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Supply Chain Optimization of Blood Products

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by

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DEDICATION

I dedicate this dissertation to my beloved parents, Mustafa and Mukerrem Gunpinar, and to my dear brother, Erkan Gunpinar, who have supported me both financially and emotionally not only throughout my study but also in my entire life. Even though we were apart from each other and living in three different continents, they have always been there for me when I was in need of help. I could not have accomplished it without their full support.

I also would like to dedicate my dissertation to my grandmother, Muhsine Guler, who recently passed away. I was feeling her prayers next to me when she was alive. May she rest in peace!

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ABSTRACT

Major challenges in the management of blood supply chain are related to the shortage and wastage of the blood products. Given the perishability characteristics of blood which can be stored up to a limited number of days, if hospitals and blood centers keep an excessive number of blood units on inventory, wastages may occur. On the other hand, if sufficient number of blood units are not stored on inventory, shortages of this resource may cause the cancellations of important activities and increase the fatality rates at hospitals. Three mathematical models have been developed with the goal to improve the efficiency of blood related activities at blood centers and hospitals. The first model uses an integer programming (IP) approach to identify the optimal order levels that minimizes the total cost, shortage and wastage levels of blood products at a hospital within a specified planning horizon. The IP model explicitly considers the age of blood inventory, uncertain demand, the demand for two types of patients and crossmatch-to-transfusion ratio. The second model formulates the different shortage and inventory distribution strategies of a blood center supplying blood products to multiple hospitals. The third model develops a vehicle routing problem for blood centers to minimize the daily distance travelled by bloodmobiles during the blood collection process. Optimal routing for each bloodmobiles is identified using CPLEX solver, branch & bound and column generation algorithms and their solution times are compared.

CHAPTER 1: INTRODUCTION

Human blood is a scarce resource. It is only produced by human beings and there are currently no other products or alternative chemical process that can be used to generate blood. The blood carries substances such as nutrients and oxygen to the cells and delivers waste away from the cells.

Blood is usually drawn as "whole blood" but then it could be mechanically separated into other useful components. These components are then used to meet the specific transfusion demands of patients. One unit of whole blood can be divided into five different blood products: red blood cells (RBCs), plasma, white blood cells, serum or platelets. Red blood cells are the most abundant cells in blood and contain a protein called hemoglobin that moves oxygen to our cells. Plasma is a yellowish liquid component and is obtained by removing RBCs from whole blood. White blood cells are part of the immune system and defend the body against infectious agents. Serum is a blood plasma without clotting factor, white and red blood cells. Finally, platelets are the clotting factors that are contained in the plasma and relate to the process of coagulation which repairs the body when a wound and bleeding occurs.

Platelets can also be drawn directly from a person through the use of an apheresis device. All blood components except for plasma can become outdated. Platelets, especially, are considered highly perishable since they can only be stored up to five days before deteriorating. The second most perishable blood component, RBC, can be kept for up to forty two days on inventory.

Figure 1.1 shows the general process related to the supply chain of blood products. It starts with the collection process. Blood units for transfusion purposes are collected from donors either at a blood center or through bloodmobiles on remote

locations. After a thorough process of rules and regulations for compliance of donors, units are tested. Then, the whole blood units are either stored or mechanically separated (extracted) into components. Hospitals place orders to blood center based on the forecasted demand for the various procedures scheduled. A recipient's blood is tested against a donor's blood (this process is known as crossmatching) and, when compatible, blood units are reserved for the specific patient for the period known as crossmatch release period.

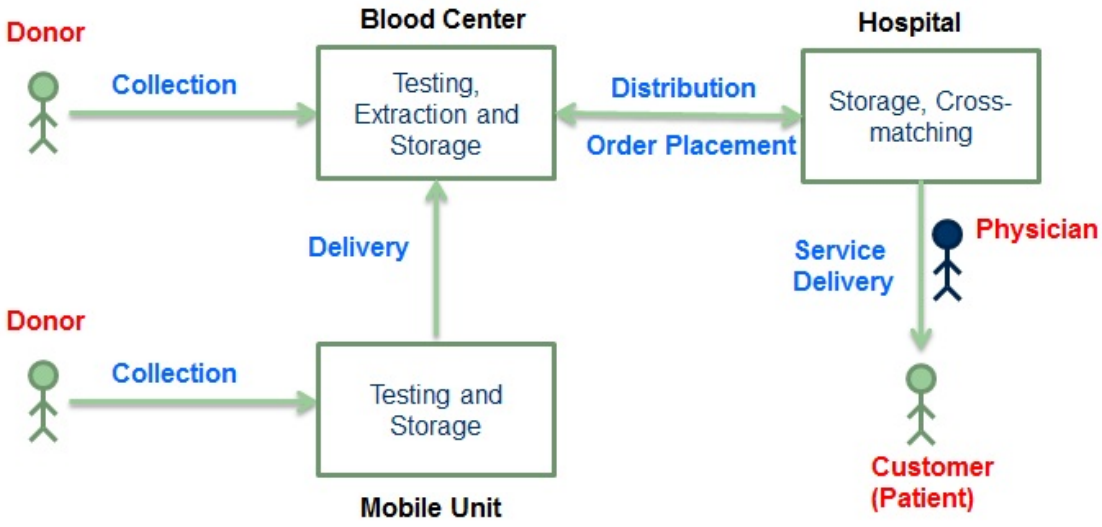

Figure 1.1: Supply Chain of Blood Products

When comparing blood products with any other item several differences directly connected to the supply chain become evident. First, supply of blood is volunteer-based whereas there is a cost associated with most products. Second, the structure of the blood supply chain is considered as reverse to the majority of traditional products since the whole blood produced by the living beings is mechanically separated into components in many cases before it is used. However, in traditional supply chain, parts are manufactured and then assembled to create a finished product. Third, the price associated with the acquisition of the blood is always linear, that is, no economies of scale are present. Finally, the most significant difference is about the inventory

issuing policy as shown in Figure 1.2. When the hospital blood bank receives a blood request for a specific patient, the crossmatched blood is moved from unassigned (free) inventory to assigned (reserved) inventory and kept for this patient until the blood is transfused or the crossmatch release period is over. If the blood is not transfused and the crossmatch release period is over, it could be returned to the unassigned inventory to be used for other purposes. Since the amount of blood needed for a medical procedure is uncertain, physicians tend to overestimate units required for safety issues. Approximately 50% of blood units requested by physicians are returned to the unassigned inventory without being transfused [1]. Depending on the patient, organizational policy and types of procedures, the crossmatch-to-transfusion ratio (C/T ratio) varies. This ratio is typically higher for the cases in emergency rooms [2].

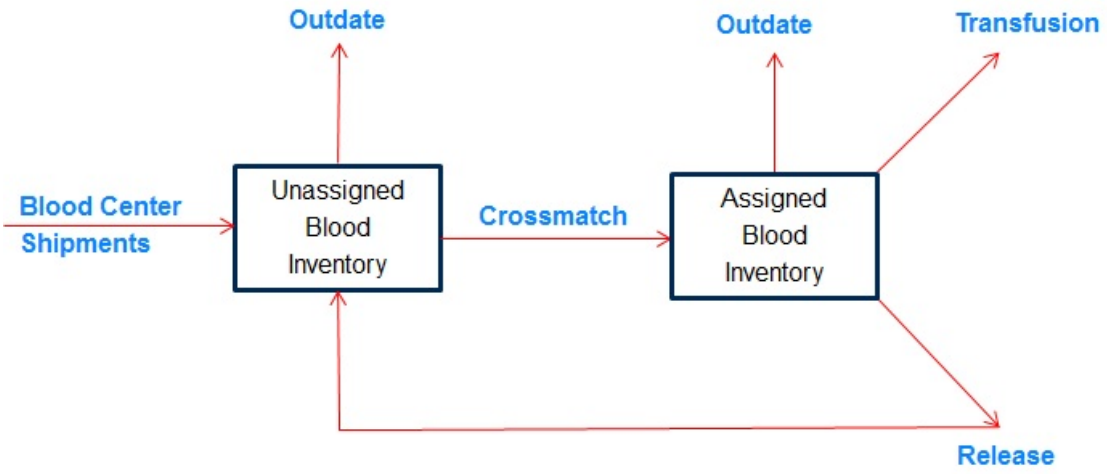

Figure 1.2: Blood Inventory Model

Minimizing shortage and wastage of the blood is the major challenge related to the management of blood both at a hospital and at a blood center. Due to the perishable characteristic of blood (it becomes outdated if not used during its predetermined shelf life) it is critical to avoid storing an excessive number of blood units. At the same time, insufficient number of blood products on inventory may increase cancellations

of the scheduled activities at a hospital and as a result increase fatality rates. In 2004, 17% of platelet units that were collected in the U.S. were outdated before being transfused [3] (wastage); and a total of 492 reported cancellations of elective surgeries on one or more days were due to blood shortages at 1700 U.S. hospitals participating in a survey in 2007 [4]. Thus, managing outdates and shortages of blood products continues to pose a challenging problem for hospitals.

1.1. Intellectual Merit

This research considers the supply chain of blood products and presents models to improve the efficiency of both blood collection process at blood centers as well as blood ordering policies at hospitals.

The existing literature shows the need for models that incorporate the age of blood units into the formulations. In addition, the majority of the models do not differentiate demand among patient groups with specific blood age requirements. We develop stochastic integer programming models that explicitly consider age of blood units on inventory as well as the demand for two types of patients (one which requires fresh blood). Furthermore, due to the impact of unique blood characteristics on blood ordering policy, unlike other models, we take C/T ratio and crossmatch release period into consideration and propose a deterministic integer programming model to investigate their effects on the operational efficiency of the hospitals.

Most problems discussed in the literature analyze a decentralized hospital network where each hospital controls and being responsible for its own blood inventory. However, in real life practices, many blood centers have informational access to the hospitals' inventory levels through online inventory control system and are responsible for replenishing and maintaining certain blood levels at hospitals. That is, the blood center takes into account the availability of blood both at the blood center and at hospitals in its network before making any decision related to the number of units to be distributed to each hospital. Using integer programming approach, our models

explicitly incorporate centralized decision making to minimize the cost and shortage levels in overall system.

1.2. Broader Impact

In this study, a decision support mechanism is developed for hospitals to manage blood resources more efficiently which will ultimately result in both cost reduction and improved service to hospitals' patients. An extensive computational study is provided to analyze the effects of several factors such as average age of blood in blood shipments, C/T ratio, and the length of crossmatch release period. The obtained results will be beneficial to hospital administrators and will aid in the process of determining adequate order sizes to minimize shortage, wastage and total costs.

Another decision support tool developed in our study selects a set of locations from among a group of potential locations to collect blood units each day. Using the formulation of vehicle routing problem we design blood mobile routes that minimize the total distance travelled during blood collection process while satisfying the daily blood demand at the blood center.

1.3. Dissertation Outline

This dissertation is organized as follows: Chapter 2 reviews the existing literature. Chapter 3 analyzes a decentralized hospital network consisting of a single hospital and a blood center. Chapter 4 outlines the different shortage and inventory distribution strategies of a centralized hospital network managed by a blood center. Chapter 5 presents a vehicle routing problem for a blood center in order to improve its efficiency in blood collection process. Chapter 6 concludes the dissertation and provides the opportunities for future work.

CHAPTER 2: LITERATURE REVIEW

This section discusses the research related to the supply chain problem of blood products including inventory management and decision models. In addition, the literature associated with column generation algorithms and their application to solve vehicle routing problems discussed.

2.1. Overview of the Literature related to Supply Chain of Blood Products

The research related to supply chain management of perishable products in general and blood products in particular was initiated in the 1960's by Van Zyl [5]. The paper written by Nahmias [6] in 1982 focuses on the perishable inventory and provides a brief review for the applications of the models to the blood bank management. In 1984, Prastacos [7] overviews the theory and practice of blood inventory management. Since then, close to one hundred blood related publications have become available in the literature. Two peaks in the publication history of blood products are observed [8] as shown in Figure 2.1; one in the period between 1976 and 1985 and more recently in the period between 2001 and 2010.

Supply chain problems of blood products have been modeled using a variety of analytical decision models. In particular, simulation methodology, dynamic programming, integer programming, goal programming and multi objective approaches are some of the most common solution methods in the literature. These approaches are either used alone or in combination with other methods to analyze and solve real-life problems.

As can be seen in Figure 2.2, most of the researches are focused on the problems in either individual hospital level or regional blood center level. Only small number of researches have considered the complete supply chain network of blood products.

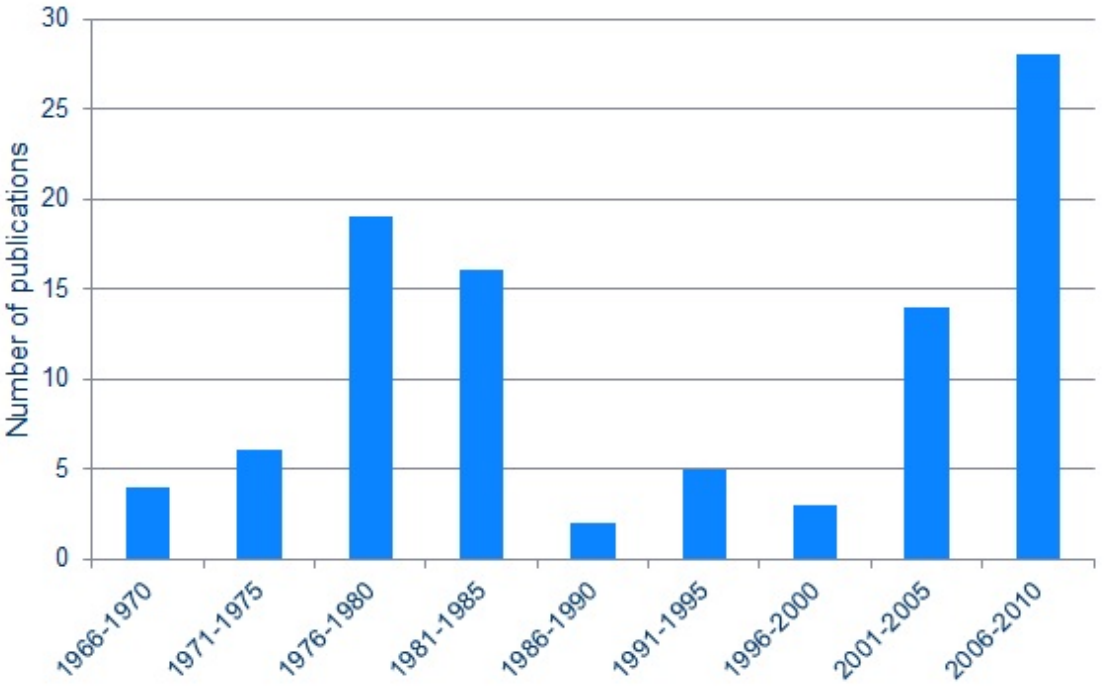

Figure 2.1: Publication History ([8])

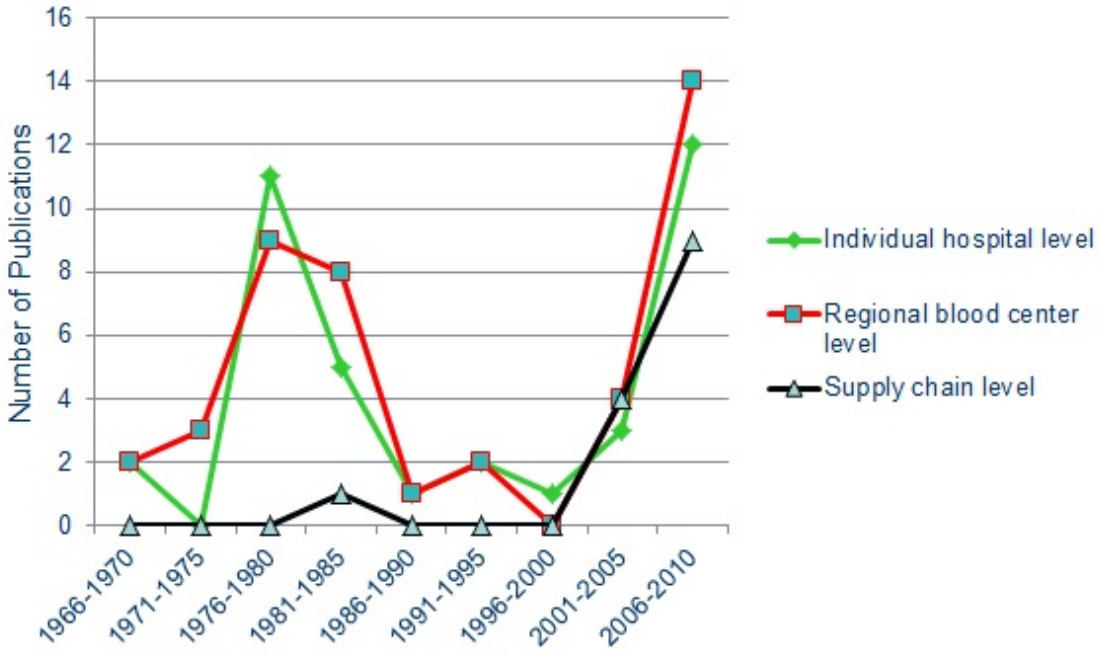

Figure 2.2: Trends in Hierarchical Level ([8])

Trends have been towards total inventory management and a limited number of studies are available in planning for blood collections (Figure 2.3).

