A Mechanized Horseback Riding Simulator as an Aid to Physical Therapy

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A Mechanized Horseback Riding Simulator as an Aid to Physical Therapy

by

Jennifer Lott

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering
Department of Mechanical Engineering
College of Engineering
University of South Florida

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Date of Approval:
July 11, 2006

Keywords: disability, equestrian, hippotherapy, kinematics, rehabilitation

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Acknowledgments

Many individuals deserve acknowledgment for their support and assistance throughout the development of this thesis. Dr. Dubey, my Major Professor, allowed me to work at my own pace and trusted that I would eventually complete a thesis that was worthwhile. Thank you, Dr. Dubey, for giving me the opportunity to pursue research in an area that I have a passion for and that has the capability of helping so many people. Dr. Ying guided me every step of the way, from my initial concept through the design process and finally to the completion of this thesis. He supported and helped to improve my ideas and made me feel good about what I was doing. Dr. Lusk helped break down into manageable pieces the seemingly insurmountable amount of work that had to be done. Lisa Payne-Hyslop, my dressage trainer and good friend, brainstormed ideas and offered professional guidance on everything horse related. Lastly, this thesis could not have been completed without the emotional and financial support of my parents, family, and friends.
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A Mechanized Horseback Riding Simulator as an Aid to Physical Therapy

Jennifer Lott

ABSTRACT

Equine-assisted therapy is a nontraditional form of physical therapy that involves riding horses as a form of rehabilitation. Limited access to these riding programs justifies a need to develop a horseback riding simulator capable of simulating the gaits, bend, and collection of the horse. Research involving the development of horseback riding simulators is limited, but the available research does show promising results in the ability to aid in physical therapy.

A two-dimensional model and simulation was developed using MATLAB. Using the results from the simulation, a horseback riding simulator was designed, fabricated and tested. The physical simulator was capable of simulating a walk, trot, and canter, bend to the left or right, and collection of the gait. The purpose of the testing of the horseback riding simulator was to evaluate the similarity of the physical simulator to the gaits of the data collected from a real horse. The results from the testing are compared with the kinematic data from the MATLAB simulation. The biomechanical effect on the hip flexion angle is also evaluated when the system simulates bend and collection of the horse’s back.

The motion data was collected using a Vicon system. Four cameras were set up to collect the data from the five reflective markers that were placed on the
rider. The kinematic results of the horseback riding simulator were compared to the computer simulation using the measurements of the inclination of the ellipse, the major axis of the ellipse, and the frequency. The results from the hip flexion angles shows that the test that simulated bend only results in a significant increase in the hip flexion angle compared to the tests without bend. Simulated collection does not change the hip flexion angles of the rider.

Future work on the horseback riding simulator is needed in order to increase the safety so that a person with a disability would be able to use it as part of their physical therapy. Adaptive programming of the system is also necessary to make the horseback riding simulator more similar to that of a real horse.
Chapter 1 – Introduction

1.1 Motivation

Physical therapy exists to try to optimize a person’s health, well-being and quality of life. The number of people that can be helped through the aid of physical therapy increases each year as the population with disabilities or limited function escalates because of an aging population and new technology saving lives of children with birth defects. Without physical therapy to rehabilitate these individuals, recurring injuries may occur; muscle tone may not fully develop; proper posture may never be seen; the individuals may not have the capabilities to perform everyday tasks. For persons with disabilities or limited function, there is an extensive list of benefits from physical therapy. While standard physical therapy equipment and procedures are used to help many people, new types of nontraditional therapies are also being developed to try to personalize programs for the greatest individual benefit.

Equine-assisted therapy involves riding horses as a form of physical therapy. Therapeutic riding, therapeutic vaulting, and hippotherapy are three nontraditional forms of physical therapy classified as equine-assisted therapy. Although they are considered nontraditional forms of physical therapy, both the American Occupational Therapy Association and the American Physical Therapy Association recognize the programs as beneficial to participants and support the
programs. Some of the benefits from equine therapeutic riding include improved mobility, muscle tone, posture, balance, and self-esteem. Many individuals benefit from these programs including those with cerebral palsy, multiple sclerosis, developmental delay, traumatic brain injury, stroke, autism, learning or language disabilities, emotional problems, and behavioral problems.

The North American Riding for the Handicapped Association (NARHA) certifies equine-assisted therapy programs. Some organizations that support and participate in NARHA include the Muscular Dystrophy Association, Multiple Sclerosis Society, Special Olympics, Spina Bifida Association and United Cerebral Palsy. If research has shown this type of physical therapy to be beneficial and it is recognized and supported by so many organizations, why aren’t more disabled individuals involved? There are many answers to this question including:

- These programs require donations of time, money, supplies, and equine participants.
- Many individuals are tentative about getting involved with large unpredictable horses.
- This type of therapy is not offered as part of the standard in physical therapy.
- Each center has to limit the number of individuals that can be serviced.

Currently there are 692 NARHA centers helping over 35,000 children and adults reap the benefits of this therapy. Unfortunately, there are also thousands on
waiting lists that these centers cannot accommodate. And this number continues to increase each year [1]. A horseback riding simulator that could be used as a physical therapy device that simulates the gaits most commonly used during equine-assisted therapy, the walk and trot, would be beneficial in involving more individuals with disabilities in these programs. Also it would be useful in the initial training program to help the riders overcome any fear of the horses.

There has been a limited amount of research in developing simulated horseback riding equipment. Youichi Shinomiya et al. [2] reproduced the walking gait of the horse and evaluated how realistic the riders felt the walk was. Massaaki Yamaguchi and Nobuhiro Iguchi [3] created a horseback riding simulator that was able to reproduce the walk, trot, canter of the horse. This simulator also responded to aids from the rider such as starting and stopping. These two horseback riding simulators only simulated the horse moving in a straight line, not around curves. Patti Koenig and George Bekey at the University of Southern California [4] created an animated horse that responded to commands given by a rider sitting on a stationary horse-like device. The commands that the animation were capable of were advanced, but the rider was only able to see the results, not feel the results.

The forward direction of this research is to design a horseback riding simulator where the movements of the simulator feel realistic when compared to those of a horse, and to allow the rider more control over the actions of the simulator. Going around a curve, called bending, or asking the horse to shorten or collect its gait changes the kinematics of the horse’s spine and in turn has an
effect on the rider. The positive benefits seen from riding a real horse may be reproduced more fully on a horseback riding simulator when it is able to replicate the gaits, bend, and collection of the horse.

A horseback riding simulator that can replicate the gaits, bend, and collection of the horse can also be used as an initial evaluation and training for those individuals interested in therapeutic equine activities. Evaluation and training in a controlled environment on a controlled device has many benefits including:

- Reducing the stress of the people with disabilities by simulating the activity and allowing them to experience what they will feel before having to get on a large, unpredictable animal.
- Increasing the safety of everyone involved because the individual can be harnessed into this stationary device.
- Allowing the people with disabilities to experience more than just the walking gait, as in other horseback riding simulators.

1.2 Thesis Objectives

There are four main objectives of this research work. These are described below:

1. Model a two-dimensional simulation of a horseback riding simulator using MATLAB. Obtain data from this simulation relating to position, velocity, and acceleration.
2. Design and fabricate a horseback riding simulator, a physical therapy device that replicates the horse’s walk, trot, bend, and collection.

3. Test and evaluate the similarity of the mechanized horseback riding simulator to the gaits of the data collected from a real horse. The results from this testing are compared with the position, velocity, and acceleration data from the MATLAB simulation.

4. Test the biomechanical effect on hip angles when the horse simulates bending and collection of the back. Motion data using cameras are used to collect the data.

This device is designed as a prototype, and with further research would be capable of being used in rehabilitation therapy centers as part of a patient’s standard therapy. The creation of a horseback riding simulator does not intend to replace the current therapies used, or phase out the programs that use real horses. The intent is to make the rehabilitative effects of the movement of the horse more readily available to those that can benefit and to determine those that can benefit by performing further research.

1.3 Thesis Outline

This work begins with the background and brief history of equine-assisted therapy activities and a description of the relevant anatomy and kinematics of the horse in Chapter 2. Also included in Chapter 2 is a description of past research and results of persons with disabilities riding horse simulators and real horses. Chapter 3 describes the modeling and simulation of the two-dimensional
horseback riding simulator using MATLAB. The derivation of the kinematic equations used in the simulation is explained. This chapter also includes the position, velocity, and acceleration analysis from the MATLAB simulation. The design specifications, design and fabrication of the horseback riding simulator will be described in Chapter 4. Chapter 5 shows the results of the testing done on the horseback riding simulator. These results include comparisons of the horseback riding simulator to a real horse’s gaits and the biomechanical analysis of the rider's pelvic position while using this device. Conclusions and future recommendations for this research are explained in Chapter 6.
Chapter 2 – Background

2.1 Equine-Assisted Therapy

Equine-assisted therapy, sometimes called equine-facilitated therapy, is a form of physical therapy that involves both riding and interacting with horses. This type of physical therapy using the horse has existed for centuries. In the past fifty years, the therapy has become much more widespread, with hundreds of therapy centers opening, thousands of people participating, and many universities and private firms conducting research studying the benefits of the therapy and seeking to improve the therapy. The explosion of equine-assisted therapy in the last century is often credited to Liz Hartel, whom after being diagnosed with poliomyelitis, rehabilitated herself using therapeutic riding and won the Silver Medal for dressage at the 1952 Olympics [5].

The benefits of equine-assisted therapy are multifaceted. There is the physical aspect, where the participant can experience improved mobility, posture, muscle tone, balance, motor skills and more as a result of the rocking, swaying movement of the horse's gait and the coordination required to control the animal. The movement of the horse's back is transferred to the rider’s pelvis and trunk in a controlled way, and during the walk, the motion transferred to the rider is similar to the walking motion of an able-bodied person. There is also a psychological aspect, a feeling of empowerment, which may come to the
participant when riding and interacting with the horse. The participant is riding a higher view and is moving at speeds faster than what they would normally experience. Also, the participant has to communicate with the horse, using the appropriate signals so the animal will perform as expected. The height, speed, and necessary communication often results in an invigorating feeling because of the new kinds of stimuli supplied to the mind through this form of therapy [6].

Equine-assisted therapy is classified into three areas, therapeutic vaulting, therapeutic riding, and hippotherapy.

Therapeutic vaulting combines gymnastics and dance on a moving horse. Professional vaulting is often performed in circuses, where the rider attempts daring moves on a horse that is trotting or cantering in a circle. The therapeutic version of vaulting is not as daring or dangerous as what is performed in the circus. Even so, therapeutic vaulting is more commonly used with individuals that have behavioral or emotional problems rather than physical disabilities. This is because vaulting is a difficult sport that requires strength and balance. Individuals with behavioral or emotional problems benefit from this type of therapy because it requires them to form a bond and to trust the horse while they perform bold maneuvers. The constant response, either positive or negative, from the horse during the vaulting exercises is non-judgmental and helps to raise the rider’s self-esteem.

Therapeutic vaulting is occasionally used as a form of therapy for persons with disabilities. The vaulting exercises can be performed on a real horse, which could be stationary or moving, or on a plastic horseback. The vaulting surcingle
is used in all vaulting activities, therapeutic and professional, and consists of a padded girth, which ties around the body of the horse and two handgrips. Depending on the capabilities and extent of the physical disabilities of the rider, many persons with disabilities are able to learn the basic positions of vaulting, and gain the self-esteem and physical benefits from the activity. Figure 2-1 shows an image of a therapeutic vaulter being escorted on a horse by a volunteer.

