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Internal Combustion Engines: Modeling Internal Temperature as a Function of Time

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Internal Combustion Engines: Modeling Internal Temperature as a Function of Time

Abstract

Just like any thermodynamic system, combustion engines must be cooled to eliminate friction due to heat. Without proper cooling, internal components, such as connecting rods, rod bearings, and pistons can be severely damaged due to thermal expansion, leading to severe damage to the engine block or outright catastrophic failure. Modern engines are cooled using coolant, which flows through internal passageways within the engine block to pull heat away from the system. The use of coolant and external components, such as a water pump, radiator, and thermostat allow an engine to efficiently warm to standard operating temperature and remain at said temperature. Using standard calculus and differential equations, the model for the idle coolant temperature increase inside of an engine block as it approaches a steady state can be obtained. For overall application purposes, this model includes an arbitrary number of inlets and exits for coolant to flow through. This model can also be applied to air cooled or electric systems, such as small engines or electrical motors which produce a quantifiable amount of heat.

Keywords

combustion engines, heat, coolant, water pump, radiator, thermostat

PROBLEM STATEMENT

Prior to the early 1980's, the vast majority of engine and vehicle manufacturers produced air cooled engines, which functioned exactly how they were described. These engines were cooled using passive airflow and were considered highly efficient. By today's standards, air cooled engines have been almost entirely phased out. Due to higher power outputs, modern engines require more sophisticated and efficient means of cooling to remain at ideal operating temperature. This is achieved using liquid coolant, which flows through the engine. By modeling the thermodynamic relationship as an engine approaches standard operating temperature, this model can be easily applied to any system which produces heat and is cooled. As vehicle manufacturers move towards the production of electric vehicles, these modern electric vehicles still need to be cooled, as their electronic components still produce a considerable amount of heat which can be detrimental to internal components. The model obtained from combustion engines can be directly applied to these modern systems and it is integral in understanding how to cool electric vehicles.

MOTIVATION

As modern machinery becomes more and more complex, there is an increasing need to be able to efficiently keep things cool, specifically, electric vehicles. Naturally, electric vehicles benefit from being in cooler environments. Ideal operating temperature for modern electric vehicle batteries falls somewhere between 0-30° C. Above 30° C, battery performance starts to degrade, effectively reducing the overall output capability of the motor. At around 40° C, irreversible damage to the battery can occur. These same general guidelines can be applied to traditional combustion vehicles, which generally run hotter than electric vehicles. Knowing this, engineers have found different ways to efficiently cool thermal components of large battery packs. Using a combination of passive air cooling and liquid cooling, finding the right combination ultimately comes down to cost effectiveness and overall ergonomics. Interestingly, active cooling solutions including liquid cooling or refrigerants can be modeled using the same mathematical formula.

