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The effect of transportation subsidies on urban sprawl

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The Effect of Transportation Subsidies on Urban Sprawl

by

Qing Su

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
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ABSTRACT

This dissertation investigates transportation subsidies as sources of urban sprawl. Apart from tolls, motorists do not pay highway user-fees, but they do pay gasoline taxes. Gasoline tax revenues are insufficient to cover the U.S. highway costs. Government, therefore, uses general tax revenues to cover highway expenditures. Since users do not pay the full cost of their travel, they have an incentive to travel longer commuting distances. Highway subsidies are, therefore, a potential contributor to urban sprawl. A similar argument applies to public transit.

To capture the effects of subsidized automobile and public transit travel, we extend the standard urban spatial single-mode model (Brueckner, 1987) to incorporate public subsidies for both one and two modes. Comparative static analysis of both models produces empirically testable hypotheses. Our most important theoretical result is that transit subsidies are inversely related to urban sprawl while auto subsidies are directly related to urban sprawl.

The empirical analysis focuses on tests of the two-mode model. For consistency with the monocentric assumption of our models, our sample consists of urbanized areas located within a single county and having only one central city. Spatial size of the urbanized area is the dependent variable. Following our theory, explanatory variables comprise the transit subsidy, the highway subsidy, number of households, agricultural land rent,

mean household income, and fixed and variable costs for transit and auto. We find that the spatial size of the urbanized area shrinks with an increase in the transit subsidy. The effect of highway subsidies, however, is ambiguous. We apply both ordinary least squares and two-stage least square regression analyses, and the results are qualitatively the same for both methods of estimation.

CHAPTER 1 INTRODUCTION

Urban sprawl is a topic that has generated much debate in recent years and has become an important policy issue in the United States. The term *urban sprawl* was first used in 1937 by Earle Draper, one of the first city planners in the southeastern United States (Black 1996). In the years since Draper introduced the term, popular concern with urban sprawl has continued and grown. In 1998, more than 150 ballot measures were introduced to restrict urban sprawl in one way or another, and more than 85 percent of those measures were passed (Samuel 1998). By the year 2000, 14 states had adopted substantive land-use regulations to reduce urban sprawl (NARC 2000). The purpose of this chapter is to define urban sprawl and to discuss its consequences and causes.

1.1 What is urban sprawl?

Osborn (Williams 2000), the British advocate of city planning, who summarized the major debate of urban sprawl in the 1940s, describes urban sprawl as a type of urban growth that is economically wasteful and socially disadvantageous. It is economically wasteful because rising income and rapid transport induce people to move away from the central cities to suburbs where they find desirable residential surroundings at the expense of long and costly daily commutes. It is socially disadvantageous because local community life is weakened or even destroyed, and access to the countryside is made more difficult for those people left in the central city (Nechyba and Walsh 2004). Brueckner (2000) defines urban sprawl as excessive spatial expansion of cities. He notes that urban sprawl is a pejorative term in that it implies the normative judgment that urban growth is

excessive. Mills (1999) prefers to use the non-pejorative term *suburbanization* to describe the phenomenon of continuous and unfettered urban growth in the United States. Nelson and Foster (1999) and Pendall (1999) define urban sprawl as unplanned, uncontrolled, and uncoordinated single-use development that does not provide a functional mix of uses and/or is not functionally related to surrounding land uses and which variously appears as low-density, ribbon or strip, scattered, leapfrog, or isolated development.

Generally, it is convenient to consider three different forms that urban sprawl can take. It may involve relatively low-density residential and employment areas coupled with low-density suburbanization at the urban fringe (Glaeser and Kahn 2003), planned communities that have their own “downtowns” perhaps located near a lake or park (Nechyba and Walsh 2004), or residences interspersed among rural areas.

There is no doubt that U.S. population has increasingly urbanized. The first census in 1790 found 5 percent of U.S. population living in urban areas; by the 2000 census, nearly 80 percent did so (Williams 2000). In addition, the urban population has decentralized, most notably since 1950, when nearly 70 percent of the population of urbanized areas lived in central cities while slightly over 30 percent lived in suburbs (Nechyba and Walsh 2004). The last half of the twentieth century has witnessed an accelerated growth of suburbs within the urban areas as Americans moved to new homes and shopping centers developed on the fringes of metropolitan areas. Even in areas of the country that have seen less dramatic population growth, urban sprawl has proceeded at a tremendous pace without letup. For instance, from 1990 to 1996, Little Rock, Arkansas, almost doubled its urban area from 109 to 199 square miles, while its population remained unchanged. Akron, Ohio, which had a population increase of only 3.5 percent, increased its urban area

by 65 percent in the same period. Austin, Texas, experienced a population increase of 50 percent between 1990 and 1996 and expanded its urban area by 160 percent (Williams 2000).

To most Americans, there is nothing particularly unusual about the sight of new housing developments with business and public facilities quickly going up around them. In the eyes of critics, however, this kind of real estate development represents an unrecognized “silent crisis” in which America’s finite resources are wasted at unsustainable rates (Williams 2000). In recent years, an increasingly heated debate has been under way between advocates of private urban development in a free market and advocates of government regulations against urban sprawl. Before turning to the debate itself, we need to consider the consequences and causes of urban sprawl.

1.2 Consequences of Urban Sprawl

Urban sprawl has several consequences that are widely criticized by economists. Generally speaking, so-called scattered, untimely, and unplanned urban development often occurs in urban fringe and rural areas, invading lands important to the environment for its open space and rich in natural resources (Nelson and Foster 1999, Brueckner 2000). Other consequences of urban sprawl are the dominance of the private automobiles for transportation and the large income disparities between city and suburbs (Downs 1999, Brueckner 2000). Additionally, unfettered urban growth contributes to the decay of downtown areas, for it lowers developers’ incentive to redevelop land closer to the central cities and lowers the tax base of central cities.

On the other hand, Gordon and Richardson (1997) argue that urban sprawl is a result of the fulfillment of consumer preferences. People choose to live in suburban areas,

thereby enjoying relatively lower land prices and larger amounts of living space, better schools for their children, a safer community, and better access to recreational facilities.

Although there are many alleged problems associated with urban sprawl, we focus on three that are widely criticized by both urban and public finance economists: traffic congestion, loss of open space, and income-level and racial segregation.

1.2.1 Traffic Congestion

Empirical studies show that traffic congestion in the United States generates huge costs to urban residents. Shrank and Lomax (2003) find that the total increase in commuting costs from congestion in 75 urban areas is \$69.5 million, of which \$60 million are time costs associated with longer commute time and \$9.5 million are spent on additional fuel consumption resulting from congestion-induced decreases in transport efficiency. These figures translate into \$520 per year for each resident in those urban areas.

Urban economists argue that peak-load taxes or toll roads are appropriate methods to internalize congestion externalities. Downs (1999) notes, however, that Americans may not be willing to incur the levels of congestion taxes that would be required to make a meaningful decrease in peak-hour traffic. Furthermore, the impact of congestion taxes on urban structure is not clear. Yinger (1993) argues that the imposition of congestion tolls or peak-load taxes would increase population density, thus reducing urban sprawl. On the other hand, Rufolo and Bianco (1998) argue that the internalization of congestion costs will lead to an increase in sprawl in the long run. They believe that the location decisions of both businesses and households are endogenous in the long run. Congestion pricing would raise commuting costs from suburbs to the CBD but not commuting costs

between suburbs. This disparity, they contend, will induce a decentralization of activity and further increase urban sprawl.

Another issue associated with congestion is air pollution. According to the U.S. Environmental Protection Agency's National Emission Inventory, in 2001, on-road vehicles in the United States accounted for 37 percent of total nitrogen oxides (a major contributor to the formation of ground-level ozone, particulate matter, haze, and acid rain), 27 percent of volatile organic compounds (which react with nitrogen oxides to form ground-level ozone), and 62 percent of total emissions of carbon monoxide (a particular threat for individuals who suffer from cardiovascular disease). Between 1970 and 2001, total vehicle miles traveled increased 151 percent from 1.1 trillion miles to 2.8 trillion miles. Over the same period, miles traveled by passenger cars and motorcycles increased by over 75 percent from 920 billion miles to 1.63 trillion (Federal Highway Administration, 2004).

Empirical studies yield no strong results on the link between air pollution and urban sprawl. The majority of the empirical studies claiming to document a clear link between density and pollution fail to account for other important variables such as income and household demographics (Nechyba and Walsh, 2003). Those studies that use micro-level data and attempt to control for these other factors generally conclude that the relationship between density and travel behavior is weaker and less certain than is often claimed in the popular press (Crane, 2001). Glaeser and Kahn (2003) also point out that U.S. urban air pollution has been on the decline since the 1970s, with more newer, cleaner cars replacing older polluting cars.

1.2.2 Loss of Open Space

Loss of open space is often identified as one of the key problems associated with sprawl. According to Burchfield, *et al.* (2003), land area in urban areas increased by 47.7 percent between 1976 and 1992, an annualized rate of 2.48 percent. As the authors point out, however, these numbers can be misleading because, although urbanization is increasing rapidly, only 1.92 percent of the entire U.S. land area as of 1992 is urbanized. Given this fact, why is the loss of open space receiving such prominence in the debate over sprawl?

Empirical studies by public finance economists suggest that open space within suburbs is significantly more important to households than open space at the urban fringe. Their work shows that housing prices are positively related to open space, such as public parks, privately owned open space, and the natural land cover immediately surrounding household locations. Geoghegan, *et al.* (1997) find that housing prices increase with the percentage of open space within a 0.1-kilometer ring surrounding a house and decrease with the percentage of open space within a 1-kilometer ring surrounding the house. Acharya and Bennett (2001) find that housing prices increase with the percentage of open space at a decreasing rate within both a quarter-mile and a 1-mile radius. The evidence seems to suggest that households prefer open space in the immediate vicinity of their residence but they do not place great value on open space on the urban fringe. Nechyba and Walsh (2003) argue that open space is a local public good. The more local the nature of the open space amenities and the larger the scale of development, the stronger will be the incentives for developers to provide efficient levels of open space.

1.2.3 Income-Level and Racial Segregation

Generally speaking, people's ability to move out of central cities into suburbs and their ability to choose among suburbs are not uniform across races and income groups. Within U.S. urbanized areas, the poor generally live in central cities while middle-income and high-income individuals live in suburbs (Mieszkowski and Mills, 1993; Glaeser, et al., 2000). Glaeser et al., (2000) suggest that for those using public transit, the auto-centered suburbs may simply not be a choice. In addition, politically created distortions through zoning leads to further segregation based on income and race (Glaeser et al., 2000). The geographic segregation and lower mobility of poorer households are very likely to result in a variety of social problems in poor areas (Glaeser and Kahn, 2003).

In the case of education, for example, it is well known that public school quality differs across neighborhoods and districts even when observable school inputs such as per pupil spending are equalized (Vigdor and Nechyba, 2003). Analogously equalized spending on public safety does not lead to equal levels of protection from crime, nor does equal public investment in basic infrastructure result in uniformly functional neighborhoods (Katz et al., 2001). In all these cases, the level of the public good depends critically on the characteristics of the local population that is being served by public expenditures on the good. Bayer, McMillan, and Reuben (2002) argue that housing markets, employment centers, and preferences for residential homogeneity rather than differences in tastes for quality of education represent the crucial explanatory forces for the racial school segregation patterns observed in the United States. This result highlights the fact that urban economies arise from a blend of decisions about housing, employments, schooling, and neighborhood and of public and private institutions that shape each of these decisions.

Further research on the impact of urban sprawl on neighborhoods within cities is therefore necessary.

Given the alleged consequences of urban sprawl, public officials and others seek ways of controlling it; knowledge of what causes urban sprawl is therefore important in designing measures to control it.

1.3 What Causes Urban Sprawl?

Why has urban development in the United States followed such patterns of suburbanization? Two groups of economists attempt to explain this trend. Public finance economists focus on the role of households' preferences for local taxes and amenities on households' location decisions. Urban economists emphasize the role of population growth, rising income, and lower transportation costs with contributing forces from government regulations and taxation, including federal tax and local government zoning policies.

1.3.1 Local Public Finance Model

The local public finance model stems from Tiebout's "voting with one's feet" theory in 1956 (Tiebout 1956). Tiebout shows that local public goods could be provided without resort to politicians. Instead of voting for politicians who would then set expenditures and taxes, households vote with their feet to select the community that happens to offer the right level of public goods. In other words, people sort themselves into different local jurisdictions based on their preferences for local public goods.

These sorting effects are divided into two broad categories for purpose of our study of urban sprawl: those that pull people out of central cities because of attractive

features of suburban areas and those that push people out of central cities because of inner city decay (Nechyba and Strauss, 1998, Nechyba and Walsh, 2004).

The pull side of the public finance model emphasizes a group of households with similar preferences willing to pay for the provision of local public goods in a suburb. These like-minded households form a new community. People are attracted to the new community from the central city because they can enjoy some features they desire, such as a high-quality school system, low crime rate, a reasonable tax rate, and clean streets. When combined with zoning policies enacted by local government, people in the new community could exclude those who are considered to create negative fiscal externalities, thus avoiding the free-rider problem (Fischel 2001)

Carruthers and Ulfarsson (2002) identify another possible contributor to urban sprawl, namely, governmental fragmentation, which falls under the category of the pull side of the public finance model. They argue that political fragmentation contributes to urban sprawl through Tiebout mechanisms. That is, in metropolitan areas, households choose locations that offer a desirable combination of amenities at a price they can afford. The price of entry is given by the cost of housing and the property tax. This plays an important role in forcing residents to balance their preferences and their budget constraints. Since real estate markets in most metropolitan areas are highly segmented, potential residents can easily be priced out. Consequently, growth at the urban fringe increases because land prices there are comparatively low. Empirical evidence reveals that fragmentation is associated with lower densities and higher property values but has no direct effect on public service expenditures. This suggests that fragmentation may be a possible cause of urban sprawl.

The push side of the public finance model focuses on inner city problems. Relatively high-income people are forced to leave the central city because of inner city problems such as a high crime rate, a high tax rate, lower quality schools, and general fiscal distress within the central city. With more people leaving the central city, those remaining face higher and higher tax rates. With the decrease in tax revenues, the local government experiences fiscal distress. As a result, public goods provision deteriorates, e.g., the crime rate increases and school quality decreases. Thus, people leave the central cities not because of attractive features in suburbs but to avoid inner city problems. There is some empirical support for this view (Cullen and Levitt 1999).

The empirical studies on sorting effects and urban sprawl mainly involve racial segregation. The results are, however, inconclusive. Some researchers believe that minorities are segregated in central cities because of their relatively lower income and exclusionary suburban policies. Yinger (1993) finds that urban development and racial segregation are positively related. Bayer et al., (2002) find that households tend to reside close to others with similar race and ethnicity, which suggests that urban sprawl may contribute to racial segregation. Some researchers argue, however, that with the spatial expansion of urban areas and increase in income, an emerging minority middle class has appeared that can afford to move to suburbs, thus decreasing racial segregation. Glaeser and Kahn (2003) find that fast growing urbanized areas have experienced a sharper decline in racial segregation than have other areas.

In conclusion, public finance economists have explored the connection between urban sprawl and Tiebout sorting. Both push and pull sorting play an important role in determining household residential location decisions. The public finance economists,

however, fail to answer a key question, that is, whether Tiebout sorting is the cause or the consequence of urban sprawl.

1.3.2 Urban Economic Theories on Urban Sprawl

Much of our understanding of urban growth can be derived from the “monocentric urban model,” early developers of which are Alonso (1964), Mills (1967), and Muth (1969). This model finds three fundamental forces responsible for spatial growth of cities: population growth, rising real income, and lower real transport costs. Population growth and rising real income increase the demand for housing. Since land prices decrease with distance from the city center, people choose housing in the suburbs, which causes the city to expand spatially. Lower real transport costs are largely due to highways that enable people to travel faster and cheaper, while facilitating suburban living.

Glaeser and Kahn (2003) identify automobiles and their accompanying lower transportation costs as the primary catalyst of sprawling cities through much of the twentieth century. By 1910, car registrations in the United States had exceeded 500,000 (Nechyba and Walsh 2004), and in 1920, it reached eight million. By 1952, a majority of households in America owned at least one car (Glaeser and Kahn (2003)). In 1960, 64 percent of commuters drove to work, growing to 78 percent in 1978, 84 percent in 1980, and 88 percent in 2000 (Center of Urban Transportation Research, 2003). It is hard to imagine urban sprawl without the rise of the automobile. Although Mills (1969) finds suburbanization occurring as far back as 1890, the concern with urban sprawl is a fairly recent phenomenon.

The monocentric model also predicts that rising incomes contribute to decreasing population densities in cities if the income elasticity of demand for housing is greater

than the income elasticity of marginal commuting costs. Margo (1992) suggests that as much as a 50 percent increase in suburbanization between 1950 and 1980 could be explained by rising incomes. Brueckner and Fansler (1983) find that a 1 percent increase in income results in a 1.5 percent increase in the spatial size of urban areas.

The real question is whether or not the spatial expansion of cities is excessive. In other words, do free markets allocate resources efficiently in urban areas or do market failures result in inefficient resources allocation? Brueckner (2000) identifies three market failures that may lead to excessive spatial growth of cities. The first involves the social value of open space. During the process of suburbanization, open space near the city fringe is converted to urban use. People living in suburban areas do not, however, shoulder the full cost of loss of the social value of the open space. This market failure results in over-conversion of rural land into urban uses. The second market failure concerns commuting. Commuters bear only the cost of vehicle operation and their time as well as the average cost of congestion and pollution, but not the costs of congestion and air pollution they impose on others. Consequently, the cost of commuting is underpaid. As a result, households choose to live farther away from CBD than socially desirable. The third market failure concerns public infrastructure cost generated by new development at the urban fringe. When new living areas are built, public infrastructure, such as roads, sewer systems, schools, and recreation centers, are needed. In the U.S., most infrastructure costs are paid through the property tax, which results in homeowners with equal assessed values in low- and high-density areas paying the same amount. That is, households' private costs are only the average infrastructure costs, which may be less than the marginal costs they generate. This also means that residents living in high-density areas subsidize those

living in low-density areas, generally found in suburbs. Perversely, in most urban areas, this means poor people subsidize rich people who live in suburban areas. As a result of the failure to fully charge for suburban infrastructure costs, developers can bid higher prices for the undeveloped land than would otherwise be the case, thus converting more rural land into urban use, contributing to urban sprawl (Mills 1978, Brueckner 2000).

The effects of these three market failures are exacerbated by public policies such as land-use planning, which often creates situations where regulations are more restrictive in some areas than in others. As a result, new development is built in less restrictive areas generally located near the urban fringe.

Many public policies have been suggested as potential contributors to urban sprawl. For example, Jackson (1985) argues that urban developments have been greatly affected by the New Deal's creation of the Home Owners Loan Corporation and the Federal Housing Administration (FHA) to provide decent housing for poorer Americans. Williams (2000) identifies generous mortgage insurance and loan programs through the FHA and the Veterans Administration (VA) as potential contributors to urban sprawl. The FHA provides federal guarantees to private mortgage lenders, lowering the minimum down payment to just 10 percent for homebuyers, and payback periods are extended to 20 or 30 years. The VA offers low-interest mortgages without down payment to all qualified veterans. The federal income tax allows deductions for mortgage interest and property tax. All of these public policies worked to generate a massive expansion of suburban growth after 1945.

Brueckner and Kim (2003) explore the connection between urban spatial expansion and the property tax. The property tax is levied at equal rates on land and improve-

ments. However, the portion levied on land has quite different effects from the portion levied on improvements. The property tax on land, as generally acknowledged, cannot be shifted and therefore is a neutral tax in that it has no effect on resource allocation. The underlying reason is that land is in fixed supply. A property tax on improvements, on the other hand, is not generally regarded as neutral. Such a tax tends to lower the equilibrium level of improvements chosen by the developer. Thus, land is developed less intensively under the property tax, which may contribute to the spatial expansion of cities.

Brueckner and Kim show that while the tax's depressive effect on improvements reduces population density, spurring the spatial expansion of cities, a countervailing effect from lower dwelling sizes may dominate, raising densities and making cities shrink. Numerical examples suggest that the property tax may encourage urban sprawl. Based on their findings, Brueckner and Kim suggest that the distortions generated by the property tax may include inefficient spatial expansion of cities.

Other papers dealing with the issue generate mixed results. Sullivan (1985) analyzes the spatial effect of the property tax using a model including both business and residential property. In view of the complexity of the model, however, the linkage between the property tax and urban growth is not clearly identified. Arnott and MacKinnon (1977a) undertake a general equilibrium simulation of the spatial effects of the property tax and find that if there is an increase in the property tax, the city shrinks as a response. Pasha and Ghaus (1995) note, however, that this result might not hold in a more general model.

Persky and Kurban (2003) identify federal spending as a possible contributor to urban sprawl. They consider the influence on urban expansion of recent federal subsidies

in Chicago. They believe that federal spending influences metropolitan expansion in at least three ways. First, subsidies can change the relative price of land and other goods at the urban fringe, which induces consumers to substitute land for other goods. Second, federal programs or subsidies can increase people's disposable income by reducing their need for local government or private spending, which increases the demand for living space. The last factor Persky and Kurban discuss concerns the employment opportunities generated by federal spending, which in turn influences employees' residential locations. Specifically, if federal spending is dispersed rather than concentrated, it may encourage urban sprawl. They find that in Chicago federal spending aimed at alleviating poverty and supporting the elderly strongly stimulates land absorption in suburban areas. They estimate that residential land use in the outer ring of the Chicago area has been increased by 20 percent as a result of federal spending. They also believe that the income tax subsidy for housing has a larger effect on urban sprawl in Chicago than federal spending. This is consistent with the findings of James and Lin (1991), who suggest that the federal tax subsidy for housing may be a possible cause of urban sprawl.

Brueckner (2004) identifies transportation subsidies as another contributor to urban sprawl. According to economic theory, individuals' transportation and location decisions would be efficient if the price paid for transportation closely matched the costs incurred by the user. Although users do not pay a direct fee every time they use a highway, they do pay a user-fee in the form of the gasoline tax. Revenues from gasoline taxes have, however, been insufficient to cover the construction, maintenance, and administration of highways. Various levels of government must use general tax revenues to augment highway expenditures, which indicates that governments provide highway subsidies to users.

Given the fact that users do not pay the full cost of their travel, they have an incentive to travel longer and more often. Highway subsidies not only foster increased travel and congestion, they also contribute to urban sprawl. Suburban areas traversed by new highways tend to attract residents because transportation is improved while the full costs of the construction are not borne by those who benefit.

User-fees only covered 68 percent of the highway systems' capital and maintenance costs during the period 1956-1986 (Voith 1989). The share of total highway expenditure covered by user-fees has fluctuated considerably over time and across regions. Voith (1989) examines thirteen major highway construction projects ranging in cost from \$97 million to \$581 million. The cost of construction varies widely, from a low of \$6.8 million per mile to a high of \$133.3 million per mile. None of the projects generates sufficient user-fees to cover the infrastructure investment. As a matter of fact, user-fees covers 54 percent of the investment at best and 2.5 percent at worst. On a per-car basis, the subsidy ranges from \$0.16 to \$4.50. On a vehicle-mile-traveled basis, the subsidy ranges from less than 1 cent per vehicle to 41 cents (Voith 1989).

1.4 Summary

This chapter has defined urban sprawl and discussed some of its consequences and causes. Although there are many potential causes of urban sprawl, in this dissertation, we concentrate on one: subsidized transportation. In Ch. 2, we modify the monocentric model to suit our purpose by introducing transportation subsidies into a single-mode model; in Ch. 3, we do the same for a two-mode model. Ch. 4 presents empirical analysis, including research hypotheses, data sources, estimation methods, and estimation re-

sults. Ch. 5 contains a discussion of the research findings, focusing on policy recommendations.

CHAPTER 2 SINGLE-MODE URBAN SPATIAL MODEL

We now present our theory of the relationship between urban sprawl and governmental transportation subsidies. We begin with some definitions and assumptions followed by a detailed discussion of the behavioral equations of the model. Finally, overall equilibrium conditions of the urban area are provided. Following standard practice in urban economics since Mills (1972), we use the term *urban area* to refer to the legal city and its suburbs which may or may not be part of the legal city.

2.1 Definitions and Assumptions

The basic framework employed is the Muth-Mills urban residential land use model explicated by Brueckner (1987).

The following notation is used:

y = an urban household's money income, including both wage and non-wage income

c = quantity of an urban household's expenditures on all commodities except housing and transportation

q = quantity of housing service consumed by an urban household

p = price per unit of housing service

t = money travel cost per round-trip mile

α = proportion of money travel cost per round-trip mile paid by household
($0 < \alpha < 1$)

x = distance from home to CBD

u = utility indicator

θ = local tax rate

N = capital, an input to housing production

l = land, an input to housing production

r = rent per acre of land, the price (rental rate) of the land input

i = interest rate, the price (rental rate) of the capital input

L = population of the urban area

r_a = agricultural land rent at the urban area's boundary

$\bar{x}$ = urban area boundary

β = proportion of urban-area tax revenue used for transportation subsidies, which we refer to as the tax-share variable

G = intergovernmental grants used for transportation

The urban area is located on a featureless plain containing a central business district (CBD) to which every urban resident travels for work, shopping, and leisure activities. The urban area has two sectors: consumption and production. Consumers choose the quantity of housing service and other expenditures based on their preferences and budget constraints. Housing service is produced using two inputs, capital and land.

All urban residents earn the same exogenously determined income and possess the same utility function defined over housing and the composite good, whose price we normalize. An urban resident's utility function is strictly quasi-concave, and housing service is assumed to be a normal good. Although local governments generally do not levy income tax, we assume local tax revenues from property taxes and sales taxes are proportional to income, therefore, local tax revenues are θy .

Housing service, q , as originally defined by Muth (1969), captures all characteristics of a dwelling unit and the land on which it is located except location. We recognize that housing is a multi-attribute commodity with such characteristics as floor space, number of bedrooms, etc., and that households may have preferences for these attributes. The assumption of a single variable representing service from a dwelling unit, however, considerably simplifies the analysis and has been used since it was introduced by Muth. Housing price, p , is the price per unit of housing service. For simplicity, we often refer to q as square feet of floor space and to p as the price (rental rate) per square foot. There is no distinction between ownership and rental of housing. If a household owns its house, housing price is the implicit cost of a unit of housing service. The quantity pq , therefore, is the observable or imputed rental value of a dwelling unit. We assume single-person households, so that the number of households equals the population of the urban area.

Assumptions concerning transportation deal with the transportation system, travel cost, and the travel subsidy. Urban residents only travel between home and the CBD. The transportation system is sufficiently developed to allow straight-line travel. We begin by assuming a single transportation mode, or alternatively, a single mode-combination, such as auto travel to a transit station. Later, we incorporate mode choice.

The urban area residents' private travel cost per round trip mile, αt , is less than the total travel cost per round trip mile, t , because urban area government spending on transportation is higher than fees collected from residents. In other words, we assume government gives commuters a subsidy of $(1 - \alpha)t$ per round trip mile of travel. This assumption has empirical support. In the United States, only around 38 percent of transit operating costs and 25 percent of combined operating and capital costs of transit systems are

covered by the total fares collected from users. For highway systems, around 60 percent of highway disbursements are covered by gas taxes and other user-fees (Brueckner 2003).

In this model, households' private travel cost is proportional to distance traveled, which implies that the urban transport system exhibits constant returns to scale. Although some empirical evidence suggests that transportation systems, such as bus or rail, exhibit increasing return to scale, the increasing returns in those systems may stem from the existence of underutilized capacity. Passenger-miles could be increased with little cost by simply filling half-empty buses or rail cars with additional riders. But this possibility implies that the original capacity of the system was too large. If system capacity is instead adjusted to match demand, then costs should increase roughly in step with passenger-miles. Adding a passenger-mile means adding a seat mile, which involves a constant additional cost (Brueckner 2004).

2.2 Demand Side of Housing Market

We start our analysis from the demand side of the housing market. The household's utility function is $V(c, q)$, where c is expenditure on non-housing, non-transportation goods and q is housing consumption. The price of the composite good (normalized to unity) is the same everywhere within the urban area, but housing price varies with location. Given the assumption that all urban residents have identical preferences, urban spatial equilibrium implies the same level of satisfaction, u , for all residents. The equilibrating factor is housing price so that those living far from the CBD and therefore paying a large transportation cost are compensated with a lower housing price, while those living close to the CBD pay a higher housing price. Households face a budget constraint represented by $y = c + pq + \theta y + \alpha tx$. This budget constraint tells us that urban

residents' income is spent on the composite good, housing, urban-area taxes, and transportation.

Solving the budget constraint for c and substituting that into the utility function, the household's problem is to maximize

$$V[(1-\theta)y - \alpha tx - pq, q] = u \quad (2.1)$$

with respect to q . This maximization problem produces the following first-order condition

$$\frac{V_2[(1-\theta)y - \alpha tx - pq, q]}{V_1[(1-\theta)y - \alpha tx - pq, q]} = p \quad (2.2)$$

where subscripts denote partial differentiation with respect to the first and second arguments of the utility function. Equation (2.2) is the familiar condition that the marginal rate of substitution equals the price ratio where $\frac{V_2}{V_1} \equiv MRS$, and, because of normalization, the price ratio is p .

All households must have the same utility for spatial equilibrium, so we have the equation:

$$V[(1-\theta)y - \alpha tx - pq, q] = u \quad (2.3)$$

where u is the urban-area-wide utility level in spatial equilibrium.

The simultaneous system composed of (2.2) and (2.3) yields solutions for the endogenous variables p and q as functions of exogenous variables x , y , t , α , u and θ . Totally differentiating (2.3), we have

$$V_1 d[(1-\theta)y - \alpha tx - pq] + V_2 dq = du$$

$$V_1 [(1-\theta)dy - yd\theta - \alpha tdx - \alpha xdt - txd\alpha - pdq - qdp] + V_2 dq = du$$

Given $V_1 p = V_2$, from (2.2), we have

$$-V_1 qdp = du - V_1 (1-\theta)dy + V_1 yd\theta + V_1 \alpha tdx + V_1 \alpha xdt + V_1 txd\alpha \quad (2.4)$$

From (2.4), we derive

$$\frac{\partial p}{\partial x} = -\frac{\alpha t}{q} < 0 \quad (2.5)$$

Housing price is a decreasing function of distance from the CBD. Given that u is constant for change in all other exogenous variables, we derive the effect of x on q as follows

$$\frac{\partial q}{\partial x} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial x} < 0,$$

where $(\partial q / \partial p)_u$ denotes the Hicksian demand slope.

The explanation of the spatial behavior of p and q is simple. Urban residents living farther away from the CBD are compensated by lower housing price for their higher travel cost. Since housing service is a normal good, the decrease of housing price with distance from the CBD leads to more consumption of housing service, or larger dwellings.

The other parameters' influences on housing price and housing service are derived below because they play a very important role in the comparative static analysis presented in the next section.

