2011

Decision Support Models for Design of Fortified Distribution Networks

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by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Industrial and Management Systems Engineering
College of Engineering
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Date of Approval:
May 17, 2011

Keywords: Lagrangian relaxation, disruption, fortification, heuristics, nonlinear

Acknowledgements

I would like to immensely thank my advisor Dr. Alex Savachkin for his encouragement and guidance. Without his support, it would not have been possible for me to finish my study. I want to thank him for his generosity and caring, both academically and personally. He has always been willing to share his knowledge, ideas, and wisdom. He has always managed to clarify my doubts and help me with great patience. His faith in my capabilities has kept pushing me to reach higher levels in both my research agenda and personal life. It has been a great fortune of my life to have him as my advisor.

I am particularly grateful to Dr. Bo Zeng, for sharing his rich experience and enthusiasm that have tremendously helped with my research. I also want to thank Dr. Kingsley Reeves for sharing his extensive knowledge on supply chain disruptions. I also deeply appreciated Dr. Kaushal Chari for his kindness and his comments that helped me to improve my work. He always found time for me despite of his busy schedule. I would like to thank Dr. Alex Volinsky for his valuable suggestions and for sacrificing his sleeping time to attend both my proposal and dissertation defense. I would like to thank all the faculty and students from the IMSE department, especially Dr. Tapas Das. I learned a lot about his persistence, hardworking, and professionalism from him. His recommending me to my advisor helped me start this long and fruitful journey.

Finally, my most heartfelt thanks go to my family. Without their love and support, I would not have come to this point of my life.

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Abstract

Lean distribution networks have been facing an increased exposure to the risk of unpredicted disruptions causing significant economic forfeitures. At the same time, the existing literature contains very few studies that examine the impact of fortification of facilities for improving network reliability. This dissertation presents three related classes of models that support the design of reliable distribution networks. The models extend the uncapacitated *P*-median and fixed-charge location models by considering heterogeneous facility failure probabilities, supplier backups, and facility fortification within a finite budget. The first class of models considers binary fortification via linear fortification functions. The second class of models extends binary fortification to partial (continuous) reliability improvement with linear fortification. This extension allows a more efficient utilization of limited fortification resources. The third class of models generalizes linear fortification to nonlinear to reflect the effect of diminishing marginal reliability improvement from fortification investment. For each of the models, we develop solution algorithms and demonstrate their computational efficiency. We present a detailed discussion on the novelty of the proposed models. The models are intended to support corporate decisions on the design of robust distribution networks using limited fortification resources.

Chapter 1: Introduction

As a part of supply chains, distribution networks (DNs) are referred to the entire chain of intermediaries and transportation logistics for distribution of goods and services from the suppliers to the consumers. Modern distribution networks are complex engineered systems due to their size, span, the nature of customer assignation, and the network flow. At the same time, more and more enterprises have been embracing the philosophy of lean manufacturing with an ever increasing reliance on consolidated suppliers, outsourcing, slim inventories, and just-in-time production and delivery. Inasmuch as such reductionism has boosted the operational efficiency of the companies, it has also elevated their risk exposure to unpredicted disruptions. Such disruptions, as triggered by forces of nature, process hazards, and human intervention, can have a potential to entail staggering economic ramifications. This is evidenced by the following sample of recent multi-billion enterprise forfeitures lost to disrupted distribution networks.

In March of 2000, a fire event halted a Philips's semiconductor fabrication plant in New Mexico, U.S. for nine months, causing a $40 million direct sales loss to Philips and an indirect loss of $2.34 billion to Ericsson's mobile phone division (Sheffi, 2005). In March of 2001, the U.S. banned the meat import from the European Union in fear of potential spread of the foot-and-mouth disease originated in the U.K. The ban was applied to 15 countries and affected four percent of the U.S. pork supply (Marquis, 2001; Reuters, 2001). In September 11, 2001, following the terrorist attack, all U.S. boarders were closed and all flights canceled for several days. This lockdown forced Ford Motors to idle several assembly lines due to the lack of components supplied from overseas (Sheffi and Rice, 2005;

Cundari et al., 2008). Two years later, a deadly SARS outbreak disrupted among many other industries the furniture manufacturing sector of China, which accounted for about 15 percent of all furniture sold in the U.S. (Koncius, 2003b,a). More recently, in 2005, the aftermath of hurricane Katrina caused a severe disruption to the crude oil production in the Gulf of Mexico amounting nearly 1.4 million barrels a day (Mouawad and Romero, 2005; Strahan and Smith, 2005; CNN, 2008; Kotak, 2005).

Snyder and Daskin (2005) illustrated the need for reliable distribution network design in the following example. For a supply network that serves 49 cities, consisting of all state capitals of the United States and Washington DC., the optimal design of an undisrupted network is shown in Figure 1.1. This solution yielded a fixed cost of $348,000 and a transportation cost of $509,000.

Figure 1.1: A schematic of the optimal undisrupted distribution network (Snyder and Daskin, 2005)

The authors also studied the impact of disruptions on the optimal network design. If the facility in Sacramento, CA became unavailable due to disruptions (see Figure 1.2), customers assigned to this facility would have to seek their demand from the facilities in Springfield, IL and Austin, TX. As a result, the transportation cost increased to $1,081,000 (112% increase). The authors also studied several disruption scenarios, failing one facility at a time. The transportation costs associated with each scenario are shown in Table 1.1.

Figure 1.2: A schematic of the resulting optimal network with a failed facility in Sacramento (Snyder and Daskin, 2005)

Table 1.1: Transportation costs in the disrupted scenarios (Snyder and Daskin, 2005)

Location	Failure cost	% increase
Sacramento, CA	1,081,229	112%
Harrisburg, PA	917,332	80%
Springfield, IL	696,947	37%
Montgomery, AL	639,631	26%
Austin, TX	636,858	25%
Transportation cost without failures	508,858	0%

The above and some other examples (Sheffi, 2001; Christopher and Peck, 2004; Wilson, 2005; Glionna and Rotella, 2009; Carpenter, 2010) reveal the acute need for distribution networks designed to effectively balance the lean and robust requirements.

Enhancing the reliability of distribution networks can be achieved by implementation and integration of both proactive and *reactive* mitigation options, including incorporation of backup and redundancy measures, investment in reliability improvement of existing facilities (fortification), and assuring rapid post-disruption recovery. In our work, we implement disruption hedging in the form of both facility backup and fortification.

The objective of this dissertation is to contribute to addressing the issue of designing robust and agile distribution networks. To this end, we have developed three classes of mathematical models for optimal facility location, assignment of customers, and allocation of limited fortification resources for DNs exposed to facility disruptions.

The remainder of this dissertation is organized as follows. In Chapter 2, we reviewed the existing related literature on reliable network design and identified some significant limitations. We first reviewed the papers which considered localized supply rate disruptions, which were one of the most common types of disruptions found in the literature. We then explored the papers on localized production rate disruptions and disruptions caused by temporary price changes. Finally, we reviewed the literature on network-wide disruptions and network design.

In Chapter 3, we presented two related models for design of reliable distribution networks: a reliable P-median problem with binary fortification (RPMP-BF) and a reliable uncapacitated fixed-charge location problem with binary fortification (RUFL-BF). Both models consider heterogeneous facility failure probabilities, one layer of supplier backup, and facility fortification within a finite budget. We assumed that once fortified, a facility would become totally reliable (this is called binary fortification). For both models, we developed Lagrangian relaxation-based (LR) solution algorithms and demonstrated their computational efficiency. We compared the effectiveness of the LR-based solutions to that of the solutions obtained by a myopic policy which aimed to first fortify most reliable facilities regardless of the network demand topology. Finally, we discussed an alternative way to assess the effectiveness of the design solutions by using the rate of return on fortification investment.

In Chapter 4, we extended the RUFL-BF and RPMP-BF models with fortification developed in Chapter 3 by considering partial fortification whereby reliability improvements of fortified facilities were defined as continuous variables. The problems were formulated as nonlinear mixed integer programming models, RUFL-PF and RPMP-PF. For both models, we developed Lagrangian relaxation-based heuristic solution algorithms and demonstrated their computational efficiency for solving large-scale problems.

Chapter 5 extends the RUFL-PF with linear fortification to the case when the amount of reliability improvement is a nonlinear function of fortification investment. The approach developed below can also be applied to the RPMP-PF model. Finally, Chapter 6 presents a summary of main results, main contributions, and directions for future research.

Chapter 2: Literature Review

In this chapter, we review the existing relevant literature on reliable network design. We classify the literature based on the type of disruptions (supply rate, production rate, and price changes) as well as their scope (localized and network-wide). Papers which consider localized supply rate disruptions are discussed in § 2.1, followed by a review of the literature on localized production rate disruptions in § 2.2 and disruptions due to price changes in § 2.3. Papers considering localized disruptions establish a foundation for the literature on network-wide disruptions which is discussed in detail in § 2.4.

2.1 Supply Rate Disruptions

One of the most common types of disruption appearing in the production/inventory control and supply chain literature is that of supply rate changes. The work was pioneered by Meyer et al. (1979) who offered a model of a single-stage production with a constant demand where the supply was subject to a random failure. Under the assumption of Poisson machine failures, a fixed storage capacity and no setup time and/or setup cost, the authors derived performance measures, such as average inventory level and the fraction of time demand was met, for either exponentially distributed or constant repair times. Backordering was not allowed and the issue of cost minimization was not addressed. Posner and Berg (1989) extended this work to the case where demand followed a compound Poisson distribution. An explicit closed form solution for the steady-state distribution of the inventory level was derived, and this result was then used to compute system performance indices of interest related to service level to customers and machine utilization.

More recently, Arreola-Risa and DeCroix (1998) explored the management of inventory for stochastic-demand systems, where the product's supply was randomly disrupted for periods of random duration. The source of supply disruptions could be process-related or market-related. Demands that arrived when the inventory system was temporarily out of stock became a mix of backorders and lost sales. The stock was managed according to a modified (s, S) policy. The analysis yielded the optimal values of the policy parameters, explored the impact on the optimal values of the policy parameters of variations in the average frequency and duration of supply disruptions, and of variations in the fraction of stockouts that were backordered, and provided insight into the optimal inventory strategy when there were changes in the severity of supply disruptions or in the behavior of unfilled demands. Weiss and Rosenthal (1992) determined the optimal inventory policy when the timing (but not the duration) of supply disruptions was known in advance.

Parlar and Berkin (1991) studied the classic economic order quantity (EOQ) problem with supply disruptions, and Parlar and Perry (1996) considered a order-quantity/reorder-point inventory models with two suppliers subject to independent disruptions. Concepts from renewal reward processes were used to develop average cost objective function. In the case of two suppliers, spectral theory was used to derive explicit expressions for the transient probabilities of a four-state continuous-time Markov chain representing the status of the system. These probabilities were used to compute the exact form of the average cost expression. For the multiple-supplier problem, assuming that all the suppliers had similar availability characteristics, the authors developed a simple model and showed that as the number of suppliers became large, the model reduced to the classical EOQ model.

Mohebbi (2003) presented an analytical model for computing the stationary distribution of the on-hand inventory in a continuous-review inventory system with compound Poisson demand, Erlang distributed lead time, and lost sales, where the supplier can assume one of the two ”available” and ”unavailable” states at any point in time according to a continuous-time Markov chain. Exact analytical expressions were derived for the special case where demand sizes were exponentially distributed, and some cost minimization numerical results were presented. A similar approach was used in Gurler and Parlar (1997) who considered deterministic demand with two suppliers where both of them could experience disruptions with *on* and *off* periods. The *on* periods had an Erlang distribution while the *off* periods were generally distributed.

Papers addressing both supply disruptions and random demand include Chao (1987); Parlar (1997); Song and Zipkin (1996). Chao (1987) proposed a dynamic model concerning optimal inventory policies in the presence of market disruptions, which were often characterized by events with uncertain arrival time, severity and duration. The model allowed the rate of inventory accumulation or reduction to be continuously adjusted. Under a linear cost structure and the framework of a continuous-time Markov decision process with a finite state space the author developed a formulation that characterized the optimal control policy using a single inventory target. Parlar (1997) considered a continuous-review stochastic inventory problem with random demand and random lead-time with the supplier availability modeled as a semi-Markov process. The standard *(q, r)* policy was used when the supplier was available. The form of the policy changed when the supplier became unavailable in which case orders cannot be placed when the reorder point was reached. Parlar constructed the average cost per time objective function using the renewal reward theorem. Finally, Song and Zipkin (1996) explored an inventory-control model which included a detailed Markovian model of the resupply system. A number of papers on supply and demand changes have been developed in the field of oil stockpiling, as there has been grave concern over the oil supply from the Middle East. For examples, see Teisberg (1981); Chap and Manne (1982); Murphy et al. (1987).

2.2 Production Rate Disruptions

Modeling production rate disruptions (like machine failures) challenged many researchers for several decades, and numerous research efforts have been devoted to extending classical economic manufacturing quantity (EMQ) models. Rosenblatt and Lee (1986) derived an EMQ model when the production process was subject to a random deterioration from an in-control state to an out-of control state. A somewhat similar approach was taken in Porteus (1986) who proposed a model to determine an optimal lot size under the following assumptions: while producing a lot, the process could go out-of control with a given probability, and the process continued to produce defective items until the entire lot was produced. The process was presumed to be in control before starting production of a new lot. Lee (1992) modeled the defect-generating process in the semiconductor wafer probe process to determine an optimal lot size, which reduced the average processing time on a critical resource.

Abboud (1997) presented a simple approximation of the EMQ model with Poisson machine breakdowns and low failure rate. Groenevelt et al. (1992b) studied an unreliable production system with constant demand and random breakdowns, with the focus on the effects of machine failure and repair on optimal lot-sizing decisions. Assuming exponentially distributed time between failures and instantaneous repair of the machine, the authors derived some unique properties of their model compared to the classical EMQ model. Since it was assumed that machine restoration times were negligible, Groenevelt et al. (1992b) only addressed the lot-sizing problem. Groenevelt et al. (1992a) extended their earlier work in Groenevelt et al. (1992b) to the case where repair times were randomly distributed and excess demand was lost.

Kim and Hong (1997) presented an extended EMQ model which determined an optimal lot size for a failure prone machine. It was assumed that time between failures of a machine was generally distributed, and a machine was repaired instantaneously when it failed. Variations of an EMQ depending on repair cost were also examined. Extensive numerical investigations were carried out on the effects of repair cost and setup cost to an EMQ as well as average cost. Hopp et al. (1989) presented a model that assumed the (s,S) control policy. With Poisson failures and exponential repair times, a cost function was derived when backordering was not allowed. Among other notable examples of such works are Henig and Gerchak (1990) and Buzacott and Shantikumar (1993).

Nearly all the work done in this area follows the traditional operations management approach: 1) identify and characterize the disruption, and 2) formulate the situation that was caused by it in a model that would prescribe optimal policies to the situation that was modeled originally. In contrast, there was a proposed methodology which took a different approach to such situations. Rather than optimize the system using the pre-disruption objective function, the methodology attempted to preserve, as much as possible, the original operational plan even if this meant non-optimal solutions to the original objective function. The rationale behind this approach was that in many cases there were significant implicit costs that were involved in breaking away from the original plan. These implicit costs were difficult to formulate and estimate, and naturally, they were not represented in the original model. The literature on the approach is still rather scant (Golany et al., 2002; Xia et al., 2001; Yang et al., 2004).

2.3 Temporary Price Changes and Financial Metrics

Temporary price changes (disruptions) have also attracted interest among operations management researchers. Basic price discount models were formulated in the 1960s (e.g., Naddor (1966)). Taylor and Bradley (1985) extended the basic model to situations in which the price change became effective at any time in the future (originally - at the end of the next cycle). Aull-Hyde (1992) extended the model to situations in which there were limits on the quantities that could be purchased at the discounted price. Arcelus and Srinivasan

(1995) analyzed the price disruption interval by looking at a minimal order quantity on discounted purchases and determined optimal policies for various cases. Tersine and Barman (1995) focused on a short disruption period that allowed only one special purchase. Ardalan (1995) emphasized the differences between a net present value model as opposed to a no-discount model for temporary price reduction.

The risk of supply chain disruptions, as an indication of an enterprise's inability to match demand and supply, has been receiving increased attention. Recent supply chain woes at Cisco, Sony, Nike, and Ericsson and others have been written about (see, for example Thurm (2001); Tran (2000); Latour (2001); Engardio (2001)). A number of researchers proposed some financial metrics in an attempt to *empirically* estimate the economic impact of unreliable and unresponsive supply chains, in particular the impact of supply chain disruptions on the shareholder value lost; the rationale was that unreliable and unresponsive supply chains were more prone to be affected by supply and demand disruptions (glitches), and if supply chains were more reliable and responsive they would not have experienced the glitches, and hence, would not have experienced the loss in shareholder value. Hendricks and Singhal (2003) suggested an estimate of the effect of supply chain disruptions on shareholder wealth (abnormal stock returns) and long-run stock price performance and equity risk. Among others, Radjou (2002); Billington et al. (2001); Lee et al. (1997); Fisher (1997) contributed to widespread recognition that supply chain disruptions had the potential to trigger significant economic impacts.