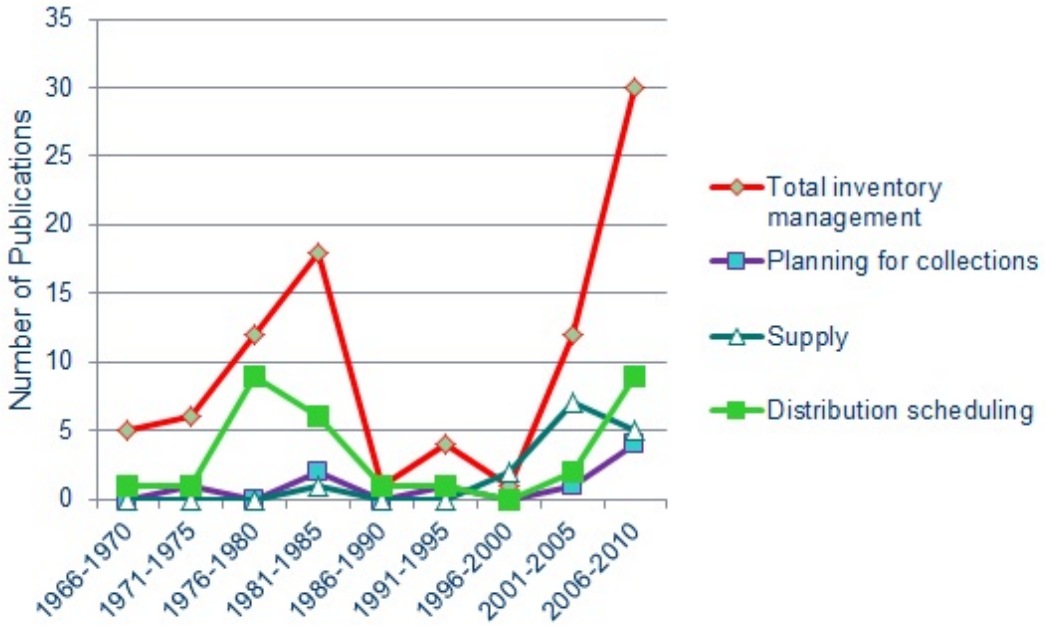

Figure 2.3: Trends in Problem Categories ([8])

Haijema et al. [9] applies markov dynamic programming and simulation approach to a real life case of a Dutch blood bank. Their paper focuses on the production and inventory management of platelets where they only consider costs that are directly related to the production and inventory of platelets. Zhou et al. [10] analyzes a platelet inventory problem assuming a fixed life span of three days and considering stochastic demand. The problem is formulated using dynamic programming approach where dual sourcing alternative is available and the decision maker has the option of placing an expedited order besides the regular order. Alfonso et al. [11] address the blood collection problem in France considering both fixed site and mobile blood collection. They use Petri net models to describe different blood collection processes,

donor behaviors, human resource requirements and apply simulation approach to identify appropriate human resource planning and donor appointment strategies.

Hemmelmayr et al. [12] develop integer programming models to decide which hospitals a vendor (through vehicles from blood centers) should visit each day given that the routes are fixed for each region. Authors consider recourse action in order for hospitals to be hedged against the uncertainty associated with blood product usage. Both, integer programming and variable neighborhood search approaches are used and compared in terms of their efficiencies. Sahin et al. [13] formulate three problems using integer programming to address the location-allocation aspects of regionalization of blood services. The experimental results obtained using real data for Turkish Red Crescent blood services were reported. Jacobs et al. [14] build two integer programming models to investigate a facility relocation problem for the mid-Atlantic region of the American Red Cross in Norfolk, Virginia. They provide insights into the current scheduling activities of blood collections and distributions. The integer programming model explained in [15] considers the orders for fresh blood separately and allocates blood units from regional blood transfusion service to the hospitals. The objective is to minimize the total expected number of units that are sent back to the blood transfusion service. Ghandforoush and Sen [16] formulates a nonlinear integer programming model to determine the minimum cost platelet production schedule for the regional blood center. Since the initial formulation carries a non-convex objective function that is difficult to solve and would not guarantee convergence to optimality, the formulation is simplified to achieve a better structure. As both objective function and constraints of the revised formulation include quadratic terms, a two-step transformation called linear 0-1 integer alternative is proposed to guarantee optimality.

Kendall and Lee [17] develop a goal programming model to attain multiple goals related to inventory levels, the availability of fresh blood, blood outdating, the age

of blood, and the cost of collecting it. The data for a large urban-rural blood region in the Midwest are collected for a period of one year; computational results of the model are reported. Cetin and Sarul [18] use a hybrid mathematical programming model that is the integration of gravity model of continuous location models and set covering model of discrete location approaches. The objective function of the problem is formulated using binary nonlinear goal programming technique and the goals are to minimize the total traveled distance between the blood banks and hospitals, the total fixed cost of locating blood banks, and the cost associated with an inequality index that is a type of fairness mechanism for the distances.

Nagurney et al. [4] analyze the complex supply chain of human blood consisting of collection sites, testing and processing facilities, storage facilities, distribution centers, as well as demand points. Authors develop a generalized network optimization model where multi criteria system-optimization approach enables decision makers to minimize both total operational cost and total risk function. Computational results are obtained by utilizing variational inequality method. The analytical model described in [19] is a tool for blood centers to model trade-offs between multiple demand levels, service levels, costs, as well as the shortages and expiration. The paper uses queuing model and level crossing techniques to determine an optimal policy. The results are validated with a simulation model using real data obtained from Canadian Blood Services.

2.2. Overview of the Literature Using Column Generation Algorithm to Solve Vehicle Routing Problem

The column generation (CG) algorithm is a widely used approach to solve vehicle routing problems (VRPs). Papers in the literature ranges from classical VRPs to more sophisticated VRPs which includes the options of time window limitations, heterogeneous vehicles and multi-depot locations.

Chabrier [20] formulates a VRP with time windows and the different limitations on vehicles capacities. The original problem is then modified using Dantzig-Wolfe decomposition [21] and column generation algorithm is applied under branch and bound framework. Labeling algorithm is generated to solve the subproblems and cuts are used to improve the solution obtained from the relaxed problem. Righini et al. [22] presents a branch-and-cut-and-price algorithm to solve multi-depot problem with time windows. The study considers heterogeneous vehicles with different capacities and fixed costs. Tcha and Choi [23] analyze a VRP using integer programming model with time windows and a fleet of vehicles having various capacities and costs. The linear programming relaxation is solved by column generation algorithm and several dynamic programming schemes are developed to generate feasible columns.

The livestock collection problem in [24] is formulated as a rich VRP with inventory and vehicle capacity constraints where the capacity depends on the loading sequence. The goal is to design a set of vehicle routes to collect animals from farms while satisfying certain constraints related to animal welfare. The paper presents a column generation based exact solution algorithm to solve richer model with much larger instances compared to the previously published studies. Vanderbeck and Mourgaya [25] builds a periodic VRP to optimize the vehicle routes while satisfying some service levels during a given time horizon. The tactical planning model schedules vehicle visits and operational model identifies sequences of each vehicles. The objective is to specialize each routes with geographical area and to evenly distribute the workload between vehicles. Ledesma and Gonzales [26] describes a school bus routing problem that aims to select a set of bus stops among a group of potential ones and to design their visiting sequences. The problem constraints include minimum number of students to be picked up, maximum number of stops to be visited and maximum distance travelled by students. The paper proposes branch-and-price algorithm based on a set partitioning formulation. Batta et al. [27] address the simultaneous sensor

selection and routing problem of unmanned aerial vehicles (UAVs). The goal is to assign sensors to UAVs so as to maximize the intelligence gain while not exceeding flight time limitations and the number of sensors that can be hold by aircraft. Heuristics and column generation algorithms are used to find good and improved solutions respectively.

CHAPTER 3: DECENTRALIZED HOSPITAL NETWORK CONSISTING OF ONE BLOOD CENTER AND ONE HOSPITAL

We consider a two-level supply chain of blood products consisting of one hospital and one blood center. The bolded red line in Figure 3.1 shows the point of this section's interest in complete blood supply chain. The hospital faces blood demands that need to be satisfied in order to perform its daily operation related to blood supply. Thus, optimal blood order levels should be identified over multiple periods.

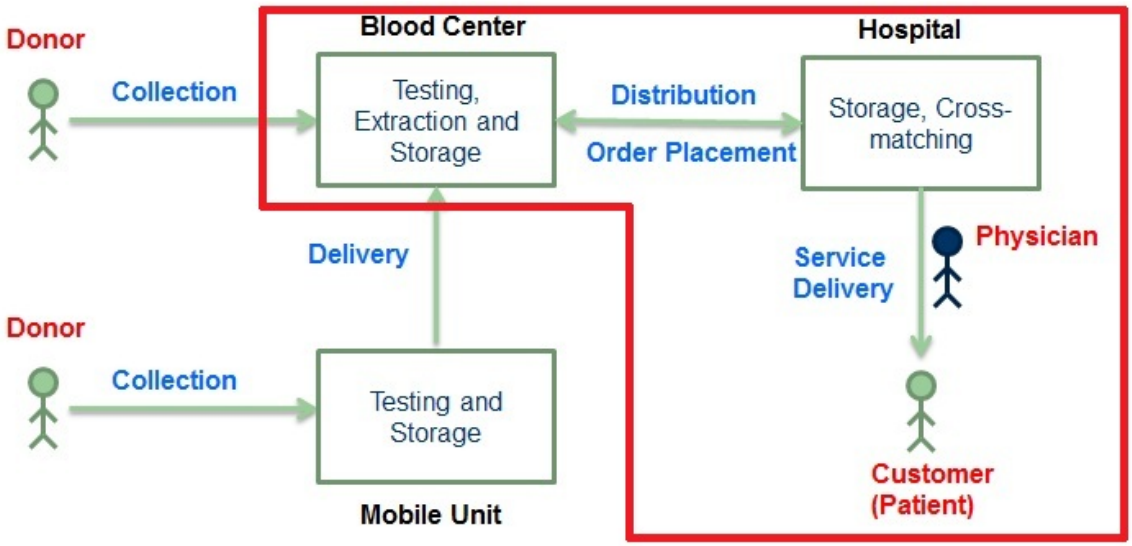

Figure 3.1: Two Level Supply Chain with One Hospital and One Blood Center

In this research, the following assumptions have been made:

- The capacity of the blood center is limited.
- Lead times for blood supply are zero.
- The age of blood units received from the blood center is known and varies over time.

- The lifetime of platelets is limited to five days including two days of testing [3].
- The lifetime of red blood cells is limited to forty two days including two days of testing [19].
- General blood issuing policy for the hospital is FIFO where oldest units on inventory are issued first when the blood units are requested by physicians for patient needs [6].
- If demand is not satisfied due to the unavailability of blood units, a shortage cost is incurred.
- If a blood unit expires, a wastage cost is incurred associated with discarding blood units.

3.1. Formulation of Random Blood Demand

Hospitals usually face two types of uncertainties associated with the use of blood products. The first relates to the uncertainty of emergency cases which are difficult to anticipate. Unlike scheduled procedures, emergency cases are unexpected and random. Thus, the amount of blood units needed is unknown in advance. The second uncertainty relates to the C/T ratio. Prior to a procedure, blood is requested by the physician for a specific patient and the number of blood units cross-matched is typically overestimated for safety issues. Thus, some blood may be returned back to inventory after the crossmatch release period is over. To address these challenges, we use stochastic programming to handle demand uncertainty and build integer programming models that explicitly consider age of blood on inventory.

Table 3.1-3.3 summarize the indices, the parameters and the variables that are used in the models. It is valuable to note that $t = 1$ refer to a Monday.

Table 3.1: Indices for Models

Index	Description
s	Demand scenario, s=1,2,...,S
i	Age of blood, i=1,2,...,I (days)
t	Time period, t= 1,2,...,T (days)
a	Age group of blood ('young' (0) or 'old' (1)), $a \in \{0, 1\}$

Table 3.2: Parameters for Models

Parameter	Description
S	Number of scenarios
I	Lifetime of blood product
T	Length of planning horizon
b	Unit shortage cost of blood at the hospital
c	Unit purchasing cost of blood
c_a	Unit purchasing cost of 'young' blood (0) and 'old' blood (1)
h	Unit holding cost of blood at the hospital
M	Big M (Big Number)
p_s	Probability of scenario s, $\sum_{s=1}^{S} p_s = 1$
w	Unit wastage cost of blood at the hospital
θ_{it}	Proportion of i days old blood in blood shipments from blood center in time period t, $0 \leq \theta_{it} \leq 1, \sum_{i=3}^{I} \theta_{it} = 1 \qquad \forall t$
θ_{ait}	Proportion of i days old blood in 'young' blood shipments (a=0) and in 'old' blood shipments (a=1) from blood center in time period t, $0 \leq \theta_{ait} \leq 1$, $\theta_{03t} = 1$, $\sum_{i=4}^{I} \theta_{1it} = 1 \qquad \forall t$
$d_t^{(s)}$	Blood demand at the hospital in time t (for scenario s)
$d_{at}^{(s)}$	'Young' blood (a=0) demand and 'any' blood (a=1) demand at the hospital in time t (for scenario s)
CAP_t	Capacity of the blood center (allocated to the hospital) in time period t
CRP	Crossmatch release period at the hospital
CT	Average C/T ratio at the hospital

Using the indices, parameters and decision variables in Table 3.1-3.3, the non-linear stochastic integer programming model is formulated as follows:

Table 3.3: Decision Variables for Models

Decision Variable	Description
$m_{it}^{(s)}$	Auxiliary variable associated with age group i in time t (for scenario s). It captures the number of blood units in an age group left to be used for the next period if all available blood in this age group is not fully used to satisfy the demand in current period
$r_t^{(s)}$	Number of blood shortage at the end of time t (for scenario s) at the hospital
$\pi_t^{(s)}$	Number of 'old' blood demand in time t (for scenario s) that are not satisfied by older blood units on inventory due to unavailability
$u_t^{(s)}$	Number of blood wastage at the end of time t (for scenario s) at the hospital
$v_{it}^{(s)}$	Inventory level of i days old blood at the end of time t (for scenario s) at the hospital
x_t	Number of blood ordered by the hospital from the blood center at the beginning of time t
x_{at}	Number of 'young' blood (0) and 'old' blood (1) ordered by the hospital from the blood center at the beginning of time t
y_{it}	Number of i days old blood received by the hospital at the beginning time t
$z_{it}^{(s)}$	1 if i days old blood used to satisfy the demand in time period t (for scenario s), 0 otherwise
$\beta_{it}^{(s)}$	Number of i days old blood returned from assigned inventory to unassigned inventory at the beginning of time t (for scenario s)

$$\text{Minimize} \sum_{t=1}^{T} c \cdot x_t + p_s \cdot (\sum_{s=1}^{S} \sum_{i=3}^{I} \sum_{t=1}^{T} h \cdot v_{it}^s + \sum_{s=1}^{S} \sum_{t=1}^{T} w \cdot u_t^s + \sum_{s=1}^{S} \sum_{t=1}^{T} b \cdot r_t^s) \quad (3.1)$$

subject to:

$$x_t \leq CAP_t \qquad \forall t \quad (3.2)$$

$$y_{it} = 0, \qquad i = 1, 2, \forall t \tag{3.3}$$

$$y_{it} = x_t \theta_{it}, \qquad i = 3, 4, ..., I, \forall t \tag{3.4}$$

$$z^s_{it} \geq z^s_{(i-1)t} \qquad i = 3, 4, ..., I, \forall s, t \tag{3.5}$$

$$d^s_t = \sum_{i=3}^{I} ((v^s_{(i-1)(t-1)} + y_{it}) z^s_{it} - m^s_{it}) + r^s_t, \forall s, t \tag{3.6}$$

$$(z^s_{it} - z^s_{(i-1)t})(v^s_{(i-1)(t-1)} + y_{it}) \geq m^s_{it} \qquad i = 3, 4, ..., I, \forall s, t \tag{3.7}$$

$$z^s_{2t} = 0 \qquad \forall s, t \tag{3.8}$$

$$d^s_t - \sum_{i=3}^{I} (v^s_{(i-1)(t-1)} + y_{it}) \leq r^s_t \qquad \forall s, t \tag{3.9}$$

$$v^s_{it} = (1 - z^s_{it})(v^s_{(i-1)(t-1)} + y_{it}) + (z^s_{it} - z^s_{(i-1)t}) m^s_{it} \qquad i = 3, ..., I, \forall s, t \tag{3.10}$$

$$v^s_{(2)t} = 0 \qquad \forall s, t \tag{3.11}$$

$$v^s_{i(0)} = 0 \qquad \forall s, i \tag{3.12}$$

$$u^s_t = v^s_{(I)t} \qquad \forall s, t \tag{3.13}$$

$$x_t \in \mathbb{Z}^+ \qquad \forall t \tag{3.14}$$

$$r_t^s, u_t^s \in \mathbb{Z}^+ \qquad \forall s, t \tag{3.15}$$

$$y_{it} \in \mathbb{Z}^+ \qquad \forall i, t \tag{3.16}$$

$$m_{it}^s, v_{it}^s \in \mathbb{Z}^+ \qquad \forall s, i, t \tag{3.17}$$

$$z_{it}^s \in \{0, 1\} \qquad \forall s, i, t \tag{3.18}$$

The objective function (3.1) seeks to minimize the purchasing cost and the expected inventory, wastage and shortage costs during the planning horizon. Constraint (3.2) is the capacity constraint of the blood center (supplier). Constraint (3.3) ensures that the hospital never receives one or two days old blood units from the blood center as two days are required for testing after the blood is collected. Constraint (3.4) allocates blood units to each age group. Constraint (3.5) guarantees the FIFO blood issuing policy. Constraint (3.6) requires demand to be fully satisfied when blood supply exceeds demand. Otherwise, the hospital faces a shortage issue. m_{it}^s is an auxiliary variable that captures the number of blood units in an age group left on inventory when at least one unit is absorbed from inventory during the given time period. Otherwise, it would be equal to zero. Note that there is at most one age group having m_{it}^s with non-zero value in each period. Constraint (3.7) assures that the values of auxiliary variable, m_{it}^s, do not exceed the number of blood units available in their age groups. Constraint (3.8) ensures that two days old blood units are not used to satisfy the demand as the hospital only receives blood units older than two days old from the blood center. Constraint (3.6) and Constraint (3.9) capture the number of blood shortages. Constraint (3.10) updates end-period blood inventory levels for each age group. Constraint (3.11) assures two days old blood is never

available on inventory. Constraint (3.12) states that there is no inventory available at the beginning of the analysis period. Constraint (3.13) identifies the wastage levels of the hospital at the end of each period. Constraints (3.14)-(3.17) show r_t^s , u_t^s , x_t, m_{it}^s, v_{it}^s and y_{it} are non-negative discrete variables since the blood units are received in blood bags. Constraint (3.18) states that z_{it}^s is a binary variable.

