative fit index (CFI; Bentler, 1989). RMSEA is the root mean square error of approximation.

Table 40

Standardized Regression Weights: Standardized Coefficients and R^2 for Final Model of OTL With Technology on Posttest 2 Achievement

Path		Estimate	R^2
HadCAS	→ OTLLessons	-.64	.41
OTLLessons	→ OTLPost2	.30	.28
DidUseStrategy	→ OTLPost2	.14	
Pre2FcnScore	→ OTLPost2	.37	

Note: *N* = 270. Pre2FcnScore is the percentage score each student received on pretest 2 for only the function items. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes. OTL Post2 is the percentage of the 16 function problems on posttest 2 for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the seven calculator neutral function items on posttest 2 and ranges from 0 to 7.

In the final model, the use of CAS had a moderate negative indirect relationship to achievement, meaning students who did not have CAS scored, on average 0.64 standard deviations higher on posttest 2 function items than students who did have access to CAS. This relationship was mediated through OTL Lessons. There was also a strong positive correlation between OTL Lessons (material covered) and OTL posttest 2, suggesting teachers who reported covering more lessons also reported covering more of the material necessary for students to answer the assessment questions. OTL Lessons had a moderate positive effect on achievement.

The effect sizes for the paths are shown in Table 41. CAS had a strong negative effect on the material covered, meaning teachers reported covering less material in classes where students had CAS. OTL Lessons and achievement on the pretest had a positive but moderate effect on posttest 2 achievement. The use of strategies had a small direct effect on achievement. OTL Lessons accounts for 41% of the variation in the model.

Table 41

Standardized Effects of OTL for Achievement on Posttest 2

	HadCAS		OTLLessons		DidUseStrategy		Pre2FcnScore	
	Direct	Indirect	Direct	Indirect	Direct	Indirect	Direct	Indirect
OTLLessons	-.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Post2FcnScore	.000	-1.91	.30	-.07	.14	-.04	.37	0.00

Note: *N* = 270. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the seven calculator neutral function items on posttest 2 and ranges from 0 to 7.

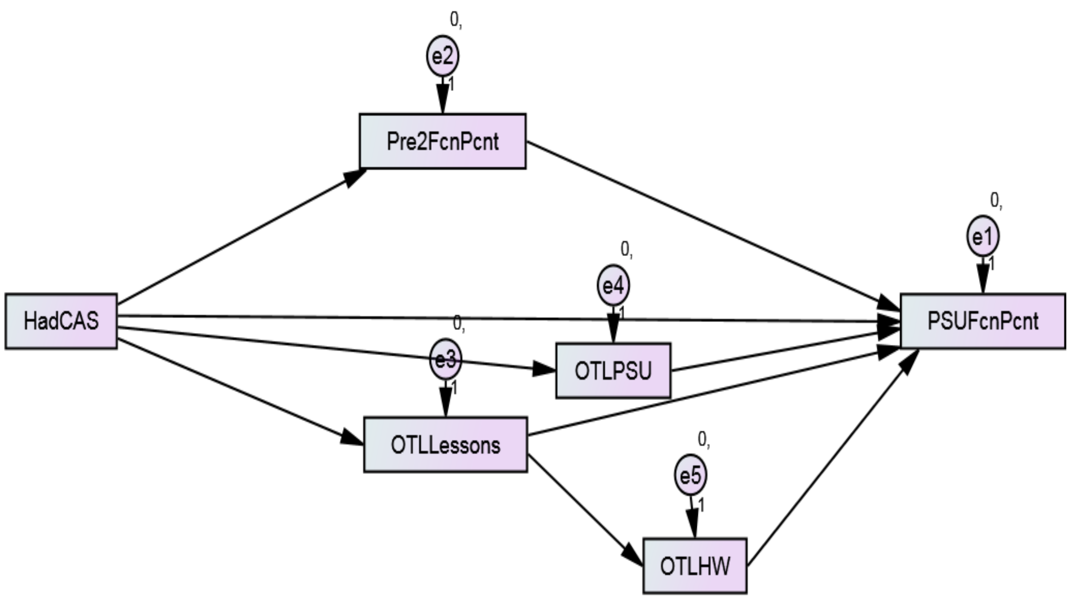


Figure 16. Use of Technology and OTL Measures With Achievement on the Problem Solving Test: Initial Model. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. Pre2FcnPcnt is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100.

Use of technology and OTL measures with achievement on the problem solving test: Path analysis. I used the same procedures to create a similar theoretical model using the achievement data for the problem solving test (See Figure 16). The analysis of the initial path model as shown in Appendix J revealed a significant model chi-square value, $\chi 2$ (6) = 310.21, and non-acceptable values for Normed Fit Index (NFI) and Comparative Fit Index (CFI). The chi-square value and the indices indicated the model was not a good fit. The model summaries are shown in Table 42.

Table 42

Use of Technology and Opportunity to Learn on Achievement for Problem Solving Test: Comparison of Initial and Final Models

Model	χ^2	*d.f.*	*p*	NFI	CFI	RMSEA
Initial model	310.21	6	0.00	.48	.47	.43
Final model	.06	1	.81	1.0	1.0	.00

Note: *N* = 271. NFI is normed fit index (NFI; Bentler & Bonett, 1980). CFI is comparative fit index (CFI; Bentler, 1989). RMSEA is the root mean square error of approximation.

First, as Hatcher (2007) recommended, the path coefficients were reviewed to identify the non-significant paths. The direct path coefficients that were not significant are shown in Table 43 and were removed from the final model. Next I examined the normalized residual matrix to determine if any paths contained an absolute value of 2.58 or greater in the intersection of the following paths, which would indicate a specification problem with the theoretical model. None of the residuals were greater than 2.58 (Hatcher, 2007).

Table 43

Regression Weights of Non-Significant Path Coefficients for Problem Solving Test Achievement

			Estimate	S.E.	C.R.	P
HadCAS	→	OTLPSU	1.53	1.50	1.03	.31
OTLPSU	→	PSUFcnPcnt	-.17	.10	-1.78	.08
OTLLessons	→	PSUFcnPcnt	.22	.19	1.16	.25
OTLHW	→	PSUFcnPcnt	.10	.11	.90	.37
HadCAS	→	PSUFcnPcnt	1.76	3.83	.46	.65

Note: *N* = 271. PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material needed to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTL HW is the percentage of function problems an individual teacher assigned and is based only on the function lessons he/she taught and ranges from 0 to 100.

After reviewing all model modification suggestions from the initial analysis, I removed the non-significant paths one at a time. After making modifications, I ran a new analysis on the revised model. The final model is shown in Figure 17. A chi-squared difference test between the original and the final models (310.21 - 0.06 = 310.15) was significant: $\chi 2$ (5) = 16.75, $p < 0.05$ indicating the final model was significantly better than the original model. The standardized coefficients are shown in Table 44.

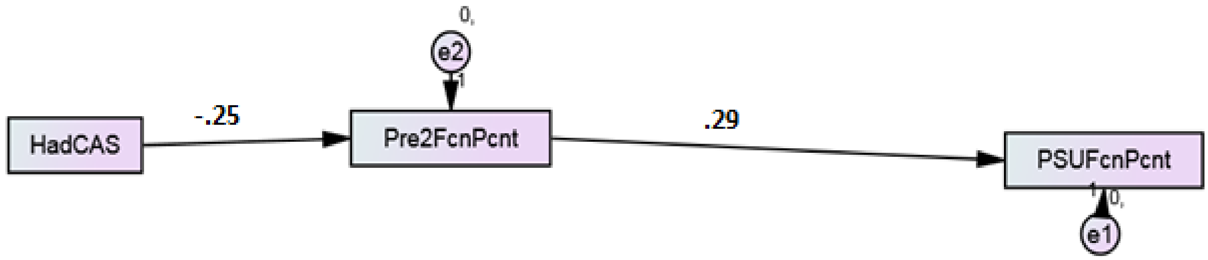


Figure 17. Final Path Analysis Model for Achievement on Problem Solving Test With OTL and Technology Measures. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes. Pre2FcnPcnt is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100.

Table 44

Standardized Regression Weights: Standardized Coefficients and R^2 for Final Model of OTL With Technology on Problem Solving Test

Path			Estimate	R^2
HadCAS	→	PreScoreFcn2	-.25	.06
Pre2FcnScore	→	PSUFcnPcnt	.29	.08

Note: *N* = 271. PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes.

The effect sizes for the problem solving model as reported in Table 45 indicate CAS had no direct effect on achievement but had a strong negative indirect effect on achievement when mitigated by pretest 2, meaning students who had access to CAS scored lower on the pretest , but overall saw a small positive effect on achievement. Pretest 2 scores had the only significant direct effect on achievement for the problem solving test. The low values for R^2, however, suggest the model does not explain much of the achievement on the problem solving test.

Table 45

Standardized Effects OTL and Technology for Problem Solving Test

	HadCAS		Pre2FcnScore	
	Direct	Indirect	Direct	Indirect
PSUScore	0.00	-.071	.29	0.00
Pre2FcnScore	-.25	0.00	0.00	0.00

Note: *N* = 271. PSUScore is the percentage score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes.

Chapter 5: Discussion

In this chapter I discuss the interpretations and implications of the results presented in Chapter 4 as well as suggestions for future research. I organized the discussion into four sections. First, I revisit the research questions from Chapter 1 and discuss the extent to which the results answered these questions and how these results compare with, or are in contrast to, results from other studies. Second, I provide implications for future research. Third, I discuss the implications from the study as they apply to teachers. Finally, I offer concluding remarks.

Precalculus Students' Achievement and the Learning of Function Items

There were four research questions that guided the design of this study. These research questions were:

1. What are students' opportunities to learn about functions in a precalculus course?
2. What calculator strategies do Precalculus students use when solving function problems? In particular, in what ways do students use these strategies when using a graphing calculator to solve function problems from both teachers' and students' perspectives?
3. How is Precalculus students' achievement in solving function problems related to their use of calculator strategies? In particular, what relationship, if any, exists among opportunity to learn, achievement and calculator strategies students use when solving function problems?
4. What effect does the use of technology, including calculator strategies, and opportunity to learn have on achievement when technology usage is reported from the students'

perspective on a multiple choice assessment and from the teachers' perspective on a free response assessment?

I address each research question and discuss the findings in relationship to how they answer the research questions and how they relate to literature.

Students' opportunities to learn function items. There were two sources of data designed to measure students' opportunities to learn functions from the *Precalculus and Discrete Mathematics* textbook: the chapter evaluation/chapter coverage forms, and Teacher Opportunity-to-Learn (OTL) posttest forms.

Opportunity to learn lessons. Many studies report content coverage is the most frequently studied indicator and one of the most prominent indicators of opportunity to learn (Porter, 2002). The textbook provided ample opportunities for students to learn functions. Fifty-five out of the 111 lessons (50%) in the Third Edition of the *Precalculus and Discrete Mathematics* textbook provided students with opportunities to learn and use functions; in the Second Edition 55 out of 116 lessons (47%) provided students with comparable opportunities to learn. Teachers who taught using the Third Edition reported teaching at least 71% of all function lessons and one teacher reported having taught 98% of the included function lessons. The two teachers who taught using the Second Edition materials reported teaching at least 82% of the function lessons.

OTL homework. Although teachers taught a majority of function lessons provided in the textbook, they were less likely to assign all of the function homework problems in the lessons they taught. The curriculum was designed with a small set of targeted homework problems with the intention that teachers would assign most, if not all of the problems. In this field study one teacher using Third Edition materials only assigned 52% of the available problems in the functions lessons he/she taught, while other teachers assigned 94% of the available lessons. The

Second Edition teachers assigned far fewer of the textbook homework problems (57%), on average, than Third Edition teachers (78%).

The relationship between OTL Lessons and OTL Homework was negative or non-significant for all three posttests. These results suggest students had fewer opportunities to learn the content of function through homework. However, there were no data in my study regarding the extent to which students actually completed homework (the percent completed), or the extent to which they were correct when completing homework. Although homework problems from standards-based curriculum are generally viewed as cognitively more difficult than homework problems from other curricula (Senk & Thompson, 2006), there were no data in this study from which to gauge the cognitive level or complexity of the homework sets as recommended by some researchers (Dettmers, Trautwein, Lüdtke, Kunter, & Baumert, 2010).

In the OTL literature, one hypothesis posits learning occurs when the time spent on learning (influenced by time constraints and effort) intersects with the time needed to learn (influenced by academic aptitude, ability to follow instructions, and the quality of instruction) (Carroll, 1963, 1984; Paschal, Weinstein, & Walberg, 1984). This suggests time spent on homework should yield positive learning rewards, but only up to the amount of time needed to learn the material, which is shorter for students with well-developed cognitive abilities (Daw, 2012). This could imply students who started the school year with more understanding of the function concept may have needed less homework to master the material (Daw, 2012).

OTL posttests. Another factor in assessing content OTL is the use of questionnaires to determine if students had an opportunity to learn the material necessary to answer assessment questions. There was less variability in OTL posttest variables than OTL Lessons or OTL Homework variables. All teachers reported 100% coverage of posttest 1 function items; the average coverage of posttest 2 items was similar for teachers using Second Edition materials

(97%) compared to those using Third Edition materials (94%). Only two teachers, one using Second Edition materials and one using Third Edition materials, reported not teaching or reviewing 100% of the material needed to answer the function items on the problem solving test.

OTL posttests should be positively correlated to OTL Lessons, because one would expect students to be successful on content they have had the change to study or review. In this study the variable entitled OTL Lessons was positively correlated to OTL posttest 2 coverage. However, there was no significant correlation for OTL PSU coverage; because OTL posttest 1 was 100%, there was no variance to correlate to achievement for posttest 1. As data from this study suggest, OTL posttest and achievement are not always positively correlated. A teacher might indicate teaching a particular lesson, but may not have covered the material necessary to answer a particular question. For example, a teacher might cover quadratics and the quadratic equation but may not have taught students to solve word problems involving quadratics. Collecting data on both lessons covered and correlating with the items assessed provide a more complete picture of opportunity to learn.

Succinctly stated, results from this study show the OTL variables played an important role in student achievement. The OTL Lessons variable was highly correlated to OTL posttest variables for posttests 1 and 2. Opportunity to learn Homework was not correlated to OTL Lessons or OTL posttests. Finally, OTL posttest coverage was only positively related to achievement for posttest 2.

Students' use of technology on function items. Data from the posttest calculator usage form were used to determine how students used their graphing calculator when solving function items on posttest 2.

Students reported *not* using a calculator, on average, 8.2 times on the 9 calculator inactive function items (See Table 13 in Chapter 4), though some used the calculator occasionally for arithmetic. On the calculator neutral items, for which students could have used a calculator to obtain a correct solution, students reported using a calculator, on average, three times when completing the seven items. These results do not demonstrate a pattern of low or high calculator usage. Instead, these data suggest the use of calculator strategies appears to relate to the assessment items themselves. Furthermore, the data suggest students are discriminating when they should and should not use calculators to solve function problems. This finding might reassure teachers. That is, students do not necessarily become dependent on calculators, which is one of the major objections some teachers have against implementing the NCTM (1989, 2000) recommendation that calculators should be used in all aspects of classroom instruction. These results are promising and might indicate a move in a positive direction away from earlier reports (Hembree & Dessart, 1986) that suggest students who have access to a calculator misuse it, and therefore, lose basic computational skills.