2.4 Network-Wide Disruptions and Network Reliability

The topic of system unreliability in the production/inventory context has also attracted interest among operations management researchers as represented in the sample of works we describe here. Gallego (1988a,b) examined the classical economic lot-sizing model with single and multiple disruptions. Posner and Berg (1989) superimposed the reliability feature comprising the machine failure process and the ensuing repair actions. Bielecki and Kumar (1988) investigated the optimality of zero-inventory policies in production systems with uncertain manufacturing capacity. They showed that under constant demand rate there were ranges of parameter values describing an unreliable manufacturing system for which zero-inventory policies were exactly optimal even when there was uncertainty in manufacturing capacity.

Gallego (1990) investigated the problem of scheduling the production of several items in a single facility where demands were random with constant expected rates. Items were produced at continuous constant rates. With demands replaced by their expectations an optimal or near-optimal target cyclic schedule was computed, and the problem of scheduling the facility after a single perturbation was analyzed. Moinzadeh and Aggarwal (1997) analyzed a single localized unreliable bottleneck facility with a constant production and demand rate that was subject to random disruptions. The time between breakdowns was assumed to be exponentially distributed while the restoration times were constant. The authors employed an (s,S) production policy and developed expressions for evaluating the probability distribution of the number of production runs in a cycle together with its first two moments, the average cycle time, the average on-hand inventory and backorder levels, and the expected total cost rate of the system. They also investigated the properties of the average total cost rate and the policy parameters with changes in reliability and other system parameters. However, the authors left to future work the case of random demand and/or production rates and a stochastic duration of the disruption period.

Abboud (2001) examined a single machine production and inventory system with a deterministic production and demand rate, when the machine was subject to random failures. The machine times to failure and repair times were random, and during repairs, demand was backordered as long as the backordering level does not exceed a prescribed amount, after which demand was lost. Considering time in discrete units and the times to failure and repair times to be geometrically distributed, the author modeled the production/inventory system as a Markov chain and developed an algorithm to compute the potentials that were used to formulate the cost function. Later, Rahim (1994) presented a model for determining an economic manufacturing quantity, inspection schedule and control chart design of an imperfect production process, where he assumed that the process was subject to the occurrence of a non-Markovian shock having an increasing failure rate.

More recently, there have been some efforts on expanding the classical *P*-median problem and uncapacitated fixed-charge facility location problem. In what follows, we first give a short description of the two models. The *P*-median problem is to locate up to *P* facilities in order to satisfy the demand of the customers so as to minimize the total transportation cost. The costs for opening facilities are disregarded (Narula et al., 1977). The uncapacitated fixed charge problem is different from the *P*-median problem in that it additionally considers a fixed facility construction cost in the objective function and relaxes the stipulation that dictates the number of facilities to be located (Owen and Daskin, 1998). The resulted problem seeks to determine the number of facilities to locate so as to minimize total (construction plus transportation) costs.

The recent literature features a number of studies on facility location problem in the presence of random disruptions. A comprehensive review of these works can be found in Snyder (2006). Below we present an up-to-date summary of the most relevant papers in this area (see also Table 2.1). The rest of the section follows a literature review summary presented in our work (Li et al., 2011).

Snyder and Daskin (2005) presented two reliability models for facility location: a reliable *P*-median and a reliable uncapacitated fixed-charge location model. Facility failure probabilities were assumed to be equal and mutually independent. Simultaneous failure of multiple facilities was allowed. Each customer had a constant demand. In both models, each customer was assigned a primary supplier and a number of backup suppliers, of which at least one was required to be totally reliable. If the current supplier failed, the customer was served by the next available backup supplier. The objective was to minimize the expected total cost. The main contribution of this paper is that the models not only determine the optimal facility locations but also assign r levels of backup facilities to each demand node. The main limitation of this paper, however, is the assumption that each facility fails with the same probability.

Cui et al. (2010) relaxed the assumption of homogeneous failure probabilities in Snyder and Daskin (2005) to the case of location specific probabilities. The authors built a mixed integer program formulation and a continuum approximation model to minimize the network initial setup costs and expected transportation costs under normal and failure scenarios. Li and Ouyang (2010) further expanded this direction by considering correlated and site specific failure probabilities.

A few recent papers have taken the analysis one step further and examined the impact of facility fortification for reliability improvement of the network. Church et al. (2004) examined two related network interdiction problems: the r-interdiction median and the r-interdiction covering problem. Both models were based on the P-median problem. The r-interdiction median problem sought to find a subset of $r \leq P$ facilities, which if removed from the network, would cause the highest loss of the network throughput. Whereas, the r-interdiction covering problem aimed to find such a subset which would result in the maximal network coverage loss. In both models, once the critical subset was identified, some of its members could be fortified, as was done in later papers by Church and Scaparra (2007); Scaparra and Church (2008a,b).

So far, to the best of our knowledge, the only effort discussing the problem of network design with fortification is by Lim et al. (2010), see Table 2.1. The authors analyzed the uncapacitated fixed-charge facility location model with two types of facilities: unreliable and totally reliable or "hardened". The facility failure probabilities were assumed to be independent and location specific. The model assumed one primary supplier and one *totally reliable* backup supplier for each customer. The objective of the model was to determine the optimal number and location of both types of facilities as well as the customer assig-

Table 2.1: Summary of the reliable facility location literature (Li and Savachkin, 2011)

Research component	Snyder & Daskin (2005)	Cui et al. (2010)	Li & Ouyang (2010)	Church et al. (2004-2008)	Lim et al. (2010)
Modeling approach	RPMP, RFLP	RUFL	RUFL	RIM, RIMF	FRP
Failure events	independent	independent	correlated	-	independent
Failure probability	homogeneous	SS	SS	-	SS
Layers of backup	multiple	multiple	-	-	one
Fortification budget	-	-	-	-	-
Solution approach	LR	CA	CA	IE	LR

RPMP - reliable P-median problem; RFLP - reliable fixed charge location problem; RUFL - reliable uncapacitated facility location; RIM - *r*-interdiction median; RIMF - RIM with fortification; FRP - facility reliability problem; SS - site-specific; LR - Lagrangian relaxation; CA - continuum approximation; IE - implicit enumeration

nation. The model was formulated as an integer programming model and a Lagrangian relaxation-based solution algorithm was developed. Although the authors incorporated the fixed cost of locating a reliable facility in the objective function, the available fortification budget was not considered. As such, the formulation essentially assumed an unlimited budget. Since this assumption does not restrict the number of reliable facilities, the optimal solution may not fit available fortification resources.

To summarize, the literature addressing the impact of random disruptions on distribution networks and supply chains is still rather scant but is growing. A significant body of works that explicitly considers random disruptions is on traditional issues of inventory management, production lot sizing, production scheduling, ordering policies, and cost management of inventory, setup, and backorder costs. A handful of papers that analyzes network-wide disruptions have a number of limitations including the following: (i) in most cases, failure scenarios are somewhat simplified; (ii) the discussion on fortification strategies is very limited and does not analyze the efficiency of the strategies; (iii) the distribution centers are considered to have unlimited capacity and hence, customer assignation is virtually unrestricted; (iv) customer demands are deterministic; (v) most models consider only single-commodity networks. It therefore appears that there exist some gap in the decision support capabilities that the existing literature can provide for design of modern distribution networks. This dissertation attempts to fill some of the vacuum, particularly in integration of facility fortification into network design decisions. The developed models are intended to provide more realistic solutions to the existing issues and ultimately support corporate decision makers in the design of reliable distribution networks using limited fortification resources.

Chapter 3: Reliable Distribution Networks with Binary Fortification

In this chapter, we develop two related models for facility location design under the risk of disruptions: a reliable P-median problem with binary fortification (RPMP-BF, Section 3.1) and a reliable uncapacitated fixed-charge location problem with binary fortification (RUFL-BF, Section 3.2). Similar to Cui et al. (2010); Lim et al. (2010), in both our models, we assume that the facility failure probabilities are independent and location specific. As in Lim et al. (2010), we also assume one layer of supplier backup. To further enhance the network reliability, we incorporate fortification of selected facilities. As a result of fortification, the facility reliability is improved at some cost. The cost of facility fortification is considered to be location specific and made up of two components: a fixed setup cost and a variable cost for reliability improvement. In both models, we assume that if fortified, the facility becomes totally reliable. Both models incorporate a finite fortification budget constraint. Both models seek to choose the optimal facility location and fortification strategy as well as the assignment of customers.

Both the RPMP-BF and RUFL-BF problems are formulated as nonlinear integer programming models which are shown to be $\mathscr{N}\mathscr{P}$-hard. For both models, we develop Lagrangian relaxation-based solution algorithms (Sections 3.1.2 and 3.2.2). We present computational results demonstrating the efficiency of the developed algorithms (Sections 3.3.2 and 3.3.3). We compare the effectiveness of the LR-based solutions to that of the solutions generated by a myopic policy which aims to fortify most reliable facilities regardless of the demand topology (Section 3.3.4). The comparison is done at different levels of the fortification budget. We also discuss a way to assess the effectiveness of the design solutions by determining the rate of return on fortification investments (Section 3.3.5).

Comparing to Lim et al. (2010), our work presents the following *main advances*:

(i) Our model incorporates the fortification budget constraint. As a result, the model provides a more realistic decision support for network design where the optimal solution is matched to the available reliability improvement resources, no matter how scarce or abundant they are.

(ii) Our formulation enables a decision maker to assess the rate of return on fortification investment and compare it to that of alternative investment opportunities. For example, a company may choose to invest in network fortification only if the rate of return exceeds the minimum acceptable rate of return (MARR, Rogers (2001)).

(iii) Our model allows periodic fortification upgrades whereby reliability of an existing network can be improved as additional fortification budget becomes available. Examples include gradual release of fortification resources or availability of excess cash flow which can be channeled into fortification. To allocate additional fortification budget for an existing network, the model has to be re-solved with fixed facility location decision variables. This ability to support gradual fortification results from incorporation of the budget constraint and separation of the location selection and fortification decision variables, which are combined in Lim et al. (2010).

3.1 The Reliable *P*-Median Problem with Binary Fortification (RPMP-BF)

The model extends the reliable P-median facility location problem introduced in Snyder and Daskin (2005) by considering heterogeneous facility failure probabilities and facility fortification. The model seeks to minimize the total expected transportation cost by optimally locating P facilities, allocating a finite fortification budget, and assigning the customers. We first formulate this problem as a nonlinear integer programming model and then develop a Lagrangian relaxation-based solution algorithm. The rest of the section follows our work in Li et al. (2011).

3.1.1 Problem Formulation

We define I to be the set of customers, J the set of potential facility locations, and P the number of facilities to open. Each customer $i \in I$ has demand h_i. Let $d_{ij} \geq 0$ be the cost of transporting one unit of demand from facility location $j \in J$ to customer i (with the convention that $d_{ii} = 0\ \forall i$). Associated with each facility j is the failure probability $0 \leq q_j \leq 1$. The events of facility failures are assumed to be independent, as in Snyder and Daskin (2005); Cui et al. (2010); Lim et al. (2010). Once a facility fails, it becomes unavailable. Each customer is assigned a primary supplier and a different backup supplier (as in Lim et al. (2010)). While Lim et al. (2010) required each backup facility to be "totally reliable" (i.e., available at all times), we stipulate that for any customer, the probability of a simultaneous failure of its primary and backup supplier is negligible. Hence, we assume that for any customer, if its primary supplier fails, the backup supplier is available.

Our model incorporates facility fortification whereby reliability of facilities can be improved at some cost. We assume that if a facility is fortified, it becomes non-failable. The total cost of fortifying facility j includes the setup cost and the variable cost components. The setup cost S_j is a fixed cost required to implement facility fortification (examples include the costs of R&D, contract negotiation, overhead, personnel training, etc.). The variable fortification cost varies with the amount of reliability improvement of the facility. Examples include the cost of acquiring and installing the units of protective measures, the cost of procurement and storage of backup inventory, and the cost of hiring extra workforce. We define r_j as the cost associated with the unit reduction in the failure probability of facility j. Our model incorporates a total available fortification budget B. Finally, the facilities are assumed to have unlimited capacity to guarantee that all demands will be met (as in Snyder and Daskin (2005); Cui et al. (2010); Lim et al. (2010)).

Our model incorporates the following *decision variables*:

$$X_j = \begin{cases} 1, & \text{if a facility is opened at location } j; \\ 0, & \text{otherwise,} \end{cases}$$

$$Y_{ij0} = \begin{cases} 1, & \text{if customer } i \text{ has facility } j \text{ as its primary supplier;} \\ 0, & \text{otherwise,} \end{cases}$$

$$Y_{ij1} = \begin{cases} 1, & \text{if customer } i \text{ has facility } j \text{ as its backup supplier;} \\ 0, & \text{otherwise,} \end{cases}$$

$$Z_j = \begin{cases} 1, & \text{if facility } j \text{ is fortified;} \\ 0, & \text{otherwise.} \end{cases}$$

We formulate the problem as follows.

(RPMP-BF)

minimize $\sum_{i\in I}\sum_{j\in J}\left[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\in J,\, r\neq j} q_r Y_{ir0}(1-Z_r)\right]$

subject to

$$\sum_{j\in J} Y_{ij0} = 1, \ \forall i \in I \tag{3.1a}$$

$$\sum_{j\in J} Y_{ij1} = 1, \ \forall i \in I \tag{3.1b}$$

$$Y_{ij0} + Y_{ij1} \le X_j, \ \forall i \in I, \, j \in J \tag{3.1c}$$

$$\sum_{j\in J} X_j = P \tag{3.1d}$$

$$\sum_{j\in J} (S_j + r_j q_j) Z_j \le B \tag{3.1e}$$

$$X_j, Z_j \in \{0,1\}, \ \forall j \in J \tag{3.1f}$$

$$Y_{ij0}, Y_{ij1} \in \{0,1\},\ \forall i \in I,\ j \in J. \tag{3.1g}$$

The objective function above is the expected total transportation cost associated with satisfying the demands of all customers. The term $\sum_{j\in J} h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j))$ represents the part of the expected transportation cost associated with customer i served by its primary supplier, where $(1-q_j(1-Z_j))$ is the probability that the supplier is available. The term $\sum_{j\in J} h_i d_{ij} Y_{ij1} \sum_{r\neq j} q_r Y_{ir0}(1-Z_r)$ is the cost of customer i served by its backup supplier, where $\sum_{r\neq j} q_r Y_{ir0}(1-Z_r)$ is the probability that the primary supplier failed (recall that in this case, the backup facility is assumed to be available).

Constraints (3.1a) and (3.1b) respectively assure that each customer is assigned only one primary and one backup supplier. Constraint (3.1c) serves two purposes. First, it guarantees that only an open facility can serve as a supplier. It also assures that for each customer, its primary and backup suppliers are different facilities. Constraint (3.1d) demands P facilities to be opened. Constraint (3.1e) is the total fortification budget constraint. Finally, (3.1f) and (3.1g) are the integrality constraints.

The next theorem shows that the model above is $\mathscr{NP}$-hard.

Theorem 1 *The RPMP-BF is $\mathscr{NP}$-hard.*

Proof. We prove this by showing that a special case of the RPMP-BF is $\mathscr{NP}$-hard. Consider $I = J$ and $P = ||I||$. It then follows that there will be a facility open at each customer location. Assume that one of the facilities is totally reliable; assign index s to it (i.e., $q_s = 0$). We also assume that for each customer location $i \neq s$, $d_{is} < \infty$ and $d_{ij} = \infty$ for $j \neq s$ (recall that $d_{ii} = 0\ \forall i$).

Note that since $q_s = 0$ and $d_{ss} = 0$, customer s is assigned facility s as the primary supplier and no backup supplier is needed in this case. Also note that for each customer $i \neq s$, its primary and backup supplier will be assigned as facility i or facility s. In the case when facility i is chosen as the primary supplier and facility s as the backup, the expected transportation cost associated with customer i becomes $h_i d_{is} q_i$. In the other case, the cost is

$h_i d_{is}$. Since $h_i d_{is} q_i \leq h_i d_{is}$, supplier i and s will be chosen as the primary and backup supplier, respectively.

At this point, since facility locations and customer assignations are determined, the problem reduces to selecting facilities for fortification. Note that fortification of facility i eliminates the need for its backup supplier, which results in the expected reward $h_i\, d_{is}\, q_i$. The problem then is to maximize the total expected reward gained from fortification subject to the fortification budget availability:

max $\left\{ \sum_{i \in I} h_i\, d_{is}\, q_i\, Z_i : \sum_{i \in I} (S_i + r_i\, q_i)\, Z_i \leqslant B, Z_i \in \{0,1\} \right\}$, which is the 0-1 knapsack problem.

3.1.2 RPMP-BF: Lagrangian Relaxation

As shown above, the RPMP-BF is $\mathcal{NP}$-hard and has a nonlinear objective function. One possible solution is to linearize the model by introducing new variables, $U_{ij0} = Y_{ij0} Z_j$, $V_{ijr} = Y_{ij1} Y_{ir0}$, and $W_{ijr} = V_{ijr} Z_j = Y_{ij1} Y_{ir0} Z_j$, with necessary constraints. However, the resultant problem becomes excessively large even for moderately sized networks, which makes solving such cases using commercial solvers challenging (see also § 3.3). This motivates us to develop a Lagrangian relaxation-based algorithm.

