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Revenue management techniques applied to the parking industry

Daniel Rojas
University of South Florida

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Revenue Management Techniques Applied to the Parking Industry

by

Daniel Rojas

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering
Department of Industrial and Management Systems Engineering
College of Engineering
University of South Florida

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     Edward Mierzejewski, Ph.D.
     Kingsley Reeves, Ph.D.

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November 2, 2006

Keywords:  parking modeling, logistic regression, yield management, pricing, neural network, prediction

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DEDICATION

To my Mom, Dad, Mauricio, Juan Alberto, Andres, Fernando, Christine and Valeria
ACKNOWLEDGEMENTS

I would like to thank Dr. Grisselle Centeno for her incredible support and help throughout these years. Her encouragement and motivation was essential to the completion of this work.

Also, I would like to thank my thesis committee Doctors Kingsley Reeves and Edward Mierzejewski whose guidance was extremely important during this journey. Also, I want to thank Aldo Fabregas for his support throughout all these years.

I also want to recognize my family for their support through all these years. Mom you are my hero and you will always be.

Finally, heartfelt thanks go to the two people that have changed my life for the best: Christine (my wife) and Valeria (my beautiful daughter).
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REVENUE MANAGEMENT TECHNIQUES APPLIED TO THE PARKING INDUSTRY

Daniel Rojas

ABSTRACT

The time spent searching for a parking space increases air pollution, driver frustration, and safety problems impacting among other issues, traffic congestion and as consequence the environment. In the United States, parking represents a $20 billion industry (National Parking Association, 2005), and research shows that a car is parked on average 90 percent of the time.

To alleviate this problem, more parking facilities should be built or intelligent models to better utilize current facilities should be explored. In this thesis, a general methodology is proposed to provide solutions to the parking problem. First, stated preference data is used to study drivers’ choice/behavior. Parking choices are modeled as functions of arrival time, parking price, age, income and gender. The estimated values show that choice is relatively inelastic with respect to distance and more elastic with respect to price. The data is used to estimate the price elasticity that induces drivers to change their behavior. Second, neural networks are used to predict space availability using data provided by a parking facility. The model is compared with traditional forecasting models used in revenue management.
Results show that neural networks are an effective tool to predict parking demand and perform better than traditional forecasting models. Third, the price elasticity that induces drivers to change their choice or behavior is determined. Finally, taking as an input the forecasting results obtained from the neural network and the price elasticity, parking spaces are optimally allocated at different price levels to optimize facility utilization and increase revenue.

This research considers a parking facility network consisting of multiple parking lots with two, three and four fare classes and utilizes revenue management techniques as a mean to maximize revenue and to stimulate and diversify demand. The output indicates the number of parking spaces that should be made available for early booking to ensure full utilization of the parking lot, while at the same time attempting to secure as many full price parking spaces to ensure maximization of revenue.
CHAPTER 1

INTRODUCTION

More than 41,000 Americans die as a result of 6 million traffic accidents on the nation roadways system each year. This is the equivalent of 115 people each day, or one every 13 minutes. Traffic accidents injured 3.2 million Americans in 2000. Most crash survivor remains with multiple injuries who account for $150 billion a year on Nation’s Health care costs (Intelligent Vehicle Initiative, 2000). Congested roadways which slow transit vehicles are also another reason why the statistics shown above are so high. The United States Federal Highway Administration forecasts that the severity of traffic congestion will continue to increase at significant rates in all U.S. urban areas, unless specific actions are taken.

Demand for highway travel by Americans continues to grow as population increases, particularly in metropolitan areas. Between 1980 and 1999, route miles of highways increased 1.5 percent while vehicle miles of travel increased 76 percent. The Texas Transportation Institute showed that, in 2000, the 75 largest metropolitan areas experienced 3.6 billion vehicle-hours of delay, resulting in 21.6 billion liters (5.7 billion gallons) in wasted fuel and $67.5 billion in lost productivity. Traffic volumes also are projected to continue to increase. The volume of freight movement alone is forecast to nearly double by 2020 (United States Department of Transportation, 2003). Without intervention, we can only expect these costs to increase as more and more drivers occupy our roads.
The traditional approach to relieve congestion was to invest on the expansion of highway capacity. However, highway capacity has not kept pace with the growth in vehicles miles traveled. As a result congestion has grown steadily worse. Also, highway expansion results in considerable disruptions on traffic. Furthermore, large highway construction projects are expensive, and they do not offer long term solutions. Construction of a large highway may be enough to alleviate congestion for a couple of years; however, after certain periods of time the highway would need another expansion to keep up with the traffic demand. The highway system would reach a point where expansion of new roads and highways would not be possible due to space and cost limitations.

Currently, the United States transportation agencies are changing from the traditional expansion strategy. Agencies are focusing on the optimization of existing infrastructures. The U.S. Department of Transportation (DOT) is one of the leading institutions on the research of new optimization technologies that would reduce the number of traffic accidents in the United States each year. This agency is studying the development and applications of Intelligent Transportation Systems.

Intelligent Transportation Systems (ITS) is one of the leading technologies in the reduction of traffic congestion. Intelligent transportation encompasses the full scope of information technologies used in transportation, including control, computation and communication, as well as the algorithms, databases and human interfaces within intelligent transportation systems (ITS Journal, 2002). Joining these technologies to the transportation system is expected to reduce the number of traffic accidents, deaths, time, and money. The future of ITS is very promising, and already many states around the
United States are implementing this technology to their highway systems. The innovative prepaid toll program, and the 511 real-time traffic information are clear examples of the application of ITS.

1.1 The Parking Industry

Parking plays an important role in the traffic system since all vehicles require a storage location when they are not being used to transport passengers. Most major cities continually struggle with parking limitations, violations and cost. Its availability influences where people travel and how they commute, impacting among other issues, air pollution, driver frustration, traffic safety, and especially congestion which continues to be one of the most critical problems faced by urban America (Axhausen and Polak, 1991) and (Innovative Mobility, 2002). For over a decade, European cities have been investigating intelligent parking mechanisms and are finding substantial benefits. In addition, several German cities that have intelligent parking, such as dynamic parking signs that direct drivers to the nearest vacant parking structure, have reported 15 percent less traffic in their downtowns when compared to cities that do not used advance technology for parking routing (Axhausen and Polak, 1991). The US infrastructure needs to be fortified by advancing knowledge on parking modeling and integrating advances in technology to better plan for capacity needs (Centeno and Rojas, 2006).

In parking terms, capacity planning can be defined as the "science" of predicting the quantity and specific attributes of parking facilities and spaces needed to satisfy the forecasted demand. Currently, capacity planning methods do not provide efficient results because most of the time the huge amount of dynamic input data is ignored and not many
demand scenarios are considered even though a high uncertainty in the forecasts typically exists.

1.2 Parking Problem Overview

It is extremely important to define a “parking architecture” that would combine different technologies to solve the parking problem. The parking problem can be described from two perspectives: drivers’ point of view and parking management point of view. The objective for drivers is to find the closest parking space to their destination at the lowest possible cost and as fast as possible. The objective for managers is to maximize their revenues. An ideal parking architecture must consider these perspectives to find alternative solutions to the parking problem.

Figure 1 presents a general overview of the parking problem and a proposed approach to provide drivers with reliable information on the parking lot state. A parking management system will inform drivers with alternatives on where and when to park.

During the last years, parking reservations systems are becoming more popular especially in large metropolitan areas such as San Francisco, Chicago, Los Angeles, and Philadelphia. Parking reservation systems provides drivers with real-time information on the availability of parking spaces for facilities that provide the service. The basic idea of this type of system is that drivers would reserve a parking space in advance through the internet or cell-phone. Other companies that provide traffic information for navigation systems such as XM Satellite Radio Holdings Inc have introduced parking reservation systems as part of their services. Drivers can search for available parking spaces by looking at the navigation system that provides real-time information of parking occupancy.
The question might be why companies are allocating so many resources to develop this type of systems. The answer could be that parking is actually a huge business. In the United States, parking represents a $20 billion industry (National Parking Association, 2005). This among all the previous statistics presented before make the parking problem very attractive for research purposes. The response of the public for this type of reservation systems is very positive as revealed by a female user from San Francisco to the Wall Street Journal on March 2006 during an interview. She uses the system to reserve parking spaces in the train station that she transfers to commute to work. Without this option, she would have opted to drive her car to work, a non-desirable alternative since public transportation alleviates congestion.

Unfortunately, the problem with reservation systems currently in the market is that they have increased the cost of parking since drivers have to pay a higher fare when they reserve a space in advance. This has provided critics of such systems with facts to diminish the use of them. However, parking reservation systems can be extremely useful if they are used to control parking demand.

For example, managers/planners can control the demand of drivers on certain facilities with high utilization by diverting drivers to facilities with low utilization. Drivers that reserve a parking space in advance would be “rewarded” with lower fares since they have provided parking manager/planners with information in advance. On the

Figure 1: Parking Problem Broader Context
other hand, drivers who do not reserve a space and just show up on the parking facility would be charged a higher fare. This could be achieved through the introduction of pricing strategies. However, to determine optimal pricing strategies, it is necessary to first study the state of the system and to predict future states of parking facilities. For a reservation system to work efficiently, parking managers/planners need a prediction model to determine the number of parking spaces available. After the various states of the system are well known, the next step is to efficiently allocate the parking spaces to demand.

These ideas are the basis for revenue management. Revenue management also known as yield management has mainly been used in the airline and hotel industries. The principle of revenue management is to sell the right product to the right customer at the right time and for the right price. In the parking problem this can be translated to selling the right parking space to the right driver at the right time and for the right price. The previous statements assume that the same product could be sold at different prices and that there are several types of customers for the same product. For example, in the airline industry there are business travelers and leisure travelers. The later being a customer segment that would prefer to pay less for their seats by sacrificing changes in their schedules. Business travelers, on the other hand, would pay higher rates for the same seat because they do not have much flexibility on their schedules. These characteristics are also present in the parking problem setting since a parking space can be sold at different price to different customer segments demonstrating that revenue management techniques are relevant for the parking problem solution.
1.3 General Problem Description and Approach

Figure 2 presents the general problem description and overview to provide solutions to the parking problem. At the top level of the methodology is the parking reservation system. This type of system, as previously described, is already in place in various cities around the US. The main ingredient to have a reservation system in place is a well designed information system that allows parkers to reserve a parking space in advance. In this thesis, revenue management techniques are proposed as an input to parking reservation systems. However, information systems are out of scope. The revenue management process would be applied to the parking problem. First, market segmentation would be studied through a parking behavior/choice survey. The objective of market segmentation is to determine if drivers are willing to pay higher fares under certain factors such as arrival time, time to destination, and price. After drivers have been segmented, the next step is to predict parking space availability. This thesis proposes a neural network model as an alternative to other traditional forecasting models such as moving average, exponential smoothing, Holt’s model, and Winter’s model to predict parking space availability. The results of the prediction model are extremely useful since they would be used as an input later to optimally allocate available parking spaces. Revenue management theory states that a parking space could be sold at different fare rates. The goal is to determine what is the price difference that would influence drivers to change their parking choice. This will be studied through parking behavior/choice models. Finally, these results and the ones obtained from the prediction model will be used to determine how many parking spaces should be reserved for each drivers segment.
1.4 Thesis Organization

This thesis is organized as follows: Chapter 2 identifies the most important studies related to parking studies. Chapter 3 describes the problem statement and the motivation for this research. Chapter 4 introduces revenue management and how it can be adapted to suit the parking problem. Chapter 5 presents market or drivers’ segmentation and presents drivers characteristics regarding their willingness to pay higher fares for the same parking space under certain factors such as arrival time, time to destination, and price. Chapter 6 presents a comparison of different forecasting models to predict parking demand with a proposed neural network model. Chapter 7 presents the capacity control model where parking spaces are optimally allocated to different pricing strategies to maximize revenue. Finally, Chapter 8 presents the conclusions and future research of this study, as well as future research opportunities.
CHAPTER 2
LITERATURE REVIEW

The research methods used for modeling parking systems have varied in complexity, ranging from simple empirical studies and heuristics to advanced techniques for mapping complex parking non-linearity. In the following subsections, a brief summary of the main parking components addressed in the literature, and the models developed are presented.

The reviewed articles have been classified according to the parking factor under study, that is, policy, pricing, choice/behavior, technology and parking design. Table 1 presents an overview of how the parking literature has been organized and the most significant articles reviewed under each category.

2.1 Parking Policy

Parking policy has been studied to provide tools for effective policy decisions such as changes in the number of parking spaces, number of parking facilities or new traffic enforcement. Feeney (1989) presents a review of quantitative results relating to the impact of parking policy on the parking and travel demand. Disaggregated modal choice models; disaggregate parking location models and site-specific studies of parking behavior were examined.
<table>
<thead>
<tr>
<th>Title</th>
<th>Year</th>
<th>Policy</th>
<th>Choice</th>
<th>Pricing</th>
<th>Design</th>
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<tr>
<td>A Review of the Impact of Parking Policy Measures on Travel Demand</td>
<td>1989</td>
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<td>A Parking Model Hierarchy</td>
<td>1991</td>
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<td>The Effects of Parking Measures on Traffic Congestion</td>
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<td>A Stochastic User Equilibrium Assignment Model for the Evaluation of Parking Policies</td>
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<td>A nested logit model of parking location choice</td>
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<td>Mixed Logit Estimation of Parking Type Choice</td>
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<td>Development of parking choice models for special event</td>
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<td>Modeling time. Dependent travel choice problems in road networks with multiple user classes</td>
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<td>Modeling Parking</td>
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<td>A Probabilistic Approach to Evaluate Strategies for Selecting a Parking Space</td>
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<tr>
<td>The Impact of the Parking Situation in Shopping Centers on Store Choice Behavior</td>
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<td>Raising Commuter Parking Prices-An Empirical Study</td>
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<td>Parking Subsidies and Travel Choices: Assessing the Evidence</td>
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<td>An Opportunity to Reduce Minimum Parking Requirements</td>
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<td>The Economics of Regulatory Parking Policies: The (im)possibilities of Parking Policies</td>
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<td>PARKSIM/1: A Network Model for Parking Facility Design</td>
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<td>The Research on the Key Technologies for Improving Efficiency of Parking Guidance System</td>
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<tr>
<td>Parking Guidance and Information Systems: Performance and Capability</td>
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<td>Understanding the Demand for Access Information</td>
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<td>Behavioral Impact of A Broadcast Parking Information Service in Nottingham</td>
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It was found that disaggregated models of parking location choice were highly correlated with parking prices and supply restrictions.

Young and Taylor (1991) developed a hierarchy of microcomputer models and information systems that can investigate parking policy and study the “level of service” provided by parking systems. It outlines six parking models that can be used to address parking issues from an urban to a parking lot level. The most important feature of this hierarchy is that it allows data to be passed from one level to another, enabling a realistic representation of the total parking system. Scholefield, Bradley, and Skinner (1997) developed a computer simulation model TRAM for testing policies to control parking, as well as other types of traffic restraint. The policies of interest in this study were: pricing and capacity reductions. The objective of this study was to determine the extent to which parking controls can be useful in reducing traffic congestion.

Parking policy has a significant impact on urban management. Several authors including Visser and Van der Mede (1986) have concluded that parking policy has an influence on the parking, the transportation, and the socio-economic systems. Despite the significance of parking policy, only a small number of models related to evaluation of parking policy have been built.