When solving calculator neutral items, the most frequently reported strategy was the use of a graph, which students stated as being used, on average, 2.2 times on the seven items. A review of the neutral function items indicates three of the items (31, 46, and 59) could be efficiently solved using a graph. Results regarding students' use of graphing calculators are encouraging for two reasons. One, it perhaps indicates students choose a strategy that seems most viable in a particular situation. This may indicate students use graphs to solve function items more often than reported in past research (Huntley, Marcus, Kahan, & Miller, 2007; Walen, Williams, & Garner, 2003), and perhaps indicates students no longer focus on the procedural aspects of the function concept (Kollöffel & de Jong, 2005) as they did previously.

The second promising result with regard to the strategies students chose to solve function problems is the indication that they moved more easily between algebraic and graphical representations of a function. Prior research indicates this is a particularly complicated skill for students to master. In four (31, 36, 38, 42) of the seven calculator neutral items, students used both algebraic and graphical representations of a function to solve the item (see Table 15 in Chapter 4). For example, in item 31, more students reported using a graph to solve the item, which was presented in algebraic form, than any other calculator strategy.

Data presented above support many researchers who believe the use of multiple representations help students master the concept of functions (Abdullah, 2010; Artique, 1992; Gagatsis, 2004; Hitt, 1998; NCTM, 2000). Although this study provides no direct evidence regarding students' use of multiple representations, the results may indicate students are making positive progress in the use of multiple representations when solving function items. On five of the seven calculator neutral items (31, 37, 38, 46, 52) students chose to use a different representation of the item than was presented, or made a direct connection between a graph and an algebraic representation (see Table 21 in Chapter 4). These results are encouraging because past studies demonstrate a correlation between translation ability and problem solving ability (Gagastis & Shiakalli, 2004). Data from prior research also show students often have difficulties going from the algebraic representation to a graphical representation (Abdullah, 2010).

The results of the study are not as encouraging with regards to the use of graphing calculators equipped with computer algebra systems (CAS). Three items on posttest 2 (items 31, 38, and 42) might have been solved using a CAS capable calculator, but few students reported using CAS to solve those items. Students who had access to CAS calculators reported using CAS for 1 item, on average. What is promising, though, is that students did not appear to have used CAS indiscriminately and few students indicated they attempted to use CAS on calculator inactive items. One item on posttest 2 (Item 38), however, does provide some encouragement that students are perhaps beginning to use CAS appropriately (see Table 15 in Chapter 4);

students who had access to CAS, used CAS features on that item more than any other calculator strategy.

Precalculus students' achievement on function items. Three sources of data were designed to measure students' achievement on function items from the Second and Third Editions of *Precalculus and Discrete Mathematics*: pretest 1/posttest 1, pretest 2/posttest 2, and the problem solving test. Achievement on posttest 1, on which students were not permitted to use calculators, was higher than on pretest 1, on average, for six of the ten classes using Third Edition materials and for both classes using Second Edition Materials. There were significant differences in posttest 1 achievement scores across curricula, with students who used Second Edition materials scoring, on average, higher than those using Third Edition materials. For posttest 2, on which students were permitted to use graphing calculators, all classes scored higher on the posttest than on the pretest. There were significant differences across classes but none between the different curricula.

These results suggest there is some relationship between achievement and other variables, such as opportunity to learn and use of technology, including calculator strategies. In the following sections I discuss the relationships between student achievement on function items, opportunity to learn, access to technology, and students' use of calculator strategies.

Opportunity to learn and student achievement on function items. Teachers who reported teaching more of the intended curriculum in regards to functions also reported their students achieved higher scores on function assessments, even when controlling for preexisting knowledge. The results indicate OTL had a positive effect on achievement, consistent with results of other studies (Boscardin, 2005; Senk & Thompson, 2006).

OTL was positively correlated and significant when compared to achievement on two of the three posttests. On the problem solving test, however, there was no relationship between lesson coverage and achievement. This result should be interpreted with caution. The problem solving test contained only three function problems out of five problems. The small sample size could affect the association between the variables.

There was either a negative or non-significant correlation between OTL Homework and achievement on function items for two of the posttests (posttest 1 and posttest 2). This suggests the more homework teachers assigned students the lower students scored on the posttests. One possible explanation for this result is supported by research that suggests homework associated with standards-based curricula may be cognitively more challenging for some students than homework in traditional based curricula (Senk & Thompson, 2003). Although there are no data from this study that details the cognitive level of the homework problems in this curriculum, if students found the homework items to be difficult, the negative relationship between achievement and homework would be consistent with the findings from at least one study (Dettmers, Trautwein, Lüdtke, Kunter, & Baumert, 2010). Another explanation might be the method I employed to calculate the OTL Homework variable. Teachers using Second Edition materials were more likely to report using outside sources for homework and also reported being dissatisfied with some of the problems included in the textbook. The teachers using the Third Edition of the textbook reported being more satisfied with the problems in the textbook and were therefore more likely to use the included problems. I calculated the OTL Homework variables using only the homework problems assigned from the textbook. In some cases, as I reported previously, teachers provided outside sources for homework, but there was no way to incorporate these into the calculated OTL variable.

Use of technology, calculator strategies, and student achievement on function items. A recent analysis of data from the National Assessment of Educational Progress (NAEP) found calculator usage was a significant and positive factor in student achievement (Cawthon, Beretvas, Kaye, & Lockhart, 2012). In this study, approximately 85% of students who reported using a graph, obtained the correct solution to the item. This is significantly higher than the percentage of students who reported not using a calculator strategy (48% for students with CAS and 57% for students with graphing calculators) and who obtained the correct solution.

Students' use of strategies and the relationship between calculator strategies and achievement on function items is, perhaps, the most important finding of this study. On posttest 1, access to CAS had no impact on achievement. When students used CAS capable graphing calculators in the classroom, there was no effect on achievement scores when calculators were not available on an assessment. This result is consistent with findings from numerous studies on calculator usage that suggest students do not lose their abilities to do computational mathematics on assessments when taught using calculators (Zbiek & Hollebrands, 2008). However, when students did use calculator features to solve function items on posttest 2, there was a direct correlation to achievement. The HLM model showed students who had access to CAS scored, on average, 5% higher on the posttest when controlling for prior knowledge. Students who used calculator features/strategies to solve function items also scored one percentage point higher each time they used the calculator. The data seem to suggest students are more successful when they use calculator strategies. This may imply students are experiencing fewer difficulties moving from one representation to another and are becoming better problem solvers, which in turn can result in higher student achievement (Elia, Panaoura, Gagatsis, Gravvani, & Spyrou, 2006; Gagatsis & Shiakalli, 2004; Herman, 2007; Hitt, 1998; Kaput, 1987; NCTM, 2000).

Students' use of strategies, access to CAS, and their effects on achievement is a promising finding. Examining the relationship between the use of technology and achievement on posttests alone seemed to indicate the relationships were negative, or non-significant. However, when examining both OTL and technology together, and their effects on achievement for posttest 2, the use of strategies and access to CAS were both positive and significant. On the HLM model the technology variables were more significant than prior knowledge and OTL. This suggests the use of graphing calculators and calculator strategies, within a standards-based curricula has a positive effect on achievement of function items. The findings are consistent with results from other studies that report a positive association between the use of graphing calculators and achievement (Dunham, 2000; Kastberg & Leatham, 2005).

The results of student achievement using CAS features, however, were not as encouraging. Of the three items on posttest 2 on which students could have used CAS to solve the problem (items 31, 38, and 42), data on only one item (item 38) indicated more students with access to CAS reported using CAS to solve this item and were more successful than students using any other strategy.

The results from the self-reported data regarding students using CAS features should, however, be interpreted with some caution for several reasons. First, there is no indication as to the extent teachers taught students how to use CAS features. Student survey data in four classes (412, 413, 416, and 417) indicated only one student had access to CAS capable graphing calculators. Of these four students, only two reported using any CAS features on posttest function items. There are no data to indicate how often students or teachers used CAS in lessons, or homework, but the research clearly shows the mere presence of CAS is not an indication of its use (Pierce & Stacey, 2004). Ertmer (1999) and others (Lim & Khine, 2006) documented barriers for technology integration that include difficulty using technology unless all students have access to the technology. For example, when only one student in a class has a CAS capable calculator, it is unclear how much class instruction or time a teacher would devote to using CAS.

The steep learning curve using CAS for both teachers and students is well documented in the literature (Marshall, Buteau, Jarvis, & Lavicza, 2012; Pierce & Stacey, 2013). The presence of a learning curve might affect instruction in multiple ways. Some teachers may not be comfortable using features they have not mastered themselves and may restrict students' use of CAS until they master it themselves (Ertmer, 1999; Pierce & Stacey, 2013) In other cases, teachers may quit using CAS because it takes too much time to learn (Ertmer, 1999). However, as teachers become more comfortable using CAS themselves, their use of CAS features in the classroom may increase over time. It is reasonable to expect teachers were at differing levels on the technology learning curve and might be more comfortable teaching with CAS in subsequent years.

The effects of OTL and technology on achievement. Data used to examine the relationships between achievement, OTL, and calculator strategies include achievement scores as percent correct on two pretests and three posttests, teachers' reported OTL data, and students' calculator usage data. In the following section, I discuss and interpret the results from the path analyses of the posttests.

For all of the posttests, I began with initial models that included all OTL variables and technology variables. I hypothesized technology variables might have both direct and indirect effects on achievement based on the results of HLM models. I also hypothesized OTL Homework and OTL Posttests might have indirect effects though OTL Lessons. In particular, the amount of homework a teacher assigns as well as the amount of material on an assessment the teacher taught or reviewed might be related to how many lessons the teacher taught. I also controlled for prior knowledge in the models.

There were some major differences between the results of the path analyses and the HLM models. For example, on posttest 2 the overall effects of OTL Lessons and the use of strategies were positive but both had small negative indirect effects. These relationships are easier to see in a path analysis and are not demonstrated in the HLM models. It is possible for two variables to have a negative correlation, such as between HadCas and achievement on posttest 2, but for the beta coefficient to be positive. This can occur because the correlation cannot control for the effect of other variables whereas the HLM models and the path analysis can control for the effect of other variables.

The significant differences between the two models are to be expected. The path analysis is equivalent to doing multiple linear regressions that contain multiple dependent variables, whereas the HLM model has only one dependent variable. Both models, however, show access to graphing calculators and use of calculator strategies are significant factors in predicting achievement.

Limitations

As is often the case in quantitative research, confounding and lurking variables might be responsible for my findings differing from other findings from similar research. The OTL Lessons variable was strongly related to achievement on both posttest 1 and posttest 2. The impact of OTL Lessons may have masked or mitigated the effect of homework on achievement. It is also possible the complexity level of homework problems might have masked other relationships between homework and achievement. Additional studies in which students' opportunity to learn the content is similar might allow more insight into the role OTL Homework plays when compared to student achievement. It would also be beneficial to collect more data on the complexity of homework problems and incorporate this data into OTL Homework variables.

The sample size for the problem solving test is another limitation of this study. There were only three function problems and five total problems. Results of analyses using these data may not be as reliable as the data from the multiple-choice posttests. HLM analysis works best with at least 50 classes (Niehaus, Campbell, & Inkelas, 2014).). There were only 14 classes in this study. Additional research, which includes more classes, might strengthen the results of this study.

Finally, students were not graded on any of the assessments used in this study. They were not high-stake assessments, and therefore, students might not have been motivated to do their best work. For instance,scholars who study motivation and achievement report there is wide variability in achievement when students take no-stakes assessments, assessments in which there are no consequences for student performance. In such cases, student achievement is often under estimated, (Wise & DeMars, 2005) and therefore the results from this study might be higher than reported.

Implications for future research

As in all studies, new questions are discovered or new questions arise that might be addressed in future research. Toward that end in this section I discuss ways to increase the generalizability of the study and ways to tease out the contributions of OTL and the use of technology on student achievement when learning functions.

Increasing generalizability. I addressed many of the concerns researchers have documented in studies on the use of technology and CAS by collecting data about students' use of technology at home, by having technology used over an entire school year, and by having technology embedded within the curriculum. However, there are still opportunities to further increase generalizability. This study utilized data from six schools and 14 classes. I used HLM to analyze the data because students are nested in classes. Future studies might benefit from the inclusion of additional classes in the HLM analysis. Research that documents teachers' level of experience and includes teachers with varying degrees of experience in teaching with CAS might increase the variability of the technology data, and therefore, might increase the generalizability.

Most curriculum studies are conducted over a short period of time, sometimes weeks (Senk & Thompson, 2003). This study was conducted over an entire academic year. However, studies on the use of technology indicate teachers need several years to incorporate calculators into the curriculum (Ertmer, 1999, Hew & Brush, 2007). Additional research that examines how teachers incorporate technology in the same course they have been teaching for several years would add some valuable insights into the variables that affect the relationship between students' use of graphing calculators and their achievement.

Furthermore, overall generalizability might be increased by the inclusion of qualitative data to provide insights beyond the quantitative data. Students self-reported if they used a calculator and, if so, what strategy they used to solve a problem. Interviews with students to gain insight into why they chose a strategy, what their thinking process was in using the strategy, and how their use of a strategy connects to their understanding in multiple representations would be valuable. There have been some small studies that have included qualitative data in students' use of problem solving of functions and the use of multiple representations (Herman, 2007; Huntley & Davis, 2008; Huntley, Marcus, Kahan, & Miller, 2007), but conducting a larger study that included qualitative data to support the use of technology would be helpful to extending the extant literature.

Greater insight into OTL and the use of technology in achievement. Although studies have long stressed the importance of content coverage on OTL and its impact on achievement, until now there has been no consistency across classrooms with regards to OTL of content coverage because there has been no consistency on what content is covered in different classrooms. Nationwide standards related to college and career readiness might help establish a minimum level of content coverage, and therefore, provide a baseline for future studies on OTL.

There have been few studies investigating the impact of homework on OTL, especially at the high school level, and the existing literature shows mixed results on the influence of homework and achievement (Daw, 2012 Dettmers, Trautwein, Lüdtke, Kunter, & Baumert, 2010;). Therefore, caution must be exercised when interpreting OTL Homework results. More research is necessary to draw conclusions. A study in which teachers covered identical lessons but varied the amounts and cognitive levels of homework problems assigned, accounting for student variables, might be useful in shedding more light on the relationships between homework, OTL, and achievement.

In this study, the OTL Lessons variable was calculated using the percentage of lessons a teacher reported teaching during the year. Data were collected on how many days teachers spent on each lesson, but there was no indication of how much emphasis teachers placed on an individual lesson. Additional analysis of the data might include a weighted OTL Lessons variable, which includes a difficulty factor to account for the amount of emphasis placed on individual lessons. Using a weighted OTL variable might provide more insight into the relationship between OTL Lessons, which was a significant variable in this study, and its effect on student achievement of functions.

The data collected for OTL Homework consisted of the problems teachers assigned for students to complete from the textbook. There was limited information regarding the use of outside sources of homework problems or completion rates. OTL Homework might be *weighted* to show how much homework was assigned from outside sources or sub variables might be created to demonstrate student completion rates and/or accuracy rates. OTL Homework variables might be weighted to account for complexity, which some researchers think is lacking in many current studies (Dettmers, Trautwein, Lüdtke, Kunter, & Baumert, 2010). Including additional data on the role homework plays might provide more insights into the currently sparse body of research on homework and its relationship to achievement and OTL.