There is not as clear of a distinction between therapeutic riding and hippotherapy as there was a few decades ago. Both therapies involve riding horses and utilize the benefit of the horse's motion that is transferred to the
rider’s pelvis and trunk. At one time, therapeutic riding was used solely as a form of exercise for the rider on the horse. Now that the therapeutic benefits of horseback riding are proven more scientifically, the definition of therapeutic riding has changed. The clear distinction of these two forms of therapy, however, involves insurance coverage. Hippotherapy is considered a type of physical therapy, occupational therapy, or speech therapy using the horse as a tool and involves a team of therapists during the riding to achieve the goals of the rider. Insurance plans that cover rehabilitative benefits will cover hippotherapy. This is not the case of therapeutic riding, which is not covered by insurance plans because often there is not a licensed physical therapist on the staff.

Therapeutic riding is used to positively contribute to the physical, cognitive, emotional, and behavioral well-being of people with disabilities through learning how to ride a horse. The rider has an active role in controlling the horse and learns basic to advanced riding skills, depending on their commitment to the sport and their ability to perform. Basic skills that the rider learns involve starting, stopping, and turning the horse. More advanced skills that the rider learns include asking the horse to transition from one gait to another, such as from a walk to a trot, or to slow down or speed up a gait. The way that a rider asks the horse to perform an action is physical or verbal and is referred to as an aid. The rider’s position, center of gravity, and aids will determine the response from the horse. Therefore, the rider may need to learn to relax his body, shift his weight in one direction, only pull on one rein, or only squeeze with one leg in order to get the desired result. If the rider does not do this, the horse will not respond. Using
the horse as a tool, the rider will learn to perfect his coordination, balance, and motor function. Figure 2-2 shows an image of two riders involved in therapeutic riding reaching out to touch hands while still maintaining their balance on the horse.

![Figure 2-2: Therapeutic Riders Test their Balance](image)

During therapeutic riding, riders often are able to experience different types of horseback riding, such as dressage, barrel racing, poles, and trail riding. Horse shows promote competition among the riders to do their best, while having fun. Improving the well-being of the rider is always the ultimate goal.

The term hippotherapy means treatment with the help of the horse. As stated earlier, hippotherapy is considered a type of physical therapy, occupational therapy, or speech therapy using the horse as a tool and involves a
team of therapists during the riding to achieve the goals of the rider. Hippotherapy is the medical approach in equine-assisted therapy. Therapeutic riding is the exercise approach to equine-assisted therapy. The team of therapists used in hippotherapy is composed of medical professionals including physical therapists, occupational therapists, speech-language pathologists, and psychologists or psychotherapists. The goals for the rider may be physical, cognitive behavioral or emotional. As opposed to therapeutic riding, where the rider influences the horse, in hippotherapy the horse influences the rider. Specific riding skills are not taught. The horse is controlled and its speed is regulated by a therapist or a volunteer.

The transferred motion from the horse to the rider allows therapists to see how the rider physically responds during different motions. The therapist can therefore regulate the speed, direction, or gait of the horse in order to get different physical responses. In this way, the rider builds a foundation in which the rider’s individual goals are realized. During a hippotherapy session, the therapist may position the rider forwards, backward, supine, prone, or standing. Each position will have a different affect on the rider and the way that their body reacts to the movement of the horse. The therapist may help support the rider by walking next to the horse or by sitting behind the rider on the horse. With severely disabled people, it is often necessary for the therapist to sit behind the rider and have two or more people supporting the sides of the rider from the ground. Figure 2-3 shows an image where a physical therapist adjusts a young boy’s position during a session of hippotherapy.
2.2 Anatomy and Kinematics of the Horse

Each horse is different. The conformation or proportions of the horse’s body have a great deal of influence on how the horse will move and how this movement will be transmitted to the rider. Different horse sports strive to breed for characteristics that will optimize the horse’s potential in that particular sport. Although each horse has different conformation, the fundamental anatomy and gaits are the same. This section will begin with a review of the basics of the anatomy of the horse. The three basic gaits of the horse will then be described, and an in depth look at the kinematics of the horse and spine will be discussed. The section will conclude with a description of how the movement of the horse influences the rider, and how the rider influences the movement of the horse.
The basic anatomy of the horse is shown in Figure 2-4. Some points of interest in this figure are the back and withers, which determines the placement of the saddle, and the girth, where the saddle is cinched about. The poll, crest, croup, and dock connect the vertebral column along the horse’s back. The hind end of the horse, the hock, gaskin, thigh, stifle, and buttock typically have the largest muscle mass because this area creates the most power during the horse’s movement.

![Figure 2-4: Anatomy of the Horse](image)

The four natural gaits of most breeds of horses are the walk, trot, canter, and gallop. The walk, trot, and canter are described in detail below. The walk is a four beat lateral gait and is the slowest of the four gaits. Four distinct hoof beats can be heard while the horse is walking. The sequence that the hooves
touch the ground is right hind, right front, left hind, and left front. The sequence that the hooves touch the ground for the walk is shown in Figure 2-5.

The trot is a two beat diagonal gait. The gait begins with a period of suspension where all four legs are off the ground. Next, a diagonal pairs of legs, the left front and right hind work together, touch the ground at the same time, and then push off at the same time, followed by another period of suspension. The opposite pair of diagonal legs, the right front and left hind, then touch the ground at the same time and push off together. The rider will often rise up and down every other beat, called posting, which uses less energy than sitting the trot and doesn’t affect the natural movement of the horse as much. Figure 2-6 shows this sequence of diagonal pairs of legs moving together during the trotting of a horse.
The canter is a three beat movement and has two possible leads, the right lead and the left lead. The lead refers to which foreleg hits the ground in front. In the left lead canter, the sequence begins with the right hind touching the ground while all the other legs are suspended. In the second beat, the left hind and right front touch the ground together, and the left front touches the ground in the third beat. After the horse pushes off with its legs, there is a period of suspension. The right lead canter has the same sequence, but begins with the left hind touching the ground followed by the right hind and left front touch the ground simultaneously. The right front would then touch the ground in front of the left front and the sequence ends with a period of suspension. The canter creates a rocking motion very comfortable to the rider. Figure 2-7 shows the sequence of a horse in a left lead canter.
As stated at the beginning of this chapter, each horse is different. Modeling the motion of the horse therefore, is difficult because there is not just one standard motion for the three gaits. Analyzing the motion of the horse normally involves using high-speed photography and placing markers at key points on the horse. A trace is then made of the horse’s motion, where movement at constant speed is subtracted out. Averaging trace results will result in distinct paths for the horse’s walk, trot, and canter. These traces are normally at the center of gravity of the horse when the overall movement of the horse is desired, but can be done on any spot of the horse’s body.

Masaaki Yamaguchi and Nobuhiro Iguchi analyzed the movement of a horse prior to designing their horseback riding simulator. Traces at the walk, trot, and canter were developed from the motion data that was collected. Table 2-1

Figure 2-7: Hoof Sequence for Horse in Left Lead Canter
shows the motion data that was at the horse’s center of gravity at the walk, trot, and canter. The data includes ranges of values for the frequency, horizontal and vertical amplitudes, inclination of ellipse, and pitching angle.

Table 2-1: Motion Data at the Horse’s Center of Gravity [3]

<table>
<thead>
<tr>
<th></th>
<th>Walk</th>
<th>Trot</th>
<th>Canter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>1.0 – 2.0</td>
<td>2.0 – 3.0</td>
<td>1.2 – 1.8</td>
</tr>
<tr>
<td>Horizontal Amplitude (mm)</td>
<td>20 – 50</td>
<td>20 – 40</td>
<td>40 – 70</td>
</tr>
<tr>
<td>Vertical Amplitude (mm)</td>
<td>20 – 40</td>
<td>40 – 60</td>
<td>60 – 110</td>
</tr>
<tr>
<td>Inclination of Ellipse (Degrees)</td>
<td>10 – 40</td>
<td>90 – 100</td>
<td>100 – 130</td>
</tr>
<tr>
<td>Pitching Angle (Degrees)</td>
<td>6 – 9</td>
<td>5 – 8</td>
<td>10 - 14</td>
</tr>
</tbody>
</table>

Using the data in Table 2-1, Yamaguchi and Nobuhiro developed ellipses for each of the three gaits of the horse, the walk, trot, and canter. These ellipses are shown in Figure 2-7. All the units listed in this figure are in millimeters. With the knowledge of the elliptical motion and frequency of the gaits, a horseback riding simulator can be developed that will simulate these movements.

Figure 2-8: Elliptical Movement of the Horse at the Walk, Trot and Canter [3]
Since the rider sits on the horse’s back while riding, it is necessary to take a brief look at the anatomy and kinematics of the horse’s spine. Horses have 56 vertebrae extending from their poll, through their neck, back, croup, and down to their tail. There are 7 cervical vertebrae in the neck region from the poll down the crest, 18 thoracic vertebrae in the withers and back, 6 lumbar vertebrae in the loin, 5 sacral vertebrae in the croup and 20 caudal vertebrae located in the tail of the horse. The degrees of motion shown in Figure 2-9 relates to the intervertebral motion, or the amount of motion that can occur between two adjacent vertebrae. This motion is very important when discussing how the horse’s motion influences the rider and how the rider can influence the horse’s motion.

The vertebral column of the horse can rotate in three-dimensions and are described by flexion-extension, lateral bending, and axial rotation. Flexion-extension or pitch of the spine allows the neck and back to round or hollow.
Lateral bending or yaw of the spine allows the neck and back to bend to the left or right. Axial rotation or tilt of the spine allows the vertebral column to twist. Figures 2-10, 2-11, and 2-12 show examples of flexion extension, lateral bending, and axial rotation, respectively.
The degree to which the vertebral column can bend in these three ways varies with the region of the vertebral column and the type of rotation. There is also a large variation with individual horses. Like humans, most horses are one sided and are stronger in one direction and capable of stretching more in one direction. These variations are excluded from most studies, and the average data of a number of horses is used. The cervical vertebrae in the head and neck region have the highest range of motion, which allows the horse mobility in order to graze, clean themselves, and balance themselves. The caudal vertebrae in the tail region have the next highest range of motion. The tail needs this mobility in order to allow the horse to swat at bugs that land on their body or to show their mood. Switching the tail back and forth shows unhappiness and raising their tail high in the air shows excitement. When developing a horse simulator, these regions are not as important as the thoracic region. This is the region where the
rider will be able to feel the effect of the flexion-extension, lateral bending, and axial rotation.

In the thoracic region of the vertebral column, the range of motion varies with the gait that the horse is moving in and the type of rotation of the vertebral column. Marjan Faber et al. [8, 9, 10] performed an *in vivo* study of the three-dimensional kinematics of horses' vertebral column that determined the range of motion of the thoracic region.

**Table 2-2: Range of Motion of the Thoracic Region of the Horse [8, 9, 10]**

<table>
<thead>
<tr>
<th></th>
<th>Walk</th>
<th>Trot</th>
<th>Canter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexion-Extension</td>
<td>4.2° - 8.5°</td>
<td>2.8° - 4.9°</td>
<td>7.8° - 12.2°</td>
</tr>
<tr>
<td>Lateral Bending</td>
<td>2.6° - 5.3°</td>
<td>3.6° - 4.9°</td>
<td>3.1° - 5.6°</td>
</tr>
<tr>
<td>Axial Rotation</td>
<td>4.3° - 11.0°</td>
<td>3.1° - 5.5°</td>
<td>6.1° - 8.5°</td>
</tr>
</tbody>
</table>

These results show that a horse has less flexion-extension at a trot than it does at a walk or canter, probably due to the trot being a diagonal gait rather than a lateral gait like the walk and canter. The lateral bend seen in all three gaits is about equal, showing that the gait does not affect the amount that the horse can bend laterally. In a walk, the horse has the greatest range of axial rotation. The range and magnitude of axial rotation at a trot and canter is much smaller than at a walk.

