January 2012

Statistical Content in Middle Grades Mathematics Textbooks

Maria Consuelo (suzie) Capiral Pickle
University of South Florida, spickle777@yahoo.com

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by

Maria Consuelo (Suzie) Capiral Pickle

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Secondary Education
College of Education
University of South Florida

Major Professor: Denisse R. Thompson, Ph. D.
Richard Austin, Ph. D.
Gladis Kersaint, Ph. D.
Jeffrey Kromrey, Ph. D.

Date of Approval:
April 5, 2012

Keywords: statistical content, middle school, mathematics textbooks, content analysis

Dedication

This dissertation is dedicated to my parents and grandparents, who instilled in me the value of an education, a love of knowledge and a can do attitude. To my husband, Robert, whose support, love and understanding enabled me to finish this doctorate.

Acknowledgements

I want to thank the following people for all the assistance and encouragement they gave me during my program of study and throughout the writing of this dissertation. Dr. Denisse R. Thompson, my major professor, for the support, caring and considerable time spent helping me polish my dissertation. Dr. Rick Austin, Dr. Gladis Kersaint, and Dr. Jeffrey Kromrey, my committee members, for their encouragement and critiquing of my manuscript.

I also want to thank the doctoral students who I have met during the course of my study, especially Barbara Zorin and Matt Kellogg, who have provided me with enduring support; and to Sarah Bleiler who generously shared her time. I also want to thank my friends and coworkers at St. Petersburg College for their support. Again, I offer my most heartfelt and sincere thanks.

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Abstract

This study analyzed the treatment and scope of statistical concepts in four, widely-used, contemporary, middle grades mathematics textbook series: *Glencoe Math Connects*, *Prentice Hall Mathematics*, *Connected Mathematics Project*, and *University of Chicago School Mathematics Project*. There were three phases for the data analysis. Phase 1 addressed the location and sequence of the statistical concepts. Phase 2 focused upon an examination of the lesson narrative, its components and scope. Phase 3 analyzed the level of cognitive demand required of the students to complete the exercises, and the total number of exercises per statistical concept. These three phases taken together provided insight into students' potential *opportunity to learn* statistical topics found in middle grades mathematics textbooks.

Results showed that concepts, such as measures of central tendency, were repeated in several grades while other topics such as circle graphs were presented earlier than the recommendations in documents such as the National Council of Teachers of Mathematics Principles and Standards (2000) and the Common Core State Standards (2010). Further results showed that most of the statistical content was found in a chapter near the end of the book that would likely not be covered should time run short. Also, each textbook had a particular lesson narrative style. Moreover, most of the statistical

exercises required low level cognitive demand of the students to complete the exercises, potentially hindering the development of deep understanding of concepts.

Chapter 1: Introduction and Background

Our world has become more complex over the last quarter century so that the old curriculum of the three "R"s of reading, writing and arithmetic is no longer sufficient for students to thrive in today's society. Since the late 1970s and early 1980s, numerous organizations have espoused the need for statistics to play a larger role in the curriculum. In 1975, the Conference Board of the Mathematical Sciences National Advisory Committee on Mathematical Education (NACOME) recommended that statistical concepts be given more prominence in K-12 mathematics programs; this recommendation contrasted with prior stances that statistical content was primarily for the college curriculum. Recognizing the need to include statistics in the K-12 mathematics agenda, various organizations have continued to recommend changes in the curriculum. In 1989, the National Council of Teachers of Mathematics (NCTM) created the *Curriculum and Evaluation Standards for School Mathematics* as a guide upon which schools could build their mathematical curricula, and data analysis was identified as one of the content strands to be included. NCTM expanded this guide in 2000 with the *Principles and Standards for School Mathematics* and further refined the K-8 guide in 2006 in the *Curriculum Focal Points for Pre-kindergarten through Grade 8 Mathematics: A Quest for Coherence.* In this most recent document, NCTM recommended instructional emphasis be placed upon statistical topics in the middle grades to better prepare students for higher level mathematics.

In addition to the recommendations from mathematics education organizations (NACOME, 1975; NCTM, 1989, 2000, 2006), the American Statistical Association developed *Guidelines for Assessment and Instruction in Statistics Education* (*GAISE*) as a guide for the assessment and instruction of statistical topics in the K-12 curriculum. The GAISE document states, "Every high-school graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy, happy, and productive life" (Franklin et al., 2005, p. 1). The recommendations in GAISE reiterate the importance of statistics voiced in the early 1990's: "There is perhaps no other branch of the mathematical sciences that is as important for *all* students, college bound or not, as probability and statistics" (Shaughnessy, 1992, p. 465).

In 2010, the Council of Chief State School Officers supported the development of the Common Core State Standards to provide a more uniform set of guidelines across the country. As of October 2011, all but six states had adopted the Common Core State Standards (www.corestandards.org/in-the-state), to replace their state standards. The statistical concepts recommended by the Common Core State Standards for the middle grades are essentially the same as those recommended by the NCTM *Standards*, but perhaps with differences in the grade placement of topics.

Statement of Problem and Rationale

Textbooks are an integral part of the curriculum intended for the classroom (Grouws & Smith, 2000; Robitaille & Travers 1992; Tyson-Berstein & Woodward 1991; Weiss, Banilower, McMahon, & Smith, 2001). In fact, textbooks are the primary tools of instruction for most teachers, with many teachers reporting daily use of their textbooks

(Grouws & Smith, 2000; Weiss, Banilower, McMahon, & Smith, 2001). Teachers frequently assign the problems found in the textbooks to their students and rarely introduce outside sources besides the textbook into the classroom (Braswell et al., 2001). Moreover, the farther a topic is from the front of the textbook, the less likely it will be covered should instructional time run short (Grouws & Smith, 2000; Robitaille & Travers, 1992). This suggests that if a concept is not found in the textbook, students do not have an *opportunity to learn* the concept and the most important factor in student success is the *opportunity to learn* (Heibert & Grouws, 2007). In addition, research has shown that students' learning is enhanced when they are engaged in higher level cognitive demand tasks, such as finding patterns or trends, explaining mathematical ideas, or justifying strategies used in constructing conclusions (Henningsen & Stein, 1997). Thus the two factors, namely *opportunity to learn* in conjunction with higher level cognitive demand tasks, are the basic foundation to foster the best learning environment for students.

In light of the emphasis for the inclusion of statistical content into the curriculum, it is important to determine whether or not the statistical content that students are expected to know is present in the textbook. Even though recommendations for inclusion of statistics in the K-12 curriculum were made as early as the 1970's, by the early 1990's little implementation was evident. The National Research Council (2004) reported that no studies were found in the early 2000s that examined statistical content in the K-12 mathematics curriculum. To determine if such studies were conducted later, I reviewed the literature for studies published after 2004. I found 15 studies that examined curricula, but again, none were found that examined statistical content in middle grades mathematics textbooks.

The American Statistical Association (GAISE, 2005), the Council of Chief State School Officers (2010), and NCTM (2000) all agree that statistics should be a part of the curriculum to help prepare students to function in the modern world. But many students and adults demonstrate misconceptions about statistical information. Much of the exhibited statistical reasoning is "inappropriate reasoning (and) is … widespread and persistent…" (Garfield & Ahlgren, 1988, p. 58). Because the textbook is an important factor impacting student learning (Begle, 1973; Grouws et al., 2004; Valverde et al., 2002) and represents a key element that can readily be manipulated, an examination of the statistical content found in contemporary mathematics textbooks in use can provide valuable insight into the potential for statistical content to be incorporated into mathematics instruction across the nation.

The Purpose of Study

The primary goal of this study was to determine the nature of the treatment of statistical concepts as exhibited in contemporary middle grades mathematics textbooks. The treatment of the statistical concepts included the scrutiny of the location of the statistical topics, the scope and sequence of the statistical lessons, and the cognitive demand of the corresponding statistical exercises.

Research Questions and Hypothesized Findings

Research Questions. The study investigated the following three questions.

1. What is the nature of the statistical content of the curriculum as exhibited in contemporary student mathematics textbooks in use in the middle grades

(grades 6, 7, and 8) in the United States? How much of the textbooks are devoted to statistical topics? Where are these topics located? Which statistical concepts are individual lessons and which are embedded with other non-statistical topics?

2. What are the sequences of the statistical lesson narratives in contemporary student mathematics textbooks in use in the middle grades (grades 6, 7, and 8) in the United States? What is the typical sequence of lesson narratives for each textbook series? What are the differences and similarities of these lesson narratives across the textbook series?
3. What is the nature of the exercises on statistics in contemporary student mathematics textbooks in use in the middle grades (grades 6, 7, and 8) in the United States? What is the level of cognitive demand exhibited by these exercises? How does the level of cognitive demand vary among the different textbook series, and across the grade levels?

Hypothesized Findings. In preparation for this research study, a pilot study analyzing two older versions of one grade from two different textbooks series was conducted (See Appendix A). The 1st edition of *Connected Mathematics* for grade 6 (2004) represents a National Science Foundation funded curriculum developed in response to the NCTM Standards; this curriculum is student centered and includes more applications and investigations than a typical commercial curriculum. The *Mathematics Applications and Concepts: Course 3* (2004) represents a commercial or publisher developed curriculum, with a typical textbook style lesson with examples followed by practice problems. The results from the pilot study helped me formulate the following hypotheses for the results of the actual study:

1. Less than 10% of each textbook will be devoted to statistical concepts and the statistical concepts will be confined to a single chapter. Furthermore most of the statistical content will be in the last quarter of the textbook.
2. The lesson narrative for the commercial publisher developed textbooks will be composed mainly of definitions, examples, and then exercises. In contrast, the curricula written in response to reform efforts will contain lesson narratives that exhibit a more conceptual approach to the statistical material with fewer examples and more application lessons or problems.
3. More low-level cognitive demand exercises will exist in the commercial textbook series than in the reform textbooks which will be more problem centered. Furthermore, higher level cognitive demand exercises will be in the higher grades of each textbook series to support more complex statistical concepts.

Significance of Study

Research has shown that the typical classroom in the United States is textbook driven (Grouws & Smith, 2000; Robitaille & Travers, 1992). Thus the lessons and exercises found within mathematical textbooks are what the majority of the students will have an opportunity to study. An examination of the extent to which statistical concepts are present in the mathematical textbooks will bring to light whether the statistical concepts espoused by various educational documents (e.g., GAISE 2005; NCTM Standards 2000; Common Core State Standards, 2010) are present. Also, the analysis of the treatment of the statistical content will help determine if the lessons and exercises encourage higher levels of cognitive demand to facilitate student understanding (Stein et al., 2000).

Jones (2004) analyzed the probability content of middle grades mathematics textbooks over a fifty year time span. The current study is similar to Jones (2004) but analyzes the treatment of the statistical content of contemporary middle grades mathematics textbooks. Jones found the majority of probability content to be present in textbooks from the most recent era (1990s-2000) as topics in probability and statistics are more recent additions to the K-12 curriculum (NACOME, 1974). Thus, contemporary textbooks were analyzed.

For the modern era, Jones examined two widely used middle grades mathematics textbook series. The first was the "Standards Based" (NSF funded) first edition of *Connected Mathematics.* The other was *Mathematics: Applications and Connections,* a widely used publisher developed or commercial series textbook. Since Jones' study the publisher of *Mathematics: Applications and Connections* has been absorbed into another publishing company and this version of the textbook has changed.

In my study, two widely used commercial textbook series were analyzed along with the second edition of the *Connected Mathematics* textbook series and textbooks developed by the University of Chicago School Mathematics Project (UCSMP) to represent different educational philosophies and foci. Jones analyzed the mathematical tasks and cognitive levels required to complete each probability exercises. Although Jones examined the lesson components of each probability lesson for each textbook, he did not analyze if the presentation of the same concept changed in depth across the grade levels nor did he analyze the textbook series to determine any lesson narrative patterns. Looking at what statistics is covered in more recent textbooks for the same grades provides a glimpse into the statistical half of the probability and statistics strand. Discovering how statistics (CCSS, 2010; NCTM, 1989, 2000, 2006) is presented helps ascertain whether or not the material included in these textbooks covers the recommended topics (CCSS, 2010; Franklin et al., (GAISE), 2005; NCTM, 1989, 2000, 2006).

Definition of Terms

A definition of several terms used throughout this study is provided below.

Cognitive Level: Cognitive level is the level of thinking required of the student to complete an exercise. This study used the Mathematical Tasks Framework used by the QUASAR Project team (Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996; Stein & Smith 1998; Stein et al., 2000). The Mathematical Tasks Framework has four categories of cognitive demand. Low-level cognitive thinking requires memorization or application of definitions. Higher-level cognitive thinking involves making connections with a procedure and the real world.

Content Analysis: Content analyses primarily focus upon curriculum materials, such as textbooks, and the curriculum materials are evaluated for accuracy, depth of coverage, or the logical sequencing of topics using the researcher's set of criteria (National Research Council, 2004). This study is a content analysis of the statistical content of several middle grades mathematics textbooks.

Commercial or Publisher Generated Textbooks: Commercial or publisher generated textbooks are the textbooks that are produced by one of the major publishing companies. Examples of such companies are Holt, Rinehart and Winston, Pearson

Prentice Hall, McGraw Hill, and others. The distinction is that the textbook was not developed with outside funding such as the National Science Foundation or created by university curriculum projects.

Graphicacy: Graphicacy is the ability to understand, use and create graphical images (Curio, 1987; Wainer, 1992).

Intended Curriculum: The intended curriculum (McKnight et al., 1987) is the material that is to be covered in the course for a particular grade level.

National Science Foundation Funded Textbooks: NSF-funded textbooks are those created using funds provided by the National Science Foundation, most notably in response to the 1989 NCTM Standards. These textbooks differ from typical commercial textbooks in several ways. Instead of worked examples and practice problems, these textbooks present realistic problems that students work in small groups to solve (Senk & Thompson, 2003). A more holistic view of mathematics is presented and real world applications are emphasized.

Non-NSF and Non-Commercial: The University of Chicago School Mathematics Project (UCSMP) textbooks (Senk, 2003) are one example in this category. The UCSMP textbooks were created by a university curriculum project using funds from the AMOCO Foundation (now British Petroleum (BP)). This textbook series was created before the NCTM Standards were created, but was in response to the same national recommendations that gave rise to the NCTM Standards.

Middle Grades: Middle grades or middle school are the grades between elementary school and high school. Schools may separate the middle grades as sixth, seventh, and eighth or as seventh, eighth, and ninth. For the purpose of this study, the middle grades are defined to be sixth, seventh, and eighth grades. This is consistent with what NCTM (1989, 2000) has designated as the middle grades.

Potential Opportunity to Learn: (POTL) For this study, potential *opportunity to learn* is defined as what concepts are found in the textbook. This includes the length of the lesson presentation of the concept and the cognitive demand required of the students to complete the corresponding exercises.

Statistical Exercise: A statistical exercise is an exercise whose primary purpose is to provide opportunity for students to practice statistical content.

Statistical Lesson: A statistical lesson is a lesson narrative in which the primary intent of the lesson narrative is to teach a statistical concept.

Statistical Literacy: Statistical literacy (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008) is the ability to understand statistical vocabulary and symbols and to critique statistical claims.

Statistical Reasoning: Statistical reasoning is the process by which students reason with statistical ideas and make connections among statistical concepts (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008). Statistical reasoning means being able to understand, explain statistical processes and interpret statistical results.

Statistical Thinking: Wild and Pfannkuch (1999) define statistical thinking as the way professional statisticians think. Statistical Thinking involves a deeper understanding of statistical concepts and processes, including the statistical limitations of said processes. Statistical thinking is the process of gathering data, knowing which tools to utilize to analyze the data, and being able to use the results to infer about the population.

Summary

As the world around us becomes more complex, more knowledge is needed to prepare students for success in this modern world. One such area that was not part of the curriculum in the past is the area of statistics. Although it is a vital part of the curriculum, students have a difficult time learning concepts in probability and statistics and often these concepts contradict their own beliefs (delMas et al., 2007; Jones et al., 2007; Shaughnessy, 1992, 2007). Misconceptions that are strongly embedded in students' thinking are difficult to correct and overcome (Bransford et al., 2000; Garfield & Ahlgren, 1988; Mevarech & Kramarsky, 1997). The first step in finding ways to improve student learning of statistics is an examination of the statistical content of the curriculum as textbook analysis in this area is sparse. Part of this examination should analyze the cognitive levels required of the students to complete the exercises as higher cognitive demand facilitates better student understanding (Doyle, 1983; Stein & Smith, 1998; Smith & Stein, 1998). Thus a study of both the lesson components and the cognitive level required of students to complete the exercises supporting statistical concepts could provide valuable insight into what topics are included in the curriculum, the nature of the treatment of the topics, and the cognitive level of exercises.

Chapter 2: Literature Review

The focus of this study is the treatment and scope of the statistical content in middle grades mathematics textbooks. Therefore, I considered four main areas of relevant literature: curriculum and the role of the textbook in the curriculum, including key elements that form a framework for effective content (textbook) analysis; research on mathematical textbooks, including different levels of cognitive demand required by students to complete the tasks; the historical development related to the inclusion of statistical content in the middle grades curriculum, including a discussion of the NCTM and Common Core State Standards and their recommendation in the area of statistics; and research about students learning of statistical concepts in the middle grades. Each of these areas has the potential to influence the nature of the content analysis to be conducted in this study.

Literature selection. To conduct the literature review, research articles were located through ERIC and a meta search engine. This particular meta search engine used Academic Search Premier, Wilson Omnifile, General OneFile, and CQ Researcher. The key words used to find the articles that examined textbooks were “textbook evaluation”, “content analysis”, “statistics”, “middle grades mathematics textbooks”, “textbook analysis”, “curriculum” and “mathematics textbooks”. A preliminary search yielded few research articles that analyzed mathematical textbooks for the middle grades specifically, so, the search expanded to grades K-12. National Council of Teachers of Mathematics

(NCTM) journals, handbooks, standards documents and yearbooks that covered statistical topics were also examined for relevant articles and research.

Curriculum and the Textbook

Discussed below are several types of curriculum, the role textbooks maintain in the classroom and how textbooks impact the curriculum. Different researchers identify different types of curriculum and use different terminology (Jones, 2004; Klein, Tye, & Wright, 1979; Porter, 2002, 2006; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Stein, Remillard, & Smith, 2007; Venezky, 1992), but essentially curriculum is divided into several distinct categories of what is needed, what is desired, what is intended, and what is enacted.

Types of Curriculum. In *The Handbook of Research on Curriculum,* Venezky (1992) discussed the needed curriculum, desired curriculum, prescribed curriculum, and delivered curriculum. Venezky describes the "needed curriculum" as the most general set of ideas or concepts that students should learn as recommended by experts. The "needed curriculum" provides the beginning outline of what is needed and is used to guide the "desired curriculum." Thus the "desired curriculum" brings to fruition the basic general ideas of the "needed curriculum." The "desired curriculum" incorporates the NCTM recommendations and other district, state, or federal guidelines.

The textbook and other written materials comprise the "prescribed curriculum." Porter (2006) uses the "intended curriculum" synonymously with the "prescribed curriculum." McKnight et al. (1987) also use the term "intended curriculum." The "intended" or "prescribed curriculum" encompasses the content and sequence of the content of the courses. Instead of the "intended curriculum," the term "formal" has also been used (Klein et al., 1979; Stein, Remillard, & Smith, 2007; Usiskin, 1999) to refer to the written curriculum. The "enacted curriculum" refers to how the "intended curriculum" is realized in the classroom (Porter, 2006; Stein, Remillard, & Smith, 2007).

Textbooks or "intended curriculum" serve a critical role in the overall context of the curriculum that is utilized in the classroom. Textbooks are the primary vehicle by which the content of the curriculum is relayed to the student. The textbook contains both the scope and sequence of the course content and what topics should be covered.

The role of the mathematics textbook in the classroom. This great dependence upon the textbook "is perhaps more characteristic of the teaching of mathematics than of any other subject in the curriculum" (Robitaille & Travers, 1992, p. 706). Textbooks have been found to greatly influence what is taught in the mathematics classroom (Begle, 1973; Driscoll, 1980; Grouws & Smith, 2000; Haggarty & Pepin, 2002; Porter, 1995; Reys, Reys, & Lapan, 2003; Tornroos, 2005; Tyson-Berstein & Woodward, 1991). Furthermore in the Third International Mathematics and Science Study, mathematics teachers reported daily use of the textbook in their classrooms (Robitaille & Travers, 1992) and typically courses cover the first 75% of the textbook (Flanders, 1987; Tarr et al., 2006a; Tarr et al., 2008). Moreover, teachers rarely introduce material not found in the textbook (Reys, Reys, & Lapan, 2003). Thus, research suggests that the content within textbooks greatly influences the mathematics curriculum that is taught in the classroom.

Concerns about mathematics textbooks. The majority of textbooks used in classrooms across the United States are publisher generated textbooks (Educational Market Research, 2001; Weiss et al., 2001). Although guidelines or standards are provided for mathematics curricula in the NCTM Standards documents (1989, 2000, 2006), and in documents by other national bodies (e. g. GAISE by the American Statistical Association, 2005; Common Core State Standards by National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) the ultimate task of developing curricula and publishing textbooks falls to textbook authors and publishing personnel. Publishers advertise that textbook content satisfies recommended guidelines but something is lost when trying to satisfy too many guidelines, and "the educational vision is, at best diluted" (Clements, 2008, p. 599). Many concerns about mathematics textbooks have been voiced in the research. Below is a list of concerns about the mathematics curriculum that this literature review yielded.

- Mathematical textbooks contain too many topics (Clements & Battista, 1992; Ginsburg, et al., 2005; Jones, 2004; Porter, 1989; Valverde et al., 2002)
- Mathematical textbooks provide little opportunity for in depth study of topics (Jones, 2004; McKnight et al., 1987; Schmidt, McKnight, & Raizem, 1997; Tarr, Reys, Barker & Billstein, 2006; Valverde et al., 2002).
- Topics tend to be repeated for several years without changing the depth of the coverage (Flanders, 1987; McKnight et al., 1987; Senk & Thompson 2003; Usiskin 1987).
- Many mathematical topics are presented in small segments without connections at different times and at different grade levels (Flanders, 1994, McKnight et al., 1987; U. S. Dept. of Education, 1996, 1997, 1998).

- There is a lack of high levels of cognitive demand required of the students when completing exercises (Jones, 2004; Li, 2000; Senk & Thompson, 2003; Smith & Stein, 1998).
- "Many textbooks provide (d) little development in sophistication of mathematical ideas from grades 6 to 8" (American Association for the Advancement of Science: Project 2061 [AAAS], 2000. p. 9).

Relationship between textbooks and curriculum. Research has shown that the majority of middle grade teachers use commercially published textbooks (National Assessment of Educational Progress (NAEP), 2000) and textbooks greatly influence what mathematical content teachers will present in the classroom (Begle, 1973). According to NAEP survey results, seventy-five percent of eighth graders used their textbooks almost everyday (Braswell et al., 2001; Grouws & Smith 2000; Grouws et al., 2004). For all grade levels and content areas, teachers frequently use textbooks as the primary resource for instruction (Tyson-Berstein & Woodward, 1991). This dependence upon textbooks is not unique to the United States but has been documented internationally (Robitaille & Travers, 1992). Moreover, teachers rarely present material not found in the textbook and material in the last half of the textbook is more likely to be omitted should time run short (Begle, 1973; Jones, 2004; Reys, Reys, & Lapan, 2003). These research articles and reports suggest that textbooks are the major component of what happens in the classroom curriculum (Hummel, 1988) and determine what material students have the potential *opportunity to learn* (Down, 1988). Here the *opportunity to learn* specifically relates to the intersection of content in the textbook that is also taught in the classroom.

Curriculum content analysis. The process of curriculum content analysis is relatively new in the form of a thoughtful planned design of textbooks and was not considered until the 1970s (Senk & Thompson, 2003; Woodward, Elliott & Nagel, 1988). The completion of several curriculum projects in the 1970s thru the 1980s brought forth a need to formally analyze textbook content. Kilpatrick states that "the job of curriculum analyzer, like the job of the curriculum developer, is a 20th century invention" (Kilpatrick, 2003, p. 182). Curriculum content analysis started with the comparison of developed curriculum: publisher or commercially generated and standards based. Publisher or commercial textbooks are produced by a major publishing house without funding from either the National Science Foundation or created by university curriculum projects. National Science Foundation funded textbooks were created in response to the NCTM Standards, are problem centered, and try to present a holistic view of mathematics. University curriculum project textbooks were created under the aegis of a university with a particular philosophy in mind and funded privately (non-NSF funded). Today, however, almost all publishers have embedded elements of Standards-based instruction into their materials; so researchers have advocated the use of *publisher developed* or *NSF-funded* to distinguish between the two types of textbooks (Reys, Reys, & Rubenstein, 2010). There are still many questions concerning textbooks that have yet to be answered, such as how these curricula are similar or different from each other (Chappell, 2003) and how different philosophies or priorities manifest in the different textbook series (Reys et al., 2003).

