Motion Analysis of Fluid Flow in a Spinning Disk Reactor

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Motion Analysis of Fluid Flow in a Spinning Disk Reactor

by

Valentina N. Korzhova

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Computer Science and Engineering
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Keywords: image processing, fluid-flow tracking, wave detection, wave velocity, wave inclination, mathematical modeling

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DEDICATION

This thesis is dedicated, especially, to

my parents

and, also, to

my husband, son, and daughter for

their love, support, guidance, and understanding.
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MOTION ANALYSIS OF FLUID FLOW IN A SPINNING DISK REACTOR

Valentina N. Korzhova

ABSTRACT

The flow of a liquid film over a rapidly rotating horizontal disk has numerous industrial applications including pharmaceuticals, chemical engineering, bioengineering, etc. The analysis and control of complex fluid flows over a rapidly rotating horizontal disk is an important issue in the experimental fluid mechanics. The spinning disk reactor exploits the benefits of centrifugal force, which produces thin highly sheared films due to radial acceleration. The hydrodynamics of the film results in excellent fluid mixing and high heat or mass transfer rates.

This work focuses on developing a novel approach for fluid flow tracking and analysis. Specifically, the developed algorithm is able to detect the moving waves and compute controlling film flow parameters for the fluid flowing over a rotating disk. The input to this algorithm is an easily acquired non-invasive video data. It is shown that under single light illumination it is possible to track specular portion of the reflected light on the moving wave. Hence, the fluid wave motion can be tracked and fluid flow parameters can be computed. The fluid flow parameters include wave velocities, wave inclination angles, and distances between consecutive waves. Once the parameters are computed, their accuracy is analyzed and compared with the solutions of the mathematical fluid dynamics models based on the Navier-Stokes equations for the case of a thin film. The fluid model predicts wave characteristics based on directly measured controlling parameters, such as disk rotation speed and fluid flow rate. It is shown that
the calculated parameter values approximately coincide with the predicted ones. The average computed parameters were within 5 – 10\% of the predicted values.

In addition, given recovered fluid characteristics and fluid flow controlling parameters, full 3D wave description is obtained. That includes 3D wave location, speed, and distance between waves, as well as approximate wave thickness.

Next, the developed approach is generalized to model-based recovery of fluid flow controlling parameters: the speed of the spinning disk and the initial fluid-flow rate. The search in space for model parameters is performed as to minimize the error between the flow characteristics predicted by the fluid dynamics model (e.g. distance between waves, wave inclination angles) and parameters recovered from video data. Results demonstrate that the speed of a disk and the flow rate are recovered with high accuracy. When compared to the ground truth available from direct observation, we noted that the controlling parameters were estimated with less than 10\% error.
1.1 Overview of the Relative Works

The analysis of fluid flow is an important issue in the experimental fluid mechanics, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines, and understanding nebulae in interstellar space. Understanding atmospheric dynamics, oceanographic streams, and cloud motion is of great importance for weather, climate prediction, etc. Rotating flows have been used to study a variety of physical processes including geostrophic turbulence, baroclinic instability, convection, and chaos.

The flow of a liquid film over a rapidly rotating horizontal disk has numerous industrial applications (pharmaceutical, chemical engineering, bioengineering, etc.), ranging from spin-coating of silicon wafers to the atomization of liquids. One of the most important applications in present time is the transfer gases, for instance of carbon dioxide, into liquids. Under certain conditions, this flow is accompanied by the formation of non-linear waves leading to remarkably large increases in the rates of heat and mass transfer. Therefore, the analysis and control of complex fluid flows over a rapidly rotating horizontal disk is a major scientific issue. Here, we propose a novel video-based technique for extraction of fluid flow characteristics and estimation of their accuracy.

The study [2, 39, 63] has investigated the heat and mass transfer characteristics of the film considered as a steady state, giving rise to local heat and mass transfer coefficients as a function of radius, flow rate, and rotation speed. It has been shown that the presence of surface waves leads to a significant enhancement in transfer processes
at the film surface, and as such are desirable features of the flow. Dependence of
the heat transfer coefficient on the radius of the disk and the speed of the disk is
demonstrated in Figure 1.1. To intensify these processes, control mechanism needs

to adjust hydrodynamic parameters of flow with relevant transfer of processes. So,
the controlled film flow is requested. Since various regimes of film flow in a spinning
disk reactor strongly influence these processes, it is important to control the formation
of the regimes to increase process productivity [33]. Video observation can provide a
cost-effective way to observe the film flow and to determine the actual flow parameters.

Recent review [33] summarizes experimental and theoretical studies of film flow over
a rotating disk. Most experimental investigations of flow over a spinning disk attempt
to measure the local maximum or the mean of a film thickness in order to obtain
information about the surface waves [2]. Various mechanical [6], electrical [3, 38], and
optical [30, 63] techniques were employed. The most promising was the optical technique
that was used to collect information about the waves observed. In the considered
experiments, a camera was placed below the disk and connected to a computer that
provided video imaging hardware and software. To measure the film thickness over a
disk domain, calibration of mechanical or optical tools and estimation of absorption

![Variation in heat transfer coefficient with radius](image_url)
coefficients were performed. All three techniques (mechanical, electrical, and optical) gave insufficient information to classify wave regimes and select most efficient regime for specific technological applications.

Early experimental investigations [6] provided some qualitative and quantitative understanding of the effect of flow rate and rotational speed on the flow characteristics for a given set of physical parameters. Experimental observations [2, 3, 30, 55, 63] have demonstrated that at a small flow-rate, a smooth film is formed, and at a moderately higher flow-rate, circumferential waves moving from the disk center to the disk periphery are formed. Further increase in flow rate leads to the appearance of spiral waves unwinding in the direction of the rotation [6]. It is shown in [63] that the initially uniform film breaks down into well-defined spiral waves, which then break down further into a confused assembly of wavelets. Circumferential and spiral waves were found to decay at large disk radii. Comparison of those observed waves and their associated parameters with the waves observed in falling films shows their similarity. The waves in the falling films were first studied experimentally in the seminal work [22]. The monograph [4] had collected most important results concerning falling film. In [31] the authors present measurements of the instabilities of thin liquid film flowing down an incline. The results are in good agreement with linear stability prediction. The growth rates and wave velocities have been measured as a function of wavenumber. They also estimated the film thickness in real time with high accuracy using fluorescence imaging.

Theoretical explanation of experimental results has received increasing attention in recently published research. There are three main directions for theoretical investigation of a film flow over a rotating disk: calculation of waveless flow, analysis of its linear stability, and non-linear simulations of finite-amplitude waves. The waveless solutions, asymptotic and numerical, were investigated in [13, 40, 52]. These studies have been successful in determining self-similar and asymptotic solutions in the limit of large Eckman numbers [40, 45] as well as numerical solutions for finite Eckman numbers [13, 52].
The linear stability analysis was examined and performed using asymptotic methods [6] and the full Navier Stokes equation for finite Eckman numbers [50, 51].

In recent papers [32, 47, 48, 49] an evolution system of equations was derived and analyzed to model axisymmetric finite-amplitude waves; this model may be extended for non-axisymmetric flows to explain the experimental results. Nevertheless, there are problems that could be treated by parallel application of experimental and theoretical approaches: wave regime sensitivity to flow conditions and three-dimensional structures observed in experiments.

In the last decade, there has been significant work in image processing related to the motion analysis of non-rigid objects [8, 11, 10, 21, 36, 69]. Most of the works have concentrated on articulated and elastic motion [1]. In [21], the authors demonstrate that the Finite Element Modeling allows to realize the motion analysis of biological objects. The simulation of deformable objects is essential for many applications. Computer Aided Design uses deformable models to simulate the deformation of industrial materials and tissues. In image analysis, deformable models were used for fitting curved surfaces, boundary smoothing, registration, and image segmentation. Later, deformable models were deployed in character animation and computer graphics for the realistic simulation of skin, facial muscles, clothing, and human or animal characters. The modeling of deformable soft tissue is of great interest for a wide range of medical imaging applications. A comprehensive review of deformable models for medical image analysis and clothing modeling applications can be found in [34, 46] and [19], respectively. More recently, such modeling techniques have been used for tasks such as age estimation [29] and person identification [65].

Efforts have been made to assist face animation and recognition using a highly accurate model that takes into account anatomical details of a face such as bones, musculature, and skin tissues [66]. This is based on the premise that the nuances recognizable by humans can be synthesized (and fully explained) only by an elaborate
biomechanical model. The early studies in this direction developed models with a hierarchically biomechanical structure that were capable of simulating linear and sphincter facial movements [37]. For example, Zhang et al. [36, 38] studied a model that incorporates a more detailed 3-layer skin module to characterize the behavior and interaction among the epidermis, dermis, hypodermis, and muscular units. Analysis of fluid-like motion was also attempted [11, 36, 69].

Recently, work was begun in an effort to combine precise experimental setup, theoretical derivation, and basic image analysis techniques [56, 55, 57, 63]. The equations constitute a set of physical constraints that are different [21] from those commonly used in the study of solid motion (e.g., the rigidity constraint). For the fast fluid-like motion in the air, having wavy or turbulent character, detecting interface between fluid and air is important. A special so-called particle image velocimetry (PIV) technique was developed [44] to measure the kinematics of turbulent fluid flow in controlled laboratory experiments. Given a typical ensemble of PIV images, the aim is to calculate the instantaneous interface. This, however, requires specialized imaging devices.

A novel approach dedicated to measuring velocity in fluid experimental flows through image sequences was developed in [10]. The proposed technique is the extension of optical-flow schemes that includes a specific enhancement for fluid mechanics applications. In paper [8], the authors defined a complete framework for processing large image sequences for a global monitoring of short range oceanographic and atmospheric processes. They used a novel regularization technique in optical flow computation that preserves flow discontinuities. The study [16] confirms that the optical flow-based dense vector fields show motion basically consistent with traditional atmospheric motion vector calculation methods. The motion information demonstrates that some areas are not covered by ‘traditional’ vectors. Differences are observed in areas of strong winds (such as jet-streams) where the optical-flow method tends to underestimate the strength of winds, including instantaneous velocity on the surface of fluid and air contact. Using
the novel regularization techniques, the strength of winds is estimated efficiently and with a reasonable degree of accuracy. Algorithms used are typically based on filter-like motion. In the paper [5] a novel approach for estimating fluid motion fields is presented. First, a local flow probability distribution function at each pixel was estimated using the STAR model and the data from a spatio-temporal neighborhood. Then, the set of distribution functions was fed into a global optimization framework. Experiments with real fluid sequences show that this method can successfully estimate their motion fields.

Analysis of fluid-like motion was also attempted [11, 36, 69]. In [69], authors proposed a new method for recovering nonrigid motion and structure of clouds under affine constraints using time-varying cloud images obtained from meteorological satellites. This problem is challenging not only due to the correspondence problem but also due to the lack of depth cues in the 2D cloud images (scaled orthographic projection). In this paper, affine motion was chosen as a suitable model for small local cloud motion. In [11], authors addressed the problem of estimating and analyzing the motion of fluids in image sequences. They investigated a dedicated minimization-based motion estimator and demonstrated the performance of the resulting fluid flow estimator on meteorological satellite images. The paper [36] presents a physics-based method to compute the optical flow of a fluid. Authors proposed a method in which physical equations describing the object are used as supplementary constraints. The physical model employed is a combination of the continuity equation and the Navier-Stokes’ equations. The authors demonstrated the effectiveness of the proposed method by presenting experimental results of simulated and real Karman flows.

In the paper [41], a fluid flow estimation method for ocean/river waves, clouds, and smoke based on the physical properties of waves such as the velocity-frequency relationship and a wave statistical property was developed assuming that many fluid-like motion changes are due to wave phenomena that lead to a brightness change. The author shows that the results of the experiments with synthetic and real images are
improved compared to the works [11, 35, 62]. In the paper [37], authors investigate estimation of velocity, water elevation and contaminant concentration in a river current using the Kalman filter finite element method (KE-FEM). Close agreement between the observed and the computed results was obtained.

The described work in this dissertation focuses on developing imaging techniques and image analysis algorithms to detect the traveling waves, determine the wave regimes, and compute physical and controlling film flow parameters. The input to these algorithms is an easily acquired non-invasive video data. The production of thin films over a spinning disk and the formation of waves in realistic conditions are of interest here. In this work, a two part algorithm is proposed. The first part includes image analysis, detecting, tracking, and reconstructing of measuring wave shape and wave propagating speed. Based on the image intensities and geometrical constraints of disk and surface waves, the algorithm is proposed. The fluid flow parameters and characteristics are calculated and compared with the solutions of the relevant mathematical models. Calculations of radial velocity and inclination angle are obtained with the asymptotically optimal steps. New results for step selection are derived. Initial version and further version of algorithms and analysis of spiral waves in a spinning disk reactor are presented in [25, 26, 27, 28].

The second part of the algorithm concentrates on model-based recovery of fluid flow controlling parameters. Here the search in the space of model parameters is performed to minimize the difference between the predicted flow characteristics (e.g. distances between waves, wave inclination angles) and the ones measured from video data.

The overall purpose of this work is to develop a system of visual scanning, recording, and tracking of the film flow over a spinning disk with the intention of detecting regimes of the fluid flow with regard to different conditions using a single camera system, of calculating fluid flow parameters and characteristics, and of comparing them with the solutions of the relevant mathematical models.
1.2 Contributions of This Dissertation

This dissertation has the following contributions:

1. A novel approach and analysis were developed for fluid wave detecting and tracking over a spinning disk. The novelty consists of wave studying in real conditions, with regard to certain disk surface frictions and air resistances.

2. It was shown that under single light illumination it is possible to track specular portion of the reflected light on the moving wave.

3. A novel approach for computing fluid flow parameters (wave velocity components, wave inclinations, thickness of film, and distances between consecutive waves) from observed wave patterns was developed. The novelty consists of developing new model and video based algorithms and their accuracy analysis.

4. For practical realization, the optimal methods (asymptotically optimal and quasi-optimal) are used for estimating the velocities and the inclination angles.

5. An arbitrary step along the azimuthal angle is used for the experimental estimation of wave inclinations.

6. The developed approach is generalized to a model-based recovery of fluid flow controlling parameters: the speed of spinning disk and the fluid-flow rate. The search in space for model parameters is performed to minimize the error between the predicted flow characteristics predicted by the fluid dynamics model (e.g. distance between waves, wave inclination angles) and the parameters recovered from video data.

Portions of the work in this dissertation have been presented on the Seminar of Image Processing, June 2005 and Graduate Student Research Competition, November 2005 at USF, Department of Computer Science and Engineering. Also, parts are published
in the ICPR-2006, Ukrobraz 06 Conferences, 59th Annual Meeting of the Division of fluid Dynamics, and Optical 3-D Measurement Techniques Conference 2009 [25, 26, 27, 28]. The detailed paper is submitted in the journal ”International Journal Pattern Recognition and Artificial Intelligence” [24].

