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Optimum Gear Ratios for an Electric Vehicle

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Optimum Gear Ratios for an Electric Vehicle

Abstract

The goal of this project is to determine the optimal gear ratios for a vehicle containing a four-speed transmission. This vehicle is required to reach a speed of 30 m/s in the minimum time possible. Equations for the velocity at each shift point were found. An equation for the total time that the vehicle took to reach 30 m/s was then derived and equations for the times spent in each gear were found through integration of the provided formula for acceleration. The optimal gear ratios were then found by taking the partial derivatives of the total time equation with respect to each gear ratio. These equations were set equal to zero and solved by the method of substitution to obtain the optimum value for each gear ratio. These results would provide engineers with the information required to efficiently develop transmissions to reach particular speeds in a minimum amount of time.

Keywords

transmission, gear ratios, optimization

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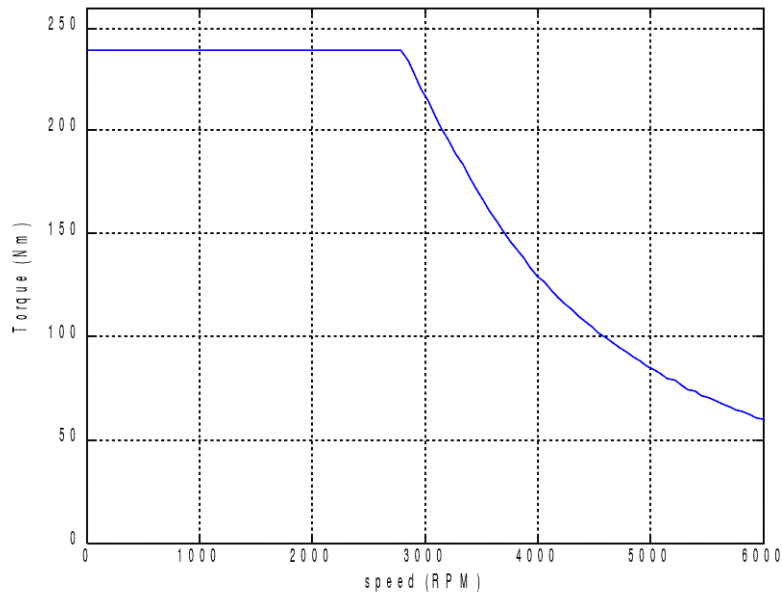


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PROBLEM STATEMENT

A vehicle powered by an electric motor has a mass M (car + driver) of 600 kg and a wheel radius of 0.25 m. It is to have a four-speed transmission and the goal of this project is to find the optimal gear ratios GR_1 , GR_2 , GR_3 and GR_4 for the transmission.

A graph of engine torque versus engine (angular) speed is shown below for the electric motor. Note that the engine can maintain a nearly constant torque until the rotational speed reaches a certain value, at which point the torque produced begins to fall off. A reasonable approach to shifting is to shift to the next gear just before the torque begins to fall off. We will assume here that the engine can maintain a torque of 240 N·m to a speed of 2500 RPM and that the driver will shift to the next gear at 2500 RPM.



The equations for the velocity v and acceleration dv/dt of the vehicle are:

$$v = \frac{R\omega_E}{(GR_i) \cdot FDR} \quad \text{and} \quad \frac{dv}{dt} = \frac{\tau \cdot FDR \cdot GR_i}{M \cdot R}$$

where ω_E is the engine speed in **radians/s**, and τ is the engine torque produced. M and R are the vehicle mass and wheel radius and GR is the gear ratio. Its value depends on what gear ($i = 1, 2, 3$ or 4) the transmission is in. FDR is the final drive ratio, which will be 3.0 .

(a) One goal of the design is for the vehicle to reach a velocity of 30 m/s when the engine is turning 2500 RPM in 4th gear. Use this criterion to calculate GR_4 .

(b) Another goal of the design is that the vehicle reaches the speed of 30 m/s *in the minimum total time*. Find the gear ratios GR_1 , GR_2 and GR_3 that will minimize the total time needed to reach a velocity of 30 m/s. You may assume that the vehicle starts from rest and that the velocity v of the vehicle is continuous at the shift points.

(c) In addition to finding the optimum gear ratios, calculate the corresponding total time needed to reach 30 m/s, the times t_1 , t_2 , t_3 and t_4 that the vehicle spends in each gear, and the velocities v_1 , v_2 , and v_3 at the shift points from first to second gear, from second to third gear, and from third to fourth gear.