The range of motion of the horse’s vertebral column during flexion-extension, lateral bending, and axial rotation has a great influence of the capabilities of the horse in many sports. When riding dressage, a type of horse
sport, common terms that are heard while in training include “collection”, and “bend”.

Collection refers to the horse rounding and shortening its entire frame, from the poll to the croup, shifting its weight more to its hind end and shortening the length of the stride of the gait. Collection requires a great deal of flexion-extension of the vertebral column when done correctly. The humping or rounding of the horse’s back during collection, changes the angle of the rider’s pelvis causing them to sit more on their front seat bones. The movement of the horse will often feel much smoother, controlled and balanced.

Bending the horse refers to curving the neck and spine around a curve. The horse is trained to bend around pressure from the rider’s leg given at the girth. A correctly bent horse will bend from their poll to their tail evenly. Bending requires both lateral bend and axial rotation of the vertebral column. The rider’s pelvis position and weight needs to be adjusted to accommodate a horse bending. The weight of the rider needs to be slightly to the inside of the curve and the rider’s pelvis needs to follow the same curve that the horse’s pelvis does. This requires the rider to rotate their pelvis so that the side of the pelvis on the outside of the curve is pushed more forward than the side of the pelvis on the inside of the curve.

The riders are required to balance themselves in order to allow the horse to perform collection, bend, and a specific gait. A highly trained dressage horse will respond to a slight shift in weight, pull on the rein or squeeze with the leg. The response may be to speed up, slow down, collect, extend, bend to the right
or left, or to just increase the balance or impulsion of the gait. Even with a horse that is not highly trained, a shift in weight may not signal the horse to do something specific, but may throw it off balance and end up in an undesired result. The rider can influence the horse as much as the horse can influence the rider. The rider influences the horse by giving aids that result in specific actions, and the horse influences the rider through the motion transmitted through its body to the rider.

2.3 Equine-Assisted Therapy Effects on Children with Cerebral Palsy

Equine-assisted therapy has positive effects, physically, cognitively, behaviorally, and emotionally, on people with disabilities. This section will focus on the physical improvements that are seen in children with cerebral palsy. Cerebral palsy (CP) is caused by damage or abnormal development in the brain. Symptoms for this disease are normally seen in infancy or early childhood. The physical symptoms exhibited may include muscle stiffness, muscle spasticity, poor muscle tone, uncontrolled movements, and problems with posture, balance, coordination, walking, speech, and swallowing. The three main types of CP are spastic CP, dyskinetic CP, and mixed CP. Spastic CP is the most common form of CP and the defining characteristic of this form is increased muscle tone that causes stiff, jerky movements. Dyskinetic CP affects the coordination of movements, and mixed CP is a combination of more than one type of CP. Cerebral palsy can be mild to severe, but almost all children with cerebral palsy require some form of therapy. Equine-assisted therapy has been proven as an
effective therapy to increase the physical well-being of children with CP, since a large portion of pediatric physical therapy involves children with CP, developing and using a therapy that is effective and that the children enjoy is valuable.

Dolores Bertoti [11] performed one of the first objective clinical analyses on the effectiveness of equine-assisted therapy for children with CP. Her study focused on measuring postural changes in children with spastic cerebral palsy. She developed a posture assessment scale, which rated the alignment and symmetry of body parts and control of the muscles around the joints. The children participated in a 10-week hippotherapy program, riding twice weekly for one hour. Riding was done while in the prone, sitting, squatting, standing, and lying positions. The goal of the riding sessions was to reduce spasticity and postural compensations, and to improve the children’s movements including trunk control, weight shift, rotation, and isolating movements of the pelvis and shoulders. The horse was directed in circles and up or down small grades to challenge the children’s balance, strength, and stability.

The study showed that the children’s posture significantly improved following participation in hippotherapy sessions. The improvements were included increased midline head control, decreased neck hyperextension, decreased scapular retraction, more developed scapular musculature, improved symmetry at the trunk, decreased lateral trunk flexion, decreased postural scoliosis, decreased exaggerated lumbar lordosis, increased trunk elongation, decreased anterior pelvic tilt, increased alignment of the pelvis and a more erect posture. Other positive benefits that were observed during the course of this
study were improvements in self-confidence, and a decreased fear of movement and changes in position.

John Sterba et al. [12] conducted a study measuring the effects of therapeutic riding on gross motor function in children with spastic diplegic, spastic quadriplegic and spastic hemiplegic CP. The Gross Motor Function Measure (GMFM), an accepted and validated way of measuring gross motor function was used to measure the physical effect of the therapeutic riding. The children participated in three 6-week sessions, for a total of 18 weeks, and rode for one hour per week. Some of the exercises that the children performed while riding the horses involved reaching for certain parts of the horse’s body while sitting or lying on the horse, holding a stick horizontally with both hands and moving this stick up and down while maintaining correct posture, and tossing cones or bags at traffic cones set on the ground.

The results of this study showed that after 12 weeks of involvement in the therapeutic riding, there was a significant increase in the GMFM Dimension E, which included walking, running and jumping. This increase remained through the last 6-week session, but dropped slightly six weeks following the completion of the therapeutic riding program. The GMFM Dimension E score six weeks after the completion of the program was still 2% higher than before the therapeutic riding program. The total score of the GMFM had a statistically significant increase of 7.6% after the completion of the 18 weeks of the program, but returned to pre-riding levels 6 weeks after stopping riding. The results from the study do show that therapeutic riding has a positive effect in gross motor function.
for the participants. Since CP often requires life-long physical therapy, the completion of the therapeutic riding program meant that some of the positive effects that it did have on the gross motor function diminished.

Michal Kuczynski and Karina Slonka [13] performed a study on the postural stability in children with CP using a microprocessor-driven artificial saddle. The artificial saddle provided stimuli that were similar to that of real horse’s movement only at the walk. The dynamics of postural balance were measured quantitatively by stabilography, which computed the centre-of-pressure (COP). COP gives a measurement of the external characteristics of body balance. The testing involved riding on the artificial saddle for 20 minutes twice a week for 3 months. COP measurements were taken before and after the ride on the artificial saddle. The measurements involved standing on a platform in both the anteroposterior and mediolateral directions.

The results of this study showed that following the therapy, the children with CP exhibited reduced muscle stiffness, and decreased spasticity of the muscles. After single sessions, the patients’ ankle stiffness lessened, allowing them to stand more stable without swaying. It was concluded that the artificial saddle riding contributed to a significant improvement in the postural performance of the children with CP, and that this type of treatment is recommended for children with CP.

All types of equine-assisted therapy; therapeutic riding, therapeutic vaulting, and hippotherapy are capable of improving the physical well-being of children with CP. Increased gross motor function and improved postural stability,
show that equine-assisted therapy is a great form of physical therapy for children with CP. Whether it is on a real horse or a simulated horse, it is the motion that is transferred to the rider that delivers these results. Therefore, developing these programs and making them more widely available is invaluable to those children and adults that can benefit from them.
Chapter 3 – Computer Modeling and Simulation

3.1 Model and Simulation Description

A two-dimensional computer model and simulation of a horseback riding device was developed using MATLAB. Figure 3-1 shows the image of the computer model. The links are labeled and lengths of the links are shown in the starting position.

Figure 3-1: MATLAB Computer Model of Horseback Riding Simulator
The link lengths were chosen to be the same as the lengths used in the physical horseback riding simulator that will be discussed in Chapter 4. The model is a two-dimensional, two degree of freedom system. Link $R_1$ and $R_5$ are the inputs for the system and change length with time. Link $R_2$, $R_3$, $R_4$, and $R_p$, have constant length. The angles $\Theta_4$ and $\Theta_5$, shown in Figure 3-2, change with time and are dependent on the link length of $R_5$. Point $P$ is the point of interest of the system, and has a fixed angle of 30˚ relative to link $R_4$. Point $P$ represents the point where the rider’s hip is, and established the movement that is transmitted from the horseback riding simulator to the rider. Table 3-1 shows the initial link lengths for the simulation.

<table>
<thead>
<tr>
<th>Link</th>
<th>Length (Inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>37.525</td>
</tr>
<tr>
<td>$R_2$</td>
<td>22.2875</td>
</tr>
<tr>
<td>$R_3$</td>
<td>26.75</td>
</tr>
<tr>
<td>$R_4$</td>
<td>22.2875</td>
</tr>
<tr>
<td>$R_5$</td>
<td>26.75</td>
</tr>
</tbody>
</table>

The computer model is capable of simulating the three gaits of the horse, the walk, trot, and canter, by having different input values and phase shifts for link $R_1$ and link $R_5$. An animation of the horseback riding simulator, and the position, velocity and acceleration of point $P$ during the simulation are the outputs. The input values of links $R_1$ and $R_5$ for the simulation were determined based on the kinematics of the walk, trot, and canter discussed in section 2.2.
A position, velocity, and acceleration analysis of the system was done in order to calculate the kinematics of the system. The derivations of the kinematic equations and results from the MATLAB simulation will be shown in the following sections of this chapter. The MATLAB code for the simulation may be viewed in Appendix A.

### 3.2 Position Analysis and Results

Figure 3-2 shows the kinematic drawing of the horseback riding simulator.

A position analysis of the vector loop that consists of \( \vec{r}_2, \vec{r}_3, \vec{r}_4, \) and \( \vec{r}_5 \) was done in order to determine the unknown angles \( \Theta_4 \) and \( \Theta_5 \), as a function of the link lengths. This derivation is shown below.
The vectors $\vec{r}_2$, $\vec{r}_3$, $\vec{r}_4$, and $\vec{r}_5$ shown in Figure 3-2 create a closed loop and will be used to solve for the angles $\Theta_4$ and $\Theta_5$.

$$\vec{r}_2 + \vec{r}_3 + \vec{r}_4 = \vec{r}_5 \quad (3.2-1)$$

Writing Equation (3.2-1) in complex form yields:

$$r_2 e^{i\Theta_2} + r_3 e^{i\Theta_3} + r_4 e^{i\Theta_4} = r_5 e^{i\Theta_5} \quad (3.2-2)$$

By Euler’s Formula:

$$e^{i\Theta} = \cos(\Theta) + i \sin(\Theta) \quad (3.2-3)$$

Equation (3.2-3) is substitutes into Equation (3.2-2).

$$r_2 (c_2 + i s_2) + r_3 (c_3 + i s_3) + r_4 (c_4 + i s_4) = r_5 (c_5 + i s_5) \quad (3.2-4)$$

where $\cos(\Theta_x)$ and $\sin(\Theta_x)$ are denoted by $c_x$ and $s_x$, respectively.

Equation (4) is separated into the real and imaginary components

Real: \[ r_2 c_2 + r_3 c_3 + r_4 c_4 = r_5 c_5 \quad (3.2-5a) \]

Imaginary: \[ r_2 s_2 + r_3 s_3 + r_4 s_4 = r_5 s_5 \quad (3.2-5b) \]

Known, constant values are substituted into Equations (3.2-5a) and (3.2-5b).