Bifulco (1993) develops an interaction model of supply and demand to evaluate various parking policies. The model is applied at Avellino, a small town in southern Italy. A generalized random utility choice model is used to represent such demand. Additionally, a supply model is created in four zones considering parking types such as free on street parking, metered on street parking, on street parking with limited duration, on street metered and limited parking, off street parking, and illegal parking. Model
capabilities and characteristics are compared to other model types including Eldin and CLAMP. Unlike other models, this model can be dynamic and multimodal while allowing multiple users and considering feedbacks on path, mode, destination, timing, and the demand/supply interaction. The model designed proves that, through the parking supply of neighboring zones, a high parking demand in a specific zone is satisfied.

2.1.1 Parking Pricing

Pricing has been proposed as an effective policy option to minimize the parking problem. Miller and Everett (1982) presented an empirical study to determine the impact of parking price increase on commuting behavior at a sample of 15 worksites in metropolitan Washington, DC. The study revealed that removing free parking and raising parking rates “influenced significant shifts to higher-occupancy modes, but that the shifts were not uniform in direction or magnitude across the sites.” Furthermore, the authors provide a discussion of policy implications derived from the study such as:

- “Parking pricing strategies can be effective in reducing the number of work commute automobile”
- “The effectiveness of new parking rates depends on many factors (external and site specific)”
- “Under certain parking supply conditions, parking pricing strategies can have adverse impacts, such as increasing the use of single-occupant autos”
- “Some carpoolers may shift to transit as parking rates increase”
- “Unlike most other transportation system management strategies, imposing parking prices can result in significant revenues”
Willson and Shoup (1990) reviewed several empirical studies of how employer-paid parking affects employees’ travel choice. It was concluded that parking subsidies increase solo driving. It was also found that when parking subsidies were removed, a significant amount of solo drivers shift to carpools and/or transit. The case studies reviewed reveal that ending employer-paid parking reduces the number of solo drivers by between 19 and 81 percent, and reduces the number of autos driven to work by between 15 and 38 percent.

Shoup (1995) studied the effects legislation passed in California with the objective of reducing traffic congestion and air pollution. The legislation required employers who subsidize employee parking to offer employees the option to take the cash value of parking subsidy, in lieu of the parking itself. The legislation also required cities to reduce the parking requirements for developments that implement a parking cash-out program. The hypothesis is that by shifting subsides from parking to people will encourage drivers to carpool, ride mass transit, bicycle, or walk to work. A study of how the option to cash out employer-paid parking will reduce parking demand and recommends a reduction in minimum parking requirements was presented.

Several economists have advocated that drivers are not paying the true cost for commuting and most drivers park for free. The studies presented above revealed that removing free parking is an efficient tool to influence parking demand and reduce levels of congestion and air pollution. Calthrop, Proost and Dender (2000) used a numerical simulation model to study the efficiency gains from different parking policies with and without a simple cordon system. The authors show that it is necessary to simultaneously determine the pricing of parking and road use. This study considers an analysis of the
welfare gains by combining both parking and road pricing simultaneously. The model shows that increasing parking prices produces higher welfare gains than the use of a single-ring cordon scheme. However, this result is lower than a combination of a cordon charge with pricing parking spots.

It has been shown in the literature quantitative and qualitative the many advantages of pricing strategies when parking supply is limited. Researchers have studied the impact of pricing on the workplace and how employees’ react to these changes. The expectation is that employees’ will react to these changes by opting for different transportation modes. Also, it has been shown that pricing strategies can be made for each parking spot or a combination with road use. Vehoef, Nijkamp, and Rietveld (1994) present an economic analysis of regulatory parking policies as a substitute to road pricing. Three reasons of why parking fees are superior to physical restrictions in parking space supply are discussed. The disadvantages of regulatory parking policies in comparison to a system of road pricing are also stated: regulatory parking policies will always remain a second-best option by nature, the risk for spillover (drivers who park in adjacent areas), enforcement of the policy may by more expensive in the long run.

2.1.2 Parking Choice/Behavior

Over the years several models have been developed to investigate drivers’ choice when deciding upon a parking location or a parking space. The literature on parking choice assist parking policy makers to better understand the behavior of drivers at the time of choosing a parking space. Hunt and Teply (1993) provided a nested logit model of parking location choice using revealed preference data. The model was evaluated
using data for a central business district (CBD). The authors conclude that parking location choice is influenced by factors other than money cost and proximity to final destination. Other factors that influence parking location choice are: position relative to the trip being made, nature of the parking surface, and time for searching a parking space.

Hess and Polak (2004) presented the results of a study of parking choice behavior, based on stated preference data, collected in various city centre locations in the UK. The authors presented a mixed multinomial logit (MMNL) model. The model is capable of including the random variation in preferences within groups of drivers that has been previously ignored in the literature. Through the inclusion of this factor other relevant factors such as access, search and egress time were identified.

Sattayhatewa and Smith (2002) present a study on how drivers choose a parking lot during a special event. The authors present a lot choice model (using logit function) and the joint parking lot destination choice and assignment model (using user equilibrium traffic assignment and entropy maximization). Results reveal that walking time and driving cost are very important for drivers.

Other important factors that affect parking choice are presented in Lam et al. (2005). The authors found that parking behavior is influenced by travel demand, walking distance, parking capacity, and parking price.

Various parking choice models have been built to analyze and understand the decision process that drivers experience on a daily basis when selecting a parking space. For example, Arnott and Rowse (1999) developed four models of parking including a structural model, an extended model incorporating several realistic complications, a general equilibrium model used for welfare analysis, and a model to study stochastic
characteristics of parking. For simplicity, a city lying on the circumference of a circle is used to explore parking on a spatially symmetric area and disregarding flow congestion.

In addition, Cassady and Kobza (2000) study the driver’s parking decision process in a stochastic environment. By representing the results of stated preference surveys through network models, parking demand is analyzed. Parking strategies that drivers use are analyzed: 1) pick a row, closest space and 2) cycling. In the first strategy, the driver chooses a row and the closest available space from that row. In the second strategy, the driver chooses a row and, only if any of the closest 20 spaces are available, the closest space is selected. If not, the driver continues to the next row. If any of the closest 40 spaces is available, the closest space is selected. Otherwise, the customer comes back to the other row and chooses the closest available space.

Performance measures studied include total walking distance, search time, and the sum of these two values. Conclusions regarding the most accurate strategy to predict performance measures are made. Indeed, the first strategy yields more accurate results for search time as well as the combination of search time and walking time. In contrast, the second strategy provides more accurate results for walking time.

Van der Waerden, Borgers, and Timmermans (1998) designed and validated a hierarchical logit model of parking lot and store choice behavior to analyze how parking policy affects driver’s behavior. Data is collected at City-Centre Veldhoven, a shopping center in the Eindhoven Metropolitan area at the Netherlands. Conclusions made after the study include: 1) walking distance has an impact on driver’s choice, 2) the driver’s choice process could be accurately represented by a sequential decision making process,
and 3) the probability of selecting a parking space decreases as the size of the parking lot increases indicating that drivers tend to avoid long walking distances.

2.1.3 Parking Design and Technology

Parking Design models are extremely important for transportation planners since they can assist to determine the location of parking facilities and to evaluate design alternatives of parking facilities. The interaction among the parking facilities component and traffic systems was presented in Young (1986) through a simulation model (PARKSIM/1). Planners can use this model to determine the efficiency of a particular parking lot layout. Another discrete simulation model was presented by Le and Young (1997). This model allows planners to understand the interaction among location of shops and the design of parking and traffic systems.

Technology is playing an important role in the new design of parking and traffic systems. New developments of parking technology have opened the door for researchers and planners to study and understand the effect of these technologies into the parking system. Maccubin and Hoel (2000) developed a methodology to evaluate the different alternatives in technology for improving parking management at change-mode facilities. The authors tested the methodology developed using a computer simulation model to identify the benefits of different intelligent transportation systems solutions.

Parking Guidance Information Systems is one of the technology alternatives used around the world to alleviate congestions. Yang, Liu and Wang (2003) presented a study on the key technologies needed for a successful implementation of a Parking Guidance and Information System in the city of Beijing, China. They also presented some of the problem that raised during the installation and running phase such as: (1) parking fees are
not the same throughout the network; therefore, drivers do not move to parking lots as directed by the system; (2) users have difficulty understanding the meaning of the messages on the boards; (3) a survey was distributed an 20 percent of the respondents were aware of the system, but had not used it.

After a PGI system is implemented, statistical analysis of the effects of the PGI system on driver’s behavior are developed from surveys. Elements studied in such surveys include level of awareness, understanding, usage of PGI systems, and stated preference of information displayed. In a number of papers, surveys are designed to differentiate results according to physical, trip purpose, and service time characteristics. Additionally, to identify the most crucial information to be displayed for a wide range of real time information such as park location, availability, waiting times, and prices are being studied. Conclusions regarding possible improvements to PGI systems are made based on the results of the survey. Suggestions to improve the implementation of PGI systems include increasing awareness and displaying new messages with traffic information appealing to drivers. To study the PGI system, tools used include data collection strategies (location, survey technique, survey method, sample size, etc.), statistical inference, and logit models of parking choice.

A simplified system architecture of a PGI system is provided by Polak, et al. (1990) to individuals who are not familiar with the operations of PGI systems. Other topics discussed include benefits, components, data collection, data transmission, and data processing of PGI systems. In addition, the alternatives available for displaying information such as techniques, locations, and the information on the signs are studied. Approaches to the design and control of PGI systems are described as well. Results of
different case scenarios from previous impact and behavioral studies to analyze and evaluate PGI systems are briefly reviewed. Finally, some implications of current developments for future PGI systems are discussed.

A more detailed study of the impact of PGI systems which evaluates the effects and driver reactions of PGI systems in several cities in Japan is presented by Thompson and Takada (1995). The driver’s cognitive information transmission process consisting of awareness, observation, understanding, belief, and usage of PGI systems is used to understand the most critical nature of driver’s reactions.

A questionnaire survey was distributed, and a statistical analysis to measure the impact of PGI signs was provided based on the survey results. The information requested in the survey was based on revealed preference data. In addition, drivers were asked to provide information such as purpose of the trip, trip origin, trip frequency, trip duration, vehicle type, gender, and age. The results of the survey suggest that different types of drivers want different types of information to be displayed. The most requested parking information type by drivers is availability of car parks (61.1%), followed by waiting time at car parks (34.3%), location of car parks (29.4%), and how to find available car parks (22.0%). The study also shows which types of drivers are most likely to use PGI signs and that there is still a lack of believe in PGI systems.

An alternative to PGI systems is reviewed and evaluated by Polak, Vythoulkas, and Chatfield. This paper determines the causes of parking congestion and explains the arguments of why a broadcast parking information was implemented in Nottingham. Furthermore, this article analyses the effects of the broadcast parking system on driver’s behavior through survey. Thus, regular users of the system according to attributes such
as gender, frequency of parking and search behavior are identified. The analysis developed is used mainly to monitor the progress after the implementation of the broadcast parking system. Conclusions regarding possible improvements to the system including increasing awareness and displaying new messages with traffic information are suggested.

2.2 Summary

In this chapter, the parking literature has been reviewed for the parking factors considered. This thesis touches upon two of these: parking choice/behavior and parking pricing. As previously described, parking choice/behavior modeling approaches consider how drivers would react to changes in the availability or location of the parking facilities. Impact would be reflected on the day/time of the trip by changing destination or discarding the trip due to parking concerns. These models are typically formulated as mode choice. Traditional mode choice models study how drivers respond to changes in the supply and operation of parking facilities. These responses are typically studied using logit models (logit models study how drivers made choices among a finite set of alternatives) based on stated (hypothetical scenarios) and revealed preference (actual data) data. Researchers have concluded that several factors such as parking price, walking distance, driving distance, parking surface, parking location, etc. influence parking choice/behavior. However, to our knowledge there is no evidence that the arrival time factor have been previously studied.

Arrival time represents how much time in advance a driver has arrived to his/her destination. For example, a driver may arrive 5 or 15 minutes early to a meeting, class, or flight. Taking into account the time that it would take him/her to find a parking space
and walk to the final destination, the driver has to make the decision between parking close to the final destination (typically higher fares) or further away (lower fare). Therefore, in this thesis the arrival time factor is studied to determine the impact of time arrival on drivers’ parking choice behavior.

The other parking factor that studied in this research is parking pricing, which has received attention from transportation researchers and economists. Many models have been developed to analyze how increasing parking price affects space utilization, transit service, work trips and single-occupancy vehicle (SOV). The hypothesis is that arrival time and willingness to pay are highly correlated. In other words, drivers are willing to pay higher fares when they have an urgency to reach their destination. This research will explore this hypothesis and will attempt to represent price elasticity.
CHAPTER 3

PROBLEM STATEMENT

3.1 Introduction and Motivation

The problem considered in this research is revenue management in a parking facility network consisting of multiple parking lots with different number of fare classes. The objective is to maximize revenue and to stimulate and diversify demand. The manager in a parking facility should decide how many parking spaces to reserve for customers or organizations willing to pay higher fares for spaces located closer to their destination. A parking reservation system will identify customers who book in advanced or individual early bookings who will receive a discounted fare for their early or extended booking. Therefore, the decision is to determine how many parking spaces should be made available for early booking to ensure full utilization of the parking lot, while at the same time attempting to secure as many full price parking spaces to ensure maximization of revenue. This is a complex problem because in most instances demand cannot be determined with certainty. Also, some customers who book in advanced may not show up, and the duration of stay for those who arrive as planned will vary; that is, some drivers will stay longer than others.

The parking problem is one of matching a probabilistic and sometimes unknown demand to a set of finite resources in a manner which will optimize profits or utilization of parking facilities. Parking facilities experience peak and low demand periods. The main problem is that during peak periods it is impossible to find an available parking
space which results in drivers entering the parking lot to search a space that is not available. Therefore, parking managers need tools to advice drivers that there are not available parking spaces. There are two ways to approach this problem. One is to present drivers with current information of the state of the parking facilities, which can be accomplished with the introduction of Parking Guidance Information Systems (PGI) that show drivers where they can or cannot find a parking space. This is an alternative currently in place; however, there are some problems with PGI signs because drivers tend to not follow the information provided due to a lack of believe on the accuracy of it. The other alternative is to stimulate and diversify the demand with the introduction of pricing strategies.

Usually, parking facilities form a network of resources with the objective of providing storage space for a final destination. For example, a university parking network is formed with a large number of parking lots and garages. Each parking lot has the objective of providing storage for cars for a specific building (final destination). The problem with this network is that certain parking facilities are utilized more than others. According to the previous definition of revenue management, this tool would allow managers to shift some of the high demand for certain parking facilities to other lower demand facilities. This can be done by setting different price schemes. It would allow managers to control and shift the demand and it also provides a source of revenue.

3.2 Research Objectives

The objectives of this research are as follows:

- To develop a general methodology which extends revenue management techniques to the parking problem
To study and model parking choice/behavior in terms of parking prices, time to
destination (walking time + driving time) and arrival time using stated
preference data

To explore and compare neural networks as an alternative to traditional
forecasting models to predict parking demand

To compare different revenue management models to optimally allocate parking
spaces

Specifically, the following questions will be answered in this thesis:

- Can revenue management techniques be applied to the parking problem?
- Can neural network be used to predict parking availability? Does neural network
  perform better than other traditional forecasting models?
- What factors affect drivers’ behavior? Is it arrival time, price, and/or time to
destination (walking time + driving time)
- Will drivers pay higher rates when they are under a time constraint? That is, will
drivers pay a higher fare for parking closer to their final destination because they
might be late to their meeting, class, flight, etc.?
- What is the price difference that would induce drivers to change their parking
behavior?
- How many spaces should be made available initially at various price levels (or,
alternatively, for a given allocation scheme, what are the optimal pricing levels)?