The influence of income on housing price and housing service is determined from (2.4) as follows

$$\frac{\partial p}{\partial y} = \frac{1-\theta}{q} > 0 \quad (2.6)$$

$$\frac{\partial q}{\partial y} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial y} < 0$$

The influence of money cost of travel on p and q is determined from (2.4) as follows

$$\frac{\partial p}{\partial t} = \frac{-\alpha x}{q} < 0 \quad (2.7)$$

$$\frac{\partial q}{\partial t} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial t} > 0$$

The influence of the private transport cost share on housing price and housing consumption is derived from (2.4) as follows

$$\frac{\partial p}{\partial \alpha} = -\frac{tx}{q} < 0 \quad (2.8)$$

$$\frac{\partial q}{\partial \alpha} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial \alpha} > 0$$

The influence of the urban-area tax rate on housing price and housing consumption is derived from (2.4) as follows

$$\frac{\partial p}{\partial \theta} = -\frac{y}{q} < 0 \quad (2.9)$$

$$\frac{\partial q}{\partial \theta} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial \theta} > 0$$

The influence of the utility level on housing price and housing consumption is derived from (2.4) as follows

$$\frac{\partial p}{\partial u} = -\frac{1}{qV_1} < 0 \quad (2.10)$$

We cannot determine $\partial q / \partial y$ by the method previously used, however, because u is now varying. Brueckner (1987, p. 825, n.6) provides the result as

$$\frac{\partial q}{\partial u} = \left(\frac{\partial p}{\partial u} - \frac{\partial MRS}{\partial c} \right) \left(\frac{\partial q}{\partial p} \right)_u > 0$$

In this model, $\partial q / \partial y$ cannot be used to define a normal good because utility is held fixed. Instead, following Wheaton (1974) and Brueckner (1987), we assume $\partial MRS_{qc} / \partial c > 0$ as the definition of a normal good. This implies that indifference curves become steeper as c increases holding q constant. Coupled with the concavity of indifference curves (which follows from the assumption of strict quasi-concavity of the utility function), this assumption ensures that when u increases, the resulting decrease in p produces an increase in q .

2.3 Supply Side of Housing Market

We now turn to the supply side of the housing market. Housing is produced with two inputs land, l , and capital, N . The production function, $H = H(N, l)$, is strictly concave and exhibits constant returns to scale. We assume positive marginal products, namely, $H_l > 0$, $H_N > 0$. Strict concavity implies that the marginal products of capital and land diminish, which is supported by the fact that as the height of a building increases, more and more non-land input is consumed in non-productive uses such as foundations, elevators, and stairways.

To simplify notation, let S denote the capital-land ratio, N/l , and, following Brueckner, we refer to S as structural density. It is an index of the height of a building. Since the housing production function exhibits constant returns to scale, we rewrite it as follows:

$$\frac{H}{l} = H\left(\frac{N}{l}, 1\right) = H(S, 1) \text{ or } H = lh(S)$$

A housing producer's profit is

$$\pi = l(ph(S) - iS - r) \quad (2.11)$$

where r is urban land rent per acre and i is the interest rate or rental rate per unit of capital.

The producer determines S to maximize profit per acre of land, and the first-order condition for profit maximization is given as

$$ph'(S) = i \quad (2.12)$$

In long-run competitive equilibrium, land rent must absorb profit, so that

$$ph(S) - iS = r \quad (2.13)$$

This constitutes the spatial equilibrium condition for housing production, i.e., housing producers earn the same profit everywhere in the area.

By (2.4), we know that housing price is a function of distance, x ; income, y ; money travel cost, t ; private share of commuting cost, α ; tax rate θ ; and the spatial equilibrium utility level, u . From (2.12) and (2.13), it is clear that the capital-land ratio, S , and urban land rent, r , are functions of these same variables as well as the rental price per unit of capital, i . In what follows, we suppress the variables θ and i as they play no important role in our model.

Totally differentiating (2.12) and (2.13) with respect to $\varphi = x, y, t, u, \alpha$, yields

$$\frac{\partial p}{\partial \varphi} h' + ph'' \frac{\partial S}{\partial \varphi} = 0 \quad (2.14)$$

$$(ph' - i) \frac{\partial S}{\partial \varphi} + \frac{\partial p}{\partial \varphi} h = \frac{\partial r}{\partial \varphi} \quad (2.15)$$

From (2.12), we know that $ph'(S) = i$, so (2.15) becomes

$$\frac{\partial r}{\partial \varphi} = h \frac{\partial p}{\partial \varphi} \quad (2.16)$$

Equation (2.14), upon rearrangement, becomes

$$\frac{\partial S}{\partial \varphi} = -\frac{h'}{ph''} \frac{\partial p}{\partial \varphi} \quad (2.17)$$

From (2.16) and (2.17), we have the following results

$$\begin{aligned} \frac{\partial r}{\partial x} &= -\frac{h\alpha t}{q} < 0 & \frac{\partial S}{\partial x} &= \frac{h'}{ph''} \frac{\alpha t}{q} < 0 & \frac{\partial r}{\partial \alpha} &= -\frac{hxt}{q} < 0 & \frac{\partial S}{\partial \alpha} &= \frac{h'}{ph''} \frac{xt}{q} < 0 \\ \frac{\partial r}{\partial y} &= \frac{h(1-\theta)}{q} > 0 & \frac{\partial S}{\partial y} &= -\frac{h'}{ph''} \frac{1-\theta}{q} > 0 & \frac{\partial r}{\partial \theta} &= -\frac{hy}{q} < 0 & \frac{\partial S}{\partial \theta} &= \frac{h'}{ph''} \frac{y}{q} < 0 \\ \frac{\partial r}{\partial t} &= -\frac{h\alpha x}{q} < 0 & \frac{\partial S}{\partial t} &= \frac{h'}{ph''} \frac{\alpha x}{q} < 0 & \frac{\partial r}{\partial u} &= -\frac{h}{qV_1} < 0 & \frac{\partial S}{\partial u} &= \frac{h'}{ph''} \frac{1}{qV_1} < 0 \end{aligned} \quad (2.18)$$

From (2.18), we see that both urban land rent and the capital-land ratio are decreasing function of distance from the CBD. These results are consistent with observations that in the CBD land rents are higher and buildings are taller than in suburban areas. In our model, lower land rent compensates housing producers for lower housing price farther away from the CBD. As a result, housing produced farther from the CBD uses less capital per unit of land, leading to shorter buildings in suburban areas.

Another variable of importance in the comparative static analysis is urban population density. Under our assumption of single-person households, urban population density

is $D=h(S)/q$. Population density is a decreasing function of distance from the CBD because $\partial S/\partial x < 0$ and $\partial q/\partial x > 0$. Much empirical evidence supports this result (e.g., Muth (1969)). Households living farther from the CBD live in larger dwelling units and shorter buildings than those living close to the CBD. Thus fewer and fewer people fit on each acre of land the farther from the CBD they live. Declining population density is the combined result of consumer choice over housing service and producer decisions over housing production.

2.4 Overall Equilibrium of Urban Area

To complete the model, we need three equilibrium conditions. The first one is the boundary condition, which requires housing producers to outbid agricultural users for all urban land used in housing production. At the city boundary, $\bar{x}$, urban land rent equals agricultural land rent, r_a . This condition is

$$r(\bar{x}, y, t, u, \alpha) = r_a \quad (2.19)$$

The second condition states that the urban population must exactly fit inside the urban area. Let δ represent the number of radians of land available for housing at distance x , where $0 < \delta < 2\pi$. The population condition is

$$\int_0^{\bar{x}} \delta x D(x, y, t, u, \alpha) dx = L \quad (2.20)$$

Transportation spending by government exceeds the fees collected from commuters. In addition, the part of transportation investment not covered by user-fees is covered by part of the local tax collected from urban residents and by intergovernmental grants.

The balanced budget condition is

$$\beta \theta y L + G = (1 - \alpha) \int_0^{\bar{x}} \delta t x^2 D(x, y, t, u, \alpha) dx \quad (2.21)$$

where the left-hand side is urban-area taxes and intergovernmental grants used to cover the subsidized portion of urban-area transportation costs, which is the right-hand side.

The interpretation of the urban equilibrium conditions depends on whether or not the city is a *closed city* or *open city*. A *closed city* is one in which population migration does not occur, so population, L , is exogenous but utility, u , is endogenous. An *open city* is one in which population migration occurs, so population is endogenous and utility is exogenous. We adopt the *closed-city* formulation.

In summary, for a closed city, the single mode urban spatial model is composed of the following equations.

Consumption sector

$$\frac{V_2[(1-\theta)y - \alpha tx - pq, q]}{V_1[(1-\theta)y - \alpha tx - pq, q]} = p \quad (2.22)$$

$$V[(1-\theta)y - \alpha tx - pq, q] = u \quad (2.23)$$

Equations (2.22) and (2.23) imply that

$$p = p(x, y, u, t, \alpha)$$

$$q = q(x, y, u, t, \alpha)$$

Production sector

$$ph'(S) = i \quad (2.24)$$

$$ph(S) - iS = r \quad (2.25)$$

Equations (2.24), together with (2.22), (2.23), and (2.25) imply that

$$S = S(x, y, u, \alpha, t)$$

$$r = r(x, y, u, \alpha, t)$$

Urban boundary

$$r(\bar{x}, y, t, u, \alpha) = r_a \quad (2.26)$$

Urban population

$$\int_0^{\bar{x}} \delta x D(x, y, t, u, \alpha) dx = L \quad (2.27)$$

Balanced budget condition

$$\beta \theta y L + G = (1 - \alpha) \int_0^{\bar{x}} t x^2 D(x, y, t, u, \alpha) dx \quad (2.28)$$

In a closed city model, the equilibrium conditions (2.26), (2.27), and (2.28) constitute simultaneous equations that determine equilibrium values for utility, the urban boundary, and the tax-share variable, or

$$\bar{x} = \bar{x}(L, y, t, r_a, \alpha)$$

$$u = u(L, y, t, r_a, \alpha)$$

$$\beta = \beta(L, y, t, r_a, \alpha)$$

2.5 Comparative Static Analysis of Single-Mode Urban Spatial Model

This section provides a comparative static analysis of the single-mode urban spatial model presented above. We begin by totally differentiating the three urban equilibrium conditions. This enables us to derive the effect of L, y, r_a, t, α on the urban area boundary, $\bar{x}$ spatial equilibrium utility level, u , and the tax-share variable β . We then derive the effect of the exogenous variables on the remaining endogenous variables, namely, p, q, r , and S . Some variables, such y, t , and α , have both a direct effect on p, q, r , and S as well as an indirect effect through the endogenous utility. Other variables have only an indirect effects. The comparative static analysis provided in this section follows

Brueckner (1987) except for adding the variable α and a balanced-budget equation for transportation subsidies. Many of our results are identical to those obtained by Brueckner. Since our main goal is to explore the relationship between transportation subsidies and urban sprawl, our comparative static analysis focuses on the effects of the relevant exogenous variables L, y, r_a, t, α , while neglecting the variables θ, G, δ , and i .

Before presenting our comparative static analysis, we summarize those results obtained earlier that are used in this section.

$$\frac{\partial p}{\partial x} = -\frac{\alpha t}{q} \quad \frac{\partial r}{\partial x} = -\frac{h\alpha t}{q} \quad (2.29)$$

$$\frac{\partial p}{\partial y} = \frac{1-\theta}{q} \quad \frac{\partial r}{\partial y} = \frac{h(1-\theta)}{q} \quad (2.30)$$

$$\frac{\partial p}{\partial t} = -\frac{\alpha x}{q} \quad \frac{\partial r}{\partial t} = -\frac{h\alpha x}{q} \quad (2.31)$$

$$\frac{\partial p}{\partial \alpha} = -\frac{xt}{q} \quad \frac{\partial r}{\partial \alpha} = -\frac{hxt}{q} \quad (2.32)$$

$$\frac{\partial p}{\partial \theta} = -\frac{y}{q} \quad \frac{\partial r}{\partial \theta} = -\frac{hy}{q} < 0 \quad (2.33)$$

$$\frac{\partial p}{\partial u} = -\frac{1}{qV_1} \quad \frac{\partial r}{\partial u} = -\frac{h}{qV_1} < 0 \quad (2.34)$$

$$\frac{\partial S}{\partial \varphi} = -\frac{h'}{ph''} \frac{\partial p}{\partial \varphi} \quad (\text{where } \varphi = x, u, y, \alpha, t,) \quad (2.35)$$

$$r(\bar{x}) = r_a \quad (2.36)$$

$$\int_0^{\bar{x}} \delta x \frac{h(S)}{q} dx = L \quad (2.37)$$

$$\beta\theta yL + G = (1 - \alpha) \int_0^{\bar{x}} tx^2 \frac{h(S)}{q} dx \quad (2.38)$$

The goal of the analysis is to deduce the impact of changes in L , r_a , y , t , and especially α on the size of the urban area, $\bar{x}$ housing price, p , land rent, r , dwelling size, q , structural density, S , utility level, u , and the tax-share variables, β .

2.5.1 Effects of Change in Population

2.5.1.1 Effect of Population on Utility

Recall that population density is defined as $D = h/q$. From (2.29) and (2.32) we

have $D = \frac{h}{q} = \frac{\left(-\frac{\partial r}{\partial x}\right)}{\alpha t}$. Given this result, (2.37) is rewritten as

$$-\int_0^{\bar{x}} x \frac{\partial r}{\partial x} dx = \frac{\alpha t L}{\delta} \quad (2.39)$$

Integrating by parts, where $u = x$ and $dv = \int_0^{\bar{x}} \frac{\partial r}{\partial x} dx$, we have

$$-xr \Big|_0^{\bar{x}} + \int_0^{\bar{x}} r dx = \frac{\alpha t L}{\delta}, \text{ or} \quad (2.40)$$

$$-\bar{x}r_a + \int_0^{\bar{x}} r dx = \frac{\alpha t L}{\delta} \text{ (since } r(\bar{x}) = r_a \text{)} \quad (2.41)$$

Totally differentiating (2.41) with respect to $\lambda = L, r_a, y, t, \alpha$, we have

$$-r_a \frac{\partial \bar{x}}{\partial \lambda} - \bar{x} \frac{\partial r_a}{\partial \lambda} + \int_0^{\bar{x}} \left(\frac{\partial r}{\partial \lambda} + \frac{\partial r}{\partial u} \frac{\partial u}{\partial \lambda} \right) dx + r_a \frac{\partial \bar{x}}{\partial \lambda} = \frac{\alpha t \frac{\partial L}{\partial \lambda} + \alpha L \frac{\partial t}{\partial \lambda} + tL \frac{\partial \alpha}{\partial \lambda}}{\delta} \quad (2.42)$$

After rearranging (2.42), we obtain

$$\int_0^{\bar{x}} \left(\frac{\partial r}{\partial \lambda} + \frac{\partial r}{\partial u} \frac{\partial u}{\partial \lambda} \right) dx = \frac{\alpha t \frac{\partial L}{\partial \lambda} + \alpha L \frac{\partial t}{\partial \lambda} + tL \frac{\partial \alpha}{\partial \lambda}}{\delta} + \bar{x} \frac{\partial r_a}{\partial \lambda} \quad (2.43)$$

or, since u is not a function of x

$$\int_0^{\bar{x}} \frac{\partial r}{\partial \lambda} dx + \frac{\partial u}{\partial \lambda} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx = \frac{\alpha t \frac{\partial L}{\partial \lambda} + \alpha L \frac{\partial t}{\partial \lambda} + tL \frac{\partial \alpha}{\partial \lambda}}{\delta} + \bar{x} \frac{\partial r_a}{\partial \lambda}$$

Then solving for $\partial u / \partial \lambda$, we have

$$\frac{\partial u}{\partial \lambda} = \frac{\frac{\alpha t}{\delta} \frac{\partial L}{\partial \lambda} + \frac{\alpha L}{\delta} \frac{\partial t}{\partial \lambda} + \frac{tL}{\delta} \frac{\partial \alpha}{\partial \lambda} + \frac{\partial r_a}{\partial \lambda} \bar{x} - \int_0^{\bar{x}} \frac{\partial r}{\partial \lambda} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \quad (2.44)$$

When $\lambda = L$, (2.44) becomes

$$\frac{\partial u}{\partial L} = \frac{\frac{\alpha t}{\delta} - \int_0^{\bar{x}} \frac{\partial r}{\partial L} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \quad (2.45)$$

From (2.34) $\frac{\partial r}{\partial u} = -\frac{h}{qV_1} < 0$. We see that $\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx < 0$, and because $\frac{\partial r}{\partial L} = 0$

$$\frac{\partial u}{\partial L} = \frac{\frac{\alpha t}{\delta}}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} < 0 \quad (2.46)$$

2.5.1.2 Effect of Population on Urban Area Boundary

Totally differentiating (2.36), we have

$$\frac{\partial \bar{r}}{\partial x} \frac{\partial \bar{x}}{\partial \lambda} + \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial \lambda} + \frac{\partial \bar{r}}{\partial \lambda} = \frac{\partial r_a}{\partial \lambda} \quad (2.47)$$

where bars mean the function is evaluated at $\bar{x}$. By rearranging the terms of (2.47), we have

$$\frac{\partial \bar{x}}{\partial \lambda} = \frac{\frac{\partial r_a}{\partial \lambda} - \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial \lambda} - \frac{\partial \bar{r}}{\partial \lambda}}{\frac{\partial \bar{r}}{\partial x}} \quad (2.48)$$

Recall from (2.29) that $\frac{\partial \bar{r}}{\partial x} < 0$. Since $\frac{\partial r_a}{\partial \lambda} = 0$, therefore, when $\lambda = L$

$$\begin{aligned} \text{Sign}\left(\frac{\partial \bar{x}}{\partial L}\right) &= -\text{Sign}\left(\frac{\partial r_a}{\partial L} - \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial L} - \frac{\partial \bar{r}}{\partial L}\right) \\ &= -\text{Sign}\left(-\frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial L} - \frac{\partial \bar{r}}{\partial L}\right) \\ &= \text{Sign}\left(\frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial L} + \frac{\partial \bar{r}}{\partial L}\right) \end{aligned}$$

Because $\frac{\partial \bar{r}}{\partial u} < 0$, $\frac{\partial u}{\partial L} < 0$, and $\frac{\partial \bar{r}}{\partial L} = 0$, we have

$$\frac{\partial \bar{x}}{\partial L} > 0 \quad (2.49)$$

2.5.1.3 Effects of Population on Housing Price, Housing Consumption, Urban Land

Rent, and Structural Density

These effects operate through u , so

$$\frac{dp}{dL} = \frac{\partial p}{\partial u} \frac{\partial u}{\partial L} > 0 \quad (2.50)$$

$$\frac{dr}{dL} = \frac{\partial r}{\partial u} \frac{\partial u}{\partial L} > 0 \quad (2.51)$$

$$\frac{dq}{dL} = \frac{\partial q}{\partial u} \frac{\partial u}{\partial L} < 0 \quad (2.52)$$

$$\frac{dS}{dL} = \frac{\partial S}{\partial u} \frac{\partial u}{\partial L} > 0 \quad (2.53)$$

2.5.1.4 Effect of Population on the Tax-Share Variable

Totally differentiating (2.38) with respect to $\lambda = L, r_a, y, t, \alpha$ we have

$$\begin{aligned} \theta y L \frac{\partial \beta}{\partial \lambda} + \beta \theta y \frac{\partial L}{\partial \lambda} + \beta \theta L \frac{\partial y}{\partial \lambda} + \beta y L \frac{\partial \theta}{\partial \lambda} &= (1 - \alpha) t \bar{x}^2 \bar{D} \frac{\partial \bar{x}}{\partial \lambda} \\ &+ (1 - \alpha) \frac{\partial t}{\partial \lambda} \int_0^{\bar{x}} x^2 D dx - t \frac{\partial \alpha}{\partial \lambda} \int_0^{\bar{x}} x^2 D dx + t(1 - \alpha) \int_0^{\bar{x}} x^2 \left(\frac{\partial D}{\partial \lambda} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial \lambda} \right) dx \end{aligned} \quad (2.54)$$

When $\lambda = L$, (2.54) becomes

$$\frac{\partial \beta}{\partial L} = \frac{(1 - \alpha) t \bar{x}^2 \bar{D} \frac{\partial \bar{x}}{\partial L} + t(1 - \alpha) \int_0^{\bar{x}} x^2 \frac{\partial D}{\partial u} \frac{\partial u}{\partial L} dx - \beta \theta y}{\theta y L} \begin{matrix} > \\ < \end{matrix} 0 \quad (2.55)$$

We showed earlier that the urban area boundary, housing price, urban land rent, and structural density were increasing functions of population, while utility and housing consumption were decreasing function of population. This means that an increase in population expands the urban area spatially and leads to a lower urban utility. Population increase also results in a higher housing price, higher land rent, and taller buildings everywhere within the urban area. As a result, urban residents consume less housing service. However, the effect of population on the tax-share variable is ambiguous.

The logic behind these results is that an increase in population creates an excess demand for housing, which causes housing price to rise throughout the urban area, inducing urban residents to consume less housing. This expands the urban area because urban-area residents can now outbid farmers for agricultural land at the old urban area boundary.

On the supply side of the market, rising housing price causes housing producers to bid up urban land for housing production. Higher urban land rent in turn forces housing producers to build taller buildings. When buildings are taller and housing consumption is

lower, population density increases everywhere. The increase in urban land rent everywhere also means that at the old urban-area boundary, urban land rent exceeds agricultural land rent, the urban area's boundary therefore expands until urban land equals agricultural land rent again. Spatial expansion of the urban area and increase in urban population density both contribute to the elimination of excess housing demand, bringing the urban area to a new equilibrium with a larger urban area, higher population density, higher housing price, higher land rent, taller buildings, lower housing consumption, and lower utility level. The effect of an increase in population on the tax-share variable is ambiguous because when population increases, more revenue is collected, but more miles are driven because of the urban-area's spatial expansion.

2.5.2 Effects of Changes in Agricultural Land Rent

2.5.2.1 Effect of Agricultural Rent on Utility

When $\lambda = r_a$ (2.44) becomes

$$\frac{\partial u}{\partial r_a} = \frac{\bar{x} - \int_0^{\bar{x}} \frac{\partial r}{\partial r_a} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} = \frac{\bar{x}}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} < 0 \quad (2.56)$$

since $\partial r / \partial r_a = 0$ because r is not a function of r_a .

2.5.2.2 Effect of Agricultural Rent on Urban Area Boundary

When $\lambda = r_a$, (2.48) becomes

$$\frac{\partial \bar{x}}{\partial r_a} = \frac{1 - \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial r_a}}{\frac{\partial \bar{r}}{\partial x}} \quad (2.57)$$

Because $\frac{\partial r}{\partial x} < 0$

$$\text{Sign}\left(\frac{\partial \bar{x}}{\partial r_a}\right) = -\text{Sign}\left(1 - \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial r_a}\right)$$

Recall that $\frac{\partial u}{\partial r_a} = \frac{\bar{x}}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}$, so

$$1 - \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial r_a} = 1 - \frac{\partial \bar{r}}{\partial u} \left(\frac{\bar{x}}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \right) = \frac{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx - \bar{x} \frac{\partial \bar{r}}{\partial u}}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}$$

From (2.34), we know that $\frac{\partial r}{\partial u} = -\frac{h}{qV_1} < 0$, therefore,

$$\text{Sign}\left(\frac{\partial \bar{x}}{\partial r_a}\right) = \text{Sign}\left(\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx - \bar{x} \frac{\partial \bar{r}}{\partial u}\right)$$

Integrating by parts, where $u = \partial r / \partial u$ and $dv = dx$, then

$$\begin{aligned} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx - \bar{x} \frac{\partial \bar{r}}{\partial u} &= \bar{x} \frac{\partial \bar{r}}{\partial u} - \int_0^{\bar{x}} x \frac{\partial}{\partial x} \left(\frac{\partial r}{\partial u} \right) dx - \bar{x} \frac{\partial \bar{r}}{\partial u} = - \int_0^{\bar{x}} x \frac{\partial}{\partial x} \left(\frac{\partial r}{\partial u} \right) dx \\ &= \int_0^{\bar{x}} x \frac{\partial}{\partial x} \left(\frac{h}{qV_1} \right) dx \end{aligned}$$

Upon differentiating the integrand, we have

$$\frac{\partial}{\partial x} \left(\frac{h}{qV_1} \right) = \frac{\partial D}{\partial x} \frac{1}{V_1} + \frac{\partial}{\partial x} \left(\frac{1}{V_1} \right) D \quad (\text{since } D=h/q)$$

To determine the sign of this expression requires determining the sign of its partial derivative, since $V_1 > 0$ and $D > 0$

$$\frac{\partial D}{\partial x} = \frac{h' \frac{\partial S}{\partial x} q - h \frac{\partial q}{\partial x}}{q^2} < 0$$

since $h' > 0$, $\frac{\partial S}{\partial x} = -\frac{h'}{ph''} \frac{\partial p}{\partial x} < 0$, and $\frac{\partial q}{\partial x} > 0$

$$\frac{\partial}{\partial x} \left(\frac{1}{V_1} \right) = -\frac{1}{V_1^2} \frac{\partial V_1}{\partial x} < 0$$

since $\frac{\partial V_1}{\partial x} > 0$ for x a normal good (see Wheaton (1974) p. 227).

Therefore, we have

$$\frac{\partial \bar{x}}{\partial r_a} < 0 \quad (2.58)$$

2.5.2.3 Effects of Agricultural Rent on Housing Price, Housing Consumption, Urban

Land Rent, and Structural Density

$$\frac{dp}{dr_a} = \frac{\partial p}{\partial u} \frac{\partial u}{\partial r_a} > 0 \quad (2.59)$$

$$\frac{dq}{dr_a} = \frac{\partial q}{\partial u} \frac{\partial u}{\partial r_a} < 0 \quad (2.60)$$

$$\frac{dr}{dr_a} = h \frac{\partial p}{\partial r_a} > 0 \quad (2.61)$$

$$\frac{dS}{dr_a} = \frac{\partial S}{\partial u} \frac{\partial u}{\partial r_a} > 0 \quad (2.62)$$

(Note we use total derivatives here since the corresponding partial derivatives are all zero.)

2.5.2.4 Effect of Agricultural Rent on the Tax-Share Variable

When $\lambda = r_a$, (2.54) becomes

$$\frac{\partial \beta}{\partial r_a} = \frac{1}{\theta y L} \left((1-\alpha) \bar{t} \bar{x}^2 \bar{D} \frac{\partial \bar{x}}{\partial r_a} + (1-\alpha) t \int_0^{\bar{x}} x^2 \left(\frac{\partial D}{\partial u} \frac{\partial u}{\partial r_a} \right) dx \right) \quad (2.63)$$

Upon totally differentiating (2.37) with respect to r_a , we have

$$\bar{x}\bar{D}\frac{\partial\bar{x}}{\partial r_a} + \int_0^{\bar{x}} x \frac{\partial D}{\partial u} \frac{\partial u}{\partial r_a} dx = 0, \text{ or } \frac{\partial\bar{x}}{\partial r_a} = -\frac{\int_0^{\bar{x}} x \frac{\partial D}{\partial u} \frac{\partial u}{\partial r_a} dx}{\bar{x}\bar{D}} \quad (2.64)$$

Substituting (2.64) into (2.63), we have

$$\frac{\partial\beta}{\partial r_a} = \frac{1}{\theta_y L} \left((1-\alpha)t \int_0^{\bar{x}} (x^2 - \bar{x}x) \left(\frac{\partial D}{\partial u} \frac{\partial u}{\partial r_a} \right) dx \right) < 0 \quad (2.65)$$

An increase in agricultural rent reduces the urban area spatially and lowers urban-area residents' utility. It also increases housing price, urban land rent, and structural density everywhere. As a consequence, urban population density increases, but housing consumption and the tax-share variable decrease.

The logic behind these results is similar to that of an increase in population. When agricultural rent increases, urban land rent at the old urban boundary is lower than agricultural land rent, which reduces the urban area spatially. This change, however, creates excess demand for housing because the unchanged population must fit within the new urban boundary. Excess demand for housing increases housing price and decreases housing consumption everywhere within the urban area. Both rising housing price and rising agricultural rent push urban land rent up; as a consequence, housing producers build higher buildings. Since the urban area shrinks spatially and population density increases everywhere, people drive fewer miles from home to work, which lowers the proportion of local tax revenues used for the transportation subsidy.