1.3 Layout of the Dissertation

The organization of this paper is depicted in Figure 1.2.

Chapter 2.1 offers description of the general theory of fluid flow based on the equations of Navier-Stokes, including the important particular case of the thin film flow, which are later compared with the respective experimental data and uses for the proper estimations of wave parameters such as wave velocity components, wave inclinations, and distances between consecutive waves. Also, this chapter presents the derivations of the camera calibration accuracy, which is used in Section 5 under estimation of errors of experimental velocities of waves and inclination angles.

In Chapter 3, a video based algorithm for detecting and tracking of waves in a spinning disk reactor is presented. The input to this algorithm is an easily acquired non-invasive video data. Details of the video input information are analyzed. It is shown that under a single light illumination assumption it is possible to track wave motion by observing specular portion of the reflected light on the moving wave. Hence, the fluid wave motion can be tracked and fluid flow parameters can be computed. The fluid flow parameters include wave velocities, wave inclination angles, and distances between consecutive waves. Determination of velocity components and inclination angles is an ill-posed problem. So, the vision-based asymptotically optimal by accuracy method was applied.

Chapter 4 is devoted to data acquisition for the experimental disk reactor used in this study.
Chapter 5 presents experimental results for estimating parameters of wave spirals such as their velocity components, inclination angles, and distances between consecutive waves and their estimated accuracy. Those results are compared with the related results of theoretical models introduced in Section 2.1. It was shown that no statistically significant difference exists between theoretical and calculated values of wave parameters at the significant level $\alpha = 0.05$.

Variations of the rotational speed and the flow rate lead to the modification of the shape, amplitude, and velocity of the observed waves. So, the computerized system for estimation of speed of a spinning disk and flow rate using the video data and the relevant system of evaluation equations are developed and described in Chapter 6. Conclusion chapter follows.

It should be noted that the presented work focuses on computerized processing of the input video data in order to extract the fluid flow characteristics and their analysis, including estimation of errors of numerical results. Qualitative and quantitative comparison shows good coincidence (within 5-10%) of experimental and theoretical results. Also, specifics of video input information are analyzed. It is shown that under a single light illumination assumption it is possible to track wave motion by observing specular portion of the reflected light on the moving wave.
Figure 1.2 Dissertation organization flowchart.
2.1 Theory of Fluid Flow

2.1.1 General Case

Considering an inviscid fluid, described by the following variables: the fluid density $\rho(X, t)$, the velocity vector field $\mathbf{u}(X, t)$, and the pressure $p(X, t)$; $X \in \mathbb{R}^3$ is the spatial coordinate.

In a Cartesian system coordinate, an incompressible viscous fluid can be described with the Navier-Stokes equations that are four coupled nonlinear partial differential equations for four unknown functions (the three components of $\mathbf{u}$ and the pressure $p$):

$$\begin{align*}
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_1}{\partial y} + u_3 \frac{\partial u_1}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial x} &= \nu \left( \frac{\partial u_1}{\partial x}, \frac{\partial u_1}{\partial y}, \frac{\partial u_1}{\partial z} \right), \\
\frac{\partial u_2}{\partial t} + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_2}{\partial y} + u_3 \frac{\partial u_2}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial y} &= \nu \left( \frac{\partial u_2}{\partial y}, \frac{\partial u_2}{\partial y}, \frac{\partial u_2}{\partial z} \right), \\
\frac{\partial u_3}{\partial t} + u_1 \frac{\partial u_3}{\partial x} + u_2 \frac{\partial u_3}{\partial y} + u_3 \frac{\partial u_3}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial z} &= \nu \left( \frac{\partial u_3}{\partial z}, \frac{\partial u_3}{\partial y}, \frac{\partial u_3}{\partial z} \right), \\
\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} &= 0,
\end{align*}$$

where $\nu$ is the kinematics’ viscosity. Also, there is the matter of boundary conditions [12].

Using the cylindrical system coordinates $(r, \theta, z)$, the steady Navier-Stokes equation for a radial velocity $u$ is:

$$u \frac{\partial u}{\partial r} + \frac{u}{r} \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u^2}{r} \right), \quad (2.1)$$
where \( u, v, w \) are the components of velocity in the \( r, \theta, z \) direction.

The authors [40] show that for a thin film, \( h/r \ll 1 \), where \( h \) is the film thickness, the radial flow can be obtained from (2.1) to lowest order with respect to \( h/r \) by solving the equations

\[-\Omega^2 r = \nu \frac{\partial^2 u}{\partial z^2},\]

where \( \Omega \) is the angular velocity of the disk.

Later, the last two formulas above are used to estimate the wave amplitude in the case of thin film flow.

### 2.1.2 Fluid Flow over Rotating Disk - The Karman’s Problem

Considering an infinite plane disk, rotating at constant angular velocity \( \Omega \) with an unbounded fluid and taking cylindrical polar coordinates \((r, \theta, z)\), the steady axisymmetric solution for velocity components \((u, v, w)\) and pressure \((p)\) can be found in the form [64]:

\[
\begin{align*}
    u &= \Omega r f(\zeta), \quad v = \Omega r g(\zeta), \quad w = (\nu \Omega)^{1/2} h(\zeta), \\
    p &= \rho \nu \Omega P(\zeta), \quad \zeta = z \left( \frac{\nu}{\Omega} \right)^{1/2} 
\end{align*}
\]

(2.2)

under the following boundary conditions:

\[
    u = 0, \quad v = \Omega r, \quad z = 0; \quad u, v \to 0, \quad z \to \infty. \quad (2.3)
\]

Substituting (2.2) and (2.3) in the Navier-Stokes equations written in the cylindrical polar coordinates, the functions \( f(z), g(z), h(z) \), and \( P(z) \) can be found [64] as solution of a set of ordinary differential equations:
\[ 2F + \frac{dH}{d\zeta} = 0, \quad F^2 - G^2 + H \frac{dF}{d\zeta} = \frac{d^2 F}{d\zeta^2}, \]
\[ 2FG + H \frac{dG}{d\zeta} = \frac{d^2 G}{d\zeta^2}, \quad H \frac{dH}{d\zeta} = -\frac{dQ}{d\zeta} + \frac{dH}{d\zeta}, \] (2.4)

with boundary conditions \( F = H = 0, \ G = 1, \) when \( \zeta = 0, \ F \to 0, \ G \to 0, \) as \( \zeta \to \infty. \)

For small values of \( \zeta, \) a solution of (2.4) can be written in powers of \( \zeta. \) For large values of \( \zeta, \) a solution can be written in the exponential form (see [64]).

### 2.2 Mathematical Modeling

This section describes the mathematical models of the film flow over a disk rotating with angular velocity \( \Omega. \)

#### 2.2.1 Evolution Equations

A model derivation given below follows to [47] with accounting non-axisymmetric terms. The authors [48] consider the flow of a thin, Newtonian, incompressible liquid film of the density \( \rho, \) kinematic viscosity \( \nu, \) surface tension \( \sigma, \) and the flow is described by the velocity components \( u, \ v, \ w, \) and the pressure \( p, \) depending on the the cylindrical coordinates \( r, \ \theta \) and \( z \) and time \( t. \) The liquid film is bounded from above by an essentially inviscid gas; the gas–liquid interface is located at \( z = h \) while the underlying solid disc is situated at \( z = 0. \) The full Navier-Stokes system accompanied by the boundary conditions (no-slip and no–penetration at the disc surface, the kinematic boundary condition, shear and normal stress balances at the film surface) on the disk and the free surface is formulated [47]. Analysis of experiments, carried out in [47], revealed that a relation \( \varepsilon^2/\kappa^2 \ll 1 \) is satisfied in all data available when capillary waves
are observed, where $\kappa$ and $\varepsilon$ are determined by

$$\kappa = \left( \frac{\sigma H_c}{\rho \Omega^2 R_c^4} \right)^{\frac{1}{3}}, \quad \varepsilon = \frac{H_c}{\bar{r}},$$

(2.5)

where $H_c$ and $R_c$ are scales of a thickness and a radius.

After omitting terms of $O(\varepsilon^2/\kappa^2)$ in the problem statement, the pressure may be eliminated and the approximate model follows in the form:

$$\frac{\partial u}{\partial x_\kappa} + \frac{\partial w_\kappa}{\partial z} + \kappa \left( 2u + \frac{\partial v}{\partial \vartheta} \right) = 0,$$

$$\frac{\partial u}{\partial t_\kappa} + u \frac{\partial u}{\partial x_\kappa} + w_\kappa \frac{\partial u}{\partial z} + \kappa \left[ v \frac{\partial u}{\partial \vartheta} + u^2 - (v + E)^2 \right] =$$

$$\frac{1}{45\delta} \left\{ e^{-2\kappa \vartheta} \frac{\partial}{\partial x_\kappa} \left[ e^{-2\kappa \vartheta} \left( \frac{\partial^2 h}{\partial x_\kappa^2} + \kappa^2 \frac{\partial^2 h}{\partial \vartheta^2} \right) + \frac{\partial^2 u}{\partial z^2} \right] \right\},$$

$$\frac{\partial v}{\partial t_\kappa} + u \frac{\partial v}{\partial x_\kappa} + w_\kappa \frac{\partial v}{\partial z} + \kappa \left[ v \frac{\partial v}{\partial \vartheta} + 2u (v + E) \right] =$$

$$\frac{1}{45\delta} \left[ \kappa e^{-4\kappa \vartheta} \left( \frac{\partial^3 h}{\partial x_\kappa^2 \partial \vartheta} + \kappa^2 \frac{\partial^3 h}{\partial \vartheta^3} \right) + \frac{\partial^2 v}{\partial z^2} \right],$$

where $\vartheta = \theta - Et$ is the azimuthal angle related to the spinning disc; and the similarity parameter $\delta$ and the Eckman number $E$

$$\delta = (45 \kappa E^2)^{-1} = \frac{1}{45 \nu^2} \left( \frac{\rho \Omega^8 R_c^4 H_c^{11}}{\sigma} \right)^{\frac{1}{3}},$$

(2.7)

$$E = \nu / (\Omega H_c^2)$$

(2.8)

have been introduced. Here

$$x_\kappa = \frac{x}{\kappa}, \quad t_\kappa = \frac{t}{\kappa}, \quad w_\kappa = \kappa w,$$

(2.9)
These equations are rendered dimensionless via the following scaling:

\[ t \rightarrow \frac{Et}{\Omega}, \quad r \rightarrow R_c e^x, \quad z \rightarrow H_c z, \quad u_r \rightarrow \frac{\Omega ru}{E}, \]

\[ u_\theta \rightarrow \Omega r \left(1 + \frac{v}{E}\right), \quad u_z \rightarrow \frac{\Omega H_c w}{E}, \quad p \rightarrow \rho \Omega^2 r^2 p, \quad h \rightarrow H_c h, \quad (2.10) \]

where \(H_c\) is chosen so that the dimensionless radial flow rate is equal to unity for a given value of \(R_c\) under steady conditions:

\[ H_c = \left(\frac{\nu Q_c}{2\pi \Omega^2 R_c^2}\right)^{\frac{1}{3}}. \quad (2.11) \]

The observed waves have a characteristic length scale, which is much smaller than \(R_c\). In experiments \(\kappa\) is a small coefficient. Thus, the problem (2.6) may be considered as depending on two parameters: the film parameter \(\delta\) that also appears in a falling film problem [4] and the Eckman number \(E\); then \(\kappa = (45\delta E^2)^{-1}\).

Using the parabolic velocity profile, an approximate system of evolution equations for the film thickness \(h\) and two values, \(q^{(u)}\) and \(q^{(v)}\), characterizing flow rates in the radial and azimuthal directions, is derived:

\[
\frac{\partial h}{\partial t} + \frac{\partial q^{(u)}}{\partial x} + 2q^{(u)} + \frac{\partial q^{(v)}}{\partial \theta} = 0, \quad q^{(u)} = \int_0^h u_r \, dz, \quad q^{(v)} = \int_0^h u_\theta \, dz, 
\]

\[
\frac{\partial q^{(u)}}{\partial t} + a_{11} \frac{\partial}{\partial x} \left( \frac{(q^{(u)})^2}{h} \right) + a_{12} \frac{\partial}{\partial \theta} \left( \frac{q^{(u)} q^{(v)}}{h} \right) + a_{13} \frac{(q^{(u)})^2}{h} + a_{14} \frac{(q^{(v)})^2}{h} = 
\]

\[
E^2 \left\{ \kappa e^{-2x}h \frac{\partial}{\partial x} \left[ \kappa^2 e^{-2x} \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial \theta^2} \right) \right] - b_1 \frac{q^{(u)}}{h^2} + h + \frac{2}{E} q^{(v)} \right\}, 
\]

\[
\frac{\partial q^{(v)}}{\partial t} + a_{21} \frac{\partial}{\partial x} \left( \frac{q^{(u)} q^{(v)}}{h} \right) + a_{22} \frac{\partial}{\partial \theta} \left( \frac{(q^{(v)})^2}{h} \right) + a_{23} \frac{q^{(u)} q^{(v)}}{h} = 
\]

\[
E^2 \left[ \kappa^3 h e^{-4x} \left( \frac{\partial^3 h}{\partial x^2 \partial \theta} + \frac{\partial^3 h}{\partial \theta^3} \right) - b_2 \frac{q^{(v)}}{h^2} - \frac{2}{E} q^{(u)} \right]. \quad (2.12) \]
The system (2.12) includes the constant coefficients

\[ a_{11} = \frac{6}{5}, \quad a_{12} = \frac{17}{14}, \quad a_{13} = \frac{18}{5}, \quad a_{14} = -\frac{155}{126}, \quad b_1 = 3, \]
\[ a_{21} = \frac{17}{14}, \quad a_{22} = \frac{155}{126}, \quad a_{23} = \frac{34}{7}, \quad b_2 = \frac{5}{2}. \]

The axisymmetric version of (2.12) was derived in [47].

System (2.12) has a steady axisymmetric solution of spiral-type slowly varying along the radius; stability of this solution is investigated to small perturbations

\[
\left( \hat{h}, \hat{q}^{(u)}, \hat{q}^{(v)} \right) = \left( \hat{h}, \hat{q}^{(u)}, \hat{q}^{(v)} \right) \exp i (\alpha x + n \theta - \omega t),
\]

where \( \alpha \) and \( n \) are given real wave numbers, and \( \omega \) is an unknown complex frequency determining stability or instability of the basic flow.