The path analyses and the HLM models indicate use of calculators and calculator strategies have a significant impact on achievement. However, the relationship between the technology variables is not as clear. There is some comingling of the effects, which are demonstrated by the path analysis. When examining achievement with both technology and OTL variables, the effects are significant and positive for both access to CAS and the use of strategies. Yet, when examining achievement and technology without including OTL variables the results are either negative, or non-significant. This suggests more research is needed to understand the interconnected relationship between achievement, OTL variables, and technology variables such as access and use of calculator strategies.

Implications for teachers, mathematics teacher educators, and mathematics coaches

In the previous section I addressed the implications of this research as it pertains to the existing literature. In this section I discuss the implications of this research from the perspective of teachers, mathematics teacher educators, and mathematics coaches.

Although acceptance is increasing regarding teachers' use of graphing calculators in their mathematics lessons, many teachers still do not use calculators and, if they do, many do not use them to their full potential as suggested by the data from this study. In the teaching of functions, use of technology, when integrated with a curriculum that is consistent with college and career ready standards and inquiry based teaching, might offer teachers the ability to provide an enriched learning environment. In such an environment, students would have opportunities to synthesize and make connections between and among the different representations of functions. The positive outcomes from this study regarding the use of technology and achievement suggest teachers do not necessarily have to be concerned that implementing technology might result in short term decreases in students' mathematics achievement. Although it is expected the relationship between use of calculator strategies and achievement is not linear and will plateau at some point, it is reasonable to assume achievement might increase as teachers become more comfortable using and teaching with CAS.

If the use of strategies is positive with achievement, then it follows teaching different strategies in the form of different representations might have a positive impact on achievement. In stands to reason, teachers who are comfortable using different representations to teach functions are more effective when teaching functions. Thus, teacher educators and mathematics coaches might consider providing teachers and preservice teachers opportunities to integrate calculators and multiple representations into the teaching and learning of functions.

Many consider the textbook to be the most influential part of the curricula (Begle, 1973) and the most important factor in students' opportunities to learn (Porter, 2002). Teachers who have the responsibility to choose or make recommendations about curricula materials should consider materials that are congruent with college and career ready standards. In addition, teachers should consider if the materials are inquiry based, and have thoroughly integrated the use of technology, including graphing based or CAS calculators, throughout the curricula. Teachers who do not have a voice in the selection of curricula materials may consider supplementing their materials with readymade technology lessons that meet the above criteria. There are currently several websites, including www.education.ti.com and www.casioeducation.com, that contain lessons teachers can use.

These findings might be useful for teacher educators, mathematics coaches and policy makers who are considering adopting the use of CAS capable calculators on state and national level assessments. Use of CAS suggests students can solve more complicated problems than they would otherwise have been able to do by hand, and enables them to generate and manipulate symbolic expressions and to translate between and among different representtions that were otherwise too time-consuming and complicated (Heid &Edwards, 2001; Heid, 2002).

Conclusion

The concept of function is a common thread that links upper level mathematics courses (Dubinsky & Harel, 1992). This concept is essential to most high school curricula. In fact, the NCTM (2000) states mathematical instructional programs should "enable all students to understand patterns, relations, and functions" (p. 296). Results from this study suggest there is a positive association among the use of calculator strategies, opportunity to learn, and achievement when solving function items. These results are important to educators, administrators, curriculum developers, textbook writers, students, parents, and anyone else involved with the teaching and learning of functions. If technology and opportunity to learn can both play significant roles in student achievement when solving function items, it makes sense that more curricula materials need to be developed that integrate the important aspects of the NCTM's recommendations, including emphasizing conceptual understanding, problem solving, reasoning, use of multiple representations; integrating the use of technology, real-world applications; and deemphasizing memorization of rules and procedures (Marzano & Kendall, 1996; McLaughlin & Shepard, 1995; NCTM, 2000; Senk & Thompson, 2003).

The ability to use the graphing calculator to enhance learning while learning the concept of function is what Gagatsis and Shiakalli (2004) refer to as students' translation ability. They reported students showed a higher success in problem solving when they were able to translate among and between verbal and algebraic representations of a function. Because a function can be represented in multiple ways, if students can use calculator strategies to translate between the verbal, graphical, and algebraic representations of a function, their problem solving skills will most likely improve, and in turn so will their achievement on function assessments. In summary all of these recommendations need to be included in preservice teacher programs and continue through graduate studies. It is only when teachers are familiar and comfortable solving function problems using different representations that their high school students, in turn, have the potential to become more competent and accomplished problem solvers.

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Appendices

Appendix A

PDM Pretest One

UCSMP

The University of Chicago School Mathematics Project

Test Number ________

Precalculus and Discrete Mathematics Pretest One

Do not open this booklet until you are told to do so.

This test contains 35 questions. You have 40 minutes to take the test.

1. All the questions are multiple-choice. Some questions have four choices and some have five. There is only one correct answer to each question.

2. Using the portion of the answer sheet marked **TEST 2**, fill in the circle • corresponding to your answer for questions 1-35.

3. If you want to change an answer, completely erase the first answer on your answer sheet.

4. If you do not know the answer, you may guess.

5. Use the scrap paper provided to do any work. **DO NOT MAKE ANY STRAY MARKS IN THE TEST BOOKLET OR ON THE ANSWER SHEET.**

6. You may **NOT** use a calculator on this test.

DO NOT TURN THE PAGE until your teacher says that you may begin.

the conditions in the release of these items. Other items have been used on previous studies conducted by UCSMP. Reprinted with permission.

1. Which point is on the graph of $y = 5^x$?

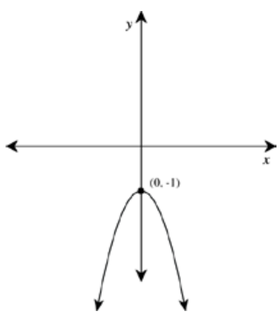
A. (0, 0)
B. (0, 1)
C. (2, 10)
D. (5, 25)
E. (0.5, 2.5)

2. Which is equivalent to $\frac{1}{\sqrt{3}}$?

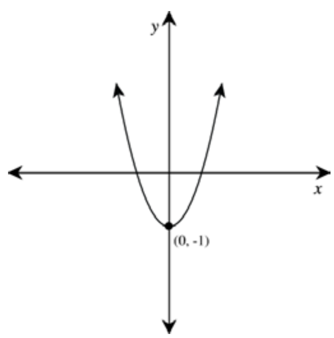
F. $\frac{1}{9}$ G. $\frac{\sqrt{3}}{3}$ H. $\sqrt{3}$ J. $\frac{\sqrt{3}}{9}$

3. Which is the graph of the set of all points satisfying $y = x^2 - 1$?

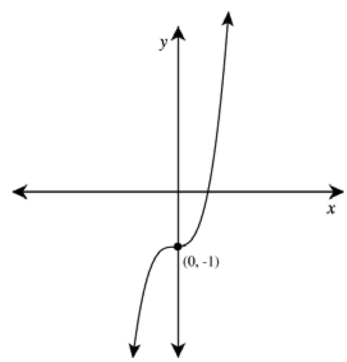
A.



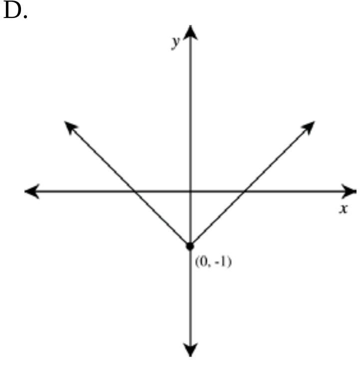
B.



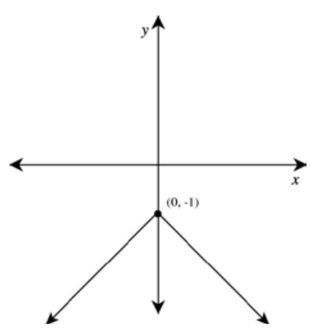
C.



D.



E.



4. Suppose x is between 0 and 2π. For what values of x is $\sin x$ positive?

F. $0 < x < \pi$

G. $\frac{\pi}{2} < x < \frac{3\pi}{2}$

H. $\pi < x < 2\pi$

J. $0 < x < \frac{\pi}{2}, \frac{3\pi}{2} < x < 2\pi$

K. $\frac{\pi}{2} < x < \pi, \frac{3\pi}{2} < x < 2\pi$

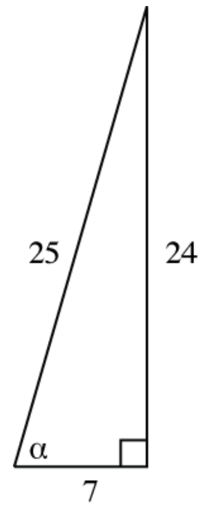
5. Consider the three arguments below:

I.	Given:	*If John reads the comics, then John reads Peanuts.*
		John reads the comics.
	Conclusion:	*John reads Peanuts.*
II.	Given:	*If Rudolph has a red nose, then he guides the sled.*
		If Rudolph guides the sled, then the night is stormy.
	Conclusion:	*If Rudolph has a red nose, then the night is stormy.*
III.	Given:	*If Jennifer wears the blue dress, then she is going to a party.*
		Jennifer is going to a party.
	Conclusion:	*Jennifer wears the blue dress.*

Which of these arguments has a valid conclusion?

A. I and II only
B. I and III only
C. II and III only
D. None has a valid conclusion.
E. All have valid conclusions.

6. $\sin \alpha =$



F. $\frac{7}{25}$ G. $\frac{7}{24}$ H. $\frac{24}{25}$ J. $\frac{25}{24}$ K. $\frac{24}{7}$

7. If you invest $100 for 8 years at 7% annual yield, then how many dollars will you have at the end of this time?

A. 100(1.56)
B. 100(8.56)
C. $100(0.07)^8$
D. $100(1.08)^7$
E. $100(1.07)^8$

8. What are the solutions to $5x^2 - 11x - 3 = 0$?

F. $x = \dfrac{11 \pm \sqrt{181}}{5}$

G. $x = \dfrac{-11 \pm \sqrt{181}}{10}$

H. $x = \dfrac{11 \pm \sqrt{181}}{10}$

J. $x = \dfrac{11 \pm \sqrt{61}}{10}$

K. $x = \dfrac{-11 \pm \sqrt{61}}{10}$

9. The equivalent resistance, R, of two resistors, R_1 and R_2, connected in parallel, is given by the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Which of the following represents the value of R?

A. $\dfrac{2}{R_1 + R_2}$

B. $\dfrac{R_1 R_2}{R_1 + R_2}$

C. $\dfrac{R_1 + R_2}{R_1 R_2}$

D. $\dfrac{R_2 + 1}{R_1 R_2}$

E. $\dfrac{R_1 + R_2}{R_1^2}$

10. When $a \neq 0$, in simplified form $\left(a + \dfrac{1}{a}\right)^3 =$

F. 1

G. $a^3 + \dfrac{1}{a^3}$

H. $a^3 + 3a + \dfrac{3}{a} + \dfrac{1}{a^3}$

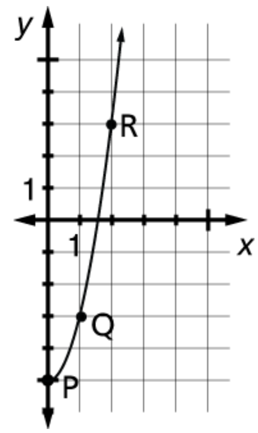
J. $a^3 + 2 + \dfrac{1}{a^3}$

K. $a^3 + \dfrac{3}{a^3}$

11. How are the solutions to $(x + 7)^2 = 65$ related to the solutions to $x^2 = 65$.

A. They are 7 larger.
B. They are 7 smaller.
C. They are $\sqrt{7}$ larger.
D. They are $\sqrt{7}$ smaller.

12. Refer to the graph at right. The average rate of change from *P* to *R* is



F. -4

G. $\frac{1}{4}$

H. 4

J. 8

K. impossible to determine

13. Given *x* = 3*t* and *y* = *t* + 4. Find *x* when *y* = 8.

A. 4
B. 12
C. 24
D. 28
E. 36

14. In the interval $2\pi \le x \le 4\pi$, the solutions to $\cos x = \frac{1}{2}$ are

F. $\frac{7\pi}{3}$ and $\frac{8\pi}{3}$.

G. $\frac{7\pi}{3}$ and $\frac{11\pi}{3}$.

H. $\frac{9\pi}{4}$ and $\frac{15\pi}{4}$.

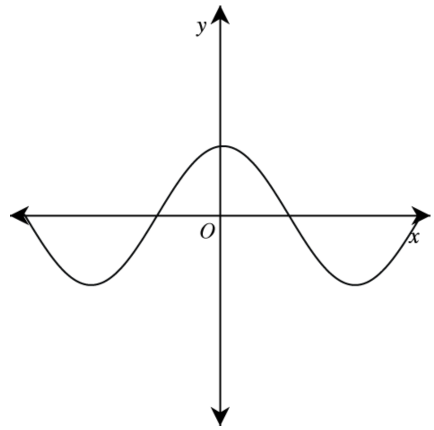
J. $\frac{13\pi}{6}$ and $\frac{23\pi}{6}$.

K. $\frac{13\pi}{6}$ and $\frac{17\pi}{6}$.

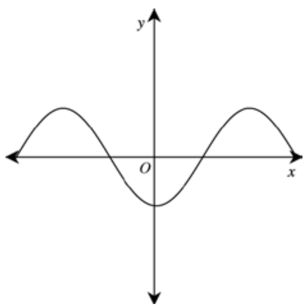
15. Suppose the following statement is true: *If Polly is a wog, then Polly is a twiddle*. Which other statement must also be true?

A. *If Polly is a twiddle, then Polly is a wog.*
B. *If Polly is not a twiddle, then Polly is a wog.*
C. *If Polly is not a wog, then Polly is a twiddle.*
D. *If Polly is not a wog, then Polly is not a twiddle.*
E. *If Polly is not a twiddle, then Polly is not a wog.*

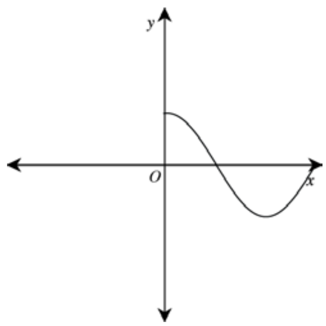
16. The figure at right shows the graph of $y = f(x)$. Which of the following could be graph of $y = |f(x)|$?



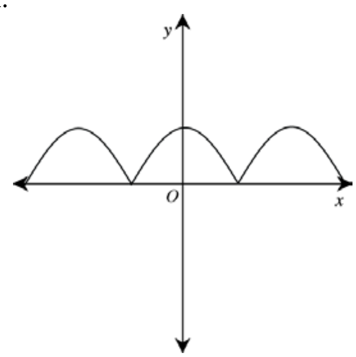
F.



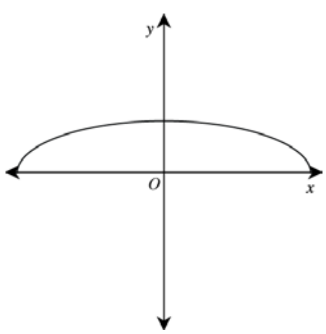
G.



