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Volume of an Industrial Autoclave

Nicholas Madaffari
University of South Florida

Advisors:
Thomas Bieske, Mathematics and Statistics
Scott Campbell, Chemical & Biomedical Engineering

Problem Suggested By: Salvatore Madaffarri

Field of Study for Problem Suggester: Sales Representative, Stericycle Incorporated

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Volume of an Industrial Autoclave

Abstract

We were able to determine the volume of an industrial autoclave sterilization tank using a technique learned in calculus. By measuring the dimensions of the tank and roughly estimating the equation of curvature at the ends of the tank, we were able to revolve half of the end of the tank around the x axis to get its fluid volume. Adding the two volumes of the ends and the volume of the cylindrical portion on the tank yielded the total volume.

Keywords

Disk Method, Volume Measurement, Cylindrical Tank

Erratum

This article was previously called Article 23.

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PROBLEM STATEMENT

An autoclave is a device used to sterilize medical waste such as used syringes by applying intense pressure and hot steam. We seek to find the volume of an industrial autoclave pictured below (Fig 1).

MOTIVATION

Finding volumes of irregular solids is a natural engineering problem. Further, engineers are frequently tasked to design containers under the constraints of size, cost, and availability of materials. The autoclave tank we are discussing was most likely designed with these and other factors in mind.

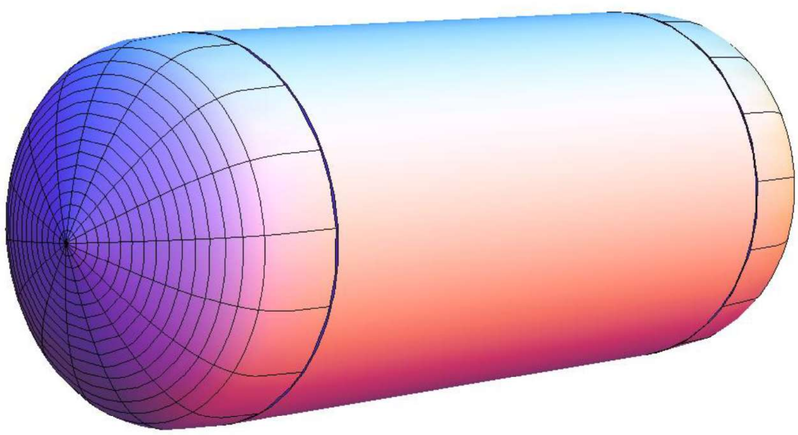

Figure 1: Shape of the industrial autoclave.

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MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Based on the overall shape of the autoclave (Fig. 1), it is easiest to separate the tank into three pieces and calculate the volume of middle cylinder and the ends separately. By adding the volume of these three components, we obtained the total volume of the tank.

We begin by calculating the volume of one of the ends. Note that the shape of the end has a rotational symmetry. Thus we may calculate the volume by estimating the shape of the cross section and rotating it about a central axis. By hand measurements we obtained the approximate shape plotted in Figure 2.

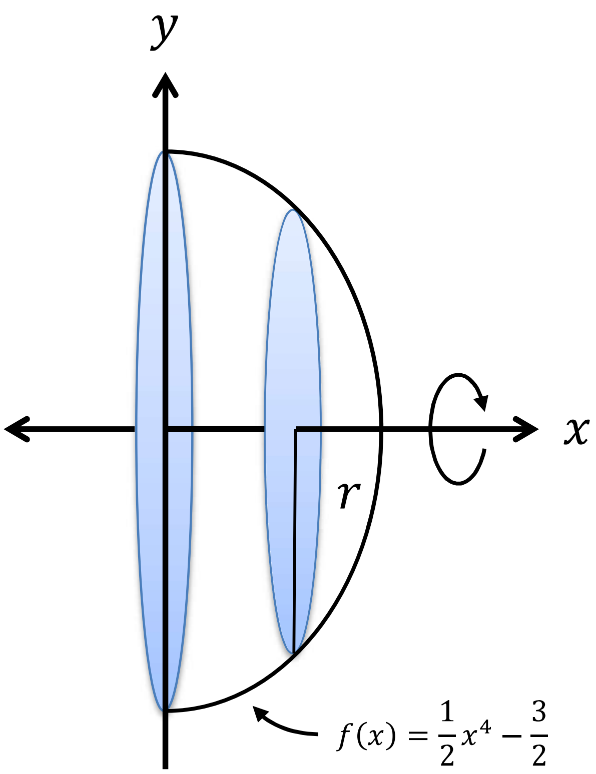

Figure 2: Diagram of the rotated end around the x-axis

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Using the disk method, the volume obtained by rotating $f(x)$ about the x-axis is given by the equation

$$V = \pi \int_a^b r(x)^2 \, dx \tag{1}$$

where $r(x)$ is the distance from $f(x)$ to the x-axis over the interval (a, b).

Our estimation for the curvature of the tank is given by $f(x) = \frac{1}{2}x^4 - \frac{3}{2}$. However, we consider only $x \geq 0$ such that $f(x) \leq 0$, so

$$r(x) = |0 - f(x)| = \frac{3}{2} - \frac{1}{2}x^4. \tag{2}$$

Also note that x ranges from $a = 0$ to the x-intercept of $f(x)$. It is straightforward to show that the x-intercept is given by $b = \sqrt[4]{3}$. There we can simply evaluate the volume for one end as

$$\begin{aligned} V_{end} &= \pi \int_0^{\sqrt[4]{3}} \left(-\frac{1}{2}x^4 + \frac{3}{2}\right)^2 dx \\ &= \pi \left(\frac{1}{36}x^9 - \frac{3}{10}x^5 + \frac{9}{4}x\bigg|_0^{\sqrt[4]{3}}\right) \\ &= \frac{8\sqrt[4]{3}}{5}\pi \text{ ft}^3. \end{aligned} \tag{3}$$

Since the middle portion is cylindrical, its volume is given by

$$V_{middle} = \pi r^2 l = \pi \left(\frac{3}{2}\right)^2 (5) = \frac{45}{4}\pi \text{ ft}^3. \tag{4}$$

Thus, the total volume of the tank is found by adding the volume of the middle portion to twice the volume of the end, i.e.,

$$V_{total} = V_{middle} + 2\,V_{end} = \frac{255 + 64\sqrt[4]{3}}{20}\pi \approx 48.57 \text{ ft}^3 \tag{5}$$

DISCUSSION

The overall result of these calculations tell us that the industrial autoclave tank with the dimensions of 5 ft in length, a radius of $\frac{3}{2}$ ft, and having the equation of $f(x) = \frac{1}{2}x^4 - \frac{3}{2}$ for the estimated curvature of the ends has a total fluid volume of 48.57 ft^3. More accurate estimations of the curvature of the autoclave ends will increase the accuracy of the volume, but we did not have the means or equipment for proper measurement.

CONCLUSION AND RECOMMENDATIONS

Using calculus, we were able to find that the volume of the industrial autoclave. By using the disk method to calculate the volume of the ends and adding the volumes of each separate component, we were able to find that the total volume of the autoclave is roughly 48.57 ft^3.

The volume we obtained is only a rough estimate of the actual volume. The curvature of the ends of the tank had to be estimated by hand. Though the blueprints of the machine could be used to obtain an exact measurement, we did not have access to this information. Someone wishing to emulate this problem in the future may take this into consideration.

NOMENCLATURE

Symbol	Description	Units
V	Volume	ft^3
f	Curvature of Ends	ft
r	Radius of cross section	ft
l	Length of the center section	ft

REFERENCES

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