Due to the interactions between binary and discrete variables, the optimization problem includes non-linear terms in the above formulation. After the first linearization technique is applied which is detailed in Appendix A, the interactions between v, y, m and z variables in constraints (3.6), (3.7) and (3.10) are replaced with the corresponding linearization variables from which constraints (3.19)-(3.21) are obtained. In addition, constraints (A.1)-(A.26) are added into the new formulation.

$$d_t^s = \sum_{i=3}^{I} (\gamma_{it}^s + \alpha_{it}^s - m_{it}^s) + r_t^s \qquad \forall s, t \tag{3.19}$$

$$\gamma_{it}^s + \alpha_{it}^s - \mu_{(i-1)t}^s - \psi_{it}^s \geq m_{it}^s \qquad i = 3, 4, ..., I, \forall s, t \tag{3.20}$$

$$v_{it}^s = v_{(i-1)(t-1)}^s + y_{it} - \gamma_{it}^s - \alpha_{it}^s + \lambda_{it}^s - \delta_{it}^s \qquad i = 3, 4, ..., I, \forall s, t \tag{3.21}$$

In summary, constraints (3.2)-(3.5), (3.8), (3.9), (3.11)-(3.21), (A.1)-(A.26) are present in the model discussed in this section.

3.1.1. Model Extension: Formulation of Blood Demand for Two Types of Patients

Freshness of the blood products (a.k.a. young blood) may be critical and required for certain types of patient groups. Fresh blood (specifically RBC) is highly preferable for operations such as open-heart surgeries [17]. Also, according to the study from University of Texas Southwestern [28], it is suggested to have younger blood

units for the transfusion of pediatric heart surgery patients, open heart pediatric and cardiopulmonary bypass surgeries. In the absence of fresh blood units, some of the blood related procedures may need to be postponed to a later date. Similarly, young platelets are highly preferred for oncology and hematology patients. However, platelets of any age (up to the shelf life) can be used for traumatology or other general surgery patients [9].

Based on these requirements, two types of patients are defined: Type 1 which requires fresh/young blood and Type 2 which could use blood of any age (young or old). Young platelets are defined as being younger than three days old [9]. Young red blood cells are younger than 5 days old [17].

The general procedure to order and manage blood units at a hospital is as follows: The hospital places separate orders for units of any age and/or specifically young units to the blood center. If no age is specified, blood centers will typically send only old age units. Upon arrival, units are grouped and stored based on their age group; then, they are allocated to the procedures using the FIFO policy. The demand for Type 1 patients can only be satisfied with young products. If demand for young products exceeds supply, the hospital faces a shortage issue. On the other hand, demand for Type 2 patients can be satisfied by units of any age. That is, young units of blood can also be used to satisfy demand for Type 2 patients but only when old blood units are not available on inventory. To incorporate the demand for two types of patients into our formulation, Constraints (3.4)-(3.7) and (3.9) of the initial model are replaced with the following constraints.

$$x_{0t} \cdot \theta_{0it} = y_{it} \qquad i = 3, \forall t \tag{3.22}$$

$$x_{1t} \cdot \theta_{1it} = y_{it} \qquad i = 4, ..., I, \forall t \tag{3.23}$$

$$z^s_{it} \geq z^s_{(i-1)t} \qquad i = 3, \forall s, t \tag{3.24}$$

$$z^s_{it} \geq z^s_{(i-1)t} \qquad i = 5, ..., I, \forall s, t \tag{3.25}$$

$$(z^s_{it} - z^s_{(i-1)t})(v^s_{(i-1)(t-1)} + y_{it}) \geq m^s_{it} \qquad i = 3, \forall s, t \tag{3.26}$$

$$(z^s_{it} - z^s_{(i-1)t})(v^s_{(i-1)(t-1)} + y_{it}) \geq m^s_{it} \qquad i = 5, ..., I, \forall s, t \tag{3.27}$$

$$d^s_{0t} + \pi^s_t = \sum_{i=3}^{3}((v^s_{(i-1)(t-1)} + y_{it}) \cdot z^s_{it} - m^s_{it}) + r^s_t \forall s, t \tag{3.28}$$

$$d^s_{1t} = \sum_{i=4}^{I}((v^s_{(i-1)(t-1)} + y_{it}) \cdot z^s_{it} - m^s_{it}) + \pi^s_t, \forall s, t \tag{3.29}$$

$$d^s_{0t} + d^s_{1t} - \sum_{i=3}^{I}(v^s_{(i-1)(t-1)} + y_{it}) \leq r^s_t \qquad \forall s, t \tag{3.30}$$

$$x_{at} \in \mathbb{Z}^+ \qquad a \in \{0, 1\}, \forall t \tag{3.31}$$

$$\pi^s_t \in \mathbb{Z}^+ \qquad \forall s, t \tag{3.32}$$

Constraints (3.22) and (3.23) allocate young and old units to each age group. Constraints (3.24) and (3.25) enforce the FIFO policy. To guarantee that young units are not prevented from being allocated to meet demand for fresh blood, some links are removed from the formulations; otherwise, young units cannot be used before all of the old units on inventory are used. Constraint (3.26) and (3.27) guarantee that the values of m^s_{it} do not exceed the number of units available in their age groups. Constraints (3.28) and (3.29) are demand constraints and ensure the usage of young units for Type 2 patients when all old units on inventory are depleted. Constraint

(3.28) in conjunction with constraint (3.30) identifies the shortage levels. Finally, constraints (3.31) and (3.32) force x_{at} and π_t^s to take non-negative discrete values.

Since the purchasing cost of young units exceeds the cost of old units, the first term of the objective function in the first formulation is replaced by the following revised term.

$$\sum_{t=1}^{T} c_0 \cdot x_{0t} + \sum_{t=1}^{T} c_1 \cdot x_{1t} \tag{3.33}$$

Similar to the model described by (3.1)-(3.18), the first linearization technique is applied to the constraints (3.26)-(3.29) as they indicate non-linear terms. After the interactions between v, y and z are replaced with the corresponding linearization variables, constraints (3.34)-(3.37) are obtained as follows:

$$d_{0t}^s + \pi_t^s = \sum_{i=3}^{8} \gamma_{(i-1)(t-1)}^s + \alpha_{it}^s - m_{it}^s + r_t^s \qquad \forall s, t \tag{3.34}$$

$$d_{1t}^s = \sum_{i=9}^{I} \gamma_{(i-1)(t-1)}^s + \alpha_{it}^s - m_{it}^s + \pi_t^s \qquad \forall s, t \tag{3.35}$$

$$\gamma_{it}^s + \alpha_{it}^s - \mu_{(i-1)t}^s - \psi_{it}^s \geq m_{it}^s \qquad i = 3, 4, ..., 8, \forall s, t \tag{3.36}$$

$$\gamma_{it}^s + \alpha_{it}^s - \mu_{(i-1)t}^s - \psi_{it}^s \geq m_{it}^s \qquad i = 10, 11, ..., I, \forall s, t \tag{3.37}$$

In summary, constraints (3.2)-(3.3), (3.8), (3.10)-(3.18), (3.22)-(3.25), (3.30)-(3.32), (3.34)-(3.37), (A.1)-(A.26) are present in the model discussed in this section.

3.2. Formulation of C/T Ratio and Crossmatch Release Period

According to [29], crossmatch-to-transfusion ratio should ideally be 1:1. They also mentioned a C/T ratio less than 2.5 to be acceptable; that is, out of five units crossmatched only two are used. In order to incorporate C/T ratio and crossmatch release period into our formulation, the problem is modified so as to obtain deterministic integer programming formulation as follows:

$$\text{Minimize} \sum_{t=1}^{T} c \cdot x_t + \sum_{i=3}^{I} \sum_{t=1}^{T} h \cdot v_{it} + \sum_{t=1}^{T} w \cdot u_t + \sum_{t=1}^{T} b \cdot r_t \tag{3.38}$$

subject to:

$$x_t \leq CAP_t \qquad \forall t \tag{3.39}$$

$$y_{it} = 0, \qquad i = 1, 2, \forall t \tag{3.40}$$

$$y_{it} = x_t \theta_{it}, \qquad i = 3, 4, ..., I, \forall t \tag{3.41}$$

$$z_{it} \geq z_{(i-1)t} \qquad i = 3, 4, ..., I, \forall t \tag{3.42}$$

$$d_t = \sum_{i=3}^{I} ((v_{(i-1)(t-1)} + y_{it}) z_{it} - m_{it}) + r_t \qquad \forall t \tag{3.43}$$

$$(z_{it} - z_{(i-1)t})(v_{(i-1)(t-1)} + y_{it}) \geq m_{it} \qquad i = 3, 4, ..., I, \forall t \tag{3.44}$$

$$z_{2t} = 0 \qquad \forall t \tag{3.45}$$

$$d_t - \sum_{i=3}^{I} (v_{(i-1)(t-1)} + y_{it}) \leq r_t \qquad \forall t \tag{3.46}$$

$$v_{it} = (1 - z_{it})(v_{(i-1)(t-1)} + y_{it}) + (z_{it} - z_{(i-1)t}) m_{it} + \beta_{it} \qquad i = 3, ..., I, \forall t \tag{3.47}$$

$$v_{it} = \beta_{it} \qquad i = I + 1, ..., I + CRP, \forall t \tag{3.48}$$

$$v_{(2)t} = 0 \qquad \forall t \tag{3.49}$$

$$v_{i(0)} = 0 \qquad \forall i \tag{3.50}$$

$$\beta_{it} = \lfloor((v_{(i-CRP-1)(t-CRP-1)} + y_{(i-CRP)(t-CRP)}) \cdot z_{(i-CRP)(t-CRP)} - m_{(i-CRP)(t-CRP)})$$

$$\cdot(1 - CT^{-1})\rfloor \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T \tag{3.51}$$

$$\beta_{it} = 0 \qquad i = 3 + CRP, ..., I + CRP, t = 1, ..., CRP \tag{3.52}$$

$$\beta_{it} = 0 \qquad i = 1, ..., 3 + CRP - 1, \forall t \tag{3.53}$$

$$u_t = \sum_{n=0}^{CRP} v_{(I+n)t} \qquad \forall t \tag{3.54}$$

$$x_t \in \mathbb{Z}^+ \qquad \forall t \tag{3.55}$$

$$r_t, u_t \in \mathbb{Z}^+ \qquad \forall t \tag{3.56}$$

$$y_{it} \in \mathbb{Z}^+ \qquad \forall i, t \tag{3.57}$$

$$m_{it}, v_{it}, \beta_{it} \in \mathbb{Z}^+ \qquad \forall i, t \tag{3.58}$$

$$z_{it} \in \{0, 1\} \qquad \forall i, t \tag{3.59}$$

The objective function (3.38) captures the deterministic version of the objective function presented in (3.1). Similarly, it seeks to minimize the purchasing, inventory, wastage and shortage costs during the planning horizon. Constraints (3.39)-(3.46) and (3.49)-(3.50) are the deterministic formulations of Constraints (3.2)-(3.9) and (3.11)-(3.12) respectively. Constraints (3.47) and (3.48) update end-period blood inventory levels for each age-group. Furthermore, right hand-side of Constraint (3.48) only indicates the variable associated with returned blood units from assigned inventory since the hospital would not receive or keep blood units that are expired. Constraints (3.51)-(3.53) are used to compute the number of blood units returned back to unassigned inventory in each period. The number of returned units is computed by multiplying the number of blood units in assigned inventory with the blood return ratio derived from subtracting the inverse of hospital's average C/T ratio from one. For each age group it is assume that the same proportions of blood units in assigned inventory are returned to unassigned inventory. Since it takes CRP time periods for reserved blood units to be returned to the unassigned inventory, the value of β_{it} in Constraint (3.52) is set equal to zero for the first CRP periods. Similarly, as the youngest blood units received by the hospital are three days old, the value of β_{it} is set equal to zero for blood units that are younger than $3 + CRP$ days old in Constraint (3.53). Constraint (3.54) identifies the wastage levels of hospital at the end of each period. Constraints (3.55)-(3.58) are the non-negativity constraints. Finally, Constraint (3.59) forces z_{it} to assume binary values.

Similar to the first two models, linearization techniques (Appendix A) are applied to the non-linear terms impacting Constraints (3.43), (3.44), (3.47) and resulting on modified Constraints (3.60)-(3.62). In addition, Constraints (A.1)-(A.26) are added into the new formulation.

$$d_t = \sum_{i=3}^{I} (\gamma_{it} + \alpha_{it} - m_{it}) + r_t \qquad \forall t \tag{3.60}$$

$$\gamma_{it} + \alpha_{it} - \mu_{(i-1)t} - \psi_{it} \geq m_{it} \qquad i = 3, 4, ..., I, \forall t \tag{3.61}$$

$$v_{it} = v_{(i-1)(t-1)} + y_{it} - \gamma_{it} - \alpha_{it} + \lambda_{it} - \delta_{it} + \beta_{it} \qquad i = 3, 4, ..., I, \forall t \tag{3.62}$$

Finally, the floor function shown in Constraint (3.51) is modified using techniques described in Appendix B and resulting in Constraints (3.63) and (3.64).

$$\beta_{it} \geq ((\gamma_{(i-CRP-1)(t-CRP-1)} - \alpha_{(i-CRP)(t-CRP)}) - m_{(i-CRP)(t-CRP)})$$

$$\cdot(1 - CT^{-1}) - 1 + TOL \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T \tag{3.63}$$

$$\beta_{it} \leq ((\gamma_{(i-CRP-1)(t-CRP-1)} - \alpha_{(i-CRP)(t-CRP)}) - m_{(i-CRP)(t-CRP)})$$

$$\cdot(1 - CT^{-1}) + TOL \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T \tag{3.64}$$

3.3. Computational Study

In this section, we present the data used in our analysis and discuss the numerical results obtained from above models using IBM ILOG CPLEX 12.1 on Dell OPTIPLEX 755 with 2.20 GHz CPU and 2GB of RAM.

3.3.1. Data

Table 3.4 summarizes the values of cost parameters that are used in our models. Most of the cost parameters are obtained from the literature as shown in [16], [9], [4] and [10]. Inventory costs were obtained from real data from a Regional Medical Center (RMC).

Table 3.4: Cost Parameters

Parameters	Value	Units	Reference
Purchase Cost (Platelet)-c	538	\$/unit	[16]
Purchase Cost (RBC)-c	180	\$/unit	[4]
Shortage Cost-b	1500	\$/unit	[10]
Wastage Cost-w	150	\$/unit	[9]
Holding Cost-h	1.25	\$/unit*day	RMC

Demand distributions from [9] were used for estimating daily demand values of platelet units. Two studies [9] and [10] provide daily demand distribution of platelets at a hospital blood bank. Two types of demand are considered in [9]. Only one type of demand for weekly platelet production is considered in [10]. The mean values of platelet demands obtained from Sanquin Blood Bank and used in both references are 24, 16, 32, 16, 24, 0, 8 for Monday through Sunday respectively. The demand data are assumed to be purely random and distributed around the mean; thus, the authors used a poisson distribution in [9] and gamma distribution in [10].

In [30], daily fluctuations of red blood cells in terms of percentages are provided for the months between April 2003 and March 2004 at Southampton Center. They are: 100%, 93%, 56%, 59%, 44%, 18% and 17% for Monday through Sunday respectively where 100% value relates to the day of the week with highest average number of units. Mean demand values for each day were computed by multiplying percentages by 100.

Matlab 2010a was used to generate a total of 35 datasets for numerical study. Three categories were selected to group them based on the purpose they serve.

1. To test the stochastic integer programming model 15 datasets of various sizes were generated. For each of the demand scenarios in a given time period (t=30 days), data are generated from a poisson distribution using the mean values of platelet

discussed above. There are five datasets (F1-F5) with 4 scenarios, five datasets (E1-E5) with 8 scenarios and five datasets (S1-S5) with 16 scenarios.

2. To analyze the extension of stochastic integer programming model that considers the blood demand for two types of patients, another 15 datasets of three groups with same sizes were generated. For all groups in a given time period (t=30 days), scenario data were generated from a poisson distribution for 8 scenarios. Five of these datasets (SY1-SY5) were generated similar to the datasets in first category, but 1/8 and 7/8 of the average daily demand was taken as the mean daily demand of young and any platelet respectively. The remaining ten datasets (SY6-SY10 and SY11-SY15) were generated taking 1/4 and 1/2 of average daily demand as the mean daily demand of young platelet.

3. To evaluate the effect of C/T ratio and crossmatch release period 5 datasets (D1-D5) were generated for each time period (t=10 days) from a poisson distribution using the average demand values of red blood cells discussed above.

3.3.2. Numerical Results

The problem described by (3.1)-(3.18) was solved with datasets F1-F5, E1-E5, and S1-S5 using CPLEX 12.1. Table 3.5 summarizes the solution times for solving these 15 problems given daily blood center (platelet) capacity of 30 units and average platelet age of 4 days old in blood shipments. Solution Time in Table 3.5 is the time it takes CPLEX to solve the problem within an optimality gap which are selected as 1.5% in the stochastic formulation and 2.5% in its extension. In addition, the gap is chosen as 3.0% in the deterministic formulation.

