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Temperature of the Central Processing Unit

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Temperature of the Central Processing Unit

Abstract
Heat is inevitably generated in the semiconductors during operation. Cooling in a computer, and in its main part – the Central Processing Unit (CPU), is crucial, allowing the proper functioning without overheating, malfunctioning, and damage. In order to estimate the temperature as a function of time, it is important to solve the differential equations describing the heat flow and to understand how it depends on the physical properties of the system. This project aims to answer these questions by considering a simplified model of the CPU + heat sink. A similarity with the electrical circuit and certain methods from electrical circuit analysis are discussed.

Keywords
central processing unit, electrical circuit analysis, thermal model

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Problem Statement

A simple model is presented to estimate the temperature of the Central Processing Unit (CPU) based on the power consumption and time. The problem is analyzed and solved using two concepts: heat capacity and heat current or flow. For simplicity the analyzed model consists of the CPU and the heat sink only. For a given heat pulse, two situations are considered: when heat is added to the system (power on), and when the system is cooling (power off).

Motivation

As mentioned thermal management of the CPU is an important aspect in the computer design. In the computer engineering field, keeping the temperature within operational limits is needed to maintain performance and preservation of the CPU itself. Therefore it can be extremely useful to have a model that can estimate the behavior of these systems. Heat transfer is a complicated process; here we attempt to provide a simple and phenomenological description of a CPU thermal behavior. As the number of components with different materials and geometries in the CPU structure make the thermal analysis complicated, we use an analogy with electrical circuits (i.e. RC-circuits) which satisfy similar differential equations. The analysis of more complex models of thermal management in a CPU could be considerably simplified if equivalent electrical circuits are designed and correlations between the thermal and electrical quantities are found.

Mathematical Description and Solution Approach

We obtain a relatively simple model using the four important assumptions.

1. Heat flow is unidirectional (the heat flows along one direction, in a straight line from the hot part to the cold one).
2. Temperature of the environment – air, is constant.
3. Complex geometry of the heat sink is neglected and we assume that it only contributes to an increase of the area available for heat dissipation.
4. Heat is added in pulses of duration t_p.

It would be easy to find the temperature based on the amount of energy dissipated as heat if CPU is completely isolated. The relationship between the obtained energy Q and the temperature of the body is

$$Q = mc\Delta T \iff \Delta T = \frac{Q}{mc} \tag{1}$$

where c is the specific heat of the body and m is the mass for the CPU.

However the CPU is also constantly cooled by the heat sink, which takes energy away and transfers it to the environment – air, and this rate is not constant and depends on the difference between the current temperature of the CPU and the environment.

Heat flow per unit time through a body is proportional to the difference of temperature at the ends, thermal conductivity – material specific constant k (Appendix, Table 3), cross-sectional area A, and it is inversely proportional to the length of the considered body L:

$$H = kA\frac{T_H - T_C}{L} \tag{2}$$

With non-constant temperature boundaries, the above equations become:

$$Q(t) = mc[T_H(t) - T_C(t)] \tag{3}$$

$$H(t) = kA\frac{T_H(t) - T_C(t)}{L} \tag{4}$$

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If the CPU dissipates P heat, which can be considered to be equal to the power consumption, then a part of it is being stored as a rise of temperature and another part is being transferred:

$$P = \frac{dQ}{dt} = mc\frac{d}{dt}[T_H(t) - T_C(t)] + kA\frac{[T_H(t) - T_C(t)]}{L} \quad (5)$$

We can define $[T_H(t) - T_C(t)] = T(t)$ as the temperature difference, or (with a constant $T_C = T_{environment}$) as the temperature above the temperature of the environment –air.

$$mc\frac{d}{dt}[T(t)] + \frac{kA}{L}[T(t)] - P = 0 \quad (6)$$

The above equation only reflects the conservation of energy, part of the thermal energy generated during normal operation (P) is absorbed by the CPU (first term in the equation) and the other part is transferred to the environment (second term).

I. Heating

During normal operation (heat pulse on), P has a non-zero value. Equation (6) is a differential equation, which has the following solution (see Appendix).

$$T(t) = Be^{-\frac{kA}{mcL}t} + \frac{PL}{kA}\left[e^{-\frac{kA}{mcL}t} - 1\right] \quad (7)$$

$T(t)$ is a function of temperature difference which is equal to 0 if t = 0:

$$B = T(0) = 0, \;\; \frac{L}{kA} = \theta, \;\; mc = C_t \Rightarrow \quad (8)$$

$$T(t) = \frac{PL}{kA}\left(1 - e^{-\frac{t}{\theta C_t}}\right) = \frac{P}{\theta}\left(1 - e^{-\frac{t}{\theta C_t}}\right) \quad (9)$$

$$Q(t) = P * t_p(1 - e^{-\frac{t}{\theta C_t}}) \quad (10)$$

Here θ and C_t are heat resistance and heat capacitance respectively, t_p is time when the heat pulse is active (regular CPU operation).