From the design parameters, $\Theta_2$ is 180˚ and $\Theta_3$ is 90˚.

$$r_2 \cos(\pi) + r_3 \cos\left(\frac{\pi}{2}\right) + r_4 c_4 = r_5 c_5 \quad (3.2-6a)$$

$$r_2 \sin(\pi) + r_3 \sin\left(\frac{\pi}{2}\right) + r_4 s_4 = r_5 s_5 \quad (3.2-6b)$$

Equations (3.2-6a) and (3.2-6b) are simplified.

$$-r_2 + r_4 c_4 = r_5 c_5 \quad (3.2-7a)$$

$$r_3 + r_4 s_4 = r_5 s_5 \quad (3.2-7b)$$
Equations (3.2-7a) and (3.2-7b) are squared to give:

\[ r_2^2 - 2r_2 r_4 c_4 + r_4^2 c_4^2 = r_5^2 c_5^2 \] (3.2-8a)

\[ r_3^2 + 2r_3 r_4 s_4 + r_4^2 s_4^2 = r_5^2 s_5^2 \] (3.2-8b)

Equations (3.2-8a) and (3.2-8b) are added and simplified.

\[ r_2^2 + r_3^2 + r_4^2 - 2r_4 (r_2 c_4 - r_3 s_4) = r_5^2 \] (3.2-9)

Equation (3.2-9) is rearranged.

\[ r_2 c_4 - r_3 s_4 = -\frac{r_3^2 + r_2^2 + r_3^2 + r_4^2}{2r_4} \] (3.2-10)

The right hand side of Equation (3.2-10) is set equal to C.

\[ C = \frac{r_2^2 + r_3^2 + r_4^2 - r_5^2}{2r_4} \] (3.2-11)

Equation (3.2-10) can now be written as:

\[ r_2 c_4 - r_3 s_4 = C \] (3.2-12)

The following trigonometric identities are substituted into Equation (3.2-12).

\[ u = \tan \frac{\theta_4}{2} \] (3.2-13a)

\[ \cos \theta_4 = \frac{1-u^2}{1+u^2} \] (3.2-13b)

\[ \sin \theta_4 = \frac{2u}{1+u^2} \] (3.2-13c)

\[ r_2 \left( \frac{1-u^2}{1+u^2} \right) - r_3 \left( \frac{2u}{1+u^2} \right) = C \] (3.2-14)

Equation (3.2-14) is multiplied by 1+u^2, to yield:
\[ r_2(1-u^2) - 2r_5u = C(1 + u^2) \]  
(3.2-15)

Powers of \(u\) are collected.

\[(C + r_2)u^2 + 2r_5u + (C - r_2) = 0 \]  
(3.2-16)

The quadratic formula is used to solve Equation (3.2-16).

\[ u = \frac{-2r_5 \pm \sqrt{4r_5^2 - 4(C + r_2)(C - r_2)}}{2(C + r_2)} \]  
(3.2-17)

Equation (3.2-17) simplifies to:

\[ u = \frac{-r_3 \pm \sqrt{r_3^2 - C^2 + r_2^2}}{C + r_2} \]  
(3.2-18)

Identity (3.2-13a) is substituted into Equation (3.2-18) to solve for \(\Theta_4\).

\[ \theta_4 = 2\tan^{-1}\left(\frac{-r_3 \pm \sqrt{r_3^2 - C^2 + r_2^2}}{C + r_2}\right) \]  
(3.2-19)

where \(C\) is defined as Equation (3.2-11).

Equation (3.2-7a) can be solved for \(\Theta_5\).

\[ \theta_5 = \cos^{-1}\left(\frac{-r_2 + r_4c_4}{r_5}\right) \]  
(3.2-20)

Once the equations for \(\Theta_4\), and \(\Theta_5\) were determined, the equations could be included in the computer simulation along with the input functions for \(r_1\) and \(r_5\) at the walk, trot, and canter. The positive result from Equation (3.2-19) is used in the computer code to calculate angle \(\Theta_4\) because since the initial length of \(r_5\) is the shortest length that \(r_5\) can be, angle \(\Theta_4\) can only increase in the positive direction. There is also no concern about the small sensitivity that may occur in Equation (3.2-20) by using the cosine function for the angle. The small sensitivity
would occur when $\Theta_5$ would approach 0° or 180°. The range for $\Theta_5$ in the simulation is between 90° and 92°.

The input functions for $r_1$ and $r_5$ are changed for each gait and are shown below:

- $r_1 = R_1 + A_1 \times (1 - \cos(2\pi f \times \text{time}))$
- $r_5 = R_5 + A_5 \times (1 - \cos(2\pi f \times \text{time} + \phi))$

The different values for the two input functions for the walk, trot, and canter are shown in Table 3-2 below.

<table>
<thead>
<tr>
<th>$r_1$</th>
<th>$r_5$</th>
<th>$A_1$</th>
<th>$A_5$</th>
<th>$f$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td>Trot</td>
<td>Canter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td>37.525 in</td>
<td>37.525 in</td>
<td>37.525 in</td>
<td>1 cycle/s</td>
<td>$-37/180 \times \pi$</td>
</tr>
<tr>
<td>$R_5$</td>
<td>26.75 in</td>
<td>26.75 in</td>
<td>26.75 in</td>
<td>2 cycles/s</td>
<td>$-\pi/3$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>1.14 in</td>
<td>0.725 in</td>
<td>0.41 in</td>
<td>1.2 cycle/s</td>
<td>$-40/180 \times \pi$</td>
</tr>
<tr>
<td>$A_5$</td>
<td>1.55 in</td>
<td>1.78 in</td>
<td>2.49 in</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For all six of the equations listed above, $R_1$ and $R_5$ represent the initial lengths of links $r_1$ and $r_5$. $A_1$, and $A_5$, are the largest increase of each link, and these values are added to the function first to avoid the link decreasing below its minimum length. Inside the cosine function, the $2\pi$ is the period length, the third term is the frequency, $f$, with units of cycles per second, and the fourth term is the time, which changes through the cycle. The frequency data is taken from Table 2-1. There is a fifth term in the input functions for $r_5$, which is the phase shift, $\phi$, and is used to create the inclination of the ellipse for each gait.
The result from the position analysis is the path of point P during one cycle at the walk, trot, and canter. Point P is located at a fixed 30° relative angle to \( r_4 \), and its horizontal component is 11 inches to the right of the end of \( r_4 \). This position represents the point of the hip on a person riding on the simulator. The path of point P at the walk, trot and canter are shown in Figures 3-3, 3-4, and 3-5, respectively.

![Figure 3-3: Path of Point P at the Walk](image)
The path of Point P at the walk has a horizontal amplitude of 1.63 inches, a vertical amplitude of 1.5 inches and an inclination of the ellipse of 22°. At the trot, the path of Point P has a horizontal amplitude of 1.32 inches, a vertical
amplitude of 1.67 inches and an inclination of the ellipse of 135°. At the canter, 
the path of Point P has a horizontal amplitude of 1.15 inches, a vertical amplitude 
of 2.3 inches, and an inclination of the ellipse of 125°. Besides the trot, the 
inclinations of the ellipse are all within the range given in Table 2-1. The trot is 
not within the range because of the constraints of the system. Independent 
horizontal and vertical motion would be required to create the inclination of the 
ellipse for the trot. This does not occur in the current system because as the length of \( r_5 \) increases, angle \( \Theta_5 \) increases since the length of link \( r_4 \) is constant. 
This results in link \( r_5 \) creating both horizontal and vertical motion.

### 3.3 Velocity Analysis and Results

The velocity analysis of the horseback riding simulator was done in a 
similar manner to that of the position analysis. A velocity analysis of the vector 
loop \( \vec{r}_2, \vec{r}_3, \vec{r}_4, \) and \( \vec{r}_5 \), was done in order to determine the unknowns \( \omega_4 \), and \( \omega_5 \). This derivation is shown below:

The vectors \( \vec{r}_2, \vec{r}_3, \vec{r}_4, \) and \( \vec{r}_5 \) shown in Figure 3-2, create a closed loop and will 
be used to solve for the angular velocities \( \omega_4 \) and \( \omega_5 \).

\[
\vec{r}_2 + \vec{r}_3 + \vec{r}_4 = \vec{r}_5 \tag{3.3-1}
\]

Equation (3.3-1) is written in complex form and yields:

\[
r_2 e^{i\theta_2} + r_3 e^{i\theta_3} + r_4 e^{i\theta_4} = r_5 e^{i\theta_5} \tag{3.3-2}
\]

The derivative of Equation (3.3-2) with respect to time is:

\[
ir_2 \frac{d\Theta_2}{dt} e^{i\theta_2} + \frac{dr_2}{dt} e^{i\theta_2} + ir_3 \frac{d\Theta_3}{dt} e^{i\theta_3} + \frac{dr_3}{dt} e^{i\theta_3} + ir_4 \frac{d\Theta_4}{dt} e^{i\theta_4} + \frac{dr_4}{dt} e^{i\theta_4} = \]

38
\( i r_3 \frac{d\Theta_5}{dt} e^{i\Theta_3} + \frac{dr_5}{dt} e^{i\Theta_3} \)  

(3.3-3)

From the design parameters of the mechanism, \( r_2, r_3, r_4, \Theta_2, \) and \( \Theta_3 \) are constant.

Equation (3.3-3) is simplified.

\[
ir_4 \frac{d\Theta_4}{dt} e^{i\Theta_4} = ir_3 \frac{d\Theta_5}{dt} e^{i\Theta_3} + \frac{dr_5}{dt} e^{i\Theta_3}
\]

(3.3-4)

By definition, \( \frac{d\Theta_4}{dt} = \omega_4, \frac{d\Theta_5}{dt} = \omega_5, \) and \( \frac{dr_5}{dt} = \dot{r}_5. \) Substituting these values into Equation (3.3-4) yields:

\[ ir_4 \omega_4 e^{i\Theta_4} = ir_3 \omega_5 e^{i\Theta_3} + \dot{r}_5 e^{i\Theta_3} \]

(3.3-5)

By Euler’s Formula:

\[ e^{i\Theta} = \cos(\Theta) + i \sin(\Theta) \]

(3.3-6)

Equation (3.3-6) is substituted into Equation (3.3-5).

\[ r_4 \omega_4 (i c_4 - s_4) = r_3 \omega_5 (i c_5 - s_5) + \dot{r}_5 (c_5 + i s_5) \]

(3.3-7)

where \( \cos(\Theta_x) \) and \( \sin(\Theta_x) \) are denoted by \( c_x \) and \( s_x \), respectively.

Equation (3.3-7) is separated into the real and imaginary components.

