Undergraduate Journal of Mathematical Modeling: One + Two

Volume 12 | 2022 Spring 2022 Article 5

2022

The Use of Calculus to Determine Efficient Fertilizer Levels for Crop Production

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Recommended Citation
Loadholtz, Cole (2022) "The Use of Calculus to Determine Efficient Fertilizer Levels for Crop Production," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 12: Iss. 2, Article 5.
DOI: https://doi.org/10.5038/2326-3652.12.2.4947
Available at: https://digitalcommons.usf.edu/ujmm/vol12/iss2/5

The Use of Calculus to Determine Efficient Fertilizer Levels for Crop Production

Abstract

For this project, I wanted to incorporate calculus into agriculture and environmental science methods. More in detail, the problem used asked for the maximum levels of nitrogen (N) and phosphorus (P) that would be best for a current crop yield. This allowed incorporating partial derivatives, and critical points to find the maximum values for the equation. The results show that in order to demonstrate maximum crop yield production, the levels of nitrogen (N) and phosphorus (P) were to be both at 2, with the correct corresponding units. The drawback from this problem is that although the problem showed effective nitrogen and phosphorus levels, it should be determined as to whether those levels, typically in synthetic fertilizers, are more, less, or equal to the suggested amount per bushel. Adding excess nutrients onto a crop can result in nutrient in our waterways due to surface water runoff. The overall purpose would be a moral question for the farmer: Does the farmer add the levels of N and P for the maximum crop yield success, regardless of it being more than the suggested bushel amount?

Keywords

crop, nitrogen (N), phosphorus (P), optimization

PROBLEM STATEMENT

Using a model to determine the yield of an agricultural crop with the best nitrogen level N and phosphorus level P in the soil. The model below is used:

$F(N, P) = 3N^2+2P^2-12N-8P-10$

Using calculus, what levels of nitrogen and phosphorus result in the best yield for this model?

MOTIVATION

The question being solved helps to show the maximum levels that nitrogen and phosphorus would need to be at in order for a crop to have the most success in growing. Although there are other variables that may alter this success (rainfall, direct sunlight, introduction of invasive species), the levels are said to be for optimal conditions. In the realm of farming and agriculture, one way that farmers change levels in nitrogen and phosphorus rapidly is adding synthetic fertilizers to the crop. This allows for a fast uptake in nutrients for plants, in hopes that a crop will grow healthy and plentiful. As an environmental science student, I have also learned that synthetic fertilizers can be harmful to the outside environment once the fertilizer is exceeding the suggested amount or is removed from its applied area. The fertilizer can be swept away from the crop field due to a sudden large amount of rainfall, which results in surface water runoff. The nutrients can end up in a local waterway, which gives excess nutrients to marine plants and algae. When large amounts of plant matter are suddenly grown, the marine plant uses all the excess nitrogen and phosphorus until it becomes unsustainable and then begins to decay. The decaying process results in the depletion of oxygen in the water, resulting in large fish kills and overall marine ecosystem disturbance.

From the overall objective of this project is to determine the amount of nitrogen and phosphorus levels are needed to have a successful maximum crop yield, and to raise awareness on the effects of excess nitrogen and phosphorus in the environment.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Solution Approach:
With the equation:
$F(N, P) = 3N^2+2P^2-12N-8P-10$
We must treat this like a normal multivariable equation. Rather than using F(N,P), we want changed the variables to F(x,y) that will represent nitrogen and phosphorus levels.
In order to solve this, the first process is to use partial derivates with respect to x, f_x.

$f_x = \partial/\partial x\ (3x^2+2y^2-12x-8y-10)$
$3\ \partial/\partial x\ (x^2) = 3 \bullet 2x = 6x$
Any y term is zero;
$12\ \partial/\partial x\ (1x) = 12(1) = 12$
-10 is zero with no variable;

Final answer: $6x – 12 = x-2$

Secondly, we must look to solve the partial derivate with respect to y, f_y.

$f_y = \partial/\partial y\ (3x^2+2y^2-12x-8y-10)$
$2\ \partial/\partial y\ (y^2) = 2 \bullet 2x = 4y$
Any x term is zero;
$8\ \partial/\partial y\ (1y) = 8(1) = 8$
-10 is zero with no variable;

Final answer: $4y – 8 = y-2$

Then, we must find the critical values. This requires setting the two values to zero.

$x-2 = 0$;
Critical value of x; $x = 2$
$y-2 = 0$
Critical value of y; $y = 2$

Lastly, we replace the variables of F(x,y) back into the original values of F(N,P)
$N = 2$
$P = 2$
Source: (Stewart & Day, 2016)

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DISCUSSION

After completing the mathematical equation that is necessary for the problem at hand, the best levels of nitrogen and phosphorus are both 2 units. These units can be manipulated, when necessary, in this case it is most likely pounds (lbs.). To make the answer clearer, if we use the crop production of corn as an example, it will take 2 lbs. of nitrogen (N), as well as phosphorus (P), per bushel of corn. This question and topic allowed me to incorporate partial derivatives and critical values into a real-life circumstance. This helped to show that the methods and mathematical objectives taught in Life Science Calculus II can be used in environmental science scenarios. Being able to derive multivariable equations can help to show the comparison of two variables within the same equation. For this reason, the overall project allowed me to meet the objective of finding the optimal levels of nitrogen and phosphorus for best crop yield. The results can be used with any scenario that requires the discovery of a derivative equation. In regards to nitrogen and phosphorus levels, it is important to have these levels, although optimal for crop yield, to follow surface area application. For example, if an area of crop demands for only 1 lb. of nitrogen and phosphorus per bushel, the application should not exceed that. In order to apply the most accurate nitrate levels, pre-application soil testing should be done to test the nitrate levels within the soil before the fertilizer is implemented. The soil testing should be done with a large sample group, each at different areas of the crop field. By creating a larger sample size of nitrate level data, a more accurate reading of nitrates in the soil will be recorded (The Board of Regents, n.d.). Overall, by exceeding the necessary levels of a given crop yield, environment impacts and overall negligence would arise if not soon, then ultimately in long-term observation.