The average solution times of stochastic model by problem size are 14.600, 83.419 and 449.837 seconds for datasets with 4, 8 and 16 scenarios respectively. As the number of scenarios increases, the problem size grows and the solution time to reach the solution increases.

Table 3.5: Solution Times of Datasets for Stochastic Model

Number of Scenario	Number of Variables	Number of Constraints	Dataset	Solution Time (s)
4	3720	2832	F1	12.016
			F2	14.926
			F3	13.808
			F4	17.728
			F5	14.523
8	7320	5364	E1	56.082
			E2	92.215
			E3	125.088
			E4	70.516
			E5	73.198
16	14520	10428	S1	415.641
			S2	412.252
			S3	413.073
			S4	522.045
			S5	486.178

Table 3.6 shows the solution times of the problem explained by (3.38)-(3.59) given the average C/T ratio of 4/3 and the total blood center (RBC) capacity of 120-110-80-80-60-40-40 units for Monday through Sunday (where the beginning of the analysis period, t=1, starts with Monday). Length of CRP column refers to the length of crossmatch release period (in days) for RBC units that is determined by hospital policy. The average solution times of deterministic model by length of CRP are 265.651, 16.508 and 57.110 seconds. The longer the crossmatch release period, the shorter the time required to solve the problem.

A variety of situations were considered to determine the sensitivity of outcomes to the model parameters. We considered the effect of the average platelet age in blood shipments and daily blood center capacity on the solution of the problem described by (3.1)-(3.18). This was done through the use of datasets E1-E5. The results are shown in Table 3.8-3.10. In addition, the abbreviations shown in Table 3.7 are used in the rest of the paper for the daily platelet capacity of the blood center (Monday through Sunday).

Table 3.6: Solution Times of Datasets for Deterministic Model

Number of Variable	Number of Constraints	Length of CRP (days)	Dataset	Solution Time (s)
1980	3400	1	D1	171.784
			D2	418.684
			D3	247.187
			D4	350.872
			D5	140.79
1980	3400	2	D1	58.132
			D2	190.552
			D3	94.447
			D4	96.82
			D5	42.411
1980	3400	3	D1	66.319
			D2	35.099
			D3	58.025
			D4	70.725
			D5	55.385

Capacity of the blood center allocated to the hospital has a noteworthy effect on the solution values. As the capacity is increased to the $CAP2$ level, blood shortages and total cost related to blood operations decrease significantly. However increasing capacity beyond this level does not have a major impact on the outcomes as shown in Table 3.8-3.10. It was noted that the average platelet age in blood shipments has some effect on both total cost and wastage levels at the hospital. Receiving older platelet units (average age of 4.5 days) causes higher levels of wastage and increases total cost. Whereas raising age of platelet units in blood shipments from 3.0 to 3.5 does not have an apparent impact. In the latter case, the changes in wastage levels show both downward and upward trend depending on the dataset used for testing. However, when $CAP1$ or $CAP2$ capacity is selected, the total cost for the 3.5 average age is higher than the 3.0 average age. For other capacity levels, raising age of platelet units in blood shipments from 3.0 to 3.5 does not have any effect on the total cost.

Table 3.11 shows the sensitivity of outcomes with the changes in unit shortage cost. The daily platelet capacity of blood center is assumed to be 25-15-35-15-25-10-

Table 3.7: The Abbreviations of the Daily Capacity Levels of the Blood Center

Abbreviation	Daily Capacity Levels (units)
CAP1	20-10-30-10-20-5-5
CAP2	25-15-35-15-25-10-10
CAP3	30-20-40-20-30-15-15
CAP4	35-25-45-25-35-20-20
CAP5	40-30-50-30-40-25-25
CAP.E1	2-1-3-1-2-1-1
CAP.E2	3-2-4-2-3-1-1
CAP.E3	5-4-6-4-5-3-3
CAP.E4	7-6-8-6-7-5-5
CAP.E5	9-8-10-8-9-7-7
CAP.E6	12-11-13-11-12-10-10
CAP.F1	4-2-6-2-4-1-1
CAP.F2	6-4-8-4-6-2-2
CAP.F3	8-6-10-6-8-4-4
CAP.F4	10-8-12-8-10-6-6
CAP.F5	13-11-15-11-13-9-9
CAP.F6	16-14-18-14-16-12-12
CAP.T1	9-5-13-5-9-1-1
CAP.T2	12-8-16-8-12-4-4
CAP.T3	15-11-19-11-15-7-7
CAP.T4	18-14-22-14-18-10-10
CAP.T5	21-17-25-17-21-13-13
CAP.T6	24-20-28-20-24-15-15

10 units for Monday through Sunday and the average age of platelet units in blood shipments is 3.5 days old. As a result of penalizing the hospital with higher unit shortage cost, more units are placed to blood center in order to avoid shortages. Increasing number of blood units received in shipments reduces the shortages and inversely increases the wastages. Thus, higher shortage cost and increasing inventory levels causes increased cost in blood related operations at the hospital.

Datasets SY1-SY15 were used to examine the effects of blood center capacity for young platelets on the solution of the problem discussed in Section 3.1.1. Table 3.12-3.14 show the results of these trials based on different demand values of young platelet units. The total capacity of blood center is assumed to be 30-20-40-20-15-15

Table 3.8: Effects of Blood Center Capacity on Outcomes Given θ_{avg} of 3

Capacity	Dataset	Total Expected Shortage (units)	Total Expected Wastage (units)	Total Expected Cost ($)
CAP1	E1	96.75	0.375	374991
	E2	92	2.25	369221
	E3	98.62	0	378811
	E4	80.25	0	350173
	E5	113.12	0	401087
CAP2	E1	12.62	10.375	304291
	E2	19.62	14	312087
	E3	14.5	11.5	310010
	E4	7.5	11.125	289741
	E5	17	7.5	315817
CAP3	E1	9.25	9.625	300728
	E2	12.37	18.125	308359
	E3	7.37	17.75	307273
	E4	6.75	9.5	288325
	E5	6.5	14.75	310373
CAP4	E1	9	10	300425
	E2	12.5	18.375	308592
	E3	11.62	12.375	307434
	E4	6.5	9	287902
	E5	10.37	10.875	310658
CAP5	E1	9.62	9.25	300747
	E2	14.5	17	308640
	E3	10.62	15.25	306922
	E4	6.62	8.75	287569
	E5	5.62	15	309110

units for Monday through Sunday. Column titled "Young Platelet Cap." refers to the maximum amount of blood units that the blood center has agreed to supply to the hospital. "Total Exp. $\pi - r$" column shows the expected number of young platelet units on inventory used to satisfy platelet demand.

As can be noted, young platelet capacity of blood center has a significant impact on the solution of the problem. Increasing the capacity up to $CAP.E3$, $CAP.F3$ and $CAP.T3$ results in significant cost savings and reduction of shortage levels at the hospital. Further increases from these capacity levels provide slight improvements on

Table 3.9: Effects of Blood Center Capacity on Outcomes Given θ_{avg} of 3.5

Capacity	Dataset	Total Expected Shortage (units)	Total Expected Wastage (units)	Total Expected Cost ($)
CAP1	E1	104.875	1.5	383568
	E2	101.25	0.75	376930
	E3	107.625	0	387450
	E4	85.875	0.625	356002
	E5	121.125	0	408767
CAP2	E1	18.625	10.375	309663
	E2	25.125	11.375	316502
	E3	25.75	13.5	316418
	E4	11.625	6.875	292398
	E5	23.5	9.375	321876
CAP3	E1	7.5	12.25	300721
	E2	12.5	20.5	309475
	E3	9.75	17	308006
	E4	6.625	9.875	288240
	E5	8.25	11.875	310329
CAP4	E1	8.875	12.375	300607
	E2	14.5	17.25	308752
	E3	10.625	14.75	307896
	E4	5.625	11.125	287988
	E5	9.125	12	310587
CAP5	E1	8.125	12.875	300649
	E2	13.5	18	308560
	E3	12.125	13.5	307817
	E4	5.625	11	288006
	E5	9.625	11.5	310201

aforementioned outcomes. Nevertheless, average values of total expected wastages for five datasets of given young platelet capacities are presented. As anticipated, these values show, in general, a downward trend when increasing young platelet capacity. When young platelet capacity is higher, as a result of increasing availabilities for young platelet units, the number of young platelet units that are used to satisfy any platelet demand increases. Moreover, due to higher purchase cost associated with young platelet units, when more young platelet units are needed, the cost will increase.

Table 3.10: Effects of Blood Center Capacity on Outcomes Given θ_{avg} of 4.5

Capacity	Dataset	Total Expected Shortage (units)	Total Expected Wastage (units)	Total Expected Cost ($)
CAP1	E1	117.875	11.125	402335
	E2	111.375	8.875	392243
	E3	120.875	9.25	406545
	E4	108.375	3.125	379329
	E5	140.25	1.125	427909
CAP2	E1	28.875	24.75	326927
	E2	40.5	18.75	332687
	E3	39.75	16.875	335570
	E4	24	15.75	308555
	E5	44.375	12	343904
CAP3	E1	12.875	28.375	314278
	E2	17.875	32.375	320223
	E3	17.625	27	321173
	E4	12.875	20.375	301204
	E5	13.375	23.875	322939
CAP4	E1	12	29.75	314261
	E2	17.625	32.125	319813
	E3	14.75	30.125	320576
	E4	10.625	22.125	300252
	E5	13.5	24	323155
CAP5	E1	12.75	28.25	314063
	E2	17.625	32.125	319810
	E3	14.75	30.125	320567
	E4	13.625	19.125	301063
	E5	13	25.625	323735

The problem described by (38)-(59) was solved with datasets D1-D5 and the results are displayed in Table 3.15 to show the effects of different C/T ratios and crossmatch release periods on outcomes. The daily RBC capacity of blood center is assumed to be 120-110-80-80-60-40-40 units for Monday through Sunday and the average age of RBC units in blood shipments is 39.5 days old. Average C/T Ratio column refers to the average value of C/T ratio in blood related procedures at the hospital.

Table 3.11: Effects of Unit Shortage Cost on Model Outcomes

Shortage Cost ($/unit)	Dataset	Total Expected Shortage (units)	Total Expected Wastage (units)	Total Expected Cost ($)
1000	E1	31.25	4.5	300024
	E2	43.125	7.25	304767
	E3	39.25	3.625	305753
	E4	22.5	5.25	288188
	E5	38.75	2.625	312631
1250	E1	20.125	10.125	305603
	E2	32.125	11	311009
	E3	30.625	4.25	311331
	E4	13.375	9.375	290615
	E5	27.75	5.625	317662
1500	E1	18.625	11.375	309663
	E2	25.125	13.5	316502
	E3	25.75	6.875	316418
	E4	11.625	9.375	292398
	E5	23.5	8.25	321876
1750	E1	14.25	14.125	311401
	E2	21.625	15.125	320147
	E3	19.5	12.375	320360
	E4	11.625	9.375	295304
	E5	19.5	11.125	325507
2000	E1	13.875	15.125	315457
	E2	21	17.5	325784
	E3	17.875	13.375	323219
	E4	9.875	11.125	297178
	E5	20.125	10.875	330536

Table 3.12: Effects of Blood Center Capacity for Young Platelet on Outcomes Given Mean Demand Value of Young Platelet is 1/8 of Total Platelet Demand

Young Platelet Cap.	Dataset	Total Exp Short. (units)	Total Exp Wast. (units)	Total Exp $\pi - r$	Total Exp Cost ($)
CAP.E1	SY1	46.5	5.75	3.62	336954
	SY2	54.37	3.75	2.87	355534
	SY3	52.5	2.37	2.37	356203
	SY4	48.37	5.5	3.5	339234
	SY5	49.75	6	3.37	341434
			avg.: 4.67		
CAP.E2	SY1	35.12	4	9.75	324446
	SY2	40.5	4.37	8.25	342881
	SY3	42.5	3.75	6.12	342529
	SY4	34.37	4.12	7	327706
	SY5	33.37	5.5	8.37	328517
			avg.: 4.34		
CAP.E3	SY1	20.62	3.12	29.25	310395
	SY2	21.75	4	32.37	327844
	SY3	25.37	3	23.25	327024
	SY4	18.62	1.75	31.12	313017
	SY5	15.75	4.87	31.87	311385
			avg.: 3.34		
CAP.E4	SY1	16.62	2.25	58.25	305829
	SY2	21.12	4.25	48.5	323156
	SY3	18.75	0.87	51.5	319750
	SY4	14.87	0.62	52.25	306359
	SY5	16.5	3.5	46.5	307413
			avg.: 2.87		
CAP.E5	SY1	13.87	1.75	76.5	301444
	SY2	14.12	1.25	83.75	316057
	SY3	15.5	2.5	74.75	317589
	SY4	13.12	3.75	73.12	305500
	SY5	12.37	2.62	82.25	306444
			avg.: 2.37		
CAP.E6	SY1	12.37	4.25	95.12	301538
	SY2	12.5	1.62	112.75	315113
	SY3	11.37	0.75	106.12	314531
	SY4	10.5	1.62	108.62	301359
	SY5	9.12	2.12	117.87	303464
			avg.: 2.07		

Table 3.13: Effects of Blood Center Capacity for Young Platelet on Outcomes Given Mean Demand Value of Young Platelet is 1/4 of Total Platelet Demand

Young Platelet Cap.	Dataset	Total Exp. Short. (units)	Total Exp. Wast. (units)	Total Exp. $\pi - r$	Total Exp. Cost ($)
CAP.F1	SY6	73.87	9.25	3.37	364461
	SY7	74.62	5.75	2.5	375845
	SY8	71.12	5.87	2.62	359784
	SY9	73.5	8.75	5.62	378926
	SY10	74.5	8.5	3	379767
			avg.: 7.62		
CAP.F2	SY6	39.62	9.25	17.25	336917
	SY7	43.75	5.62	15.12	347910
	SY8	40.37	4.75	13.75	329701
	SY9	42	7.25	16.25	348271
	SY10	42.25	5.87	15.87	350527
			avg.: 6.54		
CAP.F3	SY6	28.5	6.37	38.37	322079
	SY7	29	3.5	35.87	332712
	SY8	21.37	5.12	38	311633
	SY9	21.12	5.12	38.62	330798
	SY10	26.5	5.62	39.5	333025
			avg.: 5.14		
CAP.F4	SY6	17.5	3.75	63.12	314322
	SY7	20.62	3	64.5	325231
	SY8	16.87	3.37	55.62	304437
	SY9	14.37	5.87	60.75	325050
	SY10	16.25	2.62	63.25	322780
			avg.: 3.72		
CAP.F5	SY6	16.125	3.25	101.75	310222
	SY7	17.75	1.25	93	319065
	SY8	10.87	0.62	90.5	297329
	SY9	9.62	4.62	95.37	319504
	SY10	16.25	2.62	93.5	322052
			avg.: 2.47		
CAP.F6	SY6	8	2	124	306131
	SY7	15.125	3.5	123.12	318681
	SY8	8.5	0.87	106.37	294536
	SY9	10.37	3.12	128.37	318882
	SY10	14.12	3.75	111	322037
			avg.: 2.64		

Table 3.14: Effects of Blood Center Capacity for Young Platelet on Outcomes Given Mean Demand Value of Young Platelet is 1/2 of Total Platelet Demand

Young Platelet Cap.	Dataset	Total Exp. Short. (units)	Total Exp. Wast. (units)	Total Exp. $\pi - r$	Total Exp. Cost ($)
CAP.T1	SY11	97	10.12	3.37	387631
	SY12	111.75	9.25	3.5	415000
	SY13	96.75	12	6.25	390201
	SY14	92.25	7.75	3.62	386602
	SY15	103.62	9.62	3.5	400722
			avg.: 9.74		
CAP.T2	SY11	42.62	9.12	21	338642
	SY12	55.75	10.75	17.37	364312
	SY13	45.75	7.62	24.75	341904
	SY14	45.12	3.62	22.87	340368
	SY15	48.37	9.25	18.25	348860
			avg.: 8.07		
CAP.T3	SY11	21.12	3.37	46.75	316838
	SY12	27.12	5.37	44.87	334990
	SY13	24	5.12	54.62	321349
	SY14	26	4.75	55.75	321842
	SY15	26.5	3.62	53.62	328436
			avg.: 4.44		
CAP.T4	SY11	11.62	0.75	83.5	306194
	SY12	17.5	4.62	67.62	327556
	SY13	14.62	2.25	78.5	315660
	SY14	17.5	2.37	81.75	316398
	SY15	14.25	2.62	78.37	316722
			avg.: 2.52		
CAP.T5	SY11	7.5	4.25	110.12	304274
	SY12	15.75	3.25	88.75	324766
	SY13	14.62	2.12	110.25	314279
	SY14	14.75	0.5	104.25	311938
	SY15	16.87	4.12	115.75	320107
			avg.: 2.84		
CAP.T6	SY11	8.62	1.87	116.87	304248
	SY12	17.62	1.87	114.5	325589
	SY13	15.62	1.25	124.75	316223
	SY14	13.25	2.37	125.12	312996
	SY15	22.12	0.25	124.12	321718
			avg.: 1.52		

Table 3.15: Effects of C/T Ratio and Crossmatch Release Period on Outcomes

Avg. C/T Ratio	Length of CRP (Days)	Dataset	Total Short. (units)	Total Wast. (units)	Total Cost ($)
4/3	**1**	D1	0	12	103158
		D2	0	11	101059
		D3	0	16	107542
		D4	0	15	97734
		D5	0	14	107135
	2	D1	0	23	110045
		D2	0	20	107713
		D3	0	19	113096
		D4	0	18	103811
		D5	0	24	113744
	3	D1	0	25	112029
		D2	0	24	110003
		D3	0	21	115131
		D4	0	21	106048
		D5	1	28	117475
8/5	**1**	D1	0	18	96888
		D2	0	20	93550
		D3	0	20	99657
		D4	0	19	89751
		D5	0	29	100675
	2	D1	0	30	107405
		D2	0	31	103996
		D3	0	32	109599
		D4	0	29	100020
		D5	0	38	110409
	3	D1	0	40	110600
		D2	0	35	108068
		D3	0	37	113760
		D4	0	47	104535
		D5	0	42	114663
2/1	**1**	D1	0	27	89168
		D2	0	28	87583
		D3	0	27	92643
		D4	1	29	83730
		D5	1	32	93170
	2	D1	0	53	103775
		D2	0	50	101466
		D3	0	49	106739
		D4	0	48	97606
		D5	0	57	107875
	3	D1	0	59	110276
		D2	0	61	106724
		D3	0	61	112023
		D4	1	55	103788
		D5	0	62	112194

Finally, the numerical study shows that the changes in C/T ratio and length of CRP have significant effects on the solution values. Longer length of CRP may lead a long stay of RBC units in reserved inventory without being transfused. As a result, the lifetime of RBC units diminishes and increased RBC wastages cause higher cost in blood related operations. In addition, higher C/T ratios increase the number of RBC units returned to free inventory and, at the same time, cause higher levels of wastages at the hospital. The increase on returning units decrease the amount of RBC placed to blood center and thus results in reduced total cost.