Lagrangian relaxation is considered an important computational technique in mathematical programming. The main idea is to relax the hard constraints of a mathematical programming problem and put them into the objective function with assigned weights (the Lagrangian multipliers) . The relaxed problem provides a lower bound (for minimization problem) on the optimal value of original problem. Integrated with other techniques (e.g., subgradient optimization), Lagrangian relaxation approach has led to dramatically improved algorithms for a number of operations research problems (Fisher, 2004).

3.1.2.1 Lower Bound

Relaxing the set of constraints (3.1c) using Lagrange multipliers u_{ij} yields the following subproblem.

(RPMP-BF-LG)

$$\min \sum_i \sum_j \left[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\neq j} q_r Y_{ir0}(1-Z_r)\right] + \sum_i \sum_j u_{ij}(Y_{ij0}+Y_{ij1}-X_j)$$

subject to (3.1a), (3.1b), (3.1d) – (3.1g), and $Y_{ij0}+Y_{ij1} \leq 1$.

The objective function above can be rewritten as follows.

$$\begin{aligned}
&\sum_i \sum_j \left[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\neq j} q_r Y_{ir0}(1-Z_r)\right] + \sum_i \sum_j u_{ij}(Y_{ij0}+Y_{ij1}-X_j) \\
&= \sum_i \sum_j \left[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\neq j} q_r Y_{ir0}(1-Z_r) + u_{ij}(Y_{ij0}+Y_{ij1})\right] - \sum_j \sum_i u_{ij} X_j \\
&= \sum_i \sum_j \left\{ \left[h_i d_{ij}(1-q_j(1-Z_j)) + u_{ij}\right] Y_{ij0} + \left[h_i d_{ij} \sum_{r\neq j} q_r Y_{ir0}(1-Z_r) + u_{ij}\right] Y_{ij1} \right\} - \sum_j \sum_i u_{ij} X_j.
\end{aligned}$$

For a given $\mathbf{u}$, the optimal value of $\mathbf{X}$ can be found by ranking the values of $(-\sum_i u_{ij})$ for all j and setting $X_j = 1$ if $(-\sum_i u_{ij})$ is among the P smallest ranked values, and setting $X_j = 0$ otherwise.

To solve the rest of the problem, we first consider the case $B = 0$. Then **Z = 0** and constraint (3.1e) can be eliminated. The simplified problem is shown below.

(M1)

$$\min \sum_i \sum_j \left\{ \left[h_i d_{ij}(1-q_j) + u_{ij}\right] Y_{ij0} + \left[h_i d_{ij} \sum_{r\neq j} q_r Y_{ir0} + u_{ij}\right] Y_{ij1} \right\}$$

subject to (3.1a), (3.1b), (3.1g).

Note that relaxing constraints (3.1c) allows a customer to be assigned to a facility which is not open. Constraints (3.1a) and (3.1b) still assure that each customer is assigned only one primary and one backup supplier. Note that (M1) is separable in i, so that in order to solve the problem, it suffices to optimally assign a primary and a backup supplier to each customer. For a given customer i, if facility v and w are selected as the primary and backup supplier, respectively, the objective function of (M1) associated with customer i becomes

$\Phi_i(v,w) = h_i d_{iv}(1-q_v) + u_{iv} + h_i d_{iw} q_v + u_{iw}$. To find the optimal assignment, we enumerate the values of $\Phi_i(v,w)$ for all $v,w \in J$ to find $\Phi_i^* = \min_{v,w}\{\Phi_i(v,w)\}$.

Now we consider the case $B > 0$. Suppose again that customer i is assigned facility v and w as the primary and backup supplier, respectively. Suppose now that facility v is fortified (i.e., $Z_v = 1$). The objective function of the simplified problem (for customer i) then becomes $\Psi_i(v,w) = h_i d_{iv} + u_{iv} + u_{iw}$. Let $\Psi_i^*(v) = \min_{w}\{\Psi_i(v,w)\}$.

We now let $E_i(v) = \max\{\Phi_i^* - \Psi_i^*(v), 0\}$. In other words, $E_i(v)$ is an improvement, if any, gained from fortifying facility v, for customer i. Then the objective is to maximize the utilization of the fortification budget over all $v \in J$, for all customers $i \in I$. For this purpose, we first introduce a new variable K_{ij} as follows.

$$K_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is assigned a fortified primary supplier } j; \\ 0, & \text{otherwise.} \end{cases}$$

Then the problem becomes as following.

(M2)

max $\sum_i \sum_j E_i(j) K_{ij}$

subject to

$$K_{ij} \le Z_j \ \ \forall i \in I,\ \forall j \in J \tag{3.2a}$$

$$\sum_j K_{ij} \le 1 \ \ \forall i \in I \tag{3.2b}$$

$$\sum_j (S_j + r_j q_j) Z_j \leqslant B \tag{3.2c}$$

$$K_{ij} \in \{0,1\} \ \ \forall i \in I,\ \ \forall j \in J \tag{3.2d}$$

$$Z_j \in \{0,1\} \ \ \forall j \in J. \tag{3.2e}$$

The objective function of (M2) is to maximize the total improvement from fortification when compared to the optimal objective function value of (M1). Subtracting the optimal objective function value of (M2) from the optimal objective function value of (M1) gives the optimal objective function value of the (RPMP-BF-LG). The set of constraints (3.2a) assures that facility j, as the primary supplier of customer i, must be fortified in order to realize improvement $E_i(j)$. The set of constraints (3.2b) guarantees that customer i gets assigned to no more than one fortified supplier. Constraint (3.2c) is the fortification budget constraint. Constraints (3.2d) and (3.2e) are standard integrality constraints. (M2) can be solved by using CPLEX.

3.1.2.2 Upper Bound

At each iteration of the Lagrangian procedure, a lower bound and an upper bound for (RPMP-BF) are obtained. The solution to (RPMP-BF-LG) provides a lower bound. If the solution is feasible, it also provides an upper bound, which is then optimal for (RPMP-BF). Otherwise, if the solution is infeasible, we construct a feasible solution which becomes an upper bound. We apply the following *heuristic*.

In the solution of (RPMP-BF-LG), exactly P facilities are open. For each customer, we select the closest and second closest open facility as the primary and backup supplier, respectively. To decide which facilities to fortify, we let $G(j)$ be the set of customers who have facility j as the primary supplier.

For each customer $i \in G(j)$, if facility j is not fortified, the corresponding expected transportation cost is $h_i d_{ij}(1-q_j) + h_i d_{ir} q_j$, where r represents its backup supplier. If facility j is fortified, the expected cost becomes $h_i d_{ij}$. The total expected cost reduction from fortifying facility j is then $\varphi_j = \sum_{i \in G(j)} h_i(d_{ir} - d_{ij}) q_j$. Thus, the objective is to maximize the utilization of the fortification budget over P open facilities. The problem becomes: $\max\left\{\sum_j \varphi_j Z_j : \sum_j (S_j + r_j q_j) Z_j \leqslant B, Z_j \in \{0,1\}\right\}$.

This is a knapsack problem which can be solved by CPLEX rather easily. The described heuristic performed well in the computational tests (see § 3.3.2).

3.1.2.3 Multiplier Initiation and Updating

As discussed in Snyder and Daskin (2005), the performance of Lagrangian relaxation algorithms can be sensitive to the choice of initial multipliers. In order to obtain a good initial multiplier, we examined the final multipliers of the cases where (RPMP-BF) was solved to optimality. We found that the formula $u_{ij} = h_i / \|I\|$ generated efficient initial multipliers for our problem.

Once the algorithm starts running, at each iteration k, we use subgradient optimization Fisher (2004) to update $\mathbf{u}$ by setting

$u_{ij}^{k+1} = u_{ij}^{k} + t_k \left(Y_{ij0} + Y_{ij1} - X_j\right),$

where t_k is a step size, $t_k = \dfrac{\lambda_k (z^* - z(\mathbf{u}^k))}{\| Y_{ij0} + Y_{ij1} - X_j \|^2}$.

In the formula above, λ_k is a constant at iteration k, initially set to $\lambda_0 = 2$, as in Fisher (2004). We divide the values of λ_k by 2 when every 60 consecutive iterations fail to improve the lower bound. Also, z^* is the best known upper bound, and $z(\mathbf{u}^k)$ is the lower bound when the multipliers are equal to $\mathbf{u}^k$.

The algorithm terminates when any of the following criteria are met:

- $(z^* - z(\mathbf{u}^k))/z^* \leq \varepsilon$, for some optimality tolerance ε, specified by the user,
- $k > k_{max}$, for some iteration limit k_{max}.

3.2 The Reliable Uncapacitated Fixed-Charge Facility Location Model with Binary Fortification (RUFL-BF)

The RUFL-BF model can increase the network reliability of the RPMP-BF-based solutions by relaxing the restriction on the number of open facilities. Our RUFL-BF model extends the reliable fixed-charge facility location problem introduced by Snyder and Daskin (2005) by considering heterogeneous facility failure probabilities and facility fortification. The model seeks to minimize the sum of the total facility construction cost and the expected transportation cost by optimally selecting facility locations, allocating a finite fortification budget, and assigning the customers. RUFL-BF is formulated as a nonlinear integer programming model.

3.2.1 Problem Formulation

The formulation is similar to RPMP-BF but adding cost f_j of constructing facility j.

(RUFL-BF)

minimize $\sum_{j\in J} f_j X_j + \sum_{i\in I}\sum_{j\in J}\left[h_i d_{ij} Y_{ij0}(1-q_j(1-Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r\in J,\, r\neq j} q_r Y_{ir0}(1-Z_r)\right]$

subject to

$$\sum_j Y_{ij0} = 1,\ \forall i \in I \tag{3.3a}$$

$$\sum_j Y_{ij1} = 1,\ \forall i \in I \tag{3.3b}$$

$$Y_{ij0} + Y_{ij1} \le X_j,\ \forall i \in I, j \in J \tag{3.3c}$$

$$\sum_j (S_j + r_j q_j) Z_j \leqslant B, \tag{3.3d}$$

$$X_j, Z_j \in \{0,1\},\ \forall j \in J \tag{3.3e}$$

$$Y_{ij0}, Y_{ij1} \in \{0,1\},\ \forall i \in I,\ j \in J. \tag{3.3f}$$

The formulation is similar to (RPMP-BF) except that in (RUFL-BF), the total construction cost is included in the objective function and the number of facilities to be opened is not restricted to P. Similar to Snyder and Daskin (2005); Cui et al. (2010); Lim et al. (2010), our formulation does not consider a construction budget.

Theorem 2 *The RUFL-BF is $\mathcal{NP}$-hard.*

Proof: Note that when $B = 0$ and $q_j = 0$ for all $j \in J$, the RUFL-BF becomes the classical uncapacitated fixed-charge location problem.

3.2.2 Solution Method

Lower bound. We relax constraints (3.3c) to obtain the following subproblem.

(RUFL-BF-LG)

$$\min \sum_j f_j X_j + \sum_i \sum_j \left[h_i d_{ij} Y_{ij0} (1 - q_j(1 - Z_j)) + h_i d_{ij} Y_{ij1} \sum_{r \neq j} q_r Y_{ir0} (1 - Z_r) \right]$$
$$+ \sum_i \sum_j u_{ij} (Y_{ij0} + Y_{ij1} - X_j)$$

subject to (3.3a), (3.3b), (3.3d) – (3.3f), and $Y_{ij0} + Y_{ij1} \leq 1$.

The objective function can be rewritten as follows.

$$\min \sum_j (f_j - \sum_i u_{ij}) X_j + \sum_i \sum_j \Big(\big[h_i d_{ij} (1 - q_j(1 - Z_j)) + u_{ij} \big] Y_{ij0} +$$
$$\big[h_i d_{ij} \sum_{r \neq j} q_r Y_{ir0} (1 - Z_r) + u_{ij} \big] Y_{ij1} \Big)$$

For a given $\mathbf{u}$, the optimal value of $\mathbf{X}$ is found by setting $X_j = 1$, if $(f_j - \sum_i u_{ij}) < 0$ and $X_j = 0$ otherwise. Customer assignation and facility fortification are solved as in § 3.1.2.1. A simple heuristic is used to obtain an initial upper bound. Starting with facilities opened at all locations, an iterative procedure is used to drop one facility at a time in a greedy manner. At each dropping iteration, customer assignation and facility fortification are done as in § 3.1.2.2. The solution with the minimum objective function value is then used as the initial upper bound. Once the Lagrangian procedure starts, at each iteration, an upper bound is obtained using the same heuristic as in § 3.1.2.2. The multiplier updating is conducted in a manner described in § 3.1.2.3.

3.3 Computational Results

3.3.1 Experimental Design

We tested the performance of both the RPMP-BF and RUFL-BF solution algorithms on four datasets containing 30, 49, 100, and 150 nodes, respectively. The last three datasets were adapted from Snyder and Daskin (2005) whereas the 30-node dataset was generated by arbitrarily selecting 30 nodes from the 49-node dataset. Demands h_i were taken from Snyder and Daskin (2005). The Euclidean distance between nodes i and j was used as the transportation cost d_{ij}. We let the sets I and J be equal. The failure probabilities q_j were randomly generated from $U \sim [0, 0.05]$. The fortification setup cost S_j was set to 30. The variable fortification cost r_j (associated with the unit reduction in the failure probability) was randomly generated from $U \sim [0, 3000]$. We tested the RPMP-BF algorithm for $P = 5$ and $P = 8$. To test the RUFL-BF algorithm, the facility construction cost f_j was randomly drawn from $U \sim [500, 1500]$. Both algorithms were tested for the values of fortification budget B ranging between 0 and 360. A fragment of the 30-node dataset is shown in in Table 3.1. The full 30-node dataset can be found in Table A.1 of Appendix A.

The algorithms were coded in C++ and were run on a Windows XP SP3 PC with a 2.2 GHz Duo core CPU and 2.0 GB of physical RAM. The gap tolerance and the maximum number of iterations were set to 0.5% and 3000, respectively.

Table 3.1: A fragment of the 30-node dataset (see also appendix A)

Nodes	1	2	3	$\cdots$	29	30
Demand	297.60021	179.90455	169.8651	$\cdots$	43.75099	42.19973
Longitude	38.56685	42.66575	30.30588	$\cdots$	44.947744	30.448967
Latitude	121.46736	73.799017	97.750522	$\cdots$	93.103686	91.126043
q_j	0.014	0.045	0.015	$\cdots$	0.026	0.005
r_j	620.79	459.02	325.73	$\cdots$	44.23	546.00

3.3.2 Performance of the RPMP-BF and RUFL-BF Algorithms

Results for the RPMP-BF algorithm for $P = 5$ and $P = 8$ using four datasets are listed in Table 3.2 and Table 3.3, respectively. Results for the RUFL-BF algorithm using four datasets are listed in Table 3.4. The abbreviations LB and UB stand for the lower and upper bound, respectively. The gap is the difference between the upper and lower bounds. The algorithms solved 87 out of a total of 96 cases to 0.5% optimality. For both algorithms, the computing time increases substantially with the size of the problem. This can be partially explained by noting that at each iteration, the number of enumerations required for solving **M1** increases quadratically with size of the problem and so does the size of the resulted **M2**. In general, solving the RUFL-BF model is more time demanding which can be attributed to the fact that its underlying fixed-charge facility location problem is harder to solve. This can also be the reason that the gaps for the 150-node cases did not improve after the algorithm exhausted 3,000 iterations.

3.3.3 Comparison with CPLEX Solver

To compare the performance of our algorithms to the that of the CPLEX solver, the RPMP-BF and RUFL-BF models were linearized using the method described in § 3.1.2. The CPLEX code was written in C++ using the CPLEX Concert Technology. For the purpose of comparison, only the CPLEX solver CPU time was measured. The comparison of the performance of both algorithms and the CPLEX solver was done on the same computer with version 10.1 of CPLEX.

We used a total of fourteen 30-node cases solvable by CPLEX (CPLEX failed to solve larger size cases due to insufficient memory). The comparison of the computation times between CPLEX and both algorithms is shown in Table 3.5. It can be observed that both algorithms are significantly faster than CPLEX where the total computation time for all

Table 3.2: Testbed performance results for the RPMP-BF algorithm (*P*=5)

		P=5			
Nodes	B	LB	UB	Gap, %	Time, s
30	0	3694.2	3694.2	0.00	0.70
30	30	3694.2	3694.2	0.00	0.71
30	60	3573.8	3573.8	0.00	0.80
30	120	3502.5	3502.5	0.00	0.67
30	180	3366.1	3382.1	0.47	0.78
30	240	3327.9	3344.4	0.50	0.97
30	300	3309.7	3309.7	0.00	0.94
30	360	3283.5	3299.2	0.48	0.46
49	0	8826.3	8870.4	0.50	3.47
49	30	8826.3	8870.4	0.50	3.36
49	60	8704.7	8736.3	0.36	8.80
49	120	8625.0	8653.5	0.33	4.94
49	180	8538.4	8538.5	0.00	29.31
49	240	8417.0	8459.0	0.50	45.11
49	300	8360.0	8401.9	0.50	24.35
49	360	8325.4	8366.9	0.50	10.20
100	0	17594.4	17682.8	0.49	56.37
100	30	17594.4	17682.8	0.49	59.79
100	60	17328.8	17380.3	0.30	27.10
100	120	17086.4	17172.2	0.50	146.43
100	180	16977.0	17061.6	0.50	64.17
100	240	16897.5	16927.0	0.17	66.72
100	300	16847.2	16847.2	0.00	48.7
100	360	16810.0	16886.6	0.45	31.10
150	0	20136.7	20237.0	0.49	98.37
150	30	20136.7	20237.0	0.49	104.40
150	60	19881.7	19968.5	0.43	120.80
150	120	19760.8	19829.1	0.34	208.42
150	180	19653.5	19747.2	0.47	221.30
150	240	19494.4	19592.3	0.50	188.95
150	300	19451.0	19547.3	0.49	189.21
150	360	19512.0	19589.9	0.40	171.70

fourteen cases is 20 seconds versus 4344 seconds. Compared to the optimal solutions obtained by CPLEX, the final feasible solutions yielded by our algorithms are also optimal.