Framework for an Effective Content (Textbook) Analysis.

Another line of inquiry that arose from research about curriculum content analysis is what constitutes a good content analysis. The National Research Council created a team that examined the research on "content analysis [that] focus [es] almost exclusively on examining the content of curriculum materials" (National Research Council (NRC), 2004, p. 2), such as textbooks, and created a framework for high quality content analysis. "…These analyses usually rely on expert review and judgments about such things as accuracy, [and] depth of coverage, or the logical sequencing of topics" (National Research Council (NRC), 2004, p. 2). A committee composed of educators and mathematicians recommended that a high quality content analysis should identify the credentials of the reviewer, the standard on which the review is based, and be transparent.

Credentials and background of the reviewer. Stating the credentials and background of the reviewer gives a study more credibility and shows the educational philosophy the researchers espouse. For example, Adams et al. (2000, unpublished) compared middle school mathematics in Singapore curriculum and in two American textbook series, Connected Mathematics and Mathematics in Context. The researchers reported that the reviewers had backgrounds in pure mathematics with an interest in K-12 education, but did not have experience in teaching K-12. Because reviewers bring different viewpoints and different ideas of what they consider important to an analysis depending on their background, the credentials of each reviewer need to be identified.

Clearly defined set of standards. The second criteria that the authors of *On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations* recommend is that a content analysis clearly state the set of standards the researcher is using to evaluate materials. A clearly defined set of standards helps the researcher show how the material ranks according to the set of standards, and also makes it easier to identify the similarities and differences with a contrasting textbook or textbooks.

Transparency. The most important aspect of a quality content analysis is transparency. The criteria used and the rationale for the decisions should be clearly defined so that the analysis could be replicated with the same outcomes for a particular set of criteria. The outcomes for the same textbook content analysis can be varied according to what criteria are used and what aspect of the textbook content is being analyzed as long as the process of the evaluation is clearly delineated and supported. In fact, the National Research Council acknowledged that evaluations of curriculum materials are "connoisseurial assessments", which are closely tied to the reviewers' judgments, value system, and background.

Research on Mathematics Textbooks

Some researchers conducting content analysis on mathematics textbooks have followed a single idea or concept throughout several textbooks, while comparing and contrasting the treatment of this concept or idea (Charalambous et al., 2010; Dowling, 1996; Johnson, Thompson & Senk, 2010; Jones, 2004; Jones & Tarr, 2007; Li, 1998; Mesa, 2004; Oner, 2008; Sood & Jitendra, 2007; Stacey & Vincent, 2009; Stylianides, 2007; Tornroos, 2005; Xin, 2007). Others have evaluated textbooks for historical trends (Baker et al., 2010; Jones & Tarr, 2007) and still others for textbook adoption procedures for use in the school system or districts (Mauch & McDermott, 2007).

In the 1980s, Flanders (1987) conducted a study of elementary and middle grades mathematics textbooks. He recorded where the concept was located within the textbook pages, the total number of pages devoted to the concept, and at what grade level the concept was introduced. His study showed that the amount of new content presented steadily decreased from 100% in kindergarten to about 30% by grade 8 then abruptly increased to 90% in Algebra 1 in grade 9; the new content tended to be placed near the end of the textbook.

Xin (2007) suggested that Chinese middle grades mathematics textbooks were more balanced in the types of problems presented for student practice than their American counterparts and that students perform better on certain problem types when they have had an *opportunity to learn* those problem types. Thus, his research reinforces earlier findings by Tornroos (2005) that textbooks in alignment with international tests showed a strong correlation between students' *opportunity to learn* and student achievement. Stylianides (2005) used a framework to analyze the *opportunity to learn* reasoning and proof in a National Science Foundation funded middle grades textbooks series. He found that opportunities were unevenly distributed among the middle grade levels in the units presenting algebra, geometry, and number theory content. Tarr et al. (2006a, 2008) reported that consistency of topic content within a textbook and across a textbook series facilitates student *opportunity to learn*.

Research on Mathematical Tasks

Doyle (1983) categorized students' work into academic tasks and provided the basis for the Mathematical Tasks Framework used by the QUASAR Project team (Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996; Stein & Smith 1998; Stein et al., 2000). Academic tasks were divided into four categories: memory, procedural or routine, comprehension or understanding, and opinion. Categorizing the tasks in this way allowed the researchers to show the differences in the levels of cognitive demand required for students to complete the items. Doyle stated:

> For example, problem solving in one class might involve computing answers to well-structured multiplication exercises, whereas in another it might require that students decide which mathematical operation applies to a particular real-world problem. The cognitive demands of these two 'problems' are clearly different. (p. 171).

Based upon Doyle's work, the Mathematical Task Framework (Smith & Stein, 1998; Stein et al., 1996; Stein & Smith 1998; Stein et al., 2000) was created and identifies three "phases of task implementation" in relation to student learning. The first phase is the curricular content of the textbooks, ancillary material, and the like; the second phase is how the teacher presents the materials; and the third phase is how the students actually implement the tasks. The framework further defines four levels of cognitive demand: Lower-level (memorization); Lower-level Procedures without Connections to Understanding, Meanings or Concepts; Higher Level Procedures with Connections to Understanding, Meaning, or Concepts; and Higher Level Doing Mathematics. Stein and Lane (1996) emphasized the importance of examining the levels of cognitive demand required of students as the cognitive demand greatly influences student learning.

> The mathematical tasks with which students become engaged determine not only what substance they learn but also how they come to think about, develop, use, and make sense of mathematics. …an important distinction… is the difference between tasks that engage students at a surface level and tasks that engage students at a deeper level by demanding interpretation, flexibility, the shepherding of resources, and the construction of meaning. (Stein & Lane, p. 459)

Historical Treatment of Probability and Statistics in the Curriculum

Historical treatment of probability in middle grades mathematics textbooks.

The NCTM Standards combines the topics of probability and statistics into a single strand of study as the two areas are closely related. By convention, probability and statistical topics are found in close proximity and findings about probability will usually hold true for statistics. So a search for research on probability topics in the middle grades was conducted and Jones' (2004) study was the only research found that analyzed probability content in middle grades mathematics textbooks. Jones analyzed the historical treatment of probability from four historical eras. For each era, a widely used popular textbook series was chosen along with an alternative textbook series representing a different educational philosophy in an attempt to gain a broader view on the treatment of probability concepts within each era. He examined the components of lesson narrative and then the cognitive demand (Jones & Tarr, 2007) of the corresponding exercises. He created a lesson component system thru open coding (Strauss & Corbin, 1990). The process of open coding requires the coder to create the categories of what he/she is coding as the categories emerge in the process of the coding. He also applied the levels of cognitive demand from the Mathematical Task Framework (Smith & Stein, 1998; Stein et al., 1996; Stein & Smith 1998; Stein et al., 2000) to the corresponding exercises.

Jones' research showed that the majority of the probability content in the middle grades mathematics textbooks was found in the most recent "Standards Era" (Jones, 2004). Jones analyzed *Mathematics: Applications and Connections* (Collins et al., 1998) and the *Connected Mathematics Project* series (Lappan et al., 1998) for this era. In the probability lessons, Jones found that the Standards (NSF funded) alternative series gave few definitions and formulas, but instead guided students thru the lesson to formulate their own definitions and formulas. The discovery process prompted the students to use higher order thinking. Jones also found that the Standards (NSF funded) alternative series contained more exercises that required students to think at higher cognitive levels to complete the exercises. Moreover, the other (commercial) textbooks Jones analyzed across all three grade levels contained mainly exercises that required low level cognitive demand of the students to complete the exercises and most of the lessons were composed of definitions and worked examples. A typical lesson narrative from a commercial textbook would start with a list of definitions followed by several worked examples and end with the exercises. In contrast, the Standards (NSF funded) alternative series contained few worked examples but instead posed situations in which the students had to work out the solutions themselves.

History of Statistics Content in the Curriculum. In 1975, the Conference Board of the Mathematical Sciences National Advisory Committee on Mathematical Education (NACOME) published a report that stated "recent curriculum planning conferences and resultant development projects have given prominent attention to statistics… throughout elementary and secondary mathematics programs" (NACOME, 1975, p. 45) and recommended that statistical concepts should be incorporated in both the elementary and secondary curricula. So in comparison to other mathematical topics, such as algebra or fractions that have been a part of the curriculum for centuries, the inclusion and emphasis on statistics as part of the curriculum is fairly recent. About a decade later, *A Nation At Risk* (National Commission on Excellence in Education, 1983) recommended introducing statistical content in the middle grades.

In response to these recommendations, the NCTM (1989) Standards incorporated the Statistics strand into the curriculum for the middle grades. Refinement and updating of these Standards resulted in the publication of *Principles and Standards for School Mathematics* (2000) and the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006). Since the publication of the NACOME (1975) report, statistics has been included in the curriculum. More recent documents (NCTM Standards, 2000; NCTM Focal Points, 2006; Common Core State Standards, 2010) also recommend the inclusion of statistical topics in the middle grades and identify the middle grade levels as the starting point where in-depth coverage of the statistics strand should begin.

Statistical Literacy, Reasoning and Thinking

The focus on statistical content in the curriculum has led statisticians to carefully define statistical literacy, statistical thinking, and statistical reasoning. Society expects citizens to be statistically literate after they graduate from high school and statistical literacy is a necessary skill of a productive and informed member of the modern world (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008). Thus each citizen needs a basic understanding of statistical vocabulary, statistical symbols, and the ability to critique statistical claims in the media. Statistical Reasoning is the process of how students reason with statistical ideas and make connections of statistical concepts (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008). Statistical reasoning means being able to understand, explain statistical processes, and interpret statistical results. Statistical thinking is the way professional statisticians think (Wild & Pfannkuch, 1999). It involves a deeper understanding of statistical concepts and processes, including the statistical limitations of said processes. Statistical Thinking is the process of gathering data, knowing which tools to utilize to analyze the data and being able to use the results to infer about the population. A citizen of modern society cannot be statistically literate without the ability to think and reason statistically. Each of the areas overlap and elements of each are essential to statistical literacy.

Guiding Documents for Statistical Content Analysis.

The NCTM Standards (2000), NCTM Focal Points (2006), and Common Core State Standards (2010) all recommend that the majority of the statistical concepts recommended for the middle grades are covered in the 8th grade. Because the NCTM Standards (2000) did not specify by grade level the statistical topics to be included, the NCTM Curriculum Focal Points (2006) were created as a supporting document to the NCTM Standards (2000) to clarify three areas that each grade level should emphasize along with supporting mathematical concepts. The Common Core State Standards define in general terms for each grade level what statistical topics should be included for the middle grades. Both the NCTM Standards and the Common Core State Standards agree on the statistical concepts that should be included but only the NCTM Focal points delineate specifically what statistical objectives the student should master for each grade level. Table 1 summarizes the statistical concepts to be included for each grade level from all the documents.

Table 1

Comparison of Statistical Concepts for the Middle Grades from the NCTM Standards, NCTM Focal Points, and Common Core State Standards.

The NCTM Standards (2000)	**NCTM Focal Points (2006)**	**The Common Core State Standards (2010)**
Grade 6 - 8 • Recognize a statistical question. • Develop a basic understanding of statistical variability. • Describe and summarize distributions. • Basic understanding of the use of random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots. • Find, use, and interpret measures of center and spread, including mean and interquartile range.	Grade 6 • Statistics is not a focal point for this grade level. However, in grade 5 students created and analyzed line and double bar graphs as well as used ordered pairs on (x,y) grids. Grade 7 • Use samples and proportions to extend finding about the sample to the population. • Apply percentages in order to interpret and create histograms and circle graphs. Grade 8 • Based on previous work with organizing and displaying data, students should be able to see numerical data as an aggregate. • Summarize data using central measures of tendency.	Grade 6 • Develop understanding of statistical variability. Summarize and describe distributions. Grade 7 • Use random sampling to draw inferences about a population. Draw informal comparative inferences about two populations. Grade 8 • Investigate patterns of association in bivariate data.

Table 1 Continued

The NCTM Standards (2000)	**NCTM Focal Points (2006)**	**The Common Core State Standards (2010)**
• Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots. • Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken. • Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit. • Use conjectures to formulate new questions and plan new studies to answer them.	• Find quartiles 1, 2 and 3 and obtain information about the spread of data. • Construct box and whisker plots to convey information about the spread of data. • Create scatter plots to display bivariate data. • Informally estimate lines of best fit and make predictions.	

Although the Common Core Standards for Mathematics (CCSSO, 2010) is more succinct, the statistical objectives for each grade are essentially the same as in the NCTM Standards (2000). The NCTM Standards (2000) identifies the statistical concepts to be covered for the middle grades and the NCTM Curriculum Focal Points (2006) provide a more detailed description of the statistical concepts that are to be emphasized for each grade level. Although the majority of states have already adopted the Common Core State Standards, the current mathematical textbooks in use today were written in response to the NCTM Standards (2000) and the NCTM Curriculum Focal Points (2006). Therefore, this study used the NCTM Standards (2000) and the NCTM Curriculum Focal Points (2006) as the primary reference as to which statistical concepts should be included in the middle grades curriculum.

Importance of Statistics.

H. G. Wells once stated "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write" (Huff, 1954). Decades later his prediction has come to fruition. "Statistical literacy is essential in our personal lives as consumers, citizens and professionals. Statistics plays a role in our health and happiness" (Franklin et al., 2005, p. 3). But statistical literacy does not happen overnight or during a single statistical course. Statistical literacy is a gradual process that needs to be nurtured and expanded through out the middle school and high school years (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008).

Research on Statistical Concepts

The following research focuses specifically on how students learn statistical concepts and the problems that have been found with students' statistical proficiency.

There are several areas in research that are pertinent to the middle school statistical curriculum, namely measures of central tendency (mean, median and mode), measures of dispersion (variation and standard deviation), graphical representations, the use of technology to enhance statistical learning, how middle school students' learn statistical concepts and misconceptions about statistical concepts.

Research on Measures of Central Tendency. Initially, researchers who delved into students' statistical thinking examined the average or mean. Several research studies examined how students think about measures of central tendency, specifically the average (mean), mode, and median (Mokros & Russell, 1995; Watson & Moritiz, 2000a, 2000b; Strauss & Bichler, 1988). Strauss and Bichler (1988) conducted a study to ascertain the development of children's understanding of arithmetic mean by categorizing its properties. Their study involved 8, 10, 12 and 14 year old Israeli children and delineated the following properties:

- *the average is located between extreme values (the average value lies between the maximum and minimum of the data)*
- *the sum of deviations from the mean (average) is zero*
- *the average is influenced by values other than the average*
- *the average does not necessarily equal one of the values that was summed*
- *the average can be a fraction with no counterpart in reality*
- *the average is representative of the values that were averaged*
- *when calculating the average take a value of zero into account.*

Their results showed that the idea that the average value lies between the maximum and minimum of the data was understood by all but the 8 year olds. Similarly the majority of students solved the tasks associated with the ideas that the data values influence the average and the average does not have to equal a datum value. Younger students had difficulty with the tasks associated with the idea that the average can be a fraction with no datum counterpart and the idea that the average is representative of the data. The majority of the students had difficulty with the tasks associated with the ideas that when calculating the average a value of zero must be taken into account and that the sum of deviations from the mean is zero.

Mokros and Russell (1995) found that students had difficulty when given an average value and asked to construct the possible data attached to a particular mean. They found five different ways students view the mean (average): as mode, as a calculation, as a reasonable value, as midpoint, and as a point of balance. Mokros and Russell (1995) suggest that students first have to see the idea of distribution before the students can view mean, median and mode as representing the distribution.

Watson and Moritiz (2000a, 2000b) followed the development of students' concept of the mean (average) from the third to the ninth grade and their results supported the earlier finds of Strauss and Bichler (1988) that children's understanding of arithmetic average has several properties or categories. Watson and Moritiz (2000a, 2000b) suggest students have multiple conceptions of the average which emerge depending upon the type of task used. Students progress in their understanding of average from the common vernacular meaning of average to the modal (most frequent) to the midpoint and finally to the idea of an average as "representative" of a data set.

Konold and Pollatsek (2002) reflected upon Watson and Moritiz's (2000a, 2000b) research and postulated a hierarchical four step conceptual view of the mean (average): as a typical value, as a fair share, as a way to reduce data, and as a "signal amid noise." A typical value is viewed by the students as the most frequent or most central data value. A fair share is the view most commonly found in elementary school age children and is the calculated average. In the data reduction viewpoint, there is so much data that a single data value is needed to represent the others. For the "signal amid noise" viewpoint "…each observation (datum value) is an estimate of an unknown but specific value. A prototypical example is repeatedly weighing an object to determine its actual weight. Each observation is viewed as deviating from the actual weight by a measurement error, which is viewed as 'random'. The average of these scores is interpreted as a close approximation to the actual weight" (Ben-Zvi & Garfield, 2004, p. 179). Konold and Pollatsek (2002) postulated that, from a statistician's viewpoint, the "signal amid noise" conceptual view of the mean was the most meaningful when examining two or more sets of data; they recommended that students be first introduced to the mean in the context of comparing data sets. Furthermore, Konold and Pollatsek (2002) stated that "the notion of an average understood as a central tendency is inseparable from the notion of spread" (p. 263) and students need to view statistics as "the study of noisy processes … that have a signature, or signal" (p. 260).

Research on Measures of Dispersion. Despite the intertwining of the measures of central tendency and the measures of dispersion or variability, research into variability is just beginning. For clarification, variability is defined as "the propensity for something to change, whereas variation is a measurement of that change" (Shaughnessy, 2007, p.

972). "…Research on students' thinking and understanding about variability could focus on variation in data, or on students' conceptions of variability in samples, or on the variability across several distributions of data that are being compared" (Shaughnessy, 2007, p. 7).

Shaughnessy and Pfannkuch (2002) presented students with a table containing the wait times over three days for the eruption of the geyser Old Faithful at Yellowstone National Park. The data set was chosen because it is bimodal, that is, it has two most frequent values. Students were asked to predict how long a wait a tourist would have before Old Faithful erupted again. Most gave single number answers by calculating the mean or means for each day or for all three days. Students were then asked to graph the data and again predict the wait time. After graphing, students could see the bimodal aspect of the data and some gave a range of values instead of a single value. Examples like these help to solidify the idea of variation in data so students are not apt to calculate the mean without first examining the data.

Another area of difficulty for students is the conflict between the ideas of variability and representativeness (Rubin, Bruce, & Tenney, 1991). Students with some probability knowledge can predict the most likely outcome for a single trial but have a difficult time predicting likely outcomes for several repeated samplings. Recall that probability and statistics are intertwined like the two faces of the same coin. Probability is involved in predicting a single trial but the concept of repeated samplings falls into the realm of statistics. Students had a difficult time reconciling the two views or the idea of variability within reason around an expected value (confidence interval).

Further research by Shaughnessy, Ciancetta, and Canada (2004) revealed a hierarchy of reasoning for learning variability, consisting of *additive* (most frequent data values) to *proportional* (mean or center) and ultimately to *distribution* (center and spread). For example, in a jar were 100 candies, with 50% red and 50% a mixture of other colors. Six samples of 10 candies each were drawn and students where asked to predict the number of red candies in each of the six samples. (Recall that repeated sampling is a statistical concept.) Students at the *additive* level based their answers purely on the number of reds in the mixture (example: number of reds in each sample: 8, 7, 6, 9, 10) while students at the *proportional* level based their answers upon the means or proportions of reds. Based upon the information that 50% were red, students at the proportional level gave the number of reds in each sample: 5, 5, 5, 5, 5, 5. At the *distribution* level students' answers took into account both the center and spread of the colored candies, reflecting a balance between the ideas of representativeness and variability.

Research on Graphical Representations. Research has shown that there are three levels of graphicacy (Curcio, 1987; Wainer, 1992): *reading the data* (directly from the graph), *reading between the data* (or identifying the patterns or trends), and *reading beyond the data* (making inferences for future or missing data values). Students had difficulty with line graphs (Berg & Phillips, 1994) as well as with line plots and histograms (Friel & Bright, 1995, 1996). Also, students were more inclined to use linear representations of data (Mavarech & Kramarsky, 1997), tended to focus on the procedure of creating the graph rather then the interpretation of the graph (Capraro, Kulm & Capraro, 2005), matched certain graph types to certain questions on data (Capraro, Kulm

& Capraro, 2005), and ignored the graph altogether if the question involved a real-world application that was part of their life experiences (Lowrie, Deizamann & Logan, 2011). Researchers (Berg & Phillips, 1994; Friel & Bright, 1995; 1996) suggested that more open ended graph problems and more discussions concerning the process of graphing can help facilitate better understanding of graphing.

Research on Technology Enhanced Statistical Learning

Several studies have analyzed how software programs, such as tinkerplots, mini tab, and other technology can enhance student statistical learning (delMas & Garfield, 1999; Fitzallen & Watson, 2010; Paparistodemou & Meletiou-Mavrotheris, 2008; Ben-Zvi & Arcavi, 2001). The technology enables students to explore and discover for themselves patterns in large sets of data in a timely manner.

Paparistodemou and Meletiou-Mavrotheris (2008) conducted a study with 3rd graders in Cyprus that was designed to ascertain ways in which the basics of inferential reasoning can be incorporated in the curriculum at a very young age thru the use of Tinkerplots. Tinkerplots is a statistical software program that creates visual displays, runs probability simulations and can automatically update representations to show patterns in data (www.keypress.com). The class as a whole conducted an anonymous survey about items that interested them, such as age, grade, number of hours sleeping, food eaten for breakfast. The data collected were entered into the Tinkerplots software and discussed. The results showed that young students begin to reason about informal inference when the activity is of interest to them and involves topics to which they can relate.

Fitzallen and Watson (2010) used Tinkerplots with 10 and 12 year olds who had not had previous data handling experiences. The students were taught how to use Tinkerplots thru an activity on ways to change their heart rate. After a few weeks without use of Tinkerplots, the students were given several statistical problems to work thru with the aid of the software. The results showed that in a very brief time period the students shifted from the procedures taught during the initial activity and were manipulating the Tinkerplots software in a variety of ways to help them answer the statistical problems. The students used the Tinkerplots software to create plots of the data that supported their thinking. Another advantage of the Tinkerplots software was the ability to quickly move back and forth from a hypothesis to a plot of the data, thereby allowing students to test several hypotheses quickly in order to make sense of the data.

As several research results have shown, the right technological tools can facilitate statistical learning even at a young age. So, inclusion of these technological tools into the curriculum is essential. The next aspect to consider is how middle school students think statistically.

Mooney's Middle School Student Statistical Thinking

Shaughnessy, Garfield and Greer (1996) proposed that handling data involves the processes of organizing, describing, representing, and analyzing data. Mooney (2002) modified these processes and developed the Middle School Student Statistical Thinking (M3ST), a model that describes the steps by which students learn statistics. Mooney categorized a four step hierarchical process that starts with 1) describing data, 2) organizing and reducing data, 3) representing data and 4) analyzing and interpreting data. Students have been reported to have the greatest difficulty in "analyzing and interpreting data" because this process involves identifying trends and generalizing these trends to infer or predict (Mooney et. al., 2001).

Students' Misconceptions about Statistical Concepts

Students have many misconceptions about statistical concepts (delMas & Garfield, 1999; Garfield & Ahlgren, 1988; Garfield & Ben-Zvi, 2008; Garfield & Gal, 1999) that can be difficult to overcome. Student learning is facilitated if the misconceptions are confronted and students are shown their errors in reasoning (delMas & Garfield, 1999; Garfield & Ahlgren, 1988; Garfield & Ben-Zvi, 2008; Garfield & Gal, 1999). Garfield and Ben-Zvi (2008), Garfield & Gal (1999) and delMas & Garfield (1999) espoused an "instructional" approach based on the model of conceptual change that has students make predictions and perform trials to collect data in order to see if the students' predictions were correct. Shaughnessy, Ciancetta, and Canada (2004) found that in understanding variability "[middle-grade] students need considerable hands on experience in first predicting the results of samples, and then drawing actual samples, graphing the results, comparing their predictions to the actual data, and discussing observed variability in the distribution." Students experience misunderstandings when it comes to variability. The way to overcome these misconceptions is by showing students repeatedly their mistakes in statistical reasoning.