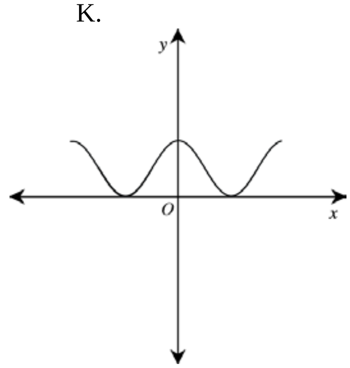
H.



J.



K.



17. In the 20th century, the world record t (in seconds) for the men's mile run in the year y can be estimated by the equation

$$t = 914.2 - 0.346y.$$

According to this estimate, how did the record change over time?

A. It decreased by about $\frac{1}{3}$ second per year.

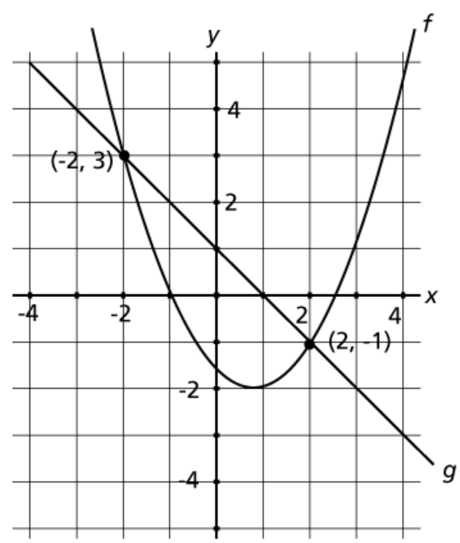
B. It decreased by about $\frac{1}{4}$ second per year.

C. It increased by about $\frac{1}{3}$ second per year.

D. It increased by about $\frac{1}{4}$ second per year.

E. It neither increased nor decreased in any regular fashion.

18. The graphs of two functions f and g are shown at the right. For what values of x is $g(x) > f(x)$?



F. $-2 < x < 2$
G. $x < -2$ or $x > 2$
H. $-1 < x < 3$
J. $x < -1$ or $x > 3$
K. $x < 2$

19. Given a sequence defined as follows:

$a_1 = 17$
$a_n = 2a_{n-1} + 3$, for $n > 1$

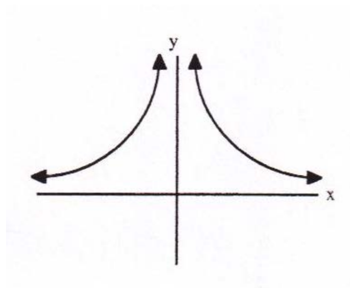
What is a_4?

A. 26
B. 37
C. 157
D. 317
E. 909

20. The functions *f* and *g* are defined by $f(x) = x^2 – 1$ and $g(x) = x + 4$. Then $g(f(x)$ is equal to

F. $(x^2 – 1)(x + 4)$
G. $(x + 4)^2 – 1$
H. $x^2 + 3$
J. $x^2 + 15$
K. $x^2 + x + 3$

21. Given that *k* is a constant and $k > 0$, which of these equations is graphed at the right?



A. $y = kx$

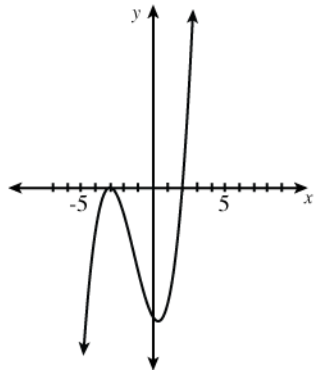
B. $y = kx^2$

C. $y = \frac{k}{x}$

D. $y = \frac{k}{x^2}$

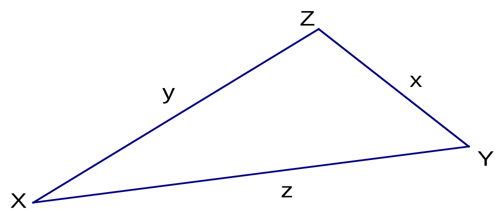
E. $y = k + x$

22. Which of the following equations best describes the graph at right?



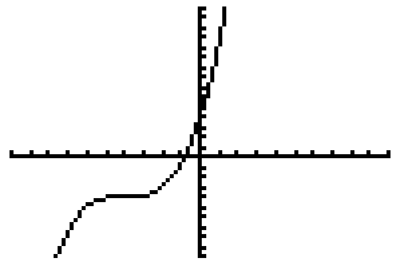
F. $f(x) = (x + 3)(x - 2)$
G. $f(x) = (x - 3)(x + 2)$
H. $f(x) = (x + 3)^2(x - 2)$
J. $f(x) = (x - 3)^2(x + 2)$

23. According to the Law of Cosines, in ΔXYZ at right



A. $x^2 = y^2 + z^2 - yz \cos X$.
B. $x^2 = y^2 + z^2 - 2yz \cos X$.
C. $x^2 = y^2 + z^2 + yz \cos X$.
D. $x^2 = y^2 + z^2 + 2yz \cos X$.
E. none of these

24. At right is the graph of a function *f* on a window $-10 \le x \le 10$ and $-10 \le y \le 15$, with tick marks by 1. Which of the following is an estimate for the zero of the function *f*?



F. -3.6
G. -0.9
H. 0
J. 4
K. cannot be estimated from the graph

25. In every arithmetic sequence, $a_n = a_1 + (n - 1)d$. Use this formula to find a_{40} for the arithmetic sequence 75, 71, 67, 63, ….

A. -85
B. -81
C. -77
D. 35
E. 1.875

26. Which of the following describes how to obtain the graph of $y = (x + 5)^3 - 4$ from the graph of $y = x^3$?

F. Translate the graph of $y = x^3$ by5 units to the right and 4 units down.
G. Translate the graph of $y = x^3$ by 5 units to the left and 4 units down.
H. Translate the graph of $y = x^3$ by 5 units to the left and 4 units up.
J. Translate the graph of $y = x^3$ by 4 units to the left and 5 units down.
K. Translate the graph of $y = x^3$ by 4 units to the right and 5 units up.

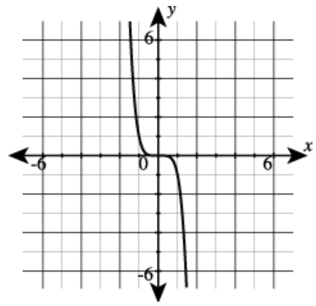
27. $\frac{2\pi}{3}$ radians is equivalent to

A. 30°
B. 60°
C. 120°
D. 150°
E. 210°

28. $p \Rightarrow q$ is false

F. when *p* is false regardless of the truth value of *q*.
G. when *q* is false regardless of the truth value of *p*.
H. only when *p* is false and *q* is false.
J. only when *p* is true and *q* is false.
K. only when *p* is false and *q* is true.

29. Which of the following could be an equation for the function graphed at the right?



A. $f(x) = x^3$
B. $f(x) = -x^4$
C. $f(x) = -x^5$
D. $f(x) = x^6$

30. Factor $25m^3 - 4m$ completely.

F. $m(25m^2 - 4)$

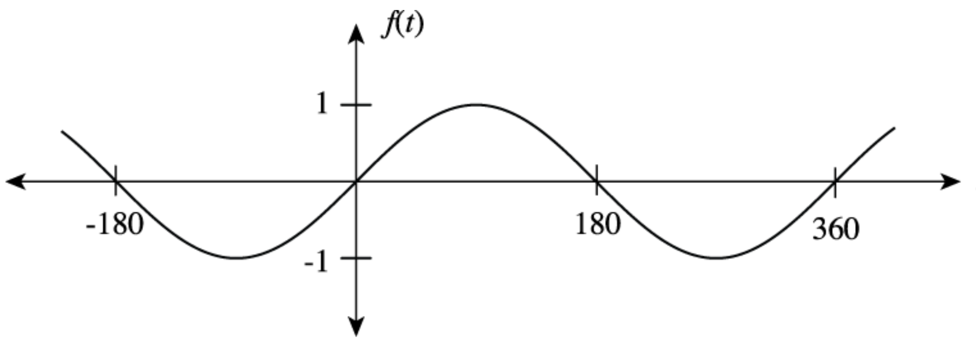
G. $m(5m - 2)^2$

H. $(5m^2 - 2m)(5m + 2)$

J. $m(5m - 4)(5m + 4)$

K. $m(5m - 2)(5m + 2)$

In 31 and 32, use the graph of the periodic function *f* at the right.



31. What is the range of *f*?

A. the set of all real numbers
B. $\{y: -1 \leq y \leq 1\}$
C. $\{x: -180 \leq x \leq 360\}$
D. $\{y: y \geq 0\}$
E. $\{x: x \geq 0\}$

32. Which is a possible equation for f?

F. $f(t) = \sin t$
G. $f(t) = \cos t$
H. $f(t) = \tan t$
J. $f(t) = \log t$
K. $f(t) = e^t$

33. Consider the function h defined by $h(x) = \dfrac{5x}{(x+3)(x-2)}$. As x gets closer and closer to 2 but remains larger than 2, the value of the function

A. gets close to 0.
B. gets close to 2.
C. gets close to 10.
D. gets smaller and smaller without bound.
E. gets larger and larger without bound.

34. From a group of 15 people, 3 are to be selected to serve on a committee. How many different committees are possible?

F. 45

G. 15^3

H. 3^{15}

J. $15 \bullet 14 \bullet 13$

K. $\dfrac{15 \bullet 14 \bullet 13}{3 \bullet 2 \bullet 1}$

35. Which of the following is equivalent to $1.2^{2.5} \approx 1.58$?

A. $\log_{1.2} 1.58 \approx 2.5$
B. $\log_{2.5} 1.58 \approx 1.2$
C. $\log_{1.58} 2.5 \approx 1.2$
D. $\log_{2.5} 1.2 \approx 1.58$
E. $\log_{1.2} 2.5 \approx 1.58$

Appendix B

PDM Pretest Two

UCSMP

The University of Chicago School Mathematics Project

Test Number ________

Precalculus and Discrete Mathematics Pretest Two

<u>Do not open this booklet until you are told to do so.</u>

This test contains 25 questions. You have 40 minutes to take the test.

1. All the questions are multiple-choice. Some questions have four choices and some have five. There is only one correct answer to each question.

2. Using the portion of the answer sheet marked **TEST 2**, fill in the circle • corresponding to your answer for questions 36-60.

3. If you want to change an answer, completely erase the first answer on your answer sheet.

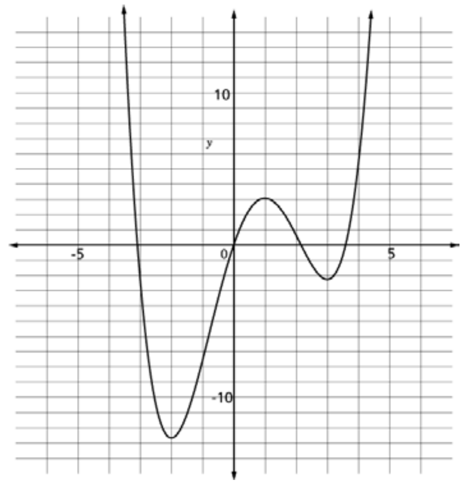
4. If you do not know the answer, you may guess.

5. Use the scrap paper provided to do any work. **DO NOT MAKE ANY STRAY MARKS IN THE TEST BOOKLET OR ON THE ANSWER SHEET.**

6. You **MAY** use a calculator on this test, including a graphing calculator with or without computer algebra systems.

DO NOT TURN THE PAGE until your teacher says that you may begin.

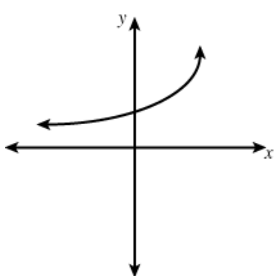
36. Refer to the graph of function *f* at right. On which of the following intervals is *f* increasing?



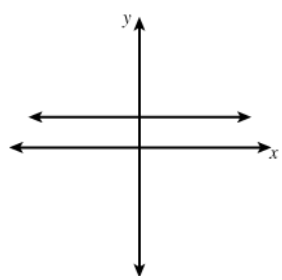
F. $x \le -2$
G. $x \ge 0$
H. $x \le -3$
J. $-2 \le x \le 0$
K. $1 \le x \le 3$

37. Which of the following is a graph of a function that has an inverse that is also a function?

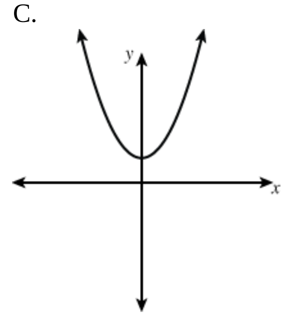
A.



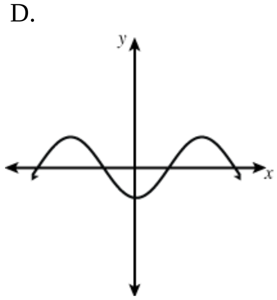
B.



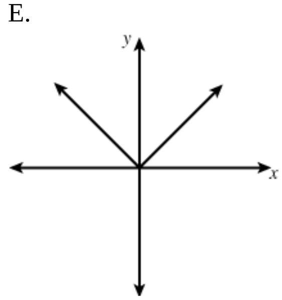
C.



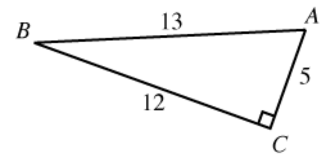
D.



E.



38. In ΔABC at right, find $m\angle B$ to the nearest degree.



F. 21°
G. 23°
H. 25°
J. 30°
K. 38°

39. When $2x^3 + 3x^2 - 32x + 27$ is divided by $x + 5$, the result is

A. quotient: $2x^2 + 7x + 3$, remainder: 12
B. quotient: $2x^2 - 7x + 3$, remainder: 12
C. quotient: $2x^2 + 13x + 33$, remainder: 192
D. quotient: $2x^2 - 7x - 67$, remainder: -308
E. none of these

40. Five persons whose names begin with different letters are placed in a row, side by side. What is the probability that they will be placed in alphabetical order from left to right?

F. $\frac{1}{720}$

G. $\frac{1}{625}$

H. $\frac{1}{120}$

J. $\frac{1}{15}$

K. $\frac{1}{5}$

41. Which of the following is closest to the value of $\log_3 7$?

A. 0.40
B. 0.56
C. 1.77
D. 343
E. The value cannot be determined.

42. If $2x^{3/4} = 5$, then

F. $\frac{1}{2}x^{4/3} = \frac{1}{5}$

G. $2x^{-4/3} = \frac{1}{5}$

H. $\frac{1}{2}x^{-3/4} = \frac{1}{5}$

J. $2x^{4/3} = \frac{1}{5}$

K. $-2x^{-4/3} = \frac{1}{5}$

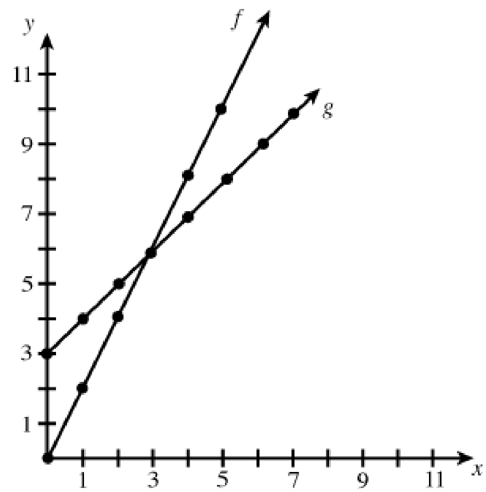
43. Suppose $f(x) = x^{1/2}$. What is the set of all values of x for which $f(x)$ is a real number?

