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The Dangers of Lift on Parked Planes: General Aviation Airport Safety

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The Dangers of Lift on Parked Planes: General Aviation Airport Safety

Abstract

The focus of this paper is to investigate proper aircraft management for safety on an airfield. This is accomplished by looking at lift caused by powerful winds to the aircrafts stored on an airfield, and the tension it places on the rope that secures them. This work could be used to determine when aircrafts are in high-risk and need to be stored either in hangars or moved to other airports prior to storms. The calculations used to determine these conclusions are also explored in the paper.

Keywords

aircraft management, safety on an airfield, airfoil, Bernoulli's Principle, Cessna 172

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PROBLEM STATEMENT

To understand the limitations of tying down aircraft in strong weather conditions so aircraft can be safely secured at general aviation airports.

MOTIVATION

Background

General aviation airports – which are designated as airports that do not have airliners delivering passengers, but instead are for primarily civilian use and privately owned aircraft – are tasked with keeping tenants' airplanes safe. Civilian aircrafts are stored in 1 of 3 different modes of storage being: Hangars (fully enclosed), Shade hangars (only has a roof), and ramp space (out in the open). Planes in shade hangars and ramp space are at the most risk due to their exposure to the elements. To prevent damage to the aircrafts they are tied down using rope connecting the plane to the ground (see Figure 1), but even with this precaution the aircrafts still can get damaged depending on weather conditions and strength of the rope.

Problem

The purpose of finding the limitations of rope in securing airplanes is to ensure that property damage is made a minimum in strong wind conditions such as tropical storms. These events can cost millions in damages [1] due to their very nature but it is important to reduce the rebuilding costs following them. By knowing the limitations of tie-downs in storms, aviators can have confidence in their aircraft's safety. This is important because aircrafts are quite the investment as they are quite expensive.

As there is a wide variety of aircrafts ranging from single-engine piston to twin-engine turbos, this paper will focus on Cessna 172s (C172), which are prominently used in flight training and personal use throughout the aviation industry [2]. Given that, all the equations used will be under the assumption that the plane has the specs of a C172 to find the optimal conditions for a C172, but the same process can be using the information of other planes to solve for theirs.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Prior Knowledge of Cessna 172

As this paper explores how a C172 will respond to lift in strong weather conditions, it is important to gather information on this type of aircraft. The important information to gather is related to the lift equation and other factors that may have forces on the plane. Since C172s have been developed since the 50s, there are many different models. To address this issue the specifications will come from the official Textron Aviation website. The wing area is an important component in calculating the lift generated as it is the area that the pressure difference acts upon. The wing area of a C172 is 16.17 m^2 . The mass of a C172 is 762 kg, which means that F_g is about 7,475N [3]. With all this information it is enough to begin finding the amount of lift generated by the winds and using that to find a suitable rope for smaller aircraft like the C172.

Bernoulli's Principle and Lift

Bernoulli's Principle is an important concept to understand how to analyze the lift of a plane. A plane's airfoil is designed to have the air above the wing move at a higher velocity than the air at the bottom. According to Bernoulli, due to this difference in velocity it creates a differential pressure because faster moving air creates a low pressure area on the top while slow moving air

creates a high pressure below the wing creating lift on the airfoil. This means that if the function of pressure above the airfoil is greater than the function of pressure below the airfoil there is a positive force of lift for each unit of area [4]:

$$\int (P_L(A) - P_U(A))dA = F, \quad \text{Appendix: (1)}$$

Lift is the force which an airfoil experiences due to certain factors that cause it to move upwards. The variables used in the lift equation are the lift coefficient (Cl), density of air (ρ), velocity of the wind in this case (v) – normally it is the aircraft speed but since it is assumed the planes are tied down it is only in relation to wind speed, and A is the wing area of the airfoil. The lift coefficient is determined by the angle of attack (AOA) which is the angle the wind interacts with the airfoil and, since the plane is grounded and assumed to be parallel to the ground, the wind will be interacting with the wing at about a 3 degree angle [5] [6] and the shape of the airfoil but this is constant for a C172; therefore, at the given AOA the lift coefficient will be about 0.5 (See Figure 3) [6]. For the purpose of this paper, it is assumed that the density of air is at sea level since it varies at different altitudes, but at sea level it is 1.225 kg/m^3 [7]. Lastly, A , the area of the airfoil is 16.17 m^2 as given by the manufactures of Cessna [3]. This is the amount of lift a C172 would need to experience for it to generate enough lift to be able to move. The integral solving for the force generated by pressure on the airfoil will be set equal to L in the lift equation:

$$F \geq L, \quad \text{Appendix: (2)}$$

The equation for lift is as follows [8]:

$$L = Cl \times \frac{\rho v^2}{2} \times A, \quad \text{Appendix: (3)}$$

Substituting all the values and isolating v to the other side of the equation gives (further math is in Appendix):

$$v = \sqrt{\frac{2L}{(Cl)(\rho)(A)}}$$

$$v = 38.852 \frac{m}{s} \text{ or } 75.522 \text{ kts}$$

These wind speeds are comparable to that of a category 1 hurricane. This may seem quite a lot however in climates susceptible to these winds such as the Caribbean that sees winds like these multiple times each summer, it is important to keep an eye out for this. This is if it concerns the sustained winds also, which would cause the plane to be lifted, but wind gusts can also reach these speeds in tropical storms or microbursts causing the plane to be thrown around causing damage to itself and others.

Relation to Rope

After finding the lift generated on the airfoils by the wind, next is to look at the optimal use of rope to tie down these aircrafts. For this we will need to find the needed amount of the rope to be able to be conventionally used. The rope's diameter is limited by the diameter of the anchor points on the C172 that the rope passes through, which is about 2.56cm in diameter. The length will be about 3 meters which allows the rope to go both through the tie down and then be able to be tied using a hurricane knot (seen in Figure 2). The cost of nylon rope of about 1.28cm that meets the needed dimensions is about \$13.29 (See Appendix Equations for more details), which is the most inexpensive option for the safety it provides to the plane. Ropes are effective in doing

the job of keeping the plane secured and unmoving. However, there is one issue that cannot be overlooked.

Sometimes the tie down ropes can come undone because they are not secured correctly, and this causes the aircraft to be exposed to the dangers as mentioned in the Bernoulli's Principle and Lift section. As a solution to this problem, a device that keeps the rope secured even if its knot loosens is constructed. The idea of this design is to make something with an abnormal, hollow dumbbell shape, so the rope can pass through it and also be connected to the anchor point so that the rope cannot move forward nor backward from the anchor point until the device is taken apart. This could be made affordable and would be helpful to anyone with a plane. The material used could be a sturdier material like a steel alloy or a less expensive but still useful 3D printed filament. The equation used to represent the shape of this design is (seen in Figure 4):

$$f(x) = (\cos(x/2))^4 + 1$$

To fill in the 3D shape of the object is to rotate the equation along the x-axis. To create a thick durable material but also a hollow one, the following equation is going to be using the washer method (seen in Figure 5):

$$V = \pi \int_{-\frac{2\pi}{2}}^{\frac{5\pi}{2}} [(\cos(x/2))^4 + 1]^2 - (\cos(x/2))^4 + 3/4)^2] dx, \quad \text{Appendix: (4)}$$

The generated solid has a volume $V \approx 20.076 \text{ cm}^3$ (See Appendix Equations for more details), knowing this volume we can find the cost for both the steel alloy – a cost of \$5.90 per kg – and the 3D printed filament (PLA) – a cost of \$0.05 per cm^3 . For the steel alloy it is \$0.96 (See Appendix Equations for more details). The 3D printed filament costs \$1.00 (See Appendix Equations for more details). This shows that the 3D printed filament actually comes out to be

more expensive to manufacture than the steel alloy, but with the availability of 3D printers, in the long run it may be cheaper and easier to use the 3D printed filament because you don't have to make it malleable. However, knowing this makes the steel alloy an available option if the manufacturer has the skills and tools to form a more stable device.

