2021

Mathematical Reconstruction of a Traffic Crash

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Recommended Citation

Covert, Benjamin (2021) "Mathematical Reconstruction of a Traffic Crash," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 11: Iss. 2, Article 2.

DOI: <https://doi.org/10.5038/2326-3652.11.2.4930>

Available at: <https://digitalcommons.usf.edu/ujmm/vol11/iss2/2>

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Abstract

The objective is to find the impact speeds of two motor vehicles involved in a traffic crash. The calculations take into account the approach (pre-collision) and departure (post collision) angles, as well as weights and post collision speeds of both vehicles. The data is provided by Timothy Sleyzack who investigated this traffic collision. The conclusions confirm the validity of the use of Conservation of Linear Momentum in the field of traffic crash reconstruction.

Keywords

traffic collision, Conservation of Linear Momentum, impact speed, post collision speed, pre-collision and post collision angles, coefficient of restitution

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PROBLEM STATEMENT

Use Conservation of Linear Momentum to reconstruct a traffic crash mathematically and accurately.

MOTIVATION

In the field of traffic crash reconstruction, one of the most commonly asked questions of the investigator is: What is the speed of each vehicle, especially the striking vehicle, at the time of the collision? It is very important for the investigator to have an accurate account of speeds of vehicles as it helps to adequately analyze and conclude on events and causes in a collision. Investigators commonly find witness accounts as to a vehicle's speed to be conflicting. To address this problem effectively, a mathematical approach is preferred to witness testimony.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

The solution approach is based on **Conservation of Linear Momentum**. **Linear momentum** is a product of the mass and velocity of an object. Momentum is a vector which points in the same direction as velocity. The total amount of **momentum** never changes, and this property is called **conservation of momentum**.

While reconstructing the traffic crash, I first determine the approach (pre-collision) and departure (post collision) angles for each vehicle from a scale diagram of the crash scene which is provided by Timothy Sleyzack who investigated the crash (see Appendices). So the first thing that needs to be done is to find all four of the angles ($\alpha, \psi, \theta, \phi$) that are required for the mathematical equations (A) and (B) below. The next thing is to obtain the weights of the vehicles and their post collision speeds. After that, it is a matter of plugging in the variables to get the desired values for the pre-collision speeds of the vehicles. The final step is to calculate the coefficient of restitution.

After determining each vehicle's approach and departure angle, I calculate the sine and cosine values for each angle. I then use the weight of both vehicles and their post collision energy loss into the following mathematical equations.

$$(A) \quad S_2 = \frac{(W_1)(S_3)(\sin\theta)}{(W_2)(\sin\psi)} (+) \frac{(S_4)(\sin\phi)}{\sin\psi}$$

For equation (A), the pre-collision speed of vehicle-2 (S_2) is equal to the weight of vehicle-1 (W_1), multiplied by the post collision speed of vehicle-1 (S_3), multiplied by the sine of the departure angle of vehicle-1 (θ); divided by the product of the weight of vehicle-2 (W_2) and the

sine of the approach angle of vehicle-2 (ψ). That calculation is then added to the product of the post collision speed of vehicle-2 (S_4) and the sine of the departure angle of vehicle-2 (ϕ), divided by the sine of the approach angle of vehicle-2 (ψ).

$$(B) \quad S_1 = \frac{(S_3)(\cos\theta)}{\cos\alpha} (+) \frac{(W_2)(S_4)(\cos\phi)}{(W_1)(\cos\alpha)} (-) \frac{(W_2)(S_2)(\cos\psi)}{(W_1)(\cos\alpha)}$$

For equation (B), the pre-collision speed of vehicle-1 (S_1) is equal to the post collision speed of vehicle-1 (S_3), multiplied by the cosine of the departure angle of vehicle-1 (θ); divided by the cosine of the approach angle of vehicle-1 (α). That calculation is then added to the product of the weight of vehicle-2 (W_2), the post collision speed of vehicle-2 (S_4) and the cosine of the departure angle of vehicle-2 (ϕ); divided by the weight of vehicle-1 (W_1) multiplied by the cosine of the approach angle of vehicle-1 (α). That calculation is then subtracted by the product of the weight of vehicle-2 (W_2), the pre-collision speed of vehicle-2 (S_2) and the cosine of the approach angle of vehicle-2 (ψ); divided by the product of the weight of vehicle-1 (W_1) and the cosine of the approach angle of vehicle-1 (α).