3.3 Research Methodology

As shown in Chapter 1, (Figure 2 – Level 2), a general methodology is proposed
to provide solutions to the parking problem. The revenue management process will be
applied to the parking problem. First, stated preference data will be used to study drivers’ choice/behavior. The data collected will be analyzed through logistic regression. Second, neural networks will be used to predict space availability using data provided by a major parking facility. The model will be compared with traditional forecasting models used in revenue management. Third, the price elasticity that induces drivers to change their choice or behavior will be determined. Finally, taking as an input the forecasting results obtained from the neural network and the price elasticity, parking spaces will be optimally allocated at different price levels to optimize facility utilization and increase revenue.
4.1 Introduction

Revenue Management originated with the deregulation of the US airline industry in the late 1970’s. The entrance of new airlines offering extremely low fares created a complex challenge for major airlines. Revenue Management was introduced as a competitive tool to respond to the new challenges. It allowed airlines to compete on all levels of the market without compromising or decreasing revenues. In addition, it enabled companies to better match the supply and demand by introducing pricing strategies. Today, revenue management has increased in popularity and is used not only in the airline industry but in firms with constrained capacity such as hotels, cruise lines, car rentals, railways, and hospitals.

The application of a revenue management system is not appropriate for all industries. Businesses that have successfully embraced revenue management have many or all of the following characteristics:

- **Limited capacity or resources** – only a fixed amount of products/resources is available, and additional inventory cannot be added without a significant capital investment.

- **Variable Demand** – low demand and high demand times can be identified.

- **Perishable Product/Service** – at certain point in time the product or service will become worthless and it can no longer be sold.
• *Market segmentation* – some customers are willing to pay different prices for the same product or service.

• *Advanced sales* – through reservation systems combined with other technologies, selling products or services in advance.

The parking industry has some similarities with the airline and hotel industry where revenue management is mostly used. Table 2, compares each characteristic previously described in the context of the airline, hotel, rental car, and the parking industry.

<table>
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<th>Table 2: Revenue Management Characteristics and Examples</th>
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<tr>
<td><strong>Airline</strong></td>
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<td><strong>Limited Capacity</strong></td>
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<td><strong>Variable Demand:</strong></td>
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<td><strong>Perishable product Service</strong></td>
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<td><strong>Market Segmentation:</strong></td>
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Parking garages have a fixed number of parking spaces to sell (limited capacity). Also, some cars may stay longer in a parking space than others (variable demand). Spaces have daily opportunities to be sold (perishable product). When traveling by air, a customer who is late for a flight may be willing to pay more for a closer parking space or any available space easy to find (market segmentation). Parking Reservation Systems are being implemented in the United States. This type of systems allows drivers to reserve a parking space in advance (advance sales). In 1998, the revenue management problem was identified by Chatwin (1998) as the hottest area of new research in traffic management. Evidently, the parking industry represents a potential area to apply revenue management techniques for improvement.

Parking plays an important role in the traffic system since all vehicles require a storage location when they are not being used to transport passengers. Most major cities continually struggle with parking limitations, violations and cost. Parking facilities experience peak and low demand periods. The problem increases during peak periods when it becomes challenging to find an available parking on a particular parking lot location. One alternative to this problem is to stimulate and diversify the demand with the introduction of pricing strategies. Pricing is an important element that can be used to increase profits by better matching supply and demand. The use of pricing strategies to increase the profit of a limited supply of assets is a common practice of Revenue Management.

The manager in a parking facility can use several revenue management concepts to stimulate and diversify the parking demand. For example:
• The parking manager can charge a lower price for drivers willing to reserve their space far in advance and a higher price for drivers looking for a space at the last minute.

• The parking manager can also charge a lower price for drivers with long-term contracts and a higher price for customers looking for a space at the last minute.

• The parking manager can charge a higher price during periods of high demand and lower prices during periods of low demand.

All of these are strategies that can be used to stimulate and diversify the parking demand. However, before they are applied, a sounded procedure must be developed. The following section presents a specific methodology designed to implement revenue management techniques into the parking problem.

4.2 Parking Revenue Management Process

A necessary characteristic to implement revenue management is a reservation system. Following examples from the airline, restaurants and car rentals, more companies and cities are offering reservation systems that allow people to reserve parking spaces online or by cell phone. Some cities that are offering this service include Baltimore, Chicago, New York, Philadelphia, Boston and San Francisco. Parking reservation systems are seen as a competitive tool for parking companies who want to provide a better service to their customers. However, there are two major problems with the system. First, the reservation system shows drivers the availability of parking spaces through sensors embedded in the parking facilities. The problem is that this information only shows the availability at “a point in time”; therefore, drivers can reserve a space only for that period of time. In other words, drivers are not able to reserve a parking
space two or three days in advance because the information is unavailable. The second problem is that customers usually have to pay more for the parking space if they reserve it in advance. Through revenue management these problems are addressed, and solutions to enhance parking reservation systems are developed.

Figure 3 shows the parking revenue management process that should be followed at the time of incorporating revenue management into parking reservation systems.

Figure 3: Parking Revenue Management Process

First, parking managers must identify market segments for the same parking spaces. In the case of parking, market segmentation can be described as follows. There are drivers who have an emergency to arrive at a final destination; therefore, they are willing to pay a higher cost for a space closer to their destination. On the other hand, some customers may not be concerned with closeness to their final destination; they are in this case more concerned with the cost of parking. These drivers are willing to walk a longer distance for a lower parking price. To accomplish this, we developed a stated preference parking survey which is discussed in detail in Chapter 5.

After the market has been segmented, the next step is to predict customers demand (for every market segment) and space availability. In the case of the airline industry, airlines need to forecast only the demand of customer for different price fares. Since the capacity of an airplane varies from “full” and “empty” after a trip, airlines do not need to forecast the seat availability. On the other hand, hotels need not only to
forecast demand for different customer segments, but also room availability. It is necessary to forecast room availability because one customer may stay in a room for one day or for three, four, five days. This is very similar in the case of parking where some spaces may be occupied for minutes while others may be occupied for hours. Therefore, for parking systems it is necessary to forecast customers demand and space availability. Traditionally, companies that have implemented revenue management use several forecasting models to predict customer demand. These companies usually do not share their forecasting models for obvious reasons. However, it is known that the most popular models used in practice are linear regression, moving average, and exponential smoothing. All these models can be used to predict drivers’ demand and space availability. In this thesis we explored these models and several others and test which one is better and if any outperforms the others for the parking scenario. Moreover, we explored neural network as a tool to predict drivers demand and space availability. In Chapter 6, we discussed neural network and compared it with other traditional forecasting model.

The forecasting model is the base of a successful revenue management model because this information will be latter used by managers/planners to implement an adequate pricing strategy. This is the third step of the parking revenue management process. Parking managers/planners must decide how many spaces to reserve for full paying customers, and how many spaces to reserve at a discount. This can be seen as an inventory control problem. In Chapter 7, we present a model that addresses the space inventory problem.
The last step in the implementation of the revenue management systems is to dynamically recalibrate the models. This means to monitor and control performance and update market response. It is necessary to constantly collect market data to update and recalibrate the forecasting and pricing models.

4.3 Revenue Management Literature

Research on revenue management started in 1960’s with the problem of overbooking. After airlines adopted the policy that a customer can cancel a ticket at any time without penalty, airlines were faced with the problem of overbooking and bumping. Researchers have studied the revenue management problem using a variety of approaches. The important elements of the revenue management problem that have been investigated are: seat inventory control, demand forecasting, overbooking and pricing.

4.3.1 Seat Inventory Control

The objective of seat inventory control models is to optimally allocate seat inventory to passenger demand before the flight departs. The objective is to find the optimal number of seats that should be sold to each passenger segment. This will create a booking control policy that determines if a passenger request should be accepted or rejected at different periods of time before the flight departs.

This problem has been studied as a single leg seat inventory control problem and as a network seat inventory control problem. In the single leg seat inventory control problem every flight is optimized separately. The booking policy for each flight is determined and optimized independently of all other flights. There are two categories of single leg solution methods: static and dynamic solution methods.
Static solution optimizes the seat inventory taking into account static data. Littlewood (1972) introduced the idea of marginal revenue. The objective is to equate the marginal revenue in each of the two fare classes. The idea is to not accept a low fare request when the expected revenue of selling the same seat at the higher fare is high. This model is known as Littlewood’s rule. In 1987, Belobaba extended Littlewood’s rule to multiple nested fare classes and introduced the term expected marginal seat revenue (EMSR). The main disadvantage of this method is that it does not yield optimal booking limits when more than two fare classes are considered. To overcome this difficulties, Curry (1990), Brumelle and McGill (1993) and Wollmer (1992) introduced optimal policies for more than two classes.

Dynamic solution methods for the seat inventory control problem monitor the state of the booking process over time and decide to accept or reject a request based on the state of the system at that point in time. Some of the methods used to solve this problem include: discrete-time dynamic programming model -- where demand for each fare class is modeled by a no homogeneous Poisson process, and dynamic and stochastic knapsack problem.

In network seat inventory control, the complete network of flights offered by the airline is optimized simultaneously. One way to do this is to distribute the revenue of an origin-destination itinerary control to the individual legs. Williamson (1992) investigates different prorating strategies, such as prorating based on mileage and on the ratio of the local fare levels.
4.3.2 Demand Forecasting

Demand forecasting is of critical importance in revenue management because booking control policies make use of demand forecasts to determine the optimal booking control strategy which performs badly.

Beckmann and Bobkowski (1958) compare different distributions to fit passenger arrival data (demand distributions). The authors compare Poisson, Negative Binomial, and Gamma distributions. Results show that the Gamma distribution is a good fit for passenger arrival data. Lyle (1970) models passenger demand distributions as composed of a Gamma systematic component with Poisson random errors which leads to a negative binomial distribution. Other studies such as Shlifer and Vardi (1975) and Belobaba (1987) reveal that the normal distribution is a good approximation for passenger demand distributions.

The arrival process of individual booking request has been studied as a Poisson process. However, demand has also been studied using historical data. Taneja (1978) described the use of traditional regression techniques for aggregate airline forecasting. Furthermore, Sa (1987) used regression techniques and concluded that regression techniques outperform traditional time series models or historical averages. McGill (1995) developed a multivariate multiple regression to test the correlation in multiple booking classes. Several researchers have also used simple smoothing techniques as a forecasting tool.

Other researchers such as Ben-Akiva (1987) have opted for forecasting demand using discrete choice behavior models which are typically model through logistic regression.
4.3.3 Overbooking

Airlines have to deal with no-shows, cancellations, and denied boarding. Therefore, in order to prevent a flight from taking off with vacant seats, airlines tend to overbook a flight. The objective is to find the optimal level of seats that should be sold over the capacity of the flight. Therefore, overbooking a flight reduces the probability that a seat will depart empty; however, it creates a risk of having more passengers than available seats. Overbooking is the oldest and most successful of the revenue management techniques. “It has been estimated that in the airline industry 50 percent of reservations result in cancellations and no-shows and 15 percent of all seats will go unsold without some form of overbooking” (Talluri and Van Ryzin, 2004 pp. 130). The overbooking problem has been studied from two approaches: static overbooking models and dynamic overbooking models. Static overbooking models do not take into account the dynamics of customer reservation and cancellation requests over time. The model find the optimal number of seats to overbook taken as an input the estimates from the current time until the day of departure. These optimization models were studied by Beckman (1958), Thompson (1961), and Taylor (1962). These models find the maximum number of seats to overbook for one fare class. Shlifer and Vardi (1975) extended the models to allow two fare classes and a two-leg flight.

The dynamic overbooking models take into account the dynamics of customer reservations and cancellations over time. Rothstein (1968) presented a dynamic programming model for the overbooking problem. Alstrup et al. (1986) showed a dynamic programming model for two fare classes.
4.3.4 Pricing

Pricing strategies are used in revenue management as a mechanism to respond to market demand. The objective of pricing models is to find the optimal combination of price adjustments to maximize revenue. Economists have long advocated that pricing is an effective strategy for strategic and marketing decisions. Dana (1996) concluded that firms who offer products at different prices and control the capacity for the low prices, are in a unique competitive equilibrium. Gallego (1996) proposed a deterministic model to study pricing and market segmentation. The model is able to capture demand dispersion and demand recapture. Watherford (1994) presented a model that assumes normally distributed demands. The mean demand is modeled as a linear function of price.
CHAPTER 5
MARKET SEGMENTATION

5.1 Introduction

The goals of market segmentation are to understand how customers are buying, what they value, and how much are they willing to pay (Talluri and Van Ryzin, 2004). To differentiate between the various segments, the firm must define a set of product attributes or customer characteristics for a given segment. For example, in the airline industry businesses travelers are willing to pay higher rates for a seat than leisure travelers. Businesses travelers would pay higher fares for a flight that matches his/her schedule. On the other hand, leisure travelers would vary their schedule for a lower fare. Some examples of market segments in the airline industry include business, leisure, students, children, youth, seniors, and military.

Market segmentation can also be applied to the parking industry. The question is if there are different customer segments in parking, and if true, how to differentiate among them. To answer these questions, we would use an explicit screening mechanism based on observable characteristics. In other words, we would study drivers’ behavior and then we will classify them according to the observed type. Figure 4 shows some common segments bases for the parking industry.
A brief explanation of each one of these segments follows.

- *Demographics*-(age-based, gender-based, etc). For example, younger people may have less income; therefore, they are willing to pay less.

- *Time of purchase*-drivers who reserve or buy a space in advance want to pay less. On the other hand, drivers who do not reserve in advance and just show up into the parking lot are willing to pay higher fare.

- *Day of the week*-parking lots have peak demand during certain days of the week. For example, a parking lot at the airport would have peak on Mondays and lower demand on Saturday.

- *Time of the day*-parking lots also have peak demand during certain hours of the day. For example, in a university parking lot demand would be low during early morning hours (5am-7am); however, demand would be at a peak during the middle of the morning (10am-1pm). Then, demand would decrease in the afternoon hours.
• *Length of stay-*some drivers may use a parking space for a few minutes while others may use the parking space for hours.

Each one of these segments represents an opportunity for manager/planners to apply different pricing strategies for each segment.

We conducted a stated preference survey to study drivers’ behavior. The objective of the survey is to identify different segments in the parking industry. Latter, we would apply different pricing strategies that would allow us to maximize parking revenue and control parking demand. The next section describes in detail the survey and its results.

5.2 Stated Preference versus Revealed Preference Survey

Previous parking demand modeling approaches have considered how parking demand (drivers) would react to changes in the availability or location of the parking facilities. Impact would be reflected on the day/time to do the trip or even changing destination or discarding the trip due to parking concerns. These models are typically formulated as mode choice. Traditional mode choice models study how drivers respond to changes in the supply and operation of parking facilities. These responses are typically studied using logit models. Logit models study how drivers made choices among a finite set of alternatives based on stated (hypothetical scenarios) and revealed preference (actual data) data. Figure 5 is an illustration that compares a stated preference survey versus a revealed preference survey.
In the revealed preference survey, the participant would have to choose (or reveal) the option that he/she currently uses to commute to work. In the stated preference, the participant would have to choose among three alternatives (the rapid train is not currently working). The objective of this survey is to study the impact of the introduction of a rapid train into the current transportation network and to forecast its utilization.

Stated preference surveys are used to develop choice models and to estimate the impact of each factor (i.e. car-bus-rapid train, price, and time). The advantage of stated preference surveys over revealed preference is that they are extremely useful to study the impact of new options into the actual market. The other advantage of stated preference surveys is that the researchers can control the factors under study. The disadvantage of
stated preference surveys is that respondents can answer one way under a hypothetical situation and respond another way under the real situation. It is necessary to take this into account at the time of producing conclusions that may overestimate the response of individuals.