DISCUSSION

The project theoretically produces the results showing that it is extremely dangerous to have a Cessna 172 stored on ramp space in winds that exceed 70 knots. Those speeds would cause the plane to become a threat to itself and other property in the vicinity. Since it is a common plane with quite the basic airfoil, it may not be representative to all planes, but it gives an idea of what to worry about in other situations. Since these types of winds are not every day, it can be said that it is safe to store the plane using rope tie-downs as it keeps the plane from lifting.

As we see it is important to reduce the risk of storing aircraft as much as possible since it can accrue a lot of property damage if not stored correctly, as well as to look for more solutions to keep the plane secure and avoid human error. A device that can be placed on the anchor points is designed to keep the rope from becoming loose and becoming ineffective in strong winds. It is also seen how inexpensive this would cost in total being around \$14.29 by adding the cost of the rope to the device.

CONCLUSION AND RECOMMENDATIONS

Conclusion

Because of the winds it is important to put in place the proper safety measures to secure the planes. As seen by the Cessna 172, conditions that can threaten the safety of the plane begin around 70 kt. This may seem like a lot but in the areas with the weather conditions such as in Florida or in the Midwest it is practical to take every precaution in keeping an expensive vehicle secure [9]. Therefore, it may be useful to tie down the aircraft using a tool that would prevent the ropes from being undone and still useful if the knots are not tightened.

Recommendation

For those making future observations on this topic, it is recommended to incorporate different specifications for airplanes such as Piper Cherokees or even larger planes like a Beechcraft King Air 200. This will give a wider array of data as not each plane behaves the same due to different designs in the airfoil generating different amounts of lift and the mass of the plane. To do this will give a more complete view to FBOs and plane owners in safely storing planes and set their minds at ease. By doing this it could even be that some planes could require different types of rope to properly secure them and could begin manufacturing targeted ropes for clientele with private planes.

The most widely used material for tying down aircraft at general aviation airports is rope, but another aspect that could be investigated is different materials. The material would have to be flexible, durable, and strong, which may prove challenging in finding an alternative but could make it even safer for aircrafts or cheaper and easier to produce.

Lastly, another recommendation following this project would be to run real-world experiments inside of a wind tunnel. A wind tunnel is recommended because it is safer and easier to generate the required conditions to test this report's conclusion. There are difficulties in doing this such as getting a C172 for the experiment, a proper wind tunnel to use, and the permit to test it. After all of this, real-world applications of this paper can be applied with certainty, as there may be compounding variables not looked at here.

NOMENCLATURE

Symbol	Meaning	Units
v	Wind Speed	kt
P	Pressure	kPa
F	Force	N
F_g	Force of Gravity	N
A	Surface Area	m^2
ρ	Density	kg/m^3
C_l	Coefficient of Lift	N/A
r	Radius	cm
m	Mass	kg
V	Volume	cm^3
C	Cost in USD	\$

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APPENDIX

Equations:

$$(1) \int (P_L(A) - P_U(A)) dA = F$$

Bernoulli's principle in math states that the pressure for each unit of area on the lower side has to be greater than the pressure for each unit of area on the upper side.

$$(2) F \geq L$$

This simple statement just says that the Force acting on the bottom has to be greater than or equal to the lift of the plane.