Further, distances and velocities of most unstable perturbations which possess largest amplification factors are obtained. These perturbations are compared with the experimental data. The corresponding instant local inclination of the linear wave spiral is:

\[
\tan \beta = \frac{1}{r} \frac{dr}{d\theta} = -\frac{n}{\alpha}, \quad (2.13)
\]

where \( \beta \) is the angle between direction of the spiral and tangent, \( \alpha \) and \( n \) are wave numbers along radial and azimuthal directions. These numbers are compared with experimental measurements. The definitions of \( \beta \) and \( \theta \) are shown in Figure 2.1.
2.2.2 The Experimental Model

Let \((x, y)\) be the Cartesian system coordinates in the plane of an observed disk with the origin at the center of that disk; and let \((r, \theta)\) be the respective polar system coordinates. Using the theory \([43]\), the following spiral equations were utilized with regard to the fluid friction and air resistance \([42, 58, 60]\):

\[
a_r = r''(t) = \Omega^2 r - 8\pi c_f r'(t) - \frac{c_{res}}{2} \sin \beta r^2(t),
\]
\[
a_s = v'_s(t) = 8\pi f(\Omega r - v_s(t)) - \frac{c}{2} \cos \beta v_s^2(t),
\]
\[
\tan \beta = \frac{r'(t)}{v_s(t)} = (y'_x + \frac{y}{x})(1 + y'_x y_x) = \frac{1}{r} \frac{dr}{d\theta},
\]
\[
\theta'(t) = \frac{r'(t)}{r(t)} \frac{1}{\sin \beta} \frac{dr}{d\theta} = \frac{r(t)r'(t)}{v_s(t)},
\]
\[
\frac{d^2r}{d\theta^2} = \left( \frac{r'^2 + r(t)r''(t)}{v_s(t)} - \frac{v'_s(t)r'(t)r(t)}{v_s^2(t)} \right) \frac{1}{\theta'(t)} ,
\]
\[
x(t) = r(t) \cos \theta, y(t) = r(t) \sin \theta,
\]
\[
y'_x = \frac{r' \sin \theta + r \cos \theta \cdot \theta'}{r' \cos \theta + r \sin \theta \cdot \theta'}, \quad v(t) = (r'^2(t) + v_s^2(t))^{\frac{1}{2}}
\]
\[
r(0) = 0, \quad r'(0) = r'_0, \quad r(T) = 100, \quad v_s(0) = 0,
\]

Figure 2.1 Definition of \(\beta\) and \(\theta\).
where $a_r$ and $a_s$ are accelerations of fluid respectively along radius and perpendicular to radius, $c_f$ is a coefficient of fluid friction, and $c_{res}$ is resistance of air to fluid. In the case of water we have

$$8\pi c_f = 0.4 \text{ (mm}^{-1}), \quad c_{ref} = 2 \text{ (mm}^{-1}).$$

The formula (2.14) is not valid in the vicinity of $r(0) = 0$. Therefore, we need to use another model for $\theta(t)$ on the segment $(0, b)$ for small $b$. Let $\theta(t) = a \cdot t$ on this segment. Then

$$\theta'(0) = 0, \quad a = \theta'(b) = \frac{r'(b)}{r(b)} \tan \beta(b).$$

It is clear that $\beta$ is the angle between direction of the spiral and tangent of the circle. To find $\theta(t)$, the angle between the radius at the moment $t$ and the axes $x$, the right triangle $ABC$, where legs $AB = dr = r'(t)dt$, $BC = rd\theta = r\theta'(t)$ (angle $\beta$ is opposite to $rd\theta$) (see Figure 2.1) is considered.

Note that the second and the third formulas for $\tan \beta$ above can be used by image algorithms, since they do not depend on the time but only on geometric property of the respective solution.

The approximate solution for this non-linear system is found using Euler’s numerical method with decreasing steps of computation until approximate solutions are stabilized.

In our experiments, in the case of water, $\Omega$ is obtained from the respective graph for the reactor used, $r'(0)$ is obtained from the experiment and equals:

$$r'_0 = \frac{3.8 \cdot 10^6}{285 \cdot 2\pi \cdot 2.5 \cdot 3} \approx 280 \text{ (mm/s)}.$$
where $3.8 \times 10^6 \ mm^3$ is volume of a gallon of water, 285 $s$ is the time for the water to run out of the respective capacity, 2.5 $mm$ is the radius of the tube, and 3 $mm$ is the gap between the end of the tube and the disk surface.

2.3 Camera Calibration Accuracy Analysis

2.3.1 Overview of Camera Calibration

Camera calibration is a necessary step in 2D and 3D computer vision in order to extract metric information from video images; and it is important for accuracy in 2D and 3D reconstruction. Much work has been done, starting in the photo-gram-metric community [59], and more recently in computer vision [7, 14, 18, 23, 54, 67, 68]. Camera calibration is the process of relating the ideal model of the camera to the actual physical device and determining the position and orientation of the camera with respect to a world reference system. Depending on the model used, there are different parameters to be determined. For the pinhole camera model the parameters to be calibrated are classified into two groups:

1. Internal (or intrinsic) parameters. Internal geometric and optical characteristics of the lenses and the imaging device.

2. External (or extrinsic) parameters. Position and orientation of the camera in a world reference system.

The relationship between a point $(X, Y)$ and its image projection $(x, y)$ is given [68] by

$$
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix} = A \cdot \begin{bmatrix} r_1 \\
r_2 \\
t \end{bmatrix} \cdot \begin{bmatrix} X \\
Y \\
1 \end{bmatrix}^T,
$$

$$
A = \begin{bmatrix} s & \gamma & c_x \\
0 & s & c_y \\
0 & 0 & 1 \end{bmatrix}
$$

(2.15)
where \([r_1, r_2, t]\) are the extrinsic parameters (the rotations and translation) that relate the world coordinate system to the camera coordinate system; \(A\) is the camera intrinsic matrix, in which \((c_x, c_y)\) is the principal point, \(s = f/s_x = f/s_y\), \(f\) is the focal length, \(s_x = s_y\) is the effective size of the pixel, \(s\) is the scale factor (accuracy of which to be accounted for any uncertainty due to imperfections in the viewing camera), and \(\gamma\) is the skewness of the image axes.

### 2.3.2 Camera Calibration Accuracy

The relations between observed \((x_d, y_d)\) coordinates and the ideal (distortion-free) pixel coordinates \((x, y)\) are:

\[
x_d = x + k(x - c_x), \quad y_d = y + k(y - c_y),
\]

\[
k = k_1r_d^2 + k_2r_d^4, \quad r_d^2 = x_d^2 + y_d^2,
\]

where \(k_1\) and \(k_2\) are the coefficients of the radial distortion. These relations allow us to calculate \(x\) and \(y\), after which values \(X\) and \(Y\) are calculated using the following formulae (2.16).

\[
[X, Y, 1]^T = [r_1, r_2, t]^{-1} \cdot A^{-1} \cdot [x, y, 1]^T,
\]

\[
x = \frac{x_d + c_x \cdot k}{1 + k}, \quad y = \frac{y_d + c_y \cdot k}{1 + k}.
\]  
(2.16)

Due to the fact that the main error of the calibration method [14] used in this work is determined by values of distortion, the relative deviations for \(x\), \(y\) and \(X\), \(Y\) are estimated, assuming for simplicity that \(c_x = c_y = 0\). In this regard, the result in [68], page 13, is used. The relative deviations for the estimates of \(k_1\) and \(k_2\) do not exceed
3-4%. Since in the case considered

\[ x = \frac{x_d}{1 + k}, \quad y = \frac{y_d}{1 + k}, \]

with regard to the main terms (assuming \( k \) is small, \( k \leq 0.03 \), and \( k_1 = k_2 \)), the absolute values of variations

\[ |\Delta x| \leq |\Delta x_d| + x_d \cdot |\Delta k|, \quad |\Delta y| \leq |\Delta y_d| + y_d \cdot |\Delta k|, \]

\[ \Delta k = \Delta k_1 \cdot (r_d^2 + r_d^4), \quad \frac{\Delta k}{k} = \frac{\Delta k_1}{k_1} \leq 0.03, \]

(2.17)

from where,

\[ \frac{|\Delta x|}{x} \leq \frac{|\Delta x_d|}{x_d} + k \cdot \frac{|\Delta k|}{k}, \quad \frac{|\Delta y|}{y} \leq \frac{|\Delta y_d|}{y_d} + k \cdot \frac{|\Delta k|}{k}, \]

\[ \Delta r_d = \Delta r \leq \frac{x \cdot |\Delta x| + y \cdot |\Delta y|}{(x^2 + y^2)^{1/2}}. \]

Using the fact that \( \frac{|\Delta x_d|}{x_d}, \frac{|\Delta y_d|}{y_d} < \frac{0.3}{720} \), where 720 is maximal value of the number of pixels and 0.3 is the upper bound for \( \Delta x_d \) and \( \Delta y_d \), it is easy to estimate that

\[ \frac{\Delta x}{x} < \frac{0.3}{720} + 0.03 \cdot 0.03 \approx 0.0013, \quad \frac{\Delta y}{y} \approx 0.0013. \]

In the case when \( r = \sqrt{x^2 + y^2} \), we have

\[ \frac{\Delta r}{r} \leq \frac{\Delta x}{x} + \frac{\Delta y}{y} \approx 0.0026. \]

Similarly, in the case when \( R = \sqrt{X^2 + Y^2} \) with regard to

\[ R = \frac{1}{s} (x^2 + y^2)^{1/4}, \quad \frac{\Delta s}{s} < 0.001, \]
we have

\[ \epsilon = \frac{\Delta R}{R} \leq \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta s}{s} \cdot \frac{1}{s \cdot R} \leq 0.0026 + 0.000001 < 0.003. \] (2.18)

The estimates of the relative errors of the initial data are calculated and used for the experimental estimation of the radial velocity components and the spiral wave inclination angles of the film flow (see Subsections 3.3.3 and 3.3.4).
CHAPTER 3
ESTIMATION OF FLUID FLOW PARAMETERS

In this section video-based algorithms for detecting and tracking of waves in a spinning disk reactor are presented. Using experimental video data and the developed models and algorithms, the characteristics of wave regimes are estimated. Therefore, in order to calculate the fluid flow parameters the following steps need to be performed: 1. Detection of points on the peaks of waves along the same radius, 2. Estimation of radial velocities, 3. Estimation of wave inclination angles, and 4. Estimation of distances between the consecutive peaks of waves or the respective wave spirals. The algorithms use the polar system coordinates and Cartesian system coordinates with the origin in the center of the disk. The block-scheme and the respective schematic view are given in Figure 3.1. Prior to the process of detecting projected points with maximal intensity on waves (with the fixed camera position relatively to the disk), the preprocessing of images is performed. This is done using local contract enhancement, which improves the visual appearance of the wavefront for human observation and normalizes the intensity values. That follows by local thresholding operation which detects points at maximal intensity along the radial (in respect to the spinning disk) direction.

In the experimental setup, videos were taken from a side view because of the better visibility of the waves. Moreover, in many industrial applications the imaging must be from a side view such as in the synthesis of aero gels. In such applications the rotating disk is inside the closed cylinder with a window on the side of the cylinder.
3.1 Tracking Specular Reflection Patterns

The total reflection is an additive of several components: diffuse reflection \( (L_d) \), specular reflection \( (L_c) \) from the front surface [23], scattering \( (L_h) \) from the volume of
liquid of thickness \((h)\), and reflection \((L_b)\) from the bottom [61],

\[
L = L_d + L_s + L_h + L_b. \tag{3.1}
\]

The optical path of the reflected light is illustrated in Figure 3.2.

We show that at each time instant \((t_0)\) there is only one point on cross-section on the wave with the specular reflection component \(L_s\), which dominates when non-clear fluid is imaged. Consequently, this specular point on the wave has the highest intensity in the image. At this point the incident angle (the angle between the viewer direction and the surface normal orientation at the given point) and the emittance angle (the angle between the illumination direction and the surface normal orientation at the given point) are equal due to geometrical optics (reflection law) [15, 17] (see Figure 3.3).

Since the approximation of a fluid wave has a sine-like shape [35], the projection of wave on the plane \(xoz\) perpendicular to wave motion of spatial structure of fluid waves in a two-dimensional space is approximated by the parabolic equation. Within the interval \([-\delta, \delta]\) the projected sine function and parabolic function are similar as shown in Figure 3.4.

Thus, the approximation of \(A \sin(\omega(\frac{\pi}{L_0} - x)) - A\) with the parabola on the interval \([-\delta, \delta]\) has the form
Figure 3.3 The incident angle $\alpha$ and the emittance angle $\beta$.

Figure 3.4 The graph $z = \sin\left(\frac{\pi}{2\omega} - x\right) - A$, $-\frac{\pi}{2\omega} \leq x \leq \frac{\pi}{2\omega}$ and $z = -A\left(\frac{\omega x}{2}\right)^2$, $-\delta \leq x \leq \delta$.

\[
A\frac{\omega^2}{2} \delta^2 - A\sin(\omega\left(\frac{\pi}{2\omega} \pm \delta\right)) + A =
\]
\[
A\frac{\omega^2}{2} \delta^2 + A(1 - \cos(\omega\left(\frac{\pi}{2\omega} \pm \delta\right))) =
\]
\[
A\frac{\omega^2}{2} \delta^2 - 2A\sin^2(\pi \pm \frac{\pi\omega}{2}) = A\frac{\omega^2}{2} \delta^2 - 2A\sin^2\left(\frac{\pi\omega}{2}\right) =
\]
\[
A\frac{\omega^2}{2} \delta^2 - 2A\left(\frac{\omega\delta}{2} - \left(\frac{\omega\delta}{2}\right)^3 + \ldots\right)^2 =
\]
\[
A\frac{\omega^2}{2} \delta^2 - 2A\left(\frac{(\omega\delta)^2}{4} - \frac{(\omega\delta)^4}{48} + \frac{(\omega\delta)^6}{24^2} - \ldots\right) \approx A\frac{(\omega\delta)^4}{24}, \quad (3.2)
\]

Using parabolic approximation of wave profile, we consider, first, a special case when two coordinates of a camera and light are the same, $x_l = x_c$ and $z_l = z_c$, (see Figure
having \( z_l \gg R \), where \( R \) is a radius of a disk, we derive the following equations:

\[
\frac{z_l - z_0}{x_l - x_0} = -\frac{1}{\frac{dz_0}{dx_0}}, \quad z_0 = -A\omega^2\frac{2}{x_0^2}, \quad \frac{dz_0}{dx_0} = -A\omega^2 x_0,
\]

\[
x_0^3 + 2\frac{A\omega^2 z_l + 1}{A^2\omega^4} x_0 - 2\frac{x_l}{A^2\omega^4} = 0,
\]

(3.3)

where \((x_0, z_0)\) is the specular point at the time instant \((t_0)\) on the wave. This equation has one real root, since the derivative \(3x_0^2 + 2\frac{A\omega^2 z_l + 1}{A^2\omega^4}\) does not change the sign. The solution of (3.3) is shown below.

\[
x_0 = \frac{(27A\omega^2 x_l + 3\sqrt{c_0})^{1/3}}{3A\omega^2} - \frac{2(A\omega^2 z_l + 1)}{A\omega^2(27A\omega^2 x_l + 3\sqrt{c_0})^{1/3}},
\]

\[
c_0 = 24A^3\omega^6 z_l^3 + 72A^2\omega^4 z_l^2 + 72A\omega^2 z_l + 24 + 81A^2\omega^4 x_l^2,
\]

(3.4)

where \(x_0\) is the distance from the top of the wave to the specular point on the wave.

