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On Classification of Groups Generated by Automata with 4 States over a 2-Letter Alphabet

by

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A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Arts in Mathematics
Department of Mathematics & Statistics
College of Arts and Sciences
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Abstract

The class of groups generated by automata have been a source of many counterexamples in group theory. At the same time it is connected to other branches of mathematics, such as analysis, holomorphic dynamics, combinatorics, etc. A question that naturally arises is finding the ways to classify these groups. The task of a complete classification and understanding at the moment seems to be too ambitious, but it is reasonable to concentrate on some smaller subclasses of this class. One approach is to consider groups generated by small automata: the automata with k states over d-letter alphabet (so called, (k, d)-automata) with small values of k and d. Certain steps in this directions have been made already: in [11] all groups generated by $(2, 2)$-automata have been classified, and in [4] groups generated by $(3, 2)$-automata were studied. In this work we study the class of groups generated by $(4, 2)$-automata. More specifically, we partition all such automata into equivalence classes up to symmetry and minimal symmetry (symmetric and minimally symmetric automata naturally generate isomorphic groups) and classify completely all finite groups generated by automata in this class. We also list all classes generating abelian groups. Another important result of the project is developing a database of $(4, 2)$-automata and computational routines that represent a new effective tool for the search for $(4, 2)$-automata generating groups with specific properties, which hopefully will lead to finding counterexamples of certain conjectures.

Chapter 1
Introduction

Groups generated by automata were first considered in the 1960's [17], but gain serious attention in the 1980's after they provided counterexamples to famous long-standing conjectures in group theory. For example, in 1902, William Burnside proposed the question, "Is a finitely generated torsion group necessarilly finite?" Golod and Shafarevich were the first to give a negative answer to this problem in 1962 [10]. However, the simplest examples up to our days happen to be in the class of groups generated by automata. The most elegant example was constructed by Grigorchuk in [15] in 1980. Aleshin in [2] and Sushchasky in [24] suggested somewhat more complicated examples. Further, developing of the language of the action on trees by Gupta and Sidki [16] ignited the interest in the area even more. In 1983 Grigorchuk proved that the group he constructed in [15] was a counterexample to another famous problem suggested by Milnor in [19] about the existence of groups of intermediate growth. This group also solved Day's question concerning the existence of groups that are amenable, but not elementary amenable [6].

On the other hand, groups generated by automata not only serve as weird counterexamples to various conjectures, but also naturally appear in other areas of mathematics. For example, one of the brightest discoveries of the last two decades in this area was, perhaps, realizing its connection of holomorphic dynamics via, so-called, iterated monodromy groups [21]. This new approach allowed to solve some open conjectures in complex dynamics, such as Hubbard's twisted rabbit problem [3]. One of the links to combinatorics suggested by Grigorchuk and Šunić [13] explores the description of Hanoi towers game in terms of groups generated by automata. The strong Atiyah conjecture from analysis was also disproved using automata groups in [14] using the analysis on Schreier graphs arising from the action of lamplighter group on the levels of the tree.

The fundamental question in most areas of mathematics is classification of objects of study. The classification of all groups generated by automata is not a realistic task with the current techniques. The main reason for this is the fact that many of these groups did not appear in mathematics before and automata language is simply the easiest way to describe them. However, some information about each such group can always be obtained, and it makes sense to classify the classes of groups generated by small automata. Certain steps in this directions have been made already. The number of states of an automaton and the cardinality of the alphabet represent a natural complexity parameters of automata. Throughout the thesis we will denote automata with k states over d-letter alphabet by (k, d)-automata, and the groups they generate by (k, d)-groups. In [11] all groups generated by $(2, 2)$-automata have been classified, and in [4] groups generated by $(3, 2)$-automata were studied. In this work we study the class of groups generated by $(4, 2)$-automata.

In this work, we set up the basis for the classification of groups generated by 4-state automata over a 2-letter alphabet (i.e., $(4, 2)$-groups). As the main application, we classify all finite groups in this class. The project substantially uses the the classification of $(3, 2)$-groups from [4], but has much larger scale. There are 1,048,576 $(4, 2)$-automata, versus 5,832 $(3, 2)$-automata. As a result, we did not set up the goal for the project to describe all $(4, 2)$-groups in as many details as in [4]. One of our main goals was to create the tools for effective work with the class. In particular, we partition all $(4, 2)$-automata into classes of symmetry and minimal symmetry such that any two automata from the same class generate isomorphic groups. We show that there are 7,471 equivalence classes up to minimal symmetry (compare it to 194 classes of $(3, 2)$-automata). We also develop routines to work with these classes that allow an effective search (using computer software) over the whole class for groups with pre-specified properties. Hopefully these routines will help to provide answers to some of the questions formulated in Chapter 6.

As the main application, we focus on the 231 classes that generate finite groups and prove that automata from all other classes generate infinite groups. Note, that this is not a trivial task as there is no algorithm deciding whether a given automaton group is finite. It was proved very recently by Gillibert that the finiteness problem for automaton semigroups is undecidable [8] and there is a common belief that this should also be the case for groups generated by automata. Basically, if the group generated by an automaton is finite, it can be usually proved quite easily: if the G has n elements, then it will act faithfully on the level $[\log_2 n]+1$ of the corresponding tree, which is a finite set. The hard part usually is to prove that the group is infinite. We use several techniques and methods described in Chapter 4 that altogether cover all our groups.

We find that there are in fact 14 finite groups generated by automata in this class. They include the Trivial Group, C_2, $C_2 \times C_2$, $C_2 \times C_2 \times C_2$, D_8, $C_2 \times D_8$, $C_2 \times C_2 \times C_2 \times C_2$, $(C_2 \times C_2 \times C_2 \times C_2) : C_2$, $C_2 \times C_2 \times D_8$, $((((C_4 \times C_2) : C_2) : C_2) : C_2)$, $C_2 \times ((((C_4 \times C_2) : C_2) : C_2) : C_2)$, $(D_8 \times D_8) : C_2$, $C_2 \times ((D_8 \times D_8) : C_2)$, and $((C_2 \times C_2 \times ((C_2 \times C_2 \times C_2 \times C_2) : C_2)) : C_2) : C_2$, where C_2 dentoes the cyclic group of order 2, and D_8 represents the dihedral group of order 8. Further, we find 90 classes of automata that generate abelian groups. We also determine automata classes 71312, 206974, 460219, 460223, 460478, and 460538 that could possibly generate infinite torsion groups and may serve as candidates for new solutions to the Burnside problem. However, our computations so far indicate that this is not the case for each of this groups, which would prove that the automaton generating Grigorchuk group is the smallest automaton over 2-letter alphabet that generates a Burnside group. We hope to handle all these cases before submitting the paper [5], which is currently under preparation.

Most of the research in this paper was done using GAP (Groups, Algorithms, Programming) [7]. The specific package used was `AutomGrp` [20], authored by Dmytro Savchuk and Yevgen Muntyan. This package contains many algorithms used for determining properties of automata groups. We describe many of these algorithms later in the paper.

The structure of the thesis is as follows. We start from introducing necessary notions in Chapter 2, including finite automata, regular rooted trees, and wreath products. In Chapter 3, we discuss a way to enumerate all (4,2)-automata. We assign to each (4,2)-automaton a number using lexicographic order and partition them into classes up to minimal symmetry using methods in GAP. In Chapter 4, we discuss different algorithms in the GAP package `AutomGrp` [20] which can be used to effectively determine the order of groups generated by automata in this class. In Chapter 5, we discuss the main application of this research, which is the description of all finite groups generated by automata in this class.

Finally, Chapter 6 gives a list of open problems to which the current research naturally leads. Two appendices containing detailed information concerning infinite groups conclude the thesis.

Chapter 2
Automaton Groups

2.1 Finite Automata and Moore Diagrams

We start from introducing necessary definitons.

Definition 2.1.1. A *finite (Mealy) automaton* is a quadruple $A = (S, X, \pi, \tau)$ where S is a finite set of states, $X = \{0, 1, ..., d-1\}$ is an alphabet over d letters ($d \geq 2$), $\pi : S \times X \to X$ is called the *output map*, and $\tau : S \times X \to S$ is called the *transition map*.

Definition 2.1.2. If for every $s \in S$ the map $\pi_s : X \to X$ defined by $\pi_s(x) = \pi(s, x)$ is a permutation of X then we call A *invertible*.

For the rest of this paper, we will only be concerned with finite, invertible automata with 4 states over a 2 letter alphabet. We will often use the term "(4,2)-automaton" or simply "automaton" to describe these automata.

There is a way to describe an automaton visually using a Moore Diagram. A Moore Diagram looks quite similarly to a directed graph, where the finite set of states S represents the vertex set. The way the edges are placed is as follows: for $a, b \in S$, there is a directed edge from a to b labeled by i ($i \in X$) if $\tau(a, i) = b$ We also label each vertex s with an element π_s of $Sym(X)$, the symmetric group on X. In the case of automata over a 2-letter alphabet, $Sym(0, 1)$ consists solely of the trivial permutation, and the nontrivial permutation swapping the two letters. We call a state $s \in S$ an active state if $\pi_s \neq id$. For all Moore diagrams in this paper, active states are shaded in red.

It is important to note, that the focus of this paper is on Mealy Automata. The primary goal is the transduction of strings and not the accepting of a language. So, initial and accepting states are not specifically indicated, but rather, each state is acts as the initial state, producing a so-called "initial automaton".

Some examples of Moore Diagrams are given in Section 4.7.

2.2 Rooted Trees and Automata Groups

Since an automaton has an alphabet of d letters (namely $X = \{0, 1, ...d - 1\}$), the set of finite words X^* over X takes the form of a regular rooted tree T_d. In this tree, the empty word corresponds to the root vertex, and vertex v has exactly one parent (exluding the root vertex) and exactly d children. Hence, v is adjacent to vx for $v \in X^*$ and $x \in X$. The n^{th} level of the tree consists all words over X of length n. We are interested in the action of each (4,2)-automaton on T_d. We will describe how each state $s \in S$ acts on infinite and finite words over X, preserving the length and adjacency in T_d. Because we are focusing specifically on invertible automata, we observe that each state of the automaton defines an automorphism of T_d.

More formally, each tree automorphism defined by state $s \in S$ maps a word xw for $x \in X$ and $w \in X^*$ to another word of equal length in X^* in the following way:

$$s(xw) = \pi_s(x)\tau(s, x)(w)$$

In other words, when the automaton is in the state s and reads the letter $x \in X$, it outputs letter $\pi_s(x)$ and shifts to the state $\tau(s, x)$.

Example 2.2.1. We can see how state a in the automaton in Figure 1 transduces the string 1011 to 0011.

$$a(1011) = \pi_a(1)\tau(a, 1)(011)$$

$$= 0d(011) = 0\pi_d(0)\tau(d, 0)(11)$$

$$= 00c(11) = 00\pi_c(1)\tau(c, 1)(1)$$

$$= 001c(1) = 001\pi_c(1)\tau(c, 1)(1)$$

$$= 0011$$

Definition 2.2.2. The group of tree automorphisms generated by all states of a given automaton A is called the *automaton group* defined by A, denoted $G(A)$.

2.3 Wreath Products

The group $Aut(T_d)$ is naturally isomorphic to

$$Sym(\{0, 1, \ldots, d-1\}) \wr Aut(T_d)$$

where $\wr$ is the *permutational wreath product*, and the isomorphism is induced by

$$g \mapsto \pi_g(g|_0, ..., g|_{d-1}).$$

This language allows us to define groups generated by automata by, so-called, "wreath recursion", where each state s of an automaton is defined by

$$s = \pi_s(s_0, ..., s_{d-1}),$$

where $s_i = \tau(s, i)$ are called *sections* of s and π_s is called the *root permutation* of X. More generally, for each $v \in X^*$ and $g \in Aut(T_d)$, define $g|_v$ by the rule: if $g(vw) = v'w'$, then $g|_v(w) = w'$.

This language gives an effective way to compute products and inverses of transformations defined by states. For the product of arbitrary automorphisms $f, g \in Aut(T_d)$ we have

$$\pi_f(f_0, ..., f_{d-1})\pi_g(g_0, ..., g_{d-1}) = \pi_f\pi_g(f_{g(0)}g_0, ..., f_{g(d-1)}g_{d-1}).$$

And for inverses of transformations, we have

$$(\pi_s(s_0, ..., s_{d-1}))^{-1} = \pi_s^{-1}(s^{-1}_{\pi_s(0)}, ..., s^{-1}_{\pi_s(d-1)}).$$

Definition 2.3.1. A group G of tree automorphisms is *self-similar* if every section of every automorphism in G is an element of G.

We notice that the tree T_d is a self-similar object, as the subtree representing uX^* consisting of all words prefixed by u is canonically isomorphic to the tree T_d itself. We also note that groups generated by automata are self-similar. The converse is also true if one allows for automata with an infinite number of states.

Chapter 3

Classification Guide

There exist $2^{20} = 1,048,576$ 4-state automata over the 2 letter alphabet $X = \{0,1\}$. Let the set of states S={**0,1,2,3**}. We label each automaton uniquely using the following method. Given the wreath recursion

$$\mathbf{0} = \sigma^{a_{11}}(a_{12}, a_{13})$$

$$\mathbf{1} = \sigma^{a_{21}}(a_{22}, a_{23})$$

$$\mathbf{2} = \sigma^{a_{31}}(a_{32}, a_{33})$$

$$\mathbf{3} = \sigma^{a_{41}}(a_{42}, a_{43})$$

where $a_{ij} \in S = \{\mathbf{0,1,2,3}\}$ for $j \neq 1$ and for $a_{i1} \in \{\mathbf{0,1}\}$, $i = 1,2,3,4$, assign the number

$$Number(A) = a_{12} + 4a_{13} + 16a_{22} + 64a_{23} + 256a_{32} + 1024a_{33} + 4096a_{42} +$$

$$16384a_{43} + 65536(a_{11} + 2a_{21} + 4a_{31} + 8a_{41}) + 1$$

Using GAP code shown below, we can produce an automaton A by inputting $Number(A)$ as an argument in the function below.

```
AutNum4x2:=function(n)
  local i, j, col,row,aut,m;
  m:=n-1;
  aut:=[[,,],[,,],[,,],[,,]];
  row:=(m-(m mod 2^16))/2^16;
  col:=m-row*2^16;
  for i in [1..4] do
```

```
      aut[i][3]:=(1,2)^(row mod 2);
    row:=(row-(row mod 2))/2;
  od;
  for i in [1..4] do
    aut[i][1]:=(col mod 4)+1;
    col:=(col-(col mod 4))/4;
    aut[i][2]:=(col mod 4)+1;
    col:=(col-(col mod 4))/4;
  od;
  return aut;
end;
```

For example, to find automata number 1 and 345652:

```
gap> AutNum4x2(1);
[ [ 1, 1, () ], [ 1, 1, () ], [ 1, 1, () ], [ 1, 1, () ] ]
gap> AutNum4x2(345652);
[ [ 4, 1, (1,2) ], [ 4, 1, () ], [ 3, 2, (1,2) ], [ 1, 2, () ] ]
```

Note: (1,2) represents 1 and () represents 0 in the numbering system. Also, GAP requires us to use {1,2,3,4} as the set of states, which correspond directly to {0,1,2,3} in our numbering system.

We now have a natural way of uniquely numbering each (4,2)-automata using lexicographic order. If our goal is to classify each automaton group up to isomorphism, our next logical step would be to begin partitioning these groups into equivalence classes. To do this we rely on the notions of *symmetry* and *minimal symmetry*.

For a given automaton, the three operations

- passing to inverses of all generators
- permuting the states of the automaton
- permuting the alphabet letters

do not change the ismorphism class of the group generated by that said automaton.

Definition 3.0.2. Two automata A and B that can be obtained from one another by using a composition of the three above operations, are called *symmetric*.

We can use GAP code to find the list of automata symmetric to a given automaton:

```
EquivList4x2:=function(A)
  local list,i,n,tmp,pi,S3,t,l,j,B;
  list:=[];
  S4:=SymmetricGroup(4);
  for pi in S4 do
# Print("pi=",pi,"\n");
    tmp:=StructuralCopy(A);
    tmp:=tmp^pi;
# Print("tmp=",tmp,"\n");
    for i in [1..4] do
      tmp[i][1]:=tmp[i][1]^pi;
      tmp[i][2]:=tmp[i][2]^pi;
    od;
    if not [tmp] in list then Add(list,[tmp]); fi;
  od;
  #inverse automata
  l:=Length(list);
  for i in [1..l] do
    tmp:=StructuralCopy(list[i][1]);
    for j in [1..4] do
      if tmp[j][3]=(1,2) then
        t:=tmp[j][1];
        tmp[j][1]:=tmp[j][2];
        tmp[j][2]:=t;
```

```
      fi;
    od;
    if not [tmp] in list then Add(list,[tmp]); fi;
  od;
  #swap 0/1
  l:=Length(list);
  for i in [1..l] do
    tmp:=StructuralCopy(list[i][1]);
    for j in [1..4] do
      t:=tmp[j][1];
      tmp[j][1]:=tmp[j][2];
      tmp[j][2]:=t;
    od;
    if not [tmp] in list then Add(list,[tmp]); fi;
  od;
  return list;
end;
```

For example, to find the list of automata symmetric to automaton 196610, or

[[2, 1, (1,2)], [1, 1, (1,2)], [1, 1, ()], [1, 1, ()]]

we use the following function:

```
gap> EquivList4x2([ [ 2, 1, (1,2) ], [ 1, 1, (1,2) ], [ 1, 1, () ], [ 1, 1, () ] ]);
[ [ [ [ 2, 1, (1,2) ], [ 1, 1, (1,2) ], [ 1, 1, () ], [ 1, 1, () ] ] ],
  [ [ [ 4, 4, () ], [ 4, 4, (1,2) ], [ 4, 4, () ], [ 2, 4, (1,2) ] ] ],
  [ [ [ 2, 2, () ], [ 4, 2, (1,2) ], [ 2, 2, () ], [ 2, 2, (1,2) ] ] ],
  [ [ [ 3, 3, () ], [ 3, 3, (1,2) ], [ 2, 3, (1,2) ], [ 3, 3, () ] ] ],
  [ [ [ 4, 1, (1,2) ], [ 1, 1, () ], [ 1, 1, () ], [ 1, 1, (1,2) ] ] ],
  [ [ [ 4, 4, (1,2) ], [ 4, 4, () ], [ 4, 4, () ], [ 1, 4, (1,2) ] ] ],
  [ [ [ 2, 2, (1,2) ], [ 1, 2, (1,2) ], [ 2, 2, () ], [ 2, 2, () ] ] ],
  [ [ [ 3, 3, (1,2) ], [ 3, 3, () ], [ 1, 3, (1,2) ], [ 3, 3, () ] ] ],
  [ [ [ 3, 1, (1,2) ], [ 1, 1, () ], [ 1, 1, (1,2) ], [ 1, 1, () ] ] ],
```

```
[ [ [ 4, 4, () ], [ 4, 4, () ], [ 4, 4, (1,2) ], [ 3, 4, (1,2) ] ] ],
[ [ [ 2, 2, () ], [ 3, 2, (1,2) ], [ 2, 2, (1,2) ], [ 2, 2, () ] ] ],
[ [ [ 3, 3, () ], [ 3, 3, () ], [ 4, 3, (1,2) ], [ 3, 3, (1,2) ] ] ],
[ [ [ 1, 2, (1,2) ], [ 1, 1, (1,2) ], [ 1, 1, () ], [ 1, 1, () ] ] ],
[ [ [ 4, 4, () ], [ 4, 4, (1,2) ], [ 4, 4, () ], [ 4, 2, (1,2) ] ] ],
[ [ [ 2, 2, () ], [ 2, 4, (1,2) ], [ 2, 2, () ], [ 2, 2, (1,2) ] ] ],
[ [ [ 3, 3, () ], [ 3, 3, (1,2) ], [ 3, 2, (1,2) ], [ 3, 3, () ] ] ],
[ [ [ 1, 4, (1,2) ], [ 1, 1, () ], [ 1, 1, () ], [ 1, 1, (1,2) ] ] ],
[ [ [ 4, 4, (1,2) ], [ 4, 4, () ], [ 4, 4, () ], [ 4, 1, (1,2) ] ] ],
[ [ [ 2, 2, (1,2) ], [ 2, 1, (1,2) ], [ 2, 2, () ], [ 2, 2, () ] ] ],
[ [ [ 3, 3, (1,2) ], [ 3, 3, () ], [ 3, 1, (1,2) ], [ 3, 3, () ] ] ],
[ [ [ 1, 3, (1,2) ], [ 1, 1, () ], [ 1, 1, (1,2) ], [ 1, 1, () ] ] ],
[ [ [ 4, 4, () ], [ 4, 4, () ], [ 4, 4, (1,2) ], [ 4, 3, (1,2) ] ] ],
[ [ [ 2, 2, () ], [ 2, 3, (1,2) ], [ 2, 2, (1,2) ], [ 2, 2, () ] ] ],
[ [ [ 3, 3, () ], [ 3, 3, () ], [ 3, 4, (1,2) ], [ 3, 3, (1,2) ] ] ] ]
```

So, if A is automaton 196610 and A generates a group G, then any automaton in the list above is symmetric to A and would generate a group isomporphic to G.

After running this code over all 1048576 (4,2)-automata, we partition the list of all automata into equivalence classes with respect to symmetry. The automaton in each class with the minimum number is selected as a representative for that class. The above code proves the following theorem:

Theorem 3.0.3. *There are 14,960 equivalence classes of (4,2)-automata with respect to symmetry.*

However, these symmetry class representatives may not be minimized. We introduce the notion of minimal symmetry.

Definition 3.0.4. If the minimization of automaton A is symmetric to the minimization of automaton B, then A is said to be *minimally symmetric* to B, denoted $A \sim B$.

After minimizing the symmetry class representatives, we can use GAP to find minimal symmetry classes. The process in GAP is very similar to the one used above to find symmetry classes, yet some issues present themselves in the minimization. Because a minimized (4,2)-automaton has as many as four and as little as one state, we must refer to previous research already completed on (3,2) and (2,2)-automata [4]. To do this, we use the follwing GAP code:

```
#SymClass is the list of numbers of the 14960 symmetry representatives
#MinSym4x2 is a function that outputs the minimized automaton for an
#inputted automaton number
Min_Sym_Class:=function()
local l, m, holder;
m:=[]; holder:=[];
for i in [1..Length(SymClass)] do
 l:=[[],[],[],[]];
 l[1]:=SymClass[i];
 l[2]:=Length(MinSym4x2(l[1]));
    if l[2]=1 then
     if MinSym4x2(l[1])[1][3]=() then
      l[3]:=1;
     else l[3]:=2;
          fi;
    fi;
   if l[2]=2 then
    l[3]:=EquivToNum2x2([MinSym4x2(l[1])]);
   fi;
 if l[2]=3 then
  l[3]:=list[NumOfAut([MinSym4x2(l[1])])][12];
 fi;
 if l[2]=4 then
```

```
  l[3]:=list4x2[l[1]][2];
 fi;
  Add(m,l);
od;
for i in [1..Length(m)] do
 if not [m[i][2], m[i][3]] in holder then
  m[i][4]:=m[i][1];
   Add(holder, [m[i][2], m[i][3]]);
  else
   for j in [1..Length(m)] do
     if [m[j][2], m[j][3]]=[m[i][2], m[i][3]] then
      m[i][4]:=m[j][1];
       break;
     fi;
    od;
   fi;
od;
return m;
end;
```

The above code works in the following way: It pours over all 14960 symmetry representatives, checking the length of its minimized automaton. Noticing that if a (4,2)-automaton is minimized to (1,2)-automaton, there are only two possible automata. We represent them using wreath recursions:

$$a = (a, a) \text{ or } a = \sigma(a, a)$$

The code will label minimized automata of this kind as 1 or 2 respectively. For the automata whose minimization has two or three states, they are partitioned accordingly, and identified into classes based on symmetry using previous libraries already generated for (2,2) and (3,2)-automata. Lasty, if the minimization has 4 states, then the minimal symmetry class is equal to the symmetry class.