2.5.3 Effects of Change in Income

2.5.3.1 Effect of Income on Utility

When $\lambda = y$, (2.44) becomes

$$\frac{\partial u}{\partial y} = \frac{\frac{\alpha t}{\delta} \frac{\partial L}{\partial y} + \frac{\alpha L}{\delta} \frac{\partial t}{\partial y} + \frac{tL}{\delta} \frac{\partial \alpha}{\partial y} + \bar{x} \frac{\partial r_a}{\partial y} - \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \quad (2.66)$$

Because $\frac{\partial L}{\partial y} = \frac{\partial t}{\partial y} = \frac{\partial \alpha}{\partial y} = \frac{\partial r_a}{\partial y} = 0$ and, from (2.30) and (2.34), $\frac{\partial r}{\partial y} > 0$ and $\frac{\partial r}{\partial u} < 0$, then

$$\frac{\partial u}{\partial y} = - \frac{\int_0^{\bar{x}} \frac{\partial r}{\partial y} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} > 0 \quad (2.67)$$

2.5.3.2 Effect of Income on Urban Area Boundary

When $\lambda = y$, (2.48) becomes

$$\frac{\partial \bar{x}}{\partial y} = \frac{\frac{\partial r_a}{\partial y} - \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial y} - \frac{\partial \bar{r}}{\partial y}}{\frac{\partial \bar{r}}{\partial x}} \quad (2.68)$$

Because $\frac{\partial \bar{r}}{\partial x} < 0$ and $\frac{\partial r_a}{\partial y} = 0$

$$\text{Sign} \left(\frac{\partial \bar{x}}{\partial y} \right) = \text{Sign} \left(\frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \bar{r}}{\partial y} \right)$$

$$\begin{aligned}
\frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \bar{r}}{\partial y} &= \frac{\partial \bar{r}}{\partial u} \left(\frac{-\int_0^{\bar{x}} \frac{\partial r}{\partial y} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \right) + \frac{\partial \bar{r}}{\partial y} \\
&= \frac{\frac{\partial \bar{r}}{\partial y} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx - \frac{\partial \bar{r}}{\partial u} \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}
\end{aligned} \tag{2.69}$$

Because $\frac{\partial r}{\partial u} = \frac{-h}{qV_1} < 0$

$$\text{Sign} \left(\frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \bar{r}}{\partial y} \right) = -\text{Sign} \left(\frac{\partial \bar{r}}{\partial y} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx - \frac{\partial \bar{r}}{\partial u} \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx \right)$$

From (2.30) and (2.34), we have

$$\frac{\partial r}{\partial u} = \frac{-h}{qV_1} \text{ and } \frac{\partial r}{\partial y} = \frac{h(1-\theta)}{q}$$

so, upon evaluating variables at $\bar{x}$, we have

$$\frac{\partial \bar{r}}{\partial u} = \frac{-\bar{h}}{\bar{q}\bar{V}_1} \text{ and } \frac{\partial \bar{r}}{\partial y} = \frac{\bar{h}(1-\theta)}{\bar{q}}$$

Therefore,

$$\begin{aligned}
\frac{\partial \bar{r}}{\partial y} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx - \frac{\partial \bar{r}}{\partial u} \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx &= \int_0^{\bar{x}} -\frac{\bar{h}(1-\theta)}{\bar{q}} \frac{h}{qV_1} dx + \int_0^{\bar{x}} \frac{\bar{h}}{\bar{q}\bar{V}_1} \frac{h(1-\theta)}{q} dx \\
&= \int_0^{\bar{x}} \frac{\bar{h}h(1-\theta)}{q\bar{q}} \left(\frac{1}{\bar{V}_1} - \frac{1}{V_1} \right) dx
\end{aligned}$$

Recall that V_1 is the first partial derivative of the utility function with respect to the com-

posite good, so $V_1 > 0$, $\frac{\partial V_1}{\partial x} = -V_{11}\alpha > 0$, $\bar{V}_1 > V_1$, and $\frac{1}{\bar{V}_1} < \frac{1}{V_1}$. Given these results,

we derive

$$\int_0^{\bar{x}} \frac{\bar{h}h(1-\theta)}{q\bar{q}} \left(\frac{1}{\bar{V}_1} - \frac{1}{V_1} \right) dx < 0$$

Therefore, $\text{Sign}\left(\frac{\partial \bar{x}}{\partial y}\right) = -\text{Sign}\left(\int_0^{\bar{x}} \frac{\bar{h}h(1-\theta)}{q\bar{q}} \left(\frac{1}{\bar{V}_1} - \frac{1}{V_1} \right) dx\right)$, so

$$\frac{\partial \bar{x}}{\partial y} > 0 \quad (2.70)$$

2.5.3.3 Effects of Income on Housing Price, Housing Consumption, Urban Land Rent, and Structural Density

The effect of an increase in income on housing price is complicated because income not only directly affects housing price, but also indirectly affects it through the utility level. Therefore, the relationship between income and housing price can be written as

$$\frac{dp}{dy} = \frac{\partial p}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial p}{\partial y} \quad (2.71)$$

At a given distance, $\hat{x}$, let housing price be $\hat{p}$, where $0 < \hat{p} < \bar{p}$, i.e., p evaluated at $0 < \hat{x} < \bar{x}$. From (2.30), (2.34), and (2.67), we have

$$\frac{\partial \hat{p}}{\partial u} = -\frac{1}{\hat{q}\hat{V}_1}, \quad \frac{\partial \hat{p}}{\partial y} = \frac{1-\theta}{\hat{q}}, \quad \text{and} \quad \frac{\partial u}{\partial y} = \frac{-\int_0^{\bar{x}} \frac{\partial r}{\partial y} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}$$

Substituting these into (2.71), we obtain

$$\begin{aligned} \frac{d\hat{p}}{dy} &= -\frac{1}{\hat{q}\hat{V}_1} \frac{\partial u}{\partial y} + \frac{1-\theta}{\hat{q}} = \frac{1}{\hat{q}\hat{V}_1} \frac{\int_0^{\bar{x}} \frac{\partial r}{\partial y} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} + \frac{1-\theta}{\hat{q}} \\ &= \frac{\frac{1}{\hat{q}\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx + \frac{1-\theta}{\hat{q}} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \end{aligned} \quad (2.72)$$

Because $\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx < 0$

$$\text{Sign}\left(\frac{d\hat{p}}{dy}\right) = -\text{Sign}\left(\frac{1-\theta}{\hat{q}} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx + \frac{1}{\hat{q}\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx\right) \quad (2.73)$$

Substituting $\frac{\partial r}{\partial u} = -\frac{h}{qV_1}$ and $\frac{\partial r}{\partial y} = \frac{h(1-\theta)}{q}$ into the above equation, we have

$$\begin{aligned} -\left(\frac{1-\theta}{\hat{q}} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx + \frac{1}{\hat{q}\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial y} dx\right) &= \int_0^{\bar{x}} \frac{h(1-\theta)}{q\hat{q}V_1} dx - \int_0^{\bar{x}} \frac{h(1-\theta)}{q\hat{q}\hat{V}_1} dx \\ &= \int_0^{\bar{x}} \frac{h(1-\theta)}{q\hat{q}} \left(\frac{1}{V_1} - \frac{1}{\hat{V}_1}\right) dx \end{aligned} \quad (2.74)$$

At the urban area boundary ($x = \bar{x}$), $\frac{1}{V_1} - \frac{1}{\hat{V}_1} > 0$ since $\frac{\partial V_1}{\partial x} > 0$. Hence, $\frac{\partial \bar{p}}{\partial y} > 0$

at the urban area boundary. At the CBD ($x = 0$), $\frac{1}{\hat{V}_1} - \frac{1}{V_1} > 0$. Hence $\frac{dp_0}{dy} < 0$ at the CBD,

where the subscript means evaluation at $x = 0$. Thus, the housing price function pivots counterclockwise. Because of the dependence of urban land rent and structural density on housing price, these functions pivot counterclockwise. The effect of income on housing consumption is more complicated, however. Breuckner (1987, p. 835) shows that q falls with y for $p < \hat{p}$, but is ambiguous for $p > \hat{p}$

2.5.3.4 Effect of Income on the Tax-Share Variable

Since

$$\begin{aligned} \frac{\partial \bar{x}}{\partial y} &> 0 \\ \frac{\partial D}{\partial y} &= \frac{\partial}{\partial y} \left(\frac{h}{q} \right) = \frac{h' \frac{\partial S}{\partial y} q - h \frac{\partial q}{\partial y}}{q^2} > 0, \quad \frac{\partial u}{\partial y} > 0 \\ \frac{\partial D}{\partial u} &= \frac{\partial}{\partial u} \left(\frac{h}{q} \right) = \frac{h' \frac{\partial S}{\partial u} q - h \frac{\partial q}{\partial u}}{q^2} \\ &= \frac{-h'^2 \frac{\partial p}{\partial u} q - h \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial q}{\partial u} + h \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial MRS}{\partial c} \frac{1}{V_1}}{q^2} \\ &= - \left(\frac{h'^2}{ph''q} + \frac{h \left(\frac{\partial q}{\partial p} \right)_u}{q^2} \right) \frac{\partial p}{\partial u} + \frac{h}{q^2} \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial MRS}{\partial c} \frac{1}{V_1} \end{aligned} \quad (2.75)$$

Following Brueckner (1987), let

$$\eta \equiv \left(\frac{\partial q}{\partial p} \right)_u, \quad \Gamma \equiv - \left(\frac{h'^2}{ph''q} + \frac{h\eta}{q^2} \right), \quad \text{and} \quad \Lambda \equiv \frac{h}{q^2} \eta \frac{\partial MRS_{qc}}{\partial c} \frac{1}{V_1}$$

Then

$$\frac{\partial D}{\partial u} = \Gamma \frac{\partial p}{\partial u} + \Lambda < 0 \quad \text{since} \quad \Gamma > 0, \quad \frac{\partial p}{\partial u} < 0, \quad \text{and} \quad \Lambda < 0 \quad (2.76)$$

Therefore, we have

$$\frac{\partial \beta}{\partial y} = \frac{(1-\alpha)t\bar{x}^2 \bar{D} \frac{\partial \bar{x}}{\partial y} + t(1-\alpha) \int_0^{\bar{x}} x^2 \left(\frac{\partial D}{\partial y} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial y} \right) dx - \beta \theta L}{\theta y L} > 0 \quad (2.77)$$

The effects of an increase in income on endogenous variables are a little complicated. As we see from the above results, an increase in income will expand the city spatially and increase urban residents' utility level. It will also increase housing service consumption and population density when p falls, but the effects on q and D are ambiguous when p rises. Income's effect on housing price, urban land rent, and structural density is to pivot these functions counterclockwise. Its effect on the proportion of local tax revenues used for the transportation subsidy is ambiguous.

Thus, according to our model, as income increases, people want to consume more housing. Since housing is cheaper farther out, the demand for housing farther from the CBD rises, causing price to rise there and to fall closer to the CBD. Because urban land rent and structural density are directly related to housing price, an increase in income will have the same effects on urban land rent and structural density as on housing price. When housing price falls close to the CBD, housing consumption per household increases there. In the area close to the urban boundary, the effect of an increase in income on housing consumption is ambiguous. The possible explanation is that rising housing price, urban land rent and structural density there, by themselves, reduce housing consumption. This, however, may be offset by rising utility because of increase in income. The effect of income on the tax-share variable is ambiguous because, on the one hand, an increase in income raises the tax base, while, on the other hand, as people move farther away from the CBD, they drive more miles and require more subsidy.

2.5.4 Effects of Change in Commuting Cost

2.5.4.1 Effect of Commuting Cost on Utility

When $\lambda=t$, (2.44) becomes

$$\frac{\partial u}{\partial t} = \frac{\frac{\alpha L}{\delta} - \int_0^{\bar{x}} \frac{\partial r}{\partial t} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \quad (2.78)$$

Because $\frac{\partial r}{\partial t} = \frac{-\alpha h x}{q} < 0$ and $\frac{\partial r}{\partial u} < 0$

$$\frac{\partial u}{\partial t} < 0 \quad (2.79)$$

2.5.4.2 Effect of Commuting Cost on Urban Area Boundary

From (2.48), substituting t for λ , we have

$$\frac{\partial u}{\partial t} = -\frac{\frac{\partial \bar{r}}{\partial t} - \frac{\partial \bar{r}}{\partial x} \frac{\partial \bar{x}}{\partial t}}{\frac{\partial \bar{r}}{\partial u}} \quad (\text{since } \frac{\partial r_a}{\partial t} = 0)$$

Totally differentiating (2.37) respect to t and substituting $\frac{\partial u}{\partial t}$ into it, we have

$$\bar{x} \bar{D} \frac{\partial \bar{x}}{\partial t} + \int_0^{\bar{x}} x \left(\frac{\partial D}{\partial t} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial t} \right) dx = 0 \quad (\text{since } \frac{\partial L}{\partial t} = \frac{\partial \delta}{\partial t} = 0) \quad (2.80)$$

$$\bar{x} \bar{D} \frac{\partial \bar{x}}{\partial t} + \int_0^{\bar{x}} \left[x \frac{\partial D}{\partial t} + x \frac{\partial D}{\partial u} \left(\frac{-\frac{\partial \bar{r}}{\partial t} - \frac{\partial \bar{r}}{\partial x} \frac{\partial \bar{x}}{\partial t}}{\frac{\partial \bar{r}}{\partial u}} \right) \right] dx = 0$$

$$\bar{x} \bar{D} \frac{\partial \bar{x}}{\partial t} + \int_0^{\bar{x}} \left(\frac{x \frac{\partial D}{\partial t} \frac{\partial \bar{r}}{\partial u} - x \frac{\partial D}{\partial u} \frac{\partial \bar{r}}{\partial t} - x \frac{\partial D}{\partial u} \frac{\partial \bar{r}}{\partial x} \frac{\partial \bar{x}}{\partial t}}{\frac{\partial \bar{r}}{\partial u}} \right) dx = 0$$

After rearranging terms, we have

$$\frac{\partial \bar{x}}{\partial t} = \frac{\int_0^{\bar{x}} x \left(\frac{\partial D}{\partial u} \frac{\partial \bar{r}}{\partial t} - \frac{\partial D}{\partial t} \frac{\partial \bar{r}}{\partial u} \right) dx}{\bar{x} \bar{D} \frac{\partial \bar{r}}{\partial u} - \int_0^{\bar{x}} x \frac{\partial D}{\partial u} \frac{\partial \bar{r}}{\partial x} dx} \quad (2.81)$$

We need to know $\frac{\partial D}{\partial t}$ to determine the sign of $\frac{\partial \bar{x}}{\partial t}$ since $\frac{\partial D}{\partial u} = \Gamma \frac{\partial p}{\partial u} + \Lambda < 0$ from (2.76)

Recall that

$$D = \frac{h}{q}, \quad \frac{\partial S}{\partial u} = -\frac{h'}{ph''} \frac{\partial p}{\partial u}, \quad \frac{\partial q}{\partial u} = \left(\frac{\partial q}{\partial p} \right)_u \left(\frac{\partial p}{\partial u} - \frac{\partial MRS}{\partial c} \frac{1}{V_1} \right)$$

$$\frac{\partial S}{\partial t} = -\frac{h'}{ph''} \frac{\partial p}{\partial t}$$

$$\frac{\partial q}{\partial t} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial t}$$

Then

$$\begin{aligned} \frac{\partial D}{\partial t} &= \frac{\partial}{\partial t} \left(\frac{h}{q} \right) = \frac{h' \frac{\partial S}{\partial t} q - h \frac{\partial q}{\partial t}}{q^2} = \frac{-h' \frac{h'}{ph''} \frac{\partial p}{\partial t} q - h \eta \frac{\partial q}{\partial t}}{q^2} \\ &= -\left(\frac{h'^2}{ph''q} + \frac{h\eta}{q^2} \right) \frac{\partial p}{\partial t} = \Gamma \frac{\partial p}{\partial t} < 0 \quad (\text{since } \frac{\partial p}{\partial t} < 0) \end{aligned} \quad (2.82)$$

Since $\frac{\partial \bar{r}}{\partial u} < 0$, $\frac{\partial D}{\partial u} < 0$, and $\frac{\partial \bar{r}}{\partial x} < 0$, then, from (2.81) we know that

$$\text{Sign} \left(\frac{\partial \bar{x}}{\partial t} \right) = -\text{Sign} \left[\int_0^{\bar{x}} x \left(\frac{\partial D}{\partial u} \frac{\partial \bar{r}}{\partial t} - \frac{\partial D}{\partial t} \frac{\partial \bar{r}}{\partial u} \right) dx \right]$$

Recall that

$$\frac{\partial D}{\partial t} = \Gamma \frac{\partial p}{\partial t}, \quad \frac{\partial D}{\partial u} = \Gamma \frac{\partial p}{\partial u} + \Lambda, \quad \frac{\partial p}{\partial t} = -\frac{x\alpha}{q}, \quad \frac{\partial p}{\partial u} = -\frac{1}{qV_1}, \quad \frac{\partial \bar{r}}{\partial t} = \frac{-\alpha \bar{h} \bar{x}}{\bar{q}}, \quad \text{and}$$

$$\frac{\partial \bar{r}}{\partial u} = -\frac{\bar{h}}{\bar{q} \bar{V}_1}.$$

Furthermore, since $\frac{\partial V_1}{\partial x} > 0$, then $V_1 > \bar{V}_1$ and $\frac{\bar{x}}{V_1} > \frac{x}{\bar{V}_1}$ for $x < \bar{x}$.

Consequently,

$$\begin{aligned} \int_0^{\bar{x}} x \left(\frac{\partial D}{\partial u} \frac{\partial \bar{r}}{\partial t} - \frac{\partial D}{\partial t} \frac{\partial \bar{r}}{\partial u} \right) dx &= \int_0^{\bar{x}} x \left(\left(\Gamma \frac{\partial p}{\partial u} + \Lambda \right) \frac{\partial \bar{r}}{\partial t} - \Gamma \frac{\partial p}{\partial t} \frac{\partial \bar{r}}{\partial u} \right) dx \\ &= \int_0^{\bar{x}} \left(-x \Gamma \frac{1}{q V_1} \left(\frac{-\alpha \bar{h} \bar{x}}{\bar{q}} \right) - x \Gamma \left(-\frac{x \alpha}{q} \right) \left(\frac{-\bar{h}}{\bar{q} \bar{V}_1} \right) + x \Lambda \frac{\partial \bar{r}}{\partial t} \right) dx \\ &= \int_0^{\bar{x}} \left(x \Gamma \frac{\alpha \bar{h}}{q \bar{q}} \left(\frac{\bar{x}}{V_1} - \frac{x}{\bar{V}_1} \right) + x \Lambda \frac{\partial \bar{r}}{\partial t} \right) dx > 0 \end{aligned}$$

Hence

$$\frac{\partial \bar{x}}{\partial t} < 0 \quad (2.83)$$

2.5.4.3 Effects of Commuting Cost on Housing Price, Housing Consumption, Urban

Land Rent, and Structural Density

Recall

$$\frac{\partial r}{\partial t} = \frac{-\alpha h x}{q}, \quad \frac{\partial p}{\partial u} = \frac{-1}{q V_1}, \quad \frac{\partial p}{\partial t} = \frac{-x \alpha}{q}, \quad \text{and} \quad \frac{\partial u}{\partial t} = \frac{\frac{\alpha L}{\delta} - \int_0^{\bar{x}} \frac{\partial r}{\partial t} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}$$

At a given location, housing price is affected by commuting cost both directly and indirectly through the utility function. Consequently,

$$\begin{aligned} \frac{d\hat{p}}{dt} &= \frac{\partial \hat{p}}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial \hat{p}}{\partial t} \quad (\text{where } \hat{p} \text{ is some } p \text{ between } 0 \text{ and } \bar{x}) \\ &= \left(-\frac{1}{\hat{q} \hat{V}_1} \right) \frac{\partial u}{\partial t} - \frac{\hat{x} \alpha}{\hat{q}} = -\frac{1}{\hat{q}} \left(\frac{1}{\hat{V}_1} \frac{\partial u}{\partial t} + \hat{x} \alpha \right) = -\frac{1}{\hat{q}} \left(\frac{1}{\hat{V}_1} \frac{\frac{\alpha L}{\delta} - \int_0^{\bar{x}} \frac{\partial r}{\partial t} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} + \hat{x} \alpha \right) \end{aligned}$$

$$= -\frac{1}{\hat{q}} \frac{\frac{\alpha L}{\delta \hat{V}_1} - \frac{1}{\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial t} dx + \hat{x} \alpha \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}$$

Because $\frac{\partial r}{\partial u} < 0$,

$$\text{Sign}\left(\frac{\partial \hat{p}}{\partial t}\right) = \text{Sign}\left(\frac{\alpha L}{\delta \hat{V}_1} - \frac{1}{\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial t} dx + \hat{x} \alpha \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx\right) \quad (2.84)$$

Substitute the following into the parenthetical expression in the right-hand side of (2.84)

$$\frac{\partial r}{\partial u} = -\frac{h}{qV_1}, \frac{\partial r}{\partial t} = -\frac{\alpha hx}{q}, \text{ and } \frac{L}{\delta} = \int_0^{\bar{x}} x D dx = \int_0^{\bar{x}} \frac{xh}{q} dx$$

Then we have

$$\begin{aligned} \frac{\alpha L}{\delta \hat{V}_1} - \frac{1}{\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial t} dx + \hat{x} \alpha \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx &= \int_0^{\bar{x}} \frac{xh\alpha}{\hat{V}_1 q} dx + \int_0^{\bar{x}} \frac{\alpha hx}{\hat{V}_1 q} dx - \int_0^{\bar{x}} \frac{\alpha \hat{x} h}{q V_1} dx \\ &= \int_0^{\bar{x}} \frac{h\alpha}{q} \left(\frac{2x}{\hat{V}_1} - \frac{\hat{x}}{V_1} \right) dx \end{aligned} \quad (2.85)$$

At $\hat{x} = 0$, (2.85) becomes $\int_0^{\bar{x}} \frac{h\alpha}{q} \left(\frac{2x}{\hat{V}_1} \right) dx > 0$, but at $\hat{x} = \bar{x}$, (2.85) becomes

$\int_0^{\bar{x}} \frac{h\alpha}{q} \left(\frac{2x}{\hat{V}_1} - \frac{\hat{x}}{V_1} \right) dx \begin{matrix} \geq \\ < \end{matrix} 0$ because both $\frac{2x}{\hat{V}_1}$ and $\frac{\hat{x}}{V_1}$ are positive. Brueckner (1987) argues

that the boundary value of p is invariant to t . That, coupled with $\frac{\partial \bar{x}}{\partial t} < 0$, implies that

$\frac{d\bar{p}}{dt} < 0$. Since p decreases monotonically in x , then, p rises with t to the left of $\hat{x}$ and falls

to the right, i.e., the p function pivots clockwise for an increase in t .

Recall

$$\frac{\partial r}{\partial u} = -\frac{h}{qV_1}, \quad \frac{\partial p}{\partial u} = -\frac{1}{qV_1}, \quad \frac{\partial r}{\partial u} = h \frac{\partial p}{\partial u}, \quad \frac{\partial r}{\partial t} = -\frac{\alpha hx}{q}, \quad \text{and} \quad \frac{\partial p}{\partial t} = -\frac{\alpha x}{q}$$

Then

$$\frac{dq}{dt} = \frac{\partial q}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial q}{\partial t} \begin{cases} < 0 & 0 < x < \hat{x} \\ \geq 0 & \hat{x} < x < \bar{x} \\ < 0 & \end{cases} \quad (2.86)$$

$$\frac{dS}{dt} = \frac{\partial S}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial S}{\partial t} = -\frac{h'}{ph''} \left(\frac{\partial p}{\partial u} \frac{\partial u}{\partial t} + \right) \frac{\partial p}{\partial t} \begin{cases} > 0 & 0 < x < \hat{x} \\ < 0 & \hat{x} < x < \bar{x} \end{cases} \quad (2.87)$$

$$\frac{dr}{dt} = \frac{\partial r}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial r}{\partial t} = \alpha h \frac{\partial p}{\partial t} \begin{cases} > 0 & 0 < x < \hat{x} \\ < 0 & \hat{x} < x < \bar{x} \end{cases} \quad (2.88)$$

2.5.4.4 Effect of Commuting Cost on the Tax-Share Variable

When $\lambda = t$, (2.54) becomes

$$\frac{\partial \beta}{\partial t} = \frac{(1-\alpha)t\bar{x}^2 \bar{D} \frac{\partial \bar{x}}{\partial t} + (1-\alpha) \int_0^{\bar{x}} x^2 D dx + t(1-\alpha) \int_0^{\bar{x}} x^2 \left(\frac{\partial D}{\partial t} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial t} \right) dx}{\theta y L} \quad (2.89)$$

Since $\frac{\partial \bar{x}}{\partial t} < 0$, $\frac{\partial D}{\partial t} < 0$, $\frac{\partial D}{\partial u} < 0$, and $\frac{\partial u}{\partial t} < 0$, $\frac{\partial \beta}{\partial t} > 0$.

From the above results, we conclude that an increase in commuting cost reduces urban residents' utility and shrinks the urban area spatially. The effects of an increase in commuting cost on housing price, urban population, land rent, and structural density vary with location, raising these variables near the CBD and lowering them farther out. The effect of commuting cost on housing consumption and population density are more complicated. An increase in commuting cost lowers housing consumption and raises popula-

tion density near the CBD, but its effect is ambiguous farther out. Finally, the effect of commuting cost on the tax-share variable is ambiguous.

Our explanation for the results is that as transport cost per round-trip mile increases, people seek locations closer to the CBD to minimize transport costs. This movement reduces the spatial area of the city and lower residents' utility. Also, as people move closer to the CBD, they drive up land rent and housing price there while driving those variables down farther out. This, in turn, causes households to consume less housing in taller buildings near the CBD and raises the population density there, while lowering structural density farther out. The effects on housing consumption and population density farther from the CBD are, however, ambiguous. This is so because the low housing price and land rent farther out, by themselves, would tend to raise housing consumption and lower population density. But the lower utility may offset these results. The effect of commuting cost on the tax-share variable is ambiguous because, on the one hand, an increase in commuting cost per round-trip mile induces more transportation subsidy, while, on the other hand, the urban area shrinks spatially and people drive fewer miles, which reduces the transportation subsidy.

2.5.5 Effects of Change in Private Cost Share

2.5.5.1 Effect of Private Cost Share on Utility

Recall that

$$\frac{\partial r}{\partial \alpha} = -\frac{txh}{q} < 0 \text{ and } \frac{\partial r}{\partial u} = -\frac{h}{qV_1} < 0$$

Then, for $\lambda = \alpha$, (2.44) becomes

$$\frac{\partial u}{\partial \alpha} = \frac{\frac{tL}{\delta} - \int_0^{\bar{x}} \frac{\partial r}{\partial \alpha} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial \alpha} dx} < 0 \quad (2.90)$$

2.5.5.2 Effect of Private Cost Share on Urban Area Boundary

Equation (2.47), with $\lambda = \alpha$, becomes

$$\frac{\partial \bar{r}}{\partial x} \frac{\partial \bar{x}}{\partial \alpha} + \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial \alpha} + \frac{\partial \bar{r}}{\partial \alpha} = 0$$

Solving for $\frac{\partial \bar{x}}{\partial \alpha}$, we have

$$\frac{\partial \bar{x}}{\partial \alpha} = - \frac{\frac{\partial \bar{r}}{\partial \alpha} + \frac{\partial \bar{r}}{\partial u} \frac{\partial u}{\partial \alpha}}{\frac{\partial \bar{r}}{\partial x}} \quad (2.91)$$

From (2.32), (2.34), and (2.29), we have

$$\frac{\partial \bar{r}}{\partial \alpha} = - \frac{\bar{h} \bar{x} t}{\bar{q}} = - \bar{D} \bar{x} t$$

$$\frac{\partial \bar{r}}{\partial u} = - \frac{\bar{h}}{\bar{q} \bar{V}_1} = - \bar{D} \frac{1}{\bar{V}_1}$$

$$\frac{\partial \bar{r}}{\partial x} = - \frac{\bar{h} \alpha t}{\bar{q}} = - \bar{D} \alpha t$$

Substituting these into (2.91), we get

$$\frac{\partial \bar{x}}{\partial \alpha} = - \frac{- \bar{D} \bar{x} t + \left(- \bar{D} \frac{1}{\bar{V}_1} \right) \frac{\partial u}{\partial \alpha}}{- \bar{D} \alpha t} = - \frac{\bar{x}}{\alpha} + \frac{1}{\bar{V}_1 \alpha t} \frac{\partial u}{\partial \alpha} < 0 \quad (2.92)$$

2.5.5.3 Effects of Private Cost Share on Housing Price, Housing Consumption, Urban Land Rent, and Structural Density

Given the fact that α affects housing price both directly and indirectly through the utility function, we have

$$\frac{d\hat{p}}{d\alpha} = \frac{\partial \hat{p}}{\partial u} \frac{\partial u}{\partial \alpha} + \frac{\partial \hat{p}}{\partial \alpha}, \text{ where } \hat{p} \text{ is } p \text{ evaluated at } \hat{x}, 0 < \hat{x} < \bar{x} \quad (2.93)$$

Then substituting from (2.32), (2.34), and (2.90), we get

$$\frac{d\hat{p}}{d\alpha} = \left(-\frac{1}{\hat{q}\hat{V}_1} \right) \left(\frac{\frac{tL}{\delta} - \int_0^{\bar{x}} \frac{\partial r}{\partial \alpha} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx} \right) - \frac{t\hat{x}}{\hat{q}} = \left(-\frac{1}{\hat{q}} \right) \frac{\frac{tL}{\delta\hat{V}_1} - \frac{1}{\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial \alpha} dx + t\hat{x} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}{\int_0^{\bar{x}} \frac{\partial r}{\partial u} dx}$$

Because $\frac{\partial r}{\partial u} < 0$

$$\text{Sign} \left(\frac{d\hat{p}}{d\alpha} \right) = \text{Sign} \left(\frac{tL}{\delta\hat{V}_1} - \frac{1}{\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial \alpha} dx + t\hat{x} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx \right) \quad (2.94)$$

From (2.37)

$$\frac{L}{\delta} = \int_0^{\bar{x}} x D dx = \int_0^{\bar{x}} \frac{xh}{q} dx$$

From (2.32), $\frac{\partial r}{\partial \alpha} = -\frac{hxt}{\bar{q}}$, and from (2.34), $\frac{\partial r}{\partial u} = -\frac{h}{qV_1}$. Substitute these into (2.94), get-

ting

$$\begin{aligned} \frac{tL}{\delta\hat{V}_1} - \frac{1}{\hat{V}_1} \int_0^{\bar{x}} \frac{\partial r}{\partial \alpha} dx + t\hat{x} \int_0^{\bar{x}} \frac{\partial r}{\partial u} dx &= \int_0^{\bar{x}} \frac{t}{\hat{V}_1} \frac{xh}{q} dx + \int_0^{\bar{x}} \frac{1}{\hat{V}_1} \frac{txh}{q} dx - t\hat{x} \int_0^{\bar{x}} \frac{h}{qV_1} dx \\ &= \int_0^{\bar{x}} \frac{th}{q} \left(\frac{2x}{\hat{V}_1} - \frac{\hat{x}}{V_1} \right) dx \end{aligned} \quad (2.95)$$

At the CBD where $x = 0$, (2.95) is positive, which means that $\frac{dp_0}{d\alpha} > 0$. At the urban area boundary, where $\hat{x} = \bar{x}$, (2.95) is of ambiguous sign. Nevertheless, it can be shown that $\frac{d\bar{p}}{d\alpha} < 0$ and that the price function pivots clockwise about some $0 < \hat{x} < \bar{x}$. First, recall that the boundary value of p is invariant to exogenous variables other than r_a and i . Thus, p is invariant to α . But, since $\frac{\partial \bar{x}}{\partial \alpha} < 0$, then $\frac{\partial \bar{p}}{\partial \alpha} < 0$. Also, since p decreases monotonically with x , the p function must pivot clockwise about some $\hat{x}$. Thus,

$$\frac{dp}{d\alpha} \begin{cases} > 0 & 0 \leq x < \hat{x} \\ = 0 & x = \hat{x} \\ < 0 & \hat{x} < x \leq \bar{x} \end{cases} \quad (2.96)$$

From (2.16) and (2.17), we know that S and r functions exhibit the same behavior as the p function, then

$$\frac{dS}{d\alpha} = -\frac{h'}{ph''} \frac{dp}{d\alpha} \begin{cases} > 0 & 0 \leq x < \hat{x} \\ = 0 & x = \hat{x} \\ < 0 & \hat{x} < x \leq \bar{x} \end{cases} \quad (2.97)$$

and

$$\frac{dr}{d\alpha} = h \frac{dp}{d\alpha} \begin{cases} > 0 & 0 \leq x < \hat{x} \\ = 0 & x = \hat{x} \\ < 0 & \hat{x} < x \leq \bar{x} \end{cases} \quad (2.98)$$

As we have seen in other contexts, the effect on q is more complicated. Since q falls when p rises but is ambiguous when p falls, then

$$\frac{dq}{d\alpha} \begin{cases} < 0 & 0 \leq x < \hat{x} \\ \geq 0 & \hat{x} \leq x \leq \bar{x} \end{cases} \quad (2.99)$$

2.5.5.4 Effect of Private Cost Share on the Tax-Share Variable

When $\lambda = \alpha$, (2.54) becomes

$$\frac{\partial \beta}{\partial \alpha} = \frac{1}{\theta y L} \left[\bar{x}^2 \bar{D} t (1 - \alpha) \frac{\partial \bar{x}}{\partial \alpha} - t \int_0^{\bar{x}} x^2 D dx + t (1 - \alpha) \int_0^{\bar{x}} x^2 \left(\frac{\partial D}{\partial \alpha} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial \alpha} \right) dx \right] \quad (2.100)$$

Totally differentiating (2.37) with respect to α and solving for $\frac{\partial \bar{x}}{\partial \alpha}$, we have

$$\frac{\partial \bar{x}}{\partial \alpha} = \frac{- \int_0^{\bar{x}} x \left(\frac{\partial D}{\partial \alpha} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial \alpha} \right) dx}{\bar{x} \bar{D}}$$

Substituting the above equation into (2.100), getting

$$\frac{d\beta}{d\alpha} = \frac{1}{\theta y L} \left[-t \int_0^{\bar{x}} x^2 D dx + t (1 - \alpha) \int_0^{\bar{x}} (x - \bar{x}) x \left(\frac{\partial D}{\partial \alpha} + \frac{\partial D}{\partial u} \frac{\partial u}{\partial \alpha} \right) dx \right] < 0 \quad (2.101)$$

From the above results, we see that an increase in α reduces the urban area spatially, lowers utility, and lowers the tax-share variable. An increase in α increases housing price, urban land rent, and structural density in the areas close to CBD while lowering these variables in areas close to the urban-area boundary. Finally, an increase in α lowers housing consumption near the CBD but may raise, lower, or leave housing consumption unchanged near the urban-area boundary.