If the wave is moving by \(\epsilon\) in the x-direction, the equation becomes

\[
\frac{z_l - z_0}{x_l - x_0} = -\frac{1}{\frac{dz_0}{dx_0}}, \quad \frac{dz_0}{dx_0} = -A\omega^2 (x_1 - \epsilon),
\]

\[
(x_1 - \epsilon)^3 + 2\frac{A\omega^2 z_l + 1}{A^2\omega^4} (x_1 - \epsilon) - \frac{2}{A^2\omega^4} (x_l - \epsilon) = 0,
\]

(3.5)
where \((x_1, z_1)\) is the specular point at the time instant \((t_1)\) on the moved wave. Solving the equation (3.5), we have

\[
x_1 - \epsilon = \frac{(27A\omega^2x_l - 27A\omega^2\epsilon + 3\sqrt{c_0})^{1/3}}{3A\omega^2} - \frac{2(A\omega^2 z_l + 1)}{A\omega^2(27A\omega^2x_l - 27A\omega^2\epsilon + 3\sqrt{c_0})^{1/3}}.
\]

\[
c_0 = c_0 + 81A^2\omega^4\epsilon^2 - 162A^2\omega^4\epsilon.
\]

(3.6)

Estimation of difference \(|x_0 - (x_1 - \epsilon)|\) approximately equals zero when \(\epsilon \ll x_1, z_l, z_l \leq \epsilon\) and \(z_l >> R\), where \(R\) is the radius of the disk. Since the distance \(x_1 - \epsilon \approx x_0\), it is reasonable to use the specular points for tracking waves. Hence, tracking the specular points allows us to track the wave of fluid flowing over a rotating disk.

In a general case, when coordinates of a camera \((x_c, z_c)\) and coordinates of a light \((x_l, z_l)\) are not the same, we derive the following equations for the specular point positions:

\[
\begin{align*}
\frac{z_c - z_0}{x_c - x_0} - m_0 & = \frac{z_l - z_0}{x_l - x_0} - m_0, \\
1 + \frac{z_c - z_0}{x_c - x_0} m_0 & = 1 + \frac{z_l - z_0}{x_l - x_0} m_0, \\
2 \frac{z_c - z_0}{x_c - x_0} x_0^4 & + \frac{2}{A^2\omega^4} x_0^3 - \frac{3}{A^2\omega^4} x_l + \frac{2}{A^2\omega^4} x_c z_l + \frac{3}{A^2\omega^4} x_c + \\
\frac{2}{A^2\omega^4} x_l z_c x_0^2 & - \frac{4}{A^2\omega^4} z_c z_l + \frac{2}{A^3\omega^6} x_0 + \frac{4}{A^2\omega^4} x_l x_c x_0 + \\
2 x_c z_l + x_l z_c & = 0.
\end{align*}
\]

(3.7)

The equation (3.7) has only one solution which is negative if \(2x_c z_l + 2x_l z_c\) is positive and positive if \(2x_c z_l + 2x_l z_c\) is negative. Moving the wave by \(\epsilon\), the distances \(|x_0| \approx |x_1 - \epsilon|\), when \(\epsilon \ll z_c, z_l, \frac{z_l}{z_l} \leq \epsilon\) and \(A\omega^2 = const\).

Thus, under general camera and light positions, the specular points do not coincide with the peaks of the waves. However, as shown above, the distances along the radial
direction between specular points and the peaks are approximately constant for the moving wave. Hence, tracking the specular points allows us to track the wave of fluid flowing over a rotating disk.

Moreover, the conditions under which the specular points practically coincide with the peaks of waves are found (see Appendix A). This coincidence is more precise when the ratio, \( \frac{z_c}{r} \), is smaller, where \( z_c \) is a \( z \)-coordinate of a camera and \( r \) is a radius of the rotating disk.

### 3.2 Algorithm for Detecting and Tracking Specular Points on Waves

In the process of detecting points along the radius-vector the maximums of intensities are found corresponding to the specular points of the waves. We consider the spiral below as periodic functions due to their stationary property [30] with respect to the rotating disk. Let \( \Delta \Phi \) be the period of the spiral equations in the polar system of coordinates \( r = r_j(\phi) : r_j(\phi + \Delta \Phi) = r_j(\phi) \), where \( \Delta \phi \) is the angle-step of the calibration in the polar system coordinates, \( N = \frac{\Delta \Phi}{\Delta \phi} > 1 \) is an integer, and

\[
\begin{aligned}
\phi &= \phi_i = \phi_0 + i \Delta \phi, \quad i = 1, 2, ..., N; \\
r_j(\phi_0) &= r_0 = \min r_j(\phi); \\
r_0 < r_{i1} < r_{i2} < ... < r_{IS} \leq 100, \\
r_{ij} &= r_j(\phi_i), \quad i = 1, 2, ..., N, \quad j = 1, ..., S, \quad (3.8)
\end{aligned}
\]

where \( S \) is the number of spirals for each \( i \), \( r_{ij} \) are experimental data for \( \phi = \phi_i \); the points \((\phi_i, r_{ij})\) are on the respective spirals. To determine the number of spirals \( S \) the training frame for each video is used. The number of spirals \( S \) is equal to the number of intensity increases in the radial direction from the center of a disk. Then

1st spiral: \((\phi_1, r_{11}), (\phi_2, r_{21}), (\phi_3, r_{31}), ..., (\phi_N, r_{N1}); (\phi_{N+1}, r_{12}), (\phi_{N+2}, r_{22}), ..., (\phi_{2N}, r_{N2}); \)

...; \((\phi_{(S-1)N+1}, r_{1S}), (\phi_{(S-1)N+2}, r_{2S}), ..., (\phi_{SN}, r_{NS}); \)
2d spiral: \((\phi_{N+1}, r_{11}), (\phi_{N+2}, r_{21}), (\phi_{N+3}, r_{31}), \ldots, (\phi_{2N}, r_{N1}); (\phi_{2N+1}, r_{12}), (\phi_{2N+2}, r_{22}), \ldots, (\phi_{3N}, r_{N2}); \ldots; (\phi_{SN+1}, r_{1S}), (\phi_{SN+2}, r_{2S}), \ldots, (\phi_{2SN}, r_{NS}); \ldots; (\phi_{2N+1}, r_{12}), (\phi_{2N+2}, r_{22}), \ldots, (\phi_{3N}, r_{N2}); \ldots; (\phi_{SN+1}, r_{1S}), (\phi_{SN+2}, r_{2S}), \ldots, (\phi_{2SN}, r_{NS}); \ldots;\)

\((n+1)\)th spiral: \((\phi_{nN+1}, r_{11}), (\phi_{nN+2}, r_{21}), (\phi_{nN+3}, r_{31}), \ldots, (\phi_{(n+1)N}, r_{N1}); (\phi_{(n+1)N+1}, r_{12}), (\phi_{(n+1)N+2}, r_{22}), \ldots, (\phi_{(n+2)N}, r_{N2}); \ldots; (\phi_{(n+S-1)N+1}, r_{1S}), (\phi_{(n+S-1)N+2}, r_{2S}), \ldots, (\phi_{(n+S)N}, r_{NS}); \ldots;\)

Let

\[R_{ij}, \quad i = 1, \ldots, N, \quad j = 1, \ldots, S,\]

be the given data on the contracted disk in the form of the standard ellipse with the parameters \(a = R\) and \(0 < b < R\). Then

\[r_{ij} = r_j(\phi_i) = R_{ij} \frac{R}{\left[(a \cos \phi_i)^2 + (b \sin \phi_i)^2\right]^{1/2}},\]

where \(\phi_0 = 0\), and \(i = 1, \ldots, N, j = 1, \ldots, S\).

Let \(\Delta r\) be the radius-step of calibration in the polar system coordinates, and

\[(\phi_i, k\Delta r; I_{ik}), \quad i = 1, 2, \ldots; \quad k = 1, 2, \ldots;\]

be the calibration net on the contracted disk, where \(I_{ik}\) are the intensities of the points \((\phi_i, k\Delta r)\). Then

\[R_{ij} = \frac{1}{2M} \sum_{k=-M}^{j+M} I_{ik} \Delta r; \quad I_{ik} \neq 0,\]

\[k = j - M, j - M + 1, \ldots, j + M;\]

\[i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, S;\]
where $2M$ means maximal number of pixels in the vicinity of the point $(\phi_i, R_{ij})$ along the radius with the angle $\phi_i$ and with the center in that point.

This processing is continued for all frames and repeated for each video sequence. Samples of resulting images are shown in Figure 3.6.

![Figure 3.6](image)

(a) (b)

Figure 3.6 (a) Detected points of waves. (b) A detected wave.

Note: With the purpose of presenting clarity, pixel detected points are enhanced in Figure 3.6.

### 3.3 Estimation of Parameters of Spiral Waves

#### 3.3.1 Asymptotically Optimal Numerical Method of Differentiation

Estimation of the first derivatives for a given function is the ill-posed problem [20], i.e., for such problems, arbitrary small errors of the initial data can give, in general, arbitrary large errors of the respective results.

In the deterministic case, when $\tilde{X}_i$ is the vector at the time $t$, $X_i$ is its experimental value, $h$ is the step of differentiation, and $|X_i - \tilde{X}_i| < \epsilon$ for any $t$. Then, with regard to the main terms
\[
\Delta = \left| \frac{X_{i+1} - X_{i-1}}{2h} \right| = \\
\left| \frac{X_{i+1} - X_{i-1}}{2h} + \frac{(X_{i+1} - X_{i-1}) - (X_{i+1} - X_{i-1})}{2h} \right| = \\
\left| \frac{X_i''}{6} h^2 + \frac{\epsilon}{h} = \phi(h), \quad \phi'(h) = \left| \frac{X_i''}{3} \right| h - \frac{\epsilon}{h^2} = 0, \right|
\]

\[h_{opt} = \left( \frac{3\epsilon}{X_i''} \right)^{1/3}, \quad \Delta_{opt} = 2 \left( \left| \frac{X_i''}{3} \right| \right)^{1/3} \epsilon^{2/3}. \tag{3.15}\]

Instead of unknown \(X_i''\) we apply

\[X_i'' = \frac{X_{g(i+2)} - 2X_{g(i+1)} + 2X_{g(i-1)} - X_{g(i-2)}}{2g^3} \neq 0, \quad g = \epsilon^{1/5}, \]

\[h = \left( \frac{3\epsilon}{X_i''} \right)^{1/3}, \quad \Delta = 2 \left( \left| \frac{X_i''}{3} \right| \right)^{1/3} \epsilon^{2/3}. \tag{3.16}\]

Theorem 1. The estimate \(\Delta\) in (3.16) is asymptotically optimal.

Proof. We have

\[d = \left| \frac{X_i'' - X_i'''}{2h} \right| = \left| \frac{X_{g(i+2)} - 2X_{g(i+1)} + 2X_{g(i-1)} - X_{g(i-2)}}{2g^3} \right| \leq \\
\left| \frac{\left( X_{g(i+2)} - 2X_{g(i+1)} + 2X_{g(i-1)} - X_{g(i-2)} \right)}{2g^3} \right| \leq cg^2 + \frac{3\epsilon}{g^3} = \psi(g), \tag{3.17}\]

where \(c\) is a certain existing constant (fifth derivative). Using (3.17), we find

\[g = \epsilon^{1/5}, \quad d = (c + 3)\epsilon^{2/5};\]
that is under \( g \) order \( \epsilon^{1/5} \) the value of \(|X_i'''' - \tilde{X}_i''''|\) has the order \( \epsilon^{2/5} \). Then

\[
\left( \frac{\Delta_{opt}}{\Delta} \right)^3 = \frac{|\tilde{X}_i''''|}{|X_i''''|},
\]

\[
1 - (c + 3)\epsilon^{2/5} \leq \frac{|X_i''''| - |X_i'''' - \tilde{X}_i''''|}{|X_i''''|} \leq \frac{|\tilde{X}_i''''|}{|X_i''''|} \leq 1 + (c + 3)\epsilon^{2/5},
\]

that is, \( \frac{\Delta_{opt}}{\Delta} \rightarrow 1, \epsilon \rightarrow 0 \), and hence \( \Delta \) is asymptotically optimal.

The stochastic case, described in Appendix B, is applicable to the estimation of the wave velocity and wave inclination. It is based on the normal distribution of errors and estimation of a variance.

### 3.3.2 Radial Velocity Component Computation

To determine the velocity of the waves, the sequences of the images of film flows are used with the step of differentiation \( \Delta t > 0 \). Choosing the system of coordinates at the center of the rotating disk, the estimate of the radial velocity component is given by

\[
t_{exp}' = \frac{\Delta r}{\Delta t} = \frac{r_{t_{i+1}} - r_{t_i}}{\Delta t}, \quad \Delta t = |t_{i+1} - t_i|,
\]

where \( r_t \) and \( r_{t+1} \) are the values of the radii from the center to the points on the wave at the moments \( t_i \) and \( t_{i+1} \). The problem of estimating radial velocity is ill-posed, i.e., for such problems, arbitrary small errors of the initial data can give, in general, arbitrary large errors of the respective results [20]. So, the asymptotically optimal method for minimizing error of estimate under a known error of initial data is used (see Equation (3.15)).
3.3.3 Wave Thickness and Azimuthal Velocity Component Computations

Due to small thickness of the flow, estimation of the third coordinate z with high accuracy is rather difficult using traditional methods. So, we use z below from its model values. According to [30], coordinate

\[ z = \left( \frac{3Q\nu}{2\pi\Omega^2 r^2} \right)^{1/3}, \quad (3.18) \]

where \( Q \) is the constant flow rate, \( \Omega \) is the angular velocity of a disk, and \( \nu \) is viscosity of fluid. This formula is applied in [30] for the condition that the convective terms are small compared to the centrifugal terms.

The azimuthal velocity component of wave is determined by

\[ v_{a,\text{theor}} = \frac{r'(t)}{\tan \beta}, \quad (3.19) \]

(see [30]). The velocity \( r'(t) \) and the inclination angle \( \beta \) are experimentally estimated.

