Undergraduate Journal of Mathematical Modeling: One + Two

Volume 8 | 2018 Spring 2018

Article 6

2018

Tsunami Waves

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Recommended Citation
Pennino, Samantha (2018) "Tsunami Waves," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 8: Iss. 2, Article 6.
DOI: http://doi.org/10.5038/2326-3652.8.2.4893
Available at: https://digitalcommons.usf.edu/ujmm/vol8/iss2/6

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Abstract

Predicting the travel time of tsunami waves is imperative to the safety and well-being of possibly affected communities since tsunami waves can happen incredibly fast. So, knowing how long an area is until a tsunami approaches dramatically helps with mitigation strategies. As an example we calculate the travel time of a tsunami wave to Somalia and Sri-Lanka that flowed an earthquake in 2004 in the Indian ocean off the coast of Sumatra. This is done by using the velocity from the shallow wave equation of the waves and piecewise integration.

Keywords

shallow water equation, ocean depth, wavelength, piecewise integration

PROBLEM STATEMENT

Compute the travel time of tsunami wave to see how long it takes to reach Sri-Lanka and Somalia from Sumatra if the earthquake has a moment magnitude of 9.1–9.3.

MOTIVATION

Tsunamis are produced by sudden displacement of large masses of water. The most common sources of tsunami are earthquakes, volcanic eruption, or landslides. Mostly tsunamis are created during earthquakes, and are often exponentially more dangerous than the earthquake itself. Tsunamis result when subsea fault movements change the composure of the sea-floor, therefore affecting the deep-ocean water mass. Since water cannot absorb fault-movement energy, it transmits the energy throughout the ocean in enormous waves that can travel to far distant places. For instance, an earthquake which occurred in Chile, South America was followed by a tsunami that traveled to Hawaii, Philippines and Japan killing many people. So when a natural hazard that causes a tsunami occurs, it is imperative to give the potentially affected countries a warning. Without estimating the time travel of the wave the effectiveness of the warning will be decreased. In this paper, the back story is as follows: On December 28th, 2004, the epicenter of an earthquake hit off the west coast of Sumatra, Indonesia in the Indian Ocean. It generated multiple destructive tsunamis along the coasts of most landmasses bordering the Indian Ocean. Some coastal communities endured waves up to 100 ft. high. The Sumatra–Andaman earthquake was one of the deadliest natural disasters in recorded history due to the tsunamis [ET, 2017].

MATHEMATICAL DESCRIPTION AND APPROACH

A wave can be described by its wavelength, height and period [Batchelor, 1967]. Wavelength is the distance between two wave crests, i.e. the highest point of each wave. The period is defined to be the time between the passages of two successive wave crests through a given point.

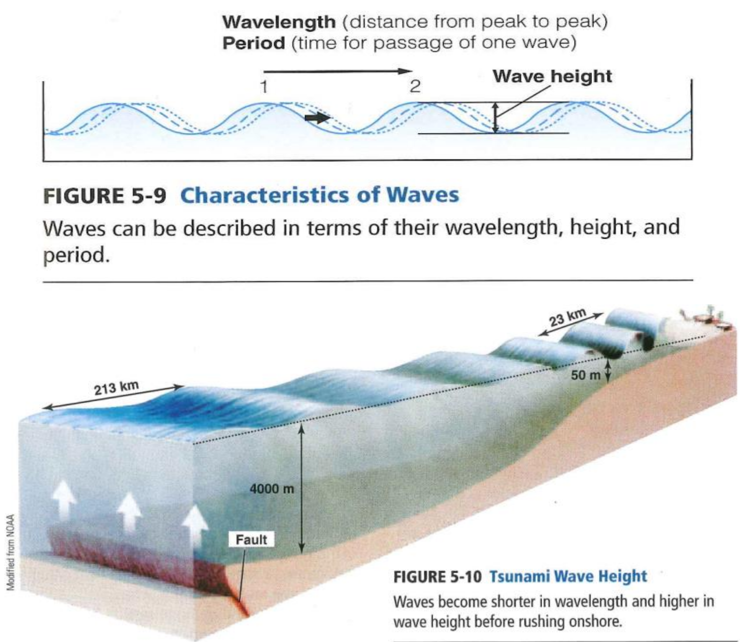

Figure 1: Natural Disasters, fifth edition McGraw Hill text book.