Table 3.3: Testbed performance results for the RPMP-BF algorithm (P=8)

		P=8			
Nodes	B	LB	UB	Gap, %	Time, s
30	0	2192.5	2201.0	0.33	0.50
30	30	2192.5	2201.0	0.33	0.51
30	60	2144.1	2150.1	0.28	0.91
30	120	2096.8	2102.7	0.28	1.63
30	180	2044.4	2053.7	0.45	0.64
30	240	2014.8	2024.7	0.49	3.02
30	300	1980.5	1990.5	0.50	1.39
30	360	1981.4	1991.3	0.49	1.22
49	0	5874.6	5903.2	0.48	2.14
49	30	5874.6	5903.2	0.48	2.06
49	60	5772.2	5801.1	0.50	3.10
49	120	5724.6	5752.8	0.49	2.36
49	180	5678.3	5705.6	0.48	3.59
49	240	5638.9	5647.3	0.14	5.26
49	300	5625.1	5627.0	0.03	8.08
49	360	5580.9	5608.1	0.48	27.07
100	0	12711.3	12775.1	0.49	27.34
100	30	12711.3	12775.1	0.49	30.39
100	60	12571.7	12593.1	0.16	32.31
100	120	12431.5	12491.3	0.47	96.43
100	180	12311.3	12372.8	0.49	27.62
100	240	12283.6	12339.3	0.45	63.90
100	300	12230.6	12252.4	0.17	44.96
100	360	12198.9	12198.9	0.00	46.85
150	0	14682.7	14755.9	0.49	143.22
150	30	14682.7	14755.9	0.49	151.71
150	60	14659.3	14685.7	0.17	190.43
150	120	14494.1	14566.7	0.49	263.91
150	180	14436.3	14508.8	0.49	406.90
150	240	14389.3	14462.3	0.50	783.34
150	300	14385.3	14440.8	0.38	159.03
150	360	14389.7	14401.5	0.66	872.23

3.3.4 Comparison with a Myopic Policy

To illustrate the effectiveness of the LR-based design solutions, we compared them to the solutions obtained by using a myopic policy which allocates the available fortification

Table 3.4: Testbed performance results for the RUFL-BF algorithm

Nodes	B	LB	UB	Gap, %	Time, s
30	0	7963.9	8003.9	0.50	0.93
30	30	7963.9	8003.9	0.50	1.41
30	60	7854.1	7886.3	0.41	1.01
30	120	7751.5	7789.8	0.49	2.45
30	180	7713.3	7751.1	0.49	2.40
30	240	7701.0	7734.3	0.43	2.92
30	300	7697.7	7734.3	0.47	2.24
30	360	7696.0	7734.3	0.50	1.70
49	0	12090.9	12151.1	0.50	4.64
49	30	12090.9	12151.1	0.50	4.98
49	60	11983.0	12042.5	0.49	5.99
49	120	11933.2	11992.4	0.49	11.62
49	180	11899.6	11959.3	0.50	12.19
49	240	11884.4	11943.3	0.49	18.12
49	300	11846.5	11903.0	0.47	13.31
49	360	11837.2	11896.6	0.50	19.87
100	0	17295.3	17380.3	0.49	62.82
100	30	17295.3	17380.3	0.49	66.31
100	60	17185.6	17271.4	0.50	63.60
100	120	17084.9	17178.5	0.54	98.99
100	180	17056.0	17140.5	0.49	187.87
100	240	17031.0	17116.8	0.50	198.63
100	300	16999.8	17082.6	0.48	246.78
100	360	16971.0	17056.0	0.50	281.32
150	0	18953.7	19117.4	0.85	810.33
150	30	18953.7	19117.4	0.85	819.90
150	60	18840.4	19021.6	0.95	1199.64
150	120	18838.9	19000.3	0.84	906.48
150	180	18763.9	18916.9	0.80	1732.63
150	240	18586.8	18907.5	1.70	587.79
150	300	18600.2	18841.3	1.28	747.83
150	360	18545.7	18853.0	1.63	885.97

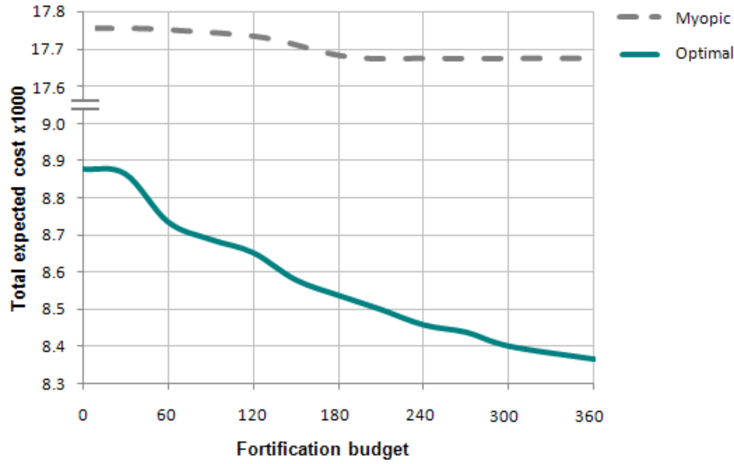
budget to the most reliable facilities first. By doing so, the policy does not account for the demand topology, hence is the name myopic.

Two myopic policies were implemented for a separate comparison with the RPMP-BF and RUFL-BF optimal (within 0.5% gap) strategies. For a RPMP-BF-type myopic policy, P most reliable facilities were open. For each customer, the closest and the second closest

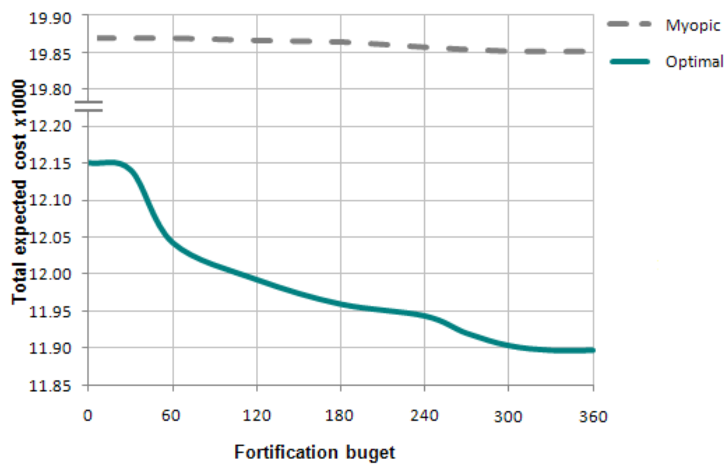
Table 3.5: Computation time comparison

Algorithm	P	B	CPLEX time, s	Algorithm time, s
RPMP-BF	5	20	243.1	0.8
	5	120	623.1	0.6
	5	180	282.8	0.7
	8	60	687.7	0.9
	8	120	948.1	1.1
	8	180	807.2	0.5
RUFL-BF		30	93.5	1.4
		60	126.1	1.0
		90	151.6	2.5
		120	148.3	2.4
		180	97.1	2.4
		240	70.4	2.9
		300	50.2	2.2
		360	108.9	1.7

open facility were assigned as the primary and the backup supplier, respectively. Facilities were fortified starting from the most reliable until either the fortification budget was used up or all open facilities were fortified. For a RUFL-BF-type myopic policy, for each level of B, the number of facilities to open N was not fixed but varied from one to $|J|$. For each value of N, facility fortification and customer assignation were done in the same way as for the RPMP-BF-type myopic policy. For a fixed B, the value of the RUFL-BF myopic policy was set as the minimum of the total expected cost over the range of values of N. Figures 3.1(a) and 3.1(b) show the results of the performance comparison of the optimal RPMP-BF and RUFL-BF strategies to that of their respective myopic policies. In both cases, the comparison was done for the values of B ranging between 0 and 360. The total expected cost was used as the measure of policy performance. As expected, in both cases, the curves for both optimal and myopic policies generally exhibit a downward trend as the fortification budget increases. For all curves, the presence of "flat" regions for the values of B between 0 and 30 is due to the fact that no facility can be fortified as the budget is consumed by the fixed fortification cost $S_j = 30$. For RPMP-BF- and RUFL-BF-type myopic policy curves, the presence of "flat" regions for the values of B exceeding 180 and



(a) RPMP-BF case ($P = 5$, 49 nodes)



(b) RUFL-BF case (49 nodes)

Figure 3.1: Comparison of the optimal RPMP-BF and RUFL-BF and their respective myopic policies

240, respectively is attributed to the fact that all facilities have been fortified and no further improvement can be made.

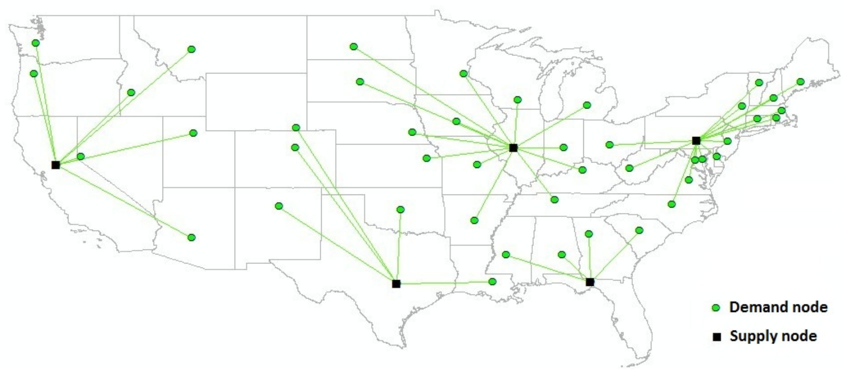
It can be observed that for both RPMP-BF and RUFL-BF optimal policies, the marginal reduction in the policy values diminishes with B. This can be explained by noting that in order to attain optimality, the model allocates the fortification budget to the facilities in the decreasing order of reduction in the total expected cost realized from fortification.

It can also be observed that in both cases, the optimal policies outperform the respective myopic policies over the entire range of B. Moreover, in both cases, the difference in policy performance widens as B increases. This can be explained as follows. As mentioned

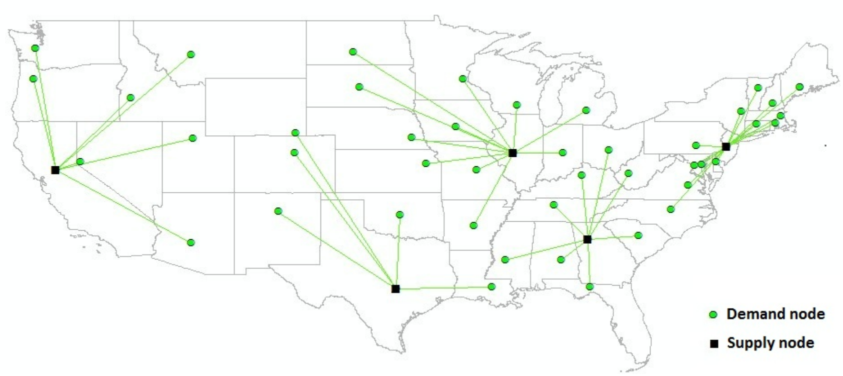
Table 3.6: Optimal locations (RPMP-BF)

B	Locations
30	0, 2, 3, 4, 5
60	0, 2, 4, 5, 10
120	0, 2, 4, 5, 10
180	0, 2, 4, 5, 10
240	0, 1, 2, 5, 10
300	0, 2, 5, 8, 10
360	0, 2, 5, 8, 10

earlier, for both myopic policies, the curves flatten starting at certain levels of B as all facilities get fortified. This "rapid" total fortification is possible because myopic policies select most reliable facilities to open and fortify. In the case of the optimal policies, selection of facilities for fortification is governed not only by facility reliability but also by topology of demand. As a result, total fortification generally requires larger levels of B.



(a) $B = 30$



(b) $B = 360$

Figure 3.2: The optimal solution to RPMP-BF ($P = 5$, 49 nodes)

Analyzing optimal solutions for both the RPMP-BF and RUFL-BF policies we can notice that facility locations were chosen from among demand-heavy nodes. Table 3.6 shows an example of optimal facility locations for the case of RPMP-BF (numbering of facilities was done in the decreasing order of demand).

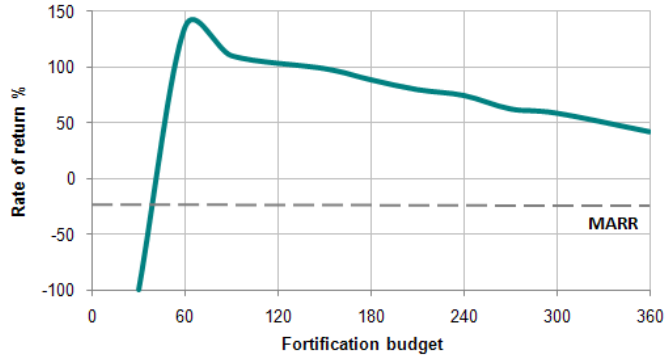
Figures 3.2(a) and 3.2(b) show the topology of the optimal solutions for the cases $B = 30$ and $B = 360$, respectively. We observe that optimal locations were chosen from among the top ten demand nodes. This can be explained by noting that in our testbed, customer sites also serve as candidates for facility locations. Since the values of facility reliabilities are chosen from a uniform distribution, the selection of facilities is primarily driven by demand topology.

3.3.5 Rate of Return on Fortification Investment

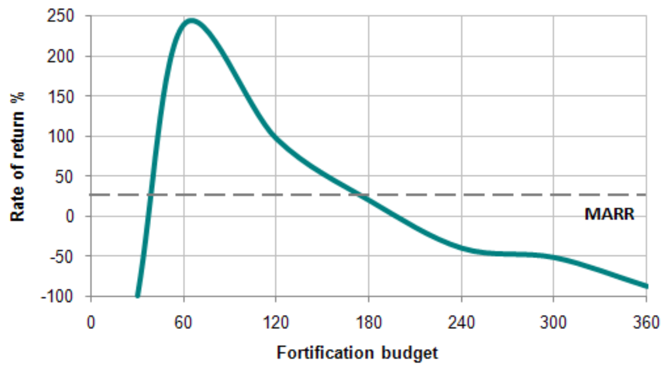
Calculation of the rate of return (ROR) on fortification investment allows a decision maker to assess the effectiveness of such investment when compared to alternative investment opportunities. For instance, a decision maker may choose to invest in network fortification only if the ROR exceeds the minimum acceptable rate of return (MARR) Rogers (2001). We illustrate such analysis for both RPMP-BF and RUFL-BF by considering the 49-node case with the values of fortification budget ranging between 0 and 360.

The optimal objective function value for $B = 0$ was used as the baseline. For each $B > 0$, the value of ROR (in %) was calculated as the overall reduction in the optimal objective function value (compared to the baseline) minus B, all divided by B.

Figures 3.3(a) and 3.3(b) depict the values of the ROR for RPMP-BF and RUFL-BF optimal policies, respectively. We can observe that for both policies, the ROR is negative for small values of B. This is due to the fact that no facility can be fortified as the budget is consumed by the fixed fortification cost. For both policies, the curves exhibit a sharp rise until reaching the maximum at $B = 60$, followed by a gradual decrease for higher



(a) RPMP-BF optimal policy ($P = 5$, 49 nodes)



(b) RUFL-BF optimal policy (49 nodes)

Figure 3.3: Rate of return (ROR) on fortification investment

values of B. The downward trend can be attributed to the fact that both models allocate the fortification budget to the facilities in the decreasing order of reduction in the optimal objective function value. Note that in the case of our study, for an arbitrarily chosen level of MARR, the RPMP-BF optimal policy has a broader range of attractive fortification investment opportunities.

Chapter 4: Reliable Distribution Networks Models with Partial Fortification

In this chapter, we extend both the RUFL and RPMP models developed in Chapter 3 to allow partial (continuous) as opposed to binary fortification of facilities. The models are called RUFL-PF and RPMP-PF, respectively. By allowing partial fortification, the reliability of fortified facilities is not only limited to "totally reliable". Instead, a facility can be fortified to *any* reliability level up to becoming totally reliable, if necessary. This option allows a more efficient utilization of limited fortification resources.

As in the case of binary fortification, both RUFL and RPMP models seek to obtain optimal facility location, fortification, and customer assignment. We again assume that events of facility failures are mutually independent and occur with location specific probabilities. We also assume one layer of supplier backup. The formulation considers continuous reliability improvement, a finite fortification budget constraint, and the facility fortification cost which is site specific.

Both models are formulated as nonlinear mixed integer programming models. The RUFL model is shown to be $\mathcal{NP}$-hard (see Section 4.1). For both models, we develop Lagrangian relaxation-based heuristic solution algorithms (Section 4.1.2 and 4.2.2). We present computational results demonstrating the efficiency of the developed heuristic for different size problems and different levels of fortification budget (Section 4.3). The rest of the section follows our work in Li and Savachkin (2011).