5.3 Stated Preference Parking Survey

A stated preference survey was conducted in this thesis to identify how drivers will react to changes on prices and which parking facility would be selected for various set of scenarios and circumstances. The results will help us to differentiate among different segments in the parking industry. The data collected from the survey is analyzed through logistic regression. Fifty one subjects were surveyed in a pencil-and-paper survey. Table 3 shows the demographics from the subjects interviewed.

<table>
<thead>
<tr>
<th>Demographics</th>
<th>Subjects</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>33</td>
<td>65%</td>
</tr>
<tr>
<td>Female</td>
<td>18</td>
<td>35%</td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 20</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>20-29</td>
<td>48</td>
<td>94%</td>
</tr>
<tr>
<td>30-39</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>40-49</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>50-59</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>60 or older</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>Employed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full-time</td>
<td>39</td>
<td>76%</td>
</tr>
<tr>
<td>Part-time</td>
<td>12</td>
<td>24%</td>
</tr>
<tr>
<td>Income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$15,000</td>
<td>49</td>
<td>96%</td>
</tr>
<tr>
<td>$20,000</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>$30,000</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>$40,000</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>$50,000</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>$60,000</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>$70,000</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>Pay more for reserved space</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YES</td>
<td>40</td>
<td>78%</td>
</tr>
<tr>
<td>NO</td>
<td>11</td>
<td>22%</td>
</tr>
<tr>
<td>Reserve space in advance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YES</td>
<td>35</td>
<td>69%</td>
</tr>
<tr>
<td>NO</td>
<td>16</td>
<td>31%</td>
</tr>
</tbody>
</table>
5.3.1 Stimulus Material

Subjects were presented with twelve parking scenarios that were arranged in random order (Appendix A). Each of the parking scenarios consists of two parking facilities labeled Lot A and Lot B, and a final destination. Below each parking lot, the time that will take to reach the destination from the parking lot selected and the price for the parking lot were presented. Furthermore, a sign indicated how early subjects are from a meeting/class or activity. Drivers would choose either Lot A or Lot B taking into account all the factors presented (arrival time, price, and time to destination). The arrival time and price were the only factors that changed among scenarios. Subjects were also asked a set of questions that would help us to identify each subject according to age, gender, and income.

5.3.2 Survey Design

Of the twelve parking scenarios, six present drivers the constraint that if they choose Lot B they may be late to their meeting/class or activity. The objective is to study if drivers are willing to pay more under this time constraint. The other six parking scenarios present drivers the option of choosing Lot A or B without a time constraint. If they choose either Lot A or B, they would be on time to their meeting/class/activity. The objective in these scenarios is to measure if drivers are more concerned about time to destination or cost. Also, we would like to measure what is the price difference that induces drivers to change their parking behavior. For example, a subject who is presented the option of choosing Lot A ($10) versus Lot B ($5) (or 100% increase in price) may choose Lot B (the lower fare). However, as the price difference decreases Lot
A ($6) versus Lot B ($5) (or 20% increase in price) the subject may be willing to choose Lot A ($6) because the price difference is not that high.

Subjects were asked to choose a parking lot where they will park by considering the time to destination, price and arrival time. The time to destination indicates the time that will take the subject to reach the final destination from the parking lot selected. The time to destination is the walking time (from the parking lot to the destination) plus the driving time (from the start sign to the parking lot). Traditionally, these factors, walking time and driving time, are studied separately. Originally, the survey was designed considering these factors separately; however, when we presented the survey to subject, it was noted that drivers would add the time of each factor and combine them into a single time. This tendency occurred because the first factor that drivers take into account is the arrival time. Their next decision would be based on this factor. In other words, subjects choose the parking lot of their preference based on the possibility of being late and based on the price. Therefore, the walking time and driving time were combined into a single factor (Time to Destination). This makes it easier for subject to fill the survey since they do not have to make the calculations to determine if they would be late to their meeting/class or activity. Price refers to the cost that drivers will have to pay if they choose either parking lot. Arrival time indicates how early drivers are from their meeting/class or activity. Table 4 shows the twelve scenarios presented to the subjects.
Table 4: Parking Survey Scenarios

<table>
<thead>
<tr>
<th>SCENARIO</th>
<th>FACTORS</th>
<th>LOT A</th>
<th>LOT B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $10</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>15 min early</td>
<td>15 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $9</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>15 min early</td>
<td>15 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $8</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>15 min early</td>
<td>15 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $7</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>15 min early</td>
<td>15 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $6</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>15 min early</td>
<td>15 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $5</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>15 min early</td>
<td>15 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $10</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>5 min early</td>
<td>5 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $9</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>5 min early</td>
<td>5 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $8</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>5 min early</td>
<td>5 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $7</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>5 min early</td>
<td>5 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $6</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>5 min early</td>
<td>5 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Time to Destination</td>
<td>10 min $5</td>
<td>5 min $5</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>5 min early</td>
<td>5 min early</td>
</tr>
<tr>
<td></td>
<td>Arrival Time</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is important to note that this is not a full experimental design. The reason is that a full experimental design will include a total of 36 scenarios. Twenty four of the possible combinations were discarded because they did not contribute with new data.
They were basically a repetition of the 12 scenarios considered. Furthermore, thirty six scenarios may fatigue subject; therefore, this may induce to unrealistic answers.

5.3.3 Survey Procedure

Subjects were asked to choose which parking lot they would prefer by taking into account all the factors presented (arrival time, time to destination, and price). A brief summary of the scope of the project and the instructions were given to the subjects (Appendix A). The instructions provided the subjects with enough information to calculate if they would be early or late to their meeting/class/activity when they choose either Lot A or B.

For example in Figure 6, the sign shows that the subject has arrived 15 minutes early. Then, the subject would compare parking Lot A versus Lot B in terms of price and time to destination. In this example, if the subject parks in Lot A, he/she would arrive 5 minutes early to his/her destination. The cost of this selection is $5.

![Figure 6: Parking Survey Example](image-url)
If the subject chooses parking lot B, he/she would 10 minutes early to his/her destination but the parking cost would be $10. Some subjects may be concerned with the price factor while other may be concerned with the time that they would spend walking.

5.3.4 Results

The raw data collected on each subject for each scenario in the survey is attached as a table in Appendix B. In the table, the leftmost column contains the scenario numbers. Each of the other columns contains the letters of the lots chosen by one subject for each of the scenarios. The results for each scenario are summarized in Appendix C, which indicates the number of subjects who choose between minimum time to destination or Lot B (MTTD) or minimum cost or Lot A (MC). The survey was analyzed in the following manner. For the first six scenarios (subject will always be early to their destination), the number of times a subject choose either lot A or lot B were counted. If a driver chooses lot A, this indicates that the driver is choosing to minimize cost. On the other hand, a driver who chooses lot B indicates that he/she is choosing to minimize the time to destination. Therefore, the rightmost column in Appendix B indicates the number of times a driver chooses either to minimize time to destination (MTTD) or minimum cost (MC).

For the other six scenarios (subject might be late to their destination if they choose lot A) the same procedure was followed. The difference is that those subjects who choose lot A will be late to their destination; therefore, they prefer to minimize cost even jeopardizing timeliness. Figure 7 shows the results of the survey.
Figure 7: Stated Preference Survey Results

On the first six scenarios (on-time) 29% choose to minimize their time to destination (Lot B) and 71% of the subjects choose the parking lot with the minimum cost (Lot A). On the other six scenarios (late) 84% of the subjects choose the parking lot that will minimize their time to destination; therefore, arrive to their meeting, class, etc on time and 16% would rather pay less.

Figure 8 shows the responses of subjects when time was not a constraint. Scenario 1, 2, and 3 represent a 100%, 80%, and 60% price difference. For these percentages most drivers choose the parking facility with the minimum cost (MC). However, scenario 4, which represents a 40 percent difference in price between the two facilities, shows a more balanced response from the subjects. That is, 55% of the subjects select the facility with the minimum cost while 45% the facility closest to the destination (higher cost). The same results can be seen on scenario 5 which represents a 20% price difference. Drivers are willing to pay for the closest parking facility and as a result a higher cost. For this scenario, 31% choose the facility with the minimum cost while 70% selected the facility closest to the destination (highest cost).
These results are helpful for strategic purposes. For example, a parking manager has two parking facilities; one of the facilities has 100% utilization while the other has only 50% utilization. Currently, both parking lots charge the same fare. However, the parking manager can implement a 40% difference in prices to balance the utilization for both facilities. This will not only increase revenue but it will improve customer satisfaction and retention. On the other hand, if the objective is to divert drivers from one facility to another, a 100% or 80% difference in prices could accomplish this.

Figure 9 shows the results from the survey when time was a constraint. The results indicate that drivers are willing to pay higher fares when they are under a time constraint since in all scenarios they choose the minimum time to destination (MTTD) while only a few choose the facility with the minimum cost (MC).
The results of the survey help to determine the parking demand. Figure 10 shows the parking demand for the closest parking lot to the destination when time is not a constraint. It is easy to see how demand decreases as price increases. This means that drivers are willing to park further away to the destination when they will not be late.

On the other hand, Figure 11 shows the parking demand for the closest parking lot to the destination when time is a constraint. The results show that drivers are willing to pay higher fares when they are under a time constraint. The demand curve appears to be more constant, and it does not decrease dramatically.
Figure 11: Demand Curve for Closest Parking Lot-Late

The results of the survey will be analyzed in more detail in the next section through a logit model.

5.4 Logit Model

In this thesis, the data collected on the stated preference survey is used to construct a parking lot choice model. The model hypothesizes that parking choices depend upon arrival time, parking price, age, gender, and income. The sample data consists of choices from 51 subjects who contributed 612 data points or choices (51 subjects times 12 parking scenarios). The parking literature states that factors such as walking distance, driving distance, parking type and parking price influence parking choice. However, to our knowledge the arrival time factor has not been studied in the literature. This study allows us to prove that drivers are willing to pay higher fares when they are under a time constraint.

The traditional methodology used by researchers to study parking choice has been through logistic regression or logit models. This type of model is appropriate when the responses take on only two possible values representing success and failure (binary data). In other words, logistic regression estimates the probability of a certain event occurring or, in our case, the probability that a drivers chooses between a two parking alternatives
depending on different factors. Generally, the dependant variable can take on only two responses such as presence/absence or success/failure. The logistic regression does not make any assumption about the distribution of the independent variables. However, it assumes a binomial distribution for the errors.

The logistic regression can be seen as a liner regression model such as \( p_i = \alpha + \beta_1 x_{i,1} + ... + \beta_k x_{i,k} \) where \( p_i \) is the probability of event \( i \) to success or fail, \( x_i \) is the independent variable for event \( i \), and \( \beta \) is a vector of regression coefficients. The problem with this model is that the probability \( p_i \) can take only take values between zero and one, but the linear term \( x_{i,k} \beta_k \) can take any real value; therefore, there is not guarantee that the predicted values will be in the correct range.

To avoid this problem a simple transformation on the probability to remove the range restrictions is performed, and model the transformation as a linear function of the covariates. This transformation is accomplished by moving from the probability \( p_i \) to the odds where

\[
odds_i = \frac{p_i}{1 - p_i}
\]

It is important to note that there is no difference between working in probabilities or odds since they are both equivalent. However, the main advantage is that odds can take on any positive value; therefore, they have not ceiling restrictions.

The next step is to eliminate the floor restrictions by calculating the logit or log-odds

\[
\text{logit} (p_i) = \log \frac{p_i}{1 - p_i}
\]
Suppose that the logit of the probability $p_i$ is a linear function of the predictors

$$\text{logit} (p_i) = \alpha + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}$$

where $x_i$ is a vector of covariates and $\beta$ is a vector of regression coefficients. The model is a generalized linear model with binomial response and link logit. By exponentiating the model logit $\text{logit} (p_i) = \alpha + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}$, it takes the form

$$p_i = \frac{e^{\alpha + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}}}{1 + e^{\alpha + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}}}$$

where $p_i$ is the probability of choosing parking lot $i$. The parameters $\alpha, \beta_1, \ldots, \beta_k$ are estimated by maximizing the log-likelihood function

$$\log L (\beta) = \sum (y_i \log (p_i) + (n_i - y_i) \log (1 - p_i))$$

This procedure helps us to test coefficients for significance. The logistic regression is almost identical to a linear regression. The main advantage of logistic regression is that the logit transformation of the probabilities to odds allows to limit the dependent variable to be a 0/1 or success/failure response. The next section will illustrate the application of these concepts to study the relationship between the independent variables (gender, income, arrival time, and price) versus the dependant variable (parking lot choice).

5.4.1 Logit Model Results

The logistic regression model described in the previous section was applied to study the relationship among the variables gender, age, income, arrival time and price with the selection of a parking facility. The objective is to find what factors influence drivers’ behavior at the time of selection among a set of parking alternatives. Most
importantly, it is important to demonstrate that the arrival time factor is significant on the selection process. Table 5 presents the results of the logit model implemented in R2.3.1. The results show the probability of selecting the parking lot with the lowest price (Lot B). The results show that gender, arrival time, and price are significant at the time of selecting a parking lot. It is important to note that the age and income factor are not significant. However, these factors are not significant because the majority of the participants have the same age and income.

| Coefficients | Estimate | Std. Error | z value | Pr(>|z|) |
|--------------|----------|------------|---------|---------|
| Intercept    | 1.13E+01 | 1.49E+00   | 7.58    | 3.46E-14 *** |
| Gender       | -8.44E-01| 3.17E-01   | -2.666  | 0.00768 **  |
| Age          | 5.61E-02 | 4.92E-02   | 1.139   | 0.25463 |
| Income       | -2.52E-05| 1.84E-05   | -1.373  | 0.16972 |
| Arrival Time | -3.14E-01| 3.55E-02   | -8.819  | <2e-16 *** |
| Price        | -9.81E-01| 1.11E-01   | -8.846  | <2e-16 *** |

Significant Codes
0 **** 0.001 *** 0.01 **

Null Deviance 533.33
Residual Deviance 314.72

The estimated values show that choice is relatively inelastic with respect to distance and more elastic with respect to price. Furthermore, the significant negative coefficient of arrival time in Table 4 indicates that when the arrival time is small; drivers choose the parking lot closer to the destination. This helps to support the previous statement that drivers are willing to pay higher parking rates when they are under a time constraint.
CHAPTER 6
PARKING DEMAND FORECASTING MODELS

6.1 Introduction

Accurate forecasts are extremely important to estimate quantities such as demand, price sensitivity, and number of bookings in a particular market and for a particular type of passenger. If the demand for each type of customer was known with certainty, the problem of optimally allocating capacity would be easy to solve. However, demands for each type of customer are never known with certainty. Historical data would help us to estimate future demand. The performance of the revenue management system depends significantly on the accuracy of the forecast of future demand.

Parking managers/planners need to be aware that forecasts are not always accurate; therefore, it is important to estimate not only the expected value of the forecast, but also a measure of the forecast error. Furthermore, short-term and aggregate forecasts are usually more accurate than long-term and disaggregate forecasts. There are two different approaches for aggregating forecasting: “top-down” and “bottom-up”.

In the “top-down” approach the total number of customer who will use the service/product is predicted and this number is divided into the demand for different parts of the facility. For example, a hotel may estimate the total number of customer who will book in a particular day and classify them into different segments, and estimate length of stay and type of room to be used.
The “bottom-up” approach is performed at a more detailed level. The end forecast is assembled by aggregating these detailed forecasts. This approach takes the lowest level unit and predicts the demand for it. For example, a hotel may predict the demand for a particular room. Then, it will aggregate the forecast of all rooms to construct the end forecast (total demand of customers). The most appropriate strategy usually depends on the type of data available or the type of outcome desired. There is a risk on both approaches of losing accuracy; however, the degree of accuracy tends to be better when using the “top-down” approach.