Several researchers (delMas et al., 2007; Jones et al., 2007; Shaughnessy, 1992, 2007) have shown that students have a difficult time learning concepts in probability and statistics and often these concepts contradict their own beliefs. Misconceptions that are strongly embedded in the students' thinking are difficult to correct and overcome (Bransford et al., 2000; Garfield & Ahlgren, 1988; Mevarech & Kramarsky, 1997).

Summary

This literature review discussed the importance of the textbook in the mathematical classroom, different types of curriculum, and major concerns about mathematics textbooks. Findings were also presented on how students learn statistical concepts and the difficulties students experience when learning statistical constructs. Few research studies were found that analyzed mathematics textbook content and none were found that analyzed the statistical content of middle grades mathematics textbooks. The next chapter presents the framework for this study and the organization for the content analysis with the methods and proposed procedures.

Chapter 3: Research Design and Methodology

Research Questions

This study examines the treatment of statistical concepts in several contemporary (student version) middle grades mathematics textbook series, in an attempt to answer the following three research questions:

1. What is the nature of the statistical content of the curriculum, as exhibited in contemporary student mathematics textbooks in use in the middle grades (grades 6, 7, and 8) in the United States? How much of the textbooks are devoted to statistical topics? Where are these topics located? Which statistical concepts are individual lessons and which are embedded with other non-statistical topics?
2. What are the sequences of the statistical lesson narratives that emerge, if any, in contemporary student mathematics textbooks in use in the middle grades (grades 6, 7, and 8) in the United States? What is the typical sequence of lesson narratives for each textbook series? What are the differences and similarities of these lesson narratives across the textbook series?
3. What is the nature of the exercises on statistics in contemporary student mathematics textbooks in use in the middle grades (grades 6, 7, and 8) in the United States? What is the level of cognitive demand exhibited by these exercises? How does the level of cognitive demand vary among the different textbook series, and across the grade levels?

Textbook Selection

Four series of textbooks for grades six, seven, and eight were examined that represented different educational philosophies. This was done to determine if there were differences in the treatment of the statistical concepts between the different educational philosophies of the textbooks. The four textbook series are composed of two commercial or publisher generated series (Glencoe and Prentice Hall), one series funded by the National Science Foundation (Connected Mathematics Project), and one series developed, with non-NSF funding, by the University of Chicago School Mathematics Project (UCSMP). All textbooks are intended for math courses for the average student, are some of the most widely used (Educational Market Research, 2001; Weiss et al., 2001) and did not include textbooks intended for honors or developmental (remedial) levels. Both Glencoe and Prentice Hall offer a choice of either the course 3 textbook or a pre-algebra textbook for grade 8, leaving the choice to individual school districts. *Connected Mathematics* and UCSMP have only one textbook offered for grade 8 and have embedded the pre-algebra and algebra concepts within the textbook series. So depending upon the school district's choice of textbook series, Glencoe and Prentice Hall offered either 6, 7 and 8 or 6, 7, and pre-algebra. For a fairer comparison and to accommodate for variations in potential *opportunity to learn* depending upon the curriculum sequence chosen, Glencoe's and Prentice Hall's 6, 7, and 8 and 6, 7, and pre-algebra were included in the analysis.

Glencoe Math Connects Series. One series was developed by McGraw Hill Publications: *Glencoe Math Connects: Concepts, Skills, and Problem Solving, Course 1* (©2009), *Course 2* (©2009), *Course 3* (©2009) and *Glencoe Pre-Algebra* (©2010). This series has been identified for research purposes as a commercial or publisher generated textbook series (Tarr et al., 2008) and is one of the most widely used series of this type (Educational Market Research, 2001; Weiss et al., 2001). This series replaced the previous middle school series *Mathematics: Applications and Connections* (1998 – 2001) and *Mathematics: Applications and Concepts* (2004 - 2006). The publisher, McGraw Hill, states on the website (http://www.glencoe.com) that the textbooks are correlated to the NCTM Standards. The textbook series has three key focus areas: building mathematics vocabulary, diversified ways to improve student achievement, and diversified instructional approaches that are aligned to different learning styles. The website also states that

> Every product developed by Macmillan/McGraw-Hill and Glencoe/McGraw-Hill has been designed specifically to meet the needs of today's teachers and students. At each stage of development, research is incorporated into the curriculum. Materials are field tested and revised based upon the input from both teachers and students. Focus groups and teacher advisory boards are essential in creating materials to meet the ever-changing needs of today's classroom. Experts in content areas and special needs (including specialists in such areas as differentiated instruction, cognitive development, and ELL) review and revise lesson manuscripts (http://www,glencoe.com).

Prentice Hall Mathematics. A second commercial textbook publisher is Pearson Publications, with *Prentice Hall Mathematics, Course 1* (©2010), *Course 2* (©2010), *Course 3* (©2008) and *Algebra Readiness* (©2010). This series has also been identified as widely used across the country (Education Market Research, 2001; Weiss et al., 2001) and is marketed by the publisher as Standards based (http://www.phschool.com). Prentice Hall hired an independent research company, Pres Associates, to conduct a study on the effectiveness of this series. In the 2007 – 2008 school year, a sample of 15 teachers and 922 seventh grade students were chosen from Colorado, Florida, Kentucky, Michigan, and New York to participate in the study. Using several assessment tools, students improved significantly in several mathematical areas after one year of use of this series. This series incorporates the use of technology in the lessons, helps build problem solving and test taking strategies.

> Capture your students' interest with exciting and easy-to-use technology. Feel confident about the program's strong authorship, proven student success, and wide range of built-in opportunities to assess your students' understanding. Know that you are preparing your students for algebra and beyond by developing their problem solving skills, improving their conceptual understanding, and providing them with the tools they'll need for ongoing success. …Guided Problem Solving strategies throughout the text provide students with the tools they need to be effective and independent learners" (http://www.phschool.com).

The website goes on to say that the students will become lifelong learners.

Connected Mathematics. *Connected Mathematics (*Lappan et al., 2007*)* is a textbook series developed with funding from the National Science Foundation and is the most widely used such textbook series for middle grades (Education Market Research, 2001; Weiss et al., 2001). It is different from the commercial publisher generated textbooks in that it is a problem-centered textbook with the main focus of "…develop[ing] student knowledge and understanding of mathematics through attention to connections: between mathematical ideas, and their applications in the world outside school; among the core ideas in mathematics; among the strands in a modern mathematics curriculum; and between the planned teaching-learning activities…" (Ridgeway et al., 2003). *Connected Mathematics* "looks at instruction in three phases: launching, exploring and summarizing" (www.connectedmath.msu.edu). The launching part sets the stage, letting students know what the problem is and what they can expect to learn, then the students explores how to solve the problems. At the end of the unit, students summarize what they have learned. This problem-centered approach generally does not show the students how to solve the problems explicitly but guides them to find the solutions on their own. The philosophy behind this problem-centered approach is the belief that "If the purpose of studying mathematics is to be able to solve a variety of problems, then students need to spend significant portions of their mathematics time solving problems that require thinking, planning, reasoning, computing, and evaluating" (www.connectedmath.msu.edu). Also, this curriculum covers the equivalent of Algebra by the 8^{th} grade (www.connectedmath.msu.edu).

University of Chicago School Mathematics Project (UCSMP). The fourth textbook series is somewhere between the commercial generated textbooks and the NSF funded series in that it is not commercial publisher generated but was developed by a university-based curriculum development project. "The University of Chicago School Mathematics Project [UCSMP] was founded in 1983 with the aim of upgrading mathematics education in elementary and secondary schools throughout the United States. UCSMP believes that, with the information explosion and advances in technology, society today demands not just basic computation skills but a more sophisticated understanding of mathematics. UCSMP seeks to raise the expectations for all students, bringing their performance in mathematics to world-class standards" (ucsmp.uchicago.edu). The textbooks "…emphasize reading, problem-solving, everyday applications, and the use of calculators, computers, and other technologies. Unnecessary repetition and review are eliminated, so that by the end of high school, the diligent average student can learn mathematics once reserved only for honors students" (ucsmp.uchicago.edu). UCSMP thoroughly tests all the material using the most current quantitative and qualitative means and their research has shown that students who use

UCSMP materials consistently do as well as or better than non-UCSMP students, both in computation and conceptual understanding. UCSMP also provides training resources for teachers to help them implement the UCSMP curriculum. The middle grades books are *Pre-Transition Mathematics*, *Transition Mathematics*, and *Algebra*. Again, the UCSMP textbook series represents a widely used non-commercial, non-NSF funded textbook series.

Analytical Framework for Data Analysis. Figure 1 illustrates the three phases of this data analysis. Phase 1 addresses the location and sequence of the statistical concepts. Phase 2 focuses on an examination of the lesson narrative, its components and scope. Phase 3 consists of the level of cognitive demand required of the students to complete the exercises, the total number of exercises per statistical concept, and the total of non-statistical exercises.

Usually, *opportunity to learn* refers to some aspect of what a student has learned as reflected in various assessments (Stylianides, 2006; Tarr et al., 2006a; 2008; Tornroos, 2005). However, because the focus of this study is the "intended curriculum" the definition of the *opportunity to learn* is slightly different. In this study the idea is more of a potential *opportunity to learn* if the concept is present in the textbook.

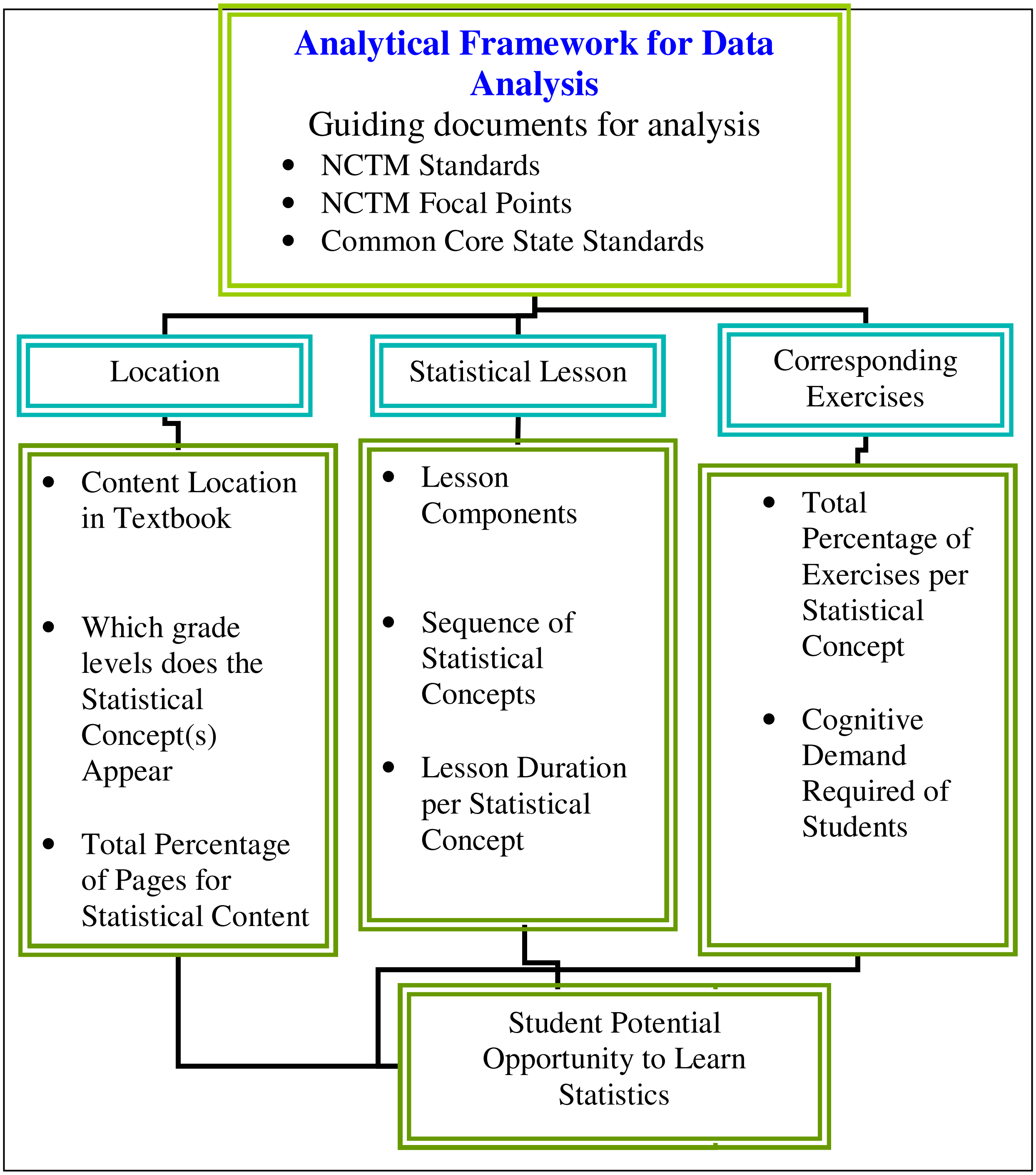

Figure 1. Analytical Framework for Data Analysis: Content Analysis of Statistical Content in Middle Grades Mathematics Textbooks.

Procedures

The analysis of the student version of each textbook series consisted of three areas, namely lesson narrative, exercises, and the amount of the textbook devoted to the statistical concepts. After the explanation of the procedures, examples are provided.

Lesson Selection. For each of the textbook series, only the lessons that taught a statistical concept were included for analysis. The determination of which sections to include was a two step process. First, in the table of contents any sections or chapters that contained statistical titles or terms were identified. Such terms and titles were not limited to but included mean, mode, median, midrange, central tendency, range, variance, standard deviation, dispersion, best fitting line, frequency distribution table, pie chart, bar graph, histogram, box and whisker plot, line graph, stem and leaf plot, outliers and other statistically related terms. (See Table 2 for complete list.) Once a section or chapter was identified from the table of contents to contain statistical terms, the section or chapter was included for analysis. Next, the index was examined and any statistical terms that fell outside of the pages already identified for inclusion in the analysis were added.

Table 2

List of Statistical Terms and Phrases Used to Examine Index

Bar Graph or Histogram
Best Fitting Line (Linear Regression)
Box and Whisker Plot
Central Tendency
Circle Graph or Pie Chart
Collect or Gather Data
Dispersion
Display Data
Frequency (Relative or Cumulative)
Frequency Distribution Table or Frequency Table
Line Graph
Maximum
Mean
Median
Midrange
Minimum
Mode
Outliers
Quartile (First Quartile, Second Quartile, Third Quartile, Upper or Lower)
Range
Standard Deviation
Stem and Leaf Plot
Variance

Note: Terms and Vocabulary from NCTM Standards (2000) and NCTM Focal Points (2006)

Similarly, all the exercises that correspond to the statistical sections identified for analysis were included in the analysis. Any exercises in the statistical sections that did not pertain to statistical concepts were omitted but the number of exercises that were non-statistical in nature was noted.

The researcher had planned to use the search feature in the e-book versions to locate statistical concepts. However, although e-books are becoming more readily available, some of the textbook series chosen for this study did not have e-book versions.

Thus to maintain consistency in how the statistical topics for analysis in all the textbooks were determined, the researcher excluded the use of the e-book search feature.

The researcher gives a word of caution about search engines. When the researcher initially explored the search function of the e-books, she was cautioned to verify with the publishers the manner in which the search is performed. Different publishers might have different ways of searching and it is important to ascertain that the manner in which the search engines perform are similar to each other before using this feature.

From the books used in the pilot study, after going thru each textbook page by page looking for statistical concepts, only two examples were missed when using the table of contents and index to locate the statistical material. The two examples were embedded in non-statistical sections and only one of the examples was about a statistical concept. The other example used a frequency table but the intent of the lesson was based upon probability. So the use of the table of contents and the index to find where the statistical concepts are located only missed one statistical example and this small amount of error was acceptable.

Excluded Sections of Problems. The researcher excluded any mid-chapter quiz, chapter test, performance assessment or standardized test prep as those items were considered as part of the assessment according to their titles. Assessment items were not part of this study. Also excluded was the student handbook that was part of both commercial textbooks but was not found in the other two textbook series. The student handbook was clearly identified as a remedial or developmental tool for students who were struggling with the material. The focus of this study was the material presented to the average student; therefore the student handbook was excluded from the study.

Textbook Coverage of Statistical Concepts. The first phase of the analysis (See Figure 1 Analytical Framework for Data Analysis) dealt with location and total percentage of statistical content, location of statistical content within the textbook, and the identification of the grade levels in which the statistical concept(s) appeared. To determine the percentage of the textbook devoted to statistics, instructional pages were counted and the total number was recorded. Instructional pages are the pages that do not include the preface, glossary, index, selected answers or table of contents pages. These pages do include the student handbook, extra practice and skills bank. Next, within each statistical lesson, the number of pages devoted to each statistical concept was determined to the nearest quarter of a page. Flanders (1987) applied this method of determining the amount of textbook content in K – 8 textbooks, including an Algebra I textbook, and also noted the subsequent location of the concept in relation to the entire textbook. The location (page number) of the statistical concept in relation to the entire textbook was noted along with the lesson number, as well as whether the material was in its own lesson or was part of another non-statistical lesson.

I will use a process similar to what Flanders (1987) used to examine the textbooks in his study where he denoted what material was new for each grade level. For each textbook series, the statistical content of grade level 6 and 7 from the same textbook series was compared, and any statistical concept that was not found in the 6th grade textbook was classified as different for 7th grade. Similarly, the statistical concepts for grade 8 were compared to both the 6th and 7th level textbooks; again, any concept not found in grades 6 or 7 was considered different. Also, the topic was considered different if the same concept was found in two different grade levels but another aspect of the concept was presented. For example, if the definition of the mean was presented in grade 6 and the property of the mean as a balance point was presented in grade 7, then the mean would be identified as different. The location of the different concept in each 7^{th} and 8^{th} level textbook was recorded. As statistical concepts are introduced in the K-5 grade levels, the material was not classified as new but different; to state confidently that the material was new, an examination of the K- 5 textbooks to verify that said material had not been introduced before would be required, which was beyond the scope of this research.

Lesson Components. The second phase of the analytical framework for data analysis (See Figure 1) dealt with the length of the lesson narrative, the sequence presentation of the statistical concepts, and the components of the statistical lesson. I noted both the length of each statistical lesson and the order in which the statistical concepts were presented. Then each lesson narrative was deconstructed into its lesson components. I used the same lesson components that Jones (2004) used in his study but focused on the statistical lessons. Jones developed the lesson component categories as he examined the probability lesson narratives. So, I started with Jones' (2004) codes, adding or deleting categories as needed as I was coding the lessons, and applied these codes to the entire statistical narrative lessons in the middle grades textbooks. In Jones' (2004) codes, the word probability was replaced with statistics.

Revision of the Lesson Component Categories. My pilot study (See Appendix A) of the first edition of *Connected Mathematics* showed a need for a few new categories not found in the study by Jones (2004). Each booklet starts with the Mathematical

Highlights, which tells the student what he/she will learn followed by the description of the Unit Project to be completed at the end. Because Jones' categories did not include these components, new categories were created: Mathematical Highlights (high), and Unit Project (proj). Also, at the end of each unit are questions to tie the content together. These questions do not fit into the definition of the question category, namely that of "questions within the development of the lesson". These questions are at the end of the unit after the exercises and are meant to summarize what the student has learned. So, a new category called refl for reflection was created.

The categories used by Jones (2004) were sufficient to analyze the 2004 version of *Course 1* by Holt, Rinehart and Winston. Table 3 contains the lesson component codes with the new codes included in italicized bold letters.

Table 3

Lesson Components for Lesson Narrative

Code	Lesson Component
desc	Description of statistical situation that establishes a reason to study or use statistics, in either narrative or pictorial form (such as a cartoon drawing)
defn	Definition of vocabulary or statement of a formula related to statistics
ques	Question(s) contained within the development portion of the lesson that refers to the description of a statistical situation
act	Activity related to statistics that instructs students to physically manipulate materials and/or work with other students
we	Worked example(s), containing questions related to statistics, suggested solution methods, and answers
orex	Oral exercises or problems related to statistics and intended to be completed in class as part of the development of the lesson
prac	Written exercises or problems related to statistics and intended to be completed in class as part of the development of the lesson (such as guided practice); these tasks were similar to the worked examples
high	***Mathematical Highlights tells the student what he/she will learn***
proj	***The description of Unit Project to be completed at the end of the unit***
refl	***Mathematical Reflections at the end of the section and not included in the lesson. Mathematical Reflections are questions designed to help summarize what the student has learned in the unit***

Note. Adapted from "Probability in Middle Grades Textbooks: An Examination of Historical Trends, " by D. L. Jones, 2004, Unpublished doctoral dissertation, University of Missouri-Columbia, Dissertation Abstracts International, AAT 3164516.

To help clarify the coding of the lesson narrative into lesson components, examples are provided.

Examples of Lesson Components. Figure 2 contains a sample lesson narrative along with the coding of its components.

Middle School Math: Course 1 (2005), p.275-276	Coding and Rationale
Players on a volleyball team measured how high they could jump. The results in inches are recorded in the table. 13 23 21 20 21 24 18 Some descriptions of a set of data are called the *range, mean, median* and *mode*.	Desc- This is considered a description of a statistical situation because it describes a statistical situation in real life.
• The **range** is the difference between the least and greatest values in the set. • The **mean** is the sum of all the items, divided by the number of items in the set. (The mean is sometimes called the average.) • The **median** is the middle value when the data are in numerical order, or the **mean** of the two middle values if there are an even number of items. • The **mode** is the value or values that occur most often. There may be more than one mode for a data set. When all values occur an equal number of times, the data set has no mode.	Defn- This is classified as definitions because several statistical terms are defined.

Figure 2 Continued

<table>
<tr>
<td>
Find the range, mean, median and mode of each data set.
A Heights of Vertical Jumps (in.)
| 13 | 23 | 21 | 20 | 21 | 24 | 18 |

Start by writing the data in numerical order.
13, 18, 20, 21, 21, 23, 24

range: 24 – 13 = 11
Subtract least value from greatest value.

mean: 13+18+20+21+21+23+24 = 140
Add all values.
140 ÷ 7 = 20
Divide the sum by the number of items.

median: 21
There is an odd number of items, so find the middle value.

mode: 21 21 occurs most often.

Think and Discuss.

1. Describe what you can say about the values in a data set if the set has a small range.
2. Tell how many modes are in the following data set. Explain your answer.
15, 12,13, 15, 12, 11.
</td>
<td>
We- The solution is clearly worked out for the student, so this would be classified as worked example.

Ques- Because these questions are part of the lesson narrative and not found in the exercises, they are classified as questions.
</td>
</tr>
</table>

Figure 2. Example of a lesson from *Middle School Math: Course 1* (2005) along with the lesson component coding, p. 275-276.
(The words and letters highlighted in yellow were also highlighted in yellow in the textbook.)

Levels of Cognitive Demand. The third phase of the analytical framework for data analysis (See Figure 1) dealt with the statistical exercises. All of the exercises corresponding to the statistical topics were categorized by statistical concept, and level of cognitive demand (Smith & Stein, 1998).

Smith and Stein (1998) proposed four categories and two levels of demand to categorize the cognitive level of mathematical exercises. I applied these categories and levels to all the statistical exercises in lessons in which statistical content was taught. The exercise items are defined as homework problems, exercises, study guides or reviews and or applications, such as “Technology Lab”, “Problem Solving on Location”, or “Math-Ables” found after the lesson components. The statistical concept corresponding to the exercise was noted, and any exercises that did not correspond to a statistical concept were omitted. Also each exercise was evaluated for its cognitive level as research has shown that higher level cognitive demand exercises help facilitate student learning (Doyle, 1983; Stein et al., 1996).

Low-level cognitive demand was divided into two categories: “Memorization” and “Procedures without Connections”. To determine whether or not an exercise problem falls under the “procedures without connections” category, the problem was compared to the examples found within the lessons. If the exercise problem was very similar to the example, then it was classified as low-level “procedures without connections”. What was meant by similar was that a student could mimic the solution to the example without really understanding the problem and still come up with the correct solution. However, if an exercise problem was similar to an example but went beyond what was required in the example, it was classified as higher level if the problem fit within the parameters of the high-level cognitive demand criteria. Examples were not coded as there was no way of determining if the student had seen a similar type before the examples were presented. There are two types of higher level cognitive demand tasks: "Procedures with Connections"; and "Doing Mathematics", which was changed to "Doing Statistics." Higher-level cognitive demand exercises require students to use conceptual ideas to determine which procedure fits the situation the best. Such exercises might also require students to extend or bridge connections between concepts. Examples of exercises of the different levels of cognitive demand follow the descriptions in Table 4.