4.1 The RUFL Model with Partial Fortification (RUFL-PF)

4.1.1 Model Formulation

The formulation of the RUFL-PF mostly follows that of the RUFL model developed in Chapter 3. For the sake of completeness, we opted to present the description of the model in its entirety.

We define I to be the set of customers and J the set of potential facility locations. Each customer $i \in I$ has demand h_i. Let $d_{ij} \geqslant 0$ be the cost of transporting one unit of demand from facility location $j \in J$ to customer i (with the convention that $d_{ii} = 0$ for all i). Let f_j denote the facility construction cost at location j. Associated with each facility j is the failure probability $0 \leqslant q_j \leqslant 1$. We assume that the events of facility failures are mutually independent. Once a facility fails, it becomes unavailable. Each customer is assigned a primary supplier and a different backup supplier. We consider that the probability of a simultaneous failure of its primary and backup supplier is negligible. Hence, we assume that for any customer, if its primary supplier fails, the backup supplier will be available.

For each facility j, we introduce u_j as the maximum reliability improvement obtainable with available technology (ideally, $u_j = q_j$). The cost of facility fortification is considered to be a function of reliability improvement. We define r_j as the cost associated with a unit reduction in the failure probability of facility j. We let B denote the total available fortification budget. Similar to the previous models, we assume unlimited capacities for all facilities to ensure that all demands are met.

Our model incorporates the following *decision variables*:

$$X_j = \begin{cases} 1, & \text{if a facility is opened at location } j; \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_{ij0} = \begin{cases} 1, & \text{if customer } i \text{ has facility } j \text{ as the primary supplier;} \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_{ij1} = \begin{cases} 1, & \text{if customer } i \text{ has facility } j \text{ as the backup supplier;} \\ 0, & \text{otherwise.} \end{cases}$$

$\Delta_j =$ reliability improvement gained from fortification of facility j ($\Delta_j \in \mathbf{R}$).

Note that as opposed to the formulations in Chapter 3, reliability improvement takes its values in the continuum of the reals for all facilities. We formulate the problem as follows:

(RUFL-PF)

$$\min \sum_{j \in J} f_j X_j + \sum_{i \in I} \sum_{j \in J} [h_i d_{ij} (1 - (q_j - \Delta_j)) Y_{ir0} + h_i d_{ij} Y_{ij1} \sum_{r \in J,\ r \neq j} (q_r - \Delta_r) Y_{ir0}]$$

subject to

$$\sum_j Y_{ij0} = 1, \ \forall i \in I \tag{4.1a}$$

$$\sum_j Y_{ij1} = 1, \ \forall i \in I \tag{4.1b}$$

$$Y_{ij0} + Y_{ij1} \leqslant X_j, \ \forall i \in I, j \in J \tag{4.1c}$$

$$0 \leqslant \Delta_j \leqslant u_j, \ \forall j \in J \tag{4.1d}$$

$$\sum_j r_j \Delta_j \leqslant B \tag{4.1e}$$

$$X_j \in \{0,1\}, \ \forall j \in J, \tag{4.1f}$$

$$Y_{ij0}, Y_{ij1} \in \{0,1\}, \ \forall i \in I, \ j \in J. \tag{4.1g}$$

The objective function of the RUFL-PF is the expected total construction and transportation cost associated with satisfying the demands of all customers. The term $\sum_{j \in J} h_i d_{ij} Y_{ij0} (1 - (q_j - \Delta_j))$ represents the part of the expected transportation cost associated with customer i served by its primary suppler, where $(1 - (q_j - \Delta_j))$ is the probability that the supplier is available. The term $\sum_{j\in J} h_i d_{ij} Y_{ij1} \sum_{r\neq j}(q_r - \Delta_r) Y_{ir0}$ is the part of the cost when customer i is served by its backup supplier, where $\sum_{r\neq j}(q_r - \Delta_r) Y_{ir0}$ is the probability that the primary supplier is unavailable.

Constraints (4.1a) and (4.1b) respectively ensure that each customer is assigned only one primary and one backup supplier. Constraint (4.1c) serves two purposes. It guarantees that only an open facility can serve as a supplier. It also ensures that for each customer, the primary and backup suppliers are different facilities. Constraint (4.1d) specifies the range of reliability improvement of facility j. Constraint (4.1e) is the total fortification budget constraint. Constraints (4.1f) and (4.1g) are standard integrality constraints.

The next theorem shows that the model above is $\mathscr{NP}$-hard.

Theorem 3 *The RUFL-PF is $\mathscr{NP}$-hard.*

Proof: Note that when $B = 0$ and $q_j = 0$ for all $j \in J$, the RUFL-PF becomes the classical uncapacitated fixed-charge location problem.

4.1.2 Lagrangian Relaxation-Based Heuristic

As shown above, RUFL-PF is $\mathscr{NP}$-hard and has a nonlinear objective function. One possible solution is to linearize the model by introducing new variables, say $U_{ij0} = Y_{ij0}\Delta_j$, $V_{ijr} = Y_{ij1} Y_{ir0}$, and $W_{ijr} = V_{ijr}\Delta_j = Y_{ij1} Y_{ir0}\Delta_j$, with necessary constraints. However, the resultant problem becomes excessively large even for networks of moderate size, which makes solving such problems using commercial solvers very time-consuming. This motivated us to develop a Lagrangian relaxation-based solution heuristic.

4.1.2.1 Heuristic for Lower Bounds

In this section, we develop a heuristic to find a solution to a relaxed model which serves as an approximated lower bound for RUFL-PF. To relax the original model, we select constraint (4.1c) as it is the only constraint that links the location variables X_j with assignation variables Y_{ij0} and Y_{ij1}. Relaxing this set of constraints using Lagrange multipliers λ_{ij} yields the following subproblem.

(LR-λ)

$$\min \sum_{j\in J} f_j X_j + \sum_{i\in I}\sum_{j\in J}[h_i d_{ij}(1-(q_j-\Delta_j))Y_{ir0} + h_i d_{ij} Y_{ij1}\sum_{r\neq j}(q_r-\Delta_r)Y_{ir0}] +$$
$$\sum_i\sum_j \lambda_{ij}(Y_{ij0}+Y_{ij1}-X_j)$$

subject to (4.1a), (4.1b), (4.1d)–(4.1g).

The objective function can be rewritten as follows to separate the decision variables.

$$\min \sum_j (f_j - \sum_i \lambda_{ij})X_j + \sum_i\sum_j\Big(\big[h_i d_{ij}(1-(q_j-\Delta_j))+\lambda_{ij}\big]Y_{ij0} +$$
$$\big[h_i d_{ij}\sum_{r\neq j}(q_r-\Delta_r)Y_{ir0}+\lambda_{ij}\big]Y_{ij1}\Big)$$

(i) Solving LR-λ for X

For a given λ, the optimal value of $\mathbf{X}$ is found by setting $X_j = 1$, if $(f_j - \sum_i \lambda_{ij}) < 0$; else $X_j = 0$.

(ii) Solving LR-λ for Y and Δ

We proceed in two steps. We first solve a special case when fortification budget is zero. We then use the solution to approach the case when fortification budget is positive.

Case 1: **B=0**

If $B = 0$, then $\Delta = 0$, and constraints (4.1d) and (4.1e) can be eliminated. The problem simplifies to:

(M1)

$$\min \sum_i\sum_j\Big\{\big[h_i d_{ij}(1-q_j)+\lambda_{ij}\big]Y_{ij0} + \big[h_i d_{ij}\sum_{r\neq j} q_r Y_{ir0}+\lambda_{ij}\big]Y_{ij1}\Big\}$$

subject to (4.1a), (4.1b), (4.1f), (4.1g).

Observe that relaxing constraint (4.1c) can result in a customer being assigned to a facility which is not open. Constraints (4.1a) and (4.1b) still assure that each customer is assigned only one primary and one backup supplier. Note that (M1) is separable in i, so that in order to solve the problem, it suffices to optimally assign a primary and a backup supplier to each customer. For a given customer i, if facility v and w are selected as the primary and backup supplier, respectively, the objective function of (M1) associated with customer i becomes $\Phi_i(v,w) = h_i d_{iv}(1-q_v) + \lambda_{iv} + h_i d_{iw} q_v + \lambda_{iw}$. To find the optimal assignment of suppliers for customer i, we enumerate the values of $\Phi_i(v,w)$ for all $v,w \in J$ to find $\min\limits_{v,w}\{\Phi_i(v,w)\}$. Once we obtain the optimal assignment of suppliers for all customers $i \in I$, we let $G(j)$ be the set of customers who have facility j as their primary supplier.

Case 2: **B>0**

Consider customer i and let j and k denote its primary and backup supplier, respectively, obtained from the case $B = 0$ (i.e., $\Phi_i(j,k) = \min\limits_{v,w}\{\Phi_i(v,w)\}$, $i \in G(j)$). Suppose that the primary supplier j is fortified and c is the amount of reliability improvement. Then the part of the objective function associated with customer i becomes $\Psi_i(j,k) = h_i d_{ij}(1 - (q_j - c)) + \lambda_{ij} + h_i d_{ik}(q_j - c) + \lambda_{ik}$. Hence, the improvement gained for customer i from fortifying supplier j is $\Phi_i(j,k) - \Psi_i(j,k)$. Then the overall improvement gained from fortifying supplier j is $\sum_{i\in G(j)}(\Phi_i(j,\cdot) - \Psi_i(j,\cdot))$.

Let $E = \{j : q_j > 0\}$. For each $j \in E$, we introduce the following measure of fortification efficiency.

$$\delta(j) = \frac{\sum_{i\in G(j)}(\Phi_i(j,\cdot) - \Psi_i(j,\cdot))}{r_j c} = \frac{\sum_{i\in G(j)} h_i c\,(d_{i\cdot} - d_{ij})}{r_j c} = \frac{\sum_{i\in G(j)} h_i (d_{i\cdot} - d_{ij})}{r_j}\cdot \quad (4.2)$$

We then find s such that $\delta(s) = \max_{j\in E}\delta(j)$. We allocate the fortification budget $B > 0$ starting with facility s. Note that fortification of facility s may change the current assignation of customers as some of them switch to supplier s and thus join the set $G(s)$. Also note that fortifying s will not change the assignation of customers already in $G(s)$ since improving the reliability of s decreases $\Phi_i(s,\cdot)$. We implement the fortification process in an iterative fashion as described in the algorithm below.

Step 1. For all $j \in E$, set $\Delta_j = 0$.

Step 2. Find facility s such that $\delta(s) = \max_{j \in E} \delta(j)$. In steps 3-5 below, we determine the level of fortification of facility s which we denote by Δ_s. Note that $\Delta_s \leqslant \min\{u_s, B/r_s\}$.

Step 3. For each customer $i \notin G(s)$, let j and k denote its current primary and backup supplier, respectively (i.e., $\Phi_i(j,k) = \min_{v,w}\{\Phi_i(v,w)\}$). We want to determine Δ_s^i, the minimal fortification level of s that causes re-assignment of i. Suppose that s is chosen as the primary supplier of i. Let $\Phi_i(s,t) = \min_{w}\{\Phi_i(s,w)\}$, where t denotes the backup supplier of i when s is chosen as the primary one. Now if the reliability of s was improved by c, then $\Phi_i(s,t)$ would decrease by $c \cdot h_i(d_{it} - d_{is})$. As the reliability of s increases, if $\Phi_i(s,t)$ becomes smaller than $\Phi_i(j,k)$, then re-assigning customer i to suppliers s and t will result in an improved objective function. We have that $\Delta_s^i = \dfrac{\Phi_i(s,t) - \Phi_i(j,k)}{h_i(d_{it} - d_{is})}$.

Step 4. Let $\tilde{\Delta}_s = \min_{i \notin G(s)} \tilde{\Delta}_s^i$. Then $\Delta_s = \min\{\tilde{\Delta}_s, \min\{u_s, B/r_s\}\}$.

Step 5. Update $G(s)$, if needed. Set $B = B - r_s\Delta_s$, $q_s = q_s - \Delta_s$, $u_s = u_s - \Delta_s$. If $u_s = 0$, $E \leftarrow E \setminus \{s\}$.

Step 6. If $B > 0$ and $E \neq \varnothing$, recalculate δ_j for all $j \in E$, and proceed to Step 1. Else, stop.

4.1.2.2 Heuristic for Upper Bounds

At each iteration of the Lagrangian relaxation process, the solution to (LR-λ) provides a lower bound to (RUFL-PF). If the solution is also feasible, it provides an upper bound as well. Else, if the solution is infeasible, we need to construct a feasible solution which will serve as an upper bound.

A feasible solution can be obtained by using the solution to (LR-λ). We first select the facilities opened in the solution to (LR-λ). For each customer, we assign the nearest and the second nearest open facility as the primary and backup supplier, respectively. To decide which facilities to fortify, we proceed in the following way. For customer i, let j and

k respectively denote its primary and backup supplier. The expected transportation cost associated with i amounts to $h_i d_{ij}(1-q_j)+h_i d_{ik} q_j$. If facility j is fortified by Δ_j, the resulting cost improvement equals to $\varphi_j = \sum_{i\in G(j)} h_i(d_{ik}-d_{ij})\Delta_j$. Thus, the objective is to maximize the utilization of the fortification budget over all open facilities. The problem is formulated as follows:

$\max \sum_{j\in E} \varphi_j$ s.t. $\sum_{j} r_j\Delta_j \leqslant B, \Delta_j \in [0,u_j]$.

This is a continuous knapsack problem which can be solved in a greedy manner.

4.1.2.3 Multiplier Initiation and Updating

From empirical analysis we found that the formula $\lambda_{ij} = h_i/\|I\|$ generated efficient initial multipliers for our problem.

Once the algorithm commences, at each iteration k, we use subgradient optimization Fisher (2004) to update λ by setting

$\lambda_{ij}^{k+1} = \lambda_{ij}^{k} + t_k(Y_{ij0}+Y_{ij1}-X_j)$,

where t_k is a step size, $t_k = \dfrac{\beta_k(z^*-z(\lambda^k))}{\| Y_{ij0}+Y_{ij1}-X_j \|^2}$.

In the formula above, β_k is a constant at iteration k, which is initially set to $\beta_0 = 2$, as in Fisher (2004). We divide the values of β_k by 2 if every 60 consecutive iterations fail to improve the lower bound. In the formula above, z^* is the best known upper bound, and $z(\lambda^k)$ is the lower bound when the multipliers are fixed to λ^k. The algorithm terminates when one of the following criteria is met:

- $(z^*-z(\lambda^k))/z^* \leqslant \varepsilon$, for some prespecified tolerance ε;
- $k > k_{max}$, for some iteration limit k_{max}.

The Lagrangian relaxation-based heuristic algorithm for RUFL-PF is presented below.

Step 0 Set $k = 0$. Initialize multipliers $\lambda_{ij} = h_i/\|I\|$.
Set lower bound $LB = 0$; set $z^* = +\infty$.

Step 1 Solve **(LR-λ)** for $z(\lambda^k)$ (§ 4.1.2.1).
Obtain a feasible solution $\hat{z}^k$ using the heuristic in § 4.1.2.2.

Step 2 Set $LB = \max\{LB, z(\lambda^k)\}$; set $z^* = \min\{z^*, \hat{z}^k\}$.
If $(z^* - z(\lambda^k))/z^* \leqslant \varepsilon$ or $k > k_{max}$, then STOP; else go to Step 3.

Step 3 Update multipliers:

$$t_k = \frac{\beta_k(z^* - z(\lambda^k))}{\| Y_{ij0} + Y_{ij1} - X_j \|^2},$$

$$\lambda_{ij}^{k+1} = \max\{\lambda_{ij}^k + t_k(Y_{ij0} + Y_{ij1} - X_j), 0\}.$$

Step 4 Set $k = k + 1$ and go to Step 1.

In the computational results section (§ 4.3), we evaluate the performance of the developed heuristic-based solution approach by comparing it to optimal solutions. The performance is evaluated based on the optimality gap and the computation time.

4.2 The RPMP Model with Partial Fortification (RPMP-PF)

The model seeks to minimize the total expected transportation cost by optimally selecting facility locations, allocating a finite fortification budget, and assigning the customers. We formulate this problem as a nonlinear mixed integer programming model and develop a Lagrangian relaxation based solution heuristic.