The final piece of code, starting with

```
for i in [1..Length(m)] do
```

finds the minimal symmetry representative for each class by selecting the automaton with the minimum number in each class.

For example, this process found that automata

69906, 69907, 69922, 69923, 69924, 70162, 70163, 70164, 70178, 70179, 70180, 70194, 70195, 70434, 78354, 205347, 205348, 205351, 205352, 205363, 205364, 205367, 205368, 205411, 205412, 205415, 205416, 205427, 205428, 205431, 205432, 205603, 205604, 205607, 205608, 205667, 205668, 205671, 205672, 206371, 206372, 206375, 206376, 206387, 206388, 206391, 206392, 206435, 206436, 206439, 206440, 206451, 206452, 206455, 206456, 206627, 206628, 206631, 206632, 206643, 206644, 206647, 206691, 206692, 206707, 209443, 209444, 209447, 209448, 209507, 209508, 209511, 209512, 210467, 210468, 210471, 210472, 210483, 210484, 210487, 210531, 210532, 210547, 471860, 471864, 471924, 471928, 471932, 471988, 471992, 471996, 472948, 472952, 472956, 473012, 473016, 473972, and 473976

minimize to symmetric (2,2)-automata, and thus create a minimal symmetry class. Automaton 69906 is selected as the class representative.

Theorem 3.0.5. *There are 7,471 equivalence classes of (4,2)-automata with respect to minimal symmetry.*

It follows that there are at most 7,471 isomorphism classes of (4,2)-groups. As the main application, we will classify all finite groups generated by (4,2)-automata. We also note that 90 of these classes generate abelian groups, 58 of which generate finite abelian groups. In Chapter 5, we show the 5 finite abelian groups generated by these 58 classes.

The classification of finite groups is not a trivial task, as there is no known algorithm deciding whether a given automaton generates a finite group.

Moreover, it has been recently shown by Gillibert [9] that the finiteness problem for automaton semigroups is undecidable. It is a common belief that the corresponding problem for groups is also undecidable. However, this conjecture is still open.

Chapter 4
Infinite Groups

Throughout this section, we describe different algorithms used for determining whether a group is infinite. These algorithms include proving the infinite order of an element by a few different methods. These methods include

- Looking at the sections of powers of group elements
- Determining if a group is self-replicating
- Strategies for reversible automata
- Using orbit techniques.

Note: These algorithms could have been used in any order, as a group may be found to be infinite by one or more of the above methods. In this paper, once a group is found to be infinite by one method, it is no longer tested by any other method. This maximizes computer efficiency. Below, we describe how each algorithm works and the results produced by each.

4.1 Method of proving the infinite order of an element by looking at the sections of its powers

The `AutomGrp` command `FindElementOfInfiniteOrder` was one of the most fruitful commands used in determining if a group is infinite. The command has three input arguments, first being the proposed group, second being maximum length (referring to elements), and third being maximum depth (referring to sections of every element up to a certain depth).

This command will return an element of infinite order if one is found; otherwise it will return `fail`.

To find an element of infinite order, GAP runs over all nontrivial elements g of $G(A)$ up to a proposed length and vertices $v \in X^*$ up to a proposed depth. It is based on the following proposition.

Proposition 4.1.1. *Let g be an element of $G(A)$. If there is $n > 1$ such g^n fixes some vertex v and $g^n|_v$ is conjugate to g, then g has infinite order.*

Proof. Suppose g has a finite order m and $g^n|_v = g^h$ for some $h \in G$. Then since g^n fixes v, for each k we have

$$(g^n)^k|_v = (g^k)^h.$$

Therefore, if $g^k \neq 1$, we have $(g^n)^k \neq 1$. Therefore, the order of g^n must be a multiple of the order of g, which is a contradiction. □

Essentially, the algorithm `FindElementOfInfiniteOrder` enumerates all elements of $G(A)$ and for each $g \in G(A)$ tries to find n and v that satisfy the above proposition. Since there are many such automata, the results of this algorithm are listed at the end of the paper in Appendix A.

4.2 Self-replicating groups

Another way of determining if a group is infinite is by using the self-replicating property. A few definitons from [4] give us insight into this notion.

Definition 4.2.1. The *n-th level stabilizer* of $G(A)$, or $Stab_{G(A)}(n)$, is the normal subgroup of G that trivially permutes the tree X^* up to (and including) level n.

Definition 4.2.2. A self-similar group G is *self-replicating* if, for every vertex u in X^*, the homomorphism $\phi_u : Stab_G(u) \to G$ from the stabilizer of the vertex u in G to G, given by $\phi(g) = g_u$ is surjective.

We rely on the following proposition to show that a group is infinite:

Proposition 4.2.3. *If a group is self-replicating and transitive on the first level, then it is infinite.*

Proof. Let G be a group. If G is self-replicating, then $Stab_G(x) \to G$ is onto, but $Stab_G(x)$ is a normal subgroup of G of finite index such that $Stab_G(x) \neq G$. If $|G| < \infty$, then $|Stab_G(x)| < |G|$, a contradiction. Hence, $|G| = \infty$ □

The algorithm works in the following way:

Let $G = \langle S \rangle$ be an automaton group. For each $x \in X$,

1. Find a generating set S' of $Stab_G(x)$ using Reidemeister-Schreier method.
2. Let $H = \langle g|_x : g \in S' \rangle$
3. Check if each $s \in S$ belongs to H.

Using this method, the following equivalence classes (with respect to minimal symmetry) were found to be infinite:

65633, 65638, 65643, 65960, 65980, 66028, 66259, 66993, 66998, 67003, 67004, 67008, 67063, 67185, 67190, 67195, 67200, 67249, 67254, 67259, 67264, 67281, 67286, 67291, 67296, 67297, 67302, 67307, 67312, 67313, 67318, 67323, 67328, 67537, 67553, 67558, 67568, 68209, 68214, 68224, 68561, 68566, 68576, 69906, 69954, 69985, 69986, 69990, 69995, 70033, 70034, 70038, 70043, 70049, 70054, 70059, 70354, 70370, 70396, 70497, 70500, 70502, 70503, 70507, 70512, 70545, 70548, 70550, 70555, 70560, 70561, 70564, 70566, 70571, 70576, 70577, 70582, 70587, 70592, 70625, 70626, 70630, 70635, 70640, 70754, 70770, 70827, 70833, 70838, 70843, 70844, 70881, 70886, 70891, 70892, 71052, 71100, 71148, 71191, 71281, 71286, 71291, 71296, 71345, 71350, 71351, 71355, 71360, 71382, 71387, 71393, 71398, 71403, 71408, 71409, 71414, 71419, 71424, 71521, 71522, 71526, 71527, 71531, 71536, 71553, 71558, 71563, 71568, 71570, 71574, 71579, 71584, 71585, 71590, 71595, 71600, 71649, 71654, 71659, 71664, 71858, 71959, 72049, 72054, 72055, 72059, 72064, 72113, 72118, 72119, 72123, 72128, 72145, 72150, 72151, 72155, 72160, 72161, 72166, 72167, 72171, 72176, 72177, 72182, 72183, 72187, 72192, 72210, 72258, 72305, 72306, 72310, 72320, 72372, 72401, 72402, 72406, 72416, 72433, 72438, 72448, 72545, 72546, 72550, 72555, 72560, 72577, 72578, 72582, 72587, 72592, 72593, 72598, 72603, 72604, 72608, 72620, 72657, 72662, 72667, 72672, 72673, 72678, 72683, 72688, 72801, 72804, 72806, 72811, 72816, 72865, 72870, 72875, 72880, 72929, 72934, 72939, 72944, 73057, 73058, 73062, 73067, 73072, 73105, 73106, 73110, 73115, 73120, 73121, 73126, 73131, 73132, 73136, 73137, 73142, 73147, 73152, 73212, 73292, 73299, 73313, 73318, 73319, 73323, 73328, 73329, 73330, 73334, 73339, 73344, 73361, 73366, 73371, 73372, 73376, 73393, 73398, 73403, 73408, 73569, 73574, 73579, 73584, 73617, 73622, 73627, 73632, 73633, 73638, 73643, 73648, 73698, 79431, 79480, 79543, 79576, 79604, 79699, 79719, 80359, 80504, 80600, 80860, 196615, 196976, 197128, 197473, 197488, 197558, 197932, 197935, 198192, 198208, 198241, 198256, 198336, 198408, 198411, 198427, 198440, 198446, 198459, 198491, 198512, 198561, 198566, 198568, 198576, 198647, 198651, 199176, 199352, 199432, 199512, 199515, 199520, 199590, 199600, 199670, 199671, 199675, 200471, 200475, 200631, 200634, 200695, 200971, 200979, 200985, 201051, 201250, 201252, 201253, 201261, 201330, 201340, 201397, 201483, 201491, 201492, 201497, 201501, 201563, 201572, 201581, 201633, 201638, 201648, 201658, 201723, 201747, 201753, 202263, 202266, 202268, 202429, 202507, 202540, 202587, 202597, 202610, 202613, 202657, 202662, 202672, 202747, 203280, 203284, 203293, 203360, 203531, 203536, 203546, 203548, 203554, 203611, 203616, 203628, 203631, 203681, 203686, 203696, 203698, 203761, 203766, 203771, 204555, 204587, 204635, 204720, 205346, 205349, 205353, 205395, 206521, 206600, 206617, 206638, 206640, 206650, 206701, 206713, 206882, 206889, 206931, 207289, 207390, 207392, 207627, 207632, 207635, 207640, 207648, 207657, 207699, 207707, 207712, 207777, 207782, 207792, 207795, 207801, 207857, 207862, 207867, 208136, 208147, 208168, 208174, 208176, 208249, 208395, 208409, 208414, 208425, 208467, 208472, 208475, 208480, 208545, 208550, 208560, 208562, 208563, 208565, 208569, 208625, 208630, 208635, 208651, 208672, 208678, 208680, 208686, 208688, 208731, 208736, 208768, 208801, 208806, 208886, 210478, 210490, 210492, 210541, 210553, 211258, 211260, 211300, 211309, 211506, 211509, 211517, 211540, 211620, 211765, 211770, 211772, 211775, 211804, 211813, 211896, 211902, 211903, 212530, 212535, 212540, 212543, 212572, 212578, 212591, 212664, 458844, 458860, 458862, 458863, 458864, 458934, 458943, 459068, 459084, 459152, 459215, 459344, 459374, 459494, 459694, 459702, 459719, 459722, 460209, 462895, 462944, 462972, 463024, 463029, 463034, 463094, 463172, 463276, 463279, 463280, 463295, 463296, 463340, 463344, 463437, 463586, 463589, 463635, 463641, 463643, 463749, 463797, 463851, 463867, 465424, 465428, 465437, 465687, 465690, 465695, 465738, 465741, 466699, 466779, 471140, 471347, 471353, 471388, 471491, 471497, 471891, 471895, 471897, 471901, 471912, 471913, 471918, 471927, 471957, 471960, 471966, 471975, 471976, 471991, 472002, 472003, 472025, 472026, 472829, 472915, 472921, 473719

4.2.1 Automaton 460219

$a = (c, c)\sigma$

$b = (d, c)\sigma$

$c = (b, b)\sigma$

$d = (a, a)$

Proposition 4.2.4. *Automaton 460219 generates an infinite group.*

Proof. Let $S = Stab_G(1) = \langle ba^{-1}, ca^{-1}, d, a^2, ab, ac, ada^{-1}\rangle$.

$$S|_0 = \left\langle dc^{-1}, bc^{-1}, a, c^2, cb, cac^{-1} \right\rangle.$$

Using GAP, we find $S|_0$ to be self-replicating, which implies that S, and in turn, G, are infinite.

We include GAP output below:

```
gap> T:=AutomatonGroup(list4x2[460219][1],["a","b","c","d"]);
< a, b, c, d >
gap> Print(T);
< a = (c, c)(1,2),
  b = (d, c)(1,2),
  c = (b, b)(1,2),
  d = (a, a) >
gap> S:=StabilizerOfFirstLevel(T);
< b*a^-1, c*a^-1, d, a^2, a*b, a*c, a*d*a^-1 >
gap> S1:=Section(S,1);
< d*c^-1, b*c^-1, a, c^2, c*b, c*a*c^-1 >
gap> IsFractal(S1);
true
```

□

4.3 Methods for reversible automata

The techniques described above are not always efficient and sometimes do not produce the desired outcome. Especially, the above methods do not perform well in the case of, so-called, reversible automata. Recently, a new partial method for deciding the finiteness of automaton (semi)groups was developed by French group consisting of Akhavi, Klimann, Lombardy, Mairesse, and Picantin in [1]. It was later proved by Klimann in [18] that this method always works in the case of invertible-reversible automata. To explain the method, we introduce several notions first.

Definition 4.3.1. Let $A = (S, X, \pi, \tau)$ be a finite automaton. The *dual automaton* of A is an automaton $D(A) = (X, S, \hat{\pi}, \hat{\tau})$, such that $\hat{\pi}\colon X \times S \to X$, given by $\hat{\pi}(x, s) = \pi(s, x)$ is the *output map* and $\hat{\tau}\colon X \times S \to S$, given by $\hat{\tau}(x, s) = \tau(s, x)$ is the *transition map.*

Definition 4.3.2. An automaton A is called *reversible* if its dual automaton $D(A)$ is invertible.

The idea here is that an automaton A describes the action of the states of A on the letters of X, while the dual automaton describes the action of the letters of X on the states of A.

We now introduce the notions MD-Triviality and MD-Reduction. *MD-Reduction* consists of minimizing an automaton A, then taking the dual of the minimization, and repeating until both the automaton and its dual are minimal. If this process terminates with the pair of two trivial Mealy automata, then A is said to be *MD-Trivial.* The fact that MD-trivial automata generate finite groups is an extension of the following proposition proved implicitly in [21] and explicitly in [23].

Proposition 4.3.3. *An automaton A generates a finite (semi)group if and only if its dual automaton $D(A)$ generates a finite (semi)group.*

The more complicated direction is the converse one developed by Klimann in [18]. The theorem below provides a sufficient and necessary condition for determining the infiniteness of groups generated by invertible-reversible Mealy automata with two states (and hence, whose dual has a 2-letter alphabet):

Theorem 4.3.4 ([18]). *Let A be a two-state invertible-reversible Mealy automaton. It generates a finite group if and only if it is MD-Trivial.*

We take the contrapositive of the above theorem, where IsIRAutomaton determines if an automaton is invertible-reversible and `IsMDTrivial` determines if an automaton is MD-Trivial. Then, the conjunction "`IsIRAutomaton and not IsMDTrivial`" should give our desired result in GAP. We use the following code to print automata generating infinite groups:

```
#unsure contains our isomorphism class representatives still to be tested

for i in unsure do
  j:=MealyAutomaton(i)
  if IsIRAutomaton(j) and not IsMDTrivial(j) then
        Print(i, "\n");
  fi;
od;
```

We found that automata 67547, 70832, 71312, 206974, 460223, 460478, and 460538 generate infinite groups using this method.

4.3.1 Existence of infinite periodic $(4, 2)$-groups

A natural question in this research is whether any infinite periodic groups exist in the class of $(4, 2)$-groups. If they exist, then it would show that the Grigorchuk Group (generated by a 5-state automaton) is the "smallest" counterexample to the Burnside problem in the class of automata over a 2-letter alphabet. From our research, we conclude the following:

Conjecture 4.3.5. *There exist no infinite periodic $(4, 2)$-groups.*

Theorem 4.3.6. *There are only 6 classes of $(4, 2)$-automata with respect to minimal symmetry (70832, 206974, 460219, 460223, 460478, and 460538) that could disprove the previous conjecture.*

Through our own research, along with some collaboration with Suzana Milea of the University of South Florida, we have shown all classes of $(4,2)$-automata (excluding the six listed in the previous theorem) have an element of infinite order. It is our general belief, though, that elements of infinite order exist for groups generated by the six classes listed above.

4.4 Examining Properties of Sub-Automata

Another fruitful procedure is locating sub-automata within our class representatives that generate infinite groups. Clearly, if automaton A contains sub-automaton B and B generates an infinite group, then A generates an infinite group. This procedure relies heavily on research previously done on (3,2) and (2,2)-automata. We use a GAP to complete the following tasks for representative $A = (S, X, \pi, \tau)$ of each class:

1. For each state $s \in S$, generate a minimal automaton B containing s (so-called, *self-similar closure* of s);

2. Let $|S|$ denote the number of states of B

3. If $|S| \leq 3$, use libraries of automata with smaller number of states to examine the properties of the group generated by B.

The following GAP code was used:

```
for i in [1..Length(unsure)] do
 Print("******* Group ",unsure[i][2],"*********************");
 G:=AutomatonGroup(unsure[i][1];
  for s in GeneratorsOfGroup(G) do
  A:=MealyAutomaton(s);
   if NumberOfStates(A)<4 then
```

```
    Print(unsure[i][2],", state ",s,":\n");
     if NumberOfStates(A)=3 then
      Print(list[list[NumOfAut([AutomatonList(A)])][2]]);
     else
      Print(A);
     fi;
    fi;
  od;
od;
```

For example, this code found the following information for automaton 65920 that uses the database of (3,2)-groups from [4]:

```
******* Group 65920*********************
65920, state a1:
a1 = (a2, a2)(1,2), a2 = (a1, a1)


65920, state a2:
[ [ [ [ 1, 1, (1,2) ], [ 3, 2, () ], [ 1, 1, () ] ] ], 775,, "$2853\\rtimes C_2$", "Non ergodic",
  [ 1, 4, 9, 17, 30, 51, 85, 140, 229, 367, 579 ], "Contracting",, "Fractal",
  [ 1, 2, 4, 6, 9, 15, 26, 48 ],
  [ "a^{2}", "b^{2}", "c^{2}", "caca", "bcbabcbacbcbabcbca", "bcbabcbcabcbabcbca",
      "bcbacbcbcacbcbcabcba", "cbcbacbcbabcbcabcbca", "cbcbacbcbcacbcbcabcbca" ],
      775, 775 ]


65920, state a4:
a1 = (a2, a2), a2 = (a1, a1)(1,2)
```

We can easily find that states $a1$ and $a4$ generate the finite group $C_2 \times C_2$, but this does not help us determine the infiniteness of the group. However, the self-similar closure of state $a2$ generates the group $C_2 \ltimes IMG((\frac{z-1}{z+1})^2)$ according to [4], which is self-replicating and, hence, infinite. The sub-automaton generated by a3 is not shown because it generates A in its entirety. Hence, A generates an infinite group. Many other groups were found to be infinite using this method, and will be described in more detail in the Appendix B.

4.5 Infinite Abelian Groups

Infinite abelian groups generated by (4,2)-automata represent a very interesting class of groups. Classifying every group in this class is no small task. Of the 90 classes generating abelian groups, 32 classes generate infinite groups. These classes are listed below:
65538, 65618, 65619, 65623, 65698, 65699, 65703, 67058, 67059, 67060, 67063, 67064, 67068, 68338, 68339, 68340, 68343, 68344, 68348, 87378, 197934, 197946, 197997, 198009, 199196, 199210, 199225, 199273, 200493, 200629, 240170, 240233

4.6 Using Orbit Techniques

Many of the methods described above were not successful with three particular automata classes, namely 68598, 460534, and 460794. `FindElementOfInfiniteOrder` and `IsFractal` both proved to be unsuccessful (after many hours of computing time). To show that these classes generate infinite groups, we rely mainly on a technique from a paper by Elder, Davis, and Reeves on non-contracting groups [22]. This technique uses the `AutomGrp` command `PrintOrbitOfVertex`. This command takes a word $v \in X^*$ and prints its orbit for a given number of iterations of a specified tree automorphism g in a group G. The goal is to locate some $g \in G$, some $v \in X^*$, and some $m > 0$ such that $g^{-m}(v) \neq g^n(v), \forall n > 0$. If this condition is satisfied, then g has infinite order.

Below, we give a more detailed proof that these three classes generate infinite groups:

4.6.1 Automaton 68598

$a = (b, b)\sigma$

$b = (d, d)$

$c = (d, c)$

$d = (a, a)$

(See Figure 1 for Moore diagram).

Proposition 4.6.1. *Element cb has infinite order.*

Proof. We begin by producing automaton cb which is the prouct of the two initial automata with initial states c and b (see Figure 2).

From 2, we see that $\forall i : a_i|_{011}, a_i|_{000} \in \{1, a_4, a_5, a_6\}$ and

$a_6(0^\infty) = (011)^\infty$ $\quad$ $a_6((011)^\infty) = (0)^\infty$

$a_5(0^\infty) = 1(011)^\infty$ $\quad$ $a_5((011)^\infty) = 11(011)^\infty$

$a_4(0^\infty) = 11(011)^\infty$ $\quad$ $a_4((011)^\infty) = 1(011)^\infty$

Therefore, $(cb)^n(1^\infty)$ always terminates with either 0^∞ or $(011)^\infty$ for $n \geq 1$. But, we see that $(cb)^{-1}(1^\infty) = 11(010)^\infty \neq (cb)^n(1^\infty), n \geq 0$. Thus, cb has infinite order. □

4.6.2 Automaton 460534

$a = (b, b)\sigma$

$b = (d, d)\sigma$

$c = (c, b)\sigma$

$d = (a, a)$

See Figure 3 for Moore diagram.

Proposition 4.6.2. *Element c has infinite order.*

Proof. First, we refer to Figure 5, which displays output for 50 positive iterations of c acting on a finite string of $1's$. This figure motivates us to define all outputs of $c^n(1^\infty)$ for $n \geq 0$.

We see in Figure 3 that states a, b, and d form a cycle, and after reading a word containing 1, the automaton will be in of the states a, b, or d. We observe:

$a(1^\infty) = (001)^\infty$ $\quad$ $a((001)^\infty) = 1^\infty$

$b(1^\infty) = 01(001)^\infty$ $\quad$ $b((001)^\infty) = 1(001)^\infty$

$d(1^\infty) = 1(001)^\infty$ $\quad$ $d((001)^\infty) = 01(001)^\infty$

This implies that $c^n(1^\infty)$ is either equal to $w1^\infty$ or $w(001)^\infty$ (depending on choice of n) for $n \geq 0$ and some word w. On the other hand, $c^{-1}(1^\infty) = 0^\infty$. Thus, c has infinite order. □

4.6.3 Automaton 460794

$a = (b, c)\sigma$

$b = (d, d)\sigma$

$c = (d, b)\sigma$

$d = (a, a)$

See Figure 4 for Moore Diagram.