Table 4

Levels of Cognitive Demand

Lower-level demands (Memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlies the facts, rules, formulas or definitions being learned or reproduced.

Lower-level demands (Procedures without Connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlies the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical (or statistical) understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (Procedures with Connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical (or statistical) concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connection to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations help develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (Doing Statistics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.

Table 4 Continued

- Require students to explore and understand the nature of mathematical (or statistical) concepts, processes, or relationships.
- Demand self-monitoring, or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experience and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Smith, M. S. & Stein, M. K. (1998). Selecting and Creating Mathematical Tasks: From Research to Practice, *Mathematics Teaching in the Middle School*, v3, p. 249.

Examples of Cognitive Demand. Figures 3, 4, and 5 contain examples of Low-level Cognitive Demand exercises from the textbooks used in the pilot study.

Find the range, mean, median and mode of Distance (mi) [5, 6, 4, 7, 3, 5].

Figure 3. Example of Low-Procedure Cognitive Demand from *Middle School Math: Course 1* (2005), p.282.

Figure 3 was considered Low-Procedure as it "[had] no connection to the concepts or meaning that underlies the procedure being used". This example involved simple calculations that used the formulas for finding range and mean and procedures that determined the median and mode.

Use the data in the table to make a line graph. Hint see example 1.

School Enrollment

Year	2000	2001	2002	2003
Students	2,000	2,500	2,750	3,500

(Below is example 1 as shown in the textbook.)

Population of American Colonies

Year	1650	1670	1690	1700
Population	50,400	111,900	210,400	250,900

Example 1 Use the data in the table above to make a line graph.

Step 1: Place years on the horizontal axis and *population* on the vertical axis. Label the axes.
Step 2: Determine an appropriate scale and interval for each axis.
Step 3: Mark a point for each data value. Connect the points with straight line.
Step 4: Title the graph.

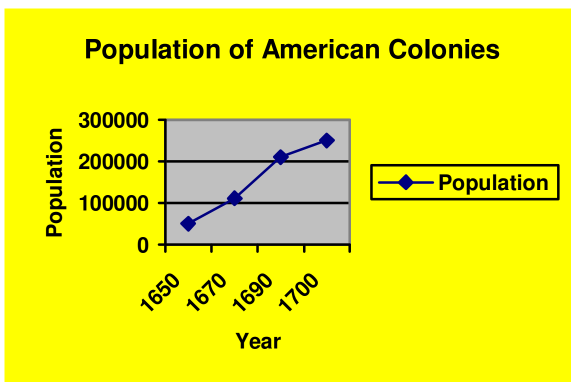

Figure 4. Example of Low-Procedure Cognitive Demand from *Middle School Math: Course 1* (2005), p.297 and 299.

Figure 4 was considered low-level because next to it was a hint to see example 1. In fact it was located under the subtitle of Guided Practice in the exercise set. This was considered low-level because of the given hint to example 1 and the fact that the exercise was very similar to example 1. “A procedure either is specifically called for or is evident from prior instruction…" A student can mimic the process used to obtain the solution by looking at example 1 without having understood the statistical concepts.

In 1-8, tell whether the answers to the question are numerical or categorical data. What is your height in centimeters?

Figure 5. Example of Low-Memorization Cognitive Demand from *Connected Mathematics* (1st edition), p. 26.

Figure 5 involved applying a definition and was considered Low-level.

Examples 6 and 7 were of High-level Cognitive Demand.

A group of middle-school students was asked this question: How many movies did you watch last month? Here …(is a) stem plot of the data: Movies Watched 0| 3 3 5 6 6 7 1| 1 5 6 8 2| B. A new value is added for Lucia, who watched 42 movies last month. This value is an outlier. How does the stem plot change when this value is added? What is the new mean? Compare the mean …(without this value) to the mean after this value is added. What do you notice?

Figure 6. Example of High-Doing Statistics Cognitive Demand from *Connected Mathematics* (1st edition), p. 60.

The exercise in Figure 6 was considered High-Doing Statistics as it "Require[d] students to explore and understand the nature of mathematical (or statistical) concepts, processes, or relationships."

Choose a Strategy. The heights of Maria, Glenn, Carol and Luis are shown in the graph, but the labels are missing. • Maria is neither the tallest nor the shortest. • Glenn is taller than Carol. • There is only one person taller than Luis. Which student's name should go with each bar in the graph?

Figure 7. Example of High-Level Cognitive Demand from *Middle School Math: Course 1*, (2005), p.287.

Figure 7 was considered high-level because “some degree of cognitive effort” was required in that the student had to organize and make sense of the facts given in order to label the graph.

Reliability

To establish reliability, two other researchers co-coded five lessons and the corresponding exercises. Also, after the coding was 75% completed, the researcher recoded a lesson from each textbook series to address coding drift.

Background of co-coders. The first co-coder holds a master’s degree in bio-statistics, has taught at the high school and college levels and has worked with other bio-statisticians. The second colleague has a Master’s in both Mathematics and Mathematics Education and has taught at middle schools, several high schools, universities and has taught AP Statistics.

Training of co-coders. The two textbooks used in the pilot study were used for training purposes. Each co-coder was given the list of statistical terms and phrases (See Table 2 List of Statistical Terms and Phrases Used to Examine Index) and was asked to identify, using the table of contents and index, what sections from the pilot textbooks were included for analysis. Any discrepancies in the lessons included were discussed until the coders agreed upon whether or not the lesson narrative covered a statistical concept. Also, the co-coders counted, within a quarter of a page, the total number of pages devoted to each statistical topic. The findings for the statistical concepts, instructional totals and textbook totals (See Appendix B: Textbook Pages Form) were compared. There were no discrepancies between the textbook totals and the instructional totals. However, when choosing what to include from the table of contents and the index, several items were in disagreement. The first was whether or not Venn diagrams were statistical concepts, and the other was the "value bar graph". The researcher explained to the co-coders that Venn diagrams were considered to be topics in set theory; although the bar graph is in the name of the "value bar graph," the content was not statistical in nature so was excluded from the study.

Next, the co-coders were supplied with the Lesson Components Table (See Table 3). The co-coders were told how the coding worked and the coding criteria. The co-coders were also given a sample coding of a lesson narrative from the pilot study and the rationale behind each coding choice was explained to them (See Appendix C: Lesson Components Worksheet for Lesson Narrative). Next, the co-coders coded a different lesson narrative from the pilot textbook to see if the coding of the same lesson components agreed or disagreed. Only one discrepancy arose concerning the question of whether or not a single sentence between worked examples was part of the previous example or had to be coded separately. Because the sentence did not define a term or describe an application, it was considered a continuation of the previous worked example.

Each co-coder was given the Smith and Stein table of cognitive demand levels (See Table 4). Then, a statistical exercise set from the pilot study that was coded earlier was examined with related discussion about why a certain level of cognitive demand was assigned to each given problem (See Appendix D: Cognitive Levels Worksheet for Exercises). Another exercise set from the pilot study was coded as a trial run and the percentage of problems in which the different coders agreed and disagreed was calculated. Discrepancies were discussed and clarification of cognitive level assigned until a high percentage of agreement (85% or higher) was achieved. Discrepancies existed between when to use LM (low-memorization) and the distinction between an HC (high-connections) and an HS (high-doing statistics) problem. According to Stein and Smith's (1998) cognitive levels, LM (low-memorization) dealt with reproducing previously learned facts, formulas, etc.; HS (high-doing statistics) required more effort and had a clear connection between the procedure and the statistical concept. Despite the discrepancies, there was 93% agreement in the coding of the cognitive levels of the pilot lesson.

Textbook selection for multiple coding. One book from each series was randomly chosen for double coding. I rolled a die for each textbook series until a number less than or equal to four was rolled. I assigned one for grade 6, 2 for grade 7, 3 for grade 8 and if needed 4 for pre-algebra. Next, I assigned a number to each statistical lesson narrative in a chosen textbook and wrote this number on pieces of paper. The numbered pieces were put into a box and shaken well. One piece of paper was drawn and the corresponding lesson narrative was coded by the colleague. This process was repeated for each selected textbook. Next, the researcher's coding for the same lesson narratives and corresponding exercises were compared with the co-coders and a percentage agreement was calculated.

To reduce coding drift, after the researcher had completed 75% of the coding, an earlier lesson from each textbook series was recoded. The discrepancies between the original coding and the re-coding were calculated to verify that the discrepancy percentage was small (less than 10%).

Summary

Chapter 3 discussed the research questions, how the textbooks and lessons were selected and the framework for the content analysis. Also discussed were the procedures for coding the lesson narratives and the corresponding exercises as well as how the researcher addressed issues concerning reliability.

Chapter 4: Findings

The purpose of this study was to determine the scope and sequence of the treatment of statistical topics in middle grades mathematics textbooks. The sample was comprised of four series of textbooks widely used in the United States. Each textbook series was composed of the textbooks slated for grades 6, 7 and 8. Because both the *University of Chicago School Mathematics Project* (UCSMP) and *Connected Mathematics* (CMP) have embedded pre-algebra or algebra within the grades 6, 7, and 8 series and *Glencoe* and *Prentice Hall* do not, the pre-Algebra textbooks for Glencoe and Prentice Hall were also examined. Although the *Connected Mathematics* series is typically provided as booklets and schools can determine the order of the presentation of the booklets, the order suggested by the publishers in a single bound version was used in this study.

Research Questions

The study investigated the following three questions.

1. What is the nature of the statistical content of the curriculum as exhibited in contemporary student mathematics textbooks in use in middle grades (grades 6, 7, and 8) in the United States? How much of the textbooks are devoted to statistical topics? Where are these topics located? Which statistical concepts are covered as individual lessons and which are embedded with other non-statistical topics?
2. What are the sequences of the statistical lesson narratives in

contemporary student mathematics textbooks in use in middle grades (grades 6, 7, and 8) in the United States? What is the typical sequence of lesson narratives for each textbook series? What are the differences and similarities of these lesson narratives across the textbook series?

3. What is the nature of the exercises on statistics in contemporary student mathematics textbooks in use in middle grades (grades 6, 7, and 8) in the United States? What is the level of cognitive demand exhibited by these exercises? How does the level of cognitive demand vary among the different textbook series, and across the grade levels?

Taken together these research questions provide insight into students' potential *opportunity to learn* statistical topics found in middle grades mathematics textbooks.

Chapter Organization

Chapter 4 delineates the results of the study to answer the research questions. For each sampled textbook, the total number of pages devoted to statistical topics, the location of the statistical topics, which topics were presented, the sequence of the lesson components for the statistical topics, the statistical concept the statistical exercises covered and the *level of cognitive demand* required by the student to complete the statistical exercises were analyzed. The chapter concludes with a summary of results. Collectively, these results will be used in Chapter 5 to address the issue of students' potential *opportunity to learn* statistical topics in middle grades mathematics textbooks currently in use in the United States.

Statistical Content Present in Middle Grades Textbooks

The following section addresses the research question: What is the nature of the statistical content of the curriculum as exhibited in contemporary mathematics textbooks in use in the middle grades (grades 6, 7, and 8) in the United States? How much of the textbooks are devoted to statistical topics? Where are these topics located? Which statistical concepts are individual lessons and which are embedded with other non-statistical topics?

Percentage of textbook devoted to statistical topics. For each of the fourteen student editions of the textbooks for the four series, the total number of instructional pages and the total number of pages devoted to statistical topics were noted. Table 5 reports the results of the instructional pages including the lesson and the corresponding exercises. *Connected Mathematics* (CMP) Grade 8 had the highest percentage (16%) of instructional pages devoted to statistical topics and the *University of Chicago School Mathematics Project: Algebra* had the lowest (4.6%). Both the *Connected Mathematics* series and the *Glencoe* series increased the percentage of statistical pages dedicated to statistical concepts in each subsequent year and the *Connected Mathematics* series had the highest percentages for each year. In contrast, in the *UCSMP* series, the percentage of pages dedicated to statistics steadily declined for each subsequent year. The *Glencoe Pre-Algebra*, *Prentice Hall Pre-Algebra*, and *UCSMP Algebra* contained the least amount for statistics, as might be expected given the focus on algebraic concepts.

Table 5

Percentage of Textbook Devoted to Statistical Topics.

Series	Grade	Statistical Pages	Total Instructional Page Count	Percentage of Statistical Pages
Glencoe	Course 1: Grade 6	66.8	668.0	10.0
	Course 2: Grade 7	72.3	664.0	10.9
	Course 3: Grade 8	80.5	664.0	12.1
	Pre-Algebra	77.3	805.0	9.6
	Total Grades 6, 7, 8	219.6	1996.0	11.0
	Total Grades 6, 7, Pre-Algebra	216.4	2137.0	10.1
Prentice Hall	Course 1: Grade 6	51.8	609.0	8.5
	Course 2: Grade 7	65.3	627.0	10.4
	Course 3: Grade 8	60.5	601.0	10.1
	Pre-Algebra	43.3	695.0	6.2
	Total Grades 6, 7, 8	183.6	1837.0	10.0
	Total Grades 6, 7, Pre-Algebra	160.4	1931.0	8.3
UCSMP	Pre-Transition Mathematics: Grade	696.0	764.0	12.6
	Transition Mathematics: Grade 7	56.8	790.0	7.2
	Algebra: Grade 8	38.5	834.0	4.6
	Total Grades 6, 7, 8	191.3	2388.0	8.0
CMP	Grade 6	68.5	602.0	11.4
	Grade 7	87.0	658.0	13.2
	Grade 8	104.0	651.0	16.0
	Total Grades 6, 7, 8	259.5	1911.0	13.6

Note: Percentage = (Statistical Pages)/ (Total Instructional Page Count) • 100

Location of statistical content. Table 6 lists the location of the statistical content for each textbook. For all the textbooks except for *UCSMP Algebra* (Grade 8), which dispersed the statistical content throughout the textbook, the majority of the statistical topics were found in a single chapter. For the sixth grade, both *Glencoe: Course 1* and *Prentice Hall: Course 1* presented statistical topics early in the textbook in the 2^{nd} chapter. Also, only *UCSMP: Pre-Transition Math* (Grade 6) had two chapters devoted to statistical topics, first in chapter 5 and then in chapter 13. The rest of the textbooks presented the statistical topics in the second half of the textbooks and many, such as CMP, presented the statistical topics in the eighth and last booklet for all three grades.

Table 6

Location of Statistical Content for Each Textbook.

	Glencoe		Prentice Hall	UCSMP		CMP
	Lesson		Lesson	Lesson		Lesson
Grade 6	2.2		2.1	5.1	13.2	BK 8 i1
	2.3		2.2	5.2	13.3	BK 8 i2
	2.4		2.3	5.3	13.4	BK 8 i3
	2.5		2.4	5.4	13.5	
	2.6		2.5	5.5	13.6	
	2.7		2.6	5.6		
	2.8		2.7	5.7		
Total # of	2.9		7.8	5.8		
Chapters	***12***		***12***		***13***	***8***
Grade 7	8.1		1.10	1.7		BK 8 i1
	8.2		7.7	1.10		BK 8 i2
	8.3		9.1	12.1		BK 8 i3
	8.4		11.1	12.2		BK 8 i4
	8.7		11.2	12.3		
	8.8		11.3	12.4		
	8.9		11.4	12.5		
	10.3		11.6	12.6		
Total # of			11.7	12.7		
Chapters	***12***		***12***	***12***		***8***
Grade 8	11.2		9.1	1.4		BK 8 i1
	11.3		9.2	1.5		BK 8 i2
	11.4		9.4	1.7		BK 8 i3
	11.5		9.5	4.2		BK 8 i4
	11.6		9.6	5.7		
	11.7		9.7	5.9		
	11.8		9.8	6.7		
	12.5		9.9	7.4		
			10.3	11.8		
Total # of			11.5			
Chapters	***12***		***12***	***13***		***8***
Pre-Algebra	1.6	13.1	12.1			
	2.5	13.2	12.2			
	7.8	13.3	12.3			
	8.9	13.4	12.4			
		13.5	12.5			
Total # of		13.7	12.6			
Chapters		***13***	***12***			

Sequence, Concept Content and Concept Sequence of Statistical Lessons

This section addresses the research question: What are the sequences of the statistical lesson narratives in contemporary student mathematics textbooks in use in middle grades (grades 6, 7, and 8) in the United States? What is the typical sequence of lesson narratives for each textbook series? What are the differences and similarities of these lesson narratives across the textbook series?

Sequence of lesson components of the statistical lessons. The lesson components for each textbook series have been compiled into a graphical display in order to ascertain if any lesson sequence patterns exist.

Glencoe sequence of lesson components. Each chapter in Glencoe opened with *Big Ideas* or key concepts for the chapter. *Key vocabulary* was followed by an application for the concepts in *Real world link* (labeled *Why?* in the pre-algebra textbook) and a *study organizer* (omitted in the pre-algebra textbook); each chapter opener concluded with a readiness quiz. Then, the actual lessons started.

Figure 8 shows the sequence of the lesson components for the Glencoe textbook series. All but one lesson started with description(s) of statistical situations followed by question(s). This introduction to the lesson was followed by either definitions of statistical terms or worked examples. The body of the lesson consisted of worked examples followed by practice problems. There were a few examples with no work shown and in some lessons (17.6%) a concept summary was present. The concept summary listed the key points or ideas pertaining to the statistical topic under discussion.

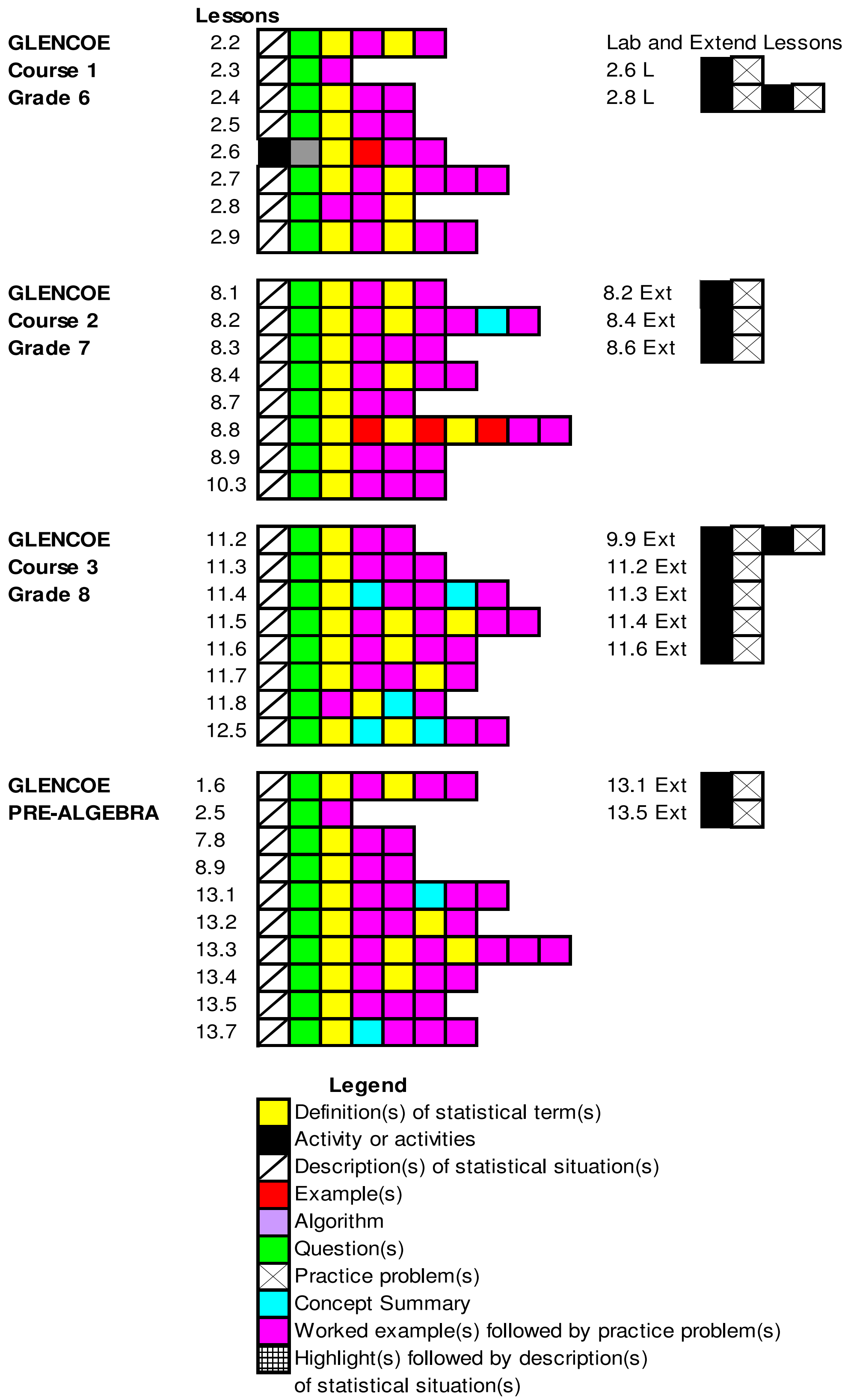

Figure 8. Instructional Sequence of Lessons for the Glencoe Series.

The Lab and Extend lessons were similar in that worked examples were followed by practice problems and these two components composed the entirety of the lesson.

Prentice Hall sequence of lesson components. In the *Prentice Hall* textbook series, each chapter started with a review entitled *What You've Learned,* followed by *Check Your Readiness*. There was then a short blurb on what was coming up and a list of vocabulary.

Figure 9 displays the lesson components for the *Prentice Hall* textbook series. Each textbook in the *Prentice Hall: Course 1, 2,* and *3* series followed a similar lesson pattern. Each lesson started with statements of what the student would learn followed by explanations about why the students should learn the concepts, including examples of real world applications of statistical situations. Any statistical definitions were followed by worked examples and practice problems. As in the *Glencoe* series, the *Prentice Hall* series ended each lesson with worked examples and practice problems. In fact, similar to the *Glencoe* series, the *Prentice Hall* series lessons were composed mainly of worked examples followed by practice problems. However, the *Prentice Hall: Pre-Algebra* lessons were different in that the lessons started either with descriptions of statistical situations or definitions of statistical terms. These were followed by worked examples and practice problems with the majority of the lesson composed of worked examples and practice problems. The length of the lesson narrative ranged from 1 to 3 pages.

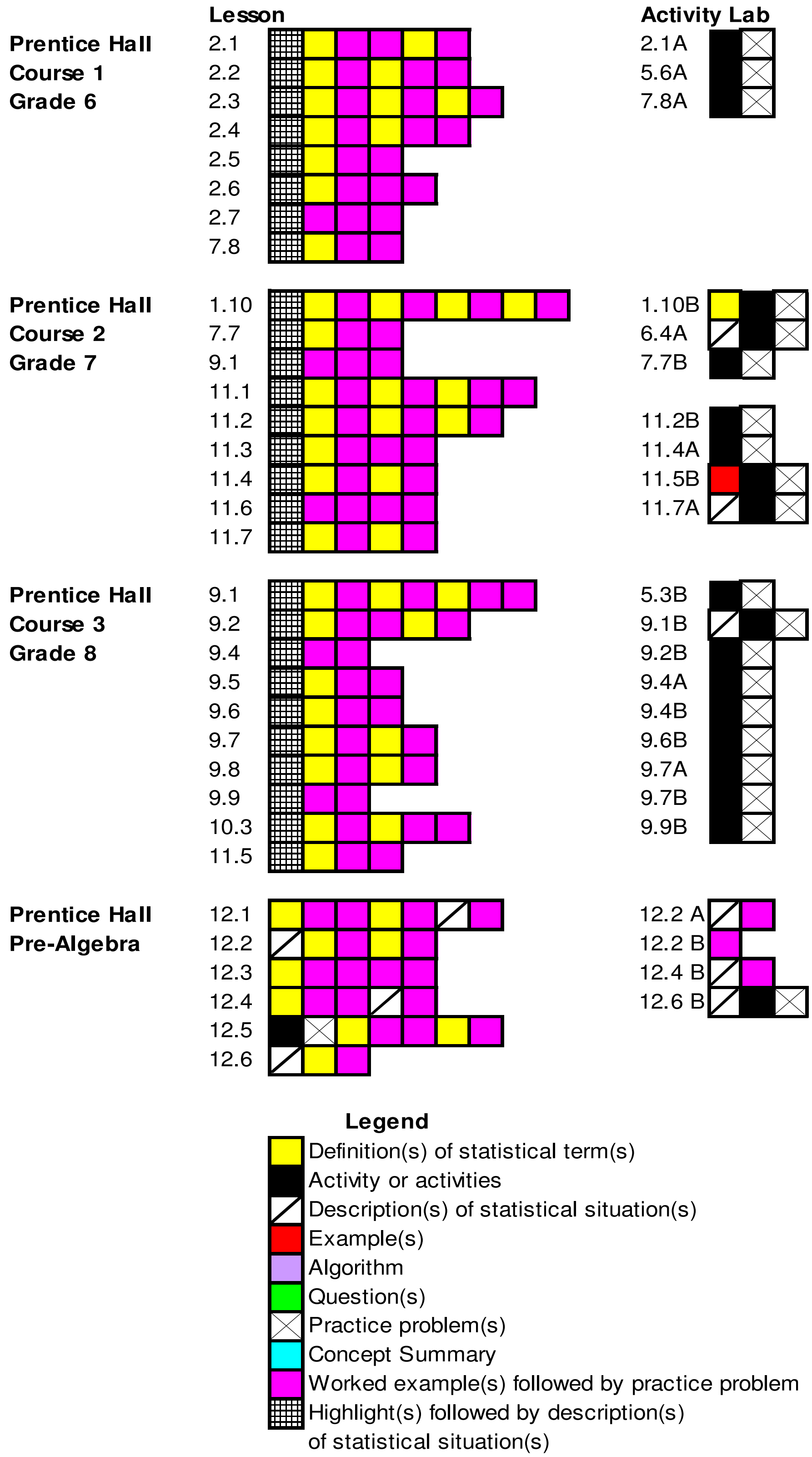

Figure 9. Instructional Sequence of Lessons for the Prentice Hall Series.