The forecasts can be estimated using qualitative or quantitative methods. Qualitative methods rely on expert opinion. This approach is useful when the data available is limited or when the experts have a critical knowledge of the market that is essential for the accuracy of the forecast. Quantitative methods can be based on the assumption that historical trends will continue on the future (i.e. Time Series methods: Moving Average, Simple Exponential Smoothing, Holt’s Model, Winter’s Model, etc), or causal models which assume that the demand forecast is correlated with certain factors in the environment (i.e. Linear Regression, Non-Linear Regression, Neural Networks, etc).

Predicting parking demand is a very complex task since it can be influenced by many factors. The relation among these factors is often non-linear which provides a challenging task for computational purposes. Previous parking demand modeling approaches have considered how parking demand (drivers) would react to changes in the availability or location of the parking facilities. Impact would be reflected on the day/time to do the trip or even changing destination or discarding the trip due to parking concerns. In Young, Thompson, and Taylor (1991) a comprehensive review of parking
models is presented; however, most of the models discussed have found limited application by only adding an understanding of the traffic system, parking system and the interaction between them. That is, previous approaches have studied parking demand trying to answer how drivers search for a parking space and how they will react to different parking scenarios, but they stop short when presenting an applicable solution to the specific cases.

It is important to note that there are differences on the forecasting needs between the parking industry and other industries such as airline and hotel. In the airline and the hotel industry there is a broad range of forecasting requirements. For example, the airline industry requires forecasts for customer demand; the way reservations for different customer types arrive during the booking period, and cancellation and no-show probabilities. In the case of the hotel industry, forecasts requirements include the total number of guests who will book to arrive on each day in each rate category, the length of time that the customer will stay in the hotel, and room occupancy. The forecast requirements for the parking industry are closer related to those in the hotel industry. For example, the parking manager/planner needs to predict the total number of drivers who will reserve a parking space for each hour and for each rate. Also, it is necessary to predict parking lot occupancy.

The objective of this chapter is to evaluate different quantitative methods and select the best technique to forecast parking demand and parking space occupancy. Traditional forecasting method including moving average, simple exponential smoothing, Holt’s model, and Winter’s model are compared against a non-traditional forecasting technique: neural networks. The next sections provide an introduction to the theory and
mathematics of the models utilized followed by an evaluation of the models in terms of their performance using parking demand data obtained from a major airport. The quality of forecasting results is measured through the mean square error, mean absolute error, mean absolute percentage error, and root mean square error.

6.2 Time Series Forecasting Methods

Time series forecasting methods assume that historical data is a good indicator of future demand. These models are typically used in revenue management because they are easy to understand, simple to code, perform well, and maintenance is relatively simple. Before proceeding to the theory of the models, it is important to define the following terminology.

- **Level**-It is the typical or average demand
- **Trend**-It is a decrease or increase in the data values over time
- **Seasonality**-It is a repeating pattern in the data values over time (i.e. day of the week, hour of the day, month of the year, etc)

6.2.1 Moving Average

The moving average method is based on a weighted average of past values. It represents the average of the $N$ most recent data points. The following is the moving average formula:

$$F_t = \frac{(D_t + D_{t-1} + \ldots + D_{t-N+1})}{N}$$

The forecast for the period $t+1$ and for the $k$ period are given by:

$$F_{t+1} = \frac{(D_{t+1} + D_t + \ldots + D_{t-N+2})}{N}$$
The moving average method is simple and fast. The fundamental idea is that the most recent observations are better predictors than older observations. As seen in the formula above, the idea is to drop the oldest observation and add the latest. The number of periods averaged \( N \) has to be determined by the analyst. It is important to note that the smaller value given to \( N \) represents a more responsive forecast. However, a value that is too small might result in a more volatile forecast. In general, the value of \( N \) ranges from 3 to 15, but this value depends on the characteristics of the data.

It is recommended to use the moving average method when demand has no observable trend or seasonality. The moving average is not adequate when the data exhibits upward and downward trend because it might under or over forecast (Chopra and Meindl, 2004).

6.2.2 Simple Exponential Smoothing

Simple exponential smoothing is very similar to the moving average. It estimates future forecasts based on a weighted average of past observations of demand. This technique weights recent observations more heavily than older observations. The exponential smoothing formula for period 1 is given by:

\[
F_1 = \frac{1}{n} \sum_{t=1}^{n} D_t
\]
The one step ahead exponentially smoothed forecast is given by:

\[ F_{t+1} = \alpha D_t + (1-\alpha)F_t \]

- \( F_1 \) = Forecast for period 1 (average of all demand periods)
- \( F_{t+1} \) = One step ahead forecasted demand
- \( D_t \) = Actual demand of empty spaces for period \( t \)
- \( \alpha \) = Smoothing constant for the level (0 < \( \alpha \) < 1)

The value of \( \alpha \) has to be determined before starting the forecast. A high value of \( \alpha \) represents more weight assigned to more recent observations; therefore, the model is more responsive to change, but it is also more susceptible to noise. In contrast, a small value of \( \alpha \) represents a smoother forecast; therefore, the model represents a more stable forecast, but it is less responsive to change. The exponential smoothing method is appropriate when demand has no observable trend or seasonality (Chopra and Meindl, 2004).

### 6.2.3 Holt’s Model

The Holt’s model is also known as Trend Corrected Exponential Smoothing. It is appropriate when demand presents either upward or downward trends (Chopra and Meindl, 2004). To account for these trends, Holt’s model decomposes the systematic component of demand into a level and a trend when making the forecast. To find the initial level \( L_0 \) and initial trend \( T_0 \), it is necessary to run a linear regression between demand \( D_t \) and time period \( t \) of the form \( D_t = at + b \), where \( L_0 = b \) and \( T_0 = a \). The forecast for period \( t+1 \) and \( t+k \) is given by:

\[ F_{t+1} = L_t + T_t \text{ and } F_{t+k} = L_t + kT_t \]
Before using the formula to find the forecast for period $t+k$, it is necessary to revise the level and trend by using the following formulas:

$$L_{t+1} = \alpha D_{t+1} + (1 - \alpha)(L_t + T_t)$$

$$T_{t+1} = \beta(L_{t+1} - L_t) + (1 - \beta)T_t$$

- $\alpha$ = Smoothing constant for the level ($0 < \alpha < 1$)
- $\beta$ = Smoothing constant for the trend ($0 < \beta < 1$)

### 6.2.4 Winter’s Model

Winter’s model, also known as Trend and Seasonality Corrected Exponential Smoothing, is appropriate to forecast data series that exhibit seasonality (i.e. hourly, daily, monthly, etc), level, and trend (Chopra and Meindl, 2004). Assume $p$ represents the periodicity of the demand. The periodicity represents the number of periods after which the seasons repeat. For example, if the seasonality is by month, $p = 12$ or by hour, $p = 24$. The first step is to determine the estimates of the initial level $L_0$, initial trend $T_0$, and seasonal factors $(S_1, \ldots, S_p)$. To obtain these values, it is necessary to deseasonalize the demand data which represent the demand without seasonal fluctuations. The next step is to run a linear regression of the form $\bar{D}_k = L + kT$ where $\bar{D}_k$ represents the deseasonalized demand. After the initial level and trend has been found, then the seasonal factor $(S_k)$ can be calculated by:

$$\bar{S}_k = \frac{D_k}{\bar{D}_k}$$

The forecast for period $t+1$ and $t+k$ is given by:

$$F_{t+1} = (L_t + T_t)S_{t+1}$$

$$F_{t+k} = (L_t + kT_t)S_{t+k}$$
and the level, trend and seasonality are updated as follows:

\[ L_{t+1} = \alpha \left( \frac{D_{t+1}}{S_{t+1}} \right) + (1 - \alpha)(L_t + T_t) \]

\[ T_{t+1} = \beta(L_{t+1} - L_t) + (1 - \beta)T_t \]

\[ S_{t+p+1} = \gamma \left( \frac{D_{t+1}}{L_{t+1}} \right) + (1 - \gamma)S_{t+1} \]

- \( \alpha \) = Smoothing constant for the level \( (0 < \alpha < 1) \)
- \( \beta \) = Smoothing constant for the trend \( (0 < \beta < 1) \)
- \( \gamma \) = Smoothing constant for the seasonal factor \( (0 < \gamma < 1) \)

### 6.3 Causal Models

Causal models assume that the demand forecast is correlated with certain factors in the environment. This makes them applicable to model parking problems since the relationship among different factors is often non-linear. In this section, neural networks would be introduced, and parking demand would be predicted using this model.

#### 6.3.1 Introduction to Neural Networks

Neural network is an effective tool at trend prediction, pattern recognition, modeling, control, signals filtering, noise reduction, image analysis, classification, and evaluation (Landau and Taylor, 1998). The neural networks are quantitative models that link together inputs and outputs adaptively in a learning process similar to that used by the human brain (Abdi, Valentin, and Edelman, 1999). The human brain consists of hundreds of billions of neurons which are connected together in a complex form. Neurons send information back and forth to each other through a series of connections.
Neurons and connections are referred as a network. This network can perform intelligent functions such as learning, analysis, prediction, and recognition.

Neural networks consist of an input layer, an output layer and one or more hidden layer as seen in Figure 12. The nodes or neurons of the network are arranged in consecutive layers (hidden layers) and the arcs are directed from one layer to the next from left to right.

This type of neural networks is called feed-forward networks or perceptrons. Basically, neural networks are built from simple units (neurons). These neurons are interlinked by a set of weighted connections \((w)\). Each node or neuron is a processing unit that contains a weight and a summation function. A weight returns a mathematical value for the relative strength of connections to transfer data from one layer to the next. On the other hand, a summation function \(y\) computes the weighted sum of all input
elements entering a neuron. In Figure 12, each neuron in the hidden layer computes the summation \( y_j \) using the following formula:

\[
y_j = \sum_{i=1}^{2} x_i w_{ij} \quad j = 1, 2, 3
\]

Furthermore, a sigmoid function \( y_T \) is used to transform the output so that it falls into an acceptable range (between 0 and 1). The objective is to prevent the output from being too large. The sigmoid function is of the following form:

\[
y_T = \frac{1}{1 + e^{-y}}
\]

As previously described, neural networks consist of neurons or nodes organized in different layers: input, hidden, and output. The input layer corresponds to the factors that would be “feed” into the network. The information is propagated through the weighted connections to the hidden layers where it is analyzed. Then, the result of this processing is propagated to the next layer and eventually, to the output layer. The output is obtained by the following function:

\[
Y = \sum_{j=1}^{3} y_j w_{kj}
\]

**6.3.1.1 Training**

After the network architecture has been defined, the next step is to train the network. A training data set is feed forward into the network to calibrate the weights and values of the threshold functions. In the forward pass the output are calculated as well as the errors of the output compared to the original values. After the forward pass, the errors of the output are back propagated and the weights are altered. This is called training the network. The goal of the network is to learn some relationship between
input and output patterns. This learning process is achieved through the modifications of the connection weights between neurons. There are several learning algorithms that are commonly used such as the Widrow-Hoff Learning Rule, the Hebbian Learning Rule, and the backpropagation algorithm (Abdi, Valentin, and Edelman, 1999). The most popular algorithms used for training purposes is the latter, error back-propagation method. The backpropagation algorithm objective is to minimize the mean square error function:

\[ E = \sum_i [y_i - F_{\text{approx}}(x_i)]^2 \]

This error functions tells us how good an approximation to the real function F is. The idea of the backpropagation algorithm is to minimize this error (threshold) by adding for each training period, small changes in the directions that minimize the error function. This minimization method is called the steepest descent method. The general learning process is described in the following steps:

1. Random numbers are assigned to the weights
2. For all data points in the data set, calculate the output using the summation functions of each neuron as described in section 7.3.1
3. Compare estimated output with actual values
4. If the results from 3 do not meet a threshold value, repeat steps 2 and 3.

6.3.1.2 Overfitting and Generalization

During the last years, many researchers have taken advantage of powerful and efficient computer systems. Neural Networks is one of the fields that have taken advantage of such advancements in technology. Its applications have increased to
numerous fields. For example, neural networks have been used by Air Canada for airport scheduling, reducing delays from flight re-scheduling, cutting fuel and other direct costs, and shortening the idle time of aircraft. Neural networks have also been used for pattern recognition, classification, reconstruction, biology, computer game playing and time series forecasting (Talluri and Van Ryzin, 2004).

However, a common problem that may occur when fitting the neural network to training data is overfitting. Overfitting occurs when the error of the training set is minimized to a very small value. As a result, when new data is introduced into the network the error becomes very large. In this situation the network has “memorized” the data set, and it is not able to “generalize” when new data is introduced into the network. Generalization refers to the ability of the model to perform well on data that has not been used to train the network.

There are two strategies that can be used to avoid overfitting: regularization and early stopping. Regularization involves modifying the performance function. Early stopping involves dividing the data set into two subsets. The first subset is the training set and the second subset is the validation set. At the beginning of the training process the error for the validation and testing sets tends to decrease; however, when the network starts to overfit the data both errors will increase. When the error for the validation set continues to increase for a specific number of iterations, then training is stopped.

This research applies neural network as a tool to predict parking demand. The traditional backpropagation algorithm is used as the learning method for our network and early stopping criteria is used to avoid overfitting. The next section describes the neural network model developed for the prediction of parking demand.
6.4 Parking Demand Predictor Model

The objective of the predictor models is to give managers a useful insight by setting up a real time mechanism of clustering the day patterns and predicting parking demand. Neural networks have proven to be an efficient tool to predict future states of a system given several relationships. Figure 13 shows an overview of the prediction model developed and the inputs that would be required. Many of these inputs have been obtained from data provided by a major airport in Florida. The data provided was analyzed using the neural network functions and tools provided in MatLab.

Figure 13: Neural Network Parking Model
The predictor model would be a helpful tool for managers that must, on a daily basis, be able to recognize the conditions that will prevail in the system to pick the appropriate strategy to implement. The design and implementation of such management plans requires the predictor model to have the following capabilities: distinguish between long, medium and short stay parkers, classify typical and atypical conditions (e.g., special events), and identify daily, hourly, and monthly patterns.

Time series data used for this study were collected at a major airport. The data obtained a four week period of demand for two parking facilities. The data obtained were studied using the layered neural network with a backpropagation least mean square error learning algorithm. To predict parking demand, a neural network with 3 input nodes (month, day, and hour), a single output node (number of cars that would enter the parking facility), and a one-layer backpropagation network has been used. There is no standard formula to calculate the number of nodes needed in the hidden layer (Wang and Sun, 1996). Basically, the number of hidden layers may be tested by trial and error. Figure 14 shows the graphical representation of the neural network used in this study.

![Neural Network Architecture](image)

Figure 14: Neural Network Architecture

The neural network developed is actually a mapping function representing the relationship between the month, day, hour and number of cars that enter the parking garage. The output obtained from the neural network is used to predict the availability at different time frames. In the next sections the results from the study are presented and
compared versus other traditional prediction techniques used to estimate demand. The comparison would be based on several performance measures that would be introduced in the next section.

6.5 Performance Measures

Forecast errors are extremely useful to determine if the forecasting model is accurately predicting demand. They can help to determine if the model is overpredicting or underpredicting. There are several performance measures of forecast error such as Mean Absolute Percentage Error (MAPE), Absolute Deviation (MAD), Mean Square Error (MSE), Root Mean Square Error (RMSE), Tracking Signal (TS), and the Mean Error. In the following sections, each one of these performance measures are describe.

6.5.1 Mean Absolute Percentage Error (MAPE)

The mean absolute percentage error (MAPE) is the average absolute error as a percentage of demand and is given by

\[
MAPE = \left( \frac{1}{n} \sum_{t=1}^{n} \frac{e_t}{D_t} \right) \times 100
\]

In practice a MAPE between 10% and 15% is excellent while a MAPE between 20% and 30% is average.