Proposition 4.6.3. *Element a has infinite order.*

Proof. We begin by noticing the following:

$a\|_{011} = a$	$a((011)^\infty) = 1(011)^\infty$
$b\|_{011} = c$	$c((011)^\infty) = 11(011)^\infty$
$c\|_{011} = c$	$b((011)^\infty) = 11(011)^\infty$
$d\|_{011} = b$	

The left column shows that by taking sections $x|_{011}$ for $x \in \{a, b, c, d\}$ we find that by reading the word $(011)^\infty$, the automaton will end up in state a, b, or c. The right column implies that the tail of $(011)^\infty$ is preserved under positive iterations of a. So we have,

$$a^n(0^\infty) = \begin{cases} 0^\infty & : n = 0 \\ w(001)^\infty & : n \geq 1 \end{cases}$$

for some word $w \in X^*$. But, $a^{-1}(0^\infty) = (1110)^\infty \neq a^n(0^\infty), n \geq 0$. Thus, a has infinite order. $\square$

4.7 Figures

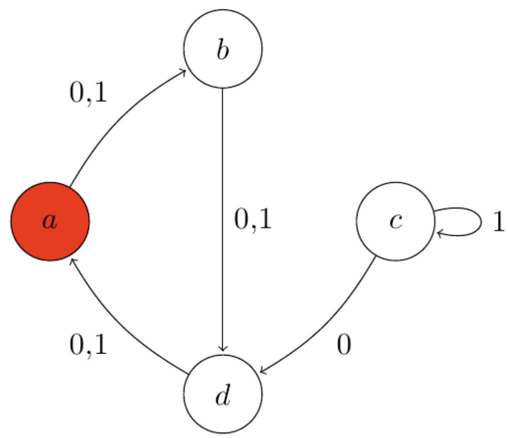

Figure 1.: Moore Diagram for automaton 68598

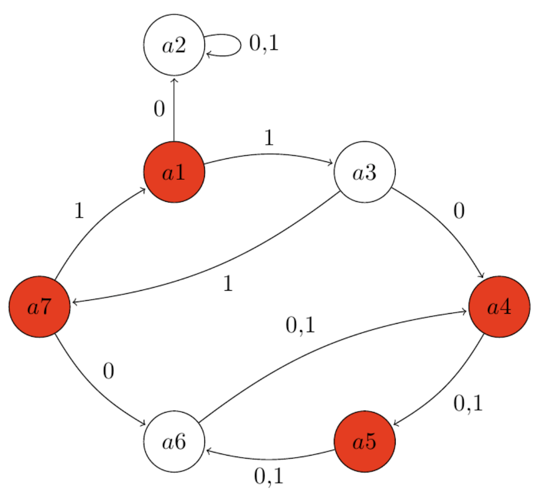

Figure 2.: Automaton for $c * b$ of automaton 68598

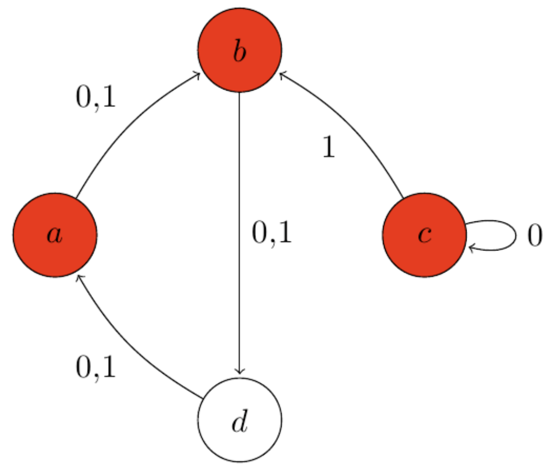

Figure 3.: Moore Diagram for automaton 460534

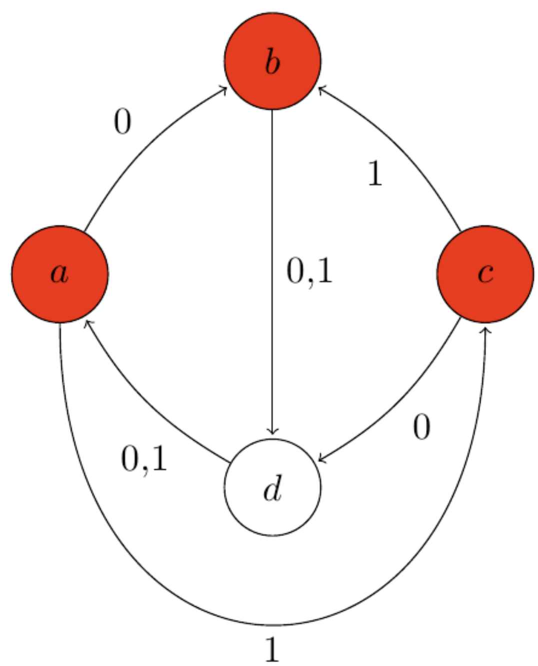

Figure 4.: Moore Diagram for automaton 460794

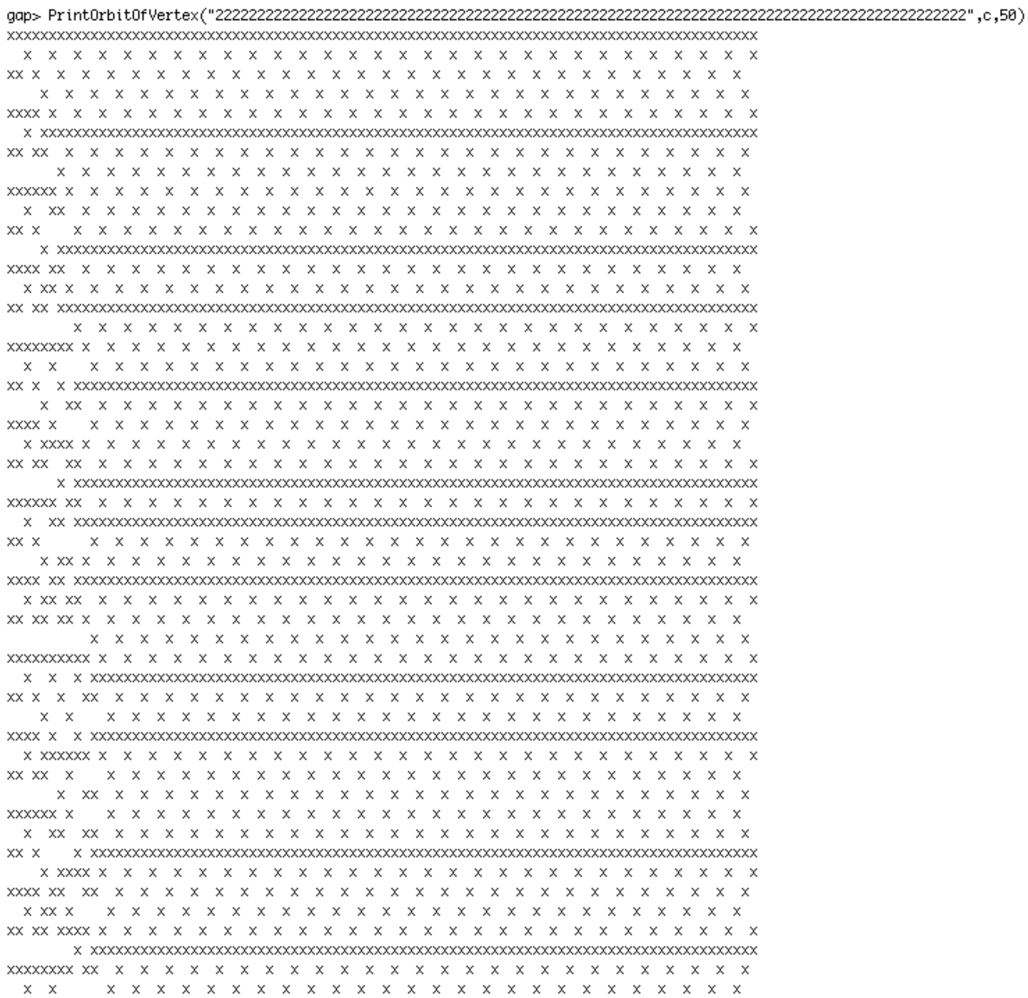

Figure 5.: c^n acting on a finite string of $1's$ for $0 \le n \le 50$.

Chapter 5

Finite Groups

After filtering out all automata equivalence classes that are known to generate infinite groups, we are left with a small pool of classes. From here, we can run an algorithm to test if these classes generate finite groups. The algorithm used is `LevelOfFaithfulAction` and is found in the GAP package `AutomGrp`.

`LevelOfFaithfulAction`($\langle G \rangle$, $\langle MaxLev \rangle$);

This procedure searches the tree up to MaxLev (words of length up to and including MaxLev) for a level n such that the $n-th$ level stabilizer of n in $\langle G \rangle$ is trivial. If such n is found, then the automaton generates a finite group. Of the 7471 equivalence classes, 231 were found to be finite. There are a total of 14 different finite groups generated by (4,2)-automata. The largest finite group, $((C_2 \times C_2 \times ((C_2 \times C_2 \times C_2 \times C_2) : C_2)) : C_2) : C_2$, is generated by automaton number 73563, and has order 512.

The following is a list of each finite group, along with some basic information.

Group: Trivial Group

Order: 1

Abelian: Yes

Automata Classes: 1

Group: C_2

Order: 2

Abelian: Yes

Automata Classes: 87377, 87382, 983041

Group: $C_2 \times C_2$

Order: 4

Abelian: Yes

Automata Classes: 65537, 65542, 65617, 65622, 65627, 196699, 196774, 199179, 199264, 199329, 199414, 240139, 240219

Group: $C_2 \times C_2 \times C_2$

Order: 8

Abelian: Yes

Automata Classes: 65697, 65702, 65707, 68337, 68342, 68347, 68352, 196619, 196769, 197899, 197984, 199184, 199259, 199334, 199344, 199409, 199419, 200464, 200539, 200544, 200614, 200689, 200694, 458848, 458998, 461558, 520288, 520368

Group: D_8

Order: 8

Abelian: No

Automata Classes: 65873, 65878, 65888, 69969, 69974, 69979, 70737, 70742, 198496, 198646, 199201, 199211, 199216, 207371, 207393, 207403, 207521, 240161, 240171, 460640, 460790, 520256, 521153, 521168, 521201

Group: $C_2 \times D_8$

Order: 16

Abelian: No

Automata Classes: 65569, 65579, 66385, 66390, 66400, 66497, 66512, 66545, 66550, 66560, 67409, 67414, 67424, 196641, 196651, 196960, 197110, 197392, 197425, 197440, 197472, 197617, 197622, 199217, 199227, 199232, 199345, 199355, 199360, 200497, 200507, 200512, 203275, 203355, 207451, 207526, 458816, 459104, 459254, 459616, 459713, 459728, 459761, 459766, 520240, 520384, 520544, 521056

Group: $C_2 \times C_2 \times C_2 \times C_2$

Order: 16

Abelian: Yes

Automata Classes: 67057, 67062, 67067, 67072, 197904, 197979, 200459, 200609, 200624, 200699, 458928, 460273, 460283

Group: $(C_2 \times C_2 \times C_2 \times C_2) : C_2$

Order: 32

Abelian: No

Automata Classes: 65883, 70747

Group: $C_2 \times C_2 \times D_8$

Order: 32

Abelian: No

Automata Classes: 66033, 66038, 66048, 67569, 67574, 67584, 196880, 197105, 198416, 198641, 200481, 200491, 200496, 200625, 200635, 200640, 458800, 458944, 459249, 460785

Group: $((((C_4 \times C_2) : C_2) : C_2) : C_2)$

Order: 64

Abelian: No

Automata Classes: 73553, 73558, 73568, 199255, 199335, 207376, 207408, 207536, 207601, 207611, 520280, 520440

Group: $C_2 \times ((((C_4 \times C_2) : C_2) : C_2) : C_2)$

Order: 128

Abelian: No

Automata Classes: 66465, 66480, 197537, 197552, 197937, 197952, 459681, 459696

Group: $(D_8 \times D_8) : C_2$

Order: 128

Abelian: No

Automata Classes: 70481, 70486, 70491, 70496, 71505, 71510, 71515, 71520, 72321, 72326, 72331, 73041, 73046, 73051, 73056, 203297, 203307, 207409, 207419, 207424, 207447, 207527, 207537, 207547, 207552, 520497, 520512, 520536, 520656, 521048, 521054, 521152, 521200

Group: $C_2 \times ((D_8 \times D_8) : C_2)$

Order: 256

Abelian: No

Automata Classes: 65841, 65856, 65985, 66000, 66481, 66496, 66529, 66544, 196913, 196928, 197409, 197424, 197553, 197568, 198449, 198464, 459057, 459072, 459201, 459216, 459697, 459712, 459745, 459760

Group: $((C_2 \times C_2 \times ((C_2 \times C_2 \times C_2 \times C_2) : C_2)) : C_2) : C_2$

Order: 512

Abelian: No

Automata Classes: 73563

Chapter 6
Open Questions

We conclude with a list of open questions motivated by the research done in this paper.

1. *Is the Grigorchuk group the smallest automaton generating an infinite Burnside group?*

2. *Develop a partial algorithm that will automate the search for an element of infinite order using orbit techniques from Chapter 4.*

3. *Describe all abelian and niloptent groups up to isomorphism.*

The research from this project will hopefully be a basis for answering some of the following questions formulated in the paper "Self-Similar Groups Acting Essentially Freely on the Boundary of the Binary Rooted Tree" [12].

4. *Is there a group generated by finite automaton that acts neither essentially freely, nor totally non-freely on the boundary of a rooted tree? (Recall that the action is totally non-free if stabilizers of different points of the set of full measure are different.)*

5. *Does the total non-freeness of an action of a group generated by a finite automaton on ∂T imply weak branchness? Observe, that the converse is true. Does the total non-freeness of an action of a group generated by finite automaton on ∂T imply weak branchness? (Observe that the converse is true [12].)*

6. *Classify all* $(4, 2)$*-groups that act essentially freely on the boundaries of corresponding rooted trees.*

7. *Is there a hereditary just-infinite group generated by finite automaton? (See Proposition 3.12 in [12] for motivation).*

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Appendix A

Groups that have elements of infinite order

The following is the detailed findings for the command `FindElementOfInfiniteOrder` in the `GAP` package `AutomGrp`. If an element of infinite order was found, it is listed below next to its minimal symmetry class representative.

65538 a
65553 ab
65555 a
65558 ab
65559 ac^{-1}
65563 ab
65570 ab^{-1}
65571 a
65575 ab
65618 a
65619 a
65623 a
65634 a
65635 a
65639 ac^{-1}
65698 a
65699 a
65703 ab^{-1}
65842 ab^{-1}
65843 a
65844 a
65847 ac
65848 ab
65851 $cacb$
65851 $cacb$
65852 ac^{-1}
65857 ab
65859 ab^{-1}
65860 a
65862 ab
65863 a
65864 ad^{-1}
65867 ab
65868 ab^{-1}
65872 ab
65874 a
65875 a
65876 a
65879 a
65880 a
65884 a
65891 a
65892 a
65895 ac
65896 ad
65900 ab^{-1}
65904 ab
65904 ab
65906 a
65907 ab^{-1}
65908 a
65911 ac
65912 ad^{-1}
65915 ab
65915 ab
65916 ac
65921 ab
65922 ab^{-1}
65924 a
65926 ab
65927 a
65928 ab^{-1}
65936 ab
65939 ab
65940 a
65943 a
65944 ad
65948 ab^{-1}
65952 $dcacdb$
65954 a
65955 ac^{-1}
65956 a
65959 a
65964 ac
65969 abc
65970 ab^{-1}
65971 ac^{-1}
65972 a
65974 ab
65975 a
65976 ab^{-1}
65979 ac
65984 abc
65986 ab^{-1}
65987 ab^{-1}
65988 a
65991 ac
65992 ab
65995 ba
65995 ba
65996 ac^{-1}
66002 a
66003 ab^{-1}
66004 a
66007 ac
66008 ad^{-1}
66011 ba
66011 ba
66012 ac
66017 abc
66018 ab^{-1}
66019 ac^{-1}
66020 a
66022 abc
66023 ac
66024 ab^{-1}
66027 abc
66032 abc
66034 a
66035 ac^{-1}
66036 a
66039 ac
66040 ab^{-1}
66043 $abcacdad$
66044 ac
66097 ac
66098 ac
66100 a
66102 ac

66103 ab^{-1}	66161 ac	66228 a	66280 ab^{-1}	66419 ab^{-1}	66498 ab^{-1}
66104 ab	66162 a	66230 ac	66283 ac	66420 a	66500 a
66107 ac	66163 ab^{-1}	66231 ad	66284 ad^{-1}	66423 ac	66503 ad^{-1}
66108 ad^{-1}	66164 a	66232 ab^{-1}	66288 ac	66424 ad^{-1}	66504 ab
66112 ac	66166 ac	66235 ac	66289 ac	66428 ac	66514 a
66113 ab	66167 ab^{-1}	66236 ad^{-1}	66290 a	66449 abc	66515 ab^{-1}
66114 ac^{-1}	66168 ac	66240 ac	66292 a	66450 ac^{-1}	66516 a
66116 a	66171 ac	66241 ac	66294 ac	66451 ab	66519 ac
66118 ab	66172 ab^{-1}	66242 ac^{-1}	66295 ab^{-1}	66452 a	66520 ad^{-1}
66119 a	66176 ac	66244 a	66296 ac	66454 abc	66524 ac
66120 ad^{-1}	66193 ac	66246 ac	66299 ac	66455 ac^{-1}	66530 ac^{-1}
66128 ab	66194 ac^{-1}	66247 ad^{-1}	66300 ad^{-1}	66456 ad^{-1}	66531 ac^{-1}
66129 ac	66195 ab	66248 ab	66304 ac	66459 abc	66532 a
66130 a	66196 a	66251 ac	66386 a	66460 ac	66534 $(ab)^2c$
66132 a	66198 ac	66252 ad^{-1}	66387 ac^{-1}	66464 abc	66535 ad^{-1}
66134 ac	66200 ad^{-1}	66256 ac	66388 a	66466 a	66536 ad
66135 a	66203 ac	66257 ac	66391 a	66467 ac^{-1}	66540 ac
66136 a	66204 ab^{-1}	66258 a	66392 a	66468 a	66546 a
66139 ac	66208 ac	66260 a	66396 ac	66470 $bcabdc$	66547 ac^{-1}
66140 ad^{-1}	66209 ac	66262 ac	66401 abc	66471 a	66548 a
66144 ac	66210 a	66263 ab^{-1}	66402 ac^{-1}	66472 ab^{-1}	66551 ac
66145 ac	66212 a	66264 ac	66403 ab	66476 ac	66552 ab^{-1}
66146 ab^{-1}	66214 ac	66267 ac	66404 a	66482 ac^{-1}	66556 ac
66147 ab	66215 ab^{-1}	66268 ad^{-1}	66406 abc	66483 ac^{-1}	66930 a
66148 a	66216 a	66272 ac	66407 ad^{-1}	66484 a	66931 a
66150 ac	66219 ac	66273 ac	66408 ad^{-1}	66486 $bcbd$	66932 a
66152 ad^{-1}	66220 ad^{-1}	66274 ab^{-1}	66411 abc	66486 $bcbd$	66935 ab^{-1}
66155 ac	66224 ac	66276 a	66412 ac	66487 ad^{-1}	66936 ad^{-1}
66156 ab^{-1}	66225 ac	66278 ac	66416 abc	66488 ad	66940 ab^{-1}
66160 ac	66226 ac^{-1}	66279 ac	66418 a	66492 ac	66994 a

66995 a	67303 ad^{-1}	67579 $(ac)^2$	69959 a	70231 a	70283 ab
66996 a	67304 ac^{-1}	67580 ab^{-1}	69963 ab	70232 a	70284 a
66999 ac^{-1}	67308 ac^{-1}	68210 a	69970 a	70235 ac	70288 ab
67000 ac^{-1}	67314 a	68211 a	69971 a	70236 a	70289 ac
67058 a	67315 a	68212 a	69975 a	70240 ac	70290 ac^{-1}
67059 a	67316 a	68215 a	69987 a	70241 ac	70291 ab
67060 a	67319 ab^{-1}	68216 ad^{-1}	69991 ac	70242 ab^{-1}	70292 ac^{-1}
67064 ab^{-1}	67320 ac^{-1}	68219 ab	70017 ab	70243 ab	70294 ac
67068 ab^{-1}	67324 ab^{-1}	68220 a	70018 ab^{-1}	70244 ab^{-1}	70296 ad
67186 a	67410 a	68338 a	70022 ab	70246 ac	70299 ac
67187 a	67411 a	68339 a	70023 a	70248 ad	70300 ad^{-1}
67188 a	67412 a	68340 a	70035 ab	70251 ac	70304 ac
67191 ab^{-1}	67415 a	68343 a	70039 a	70252 ab^{-1}	70305 ac
67192 ac^{-1}	67416 a	68344 ab^{-1}	70050 a	70256 ac	70306 a
67196 ad^{-1}	67420 a	68348 a	70051 ac^{-1}	70257 ac	70308 a
67250 a	67538 a	68562 a	70055 a	70258 ac	70310 ac
67251 a	67539 a	68564 a	70209 ab	70259 ab^{-1}	70311 ab^{-1}
67252 a	67540 a	68567 ab^{-1}	70210 ac^{-1}	70260 a	70312 ad
67255 ab^{-1}	67543 ab^{-1}	68568 ad^{-1}	70211 ab^{-1}	70262 ac	70315 ac
67256 ac^{-1}	67544 ad^{-1}	68594 a	70212 ab^{-1}	70263 ab^{-1}	70316 ad^{-1}
67260 ac^{-1}	67548 ab^{-1}	68595 a	70214 ab	70264 ad	70320 ac
67282 a	67554 a	68596 a	70215 a	70267 ac	70321 ac
67283 a	67556 a	68599 ab^{-1}	70216 a	70268 ab^{-1}	70322 ac^{-1}
67284 a	67559 ab^{-1}	68600 ac^{-1}	70219 ab	70272 ac	70323 abd
67287 ab^{-1}	67560 ab^{-1}	68604 ad^{-1}	70220 a	70273 ab	70324 ac^{-1}
67288 ac^{-1}	67570 a	69905 ab	70224 ab	70274 ac^{-1}	70326 ac
67292 ad^{-1}	67571 a	69910 ab	70225 ac	70276 ac^{-1}	70327 ab^{-1}
67298 a	67572 a	69953 ab	70226 a	70278 ab	70328 ad
67299 a	67575 ad^{-1}	69955 ab^{-1}	70228 a	70279 a	70331 ac
67300 a	67576 ab^{-1}	69958 ab	70230 ac	70280 a	70332 ad^{-1}