3.3.4 Inclination Angle Computation

Figure 2.1 shows the definition of the inclination angle \( \beta \). Again, the asymptotically optimal method for minimizing the error of estimate under a known error of initial data is used (see Equation (3.15)).

The following estimates are used:

\[ \tan \beta = \frac{1}{r} \frac{d\tilde{r}}{d\theta} \approx \frac{1}{r} \frac{r(\theta + \Delta \theta) - r(\theta - \Delta \theta)}{2\Delta \theta}, \quad |r - \tilde{r}| \leq \epsilon, \quad \Delta \theta = \left( \frac{3\epsilon}{r^m} \right)^{1/3}, \]

\[ \left| \frac{d\tilde{r}}{d\theta} - \frac{r(\theta + \Delta \theta) - r(\theta - \Delta \theta)}{2\Delta \theta} \right| \leq \left| \frac{r'''}{6} \right| \frac{\Delta \theta^2}{\Delta \theta} + \frac{\epsilon}{\Delta \theta} = \phi(\Delta \theta), \]

\[ r''' = \frac{r_{i+2} - 2r_{i+1} + 2r_{i-1} - r_{i-2}}{2g^3}, \quad g = \epsilon^{1/5}. \quad (3.20) \]
Note that formulae (3.20) do not depend on time but only on geometric property of the spirals and on resolution of the video.

### 3.3.5 Distance Between Consecutive Waves Computation

In order to calculate the distance $l$, the difference between two neighboring specular points of waves, in radial direction, was calculated:

$$ l = r_2 - r_1. $$  

(3.21)

Note that values $r_1$ and $r_2$ have to be used in the process of averaging. The part of the sequence with detected points on the front of the waves that was used for calculating the average distance is shown in Figure 3.7.

![Sample of images with detected points on waves.](image)

Figure 3.7 Sample of images with detected points on waves.
CHAPTER 4
DATA ACQUISITION

4.1 Experimental Set-up

The experimental set-up consists of: a motor, aluminum flat/round stock, reservoir, tubing, brass adapters, bunged cords, flow-meter, copper tubing, aluminum control box, switches, return pump, miscellaneous hardware (Figure 4.1).

![Experimental set-up diagram]

Figure 4.1 Experimental set-up.

The main characteristic of the motor is given by the turnable calibration. Measurements were performed in the following way. Water contained in a plastic container with an adjustment valve for the flow was drained through copper tubing at a constant starting flow rate $Q$, which can be changed in the range of 0.2-0.8 lpm (liter per minute). Liquid emerged from the nozzle as a free jet pouring out onto the center of a constantly rotating aluminum disk with a diameter of 200 mm. The rotational frequency of the disk was monitored by a motor control. Water leaving the rotating disk was collected at the bottom reservoir and recycled to the top reservoir by a pump.
The pattern of 8 x 10 squares (see Figure 4.2) was used to calibrate the camera. Three hundred twenty points (corners of squares on the experimental pattern) were chosen in the Cartesian system coordinate with the origin in the center of the disk. The technique described in [68] was used for finding the intrinsic and extrinsic parameters of the camera.

### 4.2 Data Collection

The schematic set-up for data acquisition is shown in Figure 4.3. Video data was collected for the range of flow rates from 2.0 \textit{lpm} to 8.0 \textit{lpm} and for the range of angular speed of the disk from 200 \textit{rpm} to 800 \textit{rpm}. Videos of the flow process were taken at different settings with controlling parameters (flow rate and speed of the disk), different arrangements of light illuminations, and different settings of the camera using the portable camcorder Canon Optura 20 capable of capturing images at 30 \textit{fps} (frames per second) (resolution 720x480) and high definition camera (JVC GY-HD 100), capable of capturing images at 30 \textit{fps} (resolution 1280x720, color) (see Table 4.1). Duration of
videos in time is from 1 sec to 5 sec.

Note: In the process of detecting and tracking of waves, the video data with clear visibility of waves are used. The collected videos were used as is shown in Table 4.2

Figure 4.4 shows the sample image of the liquid film that flows over a disk, rotating with the angular velocity (Ω) of 500 rpm (reverses per minute) and the flow rate (Q) of 0.8 lpm. It can be seen (Figure 4.4) that the film surface is covered by spiral waves.
Table 4.1 Collected video data.

<table>
<thead>
<tr>
<th>Camera</th>
<th>Disk rotating speed (rpm)</th>
<th>Fluid – flow rate (lpm)</th>
<th>Number of video clips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>500</td>
<td>0.8</td>
<td>10</td>
</tr>
<tr>
<td>Standard</td>
<td>300</td>
<td>0.2</td>
<td>10</td>
</tr>
<tr>
<td>High Definition</td>
<td>800</td>
<td>0.8</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>700</td>
<td>0.8</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>600</td>
<td>0.6</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>500</td>
<td>0.8</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>500</td>
<td>0.6</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>500</td>
<td>0.4</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>400</td>
<td>0.4</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>300</td>
<td>0.2</td>
<td>5</td>
</tr>
<tr>
<td>High Definition</td>
<td>200</td>
<td>0.2</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 4.2 Usage of video data.

<table>
<thead>
<tr>
<th>Disk rotating speed (rpm)</th>
<th>Fluid − flow rate (lpm)</th>
<th>Training sets of videos &amp; total # of frames</th>
<th>Testing sets of videos &amp; total # of frames</th>
<th>Ground truth sets of videos &amp; total # of frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>0.8</td>
<td>5</td>
<td>5</td>
<td>150</td>
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<tr>
<td></td>
<td></td>
<td>5</td>
<td>5</td>
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<td>700</td>
<td>0.8</td>
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<td>5</td>
<td>150</td>
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<td></td>
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<td>5</td>
<td>−</td>
</tr>
<tr>
<td>600</td>
<td>0.6</td>
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<td>5</td>
<td>150</td>
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<td>0.8</td>
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<td>15</td>
<td>15</td>
</tr>
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<td>150</td>
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<td>5</td>
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<td>−</td>
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<td>−</td>
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<td>150</td>
</tr>
<tr>
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<td>−</td>
</tr>
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<td></td>
<td>5</td>
<td>150</td>
<td>−</td>
</tr>
<tr>
<td>200</td>
<td>0.2</td>
<td>5</td>
<td>5</td>
<td>−</td>
</tr>
</tbody>
</table>
CHAPTER 5
RESULTS AND COMPARISON TO FLUID FLOW MODEL

5.1 Regimes of Fluid Flow

The fluid flow over a spinning disk of 300 $rpm$ and 500 $rpm$ are illustrated in Figure 5.1.

![Figure 5.1 The regimes of the disk rotation at (a) 300 rpm and (b) 500 rpm.](image)

5.2 Experimental Fluid-Flow Parameter Estimates

5.2.1 Estimation of Radial Velocity Component

The estimate of the step of differentiation (Equation (3.16)) is

$$\Delta t = \left( \frac{3}{f^{m(t)}} \right)^{1/3} \epsilon^{1/3} \approx \frac{1}{30},$$

(5.1)
where $\epsilon \leq 0.0026 + 0.000001 < 0.003$ (Equation (2.18)) is an error in the initial data. The video data are used for estimation of velocity components. The average experimental velocities are estimated using the derivation step equal to $\frac{1}{30}$. The radii and the average experimental velocities of waves for ten videos (taken with camera Optura) with the disk rotation of 520 rpm and the flow rate of 0.8 lpm are given in Table 5.1.

Table 5.1 Experimental wave velocities (mm/s). Video data are taken using camcorder Optura 20.

<table>
<thead>
<tr>
<th>Video numbers</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
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<tr>
<td>1</td>
<td></td>
<td>805</td>
<td>750</td>
<td>700</td>
<td>630</td>
<td>560</td>
<td>540</td>
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<tr>
<td>2</td>
<td></td>
<td>820</td>
<td>755</td>
<td>690</td>
<td>650</td>
<td>570</td>
<td>536</td>
<td>530</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>801</td>
<td>760</td>
<td>685</td>
<td>635</td>
<td>570</td>
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<td><strong>Averaged</strong></td>
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<td>9.50</td>
<td>7.76</td>
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<td>43</td>
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</table>

Using the videos taken with High Definition camera, with the disk rotating at 520 rpm and the flow rate of 0.8 lpm, the averaged velocities of waves for five videos are given in Table 5.2.

Table 5.2 Experimental wave velocities (mm/s). Video data are taken using high definition camera (JVC GY-HD 100).

<table>
<thead>
<tr>
<th>Video numbers</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
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<tr>
<td>3</td>
<td></td>
<td>800</td>
<td>735</td>
<td>680</td>
<td>625</td>
<td>570</td>
<td>524</td>
<td>515</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>805</td>
<td>746</td>
<td>690</td>
<td>630</td>
<td>575</td>
<td>530</td>
<td>510</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>790</td>
<td>730</td>
<td>670</td>
<td>625</td>
<td>570</td>
<td>525</td>
<td>510</td>
</tr>
<tr>
<td><strong>Averaged</strong></td>
<td></td>
<td>795</td>
<td>739.2</td>
<td>681</td>
<td>631</td>
<td>570</td>
<td>526.8</td>
<td>515</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>7.91</td>
<td>6.76</td>
<td>7.42</td>
<td>6.52</td>
<td>3.54</td>
<td>2.95</td>
<td>6.12</td>
<td>43</td>
</tr>
</tbody>
</table>
5.2.2 Estimation of Wave Thickness and Azimuthal Velocity Component

The formula (3.18) is applied (see [30]) for the condition that the convective terms are small compared to the centrifugal terms. Experimental set-up in [30] is similar to the considered one in this work, but different by two points: the surface of a disk was made practically without any friction and a rotating disk was placed in a vacuum chamber. That means that we have to consider in additional two forces: forces due to friction and resistance of air. It is shown below that those forces will be negligible compared to the centrifugal force for considered values of $Q$, $\Omega$, $r$.

The centrifugal force and the forces due to friction and air resistance [42], respectively, are:

$$f_1 = \frac{mv^2}{r} = m\Omega^2 r, \quad f_2 = 8\pi fmv, \quad f_3 = 0.5c\rho_a Av^2,$$

where $m$ is the mass of water, $A$ is the cross-section area perpendicular to wave velocity vector, and $\rho_a$ air density.

Under the radial velocity $v$ in the considered range of $[280, 1000]$, the radius $r$ in the range of $[0, 100]$, and the density of water $\rho$, we have

$$\frac{f_2}{f_1} = \frac{8\pi \cdot f \cdot r}{\nu} \leq \frac{8\pi \cdot 0.005 \cdot 100}{280} \leq \frac{4.5}{100} < \frac{1}{20},$$

$$\frac{f_3}{f_1} \leq \frac{0.5c\rho_a \cdot c \cdot r}{\rho} \leq \frac{0.5c \cdot 1 \cdot 100}{1009} < \frac{1}{20},$$

i.e., the forces due to friction and air resistance are negligible compared to the centrifugal force.

Using the formula (3.19), $r'(t)$ was estimated above and the inclination angle $\beta$ will be estimated below. Thus, using those two estimates, an azimuthal velocity component can be found. For instance, if the radial velocity for the radius of 60 mm is taken from Table 5.1 and the value for the inclination angle from Table 5.10, then the azimuthal
velocity $v_{a, theor} \approx \frac{r'}{\tan \beta} \approx \frac{684}{\tan 0.53} \approx 1167 \ mm/s$. Also, the film thickness $h$ can be calculated using the following formula $h = \sqrt{\frac{2\nu r'}{RQ^2}}$, where $\nu$ is kinematic viscosity.

### 5.2.3 Estimation of Inclination Angles

The asymptotically optimal step $\Delta \theta = \left(\frac{3}{r''_0}\right)^{1/3} \epsilon^{1/3} \approx \frac{1}{25} \text{(radian)}$ (see (3.20)). The obtained asymptotically optimal step is consistent with the size of the pixel for the camera used.

An error is estimated as $\Delta = 2 \left(\frac{r''_0}{3}\right)^{1/3} \epsilon^{2/3} \approx 6\%$, where $r''_0 \approx 11$ and $\epsilon = 0.003$ (see (2.18)).

The video sequences of fluid flow over a rotating disk of 500 rpm and the flow rate of 0.8 lpm are used to calculate averaged wave inclinations. The results for ten videos (taken with camera Optura) are illustrated in Table 5.3. The results for five videos (taken with High Definition camera) are illustrated in Table 5.4. The video sequences of fluid flow over a rotating disk of 500 rpm and the flow rate of 0.8 lpm are used to calculate averaged wave inclinations.

<table>
<thead>
<tr>
<th>Video numbers</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1.01</td>
<td>0.73</td>
<td>0.54</td>
<td>0.47</td>
<td>0.36</td>
<td>0.34</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.97</td>
<td>0.73</td>
<td>0.53</td>
<td>0.46</td>
<td>0.36</td>
<td>0.33</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.96</td>
<td>0.7</td>
<td>0.5</td>
<td>0.45</td>
<td>0.35</td>
<td>0.32</td>
<td>0.28</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1.01</td>
<td>0.73</td>
<td>0.54</td>
<td>0.47</td>
<td>0.34</td>
<td>0.32</td>
<td>0.29</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.96</td>
<td>0.71</td>
<td>0.5</td>
<td>0.43</td>
<td>0.33</td>
<td>0.31</td>
<td>0.28</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>1</td>
<td>0.72</td>
<td>0.54</td>
<td>0.44</td>
<td>0.35</td>
<td>0.34</td>
<td>0.3</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.96</td>
<td>0.71</td>
<td>0.5</td>
<td>0.45</td>
<td>0.33</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0.97</td>
<td>0.73</td>
<td>0.51</td>
<td>0.44</td>
<td>0.35</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>0.99</td>
<td>0.69</td>
<td>0.5</td>
<td>0.43</td>
<td>0.33</td>
<td>0.31</td>
<td>0.28</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.96</td>
<td>0.71</td>
<td>0.52</td>
<td>0.47</td>
<td>0.35</td>
<td>0.34</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td>0.979</td>
<td>0.716</td>
<td>0.518</td>
<td>0.451</td>
<td>0.346</td>
<td>0.325</td>
<td>0.291</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td></td>
<td>0.021</td>
<td>0.014</td>
<td>0.018</td>
<td>0.016</td>
<td>0.011</td>
<td>0.013</td>
<td>0.009</td>
</tr>
</tbody>
</table>


Table 5.4 Experimental wave inclinations (in radian). Video data are taken using high definition camera (JVC GY-HD 100).