6.5.2 Mean Absolute Deviation (MAD)

The mean absolute deviation (MAD) is the average of the absolute deviation over all periods. MAD measures the average distance of the sample errors from the mean of the error values. If the value of MAD is large, it is reasonably to say that the errors in the data set are spread out (variable). In contrast to MSE, the MAD is very good at detecting
overall performance of the model. It does not concentrate largely on the error of individual observations. The MAD is given by

$$MAD = \frac{1}{n} \sum_{i=1}^{n} |e_i|$$

MAD is appropriate to use when the numerical difference between the forecast value and the actual value is important.

### 6.5.3 Mean Square Error (MSE)

The Mean Square Error (MSE) can be related to the variance of the forecast error. This is extremely useful since it can be used to measure the variability or dispersion of the error. The forecast error for a particular period \( t \) is given by

$$e_t = F_t - D_t$$

where \( F_t \) is the forecasted or estimated value at time \( t \) and \( D_t \) is the actual value at time \( t \).

The Mean Square Error is given by

$$MSE = \frac{1}{n} \sum_{i=1}^{n} e_i^2$$

MSE penalizes large errors for a single observation, and it is very good at detecting if a few observations have large errors. The smaller the value of the MSE the closer the fit is to the data.

### 6.5.4 Root Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) is just the square root of the MSE. The RMSE is the distance on average of a data point from the fitted line, measured along a vertical line. The RMSE is given by
This statistic is very easy to interpret since it has the same units as the values plotted in the vertical axis.

### 6.5.5 Tracking Signal (TS)

The tracking signal (TS) is used to monitor forecast bias. If the TS exceeds a predetermined bound, this indicates an alert that the forecast is being bias one way or the other. In general, the bound of the TS is between ±6 units from the mean. If the TS is below -6 then the model is underforecasting. On the other hand, if the TS is above +6 then the model is overforecasting. This would indicate an alert for analysts who may have to decide on using another model. The TS is defined as follows

\[ TS_N = \frac{\sum_{i=0}^{N} e_i}{MAD_N} \]

### 6.5.6 Mean Error

The mean error is an estimate of the forecast bias. The mean bias should converge to zero as \( N \) increases if the forecasting is not biased one way or the other. The mean squared error is defined as follows

\[ \bar{E}_N = \frac{1}{N} \sum_{i=0}^{N} e_i \]

### 6.6 Comparison of Forecasting Techniques

Five forecasting models, namely moving average (\( ma=4 \)), simple exponential smoothing (\( \alpha = 0.7 \)), Holt’s model (\( \alpha = 0.5, \beta = 0.1 \)), Winter’s model (\( \alpha = 0.05, \beta = 0.1 \)), and neural networks were used to forecast parking occupancy at a major airport.
The data provided represents the number of cars per day that occupied the parking facilities (parking occupancy) at each hour for a period one month (the peak month), see Figure 15. There are several peak periods which make it challenging for any forecasting method to accurately predict future values of demand.

![Daily Parking Occupancy](image)

**Figure 15: Raw Data for Parking Occupancy**

The data was analyzed using the four traditional methods using Microsoft Excel, and the neural network was developed using tools and functions provided by MatLab and NeuroSolutions for MatLab (Appendix D and E). Figure 16 shows the results obtained for each forecasting method. The data plotted in this graph corresponds to one day forecast.

![Graphical Representation of Different Forecasting Methods](image)

**Figure 16: Graphical Representations for Different Forecasting Methods**
As seen in Figure 16, the neural network mimics the original data better than the other models. The neural network is able to capture more accurately the changes from peak to low periods. It is important to note that the other traditional models overforecast for periods of low demand while the neural network is able to capture very accurate this changes.

The above representation gave us a good indication of the performance of each model. However, a more detailed study was conducted using each performance measure previously described. Table 6 summarizes the results obtained for each one of the performance measures.

**Table 6: Performance Comparison for the Various Forecasting Models**

<table>
<thead>
<tr>
<th>Method</th>
<th>MAPE (%)</th>
<th>MAD</th>
<th>MSE</th>
<th>RMSE</th>
<th>TS</th>
<th>Mean Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>18.31</td>
<td>31.11</td>
<td>1,483.68</td>
<td>38.52</td>
<td>-0.000016</td>
<td>-0.0000014</td>
</tr>
<tr>
<td>Winter's Model</td>
<td>22.82</td>
<td>40.80</td>
<td>3,032.33</td>
<td>55.07</td>
<td>-1.17</td>
<td>-0.06</td>
</tr>
<tr>
<td>Holt's Model</td>
<td>70.89</td>
<td>70.74</td>
<td>8,058.62</td>
<td>89.77</td>
<td>-4.25</td>
<td>-0.40</td>
</tr>
<tr>
<td>Exponential Smoothing</td>
<td>78.23</td>
<td>61.39</td>
<td>6,191.81</td>
<td>78.69</td>
<td>-2.64</td>
<td>-0.22</td>
</tr>
<tr>
<td>Moving Average</td>
<td>106.96</td>
<td>84.64</td>
<td>11,124.65</td>
<td>105.47</td>
<td>3.53</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Experimental results in Table 6 reveal that the parking occupancy estimated by a neural network is very close to the actual values. This indicates that the estimated outputs of the neural network are very accurate with a relatively small amount of error. The low MAPE indicates that the discrepancies between the forecasted values by the neural network and the actual values are very small. The MAPE performance measure is useful for comparing performance among different time series because the errors are measured relative to the data values (Talluri and Van Ryzin, 2004). The MAPE of 18.31% obtained for the neural network is slightly over the 10%-15% range which indicates an excellent forecast; therefore, the neural network forecasts are said to be
above average. The results indicate that the MAPE values tend to increase linearly as the model complexity decreases as seen in Figure 17.

The reason may be that Holt’s model, exponential smoothing, and moving average are not able to capture the seasonality and trend of the data. On the other hand, the Winter’s model performs relatively well compared to the neural network. The reason is that this method takes into account the trend and the seasonality of the data.

It is important to note that the exponential smoothing model is performing relatively well taking into account all the performance measure. The reason can be attributed to the large value of $\alpha = 0.9$ which makes the forecast more responsive to changes in level but more susceptible to noise which in the future may lead to large forecasting errors.

Previously, the models have been compared using the MAPE performance measure. The root mean square error (RMSE) is probably the easiest performance measure to interpret since it has the same units as the demand plotted in the vertical axis (parking occupancy). In table 1, we can see that the RMSE for the neural network is 38.52. This indicates that on average the distance of the forecasted value with respect to
the actual values is 38.52 units (number of cars in the parking lot). The RMSE is an excellent performance measure for the forecast since it provides information easy to interpret that can be used for managers that can take this error into account for planning purposes.

The mean error is another important performance measure that should be discussed. The mean error is an estimate of the forecast bias. If the forecasting model is not biased, the mean bias should converge to zero as $N$ increases. In Figure 18, it can be seen that mean error for the neural network is extremely close to zero, which indicates that the neural network model is unbiased. The Winter’s model, Holt’s model, and simple exponential smoothing have a tendency to underforecast while the moving average have a tendency to overforecast.

![Figure 18: Mean Error](image)

As well as measuring forecast performance, managers need a tool that constantly monitors the forecast bias. The tracking signal (TS) is a method used to accomplish this monitoring process, see Figure 19. If the TS at any period is outside the range $\pm 6$, this indicates a signal that the forecast is overforecasting or underforecasting.
Figure 19 indicates that none of the models fall outside the upper and lower bound. However, the neural network is at zero which indicates that the model is unbiased.

6.7 Discussion

As shown in the previous section, neural networks outperforms moving average, simple exponential smoothing, Holt’s model, and Winter’s model in forecasting parking demand and utilization. A paired T-test was conducted on the Mean Square Error to determine if the performance difference between the utilized models was statistically significant (Appendix F). The test statistics revealed that the difference in performance between the neural networks and the other models is statically significant (low $p$-value). This validates the use of neural networks as an efficient tool to predict parking demand.

One advantage that neural networks have over these other methods is that its architecture does not require developing algorithms specific to problems. That is, the architecture can be easily adapted for different parking facilities where demand patterns may vary. For example, although the parking demand characteristics and the interaction among factors at a University are different to those at a major Airport, the neural network
architecture is flexible and can be modified to represent both environments. Another advantage of neural networks is that they can easily handle nonlinear functions. This is an advantage over other traditional methods since to analyze a non-linear relationship using linear regression analysis, it is necessary to first analyze the nonlinearity of the system and determine whether some input need to be squared or two input variables need to be combined. This analysis is overcome by the neural networks capabilities.
CHAPTER 7
CAPACITY CONTROL MODEL

7.1 Introduction

In parking terms, capacity control can be defined as the science of predicting the quantity and specific attributes of parking facilities and spaces needed to satisfy the forecasted demand. Currently, capacity control methods do not provide efficient results because most of the time the huge amount of dynamic input data is ignored. In addition, even when high uncertainty in the forecasts typically exists, not sufficient demand scenarios are considered. In this chapter, a model that optimally allocates parking spaces to different fare classes of demand is presented.

According to basic economic theory, it is more profitable to have more than one fare class in the same market provided that the inventory can be managed properly. The results from the parking survey discussed in Chapter 5, help to demonstrate that the introduction of more than one fare class increases revenue. As seen in Figure 20, if a space is sold at an original price - $P_0$ of $6, its revenue is $P_0D_0$ or ($6*35\text{spaces}$) $210.

![Demand Curve for Closest Parking Lot-Early](image)

Figure 20: Revenue Generated for One Segment
Figure 21 presents the revenue generated for two fare classes where $P_1>P_0$ and $P_2<P_0$, the total revenue generated with the introduction of the extra fare is $P_1D_1 + P_2(D_2-D_1)$ or $(5\times28 + 7\times(51-28)) = 301$. With this example, it is easy to see that having more prices in decreasing order may actually increase revenues. In the presence of different segments, revenue management increases the revenue from $210$ to $301$ or a 30 percent increase in total revenue.

![Demand Curve for Closest Parking Lot-Early](image)

**Figure 21: Revenue Generated for Two Segments**

The question is, how do parking managers can use parking space inventory control in such a way that the lower paying segment does not control the entire availability of the spaces. The answer is to design an inventory control system that allocates spaces between full fare drivers and discount fare drivers in an effort to maximize total revenues. To take advantage of revenue management techniques, parking managers must limit the amount of spaces reserved for a lower fare even when there might exist enough demand for the lower price segment to occupy the entire available area. In the next sections we answer how much capacity to reserve for the higher fare segment.
7.2 Capacity Control Model

This section provides some important definitions that will be carried out through this chapter. There are different strategies that can be used to control the availability of parking spaces. Examples of these strategies include booking limits, protection levels, bid prices, standard nesting, and theft nesting. In this research, booking limits and protection levels are explored to develop strategies for the capacity control model. These strategies are discussed in more detail next.

- **Booking Limits** are used to control the amount of parking spaces that may be sold to any particular class of customer at a given point in time (Talluri and Van Ryzin, 2004). For example, a booking limit of 20 spaces for low fare drivers indicates that at most 20 spaces of the total capacity will be sold to low fare drivers. Therefore, the booking limit is the maximum number of spaces that may be sold at the lowest fare. The remaining of the spaces will be sold at a full price. There are two types of booking limits: *partitioned and nested*.

  - **Partitioned booking limits** split the total capacity into blocks (one for each class), and then each block can be sold at a particular rate. For example, we have 100 parking spaces to sell. The booking limit for full fare is 20 spaces; therefore, the remaining 80 spaces will be reserved for the low fare drivers. Let say that we receive a reservation request for a full fare space and all 20 spaces previously reserved are sold (full fare is “closed”). Therefore, the revenue from the full fare will be lost because the booking limit for the full fare was closed. To
prevent full fares to be unavailable *nested booking limits* could be used.

- *Nested booking limits* make the already allocated spaces for low fare available for the full fare. This is accomplished by having the high fare class access to all the capacity reserved for the low fare class. Figure 3 illustrates this method. Consider the previously described example (total capacity = 100 spaces, 80 reserved for low fare, and 20 reserved for high fare). The high fare is allocated 80 spaces, when in reality its demand could exceed that number. A nested booking limit prevents rejecting any excess of full fare demand. In Figure 22, we can see that the nested booking limit of the full fare is 100 spaces (the total capacity), and the nested booking limit for the low fare would be 80 spaces. Therefore, we will accept at most 100 booking for full fare and discount fare, and at most 80 for low fare. The idea is that any “left over” capacity for the low fare becomes available for the full fare (Talluri and Van Ryzin, 2004).

![Figure 22: Example of Nested Booking Limits](image-url)
Protection Levels is the number of spaces that will be reserved for a particular class. The relationship of protection levels and booking limits could be described as follows. Let say that we have \( i \) classes of demand, then the booking limit \( (b_i) \) is:

\[
b_i = \text{Capacity} - y_{i-1} \quad i = 2, \ldots, n
\]

where \( y_i \) the amount of capacity to save for \( i, i-1, \ldots, 1 \) combined. This is for classes \( i \) and higher. For example, let say that there are two fare classes and a capacity of 100 spaces, then the protection level is the number of spaces that will not be sell to low fare customers because of the probability that full fare customers may book latter in time. The new challenge is on how to determine the optimal protection levels for each class. In the next section, the single-resource model for determining protection levels is presented. Furthermore, two traditional heuristics used in revenue management will be discussed.

7.2.1 Expected Marginal Revenue Model

The basic trade-off that a parking reservation system has to consider is between selling a space at a low fare or waiting for a full fare driver to arrive later on. There are two risks that have to be considered in this situation: spoilage and spill. Spoilage occurs when the capacity reserved for the full fare drivers is wasted because the demand for this class does not materialize. Spill occurs if full fare drivers have to be rejected because the capacity has already been committed to low fare drivers. Therefore, the objective is to determine the protection level for the full fare drivers so as to minimize the expected cost of spoilage and spill (Chopra and Meindl, 2004). If the demand for each fare is known with certainty then the problem would be easily solved. Unfortunately, the demands for
each fare are never known with certainty. However, historical data can provide good estimates for the demand of each fare. From historical data a distribution function of the demand for each class can be described. These demand distributions will be later used in the model to determine the optimal protection levels.

7.2.1.1 Littlewood’s Two Class Model

Littlewood’s two class model is a well-known method used in revenue management to address the problem of optimally allocating capacity to different classes. The model assumes the following:

- Two product classes with associated prices $p_1 > p_2$
- No cancellations and no overbooking
- Demand for the low fare arrives before the demand for the high fare

The demand for class $i$ is denoted by $D_i$ and its distribution is denoted by $F_i(\cdot)$. The total capacity is denoted by $C$. The problem is to determine how many of the low fare drivers to accept before seeing the realization of demand for the high fare drivers.

To illustrate this concept, assume that a request for a low fare space is received and the reservation system has to decide whether to accept or reject the request. As seen in Figure 23, the request can be either accepted or rejected. If the request is accepted then the gain in revenue is $p_2$ (low price). On the other hand, if the request is rejected, there are two possibilities. The first is that space will be sold at the full fare; therefore, the actual revenue will be $p_1$ (full price). The second possibility is that the space will not be requested by a full fare; therefore, the revenue will be zero. In other words, the decision to stop selling low fares depend on the conditional probability of selling more full fare,
which depends on the number of seats already sold and the accuracy of the forecasted demand (Talluri and Van Ryzin, 2004).