$$(3) L = Cl \times \frac{\rho v^2}{2} \times A : \text{Solving for velocity of wind}$$

$$7,475 \text{ N} = (0.5) \left(\frac{1.225 \frac{\text{kg}}{\text{m}^3} \times v^2}{2} \right) (16.17 \text{ m}^2)$$

$$v = \sqrt{\frac{2(7,475 \text{ N})}{(0.5)(1.225 \frac{\text{kg}}{\text{m}^3})(16.17 \text{ m}^2)}}$$

$$v = 38.852 \frac{\text{m}}{\text{s}} * 1.944 \frac{\text{kt}}{\frac{\text{m}}{\text{s}}} = 75.522 \text{ kt}$$

$$(4) V = \pi \int (R(x)^2 - r(x)^2) dx$$

The volume of rotating solid is found below:

$$\begin{aligned} V &= \pi \int_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \left[\left(\cos^4 \left(\frac{x}{2} \right) + 1 \right)^2 - \left(\cos^4 \left(\frac{x}{2} \right) + \frac{3}{4} \right)^2 \right] dx \\ &= \pi \int_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \left[\cos^8 \left(\frac{x}{2} \right) + 2\cos^4 \left(\frac{x}{2} \right) + 1 - \cos^8 \left(\frac{x}{2} \right) - \frac{3}{2}\cos^4 \left(\frac{x}{2} \right) - \frac{9}{16} \right] dx. \end{aligned}$$

It follows that

$$\begin{aligned} V &= \frac{\pi}{2} \int_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \left[\cos^4 \left(\frac{x}{2} \right) + \frac{7}{8} \right] dx \\ &= \frac{\pi}{2} \int_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \left[\left(\frac{1 + \cos(x)}{2} \right)^2 + \frac{7}{8} \right] dx \\ &= \frac{\pi}{8} \int_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \left(1 + 2\cos x + \cos^2 x + \frac{7}{2} \right) dx \\ &= \frac{\pi}{8} \int_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \left(5 + 2\cos x + \frac{\cos(2x)}{2} \right) dx \end{aligned}$$

and

$$V = \frac{\pi}{8} \left(\left[5x + 2\sin x + \frac{\sin(2x)}{4} \right]_{-\frac{\pi}{2}}^{\frac{5\pi}{2}} \right) = \pi(15\pi+4)/8.$$

Hence $V \approx 20.0763 \text{ cm}^3$

Conversion of volume to kg of steel alloy 304

$$20.076 \text{ cm}^3 \times 0.00806 \frac{\text{kg}}{\text{cm}^3} = 0.162 \text{ kg}$$

Conversion of kg of steel alloy 304 to dollar amount

$$0.162 \text{ kg} \times 5.90 \frac{\$}{\text{kg}} = \$0.9558$$

Conversion of volume to PLA filament dollar value

$$20.076 \text{ cm}^3 \times 0.05 \frac{\$}{\text{cm}^3} = \$1.0038$$

Conversion of dollars/ft to dollars/m

$$\$1.35 \frac{\text{dollars}}{\text{foot}} \times \frac{\text{ft}}{0.3048\text{m}} \cong \$4.4291 \frac{\text{dollars}}{\text{meter}}$$

Cost of 3 meters

$$\$4.43 \frac{\text{dollars}}{\text{meter}} \times 3 \text{ meters} = \$13.29$$

Figures:

Figure 1: Image displaying tie-down aircraft



Figure 2: Image displaying tail tie-down anchors and the knot used to hold them.

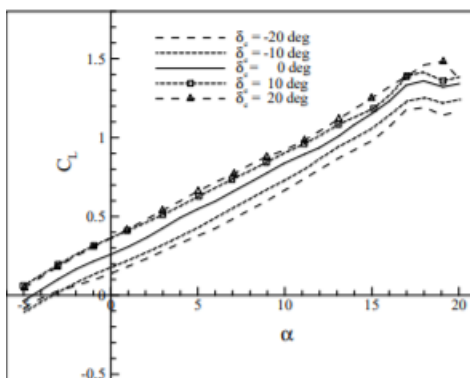


Figure 3: Graph of Coefficient of Lift for each Angle of Attack [2]