4.2.1 Problem Formulation

The formulation is similar to that of the RUFL-PF with an addition of the total number of facilities to open P. We formulate the problem as follows:

(RPMP-PF)

$$\text{minimize} \sum_{i \in I} \sum_{j \in J} [h_i d_{ij}(1 - (q_j - \Delta_j))Y_{ir0} + h_i d_{ij} Y_{ij1} \sum_{r \in J,\, r \neq j} (q_r - \Delta_r) Y_{ir0}]$$

subject to

$$\sum_{j \in J} Y_{ij0} = 1, \ \forall i \in I \tag{4.3a}$$

$$\sum_{j \in J} Y_{ij1} = 1, \ \forall i \in I \tag{4.3b}$$

$$Y_{ij0} + Y_{ij1} \leq X_j, \ \forall i \in I, \ j \in J \tag{4.3c}$$

$$\sum_{j \in J} X_j = P \tag{4.3d}$$

$$\sum_{j} r_j \Delta_j \leq B \tag{4.3e}$$

$$\Delta_j \in [0, u_j], \ \forall j \in J \tag{4.3f}$$

$$X_j \in \{0, 1\}, \ \forall j \in J \tag{4.3g}$$

$$Y_{ij0}, Y_{ij1} \in \{0, 1\}, \ \forall i \in I, \ j \in J. \tag{4.3h}$$

The objective function of the RPMP-PF is the expected total transportation cost associated with satisfying the demands of all customers. The term $\sum_{j \in J} h_i d_{ij} Y_{ij0}(1 - (q_j - \Delta_j))$ represents the part of the expected transportation cost associated with customer i served by its primary suppler, where $(1 - (q_j - \Delta_j))$ is the probability that the supplier is available. The term $h_i d_{ij} Y_{ij1} \sum_{r \neq j} (q_r - \Delta_r) Y_{ir0}$ is the cost of customer i served by its backup supplier, where $\sum_{r \neq j} (q_r - \Delta_r) Y_{ir0}$ is the probability that the primary supplier is failed (recall that in this case, the backup facility is assumed to be available).

Constraint (4.3a) and (4.3b) respectively assures that each customer is assigned only one primary and one backup supplier. Constraint (4.3c) serves two purposes. First, it guarantees that only open facility can serve as a supplier. It also assures that for each customer, the primary and backup suppliers are different facilities. Constraint (4.3d) demands P facilities to be opened. Constraint (4.3e) is the total fortification budget constraint. Finally, constraint (4.3f) specifies the continuity and the range of Δ_j, and (4.3g) and (4.3h) are the standard integrality constraints.

4.2.2 Solution Method

In this section, we develop a Lagrangian relaxation based heuristic for the RPMP-PF.

Lower bound. Relaxing the set of constraints (4.3c) and using Lagrange multipliers u_{ij} yield the following subproblem.

$$\min \sum_{i\in I}\sum_{j\in J}[h_i d_{ij}(1-(q_j-\Delta_j))Y_{ir0}+h_i d_{ij}Y_{ij1}\sum_{r\in J,\ r\neq j}(q_r-\Delta_r)Y_{ir0}]+\sum_i\sum_j u_{ij}(Y_{ij0}+Y_{ij1}-X_j)$$

subject to (4.3a), (4.3b), (4.3d) – (4.3h).

The objective function above can be rewritten as follows

$$\begin{aligned}
&\sum_{i\in I}\sum_{j\in J}[h_i d_{ij}(1-(q_j-\Delta_j))Y_{ir0}+h_i d_{ij}Y_{ij1}\sum_{r\in J,\ r\neq j}(q_r-\Delta_r)Y_{ir0}]+\sum_i\sum_j u_{ij}(Y_{ij0}+Y_{ij1}-X_j)\\
&=\sum_i\sum_j\big[h_i d_{ij}(1-(q_j-\Delta_j))Y_{ij0}+h_i d_{ij}Y_{ij1}\sum_{r\neq j}(q_r-\Delta_r)Y_{ir0}+u_{ij}(Y_{ij0}+Y_{ij1})\big]-\sum_j\sum_i u_{ij}X_j\\
&=\sum_i\sum_j\big\{\big[h_i d_{ij}(1-(q_j-\Delta_j))+u_{ij}]Y_{ij0}+[h_i d_{ij}\sum_{r\neq j}(q_r-\Delta_r)Y_{ir0}+u_{ij}\big]Y_{ij1}\big\}-\sum_j\sum_i u_{ij}X_j.
\end{aligned}$$

For a given **u**, the optimal value of **X** can be found by ranking the values of $(-\sum_i u_{ij})$ for all j and setting $X_j = 1$ if $(-\sum_i u_{ij})$ is among the P smallest ranked values, and setting $X_j = 0$ otherwise. Customer assignation and facility fortification are solved as described in § 4.1.2.1. Once the Lagrangian procedure starts, at each iteration, an upper bound is obtained using the same heuristic as in § 4.1.2.2. The multiplier updating is conducted in a manner described in § 4.1.2.3.

4.3 Computational Results

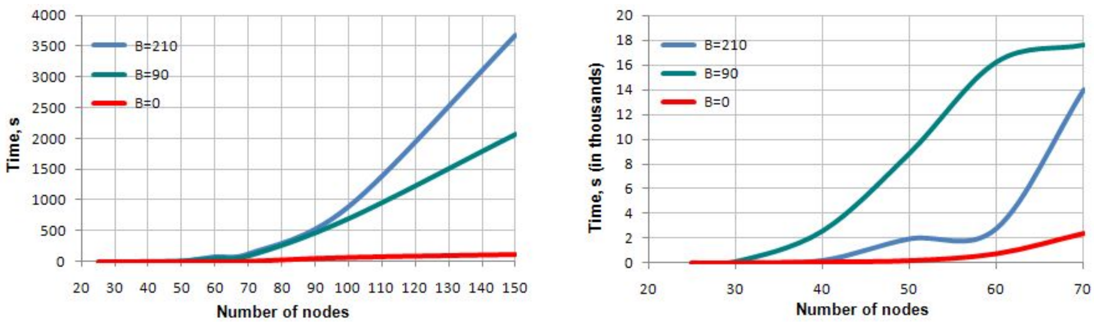
4.3.1 Experimental Design

We tested the performance of both the RUFL-PF and RPMP-PF solution algorithms on seven datasets containing 25, 30, 50, 60, 70, 100, and 150 nodes, respectively. The datasets were adapted from Snyder and Daskin (2005) and so were demands h_i. Transportation cost d_{ij} was taken as the Euclidean distance between nodes i and j. For the purpose of this testbed, we let sets I and J be equal. The facility failure probabilities q_j were randomly generated from $\text{Uni}(0, 0.05)$. The variable fortification costs r_j were generated from $\text{Uni}(0, 3000)$. The upper bounds for reliability improvement u_j were generated from $\text{Uni}(0, q_j)$. The facility construction cost f_j was drawn from $\text{Uni}(500, 1500)$. The algorithms were tested for the values of fortification budget B ranging between 0 and 210. The algorithms were coded in C++ and run on a 64-bit Linux machine with a 2.8 GHz Duo core CPU and 4.0 GB of physical RAM. The gap tolerance and the maximum number of iterations were set to 0.01% and 1000, respectively.

4.3.2 Comparison with CPLEX Solver

To compare the performance of our algorithms to the that of the CPLEX solver, the RUFL-PF and RPMP-PF models were linearized using the method described at the beginning of Section 4.1.2. The CPLEX code was written in C++ using the CPLEX Concert Technology. The comparison of the performance of the algorithm and the CPLEX solver was done on the same machine with version 12.1 of CPLEX. We used a total of 80 cases containing up to $n = 70$ (for RUFL-PF) and $n = 50$ (for RPMP-PF) nodes solvable by CPLEX. The solver failed to handle larger size cases due to insufficient memory. The performance comparison results are shown in Tables 4.1 and 4.3. Tables 4.4 and 4.5 also show performance results for our algorithms for 100 and 150 nodes. The abbreviations LB and UB stand for the lower bound and the upper bound, respectively. The gap is the difference (in %) between the upper and lower bounds.

It can be observed from Tables 4.1 and 4.3 that our algorithms are significantly faster than CPLEX where the total computation time for 80 cases is 1,774 seconds versus 344,541 seconds. The gaps between the values obtained by our algorithms and CPLEX are very small in all cases. This suggests that the solution quality of our algorithms is likely to be very good for small to medium sized problems. It can also be noted from Tables 4.4 and 4.5 that the heuristics are able to solve larger scale problems relatively fast.



(a) RUFL-PF algorithm (b) CPLEX solver

Figure 4.1: Relationship between network size and computation time

It can be observed from Figures 4.1(a) and 4.1(b) that in general, for both the RUFL-PF algorithm and CPLEX, the computation time increases with the network size. However, the rate of increase is significantly smaller for our algorithm. Table 4.6 compares the percentage of computation time increase from the case of $n = 25$ to the case of $n = 70$ (RUFL-PF). It can be seen that for each of the three levels of fortification budget, our algorithm takes relatively less time to solve larger sized case. Similar conclusions hold for the RPMP-PF algorithm.

It can also be noted from Figure 4.1(a) that for the RUFL-PF algorithm, for a fixed network size, the computation time increases substantially with fortification budget. The increase is particularly noticeable for larger sized networks. This can be explained by noting that the iterative fortification algorithm presented on page 49 uses the available fortification budget as a stopping criterion (see Step 6). From Figure 4.1(b), it also appears that CPLEX takes longer to find solutions to the cases with medium ($B = 90$) fortification

Table 4.1: Comparison results for RUFL-PF algorithm and CPLEX solver

Nodes	B	Alg. value	CPLEX value	Gap, %	Algorithm time, s	CPLEX time, s
25	0	7762.68	7762.68	0.00	0.25	6.92
25	30	7669.79	7669.79	0.00	0.51	11.76
25	60	7643.00	7643.00	0.00	0.75	10.28
25	90	7641.76	7641.76	0.00	1.01	10.80
25	120	7641.76	7641.76	0.00	0.85	12.33
25	150	7641.76	7641.76	0.00	1.34	12.33
25	180	7641.76	7641.76	0.00	1.54	12.34
25	210	7641.76	7641.76	0.00	2.22	12.44
30	0	7408.25	7408.25	0.00	1.31	8.63
30	30	7383.22	7383.22	0.00	3.31	110.64
30	60	7367.63	7367.63	0.00	3.13	114.73
30	90	7362.21	7362.14	0.00	5.19	120.81
30	120	7357.57	7357.57	0.00	4.50	117.45
30	150	7353.00	7353.00	0.00	1.38	22.44
30	180	7350.26	7350.26	0.00	1.34	20.84
30	210	7350.26	7350.26	0.00	1.30	22.27
40	0	13696.5	13696.5	0.00	0.93	89.74
40	30	13623.2	13623.2	0.00	3.26	444.6
40	60	13577.2	13576.5	0.01	3.53	2197.6
40	90	13558.3	13558.3	0.00	5.56	2605.11
40	120	13543	13543	0.00	5.79	1687.92
40	150	13534.3	13529.2	0.04	7.46	627.08
40	180	13519.4	13519.4	0.00	9.95	202.56
40	210	13519.4	13519.4	0.00	8.48	206.72
50	0	12151.10	12151.10	0.00	2.97	196.05
50	30	12023.80	12023.92	0.00	10.96	11662.46
50	60	11958.50	11958.52	0.00	8.47	14720.90
50	90	11936.70	11936.70	0.00	7.92	8900.40
50	120	11923.00	11923.00	0.00	9.06	6801.64
50	150	11916.00	11916.00	0.00	9.57	1673.88
50	180	11916.00	11916.00	0.00	12.84	603.10
50	210	11916.00	11916.00	0.00	14.30	1958.54
60	0	15262.40	15262.40	0.00	3.21	753.34
60	30	15200.50	15200.50	0.00	54.44	9430.63
60	60	15174.30	15174.30	0.00	65.22	10921.94
60	90	15148.10	15148.10	0.00	59.95	16308.94
60	120	15121.90	15121.90	0.00	38.90	7217.47
60	150	15107.11	15107.11	0.00	60.85	9484.54
60	180	15098.40	15098.35	0.00	70.09	14837.70
60	210	15091.70	15091.70	0.00	85.18	2816.82
70	0	15882.90	15882.00	0.01	10.42	2369.27
70	30	15814.90	15814.93	0.00	43.38	20998.00
70	60	15787.40	15787.40	0.00	88.02	22276.35
70	90	15759.80	15759.81	0.00	100.00	17681.44
70	120	15732.30	15732.25	0.00	67.00	22609.63
70	150	15712.30	15712.26	0.00	110.41	62501.62
70	180	15701.40	15701.38	0.00	107.63	17025.76
70	210	15693.20	15693.16	0.00	133.20	14054.24

Table 4.3: Comparison results for RPMP-PF algorithm ($P = 5$) and CPLEX solver

Nodes	B	Alg. value	CPLEX value	Gap, %	Algorithm time, s	CPLEX time, s
25	0	3488.25	3488.25	0.00	0.38	26.14
25	30	3380.31	3380.31	0.00	0.59	60.65
25	60	3360.55	3360.55	0.00	1.06	82.16
25	90	3345.43	3345.43	0.00	1.73	86.24
25	120	3337.22	3337.22	0.00	1.87	29.63
25	150	3337.22	3337.22	0.00	2.50	27.74
25	180	3337.22	3337.22	0.00	2.93	26.87
25	210	3337.22	3337.22	0.00	3.00	24.18
30	0	3694.26	3694.26	0.00	1.48	40.91
30	30	3583.91	3583.91	0.00	0.48	60.03
30	60	3564.04	3564.04	0.00	0.29	391.98
30	90	3548.92	3548.92	0.00	0.46	63.73
30	120	3540.71	3540.71	0.00	0.69	66.25
30	150	3541.71	3541.71	0.00	0.80	35.14
30	180	3542.71	3542.71	0.00	1.23	44.46
30	210	3543.71	3543.71	0.00	1.00	29.85
40	0	10828.30	10828.30	0.00	5.30	136.08
40	30	10691.40	10691.40	0.00	12.50	624.49
40	60	10581.20	10581.20	0.00	19.77	2994.91
40	90	10578.90	10523.50	0.52	30.31	283.49
40	120	10569.40	10503.30	0.63	15.69	319.40
40	150	10515.60	10497.70	0.17	21.06	349.66
40	180	10515.60	10497.70	0.17	50.27	357.07
40	210	10515.60	10497.70	0.17	28.25	294.01
50	0	8853.17	8853.24	0.00	5.85	2031.44
50	30	8683.62	8633.20	0.58	10.70	14542.66
50	60	8485.74	8485.73	0.00	27.56	4169.52
50	90	8442.81	8442.81	0.00	39.83	4952.99
50	120	8442.81	8442.81	0.00	47.14	2332.90
50	150	8442.81	8442.81	0.00	55.83	1479.38
50	180	8442.81	8442.81	0.00	67.36	1230.21
50	210	8442.81	8442.81	0.00	68.62	854.64

budget than to the cases with low ($B = 0$) and high ($B = 210$) budget. Similar conclusions hold for the RPMP-PF algorithm.

Figures 4.2(a) and 4.2(b) show how the total expected cost changes with fortification budget (for $n = 50$ and $n = 150$, RUFL-PF model). The relationship is shown for values of

Table 4.4: Performance results for RUFL-PF algorithm ($n = 100$ and $n = 150$)

Nodes	B	LB	UB	Gap, %	Alg. time, s
100	0	17303.10	17444.70	0.81	69.60
100	30	17201.30	17299.10	0.57	642.20
100	60	17204.30	17255.90	0.30	625.20
100	90	17142.10	17220.30	0.45	700.70
100	120	17110.60	17187.30	0.45	666.20
100	150	17111.80	17157.60	0.27	961.10
100	180	17056.10	17127.80	0.42	800.70
100	210	17023.00	17095.20	0.42	900.30
150	0	18903.9	19390.6	2.51	115.92
150	30	18861.01	19123.36	1.37	1278.5
150	60	18810.51	18872.54	0.33	2123.83
150	90	18808.42	18811.21	0.01	2058.84
150	120	18775	18787.61	0.07	2965.55
150	150	18740.41	18770.4	0.16	3034.89
150	180	18738.4	18760.3	0.12	3510.13
150	210	18738.4	18760.5	0.12	3679.1

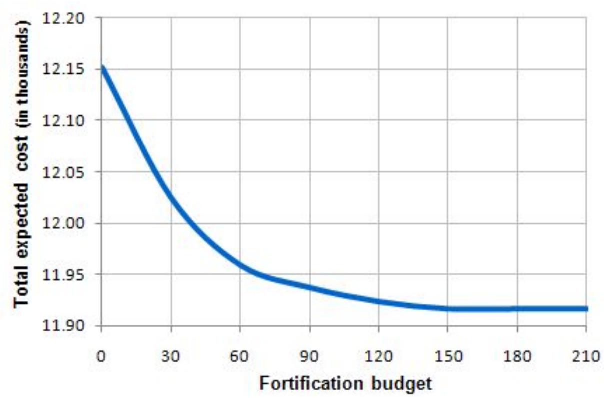
Table 4.5: Performance results for RPMP-PF algorithm ($n = 100$ and $n = 150$)

Nodes	B	LB	UB	Gap, %	Alg. time, s
100	0	12408.40	13143.20	5.59	35.17
100	15	12393.50	12849.90	3.55	283.82
100	30	12506.00	12602.80	0.77	407.59
100	45	12528.90	12732.80	1.60	455.23
100	60	12337.20	12555.70	1.74	535.29
100	75	12503.60	12514.60	0.09	539.85
100	90	12345.70	12465.70	0.96	601.80
100	105	12342.90	12426.10	0.67	758.19
150	0	14833.6	15166.1	2.19	97.97
150	15	14102.7	14978.3	5.85	1183.74
150	30	14445.8	14622.3	1.21	3521.99
150	45	14422.8	14615.3	1.32	4101.28
150	60	14504	14607.9	0.71	4242.63
150	75	14475	14604.3	0.89	5843.83
150	90	14473.1	14600.3	0.87	5884.77
150	105	14423.6	14600.3	1.21	6328.26

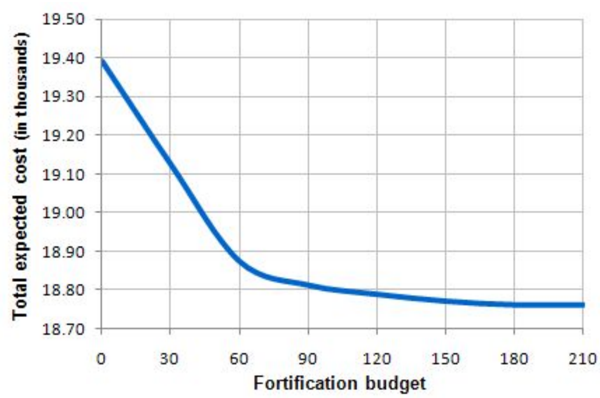
B ranging between 0 and 210. As expected, in both cases, the curves exhibit a downward trend as fortification budget increases. It can be observed that the marginal reduction in the expected total cost diminishes with B. This effect can be explained by noting that the

Table 4.6: Percentage of computation time increase from $n = 25$ to $n = 70$, in thousands % (RUFL-PF)

B	0	90	210
Algorithm	4.06	9.80	5.90
CPLEX	34.13	163.61	112.87



(a) $n = 50$



(b) $n = 150$

Figure 4.2: Relationship between fortification budget and expected total cost (RUFL-PF)

algorithm allocates fortification budget to the facilities in the decreasing order of reduction in the total expected cost gained from fortification.