Ocean waves can be distinguished between deep water and shallow water waves. While the first ones are dispersive meaning the velocity to be dependent on the wavelength, the velocity of propagation of a shallow water wave is only dependent on the depth of the water. The wavelength of a tsunami wave is of the order of hundreds of kilometers. Since all the ocean

DOI: http://doi.org/10.5038/2326-3652.8.2.4893

waves with wavelength longer than the water depth are shallow water waves, the travel speed of a tsunami wave is only dependent on the depth of the water column. The first step to calculate the travel time of the wave is to find the velocity of the tsunami wave. The velocity of tsunami waves is dependent on the depth of water and gravity. Below is the shallow water equation which is often used for the deep ocean as well [Dawson and Mirabito, 2008]:

$$V = \sqrt{gD}\ . \tag{1}$$

In (1): V is the velocity in meters per second (m/s), D is the depth in meters (m), and g is the gravitational acceleration assumed to be $9.8\text{m}/\text{sec}^2$. Thus:

$$V = 3.13\sqrt{D}\ . \tag{2}$$

So how is the velocity equation derived? Well, the shallow water equation is derived from the classical Navier-Stokes equations that express the conservation of mass and momentum [Batchelor, 1967; Dawson and Mirabito, 2008]. Since equation (2) holds and the depth of the ocean changes depending on the location, it follows that the velocity is different at each location. The travel time along a single path can reduce the problem to 1-dimension. In general it is a more complex problem, as 2-dimensional wave propagation gives interference; in our situation, we will disregard it for the sake of simplicity. Let *x* be the distance from the source along the path and let *t* be the time that occurs since the earthquake has generated the tsunami. If we assume that the path that the wave is traveling along is a great circle (the shortest distance on the surface of a sphere) then from a bathymetric map we can extract the bathymetry along the path and then read the value of the depth of the water column as function of the distance from the tsunami source.

If D in (2) is a function of the position along the path, the velocity can be written as

$$v(x) = 3.13\sqrt{D(x)}. \tag{3}$$

We have from (3):

$$\frac{dx}{dt} = v(x) \Longrightarrow \frac{dx}{dt} = 3.13\sqrt{D(x)}\ . \tag{4}$$

It is known that for a physical reason *D(x)* is greater than 0 for every value of *x* because the tsunami wave only travels in water. Thus, $v(x)$ is greater than 0 and both sides of the equation (4) can be divided by $v(x)$:

$$\frac{1}{v(x)}\frac{dx}{dt} = 1 \Longrightarrow \frac{1}{v(x)}dx = dt\ . \tag{5}$$

Equation (5) allows us to determine the travel time between two points $x=s_1$ and $x=s_2$. Since both points are along the path, the equation will be as follows:

$$\int_{s_1}^{s_2}\frac{1}{v(x)}dx = \int_{t_1}^{t_2}dt\ . \tag{6}$$

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In (6): the time variable refers to the moment in which the wave arrives at a certain point, while the variable *s* refers to the distance from the source of the tsunami. Having simplified we can assume that in the interval of integration the water depth is constant and equal to D_1. In that case the velocity is constant and $v_1 = \sqrt{gD_1}$ and the integral (6) becomes:

$$\int_{s_1}^{s_2} \frac{1}{\sqrt{gD_1}} dx = \frac{1}{\sqrt{gD_1}} \int_{s_1}^{s_2} dx = \int_{t_1}^{t_2} dt. \tag{7}$$

Equation (7) gives the solution

$$\frac{s_2 - s_1}{\sqrt{gD_1}} = t_2 - t_1 \,. \tag{8}$$

In (8): s_1 and s_2 indicate the extreme of the interval of integration for *x*, while t_1 and t_2 refer to the time interval.