70336 ac
70337 ab
70338 ab^{-1}
70339 ad^{-1}
70340 ad^{-1}
70342 ab
70343 a
70344 a
70347 ab
70348 a
70352 ab
70353 ac
70355 ad^{-1}
70356 ab
70358 ac
70359 ab^{-1}
70360 a
70363 ac
70364 ad^{-1}
70368 ac
70369 ac
70371 ad^{-1}
70372 ad^{-1}
70374 ac
70375 ab^{-1}
70376 a
70379 ac
70380 ad^{-1}
70384 ac
70385 ac
70386 a
70387 ad^{-1}
70388 ac
70390 ac
70391 ab^{-1}
70392 a
70395 ac
70400 ac
70465 ab
70467 ab^{-1}
70468 ab^{-1}
70470 ab
70471 a
70472 a
70475 ab
70476 a
70480 ab
70482 a
70483 a
70484 a
70487 a
70488 a
70492 a
70498 ab^{-1}
70499 ab
70504 ad
70508 ac
70515 ab^{-1}
70516 a
70519 ab^{-1}
70520 ad
70523 ab
70523 ab
70524 ac
70529 ab
70530 ac^{-1}
70531 ad^{-1}
70532 ab^{-1}
70534 ab
70535 a
70536 a
70539 ab
70540 a
70544 ab
70546 ac^{-1}
70547 ab
70551 ac^{-1}
70552 ad
70556 a
70562 a
70563 a
70567 ac^{-1}
70568 ad
70572 a
70578 ac^{-1}
70579 ac^{-1}
70580 ab
70583 ac^{-1}
70584 ad
70588 a
70593 ab
70594 ab^{-1}
70595 ad^{-1}
70596 ad^{-1}
70598 ab
70599 a
70600 a
70603 ab
70604 a
70608 ab
70611 ad^{-1}
70612 ab
70615 ac^{-1}
70616 a
70619 $cdcb$
70620 ac
70627 ad^{-1}
70628 a
70631 ac^{-1}
70632 a
70636 ac
70642 a
70643 a
70644 ad^{-1}
70647 ac^{-1}
70648 a
70651 $cacb$
70651 $cacb$
70652 ac
70738 a
70739 a
70743 a
70748 a
70755 ab
70756 a
70759 a
70760 a
70764 ab^{-1}
70768 $dbdcac$
70768 $dbdcac$
70771 ab
70772 a
70775 a
70776 ac
70779 ba
70779 ba
70780 ad^{-1}
70818 a
70819 ac^{-1}
70820 ac^{-1}
70823 a
70824 a
70828 ab^{-1}
70834 a
70835 ad^{-1}
70839 a
70882 a
70883 ac^{-1}
70887 a
70929 ab
70931 a
70934 ab
70935 ab^{-1}
70939 ab
70977 ab
70979 a
70980 ab^{-1}
70982 ab
70983 ab^{-1}
70984 a
70987 ab
70988 ab^{-1}
70992 ab
71027 a
71028 a
71031 ab^{-1}
71032 ad
71035 ba
71035 ba
71036 ab^{-1}
71042 ab^{-1}
71043 a
71044 ab^{-1}
71047 a
71048 ad
71056 ab
71056 $(bd)^2adcd$
71089 abd
71090 ab^{-1}

71091 a	71147 abd	71300 ab^{-1}	71399 ab^2	71556 ab^{-1}	71651 ad^{-1}
71092 ac^{-1}	71152 abd	71303 ab	71400 a	71559 ab	71652 ab
71094 abd	71154 a	71304 ad	71404 ad^{-1}	71560 ad	71655 ab^{-1}
71095 ac^{-1}	71155 a	71308 ac^{-1}	71410 a	71564 ad^{-1}	71656 a
71096 ad	71156 ad^{-1}	71346 ac	71411 a	71569 abc	71660 a
71099 abd	71159 ab^{-1}	71347 ab^{-1}	71412 ad^{-1}	71571 ab	71666 a
71104 abd	71160 a	71348 ac^{-1}	71415 ab^{-1}	71572 ab^{-1}	71667 ad^{-1}
71105 ab	71163 $cdacdb$	71352 ad	71416 a	71575 ac^{-1}	71668 ad^{-1}
71106 ab^{-1}	71164 ad	71356 ac^{-1}	71420 ad	71576 ad	71671 ab
71107 a	71185 ab	71361 ab	71489 ab	71580 ac^{-1}	71672 a
71108 ad^{-1}	71186 ac	71362 ac	71491 a	71586 a	71675 $dcbdbc$
71110 ab	71187 ab^{-1}	71363 ab^{-1}	71492 ab^{-1}	71587 ac^{-1}	71676 ab^{-1}
71111 ab^{-1}	71190 ab	71364 ac	71494 ab	71588 ab^{-1}	71697 ab
71112 a	71195 ab	71366 ab	71495 ab^{-1}	71591 ab^{-1}	71698 ac^{-1}
71115 ab	71233 ab	71367 ab^{-1}	71496 a	71592 ad	71702 ab
71116 ac^{-1}	71234 ac	71368 a	71499 ab	71596 ab^{-1}	71703 a
71120 ab	71235 ab^{-1}	71371 ab	71500 ad^{-1}	71617 ab	71761 ac
71123 a	71236 ac	71372 ab^{-1}	71504 ab	71618 ab^{-1}	71762 a
71124 ab	71238 ab	71376 ab	71506 a	71619 a	71764 a
71127 ab^{-1}	71240 a	71377 abc	71507 ac	71622 ab	71766 ac
71128 a	71243 ab	71378 ac	71508 a	71623 ac^{-1}	71767 a
71131 $bcdadc$	71244 ab^{-1}	71379 ad^{-1}	71511 a	71624 a	71768 a
71132 ad^{-1}	71248 ab	71380 ab	71512 a	71627 ab	71771 ac
71137 abd	71282 ac	71383 ab^{-1}	71516 a	71635 ad^{-1}	71772 a
71138 ab^{-1}	71283 ab^{-1}	71384 a	71523 ab	71636 ab	71776 ac
71139 a	71284 a	71388 ad^{-1}	71524 ab^{-1}	71639 ab^{-1}	71777 ac
71140 ad^{-1}	71287 ab^{-1}	71392 acd	71528 ad	71640 a	71778 ac^{-1}
71142 abd	71288 ad	71394 ac	71532 ab^{-1}	71643 $(db)^2$	71779 ab
71143 ac^{-1}	71292 ab^{-1}	71395 ad^{-1}	71554 ab	71644 a	71780 ac^{-1}
71144 a	71298 a	71396 ad^{-1}	71555 ad^{-1}	71650 ab^{-1}	71782 ac

71784 ad
71787 ac
71788 ab^{-1}
71792 ac
71793 ac
71794 acb
71795 ad^{-1}
71796 a
71798 ac
71799 ab^{-1}
71800 ad
71803 ac
71804 ad^{-1}
71808 ac
71825 ac
71826 ab^{-1}
71827 ab
71828 ac^{-1}
71830 ac
71832 ad
71835 ac
71836 ac^{-1}
71840 ac
71841 ac
71842 a
71844 a
71846 ac
71847 ab^{-1}
71848 ad
71851 ac
71852 ab^{-1}
71856 ac
71857 ac
71859 ad^{-1}
71860 ac^{-1}
71862 ac
71863 ab^{-1}
71864 ad
71867 ac
71868 ad^{-1}
71872 ac
71889 ac
71890 ac
71891 ab^{-1}
71892 ab
71894 ac
71895 ab^{-1}
71896 a
71899 ac
71900 ab^{-1}
71904 ac
71905 ac
71906 ac^{-1}
71907 ab^{-1}
71908 ac^{-1}
71910 ac
71911 ab^{-1}
71912 a
71915 ac
71916 ad^{-1}
71920 ac
71921 ac
71922 a
71923 ad^{-1}
71924 ac^{-1}
71926 ac
71927 ab^{-1}
71928 a
71931 ac
71932 ab^{-1}
71936 ac
71953 ab
71954 ac
71955 ab^{-1}
71958 ab
71963 ab
72001 ab
72002 ac
72003 ab^{-1}
72004 ac
72006 ab
72008 a
72011 ab
72012 ab^{-1}
72016 ab
72050 ac
72051 ab^{-1}
72052 a
72056 ad
72060 ab^{-1}
72066 ac
72068 ac^{-1}
72071 a
72072 ad
72076 ac^{-1}
72080 ad
72114 ac
72115 ab^{-1}
72116 ac^{-1}
72120 ad
72124 ac^{-1}
72129 ab
72130 ac
72131 ab^{-1}
72132 ac
72134 ab
72135 ab^{-1}
72136 a
72139 ab
72140 ab^{-1}
72144 ab
72146 ac
72147 ab^{-1}
72148 ab
72152 a
72156 ad^{-1}
72162 ac
72163 ab^{-1}
72164 ad^{-1}
72168 a
72172 ac^{-1}
72178 a
72179 a
72180 ad^{-1}
72184 a
72188 ad
72209 ab
72211 a
72214 ab
72215 a
72219 ab
72257 ab
72259 a
72260 ab^{-1}
72262 ab
72263 a
72264 a
72267 ab
72268 a
72272 ab
72307 a
72308 a
72311 a
72312 ad
72315 ab
72316 a
72322 a
72323 a
72324 ab
72327 a
72328 a
72332 a
72336 $(ab)^2dad$
72369 abd
72370 ab
72371 a
72374 abd
72375 a
72376 a
72379 abd
72380 a
72384 abd
72385 ab
72386 ab^{-1}
72387 a
72390 ab
72391 a
72392 a
72395 ab
72403 a
72404 ab
72407 a
72408 a
72411 ab
72412 a
72417 abd
72418 ab
72419 a
72420 ad^{-1}
72422 abd

72423 a	72540 a	72664 a	72931 ac^{-1}	73083 ba	73211 $(da)^2$
72424 a	72544 acd	72668 a	72932 a	73084 a	73281 ab
72427 abd	72547 ab	72674 ac^{-1}	72935 ac^{-1}	73107 ab	73282 ac
72428 a	72548 ac	72675 ac^{-1}	72936 a	73108 ab^{-1}	73283 ab^{-1}
72432 abd	72551 ac^{-1}	72676 ab	72940 ac	73111 ac^{-1}	73284 ac
72434 a	72552 ad	72679 ac^{-1}	73025 ab	73112 ad	73286 ab
72435 a	72556 ab^{-1}	72680 a	73027 a	73116 ad^{-1}	73287 ab^{-1}
72436 ad^{-1}	72579 ad^{-1}	72684 a	73028 ab^{-1}	73122 a	73288 a
72439 a	72580 ac	72689 acd	73030 ab	73123 ad^{-1}	73291 ab
72440 a	72583 ab	72690 a	73031 ac^{-1}	73124 ad^{-1}	73296 ab
72443 abd	72584 ad	72691 ac^{-1}	73032 a	73127 ac^{-1}	73297 acd
72444 a	72588 ad^{-1}	72692 ac	73035 ab	73128 ad	73298 a
72513 ab	72594 ac^{-1}	72694 acd	73036 ab^{-1}	73138 ab^{-1}	73300 a
72514 ac	72595 ab	72695 ac^{-1}	73040 ab	73139 ad^{-1}	73302 acd
72515 ad^{-1}	72596 ac	72696 a	73042 a	73140 a	73303 a
72516 ac	72599 ac^{-1}	72699 acd	73043 ac	73143 ac^{-1}	73304 a
72518 ab	72600 ad	72700 ad^{-1}	73044 a	73144 ad	73307 acd
72519 ab^{-1}	72609 acd	72704 acd	73047 a	73148 a	73308 a
72520 a	72610 a	72802 ab^{-1}	73048 a	73153 ab	73312 acd
72523 ab	72611 ac^{-1}	72803 ab	73052 ad	73154 ab^{-1}	73314 ab^{-1}
72524 ab^{-1}	72612 ac	72807 ac^{-1}	73059 ab	73155 a	73315 ab
72528 ab	72614 acd	72808 ad	73060 ab^{-1}	73158 ab	73316 ac
72529 acd	72615 ac^{-1}	72812 ac	73063 ac^{-1}	73159 ac^{-1}	73320 ad
72530 a	72616 ad	72866 a	73064 ad	73160 a	73324 ab^{-1}
72531 ac	72619 acd	72867 ab^{-1}	73068 ab^{-1}	73163 ab	73331 ad^{-1}
72532 a	72624 acd	72868 ad^{-1}	73075 ab^{-1}	73202 a	73332 a
72534 ac	72658 ac^{-1}	72871 ab^{-1}	73076 a	73203 ad^{-1}	73335 ab^{-1}
72535 a	72659 ad^{-1}	72872 ad	73079 ac^{-1}	73204 ad^{-1}	73336 ad
72536 a	72660 ab	72876 ac	73080 ad	73207 ac^{-1}	73340 a
72539 acd	72663 ac^{-1}	72930 ac^{-1}	73083 ba	73208 a	73362 ac^{-1}

73363 ab	73537 ab	73620 ab^{-1}	73702 abd	79287 ac^{-1}	79355 ad
73364 ac	73539 a	73623 ac^{-1}	73703 ac^{-1}	79288 ac^{-1}	79356 ab^{-1}
73367 ac^{-1}	73540 ab^{-1}	73624 ad	73704 a	79291 ad	79360 ad
73368 ad	73542 ab	73628 a	73707 abd	79292 ac^{-1}	79425 ab
73377 acd	73543 ac^{-1}	73634 a	73708 ad^{-1}	79296 ad	79426 ac
73378 a	73544 a	73635 a	73712 abd	79313 ad	79427 ab^{-1}
73379 ad^{-1}	73547 ab	73636 ac^{-1}	79169 ab	79314 ad^{-1}	79428 ac
73380 ac	73548 ad^{-1}	73639 ac^{-1}	79170 ad^{-1}	79315 a	79430 ab
73382 acd	73552 ab	73640 ad	79171 a	79316 ab	79432 a
73383 ac^{-1}	73554 a	73644 a	79172 ab^{-1}	79318 ad	79435 ab
73384 ad	73555 a	73649 abd	79174 ab	79319 ab^{-1}	79436 ac^{-1}
73387 acd	73556 a	73650 ac^{-1}	79175 ab^{-1}	79323 ad	79440 ab
73388 ad^{-1}	73559 a	73651 a	79176 a	79324 ad	79473 ad
73392 acd	73560 a	73652 ac^{-1}	79179 ab	79328 ad	79474 ac
73394 ab^{-1}	73564 a	73654 abd	79180 ab^{-1}	79329 ad	79475 ab^{-1}
73395 ad^{-1}	73570 a	73655 ac^{-1}	79184 ab	79330 ad^{-1}	79476 ab
73396 a	73571 a	73656 ad	79217 ad	79331 a	79478 ad
73399 ac^{-1}	73572 ad^{-1}	73659 abd	79218 ab^{-1}	79332 ac^{-1}	79479 ab^{-1}
73400 ad	73575 ac^{-1}	73660 ac^{-1}	79219 a	79334 ad	79483 ad
73404 a	73576 ad	73664 abd	79220 ab	79335 ab^{-1}	79484 ab^{-1}
73457 acd	73580 a	73683 a	79222 ad	79336 ab^{-1}	79488 ad
73458 a	73587 a	73684 ab	79223 ab^{-1}	79339 ad	79537 ad
73459 ad^{-1}	73588 a	73687 ac^{-1}	79227 ad	79340 ac^{-1}	79538 ac
73460 ac	73591 ac	73688 a	79228 ad	79344 ad	79539 ab^{-1}
73462 acd	73592 ad	73691 $(db)^2$	79232 ad	79345 ad	79540 ac^{-1}
73463 ac^{-1}	73595 ab	73691 $(db)^2$	79281 ad	79346 a	79542 ad
73464 a	73595 ab	73692 ac^{-1}	79282 ab^{-1}	79347 a	79544 ab^{-1}
73467 acd	73596 ab^{-1}	73697 abd	79283 a	79350 ad	79547 ad
73468 ad^{-1}	73618 a	73699 a	79284 ac^{-1}	79351 a	79548 ac^{-1}
73472 acd	73619 a	73700 ad^{-1}	79286 ad	79352 ab^{-1}	79552 ad

79569 ad	79683 a	79840 ad	79972 ab	80344 ac^{-1}	80508 a
79570 ac	79686 ab	79841 ad	79974 ac	80347 ad	80512 ad
79571 ab^{-1}	79687 ab^{-1}	79842 ad^{-1}	79975 ad^{-1}	80348 ac^{-1}	80593 ad
79572 ab	79688 a	79843 ab^{-1}	79976 ab^{-1}	80352 ad	80594 ad^{-1}
79574 ad	79691 ab	79844 ab	79979 ac	80353 ad	80595 a
79575 ab^{-1}	79697 ad	79846 ad	79980 a	80354 ac	80596 ab
79579 ad	79698 a	79847 a	79984 ac	80355 ad^{-1}	80598 ad
79580 ac^{-1}	79702 ad	79848 ab^{-1}	80097 ac	80356 ac^{-1}	80599 a
79584 ad	79703 a	79851 ad	80098 a	80358 ad	80603 ad
79585 ad	79704 a	79852 ac^{-1}	80099 ad^{-1}	80360 ac^{-1}	80604 a
79586 ac	79707 ad	79856 ad	80102 ac	80363 ad	80608 ad
79587 ab^{-1}	79708 a	79857 ad	80103 ab^{-1}	80364 ac^{-1}	80609 ad
79588 ac^{-1}	79712 ad	79858 a	80107 ac	80368 ad	80610 ad^{-1}
79590 ad	79713 ad	79859 ab^{-1}	80108 a	80369 ad	80611 a
79591 ab^{-1}	79714 ab^{-1}	79862 ad	80241 ad	80370 a	80614 ad
79592 ab^{-1}	79715 ab	79863 ad^{-1}	80242 ac	80371 a	80615 a
79595 ad	79716 ab^{-1}	79864 ab^{-1}	80243 ab^{-1}	80372 ac^{-1}	80616 a
79596 ac^{-1}	79718 ad	79867 ad	80244 ab	80374 ad	80619 ad
79600 ad	79720 ab^{-1}	79868 ab^{-1}	80246 ad	80375 a	80620 a
79601 ad	79723 ad	79872 ad	80247 ab^{-1}	80376 ac^{-1}	80624 ad
79602 a	79724 ab^{-1}	79953 ac	80248 ac^{-1}	80379 ad	80625 ad
79603 a	79728 ad	79954 a	80251 ad	80380 ac^{-1}	80626 a
79606 ad	79825 ad	79955 ad^{-1}	80252 ab^{-1}	80384 ad	80627 a
79607 a	79826 ad^{-1}	79958 ac	80256 ad	80497 ad	80630 ad
79608 ab^{-1}	79827 ab^{-1}	79959 a	80337 ad	80498 ab^{-1}	80631 a
79611 ad	79828 ab	79963 ac	80338 ac	80499 a	80632 ab^{-1}
79612 ab^{-1}	79830 ad	79964 a	80339 ad^{-1}	80500 ab	80635 ad
79616 ad	79831 a	79969 ac	80340 ab	80502 ad	80636 a
79681 ab	79835 ad	79970 ac^{-1}	80342 ad	80503 a	80640 ad
79682 ad^{-1}	79836 ac^{-1}	79971 ab	80343 ab^{-1}	80507 ad	80849 ad

80850 ad^{-1}	81399 ac^{-1}	196710 b	196910 a	196985 a	197124 a
80851 ac^{-1}	81400 ab^{-1}	196713 a	196911 b	196986 a	197127 ab^{-1}
80852 ab	81403 ad	196714 a	196912 bc	196987 b	197131 ab
80854 ad	81404 ac^{-1}	196715 b	196914 ab^{-1}	196988 b	197132 ad^{-1}
80855 a	81408 ad	196770 ab^{-1}	196917 ab^{-1}	196989 a	197136 ac
80856 ac^{-1}	81649 ad	196771 a	196918 ad^{-1}	196990 a	197139 ab^{-1}
80859 ad	81650 a	196775 bc^{-1}	196921 a	196991 b	197140 a
80864 ad	81651 ad^{-1}	196867 ab^{-1}	196922 a	196992 b	197143 a
80865 ad	81652 ac	196868 a	196923 ad^{-1}	197026 bc^{-1}	197144 bc
80866 ad^{-1}	81654 ad	196871 a	196924 ad^{-1}	197027 bc^{-1}	197145 b
80867 ac^{-1}	81655 ab^{-1}	196876 ac^{-1}	196925 a	197028 a	197146 a
80868 ab	81656 ab^{-1}	196884 a	196926 a	197031 a	197147 b
80870 ad	81659 ad	196887 b	196927 ac^{-1}	197032 ab^{-1}	197148 b
80871 ad	81664 ad	196888 ab^{-1}	196947 ba^{-1}	197041 ab^{-1}	197149 a
80872 ac^{-1}	87378 a	196890 a	196948 a	197042 ab^{-1}	197150 bc
80875 ad	196611 a	196891 ac^{-1}	196951 ac	197043 ac^{-1}	197151 ab^{-1}
80880 ad	196627 a	196892 ab^{-1}	196956 ba^{-1}	197044 a	197152 ac
80881 ad	196631 ab^{-1}	196893 a	196961 ab^{-1}	197045 ab^{-1}	197153 b
80882 a	196633 a	196894 ad^{-1}	196966 ab^{-1}	197046 ab^{-1}	197156 a
80883 ac^{-1}	196634 ac^{-1}	196895 ac^{-1}	196969 a	197047 ac	197158 b
80884 ac	196635 ab^{-1}	196896 ab^{-1}	196970 a	197048 bc^{-1}	197160 ac^{-1}
80886 ad	196642 ab^{-1}	196897 bc	196971 ac^{-1}	197049 bc^{-1}	197162 ab^{-1}
80887 ad^{-1}	196645 ab^{-1}	196898 b	196972 ad^{-1}	197050 ac	197163 b
80888 ac	196646 ac^{-1}	196901 a	196973 a	197053 a	197164 ad^{-1}
80891 ad	196649 a	196902 b	196974 a	197054 ad^{-1}	197165 a
80896 ad	196650 a	196905 a	196975 bd^{-1}	197106 ab^{-1}	197166 ad^{-1}
81393 ad	196691 a	196906 a	196977 b	197107 bc^{-1}	197167 ad^{-1}
81394 a	196705 b	196907 b	196978 b	197108 a	197168 b
81395 ad^{-1}	196706 b	196908 bc	196981 b	197111 ac	197169 ac
81398 ad	196709 b	196909 a	196982 b	197112 bd^{-1}	197170 ab^{-1}