II. Cooling

When the system is cools down, there is no heat inflow, $P = 0$ (note: when the system is being heated, it does not mean that there is no cooling):

$$P = \frac{dQ}{dt} = 0 = \frac{kA}{L}[T(t)] + mc\frac{d}{dt}[T(t)] \tag{11}$$

This differential equation is simpler and can be solved as follows:

$$mc\frac{d[T(t)]}{dt} = -\frac{kA}{L}[T(t)] \Rightarrow \int_{T_0}^{T} \frac{d[T(t)]}{T(t)} = -\int_{0}^{t} \frac{kA}{mcL}dt \tag{12}$$

$$ln\frac{T}{T_0} = -\frac{kA}{mcL}t \tag{13}$$

Exponentiation of both sides results in:

$$T(t) = T_0\left(e^{-\frac{t}{\theta C}}\right), \quad Q = C_t\Delta T \Rightarrow \tag{14}$$

$$Q(t) = C_tT_0\left(e^{-\frac{t}{\theta C_t}}\right) \tag{15}$$

DISCUSSION

Having obtained the equations, we can look at the general outline of the temperature change:

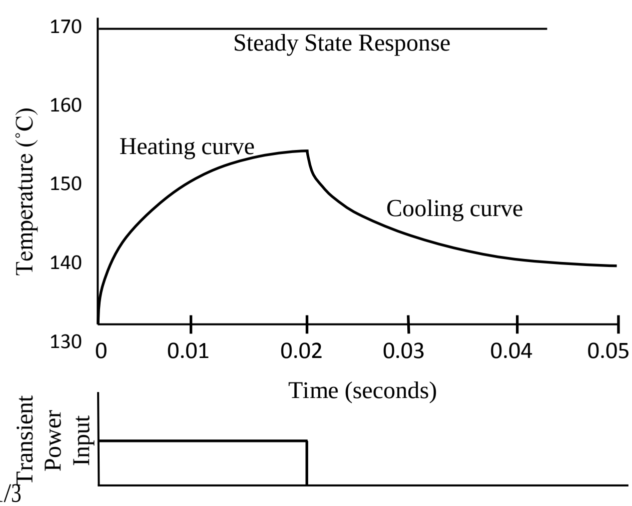

Figure 1: 20-ms Heat Pulse (Advanced Micro Devices)

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For consecutive pulses the pattern would look as follows:

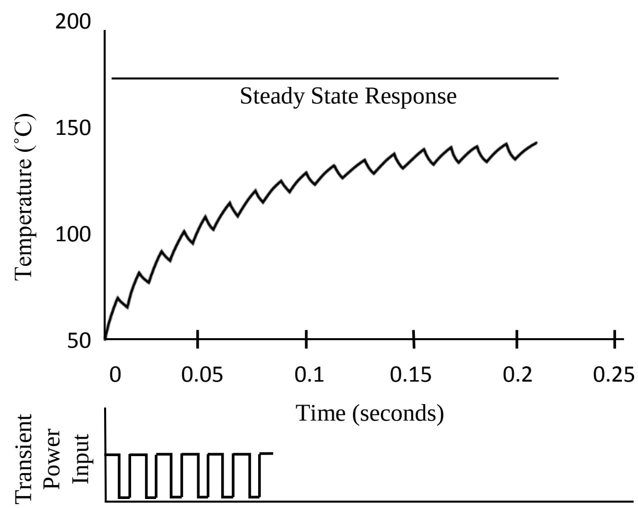

Figure 2: 9-ms Pulse with a 12-ms period (Advanced Micro Devices)

Actually the above equations and graphs are similar to those ones which define charging and discharging of the capacitor in an electrical circuit (see figure below):