Mathematical Description and Solution Approach

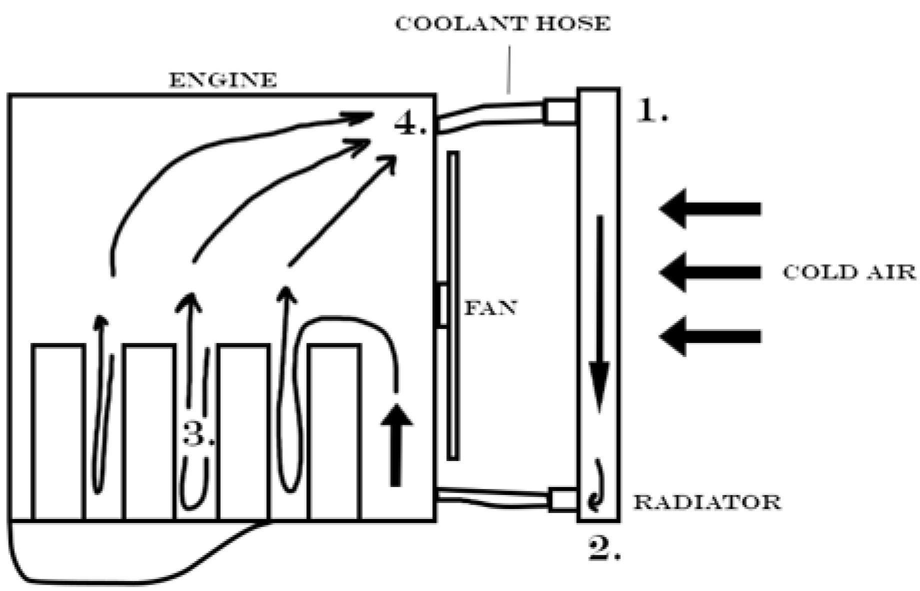

Diagram 1.0

The first step in understanding how heat is transferred is understanding how heat is represented as a derivative of time. Because heat is a form of energy, heat will be represented as E.

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Therefore,

$$dE/dt$$

represents the change in energy accumulated in a system.

Based on the Law of Conservation of Energy, Rate of change of energy accumulated in a system = (Energy in) – (Energy out) or,

$$dE/dT = (\text{Energy in}) - (\text{Energy out}).$$

Diagram 1.0 shows the path that coolant flows, in the direction of 1, 2, 3, and 4. This model focuses particularly on how fluid moves and cools from 2-4. To better understand how this process physically works, specifically how coolant temperature is regulated, diagram 2.0 shows a basic representation of a standard radiator.

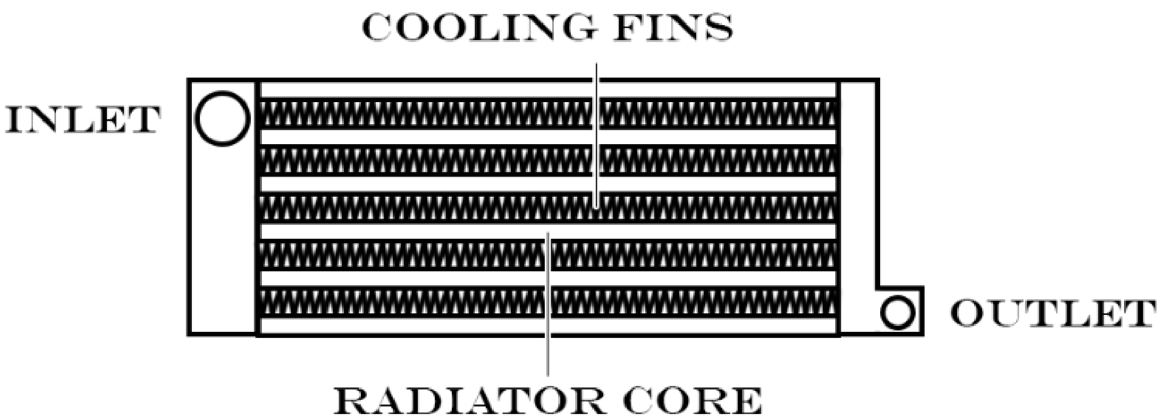
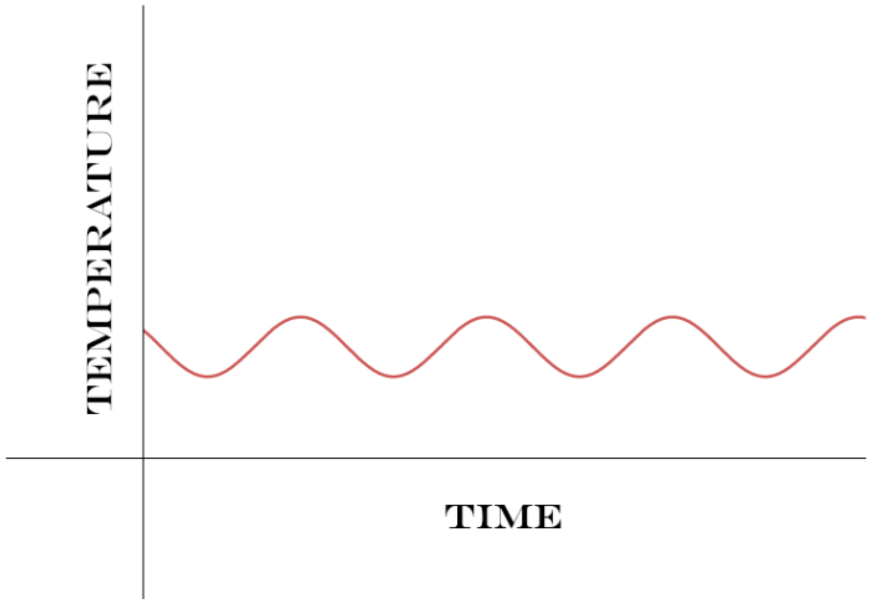