The Activity Lab lessons (Figure 9) were also similar to the *Glencoe* series lab or extend lessons in that the majority of the lessons were activities followed by practice problems. Activity Labs that were found before the actual lesson were labeled A and activity labs after the lesson were labeled B. Activity Labs (A) before the lessons were designed to prepare the student for the upcoming lesson and activity labs (B) after the lessons were designed to reinforce the concepts presented in the lesson.

Connected Mathematics sequence of lesson components. The *Connected Mathematics* series consisted of eight booklets with statistical topics found in the eighth booklet. Each booklet was composed of a *Unit Opener* or descriptions of real world applications, followed by *Mathematical Highlights* of what the student was expected to learn, and lessons called investigations. The investigations were followed by exercises labeled *Applications, Connections, Extensions* to emphasize the difference in intent of the problems. Few purely computational problems were given; instead computations were embedded into real life applications. *Connections* problems "connected" the concepts in a particular investigation with other investigations and the *Extension* problems resembled real-world activities. Also the lessons in *Connected Mathematics* were the longest of the four textbook series, ranging from 4 to 15 pages with almost no worked examples provided. After the exercises, the *Mathematical Reflections* briefly reiterated the big idea or concept of the investigation and this section was followed by questions that guided students to summarize what they had learned. Because the *Mathematical Reflections* were located after the exercises, this component was not coded into Figure 10. Only the lesson components found before the exercises were coded into the lesson component display.

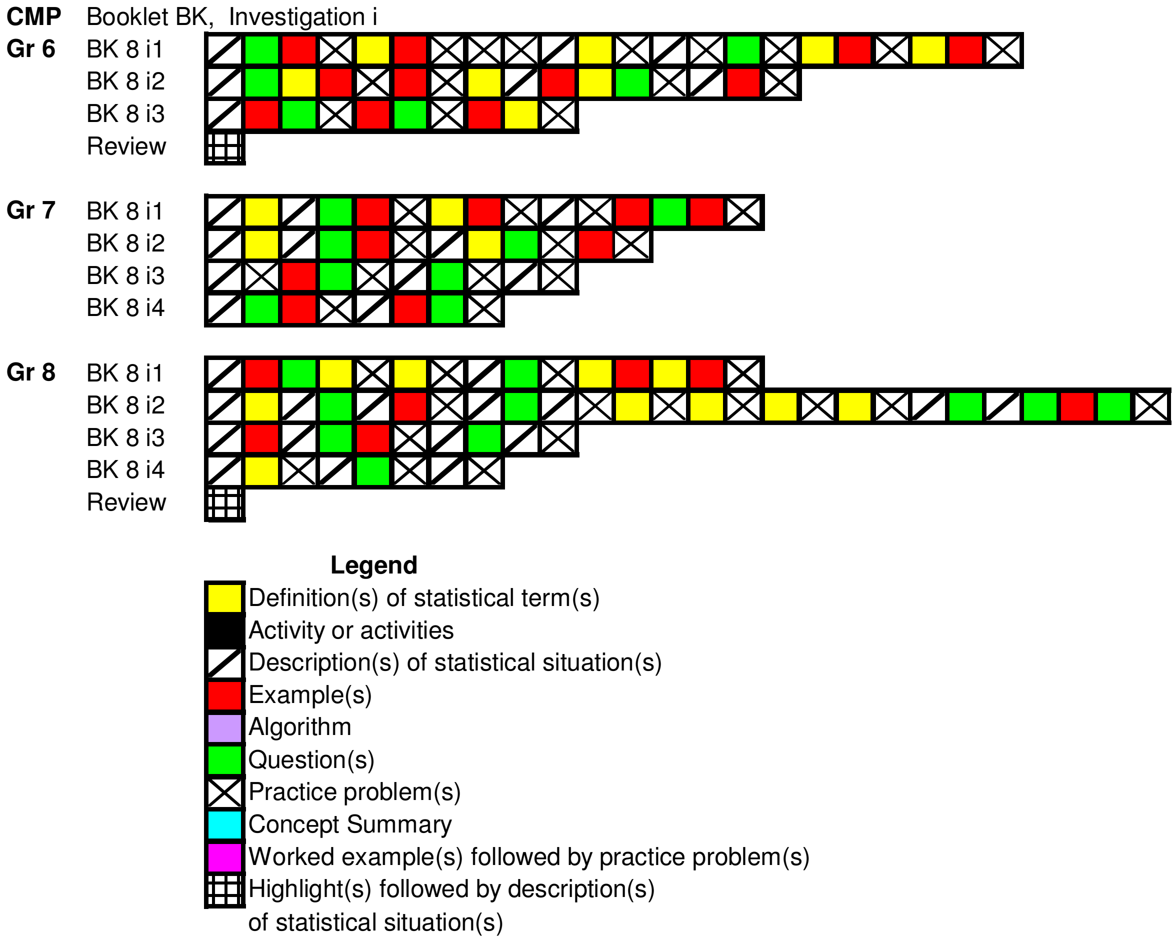

Figure 10. Instructional Sequence of Lesson for Connected Mathematics.

After the investigations there was a unit review that consisted of questions concerning what the student had learned which gave the students a preview of what lay ahead. The lesson pattern for the investigations was not as clear as in the *Glencoe* and *Prentice Hall* series. Each investigation started with descriptions of statistical situations and ended with practice problems, with the body of the investigations composed mainly of examples, questions, more descriptions of statistical applications and definitions of statistical terms. At the end of each booklet was a *Unit Project* which consisted of an activity designed to incorporate the key ideas in the booklet.

UCSMP sequence of lesson components. Each chapter in the *University of Chicago School Mathematics Project* series began with descriptions of real world applications centered on the concepts for the chapter lessons and ended with projects. The body of the lessons had the same components as the other textbook series but in a more loosely ordered pattern. The lessons contained fewer worked examples and more questions. Also, activities were found within the lessons. Most of the lessons started with either definitions of statistical terms or descriptions of statistical situations. The body of the lessons was composed of examples, questions, practice problems, activities, definitions and descriptions of more statistical applications. Also, the *University of Chicago School Mathematics Project: Algebra* textbook yielded a new lesson component category I labeled algorithms. The first statistical algorithm was found in Lesson 1-7 and was entitled *An Algorithm for the Mean Absolute Deviation*; as the title implies it gave a step by step procedure for finding the mean absolute deviation. The second statistical algorithm was in Lesson 11-8 and delineated the steps to calculate chi-square statistics. Each chapter ended with projects that encompassed the concepts taught within the chapter's lessons.

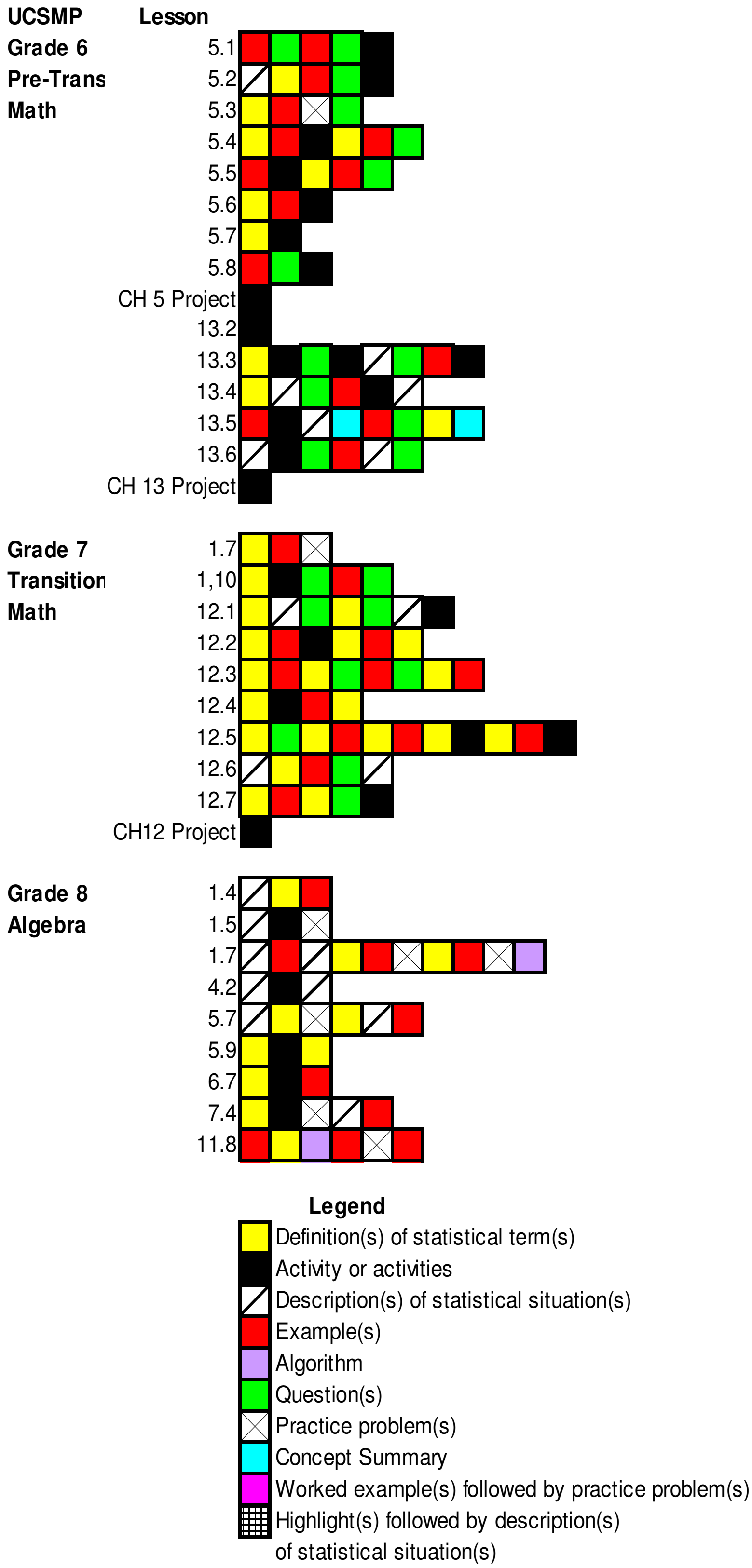

Figure 11. Instructional Sequence of Lessons for University of Chicago School Mathematics Project.

Summary of sequence of lesson components. As shown in the graphical displays, both the *Glencoe* and *Prentice Hall* series showed a distinct sequence of lesson components that were similar to each other. By contrast, both the *Connected Mathematics* and the *University of Chicago School Mathematics Project* series' lesson components were more loosely structured in that the body of the lesson was composed of certain components but not necessarily in the same order for every lesson.

Concept content of statistical lessons. The lessons were categorized into five concept categories: measures of central tendency, dispersion, graph types, best fitting line, and other. The measures of central tendency category consisted of lessons that dealt with the mean, median, mode, or the idea of central tendency. Figure 12 displays an example of an exercise that dealt with the idea of central tendency (typical) found in *Connected Mathematics: Grade 6*, booklet 8, investigation 2.

> The table below shows some of the Students Ages, Heights, and Foot Lengths data in centimeters. The table includes two new columns. Copy and complete the table to show heights and foot lengths in meters.
>
> a. Round the height for each student to the nearest tenth of a meter.
> b. Make a line plot showing the rounded height data.
> c. What is the typical height for these students in meters? Explain.

Figure 12. An example of an exercise from *Connected Mathematics* centering on the idea of typical, p. 43.

The dispersion category encompassed lessons that dealt with inter-quartile range, quartile 1, quartile 2, quartile 3, percentile, range, minimum, maximum, clusters or gaps in data, mean absolute deviation (MAD), or any other idea that touched upon dispersion. Graph types were lessons that concerned line graphs, line plots, stem and leaf plots, double bar graphs, double line graphs, tables, frequency distribution tables, circle graphs, box plots, five number summary, or anything else that involved some sort of graph. The best fitting line category consisted of lessons that looked for patterns or trends in the data to predict future or missing datum from the data set, scatter plots, or explored relationships between two variables (correlation).

These four categories were derived from the statistical content recommended for inclusion into the middle grades curriculum by the guiding documents (Common Core State Standards, 2010; NCTM Focal Points, 2006; NCTM Standards, 2000). Statistical concepts that did not fit into these four categories of central tendency, dispersion, best fitting line and graph types were classified as *other*. Statistical concepts, such as different types of sampling, randomness, and surveys were classified under the *other* category.

Figures 13 and 14 depict the total number of pages for each series by statistical concept. Across all the grades and textbook series, more lesson pages were devoted to graph types than any other statistical category.

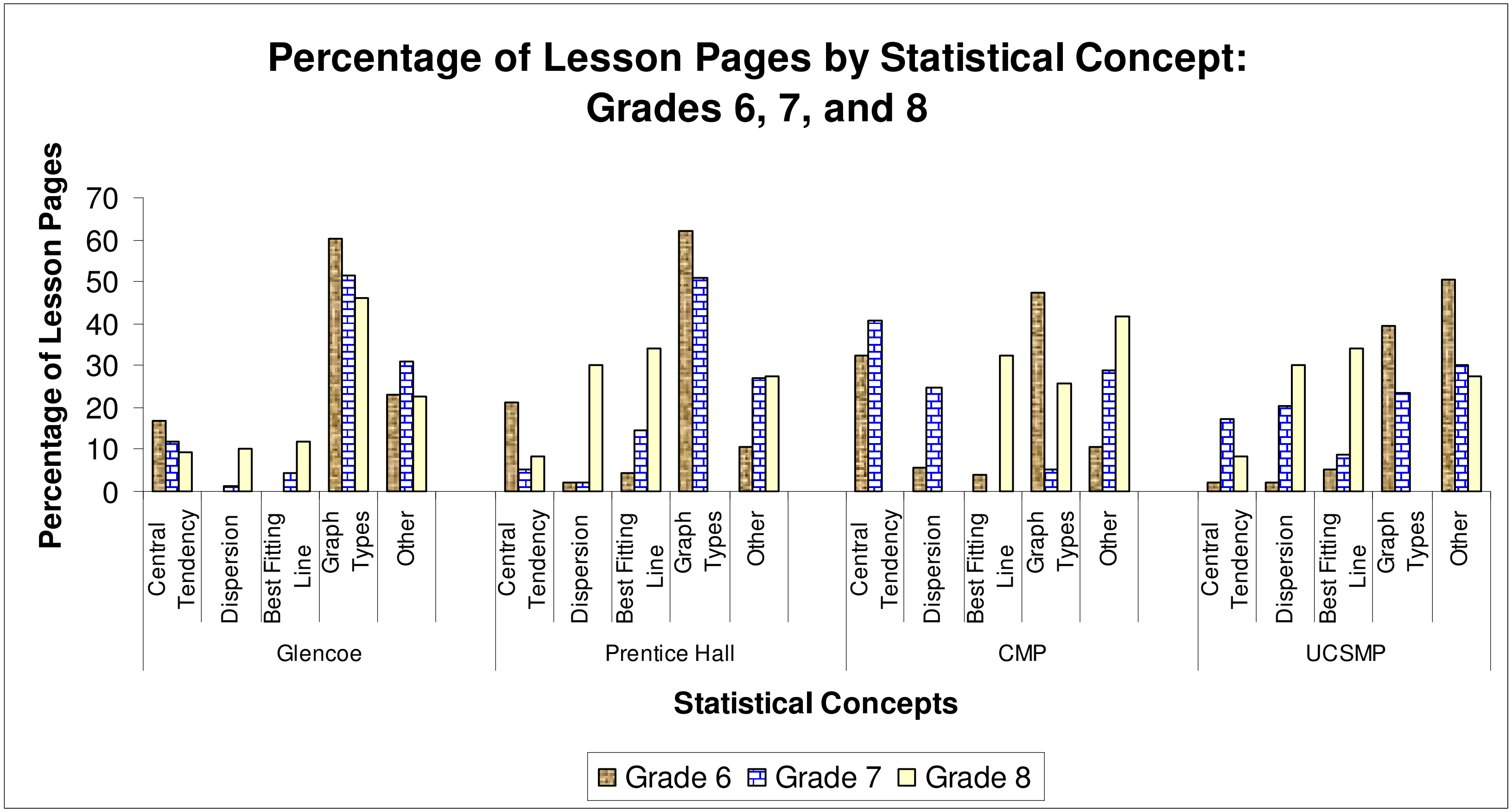

Figure 13. Percentage of Lesson Pages by Statistical Concept: Grades 6, 7, and 8.

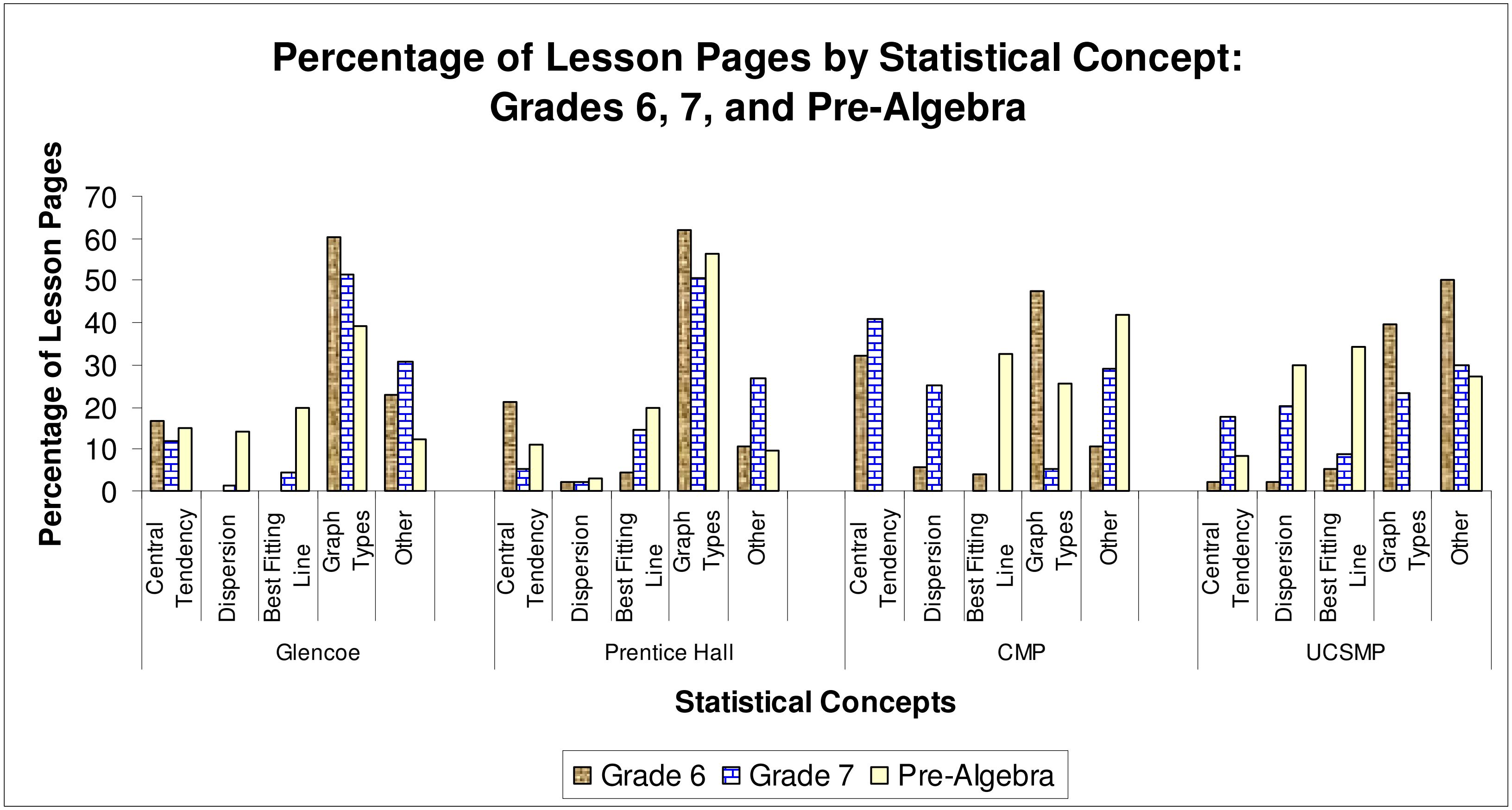

Figure 14. Total Lesson Pages by Statistical Concept: Grades 6, 7, and Pre-Algebra.

For the 6th grade, both *Glencoe: Course 1* and *Prentice Hall: Course 1* had the greatest percentage of lesson pages for the central tendency category compared to the other grades within the textbook series. By contrast, both *Connected Mathematics Grade 7* and *UCSMP: Transition Mathematics* had the highest percentage of lesson pages for central tendency within these respective textbook series. For both the dispersion and best fitting line categories, the *Glencoe* and *Prentice Hall* textbook series left those lessons for grade 8 and Pre-Algebra. *Connected Mathematics* covered dispersion in grade 7 and omitted this concept in grade 8. *UCSMP* covered dispersion in both grades 7 and 8.

Both *Connected Mathematics* and the *UCSMP* series had higher percentage of lesson pages in the *other* category than the *Glencoe* or *Prentice Hall* series. For *Connected Mathematics* the *other* category incorporated topics on categorical data, numerical data, convenience sampling, systematic sampling, voluntary response sampling, and sample size. For *UCSMP* the *other* category involved topics on collecting data, creating survey questions, conducting surveys, categorical data, deviations from the mean, tolerance, time series, mean absolute deviations, exponential regression, and chi-square statistics.

Concept Sequence of Statistical Lessons. In specific national documents, statistical concepts have been recommended for coverage for each grade level (Common Core State Standards, 2010; NCTM Focal Points, 2006; NCTM Standards, 2000). According to the NCTM Focal Points (2006), statistics is not a focal point for grade 6 but students should be able to create and analyze line and double bar graphs as well as recognize the relationship between ordered points. In grade 7, students should be able to extrapolate from a sample or proportion to the population, create and analyze histograms and circle graphs (NCTM Focal Points, 2006). By the end of grade 8, students should be able to see data as an aggregate, recognize the ideas of central tendency, create box and whisker plots, analyze the spread of the data, and determine lines of best fit from a scatter plot. Figure 15 shows the location of the statistical concept within the statistical lessons. The statistical lessons were divided into thirds and the location where each concept was presented has been plotted. The figure does not include statistical concepts that would be categorized as *other*. Some concepts were presented in several grade levels, such as the histogram, circle graph, and box and whisker plot. Only concepts of central tendency were found in all the textbooks.