Real:

\[ -r_4 \omega_4 s_4 = -r_3 \omega_5 s_5 + \dot{r}_5 c_5 \]

(3.3-8a)

Imaginary:

\[ r_4 \omega_4 c_4 = r_3 \omega_5 c_5 + \dot{r}_5 s_5 \]

(3.3-8b)

Equations (3.3-8a) and (3.3-8b) are put into matrix form.

\[
\begin{bmatrix}
-r_4 s_4 & r_3 s_5 \\
r_4 c_4 & -r_3 c_5
\end{bmatrix}
\begin{bmatrix}
\omega_4 \\
\omega_5
\end{bmatrix}
= 
\begin{bmatrix}
\dot{r}_5 c_5 \\
\dot{r}_5 s_5
\end{bmatrix}
\]

(3.3-9)

Cramer’s rule is used to solve this system of equations.
$\omega_4$ is solve for first.

$$\omega_4 = \frac{\dot{r}_5 c_5 \ r_5 s_5 - r_5 c_5 \ r_5 s_5 - r_4 s_4 \ r_5 s_5 + r_4 c_4 \ r_5 c_5}{r_4 c_4 - r_5 c_5}$$  \hspace{1cm} (3.3-10)$$

Taking the determinate of the top and bottom of Equation (3.3-10) and simplifying yields the solution for $\omega_4$.

$$\omega_4 = \frac{-\ddot{r}_5}{r_4 \sin(\Theta_4 - \Theta_5)}$$  \hspace{1cm} (3.3-11)$$

Next, $\omega_5$ is solved for using Cramer’s Rule.

$$\omega_5 = \frac{-r_4 c_4 \ r_5 s_5 - r_4 s_4 \ r_5 c_5}{r_4 c_4 \ r_5 c_5}$$  \hspace{1cm} (3.3-12)$$

Taking the determinate of the top and bottom of Equation (3.3-12) and simplifying yields the solution for $\omega_5$.

$$\omega_5 = -\frac{\ddot{r}_5}{r_5} \cot(\Theta_4 - \Theta_5)$$  \hspace{1cm} (3.3-13)$$

As was done for the position analysis, after solving for the unknown equations $\omega_4$ and $\omega_5$, the computer simulation written in MATLAB could determine the velocity of the system, including the velocity of the point of interest $P$. The simulation required the rate of change or $r_1$ and $r_5$ with time. These functions were found by taking the derivates of functions for $r_1$ and $r_5$ with respect to time. The velocity of point $P$ was determined by taking the derivative of the $x$
and $y$ components of point P with respect to time. Figures 3-6, 3-7, and 3-8 show the velocity vectors through the path of point P.

Figure 3-6: Velocity of Point P at the Walk
Figure 3-7: Velocity of Point P at the Trot

Figure 3-8: Velocity of Point P at the Canter
The velocity of point P is tangential to the path of point P at all points. The black square in the three figures indicates the start of the cycle at time equal to zero. Figure 3-9 will show a comparison of the magnitude of the velocity of point P at the walk, trot, and canter through one second.

![Figure 3-9: Velocity Magnitudes of Point P](image)

The magnitude of the velocity at the walk varies between 3.3 and 6.1 inches per second, at the trot varies between 7.1 and 11.2 inches per second, and at the canter varies between 1.7 and 9.6 inches per second. The greatest variation is in the canter because the ellipse is less round than in the trot or the walk. There is twice as much vertical displacement as horizontal displacement in
the canter, which causes a large variation in velocity as the simulation changes from going up to going down. Since the walk is the roundest, the magnitude of the velocity does not change much, resulting in the smallest variation.

3.4 Acceleration Analysis and Results

The acceleration analysis of the horseback riding simulator was completed in the same manner as the velocity analysis. The kinematic equations for the unknown angular accelerations, $\alpha_4$ and $\alpha_5$ were determined by the acceleration analysis that follows, beginning with Equation (3.3-4).

$$ir_4 \frac{d\Theta_4}{dt} e^{i\Theta_4} = ir_5 \frac{d\Theta_5}{dt} e^{i\Theta_5} + \frac{dr_5}{dt} e^{i\Theta_5}$$

(3.3-4)

The derivative of Equation (3.3-4) is taken with respect to time.

$$i \frac{dr_4}{dt} \frac{d\Theta_4}{dt} e^{i\Theta_4} + ir_4 \frac{d^2\Theta_4}{dt^2} e^{i\Theta_4} - r_4 \left( \frac{d\Theta_4}{dt} \right)^2 e^{i\Theta_4} =$$

$$i \frac{dr_5}{dt} \frac{d\Theta_5}{dt} e^{i\Theta_5} + ir_5 \frac{d^2\Theta_5}{dt^2} e^{i\Theta_5} - r_5 \left( \frac{d\Theta_5}{dt} \right)^2 e^{i\Theta_5} + \frac{d^2 r_5}{dt^2} e^{i\Theta_5} + i \frac{dr_5}{dt} \frac{d\Theta_5}{dt} e^{i\Theta_5}$$

(3.4-1)

Equation (3.4-1) is simplified, given that $r_4$ is constant.

$$ir_4 \frac{d^2\Theta_4}{dt^2} e^{i\Theta_4} - r_4 \left( \frac{d\Theta_4}{dt} \right)^2 e^{i\Theta_4} =$$

$$i \frac{dr_5}{dt} \frac{d\Theta_5}{dt} e^{i\Theta_5} + ir_5 \frac{d^2\Theta_5}{dt^2} e^{i\Theta_5} - r_5 \left( \frac{d\Theta_5}{dt} \right)^2 e^{i\Theta_5} + \frac{d^2 r_5}{dt^2} e^{i\Theta_5} + i \frac{dr_5}{dt} \frac{d\Theta_5}{dt} e^{i\Theta_5}$$

(3.4-2)

By definition, $\frac{d\Theta_4}{dt} = \omega_4$, $\frac{d\Theta_5}{dt} = \omega_5$, $\frac{d^2\Theta_4}{dt^2} = \alpha_4$, $\frac{d^2\Theta_5}{dt^2} = \alpha_5$, $\frac{dr_5}{dt} = \dot{r}_5$ and $\frac{dr_5^2}{dt^2} = \ddot{r}_5$. Substituting these values into Equation (3.4-2) yields:
\[ i r_4 \alpha_4 e^{i\Theta_x} - r_4 \omega_4^2 e^{i\Theta_x} = i r_5 \omega_5 e^{i\Theta_x} + i r_5 \alpha_5 e^{i\Theta_x} - r_5 \omega_5^2 e^{i\Theta_x} + i r_5 \omega_5 e^{i\Theta_x} \] (3.4-3)

By Euler’s Formula:

\[ e^{i\Theta_x} = \cos(\Theta_x) + i \sin(\Theta_x) \] (3.4-4)

Equation (3.4-4) is substituted into Equation (3.4-3).

\[ r_4 \alpha_4 (ic_4 - s_4) - r_4 \omega_4^2 (c_4 + is_4) = \]

\[ 2r_5 \omega_5 (ic_5 - s_5) + r_5 \alpha_5 (ic_5 - s_5) - r_5 \omega_5^2 (c_5 + is_5) + i r_5 (c_5 + is_5) \] (3.4-5)

where \( \cos(\Theta_x) \) and \( \sin(\Theta_x) \) are notated by \( c_x \) and \( s_x \), respectively.

Equation (3.4-5) is separated into the real and imaginary components.

Real:

\[ -r_4 \alpha_4 s_4 - r_4 \omega_4^2 c_4 = -2r_5 \omega_5 s_5 - r_5 \alpha_5 s_5 - r_5 \omega_5^2 c_5 + i r_5 c_5 \] (3.4-6a)

Imaginary:

\[ r_4 \alpha_4 c_4 - r_4 \omega_4^2 s_4 = 2r_5 \omega_5 c_5 + r_5 \alpha_5 c_5 - r_5 \omega_5^2 s_5 + i r_5 s_5 \] (3.4-6b)

Equations (3.4-6a) and (3.4-6b) are put into matrix form.

\[
\begin{bmatrix}
-r_4 s_4 & r_5 s_5 \\
r_4 c_4 & -r_5 c_5
\end{bmatrix}
\begin{bmatrix}
\alpha_4 \\
\alpha_5
\end{bmatrix}
=
\begin{bmatrix}
r_4 \omega_4^2 c_4 - 2r_5 \omega_5 s_5 - r_5 \omega_5^2 c_5 + i r_5 c_5 \\
r_4 \omega_4^2 s_4 + 2r_5 \omega_5 c_5 - r_5 \omega_5^2 s_5 + i r_5 s_5
\end{bmatrix}
\] (3.4-7)

Cramer’s rule can used to solve this system of equations.

\( \alpha_4 \) is solved for first.

\[
\alpha_4 = \frac{
\begin{vmatrix}
r_4 \omega_4^2 c_4 - 2r_5 \omega_5 s_5 - r_5 \omega_5^2 c_5 + i r_5 c_5 & r_5 s_5 \\
r_4 \omega_4^2 s_4 + 2r_5 \omega_5 c_5 - r_5 \omega_5^2 s_5 + i r_5 s_5 & -r_4 s_4 & r_5 s_5 \\
r_4 c_4 & -r_5 c_5
\end{vmatrix}
}{
\begin{vmatrix}
r_4 s_4 & r_5 s_5 \\
r_4 c_4 & -r_5 c_5
\end{vmatrix}
}
\] (3.4-8)

Taking the determinate of the top and bottom of Equation (3.4-8) and simplifying yields the solution for \( \alpha_4 \).
\[
\alpha_4 = \frac{-r_4 \omega_4^2 c(c_4 - s) + r_5 \omega_5^2 - \ddot{r}_5}{r_4 \sin(\Theta_4 - \Theta_5)}
\]  \tag{3.4-9}

Now \(\alpha_5\) is solved for using Cramer’s rule.

\[
\alpha_5 = \frac{|-r_4 s_4 \quad r_4 \omega_4^2 c_4 - 2\dot{r}_5 \omega_5 s_5 - r_5 \omega_5^2 c_5 + \ddot{r}_5 c_5|}{|r_4 c_4 \quad r_4 \omega_4^2 s_4 + 2\dot{r}_5 \omega_5 c_5 - r_5 \omega_5^2 s_5 + \ddot{r}_5 s_5|}
\]  \tag{3.4-10}

Taking the determinate of the top and bottom of Equation (3.4-10) and simplifying yields the solution for \(\alpha_5\).

\[
\alpha_5 = \frac{-r_4 \omega_4^2 - 2\dot{r}_5 \omega_5 \sin(\Theta_4 - \Theta_5) + r_5 \omega_5^2 \sin(\Theta_4 + \Theta_5) - \ddot{r}_5 \sin(\Theta_4 + \Theta_5)}{r_5 \sin(\Theta_4 - \Theta_5)}
\]  \tag{3.4-11}

The second derivate of links \(r_1\) and \(r_5\) with respect to time were computed in order to complete the simulation. The second derivative of the \(x\) and \(y\) components of point \(P\) were also determined in order to calculate the acceleration of the point of interest \(P\). Figure 3-10 shows a graph with the accelerations at point \(P\) for all three gaits, the walk, trot, and canter.
The acceleration ranges from 1.7 to 3.2 ft/s^2 at the walk, 6.9 to 11.8 ft/s^2 at the trot, and 0.4 to 6.1 ft/s^2 at the canter. The walk has the smoothest acceleration curve, but also moves the shortest distance at the lowest frequency. The walk, trot, and canter curves are not uniform through a complete cycle. There is slightly lower acceleration at every other valley. The peaks through these curves are also wider than the valleys. A graph of the Fourier transform showed the fundamental frequency for the system and also a 1st harmonic. The magnitude of the 1st harmonic compared to the magnitude of the fundamental frequency is very small, and therefore does not affect the sine wave much. The affect from the 1st harmonic is seen in the graph for the velocity also, but is more accentuated in the acceleration graph. The graph showing the Fourier Transform
for the walk is shown in Figure 3-11. The graphs for the trot, and canter show the same results and therefore have been omitted.

The small variation of acceleration, only 1.5 ft/s$^2$ difference, would probably not be noticed by a rider. More importantly, a jerk during the second valley of the canter is seen. Again, the change in acceleration is small, so this jerk may not be noticed in a physical simulator. Depending on the horse that is being ridden, there may be a jerk felt at one point in the canter. The average magnitude of the acceleration is less than a third of that of gravity, so no safety risks concerning high accelerations exist for this simulator.
Chapter 4 – Physical Simulator Design and Fabrication

4.1 Design

The horseback riding simulator was designed for the purpose of simulating the walk, trot, canter, flexion/extension, and lateral bend of the horse. A device of this kind can be used as a physical therapy device for persons with disabilities or as a training device at riding stables to help riders learn to ride better. This section will include a detailed look at the design of the horseback riding simulation. The following section will describe the fabrication of the simulator.