<table>
<thead>
<tr>
<th>Video numbers</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.97</td>
<td>0.72</td>
<td>0.53</td>
<td>0.47</td>
<td>0.36</td>
<td>0.33</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.01</td>
<td>0.72</td>
<td>0.52</td>
<td>0.43</td>
<td>0.34</td>
<td>0.32</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.98</td>
<td>0.71</td>
<td>0.5</td>
<td>0.45</td>
<td>0.36</td>
<td>0.32</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>0.7</td>
<td>0.52</td>
<td>0.44</td>
<td>0.35</td>
<td>0.33</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.94</td>
<td>0.72</td>
<td>0.49</td>
<td>0.45</td>
<td>0.36</td>
<td>0.31</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.98</strong></td>
<td><strong>0.714</strong></td>
<td><strong>0.512</strong></td>
<td><strong>0.448</strong></td>
<td><strong>0.354</strong></td>
<td><strong>0.322</strong></td>
<td><strong>0.29</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td><strong>0.027</strong></td>
<td><strong>0.009</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.015</strong></td>
<td><strong>0.009</strong></td>
<td><strong>0.008</strong></td>
<td><strong>0.01</strong></td>
<td></td>
</tr>
</tbody>
</table>

5.2.4 Estimation of Distances Between Consecutive Waves

The ten videos (captured with camera Optura) with the sequence of frames for the flow rate of 0.8 lpm and the rotation of disk of 500 rpm were used to determine the distance between consecutive waves (see Table 5.5).

Table 5.5 Experimental distances between consecutive waves (mm). Video data are taken using camcorder Optura 20.

<table>
<thead>
<tr>
<th>Video numbers</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.9</td>
<td>3.9</td>
<td>3.0</td>
<td>2.5</td>
<td>2.2</td>
<td>2.1</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.6</td>
<td>3.8</td>
<td>3.0</td>
<td>2.4</td>
<td>2.1</td>
<td>1.9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.8</td>
<td>3.9</td>
<td>3.1</td>
<td>2.5</td>
<td>2.2</td>
<td>2.1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.9</td>
<td>3.7</td>
<td>2.9</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.7</td>
<td>3.9</td>
<td>2.9</td>
<td>2.3</td>
<td>2.3</td>
<td>1.9</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.6</td>
<td>3.9</td>
<td>3.1</td>
<td>2.5</td>
<td>2.2</td>
<td>2.1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>4.8</td>
<td>3.7</td>
<td>3.0</td>
<td>2.4</td>
<td>2.2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4.6</td>
<td>3.9</td>
<td>2.9</td>
<td>2.5</td>
<td>2.1</td>
<td>2.1</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4.9</td>
<td>3.6</td>
<td>3.1</td>
<td>2.3</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.7</td>
<td>3.8</td>
<td>3.1</td>
<td>2.4</td>
<td>2.4</td>
<td>2.1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>4.75</strong></td>
<td><strong>3.81</strong></td>
<td><strong>3.01</strong></td>
<td><strong>2.41</strong></td>
<td><strong>2.18</strong></td>
<td><strong>2.04</strong></td>
<td><strong>1.96</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td><strong>0.127</strong></td>
<td><strong>0.110</strong></td>
<td><strong>0.088</strong></td>
<td><strong>0.088</strong></td>
<td><strong>0.063</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.070</strong></td>
<td></td>
</tr>
</tbody>
</table>

The sequence of frames for the flow rate of 0.8 lpm and the disk rotation of 500 rpm were processed to determine the distance between consecutive waves, using the High Definition camera (see Table 5.6).
Table 5.6 Experimental distances between consecutive waves (mm). Video data are taken using high definition camera (JVC GY-HD 100).

<table>
<thead>
<tr>
<th>Video numbers</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.8</td>
<td>3.9</td>
<td>3.0</td>
<td>2.5</td>
<td>2.2</td>
<td>2.0</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.8</td>
<td>3.8</td>
<td>3.1</td>
<td>2.5</td>
<td>2.3</td>
<td>2.1</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.6</td>
<td>3.9</td>
<td>3.0</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.9</td>
<td>3.7</td>
<td>3.1</td>
<td>2.5</td>
<td>2.2</td>
<td>2.0</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.6</td>
<td>3.6</td>
<td>3.1</td>
<td>2.4</td>
<td>2.1</td>
<td>2.1</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td></td>
<td>0.134</td>
<td>0.130</td>
<td>0.0548</td>
<td>0.089</td>
<td>0.071</td>
<td>0.055</td>
<td>0.1</td>
</tr>
</tbody>
</table>

5.3 Comparison to Fluid Flow Model

The system (2.12) may be applied to describe spiral wave regimes if two parameters \( \kappa^2 \) and \( (\varepsilon/\kappa) \) are small; these conditions allow one to use the boundary layer approximation. The conditions \( (\kappa^2 \) and \( (\varepsilon/\kappa) \) are small) were examined for the observed spiral waves described above. The coordinates of points on the front of the curves in the radial direction were taken at a constant increment of azimuthal angle \( \theta = \pi/6 \). The estimated radii, \( R_1 \) and \( R_2 \) are presented in Table 5.7. The parameters relevant to

Table 5.7 Radii of the first and second waves \( R_1 \) and \( R_2 \), respectively.

<table>
<thead>
<tr>
<th>( \theta ) (radian)</th>
<th>( R_1 ) (mm)</th>
<th>( R_2 ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi/6 )</td>
<td>39.94</td>
<td>43.04</td>
</tr>
<tr>
<td>( \pi/3 )</td>
<td>48.99</td>
<td>49.28</td>
</tr>
<tr>
<td>( 1\pi/2 )</td>
<td>70.03</td>
<td>72.29</td>
</tr>
<tr>
<td>( 2\pi/6 )</td>
<td>89.97</td>
<td>91.97</td>
</tr>
<tr>
<td>( 5\pi/6 )</td>
<td>98.00</td>
<td>99.91</td>
</tr>
</tbody>
</table>

Table 5.7 are shown in Table 5.8. It can be seen from Table 5.8 that the principal conditions about small values of \( \kappa^2 \) and \( (\varepsilon/\kappa) \) are fulfilled for the spiral waves as well as for the axisymmetric waves. Thus, the boundary layer approximation extended by the terms accounting for dependence on the azimuthal angle may be formulated.

To estimate the inclination angle of non-axisymmetric waves, the eigenvalues \( \omega_\kappa \) were calculated for different values of radius under experimental conditions with \( \Omega = 520 \)
rpm and the rate of initial fluid flow $Q_c$ equal to 0.8 lpm. Examples of calculations are given in Figure 5.2. It is seen that the non-axisymmetric perturbations are more unstable than the axisymmetric ones. Then for a few radius values, wave number $\alpha_k$ and inclination parameter $n$ were found in the case of maximum amplification factors.

5.3.1 Comparison of Radial Velocities

The averaged experimental velocity of waves over fifteen videos was calculated. The calculated average and the predicted results are given in Table 5.9.

The last row of this table contains the standard deviations of calculated results. The final variation of the results over different videos is estimated as the maximum of absolute differences between the averaged results over fifteen videos. Experimentally
Table 5.9 Experimental average radial velocities and predicted radial velocities (mm/s).

<table>
<thead>
<tr>
<th>Radii (mm)</th>
<th>Velocity</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental (averaged)</td>
<td></td>
<td>801</td>
<td>742</td>
<td>684</td>
<td>634</td>
<td>572</td>
<td>537</td>
<td>523</td>
</tr>
<tr>
<td>Predicted</td>
<td></td>
<td>721</td>
<td>670</td>
<td>630</td>
<td>599</td>
<td>573</td>
<td>551</td>
<td>542</td>
</tr>
<tr>
<td>Variation (max - min)</td>
<td></td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>25</td>
<td>25</td>
<td>36</td>
<td>30</td>
</tr>
<tr>
<td>Error (%)</td>
<td></td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td>13.2</td>
<td>10.8</td>
<td>7.8</td>
<td>8.8</td>
<td>7.9</td>
<td>9.9</td>
<td>9.0</td>
</tr>
</tbody>
</table>

calculated average with variations and theoretically predicted results of radial velocities of fluid flow are illustrated in Figure 5.3. The qualitative behavior of the velocity of waves, according to Figure 5.3, is in good agreement with the asymptotic theory of wave velocity [30, 47]. The error of quantitative experimental and predicted values is within 10%.

Figure 5.3 Comparison of the experimental estimation of wave velocities and the theoretical velocities of fluid flow.
5.3.2 Comparison of Inclination Angles

The sequences of ten videos of fluid flow over a rotating disk of 500 rpm and the flow rate of 0.8 lpm were processed to find the averaged inclination angles for the radii in the range of 40-100 mm. The calculated average inclination angles and predicted inclination angles are shown in Table 5.10.

Table 5.10 Calculated average wave inclinations and predicted wave inclinations (in radian).

<table>
<thead>
<tr>
<th>Radii</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>β Experimental (average)</td>
<td>0.98</td>
<td>0.71</td>
<td>0.52</td>
<td>0.45</td>
<td>0.35</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td>β Predicted</td>
<td>1.02</td>
<td>0.69</td>
<td>0.52</td>
<td>0.42</td>
<td>0.36</td>
<td>0.32</td>
<td>0.31</td>
</tr>
<tr>
<td>Variation</td>
<td>0.07</td>
<td>0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Error (%)</td>
<td>7</td>
<td>6</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.021</td>
<td>0.012</td>
<td>0.017</td>
<td>0.015</td>
<td>0.01</td>
<td>0.011</td>
<td>0.009</td>
</tr>
</tbody>
</table>

The calculated average and the theoretically predicted results for wave inclinations are shown in Figure 5.4.

![Wave inclination angles](image)

Figure 5.4 Comparison of the experimental and theoretical wave inclinations.

In accordance with experimental observations and theoretical predictions, the inclination angle $\beta$ decreases as the radius increases.
5.3.3 Comparison of Distances

The ten videos with the sequence of frames for the flow rate of 0.8 lpm and the rotation of disk at 500 rpm were used to determine the distance between consecutive waves (see Table 5.11).

The fifteen videos with the sequence of frames for the flow rate of 0.8 lpm and the disk rotation of 500 rpm were used to determine the distance between consecutive waves (see Table 5.11).

Table 5.11 Calculated average distances over fifteen videos and theoretically calculated distances (mm).

<table>
<thead>
<tr>
<th>Distances</th>
<th>Radii (mm)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental (averaged)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.7</td>
<td>3.8</td>
<td>3.1</td>
<td>2.4</td>
<td>2.1</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Predicted</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>3.9</td>
<td>3.0</td>
<td>2.6</td>
<td>2.3</td>
<td>2.2</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Variation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>0.11</td>
<td>0.077</td>
<td>0.083</td>
<td>0.062</td>
<td>0.072</td>
<td>0.077</td>
<td></td>
</tr>
</tbody>
</table>

The calculated average and the theoretically predicted results for distances are shown in Figure 5.5.

Figure 5.5 Comparison of the experimental and theoretical distances between consecutive waves.
It can be seen in Figure 5.5 that there is a correspondence between theoretical prediction of distances and experimental data as for other variables. In addition, as predicted, the distance decreases as the radius increases.
5.4 Statistical Analysis of Estimated Fluid Flow Parameters

For investigating the difference between calculated and predicted mean values of fluid-flow parameters, the method of paired comparisons, \( t \)-test, or nonparametric Wilcoxon signed rank test [9] can be used. Those methods are designed to test if the second random variable in the pair has the same mean as the first. Since two samples (experimental and predicted) are independent, it is appropriate to use those methods. Using the Wilcoxon signed rank test, the null hypothesis is \( E_D = 0 \) and the alternative hypothesis is \( E_D \neq 0 \), where \( E_D \) is the mean of differences between predicted and experimental parameters, \( D_i = Y_i - X_i \). The absolute differences \( |D_i| = |Y_i - X_i| \), \( i = 1, ..., n \), are computed for each of the \( n \) pairs. Ranks from 1 to \( n \) are assigned to these \( n \) pairs. Then the signed rank \( R_i \) is defined for each pair as follows:

\[
R_i = \text{sign}(D_i) \cdot R|D_i|, \quad i = 1, ..., n,
\]

where \( n \) is the number of pairs.

Then the test statistic is the sum of the positive signed ranks

\[
W^+ = \sum (R_i, \text{ where } D_i \text{ is positive}).
\]

Or if \( n > 30 \), the normal approximation can be used and the test statistic is:

\[
W = \frac{\sum_i^n R_i}{\sqrt{\sum_i^n R_i^2}}.
\]

The null hypothesis is rejected at the level \( \alpha \) if \( W^+ \) (or \( W \)) is less than \( \frac{\alpha}{2} \) quantile or greater than its \( 1 - \frac{\alpha}{2} \) quantile from the Table 'Quantiles of the Wilcoxon Signed Ranks Test Statistic' for \( W^+ \) and for \( W \) from Table 'Normal Distribution' [9].
The two-tailed $p$-value is twice smaller than the one-tailed $p$-values, approximated from the normal distributions as

$$lower - tailed \ p - value = P \left( Z \leq \frac{\sum_{i}^{n} R_i + 1}{\sqrt{\sum_{i}^{n} R_i^2}} \right)$$ \hspace{1cm} (5.5)$$

or

$$upper - tailed \ p - value = P \left( Z \geq \frac{\sum_{i}^{n} R_i - 1}{\sqrt{\sum_{i}^{n} R_i^2}} \right).$$ \hspace{1cm} (5.6)

For investigation of the difference between the calculated and the predicted parameter values of fluid-flow parameters, the nonparametric Wilcoxon signed rank test [9] was used. The test statistics for wave velocities (see Table 5.9) are $W = -1.183 > -1.96$ and $t = -0.7418$. So, the null hypothesis $E_D = 0$, where $E_D$ is the mean of differences between predicted and experimental parameters, is accepted at the significant level $\alpha = 0.05$. Thus, there is not a statistically significant difference between calculated and predicted radial velocities.

Using Table 5.10, the test statistic for inclination angle $W$ equals to $-0.043033$, which is higher than $-1.96$ at a significance level of $\alpha = 0.05$. So, we fail to reject the null hypothesis at that level. Also, using the t-test, the test statistic is $t = -0.0421$ and we accept the null hypothesis of no difference between the calculated and the predicted mean values at a significance level of $\alpha = 0.05$. Thus, there is not a statistically significant difference between the calculated and the predicted inclination angles.

By investigating the difference between calculated and predicted mean values of distances between consecutive waves (see Table 5.11), we compute the test statistics $W = -0.378 > -1.96$ and $t = -0.0271$. So we accept the null hypothesis of no difference between the calculated and the predicted distance mean values at a significance level of $\alpha = 0.05$. 

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CHAPTER 6
MODEL-BASED RECOVERY OF CONTROLLING PARAMETERS

The effect of the controlling parameters such as the flow rate and rotational speed on the flow characteristics for a given set of physical parameters of fluid was studied in numerous experimental and theoretical investigations (see, for example, [2, 3, 30, 33, 48, 63]).