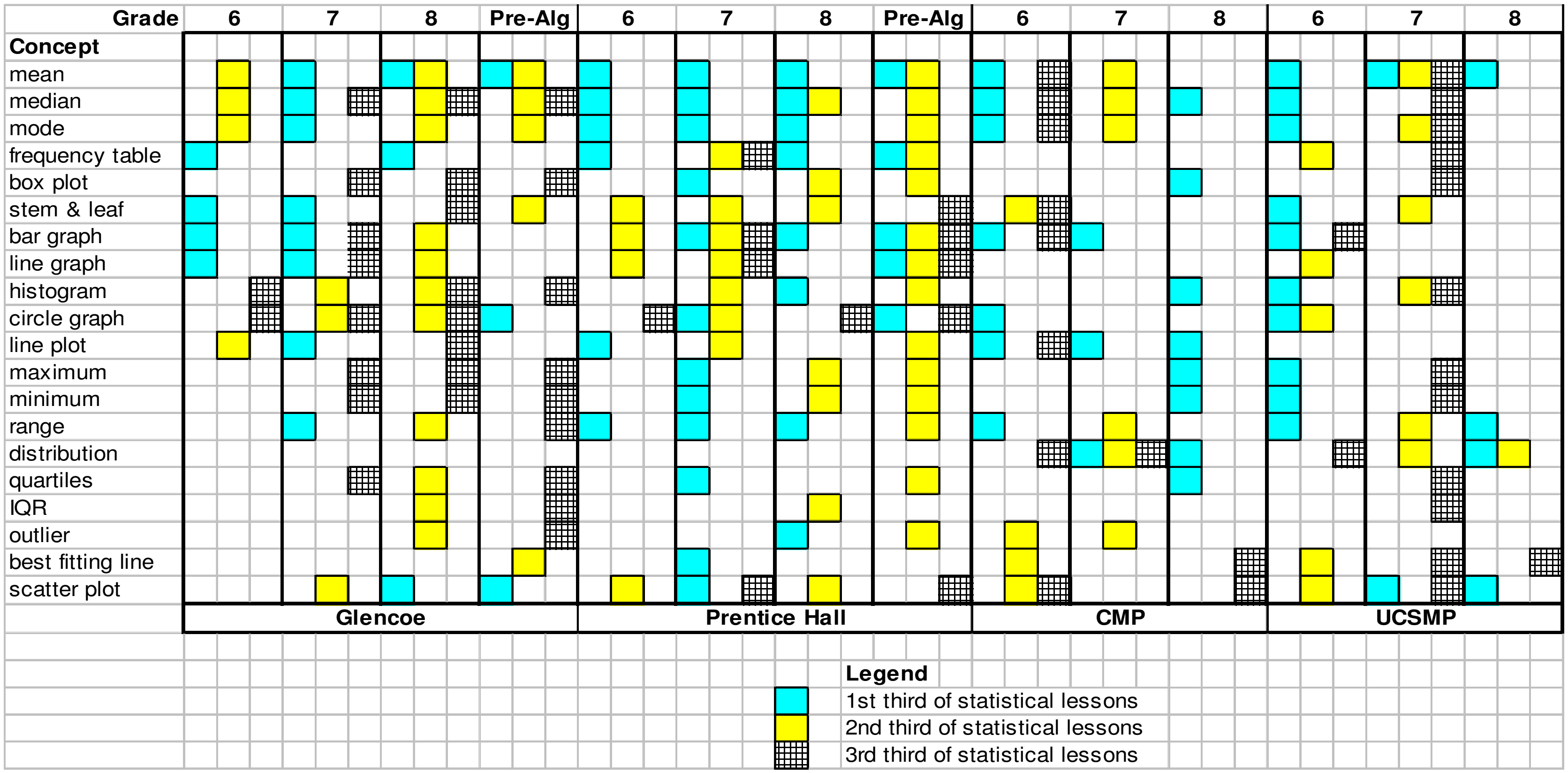

Figure 15. Location of Statistical Concept within the Statistical Lessons.

Cognitive Levels and Statistical Concept of Exercises.

This section addresses the research question: What is the nature of the exercises on statistics in contemporary student mathematics textbooks in use in middle grades (grades 6, 7, and 8) in the United States? What is the level of cognitive demand exhibited by these exercises? How does the level of cognitive demand vary among the different textbook series, and across the grade levels?

Cognitive levels of statistical exercises. The statistical exercises corresponding to the statistical lessons for each textbook have been compiled into graphical displays showing the percentage allocation of each level of cognitive demand for grades 6, 7 and 8, and for grades 6, 7 and pre-Algebra. The four levels of cognitive demand were: LM (low-memorization), LP (low-procedures without connections), HC (high-procedures with connections) and DS (high-doing statistics) (Smith & Stein, 1998; Stein & Smith, 1998).

Cognitive levels of demand of statistical exercises for grades 6, 7 and 8. Across all textbook series for grades 6, 7 and 8, the highest percentage of exercises were procedures without connections. This percentage was followed by exercises that required procedures with connections (See figure 16). The percentage of exercises involving memorization ranged from 2% for *Glencoe: Course 3* (Grade 8) to 8.8% for *UCSMP: Algebra*. Few exercises involved high cognitive level doing statistics (DS), with percentages that ranged from 0% for *Glencoe: Course 3, Prentice Hall: Course 2; Course 3* and the *Connected Mathematics* series to 8.8% for *UCSMP: Algebra*. In fact the *UCSMP* textbook series had the highest percentages of exercises overall and for each grade level for the DS category.

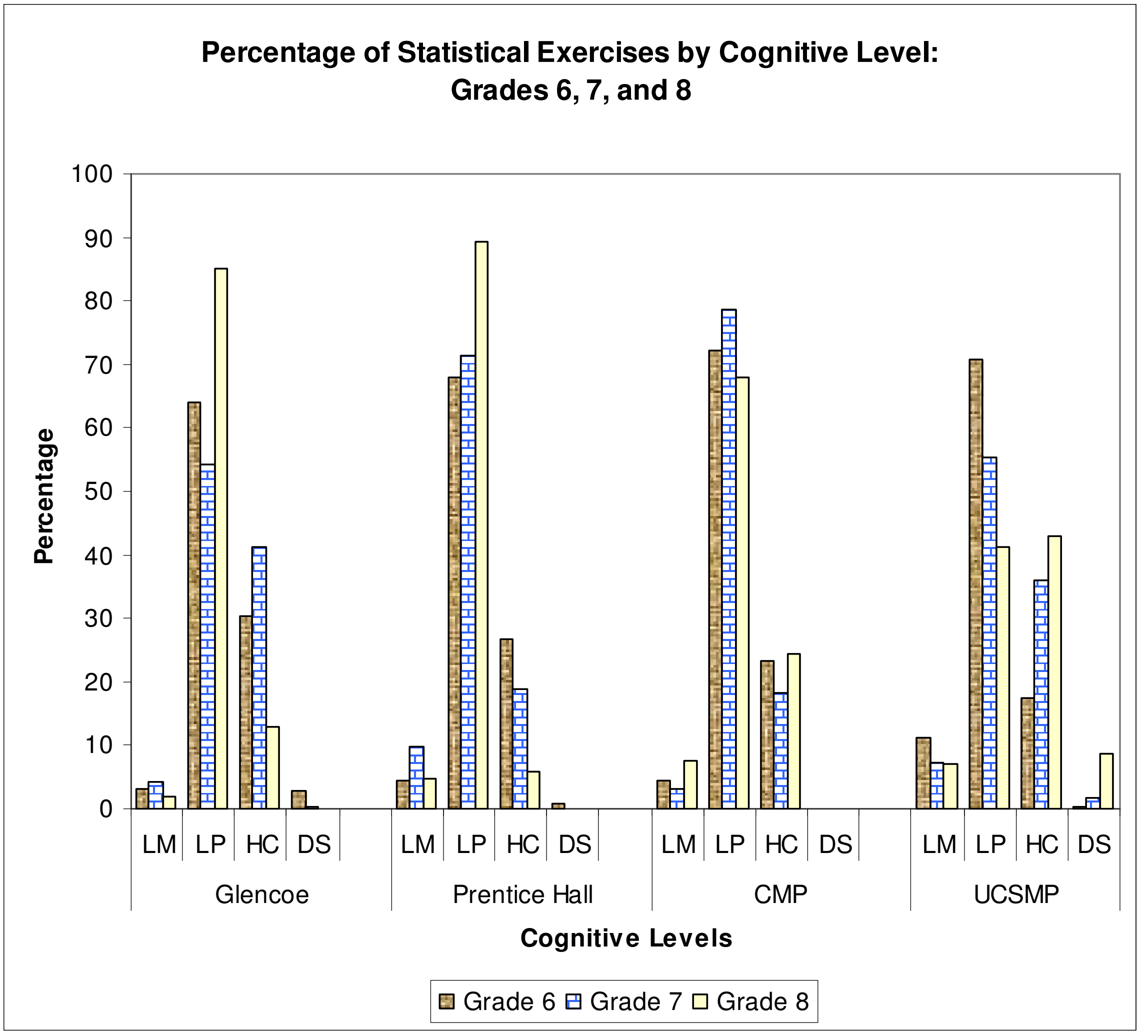

Figure 16. Percentage of Statistical Exercises by Cognitive Level: Grades 6, 7, and 8.

Cognitive levels of demand of statistical exercises for grades 6, 7 and Pre-Algebra. Figure 17 shows the compilation of the cognitive demands of the exercises for grades 6, 7, and pre-algebra. Most exercises for all the textbook series involved low cognitive level procedures without connections (LP) with percentages ranging from 41.2% for *UCSMP: Algebra* to 84.1% for *Glencoe: Pre-Algebra*. Exercises that involved high cognitive level procedures with connections (HC) had percentages that ranged from

4.2% for *Glencoe: Pre-Algebra* to 18.3% for *Connected Mathematics Grade 7* to a high of 43% for *UCSMP: Algebra.*

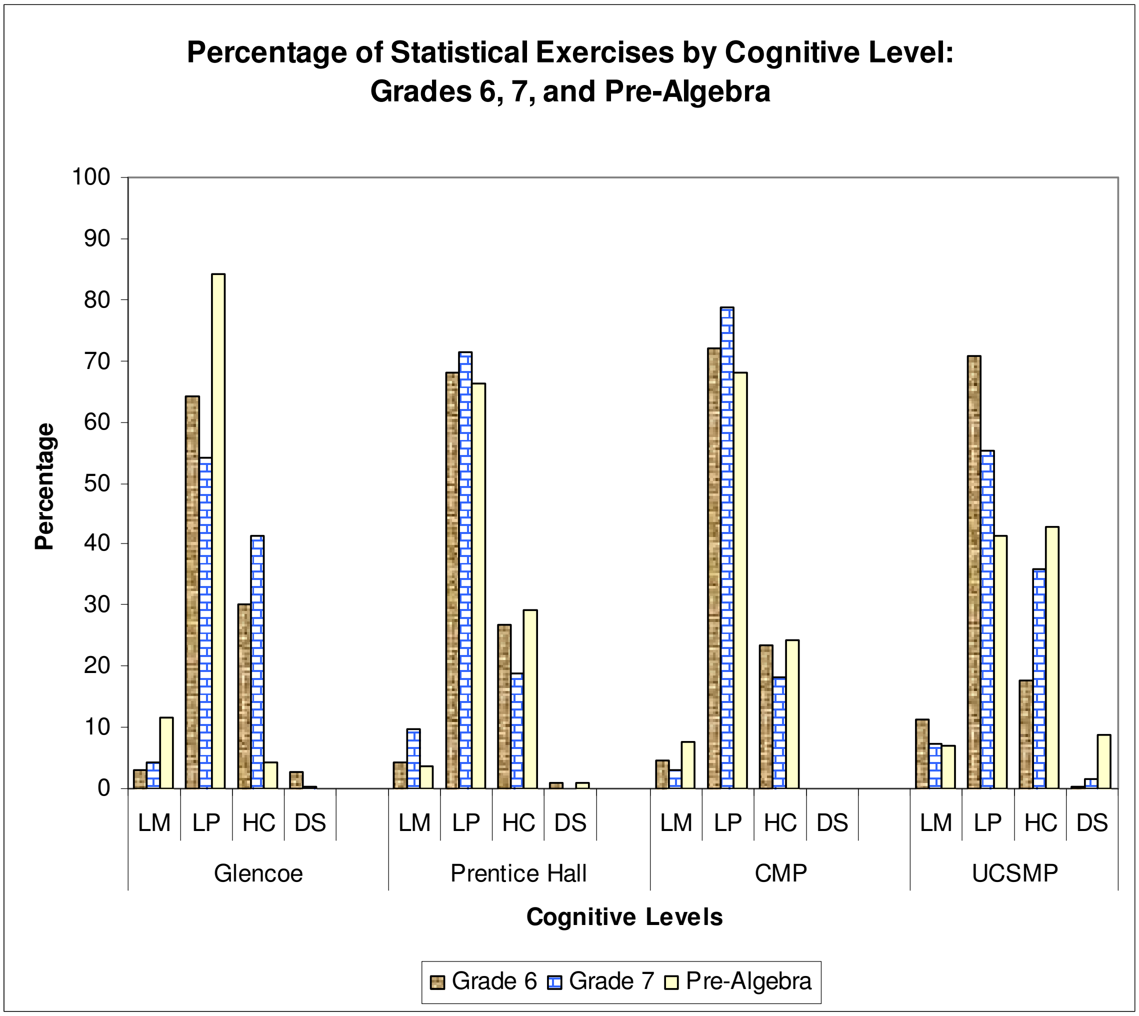

Figure 17. Percentage of Statistical Exercises by Cognitive Level: Grades 6, 7 and Pre-Algebra.

Exercises that required memorization (LM) ranged from 3.1% for both *Glencoe: Course 1* (Grade 6) and *Connected Mathematics Grade 7* to 11.2% for *UCSMP: Pre-Transition Mathematics* and 11.7% for *Glencoe: Pre-Algebra* (See figure 17). Once again the high level cognitive demand exercises of doing statistics (DS) had the lowest percentages.

Statistical concept of exercises. Figures 18 and 19 depict the total percentage of the allocation of the statistical exercises according to statistical concept. The exercises were categorized into the same five concept categories that were used in the lessons: central tendency, dispersion, graph types, best fitting line, and other. Most exercises corresponded to graph types or central tendency. *Glencoe* had no exercises for dispersion until grade 8 and pre-Algebra but *Connected Mathematics* had the most dispersion exercises in grade 6. Also, *Glencoe* did not have any exercises for the best fitting line category until grade 8 and Pre-Algebra. But the other series had best fitting line exercises for every grade level.

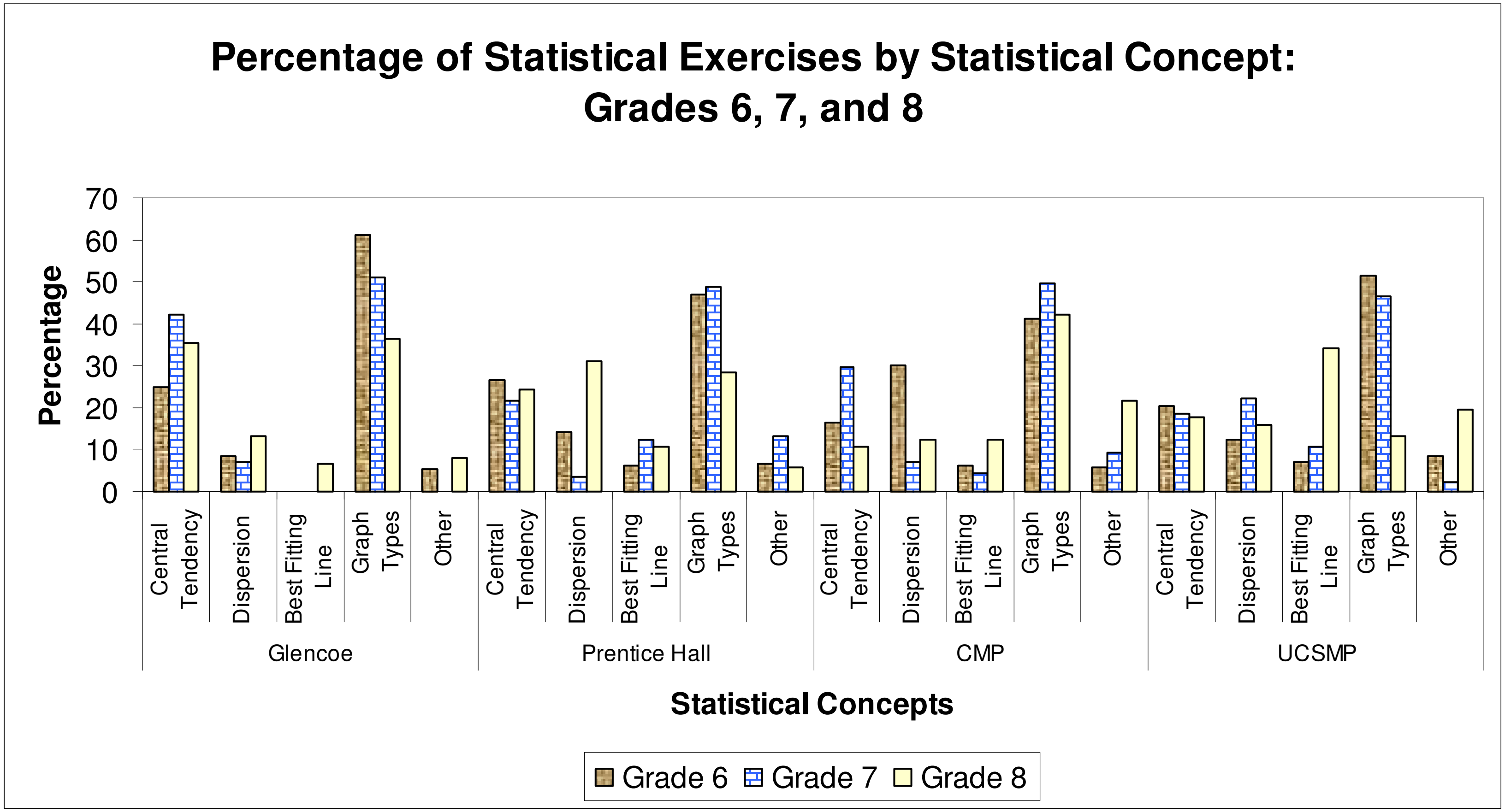

Figure 18. Percentage of Statistical Exercises by Statistical Concept: Grades 6, 7, and 8.

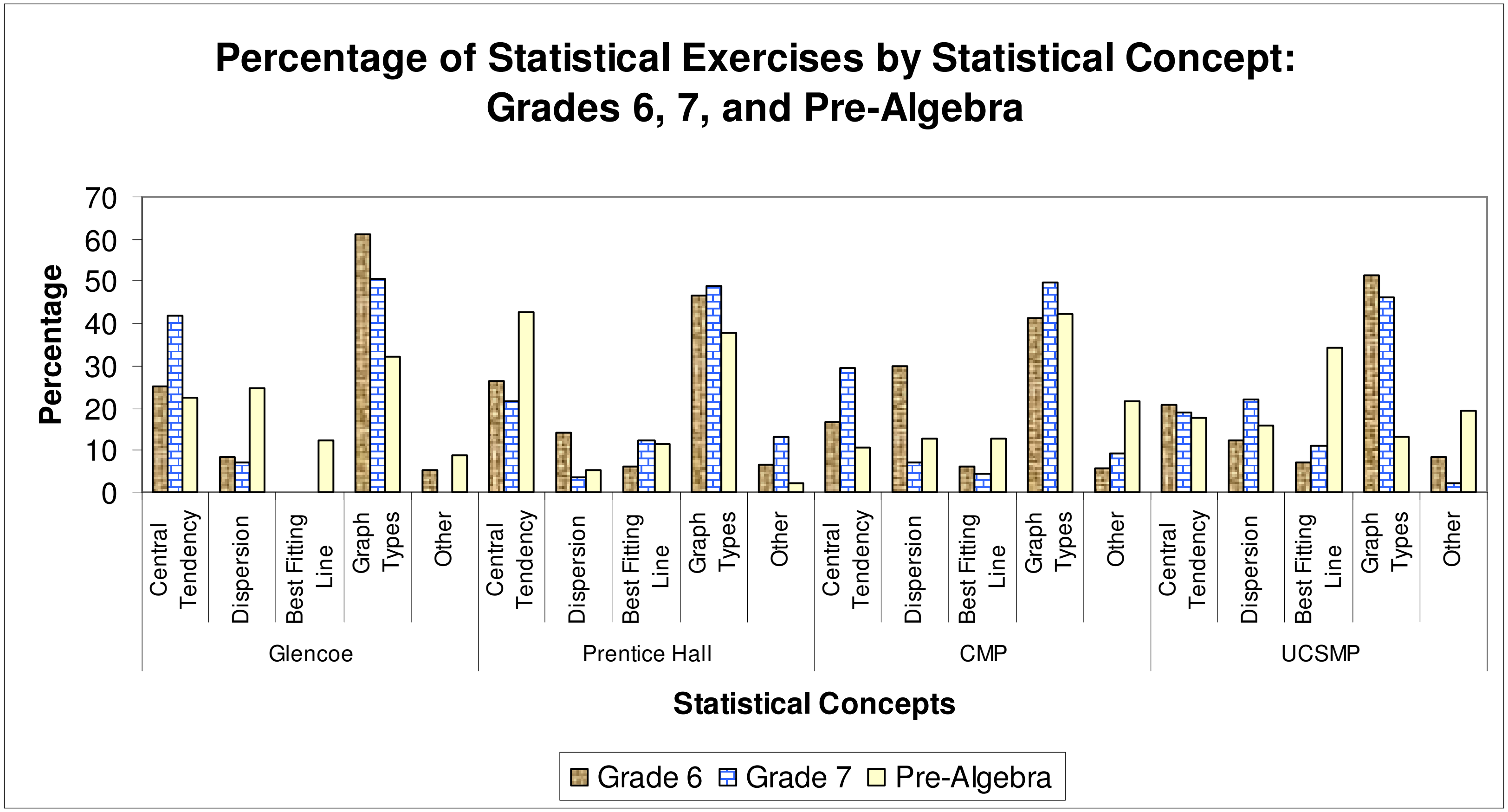

Figure 19. Percentage of Statistical Exercises by Statistical Concept: Grades 6, 7 and Pre-Algebra.

Summary.

This chapter presented the findings related to the analysis of the treatment of statistical topics in four textbook series used in the middle grades. This summary is composed of three main sections corresponding to the research questions of a) where the statistical lessons are located and how much of the textbook is devoted to statistics, b) the length and the lesson components of the lesson narrative, and the sequence of the presentation of the statistical concepts and c) the statistical concepts of the exercises and the cognitive level of demand required of the students to complete the statistical exercises.

Location and Amount of Statistical Content. For all the textbooks analyzed, the statistical lessons were contained in a single chapter with one exception. In *UCSMP: Algebra* the statistical lessons were found throughout the textbook embedded in non-statistical chapters. Furthermore, *UCSMP: Pre-Transition Mathematics* contained two chapters dedicated to statistics. For *Glencoe: Course 1* and *Prentice Hall: Course 1* the statistical material was found within the second chapter. The rest of the textbooks placed the statistical lessons near the end of the textbook and *Connected Mathematics* placed the material in the last booklet for all three grades.

The *Connected Mathematics* series had the highest total percentage of lessons dedicated to statistics and the *Glencoe: Pre-Algebra, Prentice Hall: Pre-Algebra* and *UCSMP: Algebra* books had the lowest percentage. Both the *Glencoe* and *Connected Mathematics* series increased the amount of statistical content for the upper grades while the amount in the *UCSMP* series steadily decreased. Overall, though, most of the textbooks had around 10% of the lessons dedicated to statistics.

Lesson Narrative and Statistical Lessons By Concepts. For all four textbook series the highest percentage of lesson pages was devoted to the category of graph types. Both *Connected Mathematics* and *UCSMP* had the highest percentage of pages designated for the category of "other". Also, *Glencoe* and *Prentice Hall* increased the percentage of lesson pages for both dispersion and best fitting line in subsequent grades and decreased the amount of coverage for central tendency. In contrast, both *Connected Mathematics* and *UCSMP* increased the percentage of lesson pages in subsequent grades for both dispersion and best fitting line. Also, within the series, both *Connected Mathematics* and *UCSMP* gave the highest percentage of lesson pages to central tendency in the seventh grade.