During the development of the concept of the horseback riding simulator, a list of requirements was established. This list included:

1. Replication of the horse’s walk, trot and canter.
2. Replication of the horse’s lateral bend to the right and left and flexion/extension.
3. Safety factor of at least two for all components with a rider up to 200 lbs.
4. Safe for a person with a disability to ride.

These requirements were evaluated throughout the design process, from the initial concept to the completion of the final design of the prototype.

The horseback riding simulator consists of the structure, which includes the track, base, and rider support bars, two air sprung electrically driven linear actuators (PEMRAMs), and the seat for the rider, which is a plastic horse’s back.
Detailed AutoCAD drawings with dimensions are included in Appendix C. The finished design of the horseback riding simulator is shown in Figure 4-1.

![Figure 4-1: Horseback Riding Simulator](image)

The first requirement for the horseback riding simulator is to replicate the horse’s walk, trot, and canter. The simulator has two degrees of freedom and creates the horse’s motion in the same manner as the MATLAB computer simulation discussed in Chapter 3. The horizontal and vertical PEMRAM’s change length in a periodic manner and the phase shift between the two PEMRAMs creates the elliptical motion of the horse’s walk, trot, and canter. The amplitudes of the individual PEMRAMs are set by adjusting the resistance to the amplifiers.
The next requirement for the simulator is to replicate the flexion/extension and lateral bend of the horse. These were created by adding custom shaped foam to the top and side of the plastic horse. The plastic horse without foam simulated a horse with no flexion/extension or lateral bend of the back. When the foam was in place, right or left lateral bend and flexion/extension of the back was simulated. Figure 4-2 shows the image of the horse back with the foam simulating flexion-extension and Figure 4-3 shows the image of the horse back with the foam simulating left bend.

Figure 4-2: Flexion-Extension of the Horseback Riding Simulator
A safety factor of at least two for a rider up to 200 lbs is required for the horseback riding simulator. The safety factor of the horseback riding simulator was evaluated by computing the static load analysis of the individual components. Each component is subjected to a bending, tension, compression, or a combination of the two. The most influential stress was used to evaluate the safety of the component.

The prototype of the horseback riding simulator was grossly over designed. The horizontal rod stabilizes the plastic horseback where the rider will sit. The rod is subjected to a bending stress. Using a uniformly distributed
weight of 400 lbs over the 23 inch length of the rod, the maximum displacement and bending stress that the rod is subjected to is negligible, with a safety factor well over 2. The vertical support tube is subjected to compressive stresses. Using 400 lbs and the area of the tube, the stress is calculated to be 0.4 ksi, which gives a safety factor much greater than 2 when compared to the yield stress of 30 ksi of the steel used. This calculation is shown in Equation 4.1-1.

$$\sigma = \frac{P}{A} = \frac{400lbs}{0.9024in^2} = 0.4ksi$$  \hspace{1cm} (4.1-1)

The base of the simulator was made out of steel to lower the center of gravity of the system in order to eliminate any tipping effects. The PEMRAMs are capable of moving 220 lbs each. Additional weight can be lifted if air pressure is introduced into the system, but was not necessary with the final design of the simulator.

The fourth requirement for the horseback riding simulator is that it must be safe for a person with a disability to ride. The simulator that was designed and fabricated is a prototype, and therefore all hazards and dangers of riding the simulator have not been eliminated. Some of the known hazards are listed below.

1. The plastic horse tipping to the side if the rider loses their balance.

2. Not having full control of the movement of the PEMRAMs. When the system is started, the PEMRAMs extend out to their full capacity to calibrate. The program for the simulator is then started, but the rider must mount the horse when the simulator is moving.
3. There is no external support that the rider can hold on to or be attached to that will provide additional stability.

4. At least one person must control the simulator while it is being used in order to press the emergency stop button if necessary. The rider does not have any control over the simulator.

4.2 Fabrication

The structure of the horseback riding simulator includes the track, base, and rider support bars. These are labeled in Figure 4-4.

![Figure 4-4: Horseback Riding Simulator Structure](image)

The track is made out of 90° steel angle brackets welded to steel plates. The two tracks have a steel plate welded at the front and back to create a box. The system rides on the track via four v-groove wheels. The v-groove wheels are self-aligning on the track, eliminating any misaligning that may occur with
other styles. The horizontal PEMRAM is bolted to the front plate of the track. This eliminates the need to secure the system to the floor, because since the complete system is bolted together, the weight will keep it from moving.

The base consists of 1 inch steel tubes welded in a rectangle and three reinforcing steel tubes welded across. A 1/8th inch thick steel plate is welded to the top of the base. The base weighs about 100 lbs.

The rider support bars include the support tube, support rod, and the vertical PEMRAM. The vertical PEMRAM is attached to the base by a steel bracket. The support tube is a 2-inch square tube that is welded to the base as the vertical support. A 1-inch thick rod is used as the horizontal support. The horizontal support tube is attached to the PEMRAM and to the support tube using clevis joints, which allow the small amount of pivoting motion that is necessary for the simulation.

The plastic horse is attached to the horizontal support tube by steel tubes welded together as shown in Figure 4-5.

![Figure 4-5: Attachment of Plastic Horse to Simulator](image)

Two steel bars run parallel to the horizontal support rod and three steel bars run perpendicular to the horizontal support rod. The parallel bars have a 90° angle
that has been welded to the ends, which bolt to the front and back of the plastic horse. On the underside of the plastic horse, a wooden platform was attached using epoxy and angle brackets. The wooden platform created a flat surface for the steel bars to sit. This is shown in Figure 4-6.

![Figure 4-6: Underside of Plastic Horse](image)

The fabrication of the horseback riding simulator was kept as simple and inexpensive as possible. The PEMRAMs were taken from a dynamic seat that was already at the University. Steel was used to eliminate special welding techniques. The brick red paint was applied to reduce rust on the simulator. The
system stands alone so that moving it is relatively easy. Improvements on the design are included in Chapter 6.
Chapter 5 – Motion Analysis and Results

5.1 Description of the Testing of the Horseback Riding Simulator

Following the design and fabrication of the horseback riding simulator, testing to evaluate the simulator was performed. The purpose of the testing was to evaluate the similarity of the horseback riding simulator to the gaits of the data collected from a real horse. The results from the testing are compared with the position, velocity, and acceleration data from the MATLAB simulation. The biomechanical effect on the hip angle is also evaluated when the system simulates bending and collection of the back.

The Vicon system was used to collect the position, velocity, acceleration, and hip angles of the rider on the horseback riding simulator. Four cameras with infrared lights were set up to collect the data. A Vicon 612 datastation computer was used to collect and preprocess the data from the cameras. A Dell computer took the information from the datastation and ran programs using the Vicon Bodybuilder software to analyze the data. The computer program was written using the Vicon Bodybuilder software to calculate the marker trajectories and the hip angles. Calibration of the system was completed at the beginning of each testing session using a 4-marker calibration frame with known distances and a two-marker wand to measure out the collection volume. Using direct linear transformations, the locations of the camera and the marker distances were
computed in order to collect the 2-dimensional information from each camera into one 3-dimensional set of data. Figure 5-1 shows the set up of two of the cameras. The other two cameras are just off the photo on the right and left hand side.

![Vicon Camera Configuration](image)

**Figure 5-1: Vicon Camera Configuration**

Five reflective markers were placed on the rider. Figure 5-2 shows the placement of the markers on the Anterior Superior Iliac Spine (ASIS), Posterior Superior Iliac Spine (PSIS), sacrum, thigh, and knee. The ASIS marker is the one that is placed on the hip bone and corresponds to the point P of the computer simulation. The ASIS, PSIS, and sacrum markers create the pelvis segment and were used along with the knee and thigh point that created the upper leg segment to determine the hip angle.
Four different tests were carried out. These included:

1. Horseback without simulated bend or collection.

2. Horseback with simulated collection only.

3. Horseback with simulated left bend only.

4. Horseback with simulated collection and left bend.

Eight runs of each of these tests were completed with an average data collection time of 9 seconds. The Fourier transforms of each data set was calculated in order to choose the data that would be used. The Fourier transform was used because the data with the least amount of noise had the best results. Peaks in the graph at all three gaits were seen at 60 Hz, at the fundamental frequency, at the 1st harmonic and 2nd harmonic. Peaks beyond the second harmonic are
present, but with magnitudes very low compared to the lower harmonics that these would not affect the motion much. Figure 5-3 shows the Fourier transform graph of the data collected at the walk. The trot and canter graphs are similar, and so are omitted. Note that the noise at 60 Hz is not included in the graph.

![Fourier Transform Graph at Walk](image)

**Figure 5-3: Fourier Transform Graph at Walk**

### 5.2 Kinematic Results

The kinematic results from the testing includes the position, velocity, and acceleration of the point of the rider's right ASIS, or hip. A visual inspection of the data was used to determine which graphs best represented the walk, trot,
and canter. As discussed in the previous section, the data for each gait that had the smallest amount of noise as determined from the Fourier transform graph was chosen. Differences in the kinematics between the four different tests were not seen. Figures 5-4, 5-5, and 5-6 show the graph of the position of ASIS for the walk, trot, and canter, respectively.

Figure 5-4: Horseback Riding Simulator Walk

Figure 5-5: Horseback Riding Simulator Trot
The programming capabilities of the PEMRAM's movement were limited. The data collected was run on programs that were already available for the system. This frequency, phase shifts and magnitudes were already established in these programs and could not be changed. Two discrete values for the phase shifts were available for the vertical PEMRAM relative to the horizontal PEMRAM. These discrete values for the phase shifts were only slightly offset. Also, the magnitudes of the PEMRAMs could not be adjusted.

The limitations of the programming capabilities of the system reduced the control over the system. The walk gait was simulated by using only the horizontal PEMRAM motion. The trot and canter gaits were simulated using a program that used both the vertical and horizontal PEMRAMs, but with different phase shifts. The largest problem in the motion that resulted from limited programming capabilities was in changing the individual magnitudes of the
PEMRAMs. Better motion would be created if it were possible to increase or decrease the magnitudes of the individual PEMRAMs separately.

Overall, the rider’s comments on the feel of the gaits as compared to a real horse was promising. The motion was describes as feeling a little large in the canter, but not in the walk. Also, the rider felt that there should be more force exerted in the trot and the canter.

Three different measurements are used to evaluate the position data of the horseback riding simulator. These measurements are the inclination of the ellipse, the major axis of the ellipse, and the frequency. Table 5-1 shows these three measurements for the computer simulation and the horseback riding simulator for the three gaits, the walk, trot, and canter.

| Table 5-1: Comparison of the Computer and Horseback Riding Simulators |
|-----------------|-----------------|-----------------|-----------------|
|                 | Inclination of the Ellipse | Major Axis | Frequency |
| Computer Simulation Walk | 22° | 2.59 Inch | 1 Hz |
| Horseback Riding Simulator Walk | 5° | 9.14 Inch | 0.5 Hz |
| Computer Simulation Trot | 135° | 2.14 Inch | 2 Hz |
| Horseback Riding Simulator Trot | 160° | 8.6 Inch | 0.5 Hz |
| Computer Simulation Canter | 125° | 2.86 Inch | 1.2 Hz |
| Horseback Riding Simulator Canter | 165° | 10.67 Inch | 0.5 Hz |
Comparing the frequency, length of the major axis of the ellipse, and the inclination of the ellipse between the computer simulation and the horseback riding simulator shows differences in all three measurements. The walk and canter of the horseback riding simulator have different frequencies from the computer simulation. The horseback riding simulator was run at 0.5 Hz during runs for all three gaits. This difference in the frequency does not change the feel of the motion, though, and there is a wide range of frequencies that a real horse will move. Having a lower frequency for the horseback riding simulator only shows a difference from the computer simulation, not from the range of frequencies for a real horse. Also, running the horseback riding simulator at a lower frequency was safer for the rider.