Diagram 2.0

As coolant flows through the radiator core, cold air hits the cooling fins, dropping the temperature of the coolant as it moves towards to outlet and back into the engine block. Then hot coolant enters through the inlet port, repeating this process. As the temperature of coolant constantly increases and decreases, the temperature of coolant in relation to time can be represented graphically as so:



Graph 1.0

where temperature and time are arbitrary. Of course, as the engine produces more heat, the temperature of the coolant would therefore increase. Due to this reason, I have chosen to model an engine at standard operating temperature, also known as idle temperature, where there is no excess heat or work being done.

We continue with the model,

$$dE/dt = C * (d(M_{system}T))/dt,$$

where C represents the heat capacity of the coolant, M_{system} is the mass of the coolant in the system, and T is the temperature of the coolant.

As previously stated,

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Rate of change of energy accumulated in a system = (Energy in) – (Energy out),

where

$$\text{Energy in} = w_1(h_1).$$

W_1, in this case, represents the flow rate of the stream and h_1 is the specific enthalpy of the coolant entering through the inlet port in a radiator. Applying this same formula to the outlet stream of coolant provides

$$\text{Energy out} = w_2(h_2).$$

Therefore,

$$dE/dT = (\text{Energy in}) - (\text{Energy out})$$

$$dE/dt = w_1(h_1) - w_2(h_2)$$

$$dE/dt = C * (d(M_{system}T))/dt$$

$$C * (d(M_{system}T))/dt = w_1(h_1) - w_2(h_2).$$

This model, on its own, can also be used to represent how energy changes with time inside of a system with an arbitrary number of inlets and outlets. For example, to model a system with four inlets and 7 outlets, this model can easily be adapted to

$$dE/dT = \sum (\text{Energy in}) - \sum (\text{Energy out})$$

$$C * (d(M_{system}T))/dt = \sum w_1(h_1) - \sum w_2(h_2).$$

This final model can be used anywhere liquid flows from an inlet to an outlet. However, this model is for ideal conditions where there is no outside source of heat into the system. Because of the massive amount of heat most engines produce, there will be, to some degree, an environmental factor which needs to be considered. This can be very easily done by adding another term, q_{in}.

$$C * (d(M_{system}T))/dt = q_{in} + \sum w_1(h_1) - \sum w_2(h_2).$$

Passive cooling elements, such as air cooling, can also be taken into account using this same logic:

$$C * (d(M_{system}T))/dt = q_{in} + \sum w_1(h_1) - \sum w_2(h_2) - q_{out}.$$

Discussion

The finalized model detailed meets all objectives it served to complete. The model is also easily adaptable to a wide variety of situations, including the use of gaseous coolants such as refrigerants, and can be applied to both combustion and electric vehicles. Even though the model itself does not answer any specific questions, it provides the groundwork to understanding how engineers approach practical issues analytically and it can be used to answer any questions regarding cooling efficiency. This, on its own, makes it incredibly valuable to the engineering and design process of modern vehicles.

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NOMENCLATURE

C: Heat Capacity (Joules per Degree Kelvin, J/K)

T: Temperature (Degrees Celsius, C)

w: Flow rate (Liters per minute, meters^3 per second, gallons per hour, etc.)

h: Enthalpy (Joule, J)

q: Energy due to heat (Jules or Calories, J or Cal)

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