In the case of small flow rates, the film was observed to break up into rivulets. The classification of different flow regimes along the radius at moderate flow rates is given in [3]. At higher low rates, a smooth uniform film was formed. Upon further increase in fluid flow rate, circumferential and spiral waves moving from the disk center to the disk periphery were observed. At the highest flow rates, a combination of helical and circumferential waves was observed. Variations of the rotational speed, on the other hand, led to the modification of the shape, amplitude, and the velocity of the observed waves. Photographic evidence for wave formation is presented in [63].

Therefore, the process of recovery of controlling parameters consists of two parts: determination of a specific flow regime (circular waves, spiral waves, complex chaotic waves, etc.) and recovery of specific values of controlling parameters for a specific wave regime. This is done based on the observed wave parameters. In this work we concentrate on the second part and assume manual flow regime identification. This is reasonable as the application of this work is in visual control of the processes on the spinning disk. As such, recommended values of the controlling parameters are known. The spiral wave regime is observed in our experiments, and an appropriate mathematical model of the film flow process is selected. For this regime we enumerate
observed wave parameters (wave inclination angle, distance between waves, velocity of the wave) computed from video. Given all that, algorithms for the model-based recovery of controlling parameters are proposed.

6.1 Algorithm of Recovery Controlling Parameters

The main idea is to vary controlling parameters until predicted wave parameters are within predefined error tolerance of the actual observed wave parameters. The block-scheme of such methods is given in Figure 6.1.

![Figure 6.1 A block-scheme of determining the disk speed and fluid-flow rate.](image)

The steepest descent algorithm [53] is used to find the controlling parameters. This method starts its search in the direction of the steepest slope (anti-gradient) until an
improvement in this direction is found. After each new step a new steepest slope is
determined and the process continues until the algorithm arrives at some point where
it cannot see any improvement anymore, at this point the algorithm terminates. This
algorithm is simple and fast but could get stuck in a local minimum. To overcome
this problem to some extent, the Weighted Latin Hypercube sampling (WLHS) is used
to find the starting point for the gradient-based searching algorithm. Practically we
do not need to use WHLS because, having images given, we can assume a known
certain range of the speed of the disk and the flow rate. Thus, the problem is reduced
to the case of more exact determination of the control parameters in the frame of a
priory given. Knowing of starting control parameters close to the desired ones can be
important for the convergence of the respective algorithms of search. If the starting
point is unknown, the regression model described in (C) can be used to determine the
controlling parameters close to the realistic ones.

The Newton-Raphson method is not used since it is based on the second order
derivatives, which are less reliable in noisy images.

6.2 Estimation of Absolute and Relative Errors

Obviously, to employ any optimization methods the error criterion needs to be
defined. The least-square error criterion is used.

\[
E_a = \left( \frac{1}{N} \sum_{i=1}^{N} |x_{ci} - x_{pi}|^2 \right)^{1/2}, \quad E_{rk} = \frac{\left( \sum_{i=1}^{N} |x_{ci} - x_{pi}|^2 \right)^{1/2}}{\left( \sum_{i=1}^{N} |x_{ci}|^2 \right)^{1/2}}, \quad (6.1)
\]

where \(E_a\) is an absolute error, \(E_{rk}\) is a relative error, \(x_{ci}\) and \(x_{pi}\) are the calculated and
predicted parameters respectively, and \(N\) is a data size. Specifically to this problem,
$x_i$ are fluid flow parameters ($\beta, l$)(see Equations (3.20), (3.21)) and $E_{rk}$ is a relative error corresponding to the inclination angle or the distance.

Predicted parameters are estimated using the evaluation model described in Section 2.2.

The sample absolute errors between the experimental and predicted wave inclinations and distances between consecutive waves for different rotation disk speeds ($\Omega$ rpm) and various fluid flow rates ($Q$ lpm) are shown in Table 6.1. In this sample, the averaged ground truth experimental values (over 15 video clips with sequences of 10 frames each) of wave inclinations and distances are taken under the following conditions: the speed of disk equal to 500 rpm and the initial fluid-flow rate equal to 0.8 lpm at the radius of 40 mm. The relations between the absolute error of inclination angles and distances and speeds of the rotating disk and rates of fluid-flow are illustrated in Figure 6.2. The relation between absolute errors of the inclination angles and rotation disk speeds and the fixed fluid flow rate of 0.8 lpm is shown in Figure 6.3 (a); and the relation between absolute errors of the distances and rotation disk speeds and the fixed fluid flow rate of 0.8 lpm is shown in Figure 6.3 (b). The relation between absolute errors of inclination angles and fluid flow rates and the fixed rotating disk speed of 500

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$Q$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td></td>
<td>0.86</td>
<td>0.83</td>
<td>0.79</td>
<td>0.76</td>
<td>0.81</td>
<td>3.96</td>
<td>3.88</td>
<td>3.78</td>
<td>3.74</td>
<td>3.78</td>
</tr>
<tr>
<td>300</td>
<td></td>
<td>0.74</td>
<td>0.68</td>
<td>0.64</td>
<td>0.59</td>
<td>0.63</td>
<td>2.41</td>
<td>2.36</td>
<td>2.22</td>
<td>2.15</td>
<td>2.20</td>
</tr>
<tr>
<td>400</td>
<td></td>
<td>0.65</td>
<td>0.62</td>
<td>0.59</td>
<td>0.57</td>
<td>0.59</td>
<td>1.29</td>
<td>1.24</td>
<td>1.1</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td>0.17</td>
<td>0.11</td>
<td>0.09</td>
<td>0.03</td>
<td>0.08</td>
<td>0.27</td>
<td>0.21</td>
<td>0.18</td>
<td>0.06</td>
<td>0.19</td>
</tr>
<tr>
<td>600</td>
<td></td>
<td>0.52</td>
<td>0.49</td>
<td>0.47</td>
<td>0.45</td>
<td>0.47</td>
<td>1.19</td>
<td>1.10</td>
<td>0.99</td>
<td>0.91</td>
<td>0.94</td>
</tr>
<tr>
<td>700</td>
<td></td>
<td>0.77</td>
<td>0.73</td>
<td>0.71</td>
<td>0.71</td>
<td>0.76</td>
<td>1.72</td>
<td>1.67</td>
<td>1.62</td>
<td>1.57</td>
<td>1.61</td>
</tr>
<tr>
<td>800</td>
<td></td>
<td>0.80</td>
<td>0.79</td>
<td>0.77</td>
<td>0.74</td>
<td>0.77</td>
<td>2.52</td>
<td>2.47</td>
<td>2.41</td>
<td>2.37</td>
<td>2.44</td>
</tr>
</tbody>
</table>

Table 6.1 The absolute errors of fluid flow parameters
Figure 6.2 (a) The relation between absolute errors of inclination angles, rotating disk speeds, and fluid-flow rates; (b) the relation between absolute errors of distances, rotating disk speeds, and fluid-flow rates.

rpm is shown Figure 6.4 (a); and the relation between absolute errors of distances and fluid flow rates and the fixed rotating disk speed of 500 rpm is shown Figure 6.4 (b).

Another example with the averaged ground truth experimental values (over 15 video clips with sequences of 10 frames each) of wave inclinations and distances, taken with the disk speed of 300 rpm and the initial fluid-flow rate of 0.2 lpm at the radius of 40 mm is given in Table 6.2.
Figure 6.3 The relation between absolute errors of (a) inclination angles and speeds of rotating disk; (b) distances and speeds of rotating disk.

Estimations of inclination angles have more uncertainty than measurements of the distances. So, relative errors (6.1) are calculated and different measurements of the combined relative errors are computed using the following formula,

$$E_c = \alpha E_{r\beta} + (1 - \alpha)E_{rl},$$

where $E_{r\beta}$ is a relative error of inclination angles, $E_{rl}$ is a relative error of distances, and $\alpha$ is a constant. One of the results is shown in Table 6.3 and Figure 6.5. In this example, the average ground truth fluid-flow parameters were taken from video data.
Figure 6.4 The relation between absolute errors of (a) inclination angles and fluid flow rates; (b) distances and fluid flow rates.

with the disk speed of 500 \textit{rpm} and the rate-flow of 0.8 \textit{lpm} at the radius of 40 \textit{mm}. Another example is shown in Table 6.4 and Figure 6.6. The fluid-flow parameters were take from the same videos at the radius of 60 \textit{mm}.

The average results of ground truth wave parameters for fifteen videos are compared with the predicted wave parameters and, then, the disk speed is chosen at a minimum combined relative error (see Tables 6.5 and 6.6).

As we can see from Tables 6.5 and 6.6, the controlling parameters can be estimated with the relative error less than 10%.
Table 6.2 The absolute errors of fluid flow parameters with the speed of 300 rpm and the fluid-flow rate of 0.2 lpm.

<table>
<thead>
<tr>
<th>Ω (rpm)</th>
<th>Absolute error of wave inclinations (radian)</th>
<th>Absolute error of distances (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>200</td>
<td>0.48</td>
<td>0.53</td>
</tr>
<tr>
<td>300</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>400</td>
<td>0.65</td>
<td>0.72</td>
</tr>
<tr>
<td>500</td>
<td>0.77</td>
<td>0.86</td>
</tr>
<tr>
<td>600</td>
<td>0.82</td>
<td>0.89</td>
</tr>
<tr>
<td>700</td>
<td>0.98</td>
<td>1.01</td>
</tr>
<tr>
<td>800</td>
<td>1.02</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 6.3 The relative errors of fluid flow parameters for the various rotation disk speeds at the radius of 40 mm.

<table>
<thead>
<tr>
<th>Speed of disk rotation (rpm)</th>
<th>Relative error of wave inclinations</th>
<th>Relative error of distances</th>
<th>Combined error</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.78</td>
<td>0.79</td>
<td>0.785</td>
</tr>
<tr>
<td>250</td>
<td>0.69</td>
<td>0.46</td>
<td>0.575</td>
</tr>
<tr>
<td>300</td>
<td>0.56</td>
<td>0.28</td>
<td>0.42</td>
</tr>
<tr>
<td>350</td>
<td>0.42</td>
<td>0.20</td>
<td>0.31</td>
</tr>
<tr>
<td>400</td>
<td>0.26</td>
<td>0.12</td>
<td>0.19</td>
</tr>
<tr>
<td>450</td>
<td>0.15</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>500</td>
<td>0.05</td>
<td>0.08</td>
<td>0.065</td>
</tr>
<tr>
<td>550</td>
<td>0.028</td>
<td>0.11</td>
<td>0.069</td>
</tr>
<tr>
<td>600</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>700</td>
<td>0.45</td>
<td>0.32</td>
<td>0.385</td>
</tr>
<tr>
<td>800</td>
<td>0.64</td>
<td>0.48</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Figure 6.5 Relations of fluid-flow parameter relative errors and rotation speeds of disk at the radius of 40 mm.

Table 6.4 The relative errors of fluid flow parameters for the various rotation disk speeds at the radius of 60 mm.

<table>
<thead>
<tr>
<th>Speed of disk rotation (rpm)</th>
<th>Relative error of wave inclinations</th>
<th>Relative error of distances</th>
<th>Combined error</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.69</td>
<td>0.81</td>
<td>0.75</td>
</tr>
<tr>
<td>250</td>
<td>0.58</td>
<td>0.51</td>
<td>0.545</td>
</tr>
<tr>
<td>300</td>
<td>0.44</td>
<td>0.32</td>
<td>0.38</td>
</tr>
<tr>
<td>350</td>
<td>0.32</td>
<td>0.21</td>
<td>0.265</td>
</tr>
<tr>
<td>400</td>
<td>0.2</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>450</td>
<td>0.11</td>
<td>0.08</td>
<td>0.095</td>
</tr>
<tr>
<td>500</td>
<td>0.026</td>
<td>0.06</td>
<td>0.043</td>
</tr>
<tr>
<td>550</td>
<td>0.1</td>
<td>0.13</td>
<td>0.115</td>
</tr>
<tr>
<td>600</td>
<td>0.27</td>
<td>0.19</td>
<td>0.23</td>
</tr>
<tr>
<td>700</td>
<td>0.43</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>800</td>
<td>0.61</td>
<td>0.52</td>
<td>0.565</td>
</tr>
</tbody>
</table>
Figure 6.6 Relations of fluid-flow parameter relative errors and rotation speeds of disk at the radius of 60 mm.

Table 6.5 The relative errors of recovered disk speeds (RDS) at the combined minimal relative errors (CMRE) for video segments 1 through 8.

<table>
<thead>
<tr>
<th>Video #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMRE</td>
<td>0.026</td>
<td>0.037</td>
<td>0.063</td>
<td>0.046</td>
<td>0.053</td>
<td>0.021</td>
<td>0.063</td>
<td>0.037</td>
</tr>
<tr>
<td>RDS (rpm)</td>
<td>500</td>
<td>510</td>
<td>540</td>
<td>520</td>
<td>475</td>
<td>500</td>
<td>535</td>
<td>505</td>
</tr>
<tr>
<td>Relative errors (%)</td>
<td>0</td>
<td>2</td>
<td>8</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.6 The relative errors of recovered disk speeds (RDS) at the combined minimal relative errors (CMRE) for video segments 9 through 15.

<table>
<thead>
<tr>
<th>Video #</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMRE</td>
<td>0.056</td>
<td>0.052</td>
<td>0.057</td>
<td>0.036</td>
<td>0.087</td>
<td>0.051</td>
<td>0.0534</td>
</tr>
<tr>
<td>RDS (rpm)</td>
<td>470</td>
<td>470</td>
<td>525</td>
<td>505</td>
<td>545</td>
<td>480</td>
<td>485</td>
</tr>
<tr>
<td>Relative errors (%)</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
CHAPTER 7
SUMMARY

7.1 Conclusion

This dissertation presents a novel video-based algorithm to detect moving waves, to determine wave regimes, and to compute controlling film flow parameters. The input to this algorithm is an easily acquired non-invasive video data. The first part of the algorithm includes image analysis, tracking, and reconstruction algorithms to measure the wave shape and the wave propagating speed. It is shown that it is possible to track wave motion by observing specular portion of the reflected light on the moving wave under a single light illumination assumption. The fluid flow parameters and characteristics (velocities, thickness of film, inclination angles, and distances between consecutive waves) are calculated. Velocities and inclination angles are estimated using the so-called quasi-optimal method, which minimizes error of differentiation estimate under known error of initial data. The fluid flow parameters are compared with the solutions of the relevant computation fluid dynamics models based on the Navier-Stokes equations. The fluid models predict wave characteristics based on directly measured controlling parameters (such as disk rotation speed and fluid flow rate). From the results, we observe that the average computed parameters are within 5-10% of the predicted values.