197173 ab^{-1}	197233 b	197307 b	197412 a	197478 bc^{-1}	197554 bd^{-1}
197174 ac	197234 b	197309 a	197413 b	197480 ad^{-1}	197555 ac^{-1}
197175 ab^{-1}	197235 b	197310 ac^{-1}	197414 ad^{-1}	197481 ac^{-1}	197556 a
197178 ab^{-1}	197237 b	197311 ad^{-1}	197416 ad^{-1}	197482 bc^{-1}	197557 ab^{-1}
197179 b	197238 b	197312 b	197417 ac^{-1}	197483 cb^{-1}	197559 ab^{-1}
197180 ad^{-1}	197239 b	197361 ac	197418 ac^{-1}	197484 ac	197560 ab^{-1}
197181 a	197241 b	197362 ac	197419 ab^{-1}	197485 a	197561 ac^{-1}
197182 a	197242 b	197364 a	197420 b	197486 bc^{-1}	197562 ad^{-1}
197183 ad^{-1}	197243 b	197366 ac	197421 a	197487 ac	197563 ab^{-1}
197184 ac	197244 b	197367 ab^{-1}	197422 ad^{-1}	197489 b	197565 a
197204 a	197245 a	197368 ac^{-1}	197423 b	197490 b	197566 ab^{-1}
197207 bc^{-1}	197246 a	197371 ac	197426 ab^{-1}	197491 b	197567 ac
197208 ac^{-1}	197247 b	197372 ad^{-1}	197427 ac^{-1}	197493 b	197618 ab^{-1}
197211 ab	197248 b	197379 ac^{-1}	197429 ab^{-1}	197494 b	197619 ac^{-1}
197212 ad^{-1}	197281 ab	197380 a	197430 ad^{-1}	197495 b	197620 a
197216 ac	197282 a	197383 a	197431 ad^{-1}	197497 b	197623 a
197217 b	197284 a	197388 ab	197433 ac^{-1}	197498 b	197624 bd^{-1}
197218 ab^{-1}	197286 ab	197395 ab^{-1}	197434 ac^{-1}	197499 b	197628 ac
197220 a	197287 ab^{-1}	197396 a	197435 db^{-1}	197500 b	197891 a
197221 ab^{-1}	197288 ac^{-1}	197399 ab^{-1}	197436 b	197501 a	197892 a
197222 ac	197292 ad^{-1}	197400 ab^{-1}	197437 a	197502 a	197895 bc^{-1}
197224 ac^{-1}	197296 ac	197401 ac^{-1}	197438 a	197503 b	197900 ab^{-1}
197225 ab^{-1}	197297 b	197402 ac^{-1}	197439 b	197504 b	197907 a
197226 ab^{-1}	197298 ab^{-1}	197403 ab^{-1}	197459 ac^{-1}	197538 bd^{-1}	197908 a
197227 ac	197300 a	197404 ac	197460 a	197539 ac^{-1}	197911 ab^{-1}
197228 ab^{-1}	197301 ab^{-1}	197405 a	197463 a	197540 a	197912 ab^{-1}
197229 a	197302 b	197406 ad^{-1}	197468 ac	197542 ab	197913 a
197230 ab^{-1}	197303 a	197407 b	197474 bc^{-1}	197543 a	197914 ab^{-1}
197231 ad^{-1}	197304 ad^{-1}	197408 ab^{-1}	197476 a	197544 ab^{-1}	197915 ab^{-1}
197232 ac	197306 a	197410 b	197477 ab^{-1}	197548 ac	197916 ab^{-1}

197917 a	198147 bc	198193 ac^{-1}	198258 b	198329 ab^{-1}	198433 b
197918 ad^{-1}	198148 a	198194 ac	198259 b	198330 ab^{-1}	198434 ab^{-1}
197919 ab^{-1}	198151 a	198195 ab^{-1}	198261 b	198331 bd^{-1}	198436 a
197920 ab^{-1}	198152 bc	198197 ac	198262 b	198333 a	198437 bc^{-1}
197929 a	198155 ab	198198 bc^{-1}	198263 b	198334 ac^{-1}	198438 b
197930 a	198156 bc	198199 bc^{-1}	198265 b	198335 ad^{-1}	198441 bc^{-1}
197931 ac^{-1}	198160 ac	198201 ab^{-1}	198266 b	198385 ac	198442 ab^{-1}
197933 a	198163 ab^{-1}	198202 ad^{-1}	198267 b	198386 ac	198443 b
197934 a	198164 a	198203 bc^{-1}	198268 b	198387 bc	198444 ac^{-1}
197936 ac^{-1}	198168 ab^{-1}	198204 ad^{-1}	198269 a	198388 a	198445 a
197945 a	198169 ab^{-1}	198205 a	198270 a	198390 ac	198447 ad^{-1}
197946 a	198171 b	198206 a	198271 b	198391 a	198448 b
197947 ac^{-1}	198172 ab^{-1}	198207 ad^{-1}	198272 b	198392 bc	198450 ab^{-1}
197948 ad^{-1}	198173 a	198227 a	198305 ab	198395 ac	198451 ac^{-1}
197949 a	198174 ac^{-1}	198228 a	198306 a	198396 bc	198453 ab^{-1}
197951 ab^{-1}	198175 ad^{-1}	198231 a	198307 bc	198403 ac^{-1}	198454 ad^{-1}
197971 a	198176 ac	198232 bc	198308 a	198404 a	198455 ac^{-1}
197972 a	198177 ac^{-1}	198235 ab	198310 ab	198407 ab^{-1}	198457 ab^{-1}
197980 ad^{-1}	198178 b	198236 a	198311 a	198412 ad^{-1}	198458 ab^{-1}
197993 a	198180 a	198240 ac	198312 ab	198419 ac^{-1}	198460 ab^{-1}
197995 b	198181 b	198242 a	198316 bc	198420 a	198461 a
197996 b	198182 ab^{-1}	198244 a	198320 ac	198423 a	198462 a
197997 a	198184 ac^{-1}	198245 ac	198321 ac^{-1}	198424 ab^{-1}	198463 ad^{-1}
197999 b	198185 ab^{-1}	198248 ac^{-1}	198322 b	198425 ab^{-1}	198483 ac^{-1}
198000 b	198186 ab^{-1}	198249 ab^{-1}	198323 ab^{-1}	198426 a	198484 a
198009 a	198187 ab^{-1}	198252 ad^{-1}	198324 a	198428 ab^{-1}	198487 bd^{-1}
198011 b	198188 b	198253 a	198325 a	198429 a	198492 ad^{-1}
198012 b	198189 a	198254 ac^{-1}	198326 ab^{-1}	198430 ad^{-1}	198497 ac^{-1}
198015 b	198190 ab^{-1}	198255 ad^{-1}	198327 ab^{-1}	198431 ad^{-1}	198498 ab^{-1}
198016 b	198191 b	198257 b	198328 ac^{-1}	198432 ab^{-1}	198500 a

198501 ab^{-1}	198578 a	199196 a	199265 b	199332 a	199451 ab^{-1}
198502 ab^{-1}	198579 bc	199197 a	199266 b	199336 a	199452 ab^{-1}
198504 ad^{-1}	198580 a	199198 ad^{-1}	199268 a	199340 a	199453 a
198505 ac^{-1}	198581 ab^{-1}	199199 a	199269 b	199346 a	199454 ad^{-1}
198506 ab^{-1}	198582 b	199200 ab^{-1}	199270 b	199347 a	199455 ad^{-1}
198507 ab^{-1}	198583 a	199202 a	199272 a	199348 a	199456 ab^{-1}
198508 ab^{-1}	198584 ab^{-1}	199204 a	199273 a	199349 a	199457 ab^{-1}
198509 a	198585 bc	199205 a	199274 a	199350 a	199458 ab^{-1}
198510 ab^{-1}	198586 a	199206 a	199275 b	199351 a	199460 a
198511 ab^{-1}	198587 bc	199208 a	199276 b	199353 a	199461 ac^{-1}
198513 b	198589 a	199209 a	199277 a	199354 a	199462 ac^{-1}
198514 b	198590 b	199210 a	199278 a	199357 a	199464 a
198515 b	198591 ac	199212 a	199279 b	199358 a^2	199465 a
198517 b	198592 bc	199213 a	199280 b	199359 a	199466 a
198518 b	198642 ab^{-1}	199214 a	199281 b	199410 ab^{-1}	199467 ab^{-1}
198519 b	198643 ac^{-1}	199215 a	199282 b	199411 a	199468 ad^{-1}
198521 b	198644 a	199218 ab^{-1}	199283 a	199412 a	199469 a
198522 b	198648 bd^{-1}	199219 a	199285 b	199415 b	199470 a
198523 b	198652 ac^{-1}	199221 ab^{-1}	199286 b	199416 bd^{-1}	199471 ad^{-1}
198524 b	199171 a	199222 ad^{-1}	199287 a	199420 a	199472 ab^{-1}
198525 a	199172 a	199223 a	199289 a	199427 a	199473 cb^{-1}
198526 a	199175 b	199225 a	199290 a	199428 a	199474 ab^{-1}
198527 b	199180 a	199226 a	199291 b	199431 ad^{-1}	199475 a
198528 b	199187 a	199228 a	199292 b	199436 ad^{-1}	199477 ab^{-1}
198562 ab^{-1}	199188 a	199229 a	199293 a	199443 a	199478 ac^{-1}
198563 ab^{-1}	199191 a	199230 a	199294 a	199444 a	199479 a
198564 a	199192 ab^{-1}	199231 a	199295 b	199447 ab^{-1}	199481 a
198567 ad^{-1}	199193 a	199251 a	199296 b	199448 ab^{-1}	199482 a
198572 ab^{-1}	199194 b	199252 a	199330 a	199449 a	199483 ab^{-1}
198577 bc	199195 b	199260 a	199331 a	199450 ab^{-1}	199484 ad^{-1}

199485 a	199547 b	199672 ac^{-1}	200502 ad^{-1}	200566 b	200700 ac^{-1}
199486 a	199548 b	199676 ad^{-1}	200503 a	200567 a	200963 ab^{-1}
199487 ad^{-1}	199549 a	200451 a	200505 a	200569 a	200967 a
199488 cb^{-1}	199550 a	200452 a	200506 a	200570 a	200983 b
199507 a	199551 b	200455 ad^{-1}	200508 ac^{-1}	200571 b	200986 a
199508 a	199552 b	200460 ac^{-1}	200509 a	200572 b	200987 ac^{-1}
199511 ad^{-1}	199586 ab^{-1}	200467 a	200510 a	200573 a	200993 bc
199516 ad^{-1}	199587 a	200468 a	200511 ac^{-1}	200574 a	201001 a
199521 b	199588 a	200472 ab^{-1}	200531 a	200575 b	201002 a
199522 b	199591 bd^{-1}	200473 a	200532 a	200576 b	201003 b
199524 a	199592 ac^{-1}	200474 ac^{-1}	200535 $cdbcbd$	200610 ac^{-1}	201043 ba^{-1}
199525 b	199596 ad^{-1}	200476 ab^{-1}	200540 ac^{-1}	200611 a	201065 a
199526 b	199601 ab^{-1}	200477 a	200545 b	200612 a	201067 ac^{-1}
199528 a	199602 bd^{-1}	200478 ad^{-1}	200546 b	200615 ad^{-1}	201220 ab^{-1}
199529 a	199603 a	200479 ac^{-1}	200548 a	200616 bd^{-1}	201223 ab^{-1}
199530 a	199604 a	200480 ab^{-1}	200549 b	200620 ac^{-1}	201224 a
199531 b	199605 ab^{-1}	200482 ac^{-1}	200550 b	200626 bd^{-1}	201227 ab
199532 b	199606 ac^{-1}	200484 a	200552 a	200627 a	201228 ac^{-1}
199533 a	199607 ad^{-1}	200485 bc^{-1}	200553 a	200628 a	201232 ac
199534 a	199608 ad^{-1}	200486 ac^{-1}	200554 a	200629 ab^{-1}	201235 ad
199535 b	199609 a	200488 a	200555 b	200630 ad^{-1}	201236 bc
199536 b	199610 ab^{-1}	200489 a	200556 b	200632 ad^{-1}	201239 a
199537 b	199611 ab^{-1}	200490 a	200557 a	200633 a	201240 b
199538 b	199613 a	200492 ac^{-1}	200558 a	200637 a	201241 b
199539 a	199614 ab^{-1}	200493 a	200559 b	200638 ab^{-1}	201242 a
199541 b	199615 ad^{-1}	200494 a	200560 b	200639 ac^{-1}	201243 b
199542 b	199616 ab^{-1}	200495 ac^{-1}	200561 b	200690 ab^{-1}	201244 a
199543 a	199666 ab^{-1}	200498 ab^{-1}	200562 b	200691 a	201245 bc
199545 a	199667 a	200499 a	200563 a	200692 a	201246 a
199546 a	199668 a	200501 ab^{-1}	200565 b	200696 bd^{-1}	201247 a

201248 ac	201313 b	201383 ab^{-1}	201480 a	201529 bd	201594 ab^{-1}
201249 b	201314 ab^{-1}	201384 a	201484 ab	201530 ab^{-1}	201595 ab^{-1}
201254 b	201316 ab^{-1}	201388 ad^{-1}	201495 ac^{-1}	201531 bd	201596 ac
201256 ab	201317 ab^{-1}	201392 ac	201496 b	201532 a	201597 a
201258 ab^{-1}	201318 ac	201393 ab	201498 ac^{-1}	201533 a	201598 a
201259 b	201320 ad	201394 bd^{-1}	201499 ac^{-1}	201534 a	201599 ac
201260 ac^{-1}	201321 ab^{-1}	201395 b	201500 ac	201535 a	201600 ad^{-1}
201262 ad	201322 ab^{-1}	201396 bd^{-1}	201502 a	201536 b	201634 bd^{-1}
201263 ad^{-1}	201323 ac	201398 ac	201503 b	201555 ab	201635 ad
201264 b	201324 ab^{-1}	201399 ab^{-1}	201504 ad^{-1}	201556 ba^{-1}	201636 bd^{-1}
201265 ac	201325 ab^{-1}	201400 b	201505 ab^{-1}	201559 ba^{-1}	201639 bc^{-1}
201266 b	201326 ad	201401 b	201506 bc^{-1}	201560 ad	201640 ab
201267 bd	201327 ad^{-1}	201402 ab^{-1}	201508 bc^{-1}	201564 ac	201644 ac
201269 a	201328 b	201403 ac	201509 bc^{-1}	201569 ab^{-1}	201649 ab^{-1}
201270 b	201329 ac	201405 ad^{-1}	201510 bc^{-1}	201570 a	201650 bd^{-1}
201271 ab	201331 bc^{-1}	201406 b	201512 b	201573 ac	201651 bd^{-1}
201273 bd	201333 bc^{-1}	201407 ac^{-1}	201513 bc^{-1}	201574 bc^{-1}	201652 bd^{-1}
201274 ab^{-1}	201334 ac	201408 ac	201514 ac^{-1}	201576 ad	201653 bd^{-1}
201275 ab	201335 bc^{-1}	201457 ac	201515 bc^{-1}	201577 ac^{-1}	201654 ab^{-1}
201276 b	201337 ad^{-1}	201458 ac	201516 b	201578 bc^{-1}	201655 ac^{-1}
201277 a	201338 ab^{-1}	201459 bd^{-1}	201517 bc^{-1}	201579 ac^{-1}	201656 ad
201278 a	201339 ac	201460 bd^{-1}	201518 ad	201580 a	201657 bd^{-1}
201279 bd	201341 a	201462 ac	201519 b	201582 ad	201659 bd^{-1}
201280 b	201342 a	201463 ab^{-1}	201520 bc^{-1}	201583 a	201661 bd^{-1}
201300 ba^{-1}	201343 ac^{-1}	201464 a	201521 bd	201584 ad^{-1}	201662 a
201303 bc^{-1}	201344 ac	201467 ac	201522 b	201585 ab^{-1}	201663 ac
201304 ad	201377 ab	201468 bc^{-1}	201523 bd	201587 ab^{-1}	201664 ac^{-1}
201307 ab	201378 a	201475 ad	201525 a	201590 ab^{-1}	201714 bd^{-1}
201308 ba^{-1}	201380 ac^{-1}	201476 ab^{-1}	201526 b	201591 ac^{-1}	201715 ab
201312 ac	201382 ab	201479 ac^{-1}	201527 ab	201593 ab^{-1}	201716 bd^{-1}

201719 ab^{-1}	202248 a	202293 a	202355 bc^{-1}	202426 ab^{-1}	202530 bc^{-1}
201720 a	202251 ab	202294 b	202357 ac	202427 ac^{-1}	202532 bc^{-1}
201724 ac	202252 a	202295 ab	202358 bc^{-1}	202430 ad	202533 bc^{-1}
201731 ab^{-1}	202256 ac	202297 bd	202359 bc^{-1}	202431 ac^{-1}	202534 ab^{-1}
201735 a	202259 ab^{-1}	202298 a	202361 bc^{-1}	202432 ad^{-1}	202536 ab
201740 ac^{-1}	202260 ac^{-1}	202299 b	202362 bc^{-1}	202481 ac	202537 bc^{-1}
201751 b	202264 b	202300 ac	202363 bc^{-1}	202482 ac	202538 ab^{-1}
201754 a	202265 ab^{-1}	202301 a	202364 bc^{-1}	202483 a	202539 ac^{-1}
201755 ac^{-1}	202267 b	202302 a	202365 a	202484 bc	202541 ab^{-1}
201756 ad^{-1}	202269 ac^{-1}	202303 a	202366 a	202486 ac	202542 ad
201761 bc	202270 a	202304 b	202367 bc^{-1}	202487 a	202543 bc^{-1}
201769 a	202271 ad^{-1}	202323 a	202368 ad^{-1}	202488 a	202544 bc^{-1}
201770 a	202272 ac	202324 bc	202401 ab	202491 ac	202545 ac
201771 b	202273 ac^{-1}	202327 a	202402 a	202492 a	202546 b
201772 bc	202274 b	202328 ad	202403 ad	202499 bc^{-1}	202547 bd
201773 a	202276 ab^{-1}	202331 ab	202404 bc	202500 ab^{-1}	202549 a
201774 a	202277 b	202332 a	202406 ab	202503 ab^{-1}	202550 b
201775 bc	202278 ab^{-1}	202336 ac	202407 a	202504 a	202551 a
201776 b	202280 ab	202338 a	202408 ab	202508 bc^{-1}	202553 a
201811 ba^{-1}	202281 ab^{-1}	202340 ac^{-1}	202412 a	202515 a	202554 ab^{-1}
201820 ba^{-1}	202282 ab^{-1}	202341 ac	202416 ac	202519 ab^{-1}	202555 ab
201833 a	202283 ab^{-1}	202344 ad	202417 ab^{-1}	202520 b	202556 a
201835 ac^{-1}	202284 ab^{-1}	202345 ab^{-1}	202418 ac	202521 b	202557 a
201836 ac^{-1}	202285 ab^{-1}	202348 a	202419 ab^{-1}	202522 ab^{-1}	202558 a
201837 a	202286 ad	202349 ac^{-1}	202420 ab^{-1}	202523 ac^{-1}	202559 a
201839 ac^{-1}	202287 ad^{-1}	202350 ad	202421 a	202524 ad^{-1}	202560 b
201840 ad^{-1}	202288 ab^{-1}	202351 ac	202422 bd^{-1}	202526 a	202579 ac^{-1}
202243 ad	202289 bd	202352 ad^{-1}	202423 bd^{-1}	202527 ab^{-1}	202580 ba^{-1}
202244 bc	202290 b	202353 ab^{-1}	202424 ad	202528 ad^{-1}	202583 ba^{-1}
202247 a	202291 bd	202354 ac	202425 ad^{-1}	202529 bc^{-1}	202584 ad

202588 ba^{-1}	202664 ad	203288 b	203369 a	203553 ab^{-1}	203608 ad
202593 b	202668 bc^{-1}	203289 a	203371 b	203556 a	203612 ba^{-1}
202594 ab^{-1}	202673 bc	203290 b	203372 b	203557 ac^{-1}	203617 ab^{-1}
202596 ab^{-1}	202674 a	203291 b	203373 a	203558 ab^{-1}	203618 ab^{-1}
202598 b	202675 b	203292 a	203375 b	203560 a	203620 a
202600 ad	202676 bc	203294 a	203376 b	203561 a	203621 ac^{-1}
202601 ac^{-1}	202677 ab^{-1}	203295 a	203387 bd	203562 a	203622 ac^{-1}
202602 ab^{-1}	202678 b	203296 ad^{-1}	203388 a	203563 ac^{-1}	203624 a
202603 b	202679 a	203300 a	203389 a	203564 ad^{-1}	203625 a
202604 ab^{-1}	202680 ad	203304 a	203391 a	203565 a	203626 a
202605 ab^{-1}	202681 ad	203305 a	203392 ad^{-1}	203566 a	203627 ac^{-1}
202606 ad	202682 a	203306 a	203436 a	203567 ab^{-1}	203629 a
202607 ac^{-1}	202683 bc	203308 a	203440 ad	203568 ac^{-1}	203630 a
202608 b	202685 bc	203309 a	203455 a	203569 bd	203632 ac^{-1}
202609 ab^{-1}	202686 a	203310 a	203456 b	203570 b	203633 bc
202611 ac^{-1}	202687 ac	203311 a	203523 ab^{-1}	203571 a	203634 bc
202614 ab^{-1}	202688 bc	203312 a	203524 ac	203573 a	203635 a
202615 ac^{-1}	202738 bd^{-1}	203313 bd	203527 ab^{-1}	203574 b	203637 bc
202617 ac^{-1}	202739 bd^{-1}	203315 a	203528 a	203575 a	203638 bc
202618 ac^{-1}	202740 bd^{-1}	203321 a	203532 ab^{-1}	203577 a	203639 a
202619 ad^{-1}	202743 ab^{-1}	203323 bd	203539 ac^{-1}	203578 a	203641 a
202620 ad^{-1}	202744 a	203324 a	203540 ac	203579 bd	203642 a
202621 a	202748 ad^{-1}	203325 a	203543 bc^{-1}	203580 a	203643 ab
202622 a	203267 a	203326 a	203544 b	203581 a	203644 ab^{-1}
202623 ad^{-1}	203268 ab^{-1}	203327 a	203545 ad^{-1}	203582 a	203645 a
202624 ad^{-1}	203271 b	203328 b	203547 ac^{-1}	203583 a	203646 a
202658 bd^{-1}	203272 a	203347 a	203549 ac	203584 b	203647 ab
202659 ab^{-1}	203276 a	203348 ba^{-1}	203550 a	203603 bc^{-1}	203648 bc
202660 ab^{-1}	203283 a	203356 a	203551 ab^{-1}	203604 ac	203682 bd^{-1}
202663 bd^{-1}	203287 a	203364 a	203552 ac^{-1}	203607 bc^{-1}	203683 bd^{-1}

203684 ab	204567 bc^{-1}	204636 ba^{-1}	205410 ab^{-1}	206374 ab^{-1}	206441 ab^{-1}
203687 bd^{-1}	204568 b	204644 a	205413 ab^{-1}	206377 ab^{-1}	206444 ad
203688 ad	204569 a	204649 a	205414 ac	206378 ab^{-1}	206445 ab^{-1}
203692 bd^{-1}	204570 ac^{-1}	204651 b	205417 ab^{-1}	206379 ab^{-1}	206446 ab^{-1}
203697 ab^{-1}	204571 bc^{-1}	204652 b	205418 ab^{-1}	206380 a	206447 ad
203699 bd^{-1}	204572 ad^{-1}	204653 a	205419 ac	206381 ac^{-1}	206448 ab^{-1}
203700 b	204574 a	204655 b	205473 ab	206382 ab^{-1}	206449 bd^{-1}
203701 bd^{-1}	204575 bc^{-1}	204656 b	205474 a	206383 ad	206450 ac
203702 ac^{-1}	204576 ad^{-1}	204667 ad^{-1}	205478 ab	206384 ad^{-1}	206453 ac
203703 bd^{-1}	204577 ab^{-1}	204668 ad^{-1}	205479 ab^{-1}	206385 ac^{-1}	206454 bd^{-1}
203704 ad	204580 a	204669 a	206339 bc	206386 ac	206457 bd^{-1}
203705 bd^{-1}	204584 a	204671 ad^{-1}	206340 bc	206389 ac	206458 ab^{-1}
203706 bd^{-1}	204585 a	204672 ad^{-1}	206343 a	206390 ab^{-1}	206459 ab^{-1}
203707 ac^{-1}	204586 a	204716 bd^{-1}	206344 bc	206393 ab^{-1}	206460 ad
203709 b	204588 bc^{-1}	204735 ad^{-1}	206347 ab	206394 ab^{-1}	206461 bd^{-1}
203710 a	204589 a	204736 bd^{-1}	206348 a	206395 ac^{-1}	206462 ad^{-1}
203711 ad^{-1}	204590 a	205319 ab^{-1}	206352 ab	206396 b	206463 ad
203712 ac^{-1}	204591 ac^{-1}	205323 ab	206355 ab^{-1}	206397 ab^{-1}	206464 bd^{-1}
203762 bd^{-1}	204592 ab^{-1}	205331 ab^{-1}	206356 ac^{-1}	206398 ab^{-1}	206497 ab
203763 bd^{-1}	204593 bd	205335 a	206360 ad^{-1}	206399 b	206498 a
203764 ac	204595 a	205337 b	206361 ab^{-1}	206400 ad^{-1}	206499 bc
203767 bd^{-1}	204601 a	205338 a	206363 b	206419 a	206500 bc
203768 a	204603 bd	205339 b	206364 b	206420 bc	206502 ab
203772 ad^{-1}	204604 a	205345 b	206365 ab^{-1}	206423 a	206503 a
204547 a	204605 a	205350 b	206366 ad^{-1}	206424 bc	206504 ab
204548 ab^{-1}	204606 a	205354 ab^{-1}	206367 b	206427 ab	206513 b
204551 ab^{-1}	204607 a	205355 b	206368 b	206428 a	206514 a
204552 a	204608 b	205399 bc^{-1}	206369 ac^{-1}	206432 ab	206515 ab^{-1}
204556 ab^{-1}	204627 a	205403 ab	206370 b	206434 a	206516 ac^{-1}
204563 a	204628 ba^{-1}	205409 b	206373 b	206437 ac	206517 a