Our explanation for these results is that as households' unsubsidized transport cost increases, people seek locations nearer the CBD. This movement reduces the urban area spatially and lowers residents' utility level. As people move closer to the CBD, they drive up land rents and housing prices there while driving down those variables farther from the CBD. This, in turn, causes households to consume less housing in taller buildings near the CBD, while lowering structural density farther out. The effects on housing

consumption and population density farther from the CBD area are, however, ambiguous. This is so because the lower housing price and land rent farther out, by themselves, would tend to raise housing consumption and lower population density. But the lower utility level may offset these results. Finally, because the private share of commuting cost increases, the tax-share variable is reduced.

Table 2.1 summarizes the comparative static analysis of the single-mode urban spatial model. We find that an increase in population and income results in urban expansion, while an increase in agricultural land and commuting cost shrinks the urban area spatially. Since our goal is to explain how transport subsidies affect urban sprawl, we emphasize $\partial \bar{x} / \partial \alpha$. We find that in a single-mode model, when subsidies increase, the urban area expands spatially. This is so because an increase in transport subsidies raises people's income net of transport cost, which results in a higher demand for housing in suburban areas because of lower housing price there. Consequently, housing price, urban land rent, and structural density in the suburbs increase, which leads to spatial expansion of the urban area. This finding suggests that subsidized transport is a contributor to urban sprawl in a single-mode model.

Table 2.1

Comparative Static Results of Single-Mode Model

<i>Exogenous Variable</i>		<i>Endogenous Variable</i>						
		U	$\bar{x}$	S	r	p	q	β
L		-	+	+	+	+	-	?
t	$x < x'$	-	-	+	+	+	-	?
	$x > x'$	-	-	-	-	-	?	?
y	$x < x'$	+	+	-	-	-	+	?
	$x > x'$	+	+	+	+	+	?	?
r_a		-	-	+	+	+	-	-
x		NC	NC	-	-	-	+	NC
$1-\alpha$	$x < x'$	+	+	-	-	-	+	+
	$x > x'$	+	+	+	+	+	?	+

CHAPTER 3 TWO-MODE URBAN SPATIAL MODEL

We now relax the assumption that only one transport mode prevails within a city. We begin with some new definitions and assumptions, proceed to a detailed discussion of the behavioral equations of the model, and then provide a comparative static analysis.

3.1 Modal Choice and Preliminary Results Used for Comparative Static Analysis

All definitions and assumptions from the single-mode model remain unchanged except the following: Two different transport modes, 1 and 2, are available in the urban area and cost of commuting by each mode incurred by a resident at location x is represented as follows:

$$M_i(x) = f_i + \alpha_i t_i x \quad i = 1, 2 \quad (3.1)$$

where f represents fixed cost, which includes all travel cost independent of distance traveled, and $\alpha_i t_i x$ represents private variable cost, which include all costs that vary with distance traveled. We assume that variable cost is proportional to distance. As in Ch. 2, the α 's represent the proportion of total variable cost directly paid by travelers. We also assume that $f_1 < f_2$ while $\alpha_1 t_1 > \alpha_2 t_2$. As in Ch. 2, $(1 - \alpha_i)$ represents the governmental subsidy per round-trip mile.

These assumptions imply that Mode 2 has a higher fixed cost but a lower variable cost per round-trip mile because it runs faster, more conveniently, more efficiently or receives a larger governmental subsidy. It is necessary to assume that Mode 1's fixed cost is less than Mode 2's but that its variable cost per mile is greater so that one mode will not dominate the other for all trips. In our model, a resident chooses between two modes.

He will be indifferent between the two modes if he lives at $\hat{x}$ where $\hat{x} = (f_2 - f_1)/(\alpha_1 t_1 - \alpha_2 t_2)$, will choose Mode 1 if he lives between CBD and $\hat{x}$ and will choose Mode 2 if he lives between $\hat{x}$ and the urban area boundary. Therefore we refer to Mode 1 as the central city mode and Mode 2 as the suburban mode.

Based on the new assumptions, the single-mode urban spatial model is modified as follows. Each individual chooses the amount of a composite good, c , and the amount of residential housing, q , so as to maximize utility

$$u = u(c, q)$$

subject to a budget constraint

$$y = c + pq + \theta y + M(x) \text{ , where } M(x) = \min \{M_1(x), M_2(x)\}$$

Substituting c from the budget constraint into the utility function, the requirement that the maximized utility equals u can be written as

$$\underset{q}{\text{Max}} V [(1 - \theta)y - pq - M(x), q] = u \quad (3.2)$$

Since (3.2) represents the maximized utility, two conditions must hold. First, since consumers choose q optimally conditional on p , the first-order condition must hold, that is

$$\frac{V_2[(1 - \theta)y - pq - M_i, q]}{V_1[(1 - \theta)y - pq - M_i, q]} = p \quad (3.3)$$

Also, the resulting consumption bundle must generate the spatial equilibrium utility, u , so that

$$V[(1 - \theta)y - pg - M_i, q] = u$$

Given the assumption of mode choice, the exogenous variables are $x, y, t_i, \alpha_i, \theta, i, G$ and $f_i (i = 1, 2)$. We suppress the variables i and G because they play no important role in our analysis. In presenting the results for p, q, S and r , we suppress notation identifying the mode, except for t_i and α_i , because these results hold for either mode. Later, it will become necessary to distinguish these variables by mode.

Using the same method as in Ch. 2, we get

$$\frac{\partial p}{\partial x} = -\frac{\alpha_i t_i}{q} < 0 \quad \frac{\partial q}{\partial x} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial x} > 0 \quad (3.4)$$

$$\frac{\partial p}{\partial y} = \frac{1-\theta}{q} > 0 \quad \frac{\partial q}{\partial y} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial y} < 0 \quad (3.5)$$

$$\frac{\partial p}{\partial t_i} = \frac{-\alpha_i x}{q_i} < 0 \quad \frac{\partial q_i}{\partial t_i} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial t_i} > 0 \quad (3.6)$$

$$\frac{\partial p}{\partial \alpha_i} = -\frac{t_i x}{q} < 0 \quad \frac{\partial q}{\partial \alpha_i} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial \alpha_i} < 0 \quad (3.7)$$

$$\frac{\partial p}{\partial \theta} = -\frac{y}{q} < 0 \quad \frac{\partial q}{\partial \theta} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial \theta} > 0 \quad (3.8)$$

$$\frac{\partial p}{\partial f_i} = -\frac{1}{q} < 0 \quad \frac{\partial q}{\partial f_i} = \left(\frac{\partial q}{\partial p} \right)_u \frac{\partial p}{\partial f_i} > 0 \quad (3.9)$$

$$\frac{\partial p}{\partial u} = -\frac{1}{qV_1} < 0 \quad \frac{\partial q}{\partial u} = \left(\frac{\partial p}{\partial u} - \frac{\partial MRS_{qc}}{\partial c} \frac{1}{V_1} \right) \left(\frac{\partial q}{\partial p} \right)_u > 0 \quad (3.10)$$

Except for the addition of fixed cost, nothing is changed in the housing production sector of the model, so, from Ch. 2, we have

$$\frac{\partial S}{\partial \varphi} = -\frac{h'}{ph''} \frac{\partial p}{\partial \varphi} \quad (3.11)$$

$$\frac{\partial r}{\partial \varphi} = h \frac{\partial p}{\partial \varphi} \quad (3.12)$$

where $\varphi = x, y, t_i, \alpha_i, f_i, u$.

Under our assumption, Mode 1 is used from the CBD to $\hat{x}$, and Mode 2 is used from $\hat{x}$ to $\bar{x}$. Hence, the boundary condition is more complicated than that of the single-mode model, namely,

$$r_1(\hat{x}, y, t_1, u, \alpha_1, f_1, \theta) = r_2(\hat{x}, y, t_2, u, \alpha_2, f_2, \theta) \quad (3.13)$$

$$r(\bar{x}, y, t_2, u, \alpha_2, f_2, \theta) = r_a \quad (3.14)$$

The population and balanced budget condition must also reflect mode choice. The population condition becomes

$$\int_0^{\hat{x}} \delta x D_1(x, y, t_1, f_1, \alpha_1, \theta, i) dx + \int_{\hat{x}}^{\bar{x}} \delta x D_2(x, y, t_2, f_2, \alpha_2, \theta, i) dx = L \quad (3.15)$$

where D_i represent population density. The balanced budget condition becomes

$$\beta \theta y L + G = (1 - \alpha_1) \int_0^{\hat{x}} t_1 x^2 D_1 dx + (1 - \alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 D_2 dx \quad (3.16)$$

3.2 Comparative Static Analysis

The goal of the analysis is to deduce the impact of changes in L , r_a , y , t_i , α_i , and f_i on the spatial size of the urban area, $\bar{x}$, housing price, p_i , land rent, r_i , housing consumption, q_i , structural density, S_i , utility level, u , and the tax-share variable, β . Given that the purpose of this thesis is to study the relationship between urban sprawl and transportation subsidies, we ignore the effects of the exogenous variables G , and i .

Recall

$$D_i = \frac{h_i}{q_i} = -\frac{\frac{\partial r_i}{\partial x}}{\alpha_i t_i} \text{ (note we often use } h_i \text{ as shorthand for } h(S_i))$$

Substitute this into (3.15) to get

$$-\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} x \frac{\partial r_2}{\partial x} dx = \frac{L}{\delta} \quad (3.17)$$

Integrating by parts, where $u = x$ and $dv = \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx$ or $\int_{\hat{x}}^{\bar{x}} x \frac{\partial r_2}{\partial x} dx$, we get

$$-\frac{1}{\alpha_1 t_1} x r_1 \Big|_0^{\hat{x}} + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} r_1 dx - \frac{1}{\alpha_2 t_2} x r_2 \Big|_{\hat{x}}^{\bar{x}} + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} r_2 dx = \frac{L}{\delta} \quad (3.18)$$

or

$$-\frac{1}{\alpha_1 t_1} \hat{x} r_1(\hat{x}) - \frac{1}{\alpha_2 t_2} [(\bar{x} r_a - \hat{x} r_2(\hat{x}))] + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} r_1 dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} r_2 dx = \frac{L}{\delta} \quad (3.19)$$

Because $r_1(\hat{x}) = r_2(\hat{x})$, we have

$$\left(\frac{1}{\alpha_2 t_2} - \frac{1}{\alpha_1 t_1} \right) \hat{x} r_1(\hat{x}) - \frac{1}{\alpha_2 t_2} \bar{x} r_a + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} r_1 dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} r_2 dx = \frac{L}{\delta} \quad (3.20)$$

Totally differentiating (3.20) with respect to $\lambda = L, r_a, y, t_1, t_2, f_1, f_2, \alpha_1, \alpha_2$, we have

$$\begin{aligned} & \frac{\partial}{\partial \lambda} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) \hat{x} r_1(\hat{x}) + \frac{\partial \hat{x}}{\partial \lambda} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) r_1(\hat{x}) + \frac{\partial r_1(\hat{x})}{\partial \lambda} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) \hat{x} \\ & - \frac{\partial}{\partial \lambda} \left(\frac{1}{\alpha_2 t_2} \right) \bar{x} r_a - \frac{\partial r_a}{\partial \lambda} \left(\frac{1}{\alpha_2 t_2} \right) \bar{x} + \frac{\partial}{\partial \lambda} \left(\frac{1}{\alpha_1 t_1} \right) \int_0^{\hat{x}} r_1 dx - \frac{\partial \bar{x}}{\partial \lambda} \frac{1}{\alpha_2 t_2} r_a \\ & + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial \lambda} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial \lambda} \right) dx + \frac{1}{\alpha_1 t_1} r_1(\hat{x}) \frac{\partial \hat{x}}{\partial \lambda} + \frac{\partial}{\partial \lambda} \left(\frac{1}{\alpha_2 t_2} \right) \int_{\hat{x}}^{\bar{x}} r_2 dx \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial \lambda} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial \lambda} \right) dx - \left(\frac{1}{\alpha_2 t_2} \right) r_2(\hat{x}) \frac{\partial \hat{x}}{\partial \lambda} + \frac{\partial \bar{x}}{\partial \lambda} \frac{1}{\alpha_2 t_2} r_a \\
& = \frac{1}{\delta} \frac{\partial L}{\partial \lambda} - \frac{L}{\delta^2} \frac{\partial \delta}{\partial \lambda}
\end{aligned} \tag{3.21}$$

3.2.1 Effects of Change in Population

3.2.1.1 Effect of Population on Utility

If $\lambda = L$, (3.21) becomes

$$\begin{aligned}
& \frac{\partial r_1(\hat{x})}{\partial L} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) \hat{x} + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial L} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial L} \right) dx \\
& + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial L} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial L} \right) dx = \frac{1}{\delta}
\end{aligned} \tag{3.22}$$

Since $r_1(x)$, $r_1(\hat{x})$, and $r_2(x)$ are not functions of L , (3.22) becomes

$$\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial L} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial L} \right) dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial L} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial L} \right) dx = \frac{1}{\delta}$$

Then, since u is not a function of x , we have

$$\frac{\partial u}{\partial L} = \frac{1}{\delta \left(\frac{1}{\alpha_1 t_1} \int_0^{\bar{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\bar{x}}^{\hat{x}} \frac{\partial r_2}{\partial u} dx \right)} \tag{3.23}$$

Because $\frac{\partial r_1}{\partial u} = -\frac{h(S_1)}{q_1 V_1} < 0$ and $\frac{\partial r_2}{\partial u} = -\frac{h(S_2)}{q_2 V_1} < 0$

$$\frac{\partial u}{\partial L} < 0 \tag{3.24}$$

3.2.1.2 Effect of Population on Urban Area Boundary

Totally differentiating (3.14) with respect to $\lambda = L, r_a, t_1, t_2, \alpha_1, \alpha_2, f_1$, and f_2

$$\frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial \lambda} + \frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial \lambda} + \frac{\partial \bar{r}_2}{\partial \lambda} = \frac{\partial r_a}{\partial \lambda} \tag{3.25}$$

Then

$$\frac{\partial \bar{x}}{\partial \lambda} = \frac{\frac{\partial r_a}{\partial \lambda} - \frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial \lambda} - \frac{\partial \bar{r}_2}{\partial \lambda}}{\frac{\partial \bar{r}_2}{\partial x}} \quad (3.26)$$

Since $\frac{\partial \bar{r}_2}{\partial x} < 0$, then we have

$$\text{Sign} \frac{\partial \bar{x}}{\partial L} = -\text{Sign} \left(\frac{\partial r_a}{\partial L} - \frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial L} - \frac{\partial \bar{r}_2}{\partial L} \right) \quad (3.27)$$

Since $\frac{\partial r_a}{\partial L} = 0$, this becomes

$$\text{Sign} \frac{\partial \bar{x}}{\partial L} = \text{Sign} \left(\frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial L} + \frac{\partial \bar{r}_2}{\partial L} \right) \quad (3.28)$$

From (3.24), we know $\frac{\partial u}{\partial L} < 0$, and $\frac{\partial \bar{r}_2}{\partial L} = \frac{\partial r_a}{\partial L} = 0$ because r_2 is not a function of L except

through u . Also $\frac{\partial \bar{r}_2}{\partial u} < 0$ from (3.10) and (3.12), so

$\text{Sign} \left(\frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial L} \right) > 0$, which means

$$\frac{\partial \bar{x}}{\partial L} > 0 \quad (3.29)$$

3.2.1.3 Effects of Population on Housing Price, Housing Consumption, Structural

Density and Urban Land Rent

$$\frac{dp_i}{dL} = \frac{\partial p_i}{\partial u} \frac{\partial u}{\partial L} > 0, i=1,2 \text{ (from (3.10) and (3.24))} \quad (3.30)$$

$$\frac{dq_i}{dL} = \frac{\partial q_i}{\partial u} \frac{\partial u}{\partial L} < 0, i=1,2 \text{ (from (3.10) and (3.24))} \quad (3.31)$$

$$\frac{dS_i}{dL} = -\frac{h'(S_i)}{ph''(S_i)} \frac{dp_i}{dL} > 0 \quad i=1,2 \text{ (from (3.11) and (3.30))} \quad (3.32)$$

$$\frac{dr_i}{dL} = \frac{\partial r_i}{\partial u} \frac{\partial u}{\partial L} = h(S_i) \frac{dp_i}{dL} > 0 \quad i=1,2 \text{ (from (3.12) and (3.30))} \quad (3.33)$$

3.2.1.4 Effect of Population on the Tax-Share Variable

$$\begin{aligned} \frac{d\beta}{dL} = \frac{G}{(\theta y L)^2} &+ \left(\frac{1}{\theta y L} \int_0^{\hat{x}} (1-\alpha_1) t_1 x^2 \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial L} dx - \frac{1}{\theta y L^2} \int_0^{\hat{x}} (1-\alpha_1) t_1 x^2 D_1 dx \right) \\ &+ \left(\frac{1}{\theta y L} \int_{\hat{x}}^{\bar{x}} (1-\alpha_2) t_2 x^2 \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial L} dx - \frac{1}{\theta y L^2} \int_{\hat{x}}^{\bar{x}} (1-\alpha_2) t_2 x^2 D_2 dx \right) \end{aligned} \quad (3.34)$$

Since we do not know the sign of the second and third terms of (3.34), $\frac{d\beta}{dL} \begin{matrix} > \\ < \end{matrix} 0$.

We have shown that, in a two-mode urban spatial structure model, the results are the same as in the single-mode urban spatial model. When population increases, the urban area boundary, housing price, urban land rent, and structural density also increase, while the utility level and housing consumption fall. The effect of population on the tax-share variable is ambiguous.

3.2.2 Effects of Change in Agricultural Land Rent

3.2.2.1 Effect of on Agricultural Land Rent on Utility

When $\lambda = r_a$, (3.21) becomes

$$-\frac{1}{\alpha_2 t_2} \bar{x} + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial r_a} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial r_a} \right) dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial r_a} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial r_a} \right) dx = 0 \quad (3.35)$$

Since r_1 and r_2 are not functions of r_a except through u , then $\frac{\partial r_1}{\partial r_a} = \frac{\partial r_2}{\partial r_a} = 0$, and since u is

not a function of x , (3.34) becomes

$$\frac{\partial u}{\partial r_a} \left(\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx \right) = \frac{\bar{x}}{\alpha_2 t_2}$$

After rearranging terms, we have

$$\frac{\partial u}{\partial r_a} = \frac{\frac{\bar{x}}{\alpha_2 t_2}}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx}$$

Because

$$\frac{\partial r_1}{\partial u} < 0 \text{ and } \frac{\partial r_2}{\partial u} < 0 \text{ from (3.12), then}$$

$$\frac{\partial u}{\partial r_a} < 0 \quad (3.36)$$

3.2.2.2 Effect of Agricultural Land Rent on Urban Area Boundary

Differentiate (3.15) with respect to r_a , getting

$$\hat{x} D_1(\hat{x}) \frac{\partial \hat{x}}{\partial r_a} + \int_0^{\hat{x}} x \frac{\partial D_1}{\partial r_a} dx + \bar{x} D_2(\bar{x}) \frac{\partial \bar{x}}{\partial r_a} + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial r_a} dx - \hat{x} D_2(\hat{x}) \frac{\partial \hat{x}}{\partial r_a} = 0 \quad (3.37)$$

The first and last term on the left side of (3.37) cancel because $D_1(\hat{x}) = D_2(\hat{x})$. Solving

for $\frac{\partial \bar{x}}{\partial r_a}$, we have

$$\frac{\partial \bar{x}}{\partial r_a} = - \frac{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial r_a} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial r_a} dx}{\bar{x} D_2(\bar{x})} \quad (3.38)$$

$$\text{Since } D_i = \frac{h_i}{q_i}, \text{ then } \frac{\partial D_i}{\partial r_a} = \frac{q_i h_i' \frac{\partial S_i}{\partial r_a} - h_i \frac{\partial q_i}{\partial r_a}}{q_i^2} > 0 \text{ (because } \frac{\partial S_i}{\partial r_a} = - \frac{h'(S_i)}{p_a h''(S_i)} \frac{\partial p_i}{\partial r_a} > 0)$$

and $\frac{\partial q_i}{\partial r_a} = \frac{\partial q_i}{\partial u} \frac{\partial u}{\partial r_a} < 0$. Thus, we know that (3.38) is negative; that is

$$\frac{\partial \bar{x}}{\partial r_a} < 0 \quad (3.39)$$

3.2.2.3 Effects of Agricultural Land Rent on Housing Price, Housing Consumption, Structural Density, and Urban Land Rent

$$\frac{dp_i}{dr_a} = \frac{\partial p_i}{\partial u} \frac{\partial u}{\partial r_a} > 0, i=1,2 \quad (3.40)$$

$$\frac{dq_i}{dr_a} = \frac{\partial q_i}{\partial u} \frac{\partial u}{\partial r_a} < 0, i=1,2 \quad (3.41)$$

$$\frac{dS_i}{dr_a} = -\frac{h'(S_i)}{p_a h''(S_i)} \frac{\partial p_i}{\partial r_a} > 0, i=1,2 \quad (3.42)$$

$$\frac{dr_i}{dr_a} = h \frac{\partial p_i}{\partial r_a} > 0, i=1,2 \quad (3.43)$$

3.2.2.4 Effect of Agricultural Land Rent on the Tax-Share Variable

$$\frac{\partial \beta}{\partial r_a} = \frac{1}{\theta y L} \left[(1-\alpha_1) \int_0^{\hat{x}} t_1 x^2 \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial r_a} dx + (1-\alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial r_a} dx \right] \begin{matrix} > \\ < \end{matrix} 0 \quad (3.44)$$

because $\frac{\partial D_i}{\partial u} \frac{\partial u}{\partial r_a} > 0$ while $\frac{\partial \bar{x}}{\partial r_a} < 0$.

The results of an increase in rural land rent are the same as in the one-mode urban spatial model. When agricultural land rent increases, the utility level attained by urban area residents falls and the urban area shrinks spatially. An increase in agricultural rent

also raises housing price, urban land rent, and structural density everywhere but reduces urban residents' housing consumption. Its effect on the tax-share variable is ambiguous.

3.2.3 Effects of Change in Income

3.2.3.1 Effect of Income on Utility

When $\lambda = y$, (3.21) becomes

$$\frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2} + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial y} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial y} \right) dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial y} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial y} \right) dx = 0, \quad (3.45)$$

since $\hat{r}_1 = \hat{r}_2, \bar{r}_2 = r_a$, and $M_1 = M_2$ at $\hat{x}$. Then, since u is not a function of x , we have

$$\frac{\partial u}{\partial y} = - \frac{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial y} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial y} dx + \frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2}}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} \quad (3.46)$$

Since $f_2 > f_1$, $\frac{\partial r_i}{\partial y} = \frac{h_i(1-\theta)}{q_i} > 0$, and $\frac{\partial r_i}{\partial u} < 0$, so we have

$$\frac{\partial u}{\partial y} > 0 \quad (3.47)$$

3.2.3.2 Effect of Income on Urban Area Boundary

From (3.26), and the fact that $\frac{\partial \bar{r}_2}{\partial x} < 0$, we have

$$\text{Sign} \left(\frac{\partial \bar{x}}{\partial y} \right) = - \text{Sign} \left(- \frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial y} - \frac{\partial \bar{r}_2}{\partial y} \right) = \text{Sign} \left(\frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \bar{r}_2}{\partial y} \right)$$

$$\begin{aligned} & \frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \bar{r}_2}{\partial y} \\ &= \frac{\partial \bar{r}_2}{\partial u} \left(- \frac{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial y} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial y} dx + \frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2}}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} \right) + \frac{\partial \bar{r}_2}{\partial y} = \end{aligned}$$

$$\frac{-\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial \bar{r}_2}{\partial u} \frac{\partial r_1}{\partial y} dx - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial \bar{r}_2}{\partial u} \frac{\partial r_2}{\partial y} dx - \frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2} \frac{\partial \bar{r}_2}{\partial u} + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial \bar{r}_2}{\partial y} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial \bar{r}_2}{\partial y} \frac{\partial r_2}{\partial u} dx}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} \quad (3.48)$$

The sign of $\frac{\partial \bar{x}}{\partial y}$ is the opposite of the sign of the numerator of (3.48), the numerator of

(3.48) is

$$\begin{aligned} & \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial \bar{r}_2}{\partial u} \frac{\partial r_1}{\partial y} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial \bar{r}_2}{\partial u} \frac{\partial r_2}{\partial y} dx + \frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2} \frac{\partial \bar{r}_2}{\partial u} \\ & - \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial \bar{r}_2}{\partial y} \frac{\partial r_1}{\partial u} dx - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial \bar{r}_2}{\partial y} \frac{\partial r_2}{\partial u} dx \end{aligned} \quad (3.49)$$

Substitute the following equations into (3.49)

$$\frac{\partial \bar{r}_2}{\partial y} = \frac{\bar{h}_2(1-\theta)}{\bar{q}_2}, \quad \frac{\partial r_1}{\partial u} = -\frac{h_1}{q_1 V_1}, \quad \frac{\partial \bar{r}_2}{\partial u} = -\frac{\bar{h}_2}{\bar{q}_2 \bar{V}_1},$$

$$\frac{\partial r_1}{\partial y} = \frac{h_1(1-\theta)}{q_1}, \quad \frac{\partial r_2}{\partial y} = \frac{h_2(1-\theta)}{q_2}$$

and note that $\frac{\partial \left(\frac{1}{V_1} \right)}{\partial x} < 0$ implies $\frac{1}{V_1} > \frac{1}{\bar{V}_1}$, then (3.49) becomes

$$\begin{aligned} & \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} -\left(\frac{\bar{h}_2}{\bar{q}_2 \bar{V}_1} \right) \left(\frac{h_1(1-\theta)}{q_1} \right) + \left(\frac{\bar{h}_2(1-\theta)}{\bar{q}_2} \right) \left(\frac{h_1}{q_1 V_1} \right) dx \\ & + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(-\left(\frac{\bar{h}_2}{\bar{q}_2 \bar{V}_1} \right) \left(\frac{h_2(1-\theta)}{q_2} \right) + \left(\frac{\bar{h}_2(1-\theta)}{\bar{q}_2} \right) \left(\frac{h_2}{q_2 V_1} \right) \right) dx + \frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2} \frac{\partial \bar{r}_2}{\partial u} \\ & = \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\bar{h}_2 h_1(1-\theta)}{\bar{q} q_1} \right) \left(\frac{1}{V_1} - \frac{1}{\bar{V}_1} \right) dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\bar{h}_2 h_2(1-\theta)}{\bar{q}_2 q_2 V_1} \left(\frac{1}{V_1} - \frac{1}{\bar{V}_1} \right) dx \\ & + \frac{\partial r_1(\hat{x})}{\partial y} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2} \frac{\partial \bar{r}_2}{\partial u} > 0 \end{aligned} \quad (3.50)$$

3.2.3.3 Effects of Income on Housing Price, Housing Consumption, Structural Density, and Urban Land Rent

The effect of income on price includes direct and indirect effects

$$\frac{d\hat{p}_i}{dy} = \frac{\partial \hat{p}_i}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial \hat{p}_i}{\partial y} \quad (3.51)$$

Because $\frac{\partial \hat{p}_i}{\partial u} < 0$, $\frac{\partial \hat{p}_i}{\partial y} > 0$ and $\frac{\partial u}{\partial y} > 0$,

then

$$\frac{d\hat{p}_i}{dy} > 0 \quad (3.52)$$

$$\frac{dq_i}{dy} = \frac{\partial q_i}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial q_i}{\partial y} > 0 \quad (3.53)$$

$$\frac{dS_i}{dy} = -\frac{h_i'}{p_i h_i''} \frac{dp_i}{dy} > 0 \quad (3.54)$$

$$\frac{dr_i}{dy} = h_i \frac{dp_i}{dy} > 0 \quad (3.55)$$

3.2.3.4 Effect of Income on the Tax-Share Variable

$$\frac{\partial \beta}{\partial y} = \frac{1}{(\theta y L)^2} \left(\theta y L \left[(1 - \alpha_1) \int_0^{\hat{x}} t_1 x^2 \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial y} dx + (1 - \alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial y} dx \right] - \theta L \left[(1 - \alpha_1) \int_0^{\hat{x}} t_1 \bar{x}^2 D_1 dx + (1 - \alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 D_2 dx \right] \right) \begin{matrix} > \\ < \end{matrix} 0 \quad (3.56)$$

)since $\frac{\partial \bar{x}}{\partial y} > 0$

In the two-mode urban spatial model, an increase in income increases urban residents' utility. Its effects on the urban area boundary, housing price, urban land rent, structural density, and the tax-share variable are ambiguous.