![Figure 23: Decision Tree](image)

The solution to this problem can be derived using simple marginal analysis. Suppose that there are \( x \) units of capacity remaining and a request from a low fare driver is requested. If the request is not accepted, the \( x \) unit will be sold at the high fare if and only if demand for high fare is \( x \) or higher \( D_1 \geq x \). A request from a low fare driver should be compared with the expected revenue from waiting for a high fare driver. The expected marginal revenue from the higher fare driver is given by \( p_1 P(D_1 \geq x) \). The request for the low fare driver should be accepted if the expected revenue from the higher fare driver is lower than the revenue from the lower fare driver. The following equation illustrates this concept.

\[
p_2 \geq p_1 P(D_1 \geq x)
\]

The reserved number of spaces for the high fare driver should be chosen such that the expected marginal revenue from the higher fare driver equals the current marginal
revenue from the low fare driver \( p_i P(D_i \geq x) = p_2 \). In other words, the number of spaces \( x^* \) reserved for the full fare drivers should be such that

\[
p_i P(D_i \geq x) = \frac{p_2}{p_1}
\]

If the demand for the full fare drivers is normally distributed, then the optimal protection level for the high fare drivers is

\[
x_i^* = F^{-1}(1 - \frac{p_2}{p_1})
\]

This equation is known as Littlewood’s rule. It provides the optimal booking limit for the high fare drivers. Then, the booking limit for the low fare drivers is

\[
b_2^* = \text{Capacity} - x_i^*
\]

Let’s consider an example where the demand for the high fare drivers is normally distributed with mean \( D_i \) and standard deviation \( \sigma_i \). Using the previously described concepts, the reservation quantity would be

\[
x_i^* = D_i + \text{NORMINV}\left(1 - \frac{p_2}{p_1}\right) \star \sigma_i
\]

Therefore, enough capacity will be reserved to meet the mean demand \( D_i \) plus or minus a factor that depends both on the revenue ratio and the standard deviation \( \sigma_i \). In general, the lower the ratio \( \frac{p_2}{p_1} \), the more capacity we reserve for the high fare. Consider the following cases:

<table>
<thead>
<tr>
<th>( p_i = $10 )</th>
<th>( p_i = $10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_2 = $8 )</td>
<td>( p_2 = $2 )</td>
</tr>
<tr>
<td>( \frac{p_2}{p_1} = 0.8 )</td>
<td>( \frac{p_2}{p_1} = 0.2 )</td>
</tr>
</tbody>
</table>

Case 1 | Case 2
For case 2, more spaces will be reserved for the full fare drivers than in case 1. The reason is obvious since we should be willing to reserve low prices only when the chances of selling full fare spaces are lower. This model will be used latter on in this chapter to optimally allocate spaces for two classes.

7.2.1.2 Expected Marginal Seat Revenue-version a (EMSR-a)

Littlewood’s two class model has proven to provide optimal protection and booking levels. Although the implementation of optimal policies is not complex, in practice most revenue management systems prefer to use heuristics to allocate spaces to more than two fare classes. The reason is that the optimality of Littlewood’s model for more than two classes was proven after heuristics were introduced. Therefore, heuristics gained popularity, and at that point, it was hard to convince airline management to redesign their reservation systems. Managers preferred to have a solution that was approximately right rather than having one that was precisely wrong.

There are two heuristics commonly used in revenue management introduced by Belobaba in 1987: Expected Marginal Seat Revenue-version a (EMSR-a) and Expected Marginal Seat Revenue –version b (EMSR-b).

The idea of EMSR-a is to apply Littlewood’s rule to successive pairs of classes and then add the protection levels produced. The following procedure is based on the one outlined by Talluri and Van Ryzin, 2004. Consider stage $j+1$ where demand arrives with price $p_{j+1}$. We want to determine the protection level $y_i$ for classes $j$ and higher ($j, j-1, ... 1$). Consider a single fare class $k$ among the remaining classes $j, j-1, ... 1$ and compare $k$ and $j+1$ in isolation. Taking into account these two classes, Littlewood’s rule is used to reserve capacity $y_k^{j+1}$ for class $k$, where
Therefore, the capacity for each class will be computed in isolation. Then, each one of these individual protection levels are added up to approximate the total protection level \( y_i \) for classes \( j \) and higher. The protection level \( y_i \) is

\[
y_j = \sum_{k=1}^{j} y_j^{j+1} \quad \forall j
\]

This heuristic is easy to implement. The problem is that by adding individual protection levels; it ignores the pooling effect produced by aggregating demand across classes. To avoid this problem, Belobaba introduced EMSR-b.

### 7.2.1.3 Expected Marginal Seat Revenue-version b (EMSR-b)

EMSR-b uses the same principle as EMSR-a of reducing the problem at each stage to a two class in order to apply Littlewood’s rule. To avoid the pooling effect of EMSR-a, this heuristic approximates the protection levels by aggregating demand rather than aggregating protection levels. Therefore, the demand from future classes is added and treated as revenue equal to the weighted-average revenue. The following procedure is based on the one outlined by Talluri and Van Ryzin, 2004. The heuristic works as follows. Consider that we are given estimates of the mean and standard deviation for each fare class \( j \) then the protection level \( y_i \) for class \( j \) and higher is given by Littlewood’s rule so that

\[
P(D_k > y^{j+1}_k) = \frac{p^{j+1}_k}{p_k} \quad \forall k = j, j-1, ..., 1
\]

\[
P(S_j > y_j) = \frac{p^{j+1}_j}{p_j}
\]

where \( S_j \) is the aggregate future demand for classes \( j, j-1, ..., 1 \)
\[ S_j = \sum_{k=1}^{j} D_k \]

And let the weighted-average revenue \( \bar{p}_j \) from classes \( 1, \ldots, j \) be defined by

\[
\bar{p}_j = \frac{\sum_{k=1}^{j} p_k E[D_k]}{\sum_{k=1}^{j} E[D_k]}
\]

Assuming that \( S_j \) is a normal random variable with mean \( \mu = \sum_{k=1}^{j} \mu_k \) and variance of the aggregated demand is \( \sigma^2 = \sum_{k=1}^{j} \sigma_k^2 \). Therefore, the protection level \( y_i \) for class \( j \) and higher is defined as \( y_i = \mu + z_{\alpha} \sigma \)

where \( z_{\alpha} = \phi^{-1}\left(1 - \frac{p_{j+1}}{\bar{p}_j}\right) \).

This heuristic is easy to implement and it extremely popular in many revenue management implementations. The next section will show an application of the three previously discussed capacity control methods to the parking problem.

7.3 Application to the Parking Industry

This section adapts the models discussed in the previous section to the parking problem by computing the protection levels and calculating the number of spaces that should be reserved for each fare.

The results obtained from the parking survey distributed are used as input data for the models. The parking survey was distributed to 51 subjects. Table 7 shows the demand data obtained from the parking survey. The demand is assumed to be normally distributed. It also shows the optimum protection levels for 51 parking spaces. The
optimum protection levels were obtained using Littlewood’s two class model described in section 8.2.1.1.

Table 7: Protection Levels for Two Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>P(j)</th>
<th>μ(j)</th>
<th>σ(j)</th>
<th>Opt. Protection Level</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7</td>
<td>23.0</td>
<td>5.8</td>
<td>19.7</td>
<td>$294.43</td>
</tr>
<tr>
<td>2</td>
<td>$5</td>
<td>28.0</td>
<td>2.4</td>
<td>31.3</td>
<td></td>
</tr>
</tbody>
</table>

The results indicate that twenty spaces should be reserved for the full fare segment and thirty one for the lowest fare. Littlewood’s model provided optimal protection levels for two fare classes. For more than two classes, EMSR-a and EMSR-b heuristics are used to analyze the impact on revenue by adding more than two fare classes. The heuristics implemented in Excel are discussed in more detail in Appendix G.

Table 8 shows the protection levels for three fare classes implementing EMSR-a and EMSR-b heuristics. As the results indicate the addition of a fare classes increased revenue from $294.43 to $330.32 or 11 percent. It is important to note that there is not a significant discrepancy among the computed protection levels from the heuristics.

Table 8: Protection Levels for Three Fare Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>p(j)</th>
<th>μ(j)</th>
<th>σ(j)</th>
<th>EMSR-a</th>
<th>EMSR-b</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$9</td>
<td>10.0</td>
<td>2.4</td>
<td>8.16</td>
<td>8.16</td>
<td>$330.32</td>
</tr>
<tr>
<td>2</td>
<td>$7</td>
<td>23.0</td>
<td>5.6</td>
<td>29.50</td>
<td>30.53</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$5</td>
<td>18.0</td>
<td>3.2</td>
<td>13.34</td>
<td>12.30</td>
<td></td>
</tr>
</tbody>
</table>

Table 9 illustrates the option of adding a fourth fare class. The results show that the addition of the fourth fare classes increased revenues from $330.32 (for three fare classes) to $354.65 or by 7 percent.
Table 9: Protection Levels for Four Fare Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>( p(j) )</th>
<th>( \mu(j) )</th>
<th>( \sigma(j) )</th>
<th>EMSR-a</th>
<th>EMSR-b</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$11</td>
<td>5.0</td>
<td>2.3</td>
<td>2.91</td>
<td>2.91</td>
<td>$ 354.65</td>
</tr>
<tr>
<td>2</td>
<td>$9</td>
<td>10.0</td>
<td>2.5</td>
<td>12.29</td>
<td>12.98</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$7</td>
<td>23.0</td>
<td>5.8</td>
<td>34.63</td>
<td>35.93</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$5</td>
<td>13.0</td>
<td>4.6</td>
<td>1.17</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Figure 24 summarizes the results previously discussed. It shows the relationship between the number of fare classes and the total expected revenue.

As can be seen in this figure, an increase in number of fare classes leads to an increase in total revenue. However, it is important to note that the increase in revenue can only be accomplished if the interactions among classes are minimized. This means that it is recommended that there is a significant difference in price that induces drivers to discriminate among classes. The results obtained from the parking survey show that a 20 percent difference in prices produces a more balanced demand.

The results of the scenarios using both heuristics for three and four classes are shown in Table 10. The capacity is varied from 50 parking spaces to 140 spaces. This variation on capacity illustrates how capacity will vary on a parking facility during a normal day. The results corroborate the previous conclusion that the addition of an extra
fare class increases total revenue. However, this study shows that the increase in revenue is more significant as capacity increases.

Table 10: Simulation of Revenue Performance

<table>
<thead>
<tr>
<th>Capacity</th>
<th>Revenue EMSR-a</th>
<th>%</th>
<th>Revenue EMSR-b</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-classes</td>
<td>4-classes</td>
<td>Increase</td>
<td>3-classes</td>
</tr>
<tr>
<td>50</td>
<td>$325.32</td>
<td>$349.65</td>
<td>7%</td>
<td>$327.39</td>
</tr>
<tr>
<td>60</td>
<td>$375.32</td>
<td>$399.65</td>
<td>6%</td>
<td>$377.39</td>
</tr>
<tr>
<td>70</td>
<td>$425.32</td>
<td>$452.39</td>
<td>6%</td>
<td>$427.39</td>
</tr>
<tr>
<td>80</td>
<td>$475.32</td>
<td>$522.39</td>
<td>9%</td>
<td>$477.39</td>
</tr>
<tr>
<td>90</td>
<td>$539.13</td>
<td>$600.62</td>
<td>10%</td>
<td>$539.13</td>
</tr>
<tr>
<td>100</td>
<td>$609.13</td>
<td>$690.62</td>
<td>12%</td>
<td>$609.13</td>
</tr>
<tr>
<td>110</td>
<td>$679.13</td>
<td>$780.62</td>
<td>13%</td>
<td>$679.13</td>
</tr>
<tr>
<td>120</td>
<td>$749.13</td>
<td>$870.62</td>
<td>14%</td>
<td>$749.13</td>
</tr>
<tr>
<td>130</td>
<td>$825.40</td>
<td>$977.40</td>
<td>16%</td>
<td>$825.40</td>
</tr>
<tr>
<td>140</td>
<td>$879.40</td>
<td>$1,043.40</td>
<td>16%</td>
<td>$879.40</td>
</tr>
</tbody>
</table>

This study shows how revenue management techniques increase revenues by increasing the number of fare classes. However, the objective of a parking manager is not only to increase revenue but also to maximize utilization. During low-periods of demand, parking manager could decrease prices to attract drivers during these periods. On the other hand, higher prices could be applied during peak periods of demand. This strategy is illustrated in Table 11 where Littlewood’s two class model is used to optimally allocate parking spaces to two fare classes. The demand data is normally distributed. The next analysis uses the results of the neural network model to forecast parking space availability. During low demand periods (Hours 0-9) more spaces are reserved for low fare drivers. On the other hand, all spaces are sold at the higher price during peak hours (Hours 10-17).
Table 11: RM Capacity Control Example

<table>
<thead>
<tr>
<th>Hour</th>
<th>Forecasted Availability</th>
<th>OPT CLASS1</th>
<th>OPT CLASS2</th>
<th>Expected Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>421</td>
<td>14</td>
<td>406</td>
<td>2131</td>
</tr>
<tr>
<td>1</td>
<td>442</td>
<td>14</td>
<td>428</td>
<td>2238</td>
</tr>
<tr>
<td>2</td>
<td>447</td>
<td>14</td>
<td>433</td>
<td>2265</td>
</tr>
<tr>
<td>3</td>
<td>442</td>
<td>14</td>
<td>428</td>
<td>2236</td>
</tr>
<tr>
<td>4</td>
<td>393</td>
<td>14</td>
<td>379</td>
<td>1994</td>
</tr>
<tr>
<td>5</td>
<td>246</td>
<td>14</td>
<td>232</td>
<td>1259</td>
</tr>
<tr>
<td>6</td>
<td>172</td>
<td>14</td>
<td>158</td>
<td>890</td>
</tr>
<tr>
<td>7</td>
<td>270</td>
<td>14</td>
<td>256</td>
<td>1377</td>
</tr>
<tr>
<td>8</td>
<td>243</td>
<td>14</td>
<td>229</td>
<td>1241</td>
</tr>
<tr>
<td>9</td>
<td>163</td>
<td>14</td>
<td>149</td>
<td>844</td>
</tr>
<tr>
<td>10</td>
<td>89</td>
<td>89</td>
<td>0</td>
<td>626</td>
</tr>
<tr>
<td>11</td>
<td>89</td>
<td>89</td>
<td>0</td>
<td>626</td>
</tr>
<tr>
<td>12</td>
<td>89</td>
<td>89</td>
<td>0</td>
<td>626</td>
</tr>
<tr>
<td>13</td>
<td>56</td>
<td>56</td>
<td>0</td>
<td>394</td>
</tr>
<tr>
<td>14</td>
<td>25</td>
<td>25</td>
<td>0</td>
<td>178</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
<td>25</td>
<td>0</td>
<td>177</td>
</tr>
<tr>
<td>16</td>
<td>56</td>
<td>56</td>
<td>0</td>
<td>395</td>
</tr>
<tr>
<td>17</td>
<td>56</td>
<td>56</td>
<td>0</td>
<td>395</td>
</tr>
<tr>
<td>18</td>
<td>101</td>
<td>14</td>
<td>87</td>
<td>532</td>
</tr>
<tr>
<td>19</td>
<td>192</td>
<td>14</td>
<td>178</td>
<td>989</td>
</tr>
<tr>
<td>20</td>
<td>198</td>
<td>14</td>
<td>184</td>
<td>1019</td>
</tr>
<tr>
<td>21</td>
<td>212</td>
<td>14</td>
<td>198</td>
<td>1088</td>
</tr>
<tr>
<td>22</td>
<td>221</td>
<td>14</td>
<td>207</td>
<td>1132</td>
</tr>
<tr>
<td>23</td>
<td>311</td>
<td>14</td>
<td>297</td>
<td>1581</td>
</tr>
</tbody>
</table>

Total Revenue $26,232.68

7.4 Summary

“Revenue Management is the use of differential pricing over time or customer segments to maximize profits from a limited capacity of resources.” (Chopra and Meindl, 2004). This chapter has illustrated this concept by introducing different prices for multiple fare classes. It has been demonstrated that an increase in the number of fare classes increases revenue. The idea of balance demand and supply has also been illustrated. Littlewood’s two class model has been used to optimally allocate parking
spaces for two fare classes. Furthermore, two traditional heuristics, EMSR-a and EMSR-b has been used to allocate parking spaces for three and four fare classes.
CHAPTER 8
CONCLUSIONS AND FUTURE RESEARCH

The research methods used for modeling parking systems have varied in complexity, ranging from simple empirical studies and heuristics to advanced techniques for mapping complex parking non-linearity. Most parking related literature reviewed appears to have several gaps that present opportunities associated with integration, and incorporation of technology into the approaches used for modeling. Although the models presented in the literature have a strong technical foundation, they have found limited application. Therefore, there is a need in the parking literature to provide managers and planners with tools that can be used to control parking demand. Each advance in information technology provides an opportunity for more innovative and comprehensive solutions, and greater integration with other important transportation functions.