206518 b	206625 bc^{-1}	206856 ac^{-1}	206902 b	206962 bc^{-1}	207038 ad^{-1}
206519 ab^{-1}	206633 bc^{-1}	206859 ab	206903 ab^{-1}	206963 bc^{-1}	207039 a
206520 ad^{-1}	206634 ab^{-1}	206860 a	206906 ab^{-1}	206965 ac^{-1}	207040 b
206522 ab^{-1}	206635 ac^{-1}	206864 ab	206907 b	206966 b	207089 ab
206525 ab^{-1}	206636 a	206867 ab^{-1}	206908 b	206967 ab^{-1}	207090 a
206526 ad^{-1}	206637 ab^{-1}	206868 ab^{-1}	206909 ac^{-1}	206969 bc^{-1}	207092 ac^{-1}
206577 ab	206639 ad	206871 a	206910 ac^{-1}	206970 ab^{-1}	207094 ab
206578 a	206641 ac^{-1}	206872 a	206911 b	206971 b	207095 ab^{-1}
206579 bc	206649 ac^{-1}	206873 b	206912 b	206972 a	207096 ac^{-1}
206580 bc	206651 ab^{-1}	206874 a	206932 ac^{-1}	206973 ac^{-1}	207099 ac
206582 ab	206652 b	206875 b	206935 bc^{-1}	206975 a	207100 a
206583 a	206653 ab^{-1}	206876 a	206936 ac^{-1}	206976 b	207107 bc
206584 bc	206655 b	206877 b	206939 ab	207009 ab	207108 bc
206595 ab^{-1}	206656 ad^{-1}	206878 a	206940 a	207010 a	207111 a
206596 ab^{-1}	206675 ac^{-1}	206879 a	206944 ab	207012 ac^{-1}	207112 bc
206599 ab^{-1}	206676 bc^{-1}	206880 b	206945 b	207014 ab	207115 ab
206603 ab	206683 ab	206881 b	206946 ab^{-1}	207015 ab^{-1}	207116 a
206604 a	206684 a	206884 ac^{-1}	206948 ac^{-1}	207016 ad^{-1}	207120 ab
206608 ab	206688 ab	206886 b	206949 ab^{-1}	207020 a	207123 ab^{-1}
206611 ac^{-1}	206697 ac^{-1}	206888 ad^{-1}	206950 ac	207024 ac	207124 ac^{-1}
206612 ab^{-1}	206699 ab^{-1}	206890 ab^{-1}	206952 ac^{-1}	207025 b	207128 ac^{-1}
206615 ab^{-1}	206700 ad	206891 b	206953 ab^{-1}	207026 a	207129 ab^{-1}
206616 ad^{-1}	206703 ad	206892 b	206954 ab^{-1}	207028 ac^{-1}	207131 b
206618 ab^{-1}	206704 ab^{-1}	206893 ac^{-1}	206955 ac	207029 a	207132 a
206619 b	206715 bd^{-1}	206894 ac^{-1}	206956 a	207030 b	207133 ac^{-1}
206620 b	206716 ad	206895 b	206957 ab^{-1}	207031 ab^{-1}	207134 ac^{-1}
206621 ab^{-1}	206719 ad	206896 b	206958 ab^{-1}	207032 ac^{-1}	207135 a
206622 ab^{-1}	206720 ab^{-1}	206897 b	206959 a	207034 ab^{-1}	207136 b
206623 b	206852 ac^{-1}	206898 ac^{-1}	206960 ac	207035 b	207137 ac^{-1}
206624 b	206855 ab^{-1}	206901 ac^{-1}	206961 ab	207037 ac^{-1}	207138 b

207140 ac^{-1}	207195 ab	207268 bc	207364 a	207417 a	207479 b
207141 b	207196 a	207270 ab	207367 b	207418 a	207481 a
207142 ab^{-1}	207200 ab	207271 a	207368 a	207420 b	207482 b^2
207144 ac^{-1}	207202 a	207272 ab	207372 a	207421 a	207483 b
207145 ab^{-1}	207204 ab^{-1}	207276 a	207379 a	207422 a	207484 a
207146 ab^{-1}	207205 ac	207280 ac	207380 a	207423 b	207485 a
207147 ab^{-1}	207208 ac^{-1}	207281 b	207383 a	207443 a	207486 bd^{-1}
207148 ab	207209 ab^{-1}	207282 a	207384 ad	207444 a	207487 a
207149 ab^{-1}	207212 ad	207283 ab^{-1}	207385 a	207448 a	207488 b
207150 ab^{-1}	207213 ab^{-1}	207284 ad^{-1}	207386 b	207452 a	207522 a
207151 ad	207214 ab^{-1}	207285 a	207387 b	207456 $(db)^2a$	207523 a
207152 ab^{-1}	207215 ad	207286 b	207388 a	207457 b	207524 a
207153 bc^{-1}	207216 ab^{-1}	207287 ab^{-1}	207389 a	207458 b	207528 a
207154 ac	207217 bc^{-1}	207288 ac^{-1}	207391 a	207460 b	207532 a
207155 ab^{-1}	207218 ac	207290 ab^{-1}	207394 a	207461 b	207538 a
207157 ac	207219 bc^{-1}	207291 b	207396 a	207462 b	207539 a
207158 bc^{-1}	207221 b	207293 ad^{-1}	207397 a	207464 b	207540 a
207159 bc^{-1}	207222 bc^{-1}	207294 ac^{-1}	207398 a	207465 a	207541 a
207161 ab^{-1}	207223 bc^{-1}	207295 a	207400 a	207466 a	207542 a
207162 bc^{-1}	207225 bc^{-1}	207296 b	207401 a	207467 b	207543 a
207163 bc^{-1}	207226 bc^{-1}	207345 ab	207402 a	207468 b	207544 a
207164 b	207227 bc^{-1}	207346 a	207404 a	207469 b	207545 a
207165 bc^{-1}	207228 b	207347 bc	207405 a	207470 b	207546 a
207166 ac^{-1}	207229 bc^{-1}	207348 bc	207406 a	207471 b	207549 a
207167 b	207230 ac^{-1}	207350 ab	207407 a	207472 b	207550 ad^{-1}
207168 bc^{-1}	207231 ad	207351 a	207410 b	207473 ab^{-2}	207551 a
207187 a	207232 bc^{-1}	207352 bc	207411 a	207474 ab^{-1}	207602 ab^{-1}
207188 bc	207265 ab	207355 ac	207413 b	207475 a	207603 a
207191 a	207266 a	207356 a	207414 ad^{-1}	207477 ab	207604 a
207192 bc	207267 bc	207363 a	207415 ab	207478 b^2	207606 $(bd)^2a$

207607 b
207608 a
207612 a
207620 a
207623 ab
207624 ab^{-1}
207628 ab
207636 a
207639 ac^{-1}
207641 bc^{-1}
207642 bc^{-1}
207643 cb^{-1}
207644 b
207645 a
207646 bc^{-1}
207647 ad
207649 ab^{-1}
207650 ad
207652 b
207653 a
207654 ac^{-1}
207656 ad^{-1}
207658 ab^{-1}
207659 ab^{-1}
207660 ab
207661 b
207662 ab^{-1}
207663 ad
207664 ab^{-1}
207665 ab^{-1}
207666 ac^{-1}
207669 ab^{-1}
207670 ad^{-1}
207671 ac^{-1}
207674 ac^{-1}
207675 ab^{-1}
207676 b
207677 a
207678 a
207679 b
207680 cb^{-1}
207700 a
207703 ac
207704 bc^{-1}
207708 ad
207713 ab^{-1}
207714 ac^{-1}
207716 a
207717 ac^{-1}
207718 ab^{-1}
207720 ad^{-1}
207721 ab^{-1}
207722 ab^{-1}
207723 ab^{-1}
207724 ad
207725 a
207726 ab^{-1}
207727 ad
207728 ab^{-1}
207729 bc
207730 a
207731 ab
207733 ad
207734 bc
207735 a
207737 ab^{-1}
207738 a
207739 ab
207740 b
207741 a
207742 a
207743 ab
207744 b
207778 ac^{-1}
207780 a
207783 a
207784 ad^{-1}
207788 ad
207793 ab^{-1}
207794 ac^{-1}
207796 b
207797 ac^{-1}
207798 ac^{-1}
207799 ab^{-1}
207800 ad^{-1}
207802 ab^{-1}
207803 ab^{-1}
207805 b
207806 ad^{-1}
207807 ad
207808 ab^{-1}
207858 ab^{-1}
207860 a
207863 a
207864 ac^{-1}
207868 ad
207875 ac^{-1}
207879 ac^{-1}
207883 ab
207884 a
207891 ab^{-1}
207895 a
207897 b
207898 a
207899 b
207900 a
207905 b
207906 ad^{-1}
207908 ad^{-1}
207909 ad^{-1}
207910 b
207912 ab^{-1}
207913 ac^{-1}
207914 ab^{-1}
207915 b
207916 b
207917 ad^{-1}
207918 ab^{-1}
207919 b
207920 b
207955 ac^{-1}
207959 bc^{-1}
207963 ab
207964 a
207969 ab
207970 ab^{-1}
207972 ad^{-1}
207973 ab^{-1}
207974 b
207976 ab^{-1}
207977 ac^{-1}
207978 ab^{-1}
207979 b
207980 a
207981 ab^{-1}
207982 ab^{-1}
207983 a
207984 b
208033 ab
208034 a
208035 ac^{-1}
208036 ad^{-1}
208038 ab
208039 ac^{-1}
208040 ab^{-1}
208044 a
208048 ac
208049 b
208050 a
208051 ac^{-1}
208052 ad^{-1}
208054 b
208055 ab^{-1}
208056 ad^{-1}
208059 b
208063 a
208131 ab^{-1}
208132 ad^{-1}
208135 ab^{-1}
208139 ab
208140 a
208144 ab
208148 ad^{-1}
208151 ab^{-1}
208152 ad^{-1}
208153 ab^{-1}
208154 ab^{-1}
208155 b
208156 a
208157 ad^{-1}
208158 ad^{-1}
208159 a
208160 b
208161 ac^{-1}
208164 ad^{-1}
208169 ab^{-1}
208170 ab^{-1}
208171 bc^{-1}
208172 a
208173 ab^{-1}

208175 a	208315 b	208433 cb^{-1}	208498 a	208628 ac	208689 ab^{-1}
208177 ab^{-1}	208319 a	208434 ac^{-1}	208499 b	208631 ab	208690 b
208179 ab^{-1}	208379 ac	208437 ac^{-1}	208501 ad	208632 ad^{-1}	208691 a
208183 ab^{-1}	208388 ac	208438 ac^{-1}	208502 bc	208636 a	208693 b
208185 ab^{-1}	208391 a	208439 ab^{-1}	208503 a	208643 a	208694 ad^{-1}
208186 ab^{-1}	208392 bc^{-1}	208442 ab^{-1}	208505 ab^{-1}	208644 ad^{-1}	208695 ab^{-1}
208187 ab^{-1}	208396 a	208443 ab^{-1}	208506 a	208647 ab^{-1}	208697 a
208188 b	208403 bc^{-1}	208444 b	208507 b	208648 a	208698 ab^{-1}
208189 ad^{-1}	208404 ac	208445 ac	208508 a	208652 a	208699 ab^{-1}
208191 b	208407 ab^{-1}	208446 ac	208509 ac	208659 a	208700 b
208192 ad^{-1}	208408 ac^{-1}	208447 b	208510 ac	208660 ad^{-1}	208701 ad^{-1}
208211 bd^{-1}	208410 ab^{-1}	208448 ab^{-1}	208511 a	208663 ac^{-1}	208702 ad^{-1}
208212 ad^{-1}	208411 ab^{-1}	208468 ab	208512 bc	208664 ad^{-1}	208703 b
208219 ab	208412 a	208471 a	208546 ac^{-1}	208665 a	208723 a
208220 a	208413 ac	208476 a	208548 ab	208666 bc^{-1}	208724 ad^{-1}
208224 ab	208415 a	208481 ab^{-1}	208551 ac	208667 ab^{-1}	208727 ac^{-1}
208228 ad^{-1}	208416 ab^{-1}	208482 ab^{-1}	208552 ad^{-1}	208668 a	208728 a
208233 bc^{-1}	208417 ab^{-1}	208484 ab	208556 a	208669 ad^{-1}	208732 a
208235 bc^{-1}	208418 ad	208485 ab^{-1}	208561 ab^{-1}	208670 bc^{-1}	208737 b
208236 a	208420 b	208486 ab^{-1}	208564 ac	208671 a	208738 b
208237 ab^{-1}	208421 a	208488 ad^{-1}	208566 ad^{-1}	208673 ab^{-1}	208740 b
208239 a	208422 ad^{-1}	208489 ab^{-1}	208567 ab^{-1}	208674 bc^{-1}	208741 b
208240 ab^{-1}	208424 ab^{-1}	208490 ab^{-1}	208568 ad^{-1}	208676 ad^{-1}	208742 b
208243 ac^{-1}	208426 ab^{-1}	208491 ab^{-1}	208570 ab^{-1}	208677 ac^{-1}	208744 b
208251 ac^{-1}	208427 ab^{-1}	208492 a	208571 ab^{-1}	208681 a	208745 a
208252 a	208428 a	208493 ac	208573 ab	208682 a	208746 a
208255 a	208429 b	208494 ab^{-1}	208574 ad^{-1}	208683 ab^{-1}	208747 b
208256 ad^{-1}	208430 ad^{-1}	208495 a	208575 a	208684 a	208748 b
208300 a	208431 a	208496 ab^{-1}	208576 db^{-1}	208685 ad^{-1}	208749 b
208304 ac	208432 ab^{-1}	208497 bc	208626 ad^{-1}	208687 a	208750 b

208751 b	208826 ab^{-1}	210465 ad	210955 ab	211046 ac	211207 a
208752 b	208827 db^{-1}	210473 ab^{-1}	210956 a	211048 ac^{-1}	211208 bc
208753 ab^{-1}	208829 ad^{-1}	210474 ab^{-1}	210963 ab^{-1}	211049 ab^{-1}	211211 ab
208754 ab^{-1}	208830 ad^{-1}	210475 ad	210967 a	211050 ab^{-1}	211212 a
208755 a	208831 a	210476 ab^{-1}	210969 b	211051 ac	211216 ab
208757 ad^{-1}	208882 bd^{-1}	210477 ab^{-1}	210970 a	211052 a	211219 bd
208758 ab^{-1}	208883 a	210479 ab^{-1}	210971 b	211053 ac^{-1}	211220 ab^{-1}
208759 ab^{-1}	208884 ad^{-1}	210480 ab	210972 a	211054 ab^{-1}	211223 bd
208761 a	208887 ab^{-1}	210481 b	210977 b	211055 a	211224 a
208762 bd^{-1}	208888 a	210489 ab^{-1}	210978 ad^{-1}	211056 ac	211225 bd
208763 db^{-1}	208892 a	210491 b	210980 ac^{-1}	211105 ab	211226 bd
208764 a	210435 bc	210493 ac^{-1}	210981 ad^{-1}	211106 a	211227 b
208765 ad^{-1}	210436 bc	210495 bc^{-1}	210982 b	211107 ad^{-1}	211228 a
208766 ab^{-1}	210439 a	210496 b	210984 ac^{-1}	211108 ac^{-1}	211229 b
208767 a	210440 bc	210515 a	210985 ad^{-1}	211110 ab	211230 a
208802 ab^{-1}	210443 ab	210516 bc	210986 ab^{-1}	211111 ab^{-1}	211231 a
208803 a	210444 a	210523 ab	210987 b	211112 ac^{-1}	211232 b
208804 ad^{-1}	210448 ab	210524 a	210988 b	211116 a	211233 ad
208807 ab^{-1}	210451 bd	210528 ab	210989 ac^{-1}	211120 ac	211236 ac^{-1}
208808 a	210452 ab^{-1}	210537 ab^{-1}	210990 ab^{-1}	211121 b	211240 ac^{-1}
208812 a	210455 bd	210539 ad	210991 b	211122 a	211241 ab^{-1}
208817 ab^{-1}	210456 a	210540 bd^{-1}	210992 b	211123 ad^{-1}	211242 ab^{-1}
208818 ab^{-1}	210457 bd	210543 ab^{-1}	211027 ad^{-1}	211124 ac^{-1}	211243 ad
208819 a	210458 bd	210544 ad	211031 ad^{-1}	211126 b	211244 bd^{-1}
208820 ad^{-1}	210459 b	210555 ad	211035 ab	211127 ad^{-1}	211245 bd^{-1}
208821 ad^{-1}	210460 b	210556 ab^{-1}	211036 a	211128 ab^{-1}	211246 ab^{-1}
208822 ad^{-1}	210461 b	210559 ab^{-1}	211041 b	211131 b	211247 bd^{-1}
208823 ab^{-1}	210462 a	210560 ad	211042 ab^{-1}	211135 a	211248 ab
208824 ad^{-1}	210463 b	210947 ad^{-1}	211044 ac^{-1}	211203 bc	211249 b
208825 a	210464 b	210951 ab^{-1}	211045 ab^{-1}	211204 bc	211251 bc^{-1}

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211284 bc
211291 ab
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211296 ab
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211312 ad
211315 ab^{-1}
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211328 ad
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211451 ac
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211468 a
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211476 ab^{-1}
211479 a
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211482 b
211483 b
211484 a
211485 b
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211487 a
211488 b
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211490 a
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211494 a
211496 a
211497 a
211498 a
211499 ad
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211503 a
211504 ab
211505 b
211507 a
211510 b
211511 b
211513 a
211514 b
211515 b
211516 a
211518 ab^{-1}
211519 a
211520 b
211539 a
211543 $d^{-1}bd^{-1}$
211544 bd^{-1}
211547 ad
211548 a
211552 ab
211553 b
211554 b
211556 b
211557 b
211558 b
211560 b
211561 a
211562 a
211563 b
211564 b
211565 b
211566 b
211567 b
211568 b
211569 b
211570 ab^{-1}
211571 a
211573 ab^{-1}
211574 ad
211575 ad^{-1}
211577 a
211578 ab^{-1}
211579 ad
211580 a
211581 ab^{-1}
211582 ab^{-1}
211583 a
211584 ad
211617 ad
211618 a
211619 a
211622 ad
211623 bd^2
211624 a^2
211628 a
211632 ad
211633 ab
211634 a
211635 a
211637 a
211638 a
211639 a
211640 a
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211698 a
211699 a
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211716 ac
211719 ab^{-1}
211720 ab^{-1}
211723 ad
211728 ab
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211732 ac
211735 b
211736 a
211737 bd
211738 a
211739 ad
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211742 a
211743 b
211744 b
211745 b
211746 ac^{-1}
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211750 b
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211754 ac^{-1}
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211763 ac^{-1}
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211767 ad^{-1}
211769 ac^{-1}
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211774 ac
211776 b
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211808 ab
211809 ad
211810 ab^{-1}

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211823 bd^{-1}
211824 ad
211825 b
211826 bc
211827 b
211829 bc
211830 ad
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211834 a
211835 b
211836 ab^{-1}
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211894 ad
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211899 ad
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211904 ad
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211956 ac
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211963 ad
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212499 bd
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212503 b
212504 a
212505 bd
212506 ac
212507 ad
212508 ab^{-1}
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212513 ad
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212516 b
212517 ac^{-1}
212518 ab
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212544 b
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212576 ab
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212585 ad^{-1}
212586 bd^{-1}
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212588 bd^{-1}
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212590 ab^{-1}
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212594 bc
212595 b
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212598 ad
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212643 ad^{-1}
212644 ab
212646 ad
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212648 ab^{-1}
212656 ad
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212659 ad^{-1}
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212663 ab
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212666 ac
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212672 b
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212722 a
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212724 ac
212726 ab
212727 ad^{-1}
212728 ab^{-1}
212731 ad
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212743 ad^{-1}
212744 ab^{-1}
212747 ad
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212756 ab^{-1}
212759 bd
212760 a
212761 a
212762 a
212763 ad
212764 ab^{-1}
212765 b
212766 a
212767 b
212768 b
212769 b
212770 ab^{-1}
212773 ad^{-1}
212774 ad
212776 ab^{-1}
212777 a
212778 a
212779 ad