3.2.4 Effects of Change in Mode 1's Variable Commuting Cost

3.2.4.1 Effect of Mode 1's Variable Commuting Cost on Utility

Since r_2 is not a function of t_1 and $\bar{r}_2 = r_a$, then when $\lambda = t_1$, (3.21) becomes

$$\begin{aligned} & \frac{1}{\alpha_1 t_1^2} \hat{x} r_1(\hat{x}) + \left(\frac{1}{\alpha_2 t_2} - \frac{1}{\alpha_1 t_1} \right) r_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} + \frac{\partial r_1(\hat{x})}{\partial t_1} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) (\hat{x}) - \frac{r_a}{\alpha_2 t_2} \frac{\partial \bar{x}}{\partial t_1} \\ & - \frac{1}{\alpha_1 t_1^2} \int_0^{\hat{x}} r_1 dx + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial t_1} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial t_1} \right) dx + \frac{1}{\alpha_1 t_1} r_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} \\ & + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial t_1} dx - \frac{1}{\alpha_2 t_2} r_2(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} + \frac{r_a}{\alpha_2 t_2} \frac{\partial \bar{x}}{\partial t_1} = 0 \end{aligned} \quad (3.57)$$

Upon integrating $\int_0^{\hat{x}} r_1 dx$ by parts where $u = r_1$, and $dv = dx$, we have

$$\frac{1}{\alpha_1 t_1^2} \int_0^{\hat{x}} r_1 dx = \frac{1}{\alpha_1 t_1^2} \hat{x} r_1(\hat{x}) - \frac{1}{\alpha_1 t_1^2} \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx$$

Substitute this into (3.57) and rearrange terms, noting that $\hat{r}_1 = \hat{r}_2$, then

$$\begin{aligned} & \frac{1}{\alpha_1 t_1^2} \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial t_1} dx + \frac{\partial u}{\partial t_1} \left(\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx \right) \\ & + \frac{\partial r_1(\hat{x})}{\partial t_1} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) (\hat{x}) = 0 \end{aligned} \quad (3.58)$$

Then

$$\frac{\partial u}{\partial t_1} = \frac{-\frac{1}{\alpha_1 t_1^2} \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx - \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial t_1} dx - \frac{\partial r_1(\hat{x})}{\partial t_1} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) (\hat{x})}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} < 0 \quad (3.59)$$

3.2.4.2 Effects of Mode 1's Variable Commuting Cost on Urban Area Boundary

Recall (3.15), $\int_0^{\hat{x}} x D_1(x) dx + \int_{\hat{x}}^{\bar{x}} x D_2(x) dx = \frac{L}{\delta}$. Totally differentiating (3.15) with

respect to t_1 yields

$$\begin{aligned} \hat{x} D_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} + \int_0^{\hat{x}} x \left(\frac{\partial D_1}{\partial t_1} + \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial t_1} \right) dx + \bar{x} D_2(\bar{x}) \frac{\partial \bar{x}}{\partial t_1} + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_1} dx \\ - \hat{x} D_2(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} = 0 \end{aligned} \quad (3.60)$$

Because $D_1(\hat{x}) = D_2(\hat{x})$, and after rearranging terms, (3.61) becomes

$$\frac{\partial \bar{x}}{\partial t_1} = \frac{\int_0^{\hat{x}} x \left(\frac{\partial D_1}{\partial t_1} + \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial t_1} \right) dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_1} dx}{-\bar{x} D_2(\bar{x})} \quad (3.61)$$

Recall from (3.25), since r_2 is not a function of t_1 and $\bar{r}_2 = r_a$, $\partial \bar{r}_2 / \partial t_1 = 0$, we have

$$\frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial t_1} + \frac{\partial \bar{r}_2}{\partial u} \frac{\partial u}{\partial t_1} + \frac{\partial \bar{r}_2}{\partial t_1} = 0 \quad (3.62)$$

Then

$$\frac{\partial u}{\partial t_1} = - \frac{\frac{\partial \bar{r}_2}{\partial t_1} + \frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial t_1}}{\frac{\partial \bar{r}_2}{\partial u}} = - \frac{\frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial t_1}}{\frac{\partial \bar{r}_2}{\partial u}} \quad (3.63)$$

because $\frac{\partial \bar{r}_2}{\partial t_1} = 0$.

Substituting (3.63) into (3.62), we have

$$\frac{\partial \bar{x}}{\partial t_1} = \frac{\int_0^{\hat{x}} x \left[\frac{\partial D_1}{\partial t_1} + \frac{\partial D_1}{\partial u} \left(- \frac{\frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial t_1}}{\frac{\partial \bar{r}_2}{\partial u}} \right) \right] dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \left(- \frac{\frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial t_1}}{\frac{\partial \bar{r}_2}{\partial u}} \right) dx}{-\bar{x} D_2(\bar{x})} \quad (3.64)$$

After rearranging terms of (3.64), we have

$$\frac{\partial \bar{x}}{\partial t_1} = \frac{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial t_1} \frac{\partial \bar{r}_2}{\partial u} dx}{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx - \frac{\partial \bar{r}_2}{\partial u} \bar{x} D_2(\bar{x})} \quad (3.65)$$

Because $\frac{\partial \bar{r}_2}{\partial t_1} = 0$, the sign of (3.65) depends on $\frac{\partial D_1}{\partial t_1}$ and $\frac{\partial D_i}{\partial u}$

$$\frac{\partial D_i}{\partial u} = \frac{\partial}{\partial u} \left(\frac{h(S_i)}{q_i} \right) = \frac{q_i h' \frac{\partial S_i}{\partial u} - h \frac{\partial q_i}{\partial u}}{q_i^2}$$

Recall that

$$\frac{\partial S_i}{\partial u} = - \frac{h'}{p_i h''} \frac{\partial p_i}{\partial u}$$

$$\frac{\partial p_i}{\partial u} = - \frac{1}{q_i V_1} < 0$$

$$\frac{\partial q_i}{\partial u} = \left(\frac{\partial q_i}{\partial p_i} \right)_u \left(\frac{\partial p_i}{\partial u} - \frac{\partial MRS}{\partial c} \frac{1}{V_1} \right)$$

Then

$$\frac{\partial D_i}{\partial u} = - \left[\frac{h'^2}{p_i h'' q_i} + \frac{h \left(\frac{\partial q_i}{\partial p_i} \right)_u}{q^2} \right] \frac{\partial q_i}{\partial u} + h \left(\frac{\partial q_i}{\partial p_i} \right)_u \frac{\partial MRS}{\partial c} \frac{1}{V_1} = \Gamma \frac{\partial q_i}{\partial u} + \Lambda,$$

$$\text{where } \Gamma = - \left[\frac{h'^2}{p_i h'' q_i} + \frac{h \left(\frac{\partial q_i}{\partial p_i} \right)_u}{q^2} \right], \text{ and } \Lambda = h \left(\frac{\partial q_i}{\partial p_i} \right)_u \frac{\partial MRS}{\partial c} \frac{1}{V_1}, \text{ as originally defined in Ch.}$$

2. Hence

$$\frac{\partial D_i}{\partial u} < 0$$

We also need to obtain the sign of $\frac{\partial D_1}{\partial t_1}$. Note that

$$\frac{\partial D_1}{\partial t_1} = \frac{\partial}{\partial t_1} \left(\frac{h(S_1)}{q_1} \right) = \frac{h' \frac{\partial S_1}{\partial t_1} q_1 - h(S_1) \frac{\partial q_1}{\partial t_1}}{q^2} < 0 \quad (3.67)$$

Because $\frac{\partial S_1}{\partial t_1} = -\frac{h'}{p_1 h''} \frac{\partial p_1}{\partial t_1} < 0$ and $\frac{\partial q_1}{\partial t_1} = \left(\frac{\partial q_1}{\partial p_1} \right)_u \frac{\partial p_1}{\partial t_1} > 0$, given (3.66) and (3.67), the de-

nominator and numerator in (3.65) are both positive, so we have

$$\frac{\partial \bar{x}}{\partial t_1} > 0 \quad (3.68)$$

3.2.4.3 Effects of Mode 1's Variable Commuting Cost on Housing Price, Housing Consumption, Structural Density, and Urban Land Rent

The total effect of Mode 1's transport cost on housing price in the area of Mode 1 is

$$\frac{dp_1}{dt_1} = \frac{\partial p_1}{\partial u} \frac{\partial u}{\partial t_1} + \frac{\partial p_1}{\partial t_1} \quad (3.69)$$

Because $\frac{\partial p_1}{\partial u} < 0$, $\frac{\partial u}{\partial t_1} < 0$, and $\frac{\partial p_1}{\partial t_1} < 0$, the sign of (3.69) is ambiguous. Therefore, we

have

$$\frac{dr_1}{dt_1} = h \frac{dp_1}{dt_1} > 0 \quad (3.70)$$

$$\frac{dS_1}{dt_1} = \frac{h'_1}{p_1 h''_1} \frac{dp_1}{dt_1} > 0 \quad (3.71)$$

$$\frac{dq_1}{dt_1} = \frac{\partial q_1}{\partial u} \frac{\partial u}{\partial t_1} + \frac{\partial q_1}{\partial t_1} > 0 \quad (3.72)$$

$$\frac{dp_2}{dt_1} = \frac{\partial p_2}{\partial u} \frac{\partial u}{\partial t_1} > 0 \quad (3.73)$$

$$\frac{dr_2}{dt_1} = h \frac{\partial p_2}{\partial t_1} > 0 \quad (3.74)$$

$$\frac{dS_2}{dt_1} = -\frac{h_2'}{p_2 h_2''} \frac{\partial p_2}{\partial t_1} > 0 \quad (3.75)$$

$$\frac{dq_2}{dt_1} = \frac{\partial q_2}{\partial u} \frac{\partial u}{\partial t_1} < 0 \quad (3.76)$$

3.2.4.4 Effect of Mode 1's Variable Commuting Cost on the Tax-Share Variable

$$\frac{d\beta}{dt_1} = \frac{1}{\theta y L} \left[\begin{aligned} & (1 - \alpha_1) \hat{x}^2 t_1 D_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} + (1 - \alpha_1) \int_0^{\hat{x}} t_1 x^2 \left(\frac{\partial D}{\partial u} \frac{\partial u}{\partial t_1} + \frac{\partial D_1}{\partial t_1} \right) dx \\ & + (1 - \alpha_2) \bar{x}^2 t_2 D_2(\bar{x}) \frac{\partial \bar{x}}{\partial t_1} + (1 - \alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_1} + \frac{\partial D_2}{\partial t_1} \right) dx \\ & - (1 - \alpha_2) \hat{x}^2 t_2 D_2(\hat{x}) \frac{\partial \hat{x}}{\partial t_1} \end{aligned} \right] \quad (3.77)$$

Because $\frac{\partial \hat{x}}{\partial t_1} < 0$, $\frac{\partial D_1}{\partial t_1} < 0$, $\frac{\partial \bar{x}}{\partial t_1} > 0$, and $\frac{\partial D_2}{\partial t_1} > 0$, then

$$\frac{d\beta}{dt_1} > 0 \quad (3.78)$$

In a two-mode urban spatial model, an increase in Mode 1's variable commuting cost results in a lower utility level, which is the same as in the a single-mode model. The above results show that an increase in Mode 1's commuting cost expands the urban area spatially, however, while in a single-mode model, an increase in commuting cost shrinks the urban area spatially. The effects of this variable on housing price, housing consump-

tion, urban land rent, and structural density in the area between the CBD and $\hat{x}$ are ambiguous, while it increases housing price, land rent, and structural density while decreasing housing consumption in the area between $\hat{x}$ and x . The effect of an increase in Mode 1's variable commuting cost on the tax-share variable is ambiguous.

An intuitive explanation is that when Mode 1's commuting cost increases, all other things equal, the area using Mode 1 shrinks. (Recall that $\hat{x} = \frac{f_2 - f_1}{\alpha t_1 - \alpha_2 t_2}$, so

$\frac{\partial \hat{x}}{\partial t_1} < 0$) For those residents located in the area between the new $\hat{x}$ and the original $\hat{x}$,

Mode 1's commuting cost exceeds that of Mode 2. Those residents switch from Mode 1 to Mode 2. Mode 1's higher t induces households between the old and the new $\hat{x}$ to move farther from the CBD because their income net of transportation cost increases. Their move farther from CBD bids up housing price in the areas between $\hat{x}$ and the urban area boundary, which in turn raises urban land rent and structural density but lowers per-household housing consumption. Because of the increase in t_1 , people originally located in the area between the CBD and new $\hat{x}$ have lower income net of transportation costs, which lowers the demand for housing. At the same time, however, an increase in Mode 1's commuting cost makes living closer to the CBD more desirable, which bids up housing price close to the CBD. This in turn raises urban land rent and structural density in the area close to the CBD. As a result, when there is an increase in Mode 1's commuting cost, its effects on housing price, urban land rent, and structural density in the area between the CBD and $\hat{x}$ are ambiguous. An increase in Mode 1's commuting cost reduces population density in the area between the CBD and the new $\hat{x}$, but its effect on

population density in the area between $\hat{x}$ and the urban area boundary is ambiguous.

Thus, its effect on total transportation subsidies is ambiguous. As a result, the effect of an increase in Mode 1's variable cost on the tax-share variable is ambiguous.

3.2.5 Effects of Change in Mode 1's Private Cost Share

3.2.5.1 Effect of Mode 1's Private Cost Share on Utility

When $\lambda = \alpha_1$, (3.21) becomes

$$\begin{aligned} & \frac{1}{\alpha_1^2 t_1} \hat{x} r_1(\hat{x}) + \left(\frac{1}{\alpha_2 t_2} - \frac{1}{\alpha_1 t_1} \right) r_1(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_1} + \frac{\partial r_1(\hat{x})}{\partial \alpha_1} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) (\hat{x}) - \frac{r_a}{\alpha_2 t_2} \frac{\partial \bar{x}}{\partial \alpha_1} \\ & - \frac{1}{\alpha_1^2 t_1} \int_0^{\hat{x}} r_1 dx + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial \alpha_1} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial \alpha_1} \right) dx + \frac{1}{\alpha_1 t_1} r_1(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_1} \\ & + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial \alpha_1} dx - \frac{1}{\alpha_2 t_2} r_1(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_1} + \frac{r_a}{\alpha_2 t_2} \frac{\partial \bar{x}}{\partial \alpha_1} = 0 \end{aligned} \quad (3.79)$$

Substitute $\frac{1}{\alpha_1^2 t_1} \int_0^{\hat{x}} r_1 dx = \frac{1}{\alpha_1^2 t_1} \hat{x} r_1(\hat{x}) - \frac{1}{\alpha_1^2 t_1} \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx$ into (3.79) and rearrange terms; then

we have

$$\frac{\partial u}{\partial \alpha_1} = \frac{-\frac{1}{\alpha_1^2 t_1} \int_0^{\hat{x}} x \frac{\partial r_1}{\partial x} dx - \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial \alpha_1} dx - \frac{\partial r_1(\hat{x})}{\partial \alpha_1} \left(\frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} \right) (\hat{x})}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} < 0 \quad (3.79)$$

3.2.5.2 Effect of Mode 1's Private Cost Share on Urban Area Boundary

Recall (3.15)

$$\int_0^{\hat{x}} x D_1 dx + \int_{\hat{x}}^{\bar{x}} x D_2 dx = \frac{L}{\delta}$$

Totally differentiating (3.15) with respect to α_1 yields

$$\begin{aligned} & \hat{x}D_1(\hat{x})\frac{\partial\hat{x}}{\partial\alpha_1} + \int_0^{\hat{x}} x \left(\frac{\partial D_1(x)}{\partial\alpha_1} + \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial\alpha_1} \right) dx + \bar{x}D_2(\bar{x})\frac{\partial\bar{x}}{\partial\alpha_1} \\ & + \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial\alpha_1} \right) dx - \hat{x}D_2(\hat{x})\frac{\partial\hat{x}}{\partial\alpha_1} = 0 \end{aligned}$$

Since $D_1 = D_2$ at $\hat{x}$, we have

$$\int_0^{\hat{x}} x \left(\frac{\partial D_1(x)}{\partial\alpha_1} + \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial\alpha_1} \right) dx + \bar{x}D_2(\bar{x})\frac{\partial\bar{x}}{\partial\alpha_1} + \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial\alpha_1} \right) dx = 0 \quad (3.80)$$

Recall from (3.25)

$$\begin{aligned} & \frac{\partial\bar{r}_2}{\partial x} \frac{\partial\bar{x}}{\partial\alpha_1} + \frac{\partial\bar{r}_2}{\partial u} \frac{\partial u}{\partial\alpha_1} + \frac{\partial\bar{r}_2}{\partial\alpha_1} = 0 \\ & \frac{\partial u}{\partial\alpha_1} = \frac{-\frac{\partial\bar{r}_2}{\partial\alpha_1} - \frac{\partial\bar{r}_2}{\partial x} \frac{\partial\bar{x}}{\partial\alpha_1}}{\frac{\partial\bar{r}_2}{\partial u}} = -\frac{\frac{\partial\bar{r}_2}{\partial x} \frac{\partial\bar{x}}{\partial\alpha_1}}{\frac{\partial\bar{r}_2}{\partial u}} \quad (\text{since } \frac{\partial\bar{r}_2}{\partial\alpha_1} = 0) \end{aligned} \quad (3.81)$$

Substitute (3.81) into (3.80) and rearrange terms, getting

$$\frac{\partial\bar{x}}{\partial\alpha_1} = \frac{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial\alpha_1} \frac{\partial\bar{r}_2}{\partial u} dx}{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial\bar{r}_2}{\partial x} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial\bar{r}_2}{\partial x} dx - \bar{x}D_2(\bar{x})\frac{\partial\bar{r}_2}{\partial u}} \quad (3.82)$$

Since $\frac{\partial D_i}{\partial u} < 0$ and $\frac{\partial\bar{r}_2}{\partial x} < 0$, the denominator of (3.82) is positive, so the sign of (3.82)

depends on the sign of $\frac{\partial D_1}{\partial\alpha_1}$

$$\frac{\partial D_1}{\partial\alpha_1} = \frac{\partial}{\partial\alpha_1} \left(\frac{h(S_1)}{q_1} \right) = \frac{h' \frac{\partial S_1}{\partial\alpha_1} q_1 - h \frac{\partial q_1}{\partial\alpha_1}}{q_1^2} < 0 \text{ since } \frac{\partial S_1}{\partial\alpha_1} < 0 \text{ and } \frac{\partial q_1}{\partial\alpha_1} > 0$$

Therefore, we have

$$\frac{\partial \bar{x}}{\partial \alpha_1} > 0$$

3.2.5.3 Effects of Mode 1's Private Cost Share on Housing Price, Housing Service

Consumption, Urban Land Rent, and Structural Density

$$\frac{dp_1}{d\alpha_1} = \frac{\partial p_1}{\partial u} \frac{\partial u}{\partial \alpha_1} + \frac{\partial p_1}{\partial \alpha_1} \quad (3.83)$$

Because $\frac{\partial p_1}{\partial u} < 0$, $\frac{\partial u}{\partial \alpha_1} < 0$, and $\frac{\partial p_1}{\partial \alpha_1} < 0$, the sign of (3.83) is ambiguous, so

$$\frac{dr_1}{d\alpha_1} = h \frac{\partial p_1}{\partial \alpha_1} > 0 \quad (3.84)$$

$$\frac{dS_1}{d\alpha_1} = -\frac{h_1'}{p_1 h_1''} \frac{\partial p_1}{\partial \alpha_1} > 0 \quad (3.85)$$

$$\frac{dq_1}{d\alpha_1} = \frac{\partial q_1}{\partial u} \frac{\partial u}{\partial \alpha_1} + \frac{\partial q_1}{\partial \alpha_1} > 0 \quad (3.86)$$

$$\frac{dp_2}{d\alpha_1} = \frac{\partial p_2}{\partial u} \frac{\partial u}{\partial \alpha_1} > 0 \quad (3.87)$$

$$\frac{dr_2}{d\alpha_1} = h_2 \frac{\partial p_2}{\partial \alpha_1} > 0 \quad (3.88)$$

$$\frac{dS_2}{d\alpha_1} = -\frac{h_2'}{p_2 h_2''} \frac{\partial p_2}{\partial \alpha_1} > 0 \quad (3.89)$$

$$\frac{dq_2}{d\alpha_1} = \frac{\partial q_2}{\partial u} \frac{\partial u}{\partial \alpha_1} < 0 \quad (3.90)$$

3.2.5.4 Effects of Mode 1's Private Cost Share on the Tax-Share Variable

$$\frac{d\beta}{d\alpha_1} = \frac{1}{\theta_Y L} \left[\begin{aligned} & (1-\alpha_1)\hat{x}^2 t_1 D_1(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_1} + (1-\alpha_1) \int_0^{\hat{x}} t_1 x^2 \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial \alpha_1} dx \\ & + (1-\alpha_2)\bar{x}^2 t_2 D_2(\bar{x}) \frac{\partial \bar{x}}{\partial \alpha_1} + (1-\alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial \alpha_1} + \frac{\partial D_2}{\partial \alpha_1} \right) dx \\ & - (1-\alpha_2)\hat{x}^2 t_2 D_2(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_1} \end{aligned} \right] \quad (3.91)$$

Since $\frac{\partial \hat{x}}{\partial \alpha_1} < 0$, $\frac{\partial \bar{x}}{\partial \alpha_1} > 0$, $\frac{\partial D_1}{\partial u} \frac{\partial u}{\partial \alpha_1} > 0$, and $\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial \alpha_1} > 0$, then

$$\frac{d\beta}{d\alpha_1} > 0$$

These results show that when the private share of Mode 1's variable commuting cost increases, urban residents' utility falls and the urban area expands spatially. Its effects on housing price, housing consumption, urban land rent, and structural density in the area of Mode 1's use are ambiguous. An increase in the private share of Mode 1's variable costs raises housing price, urban land rent, and structural density while lowering housing consumption in the area of Mode 2's use. The effect on the tax-share revenue is ambiguous.

The intuitive explanation is similar to that of an increase in Mode 1's variable commuting cost. When the private share of Mode 1's variable commuting cost increases, the area for which Mode 1 is cheaper shrinks (recall that $\hat{x} = \frac{f_2 - f_1}{\alpha t_1 - \alpha_2 t_2}$, so $\frac{\partial \hat{x}}{\partial t_1} < 0$). The people who originally located between the new smaller $\hat{x}$ and old $\hat{x}$ now use Mode 2. Since they now use Mode 2, which has a lower variable cost than Mode 1, they are motivated to relocate farther from the CBD because their income net of transport cost in-

creases. Their move farther from the CBD bids up housing price in the area between $\hat{x}$ and $\bar{x}$, which in turn increases urban land rent and structural density while reducing housing consumption.

For the people originally located in the area between the CBD and the new $\hat{x}$, an increase in α_1 reduces their income net of travel cost. As a result, demand for housing falls, which results in a lower housing price, urban land rent, and structural density. An increase in α_1 also makes living close to the CBD more desirable, which bids up housing price close to the CBD, and increases urban land rent and structural density there. Overall, an increase in α_1 has ambiguous effects on housing price, urban land rent, and structural density in the area between the CBD and $\hat{x}$. Its effect on the tax-share variable is ambiguous because, on the one hand, an increase in α_1 decreases the subsidies needed for Mode 1, while, on the other hand, it expands the urban area boundary, which increases the total miles driven by the residents, thus increasing the subsidies needed for Mode 2.

3.2.6 Effects of Change in Mode 2's Variable Commuting Cost

3.2.6.1 Effect of Mode 2's Variable Commuting Cost on Utility

When $\lambda = t_2$, (3.21) becomes

$$\begin{aligned}
& -\frac{1}{\alpha_2 t_2^2} \hat{x} r_1(\hat{x}) + \left(\frac{1}{\alpha_2 t_2} - \frac{1}{\alpha_1 t_1} \right) r_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} + \frac{1}{\alpha_2 t_2^2} \bar{x} r_a - \frac{r_a}{\alpha_2 t_2} \frac{\partial \bar{x}}{\partial t_2} \\
& + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial t_2} dx + \frac{1}{\alpha_1 t_1} r_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} - \frac{1}{\alpha_2 t_2^2} \int_{\hat{x}}^{\bar{x}} r_2 dx \\
& + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial t_2} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial t_2} \right) dx + \frac{1}{\alpha_2 t_2} r_2(\bar{x}) \frac{\partial \bar{x}}{\partial t_2} - \frac{1}{\alpha_2 t_2} r_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} = 0
\end{aligned} \tag{3.92}$$

Upon substituting $\frac{1}{\alpha_2 t_2^2} \int_{\hat{x}}^{\bar{x}} r_2 dx = \frac{1}{\alpha_2 t_2^2} \bar{x} r_a - \frac{1}{\alpha_2 t_2^2} \hat{x} \hat{r}_1 - \frac{1}{\alpha_2 t_2^2} \int_{\hat{x}}^{\bar{x}} x \frac{\partial r_2}{\partial x} dx$ into (3.92) and rearranging terms, we have

$$\frac{\partial u}{\partial t_2} \left(\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx \right) = - \frac{1}{\alpha_2 t_2^2} \int_{\hat{x}}^{\bar{x}} x \frac{\partial r_2}{\partial x} dx - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial t_2} dx \quad (3.93)$$

Then

$$\frac{\partial u}{\partial t_2} = \frac{- \frac{1}{\alpha_2 t_2^2} \int_{\hat{x}}^{\bar{x}} x \frac{\partial r_2}{\partial x} dx - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial t_2} dx}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} < 0 \quad (\text{since } \frac{\partial r_2}{\partial t_2} = h \frac{\partial p_2}{\partial t_2} < 0) \quad (3.94)$$

3.2.6.2 Effect of Mode 2's Variable Commuting Cost on Urban Area Boundary

Totally differentiating (3.15) with respect to t_2 yields

$$\begin{aligned} \hat{x} D_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} + \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial t_2} dx + \bar{x} D_2(\bar{x}) \frac{\partial \bar{x}}{\partial t_2} + \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_2} + \frac{\partial D_2}{\partial t_2} \right) dx \\ - \hat{x} D_2(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} = 0 \end{aligned} \quad (3.95)$$

Then

$$\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial t_2} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_2} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial t_2} dx = - \bar{x} D_2(\bar{x}) \frac{\partial \bar{x}}{\partial t_2} \quad (3.96)$$

Totally differentiating boundary condition (3.14) with respect to t_2 , we have

$$\frac{\partial u}{\partial t_2} = - \frac{\frac{\partial \bar{r}_2}{\partial t_2} + \frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial t_2}}{\frac{\partial \bar{r}_2}{\partial u}}$$

Substitute this into (3.96) and rearrange terms; then we have

$$\frac{\partial \bar{x}}{\partial t_2} = \frac{-\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial t_2} dx - \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial t_2} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial t_2} \frac{\partial \bar{r}_2}{\partial u} dx}{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx - \bar{x} D(\bar{x}) \frac{\partial \bar{r}_2}{\partial u}} \quad (3.97)$$

The sign of (3.97) is determined by the numerator because the denominator is positive.

$$\frac{\partial D_2}{\partial t_2} = \frac{\partial}{\partial t_2} \left[\frac{h(S_2)}{q_2} \right] = \frac{h' \frac{\partial S_2}{\partial t_2} q_2 - h \frac{\partial q_2}{\partial t_2}}{q^2} = - \left[\frac{h'^2}{p_2 h'' q_2} + \frac{h(\partial q_2 / \partial p_2)_u}{q^2} \right] \frac{\partial p_2}{\partial t_2} < 0$$

$$\text{since } \frac{\partial S_2}{\partial t_2} = - \left(\frac{h'}{p_2 h''} \right) \frac{\partial p_2}{\partial t_2} \text{ and } \frac{\partial q_2}{\partial t_2} = \left(\frac{\partial q_2}{\partial p_2} \right)_u \frac{\partial p_2}{\partial t_2}$$

This seems to imply that the numerator of (3.97) is ambiguous, but this is not so. Recall that

$$\frac{\partial p_2}{\partial t_2} = - \frac{\alpha_2 x}{q_2}$$

$$\frac{\partial p_2}{\partial u} = - \frac{1}{q_2 V_1}$$

$$\frac{\partial \bar{r}_2}{\partial t_2} = \frac{-h(\bar{S}_2) \alpha_2 x}{\bar{q}_2}$$

$$\frac{\partial \bar{r}_2}{\partial u} = - \frac{h(\bar{S}_2)}{\bar{q}_2 V_1}$$

$$\frac{\partial D_i}{\partial u} = \Gamma \frac{\partial q_i}{\partial u} + \Lambda \text{ where } \Gamma > 0 \text{ and } \Lambda < 0$$

$$\frac{\partial V_1}{\partial x} > 0$$

$$\frac{1}{\bar{V}_1} - \frac{1}{V_1} < 0$$

Upon substituting the above along with the previously derived expression for $\frac{\partial D_2}{\partial t_2}$

into (3.97), we have

$$\begin{aligned}
& \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial t_2} \frac{\partial \bar{r}_2}{\partial u} - \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial t_2} \right) dx = \int_{\hat{x}}^{\bar{x}} x \left[\Gamma \frac{\partial p_2}{\partial t_2} \frac{\partial \bar{r}_2}{\partial u} - \left(\Gamma \frac{\partial p_2}{\partial u} + \Lambda \right) \frac{\partial \bar{r}_2}{\partial t_2} \right] dx \\
& = \int_{\hat{x}}^{\bar{x}} x \left[\Gamma \frac{\alpha_2 x}{q_2} \frac{h(\bar{S}_2)}{\bar{q}_2 \bar{V}_1} - \Gamma \frac{h(\bar{S}_2)}{q_2 V_1} \frac{h(\bar{S}_2) \alpha_2 x}{\bar{q}_2} + \Lambda \frac{h(\bar{S}_2) \alpha_2 x}{\bar{q}_2} \right] dx \\
& = \int_{\hat{x}}^{\bar{x}} x \Gamma \frac{h(\bar{S}_2) \alpha_2 x}{q_2 \bar{q}_2} \left(\frac{1}{\bar{V}_1} - \frac{1}{V_1} \right) dx + \int_{\hat{x}}^{\bar{x}} x \Lambda \frac{h(\bar{S}_2) \alpha_2 x}{\bar{q}_2} dx < 0
\end{aligned} \tag{3.98}$$

Since $-\int_0^{\bar{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial t_2} dx < 0$, then the numerator of (3.97) is negative, so

$$\frac{\partial \bar{x}}{\partial t_2} < 0$$

3.2.6.3 Effects of Mode 2's Variable Commuting Cost on Housing Price, Housing

Service Consumption, Urban Land Rent, and Structural Density

$$\frac{dp_1}{dt_2} = \frac{\partial p_1}{\partial u} \frac{\partial u}{\partial t_2} > 0 \tag{3.99}$$

$$\frac{dr_1}{dt_2} = h_1 \frac{\partial p_1}{\partial t_2} > 0 \tag{3.100}$$

$$\frac{dS_1}{dt_2} = -\frac{h_1'}{p_1 h_1''} \frac{\partial p_1}{\partial t_2} > 0 \tag{3.101}$$

$$\frac{dq_1}{dt_2} = \frac{\partial q_1}{\partial u} \frac{\partial u}{\partial t_2} < 0 \tag{3.102}$$

$$\frac{dp_2}{dt_2} = \frac{\partial p_2}{\partial u} \frac{\partial u}{\partial t_2} + \frac{\partial p_2}{\partial t_2} \tag{3.103}$$

Because $\frac{\partial p_2}{\partial u} < 0$, $\frac{\partial u}{\partial t_2} < 0$, and $\frac{\partial p_2}{\partial t_2} < 0$, the sign of (3.103) is ambiguous. Therefore, we

have

$$\frac{dr_2}{dt_2} = h_2 \frac{\partial p_2}{\partial t_2} \frac{\partial u}{\partial t_2} > 0 \quad (3.104)$$

$$\frac{dS_2}{dt_2} = -\frac{h_2'}{p_2 h_2} \frac{\partial p_2}{\partial t_2} \frac{\partial u}{\partial t_2} > 0 \quad (3.105)$$

$$\frac{dq_2}{dt_2} = \frac{\partial q_2}{\partial u} \frac{\partial u}{\partial t_2} + \frac{\partial q_2}{\partial t_2} \frac{\partial u}{\partial t_2} > 0 \quad (3.106)$$

3.2.6.4 Effect of Mode 2's Variable Commuting Cost on the Tax-Share Variable

$$\frac{d\beta}{dt_2} = \frac{1}{\theta y L} \left(\begin{aligned} & (1-\alpha_1) \hat{x}^2 t_1 D_1(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} + (1-\alpha_1) \int_0^{\hat{x}} t_1 x^2 \left(\frac{\partial D_1}{\partial u} \frac{\partial u}{\partial t_2} \right) dx + \\ & (1-\alpha_2) \bar{x}^2 t_2 D_2(\bar{x}) \frac{\partial \bar{x}}{\partial t_2} + (1-\alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 \left(\frac{\partial D_2}{\partial t_2} + \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_2} \right) dx > 0 \\ & -(1-\alpha_2) \hat{x}^2 t_2 D_2(\hat{x}) \frac{\partial \hat{x}}{\partial t_2} + (1-\alpha_2) \int_{\hat{x}}^{\bar{x}} x^2 D_2 dx < 0 \end{aligned} \right)$$

because $\frac{\partial \hat{x}}{\partial t_2} > 0$, $\frac{\partial D_1}{\partial u} \frac{\partial u}{\partial t_2} > 0$, $\frac{\partial \bar{x}}{\partial t_2} < 0$, but $\frac{\partial D_2}{\partial t_2} + \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial t_2} > 0$.