Chapter 5: Incorporation of Nonlinear Fortification Functions

In most real-life environments, the reliability of a system can be improved at some cost but the marginal improvement generally decreases as the system becomes more reliable and ultimately reaches its maximum attainable reliability (McEachern, 2008). In this chapter, we extend the RUFL-PF with linear fortification model and handle the case when the amount of reliability improvement is a nonlinear function of fortification investment. We call this model RUFL-NF. The developed approach is also applicable to RPMP-PF.

5.1 Model Formulation

We use the same notation as for RUFL-PF (see § 4.1.1). Similar to RUFL-PF, for each facility j, we let u_j and B respectively denote the maximum obtainable reliability improvement and the total available fortification budget. With each facility j, we associate a function $g_j(\cdot)$ which maps the fortification investment onto the facility reliability. Without the loss of generality and to reflect the effect of diminishing marginal reliability improvement, we consider that functions $\{g_j\}$ are concave in the range of fortification investment under study (e.g., see Fig. 5.1).

The formulation of RUFL-NF is similar to that of the RUFL-PF with the exception that the constraint $\sum_j r_j \Delta_j \leqslant B$ is replaced with the constraint $\sum_j g_j(\Delta_j) \leqslant B$, as shown below.

(RUFL-NF)

minimize $\sum_{j \in J} f_j X_j + \sum_{i \in I} \sum_{j \in J} [h_i d_{ij}(1 - (q_j - \Delta_j))Y_{ir0} + h_i d_{ij} Y_{ij1} \sum_{r \in J,\ r \neq j} (q_r - \Delta_r) Y_{ir0}]$

subject to

$$\sum_j Y_{ij0} = 1,\ \forall i \in I \tag{5.1a}$$

$$\sum_j Y_{ij1} = 1,\ \forall i \in I \tag{5.1b}$$

$$Y_{ij0} + Y_{ij1} \leqslant X_j,\ \forall i \in I, j \in J \tag{5.1c}$$

$$0 \leqslant \Delta_j \leqslant u_j,\ \forall j \in J \tag{5.1d}$$

$$\sum_j g_j(\Delta_j) \leqslant B \tag{5.1e}$$

$$X_j \in \{0,1\},\ \forall j \in J \tag{5.1f}$$

$$Y_{ij0}, Y_{ij1} \in \{0,1\},\ \forall i \in I,\ j \in J. \tag{5.1g}$$

5.2 Lagrangian Relaxation-Based Heuristic

In this section, we develop a Lagrangian relaxation-based heuristic solution by approximating the functions $\{g_j\}$ in the constraint (5.1e) by using piecewise linear functions. The developed heuristic is based on the algorithms presented in Chapter 4.

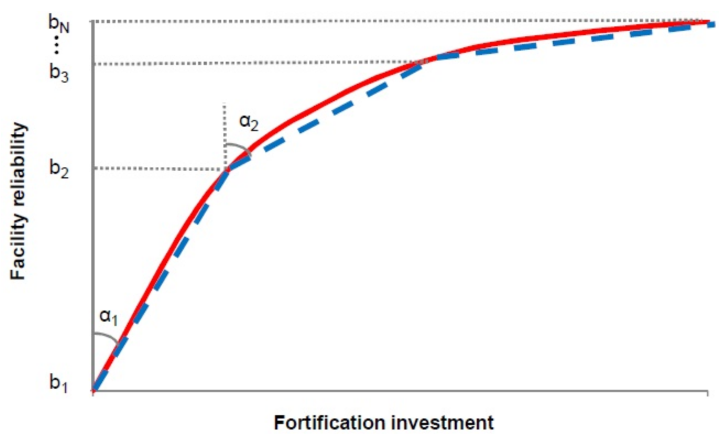


Figure 5.1: A piecewise linear approximation of a nonlinear fortification function

To approximate the nonlinear functions by piecewise linear functions, we have adopted the approach discussed in detail by Nemhauser and Wolsey (1988). As illustrated in Figure

5.1, we let $\{b_1, b_2, \dots, b_N\}$ be the set of breakpoints connecting the linear segments of the approximating function (blue dotted line). We let α_n denote the slope of the segment between breakpoints b_n and b_{n+1} (relative to the vertical axis). In other words, α_n can be interpreted as the cost associated with the unit reduction in the failure probability of facility j in the range of fortification investment between b_n and b_{n+1}. From this point, we will use the piecewise linear function to develop our heuristic solution.

5.2.1 Heuristic for Lower Bound

The process of developing a heuristic for the lower bounds is somewhat similar to the approach used in § 4.1.2.1. We proceed by relaxing the constraint (5.1c) by using Lagrange multipliers λ_{ij} (the relaxed model is called RUFL-NF-λ).

We solve RUFL-NF-λ for facility location variables X as we did in § 4.1.2.1. To solve for the customer assignment variables Y and fortification variables Δ, we proceed in two steps. We first solve a special case when fortification budget is zero. We then use this solution to handle the case when fortification budget is positive.

We solve the case $B = 0$ as in § 4.1.2.1. We then let $\Phi_i(j,k)$ denote the part of the objective function of the resulting solution associated with customer i, where j and k denote its primary and backup suppliers, respectively. Once we obtain the optimal assignment of suppliers for all customers $i \in I$, we let $G(j)$ be the set of customers who have facility j as their primary supplier.

To solve the case when $B > 0$, suppose that supplier j is fortified and c is the amount of reliability improvement. We denote the corresponding objective function as $\Psi_i(j,k)$. For customer i, the improvement gained from fortifying supplier j is then $\Phi_i(j,k) - \Psi_i(j,k)$. Hence, the overall improvement gained from fortifying supplier j is $\sum_{i \in G(j)}(\Phi_i(j,\cdot) - \Psi_i(j,\cdot))$. We let $\bar{r}_j$ denote the cost associated with the unit reduction in the failure probability of supplier j at its current reliability level. The value of $\bar{r}_j$ can be found by locating the current reliability level on the vertical axis of the approximating function (see Fig. 5.1.

We now let $E = \{j : q_j > 0\}$. For each $j \in E$, we introduce the following measure of fortification efficiency.

$$\delta(j) = \frac{\sum_{i \in G(j)}(\Phi_i(j,\cdot) - \Psi_i(j,\cdot))}{\bar{r}_j \cdot c} = \frac{\sum_{i \in G(j)} h_i c\,(d_{i\cdot} - d_{ij})}{\bar{r}_j \cdot c} = \frac{\sum_{i \in G(j)} h_i(d_{i\cdot} - d_{ij})}{\bar{r}_j} \cdot \quad (5.2)$$

We aim to find s such that $\delta(s) = \max_{j \in E} \delta(j)$ and allocate fortification budget $B > 0$ starting with this supplier. Note that since the value of $\delta(j)$ depends on the current reliability level of j, the solution algorithm will need to update the value of $\delta(j)$ whenever the fortification process improves the reliability of facility j to the next level (linear segment).

We implement the fortification process as described in the algorithm below.

Step 1. For all $j \in E$, set $\Delta_j = 0$.

Step 2. Find facility s such that $\delta(s) = \max\limits_{j \in E} \delta(j)$. Select n such that $b_n \leqslant (1 - q_s) \leqslant b_{n+1}$. In steps 3-5 below, we determine the level of fortification of facility s which we denote by Δ_s. Note that $\Delta_s \leqslant \min\{u_s, B/\bar{r}_s, [b_n - (1 - q_s)]\}$.

Step 3. For each customer $i \notin G(s)$, let j and k denote its current primary and backup supplier, respectively (i.e., $\Phi_i(j,k) = \min\limits_{v,w}\{\Phi_i(v,w)\}$). We want to determine Δ_s^i, the minimal fortification level of s that causes re-assignment of i. Suppose that s is chosen as the primary supplier of i. Let $\Phi_i(s,t) = \min\limits_{w}\{\Phi_i(s,w)\}$, where t denotes the backup supplier of i when s is chosen as the primary one. Now if the reliability of s was improved by c, then $\Phi_i(s,t)$ would decrease by $c \cdot h_i(d_{it} - d_{is})$. As the reliability of s increases, if $\Phi_i(s,t)$ becomes smaller than $\Phi_i(j,k)$, then re-assigning customer i to suppliers s and t will result in an improved objective function. We have that $\Delta_s^i = \dfrac{\Phi_i(s,t) - \Phi_i(j,k)}{h_i(d_{it} - d_{is})}$.

Step 4. Let $\tilde{\Delta}_s = \min\limits_{i \notin G(s)} \tilde{\Delta}_s^i$. Then $\Delta_s = \min\{\tilde{\Delta}_s, \min\{u_s, B/\bar{r}_s, [b_n - (1 - q_s)]\}\}$.

Step 5. Update $G(s)$, if needed. Set $B = B - \bar{r}_s\Delta_s$, $q_s = q_s - \Delta_s$, $u_s = u_s - \Delta_s$. If $u_s = 0$, $E \leftarrow E \backslash \{s\}$.

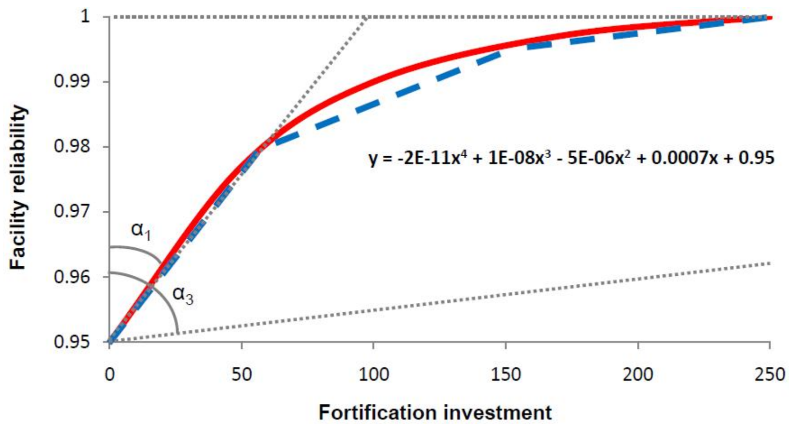
Step 6. If $B > 0$ and $E \neq \varnothing$, recalculate δ_j for all $j \in E$, and proceed to Step 1. Else, stop.

5.2.2 Heuristic for Upper Bound

A feasible solution can be obtained by using the solution to (RUFL-NF-λ). We first select the facilities opened in the solution to (RUFL-NF-λ). For each customer, we assign the nearest and the second nearest open facility as the primary and backup supplier, respectively. To decide which facilities to fortify, we proceed in the following way. For customer i, let j and k respectively denote its primary and backup supplier. The expected transportation cost associated with i amounts to $h_i d_{ij}(1-q_j) + h_i d_{ik} q_j$. If the reliability of facility j is increased by c, the resulting cost improvement equals to $\varphi_j = \sum_{i \in G(j)} h_i(d_{ik} - d_{ij}) \cdot c$. Next we determine $\bar{r}_j$, the cost associated with the unit reduction in the failure probability of supplier j, by locating its current reliability level of on the vertical axis of the approximating function. We then fortify opened facilities in a greedy manner, one at a time, in the decreasing order of $\varphi_j / \bar{r}_j$ values. Updating of the Lagrangian multipliers is conducted in a manner described in §4.1.2.3.

5.3 Computational Results

We tested the performance of the RUFL-NF solution algorithm using seven datasets containing 25, 30, 50, 60 and 70 nodes. The values for all the parameters were the same as in Section 4.3.1. In the testbed examples, we generated nonlinear fortification functions of the form $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$, where the values of the coefficients were site specific (see Fig. 5.2). We used 3-piecewise linear functions to approximate each g_j. Since the failure probabilities of the facilities were randomly drawn from $\mathrm{Uni}(0, 0.05)$, we set the first breakpoint b_1 to 0.95 for all facilities. For each facility j, the choice of the second breakpoint was done so that the value of α_1 (see Fig. 5.1) was equal to the value of r_j in § 4.3.1. The choice of b_3 was arbitrary; b_4 was set to 1.0 for all facilities.



6

Figure 5.2: A sample nonlinear fortification function

We compared the solutions obtained using the RUFL-NF algorithm with optimal objective function values for RUFL-PF for the cases when $r_j = \alpha_1$ and $r_j = \alpha_3$. The solutions to these cases provide the respective lower and upper bounds for RUFL-NF based solutions (see Fig. 5.2). The results of the comparison are shown in Table 5.1.

As it can be seen from the table, the developed algorithm performed well in terms of the computational time. As expected, the computation time increased with the size of the problem. Comparing the computational time to that of the RUFL-PF heuristic (see Table 4.1), the RUFL-NF algorithm performs somewhat slower. This can be explained by noting that at each iteration, the "nonlinear" fortification process is more complex and thus takes longer. The objective function values obtained by the algorithm were consistently in between the respective lower and upper RUFL-PF bounds.

Table 5.1: Results of RUFL-NF algorithm

Nodes	B	RUFL-PF lower bound	RUFL-NF value	RUFL-PF upper bound	RUFL-NF time, s
25	0	7762.68	7762.68	7762.68	0.08
25	30	7669.79	7676.42	7687.77	1.00
25	60	7643.00	7649.32	7678.78	0.67
25	90	7641.76	7641.76	7669.79	0.56
25	120	7641.76	7641.76	7660.80	0.62
25	150	7641.76	7641.76	7651.80	0.73
25	180	7641.76	7641.76	7643.81	0.84
25	210	7641.76	7641.76	7641.76	1.10
30	0	7408.25	7408.25	7408.25	0.38
30	30	7383.22	7390.03	7395.00	1.43
30	60	7367.63	7379.80	7388.53	1.37
30	90	7362.14	7371.83	7383.22	1.78
30	120	7357.57	7365.24	7377.90	2.09
30	150	7353.00	7360.60	7372.59	2.38
30	180	7350.26	7356.01	7367.63	3.82
30	210	7350.26	7353.05	7365.31	1.30
40	0	13696.50	13696.50	13696.50	1.44
40	30	13623.20	13664.40	13670.00	5.89
40	60	13576.50	13623.20	13650.70	3.51
40	90	13558.30	13595.40	13623.20	4.53
40	120	13543.00	13574.10	13597.70	7.53
40	150	13529.20	13559.40	13583.30	7.46
40	180	13519.40	13547.20	13575.40	5.79
40	210	13519.40	13536.90	13569.90	34.48
50	0	12151.10	12151.10	12151.10	2.97
50	30	12023.92	12051.90	12102.50	12.64
50	60	11958.52	11998.90	12057.00	40.43
50	90	11936.70	11973.50	12023.90	46.48
50	120	11923.00	11953.70	11997.60	94.02
50	150	11916.00	11943.60	11975.10	54.87
50	180	11916.00	11933.60	11958.50	53.81
50	210	11916.00	11926.40	11950.10	171.39
60	0	15262.40	15262.40	15262.40	3.86
60	30	15200.50	15209.60	15221.40	402.00
60	60	15174.30	15183.40	15209.30	355.37
60	90	15148.10	15163.50	15200.50	1332.20
60	120	15121.90	15149.80	15191.80	277.90
60	150	15107.11	15136.70	15183.00	404.52
60	180	15098.35	15123.60	15174.30	511.49
60	210	15091.70	15112.40	15165.60	188.46
70	0	15882.00	15882.00	15882.00	5.53
70	30	15814.93	15827.30	15838.50	630.82
70	60	15787.40	15799.70	15824.50	455.00
70	90	15759.81	15779.40	15814.90	479.36
70	120	15732.25	15764.50	15805.80	555.61
70	150	15712.26	15750.80	15796.60	522.92
70	180	15701.38	15737.00	15787.40	662.86
70	210	15693.16	15723.60	15778.20	483.16

Chapter 6: Summary of Main Results, Contributions and Future Research

This dissertation presents the development of three related classes of models which are intended to aid in the design of robust distribution networks and improve the utilization of limited fortification resources. The developed models extend the uncapacitated P-median and fixed-charge location models by considering random facility failures, incorporation of supplier backups, and integration of facility fortification within a finite budget. The development of the models proceeded progressively from binary fortification via linear fortification functions to partial fortification using nonlinear functions. For each of the models, we developed solution algorithms and demonstrated their computational efficiency for solving large-scale problems.