CONCLUSIONS AND RECOMMENDATIONS

In this project, I was able to use partial derivatives and the finding of critical points to determine the optimal nitrogen and phosphorus levels for best crop yield. This project allowed me to show the process of obtaining accurate data through mathematical application to benefit farming methods. Farmers and environmental scientists could use the same strategy to obtain the right nutrient levels for their crop. Being able to apply previously learned mathematics, I have a better understanding of the use of calculus in my field of study.

When doing this project or similar data analysis, there were a variety of methods that could have been done differently. With this project being more a simulation, being able to have pre-recorded data of nitrate levels could help advance this project. By having more data to present, a more accurate result could be determined. For example, since nitrate and phosphorus levels depend on number of bushels or overall exact crop yield, having records of current nitrate and phosphorus levels in the soil could help to show the exact number of levels to increase or decrease. An idea to do differently would be preforming this activity over a variety of crop categories. Below displays the variety fertilizer average, calculated in kilograms per hectare (kg/ha), of various cereal crops.

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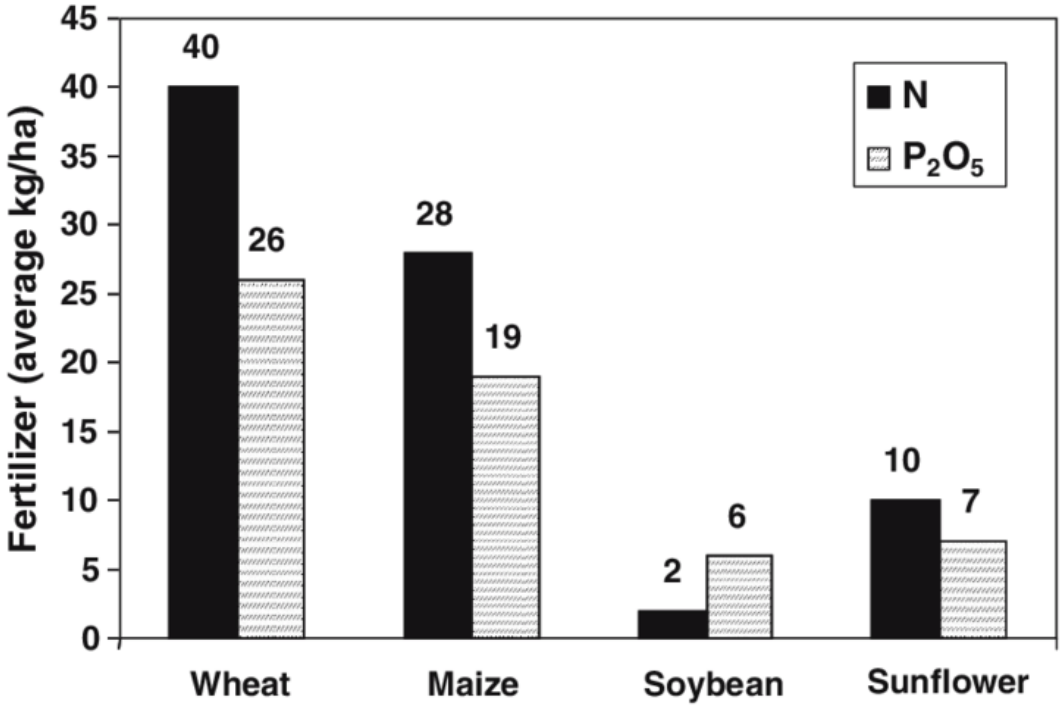

Figure 1; (Austin et al, 1970)

Whether it is corn, wheat, soybeans, or any other crop, being able to see the comparison of necessary nitrate levels per crop would be an intriguing study. Lastly, the project performed helps to emphasize the importance of accurate soil sampling and accurate nitrite application. Since most fertilizers are synthetic, they can provide negative impacts to nontargeted areas at a rapid pace. The message to overlay would be to ensure management of soil and crop on a continuous level would help to benefit the farmer's crop, as well as the local environments around the crop.

NOMENCLATURE

Symbols used in the report:

lb(s). Pound(s)

N Nitrogen

P Phosphorous

REFERENCES

Austin, A. T., Piñeiro, G., & Gonzalez-Polo, M. (1970, January 1). More is less: Agricultural impacts on the N cycle in Argentina. Figure 1. SpringerLink. Retrieved from https://link.springer.com/chapter/10.1007/978-1-4020-5517-1_3

Stewart, J., & Day, T. (2016). Chapter 9: Multivariable Calculus. In Biocalculus: Calculus, probability, and statistics for the Life Sciences (pp. 566–627). essay, Cengage Learning.

The Board of Regents of the University of Nebraska. (n.d.). How to determine the optimum rate of nitrogen fertilizer - UNL water. Irrigation and Nitrogen Management. Retrieved from https://water.unl.edu/documents/Section%20E.pdf

DOI: https://doi.org/10.5038/2326-3652.12.2.4947