From the above results, we conclude that an increase in Mode 2's variable commuting cost reduces utility and shrinks the urban area spatially. Its effects on housing price, housing consumption, urban land rent, and structural density are ambiguous in the urban area between $\hat{x}$ and $\bar{x}$. For the area between the CBD and $\hat{x}$, however, an increase in Mode 2's variable cost increases housing price, urban land rent, and structural density while decreasing housing consumption. The effect of Mode 2's variable cost on the tax-share variable is ambiguous.

The intuitive explanation is that as Mode 2's variable cost increases, the area that uses Mode 1 expands from the old $\hat{x}$ to new $\hat{x}$ (recall that $\hat{x} = \frac{f_2 - f_1}{\alpha t_1 - \alpha_2 t_2}$, so $\frac{\partial \hat{x}}{\partial t_2} < 0$).

For the people originally located in the area between the old and new $\hat{x}$, Mode 1's commuting cost is cheaper than that of Mode 2; therefore, they replace Mode 2 with Mode 1, which makes living closer to the CBD desirable. As a result, housing price rises in the area between the old and new $\hat{x}$, which in turn bids up urban land rent and structural density while causing per-household housing consumption to fall.

For those people living between the new $\hat{x}$ and $\bar{x}$, their income net of transport cost is reduced. As a result, housing price falls, which in turn reduces urban land rent and structural density while increasing housing consumption. Because urban land rent falls, the urban area shrinks spatially. On the other hand, an increase in Mode 2's variable cost makes living closer to the CBD desirable, which bids up housing price and land rent, increasing structural density and reducing housing consumption. Therefore, the overall effects of an increase in Mode 2's variable cost on housing price, housing consumption, and structural density in this area are ambiguous. Its effect on the tax-share variable is ambiguous because, on the one hand, an increase in Mode 2's commuting cost results in higher subsidies, while, on the other hand, such an increase shrinks the urban area, which reduces transport subsidies within the urban area.

3.2.7 Effects of Change in Mode 2's Private Cost Share

3.2.7.1 Effect of Mode 2's Private Cost Share on Utility

Let $\lambda = \alpha_2$ in (3.21), getting

$$\frac{\partial u}{\partial \alpha_2} = \frac{-\frac{1}{\alpha_2^2 t_2} \int_{\hat{x}}^{\bar{x}} x \frac{\partial r_2}{\partial x} dx - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial \alpha_2} dx}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} \quad (3.107)$$

Since $\frac{\partial r_2}{\partial x} < 0$, $\frac{\partial r_2}{\partial \alpha_2} < 0$, and $\frac{\partial r_i}{\partial u} < 0$, we have

$$\frac{\partial u}{\partial \alpha_2} < 0$$

3.2.7.2 Effect of Mode 2's Private Cost Share on Urban Area Boundary

Totally differentiating (3.15) with respect to α_2 and canceling terms involving

$\hat{x}D_1(\hat{x})$, we have

$$\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial \alpha_2} dx + \bar{x}D_2(\bar{x}) \frac{\partial \bar{x}}{\partial \alpha_2} + \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial \alpha_2} + \frac{\partial D_2}{\partial \alpha_2} \right) dx = 0 \quad (3.108)$$

Let $\lambda = \alpha_2$ in (3.25) and solve for $\frac{\partial u}{\partial \alpha_2}$; then we have

$$\frac{\partial u}{\partial \alpha_2} = \frac{-\frac{\partial \bar{r}_2}{\partial \alpha_2} - \frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial \alpha_2}}{\frac{\partial \bar{r}_2}{\partial u}}$$

Upon substituting this into (3.108) and solving for $\frac{\partial \bar{x}}{\partial \alpha_2}$, we get

$$\frac{\partial \bar{x}}{\partial \alpha_2} = \frac{-\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial \alpha_2} dx - \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial \alpha_2} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial \alpha_2} \frac{\partial \bar{r}_2}{\partial u} dx}{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx - \bar{x}D_2(\bar{x}) \frac{\partial \bar{r}_2}{\partial u}} \quad (3.109)$$

We know that the denominator of (3.109) is positive; therefore the sign of (3.109) is determined by the numerator. We need to know the following to determine the sign of (3.109)

$$\begin{aligned}\frac{\partial D_2}{\partial \alpha_2} &= \frac{\partial}{\partial \alpha_2} \frac{h(S_2)}{q_2} = \frac{h' \frac{\partial S_2}{\partial \alpha_2} q_2 - h \frac{\partial q_2}{\partial \alpha_2}}{q_2^2} \\ &= - \left[\frac{h_2'^2}{p_2 h'' q_2} + \frac{h \left(\frac{\partial q_2}{\partial p_2} \right)_u}{q^2} \right] \frac{\partial p_2}{\partial \alpha_2} = \Gamma \frac{\partial p_2}{\partial \alpha_2} < 0\end{aligned}$$

$$\frac{\partial p_2}{\partial \alpha_2} = - \frac{t_2 x}{q_2}$$

$$\frac{\partial p_2}{\partial u} = - \frac{1}{q_2 V_1}$$

$$\frac{\partial \bar{r}_2}{\partial \alpha_2} = - \frac{\bar{h}_2 t_2 x}{\bar{q}_2}$$

$$\frac{\partial \bar{r}_2}{\partial u} = - \frac{\bar{h}_2}{\bar{q}_2 V_1}$$

$$\frac{\partial V_1}{\partial x} > 0, \text{ which implies } \frac{1}{\bar{V}_1} - \frac{1}{V_1} < 0$$

Substitute the above into the second and third terms of the numerator of (3.109); then we have

$$\begin{aligned}& \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial \alpha_2} \frac{\partial \bar{r}_2}{\partial u} - \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial \alpha_2} \right) dx = \int_{\hat{x}}^{\bar{x}} x \left[\Gamma \frac{\partial p_2}{\partial \alpha_2} \frac{\partial \bar{r}_2}{\partial u} - \left(\Gamma \frac{\partial p_2}{\partial u} + \Lambda \right) \frac{\partial \bar{r}_2}{\partial \alpha_2} \right] dx \\ &= \int_{\hat{x}}^{\bar{x}} x \left[\Gamma \frac{t_2 x}{q_2} \frac{\bar{h}_2}{\bar{q}_2 \bar{V}_1} - \Gamma \frac{1}{q_2 V_1} \frac{\bar{h}_2 t_2 x}{\bar{q}_2} \right] dx + \int_{\hat{x}}^{\bar{x}} \Lambda \frac{\bar{h}_2 t_2 x^2}{\bar{q}_2} dx\end{aligned}$$

$$= \int_{\hat{x}}^{\bar{x}} x \Gamma \frac{\bar{h}_2 t_2 x}{q_2 \bar{q}_2} \left(\frac{1}{\bar{V}_1} - \frac{1}{V_1} \right) dx + \int_{\hat{x}}^{\bar{x}} \Lambda \frac{\bar{h}_2 t_2 x^2}{\bar{q}_2} dx < 0$$

Because the first term of the numerator of (3.109) is negative, we conclude that

$$\frac{\partial \bar{x}}{\partial \alpha_2} < 0$$

3.2.7.3 Effects of Mode 2's Private Cost Share on Housing Price, Housing Consumption, Urban Land Rent, and Structural Density

$$\frac{dp_1}{d\alpha_2} = \frac{\partial p_1}{\partial u} \frac{\partial u}{\partial \alpha_2} > 0 \quad (3.110)$$

$$\frac{dr_1}{d\alpha_2} = h_1 \frac{\partial p_1}{\partial \alpha_2} > 0 \quad (3.111)$$

$$\frac{dS_1}{d\alpha_2} = -\frac{h_1'}{p_1 h_1''} \frac{\partial p_1}{\partial \alpha_2} > 0 \quad (3.112)$$

$$\frac{dq_1}{d\alpha_2} = \frac{\partial q_1}{\partial u} \frac{\partial u}{\partial \alpha_2} < 0 \quad (3.113)$$

$$\frac{dp_2}{d\alpha_2} = \frac{\partial p_2}{\partial u} \frac{\partial u}{\partial \alpha_2} + \frac{\partial p_2}{\partial \alpha_2} \quad (3.114)$$

Because $\frac{\partial p_2}{\partial u} < 0$, $\frac{\partial u}{\partial \alpha_2} < 0$, and $\frac{\partial p_2}{\partial \alpha_2} < 0$, the sign of (3.114) is ambiguous. Therefore,

we have

$$\frac{dr_2}{d\alpha_2} = h_2 \frac{dp_2}{d\alpha_2} > 0 \quad (3.115)$$

$$\frac{dS_2}{d\alpha_2} = -\frac{h_2'}{p_2 h_2''} \frac{dp_2}{d\alpha_2} > 0 \quad (3.116)$$

$$\frac{dq_2}{d\alpha_2} = \frac{\partial q_2}{\partial u} \frac{\partial u}{\partial \alpha_2} + \frac{\partial q_2}{\partial \alpha_2} \begin{matrix} > 0 \\ < 0 \end{matrix} \quad (3.117)$$

3.2.7.4 Effect of Mode 2's Private Cost Share on the Tax-Share Variable

$$\frac{d\beta}{d\alpha_2} = \frac{1}{\theta y L} \left(\begin{aligned} & (1-\alpha_1)\hat{x}^2 t_1 D_1(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_2} + (1-\alpha_1) \int_0^{\hat{x}} t_1 x^2 \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial \alpha_2} dx \\ & + (1-\alpha_2)\bar{x}^2 t_2 D_2(\bar{x}) \frac{\partial \bar{x}}{\partial \alpha_2} + (1-\alpha_2) \int_{\hat{x}}^{\bar{x}} t_2 x^2 \left(\frac{\partial D_2}{\partial \alpha_2} + \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial \alpha_2} \right) dx \\ & - (1-\alpha_2)\hat{x}^2 t_2 D_2(\hat{x}) \frac{\partial \hat{x}}{\partial \alpha_2} - \int_{\hat{x}}^{\bar{x}} t_2 x^2 D_2 dx \end{aligned} \right)$$

Since $\frac{\partial \hat{x}}{\partial \alpha_2} > 0$, $\frac{\partial D_1}{\partial u} \frac{\partial u}{\partial \alpha_2} > 0$, $\frac{\partial \bar{x}}{\partial \alpha_2} < 0$, and $\frac{\partial D_2}{\partial \alpha_2} + \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial \alpha_2} > 0$, then

$$\frac{d\beta}{d\alpha_2} \begin{matrix} > 0 \\ < 0 \end{matrix}$$

The above results show that an increase in the private share of Mode 2's variable cost reduces urban residents' utility and shrinks the urban area spatially. For the area between the CBD and $\hat{x}$, housing price, urban land rent, and structural density increase while housing consumption per household decreases. For the area between $\hat{x}$ and the urban area boundary, the effects of the private share of Mode 2's variable cost are ambiguous.

The intuitive explanation is that when the private share of Mode 2's variable commuting cost increases, the area in which Mode 1's transportation cost is lower than Mode 2's expands (recall that $\hat{x} = \frac{f_2 - f_1}{\alpha t_1 - \alpha_2 t_2}$, so $\frac{\partial \hat{x}}{\partial \alpha_2} < 0$). Those people located between the old $\hat{x}$ and new $\hat{x}$ use Mode 1 rather than Mode 2, which makes living closer to the CBD desirable. As a result, housing price, urban land rent, and structural density in

the area increase, while per-household housing consumption decreases. For the people located between the new $\hat{x}$ and urban area boundary, an increase in the private share of Mode 2's variable cost reduces income net of transportation costs, which reduces housing price, urban land rent, and structural density in the area. Because of the decrease in urban land rent, the urban area shrinks spatially. On the other hand, because of the increase in transportation costs, people are motivated to move closer to $\hat{x}$, which bids up housing price there. Therefore, the overall effects on housing price, urban land rent, and structural density in the area beyond $\hat{x}$ are ambiguous. The effect of the private share of Mode 2's variable cost on the tax-share variable is ambiguous because an increase in the private share of Mode 2's variable cost reduces subsidies for Mode 2's use while increasing population density in the area between the CBD and $\hat{x}$, which in turn raises subsidies for Mode 1's use.

3.2.8 Effects of Change in Mode 1's Fixed Cost

3.2.8.1 Effect of Mode 1's Fixed Cost on Utility

When $\lambda = f_1$, (3.21) becomes

$$\begin{aligned}
& -\frac{1}{\alpha_1 t_1 \alpha_2 t_2} r_1(\hat{x}) + \frac{\partial r_1(\hat{x})}{\partial f_1} \left(\frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2} \right) + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \left(\frac{\partial r_1}{\partial f_1} + \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial f_1} \right) dx \\
& + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial f_1} dx = 0
\end{aligned} \tag{3.118}$$

Rearranging (3.118), we have

$$\frac{\partial u}{\partial f_1} = \frac{-\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial f_1} dx - \frac{\partial r_1(\hat{x})}{\partial f_1} \frac{f_2 - f_1}{\alpha_1 t_1 \alpha_2 t_2}}{\frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx + \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx} \tag{3.119}$$

Recall that

$$\frac{\partial r_1}{\partial u} < 0$$

$$\frac{\partial r_2}{\partial u} < 0$$

$$\frac{\partial r_1}{\partial f_1} < 0$$

Therefore, we have

$$\frac{\partial u}{\partial f_1} < 0$$

3.2.8.2 Effect of Mode 1's Fixed Cost on Urban Area Boundary

Totally differentiating (3.15) with respect to f_1 , we have

$$\int_0^{\hat{x}} x \left(\frac{\partial D_1}{\partial f_1} + \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial f_1} \right) dx + \bar{x} D_2(\bar{x}) \frac{\partial \bar{x}}{\partial f_1} + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial f_1} dx = 0 \quad (3.120)$$

Substitute $\lambda = f_1$ into (3.26) and solve for $\frac{\partial u}{\partial f_1}$; then we have

$$\frac{\partial u}{\partial f_1} = \frac{-\frac{\partial \bar{r}_2}{\partial f_1} - \frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial f_1}}{\frac{\partial \bar{r}_2}{\partial u}}$$

Substitute this into (3.120) to obtain

$$\begin{aligned} & \int_0^{\hat{x}} x \frac{\partial D_1}{\partial f_1} \frac{\partial \bar{r}_2}{\partial u} dx - \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial f_1} dx - \frac{\partial \bar{x}}{\partial f_1} \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx + \bar{x} D_2(\bar{x}) \frac{\partial \bar{r}_2}{\partial u} \frac{\partial \bar{x}}{\partial f_1} \\ & - \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial f_1} dx - \frac{\partial \bar{x}}{\partial f_1} \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx = 0 \end{aligned} \quad (3.121)$$

Rearranging the terms of (3.121), we have

$$\frac{\partial \bar{x}}{\partial f_1} = \frac{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial f_1} \frac{\partial \bar{r}_2}{\partial u} dx - \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial f_1} dx - \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial f_1} dx}{\int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx + \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx - \bar{x} D_2(\bar{x}) \frac{\partial \bar{r}_2}{\partial u}} \quad (3.122)$$

Because $\frac{\partial \bar{r}_2}{\partial f_1} = 0$, we have

$$\frac{\partial \bar{x}}{\partial f_1} = \frac{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial f_1} \frac{\partial \bar{r}_2}{\partial u} dx}{\int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx + \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx - \bar{x} D_2(\bar{x}) \frac{\partial \bar{r}_2}{\partial u}}$$

Since $\frac{\partial D_1}{\partial f_1} = \frac{\partial [h(S_1)/q_1]}{\partial f_1} = \frac{q_1 h' \frac{\partial S_1}{\partial f_1} - h \frac{\partial q_1}{\partial f_1}}{q_1^2} < 0$, then

$$\frac{\partial \bar{x}}{\partial f_1} > 0$$

3.2.8.3 Effects of Mode 1's Fixed Cost on Housing Price, Housing Consumption, Urban Land Rent, and Structural Density

$$\frac{dp_1}{df_1} = \frac{\partial p_1}{\partial u} \frac{\partial u}{\partial f_1} + \frac{\partial p_1}{\partial f_1} \quad (3.123)$$

Because $\frac{\partial p_1}{\partial u} < 0$, $\frac{\partial u}{\partial f_1} < 0$, and $\frac{\partial p_1}{\partial f_1} < 0$, the sign of (3.123) is ambiguous. Therefore, we

have

$$\frac{dr_1}{df_1} = h \frac{dp_1}{df_1} \begin{matrix} > \\ < \end{matrix} 0 \quad (3.124)$$

$$\frac{dS_1}{df_1} = - \frac{h_1'}{p_1 h_1''} \frac{dp_1}{df_1} \begin{matrix} > \\ < \end{matrix} 0 \quad (3.125)$$

$$\frac{dq_1}{df_1} = \frac{\partial q_1}{\partial u} \frac{\partial u}{\partial f_1} + \frac{\partial q_1}{\partial f_1} > 0 \quad (3.126)$$

$$\frac{dp_2}{df_1} = \frac{\partial p_2}{\partial u} \frac{\partial u}{\partial f_1} > 0 \quad (3.127)$$

$$\frac{dr_2}{df_1} = h \frac{\partial p_2}{\partial f_1} > 0 \quad (3.128)$$

$$\frac{dS_2}{df_1} = -\frac{h_2'}{p_2 h_2''} \frac{\partial p_2}{\partial f_1} > 0 \quad (3.129)$$

$$\frac{dq_2}{df_1} = \frac{\partial q_2}{\partial u} \frac{\partial u}{\partial f_1} < 0 \quad (3.130)$$

3.2.8.4 Effect of Mode 1's Fixed Cost on the Tax-Share Variable

$$\frac{d\beta}{df_1} = \frac{1}{\theta y L} \left(\begin{aligned} & (1-\alpha_1)t_1\hat{x}^2 D_1(\hat{x}) \frac{\partial \hat{x}}{\partial f_1} + (1-\alpha_1)t_1 \int_0^{\hat{x}} x^2 \left(\frac{\partial D_1}{\partial f_1} + \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial f_1} \right) dx \\ & + (1-\alpha_2)t_2\bar{x}^2 D_2(\bar{x}) \frac{\partial \bar{x}}{\partial f_1} + (1-\alpha_2)t_2 \int_{\hat{x}}^{\bar{x}} x^2 \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial f_1} dx \\ & - (1-\alpha_2)t_2\hat{x}^2 D_2(\hat{x}) \frac{\partial \hat{x}}{\partial f_1} \end{aligned} \right)$$

Since $\frac{\partial \hat{x}}{\partial f_1} < 0$, $\frac{dD_1}{df_1} > 0$, $\frac{\partial \bar{x}}{\partial f_1} > 0$, and $\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial f_1} > 0$, then $\frac{d\beta}{df_1} > 0$.

The above results show that an increase in Mode 1's fixed cost leads to lower utility but expands the urban area spatially. For the area between the CBD and $\hat{x}$, the effects of Mode 1's fixed cost on housing price, housing consumption, urban land rent, and structural density are ambiguous. For the area between $\hat{x}$ and $\bar{x}$, however, an increase in Mode 1's fixed cost increases housing price, urban land rent, and structural density while reducing housing consumption. The effect of Mode 1's fixed cost on the tax-share variable is ambiguous.

The intuitive explanation is that an increase in Mode 1's fixed commuting cost shrinks the area where Mode 1 is cheaper than Mode 2. Those people located between the new $\hat{x}$ and old $\hat{x}$ switch to Mode 2. The resulting increase in their income net of transport cost motivates them to move farther from the CBD to enjoy lower housing prices and larger dwelling sizes. As a result, housing price beyond the new $\hat{x}$ increases, which in turn increases urban land rent and structural density. Rising urban land rent expands the urban area spatially.

For the people located in the area between the CBD and the new $\hat{x}$, an increase in Mode 1's fixed cost reduces their income net of transport cost. Their demand for housing falls, leading to a decrease in housing price, lower urban land rent, and lower structural density. On the other hand, an increase in Mode 1's fixed cost makes living closer to the CBD more desirable, which counteracts the income effect. Therefore, the overall effects on housing price, urban land rent, housing consumption, and structural density are ambiguous in the area between the CBD and the new $\hat{x}$. The effect of an increase in Mode 1's fixed cost on the tax-share variable is ambiguous. An increase in Mode 1's fixed cost expands the urban area, which increases subsidies for Mode 2. The effect on the subsidies for Mode 1 is ambiguous because the effect of Mode 1's fixed cost on population density between the CBD and the new $\hat{x}$ is ambiguous.

3.2.9 Effects of Change in Mode 2's Fixed Cost

3.2.9.1 Effect of Mode 2's Fixed Cost on Utility

When $\lambda = f_2$, (3.21) becomes

$$\frac{1}{\alpha_1 t_1 - \alpha_2 t_2} \frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 t_1 \alpha_2 t_2} r_1(\hat{x}) + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} \frac{\partial u}{\partial f_2} dx + \frac{1}{\alpha_1 t_1} \frac{1}{\alpha_1 t_1 - \alpha_2 t_2} r_1(\hat{x})$$

$$+\frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \left(\frac{\partial r_2}{\partial f_2} + \frac{\partial r_2}{\partial u} \frac{\partial u}{\partial f_2} \right) dx - \frac{1}{\alpha_2 t_2} \frac{1}{\alpha_1 t_1 - \alpha_2 t_2} r_1(\hat{x}) = 0 \quad (3.131)$$

After rearranging terms, we have

$$\frac{\partial u}{\partial f_2} = \frac{-\frac{1}{\alpha_1 t_1 \alpha_2 t_2} r_1(\hat{x}) + \left(\frac{1}{\alpha_2 t_2} - \frac{1}{\alpha_1 t_1} \right) r_1(\hat{x}) \frac{1}{\alpha_1 t_1 - \alpha_2 t_2} - \frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial f_2} dx}{\frac{1}{\alpha_2 t_2} \int_{\hat{x}}^{\bar{x}} \frac{\partial r_2}{\partial u} dx + \frac{1}{\alpha_1 t_1} \int_0^{\hat{x}} \frac{\partial r_1}{\partial u} dx} < 0 \quad (3.132)$$

3.2.9.2 Effect of Mode 2's Fixed Cost on Urban Area Boundary

Totally differentiating (3.15) with respect to f_2 , we have

$$\begin{aligned} \hat{x} D_1(\hat{x}) \frac{\partial \hat{x}}{\partial f_2} + \int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial f_2} dx + \bar{x} D_2(\bar{x}) \frac{\partial \bar{x}}{\partial f_2} + \int_{\hat{x}}^{\bar{x}} x \left(\frac{\partial D_2}{\partial u} \frac{\partial u}{\partial f_2} + \frac{\partial D_2}{\partial f_2} \right) dx \\ - \hat{x} D_2(\hat{x}) \frac{\partial \hat{x}}{\partial f_2} = 0 \end{aligned} \quad (3.133)$$

Substituting $\lambda = f_2$ into (3.25) and solving for $\frac{\partial u}{\partial f_2}$, we get

$$\frac{\partial u}{\partial f_2} = \frac{-\frac{\partial \bar{r}_2}{\partial f_2} - \frac{\partial \bar{r}_2}{\partial x} \frac{\partial \bar{x}}{\partial f_2}}{\frac{\partial \bar{r}_2}{\partial u}}$$

Substitute this into (3.133) and rearrange terms; then, we have

$$\frac{\partial \bar{x}}{\partial f_2} = \frac{-\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial f_2} dx - \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial f_2} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial f_2} \frac{\partial \bar{r}_2}{\partial u} dx}{\int_0^{\hat{x}} x \frac{\partial D_1}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx - \bar{x} D_2(\bar{x}) \frac{\partial \bar{r}_2}{\partial u} + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial x} dx} \quad (3.134)$$

Since we know the denominator of (3.134) is positive, the sign of (3.134) is determined by the numerator.

Upon substituting the following into the second and third terms of the numerator of (3.134),

$$\frac{\partial D_2}{\partial f_2} = \Gamma \frac{\partial p_2}{\partial f_2}$$

$$\frac{\partial p_2}{\partial f_2} = -\frac{1}{q_2}$$

$$\frac{\partial D_2}{\partial u} = \Gamma \frac{\partial p_2}{\partial u} + \Lambda$$

$$\frac{\partial p_2}{\partial u} = -\frac{1}{q_2 V_1}$$

$$\frac{\partial \bar{r}_2}{\partial u} = -\frac{\bar{h}_2}{\bar{q}_2 \bar{V}_1}$$

$$\frac{\partial \bar{r}_2}{\partial f_2} = -\frac{\bar{h}_2}{\bar{q}_2}$$

we have

$$\begin{aligned} -\int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial u} \frac{\partial \bar{r}_2}{\partial f_2} dx + \int_{\hat{x}}^{\bar{x}} x \frac{\partial D_2}{\partial f_2} \frac{\partial \bar{r}_2}{\partial u} dx &= \int_{\hat{x}}^{\bar{x}} x \left(\Gamma \frac{1}{q_2} \frac{\bar{h}_2}{\bar{q}_2 \bar{V}_1} - \Gamma \frac{1}{q_2 V_1} \frac{\bar{h}_2}{\bar{q}_2} + \Lambda \frac{\bar{h}_2}{\bar{q}_2} \right) dx \\ &= \int_{\hat{x}}^{\bar{x}} x \left[\Gamma \frac{\bar{h}_2}{q_2 \bar{q}_2} \left(\frac{1}{\bar{V}_1} - \frac{1}{V_1} \right) + \Lambda \frac{\bar{h}_2}{\bar{q}_2} \right] dx < 0 \end{aligned}$$

Given that the first term of the numerator is negative, we have

$$\frac{\partial \bar{x}}{\partial f_2} < 0 \tag{3.135}$$

3.2.9.3 Effects of Mode 2's Fixed Cost on Housing Price, Housing Consumption, Urban Land Rent, and Structural Density

$$\frac{dp_1}{df_2} = \frac{\partial p_1}{\partial u} \frac{\partial u}{\partial f_2} > 0 \tag{3.136}$$

$$\frac{dr_1}{df_2} = h_1 \frac{\partial p_1}{\partial f_2} > 0 \quad (3.137)$$

$$\frac{dS_1}{df_2} = -\frac{h_1'}{p_1 h_1''} \frac{\partial p_1}{\partial f_2} > 0 \quad (3.138)$$

$$\frac{dq_1}{df_2} = \frac{\partial q_1}{\partial u} \frac{\partial u}{\partial f_2} < 0 \quad (3.139)$$

$$\frac{dp_2}{df_2} = \frac{\partial p_2}{\partial u} \frac{\partial u}{\partial f_2} + \frac{\partial p_2}{\partial f_2} \quad (3.140)$$

Because $\frac{\partial p_2}{\partial u} < 0$, $\frac{\partial u}{\partial f_2} < 0$, and $\frac{\partial p_2}{\partial f_2} < 0$, the sign of (3.140) is ambiguous. Therefore, we

have

$$\frac{dr_2}{df_2} = h_2 \frac{dp_2}{df_2} < 0 \quad (3.141)$$

$$\frac{dS_2}{df_2} = -\frac{h_2'}{p_2 h_2''} \frac{dp_2}{df_2} > 0 \quad (3.142)$$

$$\frac{dq_2}{df_2} = \frac{\partial q_2}{\partial u} \frac{\partial u}{\partial f_2} + \frac{\partial q_2}{\partial f_2} < 0 \quad (3.143)$$

3.2.9.4 Effect of Mode 2's Fixed Cost on the Tax-Share Variable

$$\frac{d\beta}{df_2} = \frac{1}{\theta y L} \left(\begin{aligned} & (1-\alpha_1)t_1 \hat{x}^2 D_1(\hat{x}) \frac{\partial \hat{x}}{\partial f_2} + (1-\alpha_1)t_1 \int_0^{\hat{x}} x^2 \frac{\partial D_1}{\partial u} \frac{\partial u}{\partial f_2} dx + \\ & (1-\alpha_2)t_2 \bar{x}^2 D_2(\bar{x}) \frac{\partial \bar{x}}{\partial f_2} + (1-\alpha_2)t_2 \int_{\hat{x}}^{\bar{x}} x^2 \left(\frac{\partial D_2}{\partial f_2} + \frac{\partial D_2}{\partial u} \frac{\partial u}{\partial f_2} \right) dx \\ & -(1-\alpha_2)t_2 \hat{x}^2 D_2(\hat{x}) \frac{\partial \hat{x}}{\partial f_2} \end{aligned} \right)$$

Since $\frac{\partial \hat{x}}{\partial f_2} > 0$, $\frac{dD_1}{df_2} > 0$, $\frac{\partial \bar{x}}{\partial f_2} < 0$, and $\frac{dD_2}{df_1} > 0$, then $\frac{d\beta}{df_1} < 0$.

We have shown that an increase in Mode 2's fixed cost reduces urban residents' utility and shrinks the urban area spatially. It increases housing price, urban land rent, and structural density in the area between the CBD and $\hat{x}$, while decreasing per-household housing consumption. The effects on these variables in the area between $\hat{x}$ and urban area boundary are ambiguous. The effect of an increase in Mode 2's fixed cost on the tax-share variable is ambiguous.

The intuitive explanation is very similar to that of an increase in Mode 2's variable commuting cost. An increase in Mode 2's fixed cost increases the area using Mode 1. Those people who originally located in the area between the new $\hat{x}$ and old $\hat{x}$ switch to Mode 1. Given the resulting decline in their income net of transport cost, living close to the CBD becomes more desirable. The increased demand for housing bids up housing price there, which in turn increases urban land rent and structural density while decreasing housing consumption. Those people located beyond the new $\hat{x}$ find their income net of transport cost fall, which lowers the demand for housing. This reduces housing price, urban land rent, and structural density. The decrease in urban land rent shrinks the urban area spatially. It also makes living closer to $\hat{x}$ more desirable, which increases housing price closer to $\hat{x}$. Therefore, its overall effects on housing price, housing consumption, urban land rent, and structural density are ambiguous. An increase in Mode 2's fixed cost shrinks the urban area spatially, which reduces the subsidies needed for Mode 2; however, it raises population density in the area between the CBD and the new $\hat{x}$, thus increasing the subsidies needed for Mode 1. Therefore, its overall effect on the tax-share variable is ambiguous.

3.3 Summary

Table 3.1 summarizes the comparative static analysis of the two-mode model. Rather than discuss all of these results, we emphasize those that are important for the empirical analysis to follow. Since our main goal is to explain how transport subsidies affect urban sprawl, we concentrate on $\frac{\partial \bar{x}}{\partial \alpha_1}$ and $\frac{\partial \bar{x}}{\partial \alpha_2}$.