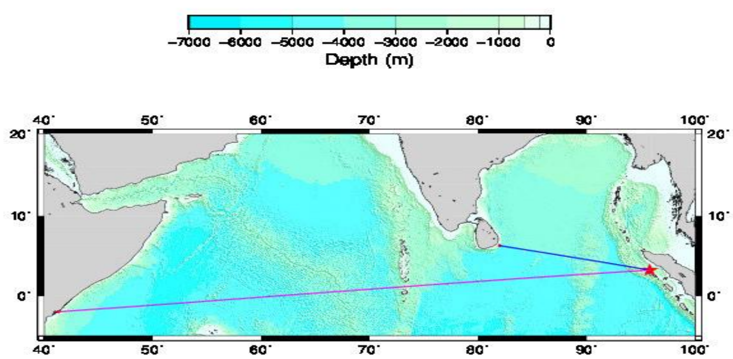

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Figure 2: Bathymetry map from the epicenter of the earthquake (star) to Somalia and Sri-Lanka to approximate the depth [GEBCO, 2017]

Piecewise integrated equation (6) and equation (8) can be used for each section of the wave to be added to the total time over a certain distance that the wave will travel.

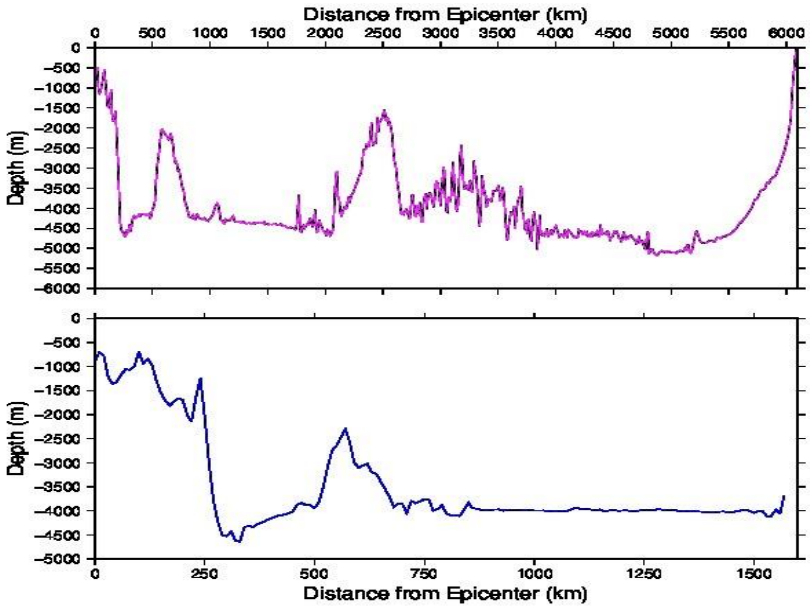

Figure 3: The distance that each wave takes to reach each location and the depth at which it occurs. Source: GEBCO (Somalia is pink line; Sri-Lanka is blue line).

Starting with Somalia we can split the wave length into 4 sections in which we can assume the depth to be constant. Taking the integral with respect to time from the epicenter, as shown above in equations (7) and (8), we get:

$$\int_0^{750} \frac{1}{v(x)}\,dx = \int_0^{t_1} dt\,, \tag{9}$$

$$\frac{750-0}{563.4} = t_1\text{-}0 \Longrightarrow t_1 = 1.33 \text{ hr},$$

where $v(x) \approx 3.13\sqrt{2500}$ = 156.5 m/s =563.4 km/hr.

Now, we take the integral with respect to time from 750 to our next point:

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DOI: http://doi.org/10.5038/2326-3652.8.2.4893

$$\int_{750}^{2100} \frac{1}{v(x)}\,dx = \int_{1.33}^{t_2} dt, \tag{10}$$

$v(x) \approx 3.13\sqrt{4000}$ = 197.96 m/s = 712.65 km/hr,

$\frac{2100-750}{712.65}$ = t_2-1.33 $\Longrightarrow$ t_2= 3.22 hr.

Doing the same steps for each integral, we get: **T** ≈ 9.5 hours.

With Sri-Lanka, we can split the wave length into 5 sections of similar depths. Taking the integral with respect to time from the epicenter we get:

$$\int_{0}^{250} \frac{1}{v(x)}\,dx = \int_{0}^{t_1} dt, \tag{11}$$

$v(x) \approx 3.13\sqrt{2000} = 139.98 \text{ m/s} = 503.92 \text{ km/hr},$

$\frac{250-0}{503.92}$ = t_1-0 $\Longrightarrow$ t_1= 0.496 hr.