$$(C) \quad e = \frac{(S_3)-(S_4)}{(S_1)-(S_2)}$$

For equation (C), the coefficient of restitution (e) is the difference of the post collision speeds of vehicle-1 (S_3) and vehicle-2 (S_4) divided by the difference of the pre-collision speeds of vehicle-1 (S_1) and vehicle-2 (S_2).

DISCUSSION

I calculated the impact speed of the target vehicle (vehicle-2) to be 21.96 miles-per-hour and that of the striking vehicle (vehicle-1) to be 40.85 miles-per-hour. To ensure the accuracy of results, two mathematical approaches are used. The first, Coefficient of Restitution, with a desired value between zero and one, produces a result of 0.15. The second mathematical approach, Vector Sum Analysis, where a similar value for each vehicles' change in momentum (ΔM) is desired, results in identical values.

To further ensure the accuracy of my calculations, two electronic software calculations are used. The first one is Rec-Tec Professional Accident Reconstruction Software. Rec-Tec requires the input of each vehicles' weight, approach angle, departure angle and post collision speed to calculate an impact speed of 40.97 miles-per-hour for vehicle-1 and 22.35 miles-per-hour for vehicle-2. The second electronic software that is used to ensure the accuracy of my results is the review of each vehicle's Crash Data Retrieval System Report (Bosch), provided by Timothy Sleyzack. The Crash Data Retrieval System is often referred to as the Airbag Control Module or

the vehicle's Black Box. This provides data such as a vehicle's speed, the time that an airbag deployed, up to five seconds prior to the deployment of the airbags or collision. The Crash Data Retrieval System denotes an impact speed of 37 miles-per-hour for vehicle-1 and a speed of 14 miles-per-hour for vehicle-2 (see below).

While reconstructing the traffic collision, my project objective, which is to calculate each vehicles' impact speed, is met. It confirms that the use of Conservation of Linear Momentum in the field of traffic crash reconstruction is a useful and accurate tool for a crash investigator.

CONCLUSION AND RECOMMENDATIONS

While the impact speed calculated for vehicle-1 is close (within 3 miles-per-hour) to the impact speed denoted in the Electronic Data Retrieval Report, the impact speed calculated for vehicle-2 is further (7 miles-per-hour) from the impact speed denoted in the Electronic Data Retrieval Report. This discrepancy can be attributed to the fact that vehicle-2's approach angle, which relies on physical evidence such as tire marks and swerve marks, can be susceptible to error, whereas the approach angle for the striking vehicle is always zero degrees.

For someone looking to do the same project I have a few recommendations. To obtain accurate results, when taking the angles into consideration, the angles should be measured multiple times. This will ensure that the angles have been measured effectively.

In conclusion, the importance of accurately determining approach and departure angles is significant when using Conservation of Linear Momentum.

NOMENCLATURE

Variables used in Conservation of Linear Momentum

W	Weight	Pounds (lbs.)
α	Approach Angle Vehicle-1	Degrees
ψ	Approach Angle Vehicle-2	Degrees
θ	Departure Angle Vehicle-1	Degrees
ϕ	Departure Angle Vehicle-2	Degrees
S_1	Pre-Collision Speed Vehicle-1	Miles-per-hour (mph)
S_2	Pre-Collision Speed Vehicle-2	Miles-per-hour (mph)
S_3	Post Collision Speed Vehicle-1	Miles-per-hour (mph)
S_4	Post Collision Speed Vehicle-2	Miles-per-hour (mph)
e	Coefficient of Restitution	N/A