The length of the major axis is different in the horseback riding simulator and the computer simulation. The length of the major axis for the horseback riding simulator is 352% higher in the walk, 402% in the trot, and 373% in the canter compared to the computer simulation. Although the magnitudes of the horseback riding simulator are larger than the computer simulation, during the motion testing the rider did not feel that the movements were exaggerated. The magnitudes of the physical simulator could be reduced, but the noise from the PEMRAMs is amplified as the magnitudes are decreased.

The inclination of the ellipse is the third measurement that was used to compare the computer and horseback riding simulators. When the inclination of the ellipse was calculated for the three gaits, they were not the same as from the computer simulation. As shown in Table 5-1, the walk did not have enough
inclination, and the trot and canter had too much inclination. To improve these values for the horseback riding simulator, the phase shifts between the two PEMRAMs would have to be adjusted.

The velocity and acceleration of the horseback riding simulator was also calculated. The magnitude of the velocity of the walk varied from 1 in/s to 13 in/s, the magnitude of the velocity of the trot varied from 1.1 in/s and 23 in/s, and the magnitude of the velocity of the canter varied from 1.3 in/s and 20.8 in/s. Unfortunately because of the noise that was apparent in the PEMRAMs the velocity range is much higher. The PEMRAM would stick and slip during the testing which caused inconsistencies in the velocities at the walk, trot, and canter.

The acceleration of the horseback riding simulator was also calculated. For the walk, the acceleration ranged from 0 ft/s^2 to 6.5 ft/s^2, for the trot ranged from 0 ft/s^2 to 16.2 ft/s^2, and for the canter ranged from 0 ft/s^2 to 9.8 ft/s^2. Jerking was felt by the rider and was seen in the data, but the jerking was determined to be from the stick and slip that occurred in the PEMRAMs. Noise that was seen in the graphs for the position of the hip during the walk, trot, and canter is greatly accentuated in the acceleration data. If the noise that was included in the horseback riding simulator was lessened, the acceleration values would have much less noise, and the ranges would be smaller. But overall, unsafe velocities and accelerations were not felt by the rider.
5.3 Biomechanical Results

Biomechanical measurements were also taken during the testing of the horseback riding simulator. Hip angles during the four tests described in section 5.1 were compared. These tests compared the hip angles when there was no simulated bend or collection, with simulated bend only, with simulated collection only, and with simulated bend and collection. Table 5-2 shows the results from the hip angles for the walk, trot, and canter.

<table>
<thead>
<tr>
<th></th>
<th>Walk</th>
<th>Trot</th>
<th>Canter</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Bend or Collection</td>
<td>49.5° - 66.5°</td>
<td>36.3° - 63.1°</td>
<td>38.4° - 66.6°</td>
</tr>
<tr>
<td>Collection Only</td>
<td>44.1° - 58.7°</td>
<td>37.0° - 64.6°</td>
<td>37.1° - 66.7°</td>
</tr>
<tr>
<td>Bend Only</td>
<td>61.2° - 84.8°</td>
<td>57.8° - 86.1°</td>
<td>60.9° - 86.0°</td>
</tr>
<tr>
<td>Bend and Collection</td>
<td>54.2° - 78.9°</td>
<td>44.1° - 79.1°</td>
<td>45.9° - 75.8°</td>
</tr>
</tbody>
</table>

The difference between the walk, trot, and canter hip flexion angles between the four different tests is minimal. The trot and canter have similar ranges, although the trot ranges are wider than the canter. The walk has the smallest range of hip flexion angles. There is no hip flexion angle difference between the test without any simulated bend or collection and the test with collection only. The test with the simulated bend only shows a significant increase in the hip flexion angle compared to the tests without bend. In the walk, the smallest hip flexion angle increases by 12° in the test with bend only compared to the test without bend or collection. The range of the hip flexion
angle also increases by 6.6°. This same trend is also seen in the trot and the canter. This shows that adding simulated collection does not change the hip flexion angles of the rider, but adding simulated bend changes the hip flexion angles significantly.
Chapter 6 – Conclusions and Recommendations

6.1 Conclusions

In the past decades, research dealing with horses has been funded and performed in order to improve the racehorse industry. Recently, research dealing with horses has expanded in all areas, including dressage, jumping, reigning, horse biomechanics, rider biomechanics, and equine-assisted therapy. It is unfortunate that with the expansion of this research, that communities in the United States continue to force horses out. It is for this reason that research involving simulating the horse’s motion is important. It will continue to become harder to have land in and near metropolitan areas where many people with disabilities live, and therefore the expansion of rehabilitation therapy centers that use horses is not expected to increase as quickly as the number of people that are interested in being involved in these programs.

A computer model of a horseback riding simulator and a physical horseback riding simulator have been developed. The computer model was developed using MATLAB and allowed the user to choose the gait and the time that the simulator ran for. The results from the computer simulation was compared with data from a real horse.

The physical horseback riding simulator was designed and fabricated using the same link lengths as the computer model. The track, frame, rider
support bars, and plastic horse seat were designed to work with the PEMRAMs and existing programs. The horseback riding simulator was designed to simulate the walk, trot, canter, bend, and collection of a real horse. Kinematic and biomechanical data was collected during the testing of the horseback riding simulator and was compared to the data from the computer simulation.

The results from the horseback riding simulator show that the simulated bend affects the rider’s hip flexion angle significantly more than the simulated collection. This result can be used during the physical therapy treatment of persons with disabilities. During the period when a person with a disability is involved in equine-assisted therapy, the rehabilitation effects will improve their mobility, posture, muscle tone, balance, motor skills. As these effects are seen, the simulated bend can be used to increase the hip flexion angle and hip flexion angle range that the rider will experience. This should result in even greater results from the therapy.

The horseback riding simulator is intended to be used as an aid for physical therapy. Clinical studies involving persons with disabilities riding the simulator would be necessary to test the effectiveness of the horseback riding simulator. These clinical studies were not performed because the current horseback riding simulator is a prototype and does not meet the safety criteria that would be necessary. Further development of the simulator to meet this requirement would allow clinical research to be performed.
6.2 Future Work and Recommendations

More research is needed in the area of equine-assisted therapy. Using a horseback riding simulator it is possible to conduct controlled studies that can quantitatively measure the improvement of a rider using this type of therapy. The prototype of the horseback riding simulator that was built for this research requires improvement of control of the movement. A graphical user interface created in the programming would easily the phase shift, magnitudes, and frequencies of the PEMRAMs to be changed. Also, adding components that give the rider control over the motion that they are feeling would improve using the horseback riding simulator. Rider control could be added by developing a new way to simulate the bend and collection of the horse. If the rider were able to put pressure with their legs on a specific part of the plastic horse in order to simulate the bend and collection, they would have more control over their movement. Also, adding components which would allow the rider to start and stop the movement is important.

There are safety issues to consider with the horseback riding simulator also. The track and frame have sharp, metal corners that would need to be covered. The rider support bars tend to rotate and unscrew, which tilts the plastic horse. Fixing this problem would require welding the bars that are screwed in or having a larger axle at the pivot point of the device. A handle for the rider to hold on to that is more substantial than a strap on the saddle is also recommended.
References


Bibliography


Appendices
function [Horse_plot] = Horse2(tot_time,gait)

    % This function will plot a 2-dimensional model of the horseback riding simulator and graphs.
    % The input for the function Horse_plot is the total time of the simulation in seconds and the gait, either walk, trot or canter that should be simulated.
    % The gait can be specified by:
    %       Walk    =  1
    %       Trot    =   2
    %       Canter  = 3

    % Revised on February 11, 2006 in order to update equations for Theta_4 and Theta_5.
    % Updated on May 22, 2006 to change the input functions and add velocity and acceleration graphs.

    __________________________________________________________________________

    % This will be the time intervals used
    dt = 0.01;
    % Total number of steps that the simulation will go through
    Steps = (tot_time/dt);

    % Initialize the length of r1
    r1 = 37.525;
    % Initialize the velocity of r1
    dr1_dt = 0;
    % Initialize the acceleration of r1
    d2r1_dt2 = 0;
    % Given the value of the length of r2
    r2 = 22.2875;
    % Given the value of the length of r3
    r3 = 26.75;
    % Given the value of the length of r4
    r4 = 22.2875;
    % Initialize the length of r5
    r5 = 26.75;
    % Initialize the velocity of r5
    dr5_dt = 0;
    % Initialize the acceleration of r5
Appendix A (Continued)

d2r5_dt2 = 0;
% Given the value of the angle delta
delta = 30*pi/180;
% Given the value of the length of rp
rp = 11/cos(delta);
% Initialize the time
time = 0;

% The initial value of r1 is needed for the input functions
R1 = r1;
% The initial value of r5 is needed for the input functions
R5 = r5;

% Simplification for the equation that is solved for the position of Theta_4
C = (r2^2+r3^2+r4^2-r5^2)/(2*r4);

% Equations for theta_4 and theta_5
Theta_4 = 2*atan((-r3+sqrt(r3^2+r2^2-C^2))/(C+r2));
Theta_5 = acos((-r2+r4*cos(Theta_4))/r5);

% Equations for omega_4 and omega_5
Omega_4 = -dr5_dt/(r4*sin(Theta_4-Theta_5));
Omega_5 = -dr5_dt/r5*cot(Theta_4-Theta_5);

% Equations for alpha_4 and alpha_5
Alpha_4 = (-r4*Omega_4^2*cos(Theta_4-Theta_5)+r5*Omega_5^2-d2r5_dt2)/(r4*sin(Theta_4-Theta_5));
Alpha_5 = (-r4*Omega_4^2-2*dr5_dt*Omega_5*sin(Theta_4-Theta_5)+r5*Omega_5^2*sin(Theta_4+Theta_5)-d2r5_dt2*sin(Theta_4+Theta_5))/(r5*sin(Theta_4-Theta_5));

% These values are initialized for the plots of these changing values
plot_r1 = r1;
plot_r5 = r5;
plot_Theta_4 = Theta_4;
plot_Theta_5 = Theta_5;
plot_p = [r1+rp*cos(Theta_4+delta),r3+rp*sin(Theta_4+delta)];
plot_dp_dt = [dr1_dt-rp*Omega_4*sin(Theta_4+delta),rp*Omega_4*cos(Theta_4+delta)];
plot_d2p_dt2 = [d2r1_dt2-(rp*Omega_4^2*cos(Theta_4+delta)+rp*Alpha_4*sin(Theta_4+delta)),Alpha_4*rp*cos(Theta_4+delta)+Omega_4^2*rp*sin(Theta_4+delta)];
plot_dr1_dt = dr1_dt;
plot_dr5_dt = dr5_dt;
Appendix A (Continued)

plot_d2r1_dt2 = d2r1_dt2;
plot_d2r5_dt2 = d2r5_dt2;

% Loop to animate simulation
for LOOP = 0:Steps-1
    % Clear the graph for the animation of the simulation
    clf;