MOTIVATION

This problem is important to engineering in that the engineers will already be provided with the information that they might need to make calculations towards achieving a goal of designing a vehicle's transmission to reach a particular speed as quickly as possible. This would more specifically appeal to racing development firms because they would be focusing on achieving the optimum gear ratios that would help improve the performance of their racing vehicle. The motor used to power this vehicle is electric. The results provided would therefore only be of major significance if the same type of motor was being used in another project. However, even if an electric motor was not being used, the information could still be used for readers to receive a greater understanding of how the gear ratios should generally be determined.

The objective of this project is to find the optimal gear ratios for each gear of a four-speed transmission that will minimize the total time required for the vehicle to reach a speed of 30 m/s.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

We use

$$\text{Equation 1: Velocity } (v) = \frac{R\omega_E}{(GR_i) \cdot FDR}$$

and

$$\text{Equation 2: Acceleration } \left(\frac{dv}{dt}\right) = \frac{\tau \cdot FDR \cdot (GR_i)}{M \cdot R},$$

where

$$M(\text{mass of car + driver}) = 600\text{kg},$$

$$R(\text{wheel radius}) = 0.25\text{m},$$

$$\tau(\text{torque}) = 240\text{Nm},$$

$$\omega_E(\text{engine speed}) = 2500\text{RPM} \times \frac{1}{2\pi} \times 60\text{s} = 261.799\text{rad/s},$$

$$FDR(\text{final drive ratio}) = 3.0,$$

$$GR_i(\text{gear ratio}); (i = 1, 2, 3 \text{ or } 4).$$

$$(a) \text{ The value of } GR_4 \text{ is calculated using the equation for velocity provided: } = \frac{R\omega_E}{(GR_4) \cdot FDR},$$

hence

$$\therefore GR_4 = 0.727\text{rad}.$$

(b) Equations for the velocities v_i ; ($i = 0, 1, 2, 3, 4$) at each of the shift points can be determined using the equation for velocity provided. The value of v_0 is 0 m/s because it is the initial velocity before the vehicle started moving. The value of v_1 can be considered to be the velocity of the vehicle at the shift point from 1st to 2nd gear. The values of v_2 and v_3 will obviously be considered to be the velocity at the shift points from 2nd to 3rd gear and 3rd to 4th gear

respectively. The value of v_4 as we know already is the final velocity of 30 m/s. The gear ratios must correspond to their respective shift point velocities. For example: GR_1 and v_1 etc.

$$v_i = \frac{R\omega_E}{(GR_i) \cdot FDR}.$$

The formula provided for acceleration can be integrated in order to obtain an equation for velocity in terms of t_i ; ($i = 1, 2, 3, 4$), where t_i is the time spent in a particular gear:

$$\int_{v_i}^{v_i} dv_i = \frac{\tau \cdot FDR \cdot (GR_i)}{M \cdot R} \int_0^{t_i} dt,$$

$$v_i - v_i = \frac{\tau \cdot FDR \cdot (GR_i)}{M \cdot R} (t_i).$$

Substituting the equations for v from above results in:

$$\frac{R\omega_E}{(GR_i) \cdot FDR} - \frac{R\omega_E}{(GR_i) \cdot FDR} = \frac{\tau \cdot FDR \cdot (GR_i)}{M \cdot R} (t_i).$$

Solving for t_i :

$$t_i = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_i)^2} - \frac{1}{(GR_i)(GR_i)} \right).$$

An equation for the total minimum time required for the vehicle to reach a velocity of 30 m/s is:

$$t_t = t_1 + t_2 + t_3 + t_4$$

Substituting for the equations for t_i from above results in:

$$t_t = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[\frac{1}{(GR_1)^2} + \frac{1}{(GR_2)^2} - \frac{1}{(GR_1)(GR_2)} + \frac{1}{(GR_3)^2} - \frac{1}{(GR_2)(GR_3)} + \frac{1}{(GR_4)^2} - \frac{1}{(GR_3)(GR_4)} \right]$$

Equations for the partial derivatives of the total minimum time with respect to each gear ratio are:

$$\frac{\partial t_t}{\partial GR_1} = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_1)^3} + \frac{1}{(GR_1)^2 (GR_2)} \right], \quad (1)$$

$$\frac{\partial t_t}{\partial GR_2} = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_2)^3} + \frac{1}{(GR_1)(GR_2)^2} + \frac{1}{(GR_2)^2(GR_3)} \right], \quad (2)$$

$$\frac{\partial t_t}{\partial GR_3} = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_3)^3} + \frac{1}{(GR_2)(GR_3)^2} + \frac{1}{(GR_3)^2(GR_4)} \right]. \quad (3)$$