Now, we take the integral with respect to time from 250 to our next point:

$$\int_{250}^{500} \frac{1}{v(x)}\,dx = \int_{0.496}^{t_2} dt, \tag{12}$$

$v(x) \approx 3.13\sqrt{4000}$ = 197.96 m/s = 712.63 km/hr,

$\frac{500-250}{712.63}$ = t_2-0.496 $\Longrightarrow$ t_2 = 0.847 hr.

Doing the same steps for each integral, we get: **T** ≈ 2 hours.

The time needed to travel a given distance L at a constant average velocity V is equal to L/V, so we use this equation to check our results. After reviewing the bathymetry map we can assume that the average depth of the ocean between Somalia and Sumatra and between Sri-Lanka and Sumatra is 3000 m. Hence the corresponding average velocity is:

$V=3.13\sqrt{3000}$ = 171.44 m/s = 617.17 km/hr. It follows that the approximate travel time for a tsunami to reach Somalia from Sumatra is: 6100km/ 617.17km/hr ≈ 9.88 hours. It seems reasonable to be off by 23 minutes because this equation takes the whole distance at the same depth. A rough estimate of the travel time for a tsunami to reach Sri-Lanka from Sumatra is: 1600 km/ 617.17 km/hr ≈ 2.59 hours. It shows that the piecewise integration is important.

DISCUSSION

Calculations for the travel time of a tsunami from Sumatra to Sri Lanka and Somalia have been obtained from the shallow wave equation $V = 3.13\sqrt{D}$, using piecewise integration. The equations showed that it would take approximately 2 hours for a tsunami to make land on Sri-Lanka from Sumatra. Also, the equations showed that it would take approximately 9.5 hours for a tsunami to reach Somalia from Sumatra. The objective of the project is to see if it is possible to predict the amount of time a certain location has till a tsunami makes land, assuming it is known where the tsunami is starting from.

The results obtained are in good concordance with the known data [GEBCO, 2017]. It has been shown that tsunamis can reach land within a matter of hours if not less. It is imperative to be able to make accurate calculations to dispatch evacuation alarms in time, in order to allow the authorities to deal with the emergency in the most efficient way.

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Damages from a tsunami are first caused by the intense force of the tidal wave impacting the shoreline. Flooding from the tsunami hangs around for several more weeks causing major losses and damages [What are the effects of a tsunami?]. The effects of a tsunami depend on the seismic event that generates the tsunami, the distance from the source, its magnitude and the depth of water in the ocean of the locations the tsunami is approaching [Tsunamis: the effects].

CONCLUSIONS AND RECOMMENDATIONS

To calculate the travel time of a tsunami wave we first use the shallow wave equation to get the volume for each section on **Figure 2** that illustrates the distance from the epicenter and the depths. Then we calculate the travel times of each section using an integral from points $s_n \ to \ s_{n+1}$ and dividing by the velocity assumed to be constant along the path of integration. It equals the integral from $t_n to \ t_{n+1}$. The total time will be the sum of all the times for each segment. We assume that each segment "n" starts at the point $x=s_n$ and ends at the point $x=s_{n+1}$ so then the wave will arrive at the beginning of the segment at the time t_n and at the end of the segment at the time t_{n+1}. By this method, the travel time of a tsunami wave to Sri-Lanka is approximately 2 hours, and to Somalia is approximately 9.5 hours.

If an earthquake occurs on a low point on the ocean floor, then the tsunami will have an immense amount of depth to increase its velocity, making the waves much higher, stronger and therefore increasing the speed and decreasing the time it necessitates to arrive.

If someone were to do the same project, we would recommend to figure out how the speed of a tsunami can affect an area with a long distance and a short distance. Also, the person conducting the project could calculate how certain heights of a tsunami wave can accumulate or change over a long period or short period. Since most tsunamis occur from earthquakes, people in the field could conduct experiments on where potentially hazardous faults are by installing sea floor sensors, and calculate how long possibly impacted areas would be before a tsunami reaches them. This would help with mitigation strategies and evacuations.

NOMENCLATURE

Symbol	Description	Units
v	Velocity	m/s, km/hr
D	Depth	M
g	Gravitational acceleration	$9.8 m/s^2$
x, s	Distance	Km
t	Time	Hr

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