For the lesson components of the lesson narratives, both the *Glencoe and Prentice Hall* series had clear lesson patterns that were similar to each other. *Glencoe* lessons started with descriptions of statistical situations followed by questions. The body of the lesson consisted of worked examples followed by practice problems and definitions. *Prentice Hall* lessons started with highlights of what will be learned followed by descriptions of statistical situations. Similar to *Glencoe*, the body of the *Prentice Hall* lessons were mostly worked examples, practice problems and definitions. Both *Glencoe* and *Prentice Hall* lessons ended with practice problems. By contrast, *Connected Mathematics* booklets had a set pattern but the investigations or lessons were more loosely structured. The body of the investigations was composed of the same lesson components but the order of the components varied. Each investigation started with definitions or descriptions of statistical situations with few worked examples. The lessons were the longest out of all the series and ended with practice problems. *UCSMP* lessons also started with definitions or descriptions of statistical situations. Here again the lesson pattern of the narrative was more varied. Activities were found within the lessons and more questions were incorporated in the body of the lessons. Also, statistical algorithms were found in the *UCSMP: Algebra*.

Cognitive Level of Demand and Statistical Concepts of Exercises. Figures 20 and 21 depict the overall percentage totals for the cognitive level of demand for the statistical exercises for each textbook series. Across all four series most of the statistical exercises exhibited low level cognitive demand of procedures without connections (LP). The next highest percentages were for exercises that required high level cognitive demand of procedures with connections (HC). Exercises with low level cognitive demand of memorization (LM) accounted for 10% or less of the statistical exercises and few exercises were found for the high level cognitive demand of doing statistics (DS). In fact, *UCSMP: Algebra* had the highest percentage of DS exercises at 8.8%, which was almost three times the next highest percentage for this category.

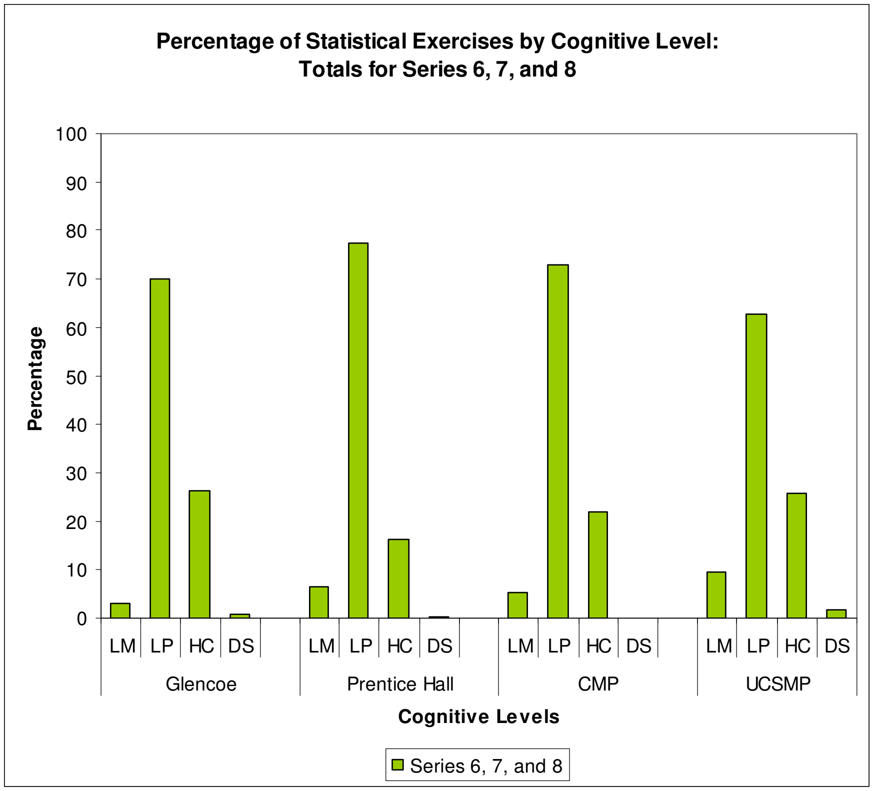

Figure 20. Percentage of Statistical Exercises by Cognitive Level: Totals for Series 6, 7, and 8.

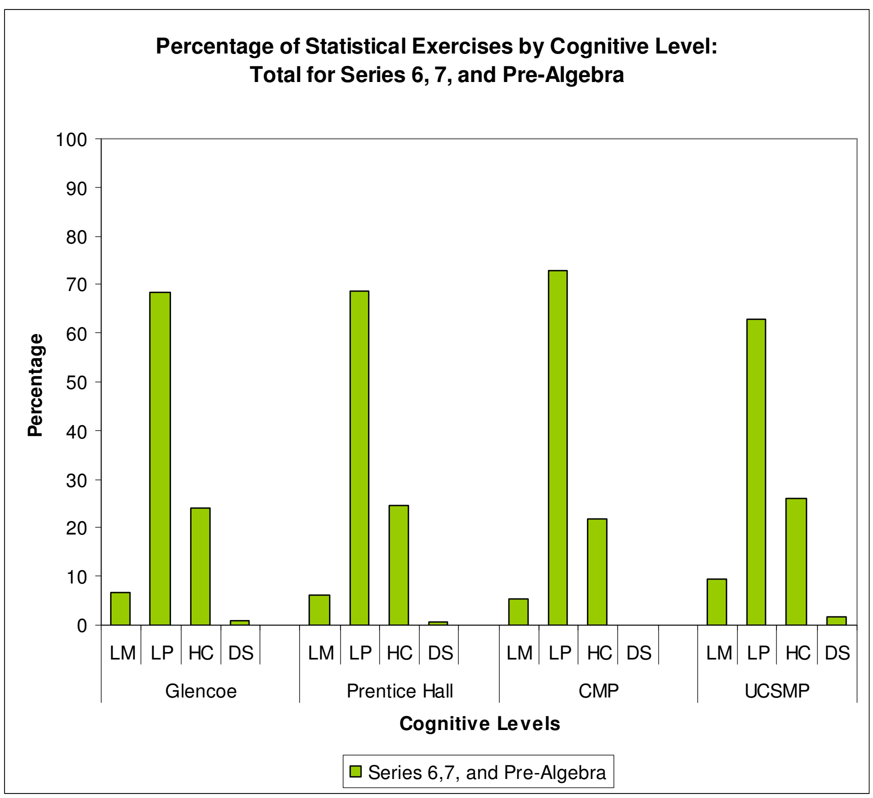

Figure 21. Percentage of Statistical Exercises by Cognitive Level: Totals for Series 6, 7, and Pre-Algebra.

Figures 22 and 23 depict the results of the totals for each series of the statistical exercises according to statistical concepts. The majority of the exercises were for graph types such as line plots, histograms, box plots and circle graphs. Few exercises were concerned with dispersion, best fitting line and the category of “other”.

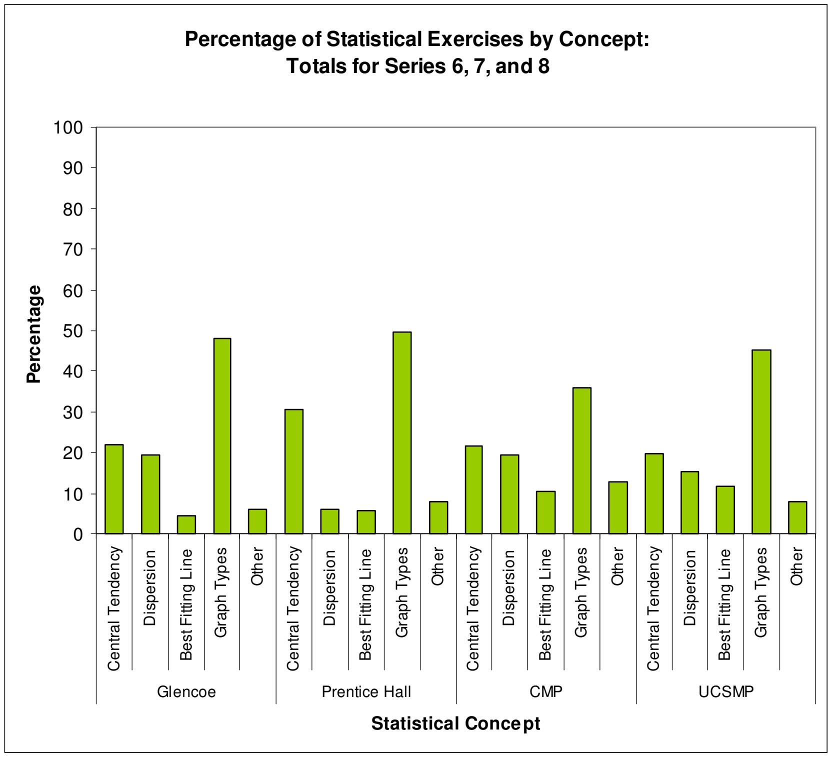

Figure 22. Percentage of Statistical Exercises by Concept: Totals for Series 6, 7, and 8.

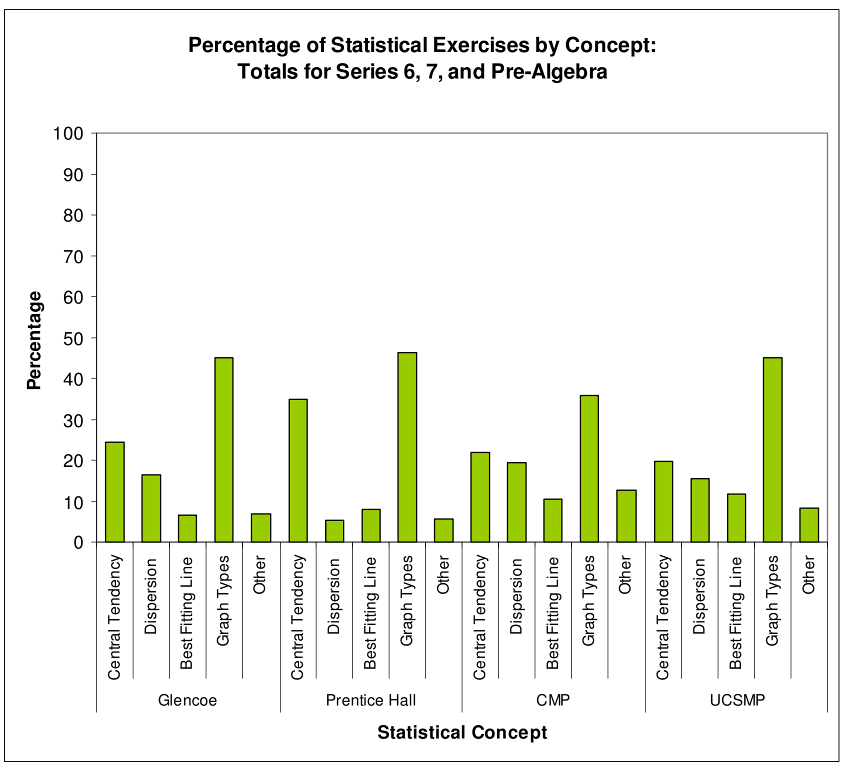

Figure 23. Percentage of Statistical Exercises by Concept: Totals for Series 6, 7, and Pre-Algebra.

Chapter 5: Discussions

This study analyzed the treatment and scope of statistical concepts in four, widely-used, contemporary, middle grades mathematics textbook series: *Glencoe Math Connects*, *Prentice Hall Mathematics*, *Connected Mathematics Project*, and the *University of Chicago School Mathematics Project*. This chapter presents a summary of the study and a discussion of the findings in relation to the research questions and the literature. Also presented are the limitations of the study, implications for curriculum (textbook) development, and possible future research.

Summary of the Findings

The results of this research indicate that each of the middle grades mathematics curricula contain statistical content, but there were differences in how much of the textbook was dedicated to the statistical content, components of the lesson narrative, what statistical concepts were covered, and the cognitive level of demand required for the corresponding statistical exercises for each textbook series. The percentage of pages containing statistical ideas ranged from 4.6% to 16.0% with a mean of 10.2% (sd 3.0%). Overall, the textbook series *Connected Mathematics* had the highest percentage of instructional pages devoted to statistics and *UCSMP* had the least.

For all the textbooks except for *UCSMP Algebra*, which dispersed the statistical content throughout the textbook, most of the statistical concepts were found within a single chapter. For both *Glencoe: Course 1* and *Prentice Hall: Course 1* the statistical material was found in the first quartile of the textbook. In *UCSMP: Pre-Transition Mathematics* the statistical content was found in quartiles 2 and 4. The rest of the textbooks placed the statistical topics in the fourth quartile of the text and research suggests that material in the last half of the textbook is more likely to be omitted should time run short (Begle, 1973; Jones, 2004; Reys, Reys, & Lapan, 2003).

Both *Glencoe* and *Prentice Hall* had similar lesson component sequences in that the lessons started with descriptions of statistical situations or definitions of statistical terms and the body of the lesson was worked examples and practice problems. The lengths of the lessons were 1 to 4 pages. For *Connected Mathematics*, the chapters were packaged as booklets and the lessons were called investigations. The lessons in *Connected Mathematics* were much longer than the other textbook series, running 4 to 15 pages long. Each investigation started with descriptions of statistical situations and the body of the investigation was composed of examples, questions, and descriptions of statistical situations and definitions of statistical terms. In the *UCSMP* series, most of the lessons started with either definitions of statistical terms or descriptions of statistical situations. The body of the lesson consisted of examples, questions, practice problems, activities, and more descriptions of statistical situations and definitions of statistical terms. *UCSMP* was the only textbook series in which the activities were embedded within the lesson. The *UCSMP* lessons were typically 1 to 5 pages long.

The statistical exercises were allocated into four levels of cognitive demand: low-memorization, low-procedures without connections, high-procedures with connections, and high-doing statistics. An analysis of the statistical exercises found memorization exercises had values that ranged from 2.0% to 11.7% and a mean of 6.0% (sd 3.1%). The majority of the exercises for all the textbook series were LP (procedures without connections). The percentages of problems of LP exercises ranged from 41.2% to 89.3% with a mean of 69.2% (sd 13.1%). The percentage of exercises coded HC (procedures with connections) varied from 4.2% to 43.0% with a mean of 23.7% (sd 11.9%). Exercises that required the highest level of cognitive demand (doing statistics) ranged from 0.0% to 8.8%. The *Connected Mathematics* series did not have any exercises for this category and the *UCSMP* series had the highest percentages of exercises rated as doing statistics. The average for the cognitive level of doing statistics was 1.1% (sd 2.4%).

Findings in Relation to Chapter 2 Hypotheses

In Chapter 2 three *hypotheses* were stated, and are answered here. (1) *Hypothesis: Less than 10% of the textbook will be devoted to statistics and the statistical material will be found in a single chapter in the fourth quartile of the textbook.* The results show that, on average, the textbooks devoted 10.1% of instructional pages to statistics. Most of the statistical concepts were located in chapters near the end of the textbooks.

(2) *Hypothesis: The publisher textbooks will have lesson narratives composed mainly of worked examples and the non-publisher textbooks will have a more conceptual approach.* This *hypothesis* was also supported within this sample. Both the publisher textbook series from Glencoe and Prentice Hall had lesson narratives mainly composed of worked examples. In contrast, both UCSMP and Connected Mathematics textbook series had fewer worked examples but included more questions and activities designed to elicit conceptual understanding.

(3) *Hypothesis: Most of the exercises will be lower level cognitive demand with the higher level cognitive demand in the higher grade levels to support more difficult statistical concepts.* The results for this *hypothesis* were more surprising. For all the textbooks series, the majority of the problems were low level (procedures without connections) but the second highest category for all grade levels was high level cognitive demand (procedures with connections). However there were few high level cognitive demand (doing statistics) exercises.

Alignments of Statistical Content with National Recommendations

The results of the analyses of these textbook series can shed some light upon the following questions: (1) How closely do these textbooks follow the topics recommended in the guidelines from various national organizations, (2) What potential opportunities to learn statistical concepts do students have, and (3) Do the cognitive levels required of students to complete the exercises facilitate conceptual understanding of the statistical concepts?

Alignment with the NCTM Focal Points. The NCTM Principles and Standards (2000) gives a general description of what statistical topics should be covered in the middle grades (6 – 8) but does not define which statistical topics should be covered at each grade level. Because there were questions about the grade placement of topics, NCTM created the Focal Points (2006) which delineated the statistical topics by grade level. This was the document used in this study to determine the alignment of topics within the textbooks to national recommendations.

Grade 6. The NCTM Focal Points (2006) state that statistics is not a focal point in grade 6 but students should have previously dealt with line graphs, double bar graphs and ordered pairs. All the 6th grade textbooks contained topics covering line graphs, bar graphs, ordered pairs, stem and leaf plots, and central tendency. In fact the majority of statistical content found in *Connected Mathematics* revolved around central tendency. However, the histogram and circle graphs, topics recommended for the seventh grade (NCTM Focal Points, 2006) were found in the sixth grade textbook. All the textbook series presented the circle graph in grade 6 and *Glencoe* and *UCSMP* also presented the histogram.

Grade 7. Both *Glencoe* and *Prentice Hall* presented the recommended circle graphs and histograms. *UCSMP* contained histograms but no circle graphs and *Connected Mathematics* had neither circle graphs nor histograms. The NCTM Focal Points (2006) recommend the box and whisker plot to be covered in grade 8, but all the textbooks presented the topic in grade 7 except for *Connected Mathematics*. Also, both *Connected Mathematics* and *UCSMP* presented not just the formula for the mean but its different properties, such as the idea of the mean as an equal share or as the balance point. *Connected Mathematics* also presented the concept of comparing distributions and *UCSMP* presented the deviations from the mean and time series.

Grade 8. For grade 8 the NCTM Focal Points (2006) recommend the inclusion of quartiles, box and whisker plots, scatter plots, lines of best fit, ideas about the spread of data and using the measures of central tendency to summarize data. All the 8th grade textbooks covered quartiles, scatter plots and lines of best fit. All but *Connected Mathematics*, which presented this idea in grade 7, explored how to use central tendency to summarize data. *Glencoe* and *Prentice Hall* repeated the presentation of the box and whisker plot and *Connected Mathematics* presented the box and whisker plot for the first time. *UCSMP* did not cover box and whisker plots as the concept was presented in the previous grade. *Glencoe* and *Connected Mathematics* included different types of sampling, such as convenience sampling, nth type of sampling and voluntary sampling. *UCSMP* also covered the idea of best fitting curve, the chi-square statistic, and mean absolute deviation.

Pre-algebra. Because the pre-algebra textbook was intended for use in the 8th grade by more advanced students, the same recommendations for grade 8 were used to check alignment with the Standards. *Glencoe* covered scatter plots, lines of best fit, quartiles, box and whisker plots and the different types of sampling. *Prentice Hall* covered scatter plots, lines of best fit, box and whisker plots and quartiles. The statistical content found in the grade 8 textbook was also present in the pre-algebra textbook.

Summary of Alignment with the Standards. Overall, the topics recommended by the NCTM Focal Points (2006) were included in the textbooks examined. But there was repetition of concepts, such as the mean, median and mode; also several concepts were introduced at earlier grade levels than recommended. For example, circle graphs were introduced one grade level earlier than recommended by the NCTM Focal Points (2006). However, to fully understand circle graphs, students need to be well versed in fractions, proportions and percents. This earlier introduction of circle graphs begs the question of whether or not students are prepared for it.

For most states the Common Core State Standards (2010) have replaced the NCTM Principles and Standards (2000), but both documents generally agree upon which statistical topics should be included in the middle grades curriculum. All the guiding documents (Common Core State Standards, 2010; NCTM Standards, 2000; Focal Points,

2006) have a holistic view of the K-12 mathematics curriculum and take into account which topics should be presented first in order to prepare students to receive more complicated concepts. Presenting a topic earlier than recommended means students might not have the pre-requisite knowledge to fully understand the concept.

Opportunity to Learn.

Research has shown a correlation between student achievement and *opportunity to learn* (Stylianides, 2005; Tornroos, 2005; Xin, 2007). Furthermore, consistency of topic content within a textbook and across a textbook series also impacts student *opportunity to learn* (Tarr et al., 2006a, 2008). The results of this study show which topics student had the potential *opportunity to learn* within a textbook and across these four textbook series. Some topics were found only in certain textbooks. For example, different types of sampling such as the convenience sample, and systematic sampling were presented in only two of the textbook series. Thus, this greatly impact students' potential opportunity to learn these topics as research has shown very little material that is not found in the textbooks is introduced into the classroom (Braswell et al., 2001).

Cognitive Levels.

Stein et al. (1996) emphasized the importance of examining the levels of cognitive demand required of students as the cognitive demand greatly influences student learning. Mathematical tasks that require students to interpret, synthesize and make sense of the mathematics facilitate student understanding and learning at a higher level than the rote calculations and memorizations of surface learning. The cognitive level of demand required of students by non-contextual computational problems is different from the cognitive level of demand required of a word problem (Doyle, 1983). The results of this study show that the majority of the statistical exercises found within these four textbook series required predominantly low level cognitive demand of students to complete the exercises. Few if any problems required students to attain the highest level of cognitive demand. Low level cognitive demand exercises do not foster a deeper understanding of the concepts (Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996; Stein & Smith 1998; Stein et al., 2000) which could explain, in part, why students have a difficult time with statistical concepts. Moreover, very few exercises were found to confront students' most common misconceptions about statistics in order to correct these misconceptions (delMas & Garfield, 1999; Garfield & Ahlgren, 1988; Garfield & Ben-Zvi, 2008; Garfield & Gal, 1999). The inclusion of such problems requiring students' higher levels of cognitive demand and exercises confronting students' misconceptions are needed in order to better facilitate student understanding of the statistical concepts.

Use of Technology

Surprisingly many of the textbook series did incorporate the use of technology to facilitate the learning of several statistical concepts. Some examples of technology used in the lessons were excel spreadsheets to display data, and calculator software to show different distributions of data.

Significance of Study

The results from this study can inform curriculum developers and evaluators of middle grades mathematics curriculum in their future endeavors to improve the textbooks for use at this level. For example, the findings of this study showed that several statistical topics were introduced in earlier grade levels than recommended (Common Core State Standards, 2010; NCTM Focal Points, 2006). Certain statistical topics were repeated in several grades, such as the circle graph and histogram. Curriculum developers can use the results of this study and others like it to help inform the best placement of the statistical material and to ensure unnecessary repetitions. Also, the results of this study showed more high-level cognitive demand exercises to better facilitate deeper understandings of statistical concepts are needed and future textbook developers can address this need.

The analysis of these middle grades textbooks contributes to the body of research by documenting students' potential opportunity to learn statistical concepts (Begle, 1973; Jones, 2004; Reys, et al., 2003; Tarr et al., 2006b; Tarr et al., 2008). For example, *UCSMP* was the only textbook series to address the concept that data can be seen to fit not only the best fitting line but also the best fitting curve. Students using the other textbook series would not have had the potential opportunity to extend the idea of best fitting line to best fitting curve and have missed the potential opportunity to learn this concept.

The findings from this study could also help illuminate some possible relationships between students' understanding of statistical concepts and their presentation in the textbook. For example, several of the textbooks presented the formula for the mean without fully exploring the different properties. This could explain why students have difficulty applying the different properties of the mean beyond the basic formula (Konold & Pollatsek, 2002; Mokros & Russell, 1995; Strauss & Bichler, 1988; Watson & Moritiz, 2000a, 2000b).

My study has extended Jones' (2004) research on middle grades probability content to the statistical realm. Jones analyzed only two textbooks for each era; in contrast, in my study I analyzed textbook series. An analysis of the textbook series provides a perspective about the statistical content across the entire middle grades curriculum and was not done in the Jones' study of probability. Based upon my literature review search, the Mathematical Task Framework, which defines the cognitive levels used to analyze the exercises, has not been used to analyze the treatment of statistical content in mathematics textbooks. My study also compared when and where statistical concepts were introduced in the middle grades textbook for a comprehensive view of the grades 6-8 statistical content. This same methodology of analyzing the location of the content, analyzing the lesson narrative and analyzing the cognitive level of demand required of the student in the corresponding exercises can be used by future researchers in other content areas.

The National Research Council (2004) recommends more content analysis carried out in mathematics to include certain criteria for effective content analysis. These recommendations include providing the background and education of the researcher, clearly defining the set of standards, and transparency. This study incorporated these criteria into the research and thus increases the body of knowledge of this type of content analysis.

Limitations

Although the textbooks used in this study are some of the more widely used textbooks, the sample size is relatively small when compared to all the textbook options available. Repetition of this study with a larger sample size or different textbook series may change the results. Also, any generalizations of the finding to other textbooks must be exercised with caution.

This study examined the intended curriculum, not the enacted curriculum. Individual teachers actually determine what will be taught in the classrooms as the presence of the material within the textbook does not guarantee that the material will be covered in the class. The teacher's decision of what to present to the class impacts the students' potential opportunity to learn those particular concepts that have been omitted. Furthermore, outside pressures such as local and state school policies, the prior preparation of students and other factors play a part in what topics will be included or excluded in a textbook and in actual classroom instruction.