212780 ac^{-1}	212844 b	212917 ab^{-1}	458772 a	458879 b	459088 ab^{-1}
212782 ab^{-1}	212845 b	212918 b	458776 ab^{-1}	458880 b	459092 a
212783 ac^{-1}	212846 b	212919 ac^{-1}	458780 ab^{-1}	458916 a	459100 abc
212784 ab	212847 b	212920 a	458781 a	458920 ad^{-1}	459101 a
212785 b	212848 b	212921 a	458782 ad^{-1}	458930 bd^{-1}	459103 abc
212787 a	212849 b	212922 ad^{-1}	458783 abc	458932 a	459108 a
212790 b	212850 ab^{-1}	212923 b	458784 ab^{-1}	458933 ab^{-1}	459112 ab^{-1}
212791 ab^{-1}	212851 a	212926 a	458788 a	458935 ad^{-1}	459116 ab^{-1}
212793 a	212853 ab^{-1}	212927 ac^{-1}	458792 ad^{-1}	458936 ad^{-1}	459117 a
212794 ad^{-1}	212854 ad	212928 b	458797 a	458938 bd^{-1}	459118 ab^{-1}
212795 b	212855 ad^{-1}	212977 ab	458798 ad^{-1}	458941 a	459119 ab^{-1}
212796 ac^{-1}	212857 a	212978 a	458802 ab^{-1}	458942 ad^{-1}	459120 ab^{-1}
212798 ab^{-1}	212858 ab^{-1}	212979 a	458805 ab^{-1}	458994 ab^{-1}	459121 b
212799 ac^{-1}	212859 ad	212982 ab	458806 ad^{-1}	458996 a	459123 b
212800 b	212860 ab^{-1}	212983 ad^{-1}	458807 ad^{-1}	458999 ad^{-1}	459125 b
212819 a	212861 ab^{-1}	212984 ab^{-1}	458810 ad^{-1}	459000 bd^{-1}	459126 b
212823 ad^{-1}	212862 ab^{-1}	212987 ad	458813 a	459059 ab^{-1}	459127 b
212824 bd^{-1}	212863 ab^{-1}	212988 ac^{-1}	458814 a	459061 ab^{-1}	459129 b
212827 ad	212864 ad	240131 a	458836 a	459062 ad^{-1}	459131 b
212828 ac^{-1}	212897 ad	240135 b	458852 a	459063 ab^{-1}	459132 b
212832 ab	212898 ad	240147 a	458856 ad^{-1}	459065 ab^{-1}	459133 a
212833 b	212899 a	240151 a	458861 a	459067 ab^{-1}	459134 a
212834 b	212902 ad	240153 a	458866 b	459069 a	459135 b
212836 b	212903 ad^{-1}	240154 b	458869 b	459070 a	459136 b
212837 b	212904 ab^{-1}	240155 b	458870 b	459071 ab^{-1}	459140 a
212838 b	212908 ac^{-1}	240169 a	458871 b	459076 a	459144 ac^{-1}
212840 b	212912 ad	240170 a	458874 b	459080 ad^{-1}	459148 ac^{-1}
212841 a	212913 b	240211 a	458876 b	459085 a	459156 a
212842 a	212914 ac^{-1}	240233 a	458877 a	459086 ab^{-1}	459160 bd^{-1}
212843 b	212915 a	240235 b	458878 a	459087 ab^{-1}	459164 ac^{-1}

459165 a	459209 ab^{-1}	459253 ab^{-1}	459368 ac^{-1}	459444 a	459490 ab^{-1}
459166 ab^{-1}	459211 ab^{-1}	459255 ab^{-1}	459372 ac^{-1}	459445 ab^{-1}	459492 a
459167 ab^{-1}	459212 ab^{-1}	459256 bd^{-1}	459373 a	459446 ac^{-1}	459493 ab^{-1}
459168 ab^{-1}	459217 b	459257 ab^{-1}	459375 ad^{-1}	459447 ab^{-1}	459495 bd^{-1}
459172 a	459219 b	459259 ab^{-1}	459376 ac^{-1}	459448 ac^{-1}	459496 ac^{-1}
459176 ab^{-1}	459220 a	459260 ab^{-1}	459378 b	459450 ac^{-1}	459498 ad^{-1}
459180 ab^{-1}	459221 b	459261 a	459381 b	459453 a	459500 ac^{-1}
459181 a	459222 b	459262 bd^{-1}	459382 b	459454 ac^{-1}	459501 a
459182 ab^{-1}	459223 b	459263 ab^{-1}	459383 b	459455 ad^{-1}	459502 ad^{-1}
459183 ab^{-1}	459224 a	459314 ab^{-1}	459386 b	459456 ac^{-1}	459504 ac^{-1}
459184 ab^{-1}	459225 b	459317 ab^{-1}	459388 b	459458 ab^{-1}	459506 ab^{-1}
459185 abc	459227 b	459318 ac^{-1}	459389 a	459460 a	459508 a
459187 ab^{-1}	459228 b	459319 ab^{-1}	459390 a	459461 ab^{-1}	459509 ab^{-1}
459188 a	459231 b	459322 ac^{-1}	459391 b	459462 ad^{-1}	459510 ac^{-1}
459189 ab^{-1}	459232 b	459324 ac^{-1}	459392 b	459463 ab^{-1}	459511 ab^{-1}
459190 abc	459233 abc	459325 a	459412 a	459464 a	459512 ac^{-1}
459191 ab^{-1}	459235 ac^{-1}	459326 a	459416 ac^{-1}	459466 ad^{-1}	459514 ac^{-1}
459192 bd^{-1}	459236 a	459327 ad^{-1}	459420 ac^{-1}	459468 ac^{-1}	459516 ac^{-1}
459193 ab^{-1}	459237 ab^{-1}	459328 ac^{-1}	459421 a	459471 ad^{-1}	459517 a
459195 ab^{-1}	459238 abc	459332 a	459422 bd^{-1}	459472 ac^{-1}	459518 ac^{-1}
459197 a	459239 ad^{-1}	459336 ac^{-1}	459423 ad^{-1}	459474 b	459519 ad^{-1}
459198 ab^{-1}	459240 ab^{-1}	459340 ac^{-1}	459424 ac^{-1}	459476 a	459603 ac^{-1}
459199 ab^{-1}	459241 ab^{-1}	459348 a	459428 a	459477 b	459607 a^2d
459200 ab^{-1}	459243 ab^{-1}	459352 ac^{-1}	459432 ac^{-1}	459478 b	459609 ac^{-1}
459203 ac^{-1}	459244 ab^{-1}	459356 ac^{-1}	459436 ac^{-1}	459479 b	459610 a^2d
459204 a	459245 a	459357 a	459437 a	459480 a	459611 ad^{-1}
459205 ab^{-1}	459246 ad^{-1}	459358 abc	459438 ac^{-1}	459482 b	459613 a
459206 ad^{-1}	459248 ab^{-1}	459359 ad^{-1}	459439 ad^{-1}	459484 b	459615 a
459207 ab^{-1}	459251 ab^{-1}	459360 ac^{-1}	459440 ac^{-1}	459487 b	459617 abc
459208 a	459252 a	459364 a	459442 ac^{-1}	459488 b	459618 ab^{-1}

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459627 ad^{-1}
459629 a
459630 ac^{-1}
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459633 b
459634 b
459635 b
459637 b
459638 b
459639 b
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459642 b
459643 b
459645 a
459646 a
459647 a
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459735 b
459736 a
459737 b
459738 b
459739 b
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460151 b
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460155 b
460156 b
460157 a
460158 a
460159 b
460160 b
460210 bd
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460212 a
460213 ab^{-1}
460215 bd^{-1}
460216 bd^{-1}
460217 ab^{-1}
460218 bd^{-1}
460221 a
460222 $ca^{-1}d^{-1}a$
460224 dcb^2
460274 ab^{-1}
460275 ab^{-1}
460276 a
460279 bd^{-1}
460280 bd^{-1}
460284 $a^2b^{-1}cd^{-1}b^{-1}$
460401 b
460402 b
460403 b
460406 b
460409 b
460410 b
460411 b
460412 b
460413 a
460414 a
460415 b
460416 b
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460499 b
460500 a
460502 b
460504 a
460505 b
460506 b
460507 b
460508 b
460511 b
460512 b
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460514 ab^{-1}
460515 ab^{-1}
460516 a
460518 bd^{-1}
460520 $aca^{-1}d^{-1}$
460521 bc^{-1}
460522 bd^{-1}
460523 cd^{-1}
460524 cd^{-1}
460525 a
460526 bd^{-1}
460528 $cabcbacd$
460529 dc^{-1}
460530 ab^{-1}
460531 cd^{-1}
460532 a

460536 cd^{-1}
460537 ac^{-1}
460539 cd^{-1}
460540 cd^{-1}
460541 a
460542 bd^{-1}
460543 cd^{-1}
460627 abc
460628 a
460631 $dbacbdbcab$
460633 ac^{-1}
460634 $bcdaca$
460635 $abcabdbacbad$
460636 $a^2bc^{-1}d^{-1}$
460637 a
460639
460753 b
460754 b
460755 b
460756 a
460757 b
460758 b
460759 b
460761 b
460762 b
460763 b
460764 a
460767 a
460768 b
460769 b
460770 b
460771 b
460772 a
460774 b
460775 b
460778 b
460780 a
460784 b
460786 ab^{-1}
460787 ab^{-1}
460788 a
460789 ab^{-1}
460791 bd^{-1}
460793 ac^{-1}
460795 $abcad^{-1}ca^{-1}cb^{-1}$
460796 ad^{-1}
460797 a
460798 bd^{-1}
460799 cd^{-1}
461426 b
461429 b
461430 b
461431 b
461434 b
461436 b
461437 a
461438 a
461439 b
461440 b
461554 ab^{-1}
461556 a
461559 $cbdabcda$
461560 bd^{-1}
461777 b
461778 b
461779 b
461782 b
461783 b
461784 a
461786 b
461792 b
461809 c
461810 c
461811 c
461813 c
461814 c
461815 c
461816 c
461817 c
461818 c
461819 c
461821 a
461822 c
461823 a
462872 b
462876 ad^{-1}
462878 a
462879 ad^{-1}
462880 ad^{-1}
462884 ab^{-1}
462888 b
462892 ab^{-1}
462893 ad
462894 b
462896 ab^{-1}
462898 b
462901 a
462902 b
462903 b
462906 b
462908 bd
462909 a
462910 a
462911 bd
462912 b
462932 ba^{-1}
462936 ac
462940 ac
462948 ab^{-1}
462952 a
462956 b
462957 ac^{-1}
462958 a
462959 b
462960 ab^{-1}
462966 ab^{-1}
462967 ad
462970 ad
462973 a
462974 a
462975 ab^{-1}
462976 ad^{-1}
463012 ab^{-1}
463016 ac
463020 ac
463026 ab^{-1}
463028 bd^{-1}
463030 b
463031 ab^{-1}
463032 ac
463037 ab^{-1}
463038 ac
463039 ab^{-1}
463040 bd
463090 bd^{-1}
463092 bd^{-1}
463095 ab^{-1}
463096 a
463100 ac
463108 ab^{-1}
463112 c
463116 ab^{-1}
463117 ab^{-1}
463118 c
463119 ad^{-1}
463120 ac^{-1}
463153 bd
463155 bd
463157 a
463159 c
463161 bd
463163 bd
463164 ac^{-1}
463165 a
463166 a
463167 c
463168 b
463176 a
463180 a
463183 b
463184 ad^{-1}
463188 ba^{-1}
463192 ac
463196 ba^{-1}
463197 ba^{-1}
463199 ba^{-1}
463200 ac^{-1}
463204 ac^{-1}
463208 b
463212 ac^{-1}
463213 cd
463215 ab^{-1}
463216 ac^{-1}
463219 cd
463223 ac^{-1}

463227 ad^{-1}	463374 a	463470 ad	463533 ac^{-1}	463576 a	463627 ab^{-1}
463228 ad^{-1}	463375 ab^{-1}	463471 ad^{-1}	463534 ad	463578 ac^{-1}	463629 ab^{-1}
463229 a	463376 ac^{-1}	463472 ad^{-1}	463535 ad^{-1}	463580 ad^{-1}	463630 c
463231 ad^{-1}	463410 b	463474 bc^{-1}	463536 ac^{-1}	463583 ad^{-1}	463631 ad^{-1}
463232 ad^{-1}	463413 a	463477 bc^{-1}	463538 bc^{-1}	463584 ad^{-1}	463632 ac^{-1}
463236 ac^{-1}	463414 b	463478 bc^{-1}	463540 bc^{-1}	463588 ad^{-1}	463639 cd
463240 b	463415 ab	463479 bc^{-1}	463541 bc^{-1}	463590 c	463640 c
463244 b	463418 b	463482 bc^{-1}	463542 c	463591 bd^{-1}	463642 c
463247 c	463420 bd	463484 bc^{-1}	463543 bc^{-1}	463592 c	463646 c
463248 ac^{-1}	463421 a	463485 a	463544 c	463594 ac^{-1}	463647 ab^{-1}
463252 ac^{-1}	463422 a	463486 a	463546 bc^{-1}	463596 ad^{-1}	463648 ad^{-1}
463256 ac	463423 bd	463487 bc^{-1}	463549 bc^{-1}	463597 bd^{-1}	463649 ab^{-1}
463260 ac^{-1}	463424 b	463488 ad^{-1}	463550 ad	463598 ad	463650 bc^{-1}
463263 ad^{-1}	463428 ac^{-1}	463492 ac^{-1}	463551 bc^{-1}	463600 bd	463653 bc^{-1}
463264 ac^{-1}	463432 c	463496 b	463552 bd	463602 bc^{-1}	463654 bd
463268 ac^{-1}	463436 c	463500 b	463554 a	463604 bc^{-1}	463656 c
463272 ac	463438 b	463501 ac^{-1}	463556 a	463605 bc^{-1}	463657 bc^{-1}
463284 bd^{-1}	463439 b	463502 b	463557 b	463606 bc^{-1}	463658 bc^{-1}
463288 ac	463440 ad^{-1}	463503 ab	463558 b	463607 bc^{-1}	463659 ad
463300 a	463444 ba^{-1}	463504 ac^{-1}	463559 ab^{-1}	463608 c	463661 bc^{-1}
463304 a	463448 c	463508 ac^{-1}	463560 a	463610 bc^{-1}	463662 c
463308 ac^{-1}	463452 ba^{-1}	463512 c	463562 b	463612 bc^{-1}	463663 bc^{-1}
463312 b	463453 ba^{-1}	463516 ac^{-1}	463564 bd	463613 bc^{-1}	463664 c
463316 a	463454 ad	463517 ac^{-1}	463567 bd	463614 a	463683 ad^{-1}
463324 ad^{-1}	463455 ba^{-1}	463518 ad	463568 b	463615 bc^{-1}	463687 cd
463328 ad^{-1}	463456 ac^{-1}	463519 ad^{-1}	463570 cd	463619 ad^{-1}	463688 c
463364 ab^{-1}	463460 ab^{-1}	463520 ad^{-1}	463572 a	463623 ab^{-1}	463689 ad^{-1}
463368 c	463464 c	463524 ac^{-1}	463573 cd	463624 c	463690 c
463372 ac^{-1}	463468 ab^{-1}	463528 c	463574 ab^{-1}	463625 ad^{-1}	463691 ad^{-1}
463373 ab^{-1}	463469 ac^{-1}	463532 ac^{-1}	463575 ab^{-1}	463626 ab^{-1}	463694 c

463695 ac^{-1}	463739 ad^{-1}	463779 bd^{-1}	463817 a	463861 bd^{-1}	465533 a
463696 ad^{-1}	463741 a	463781 bd^{-1}	463818 b	463862 ac^{-1}	465535 cd^{-1}
463699 c	463742 a	463782 ac^{-1}	463819 b	463863 ab^{-1}	465536 ad^{-1}
463703 ba^{-1}	463743 a	463783 bd^{-1}	463823 a	463864 c	465599 cd^{-1}
463704 c	463744 ad^{-1}	463784 c	463824 b	463865 ad^{-1}	465600 bd
463705 c	463745 ab^{-1}	463785 ac^{-1}	463825 ab^{-1}	463866 ac^{-1}	465667 ad^{-1}
463706 ba^{-1}	463746 ac^{-1}	463786 ac^{-1}	463827 ad^{-1}	463869 bd^{-1}	465671 ac^{-1}
463707 ad^{-1}	463747 ad^{-1}	463789 bd^{-1}	463830 ab^{-1}	463870 c	465672 a
463709 ba^{-1}	463750 ab^{-1}	463790 c	463831 ab^{-1}	463871 ad^{-1}	465673 ad^{-1}
463710 c	463751 ab^{-1}	463791 bd^{-1}	463832 a	465412 ab^{-1}	465674 ab^{-1}
463711 ba^{-1}	463752 c	463792 bd^{-1}	463833 ad^{-1}	465416 a	465675 c
463712 ac^{-1}	463755 ad^{-1}	463793 ab^{-1}	463834 ac^{-1}	465420 a	465677 ab^{-1}
463713 ab^{-1}	463757 ad^{-1}	463794 ac^{-1}	463835 ac^{-1}	465432 b	465678 a
463714 ac^{-1}	463758 c	463795 bd^{-1}	463839 a	465436 ca^{-1}	465679 ab^{-1}
463717 bc^2	463759 ac^{-1}	463798 bd^{-1}	463840 ad^{-1}	465438 a	465680 c
463718 ac^{-1}	463760 c	463799 bd^{-1}	463841 ab^{-1}	465439 ca^{-1}	465683 ac^{-1}
463720 c	463761 ab^{-1}	463801 ac^{-1}	463842 ac^{-1}	465440 ad^{-1}	465688 b
463721 ab^{-1}	463762 ac^{-1}	463802 bd^{-1}	463843 ad^{-1}	465444 cd^{-1}	465689 ac^{-1}
463722 ac^{-1}	463763 ac^{-1}	463803 bd^{-1}	463845 bd^{-1}	465448 ab	465691 c
463723 ac^{-1}	463765 bd	463805 a	463846 ab^{-1}	465452 cd^{-1}	465693 ac^{-1}
463725 ac^{-1}	463766 ac^{-1}	463806 a	463847 bd^{-1}	465453 ca^{-1}	465694 a
463726 c	463767 ac^{-1}	463807 a	463848 a	465454 ad	465696 c
463727 ac^{-1}	463768 c	463808 bd^{-1}	463849 ad^{-1}	465455 cd^{-1}	465697 ab^{-1}
463728 ad^{-1}	463771 ac^{-1}	463809 bd	463850 bd^{-1}	465456 cd^{-1}	465698 ac^{-1}
463729 ab^{-1}	463773 ac^{-1}	463810 a	463853 a	465468 bd	465704 ab
463731 ab^{-1}	463774 c	463811 ab	463854 a	465469 a	465705 c
463734 ab^{-1}	463775 ac^{-1}	463813 b	463856 bd^{-1}	465470 a	465706 c
463735 ac^{-1}	463776 ad^{-1}	463814 b	463857 ac^{-1}	465471 bd	465707 bc^{-1}
463737 ab^{-1}	463777 ab^{-1}	463815 b	463858 bd^{-1}	465472 b	465709 ac^{-1}
463738 ac^{-1}	463778 ac^{-1}	463816 a	463859 ad^{-1}	465532 ad^{-1}	465710 b

465711 bc^{-1}	465774 ad	466713 ad^{-1}	466796 a	471136 ab	471286 ab
465712 bc^{-1}	465775 bc^{-1}	466714 ac^{-1}	466797 ac^{-1}	471144 ab^{-1}	471287 a
465713 bd	465776 ad^{-1}	466715 ac^{-1}	466799 a	471148 ac^{-1}	471288 ab^{-1}
465714 b	465803 cd^{-1}	466716 ac^{-1}	466811 ac^{-1}	471149 bd^{-1}	471292 ac^{-1}
465715 bd	465805 ab^{-1}	466718 a	466812 ad^{-1}	471150 ab^{-1}	471345 b
465721 bd	465806 ab	466719 ad^{-1}	466813 a	471151 ac^{-1}	471350 b
465722 ab^{-1}	465807 ad^{-1}	466721 ab^{-1}	466815 ac^{-1}	471152 b	471351 ad^{-1}
465723 bd	465808 ad^{-1}	466724 ad^{-1}	466860 a	471154 ab^{-1}	471355 b
465725 a	465819 cd^{-1}	466728 ab	466879 cd^{-1}	471157 ab^{-1}	471356 ad
465726 a	465821 ac^{-1}	466729 ad^{-1}	471060 ab^{-1}	471158 ad	471358 ab^{-1}
465727 a	465822 ad	466730 ac^{-1}	471064 a	471159 ab^{-1}	471359 ac^{-1}
465728 b	465823 ac^{-1}	466731 bc^{-1}	471068 ab^{-1}	471162 ab^{-1}	471360 b
465731 ad^{-1}	465824 ad^{-1}	466732 cd^{-1}	471069 b	471164 ac^{-1}	471364 b
465737 ad^{-1}	465837 bc^{-1}	466733 cd	471070 a	471165 ab^{-1}	471368 a
465739 c	465838 ad	466734 ad	471071 b	471166 ab^{-1}	471372 a
465742 b	465839 cd^{-1}	466735 bc^{-1}	471072 b	471167 ab^{-1}	471373 ac
465743 cd^{-1}	465840 c	466737 bd	471080 ab^{-1}	471168 ac	471374 a
465744 c	465871 a	466739 bd	471084 ac^{-1}	471208 ab^{-1}	471375 a
465747 ba^{-1}	465872 b	466745 bd	471086 ab^{-1}	471212 ac^{-1}	471376 b
465753 ba^{-1}	465887 a	466747 bd	471087 ac^{-1}	471216 ab	471384 bd^{-1}
465754 ba^{-1}	465888 ad^{-1}	466748 a	471088 b	471218 ab^{-1}	471390 bd^{-1}
465755 c	465904 cd^{-1}	466749 a	471094 b	471221 ab^{-1}	471391 ac^{-1}
465757 ba^{-1}	466691 ad^{-1}	466750 a	471095 ad^{-1}	471222 b	471392 c
465758 ad	466692 ab^{-1}	466751 bd	471098 ad^{-1}	471223 ad^{-1}	471396 bd^{-1}
465759 ba^{-1}	466695 ab^{-1}	466771 ac^{-1}	471100 ac^{-1}	471224 ab^{-1}	471400 bd^{-1}
465760 c	466696 a	466772 ba^{-1}	471102 ab^{-1}	471226 ab^{-1}	471404 bd^{-1}
465769 c	466700 ab	466780 a	471103 ac^{-1}	471230 ad	471405 bd^{-1}
465770 c	466707 ad^{-1}	466788 ac^{-1}	471104 b	471231 ac^{-1}	471406 bd^{-1}
465771 ad^{-1}	466711 ac^{-1}	466793 ac^{-1}	471128 bd^{-1}	471232 b	471407 ac^{-1}
465773 ac^{-1}	466712 b	466795 ac^{-1}	471132 ac^{-1}	471282 a	471408 c

471409 c	471473 c	471516 ab^{-1}	471607 ab^{-1}	471676 a	471746 bc
471411 ab^{-1}	471475 ac^{-1}	471519 ac^{-1}	471610 c	471677 c	471748 ac^{-1}
471413 ab^{-1}	471476 ac^{-1}	471520 ac	471612 a	471678 c	471749 c
471414 ad	471477 ab^{-1}	471521 c	471613 c	471679 a	471750 b
471415 ab^{-1}	471478 c	471523 ac^{-1}	471614 c	471680 c	471751 a
471417 ab^{-1}	471479 ac^{-1}	471524 ac^{-1}	471615 a	471700 b	471752 c
471419 c	471480 ac^{-1}	471525 ab^{-1}	471616 b	471704 b	471754 a
471420 ab^{-1}	471481 ab^{-1}	471526 ad	471620 b	471708 a	471756 a
471421 ab^{-1}	471483 ad	471527 ab^{-1}	471624 a	471709 c	471759 a
471422 ab^{-1}	471485 ab^{-1}	471528 ac^{-1}	471628 a	471710 c	471760 b
471423 ab^{-1}	471486 ab^{-1}	471529 ab^{-1}	471632 b	471711 a	471762 bc
471424 ac	471487 ac^{-1}	471531 ad	471636 ac^{-1}	471712 c	471764 ac^{-1}
471428 c	471488 ac	471532 ac^{-1}	471640 ac^{-1}	471716 ab	471765 c
471432 a	471489 b	471533 abc^{-1}	471644 a	471720 ab	471766 c
471436 a	471494 b	471534 ab^{-1}	471645 c	471724 a	471767 a
471440 b	471495 ac^{-1}	471536 ac	471646 c	471725 c	471768 c
471444 bd^{-1}	471496 ab^{-1}	471537 c	471647 a	471726 c	471770 a
471448 bd^{-1}	471499 b	471539 c	471648 c	471727 a	471772 a
471452 bd^{-1}	471500 ad	471541 a	471652 ab^{-1}	471728 c	471775 a
471453 bd^{-1}	471503 ac^{-1}	471542 c	471656 ab^{-1}	471730 b	471776 c
471454 ab^{-1}	471504 b	471543 a	471660 a	471732 b	471778 b
471455 ac^{-1}	471505 c	471544 ab^{-1}	471661 c	471733 b	471780 b
471456 c	471507 ab^{-1}	471545 c	471662 c	471734 b	471781 b
471460 bd^{-1}	471508 ab^{-1}	471547 c	471663 a	471735 b	471782 b
471464 ac^{-1}	471509 ab^{-1}	471548 ac^{-1}	471664 c	471736 b	471783 b
471468 bd^{-1}	471510 ad	471550 ab^{-1}	471666 bc^{-1}	471738 b	471784 b
471469 bd^{-1}	471511 ab^{-1}	471551 ac^{-1}	471669 bc^{-1}	471741 b	471786 b
471470 ab^{-1}	471512 ab^{-1}	471602 bc^{-1}	471670 c	471742 b	471788 a
471471 ac^{-1}	471513 ab^{-1}	471605 bc^{-1}	471671 ab^{-1}	471743 a	471789 b
471472 c	471515 ad	471606 b	471674 b	471744 b	471790 b