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APPENDICES

Momentum Worksheet

		Degrees	Sin	Cos	Weights and Speeds
Approach Angle of V1	α	0	0	1	W1 - 6682
Approach Angle of V2	ψ	139	0.6560	-0.7547	W2 - 2678
Departure Angle of V1	θ	6.5	0.1132	0.9935	
Departure angle of V2	ϕ	17	0.2923	0.9563	
Impact Speed of V-1					S1 - 40.85
Impact Speed of V-2					S2 - 21.96
Post Collision Speed of V-1					S3 - 24.05
Post Collision Speed of V-2					S4 - 26.93
Non – Collinear Crashes (Angular)					
$S_2 = \frac{W_1 S_3 \sin \theta}{W_2 \sin \psi} + \frac{S_4 \sin \phi}{\sin \psi}$ $S_1 = \frac{S_3 \cos \theta}{\cos \alpha} + \frac{W_2 S_4 \cos \phi}{W_1 \cos \alpha} - \frac{W_2 S_2 \cos \psi}{W_1 \cos \alpha}$					
Collinear Crashes (In Line)					
Rear End: $S_1 = S_3 + \frac{W_2 S_4}{W_1} - \frac{W_2 S_2}{W_1}$ Head On: $S_1 = S_3 + \frac{W_2 S_4}{W_1} + \frac{W_2 S_2}{W_1}$					

Crash Diagram

1. Pre-collision speed vehicle - 2

$$S_2 = \frac{(W_1)(S_3)(\sin \theta)}{(W_2)(\sin \psi)} + \frac{(S_4)(\sin \phi)}{\sin \psi}$$

$$S_2 = \frac{(6682)(24.05)(0.1132)}{(2678)(0.6560)} + \frac{(26.05)(0.2923)}{0.6560}$$

$$S_2 = \frac{18191.4777}{1756.7860} + \frac{7.6114}{0.6560}$$

$$S_2 = 10.3549 + 11.6073 = \boxed{21.96 \text{ mph}}$$

2. Pre-collision speed vehicle - 1

$$S_1 = \frac{(S_2)(\cos \theta)}{\cos \alpha} + \frac{(W_2)(S_4)(\cos \phi)}{(W_1)(\cos \alpha)} - \frac{(W_2)(S_3)(\cos \psi)}{(W_1)(\cos \alpha)}$$

$$S_1 = \frac{(24.05)(0.9935)}{1} + \frac{(2678)(26.93)(0.9563)}{(6682)(1)} - \frac{(2678)(21.96)(-0.7547)}{(6682)(1)}$$

$$S_1 = \frac{23.8936}{1} + \frac{68966.9548}{6682} + \frac{44383.0617}{6682}$$

$$S_1 = 23.8936 + 10.3213 + 6.6421 = \boxed{40.85 \text{ mph}}$$

3. Coefficient of Restitution

$$e = \frac{|S_3 - S_4|}{|S_1 - S_2|}$$

$$e = \frac{|24.05 - 26.93|}{|40.85 - 21.96|}$$

$$e = \frac{|-2.88|}{18.89} = \frac{2.88}{18.89} = 0.15$$

Vector Sum Analysis My Calculations

Vector Sum Analysis Computer Analysis

Rec-Tec Data

Linear Momentum - Unit 1

Number of Fragments: 1 2 3 4

☐ Inline Collision (EBS) ☐ LHCS/H

☐ Inline Collision (V' & Ed or Dv1)

Approach Data:

Angle (1) degrees: 0

Departure Data: M/A File

Odd Fragments: *

Angle (3) degrees: 6.5

Weight lbs: 6682

Speed (D) M/H: 24.05

Enter 0 to use mu/Dist Input
Enter X to use DeltaV Inputs

Unit 2

Number of Fragments: 1 2 3 4

V2 Speed (Known) M/H:

Approach Data:

Angle (2) degrees: 139

Departure Data: M/A File

Even Fragments: *

Angle (4) degrees: 17

Weight lbs: 2678

Speed (D) M/H: 26.93

Enter 0 to use mu/Dist Input

Output - Unit 1

PDOF (A1): 9.0549 degrees*

Impulse: 5265.0291 lbf*s

DeltaV-Lat: -2.7225 M/H

DeltaV-Lat: -3.993 f/s

DeltaV-Lng: -17.0833 M/H

DeltaV-Lng: -25.0555 f/s

Delta V: 17.2989 M/H

Delta V: 25.3717 f/s

Post Impact

Speed: 24.05 M/H

Speed: 35.2733 f/s

Intercept (Pre-Impact)