Set each equation [(1), (2), (3)] = 0 and solving by method of substitution results in equations for the optimum gear ratios:

$$GR_1 = 2(GR_2); GR_2 = 2GR_4; GR_3 = \frac{2}{3}(GR_2)$$

Substituting for the value of GR_4 calculated in part (a) you can calculate the values of the optimum gear ratios:

$$GR_1 = 2.908rad; GR_2 = 1.454rad; GR_3 = 0.969rad$$

(c) The values that were found for each gear ratio can be substituted into the equations that were derived for the velocities at the shift points (v_i), the times that the vehicle spends in each gear t_i and the total time needed to reach a velocity of 30 m/s (t_t) to calculate these corresponding values.

$$v_1 = 7.502m/s; v_2 = 15.00m/s; v_3 = 22.51m/s$$

$$t_1 = 0.537s; t_2 = 1.075s; t_3 = 1.615s; t_4 = 2.148s$$

$$t_t = 5.375s$$

DISCUSSION

The gear ratios decreased for each gear from 1st to 4th. This result was as expected because the gear ratios typically decrease in this order. It was observed however, that the gear ratio for 1st gear was by far the longest when comparing the differences between the other gear ratios. The gear ratios (GR_2 , GR_3 and GR_4) could be considered to be close-ratio because of a smaller progression between the gears. This wide-ratio between 1st gear and 2nd gear is due to the

vehicle needing to spend more time in 1st gear because it was starting from rest with a low engine speed. At the shift points between the other gears, the engine speed would not drop by such a great amount and thus the other gear ratios were shorter because less time was required to reach an engine speed of 2500 RPM. As each gear is shifted at 2500 RPM, the time required for the following gear to reach that same engine speed is shorter because at each progressing shift point, the engine speed does not drop by as much as the one before it since the velocity of the vehicle is increasing.

These results provide accurate information as to how you can calculate the optimal gear ratios for a vehicle to reach a particular speed. Engineers can readily draw on these results to save them time in making accurate calculations. These would help in the field in general by providing a useful template when making similar calculations without the engineers having to spend time trying to solve such a problem.

CONCLUSIONS AND RECOMMENDATIONS

From the information gathered in this project, the results can be used to aid in the improvement of transmissions to reach particular speeds in a minimum total time. It was observed that the optimum gear ratios needed for this to occur should have relatively small progressions up until each shift point, with these progressions decreasing between each gear as the velocity of the vehicle increases. For example: GR₁ should have the longest progression before changing to 2nd gear and GR₃ should have the shortest progression before changing to 4th gear. For others working on a similar type of project, it must be noted that this problem is based on a vehicle that is being powered by an electric motor. Therefore, unless the vehicle in their project is also being powered by the same type of motor, these results would prove invalid. This is because the torque would not be constant at all engine speeds up until 2500 RPM as is the case

for this particular electric motor. An example of a different type of motor could be an internal combustion engine.

NOMENCLATURE

V	Velocity	m/s
T	Time	s
GR _i	Gear Ratio	rad
R	Wheel Radius	m
M	Mass (car + driver)	kg
ω_e	Engine Speed	RPM
τ	Torque	Nm
FDR	Final Drive Ratio	-

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APPENDIX

(a)

From Equation 1:

$$v = \frac{R\omega_E}{(GR_4) \cdot FDR}$$

$$\therefore GR_4 = \frac{R\omega_E}{v \cdot FDR} = \frac{(0.25m)(261.799rad/s)}{(30m/s)(3.0)} = 0.727rad$$

(b)

$v_0 = 0m/s$; Where v_0 is the initial velocity.