CHAPTER 4: CENTRALIZED HOSPITAL NETWORK CONSISTING OF ONE BLOOD CENTER AND MULTIPLE HOSPITALS

We focus on a two-level supply chain of blood products consisting of one blood center and multiple hospitals as shown with the bolded red line in Figure 4.1. The blood center has access to the information related to blood inventory and the demand levels at each hospital. It is responsible for making the decisions on behalf of the system players to minimize the cost and shortage levels of the whole system.

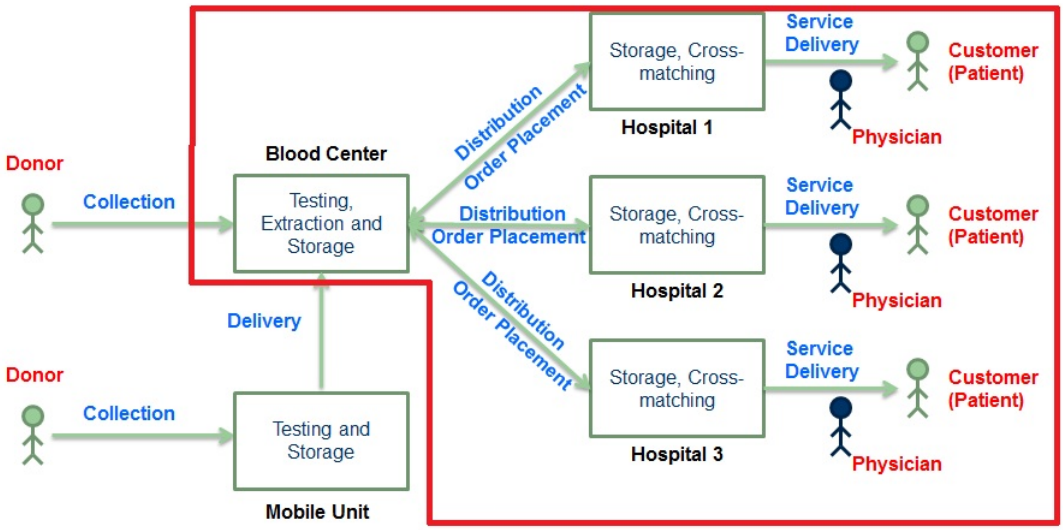

Figure 4.1: Two Level Supply Chain with Multiple Hospitals and One Blood Center

In this research, the following assumptions have been made:

- The capacity of the blood center is limited.
- Lead times for blood supply are zero.
- Hospitals carry safety stock.

- The number of blood units wasted are estimated and known for each hospital.
- A shortage cost is incurred if demand is not satisfied due to unavailability of blood units on inventory.

4.1. Formulation of Centralized Hospital Network

A non-linear integer programming model is developed to improve the efficiency of blood supply chain within the hospital network considering several distribution policies of blood products.

The indices, the parameters and the variables that are used in the models are summarized in Tables 4.1-4.3. It is valuable to note that core blood demand refer to the summation of hospitals' blood demands without considering their safety stock levels.

Table 4.1: Model Indices

Index	Description
k	Hospital, k=1,2,...,K
t	Time Period, t=1,2,...,T (days)

Using the indices, parameters and decision variables in Table 4.1-4.3, the integer programming model is formulated as follows:

$$\text{Minimize} \sum_{k=1}^{K} \sum_{t=1}^{T} (c \cdot x_{kt} + h \cdot v_{kt} + b \cdot r_{kt}) \tag{4.1}$$

subject to:

$$r_{kt} + v_{k(t-1)} + x_{kt} = d_{kt} + v_{kt} + u_{kt} \qquad \forall k, t \tag{4.2}$$

$$u_{kt} = \lceil \theta_k \cdot v_{k(t-1)} \rceil \qquad \forall k, t \tag{4.3}$$

Table 4.2: Parameters for Model (4.1)-(4.31)

Parameter	Description
K	Number of hospitals
T	Length of planning horizon
b	Unit shortage cost of blood at the hospital
c	Unit purchasing cost of blood
h	Unit holding cost of blood at the hospital
d_{kt}	Blood demand at hospital k in time t
CAP_t	Capacity of the blood center in time period t
SS_k	Safety stock level at hospital k
M	Big M (Big Number)
θ_k	Average proportion of blood at hospital k that is wasted in a period
α	Fairness index
CT	Average C/T ratio at the hospital
$Lmin_{kt}$	Minimum for lower bound of $EXCS_{kt} + z_t * (v_{k(t-1)} - u_t)$ and SS_k
U_{1kt}	Upper bound of $EXCS_{kt} + z_t(v_{k(t-1)} - u_t)$
U_{2k}	Upper bound of SS_k
TOL:	Small number

Table 4.3: Decision Variables of the Model

Decision Variable	Description
r_{kt}	Number of blood shortage at the end of time t in hospital k
u_{kt}	Number of blood wastage at the beginning of time t in hospital k
v_{kt}	Inventory level of blood at the end of time t in hospital k
x_{kt}	Number of blood ordered by the hospital k at the beginning of time t
z_t	1 if a shortage occurs at the blood center in time period t, 0 otherwise
π_t	Binary variable ro capture the relationship between r and z 1 if a shortage occurs at the blood center in time period t, 0 otherwise
EXC_t	Amount of blood inventory left at the blood center after fulfilling hospitals' core demands in time period t
$EXCS_{kt}$	Amount of available blood inventory at the blood center (after fulfilling hospitals' core demands) that can be sent to hospital k in time period t
w_{1kt}, w_{2kt}	Binary variables used in the linearization of min function

$$\sum_{k=1}^{K} x_{kt} \leq CAP_t \qquad \forall t \tag{4.4}$$

$$r_{kt}/d_{kt} \leq \sum_{m=1}^{K} r_{kt} / \sum_{m=1}^{K} d_{kt} + \alpha \qquad \forall k, t \tag{4.5}$$

$$\sum_{k=1}^{K} r_{kt} \leq M \cdot \pi_t \qquad \forall t \tag{4.6}$$

$$z_t \leq M \cdot (1 - \pi_t) \qquad \forall t \tag{4.7}$$

$$\sum_{k=1}^{K} r_{kt} + z_t \geq 1 \qquad \forall t \tag{4.8}$$

$$v_{kt} = min\{EXCS_{kt} + z_t \cdot (v_{k(t-1)} - u_t), SS_k\} \qquad \forall k, t \tag{4.9}$$

$$v_{k0} = 0 \qquad \forall k \tag{4.10}$$

$$EXC_t = (CAP_t - \sum_{t=1}^{T} d_{kt}) \cdot z_t \qquad \forall t \tag{4.11}$$

$$EXC_t/K - 1 + TOL \leq EXCS_{kt} \qquad \forall k, t \tag{4.12}$$

$$EXCS_{kt} \leq EXC_t/K + 1 - TOL \qquad \forall k, t \tag{4.13}$$

$$\pi_t, z_t \in \mathbb{B}^{\{0,1\}} \qquad \forall t \tag{4.14}$$

$$x_{kt}, v_{kt}, r_{kt} \in \mathbb{Z}^+ \qquad \forall k, t \tag{4.15}$$

$$EXC_t \in \mathbb{Z} \qquad \forall t \tag{4.16}$$

$$EXCS_{kt} \in \mathbb{Z} \qquad \forall k, t \tag{4.17}$$

The objective function (4.1) seeks to minimize the total purchasing, inventory and shortage cost during the planning horizon. Constraint (4.2) is an equilibrium constraint which is illustrated with Figure 4.1 (the amount of blood units entered into the system is equal to the amount of blood units exited from the system).

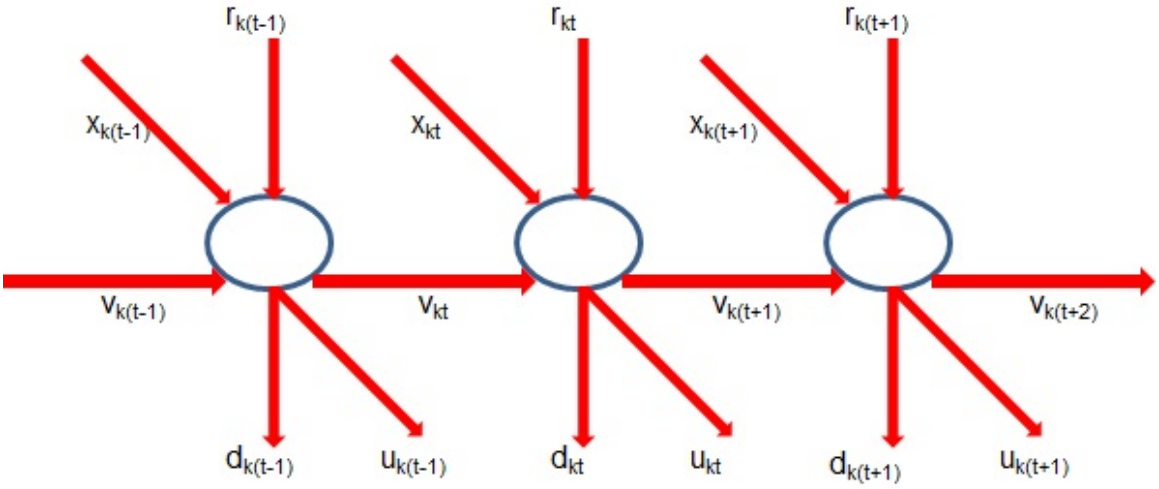

Figure 4.2: System Equilibrium

Constraint (4.3) identifies end-period wastage levels of each hospital. Constraint (4.4) is the capacity constraint of the blood center. Constraint (4.5) ensures that hospitals face roughly same amount of shortages if the blood center carries insufficient number of blood units on its inventory. When a higher fairness index (α) is selected, more shortage variations should be expected between hospitals. Constraints (4.6)-(4.8) captures the relationships between r and z variables. One of these variables takes a non-negative value. Thus, if the system face a shortage issue (r is positive), z is forced to be zero which also makes the blood inventory levels equal to zero. Otherwise (r is equal to zero), z takes a positive value and hospitals' blood inventory may increase up to the safety stock levels. Constraint (4.9) updates end-period blood

inventory levels. The blood center replenishes up to safety stock levels of the hospitals when there are adequate number of blood units. Constraint (4.10) states that there is no inventory available at the beginning of the analysis period. Constraint (4.11) identifies the number of units that are available at the blood center after hospitals' core demands are satisfied. Constraints (4.12)-(4.13) allocates these units to each hospital. Parameter TOL is a small number and its use is critical to obtain the correct value of $EXCS$ when the left hand-side value in Constraint (4.12) or the right hand-side value in Constraint (4.13) is an integer value. These constraints fairly distribute excessive units at the blood center to the hospitals. Constraint (4.14) shows that π_t and z_t are binary variables. Constraint (4.15) states that x_{kt}, v_{kt} and r_{kt} are non-negative discrete variables. Finally, Constraint (4.16)-(4.17) shows that EXC_{kt} and $EXCS_{kt}$ are unrestricted variables.

The optimization problem includes non-linear terms in above formulation. First, linearization technique in Appendix C is applied to Constraint (4.3) and modified constraints are obtained as follows:

$$u_{kt} \geq \theta_k v_{k(t-1)} + TOL \qquad \forall k, t \tag{4.18}$$

$$u_{kt} \leq \theta_k v_{k(t-1)} + 1 + TOL \qquad \forall k, t \tag{4.19}$$

Second, Constraint (4.9) indicates two types of non-linear terms: one is related to the minimum values of two terms and another one is related to the interaction of binary and discrete variables. After the fourth linearization technique is applied which is detailed in Appendix D, Constraint (4.9) is replaced with Constraints (4.20)-(4.25).

$$v_{kt} \leq EXCS_{kt} + z_t(v_{k(t-1)} - u_t) \qquad \forall k, t \tag{4.20}$$

$$v_{kt} \leq SS_k \qquad \forall k, t \tag{4.21}$$

$$v_{kt} \geq EXCS_{kt} + z_t(v_{k(t-1)} - u_t) - (U_{1kt} - Lmin_{kt})(1 - w_{1kt}) \qquad \forall k, t \qquad (4.22)$$

$$v_{kt} \geq SS_k - (U_{2kt} - Lmin_{kt})(1 - w_{2kt}) \qquad \forall k, t \qquad (4.23)$$

$$w_{1kt} + w_{2kt} = 1 \qquad \forall k, t \qquad (4.24)$$

$$w_{1kt}, w_{2kt} \in \mathbb{B}^{0,1} \qquad \forall k, t \qquad (4.25)$$

Finally, linearization techniques (Appendix A) are applied once more to the non-linear terms impacting Constraints (4.20) and (4.22) and resulting on modified Constraints (4.26) and (4.27). In addition, Constraints (A.19)-(A.21) and (A.22)-(A.27) are added into the new formulation.

$$v_{kt} \leq EXCS_{kt} + \lambda_{kt} - \mu_t \qquad \forall k, t \qquad (4.26)$$

$$v_{kt} \geq EXCS_{kt} + \lambda_{kt} - \mu_t - (U_{1kt} - Lmin_{kt})(1 - w_{1kt}) \qquad \forall k, t \qquad (4.27)$$

In summary, constraints (4.1)-(4.2), (4.4)-(4.8), (4.10)-(4.17), (4.18)-(4.19), (4.21), (4.23)-(4.25), (4.26), (4.27), (A.19)-(A.21) and (A.22)-(A.27) are present in the model discussed in this section.

4.1.1. Model Extension: Formulating Inventory Distribution Policies of Blood Center

The blood center experiences one of the three different cases when shipping blood units to the hospitals. The following assumptions on inventory distribution policies have been made to formulate our model. It is valuable to note that overall blood

demand refers to the summation of hospitals' blood demands considering their safety stock levels.

1. *Case 1*: When the blood center carries sufficient number of blood units to satisfy overall blood demands,

$$CAP_t \geq \sum_{k=1}^{K}(d_{kt} + SS_k - v_{k(t-1)} + u_t),$$

it replenishes up to safety stock levels of the hospitals.

2. *Case 2*: When the blood center carries excessive blood units after core blood demands are met but the overall blood demands are not able to be satisfied due to unavailability of blood units,

$$\sum_{k=1}^{K} d_{kt} \leq CAP_t \leq \sum_{k=1}^{K}(d_{kt} + SS_k - v_{k(t-1)} + u_t),$$

- (a) one policy is to distribute excessive units between hospitals roughly even after core demand is met.
- (b) another policy is to distribute the excessive units based on the safety stock levels of the hospitals after core demand is met.

3. *Case 3*: When the blood center faces blood shortages,

$$CAP_t \leq \sum_{k=1}^{K} d_{kt},$$

the available units:

- (a) are to be distributed among the hospitals based on hospitals' core demands.
- (b) are to be distributed between hospitals roughly even.

The model discussed incorporates the assumptions made in Case 1, Case 2 (a) and Case 3 (a). However, Case 2 (b) and Case 3 (b) can be incorporated replacing Constraints (4.12)-(4.13) and (4.5) by Constraints (4.28)-(4.29) and (4.30)-(4.31) respectively.

$$EXC_t(SS_k/\sum_{k=1}^{K} SS_k) - 1 + TOL \leq EXCS_{kt} \qquad \forall k,t \tag{4.28}$$

$$EXCS_{kt} \leq EXC_t(SS_k/\sum_{k=1}^{K} SS_k) + 1 + TOL \qquad \forall k,t \tag{4.29}$$

$$r_{kt} \geq \sum_{n=1}^{K} r_{nt}/K - 1 \qquad \forall k,t \tag{4.30}$$

$$r_{kt} \leq \sum_{n=1}^{K} r_{nt}/K + 1 \qquad \forall k,t \tag{4.31}$$

4.2. Computational Study

The data used to test the models and numerical results are presented in this section. One of the state-of-the-art solver, IBM ILOG CPLEX 12.1, in C++ platform is used on Dell OPTIPLEX 755 computer running with 2.20 GHz CPU and 2GB of RAM.

4.2.1. Data

The values of cost parameters summarized in Table 3.4 are used for the purchase and inventory costs (c = \$180 and h = \$1.25). However, we decided to change the shortage cost with a big number (b = \$100000) in order to avoid shortage issues if there are available units in blood center inventory. Depending on the hospital, safety stock levels vary from 8 to 12 units. Furthermore, 0.1 is selected for the value of θ so it is assummed that 10% of the hospitals' inventory are expired and discarded at the end of each time period.

Daily demand values are generated in Matlab 2010a using demand distributions and mean values provided in [10] from Sanquin Blood Bank (Table 4.4). The demand is assumed to be randomly distributed around the mean with gamma distribution.