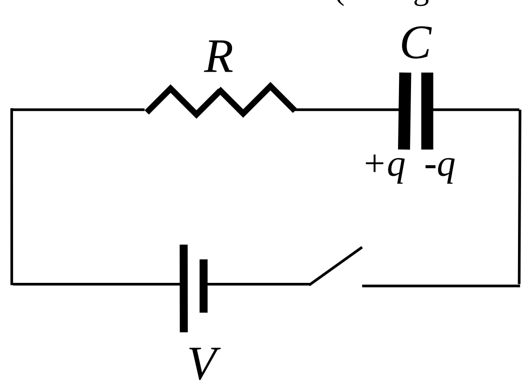

Figure 3: Electrical circuit with a resistor and a capacitor

Charging of the capacitor follows equation (16)

$$q(t) = C_e \varepsilon \left(1 - e^{-\frac{t}{RC_e}}\right) \Rightarrow i(t) = \frac{dq(t)}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{RC_e}} \quad (16)$$

and discharging follows equation (17) (Young):

$$q(t) = Q_0(e^{-\frac{t}{RC_e}}) \Rightarrow i(t) = \frac{dq(t)}{dt} = -\frac{Q_0}{RC_e} e^{-\frac{t}{RC_{cap}}} \quad (17)$$

A clear analogy can be found between thermal characteristics and electrical properties of a circuit (CPU Thermal Management).

Electrical	Thermal
$V = \Delta\varphi$	ΔT
R	θ
C_e	C_t
$i = dq / dt$	$P = dQ / dt$

Table 1. Electric and Thermal analogous characteristics.

Therefore, a conversion to an electrical circuit can be done to solve the complicated cases (Advanced Micro Devices). In a simple case the CPU can be thought to be a capacitor, thermal energy storage –C_t, and a resistor, the heat sink, connected in series. The case of several elements made of different materials can be studied as well. For example, if they are located layer on layer, their total thermal resistance $\theta_T = \theta_1 + \theta_2 + \theta_3 + \ ...$ because they can be represented as resistors in a circuit connected in series.

Conclusion and Recommendations

The project shows only one of the possible approaches to model such a complicated process as heat transfer taking into account the consecutive heating and cooling phases. The corresponding equations based on concepts of heat capacity and current are derived. This model gives a possibility to estimate how certain design choices such as dimensions and materials may impact the cooling and hence temperature of the CPU. Though this model might not return the

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exact values for the temperature, it still can be used for qualitative understanding of the CPU thermal behavior. The similarity with the electrical systems allows solving such problems through a conversion to an electrical circuit. There are various directions for development of the presented approach. One example would be to study a setup with more elements.

NOMENCLATURE

Symbol	Description	Value
t	time	seconds [s]
Q	Heat	Joules [J]
T	Temperature	Degree Celsius [°C]
T_0	Initial temperature	Degree Celsius [°C]
θ	Thermal resistance	$[\frac{°C}{W}]$
C_t	Thermal capacitance	$[\frac{J}{°C}]$
c	Specific heat	$[\frac{J}{kg \cdot °C}]$
H	Heat flow	Watts [W]
P	Heat dissipation – inflow to the system	Watts [W]
m	mass	kilograms [kg]
q	Electric charge	Coulombs [C]
V	voltage	Volts [V]
i	Electric current	Amperes [A]
R	Electrical resistance	Ohms [Ω]
C_e	Capacitance of a capacitor (electrical)	Farads [F]
Q_0	Initial charge	Coulombs [C]
L	Length	Meters [m]
A	Cross sectional area	Square meters[m^2]

Table 2. Used nomenclature.

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APPENDIX

Silicon	
K = 1.05 W/cm ˚C	@ T = 100˚C
K = 0.976 W/cm ˚C	@ T = 120˚C
K = 1.29 W/cm ˚C	@ T = 25˚C
Kovar	
K = 0.21 W/cm ˚C	@ T = 20-100˚C
K = 0.134 W/cm ˚C	@ T = 25˚C
Gold-Silicon Eutectic	
K = 2.96 W/cm ˚C	@ T = 25˚C
K = 2.16 W/cm ˚C	@ T = 25˚C
Beryllia	
K = 1.54 W/cm ˚C	@ T = 100˚C
K = 2.05 W/cm ˚C	@ T = 20˚C
K = 2.54W/cm ˚C	@ T = 25˚C
Conductive Silver Filled Epoxy	
K = 0.016 W/cm ˚C	@ T = 120˚C

Alumina (90-92%)	
K = 0.06 W/cm ˚C	@ T = 25˚C
K = 0.07 W/cm ˚C	@ T = 100˚C
Solder	
K = 0.492 W/cm ˚C	@ T = 25˚C
Sapphire	
K = 0.033 W/cm ˚C	@ T = 38˚C
K = 0.026 W/cm ˚C	@ T = 93˚C
K = 0.023 W/cm ˚C	@ T = 149˚C

Table 3. *K values for several materials (CPU Thermal Management)*

Solution for a non-homogeneous differential equation $a\frac{df(x)}{dx} + bf(x) + c = 0$ is

$$f(x) = f(0)e^{-\frac{b}{a}x} + \frac{c}{b}\left[e^{-\frac{b}{a}x} - 1\right].$$

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