From equation 1:

$$v_1 = \frac{R\omega_E}{(GR_1) \cdot FDR}; \text{ Where } v_1 \text{ is the velocity at the shift point from 1}^{st} \text{ to 2}^{nd} \text{ gear.}$$

$$v_2 = \frac{R\omega_E}{(GR_2) \cdot FDR}; \text{ Where } v_2 \text{ is the velocity at the shift point from 2}^{nd} \text{ to 3}^{rd} \text{ gear.}$$

$$v_3 = \frac{R\omega_E}{(GR_3) \cdot FDR}; \text{ Where } v_3 \text{ is the velocity at the shift point from 3}^{rd} \text{ to 4}^{th} \text{ gear.}$$

$$v_4 = \frac{R\omega_E}{(GR_4) \cdot FDR}; \text{ Where } v_4 \text{ is the final velocity (30m/s) in 4}^{th} \text{ gear.}$$

From equation 2:

$$\frac{dv_1}{dt_1} = \frac{\tau \cdot FRD \cdot (GR_1)}{M \cdot R}$$

$$\int_{v_0}^{v_1} dv_1 = \frac{\tau \cdot FDR \cdot (GR_1)}{M \cdot R} \int_0^{t_1} dt$$

$$v_1 - v_0 = \frac{\tau \cdot FDR \cdot (GR_1)}{M \cdot R} (t_1)$$

$$v_1 = \frac{\tau \cdot FDR \cdot (GR_1)}{M \cdot R} (t_1)$$

$$\text{Substituting for } v_1 \text{ from above: } \frac{R\omega_E}{(GR_1) \cdot FDR} = \frac{\tau \cdot FDR \cdot (GR_1)}{M \cdot R} (t_1)$$

$$\therefore t_1 = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \frac{1}{(GR_1)^2}; \text{ Where } t_1 \text{ is the time spent in 1}^{st} \text{ gear.}$$

From equation 2:

$$\frac{dv_2}{dt_2} = \frac{\tau \cdot FRD \cdot (GR_2)}{M \cdot R}$$

$$\int_{v_1}^{v_2} dv_2 = \frac{\tau \cdot FDR \cdot (GR_2)}{M \cdot R} \int_0^{t_2} dt$$

$$v_2 - v_1 = \frac{\tau \cdot FDR \cdot (GR_2)}{M \cdot R} (t_2)$$

$$\text{Substitute for } v_1 \text{ and } v_2 \text{ from above: } \frac{R\omega_E}{(GR_2) \cdot FDR} - \frac{R\omega_E}{(GR_1) \cdot FDR} = \frac{\tau \cdot FDR \cdot (GR_2)}{M \cdot R} (t_2)$$

$$\therefore t_2 = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_2)^2} - \frac{1}{(GR_1)(GR_2)} \right); \text{ Where } t_2 \text{ is the time spent in 2}^{nd} \text{ gear.}$$

From equation 2:

$$\frac{dv_3}{dt_3} = \frac{\tau \cdot FRD \cdot (GR_3)}{M \cdot R}$$

$$\int_{v_2}^{v_3} dv_2 = \frac{\tau \cdot FDR \cdot (GR_3)}{M \cdot R} \int_0^{t_3} dt$$

$$v_3 - v_2 = \frac{\tau \cdot FDR \cdot (GR_3)}{M \cdot R} (t_3)$$

Substitute for v_2 and v_3 from above:
$$\frac{R\omega_E}{(GR_3) \cdot FDR} - \frac{R\omega_E}{(GR_2) \cdot FDR} = \frac{\tau \cdot FDR \cdot (GR_3)}{M \cdot R} (t_3)$$

$$\therefore t_3 = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_3)^2} - \frac{1}{(GR_2)(GR_3)} \right); \text{Where } t_3 \text{ is the time spent in 3}^{rd} \text{ gear.}$$

From equation 2:

$$\frac{dv_4}{dt_4} = \frac{\tau \cdot FRD \cdot (GR_4)}{M \cdot R}$$

$$\int_{v_3}^{v_4} dv_2 = \frac{\tau \cdot FDR \cdot (GR_4)}{M \cdot R} \int_0^{t_4} dt$$

$$v_4 - v_3 = \frac{\tau \cdot FDR \cdot (GR_4)}{M \cdot R} (t_4)$$

Substitute for v_3 and v_4 from above:
$$\frac{R\omega_E}{(GR_4) \cdot FDR} - \frac{R\omega_E}{(GR_3) \cdot FDR} = \frac{\tau \cdot FDR \cdot (GR_4)}{M \cdot R} (t_4)$$

$$\therefore t_4 = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_4)^2} - \frac{1}{(GR_3)(GR_4)} \right); \text{Where } t_4 \text{ is the time spent in 4}^{th} \text{ gear.}$$

$$t_t = t_1 + t_2 + t_3 + t_4; \text{Where } t_t \text{ is the total time needed to reach 30m/s.}$$

$$t_t = \left[\frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \frac{1}{(GR_1)^2} \right] + \left[\frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_2)^2} - \frac{1}{(GR_1)(GR_2)} \right) \right]$$