Table 4.4: Mean Demand Values at Sanquin Blood Bank ([10])

Mon	Tue	Wed	Thu	Fri	Sat&Sun
24	16	32	16	24	8

Two groups of datasets were generated for numerical study and they are categorized based on the purpose they serve.

1. To analyze the integer programming model four groups of datasets with varies sizes were generated for a given time period ($t = 7$ days). There are three datasets in each group with 10 hospitals (K1-K3), 20 hospitals (K4-K6), 30 hospitals (K7-K9) and 40 hospitals (K10-K12) respectively.

2. To verify the model accuracy for different inventory distribution policies of blood center only one dataset (D1) with 3 hospitals was generated for a given time period ($t = 7$ days).

4.2.2. Numerical Results

The problem described by (4.1)-(4.17) was solved with datasets K1-K12. Given the daily capacities of blood center and the number of hospitals in the centralized hospital system, the time it takes CPLEX to solve 12 instances are summarized in Table 4.5.

The average solution times for different sizes of the centralized system (10, 20, 30 and 40 hospitals) are 0.461, 0.675, 1.22 and 2.435 seconds respectively. As the number of hospitals in the system increases, the problem size grows and it takes longer time to reach the optimal solution.

Tables 4.6-4.9 show the effects of daily blood center capacity on total cost, total shortage and the average daily blood inventory in overall system. Datasets that are selected for given sizes of the centralized system (10 through 40 hospitals) are K1, K4, K7 and K10 respectively.

Table 4.5: Solution Times of Datasets for Integer Programming Model

Capacity of Blood Center	Number of Hospitals	Dataset	Solution Time (seconds)
170	10	K1	0.394
		K2	0.602
		K3	0.388
340	20	K4	0.74
		K5	0.73
		K6	0.556
510	30	K7	1.563
		K8	1.2
		K9	1.889
680	40	K10	2.64
		K11	2.362
		K12	2.305

Table 4.6: Effect of Daily Blood Center Capacity on Model Outcomes-10 Hospitals in the System (K1)

Capacity of Blood Center	Total Cost ($)	Total Shortage (Units)	Average Daily Blood Inventory (Units/T)
0	124000000	1242	0
38	97639900	976	0
100	66404700	663	22.86
200	18481700	183	39.29
258	212960	0	68
300	218660	0	85.14
376	220900	0	100
400	220900	0	100
500	220900	0	100

Daily capacity of the blood center has a significant effect on the model outcomes and three critical capacity levels are observed during our experiments as can be seen in Tables 4.6-4.9. When the capacity is gradually increased up to these levels, similar effects were observed in all four of these hospital networks which consist of 10, 20, 30 and 40 hospitals (Table 4.10).

Table 4.7: Effect of Daily Blood Center Capacity on Model Outcomes-20 Hospitals in the System (K4)

Capacity of Blood Center	Total Cost ($)	Total Shortage (Units)	Average Daily Blood Inventory (Units/T)
0	238000000	2384	0
80	182000000	1824	0
100	172000000	1723	5.57
200	123000000	1223	44.86
300	72587100	723	56.43
400	26663900	263	79.14
496	414720	0	136.71
500	415430	0	140.43
600	423010	0	173.71
699	426800	0	200
700	426800	0	200

Table 4.8: Effect of Daily Blood Center Capacity on Model Outcomes-30 Hospitals in the System (K7)

Capacity of Blood Center	Total Cost ($)	Total Shortage (Units)	Average Daily Blood Inventory (Units/T)
0	351000000	3507	0
100	281000000	2807	0
115	271000000	2704	0
200	228000000	2277	32
300	178000000	1777	68.29
400	128000000	1277	82.57
500	82867500	824	9.43
600	46220500	457	102.43
700	7095570	65	183.14
751	616260	0	223.71
800	622420	0	245.29
900	624420	0	273.86
1000	626850	0	295.71
1078	629850	0	300
1100	629850	0	300
1200	629850	0	300

Table 4.9: Effect of Daily Blood Center Capacity on Model Outcomes-40 Hospitals in the System (K10)

Capacity of Blood Center	Total Cost ($)	Total Shortage (Units)	Average Daily Blood Inventory (Units/T)
0	483000000	4831	0
100	413000000	4131	0
150	358000000	3783	0
200	352000000	3519	12.57
300	302000000	3019	55
400	252000000	2519	89.86
500	202000000	2019	104.14
600	152000000	1519	112.71
700	109000000	1083	123.43
800	60625800	599	152
900	26686400	259	203.43
993	831890	0	254.86
1000	834510	0	260.14
1100	853080	0	326.14
1200	855240	0	357
1300	858360	0	384.43
1400	860410	0	396.57
1473	863050	0	400
1500	863050	0	400
1600	863050	0	400

Table 4.10: Critical Capacity Levels of Blood Center for Given Hospital Networks

Number of Hospitals	Capacity Level 1 (Units)	Capacity Level 2 (Units)	Capacity Level 3 (Units)
10	38	258	376
20	80	496	699
30	115	751	1078
40	150	993	1473

When more hospitals exist in the system, as anticipated, the associated cost, shortage and inventory levels are, in general, higher. By increasing the daily capacity from Level 1, shortage levels continue to decrease and hospitals' inventory start rising. An increase between Level 1 and Level 2 results in continued decrease on shortage and increased inventory levels, however, hospitals do not face any shortage issue beyond

Level 2. As a result of increasing units in hospitals' inventory, replenishing up to Level 3 increases total cost of the system. Further increase from this level does not have any effect on the system outcomes.

Finally, the model accuracy for different inventory distribution policies are tested using the formulations discussed in Section 4.1.1. Table 4.11 displays the daily demand levels of three hospitals (dataset D1) that are generated by Matlab 2010a. Incorporating these policies into model formulations daily inventory and shortage levels are listed for each hospital in Tables 4.12-4.15.

Table 4.11: Hospitals' Daily Demand Levels (Units) (D1)

Hospital	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	13	10	48	32	10	4	5
2	28	15	35	32	17	5	6
3	33	12	24	17	21	2	3

Table 4.12: Daily Inventory Levels of the Hospitals Given CAP of 89 Units and Inventory Distribution Policy of Case 2 (a) (Units)

Hospital	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	5	10	3	5	10	10	10
2	5	8	1	3	8	8	8
3	5	12	4	5	12	12	12

Table 4.13: Daily Inventory Levels of the Hospitals Given CAP of 89 Units and Inventory Distribution Policy of Case 2 (b) (Units)

Hospital	Safety Stock Levels	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	10	5	10	3	5	10	10	10
2	8	4	8	3	4	8	8	8
3	12	6	12	2	4	12	12	12

Table 4.14: Daily Shortage Levels of the Hospitals Given CAP of 50 Units and Inventory Distribution Policy of Case 3 (a) (Units)

Hospital	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	4	0	22	13	0	0	0
2	9	0	14	13	0	0	0
3	11	0	11	6	0	0	0

Table 4.15: Daily Shortage Levels of the Hospitals Given CAP of 50 Units and Inventory Distribution Policy of Case 3 (b) (Units)

Hospital	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	9	0	16	11	0	0	0
2	7	0	15	10	0	0	0
3	8	0	17	10	0	0	0

As can be noted in Tables 4.12 and 4.14, excessive units at the blood center are distributed roughly even or based on predetermined safety stock levels of the hospitals. In addition, when the blood center faces shortage issue, the units are distributed roughly even (Table 4.13) or based on demand levels of the hospitals (Table 4.15).

CHAPTER 5: BLOOD COLLECTION AT REMOTE LOCATIONS THROUGH BLOODMOBILES

Blood units are given to the blood center at either fixed or remote locations. According to quick facts related to blood donations that was published in Red Cross website [32], approximately 80% of the blood units are collected at remote bloodmobiles that are sent to community organizations, companies, high schools, colleges, places of worship or military installations. Thus, our focus in this section relates to the logistics associated to whole blood donations given at remote locations as shown in Figure 5.1.

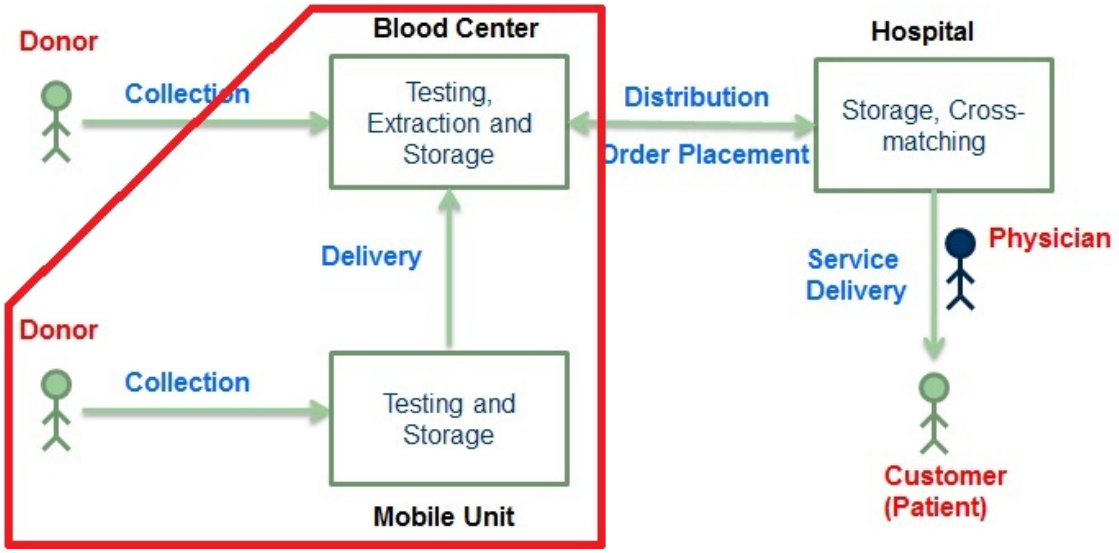

Figure 5.1: Blood Collection at Remote Locations through Bloodmobiles

The following assumptions have been made:

- The daily demand at the blood center is given.

- Each blood unit is tested for HIV, hepatits, etc. A portion of them (which is given) happen to carry at least one of these diseases and get thrown out after collection.

- Shortages are not allowed so the blood center has to collect sufficient number of blood units in order to satisfy the demand.

- Number of bloodmobiles serving to the blood center and their capacities are given.

- Each bloodmobile visits at most two or three locations per day.

- If a remote location is visited today, it can not be visited within the next couple of weeks.

5.1. Formulation of Bloodmobile Routing Problem

Every day, blood center sends bloodmobiles to remote locations (Figure 5.2) in order to collect blood units from donors. The goal is to minimize the daily distance travelled by bloodmobiles while satisfying the demand at the blood center. As in the traditional vehicle routing formulation, the integer programming approach proposed involves simultaneous decisions on the number of locations to be visited by each bloodmobile (general assignment problem) and the design of these routes (traveling salesman problem). However, unlike the traditional vehicle routing problem, it is not necessary for bloodmobiles to visit all donation that are available for blood collection. In addition, a bloodmobile can visit at most two or three different locations during the same day.

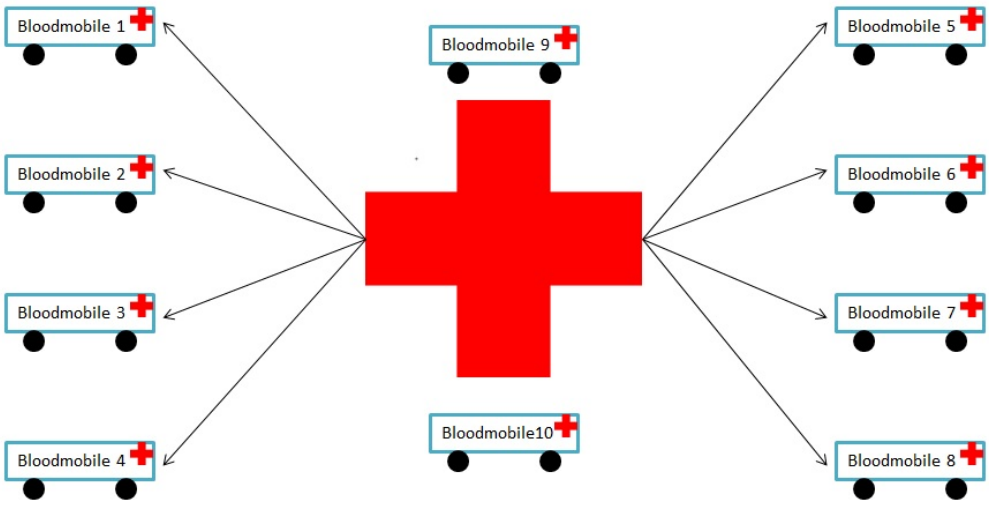

Figure 5.2: Blood Collection at Remote Locations

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ be a directed graph where $\mathcal{N}$ is a set of nodes covering initial destination (0), donation locations $(1, ...N)$ and final destination $(N+1)$. Even though initial and final destinations refer to the same location (blood center), for formulation purpose, they are indexed using different numbers. Furthermore, $\mathcal{A}$ indicates all arc pairs $(i,\ j)$ representing the travel from all i's to all j's. Finally, index k refers to bloodmobile.

The parameters that are used in our model are summarized in Tables 5.1.

Using the indices, parameters and decision variable outlined so far, the integer programming model is formulated as follows:

$$\text{Minimize} \sum_{k=1}^{K} \sum_{(i,j)\in\mathcal{A}} c_{ij} \cdot x_{ijk} \tag{5.1}$$

subject to:

$$\sum_{k=1}^{K} \sum_{j=1}^{N+1} x_{ijk} \leq 1 \qquad i = 1, ..., N \tag{5.2}$$

Table 5.1: Parameters for Model (5.1)-(5.10)

Index	Description
c_{ij}	Distance from location i to location j (miles)
d_i	The number of blood units that is collected at location i (units)
q_k	Capacity of bloodmobile k (units)
TD	Blood demand at blood center (units)
π	The number of blood units that need to be collected (units)
Inv	The number of blood units on inventory (units)
β	Percent of collected units that carry diseases and are thrown out
μ	Maximum number of visits allowed per bloodmobile
M	Big M (Big Number)

$$\sum_{i=1}^{N} d_i \sum_{j=1}^{N+1} x_{ijk} \leq q_k \qquad \forall k \tag{5.3}$$

$$\sum_{j=1}^{N+1} x_{0jk} = 1 \qquad \forall k \tag{5.4}$$

$$\sum_{i=0}^{N} x_{ihk} - \sum_{j=1}^{N+1} x_{hjk} = 0 \qquad \forall k, h = 1, ..., N \tag{5.5}$$

$$\sum_{i=0}^{N} x_{i,N+1,k} = 1 \qquad \forall k \tag{5.6}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{ijk} \leq \mu \qquad \forall k \tag{5.7}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{(i,j)\in\mathcal{A}} d_i x_{ijk} \geq \pi \tag{5.8}$$

$$\pi = (TD - Inv) \cdot (1 + \beta) \tag{5.9}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall k, (i, j) \in \mathcal{A} \tag{5.10}$$

The decision variable $x_{ijk} = 1$ if bloodmobile k travels from location i to loaction

j and 0 otherwise. The objective function (5.1) is to minimize the daily distance travelled by bloodmobiles. Constraint (5.2) ensures that each donation location is visited at most once. Constraint (5.3) is the capacity constraint stating that the number of blood units collected can not exceed the capacity of a bloodmobile. Constraints (5.4)-(5.6) are the arc flow constraints indicating that each bloodmobile must leave from the blood center; after a bloodmobile visits a donation location it has to leave for another destination; and finally, all vehicles must arrive at the blood center. Constraint (5.7) states that each bloodmobile visits at most two or three donation locations. Constraints (5.8)-(5.9) ensure that the number of blood units collected satisfies the demand at the blood center. Constraint (5.10) shows that x is a binary variable.

As can be noticed, the above formulation does not indicate any subtour elimination constraints. We assume the parameters of our problem obey triangle inequality, i.e. $c_{ii'} + c_{i'j} > c_{ij}$ for all $i, i', j \in \mathcal{N}$.

5.2. Solution Method

We implement branch & bound and column generation algorithms to solve bloodmobile routing problem and compare the quality of their solutions with the results obtained by CPLEX solver. Modified formulations are presented and the main components of these algorithms are described.

5.2.1. Branch and Bound Algorithm

Branch and bound algorithm is an approach to be used for many NP-hard problems including integer programming, traveling salesman and vehicle routing problems. The algorithm decomposes the original problem (P) into subsets ($P = P_1 \cup ...P_k$) as illustrated in Figure 5.3. Each node represents a subset of the original problem. Lower bound associated with a node is obtained by solving its linear programming (LP) relaxation. In addition, upper bound is obtained when the solution indicates

all integer values. All candidate solutions are implicitly enumerated and the subsets that are found to be fruitless are pruned using bounds. In this study, we apply three pruning techniques that are shown in Figures 5.4-5.6. The upper and lower bounds are placed at the top and bottom of each node respectively.

