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The Distribution of Energies

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The Distribution of Energies

Abstract

The application of calculus is of great importance in physics. It is used to calculate energy, acceleration, velocity, and much more. In this paper, to calculate the internal energy of an ideal gas (gas molecules) we will use Maxwell-Boltzmann statistical distribution based entirely on calculus. The technique represents the mathematical concept that depends on integration and substitution in calculus to solve a problem.

Keywords

energy, gas molecules, Maxwell-Boltzmann statistical distribution

PROBLEM STATEMENT

Maxwell-Boltzmann statistical distribution, one of the most important techniques in physics, is used to calculate gas molecules. It is the same for any given gas at any temperature, no matter the mass of the molecules.

In this paper, we use it to calculate the internal energy of an ideal gas. The standard approach uses thermodynamics, and it gives the similar results to calculations based on the kinetic theory. So, we find the statistical result for the internal energy of an ideal gas and then compare it with that of from the standard approach. The number of molecules is n(E)dE with the energy between E and E + dE. Therefore, the contribution to the internal energy of the gas is En(E)dE. According to it, we have the equation of the total of all contributions that give the internal gas. Thus, we substitute n(E), integrate the equation, and find out the result.

MOTIVATION

Physics and mathematics always have a close connection, especially in calculus. In physics, math is a tool to solve problems and questions. In mathematics, "physics can be a source of inspiration," it challenged mathematicians to come up with new methods ("Where Math Meets Physics").

Calculus was invented by Newton to describe motion in physics ("Where Math Meets Physics"). That is why when we solve a motion problem in physics, we use the techniques that we learned from calculus.

Calculus generally has two main parts, differential calculus and integral calculus ("World Web Math: Calculus Summary"). Differential calculus is a study of derivatives and integral calculus is a study of integrals. Both derivative and integral are linked because they are the concept of the limit, they are inverse operations of each other and a fundamental to modern science ("World Web Math: Calculus Summary").

To look at the motion of molecules, scientists usually look for the distribution in the speed of the molecules. Maxwell, however, was the first person to use a different technique to look at the motion of molecules. He was looking for the distribution in the energy of the molecules, known as Maxwell-Boltzmann statistical distribution (Halliday et al.). The technique he created is very useful when calculating the kinematics of gas molecules under certain environmental conditions (Halliday et al.). The technique itself is based on integration in calculus.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

In physics, calculus is used to calculate acceleration, velocity, energy, and more. In this paper, we use Maxwell-Boltzmann statistical distribution, which is a technique that exists in physics to calculate the energy of gas molecules based on integral calculus. The equation that represents the total of all contributions gives the energy of the gas (Halliday et al.):

$$E_{\text{int}} = \int_0^\infty En(E)dE \quad (1)$$

The equation that represents $n(E)$ is known:

$$n(E) = \frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}E^{1/2}e^{-E/kT} \quad (2)$$

Substituting Eq. 2 for $n(E)$ in Eq. 1, we obtain

$$E_{\text{int}} = \frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\int_0^\infty E^{3/2}e^{-E/kT}dE \quad (3)$$

To put Eq. 3 in the standard form, we substitute $u = E/kT$, which gives

$$E_{\text{int}} = \frac{2N}{\sqrt{\pi}}kT\int_0^\infty u^{3/2}e^{-u}du \quad (4)$$

The last step is the evaluation of the integral by substiting $u = x^2$ and using a definite integral from Appendix H, where the result of the integral is given as $\frac{3}{4}\sqrt{\pi}$, then the final form of the equation is

$$E_{\text{int}} = \frac{2N}{\sqrt{\pi}}kT\left(\frac{3}{4}\sqrt{\pi}\right) = \frac{3}{2}NkT$$

The result of the calculation based on kinetic theory is $E_{\text{int}} = \frac{3}{2}nRT$ (Halliday et al.). Therefore, Maxwell-Boltzmann statistical distribution is consistent with the results derived from kinetic theory.

DISCUSSION

We calculated the internal energy of an ideal gas by using Maxwell-Boltzmann statistical distribution, which was based on integral calculus. During the process, we also used a substitute method to solve the integral.

According to the result, Maxwell-Boltzmann statistical distribution is an accurate and reliable method just as kinetic theory. Both results of the Maxwell-Boltzmann statistical distribution method and kinetic theory method are consistent with each other. With Maxwell-

Boltzmann statistical distribution, the result always precisely is the same for any type of gas at a given temperature, the mass of the gas does not matter.

Maxwell-Boltzmann statistical distribution is an important method in the physics field because it helps scientists to define the distribution of energy for gas at a certain temperature. Using the result, the scientists can derive the average energy, the most probable energy, and more.

Conclusion and Recommendations

In this paper, we solve the equation for the internal energy of an ideal gas by using Maxwell-Boltzmann statistical distribution method. The technique is based on integral calculus. We discuss the usefulness of the method in the physics field, especially in gas-related problems.

The kinetic theory method can also be explained more in detail, so that it can be compared directly to Maxwell-Boltzmann statistical distribution method.

Nomenclature

- E_{int} Joules J
- T Temperature K
- N no unit
- k Boltzmann constant J K^{-1}

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