$$+ \left[\frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_3)^2} - \frac{1}{(GR_2)(GR_3)} \right) \right] + \left[\frac{R^2 \omega_E M}{\tau \cdot FDR^2} \cdot \left(\frac{1}{(GR_4)^2} - \frac{1}{(GR_3)(GR_4)} \right) \right]$$

$$\therefore t_t = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[\frac{1}{(GR_1)^2} + \frac{1}{(GR_2)^2} - \frac{1}{(GR_1)(GR_2)} + \frac{1}{(GR_3)^2} - \frac{1}{(GR_2)(GR_3)} + \frac{1}{(GR_4)^2} - \frac{1}{(GR_3)(GR_4)} \right]$$

$$\frac{\partial t_t}{\partial GR_1} = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_1)^3} + \frac{1}{(GR_1)^2(GR_2)} \right] \quad (1)$$

$$\frac{\partial t_t}{\partial GR_2} = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_2)^3} + \frac{1}{(GR_1)(GR_2)^2} + \frac{1}{(GR_2)^2(GR_3)} \right] \quad (2)$$

$$\frac{\partial t_t}{\partial GR_3} = \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_3)^3} + \frac{1}{(GR_2)(GR_3)^2} + \frac{1}{(GR_3)^2(GR_4)} \right] \quad (3)$$

Set each equation [(1), (2), (3)] = 0 and solve by method of Substitution.

$$\text{From (1): } \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_1)^3} + \frac{1}{(GR_1)^2(GR_2)} \right] = 0$$

Divide throughout by $\frac{R^2 \omega_E M}{\tau \cdot FDR^2}$ and multiply both sides by $(GR_1)^3(GR_2)$:

$$-2(GR_2) + GR_1 = 0$$

$$\therefore GR_1 = 2(GR_2)$$

$$\text{From (2): } \frac{R^2 \omega_E M}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_2)^3} + \frac{1}{(GR_1)(GR_2)^2} + \frac{1}{(GR_2)^2(GR_3)} \right] = 0$$

Divide throughout by $\frac{R^2 \omega_E M}{\tau \cdot FDR^2}$ and multiply both sides by $(GR_1)(GR_2)^3(GR_3)$:

$$-2(GR_1)(GR_3) + (GR_2)(GR_3) + (GR_1)(GR_2) = 0$$

$$\text{Sub. } GR_1 = 2(GR_2)$$

$$-2(2GR_2)(GR_3) + (GR_2)(GR_3) + (2GR_2)(GR_2) = 0$$

$$\therefore GR_3 = \frac{2}{3}(GR_2)$$

$$\text{From (3): } \frac{R^2 \omega_{EM}}{\tau \cdot FDR^2} \left[-\frac{2}{(GR_3)^3} + \frac{1}{(GR_2)(GR_3)^2} + \frac{1}{(GR_3)^2(GR_4)} \right] = 0$$

Divide throughout by $\frac{R^2 \omega_{EM}}{\tau \cdot FDR^2}$ and multiply both sides by $(GR_2)(GR_3)^3$:

$$-2(GR_2) + (GR_3) + (GR_2)(GR_3) \frac{1}{GR_4} = 0$$

$$\text{Sub. } GR_3 = \frac{2}{3}(GR_2)$$

$$-2(GR_2) + \left(\frac{2}{3}(GR_2)\right) + (GR_2) \left(\frac{2}{3}(GR_2)\right) \frac{1}{GR_4} = 0$$

Multiply both sides by 3 and divide throughout by 2:

$$-3(GR_2) + GR_2 + (GR_2)^2 \frac{1}{GR_4} = 0$$

Multiply throughout by GR_4 :

$$(GR_2)^2 - 2(GR_2)(GR_4) = 0$$

$$\therefore GR_2 \neq 0 \text{ and } GR_2 = 2GR_4$$

\therefore Optimum Gear Ratios:

$$GR_1 = 4GR_4 = 4(0.727\text{rad}) = 2.908\text{rad}$$

$$GR_2 = 2GR_4 = 2(0.727\text{rad}) = 1.454\text{rad}$$

$$GR_3 = \frac{4}{3}GR_4 = \frac{4}{3}(0.727\text{rad}) = 0.969\text{rad}$$

(c)

$$v_1 = \frac{(0.25\text{m})(261.799\text{rad/s})}{(2.908\text{rad})(3.0)} = 7.502\text{m/s}$$