A. $\{x: x > 0\}$
B. $\{x: x \geq 0\}$
C. $\{x: x > 1\}$
D. $\{x: x \geq 1\}$
E. the set of all real numbers

44. A woman is standing on a cliff 200 feet above the water. Through a set of high-powered binoculars, she sees a boat on the water off in the distance. If the angle of depression is 10°, about how far is the boat from the base of the cliff?

F. 35 feet
G. 203 feet
H. 308 feet
J. 1134 feet
K. 1151 feet

In questions 45 and 46, refer to the graphs of functions *f* and *g* at right.



45. What is the value of $g(1)$?

A. 2
B. 4
C. 5
D. 6
E. 8

46. What is the value of $f(g(1))$?

F. 2
G. 4
H. 5
J. 6
K. 8

47. The distance between (-1, 2) and (4, 5) in the plane is

A. 6
B. 8
C. $9\sqrt{2}$
D. $\sqrt{34}$
E. $\sqrt{58}$

48. Suppose $y = f(x)$. If S maps each point (x, y) in the plane to $(2x, 5y)$, what is an equation for the image of the graph of $y = f(x)$?

F. $\frac{y}{5} = f(\frac{x}{2})$

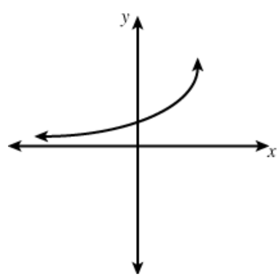
G. $5y = f(\frac{x}{2})$

H. $y = 2f(5x)$

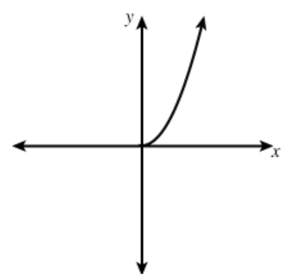
J. $y = 5f(2x)$

49. Which of the following could be the graph of $y = \log_2 x$ for $x > 0$?

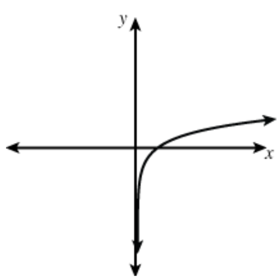
A.



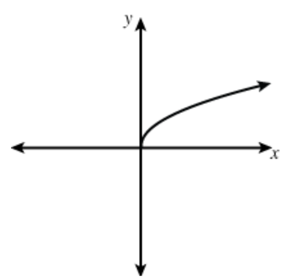
B.



C.



D.



50. $\lim_{x\to 10}\dfrac{x^2-100}{2x^2-23x+30}=$

F. 0

G. $\pm\sqrt{110}$

H. $\dfrac{3}{2}$

J. $\dfrac{20}{17}$

K. does not exist

51. Which of the following equals $(2m+1)^3$?

A. $8m^3 + 1$

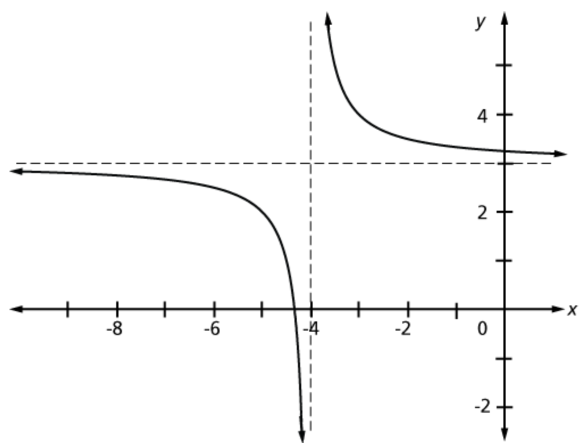
B. $8m^3 + 3m^2 + 6m + 1$

C. $8m^3 + 4m^2 + 2m + 1$

D. $8m^3 + 6m^2 + 6m + 1$

E. $8m^3 + 12m^2 + 6m + 1$

52. A function h is graphed at right. As $x \to +\infty$,



F. $h(x) \to 0$
G. $h(x) \to 3$
H. $h(x) \to +\infty$
J. $h(x) \to -\infty$
K. the values of $h(x)$ do not exist.

53. Which of the following is the negation of the statement *Some animals are horses*?

A. *Some animals are not horses.*

B. *All animals are horses.*

C. *No animals are horses.*

D. *If it is an animal, then it is a horse.*

E. *It is an animal and it is not a horse.*

54. Estimate the measure of the angle between the vectors $\vec{u}$ = (3, 2) and $\vec{v}$ = (-2, 5).

F. 164° G. 136° H. 78° J. 12°

55. Which of the following is the derivative of f at x?

A. $f(x + h) - f(x)$

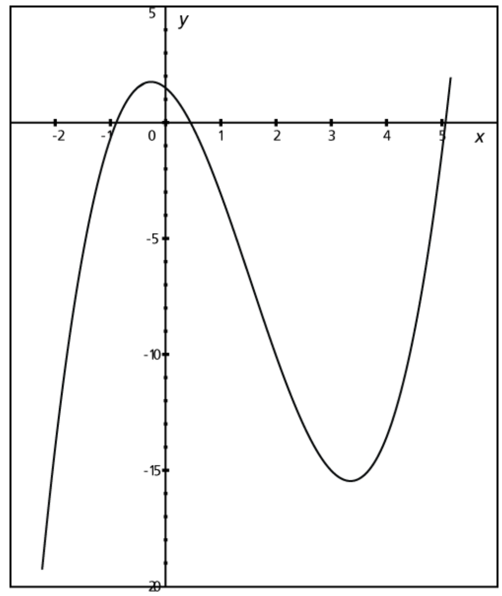
B. $\lim_{h \to 0}[f(x+h) - f(x)]$

C. $\dfrac{f(x+h) - f(x)}{h}$

D. $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

E. none of these

56. Use the graph of the function $f(x) = ax^3 + bx^2 + cx + d$ shown at right. How many real solutions are there to the equation $f(x) = ax^3 + bx^2 + cx + d = -2$?



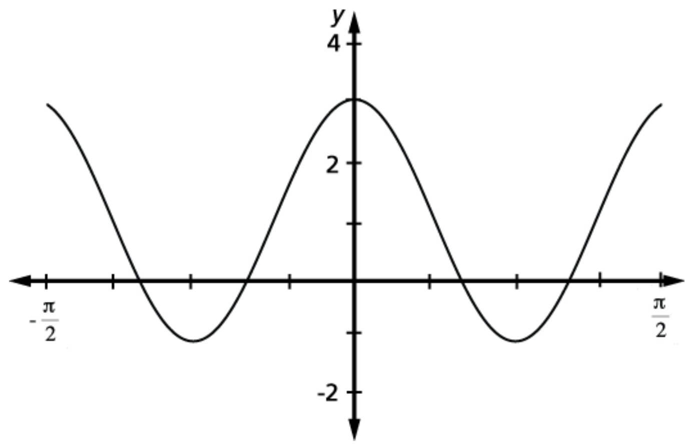
F. none
G. 1
H. 2
J. 3
K. infinitely many

57. Which of the following is (are) true for all values of θ for which the functions are defined?

I. $\sin(-\theta) = -\sin\theta$
II. $\cos(-\theta) = -\cos\theta$
III. $\tan(-\theta) = -\tan\theta$

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

58. Which equation corresponds to the graph at the right?



F. $y = -1 + 2\cos 4x$

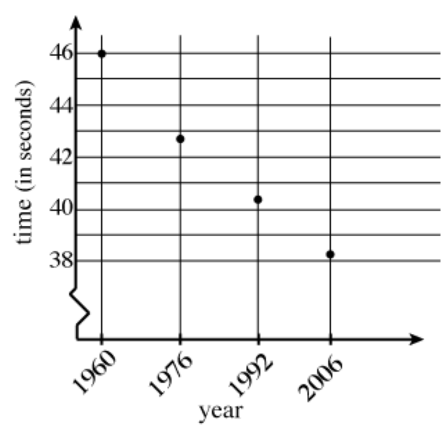
G. $y = 1 + 2\sin(4x - \frac{\pi}{2})$

H. $y = 1 + 2\sin(4x + \frac{\pi}{2})$

J. $y = 1 + 4\sin(2x + \frac{\pi}{4})$

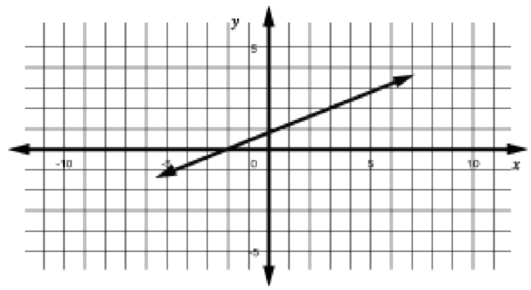
59. The table and graph below give the gold medal times for the women's 500 meter speed skating event in the Winter Olympics for four years. If y is the number of years after 1900 and t is time (in seconds), which of the following is an equation for a line that fits these data well?

Year	**Time (in seconds)**
1960	45.9
1976	42.76
1992	40.33
2006	38.23



A. $y = -0.165t + 55$
B. $y = -7.7t + 50$
C. $y = -0.165t + 370$
D. $y = 55t - 0.165$

60. The line ℓ in the figure at right is the graph of $y = f(x)$.



$$\int_{-2}^{3} f(x)dx$$

is equal to

F. 3
G. 4
H. 4.5
J. 5
K. 5.5

Appendix C

PDM Posttest One

UCSMP

The University of Chicago School Mathematics Project

Test Number ________

Precalculus and Discrete Mathematics Posttest One

<u>Do not open this booklet until you are told to do so.</u>

This test contains 30 questions. You have 40 minutes to take the test.

1. All the questions are multiple-choice. Some questions have four choices and some have five. There is only one correct answer to each question.

2. Using the portion of the answer sheet marked **TEST 2**, fill in the circle • corresponding to your answer for questions 1-30.

3. If you want to change an answer, completely erase the first answer on your answer sheet.

4. If you do not know the answer, you may guess.

5. Use the scrap paper provided to do any work. **DO NOT MAKE ANY STRAY MARKS IN THE TEST BOOKLET OR ON THE ANSWER SHEET.**

6. You may **NOT** use a calculator on this test.

DO NOT TURN THE PAGE until your teacher says that you may begin.

1. If $u = 4 - i$ and $v = 2i + 7$, then uv equals

A. $11 + i$.
B. $26 + i$.
C. $30 - i$.
D. $30 + i$.
E. 29.

2. Which is equivalent to $\frac{1}{\sqrt{3}}$?

F. $\frac{1}{9}$ G. $\frac{\sqrt{3}}{3}$ H. $\sqrt{3}$ J. $\frac{\sqrt{3}}{9}$

3. If $R(n) = \frac{(n+2)(n-4)(n+3)}{(n+2)(n-4)}$, then $R(n)$ is not defined for which of the following?

A. $n = -3$ only
B. $n = -2$ and $n = 4$ only
C. $n = 2$ and $n = -4$ only
D. $n = 2$ and $n = -4$ and $n = 3$
E. $n = -2$ and $n = 4$ and $n = -3$

4. The curve defined by $y = 3x(x - 2)(2x + 1)$ intersects the x-axis **only** at which of the following points?

F. (2, 0) and $(-\frac{1}{2}, 0)$

G. (-2, 0) and $(\frac{1}{2}, 0)$ and (0, 0)

H. (3, 0) and (-2, 0) and $(\frac{1}{2}, 0)$

J. (3, 0) and (2, 0) and $(-\frac{1}{2}, 0)$

K. (0, 0) and (2, 0) and $(-\frac{1}{2}, 0)$

5. Which of the three arguments below has a valid conclusion?

I.	Given:	*If Carlos reads English literature, then Carlos reads Shakespeare.*	
		Carlos reads English literature.	
	Conclusion:	*Carlos reads Shakespeare.*	
II.	Given:	*If David plays in the band, then he plays the clarinet.*	
		If David plays the clarinet, then he marches in the parade.	
	Conclusion:	*If David plays in the band, then he marches in the parade.*	
III.	Given:	*If Lynne wears the red swimsuit, then she is going to the beach.*	
		Lynne is going to the beach.	
	Conclusion:	*Lynne wears the red swimsuit.*	

A. I and II only
B. I and III only
C. II and III only
D. None has a valid conclusion.
E. All have valid conclusions.

6. What type of function is the derivative of a velocity function?

F. a position function
G. an acceleration function
H. another velocity function
J. a constant velocity function
K. none of F through J

7. How are the solutions to $(x + 7)^2 = 65$ related to the solutions to $x^2 = 65$?

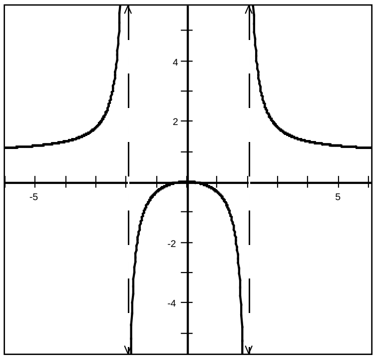
A. They are 7 larger.
B. They are 7 smaller.
C. They are $\sqrt{7}$ larger.
D. They are $\sqrt{7}$ smaller.

8. The functions f and g are defined by $f(x) = x^2 – 1$ and $g(x) = x + 4$. Then $g(f(x))$ is equal to which of the following?

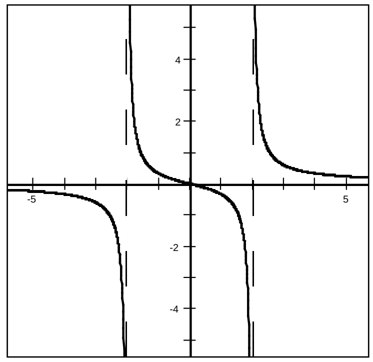
F. $(x^2 – 1)(x + 4)$
G. $(x + 4)^2 – 1$
H. $x^2 + 3$
J. $x^2 + 15$
K. $x^2 + x + 3$

9. Which of these is a sketch of the graph of the function f where $f(x) = \dfrac{x}{(x-2)(x+2)}$?

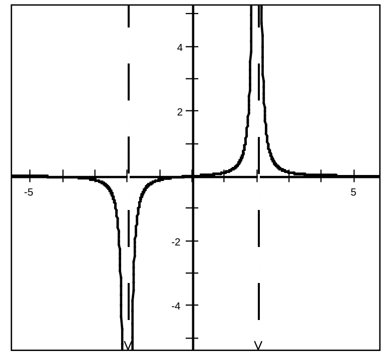
A.



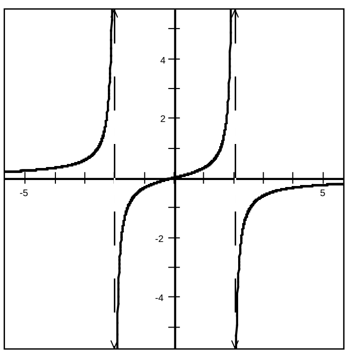
B.



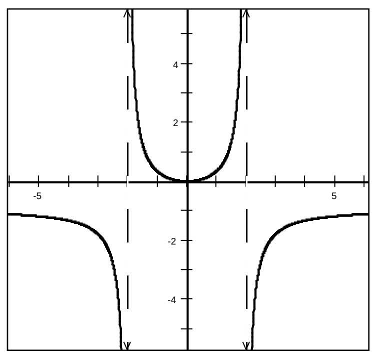
C.