Recall that α_i is the i th mode's private cost share. Since $\frac{\partial \bar{x}}{\partial \alpha_1} > 0$, an increase in the subsidized share of Mode 1's transport cost causes the urban area to contract. This result may seem counterintuitive, but when the private share of Mode 1's variable cost decreases, the area for which Mode 1 is cheaper expands (recall that $\hat{x} = \frac{f_2 - f_1}{\alpha t_1 - \alpha_2 t_2}$, so $\frac{\partial \hat{x}}{\partial \alpha_1} < 0$). The people who originally located between the old, smaller $\hat{x}$ and new, larger $\hat{x}$ now use Mode 1. Since Mode 1 has a higher variable cost than Mode 2, they are motivated to relocate closer to the CBD because their income net of transport cost increases more rapidly with Mode 1 than with Mode 2. Their move closer to the CBD reduces housing price in the area between $\hat{x}$ and $\bar{x}$, which in turn decreases urban land rent and structural density while increasing housing consumption. As a result, the urban area shrinks.

Since $\frac{\partial \bar{x}}{\partial \alpha_2} < 0$, the urban area expands if the subsidized portion of Mode 2's cost increases. This result is more appealing intuitively and happens because when the private share of Mode 2's variable cost decreases, the area in which Mode 1's transporta-

tion cost is lower than Mode 2's shrinks. Those people located between the old, larger $\hat{x}$ and new, smaller $\hat{x}$ use Mode 2 rather than Mode 1, which makes living farther from the CBD desirable since the increase in transport costs associated with a move farther out is now less. As a result, housing price, urban land rent and structural density in the area decrease, while per-household housing consumption increases. For the people located between the new $\hat{x}$ and urban area boundary, a decrease in the private share of Mode 2's variable commuting cost increases income net of transportation costs, which raises housing price, urban land rent, and structural density in the area. Because of the increase in urban land rent, the urban area expands spatially.

Table 3.1

Comparative Static Results of Two-Mode Model

<i>Exogenous Variable</i>	<i>Endogenous Variable</i>						
	u	$\bar{x}$	S_i	r_i	p_i	q_i	β
L	-	+	+	+	+	-	?
$f_1(x = \bar{x})$	-	+	+	+	+	-	?
$f_2(x = \bar{x})$	-	-	?	?	?	?	?
$t_1(x = \bar{x})$	-	+	+	+	+	-	?
$t_2(x = \bar{x})$	-	-	?	?	?	?	?
$y(x = \bar{x})$	+	?	?	?	?	+	?
r_A	-	-	+	+	+	-	-
x	NC	NC	-	-	-	+	NC
$\alpha_1(x = \bar{x})$	-	+	+	+	+	-	?
$\alpha_2(x = \bar{x})$	-	-	?	?	?	?	?

CHAPTER 4 EMPIRICAL ANALYSIS

Although there have been many empirical studies of urban sprawl (see, for example, the citations in Nechyba and Walsh (2004)), the one closest in spirit and approach to ours is Brueckner and Fansler (1983). Consequently, we provide a brief review of that paper.

Brueckner and Fansler (1983) base their empirical analysis on the theoretical model exposited by Brueckner (1987). That model is similar to, but simpler than, ours: it has only one transportation mode, no variables representing transportation subsidies, and no balanced budget equation.

They estimate the following general relationship

$$Area = f(L, r_A, y, t_1, t_2) \quad (4.1)$$

where *Area* is the land area in square miles of forty urbanized areas in 1970, *L* is 1970 urbanized-area population, *r_A* is the 1969 median agricultural land value per acre for the county in which the urbanized area is located, *y* is 1970 urbanized-area mean household income, *t₁* is the percentage of urbanized-area commuters using bus transit in 1970, and *t₂* is the percentage of 1970 urbanized-area households owning one or more autos.

Equation (4.1) is essentially the reduced-form urban-rural boundary equation from the theoretical model, $\bar{x} = \bar{x}(L, t, y, i, r_A, \delta)$, where $\bar{x}$ is the urban-area radius, which is of course directly related to *Area*. The two transportation variables are proxies for round-trip transport cost per mile. Brueckner and Fansler believe that *t₁* and transport cost should be

positively correlated because bus travel has a high time cost relative to auto travel, whereas t_2 and transport cost should be negatively correlated because auto travel involves less time cost than bus travel. The variables for structure cost, i , and radians of urban land used for residences, δ , are suppressed.

Since functional forms are not specified by the theory, the authors estimate a linear equation by ordinary least squares (OLS) and a flexible form using the Box-Cox transformation. The results of both estimation techniques are qualitatively the same. The estimated coefficients on population and income are statistically significantly positive, while the estimated coefficient on agricultural land rent is statistically significantly negative. The transportation cost proxies have the correct signs but are not statistically significant. The coefficient of determination is approximately 0.80 for both estimated equations. Brueckner and Fansler do not discuss potential endogeneity problems arising from the transportation-cost variables; we shall return to this topic in our empirical analysis.

The authors conclude that sprawl is the result of an orderly market process, that transferring farmland to urban use is not a waste, and that the empirical robustness of the theory increases confidence in the closed-city model as a policy tool. We turn now to our own empirical work.

4.1 Objectives and Hypotheses

The objective of this thesis is to explore the effect of transportation subsidies on urban sprawl. The theoretical model of Ch. 3 predicts that highway subsidies promote urban sprawl, while public transit subsidies retard it. Although these are the primary hypotheses of interest, we also test other hypotheses of the theory.

We start with OLS and flexible-form procedures, as did Brueckner and Fansler. We also investigate endogeneity, which requires us to estimate the model using two-stage least square (2SLS). Some of the variables in our model are the same as those used by Brueckner and Fansler, and some are different. In addition, we sometimes measure the same variables in different ways from those of Brueckner and Fansler. We use what we think are more accurate measures of modal commuting costs, and we add new variables for highway subsidies, transit subsidies, and the local “income tax” rate.

4.2 Data Sources

We draw all of the basic data used in our empirical work from the web sites of various organizations. From the U.S. Bureau of the Census, we have drawn data from the population census of 2000; the censuses of government of 1992, 1997, and 2002; and the census of agriculture of 1997 and 2002. From the Federal Highway Administration (FHWA), we use Highway Statistics 2000. From the Federal Transit Administration (FTA), we use the National Transit Database (NTD). From the Insurance Information Institute, we use data on average expenditures for auto insurance.

4.2.1. United States Census 2000

United States Census 2000 Summary File 3 (SF3) (U.S. Census Bureau 2000) provides number of households, population, household income, and spatial size of urbanized areas. SF3 contains some of the richest and most complete statistical data available on U.S. residents, collected from one in six households in the United States. There are 813 detailed tables available in the SF3 data products. The total number of household by urbanized area is from Table H1. Mean household income by urbanized area is derived from Table P54 and Table H1.

4.2.2. Census of Agriculture

The United States Department of Agriculture conducts a census of agriculture every five years. We obtain rural land rent from the censuses of agriculture for 1997 and 2002 (National Agricultural Statistics Service 1999, 2004).

4.2.3. Census of Government

The U.S. Census Bureau conducts censuses of government every five years. These censuses cover three major subjects: government organization, public employment, and government finances. We calculate highway subsidies from information on highway expenditures and revenues of counties in which urbanized areas are located. We detail this procedure in Section 4.3 below. We also derive the property tax rate using data provided by the census of government.

4.2.4. National Transit Database

We calculate two variables from the NTD (Federal Transit Administration 2004): transit cost per passenger-mile and transit subsidy per passenger-mile. The FTA collects and disseminates data on the state of mass transportation via the NTD program. Over 600 of the nation's transportation providers submit data to the NTD annually. Although the NTD reports data on twelve mass transportation modes, we focus on bus because most urbanized areas provide bus service, while very few provide other kinds of transit service. We define the transit subsidy as the difference between transit operating and capital expenses and transit fare revenues, expressed per passenger-mile. We explain how we estimate this variable later.

4.2.5 Highway Statistics

The FHWA annually publishes *Highway Statistics*, which contains data on motor fuel, motor vehicles, driver licensing, highway-user taxation, state and local government highway finance, highway mileage, and federal aid for highways. States submit the highway data, and the FHWA reports the data at the state level. We use *Highway Statistics 2000* (Federal Highway Administration 2003) to calculate motor fuel taxes per vehicle-mile traveled (VMT) as a proxy for highway cost. We provide details of this procedure in section 4.3.7.

4.2.6. Insurance Information Institute

The Insurance Information Institute publishes data on average expenditures for auto insurance in 2000 by state (Insurance Information Institute 2005). We use these data as part of fixed auto cost.

4.3 Variables Used in the Analysis

This section discusses each variable used in the analysis. Our two-mode theoretical model produces the following reduced form equation

$$\bar{x} = f(L, y, r_a, t_1, \alpha_1, t_2, \alpha_2, \theta) \quad (4.2)$$

where $\bar{x}$ is the urban boundary, measured by distance from the CBD; L is population; y is income per household; r_a is rural land rent at the urban boundary; t_1 is transit cost per round-trip mile; α_1 is transit subsidy per round-trip mile (please note that in Ch. 3, α_1 is used to represent the private share of transit cost per round-trip mile); t_2 is highway cost per round-trip mile; α_2 is highway subsidy per round-trip mile (in Ch. 3, α_2 is used to represent the private share of auto cost per round-trip mile); and θ is the local “income”

tax rate. Following Brueckner and Fansler, we suppress the structural input rental rate, i , in (4.2) because the most likely proxy for it, the interest rate, does not vary much nationwide. Again, following Brueckner and Fansler, we also suppress the variable for radians of urban land used for residences. This variable theoretically affects the size of the urban area, but it is very difficult to get data on it.

Of the various kinds of urban areas the census delineates—including legal cities, census designated places, urban clusters, urbanized areas, metropolitan statistical areas, primary metropolitan statistical areas, and consolidated metropolitan statistical areas—it is the opinion of urban economists that urbanized areas best conform to the generic urban area used for theoretical analysis.

Many urbanized area have more than one central city. For example, the Tampa Bay area has three: Tampa, St. Petersburg, and Clearwater. Since our model is monocentric, we eliminate those urbanized areas with more than one central city. In addition, there are sometimes more than one urbanized areas in a single county. In these cases, we are concerned that our proxy for rural land value at the urban fringe may be inflated by the presence of another nearby urbanized area. There are a total of 465 urbanized areas delineated by the 2000 census. Using GIS software, we identify 201 urbanized areas that are located within a single county and have only one central city. This set of urbanized areas constitutes our sample. We turn now to a detailed discussion of our variables.

4.3.1 The Urban Boundary, $\bar{x}$

Theoretically, the dependent variable in our estimating equation should be the radius of an urban area with the CBD as the center. To avoid having to calculate an implied radius for each of our sample urbanized area, we use instead the spatial size of the urban-

ized area, which is, of course, directly related to the implied radius. The 2000 census of population provides the spatial sizes of urbanized areas, measured in square meters, which we convert to square miles. This variable is found in Census 2000(U.S. Census Bureau 2002)

4.3.2. Total Number of Households in Urban Area, L

In our theoretical models, we assume single-person households, which is, of course, not the case in reality. Since the household is the basic unit of analysis in our models, we use households, rather than population, to measure L in our model. This variable is found in U.S. Census 2000, Table P15 (U.S. Census Bureau 2002).

4.3.3. Mean Household Income, y

The U.S. Census 2000 reports *median* household income in 1999 dollars by urbanized area, but not *mean* household income. The census does, however, report aggregate household income in 1999 dollars in Table P54. We calculate mean income per household by dividing aggregate household income by the total number of households residing in the urbanized area.

4.3.4. Agricultural Land Rent, r_a

In our theoretical models, agricultural land rent is the land rent immediately adjacent to the built-up part of the urban area. This theoretical construct is not available empirically in any published source of which we are aware. As an alternative, we use mean estimated market value of farmland per acre for the county in which the urbanized area is located. This variable is available from the Census of Agriculture (National Agricultural Statistics Service, 1999, 2004). Since the Census of Agriculture is conducted every five years and in different years from the decennial census, our variable is the mean of the

means reported for 1997 and 2002. We perform this calculation to obtain a value for the rural land variable as consistent as possible with other variables, most of which are for the year 2000.

4.3.5 Fixed Transit Cost, f_1

Fixed transit cost includes travel time between home and bus stop and waiting time at the bus stop. Since there are no data by urbanized area on travel time to and waiting time at bus stops, we use as a proxy the percentage of the working-age population taking transit to work. We expect this proxy to be negatively related to fixed transit cost.

Brueckner and Fansler (1983) use the percentage of urbanized-area commuters using bus transit as a proxy for their single-mode variable transport cost. They reason that the percentage of commuters using transit should be positively correlated with variable transport cost because bus has a high time cost. This argument is not supported by travel-behavior studies, however.

Studies of travel behavior find that transit passengers find waiting time to be more onerous than in-vehicle travel time (Federal Transit Administration 2004). We argue therefore that the longer the waiting time, the less attractive transit becomes as a means of transportation, and the fewer users it will attract. Our view is further supported by a Florida Department of Transportation report on factors affecting transit use (Hinebaugh and Zhao 1999)). The report says that transit use deteriorates exponentially with walking distance to transit stops. In addition, *Nationwide Personal Transportation Studies* (U.S. Department of Transportation 1986) reports that the maximum convenient walking distance is about 450 meters. We conclude that the longer the walking time, the fewer the people who use transit. Data on the percentage of the working age population taking transit to

work are from U.S Census 2000 Table P32. Another potential proxy we may use is the transit density, defined as the land area covered by each bus route mile. It is expected to be positively related to the fixed transit cost. The larger the land area is served by each bus route mile, the longer the waiting time, thus higher fixed transit cost. We use data from National Transit Database to derive the transit density. Its effect, however, is not statistically significant.

4.3.6. Variable Transit Cost, t_1

In our two-mode model, private transit cost is the household's money cost of transit per round-trip mile. Since no direct measure of this cost exists, we use a close substitute: private transit cost per passenger-mile. This variable is derived by dividing annual bus fare revenue by the annual total passenger-miles reported in the National Transit Database.

The National Transit Database 2002 data tables (Federal Transit Administration, 2004) provide detailed summaries of financial and operating data submitted to the FTA by the nation's mass transit agencies. For the 2002 report year, 613 transit agencies submitted data. The FTA has established a uniform system of accounts and records, as well as a reporting system for the collection and dissemination of public mass transportation financial and operating data by uniform categories. Additionally, according to federal law, all applicants and direct beneficiaries of federal assistance are subject to the reporting system and the uniform system of accounts and records. Therefore, data reported in the NTD are uniform, consistent, comparative, and reliable.

We use these data for the year 2002 rather than 2000 for a couple of reasons. First, since 2002 the FTA has required reporting of fare revenues by mode. Before 2002,

transit agencies reported only an aggregate fare revenue for all the modes they operated, which makes it impossible to identify the exact amount of fare revenue generated by a particular mode and makes the data less comparable if the modes operated by different transit agencies are different. Second, 2002 was the first year that transit agencies were required to report performance indicators, such as unlinked passenger trips and passenger-miles by mode. These data enable us to obtain fare revenue per passenger-mile by mode. Third, we compare the transit revenues and transit expenses from 2000 to 2002 and find that the amounts reported are very close, which justifies using 2002 data.

The NTD program collects data on eighteen modes, including railroad, cable car, demand response (i.e., taxi), commuter rail, ferryboat, heavy rail, inclined plane, jitney, light rail, bus, monorail, and trolleybus. Of the 536 transit agencies that report fare revenues, 456 provide bus service, 432 provide demand-response service, forty-five provide vanpools, twenty-four provide light rail service, and seventeen provide ferryboat service. Other modes have less than fifteen providers. Since bus service is widely provided and demand response is generally not considered public transit, we use bus to represent transit in our analysis. Based on the transit agency address, we match the transit agencies and the urbanized area for which they provide service. We then derive private transit cost per passenger-mile from fare revenue and passenger-miles.

4.3.7 Transit Subsidy, α_1

In our theoretical model, α_1 is used to represent the private share of transit cost. In this empirical analysis, we use α_1 to represent the transit subsidy. We derive transit subsidy per passenger-mile as follows. First, we sum bus transit operating cost (from Table 1, NTD 2002) and transit capital cost (from Table 7, NTD 2002) to get total cost. Second,

we subtract fare revenue from total cost to get the subsidy. Finally, we divide the subsidy by total passenger miles to get transit subsidy per passenger-mile.

4.3.8. Fixed Auto Cost, f_2

Fixed auto cost includes all costs invariant to distance traveled. For highway users, fixed costs generally include registration fees, license fees, motor vehicle taxes, auto insurance premiums, maintenance, and depreciation. Since data on maintenance and depreciation are not available, we use the sum of average annual motor vehicle fees and taxes and annual auto insurance premiums per household to represent fixed auto cost. Data on average annual motor vehicle fees and taxes are from Highway Statistics 2000, and data on annual auto insurance premiums are from the Insurance Information Institute.

4.3.9 Variable Auto Cost, t_2

In the two-mode model, variable auto cost is the household's auto expenditure per round-trip mile. No direct measure of this cost exists, and this is the most difficult variable for which to develop a proxy. After consulting many sources, we conclude there is no way to get an accurate estimate of this cost by urbanized area. Consequently, we use highway fuel tax per VMT to represent private variable highway cost.

To help justify the use of this variable, we summarize problems with alternative measures. The Bureau of Labor Statistics' annual consumer expenditure survey reports consumer units' annual expenditures on transportation, including vehicle purchase, gasoline and oil purchase, other related expenditures, and expenditures on public transportation. (A consumer unit comprises: (1) all members of a particular household who are related by blood, marriage, adoption, or other legal arrangement; (2) a person living alone or sharing a household with others or living as a roomer in a private home or lodging

house or in permanent living quarters in a hotel or motel, but who is financially independent; or (3) two or more persons living together who use their income to make joint expenditure decisions.) Financial independence is determined by the three major expense categories: housing, food, and other living expenses (Bureau of Labor Statistics 2000). This seems an ideal source of data with which to construct variable auto cost. Unfortunately, the consumer expenditure survey reports only the average transportation expenditures for four major regions (Midwestern, Northeastern, South, and Western) and the twenty-eight largest Metropolitan Statistical Areas. We considered using the public use micro-data files from the Bureau of Labor Statistics to calculate average highway expenditures by urbanized area or by county or MSA in which our sample urbanized areas are contained. This approach was precluded by the fact that the geographical location of the consumer units is suppressed or recoded to preserve confidentiality. Thus, it is impossible for us to obtain the desired transport cost.

The only available proxy for the variable we want to measure is average highway motor fuel taxes per VMT. Other things equal, the higher the motor fuel taxes paid, the higher are variable auto costs.

Before we explain how we obtain motor fuel taxes per VMT, which is the largest component of highway user-fees by urbanized area (Federal Highway Administration, 2003), we explain what highway user-fees are and how they are collected and apportioned. These fees play a key role in highway financing in the United States. Highway user-fees consist of highway-based exercise taxes; federal use tax and taxes on tires, trucks, and trailers; and vehicle-related license and operation taxes. The three major categories of highway taxes are federal motor fuel taxes, state motor fuel taxes, and local mo-

tor fuel taxes. Vehicle-related license and operation taxes are generally collected at the state and local level (Federal Highway Administration 2003).

Federal motor fuel taxes are specific taxes that differ by type of fuel. These taxes are not paid directly by highway users to the Internal Revenue Service (IRS). Rather, most taxes are collected by the IRS from several large corporations, typically large oil companies or distributors with storage facilities. This makes it impossible to measure fuel usage within each state. Therefore, there is no direct measure of highway taxes paid by the residents in each state.

After the IRS collects highway-based excise taxes, the Office of Tax Analysis estimates monthly highway receipts from each state. The Financial Management Service and the Bureau of Public Debt then credit the Highway Trust Fund (HTF) with the estimated revenue. Each quarter, the IRS certifies total receipts to reflect actual tax revenue collected by type of tax. The Financial Management Service and the Bureau of Public Debt then adjust the amount initially estimated based on the IRS's quarterly verification. The final information from the Department of Treasury, which documents receipts for the previous fiscal year, determines the overall level of highway user tax receipts attributed to the HTF by tax type. This amount represents the total highway program funds available for apportionment to each state. Among many criteria, state-reported motor fuel data are a critical component of the process that distributes HTF to the states (Federal Highway Administration 2003, 2005).

States, however, have very different tax legislation, tax forms, and administrative procedures. There are significant variations among the states in exemptions or refunds for off-highway use or government use. Therefore, it is not sufficient for the FHWA to sim-

ply sum up the total gallons of fuel reported by states and use those data for attribution. The FHWA has to adjust the data reported by states and separate the data into use (on-highway versus off-highway), fuel type (gasoline, gasohol, special fuels), and tax statutes based on public, private, and commercial categories (exempted, refunded, taxed-at-other-rate). As a final step, the FHWA sums up the on-highway gallons to derive a national total for each of the three motor fuel types and divides each state's gallons by the total gallons to derive each state's share (Federal Highway Administration 2003).

Highway users not only pay federal motor fuel taxes but also state motor fuel taxes. State motor fuel tax rates on gasoline vary from 7.5 cents per gallon in Georgia to 29.3 cents per gallon in New York State, with a national average of 20.17 cents in 2000 (Federal Highway Administration 2003). Based on state laws, state governments have authority to collect motor license fees, registration fees, and other motor-related taxes. Revenues from state motor fuel taxes and motor-related fees and taxes are generally used by state governments for transportation purposes and are therefore counted as a part of user-fees.

In some states, due to population growth and inflation in the early 1960's and 1970's, new demands placed on local governments were greater than their ability to raise capital for local transportation projects. Therefore, in the early 1970's, several states allowed counties to "piggyback," or add to, the state's tax on highway fuels. For example, the Florida legislature authorizes counties to collect up to 12 cents per gallon gasoline tax to be used on local transportation projects (Florida Department of Transportation 2005). The majority of states, however, do not allow local governments to collect motor fuel taxes.

Since highway users pay three layers of fuel taxes, we add them to obtain the total fuel taxes paid in each state. We derive the urbanized area's share of fuel taxes by multiplying the amount of total state fuel taxes by the urbanized area's proportion of its state's vehicles, calculated as the number of vehicles owned by urbanized-area residents divided by the total number of vehicles owned by state residents. Since states vary significantly regarding the tax exemption for public use or government use, we use the total number of privately owned vehicles in each state.

After obtaining a figure for annual fuel taxes by urbanized area, we divide that figure by annual total VMT, thus deriving highway fuel taxes per VMT, which represents variable auto cost in our empirical work. The annual VMT by urbanized area is derived by multiplying average daily VMT by 365. To get a per-vehicle figure for each urbanized area, we divide the sum of each state's share of federal motor fuel tax receipts, state motor fuel tax receipts, and local motor fuel tax receipts by the annual VMT.

Federal motor fuel taxes attributed to each state are from Highway Statistics 2000, Table FE-9. Data on state motor fuels taxes and related receipts and state motor vehicle and motor carrier tax receipts are from Table MF-1 and Table MV-2. Data on local motor fuels tax receipts are from Table LDF. Data on the total number of vehicles by state and urbanized area are from U.S. Census 2000, Table H46. Data on VMT by urbanized area are from Highway Statistics 2000 Table HM72.

4.3.10 Highway Subsidy, α_2

In our theoretical model, we use α^2 to represent the private share of auto cost. In this empirical analysis, we use the same notation to represent the highway subsidy. We present our method for measuring the highway subsidy by urbanized area in this section.

One question must be answered before we discuss how we measure the highway subsidy: What are highway subsidies or what constitute highway subsidies?

There is no official definition of highway subsidies. If user-fees collected for highway purposes are not sufficient to cover highway expenditures, governments must use other revenues to make up the difference, which amounts to subsidization. According to *Highway Statistics 2000*, total U.S. highway user-fees were \$106 billion in 2000, of which \$81.0 billion was used for highway purposes (Federal Highway Administration 2003). Total U.S. highway expenditures in 2000 were \$127.5 billion. Therefore, the subsidy was \$46.5 billion, which was contributed by all levels of government from other revenue sources. Revenues contributing to the subsidy are from a number of sources, including local property taxes and assessments, other dedicated taxes, general funds, bond issues, and other investment income.

As shown in Table 4.1, the degree to which highway programs are funded by highway-user-fees differs widely among different levels of government. At the federal level, 95.6 percent of highway revenues came from motor-fuel and motor vehicle taxes in 2000. The remainder came from general fund appropriations, timber sales, lease of Federal lands, oil and mineral royalties, and motor carrier fines and penalties. Highway user-fees provided 75.5 percent of highway revenues at the state level in 2000. Bond issue proceeds were another significant source of funding, providing 12.3 percent of highway funds at the state level. The remaining 14 percent of state highway funding came from general fund appropriations, other state taxes and fees, investment income, and other miscellaneous revenue sources (Federal Highway Administration 2003).

Many states do not permit local governments to impose motor-fuel and motor vehicle taxes, or they cap them at relatively low levels. At the local government level, only 7.5 percent of highway funding was provided by highway user-fees in 2000. Local general funds, property taxes, and other taxes and fees were the sources of 67.5 percent of local highway funding. Bond issue proceeds provided 9.8 percent of local highway funding, while investment income and miscellaneous receipts provided the remaining 14 percent (Federal Highway Administration 2003).

Since only 62.9 percent of revenue sources for highways come from highway user-fees, highway use is clearly subsidized. The question is how to use this information to form the highway subsidy variable in our model.

For consistency with our theoretical model, highway subsidies should be measured per round-trip mile per household for travel within an urban area. (Although the model assumes single-person households, we use per-household figures, rather than per-person figures, because the household is assumed to be the decision maker and the bearer of travel costs.) Highway subsidies are the difference between highway expenditures paid by various level of governments in a given urban area and the total user-fees collected from highway users in that same area. Such a measure of highway subsidies is unavailable because of complicated highway ownership and financing systems in the United States.

Table 4.1**Revenue Sources for Highways, 2000 (billions of dollars)**

<i>Revenues and Expenditures</i>	<i>Governments</i>				<i>Percent of Total Revenue</i>
	<i>Federal</i>	<i>State</i>	<i>Local</i>	<i>All</i>	
<i>User Charges</i>					
Motor-Fuel Taxes	\$25.1	\$28.7	\$1.0	\$54.8	42.5
Motor-Vehicle Taxes and Fees	\$4.6	\$15.5	\$0.7	\$20.8	16.2
Tolls	\$0.0	\$4.7	\$0.7	\$5.4	4.2
Subtotal	\$29.7	\$49.0	\$2.3	\$81.0	62.9
Other Sources of Revenue					
Property Taxes and Assessments	\$0.0	\$0.0	\$6.4	\$6.4	4.9
General Fund Appropriations	\$1.2	\$4.1	\$11.9	\$17.2	13.4
Other Taxes and Fees	\$0.1	\$2.4	\$2.8	\$5.4	4.2
Investment Income & Other Receipts	\$0.0	\$2.7	\$4.8	\$7.5	5.8
Bond Issue Proceeds	\$0.0	\$8.2	\$3.1	\$11.2	8.7
Subtotal	\$1.4	\$17.5	\$28.9	\$47.7	37.1
Total Revenues	\$31.1	\$66.4	\$31.3	\$128.7	100.0
Total Expenditures	\$27.7	\$67.0	\$32.7	\$127.5	99.0

Source: Federal Highway Administration, 2003, Table HF-10

Highways are typically classified by either ownership or purpose. Ownership is determined by the jurisdiction that has primary responsibility for a particular structure, while purpose (as well as level of service) is identified from the structure's function. Ownership is divided among federal, state, and local governments. According to the De-

partment of Transportation, roads and bridges owned by these governments are considered public, while structures owned privately are commonly considered nonpublic. States own almost 20 percent of the nation's road system. The federal government has control over about 3 percent of the network. Over 77 percent of American roads are locally owned although some intergovernmental agreements may authorize states to construct and maintain locally owned highways.

Highway financing is not solely determined by highway ownership, however. Highways owned by the federal government (including interstate highways; the strategic highway network, which is generally highways important to military mobilization; other rural and urban principal arterials; strategic highway network connectors; and intermodal connectors) are 90-percent funded by the federal government. For highways owned by states, capital outlays are largely funded by the federal government through intergovernmental grants. Even local roads are not funded solely by local governments. State and federal governments fund the majority of capital outlays for local roads (Federal Highway Administration 2002).

According to Highway Statistics 2000, although the federal government funded \$27.7 billion (21.7 percent) of total highway expenditures, the majority of the federal government's contribution to highways consisted of grants to state and local governments. Direct federal spending on capital outlay, maintenance, administration, and research amounted to only \$2.3 billion (1.8 percent). The remaining \$25.4 billion was in the form of transfers to state and local governments. State governments combined \$24.4 billion of federal funds with \$52.1 billion of state funds and \$1.3 billion of local funds to make direct expenditures of \$77.9 billion (61.1 percent). Local governments combined

\$1.0 billion of federal funds with \$14.9 billion of state funds and \$31.4 billion of local funds to make direct expenditures of \$47.3 billion.

Given the complicated highway ownership and funding systems, it is almost impossible to obtain an accurate estimate of highway subsidies by urbanized area. For example, in a particular urbanized area, urban residents may have access to interstate highways owned and mainly funded by the federal government but maintained by the state government, state-owned highways funded by the state, highways owned by counties or cities maintained and funded by the state, and county or city roads owned and funded by them. For the various roads within an urbanized area, local governments report only revenues and expenditures on the roads owned and funded by them. No financing data are available for those roads not owned or funded by local governments. We are therefore restricted to use county and city data to calculate highway subsidies for a given urban area. Unfortunately, city data are not reliable because most highway revenue data are not reported. We are therefore reduced to using only county data to calculate highway subsidies.

The highway subsidy variable used in our analysis is derived as follows. First, we obtain county highway subsidies as the difference between total county highway expenditures and total county highway revenues. The urbanized area's share of highway subsidies is calculated as the product of total county subsidies and the number of vehicles owned by urbanized-area residents divided by the number of vehicles owned by county residents. The urbanized area's share of highway subsidies then is divided by urbanized-area VMT to derive the highway subsidies per VMT. If major construction occurred in a particular year, highway expenditures and related subsidies in that year may have risen

substantially. To smooth this kind of effect, we use a three-year average for 1992, 1997, and 2002. All the three-year data are adjusted to 2000 dollars based on the CPI.

Our data are provided by censuses of government. The census of government collects detailed data on government financing every five years. We use their data for the survey year 1997 and 2002. Highway-related expenditures and revenues are reported by county. According to the census of government, highway expenditures include expenditures on maintenance, operation, repair, and construction of non-toll highways, streets, roads, alleys, sidewalks, bridges, and related structures, while highway revenues include revenues from motor fuel taxes, motor vehicle license fees, motor vehicle operation fees, transfers from state government and federal governments, and regular charges on highways (U.S Census Bureau 2004).

4.3.11 Local Property Tax Rate, θ

The local property tax rate is derived from information provided by the census of government. We deflate 1997 and 2002 county and city property tax revenues to 2000 dollars by the CPI. To derive the per-household property tax payment, we average the property tax figures and divide by the number of urbanized-area households. Then the per-household property tax payment is divided by mean household income to derive the local property tax rate. Although the urbanized areas in our model are smaller than the counties in which they are located, this should not introduce much error since the high value property is located predominantly in the urbanized areas.

4.3.12 Intergovernmental Grants, G

The variable intergovernmental grants is derived from information provided by

the census of government. We deflate 1997 and 2002 county and city intergovernmental grants for transportation purposes from state government and derive the average intergovernmental grants by urbanized area.