Coding drift was addressed through the use of a code-recode strategy. Co-coders were used to ascertain that inter-coder disagreement was minimal (less than 10 %). However, the amount of material examined was large (fourteen textbooks and thousands of problems) and human error can occur which could impact the accuracy of the results.

Future Research

This study analyzed the treatment and scope of statistical concepts of four textbook series, a relatively small sample size, in light of the many textbook options available. Future research could examine the treatment of statistical concepts of a larger sample of textbook series. The results from a larger sample can help determine the current state of affairs of how statistical concepts are presented in American textbooks.

Future researchers might also examine international textbooks to explore a global view of the treatment of statistical concepts. Students in Asia usually perform well on international tests and it would be interesting to find out if any differences in the textbooks used exist between Asia and the United States.

Another area for research would be the statistical content of the K-5 and high school textbooks to give a K-12 view of the treatment and scope of the statistical content. Examining textbook series that encompass K-12 could uncover any unique textbook styles or patterns in lesson presentations. Also, a K-12 analysis of the cognitive level of demand of the exercises can ascertain what levels of cognitive demand are found for each grade level.

Another avenue of inquiry could be the examination of the process textbook publishers use to determine what is included in a textbook and how the material is presented. What part if any does research play in the creation of textbooks? Who decides the order of the presentation of the topics?

My Research Agenda

The findings from this study have sparked several more avenues for my future research. First, with the adoption of the Common Core State Standards (2010), although the statistical recommendations for the middle grades curriculum remains essentially the same, I would like to repeat this study to ascertain what differences there are in the textbooks in response to the Common Core State Standards.

The current atmosphere of high stakes testing brings to the forefront the importance of the alignment of the assessment items to the curriculum. I would like to pursue this avenue of research by analyzing the assessment tests and the alignment to the textbook content as far as topic and cognitive level required of both the assessment items and the exercises.

The focus of this study was the “intended” curriculum and there can be differences between the “intended” and “enacted” curriculums. Analyzing the “intended” curriculum is the first step and future research could explore the “enacted” curriculum. Namely, what statistical concepts are students given the actual opportunity to learn and how teachers present these concepts? The study of the “enacted” curriculum could also extend to the exploration of the “achieved curriculum” of what statistical concepts students actually learn and make their own.

Third, there were few instances found in these textbooks that forced students to face their misconceptions about statistical topics. I would like to create and field test several activities or projects geared towards showing students some basic misconceptions about statistics.

Personal Reflection

In conducting the research in this dissertation, I learned about the different cognitive levels and what mathematical tasks these levels correspond to. Prior to this study, I could ascertain when an exercise was harder than another but I had never quantified what constitutes the difference in level of difficulty. For the first time in my teaching career, I can examine an exercise and assign a cognitive level of difficulty required of the student to complete the exercise in a concise and systematic way.

Reflecting back, I started this program of study because I felt that my background in pure mathematics and electrical engineering did not prepare me for a teaching career. I felt the need to improve my teaching so that I can better help my college students. During the course of my studies I learned how to facilitate better understanding through the use of manipulatives, activities, projects and writing. I now incorporate these items into the courses I teach. When I began this program I knew I needed help but did not know where to find it. This program of study gave me the tools to help myself. I cannot claim to know all the answers, but now I know how to find them. As the saying goes, "Give a man some fish and he will eat today, but teach him how to fish and he will eat for a lifetime." I am definitely an expert "fisherman" now.

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Appendix A

Pilot Study

A few generations ago, being able to read, write and do arithmetic was enough to prepare students for the real world. Today, with the advances in technology and the frenetic pace of our lives, the three R's of reading, writing and arithmetic are no longer sufficient; hence several topics have been added to the curriculum, including the area of statistics. The National Council of Teachers of Mathematics (NCTM, 2000), and the American Statistical Association (GAISE, 2005) recommend that statistical concepts be part of the repertoire of a well rounded and productive member of society. One way to ascertain if certain statistical concepts are in the intended curriculum is to examine the textbooks used in the classrooms because research has shown that American classrooms rely heavily upon textbooks (Grouws & Smith, 2000; Weiss, Banilower, McMahon & Smith, 2001).

Middle grades textbooks were examined as the middle grades are the bridge between grade school and the more challenging subjects found in high school and several important statistical ideas should be part of the middle grades curriculum (NCTM, 2000). This pilot study was undertaken to determine the sequence and scope of the treatment of statistical topics in two 6^{th} grade textbooks delineated in the research questions below and to ascertain if a more extensive analysis was merited.

Research Questions

1. What is the nature of the statistical content in 6th grade mathematics textbooks? How much of the textbooks are devoted to statistical topics? Where are these topics located? Which statistical concepts are individual lessons and which are embedded with other non-statistical topics?
2. What are the sequences of the statistical lesson narratives that emerge in the 6th grade mathematics textbooks? What is the typical sequence of lesson narratives for each textbook? What are the differences and similarities of these lesson narratives across the textbook from different publishers?
3. What is the nature of the exercises on statistics in 6th grade mathematics textbooks? What is the level of cognitive demand exhibited by these exercises?

Sample

For the pilot, two different textbooks for grade 6 were chosen: the National Science Foundation funded *Connected Mathematics (1st Edition)*, and the more traditional, publisher generated *Middle School Math: Course 1* by Holt, Rinehart and Winston (2004). Both are older editions than I expect to use for the dissertation.

Connected Mathematics (1st Edition), created with funding from the National Science Foundation, is actually several booklets and all the booklets together were meant for grade 6 as the textbook for that grade. *Connected Mathematics (1st Edition)* uses a discovery type of learning where students work out the problems themselves and are rarely provided worked examples. The textbook begins by letting students know what the problem is and what they can expect to learn. Students formulate how to solve the problem and at the end the students summarize what they learned. According to the Connected Mathematics Project website, the last booklet to be covered, "Data About Us", contained statistical concepts.

Middle School Math: Course 1 by Holt, Rinehart and Winston (2004) is a more traditional textbook created by a major publisher. Holt, Rinehart and Winston has since merged with McDougal Littel to become Holt McDougal. The current version of this textbook is now called *Mathematics Course 1*. *Middle School Math: Course 1* followed a more traditional lesson presentation in that the lesson contained worked examples and guided problem solving.

Procedures

My initial step, before conducting this pilot study, was to review similar content analysis research to determine what was examined and what types of coding were utilized. Jones (2004), conducted an analysis of the treatment of probability in middle grades mathematics textbooks across four eras and found the majority of the probability content in the most recent Standards Era. Because probability and statistics has been identified by NCTM (1998; 2000) as a single strand, I started with the coding schema utilized by Jones on the treatment of probability and refined the coding schema to fit the needs of my analysis of the treatment of statistics. The analysis of each textbook consisted of three areas, namely lesson narrative, exercises, and the amount of the textbook devoted to the statistical concepts.

Amount of Textbook Devoted to Statistical Concepts. To the nearest quarter of a page, the total number of pages devoted to each statistical concept was counted. Following the technique used by Flanders (1987) with K-8 algebra books, the main body of the textbook containing the lessons and corresponding exercises, "Problem Solving on Location", "Technology Lab" and "Math-Ables", were counted as instructional pages and excluded were the preface, glossary, index, selected answers, table of contents, student handbook, extra practice and skills bank. I also noted the subsequent location of the concept in relation to the entire textbook as research has indicated that teachers generally cover 75% of the book (Grouws & Smith, 2000; Robitaille & Travers, 1992), which implies that lessons in the final quartile might not be covered. I also noted if the material was in a separate lesson or was part of another non-statistical lesson. Table A1 shows the total page counts for the instructional pages, statistical pages and booklet pages since *Connected Mathematics* was divided into booklets.

Table A1

Number and Percent of Instructional Pages Overall and for Statistics

	Connected Mathematics		Middle School Math	
	N	%	N	%
Total Page Count	699		719	
Instructional Pages in Textbook	620	88.7	682	94.9
Total Pages for Statistics	69	9.9	29	4.3
Total Page Count for Booklet	82			
Location of Content	Bk 8 of 8		Ch 6 of 12	

In *Connected Mathematics (1st Edition),* the booklet "Data About Us" had a total of 82 pages of which 69 were instructional pages. *Connected Mathematics (1st Edition)* contained over twice as many pages devoted to statistical concepts in comparison to

Middle School Math: Course 1 by Holt, Rinehart and Winston (2004). For almost every statistical concept that was found in both textbooks, *Connected Mathematics* devoted more instructional pages than *Middle School Math: Course 1*. The emphasis was on representation of data and the measures of central tendency, as expected because these were 6th grade textbooks, aligned with the NCTM Standards (2000) recommendations. Little instruction was reserved for the idea of dispersion as dispersion is recommended for inclusion in the 7th grade (NCTM, 2000).

Table A2 lists the pages by statistical content and figure A1 shows the comparison of instructional pages for each statistical concept.

Table A2

Total Page Count for the Central Measures of Tendency Lesson Narrative.

Statistical Concept	Connected Mathematics Number of Pages	Connected Mathematics %	Middle School Math Number of Pages	Middle School Math %
Total Instructional Pages	620		682	
Mean	8.3	1.3	4.3	0.6
Median	4.0	0.6	3.6	0.5
Mode	1.3	0.2	3.6	0.5
Range	1.3	0.2	1.6	0.2
Bar Graph	7.3	1.2	5.0	0.7
Stem & Leaf	7.0	1.1	1.7	0.2
Scatter plot	4.3	0.7	1.5	0.2
Table	0.0	0.0	3.5	0.5
Line plot or graph	3.3	0.5	3.0	0.4
Box & Whisker	0.0	0.0	1.3	0.2
Types of Data	4.0	0.6	0.0	0.0

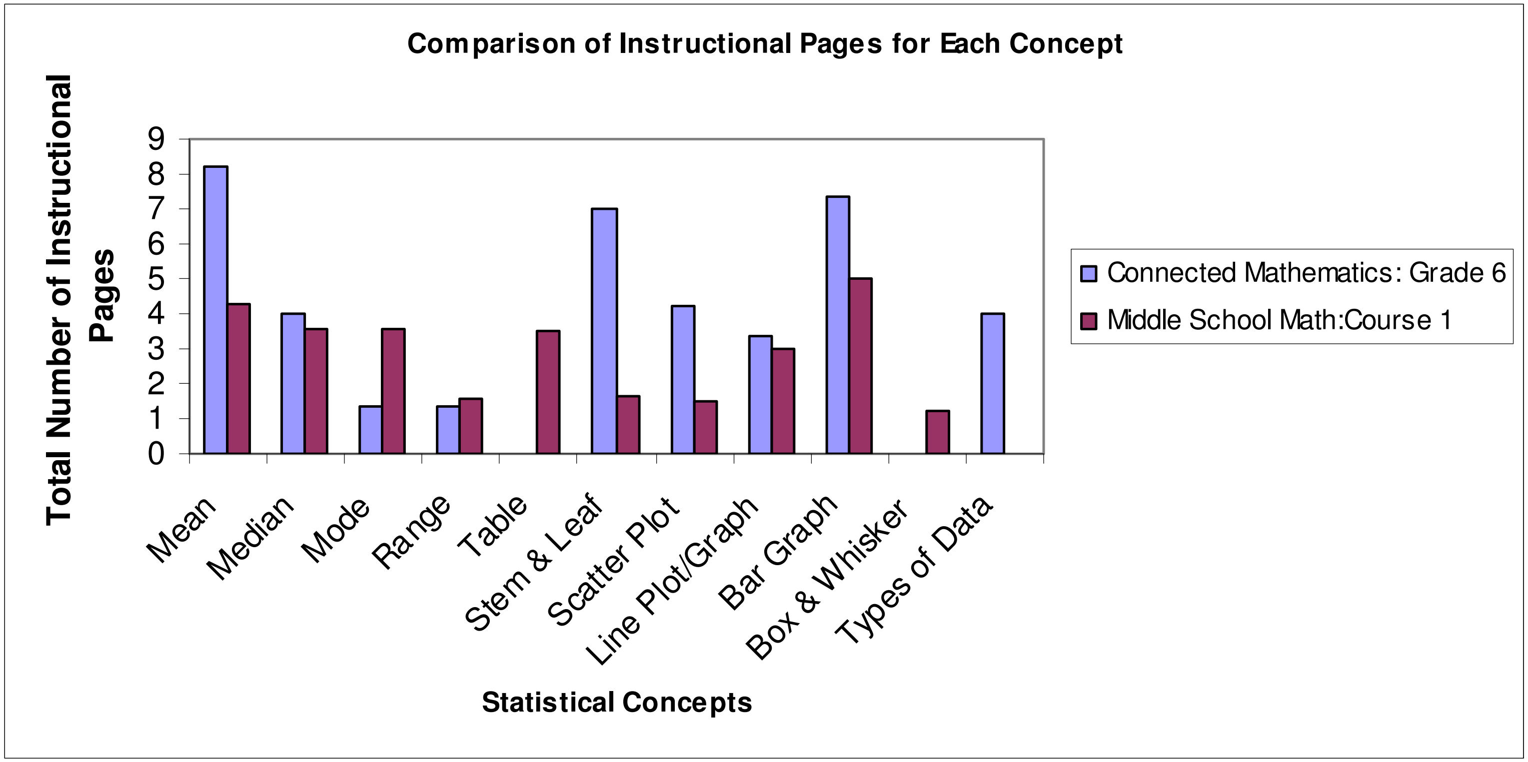
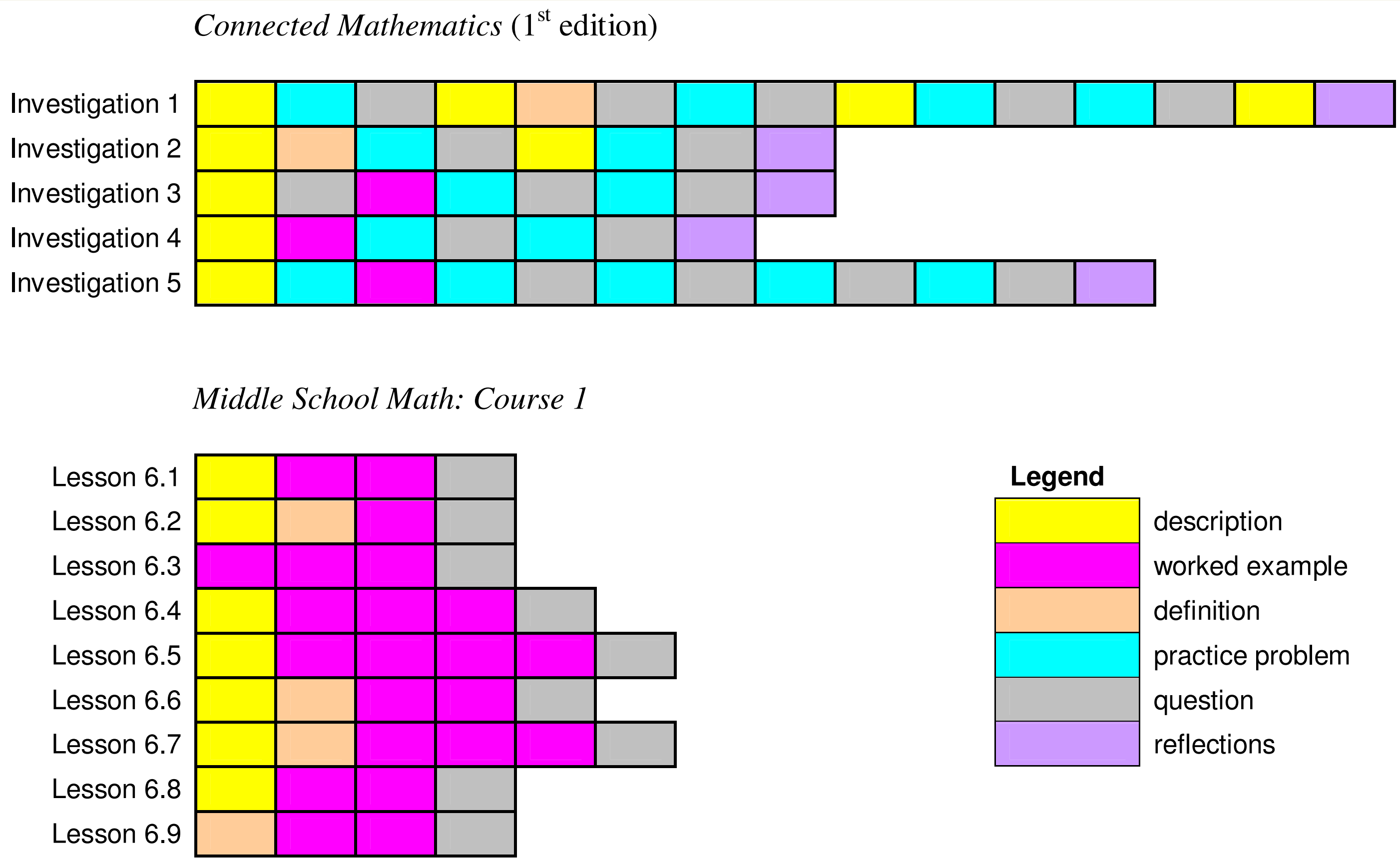


Figure A1 Comparison of Instructional Pages for Each Statistical Concept

Lesson Components of Lesson Narrative. Next the lesson components of the lesson narrative were examined. The analysis of the first edition of *Connected Mathematics* (1^{st} edition) showed a need for several new categories of lesson components not found in the study by Jones (2004). Each booklet starts with Mathematical Highlights which tells the student what he/she will learn followed by the description of the Unit Project to be completed at the end. Because Jones' categories did not include these components, two new categories were created: Mathematical Highlights (high), and Unit Project (proj). Also, at the end of each unit are questions to tie the content together and these questions do not fit into the definition of the question category. The question category is defined as questions within the development of the lesson but these questions are at the end of the unit and are more of a reflection upon what has been learned. So a new category called refl for reflection was created. The 2004 version of *Course 1* by Holt, Rinehart and Winston fit into the categories used by Jones (2004). However, there were several categories (act, orex, prac) not used during my pilot study. For now, these categories will be kept, but if after the study these remain unused, these categories will be deleted from the lesson components categories. Table A3 shows the lesson components of the statistical lesson for the two textbooks.

Table A3

Lesson Component Breakdown of Lesson Narrative for 6th Grade Mathematics Textbooks



Comparison of Lesson Narratives

As stated previously, each booklet for the *Connected Mathematics* has a general pattern: Mathematical Highlights, description of the Unit Project, Investigation, Applications-Connections-Extensions, and Mathematical Reflections. I coded the Investigation and Mathematical Reflections and deconstructed the lesson into components. Both textbooks had similarities in the order of the lesson components. Both textbooks started the lesson with descriptions of real-world statistics (desc) followed by definitions (defn), if needed, and then questions (ques). The major difference between the two textbooks was that in *Connected Mathematics* there were few worked examples (we) and the main body of the lesson narrative was composed of questions (ques) and practice (prac). In *Middle School Math: Course 1*, the main body of the lesson was worked examples (we) and the lesson ended with questions (ques).

Cognitive Level of Exercises

The last part of this pilot study examined the exercises corresponding to the statistical lessons. The exercise set from each textbook that corresponded to the lesson on the measures of central tendency was chosen to code. Each exercise was categorized by statistical concept (mean, median, and mode) and level of cognitive demand (Smith & Stein, 1998). I found both low and high level cognitive demand problems in both of the exercise sets that I coded. Figure A2 shows the tally of the different cognitive levels for each concept for each textbook.

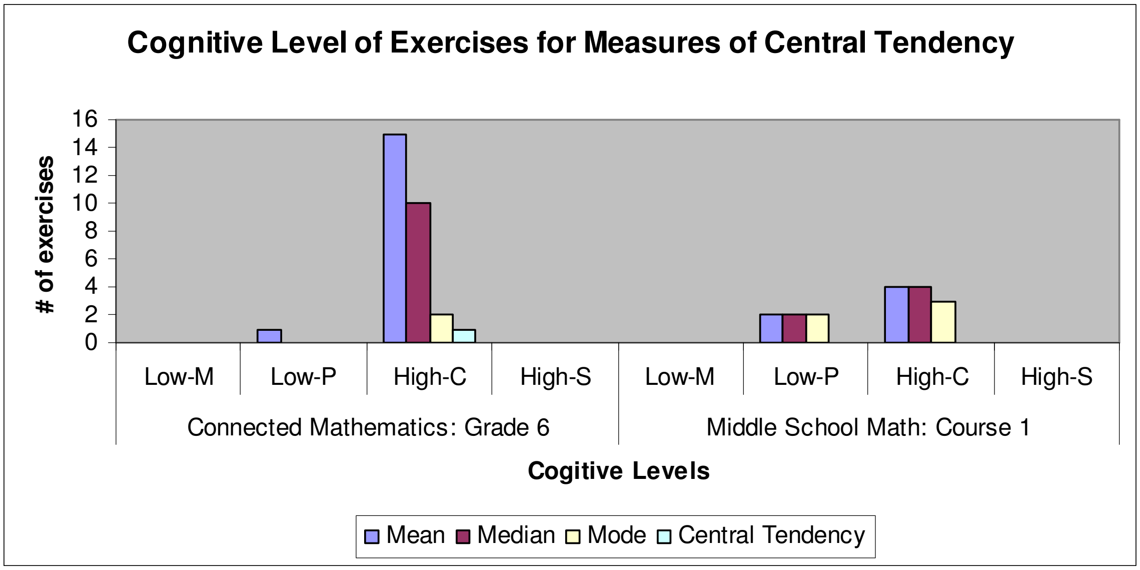


Figure A2. *Cognitive Level of Exercises for the Measures of Central Tendency*

As shown in figure A2, *Connected Mathematics: Grade 6* had more exercises dealing with the mean, median, and mode and the cognitive level of these exercises was higher than in the *Middle School Math: Course 1.*

Summary

The results of this pilot study have shown that there are distinct differences in amount of instruction devoted to statistical topics, lesson narrative style and total number and cognitive level of the exercises supporting the measures of central tendency. *Connected Mathematics (1st Edition)* devoted more instructional pages to statistical concepts than *Middle School Math: Course 1* by Holt, Rinehart and Winston (2004). Also, the two textbooks differed in some of the topics covered. *Connected Mathematics (1st Edition)* defined the difference between categorical and numerical data while *Middle School Math: Course 1* by Holt, Rinehart and Winston (2004) included the box and whisker plot and table graphs. However, both the first edition of *Connected Mathematics* and 2004 version of *Course 1* by Holt, Rinehart and Winston covered frequency distribution tables, mean, mode, median, range, histograms, line graphs, outliers, and stem and leaf plots. Although some of the differences in the lesson components style can be attributed to the problem-centered philosophy of *Connected Mathematics*, further investigation of several contemporary middle grades mathematics textbooks could uncover some interesting results.

Appendix B

Textbook Pages Form

Book Title____ Grade_________

Total Textbook Pages ______________ Total Instructional Pages _____________

Topic	Location Page #	Instructional Pages total
Central Tendency: Mean Median Mode		
Graphs:Bar Graph Stem &Leaf Scatter plot Table Line plot Box & Whisker Line Graph		
Dispersion: Range		

Appendix C

Lesson Components Worksheet for Lesson Narrative

Textbook Title ____ Grade __________

Section or Investigation	pages for lesson	Concept(s) Pg for each concept.	Lesson Components
Example: Sec 3.5	2	Mean, 1, Mode, 1	desc, defn, we, prac, prac, defn, we, prac, prac

Appendix D

Cognitive Levels Worksheet for Exercises

Title____ Grade Level_____________

Lower Level Memorization LM
Lower Level Procedures LP

Higher Level Connections HC
Higher Level Statistics DS

Location: Section	Exercise Number	Concept(s):	Cognitive Level: LM, LP, HC, DS
Example: Sec 3.5	1.a	Histogram	LP
	1.b	Histogram	LP
	2.	Stem and Leaf plot	LP

About the Author

Maria Consuelo Capiral Pickle, known as Suzie to her friends and students, grew up in Washington State. She earned both a B. S. and M. S. degree in Mathematics from the University of Nevada, Las Vegas, a M. S. degree in Electrical Engineering from the University of Southern California and a Ph. D. from the University of South Florida. She has been a faculty member at St. Petersburg College since 2001. She has presented at the Florida Council of Teachers of Mathematics several times and was an AP statistics grader. She enjoys traveling to Europe and Asia visiting relatives and learning about other cultures.