D.



E.



10. Consider the statement

$1 + 3 + 5 + \ldots + (2n - 1) = n^2$, for all integers $n \geq 1$.

To use mathematical induction to prove the statement is true, you should start by

F. verifying the statement is true for $n = 1$.
G. assuming the statement is true for $n = 1$.
H. proving the statement is true for $n = k$.
J. assuming the statement is true for $n = k$.
K. proving the statement is true for $n = k + 1$.

11. At what points does the graph of the curve $y = \frac{2x+1}{x^2-2x-3}$ intersect the axes?

A. $(-\frac{1}{2}, 0)$, $(3, 0)$, and $(-1, 0)$ only

B. $(-\frac{1}{2}, 0)$, $(0, -\frac{1}{3})$, $(3, 0)$, and $(-1, 0)$ only

C. $(-\frac{1}{2}, 0)$ and $(0, -\frac{1}{3})$ only

D. $(-\frac{1}{3}, 0)$ and $(0, -\frac{1}{2})$ only

E. $(-\frac{1}{2}, 0)$ and $(0, \frac{1}{3})$ only

12. The equivalent resistance, R, of two resistors, R_1 and R_2, connected in parallel, is given by the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Which of the following represents the value of R?

F. $\frac{2}{R_1 + R_2}$

G. $\frac{R_1 R_2}{R_1 + R_2}$

H. $\frac{R_1 + R_2}{R_1 R_2}$

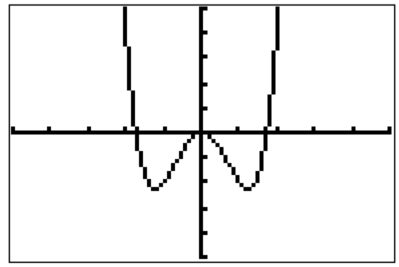
J. $\frac{R_2 + 1}{R_1 R_2}$

K. $\frac{R_1 + R_2}{R_1^2}$

13. Which of the following describes how to obtain the graph of $y = (x + 5)^3 - 4$ from the graph of $y = x^3$?

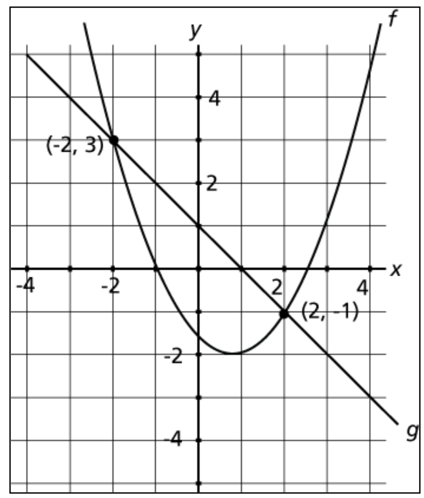
A. Translate the graph of $y = x^3$ by 5 units to the right and 4 units down.
B. Translate the graph of $y = x^3$ by 5 units to the left and 4 units down.
C. Translate the graph of $y = x^3$ by 5 units to the left and 4 units up.
D. Translate the graph of $y = x^3$ by 4 units to the left and 5 units down.
E. Translate the graph of $y = x^3$ by 4 units to the right and 5 units up.

14. The graph at the right shows a function graphed on the window $-5 \le x \le 5$ and $-5 \le y \le 5$ with tick marks by 1. Estimate the relative maximum value(s) of the function.



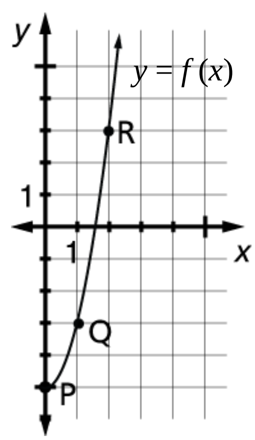
F. -2.3
G. -1.2
H. 0
J. -1.2 and 1.2
K. There is no relative maximum value.

15. The graphs of two functions *f* and *g* are shown at the right. For what values of *x* is $g(x) > f(x)$?



A. $-2 < x < 2$
B. $x < -2$ or $x > 2$
C. $-1 < x < 3$
D. $x < -1$ or $x > 3$
E. $x < 2$

16. Refer to the graph of a function $y = f(x)$ at the right. What is the average rate of change of the function from *P* to *R*?



F. -4
G. $\frac{1}{4}$
H. 4
J. 8
K. impossible to determine.

17. What are the solutions to $\cos x = \frac{1}{2}$ in the interval $2\pi \le x \le 4\pi$?

A. $\frac{7\pi}{3}$ and $\frac{8\pi}{3}$

B. $\frac{7\pi}{3}$ and $\frac{11\pi}{3}$

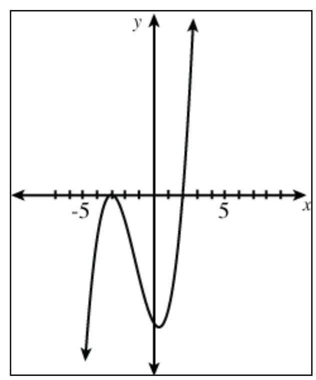
C. $\frac{9\pi}{4}$ and $\frac{15\pi}{4}$

D. $\frac{13\pi}{6}$ and $\frac{23\pi}{6}$

E. $\frac{13\pi}{6}$ and $\frac{17\pi}{6}$

18. Which of the following equations best describes the graph at right?

F. $f(x) = (x + 3)(x - 2)$
G. $f(x) = (x - 3)(x + 2)$
H. $f(x) = (x + 3)^2(x - 2)$
J. $f(x) = (x - 3)^2(x + 2)$



19. Which would be an appropriate way to begin a proof of the statement below?

If m is any odd integer and n is any even integer, then m – n is an odd integer.

A. Let $m = 2k + 1$ and $n = 2k$, where k is an integer.

B. Let $m = 2k$ and $n = 2k + 1$, where k is an integer.

C. Let $m = 2k + 1$ and $n = 2j$, where k and j are integers.

D. Let $m = 2k$ and $n = 2j + 1$, where k and j are integers.

20. Suppose the following statement is true: *If Molly is a tweedle, then Molly is a dee*. Which other statement must also be true?

F. *If Molly is a dee, then Molly is a tweedle.*
G. *If Molly is not a dee, then Molly is a tweedle.*
H. *If Molly is not a tweedle, then Molly is a dee.*
J. *If Molly is not a tweedle, then Molly is not a dee.*
K. *If Molly is not a dee, then Molly is not a tweedle.*

21. Consider the statement:

There is no largest prime number.

To prove this statement true using proof by contradiction, with what supposition should you begin?

A. There is a largest prime number.
B. There is no largest prime number.
C. There is a smallest prime number.
D. There are infinitely many primes.

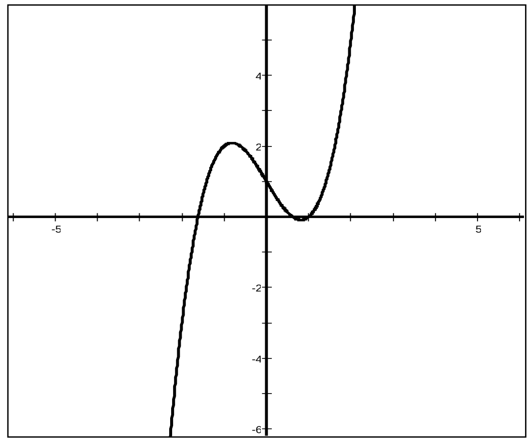
22. When is the implication $p \Rightarrow q$ false?

F. when p is false regardless of the truth value of q
G. when q is false regardless of the truth value of p
H. only when p is false and q is false
J. only when p is true and q is false
K. only when p is false and q is true

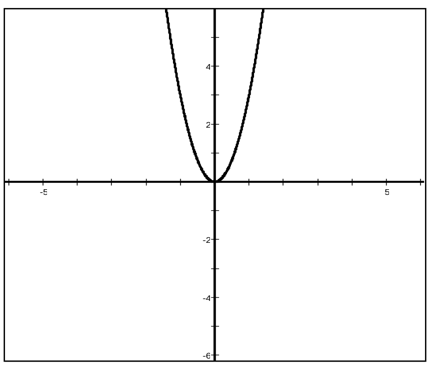
23. Which of the following is equivalent to $1.5^{3.2} \approx 3.66$?

A. $\log_{1.5} 3.66 \approx 3.2$
B. $\log_{3.2} 3.66 \approx 1.5$
C. $\log_{3.66} 3.2 \approx 1.5$
D. $\log_{3.2} 1.5 \approx 3.66$
E. $\log_{1.5} 3.2 \approx 3.66$

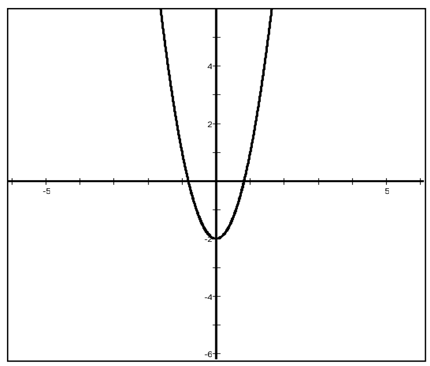
24. Consider the function graphed at the right. Which of the graphs below could be the graph of its derivative?



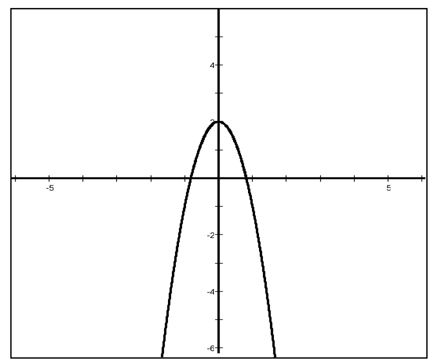
F.



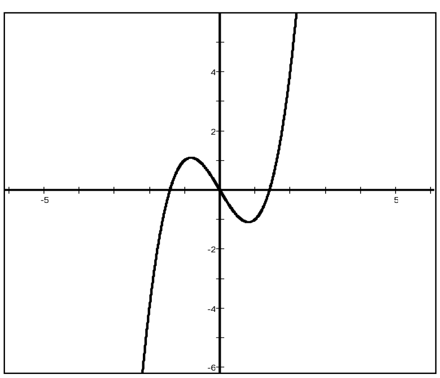
G.



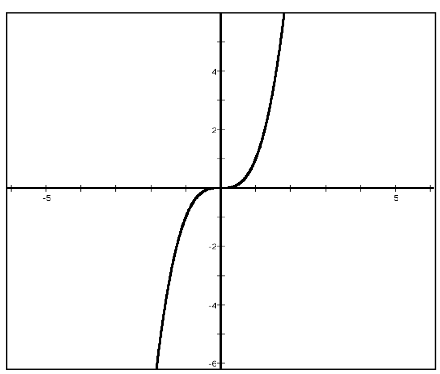
H.



J.



K.



25. Which of the following is the negation of the statement *Some animals are horses*?

A. *Some animals are not horses.*

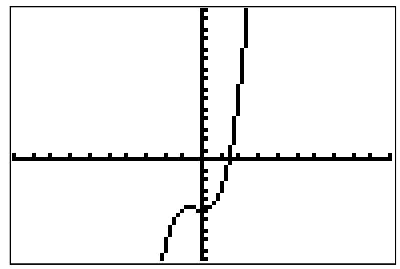
B. *All animals are horses.*

C. *No animals are horses.*

D. *If it is an animal, then it is a horse.*

E. *It is an animal and it is not a horse.*

26. At right is the graph of a function on a window $-10 \le x \le 10$ and $-10 \le y \le 15$, with tick marks by 1. Which of the following is an estimate for the zero of the function?



F. 1.4
G. 0
H. -0.7
J. -5
K. cannot be estimated from the graph

27. Factor $4m^3 - 25m$ completely.

A. $m(4m^2 - 25)$

B. $m(2m - 5)^2$

C. $(2m^2 - 5m)(2m + 5)$

D. $m(2m - 25)(2m + 25)$

E. $m(2m - 5)(2m + 5)$

28. Which step is **not** reversible?

$$3 - x = \sqrt{15 - x}$$

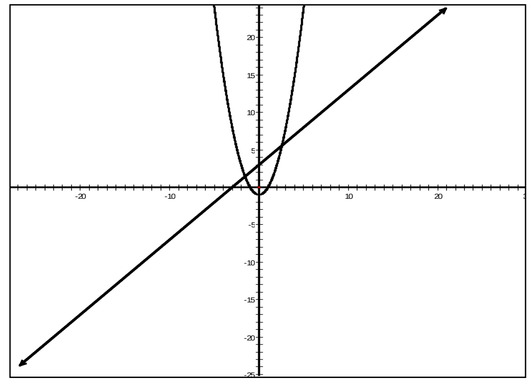
Step 1. $9 - 6x - x^2 = 15 - x$
Step 2. $x^2 + 5x - 6 = 0$
Step 3. $(x + 6)(x - 1) = 0$
Step 4. $x + 6 = 0$ or $x - 1 = 0$
Step 5. $x = -6$ or $x = 1$

F. Step 1
G. Step 2
H. Step 3
J. Step 4
K. Step 5

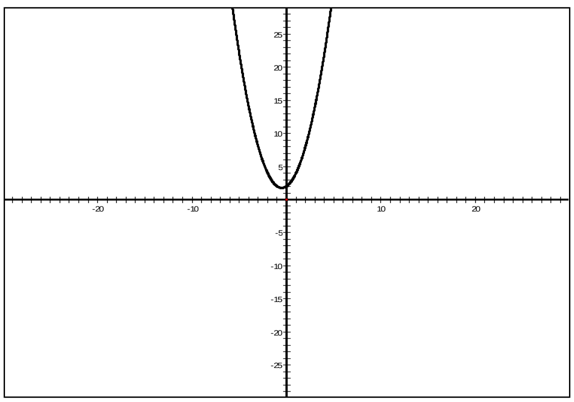
29. Consider the function h defined by $h(x) = \dfrac{5x}{(x+1)(x-4)}$. As x gets closer and closer to 4 but remains greater than 4, which of the following describes the function h?

A. h gets close to 0.
B. h gets close to 4.
C. h gets close to 20.
D. h decreases without bound.
E. h increases without bound.

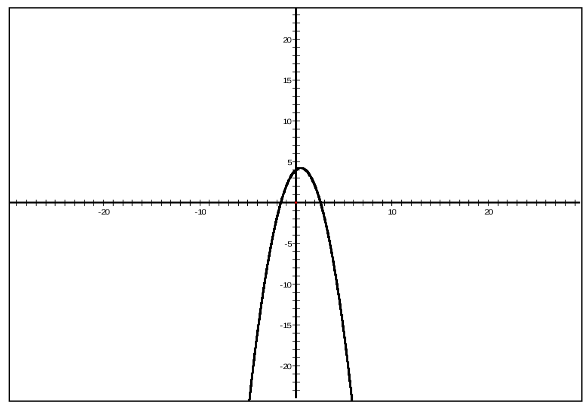
30. The graph at the right shows a linear function *f* and a quadratic function *g*. Which of the graphs below could be the graph of the product function $f \bullet g$?