4.3.13 Missing Observations

In this empirical analysis, 201 urbanized areas meet our criteria that the urbanized area be located within a single county and have only one central city. Because of missing observations, however, we end up with 93 urbanized areas. Almost all the incomplete observations are due to the absence of two key variables, the transit subsidy and the highway subsidy. Although it is fairly common for a data set to have gaps for a variety of reasons, it is nevertheless important to ensure as well as possible that these missing observations do not cause problems in the empirical analysis.

In general, there are two cases to consider, depending on why the data are missing. One is that the data are simply unavailable and not systematically related to other observations in the sample. This case is called *ignorable* (Greene 2000) because the analyst may simply ignore the problem without reducing the statistical reliability of the estimated results. Were the missing data available and not used, there would of course be a loss of efficiency.

The second case involves missing data that are systematically related to other variables in the model. This case occurs most often in surveys in which the data are self-selected. This case causes a more serious problem because the gaps in the data set represent, not only missing information, but also information that, if available, could change the estimated results. All our incomplete observations are due to the absence of data on two key variables, the transit subsidy and the highway subsidy. The transit subsidy is

missing for some urbanized areas because they simply have no public transit. This situation is not related to self-selection or non-response error. Because of the non-existence of public transit in 96 urbanized areas, our sample size falls from 201 to 107. Because of non-existence of VMT by urbanized area, our final sample is 93. A list of variables, definition of the variables in the empirical analysis, and their data sources are presented in Table 4.2. Table 4.3 provides summary statistics.

Table 4.2
List of Variables, Definitions, and Data Sources

<i>Variable</i>	<i>Definition</i>	<i>Source</i>
$Area$	Spatial size of the urbanized area in square miles	Census 2000
L	Number of households by urbanized area	Census 2000
r_a	Estimated market value of farmland per acre for the county in which the urbanized area is located	Census of Agricultural 1997, 2002
y	Mean household income by urbanized area	Census 2000
f_1	Percentage of working age population using transit	Census 2000
f_2	Sum of auto annual insurance premium, registration fee, license fee, and motor vehicle tax per household by urbanized area	Highway Statistics 2000, Insurance Information Institute
t_1	Bus fare cost per passenger-mile	National Transit Database 2002
t_2	Fuel tax payment per vehicle-mile traveled	Highway Statistics 2000
α_1	Subsidies to bus service per passenger mile	National Transit Database 2002
α_2	County subsidies to auto use per vehicle mile traveled	Census of Government 1997, 2002
G	Intergovernmental Transfer from State to local governments for transportation purposes	Census of Government 1997, 2002
θ	Local income tax rate: percentage of average household income paid as property tax payment	Census of Government 1997, 2002

Table 4.3**Summary Statistics**

<i>Variable</i>	<i>Unit</i>	<i>Mean</i>	<i>Std. Error</i>	<i>Minimum</i>	<i>Maximum</i>
<i>Area</i>	Sq. mi.	85.7978	75.01	13.6256	323.556
<i>L</i>	1000	79.839	92.85	17.87	518.61
<i>r_a</i>	\$1000	2.176	1.143	0.28	5.87
<i>y</i>	\$1000	49.43	8.036	31.38	92.86
<i>f₁</i>	Percentage	1.83	1.54	0.35	7.61
<i>f₂</i>	\$1000	1.275	0.203	0.9909	1.747
<i>t₁</i>	Cent	15.755	10.406	1.65	75.05
<i>t₂</i>	Cent	3.449	1.006	1.73	6.64
<i>α₁</i>	Cent	112.6	80.69	17.26	430.6
<i>α₂</i>	Cent	0.4004	0.3327	0.0071	1.7215
<i>G</i>	\$1000	20379.5	58908.34	0.9572	500000
<i>θ</i>	Percentage	1.642	0.6516	0.43	3.28

4.4 Model Specification and Estimation**4.4.1 Functional Form**

We estimate the following equation:

$$\ln Area = \beta_0 + \beta_1 L + \beta_2 r_a + \beta_3 y + \beta_4 y^2 + \beta_5 f_1 + \beta_6 t_1 + \beta_7 \alpha_1 + \beta_8 f_2 + \beta_9 t_2 + \beta_{10} \alpha_2 + \beta_{11} \alpha_2^2 + \beta_{12} \theta + \beta_{13} G + \varepsilon \quad (4.3)$$

We adopt this particular form for the following reasons. Following the method presented in Wooldridge (2003, pp. 208) in order to obtain a goodness-of-fit measure for the log model that can be compared with the *R*-squared from the level model, we find that the semi-log model explains more of the variation in the spatial size of urbanized areas than does the linear model. We therefore prefer the semi-log model on goodness-of-fit

grounds. We also choose the semi-log model because, except for the intercept, each estimated coefficient is the proportional change in the dependent variable per unit change in the independent variable and is, therefore, independent of the units in which the variables are measured. We then conduct specification tests that are embedded in STATA, the software we use to run the regression. An added-value plot reveals that there are significant nonlinearities in the data for income and the auto subsidy. We therefore, add two quadratic terms for these variables. This is further justified by the Lagrange multiplier test for adding variables. The chi-square value obtained by running regression of the residuals of the restricted model on all explanatory variables is 27.44, exceeding the 1 percent critical value of 9.21 for 2 degrees of freedom, which indicates that we should reject the restricted model.

4.4.2 Potential Endogeneity Issue

In our two-mode model, all the independent variables in (4.3) are theoretically exogenous because the equation is a reduced form. Econometrically, however, a variable on the right-hand side of (4.3) may be endogenous if it is correlated with the error term. If such a case or cases exist, OLS will produce inconsistent estimates. Our empirical model, following from the theoretical model, focuses on what we regard as the most important forces determining the spatial size of an urban area. We have not included all possible factors that may affect urban sprawl, and one of these omitted variables may give rise to endogeneity in one or more of the included variables. A potential candidate for endogeneity is variable highway cost, which measures the per-mile cost of driving to the CBD. Endogeneity could result from unobserved or unmeasured heterogeneity, which would occur if highway variable cost were correlated with any of the unmeasured spatial

size determinants that are buried in the error term. One of the unmeasured spatial size determinants could be congestion.

Although we do not believe congestion plays an important role in determining the spatial size of an urban area (Altmann and DeSalvo, 1981, find that it does not in their urban simulation model), it may be related to variable highway cost. For example, when congestion is severe, stop-and-go driving results in inefficient fuel usage, which causes higher variable highway cost. If this were the case, the resulting coefficient estimates in (4.3) would suffer from omitted-variable bias. This bias cannot be eliminated because the omitted variable or variables representing congestion are, by definition, not in our data set. If congestion is important and is not held constant in the regression, the coefficient on highway variable cost would be biased down, and we would therefore underestimate its effect on urban sprawl.

When potential endogeneity is involved, researchers generally use two-stage least squares (2SLS) to correct the problem and obtain consistent and unbiased estimates. To serve this purpose, researchers introduce instrumental variables (IV's) into the regression. The IV for an endogenous explanatory variable must satisfy two properties: (1) it must be uncorrelated with the error term of the structural equation (although in theory our estimated equation is a reduced form equation, here, because of the potential endogeneity, we refer to it as a structural equation); and (2) it must be correlated with the endogenous variable. When used properly, 2SLS allows us to estimate *ceteris paribus* effects in the presence of endogenous explanatory variables. When the instrumental variable and the endogenous explanatory variable are only weakly or even moderately correlated, how-

ever, IV estimates can have large standard errors or even a large asymptotic bias, which means 2SLS can be worse than OLS.

In our empirical analysis, we use two instrumental variables for the potential endogenous highway variable cost, state gas tax per gallon and urbanized-area freeway lane-miles. To satisfy the conditions for suitable instruments, state gas tax per gallon and freeway lane-miles must not affect the spatial size of the urbanized area in any way other than through the impact of these variables on highway variable cost per mile. In other words, these variables must be correlated with highway variable cost but not with the second-stage residuals. To determine if these conditions hold, we must investigate the processes that determine interstate differences in gas taxes and inter-urbanized-area differences in freeway lane-miles and evaluate whether the determinants of these variable have direct effects on the spatial size of an urbanized area.

State gas taxes are set within a complex system of institutional relationships designed to plan, finance, and operate the nation's highways. According to Dunn (1998), the system has two important components, intergovernmental transfers and the highway trust fund. Intergovernmental transfers ensure federal funds are funneled to the states that are responsible for construction and maintenance of highways. The highway trust fund ensures that the majority of revenues from federal and state gas taxes are dedicated to highway infrastructure needs. As discussed in 4.3.7, the federal government pays for 90 percent of interstate highway construction costs and over 50 percent of capital outlays for non-interstate highway construction. States are responsible for the construction and maintenance of highways owned by the federal government. The states raise funds for their share of highway costs through state gas taxes and other fuel taxes. The majority of states

have exclusive dedication provisions that protect gas tax revenues from general use other than highway financing, as shown by the fact that twenty-eight states have gas-tax earmarking provisions in either the state constitution or state law, while eleven states commit receipts to special-purpose funds to finance highway projects.

A state's gas tax is determined by the state's matching obligation for the construction of federally funded highway projects and the financing of state-initiated projects. In most states, revenues collected from state gas taxes are separated from state general funds. Combining that fact with the exclusive dedication provisions existing in most states, it seems reasonable to argue that the principal determinants of state taxes are highway project financing and the availability of federal funds. It is therefore reasonable to believe that the process determining the state gas tax is independent of the spatial size of the urbanized area. Thus, we may argue that state gas tax per gallon is a suitable instrument for highway variable cost.

To the best of our knowledge, there is no literature directly analyzing inter-urbanized-area differences in freeway lane-miles. Among roadway types, freeways provide the highest level of mobility at the highest speed for long, uninterrupted travel. They generally have higher design standards than other highways, often with multiple lanes and a high degree of access control. Freeways comprise interstate highways and other expressways with fully controlled access.

Highway functional classification is the grouping of roads, streets, and highways into systems of similar characteristics based primarily on the length of trips served. Additionally, functional classification defines the role that a particular road or street plays in serving the flow of trips through a highway network and analyzes the services provided

or that should be provided by each highway facility in serving the two principal functions of a highway, mobility and access. Two nationwide studies of highway functional classification were conducted during the period 1969–1971. The first study required the functional classification of existing (1968) highways, while the second study used the same functional classes and basic functional criteria as the first study, but provided for the classification to be based on projected 1990 facilities and usage. The Federal-Aid Highway Act of 1973 required the use of functional highway classification to update and modify the federal-aid highway systems by July 1, 1976 (Federal Highway Administration 2003). This legislative requirement is still in effect and the National Highway Functional Classification System has been in use since that time.

From the perspective of history, the most important use of functional classification has been to identify those streets and roads that are eligible for federal funds. The Federal-Aid Highway Acts specifically mandated nationwide studies in cooperation with states and local governments to bring greater consistency to highway classifications and to classify all public streets and highways according to their function. According to the Federal-Aid Highway Act of 1973, interstate highways and other expressways with access fully controlled are in high priority to receive federal funds. Therefore it is reasonable to believe that the location and lane-miles of freeways are determined by federal fund availability. From an engineering perspective, freeway capacity is generally set equivalent to engineering capacity because the through movement is the only concern for those highways while other roads' capacity is not only based on engineering capacity but also other factors such as population density and accessibility to major activities centers. Combined with the fact that the highway functional identification was done in 1970s, it is

reasonable to believe that freeway lane-miles are unlikely to be correlated with the unobservable determinants of the spatial size of urbanized area. This is confirmed by the change in freeway lane-miles over time. We analyze data from Highway Statistics on the freeway lane-miles for urbanized area. We find that from 1992–2002, at the national level, 33.7 percent of urbanized areas' freeway lane-miles decreased, 41.3 percent increased at an annual rate of less than 1 percent, and 26 percent increased at an annual rate of 2 percent or higher.

For the urbanized areas in our data set, 73 percent of freeway lane-miles either decreased or remained unchanged, while 37 percent increased, most at an annual rate of less than 3 percent. These facts support the argument that freeway lane-miles within an urbanized area are pure functional classification and unlikely to correlate with the unobserved determinants of the urbanized area's spatial size.

The data for these instrumental variables are from Highway Statistics 2000. Data on total freeway miles and state gas taxes are from also from Highway Statistics 2000, Table HW71 and Table MF-121T, respectively.

4.4.3 Empirical Results

In this subsection, we discuss the results of our estimation of (4.3). We confine our discussion to the statistical significance of the estimated coefficients and to the conformity of our estimated results with our theoretical expectations. In Ch. 5, we discuss the economic significance of our findings for urban sprawl.

Table 4.4 presents the OLS results and the second-stage parameters from the 2SLS estimation. The first-stage variable highway cost regression results are presented in Table 4.5. In the 2SLS estimation, we incorporate the explanatory variables discussed in

Section 4.2 and use the interstate variation in gas taxes and inter-urbanized-area freeway lane-miles as instruments. Comparing the estimated coefficients from the OLS and 2SLS regressions, we find the magnitudes of most are similar and the signs are the same. We conduct a Hausman specification test and find that we cannot reject the hypothesis that the differences in coefficients are not systematic. The results are therefore materially unaffected by the endogeneity correction, so we discuss only the OLS regression.

Table 4.4
OLS and 2SLS Regression Results

<i>Variable</i>	<i>OLS</i>			<i>2SLS</i>		
	<i>Coefficient</i>	<i>Std. Error</i>	<i>p</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>p</i>
L	0.0077	0.0006	0.000	0.0075	0.0006	0.000
r_a	-0.0041	0.0338	0.905	-0.0115	0.0351	0.743
y	0.1187	0.0271	0.000	0.1153	0.0277	0.000
y^2	-0.0011	0.0002	0.000	-0.0011	0.0002	0.000
f_1	-0.0956	0.0266	0.001	-0.0927	0.0271	0.002
t_1	0.0068	0.0039	0.084	0.0058	0.0041	0.162
α_1	-0.0010	0.0005	0.052	-0.0009	0.0005	0.067
f_2	-0.2936	0.1953	0.137	-0.3375	0.2027	0.100
t_2	-0.0690	0.0381	0.074	-0.1278	0.0687	0.067
α_2	0.6431	0.3575	0.076	0.6968	0.3666	0.061
α_2^2	-0.6589	0.2893	0.025	-0.6783	0.2943	0.024
θ	-0.1224	0.0588	0.041	-0.1156	0.0601	0.058
G	-4.18e-7	1.00e-06	0.677	-1.88e-7	1.04e-6	0.857
<i>Constant</i>	1.3520			1.7300		
R^2	0.8161			0.8105		

The coefficient on L , the number of households, is positive, as predicted, and statistically significant at the 1-percent level.

The coefficient on agricultural land rent, r_a , is negative, as predicted by our model, and is statistically insignificant. The statistical insignificance may be due to two factors. The mean estimated market value of rural land in a county may in fact do a poor job of capturing actual rural land value at the urban fringe. Brueckner and Fansler (1983) allow for this possibility by restricting their sample to urbanized areas in “small” counties, and their estimated coefficient is statistically significant. The statistical insignificance of our variable may also result from its small range of variation in our sample, which prevents the emergence of a precise estimate.

The estimated coefficients on y and y^2 , income and income squared, are statistically significant at the 1-percent level, and their signs indicate that the spatial size of the urbanized area increases at a decreasing rate with income. In the estimated equation, the marginal effect of income on urban size is given by

$$\frac{\partial \ln Area}{\partial y} = \beta_3 + 2\beta_4 y$$

where β_3 and β_4 are the estimated coefficients on y and y^2 , respectively. For the estimated coefficients and the mean urbanized-area income of \$49,430, the result is positive, which is in line with the prediction of the standard single-mode model, as well as ours. When income is \$54,000 or higher, however, our result is negative. Since urban-area size can increase, remain unchanged, or decrease with urban-area mean income, the effect of income on urban spatial size is, in general, ambiguous, which is what is predicted by our two-mode model.

The effect of income on urban size has generated considerable discussion in the urban economics literature, so we devote some time to how our findings comport with the findings in the literature. In partial equilibrium urban models that include the value of time spent commuting, the ambiguity of income's effect on location has been known since Muth (1969), and the empirical results find the effect variously positive or negative (e.g., Muth, 1969, Wheaton, 1977, Hekman, 1980, DeSalvo, 1985). The result depends on the relative magnitude of the income (or wage-rate) elasticity of housing demand and the income elasticity of marginal (distance) commuting cost (first noted by Muth (1969)). If the former is greater than, equal to, or less than the latter, then, as income increases, the household locates farther from, at the same distance from, or closer to the CBD, respectively. Since the value of time spent commuting rises with income (Hensher, 1976), it is possible that at incomes above \$54,000 households choose to live closer to the CBD than they would at lower incomes. Thus, urbanized areas with mean household incomes greater than \$54,000 might well be smaller than urban areas with smaller mean incomes.

Partial equilibrium urban models and empirical tests of them generally not do include mode choice. DeSalvo and Huq (2005) introduce mode choice into the partial equilibrium urban model and find that the wage-rate elasticity of marginal commuting cost varies with the wage rate in a complicated way. The effect of the wage rate on location can generate a wide variety of possibilities, including our empirical result. Although a few general equilibrium models include mode choice (e.g., Anas and Moses, 1979), we are unaware of general equilibrium urban models that include the value of time spent commuting.

In our two-mode model, if public transit's fixed cost, variable cost, or private cost share increases, the urban area expands, whereas the opposite is the case for auto commuting. In our empirical work, the percentage of people using transit is the proxy for fixed transit cost, f_1 . The estimated coefficient on f_1 is statistically significant at the 1-percent level, and its negative sign is consistent with our theoretical expectation. The coefficient on transit variable cost, t_1 , is positive, as expected, and statistically significant at the 8-percent level. The coefficient on the transit subsidy variable, α_1 , is negative, as predicted, and statistically significant at the 5-percent level, suggesting that the higher the transit subsidy, the smaller is the urbanized area. Another potential proxy variable for the fixed transit cost is transit density, which is derived as the land area of the urbanized areas divided by the total bus services miles. When we use this proxy in our regression, we find it to be insignificant.

The coefficient of fixed auto cost, f_2 , is negative, as predicted, but not statistically significant. Our measure of fixed auto cost, which is composed only of the auto insurance premium and motor vehicle taxes, may fail to capture the behavior of actual auto fixed cost because we do not have data on auto maintenance and depreciation. The coefficient on variable auto cost, t_2 , is negative, as predicted, and statistically significant at better than the 1-percent level. This means that if auto commuting cost increases, people will move closer to the CBD, thereby reducing the size of the urbanized area.

We use a quadratic polynomial to capture the nonlinearity of the highway subsidy, α_2 . The coefficient on highway subsidy and highway subsidy squared are statistically significant at the 8-percent level and 3-percent level, respectively. The coefficient on highway subsidy is positive, while that on highway subsidy squared is negative, in-

dicting that the spatial size of the urbanized area is increasing at a decreasing rate with the highway subsidy. The marginal effect of the highway subsidy is:

$$\frac{\partial \ln Area}{\partial \alpha_2} = \beta_{10} + 2\beta_{11}\alpha_2$$

where β_{10} and β_{11} are the estimated coefficients on α_2 and α_2^2 respectively. If we insert the estimated coefficients and the mean value of the highway subsidy of 0.4 cents per vehicle-mile traveled, the result is positive, which is consistent with our prediction that the higher the highway subsidy, the larger is the urbanized area. When the highway subsidies exceed 0.4880 cent per VMT, however, our result is negative. The overall effect of highway subsidies on the urban size is not consistent with our theoretical prediction. This may be due to the fact that our highway subsidy variable only includes subsidies from the county where the urbanized area is located, which fails to capture the subsidies from the cities. It may also be because our highway subsidy variable is not an accurate measurement because, given current data availability, there is no way to figure out exact highway subsidies by urbanized area.

The coefficient on the property tax rate, θ , is negative, as predicted, and statistically significant at the 6-percent level. Song and Zenou (2006) also obtain this result in a paper dedicated to determining the effect of the property tax on urban sprawl.

The coefficient on the intergovernmental grant for highway purposes, G , is negative but not statistically significant. Our model predicts that the relationship between urban size and the intergovernmental grant is ambiguous.

Before turning to the 2SLS estimates, a brief discussion of the results from the first-stage regression reported in Table 4.4 is in order. It is clear that state gas taxes exert

a positive effect on variable auto cost, while freeway lane-miles exert a negative effect. Both instruments are independently significant at the 0.001-level. The F-statistic for the test of the joint significance of the two instruments is 18.56. Hence, the instruments are strongly correlated with highway variable cost.

Returning to the results in Table 4.3, the 2SLS coefficient estimates on all variables except auto variable costs are quite close to the estimates from OLS. OLS underestimates the effect of auto variable costs: the coefficient on auto variable costs is -0.069 , compared to -0.128 using 2SLS. Nevertheless, both sets of results largely support our theoretical predictions. Since we use two instrumental variables, we are able to perform a test of the over-identifying restriction. The test fails to reject the over-identification restriction, suggesting that the dependent variables' effects on urban size are not sensitive to the choice of instruments.

4.4.4 Summary

This chapter presents the data and econometric issues in estimating our two-mode model. The discussion of our empirical findings focuses on the statistical significance of estimated coefficients and on the conformity to our of our empirical results with the two-mode theoretical model. Based on our theory, the variables explaining the spatial size of an urban area are the number of households, rural land value, household income, fixed and variable costs of public transit and auto travel, subsidies to public transit and auto travel, property taxes, and intergovernmental grants. We find that the spatial size of the urbanized area shrinks with an increase in the transit subsidy. The effect of highway subsidies, however, is ambiguous. Both OLS and 2SLS are applied, and the results are qualitatively the same.

Table 4.5**First-Stage Results from Regression of Auto Variable Costs on IV's**

<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>p</i>
L	0.0086	0.003	0.010
r_a	-0.2002	0.0840	0.020
y	-0.0359	0.0617	0.562
y^2	0.0003	0.0005	0.489
f_1	-0.0091	0.0665	0.892
f_2	-0.0006	0.0005	0.226
t_1	-0.0169	0.0095	0.079
α_1	0.0005	0.001	0.694
α_2	0.9379	0.8864	0.293
α_2^2	-0.3611	0.7177	0.616
<i>Freeway Lane Miles</i>	-0.0067	0.0018	0.000
<i>State Gas Tax Rate</i>	0.1083	0.0246	0.000
G	-2.37e-06	2.76e-06	0.393
θ	-0.0175	0.1477	0.906
R^2	0.43		
N	93		
F (Joint Significance of IV)	18.56		

CHAPTER 5 POLICIES TO CONTROL URBAN SPRAWL

In Ch. 4, we presented the econometric results of estimating (4.3) and discussed how these results conform to our two-mode theoretical model's predications. In this chapter, we asses the economic significance of our findings and relate them to the issue of urban sprawl.

5.1 The Sensitivity of Urban Spatial Size to Our Explanatory Variables

To get a sense of the sensitivity of sprawl to our explanatory variables, we calculate elasticities of the urban area's spatial size with respect to the individual explanatory variables, and we calculate their individual effects on the spatial size of the urban area due to a 1-percent change in the variable (all evaluated at the means of the data). These calculations are shown in Table 5.1. We show results only for those explanatory variables whose coefficients are statistically significant at least the 10-percent level. On this criterion, we omit rural land value, r_a , auto fixed cost, f_2 , and intergovernmental grants, G .

Perhaps the first thing to notice in Table 5.1 is that the spatial size of the urban area is not very sensitive to any single explanatory variable: all elasticities are numerically less than one. The largest in magnitude is the elasticity of area with respect to number of households, L , so that a 1-percent increase in the number of households, which is about 800 households, produces an approximately 0.6-percent increase in the spatial size of an urban area. This increases the size of the urban area by about 0.5 square miles, or about 0.4 acres per household, which is slightly larger than the average residential lot size of one-third acre. The next largest is the income elasticity of about 0.5. A 1-percent in-

crease in income, y , which is about \$500, produces an expansion of the mean urban area of 0.4 square miles.

Table 5.1
Elasticities and Area Change
(Due to 1-Percent Change in Explanatory Variables)

<i>Variable</i>	<i>Elasticity</i>	<i>Area Change</i> (sq. mi.)
L	0.615	0.53
y	0.492	0.42
f_1	-0.175	-0.15
t_1	0.107	0.09
α_1	-0.113	-0.10
t_2	-0.238	-0.20
α_2	0.046	0.04
θ	-0.201	-0.17

Variables affecting public transit use have relatively small effects on the spatial size of urban areas. A 1-percent increase in the percentage of commuters using public transit, which is an increase of about 0.02 percent, reduces the urban area by about 0.2 percent, or by about 0.2 square miles. The percentage of commuters using public transit is a proxy for our theoretical variable, and we expect it to be inversely related to the theory's transit fixed cost variable. Therefore, an increase in transit fixed cost increases urban-area size, which is consistent with our theory. A 1-percent increase in public transit

cost per passenger-mile, which is about 0.01 cent and is our proxy for transit variable cost, increases urban-area size by about 0.1 percent, or by slightly less than 0.1 square miles. Finally, a 1-percent increase in the transit subsidy per passenger-mile, α_1 , which is about \$0.01, reduces urban-area size by about 0.1 percent, or about 0.1 square mile.

Variables affecting auto usage also have small effects on urban-area size. A 1-percent increase, an increase of about 0.03 cents, in highway fuel taxes per VMT, our proxy for variable auto cost, produces a reduction in urban-area size of about 0.2 percent, or about 0.2 square miles. A 1-percent increase in county highway subsidies per VMT, an increase of about 0.004 cents, increases urban-area size by about 0.05 percent, or about 0.04 square miles.

Finally, a 1-percent increase in household property tax paid as a percentage of household income, our proxy for the urban-area “income” tax, which is about 0.02 percent, produces 0.2 percent decrease in urban-area size, or about 0.2 square miles.

One conclusion from the above is that the most powerful “policy” variable is the number of urban households. Policy makers can control this variable directly, through urban population constraints, or indirectly through land-use controls. The other policy variables are those affecting the cost of transportation. As we have seen, their effects are relatively small.

Although the individual effects of changes in policy variables are small, joint changes in those variables would produce a larger effect. To see how large an effect, we change all of the policy variables by 1 percent in the appropriate direction. Before proceeding to this exercise, we note that the proxy for fixed transit cost, namely, the percentage of working-age population using transit for commuting, cannot be changed directly

by policy action. Rather, it will change due to changes in other variables, such as transit variable cost and transit subsidy, so we omit that variable from our list. The remaining variables we treat as policy variables are transit variable cost and subsidy, auto fixed and variable costs and subsidy, and the property tax rate. Of these, policy should decrease transit variable cost and the auto subsidy while increasing transit subsidy, auto fixed cost, auto variable cost, and the property tax rate to produce a reduction in urban-area spatial size.

If we change these variables by 1 percent in the appropriate direction, the urban area decreases in size by about 0.8 percent, or 0.7 square mile. This is still a modest change, slightly larger than a 1 percent decrease in the number of households in the urban area.

5.2 Conclusion on Effect of Policy-Relevant Variables on Urban Sprawl

Although our intent was to determine the effect of transport subsidies on urban sprawl, our empirical analysis suggests that the spatial size of the urbanized area is not very responsive to any single policy-relevant variable in our model, which we have defined as variable transit cost, transit subsidy, fixed and variable auto cost, auto subsidy, and property taxes. Even used together and for small changes, namely, 1-percernt increases or decreases, their effect on urban sprawl is small.

Since this thesis focuses on the effect of transportation subsidies on urban sprawl, we emphasize the transportation policies or practices that may be effective in curbing sprawl. Our empirical analysis provides evidence that the spatial size of an urbanized area is negatively related to the transit subsidy and the percentage of the working-age popula-

tion using transit and positively related to transit cost. This suggests that improving transit use may help curb urban sprawl but that the effect may be small.

Transit ridership is influenced by a variety of factors, both internal and external to the transit system. Internal factors refer to those under the control of transit agencies, such as the level of service provided, fare structures and levels, service frequency and schedules, route design, and size of service area. External factors, on the other hand, are those outside the transit agency's control, such as population and employment growth, residential and workplace location, and factors that affect the relative attractiveness of transit to other modes, such as gas price and parking costs (Mineta Transportation Institute 1991).

One disadvantage of public transit use is its high time cost, including both collection and distribution cost and line-haul cost. Transit agencies or transportation authorities may reduce transit collection and distribution costs by adding additional routes, expanding coverage, increasing service frequency, and operating longer hours. They may reduce line-haul costs by busways, queue-jumping lanes, and intersection signal controls that give priority to public transit.

Our empirical analysis finds that transit cost and urban spatial size are positively related. This suggests that reducing transit cost may be another way to improve transit ridership, which may help curb urban sprawl, but, again, the effect is likely to be small. A deep discount fare program is a good way to reduce transit cost. Such policies generally offer a per-ride discount for purchase of a multi-ride pass or transit card, aiming to attract potential riders of low usage or high price-sensitivity. Parking price can be an effective means of increasing transit use for work trips. Since increasing parking costs increases

the relative attractiveness of transit as compared to auto, its effect on transit modal choice is significant. A higher gas price is expected to have the same impact on transit use.

Our empirical analysis suggests that auto costs are negatively related to the spatial size of urban area. Increasing auto costs also helps reduce auto subsidies, which are expected to contribute to urban expansions although our empirical analysis indicates that the spatial size of an urbanized areas increases at a decreasing rate with increase in auto subsidies.

The federal government can indirectly influence the cost of auto use by removing subsidies and tax benefits to oil and gasoline production. State governments may use road pricing to increase auto cost. Road pricing means that highway users pay directly for driving on a particular road or in a particular area. State governments may also change motor carrier regulations and vehicle insurance regulations to increase the cost of auto use. Local governments can offer local transportation demand management programs and allow more flexible zoning requirements to reduce auto use (Victoria Transport Policy Institute 2005).

5.3 Further Research

There are three extensions of the preceding models that we would like to pursue. Specifically, these are (1) including value of travel time, (2) endogenizing congestion, and (3) generalizing mode choice. Only very brief discussions of these extensions are provided.

5.3.1 Value of Time

To deal satisfactorily with the value of transportation time, it is necessary to introduce time elements explicitly into the utility function, the budget constraint, and a time

constraint. A partial equilibrium model of this type has been worked out by DeSalvo (1985). We intend to develop a general equilibrium model of this sort.

5.3.2 Endogenizing Congestion

Congestion is absent from the models discussed above, but congestion affects commuting cost. The first formulations of a general equilibrium urban spatial model incorporating endogenous congestion were those of Mills (1972) and Solow (1972, 1973). Muth (1975) also included congestion in his simulation model. In addition, simulation models (e.g., Muth (1975)) use a congestion function that expresses time cost of travel as a function of traffic density. Henderson (1985) discusses three ways to model congested systems: current demand dependent on past and future travel conditions, current travel speeds dependent on past and future conditions, and the traditional model (a static version of the first and second ways). Since most urban models with congestion use the Mills/Solow framework (Ross and Yinger, 2000), we intend to use the same framework for expositional simplicity.

5.3.3 Generalizing Mode Choice

Mode choice in our approach is determined by where a traveler lives and is dependent on only two parameters, fixed and variable transportation cost. We plan to use the more general approach proposed by DeSalvo and Huq (2005). This approach is similar to the "abstract mode" approach proposed many years ago by Quandt and Baumol (1966), in which a mode is defined by its characteristics. This approach has the advantage of being able to deal with past, present, and future modes, as well as mode combinations, rather than being restricted to modes currently in use.

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