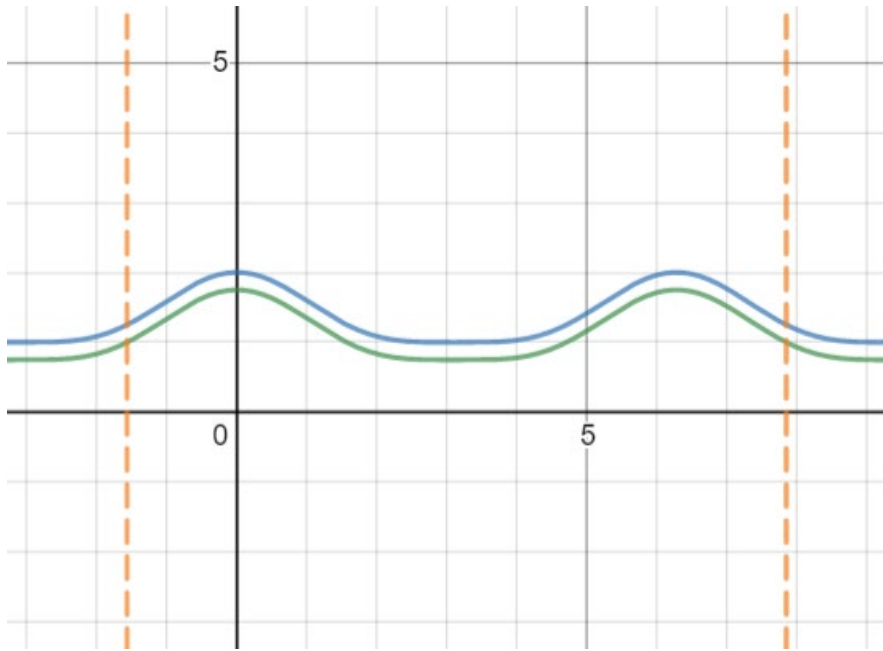


Figure 4: Displaying the outline of the wanted shape generated by equation with the boundaries of $\frac{-\pi}{2}$ and $\frac{5\pi}{2}$.

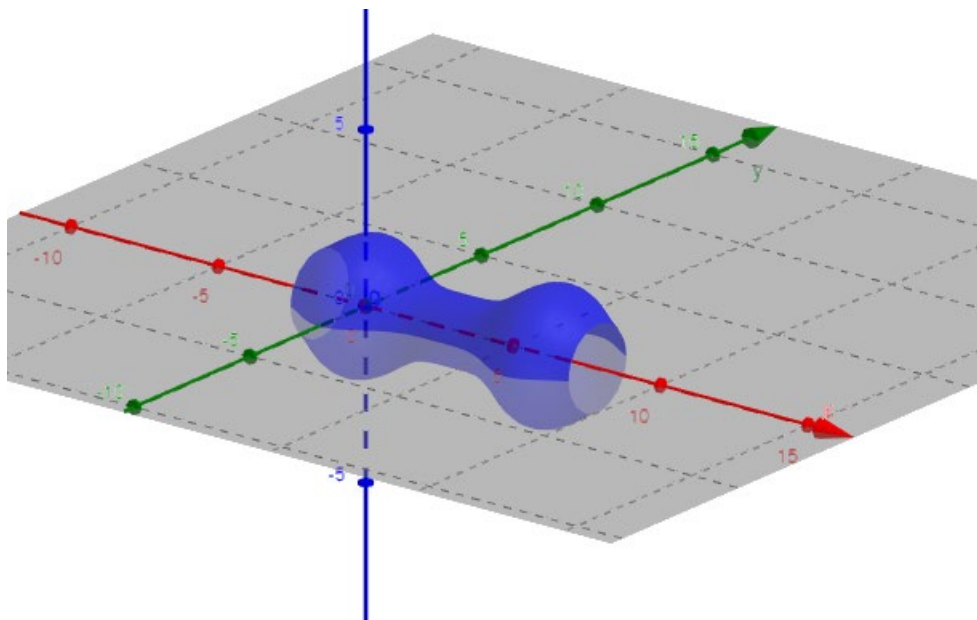


Figure 5: The generated solid of the following outline using the washer method, which rotates the solid around the x-axis.