471792 b	471925 ab^{-1}	471994 ac^{-1}	472048 c	472504 ac^{-1}	472756 cd^{-1}
471794 a	471926 c	471995 c	472049 c	472505 b	472758 ad
471796 ac^{-1}	471929 bc^2	471998 ac^{-1}	472050 c	472506 ac^{-1}	472760 cd^{-1}
471797 a	471930 ab^{-1}	471999 ac^{-1}	472053 c	472507 b	472761 ac^{-1}
471798 c	471931 c	472000 c	472054 c	472509 ac^{-1}	472762 cd^{-1}
471799 a	471933 ab^{-1}	472001 b	472055 ad^{-1}	472510 ac^{-1}	472763 ad
471800 c	471934 ab^{-1}	472006 b	472056 ab^{-1}	472511 ac^{-1}	472765 ac^{-1}
471802 a	471935 ab^{-1}	472007 b	472058 ad^{-1}	472512 b	472766 ac^{-1}
471804 a	471936 c	472008 ab^{-1}	472059 c	472561 ab	472767 a
471805 c	471953 c	472010 b	472062 ab^{-1}	472562 a	472768 ad
471806 c	471954 ac^{-1}	472016 b	472063 ac^{-1}	472563 a	472785 b
471807 a	471955 bd^{-1}	472017 c	472433 b	472567 a	472786 ab^{-1}
471896 bd^{-1}	471958 c	472018 ab^{-1}	472434 ab^{-1}	472568 ab^{-1}	472787 ab^{-1}
471898 bd^{-1}	471959 bd^{-1}	472019 ab^{-1}	472435 a	472571 ab	472788 c
471899 c	471963 c	472021 ab^{-1}	472437 ab^{-1}	472572 ac^{-1}	472790 ad
471902 bd^{-1}	471965 b	472022 c	472439 ab^{-1}	472689 b	472792 c
471903 ac^{-1}	471967 b	472023 ab^{-1}	472441 a	472690 ab^{-1}	472793 cd^{-1}
471904 c	471968 c	472024 ab^{-1}	472442 ab^{-1}	472691 cd^{-1}	472794 ab^{-1}
471905 c	471969 c	472027 c	472443 b	472694 ad	472795 ad
471906 ac^{-1}	471970 c	472031 ac^{-1}	472444 ab^{-1}	472697 cd^{-1}	472796 ab^{-1}
471909 ac^{-1}	471973 c	472032 c	472445 ab^{-1}	472698 ab^{-1}	472799 ab^{-1}
471910 c	471974 c	472033 c	472446 ab^{-1}	472699 ad	472800 ad
471914 ad^{-1}	471978 ac^{-1}	472034 ac^{-1}	472447 ab^{-1}	472700 cd^{-1}	472801 ab
471915 c	471982 ab^{-1}	472037 ad	472448 ad	472701 ac	472802 ac^{-1}
471917 b	471983 ac^{-1}	472038 c	472497 b	472702 ac	472803 cd^{-1}
471919 b	471984 c	472039 ab^{-1}	472498 ab^{-1}	472703 ab^{-1}	472804 cd^{-1}
471920 c	471985 c	472040 ab^{-1}	472499 b	472704 ad	472806 ad
471921 c	471986 ac^{-1}	472042 ac^{-1}	472500 ac^{-1}	472753 ab	472808 ac^{-1}
471922 ab^{-1}	471989 ac^{-1}	472043 c	472501 ab^{-1}	472754 ab^{-1}	472809 ac^{-1}
471923 bd	471990 c	472046 ac^{-1}	472503 ac^{-1}	472755 cd^{-1}	472810 ac^{-1}

472811 ad	473047 ad	473718 ad	474109 ac^{-1}	520301 a	520503 a
472812 a	473049 ab^{-1}	473722 ab^{-1}	474110 ac^{-1}	520302 a	520505 c
472813 ac^{-1}	473050 ad	473724 cd^{-1}	474111 ac^{-1}	520303 c	520508 c
472814 ac^{-1}	473051 ad	473725 ab^{-1}	520212 a	520304 b	520509 a
472816 ad	473052 ab^{-1}	473726 ab^{-1}	520216 a	520306 b	520510 a
472817 c	473055 ab^{-1}	473727 ab^{-1}	520220 c	520309 b	520511 a
472818 a	473056 ad	473728 ac	520221 a	520310 b	520516 a
472819 c	473057 b	473842 a	520222 b	520311 b	520520 b
472820 cd^{-1}	473058 ac^{-1}	473846 ab	520223 b	520314 b	520524 b
472822 c	473059 ab^{-1}	473847 a	520224 b	520316 b	520525 a
472824 ab^{-1}	473060 ab^{-1}	473848 ab^{-1}	520228 a	520317 a	520527 a
472825 c	473062 ad	473852 cd^{-1}	520232 a	520318 a	520528 b
472826 a	473063 ad	474065 b	520236 c	520319 b	520532 a
472827 c	473066 ad	474066 ab^{-1}	520237 a	520320 b	520540 abc
472828 ab^{-1}	473068 ab^{-1}	474067 ac^{-1}	520238 a	520360 b	520541 a
472830 ab^{-1}	473072 ad	474070 ad	520239 c	520370 a	520543 abc
472831 ab^{-1}	473073 c	474071 ad	520242 a	520372 a	520548 a
472919 bd^{-1}	473074 c	474072 ab^{-1}	520245 a	520373 b	520552 a
472922 bd^{-1}	473075 ab^{-1}	474074 ad	520246 a	520375 a	520556 a
472923 c	473077 c	474080 ad	520247 a	520376 b	520557 a
472924 bd	473078 c	474097 c	520250 a	520378 a	520559 c
472926 bd^{-1}	473079 ab^{-1}	474098 a	520252 c	520381 a	520560 c
472927 bd	473081 ab^{-1}	474099 ac^{-1}	520253 a	520382 b	520563 b
472928 c	473082 ab^{-1}	474101 a	520254 a	520383 c	520567 b
473041 b	473083 c	474102 c	520255 c	520434 a	520572 b
473042 ab^{-1}	473084 ab^{-1}	474103 ac^{-1}	520276 a	520436 a	520573 a
473043 ab^{-1}	473086 ab^{-1}	474104 ab^{-1}	520284 c	520439 a	520575 b
473044 ab^{-1}	473087 ab^{-1}	474105 ac^{-1}	520292 a	520444 c	520576 b
473045 ab^{-1}	473714 ab^{-1}	474106 ab^{-1}	520296 b	520499 a	520584 c
473046 ad	473717 ab^{-1}	474107 ad	520300 c	520501 a	520592 b

520596 a
520600 c
520604 c
520607 b
520608 c
520628 a
520632 c
520639 a^2c
520640 c
520644 a
520648 a
520652 a
520660 a
520668 b
520672 b
520684 a^2c
520688 c
520754 a
520757 a
520764 a
520765 a
520766 a
520767 c
520768 c
520772 a
520776 b
520780 b
520784 b
520788 a
520792 abc
520796 a
520797 a
520798 c
520799 c
520800 c
520804 a
520808 a
520812 a
520813 a
520814 c
520816 c
520821 b
520828 b
520829 a
520830 a
520831 b
520832 b
520856 c
520861 a
520862 c
520863 c
520864 c
520884 a
520888 a
520893 a
520894 c
520895 c
520896 c
520900 a
520904 a
520908 a
520911 c
520912 c
520916 a
520920 a
520924 b
520927 b
520928 b
520932 a
520936 c
520940 a
520941 a
520942 c
520944 c
520948 a
520952 c
520956 a
520957 a
520958 c
520959 c
521053 a
521055 a
521073 b
521085 a
521086 a
521087 a
521088 b
521149 a
521150 a
521151 a
521169 b
521184 b
521213 a
521215 a
522877 a
522880 b
523232 b

Appendix B

Automata that have proper subautomaton generating an infinite group

The following automata generate infinite groups because they contain either a (3,2) or (2,2) subautomaton that generates an infinite group. There are far too many groups listed below to give a detailed proof on why each are infinite. For this information, please consult the "Proofs" section of [4]. Below, we give a list of the (4,2) isomporphism class representative, number of corresponding (3,2) subautomaton (if applicable), and wreath recursion for said subautomaton.

65554. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

65574. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

65846. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

65858. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

65889. Contains (856), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (b, a)$

65890. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

65894. Contains (860), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (b, a)$

65899. Contains (864), Fractal

WR: $a = \sigma(c, c), b = (c, b), c = (b, a)$

65905. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

65910. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

65920. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

65937. Contains (874), Fractal

WR: $a = \sigma(a, a), b = (b, c), c = (b, a)$

65938. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

65942. Contains(878), Fractal

WR: $a = \sigma(b, b), b = (b, c), c = (b, a)$

65947. Contains (882), Fractal

WR: $a = \sigma(c, c), b = (b, c), c = (b, a)$

65953. Contains (883), Fractal

WR: $a = \sigma(a, a), b = (c, c), c = (b, a)$

65958. Contains (887), Fractal

WR: $a = \sigma(b, b), b = (c, c), c = (b, a)$

65963. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

65990. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66001. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66006. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

66016. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66099. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66131. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66151. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

66199. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

66211. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66227. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66243. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66275. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66291. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

66395. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66417. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66422. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

66427. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66432. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66475. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66491. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66502. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66513. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66518. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

66523. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66528. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66539. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66555. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

66929. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

66934. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

66944. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

67542. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

67552. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

68593. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

68603. Contains (779), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (a, a)$

68608. Contains (775), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (a, a)$

70227. Contains a (2,2)-automaton Fractal by words

WR: $a = \sigma(b, a), b = (b, a)$

70247. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

70275. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

70295. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

70307. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

70466. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

70513. Contains (856), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (b, a)$

70514. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

70518. Contains (860), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (b, a)$

70528. Contains (864), Fractal

WR: $a = \sigma(c, c), b = (c, b), c = (b, a)$

70609. Contains (874), Fractal

WR: $a = \sigma(a, a), b = (b, c), c = (b, a)$

70610. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

70614. Contains (878), Fractal

WR: $a = \sigma(b, b), b = (b, c), c = (b, a)$

70624. Contains (882), Fractal

WR: $a = \sigma(c, c), b = (b, c), c = (b, a)$

70641. Contains (883), Fractal

WR: $a = \sigma(a, a), b = (c, c), c = (b, a)$

70646. Contains (887), Fractal

WR: $a = \sigma(b, b), b = (c, c), c = (b, a)$

70656. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

70753. Contains (874), Fractal

WR: $a = \sigma(a, a), b = (b, c), c = (b, a)$

70758. Contains (878), Fractal

WR: $a = \sigma(b, b), b = (b, c), c = (b, a)$

70763. Contains (882), Fractal

WR: $a = \sigma(c, c), b = (b, c), c = (b, a)$

70769. Contains (856), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (b, a)$

70774. Contains (860), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (b, a)$

70784. Contains (864), Fractal

WR: $a = \sigma(c, c), b = (c, b), c = (b, a)$

70817. Contains (883), Fractal

WR: $a = \sigma(a, a), b = (c, c), c = (b, a)$

70822. Contains (887), Fractal

WR: $a = \sigma(b, b), b = (c, c), c = (b, a)$

70930. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

70978. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

71025. Contains (856), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (b, a)$

71026. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

71030. Contains (860), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (b, a)$

71040. Contains (864), Fractal

WR: $a = \sigma(c, c), b = (c, b), c = (b, a)$

71041. Contains (883), Fractal

WR: $a = \sigma(a, a), b = (c, c), c = (b, a)$

71046. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

71051. Contains (887), Fractal

WR: $a = \sigma(b, b), b = (c, c), c = (b, a)$

71121. Contains (874), Fractal

WR: $a = \sigma(a, a), b = (b, c), c = (b, a)$

71122. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

71126. Contains (878), Fractal

WR: $a = \sigma(b, b), b = (b, c), c = (b, a)$

71136. Contains (882), Fractal

WR: $a = \sigma(c, c), b = (b, c), c = (b, a)$

71153. Contains (883), Fractal

WR: $a = \sigma(a, a), b = (c, c), c = (b, a)$

71158. Contains (887), Fractal

WR: $a = \sigma(b, b), b = (c, c), c = (b, a)$

71168. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

71239. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

71297. Contains (856), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (b, a)$

71299. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

71302. Contains (864), Fractal

WR: $a = \sigma(c,c), b = (c,b), c = (b,a)$

71307. Contains (860), Fractal

WR: $a = \sigma(b,b), b = (c,b), c = (b,a)$

71490. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (a,b)$

71633. Contains (874), Fractal

WR: $a = \sigma(a,a), b = (b,c), c = (b,a)$

71634. Contains (875), Fractal

WR: $a = \sigma(b,a), b = (b,c), c = (b,a)$

71638. Contains (878), Fractal

WR: $a = \sigma(b,b), b = (b,c), c = (b,a)$

71648. Contains (882), Fractal

WR: $a = \sigma(c,c), b = (b,c), c = (b,a)$

71665. Contains (883), Fractal

WR: $a = \sigma(a,a), b = (c,c), c = (b,a)$

71670. Contains (887), Fractal

WR: $a = \sigma(b,b), b = (c,c), c = (b,a)$

71680. Contains (891), Fractal

WR: $a = \sigma(c,c), b = (c,c), c = (b,a)$

71763. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (a,b)$

71783. Contains (960), Fractal

WR: $a = \sigma(c,b), b = (b,c), c = (c,a)$

71831. Contains (942), Fractal

WR: $a = \sigma(c,b), b = (c,b), c = (c,a)$

71843. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (a,b)$

72007. Contains (960), Fractal

WR: $a = \sigma(c,b), b = (b,c), c = (c,a)$

72065. Contains (874), Fractal

WR: $a = \sigma(a,a), b = (b,c), c = (b,a)$

72067. Contains (875), Fractal

WR: $a = \sigma(b,a), b = (b,c), c = (b,a)$

72070. Contains (882), Fractal

WR: $a = \sigma(c,c), b = (b,c), c = (b,a)$

72075. Contains (878), Fractal

WR: $a = \sigma(b,b), b = (b,c), c = (b,a)$

73026. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (a,b)$

73073. Contains (856), Fractal

WR: $a = \sigma(a,a), b = (c,b), c = (b,a)$

73074. Contains (857), Fractal

WR: $a = \sigma(b,a), b = (c,b), c = (b,a)$

73078. Contains (860), Fractal

WR: $a = \sigma(b,b), b = (c,b), c = (b,a)$

73088. Contains (864), Fractal

WR: $a = \sigma(c,c), b = (c,b), c = (b,a)$

73201. Contains (883), Fractal

WR: $a = \sigma(a,a), b = (c,c), c = (b,a)$

73206. Contains (887), Fractal

WR: $a = \sigma(b,b), b = (c,c), c = (b,a)$

73216. Contains (891), Fractal

WR: $a = \sigma(c,c), b = (c,c), c = (b,a)$

73538. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (a,b)$

73585. Contains (856), Fractal

WR: $a = \sigma(a, a), b = (c, b), c = (b, a)$

73586. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

73590. Contains (860), Fractal

WR: $a = \sigma(b, b), b = (c, b), c = (b, a)$

73600. Contains (864), Fractal

WR: $a = \sigma(c, c), b = (c, b), c = (b, a)$

73681. Contains (874), Fractal

WR: $a = \sigma(a, a), b = (b, c), c = (b, a)$

73682. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

73686. Contains (878), Fractal

WR: $a = \sigma(b, b), b = (b, c), c = (b, a)$

73696. Contains (882), Fractal

WR: $a = \sigma(c, c), b = (b, c), c = (b, a)$

79224. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

79320. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

79348. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

79700. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

79832. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

79860. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

80612. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

80628. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

80876. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

80892. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

81396. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

81660. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

196695. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

196872. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

196875. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

196883. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

196889. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

196952. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

196955. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

196962. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

196965. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

197025. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

197030. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

197123. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

197154. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

197157. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

197161. Contains (821), Fractal

WR: $a = \sigma(b, a), b = (b, a), c = (b, a)$

197171. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

197177. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

197203. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

197283. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

197299. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

197305. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

197363. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

197384. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

197387. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

197464. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

197467. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

197627. Contains (752) Infinite: Virtually $\mathbb{Z}^3$

WR: $a = \sigma(b, b), b = (c, a), c = (a, a)$

197896. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

197921. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

198167. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

198170. Contains (2369), Fractal

WR: $a = \sigma(b,a), b = \sigma(c,a), c = (c,a)$

198246. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

198250. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

198251. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

198488. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c,b), b = \sigma(b,b), c = (a,a)$

199256. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c,b), b = \sigma(b,b), c = (a,a)$

199435. Contains (779), Fractal

WR: $a = \sigma(b,b), b = (c,b), c = (a,a)$

199440. Contains (775), Fractal

WR: $a = \sigma(a,a), b = (c,b), c = (a,a)$

199585. Contains (775), Fractal

WR: $a = \sigma(a,a), b = (c,b), c = (a,a)$

199665. Contains (775), Fractal

WR: $a = \sigma(a,a), b = (c,b), c = (a,a)$

200456. Contains (2193), Fractal

WR: $a = \sigma(c,b), b = \sigma(a,a), c = (a,a)$

200536. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c,b), b = \sigma(b,b), c = (a,a)$

201219. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

201257. Contains (821), Fractal

WR: $a = \sigma(b,a), b = (b,a), c = (b,a)$

201299. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

201379. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

201488. Contains (2277), Fractal

WR: $a = \sigma(c,c), b = \sigma(a,a), c = (b,a)$

201568. Contains (2277), Fractal

WR: $a = \sigma(c,c), b = \sigma(a,a), c = (b,a)$

201586. Contains (2284), Fractal

WR: $a = \sigma(a,c), b = \sigma(b,a), c = (b,a)$

201589. Contains (2280), Fractal

WR: $a = \sigma(c,a), b = \sigma(b,a), c = (b,a)$

201713. Contains (2277), Fractal

WR: $a = \sigma(c,c), b = \sigma(a,a), c = (b,a)$

201718. Contains (2277), Fractal

WR: $a = \sigma(c,c), b = \sigma(a,a), c = (b,a)$

201739. Contains (2277), Fractal

WR: $a = \sigma(c,c), b = \sigma(a,a), c = (b,a)$

201819. Contains (2277), Fractal

WR: $a = \sigma(c,c), b = \sigma(a,a), c = (b,a)$

202337. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b,a), b = (b,a)$

202342. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

202346. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

202347. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

202512. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

202516. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

202525. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

202592. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

202737. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

202742. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

204560. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

204564. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

204573. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

204640. Contains (2277), Fractal

WR: $a = \sigma(c, c), b = \sigma(a, a), c = (b, a)$

205315. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

205475. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

206359. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

206362. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

206433. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

206438. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

206442. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

206443. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

206851. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

206885. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

206899. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

206905. Contains (821), Fractal

WR: $a = \sigma(b, a), b = (b, a), c = (b, a)$

207011. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207027. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207033. Contains (821), Fractal

WR: $a = \sigma(b, a), b = (b, a), c = (b, a)$

207091. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207127. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

207130. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

207201. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207206. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207210. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207211. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (a, b)$

207619. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

207667. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

207673. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

207779. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

207859. Contains (857), Fractal

WR: $a = \sigma(b, a), b = (c, b), c = (b, a)$

208387. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

208400. Contains (882), Fractal

WR: $a = \sigma(c, c), b = (b, c), c = (b, a)$

208435. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

208441. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

208547. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

208627. Contains (875), Fractal

WR: $a = \sigma(b, a), b = (b, c), c = (b, a)$

208656. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

208704. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

208816. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

208832. Contains (891), Fractal

WR: $a = \sigma(c, c), b = (c, c), c = (b, a)$

208881. Contains (883), Fractal

WR: $a = \sigma(a, a), b = (c, c), c = (b, a)$

208891. Contains (887), Fractal

WR: $a = \sigma(b, b), b = (c, c), c = (b, a)$

211460. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

211492. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

211501. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

211636. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

211645. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

211700. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

211724. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

211756. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

211759. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

211884. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

211964. Contains (942), Fractal

WR: $a = \sigma(c, b), b = (c, b), c = (c, a)$

212492. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

212524. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

212527. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

212652. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

212671. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

212732. Contains (960), Fractal

WR: $a = \sigma(c, b), b = (b, c), c = (c, a)$

212740. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

212772. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

212781. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

212786. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

212789. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

212797. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

212820. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

212900. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

212916. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

212925. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

212980. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

458796. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

458799. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

458812. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

458815. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

458840. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

458924. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

459004. Contains (2193), Fractal

WR: $a = \sigma(c, b), b = \sigma(a, a), c = (a, a)$

459096. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

459102. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

459608. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

459614. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

460638. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

461564. Contains (2229), Infinite (see "Proofs" in [4])

WR: $a = \sigma(c, b), b = \sigma(b, b), c = (a, a)$

462868. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

462877. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

462962. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

462965. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

463181. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

463645. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

463693. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

463730. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

463733. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

463826. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

463829. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

466708. Contains (2280), Fractal

WR: $a = \sigma(c, a), b = \sigma(b, a), c = (b, a)$

466717. Contains (2284), Fractal

WR: $a = \sigma(a, c), b = \sigma(b, a), c = (b, a)$

471076. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471085. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471090. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

471093. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

471101. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471124. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471204. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471220. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471229. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471284. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471349. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

471357. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471380. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471389. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471492. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471493. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

471540. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

471549. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471971. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

471977. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

471981. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

471987. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

471993. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

471997. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

472035. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

472041. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

472045. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

472051. Contains (2369), Fractal

WR: $a = \sigma(b, a), b = \sigma(c, a), c = (c, a)$

472057. Contains (2371), Fractal

WR: $a = \sigma(a, b), b = \sigma(c, a), c = (c, a)$

472061. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

472564. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

472916. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

472925. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

473076. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$

473085. Infinite

WR: $a = \sigma(a, b), b = (b, a)$

473844. Contains (2,2)-automaton generating infinite group

WR: $a = \sigma(b, a), b = (b, a)$