Angle: 139 degrees

Momentum: 340245.5274 f/s*lbs

Energy: 419506.8198 ft*lb

Energy: 225538.1318 ft*lb

Speed: 59.681 M/H

Speed: 87.5322 f/s

Impact

Speed: 40.9787 M/H

Speed: 60.1021 f/s

eVelocity: .0172 - (e)

Output - Unit 2

PDOF (A2): -31.945 degrees*

Impulse: 5265.0291 lbf*s

DeltaV-Lat: 22.8379 M/H

DeltaV-Lat: 33.4956 f/s

DeltaV-Lng: -36.6264 M/H

DeltaV-Lng: -53.7188 f/s

Delta V: 43.1633 M/H

Delta V: 63.3061 f/s

Post Impact

Speed: 26.93 M/H

Speed: 39.4973 f/s

Separation (Post-Impact)

Angle: 10.5 degrees

Momentum: 340245.5274 f/s*lbs

Energy: 193968.6879 ft*lb

Energy: 227452.9183 ft*lb

Speed: 5.4758 M/H

Speed: 8.0312 f/s

Impact

Speed: 22.3557 M/H

Speed: 32.7884 f/s

rVelocity: .0917 - (Vs/Vc)

Energy Momentum **Formulae** **Formulae*** **N**

Vector Analysis **Graphics** **360LM > Vectors**

Velocity Vectors **Animation** **360LM > smac-RT**

Iteration/FDA **Open .CSV File** **Open .LMO File** **Save .LMO File**

09:56 AM Thursday - 7 May 2020 Click for Timer 1 Click for Timer 2 Law Enforcement Only

Vehicle-1 Crash Data Retrieval

Pre-Crash Data (Most Recent Event - table 1 of 3)
(the most recent sampled values are recorded prior to the event)