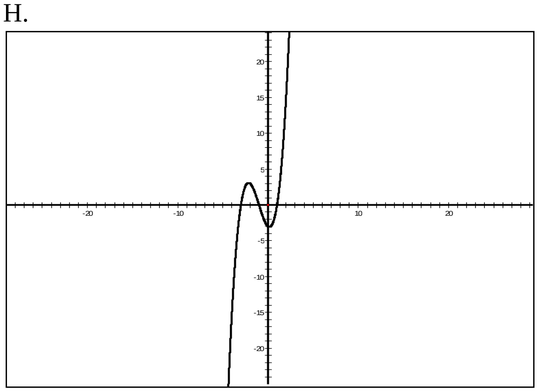
F.



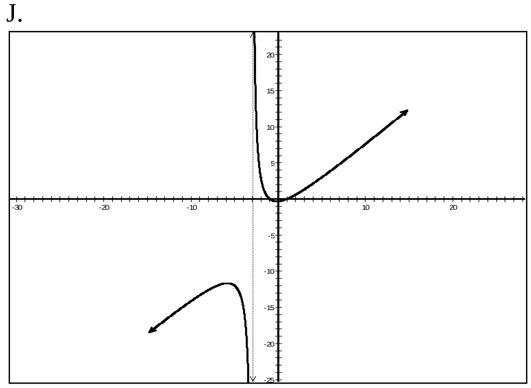
G.



H.



J.



Appendix D

PDM Posttest Two

UCSMP

The University of Chicago School Mathematics Project

Test Number ________

Precalculus and Discrete Mathematics Posttest Two

Do not open this booklet until you are told to do so.

This test contains 22 questions. You have 30 minutes to take the test.

1. All the questions are multiple-choice. Some questions have four choices and some have five. There is only one correct answer to each question.

2. Using the portion of the answer sheet marked **TEST 2**, fill in the circle • corresponding to your answer for questions 31- 52.

3. If you want to change an answer, completely erase the first answer on your answer sheet.

4. If you do not know the answer, you may guess.

5. Use the scrap paper provided to do any work. **DO NOT MAKE ANY STRAY MARKS IN THE TEST BOOKLET OR ON THE ANSWER SHEET.**

6. You **MAY** use a calculator on this test, including a graphing calculator with or without computer algebra systems.

7. After you complete the test and turn in your answer sheet, ask your teacher for the survey about calculator use on the test. You will need a copy of the test to complete the survey.

DO NOT TURN THE PAGE until your teacher says that you may begin.

the conditions in the release of these items. Other items have been used on previous studies conducted by UCSMP. Reprinted with permission.

31. Given the function h defined by $h(x)=\dfrac{(2x+4)(x-1)}{(x+2)}$. What is the behavior of the function near x = -2?

A. There is an essential discontinuity at x = -2.
B. There is a removable discontinuity at x = -2.
C. The value of $h(x)$ increases without bound near x = -2.
D. The value of $h(x)$ decreases without bound near x = -2.

32. A survey poll indicates that 38% of registered voters favor Candidate A, with a margin of error of 4%. Which of the following best describes the true percentage p of registered voters who favor Candidate A?

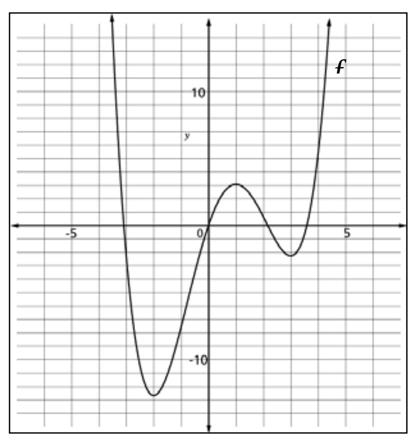
F. $p - 0.38 = 0.04$
G. $|p-0.38| = 0.04$
H. $|p-0.38| \le 0.04$
J. $|p-0.38| \ge 0.04$

33. For a function g, the derivative at 2 equals -1, that is $g'(2) = -1$. Which of the following describes the meaning of $g'(2)$?

A. The function has a value of -1 when x = 2.
B. The function g has a relative minimum value of -1 when x = 2.
C. The tangent line to the function g at x = 2 has a slope of -1.
D. The tangent line to the function g at x = 2 has equation y = -1.
E. The tangent line to the function g at x = 2 has equation x = -1.

34. Refer to the graph of function f at right. On which of the following intervals is f increasing?

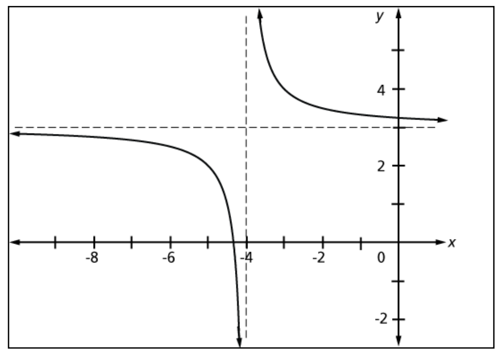
F. $x \le -2$
G. $x \ge 0$
H. $x \le -3$
J. $-2 \le x \le 0$
K. $1 \le x \le 3$



35. What is the result when $2x^3 + 3x^2 - 32x + 27$ is divided by $x + 5$?

A. quotient: $2x^2 + 7x + 3$, remainder: 12
B. quotient: $2x^2 - 7x + 3$, remainder: 12
C. quotient: $2x^2 + 13x + 33$, remainder: 192
D. quotient: $2x^2 - 7x - 67$, remainder: -308
E. none of A through D

36. A function h is graphed at right. As $x \to +\infty$, what is true about h(x)?



F. $h(x) \to 0$
G. $h(x) \to 3$
H. $h(x) \to +\infty$
J. $h(x) \to -\infty$
K. The values of h(x) do not exist.

37. Suppose $f(x) = x^{1/2}$. What is the set of all values of x for which $f(x)$ is a real number?

A. $\{x: x > 0\}$
B. $\{x: x \geq 0\}$
C. $\{x: x > 1\}$
D. $\{x: x \geq 1\}$
E. the set of all real numbers

38. Evaluate $\lim_{x \to 10} \frac{x^2 - 100}{2x^2 - 23x + 30}$.

F. 0

G. $\pm\sqrt{110}$

H. $\frac{3}{2}$

J. $\frac{20}{17}$

K. The limit does not exist.

39. Which of the following equals $(m + 2)^3$?

A. $m^3 + 8$

B. $m^3 + m^2 + m + 8$

C. $m^3 + 3m^2 + 6m + 8$

D. $m^3 + 6m^2 + 6m + 8$

E. $m^3 + 6m^2 + 12m + 8$

40. Charlie got a car loan for $30,000. Each month, interest of $\frac{1}{2}$% is added and then he makes a $600 car payment. If A_n describes the amount he owes for the car at the beginning of month n and $A_1 = 30{,}000$, which equation is true?

F. $A_n = 600(1.005)^{n-1}$

G. $A_n = 30000 - 600(1.005)^{n-1}$

H. $A_n = 30000(1.005)n - 600$

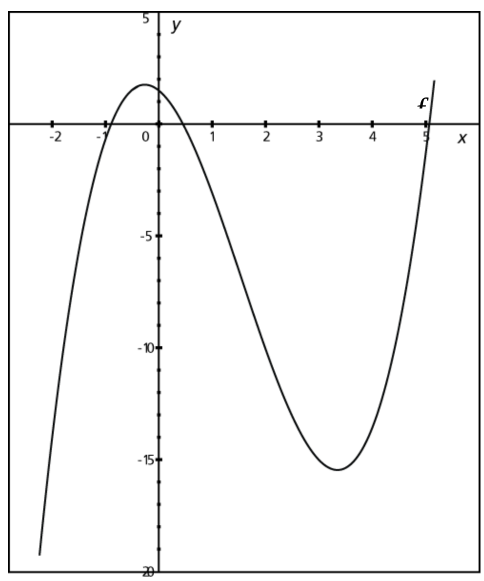
J. $A_n = A_{n-1}(1.005) - 600$

K. $A_n = A_{n-1} - 600(1.005)$

41. Estimate the measure of the angle between the vectors $\vec{u} = \langle 3, 2 \rangle$ and $\vec{v} = \langle -2, 5 \rangle$.

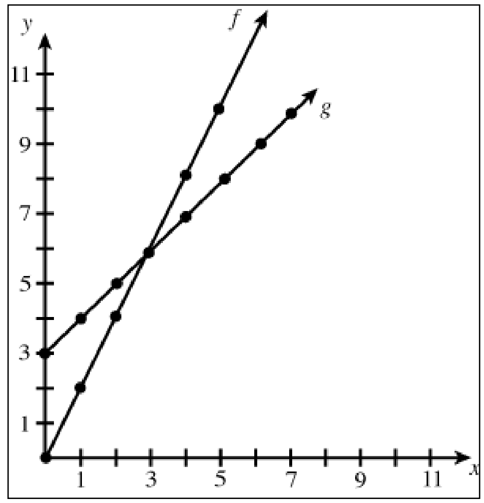
A. 164° B. 136° C. 78° D. 12°

42. Use the graph of the function $f(x) = ax^3 + bx^2 + cx + d$ shown at right. How many real solutions are there to the equation $f(x) = ax^3 + bx^2 + cx + d = -2$?



F. none
G. 1
H. 2
J. 3
K. infinitely many

In questions 43 and 44, refer to the graphs of functions f and g at right.



43. What is the value of g(1)?

A. 2
B. 4
C. 5
D. 6
E. 8

44. What is the value of $f(g(1))$?

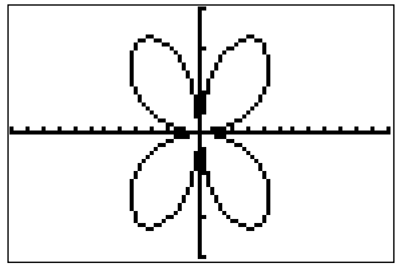
F. 2
G. 4
H. 5
J. 6
K. 8

45. Which of the following is (are) true for all values of θ for which the functions are defined?

I. $\sin(-\theta) = -\sin\theta$
II. $\cos(-\theta) = -\cos\theta$
III. $\tan(-\theta) = -\tan\theta$

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

46. Which of the following could be an equation for the graph at the right?



F. $r = 3\theta$
G. $r = 3 + \sin(2\theta)$
H. $r = 3\sin\theta$
J. $r = 3\sin(2\theta)$
K. $r = 3\cos(2\theta)$

47. A woman is standing on a cliff 200 feet above the water. Through a set of high-powered binoculars, she sees a boat on the water off in the distance. If θ represents the angle of depression, which of the following gives a formula for determining the angle of depression in terms of the distance *d* of the boat from the bottom of the cliff?

A. $\theta = \tan^{-1}\dfrac{200}{d}$

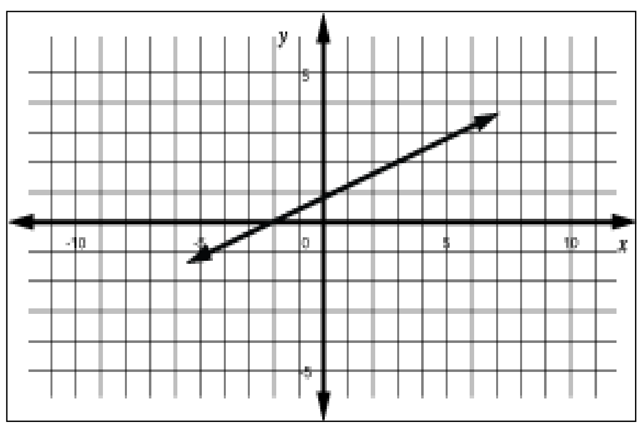
B. $\theta = \tan^{-1}\dfrac{d}{200}$

C. $\theta = \sin^{-1}\dfrac{200}{d}$

D. $\theta = \cos^{-1}\dfrac{200}{d}$

48. The line in the figure at right is the graph of y = f (x). What is the value of

$$\int_{-2}^{3} f(x)dx\,?$$



F. 3
G. 4
H. 4.5
J. 5
K. 5.5

49. How many solutions does the following system have?

$$\begin{cases} y+5=4(x+3)^2 \\ (x+3)^2+(y-2)^2=16 \end{cases}$$

A. 0
B. 1
C. 2
D. 3
E. 4

50. If $\tan\theta = \sin(2\theta)$ and $0 < \theta < \dfrac{\pi}{2}$, then what is an approximate value for θ?

F. 0 radians
G. 0.79 radians
H. 1 radian
J. 45 radians

51. Which of the following is the derivative of function f at x?

A. $f(x+h) - f(x)$

B. $\lim\limits_{h\to 0} [f(x+h) - f(x)]$

C. $\dfrac{f(x+h) - f(x)}{h}$

D. $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

E. none of these

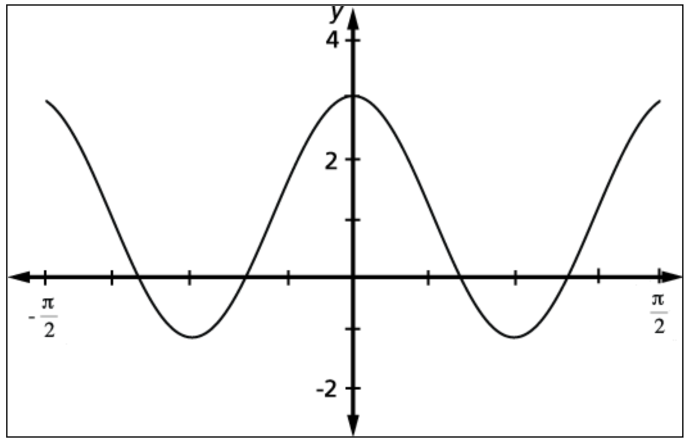
52. Which equation is graphed at the right?

F. $y = -1 + 2\cos 4x$

G. $y = 1 + 2\sin(4x - \frac{\pi}{2})$

H. $y = 1 + 2\sin(4x + \frac{\pi}{2})$

J. $y = 1 + 4\sin(2x + \frac{\pi}{4})$



Appendix E

PDM Problem Solving Test

UCSMP

The University of Chicago School Mathematics Project

Test Number ________

Precalculus and Discrete Mathematics: Problem Solving and Reasoning Test

Name (Print) ______________________________________

School ______________________________________

Teacher ______________________________________

Period ______________________________________

Do you have a calculator available for use on this test? _____ Yes _____ No

If yes, what model calculator is it? ______________________________

Which is true of your calculator?

_____ It does not graph equations.

_____ It can graph equations.

_____ It can simplify algebraic expressions. (It has a computer algebra system (CAS).)

Do not open this booklet until you are told to do so.

1. This test contains 5 questions.
2. You **MAY** use a calculator on this test, including a graphing calculator either with or without computer algebra systems.
3. There may be many ways to answer a question. We are interested in how you solve a problem, not just in the final answer. So, **be sure to show all your work** on the pages in the test booklet. If you use a calculator to solve a problem, be sure to explain what features or keys you used.
4. Try to do your best on each problem.
5. You have 30 minutes to answer the questions.

the conditions in the release of these items. Other items have been used on previous studies conducted by UCSMP. Reprinted with permission.

1. Solve the following system.

$$\begin{cases} y = x^2 - 3x + 3 \\ y = 2^x \end{cases}$$

2. A ball is thrown so that its height (in meters) after t seconds is given by $h(t) = -4.9t^2 + 18t + 15$.

   a. After how many seconds does the ball reach its maximum height?

   b. What is the maximum height reached by the ball?

   c. Find the instantaneous velocity of the ball 3.4 seconds after it is thrown. Include units.

d. Find the acceleration of the ball 3.4 seconds after it is thrown. Include units.