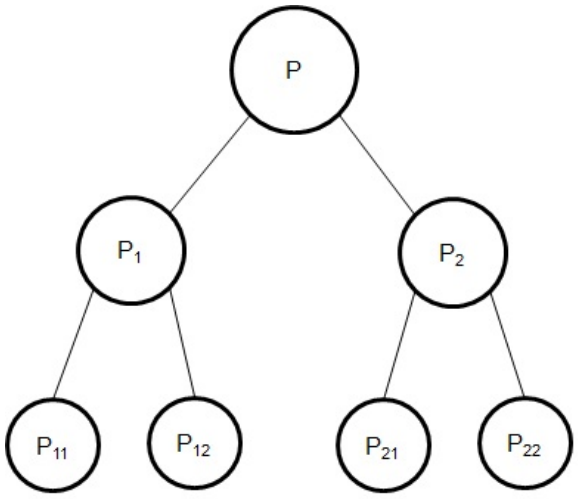

Figure 5.3: Enumeration Tree

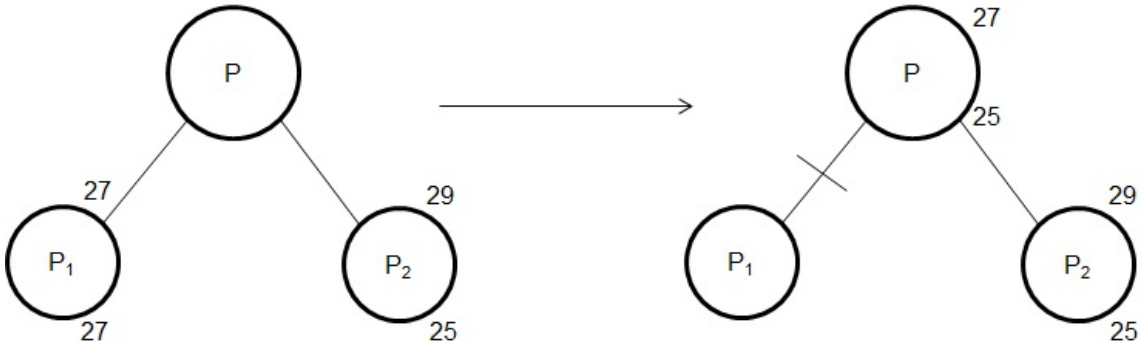

Figure 5.4: Pruning by Optimality

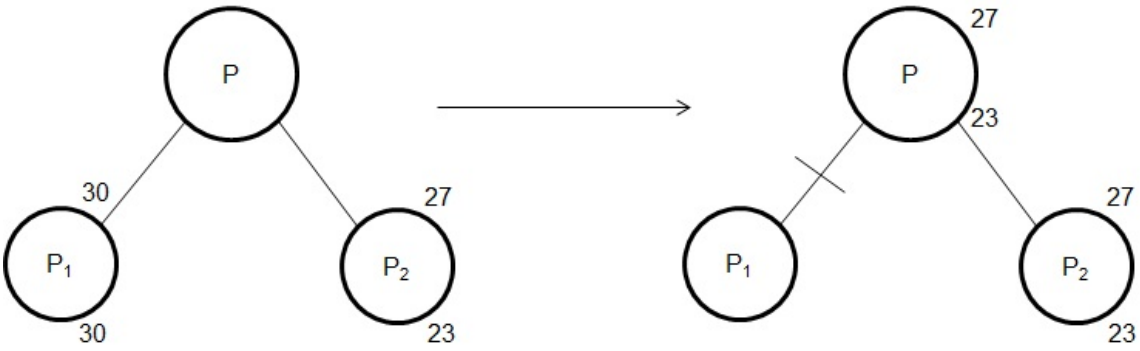

Figure 5.5: Pruning by Bound

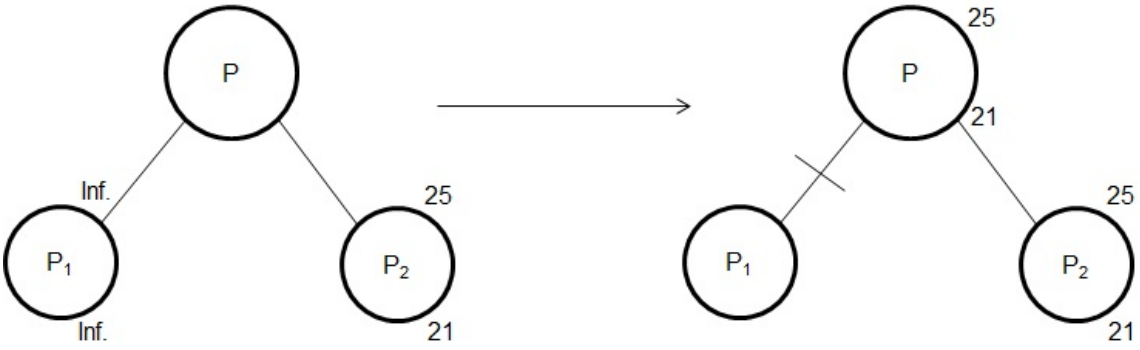

Figure 5.6: Pruning by Infeasibility

After an LP relaxation is solved, the solution usually indicates many fractional variables. One needs to decide which variable to branch on. Our algorithm branches on the variable with the most fractional value. Thus, a variable with the fraction closest to 1/2 is rounded up and down and then decomposed by adding cuts. For the complete branch and bound algorithm, please refer to Table 5.2.

Table 5.2: Branch and Bound Algorithm

```
Supply an initial feasible solution and update UB
DO{
    Solve the relaxed (LP) problem associated with the best node
    IF the solution is infeasible
        Prune by infeasibility
    IF the objective value is greater than UB
        Prune by bound
    IF there are fractional values in the solution
        Find the most fractional variable
        By rounding the fraction up and down, create two new branches (nodes)
    ELSE
        Prune by optimality
        Update UB
        Find the best node (with lowest LB)
}WHILE (There are nodes to branch)
```

5.2.2. Column Generation Algorithm

Column generation algorithm is one of the most widely used methods to solve vehicle routing problems. The appealing idea here is to generate only the variables which can improve the value of objective function. The original problem is split into the restricted master problem (RMP) and subproblem. The restricted master problem involves small subset of variables only. The subproblem is a new problem that is created to identify the variable with the most negative reduced cost.

The algorithm starts with solving an initial RMP to obtain dual prices. Using this information the subproblem is solved. If the objective function is negative, the variable with the most negative reduced cost is found and then added into RMP.

This process is repeated until the objective function value of the subproblem is non-negative. When a non-negative value is identified, we can conclude that the solution obtained from the last RMP is optimal.

The restricted master for bloodmobile routing problem is formulated in terms of column variables representing the set of bloodmobile routes that satisfy the arc flow and capacity constraints as well as the constraint related to the maximum number of visit that a bloodmobile can make. Let p be the index for routes already been generated by the subproblem and Ω be the set covering these routes. λ_k^p is a decision variable and represents the route p for bloodmobile k. x_{ijk}^p is a coefficient of λ_k^p and decodes the information for a given arc belonging to route p. Thus, the original problem can be reformulated and the restricted master problem is obtained as follows:

$$\text{Minimize} \sum_{k=1}^{K} \sum_{(i,j)\in\mathcal{A}} \sum_{p\in\Omega} c_{ij} x_{ijk}^p \lambda_k^p \tag{5.11}$$

subject to:

$$\sum_{k=1}^{K} \sum_{j=1}^{N+1} \sum_{p\in\Omega} x_{ijk}^p \lambda_k^p \leq 1 \qquad i = 1, ..., N \tag{5.12}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{(i,j)\in\mathcal{A}} d_i \sum_{p\in\Omega} x_{ijk}^p \lambda_k^p \geq \pi \tag{5.13}$$

$$\pi = (TD - Inv) \cdot (1 + \beta) \tag{5.14}$$

$$\sum_{p\in\Omega} \lambda_k^p = 1 \qquad \forall k \tag{5.15}$$

$$\lambda_k^p \geq 0 \qquad \forall p \in \Omega, \forall k \tag{5.16}$$

When the restricted master problem indicates all possible routes of the bloodmobiles and the integrality requirement of variable x is held, the objective function (5.11) and Constraints (5.12)-(5.14) are the equivalent formulations of (5.1) and (5.2), (5.8)-

(5.9) respectively. These constraints also link bloodmobiles together. In addition, Constraint (5.15) guarantees the convexity of λ. Dropping the integrality constraint and solving the linear programming relaxation of RMP, dual variables γ_i, η and α_k that are associated with Constraints (5.12)-(5.13) and Constraint (5.15) are obtained respectively.

$$\text{Minimize} \sum_{k=1}^{K}(\sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ijk} - \sum_{i=1}^{N}\sum_{j=1}^{N+1} x_{ijk}\cdot\gamma_i - \sum_{i=1}^{N}\sum_{j=1}^{N+1} d_i\cdot x_{ijk}\cdot\eta - \alpha_k) \qquad (5.17)$$

subject to:

$$\sum_{i=1}^{N}\sum_{j=1}^{N+1} d_i x_{ijk} \leq q_k \qquad \forall k \qquad (5.18)$$

$$\sum_{j=1}^{N+1} x_{0jk} = 1 \qquad \forall k \qquad (5.19)$$

$$\sum_{i=0}^{N} x_{ihk} - \sum_{j=1}^{N+1} x_{hjk} = 0 \qquad \forall k, h \qquad (5.20)$$

$$\sum_{i=0}^{N} x_{i,N+1,k} = 1 \qquad \forall k \qquad (5.21)$$

$$\sum_{(i,j)\in\mathcal{A}} x_{ijk} \leq \mu \forall k \qquad (5.22)$$

$$x_{ijk} \in \{0,1\} \qquad \forall k, (i,j) \in \mathcal{A}, \qquad (5.23)$$

Constraints (5.18)-(5.22) and (5.23) serve for the same purpose as Constraints (5.3)-(5.7) and (5.10) respectively. However, the objective function (5.17) is modified in order to identify the routes (one for each bloodmobile) that has the most negative reduced cost.

As the solution obtained by the linear programming relaxation indicates non-integer variables, we apply column generation within a branch and bound framework.

Thus, the following modifications need to be made after the problem is decomposed. Assume that we branch on x_{abc} and solve RMP with $x_{abc} = 0$. Constraint $(\sum_{p \in \Omega} x^p_{abc} \cdot \lambda^p_c = 0)$ should be added into the restricted master problem with the corresponding dual price Γ_{abc}. In addition, the subproblem should be modified by adding the term $(-\Gamma_{abc} \cdot x_{abc})$ in objective function and the Constraint $(x_{abc} = 0)$ in the constraint set.

For the embedded column generation within branch and bound algorithm, please refer to Table 5.3.

Table 5.3: Embedded Column Generation within Branch and Bound Algorithm

```
Supply an initial feasible solution and update UB
DO{
    DO{
        IF a new column is generated
            Add the column into RMP
        IF a new branching is done
            Add the associated branch constraint into RMP
        Solve RMP
        Obtain dual prices
        Modify the subproblem using dual prices & associated branch constraints
        Solve subproblem
    }WHILE(Objective function value of subproblem is negative)
    IF the solution is infeasible
        Prune by infeasibility
    IF the objective value is greater than UB
        Prune by bound
    IF there are fractional values in the solution
        Find the most fractional variable
        By rounding the fraction up and down, create two new branches (nodes)
    ELSE
        Prune by optimality
        Update UB
        Find the best node (with lowest LB)
}WHILE (There are nodes to branch)
```

5.3. Computational Study

The data and numerical results are presented in this section. All experiments are carried out on Dell OPTIPLEX 755 with 2.20 GHz CPU and 2GB of RAM. The solution time of the algorithms are reported using IBM ILOG CPLEX 12.1 solver on a C++ platform.

5.3.1. Data

OneBlood, Inc. website ([33]) is used to identify the donation locations of bloodmobiles from 05/06/2013 to 12/31/2013. 462 different locations are obtained in the City of Tampa during the 8-month period. Using these locations distance matrix indicating c_{ij}'s is computed by a macro created in Microsoft Excel. If a donation location is visited today, same location can not be visited next couple of weeks. Thus, each day, all these locations are not available for blood collection. Depending on the locations visited previously, the blood center have a different set of locations every day to consider for blood collection. We use Matlab 2010a to randomly generate donation locations from 462 locations and obtain five groups (N=20, 30, 40, 50 and 60) with each one having three instances.

According to [34], 12.6 million units of whole blood are annually collected in U.S. which consist of approximately 4% of the population. 80% of these donations are through bloodmobiles at remote locations ([32]). Thus, it can be noted that 3.2% of whole blood donations in U.S. are given at remote locations. In addition, OneBlood is the major blood center in Florida and we assume that all units donated in Tampa are collected by OneBlood. Using the U.S. census data in [35], the number of units collected by bloodmobiles in each zipcode (UB) is estimated with multiplying zipcode population by 3.2%. Furthermore, the number of visits made to each location during 8-month period by the bloodmobiles (VL) is identified in OneBlood website and UB is divided by VL to determine the number of whole blood units to be collected in each visit to a remote location (d_i).

The data for bloodmobile capacity ($q_k = 50$) and average thrown out rate of collected units ($\beta = 3\%$) are obtained from a local blood center. Finally, it is assumed that the blood center does not carry any blood inventory ($Inv = 0$) at the beginning of the collection period.

5.3.2. Numerical Results

Fifteen datasets generated by Matlab 2010a are used to test the model described by (5.1)-(5.10). Table 5.4 summarizes the solution times of Branch & Bound (BB) and Column Generation (CG) algorithms discussed in Tables 5.2-5.3 and compares them with CPLEX solver. It is assumed that the blood center aims to satisfy the daily blood demand of 100 units using three bloodmobiles with each one to visit at most three donation locations in a day.

Table 5.4: Solution Times

N	Instance	CPLEX Time (s)	BB Time (s)	CG Time (s)
20	1	0.116	1.134	2.363
	2	0.315	2.341	2.652
	3	0.266	4.476	213.976
30	1	0.275	2.583	146.01
	2	0.267	3.244	202.897
	3	0.432	39.309	929.745
40	1	0.413	5.804	130.665
	2	0.337	1.893	148.885
	3	1.24	25.562	329.69
50	1	2.642	74.808	127.963
	2	0.443	1.665	8.696
	3	0.833	69.581	23.21
60	1	1.105	2.291	543.307
	2	1.274	5.757	181.046
	3	2.329	1.782	77.95

As can be noted, the solution times are highly variable with the instances. In addition, the number of donation locations (N) that are available for blood collection does not have an apparent effect on the solution times. CPLEX solver reaches to

the optimal solution quicker than any other methods and Branch & Bound algorithm bids Column Generation algorithm in terms of solution time.

Tables 5.5-5.8 display the total distance travelled by bloodmobiles for given instances. 60 donation locations are considered for blood collection. The blood center aims to satisfy the whole blood demand ranging from 100 to 160 units. The results show the effects of model parameters on outcomes.

Table 5.5: Total Distance Travelled by Bloodmobiles (miles) - $TD = 100$

k	Instances	Distance Travelled ($\mu = 2$)	Distance Travelled ($\mu = 3$)
3	1	Infeasible	18.9
	2	Infeasible	32.4
	3	Infeasible	33.2
4	1	68.8	21.7
	2	100.9	40.2
	3	56.4	40.4
5	1	58.6	25.5
	2	85.5	49.4
	3	64.1	49.2
6	1	51.7	30
	2	86	58.8
	3	69.3	59.3
7	1	53.1	34.5
	2	72.7	68.4
	3	78.3	69.4

Finally, the following observations are made based on the testing results obtained from Tables 5.5-5.8.

- In order to satisfy increasing blood demand, bloodmobiles travel longer distances.
- When the demand rates increase, the blood center is not able to satisfy the demand with a few number of bloodmobiles. Thus, more infeasible solutions are observed.

Table 5.6: Total Distance Travelled by Bloodmobiles (miles) - $TD = 120$

k	Instances	Distance Travelled ($\mu = 2$)	Distance Travelled ($\mu = 3$)
3	1	Infeasible	24.1
	2	Infeasible	38.6
	3	Infeasible	37.2
4	1	Infeasible	23.3
	2	Infeasible	42.2
	3	76	42.6
5	1	89.9	27
	2	139.2	51.4
	3	77	51.2
6	1	79.4	31.4
	2	114.6	60.3
	3	77.8	60.8
7	1	75.3	35.9
	2	101.3	69.5
	3	86.1	70.8

Table 5.7: Total Distance Travelled by Bloodmobiles (miles) - $TD = 140$

k	Instances	Distance Travelled ($\mu = 2$)	Distance Travelled ($\mu = 3$)
3	1	Infeasible	73
	2	Infeasible	49.4
	3	Infeasible	42.6
4	1	Infeasible	25
	2	Infeasible	44.8
	3	Infeasible	46.1
5	1	Infeasible	28.4
	2	200.1	52.6
	3	96.6	52.2
6	1	120.1	32.6
	2	154.5	61.8
	3	95.7	61.7
7	1	108.4	36.9
	2	129.9	71.2
	3	98.4	71.8

Table 5.8: Total Distance Travelled by Bloodmobiles (miles) - $TD = 160$

k	Instances	Distance Travelled ($\mu = 2$)	Distance Travelled ($\mu = 3$)
3	1	Infeasible	Infeasible
	2	Infeasible	Infeasible
	3	Infeasible	Infeasible
4	1	Infeasible	37
	2	Infeasible	52.2
	3	Infeasible	50.1
5	1	Infeasible	30.1
	2	Infeasible	54.4
	3	Infeasible	55.3
6	1	Infeasible	34
	2	228.5	62.7
	3	115.3	63.7
7	1	141.7	38.4
	2	185.4	71.9
	3	116.3	73

- It is always the efficient way for bloodmobiles to visit at most three locations in a day. However, the optimal number of bloodmobiles needed by the blood center change based on the daily demand rate as shown in Table 5.9.

Table 5.9: The Optimal Number of Bloodmobiles

Demand (units)	100	120	140	160
k (units)	3 (in general)	3	4	5

CHAPTER 6: CONCLUDING REMARKS AND OPPORTUNITIES FOR FUTURE WORK

In the first part of this study, we develop stochastic and deterministic integer programming models to improve the efficiency of blood related operations at a hospital. The focus of the study is on red blood cells and platelets as they have short lifetimes and are the most scarce products among the whole blood components. The models explicitly accounts for the age of blood units on inventory and considers the demand for two types of patients, uncertain demand rates and crossmatch-to-transfusion ratio. These models could be used to identify the optimal order levels that improve the wastage levels at the hospital. For validation purpose, we crosschecked the first model with real-world data in [3]. In that study, the wastage rate during January through April 2006 at Stanford University Medical Center (SUMC) were given as 19.9% based on an age distribution of 22.5%, 30.1%, 28.3% and 19.1% for 2, 3, 4 and 5 days old platelet units respectively. Our model reflects a reduction in the wastage rates. Specifically, average of wastage dropped from 19.9% to 2.57% as shown in Table 6.1.