    % Establish the x axis of the first link r1
    fa = [0,r1];
    % Establish the y axis of the first link r1
    fb = [0,0];
    % Establish the x axis of the second link r2 (Constant length)
    fc = [r1,r1+r2];
    % Establish the y axis of the second link r2 (Constant length)
    fd = [0,0];
    % Establish the x axis of the third link r3 (Constant length)
    fe = [r1,r1];
    % Establish the y axis of the third link r3 (Constant length)
    ff = [0,r3];
    % Establish the x axis of the fourth link r4 (Constant length)
    fg = [r1,r1+r2-r5*sin(Theta_5-(pi/2))];
    % Establish the y axis of the fourth link r4 (Constant length)
    fh = [r3,r5*cos(Theta_5-(pi/2))];
    % Establish the x axis of the fifth link r5
    fi = [r1+r2,r1+r2-r5*sin(Theta_5-(pi/2))];
    % Establish the y axis of the fifth link r5
    fj = [0,r5*cos(Theta_5-(pi/2))];
    % Establish the x axis of rp (Constant length)
    fk = [r1,r1+rp*cos(Theta_4+delta)];
    % Establish the y axis of rp (Constant length)
    fl = [r3,r3+rp*sin(Theta_4+delta)];

    % Figure(1) is the animation of the position of the horse riding simulator
    figure(1)
    plot(fa,fb,'-k+',fc,fd,'-k+',fe,ff,'-k+',fg,fh,'-k+',fi,fj,'-k+',fk,fl,'-k+','LineWidth',4)
    grid off
    axis([-1 70 -0.5 40])
    title('Horse Simulation')
    xlabel('x axis (in)','FontSize',12)
    ylabel('y axis (in)','FontSize',12)
    hold on
    pause(0.01)
% Update the current time
    time = time + dt;

if gait == 1

    % Input functions for WALK
    % Input position function for r1
    r1 = R1+0.8496*1.34-0.8496*1.34*cos(2*pi*1*time);
    % Input velocity function for r1
    dr1_dt = 2.2769*pi*sin(2*pi*1*time);
    % Input acceleration function r1
    d2r1_dt2 = 4.5539*pi^2*cos(2*pi*1*time);
    % Input position function for r5
    r5 = R5+0.6209*2.5-0.6209*2.5*cos(2*pi*1*time-37*pi/180);
    % Input velocity function for r5
    dr5_dt = 3.1045*pi*sin(2*pi*1*time-37*pi/180);
    % Input acceleration function for r5
    d2r5_dt2 = 6.209*pi^2*cos(2*pi*1*time-37*pi/180);
elseif gait == 2

    % Input functions for TROT
    % Input position function for r1
    r1 = R1+0.6594*1.1-0.6594*1.1*cos(2*pi*2*time);
    % Input velocity function for r1
    dr1_dt = 2.90136*pi*sin(2*pi*2*time);
    % Input acceleration function r1
    d2r1_dt2 = 11.6054*pi^2*cos(2*pi*2*time);
    % Input position function for r5
    r5 = R5+0.8366*2.125-0.8366*2.125*cos(2*pi*2*time-pi/3);
    % Input velocity function for r5
    dr5_dt = 7.11*pi*sin(2*pi*2*time-pi/3);
    % Input acceleration function for r5
    d2r5_dt2 = 28.44*pi^2*cos(2*pi*2*time-pi/3);
elseif gait == 3

    % Input functions for CANTER
    % Input position function for r1
    r1 = R1+0.6157/1.5-0.6157/1.5*cos(2*pi*1.2*time);
    % Input velocity function for r1
    dr1_dt = 0.98512*pi*sin(2*pi*1.2*time);
    % Input acceleration function r1
    d2r1_dt2 = 2.3643*pi^2*cos(2*pi*1.2*time);
Appendix A (Continued)

% Input position function for r5
r5 = R5+2.3228/2*2-1.244*2*cos(2*pi*1.2*time-40*pi/180);
% Input velocity function for r5
dr5_dt = 5.9712*pi*sin(2*pi*1.2*time-40*pi/180);
% Input acceleration function for r5
d2r5_dt2 = 14.331*pi^2*cos(2*pi*1.2*time-40*pi/180);
end

% Simplification for the equation that is solved for Theta_4
C = (r2^2+r3^2+r4^2-r5^2)/(2*r4);
% Position equations for theta_4 and theta_5
Theta_4 = 2*atan((-r3+sqrt(r3^2+r2^2-C^2))/(C+r2));
Theta_5 = acos((-r2+r4*cos(Theta_4))/r5);
% Equations for omega_4 and omega_5
Omega_4 = -dr5_dt/(r4*sin(Theta_4-Theta_5));
Omega_5 = -dr5_dt/r5*cot(Theta_4-Theta_5);
% Equations for alpha_4 and alpha_5
Alpha_4 = (-r4*Omega_4^2*cos(Theta_4-Theta_5)+r5*Omega_5^2-2*dr5_dt^2)/(r4*sin(Theta_4-Theta_5));
Alpha_5 = (-r4*Omega_4^2-2*dr5_dt*Omega_5*sin(Theta_4-Theta_5)+r5*Omega_5^2*sin(Theta_4+Theta_5)-2*dr5_dt^2*sin(Theta_4+Theta_5))/(r5*sin(Theta_4-Theta_5));

% This will add the r1 values for the plot of r1
plot_r1 = [plot_r1;r1'];
% This will add the r5 values for the plot of r5
plot_r5 = [plot_r5;r5'];
% This will add the Theta_4 values for the plot of Theta_4
plot_Theta_4 = [plot_Theta_4;Theta_4'];
% This will add the Theta_5 values for the plot of Theta_5
plot_Theta_5 = [plot_Theta_5;Theta_5'];
% This will add the point p values for the plot of point p
plot_p = [plot_p;fk(2),fl(2)];
% This will add the point dp_dt values for the plot of dp_dt
plot_dp_dt = [plot_dp_dt;dr1_dt-rp*Omega_4*sin(Theta_4+delta),rp*Omega_4*cos(Theta_4+delta)];
% This will add the point d2p_dt2 values for the plot of d2p_dt2
plot_d2p_dt2 = [plot_d2p_dt2;dr21_dt2-(rp*Omega_4^2*cos(Theta_4+delta)+rp*Alpha_4*sin(Theta_4+delta)),Alpha_4*rp*cos(Theta_4+delta)-Omega_4^2*rp*sin(Theta_4+delta)];
% This will add the dr1_dt values for the plot of dr1_dt
plot_dr1_dt = [plot_dr1_dt;dr1_dt'];
% This will add the dr5_dt values for the plot of dr5_dt
plot_dr5_dt = [plot_dr5_dt;dr5_dt']);
Appendix A (Continued)

% This will add the d2r1_dt2 values for the plot of d2r1_dt2
plot_d2r1_dt2 = [plot_d2r1_dt2;d2r1_dt2'];
% This will add the d2r5_dt2 values for the plot of d2r5_dt2
plot_d2r5_dt2 = [plot_d2r5_dt2;d2r5_dt2'];

% if LOOP==Steps-1
if LOOP==Steps-1
    hold on
    pause(1)

    figure(2);
    plot(plot_r1,'LineWidth',2)
    title('Link r_1 vs. Time')
    xlabel('Time (Seconds*100)','FontSize',12)
    ylabel('inches','FontSize',12)
    pause(0.5)

    figure(3);
    plot(plot_r5,'LineWidth',2)
    title('Link r_5 vs. Time')
    xlabel('Time (Seconds*100)','FontSize',12)
    ylabel('inches','FontSize',12)
    pause(0.5)

    figure(4);
    plot(plot_Theta_4,'LineWidth',2)
    title('Theta_4 vs. Time')
    xlabel('Time (Seconds*100)','FontSize',12)
    ylabel('rad','FontSize',12)
    pause(0.5)

    figure(5);
    plot(plot_Theta_5,'LineWidth',2)
    title('Theta_5 vs. Time')
    xlabel('Time (Seconds*100)','FontSize',12)
    ylabel('rad','FontSize',12)
    pause(0.5)

    figure(6);
    plot(plot_dr1_dt,'LineWidth',2)
    title('dr_1/dt vs. Time')
    xlabel('Time (Seconds*100)','FontSize',12)
    ylabel('inches/Seconds','FontSize',12)
    pause(0.5)
Appendix A (Continued)

```matlab
figure(7);
plot(plot_dr5_dt,'LineWidth',2)
title('dr_5/dt vs. Time')
xlabel('Time (Seconds*100)', 'FontSize', 12)
ylabel('inches/Seconds', 'FontSize', 12)
pause(0.5)

figure(8);
plot(plot_d2r1_dt2,'LineWidth',2)
title('d^2r_1/dt^2 vs. Time')
xlabel('Time (Seconds*100)', 'FontSize', 12)
ylabel('inches/Seconds', 'FontSize', 12)
pause(0.5)

figure(9);
plot(plot_d2r5_dt2,'LineWidth',2)
title('d^2r_5/dt^2 vs. Time')
xlabel('Time (Seconds*100)', 'FontSize', 12)
ylabel('inches/Seconds', 'FontSize', 12)
pause(0.5)

plot_dp_dt
plot_d2p_dt2

for rp_loop = 1:Steps
    figure(10);
    plot(plot_p(rp_loop,1),plot_p(rp_loop,2),'k+','LineWidth',4)
    grid off
    axis('square')
    title('Position of Point p')
    xlabel('x axis (in)', 'FontSize', 12)
    ylabel('y axis (in)', 'FontSize', 12)
    hold on
end

for rp_vel = 2:10:Steps
    figure(11);
    scale = 0.1;
    axis('square')
    quiver(plot_p(rp_vel,1),plot_p(rp_vel,2),plot_dp_dt(rp_vel,1),plot_dp_dt(rp_vel,2), scale,'k')
    xlabel('x axis (in)', 'FontSize', 12)
    ylabel('y axis (in)', 'FontSize', 12)
```

82
grid off
hold on
end
end
end
# Appendix B – Bill of Materials for Horseback Riding Simulator

## Table B-1: Bill of Materials for Simulator

<table>
<thead>
<tr>
<th>ITEM</th>
<th>PART NUMBER</th>
<th>DESCRIPTION</th>
<th>QTY</th>
<th>UM</th>
<th>REF</th>
<th>VENDOR</th>
<th>PART TYPE</th>
<th>COST PER UM</th>
<th>TOTAL COST</th>
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<td>ANGLE, 90 DEGREE, STEEL 1/8 IN THICK 1-1/4 IN LEG LENGTH 6&quot; LENGTH</td>
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</table>

---

Appendix B – Bill of Materials for Horseback Riding Simulator
Appendix C (Continued)

Figure C-2: AutoCAD Support Tube Detail

A 5/8" Diameter Hole is drilled in the Side to be plined to the end of a cross joint from the aluminum horizontal rod.

A 1 3/4" x 2" area is cut from the back side of the tube to allow for rotation of this joint.
Figure C-3: AutoCAD Track Detail Drawing

90° Angle Bracket
1.25" x 1.25"
1/8" Thickness
Welded to a Steel plate
1/4" Thick 4" in Width
Appendix C (Continued)

Figure C-4: AutoCAD Cylinder Joint A Detail Drawing

Mechanized Horse Cylinder Joint A Detail

Top View

Side View

Front View

(Additional details and measurements are shown in the diagram.)
Figure C-5: AutoCAD Cylinder Joint B Detail Drawing
Appendix C (Continued)

Figure C-6: AutoCAD Base Cylinder Holder Detail Drawing

Top View

Side View

Front View
Figure C-7: AutoCAD Support Rod Detail Drawing