3. Are the functions *f* and *g* with *f*(*x*) = 3*x* + 2 and $g(x) = \frac{x+2}{3}$ inverses of each other?

a. Yes ______ No ______

b. Justify your answer.

4. Prove the following trigonometric identity.

For all real numbers x for which both sides are defined,

tan *x* + cot *x* = sec *x* • csc *x*.

5. a. Evaluate $\sum_{i-1}^{3}(3i+4)$.

b. Let *S*(*n*) be the statement

$$\sum_{i=1}^{n}(3i+4) = \frac{n(3n+11)}{2}.$$

Use the principle of mathematical induction to prove that *S*(*n*) is true for all positive integers *n*.

Appendix F

Student Information Form

UCSMP

The University of Chicago School Mathematics Project

Precalculus and Discrete Mathematics: Student Information Form

During this year, your class has been part of a study of mathematics materials. You have taken some tests throughout the year to show what you have learned in mathematics from the materials you have been using.

You are invited to answer the following 18 questions. Your answers to these questions will help us understand how you used the materials and class activities this year. Although you are not required to answer these questions, your responses can help improve mathematics materials for future students.

After you respond to the following questions, please put this form in the envelope provided and seal the envelope before returning to your teacher.

A. Were you in this class in this period at the beginning of the school year?

_____ Yes _____ No

B. Were you in this class in this period when you received your first course grade (report card) this school year?

_____ Yes _____ No

School ____________________________ Teacher________________________

Period ____________________________

1. About how much time did you spend, on the average, this year on your mathematics homework?

_____ 0-15 minutes per day
_____ 16-30 minutes per day
_____ 31-45 minutes per day
_____ 46-60 minutes per day
_____ more than 60 minutes per day

2. How often did your teacher expect you to read your mathematics textbook?

_____ almost every day
_____ 2-3 times per week
_____ 2-3 times a month
_____ less than once a month
_____ almost never

3. How often did you actually read your mathematics textbook?

_____ almost every day
_____ 2-3 times per week
_____ 2-3 times a month
_____ less than once a month
_____ almost never

4. How often did these things happen?

		Daily	Frequently	Seldom	Never
a.	Teacher read aloud in class.	_____	_____	_____	_____
b.	Students read aloud in class.	_____	_____	_____	_____
c.	Students read silently in class.	_____	_____	_____	_____
d.	Students discussed the reading in class.	_____	_____	_____	_____

5. How important do you think it is to read your mathematics text if you want to understand mathematics?

_____ very important

_____ somewhat important

_____ not very important

6. How often did you do these things when solving problems?

		Daily	Frequently	Seldom	Never
a.	write answers only	_____	_____	_____	_____
b.	write a few steps in your solutions	_____	_____	_____	_____
c.	write complete solutions	_____	_____	_____	_____
d.	explain or justify your work	_____	_____	_____	_____
e.	write proofs	_____	_____	_____	_____
f.	write in journals	_____	_____	_____	_____
g.	do a project	_____	_____	_____	_____

7. How important do you think it is to write explanations to show what you were thinking when solving mathematics problems?

_____ very important

_____ somewhat important

_____ not very important

8. Did you have a calculator available for use this year **in your mathematics class** (either that you brought to class or that was provided for you in class)?

_____ Yes (*Go to question 8a*.)

_____ No (*Go to question 13*.)

a. If yes, what model calculator did you have for use **in your mathematics class**?

____

b. Which is true of the calculator you used **during mathematics class**?

_____ It does not graph equations.

_____ It can graph equations.

_____ It can simplify algebraic expressions (It has a computer algebra system (CAS)).

9. About how often did you use this calculator **in your mathematics class**?

_____ almost every day

_____ 2-3 times per week

_____ 2-3 times a month

_____ less than once a month

_____ almost never

10. For what did you use this calculator **in your mathematics class**? (Check all that apply.)

_____ checking answers

_____ doing computations

_____ solving problems

_____ graphing equations

_____ working with a spreadsheet

_____ other (specify) ____

_____ making tables

_____ analyzing data

_____ finding equations to model data

_____ simplifying algebraic expressions

_____ other features of CAS

11. If you used the CAS (computer algebra system) features of a calculator, about how often did you use the calculator for this purpose **in your mathematics class**?

_____ almost every day

_____ 2-3 times per week

_____ 2-3 times a month

_____ less than once a month

_____ almost never

12. How helpful was the use of this calculator in learning mathematics **in your mathematics class**?

_____ very helpful
_____ somewhat helpful
_____ not very helpful

13. Did you have a calculator available for use this year **for homework**?

_____ Yes (*Go to question 13a*.)

_____ No (*Go to question 18*.)

a. If yes, which type of calculator did you have for use **for homework**?

_____ The same calculator I had for use in my mathematics class.

_____ A different calculator than I had for use in my mathematics class.

b. If you had a different calculator for use with homework than you had in class, please list the model.

____

c. Which is true of this calculator that you used **for homework**?

_____ It does not graph equations.
_____ It can graph equations.
_____ It can simplify algebraic expressions. (It has a CAS (computer algebra system).)

14. About how often did you use a calculator **for homework**?

_____ almost every day
_____ 2-3 times per week
_____ 2-3 times a month
_____ less than once a month
_____ never

15. How did you use a calculator **for homework**? (Check all that apply.)

_____ checking answers
_____ doing computations
_____ solving problems
_____ graphing equations
_____ working with a spreadsheet
_____ other features of CAS
_____ other (specify) ______________________________________
_____ drawing geometric figures
_____ making tables
_____ analyzing data
_____ finding equations to model data
_____ simplifying algebraic expressions

16. If you used the CAS (computer algebra system) features of a calculator, about how often did you use the calculator for this purpose **for homework**?

_____ almost every day

_____ 2-3 times per week

_____ 2-3 times a month

_____ less than once a month

_____ almost never

17. How helpful was the use of a calculator in learning mathematics **during homework**?

_____ very helpful

_____ somewhat helpful

_____ not very helpful

18. How helpful did you find your textbook in learning mathematics this year?

_____ very helpful

_____ somewhat helpful

_____ not very helpful

Place this form in the envelope provided, seal it, and return it to your teacher.

Appendix G

End of Chapter Evaluation Forms

University of Chicago School Mathematics Project
Precalculus and Discrete Mathematics: Third Edition

CHAPTER 4 EVALUATION FORM

Teacher ________________________________ School__________________________________

Date Chapter Began _______ Date Chapter Ended ________ No. Class Days (Including Tests) ____

1. Please complete the table below. In column A, circle the number of days you spent on each lesson. In columns B and C, rate the text and questions of each lesson using the following scale.

   1 = Disastrous; scrap entirely. (Reason?) 2 = Poor; needs major rewrite. (Suggestions?)
   3 = OK; some big changes needed. (Suggestions?) 4 = Good; minor changes needed. (Suggestions?)
   5 = Excellent; leave as is.

   In columns D and E, respectively, list the specific questions you assigned in the lesson and comment on any parts of the lesson text or questions you think should be changed. Use the other side or an additional sheet of paper if you need more space.

	A	**B**	**C**	**D**	**E**
		Rating			
Lesson	Circle the number of days you spent on the lesson	Lesson Text	Questions	Questions Assigned	Comments
4-1	0 0.5 1 1.5 2 2.5				
4-2	0 0.5 1 1.5 2 2.5				
4-3	0 0.5 1 1.5 2 2.5				
4-4	0 0.5 1 1.5 2 2.5				

4-5	0 0.5 1 1.5 2 2.5				
4-6	0 0.5 1 1.5 2 2.5				
4-7	0 0.5 1 1.5 2 2.5				
Self-Test	0 0.5 1 1.5 2 2.5				
SPUR Review	0 0.5 1 1.5 2 2.5				

2. Overall rating of this chapter. (Use the same rating scale as at the top of the page.) ___________

3. What comments do you have on the sequence, level of difficulty, or other specific aspects of the content of this chapter?

4. As we revise the student materials for this chapter,

   a. What should we definitely not change?

   b. What should we definitely change? What ideas do you have for changes that should be made?

5. As we revise the Teacher's Notes for this chapter,

a. What should we definitely not change?

b. What should we definitely change? What ideas do you have for changes that should be made?

6. Did you use any UCSMP Second Edition materials during this chapter (Lesson Masters, Technology Masters, etc.)? Yes _____ No _____

   If yes, how and when?

7. While teaching this chapter, did you supplement the text with any materials other than those mentioned in Question 6? Yes _____ No _____

   If yes, which materials did you use and when?

   Why did you use these materials? (If possible, please enclose a copy of the materials you used.)

8. a. Did you as the teacher demonstrate or use a calculator with this chapter? Yes _____ No _____

   b. If yes, how did you use the calculator?

c. What comments or suggestions do you have about the way calculator technology is incorporated into this chapter?

9. a. Did your students use a calculator with this chapter? Yes _____ No _____

   b. If yes, how did they use the calculator?

10. a. Did you as the teacher demonstrate or use a computer with this chapter? Yes _____ No _____

    b. If yes, how did you use the computer?

    c. What comments or suggestions do you have about the way computer technology is incorporated into this chapter?

11. a. Did your students use a computer with this chapter? Yes _____ No _____

    b. If yes, how did they use the computer?

12. Did you use the test for this chapter that we provided in the Teacher's Notes? Yes ______ No ______

    If yes, what suggestions do you have for improvement?

    If no, what specific reasons influenced your decision not to use the test?

13. Other comments? Attach additional sheets as needed.

Appendix H

Comparison of Achievement on Pretest 1

ANOVA Results for Pretest 1 Function Items by Class and Curricula

Curricula	Class	Mean	*SD*	*N*
2nd Edition	412	12.74	2.77	23
	413	13.00	3.55	20
	417	13.95	3.82	19
	Total	13.19	3.36	62
3rd Edition	410	13.50	2.90	16
	411	12.45	3.56	20
	414	14.59	3.89	17
	415	13.21	4.14	19
	416	14.38	2.33	13
	418	10.11	3.51	18
	419	7.73	3.32	11
	420	12.00	3.30	19
	421	12.13	4.14	24
	422	12.92	3.56	25
	423	11.89	3.78	27
	Total	12.35	3.85	209

Note. Results are for the 23 function items

Levene's Test of Equality of Error Variances for Pretest 1 Achievement by Class and Curricula[a]

F	df1	df2	Sig.
.84	13	257	.62

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + Curricula+ Class + Curricula * Class

Tests of Between-Subject effects for Pretest 1 by Class and Curricula

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	567.49[a]	13	43.65	3.46	.00	.15

Intercept	35288.72	1	35288.72	2799.34	.00	.92
Curricula	.00	0	.	.	.	.00
Class	533.40	12	44.45	3.53	.00	.14
Curricula * Class	.00	0	.	.	.	.00
Error	3239.78	257	12.61			
Total	46439.00	271				
Corrected Total	3807.26	270				

Note: Achievement based on only 23 function items.
a. R Squared = .149 (Adjusted R Squared = .106)

Appendix I

Comparison of Achievement on Pretest 2

ANOVA Results for Pretest 2 Function Items by Curricula and Access to CAS Controlling for Class

Curricula	Had CAS	Mean	SD	N
2nd Edition	No	7.4000	2.18003	60
	Yes	6.0000	2.82843	2
	Total	7.3548	2.18862	62
3rd Edition	No	7.3500	2.54335	60
	Yes	6.1678	2.38917	149
	Total	6.5072	2.48673	209
Total	No	7.3750	2.35883	120
	Yes	6.1656	2.38448	151
	Total	6.7011	2.44403	271

Note. Results are for the 13 function items

Levene's Test of Equality of Error Variances for Pretest 2 Achievement by Class, Curricula and Access to CAS[a]

F	df1	df2	Sig.
.51	3	267	.68

Note: Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + Class + Curric_dummy + HadCAS + Curric_dummy * HadCAS

Tests of Between-Subject Effects for Pretest 2 Function Items by Class, Curriculum and Access to CAS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	117.45[a]	4	29.37	5.23	.00	.07
Intercept	29.70	1	29.70	5.28	.02	.02
Class	19.56	1	19.56	3.48	.06	.01
Curric_dummy	.88	1	.88	.16	.69	.00
HadCAS	12.28	1	12.28	2.19	.14	.01
Curric_dummy * HadCAS	.32	1	.32	.06	.81	.00
Error	1495.30	266	5.62			
Total	13782.00	271				
Corrected Total	1612.79	270				

a. R Squared = .073 (Adjusted R Squared = .059)

Appendix J

Regression models using SPSS

SPSS Model for Posttest 1, OTL and Technology Variables

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-30.670	11.455		-2.678	.008
	HadCAS	-.782	2.208	-.021	-.354	.723
	Pre1FcnScore	.575	.051	.515	11.339	.000
	OTLHW	.034	.069	.028	.500	.618
	OTLLessons	.691	.102	.409	6.810	.000

a. Dependent Variable: Post1FcnPcnt

Note: Post1FcnPcnt is the percent of function items answered correctly on multiple choice posttest 1 and ranges from 0 to 100. Pre1FcnScore is the percentage score each student received the 23 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. OTLHW is the percentage of function problems an individual teacher assigned only for the function lessons he/she taught and ranges from 0 to 100.

SPSS Model Posttest 2, OTL and Technology Variables

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	5.884	14.483		.406	.685
	HadCAS	2.146	2.274	.065	.944	.346
	Pre2FcnPcnt	.390	.059	.363	6.599	.000
	OTLLessons	.611	.157	.402	3.890	.000
	OTLPost2	-.190	.219	-.076	-.867	.387
	Strat2	1.229	.489	.134	2.515	.013

a. Dependent Variable: Post2FcnPcnt

Note: Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. Post2FcnScore is the percentage score each student received on posttest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments, where 0 indicates no and 1 indicates yes. OTL Post2 is the percentage of the 16 function problems on posttest 2 for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100. DidUseStrategy is the number of times a student reported using a calculator strategy to solve the 7 calculator neutral items on posttest 2 function items and ranges from 0 to 7.

SPSS Model PSU and Technology Variables

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	42.837	4.961		8.634	.000
	HadCAS	-.699	2.976	-.014	-.235	.815
	Pre2FcnPcnt	.459	.097	.286	4.738	.000

a. Dependent Variable: PSUFcnPcnt

PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes.

SPSS Model PSU, OTL and Technology Variables

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	39.197	16.436		2.385	.018
	HadCAS	1.987	3.930	.040	.506	.614
	Pre2FcnPcnt	.449	.097	.279	4.607	.000
	OTLLessons	.155	.177	.068	.874	.383
	OTLPSU	-.105	.099	-.065	-1.065	.288

a. Dependent Variable: PSUFcnPcnt

PSUFcnPcnt is the score each student received on the problem solving test for only the 3 function items and ranges from 0 to 100. Pre2FcnScore is the percentage score each student received on pretest 2 for only the 16 function items and ranges from 0 to 100. HadCAS indicates an individual student had access to a CAS capable calculator while taking assessments where 0 indicates no and 1 indicates yes. OTLPSU is the percentage of function problems on the problem solving test for which the teacher reported having taught or reviewed the material necessary to answer the item and ranges from 0 to 100. OTLLessons is the percentage of function lessons taught by an individual teacher and ranges from 0 to 100.

Appendix K

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