$$v_2 = \frac{(0.25m)(261.799rad/s)}{(1.454rad)(3.0)} = 15.00m/s$$

$$v_3 = \frac{(0.25m)(261.799rad/s)}{(0.969rad)(3.0)} = 22.51m/s$$

$$t_1 = \frac{(0.25m)^2(261.799rad/s)(600kg)}{(240Nm)(3.0)^2} \cdot \frac{1}{(2.908rad)^2} = 0.537s$$

$$t_2 = \frac{(0.25m)^2(261.799rad/s)(600kg)}{(240Nm)(3.0)^2} \cdot \left(\frac{1}{(1.454rad)^2} - \frac{1}{(2.908rad)(1.454rad)} \right) = 1.075s$$

$$t_3 = \frac{(0.25m)^2(261.799rad/s)(600kg)}{(240Nm)(3.0)^2} \cdot \left(\frac{1}{(0.969rad)^2} - \frac{1}{(1.454rad)(0.969rad)} \right) = 1.615s$$

$$t_4 = \frac{(0.25m)^2(261.799rad/s)(600kg)}{(240Nm)(3.0)^2} \cdot \left(\frac{1}{(0.727rad)^2} - \frac{1}{(0.969rad)(0.727rad)} \right) = 2.148s$$

$$t_t = t_1 + t_2 + t_3 + t_4 = 0.537s + 1.075s + 1.615s + 2.148s = 5.375s$$

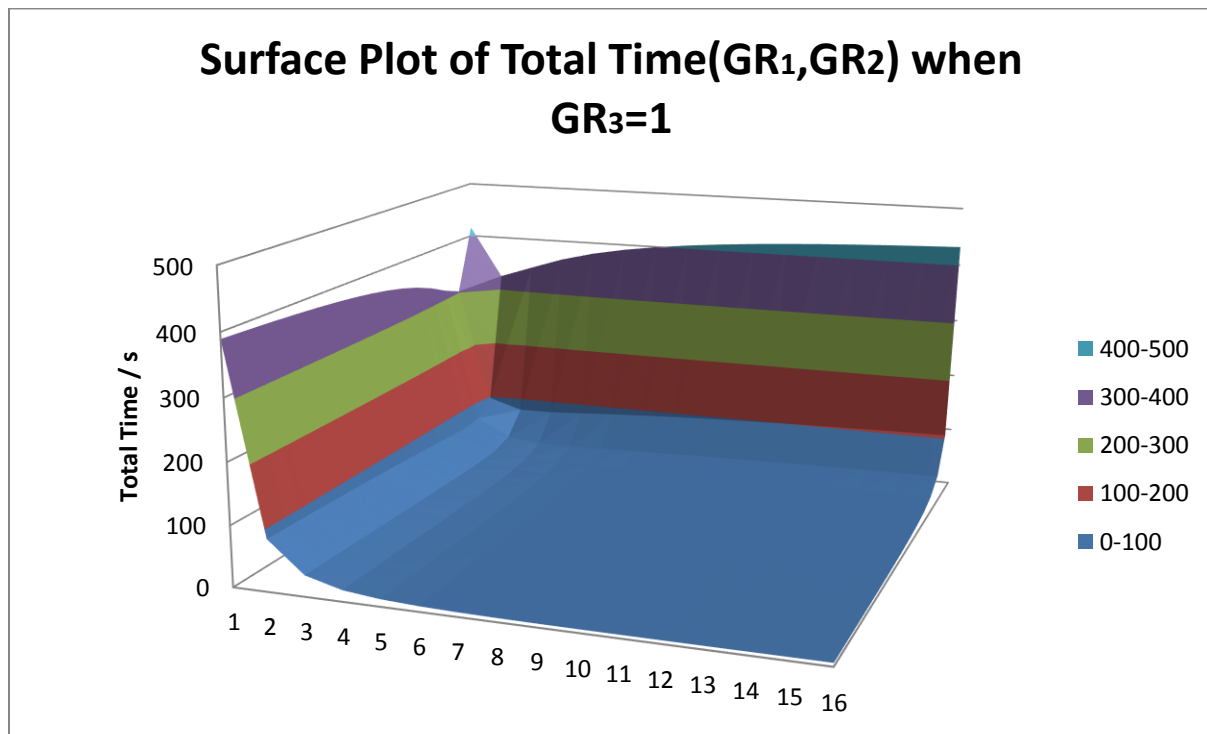


Figure 1: Shown is a surface plot for the total time of gear ratios 1 and 2 when gear ratio 3 is equal to 1.