The second part of algorithm concentrates on model-based recovery of fluid flow controlling parameters. The search in space of model parameters is performed so that the predicted flow characteristics (e.g. distance between waves, wave inclination angles) are close to those measured from video data. The aim of such algorithms is visual control
of the processes in spinning disk reactors. The method of steepest descent is used to find the controlling parameters using both the experimental video-based parameters and the theoretical model of fluid flow. Experimental results demonstrate that the speed of a disk and the flow rate are recovered with high accuracy, which supports the validity of the approach. When compared to the ground truth available from direct observation, the controlling parameters are estimated with less than 10% error.

Results presented in this work substantiate that using the developed algorithms, it is possible to accomplish the above tasks (estimation of fluid flow parameters and controlling parameters) with reasonable accuracy. We believe that the demonstrated approach will be valuable in experimental studies of wave patterns as well as suitable for practical applications of visual quality control of chemical processes.

### 7.2 Future Research

Some issues that need to be addressed in future investigations are:

1. Experiments with different sizes of the rotating disk;

2. Experiments with various physical parameters, including viscosity of liquid;

3. 3D surface recovery using multiple light illuminations;

4. Analysis of those experiments based on nonlinear solutions of the evolution system;

5. Further comparing the results of the experiments with the respective models;

6. The applicability of the theory of computer vision for the evolution systems;

7. Performing the developed system for a broader set up, for example, for the synthesis of aero gels, when a rotating disk is inside of a closed cylinder;
8. Using the developed system for fluid waves in natural condition problems including the astrophysical and geophysical problems, like global scale flow in the atmosphere, the oceans;

9. Applying this work to the medical problems, for example, for analysis of cerebral spinal fluid flow waveforms.
REFERENCES


APPENDICES
Appendix A Tracking the Peaks of Waves Using the Specular Reflection Patterns

Assuming [35] that tops of waves over a disc can be represented with enough accuracy by collections of piece-wise sinusoids, let one of those sinusoids have a form

$$Z = Asin\left(\frac{\pi}{2} - \omega X\right) - A$$

on the segment $[-\delta, \delta]$ and let $(x, z)$ be a point on that sinusoid, $-\delta \leq x \leq \delta$, ($(0, 0)$ is the coordinate of the respective top of wave projection on the plane $0XZ$), where $A$ and $\omega$ are parameters of wave approximation. Then the condition that the normal to the sinusoid in the specular point [15] $(x, z)$ crosses the point $(x_c, z_c)$ has the form

$$\frac{x_c - x}{z_c - z} = A\omega sin(\omega x),$$

from where under small $\omega \delta \sin(\omega x)$, small $A$, and large $A\omega^2 z_c$ the solution is:

$$x \approx \frac{x_c}{1 + a\omega^2(z_c - A) \approx \frac{x_c}{1 + A\omega^2 z_c} \approx \frac{x_c}{A\omega^2 z_c}.}$$

It follows from A.2 that $x$ can be close to zero under sufficiently small $\frac{x_c}{z_c}$.

Lets consider a condition when two straight lines passing through points $(x, z)$, $(x_c, Z_c)$, and $(x_l, z_l)$, have equal angles with the normal to the same sinusoid [15] in the point $(x, z)$. Using the known formula for an angle $\alpha$ between two lines,

$$\tan \alpha = \frac{k_2 - k_1}{1 + k_1 k_2},$$

where slopes of those lines, the conditions of equal angles are:

$$\left(1 - \frac{k_1}{k_2}\right) \left(\frac{1}{k_2} + k_1\right) = \left(\frac{k_3}{k_2} - 1\right) \left(\frac{1}{k_2} + k_3\right),$$

(A.3)
where $k_3$ is a slope of the second line. In our case,

\[
 k_1 = \frac{z_c - z}{x_c - x}, \quad \frac{1}{k_2} = A\omega x, \quad k_3 = \frac{z_l - z}{x_l - x}, \tag{A.4}
\]

where we can put $x_c = \delta$ and, for the simplicity, $x_c = x_l = x^*$ with the value for convenient disposition of a camera and a light. Then the obvious solution is $x = 0, z = 0$, the normal to the sinusoid is axis $0Z$, $x_l = -\delta$, $k_1 = \frac{z^*}{\delta}$ and the angle $\alpha = \arctan \frac{z^*}{\delta}$.

However, if the coordinate of $(x_c, z^*)$ and $(x_l, z^*)$ are fixed and $|x_c - x_l| \neq 2\delta$, then in the case that the segment of the size $2\delta$ under the top of the wave is in the center of the segment $[x_c, x_l]$, then $x = \frac{x_c + x_l}{2}$, $z = 0$ the normal is parallel to $0Z$, and the angle $\alpha = \arctan \left( \frac{2z^*}{x_l - x_c} \right)$.

If in addition the segment of the size $2\delta$ under the top of the wave projection is arbitrary disposed on a disk, then under condition of equality of angles above the normal in the desired point $(x, z)$ will not be parallel to $0Z$, and we have to find the solution when $(x, z)$ will be the nearest to the middle of the respective segment. Let the left side of the sinusoid have the coordinate $(x', z')$, $z' = A\cos(\omega x') - A$ and let it have maximal possible distance from $x_l$, the right end of the segment $[x_l, x_c]$. This distance does not exceed $2r$, where $r$ is the radius of a disk. The normal in the point $(x', z')$ to the sinusoid has the slope $k_2 \approx \frac{1}{A\omega x'}$, so that we have to find the condition on $z^*$ such that the lines passing through the points $(x_c, z^*)$, $(x', z')$, and the points $(x_l, z^*)$, $(x', z')$ have equal angles with the normal to the sinusoid in the point $(x', z')$. In our case,

\[
 k_1 = \frac{z^* - z'}{x_c - x'}, \quad k_3 = \frac{z^* - z'}{x_l - x'}, \quad k_2 = \frac{1}{A\omega^2 x'}. \tag{A.5}
\]
Appendix A (Continued)

Therefore, instead of (A.3) we have

\[
(z^* - z')^2 \left( \frac{1}{(x_c - x')^2} - \frac{1}{(x_l - x')^2} \right) A\omega^2 x' - \left( z^* - z' \right) \left( \frac{1}{x_c - x'} - \frac{1}{x_l - x'} \right) \left( (A\omega^2 x')^2 - 1 \right) + 2A\omega^2 x' = 0,
\]

(A.6)

from (A.6)

\[
z^* - z' = \left( \frac{(A\omega^2 x')^2 - 1}{\frac{1}{x_c - x'} + \frac{1}{x_l - x'}} \right) \pm \left( \left( \frac{(A\omega^2 x')^2 - 1}{\frac{1}{x_c - x'} + \frac{1}{x_l - x'}} \right)^2 + \frac{2}{\left( \frac{1}{x_c - x'} + \frac{1}{x_l - x'} \right)^2} \right)^{1/2}
\]

(A.7)

In the case, when \( \delta << r \) and \( x' = r - \delta, x_c = -r, x_l = r \), we have

\[
z^* - z' \approx \frac{(A\omega^2 r)^2 - 1}{\delta} - \frac{1}{2A\omega^2 r} + \left( \frac{(A\omega^2 r)^2 - 1}{\delta} - \frac{1}{2A\omega^2 r} \right)^2 + \frac{2}{\delta^2} \right)^{1/2} = \left( \frac{1}{A\omega^2 r} \right) \frac{1}{2\delta} + \frac{1}{\delta} \left( \frac{A\omega^2 r - \frac{1}{A\omega^2 r}}{4} + 2 \right)^{1/2},
\]

(A.8)

i.e., \( z^* \) will be the order \( \frac{1}{\delta} \).

Thus, arbitrary fixed disposition of \( x_c \) and \( x_l \) requires rather high disposition of \( z^* \) and certain restriction on parameters of waves. For the best accuracy, not the high disposition of \( z^* \), and without the restriction on parameters, we have to move \( x_c \) and \( x_l \) along radii in such a way that the measured tops turn out to be approximately in the middle of those coordinates.
Appendix B A Stochastic Case

Under the condition

\[ f(x) \in N(\bar{f}(x), \sigma^2), \quad P[f(x_1 \cdot x_2] = P[f(x_1)] \cdot P[f(x_2)], \quad (B.1) \]

we find estimate of \( \sigma^2 \)

\[ \sigma^* = \frac{1}{M} \sum_{\nu=1}^{M} [f(x_\nu) - \bar{f}(x_\nu)]^2, \quad |\sigma - \sigma^*| \leq \frac{c_1}{\sqrt{M}}, \quad (B.2) \]

and also

\[
\begin{align*}
|E \left[ \bar{f}'(x) - \frac{f(x+h) - f(x-h)}{2h} \right]| &= \left| \bar{f}'(x) - \frac{\bar{f}(x+h) - \bar{f}(x-h)}{2h} \right| \leq \frac{c}{6} h^2, \\
E \left[ \bar{f}'(x) - \frac{f(x+h) - f(x-h)}{2h} - \bar{f}'(x) + \frac{\bar{f}(x+h) - \bar{f}(x-h)}{2h} \right]^2 &= \\
E \left[ -f(x+h) + \bar{f}(x+h) + f(x-h) - \bar{f}(x-h) \right]^2 &= \\
\frac{1}{4h^2} [E[\bar{f}(x+h) - f(x+h)]^2 + E[\bar{f}(x-h) - f(x-h)]^2] &= \frac{1}{2} \left( \frac{\sigma}{h} \right)^2, \\
\Delta &= \left| \bar{f}'(x) - \frac{f(x+h) - f(x-h)}{2h} \right| \leq \frac{c}{6} h^2 + \left( \frac{\sigma}{h} \right)^2, \quad P \sim 0.95, \quad (B.3) 
\end{align*}
\]

from where

\[
\begin{align*}
\frac{c}{3} h - 2 \frac{\sigma^2}{h^3} &= 0, \quad h^4 = \frac{6}{c} \sigma^2, \quad h^2 = \left( \frac{6}{c} \right)^{1/2} \sigma, \\
\Delta &\leq \left( \frac{c}{6} \right)^{1/2} \sigma + \left( \frac{c}{6} \right)^{1/2} \sigma \leq 2 \left( \frac{c}{6} \right)^{1/2} \sigma \leq 2 \left( \frac{c}{6} \right)^{1/2} \sigma^* + 2 \left( \frac{c}{6} \right)^{1/2} \frac{c_1}{\sqrt{M}}. \quad (B.4) 
\end{align*}
\]
We can find $c_1$ using that $(M - 1) \frac{\sigma^2}{\sigma_2}^2$ has $\Xi^2_{M-1}$ distribution. However,

$$P \left( \left| \sigma^2 - \sigma^2 \right| \leq \frac{c_1}{M-1} \right) = P \left( \left| (M - 1) \frac{\sigma^2}{\sigma_2}^2 - (M - 1) \right| \leq c_1 \right) =$$

$$P \left( \left| \Xi^2_{M-1} - (M - 1) \right| \leq c_1 \right) = 0.95.$$  

(B.5)
Appendix C A Regression Model

The video data are used to build a regression model. The average results of experimental wave parameters (inclination angles, distances, and velocities) for various controlling parameters (speed of rotating disk and initial fluid flow rate) are calculated.

The relation between inclination angle, disk speed, and flow rate is:

\[
\frac{1}{\beta} = \beta_1 + \beta_2 x_{\text{disk}} + \beta_3 x_{\text{rate}} + \beta_4 x_{\text{disk}}x_{\text{rate}},
\]  

(C.1)

where \( \beta \) is the inclination angle and \( \beta_i, i = 1, 2, 3, 4 \), are coefficients, \( \beta_1 = 9.071e - 02 \), \( \beta_2 = 1.718e - 03 \), \( \beta_3 = -2.596e - 02 \), and \( \beta_4 = -3.598e - 04 \). The residual standard error is 0.01489 on 12 degrees of freedom, the multiple R-squared is 0.9986, and the adjusted R-squared is 0.9983.

To check if the model is correct and if the assumptions are satisfied, we plot the residuals versus the fitted values. This plot should not reveal any obvious pattern. Figure C.1 (b) plots the residuals versus the fitted values for the experimental inclination angles. No unusual structure is apparent.

An extremely useful procedure is to construct a normal probability plot of the residuals. If the underlying error distribution is normal, this plot will resemble a straight line (see Figure C.1 (c)).

Scaled residuals such as the standardized and studentized are useful in looking for outliers. Most of standardized residuals should lie in interval [-3,3], and any observation with a standardized residual outside of this interval is potentially unusual with respect to its observed response. A residual versus a leverage plot is a scatterplot of residuals against hat values. The Scale-Location plot, also called Spread-Location or S-L plot, takes the square root of the absolute residuals in order to diminish skewness (square root of the absolute residuals is much less skewed than the absolute residuals for Gaussian zero-mean residuals). The Residual-Leverage plot shows contours of equal Cook’s
distance, for values of Cook’s levels (by default 0.5 and 1) and omits cases with leverage one (see Figure C.2).
The relation of distances of speed disk and flow rate is:

\[
\frac{1}{\text{dist}} = \gamma_1 + \gamma_2 \times x_{\text{disk}} + \gamma_3 \times (1/x_{\text{rate}}) + \gamma_4 \times x_{\text{disk}} \times (1/x_{\text{rate}}),
\]

where \( \text{dist} \) distances and coefficients: \( \gamma_1 = 3.162e-02 \), \( \gamma_2 = 5.430e-04 \), \( \gamma_3 = 1.198e-03 \), and \( \gamma_4 = 2.469e-05 \). The residual standard error is 0.006687 on 12 degrees of freedom, the multiple R-squared is 0.9982, and the adjusted R-squared is 0.9978. To check if the model is correct and if the assumptions are satisfied, we plot the residuals versus the the fitted values. This plot should not reveal any obvious pattern. Figure C.3 (b) plots the residuals versus the fitted values for the experimental distances. No unusual structure is apparent. A normal probability plot of the residuals resembles the straight line (see Figure C.3 (c)). However, the underlying error distribution is normal.

The Residual-Leverage plot is shown in Figure C.4.
Figure C.3 (a) Log-Likelihood; (b) Residuals vs Fitted values; (c) Standardized Residuals; (d) Square Root of Standardized Residuals vs Fitted values.
Figure C.4 Standardized Residuals vs Leverage.
ABOUT THE AUTHOR

Valentina N. Korzhova received the MS degree in computer science from the University of South Florida (USF) in 2006, where she is currently a PhD candidate. She works as a graduate research assistant at USF, specializing in image processing and pattern recognition. Her additional interests include algorithm optimization and mathematical modeling with applications in medicine. She has published more than 28 scientific works in several conference proceedings and journals. She is a student member of the IEEE.