Time Stamp (sec)	Pre-Crash Recorder Status	Speed, Vehicle Indicated (MPH [km/h])	Accelerator Pedal, % Full	Engine Throttle, % Full	Service Brake	Engine RPM	ABS Activity	Stability Control	Steering Input (deg)
-5.0	Complete	66 [106]	63	SNA	Off	1,460	No	On	8
-4.9	Complete	66 [106]	63	SNA	Off	1,458	No	On	9
-4.8	Complete	66 [106]	63	SNA	Off	1,455	No	On	10
-4.7	Complete	66 [106]	63	SNA	Off	1,452	No	On	10
-4.6	Complete	66 [106]	63	SNA	Off	1,454	No	On	10
-4.5	Complete	66 [106]	63	SNA	Off	1,452	No	On	11
-4.4	Complete	66 [106]	63	SNA	Off	1,457	No	On	11
-4.3	Complete	66 [106]	63	SNA	Off	1,458	No	On	11
-4.2	Complete	66 [106]	63	SNA	Off	1,445	No	On	11
-4.1	Complete	66 [106]	63	SNA	Off	1,450	No	On	11
-4.0	Complete	66 [106]	63	SNA	Off	1,447	No	On	11
-3.9	Complete	66 [106]	63	SNA	Off	1,449	No	On	11
-3.8	Complete	66 [106]	63	SNA	Off	1,451	No	On	11
-3.7	Complete	66 [106]	63	SNA	Off	1,444	No	On	11
-3.6	Complete	66 [106]	63	SNA	Off	1,453	No	On	10
-3.5	Complete	65 [105]	63	SNA	Off	1,447	No	On	10
-3.4	Complete	65 [105]	63	SNA	Off	1,443	No	On	10
-3.3	Complete	66 [105]	63	SNA	Off	1,448	No	On	10
-3.2	Complete	66 [105]	63	SNA	Off	1,441	No	On	11
-3.1	Complete	65 [105]	63	SNA	Off	1,450	No	On	11
-3.0	Complete	66 [106]	63	SNA	Off	1,454	No	On	11
-2.9	Complete	66 [106]	63	SNA	Off	1,452	No	On	11
-2.8	Complete	66 [106]	63	SNA	Off	1,447	No	On	15
-2.7	Complete	65 [105]	63	SNA	Off	1,441	No	On	16
-2.6	Complete	65 [105]	63	SNA	Off	1,451	No	On	15
-2.5	Complete	66 [105]	63	SNA	Off	1,456	No	On	15
-2.4	Complete	66 [106]	63	SNA	Off	1,455	No	On	15
-2.3	Complete	66 [106]	63	SNA	Off	1,449	No	On	18
-2.2	Complete	66 [106]	63	SNA	Off	1,444	No	On	20
-2.1	Complete	66 [106]	62	SNA	Off	1,448	No	On	20
-2.0	Complete	66 [106]	61	SNA	Off	1,454	No	On	18
-1.9	Complete	66 [106]	61	SNA	Off	1,456	No	On	16
-1.8	Complete	66 [106]	61	SNA	Off	1,455	No	On	15
-1.7	Complete	66 [106]	61	SNA	Off	1,459	No	On	13
-1.6	Complete	66 [106]	61	SNA	Off	1,449	No	On	11
-1.5	Complete	66 [106]	61	SNA	Off	1,451	No	On	9
-1.4	Complete	66 [106]	8	SNA	Off	1,441	No	On	3
-1.3	Complete	65 [105]	0	SNA	Off	1,434	No	On	-8
-1.2	Complete	63 [102]	0	SNA	On	1,382	No	Engaged	-25
-1.1	Complete	60 [97]	0	SNA	On	1,319	Yes	On	-19
-1.0	Complete	57 [91]	0	SNA	On	1,232	Yes	On	-9
-0.9	Complete	56 [89]	0	SNA	On	1,195	Yes	On	-12
-0.8	Complete	54 [87]	0	SNA	On	1,227	Yes	On	-35
-0.7	Complete	51 [83]	0	SNA	On	1,118	Yes	On	-57
-0.6	Complete	49 [79]	0	SNA	On	1,031	Yes	On	-51
-0.5	Complete	45 [73]	0	SNA	On	1,025	Yes	On	-28
-0.4	Complete	43 [69]	0	SNA	On	1,035	Yes	On	4
-0.3	Complete	44 [71]	0	SNA	On	1,097	Yes	On	32
-0.2	Complete	42 [67]	0	SNA	On	949	Yes	On	46
-0.1	Complete	37 [60]	0	SNA	On	789	Yes	On	39

Skid mark on scene supports drive tire may be traveling slower than vehicle actually is.

Vehicle-2 Crash Data Retrieval

Pre-Crash Data -5 to 0 sec [2 samples/sec] (First Record)

Times (sec)	Speed vehicle indicated MPH [km/h]	Accelerator pedal, % full	Service brake, on/off	Engine RPM	ABS activity (engaged, non-engaged)	Brake Powertrain Torque Request	Driver Gear Selection	Traction Control via Brakes	Wheel Torque
- 5.0	29 [46]	0.0	On	1,228	non-engaged	No	Drive	non-engaged	-252
- 4.5	27 [44]	0.0	On	1,178	non-engaged	No	Drive	non-engaged	-252
- 4.0	26 [42]	0.0	On	1,120	non-engaged	No	Drive	non-engaged	-256
- 3.5	24 [39]	0.0	On	1,026	non-engaged	No	Drive	non-engaged	-264
- 3.0	23 [37]	0.0	On	956	non-engaged	No	Drive	non-engaged	-260
- 2.5	21 [34]	0.0	On	934	non-engaged	No	Drive	non-engaged	-164
- 2.0	19 [31]	0.0	On	854	non-engaged	No	Drive	non-engaged	-68
- 1.5	17 [28]	0.0	On	828	non-engaged	No	Drive	non-engaged	-44
- 1.0	16 [25]	0.0	On	806	non-engaged	No	Drive	non-engaged	-56
- 0.5	14 [23]	0.0	On	788	non-engaged	No	Drive	non-engaged	-84
0.0	14 [22]	0.0	Off	742	non-engaged	No	Drive	non-engaged	-80

There is a (.5) second delay from last data point and impact, therefore Vehicle could have accelerated to a higher speed.