Table 6.1: Wastage Rates Using the Model Described by (3.1)-(3.18)

Average Age	Instance	Expected Wastage Rate (%)
3.5	E1	2.47
	E2	3.46
	E3	2.59
	E4	2.11
	E5	2.21
		avg.: 2.57

As most quantitative models addressing an applied problem, the models presented in first part of this study have several opportunities for expansion. First, some hospitals use double-crossmacthing policy where same unit of blood is crossmatched for more than one patient. We only consider single crossmatching policy. It is anticipated that incorporating a double crossmatch policy will further improve results; that is, decrease wastage. Second, C/T ratio is incorporated into the model using hospitals average value. It would be valuable to explore impact on the model when C/T requirements for specific patient-groups (or procedures) are incorporated.

In the second part of this study, we analyze the centralized hospital network where the blood center has access to the information related to hospitals' blood inventory and makes decision on behalf of the system to minimize the total system cost and to reduce the total shortages. A non-linear integer programming model was developed that considers different inventory distribution policies of the blood center. It was noted that daily capacity of the blood center has significant effects on total cost and total shortages. In addition, three critical capacity levels were observed. In all testing results, model outcomes showed the same trends when the blood center capacity approaches to these levels.

The models presented under centralized hospital network consider only one supplier where there is no other option available to receive blood units from. The models can be extended to incorporate an outsourcing option which can be used as a blood source in case of a shortage issue faced by the hospital network. Also, in real-life practices, transshipment alternative is sometimes used to ship blood units between the hospitals if excessive units at a hospital are urgently needed by another hospital. Incorporating this fact will further close the gap between literature and real-life practices.

The model presented in the last part of this study formulates the bloodmobile routing problem. The focus is to minimize the total distance travelled by the blood-

mobiles while satisfying the daily blood demand at the blood center. An integer programming approach was used to model the problem and to perform sensitivity analysis of the parameters on model outcomes. The time to reach the optimal solution by CPLEX solver outperforms branch & and bound and column generation algorithms. Given the daily demand rates ranging from 100 to 160 units, the optimal number of bloodmobiles needed changes from 3 to 5.

In the formulation of bloodmobile routing problem, we consider only one type of bloodmobiles. However, this study can be extended to employ multiple bloodmobile types with different capacities. In addition, instead of a fixed number of blood units to be collected at a remote location, it will be interesting to develop a model that considers the variability in blood collection. Finally, incorporating the number of units that is changing with the waiting time of bloodmobile at a donation location will be interesting to explore.

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APPENDICES

Appendix A First Linearization Technique: Interaction between Binary and Continuous Variables

The first linearization technique is focused on the interactions between binary and discrete variables and assigns new discrete variables to replace the products of interacting variables.

As in [31], y is called the linearization variable and reflects the products of x and d in the linearization process where x is a discrete variable and d is a binary variable. Lower bound and upper bound of x are assumed to be known and take the values of L and U respectively. Then, integer programming formulation after linearization process is as follows:

$$Ld \leq y \leq Ud$$

$$L(1-d) \leq (x-y) \leq U(1-d)$$

To linearize our models, linearization technique mentioned above is applied to the non-linear terms and the interactions between binary variable z and discrete variables v, y, m and u in the original formulation are replaced with their products (called linearization variables) as shown below. Furthermore, the following constraints (that are numbered) are added into the new formulation.

$$z_{it}^{(s)} v_{(i-1)(t-1)}^{(s)} = \gamma_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t$$

$$\gamma_{it}^{(s)} \leq z_{it}^{(s)} \cdot M \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.1}$$

$$\gamma_{it}^{(s)} \leq v_{(i-1)(t-1)}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.2}$$

$$\gamma_{it}^{(s)} \geq M \cdot (z_{it}^{(s)} - 1) + v_{(i-1)(t-1)}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.3}$$

Appendix A (Continued)

$$z_{it}^{(s)} y_{it} = \alpha_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t$$

$$\alpha_{it}^{(s)} \leq z_{it}^{(s)} \cdot M \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.4}$$

$$\alpha_{it}^{(s)} \leq y_{it} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.5}$$

$$\alpha_{it}^{(s)} \geq M \cdot (z_{it}^{(s)} - 1) + y_{it} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.6}$$

$$z_{it}^{(s)} m_{it}^{(s)} = \lambda_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t$$

$$\lambda_{it}^{(s)} \leq z_{it}^{(s)} \cdot M \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.7}$$

$$\lambda_{it}^{(s)} \leq m_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.8}$$

$$\lambda_{it}^{(s)} \geq M \cdot (z_{it}^{(s)} - 1) + m_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.9}$$

$$z_{(i-1)t}^{(s)} m_{it}^{(s)} = \delta_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t$$

$$\delta_{it}^{(s)} \leq z_{(i-1)t}^{(s)} \cdot M \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.10}$$

$$\delta_{it}^{(s)} \leq m_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.11}$$

$$\delta_{it}^{(s)} \geq M \cdot (z_{(i-1)t}^{(s)} - 1) + m_{it}^{(s)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.12}$$

Appendix A (Continued)

$$z^{(s)}_{(i-1)t} y_{it} = \psi^{(s)}_{it} \qquad i = 3, 4, ..., I, \forall(s), t$$

$$\psi^{(s)}_{it} \leq z^{(s)}_{(i-1)t} \cdot M \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.13}$$

$$\psi^{(s)}_{it} \leq y_{it} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.14}$$

$$\psi^{(s)}_{it} \geq M \cdot (z^{(s)}_{(i-1)t} - 1) + y_{it} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.15}$$

$$z^{(s)}_{(i-1)t} v^{(s)}_{(i-1)(t-1)} = \mu^{(s)}_{(i-1)t} \qquad i = 3, 4, ..., I, \forall(s), t$$

$$\mu^{(s)}_{(i-1)t} \leq z^{(s)}_{(i-1)t} \cdot M \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.16}$$

$$\mu^{(s)}_{(i-1)t} \leq v^{(s)}_{(i-1)(t-1)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.17}$$

$$\mu^{s}_{(i-1)t} \geq M \cdot (z^{s}_{(i-1)t} - 1) + v^{s}_{(i-1)(t-1)} \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.18}$$

$$v_{k(t-1)} z_t = \lambda_{kt} \qquad \forall k, t$$

$$\lambda_{kt} \leq z_t \cdot M \qquad \forall k, t \tag{A.19}$$

$$\lambda_{kt} \leq v_{k(t-1)} \qquad \forall k, t \tag{A.20}$$

$$\lambda_{kt} \geq M \cdot (z_t - 1) + v_{k(t-1)} \qquad \forall k, t \tag{A.21}$$

Appendix A (Continued)

$$u_t z_t = \mu_{kt} \qquad \forall k, t$$

$$\mu_{kt} \leq z_t \cdot M \qquad \forall k, t \tag{A.22}$$

$$\mu_{kt} \leq u_t \qquad \forall k, t \tag{A.23}$$

$$\mu_{kt} \geq M \cdot (z_t - 1) + u_t \qquad \forall k, t \tag{A.24}$$

$$\gamma_{it}^{(s)}, \alpha_{it}^{(s)}, \psi_{it}^{(s)}, \mu_{(i-1)t}^{(s)} \in \mathbb{Z}^+ \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.25}$$

$$\lambda_{it}^{(s)}, \delta_{it}^{(s)} \in \mathbb{Z}^+ \qquad i = 3, 4, ..., I, \forall(s), t \tag{A.26}$$

$$\lambda_{kt}, \mu_{kt} \in \mathbb{Z}^+ \qquad \forall k, t \tag{A.27}$$

As both stochastic and deterministic models indicate nonlinear terms with the same interacting variables (for example, v and z), in order to save some space, we show their linearization in one formulation and differentiate them using (s) at the top corner of the variables.

Appendix B Second Linearization Technique: Floor Function

The second linearization technique focuses on the floor function ($x = \lfloor y \rfloor$) and used to determine the number of blood units returned to unassigned inventory.

$$x \geq y - 1 + TOL$$

$$x \leq y + TOL$$

$$x \in \mathbb{Z}^{+}$$

$$y \geq 0$$

When the second linearization technique is applied, the following set of constraints are obtained and used to replace Constraint (3.51).

$$\beta_{it} \geq \left((v_{(i-CRP-1)(t-CRP-1)} - y_{(i-CRP)(t-CRP)}) \cdot z_{(i-CRP)(t-CRP)} - m_{(i-CRP)(t-CRP)} \right)$$

$$\cdot (1 - CT^{-1}) - 1 + TOL \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T$$

$$\beta_{it} \leq \left((v_{(i-CRP-1)(t-CRP-1)} - y_{(i-CRP)(t-CRP)}) \cdot z_{(i-CRP)(t-CRP)} - m_{(i-CRP)(t-CRP)} \right)$$

$$\cdot (1 - CT^{-1}) + TOL \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T$$

Appendix B (Continued)

Parameter TOL is a small number and critical in the linearization process. Without indicating this parameter, β may take an incorrect value when the inner term (inside the floor function) of the right hand-side value in in this constraint is an integer value. Even after the second linearization technique is applied, the resulting constraints still indicates non-linear terms due to the interactions between binary and discrete variables. When the first linearization technique is applied, the following constraints are obtained as replacement for Constraint (3.51).

$$\beta_{it} \geq ((v_{(i-CRP-1)(t-CRP-1)} - y_{(i-CRP)(t-CRP)}) \cdot z_{(i-CRP)(t-CRP)} - m_{(i-CRP)(t-CRP)}).$$

$$(1 - CT^{-1}) - 1 + TOL \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T$$

$$\beta_{it} \leq ((\gamma_{(i-CRP-1)(t-CRP-1)} - \alpha_{(i-CRP)(t-CRP)}) - m_{(i-CRP)(t-CRP)}).$$

$$(1 - CT^{-1}) + TOL \qquad i = 3 + CRP, ..., I + CRP, t = CRP + 1, ..., T$$

Appendix C Third Linearization Technique: Ceil Function

The third linearization technique focuses on the ceil function ($x = \lceil y \rceil$) and used to determine the number of blood units wasted.

$$x \geq y + TOL$$

$$x \leq y + 1 + TOL$$

$$x \in \mathbb{Z}^{+}$$

$$y \geq 0$$

Applying this linearization technique, the following set of constraints are obtained and used to replace Constraint (4.3).

$$u_{kt} \geq \theta_k v_{k(t-1)} + TOL \qquad \forall k, t$$

$$u_{kt} \leq \theta_k v_{k(t-1)} + 1 + TOL \qquad \forall k, t$$

Parameter TOL is a small number and critical in the linearization process. Without indicating this parameter, u may take an incorrect value when the right hand-side value of this constraint is an integer value.

Appendix D Fourth Linearization Technique: Minimum Value of Two Variables

The fourth linearization technique is to identify the minimum values of two variables. As in [31], y is equal to min of x_1 and x_2 where both x_1 and x_2 are continuous variables. Lower bounds and upper bounds of x_1 and x_2 are L_1, U_1 and L_2, U_2 respectively. Introducing binary variables d_1 and d_2 where d_1 is 1 if x_1 is the minimum value and d_2 is 1 if x_2 is the minimum value, linearization of $min\{x_1, x_2\}$ is as follows:

$$y \leq x_1$$

$$y \leq x_2$$

$$y \geq x_1 - (U_1 - L_{min})(1 - d_1)$$

$$y \geq x_2 - (U_2 - L_{min})(1 - d_2)$$

$$d_1 + d_2 = 1$$

To linearize our model, linearization technique mentioned above is applied and the following constraints replaces Constraint (4.9)

$$v_{kt} \leq EXCS_{kt} + z_t(v_{k(t-1)} - u_t) \qquad \forall k, t$$

$$v_{kt} \leq SS_k \qquad \forall k, t$$

$$v_{kt} \geq EXCS_{kt} + z_t(v_{k(t-1)} - u_t) - (U_{1kt} - Lmin_{kt})(1 - w_{1kt}) \qquad \forall k, t$$

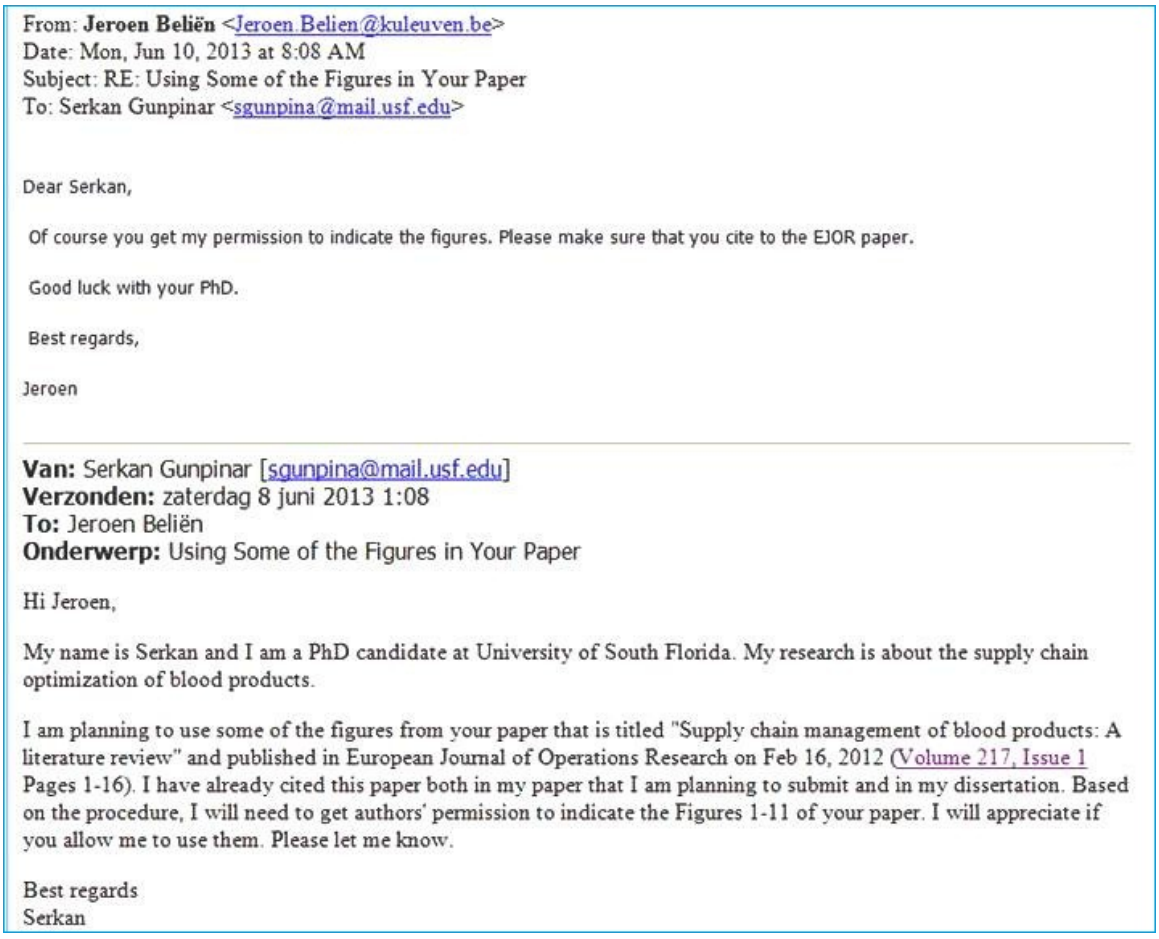
Appendix D (Continued)

$$v_{kt} \geq SS_k - (U_{2kt} - Lmin_{kt})(1 - w_{2kt}) \qquad \forall k, t$$

$$w_{1kt} + w_{2kt} = 1 \qquad \forall k, t$$

$$w_{1kt}, w_{2kt} \in \mathbb{B}^{0,1} \qquad \forall k, t$$

Appendix E Permission for Use of Figures 2.1-2.3



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Date: Mon, Jun 10, 2013 at 8:08 AM
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Of course you get my permission to indicate the figures. Please make sure that you cite to the EJOR paper.

Good luck with your PhD.

Best regards,

Jeroen

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Verzonden: zaterdag 8 juni 2013 1:08
To: Jeroen Beliën
Onderwerp: Using Some of the Figures in Your Paper

Hi Jeroen,

My name is Serkan and I am a PhD candidate at University of South Florida. My research is about the supply chain optimization of blood products.

I am planning to use some of the figures from your paper that is titled "Supply chain management of blood products: A literature review" and published in European Journal of Operations Research on Feb 16, 2012 (Volume 217, Issue 1 Pages 1-16). I have already cited this paper both in my paper that I am planning to submit and in my dissertation. Based on the procedure, I will need to get authors' permission to indicate the Figures 1-11 of your paper. I will appreciate if you allow me to use them. Please let me know.

Best regards
Serkan

ABOUT THE AUTHOR

Serkan Gunpinar earned his B.S. in Textile Engineering from Uludag University, Turkey in 2002 and his M.B.A. in Finance from Auburn University at Montgomery in 2005. He is currently pursuing his Ph.D. in Industrial and Management Systems Engineering at the University of South Florida and expecting to obtain both his second Masters and Ph.D. in August of 2013. His research interest includes analysis and modeling of supply chain systems. He has over 5 years of professional experience in industry and had a chance to work in the supply chain department of the world's largest apparel company, VF Corporation. In addition, he has been involved in a Florida Department of Transportation project with Center for Urban Transportation Research.