Summary of the main results is as follows. Both RPMP-BF and RUFL-BF were formulated as nonlinear integer programming models which were proven to be $\mathscr{N}\mathscr{P}$-hard. Our testbed results showed that the developed Lagrangian relaxation-based algorithms were computationally efficient, demonstrating a distinctively better performance than CPLEX, particularly for solving large scale problems. Our comparison study revealed that both RPMP-BF and RUFL-BF based policies outperformed their respective myopic counterparts over the examined range of fortification budget. Both RPMP-BF and RUFL-BF based policies exhibited a diminishing marginal reduction in the total expected cost as fortification budget increased. It was also shown that facility locations were generally selected from among demand-heavy nodes. In the testbed case, when compared to RUFL-BF, the RPMP-BF based policy had a broader range of attractive fortification investment opportunities.

Both RPMP-PF and RUFL-PF were formulated as nonlinear integer programming models. The RUFL-PF model was proven to be $\mathscr{N}\mathscr{P}$-hard. The developed heuristic based algorithms performed better than CPLEX for small and medium sized problems solvable by the solver. The heuristics were also found to be efficient for solving large-scale problems. For both algorithms, the computation time increased significantly with the network size but the rate of increase was substantially smaller for the algorithms than for CPLEX. For both algorithms, the computation time increased with fortification budget, especially for larger sized networks. We have also observed a diminishing marginal reduction in the total expected cost as fortification budget increased. The developed RUFL-NF solution algorithm performed relatively fast for small and medium sized networks. RUFL-NF solutions were always in between the respective lower and upper RUFL-PF based bounds.

Our main contributions are as the following. Comparing to the existing relevant literature, namely Lim et al. (2010), our models feature a number of distinguishing advances: (i) our models incorporate a finite fortification budget which enables a decision maker to match solutions to available reliability improvement resources; (ii) our formulation makes it possible to assess the effectiveness of fortification investments (by using ROR) and compare it to that of alternative investment opportunities; (iii) in our model, a facility can be fortified to any reliability level which allows a more efficient utilization of limited fortification resources; (iv) by using nonlinear fortification functions, our formulation is able to capture the effect of diminishing marginal reliability improvement from fortification investment; and (v) our formulation allows to analyze periodic fortification upgrades as additional fortification resources become available. The models are intended to support corporate decision makers in the design of robust distribution networks using constrained fortification resources.

Future research directions are discussed as the following. Our models assumed that the facilities had unlimited capacity. Although this assumption prevails in reliable facility location literature, it may be unrealistic, particularly in the case of lean manufacturing. On the other hand, the practical significance of the capacitated case is also associated with its very significant modeling complexity. Capacitated models constitute an entirely different class of methodologies since the assumption of restricted capacity requires an overhaul of backup mechanisms and customer assignation. This makes the capacitated case worthy of future study. In addition, we plan to investigate the effect of correlated and simultaneous facility failures which may be of particular importance when modeling certain types of disruptions, such as labor strikes, political unrest, and disruptions due to forces of nature.

List of References

Abboud, N. E., 1997. A simple approximation of the EMQ model with poisson machine failures. Production Planning and Control 8, 385–397.

Abboud, N. E., 2001. A discrete-time markov production-inventory model with machine breakdowns. Computers & Industrial Engineering 39, 95–107.

Arcelus, F. J., Srinivasan, G., 1995. Discount strategies for one-time only sales. IIE Transactions 27, 618–624.

Ardalan, A., 1995. A comparative analysis of approaches for determining optimal price and order quantity when a sale increases demand. European Journal of Operational Research 84, 416–430.

Arreola-Risa, A., DeCroix, C. A., 1998. Inventory management under random supply disruptions and partial backorders. Naval Research Logistics 45, 687–703.

Aull-Hyde, R. L., 1992. Evaluation of supplier-restricted purchasing options under temporary price discounts. IIE Transactions 24, 184–186.

Bielecki, R. E., Kumar, P. R., 1988. Optimality of zero inventory policies for unreliable manufacturing systems. Operations Research 36, 532–541.

Billington, C., Johnson, B., Triantis, A., 2001. A real options perspective on supply chain management in high technology. Journal of Applied Corporate Finance 15, 32–43.

Buzacott, J. A., Shantikumar, J. G., 1993. Stochastic Models of Manufacturing Systems. Englewood Cliffs, NJ: Prentice Hall.

Carpenter, C., 2010. Copper Advances as Chile Earthquake Cuts Power, Halts Mines. BusinessWeek.

Chao, H., 1987. Inventory policy in the presence of market disruptions. Operations Research 2, 274–281.

Chap, H., Manne, A. S., 1982. An Integrated Analysis of U.S. Oil Stockpiling Policies. Tech. rep., Cambridge, MA: Ballinger Publishing Company.

Christopher, M., Peck, H., 2004. Building the Resilient Supply Chain. International Journal of Logistics Management 15, 1–14.

Church, R., Scaparra, M., 2007. Protection Critical Assets: the r-interdiction Median Problem with Fortification. Geographical Analysis 39, 129–146.

Church, R., Scaparra, M., Middleton, R., 2004. Identifying Critical Infratructure: the Median and Covering Facility Interdiction Problems. Annals of the Association of American Geographers 94, 491–502.

CNN, 2008. Gulf Oil Production Still on Hold. http://edition.cnn.com/2005/BUSINESS/09/28/oil.production/, last accessed on 04/02/2008.

Cui, T., Ouyang, Y., Shen, Z.-J., 2010. Reliable Facility Location Design under the Risk of Disruptions. Operations Research 58, 998–1011.

Cundari, S., Czerwinski, J., Kitfield, J., Mason, D., Sands, C., 2008. The U.S.-Canada Strategic Partnership in the War on Terrorism. http://www.thepresidency.org/storage/documents/ScienceTechnology2008/US-and-Canada.pdf, last accessed on 04/02/2008.

Engardio, P., March 2001. Why the supply chain broke down. Business Week 41.

Fisher, M., March-April 1997. What is the right supply chain for your product? Harvard Business Review, 105–116.

Fisher, M., 2004. The Lagrangian Relaxation Method for Solving Integer Programming Problems. Management Science 50, 1861–1871.

Gallego, G., 1988a. Linear control policies for scheduling a single facility after an initial disruption. Technical report 770, School of OR and IE, Cornell University, Ithaca, NY.

Gallego, G., 1988b. Produce-up-to policies for scheduling a single facility after an initial disruption. Technical report 771, School of OR and IE, Cornell University, Ithaca, NY.

Gallego, G., 1990. Scheduling the production of several items with random demands in a single facility. Management Science 36, 1579–1592.

Glionna, J., Rotella, S., 2009. Swine Flu Moves Other Nations to Act. Los Angeles Times.

Golany, B., Xia, Y. S., Yang, J., Yu, G., 2002. An interactive goal programming procedure for operational recovery problems. Optimization and Engineering 3, 109–127.

Groenevelt, H., Seidmann, A., Pintelon, L., 1992a. Production batching with machine breakdowns and safety stocks. Operations Research 40, 959–971.

Groenevelt, H., Seidmann, A., Pintelon, L., 1992b. Production lotsizing with machine breakdowns. Management Science 38, 104–123.

Gurler, U., Parlar, M., 1997. An inventory problem with two randomly available suppliers. Operations Research 45, 904–918.

Hendricks, K. B., Singhal, V. R., 2003. The effect of supply chain glitches on shareholder wealth. Journal of Operations Management 21, 501–522.

Henig, M., Gerchak, Y., 1990. The structure of periodic review policies in the presence of random yield. Operations Research 38, 634–643.

Hopp, W. J., Pati, N., Jones, P. C., 1989. Optimal inventory control in a production flow system with failures. International Journal of Production Research 27, 1367–1384.

Kim, C. H., Hong, Y., 1997. An extended EMQ model for a failure prone machine with general lifetime distribution. International Journal of Production Economics 49, 215–223.

Koncius, J., 2003a. SARS Hits the Furniture Industry; China Is Crucial to the U.S. Market. The Washington Post.

Koncius, J., 2003b. SARS Scope Extends to 'Made in China' Furniture Industry. http://articles.sfgate.com/2003-06-04/home-and-garden/17494564-1-furniture-experts-high-end-furniture-sars, last accessed on 04/02/2008.

Kotak, R., 2005. Oil Prices Drift Lower as Demand Wanes after Katrina. http://www.forbes.com/feeds/afx/2005/09/12/afx2219126.html/, last accessed on 04/02/2008.

Latour, A., 2001. Trial by fire: A blaze in Albuquerque sets off major crisis for cell-phone giants - Nokia handles supply disruptions as Ericsson of Sweden gets burned. Wall Street Journal.

Lee, H., Padamanabhan, P., Whang, S., 1997. Information distortion in supply chain: the bullwhip effect. Management Science 43, 546–558.

Lee, H. L., 1992. Lot sizing to reduce capacity utilization in a production process with defective items, process corrections, and rework. Management Science 38, 1314–1328.

Li, Q., Savachkin, A., 2011. A Heuristic Approach to the Design of Fortified Distribution Networks. Transportation Research Part E: Logistics and Transportation Review (under review).

Li, Q., Zeng, B., Savachkin, A., 2011. Decision Support Models for Design of Reliable Distribution Networks. Naval Research Logistics (under review).

Li, X., Ouyang, Y., 2010. A Continuum Approximation Approach to Reliable Facility Location Design under Correlated Probabilistic Disruptions. Transportation Research Part B 44, 535–548.

Lim, M., Daskin, M., Bassamboo, A., Chopra, S., 2010. A Facility Reliability Problem: Formulation, Properties, and Algorithm. Naval Research Logistics 57, 58–70.

Marquis, C., 2001. Meat from Europe is Banned by U.S. as Illness Spreads. The New York Times.

McEachern, W., 2008. Microeconomics: A Contemporary Introduction. South-Western College.

Meyer, R. R., Rothkpof, M. H., Smith, S. A., 1979. Reliability and inventory in a production storage system. Management Science 25, 799–807.

Mohebbi, E., 2003. Supply interruptions in a lost-sales inventory system with random lead time. Computers and Operations Research 30, 824–835.

Moinzadeh, K., Aggarwal, P., Nov. 1997. Analysis of a production/inventory system subject to random disruptions. Management Science 43, 1577–1588.

Mouawad, J., Romero, S., 2005. Hurricane Katrina: the Oil Supply; Gas Prices Surge as Supply Drops. The New York Times.

Murphy, F. H., Toman, M. A., Weiss, H. J., 1987. A stochastic dynamic Nash game analysis of policies for managing the strategic petroleum reserves of consuming nations. Management Science 33 (4), 484–499.

Naddor, E., 1966. Inventory Systems. New York, NY: John Wiley & Sons, Inc.

Narula, S., Ogbu, I., Samuelsson, H. M., 1977. An algorithm for the p-median problem. Operations Research 25, 709–713.

Nemhauser, G., Wolsey, L., 1988. Integer and Combinatorial Optimization. Wiley-Interscience.

Owen, S., Daskin, M., 1998. Strategic facility location: A review. European Journal of Operational Research 111, 423–447.

Parlar, M., 1997. Continuous-review inventory problem with random supply interruptions. European Journal of Operational Research 99, 366–385.

Parlar, M., Berkin, D., 1991. Future supply uncertainty in EOQ models. Naval Research Logistics 38, 50–55.

Parlar, M., Perry, D., 1996. Inventory models of future supply uncertainty with single and multiple suppliers. Naval Research Logistics 43, 191–210.

Porteus, E. L., 1986. Optimal lot sizing, process quality improvement and setup cost reduction. Operations Research 34, 137–144.

Posner, M. J., Berg, M., 1989. Analysis of a production-inventory system with unreliable production facility. Operations Research Letters 8, 339–345.

Radjou, N., 2002. Adapting to supply network change. Forrester Research Inc, Cambridge, MA.

Rahim, M. A., 1994. Joint determination of production quantity, inspection schedule, and control chart design. IIE Transactions 26, 2–11.

Reuters, 2001. UK's Foot-and-mouth Crisis Deepens. http://tvnz.co.nz/view/tvnz-smartphone-story-skin/30727/, last accessed on 04/02/2008.

Rogers, M., 2001. Engineering Project Appraisal: the Evaluation of Alternative Development Schemes. Wiley-Blackwell.

Rosenblatt, M. J., Lee, H. L., 1986. Economic production cycles with imperfect production processes. IIE Transactions 18, 48–55.

Scaparra, M., Church, R., 2008a. A Bilevel Mixed-integer Program for Critical Infrastructure Protection Planning. Computers and Operations Research 35, 1095–1923.

Scaparra, M., Church, R., 2008b. An Exact Solution Approach for the Interdiction Median Problem with Fortification. European Journal of Operational Research 189, 76–92.

Sheffi, Y., 2001. Supply Chain Management under the Threat of International Terrorism. Internation Journal of Logistics Management 12, 1–11.

Sheffi, Y., 2005. The Resilent Enterprise. The MIT Press.

Sheffi, Y., Rice, J., 2005. A Supply Chain View of the Resilient Enterprise. MIT Sloan Management Rivew 47 (1), 41–48.

Snyder, L., 2006. Facility Location under Uncertainty: a Review. IIE Transactions 38, 537–554.

Snyder, L., Daskin, M., 2005. Reliability Models for Facility Location: The Expected Failure Cost Case. Transportation Science 39, 400–416.

Song, J., Zipkin, P., 1996. Inventory control with information about supply conditions. Management Science 42, 1409–1419.

Strahan, A., Smith, G., 2005. Gulf Oil, Gas Output to Take 3 Months to Recover 90 % . Bloomberg.

Taylor, S. G., Bradley, C. E., 1985. Optimal ordering strategies for announced price increases. Operations Research 33, 312–325.

Teisberg, T. J., 1981. A dynamic programming model of the U.S. strategic petroleum reserve. Bell Journal of Economics 12, 526–546.

Tersine, R. J., Barman, S., 1995. Economic purchasing strategies for temporary price discounts. European Journal of Operational Research 80, 328–343.

Thurm, S., April 2001. Missed signals: Behind cisco's woes are some wounds of its own making - Company pushed to line up more parts, kept hiring even as its sales slowed - Trouble finding the brakes. Wall Street Journal A1.

Tran, K. T., December 2000. Sony scrambles to restock Playstation 2. Wall Street Journal B11.

Weiss, H. J., Rosenthal, E. C., 1992. Optimal ordering policies when anticipating a disruption in supply or demand. European Journal of Operational Research 59, 370–382.

Wilson, M., 2005. The Impact of Transportation Disruptions on Supply Chain Performance. Transportation Research Part E: Logistics and Transportation Review 43, 295–320.

Xia, Y. S., Qi, X. T., Yu, G., 2001. Real-time production and inventory distribution management under the continuous rate economic production quantity model. The University of Texas.

Yang, M.-H., Xia, Y. S., Golany, B., Gilbert, S. M., Yu, G., 2004. Real-time disruption management in a two-stage production and inventory system. IIE Transactions 36, 111–125.

Appendices

Appendix A: The 30-Node Dataset

Table A.1: 30-Node Dataset

Nodes	Demand	Longitude	Latitude	q_j	r_j	f_j
1	297.60021	38.56685	121.46736	0.014	620.79	977
2	179.90455	42.66575	73.799017	0.045	459.02	1059
3	169.8651	30.30588	97.750522	0.015	325.73	1379
4	47.81468	38.97165	76.503033	0.035	979.00	1353
5	43.75099	44.947744	93.103686	0.026	44.23	1072
6	42.19973	30.448967	91.126043	0.005	546.00	975
7	40.40587	32.3544	86.284287	0.042	543.86	1203
8	36.85296	38.19077	84.865203	0.048	603.27	1496
9	36.65228	33.54255	112.071399	0.044	707.18	800
10	34.86703	34.039236	80.886341	0.017	486.47	618
11	32.94394	39.768035	104.872655	0.035	566.86	676
12	32.87116	41.7657	72.683866	0.038	192.05	535
13	31.45585	35.46705	97.513491	0.009	711.89	730
14	28.42321	44.9245	123.022057	0.039	204.26	1051
15	27.76755	41.576738	93.617405	0.046	506.80	1142
16	25.73216	32.3205	90.207591	0.009	444.00	828
17	24.77574	39.0379	95.691999	0.022	889.00	881
18	23.50725	34.7224	92.354076	0.035	815.14	1034
19	17.93477	38.35055	81.630439	0.014	157.07	738
20	17.2285	40.777267	111.929921	0.021	181.76	1208
21	15.78385	40.8164	96.688171	0.009	233.33	699
22	15.15069	35.678502	105.954149	0.025	608.40	1355
23	12.27928	44.330647	69.729714	0.021	940.00	597
24	12.01833	39.148328	119.743243	0.006	146.00	1467
25	11.09252	43.231594	71.560077	0.029	472.76	702
26	10.06749	43.606651	116.2261	0.025	977.84	1219
27	6.388	46.805467	100.767298	0.029	648.41	902
28	6.069	38.90505	77.016167	0.029	756.41	1173
29	5.62758	44.266482	72.571854	0.028	763.18	575
30	4.53588	41.14545	104.792349	0.017	676.29	930