The parking problem possesses distinctive characteristics where revenue management techniques may be employed for better allocation of limited resources and evaluation of issues such as parking fees. In this thesis, the traditional methodology of revenue management has been adopted and applied to the parking problem. First, market segmentation was studied through a stated preference survey. The results of the survey indicate that drivers are willing to pay higher fares under a time constraint situation. This demonstrates the concept that a parking space can be sold at different price rates. Furthermore, the parking survey helped to identify that a 20 percent difference in prices
induces drivers to change their parking choice from long walking distance to short walking distance.

The next step in the revenue management methodology is to forecast parking demand. This thesis compared neural networks versus traditional time series methods. The results show that Neural Networks are an efficient tool to predict parking demand. The major advantage of neural networks is that it is not necessary to pay major attention to nonlinearity included in the problem. Furthermore, the neural network architecture provides a framework that can be adapted to different case scenarios, and it is not necessary to develop new algorithms for specific problems. Neural networks allow to easily study complex relationship of factors which may have been difficult or impossible to model.

In this thesis, parking spaces were optimally allocated to two fare classes by Littlewood’s two class model. Furthermore, two traditional heuristics, EMSR-a and EMSR-b, were used to allocate parking spaces to three and four fare classes. The results show that an increase in the number of fare classes increases total revenue. Littlewood’s two class model was also used to optimally allocate parking spaces using the forecasted results from the neural network. The results show how revenue management techniques are effective to increase revenue and diversify demand.

The parking problem provides the opportunity for researchers to explore the creation of dynamic programming revenue management to identify optimal parking pricing strategies with real-time information. This may lead to the creation of more sophisticated information systems for drivers who will be able to know in advance where to find an available parking space. The ultimate objective should be to create an
intelligent system that allows drivers to reserve in advance where they will park and how much they are going to pay. This will provide a better balance of parking supply and demand and as a result will increase available parking inventory without the need to build additional facilities.

Some of the extensions to this research include: To develop an online parking reservation simulation system. This system will allow drivers to reserve a parking space in advance. The simulation will help to determine demand distributions for different drivers segments. The second extension of this thesis is to dynamically recalibrate the protection levels taking into account on-time reservations. It is important to further study the potential to introduce overbooking models. The stated parking survey distributed in this thesis should be applied on different demographic populations. This population should include older adults, and different income levels. There are some cases where owners of parking facility have a network of parking facilities. Therefore, it is necessary to apply the models to optimally allocate parking spaces to different fare classes for a network of parking facilities. More importantly, this thesis provides the opportunity for interdisciplinary collaboration among industrial engineering, transportation engineering and computer science. The collaboration of these disciplines will provide a more robust framework for solutions to the parking problem.
REFERENCES


Appendix A: Stated Preference Survey

DYNAMIC PARKING PRICING SYSTEM SURVEY

Project Description:
Parking plays an important role in the traffic system since all vehicles require a storage location when they are not being used to transport passengers. Most major cities continually struggle with parking limitations, violations and cost. Parking facilities experience peak and low demand periods. The problem increases during peak periods when it becomes challenging to find an available parking on a particular parking lot location. One alternative to this problem is to stimulate and diversify the demand with the introduction of pricing strategies. The goal of this survey is to identify how you as a driver would react to changes on prices and which parking facility would be selected for various set of scenarios and circumstances.

Instructions:
In the next pages you will find 12 questions. For each question, you will have the options to park on either LOT A or LOT B. Keep in mind the following assumptions when answering the questions:

1. Weather condition is about 75 degrees Fahrenheit and it is not raining.
2. When walking to destination, the walking pace is fixed and the same for all individuals -- that is, you won’t be able to walk faster in order to arrive earlier. Walking time accounts for the time it takes from the parking lot of your choice to the indicated destination.
3. Arrive time – indicates how early you are for your meeting/class or activity on a given scenario. For example indicates that you arrived 15 minutes early to your meeting/class or activity.
4. Time to Destination- (time to destination = walking time + driving time) indicates the time that will take to reach your final destination. NOTE: The majority of the time to reach your destination will be spend walking rather than driving

An example: In Figure 1, you will position yourself in the on the left side of the picture. Then you will check for the sign that indicates the time you have available from the parking lot you choose to the final destination. Sometimes you will be 15 minutes early, some others you might be 5 minutes early. In the example, the sign shows that you “ARRIVED 15 minutes EARLY” Next, you will compare parking LOT A versus parking LOT B in terms of price and time to destination. In this example, if you park in Lot A you will have 10 minutes to reach your destination (you will arrived 5 minutes early to your destination). The cost to park in lot A is $5. To park in Lot B you will have 5 minutes to reach your destination (you will be 10 minutes early), but you will have to pay $10. If you would prefer to pay less, then you will circle Lot A (as shown in the Figure). If you rather walk less time and cost is not of concern, you would choose Lot B. Remember that you have to take all the factors (arrival time, price, and time to destination) into consideration before selecting either Lot.
Appendix A: (Continued)

Figure 25: Survey Case Scenario Sample
Appendix A: (Continued)

Figure 26: Case Scenario # 1

Figure 27: Case Scenario # 2

Figure 28: Case Scenario # 3

Figure 29: Case Scenario # 4

Figure 30: Case Scenario # 5

Figure 31: Case Scenario # 6
Appendix A: (Continued)

Figure 32: Case Scenario # 7

Figure 33: Case Scenario # 8

Figure 34: Case Scenario # 9

Figure 35: Case Scenario # 10

Figure 36: Case Scenario # 11

Figure 37: Case Scenario # 12
Appendix A: (Continued)

Please answer the following questions:

1. What is your gender?
   ☐ Male ☐ Female

2. How old are you?
   ☐ Less than 20
   ☐ 20 – 29
   ☐ 30 – 39
   ☐ 40 – 49
   ☐ 50 – 59
   ☐ 60 or older

3. Are you employed?
   ☐ Yes → ☐ Full-time
   ☐ Part-time
   ☐ No

4. What is your income?
   ☐ Less than $15,000
   ☐ $15,000 – $19,999
   ☐ $20,000 – $29,999
   ☐ $30,000 – $39,999
   ☐ $40,000 – $49,999
   ☐ $50,000 – $59,999
   ☐ $60,000 – $69,999
   ☐ $70,000 or more

5. Would you pay more for a guaranteed parking space close to your destination?
   ☐ Yes ☐ No

6. Would you reserve a parking space in advance (on-line or by phone)? (Similar to the way you reserve an airline ticket)
   ☐ Yes ☐ No

7. Please rank which factor was most important to you when making your selection in the previous scenarios. Please write 1, 2 or 3 next to the factor.
   (1=very important, 2 important, 3 less important)
   Time to Destination (walking time + driving time) =
   Arrival time (being early or late) =
   Price ($) =

8. Please rank which factor was most important to you when making your selection in the
### Table 12: Survey Raw Data Results

<table>
<thead>
<tr>
<th>Sc 1</th>
<th>Sc 2</th>
<th>Sc 3</th>
<th>Sc 4</th>
<th>Sc 5</th>
<th>Sc 6</th>
<th>Sc 7</th>
<th>Sc 8</th>
<th>Sc 9</th>
<th>Sc 10</th>
<th>Sc 11</th>
<th>Sc 12</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

|      |      |      |      |      |      |      |      |      | # OT | # MTTD | # MC |
| 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11    | 12   |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| B    | B    | B    | B    | B    | B    | B    | B    | B    | B     | B     | B     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |
| A    | A    | A    | A    | A    | A    | A    | A    | A    | A     | A     | A     |

OT = On Time  
MTTD = Minimum Time to Destination  
MC = Minimum Cost  
Sc1 to Sc6= Scenario 1 to Scenario 6 = On Time (Time is not a constraint)  
Sc7 to Sc 12= Scenario 7 to Scenario 12 = Late (Time is a constraint)  
A = Parking Lot Further Away to the destination (Minimum Cost)  
B = Parking Lot Closest to the Destination (Highest Cost and Minimum Time to Destination)
Table 12: (Continued)

| Sc  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| 2   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| 3   | A  | A  | A  | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| 5   | B  | A  | B  | A  | B  | B  | A  | B  | B  | A  | B  | B  | A  | B  | B  | A  | B  | B  | A  | B  | B  | A  |
| 7   | B  | A  | A  | A  | B  | B  | B  | A  | B  | B  | B  | A  | A  | A  | B  | A  | B  | A  | B  | B  | B  | A  |

OT = On Time  
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Sc1 to Sc6= Scenario 1 to Scenario 6 = On Time (Time is not a constraint)  
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A = Parking Lot Further Away to the destination (Minimum Cost)  
B = Parking Lot Closest to the Destination (Highest Cost and Minimum Time to Destination)
Appendix C: Scenario Results

Figure 38: Case Scenario 1 Results

Figure 39: Case Scenario 2 Results

Figure 40: Case Scenario 3 Results

Figure 41: Case Scenario 4 Results

Figure 42: Case Scenario 5 Results

Figure 43: Case Scenario 6 Results

Figure 44: Case Scenario 7 Results

Figure 45: Case Scenario 8 Results

Figure 46: Case Scenario 9 Results

Figure 47: Case Scenario 10 Results

Figure 48: Case Scenario 11 Results

Figure 49: Case Scenario 12 Results
Appendix D: Forecasting Models Results

Figure 50: Moving Average Forecasting Results

Figure 51: Exponential Smoothing Forecasting Results
Appendix D: Forecasting Models Results

HOLT’S MODEL

Figure 52: Holt's Model Forecasting Results

WINTER’S MODEL

Figure 53: Winter's Model Forecasting Results
Figure 54: Neural Network Forecasting Results
Appendix E: Neural Network Code in MatLab

%#################################################################
%###NEURAL NETWORK TO FORECAST PARKING DEMAND###########
%#################################################################

%The following code forecast the parking demand using a Neural Network
%The model is tested with data provided from a major airport

%STEP 1
%The data needs to be separated into two subsets: testing data
%and validation data
%The testing data would be used to train the network and the validation
%data would be used to test network
%####################################################################
%CREATES INDEX FOR DAYS
%####################################################################
g=(unidrnd(2,31,1));% CREATES INDEX FOR SUBSAMPLING
k=0;
for i=1:31
    for c=1:24
        k=k+1;
        vec(k)=i;
        vec2(k)=g(i);
    end
end
data=[d vec' vec2'
for i=1:1 %FOR EACH VALIDATION SET
    valid=data(find(data(:,5)==i),1:3);
    clear tdata
    z=0;
    for j=1:3%TRAINING DATA SET
        if i == j
            z=z+1;
        else
            tdata=data(find(data(:,5)==j),1:3);
        end
    end
end
Appendix E: (Continued)

```matlab
%NN HERE########################################################################
p = tdata(:,2)';
t = tdata(:,3)';
val.P = valid(:,2)';
val.T = valid(:,3)';
net = newff(minmax(p),[3,1],{'tansig','purelin','trainlm'});
net.trainParam.show = 25;
net.trainParam.epochs = 300;
net = init(net);
[net,tr] = train(net,p,t);
%END NN#########################################################################```

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Appendix F: Statistical Test of MSE

**NEURAL NETWORK VS MOVING AVERAGE**

Paired T-Test and CI: NN, MA

Paired T for NN - MA

<table>
<thead>
<tr>
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<th>N</th>
<th>Mean</th>
<th>StdDev</th>
<th>SE Mean</th>
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<td>NN</td>
<td>24</td>
<td>749.5</td>
<td>78.2</td>
<td>16.0</td>
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<tr>
<td>MA</td>
<td>24</td>
<td>10513.2</td>
<td>173.4</td>
<td>35.4</td>
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<tr>
<td>Difference</td>
<td>24</td>
<td>-9763.7</td>
<td>168.3</td>
<td>34.3</td>
</tr>
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</table>

95% CI for mean difference: (-9834.8, -9692.7)

*T-Test of mean difference = 0 (vs not = 0): T-Value = -284.28  P-Value = 0.000*

**NEURAL NETWORK VS EXPONENTIAL SMOOTHING**

Paired T-Test and CI: NN, ES

Paired T for NN - ES

<table>
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<td>ES</td>
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<td>Difference</td>
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<td>-5133.7</td>
<td>128.8</td>
<td>26.3</td>
</tr>
</tbody>
</table>

95% CI for mean difference: (-5188.0, -5079.3)

*T-Test of mean difference = 0 (vs not = 0): T-Value = -195.23  P-Value = 0.000*
Appendix F: (Continued)

**NEURAL NETWORK VS HOLT'S METHOD**
Paired T-Test and CI: NN, Holts

Paired T for NN - Holts

<table>
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<tr>
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<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>24</td>
<td>749.5</td>
<td>78.2</td>
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<tr>
<td>Holts</td>
<td>24</td>
<td>8023.3</td>
<td>130.9</td>
<td>26.7</td>
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<tr>
<td>Difference</td>
<td>24</td>
<td>-7273.9</td>
<td>134.2</td>
<td>27.4</td>
</tr>
</tbody>
</table>

95% CI for mean difference: (-7330.5, -7217.2)

T-Test of mean difference = 0 (vs not = 0): T-Value = -265.54  P-Value = 0.000

**NEURAL NETWORK VS WINTER’S MODEL**
Paired T-Test and CI: NN, Winters

Paired T for NN - Winters

<table>
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<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
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<tr>
<td>NN</td>
<td>24</td>
<td>749.5</td>
<td>78.2</td>
<td>16.0</td>
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<tr>
<td>Winters</td>
<td>24</td>
<td>2635.9</td>
<td>61.9</td>
<td>12.6</td>
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<tr>
<td>Difference</td>
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<td>-1886.5</td>
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<td>10.0</td>
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</table>

95% CI for mean difference: (-1907.2, -1865.8)

T-Test of mean difference = 0 (vs not = 0): T-Value = -188.60  P-Value = 0.000
Appendix G: Capacity Control Models

Table 13: Littlewood's Two Class Model Results

<table>
<thead>
<tr>
<th>Class</th>
<th>p(j)</th>
<th>μ(j)</th>
<th>σ(j)</th>
<th>OPT</th>
<th>Revenue</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>23.0</td>
<td>5.8</td>
<td>19.7</td>
<td>$339.43</td>
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<tr>
<td>2</td>
<td>5</td>
<td>28.0</td>
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Table 14: Three Classes Data

<table>
<thead>
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<th>Class</th>
<th>p(j)</th>
<th>μ(j)</th>
<th>σ(j)</th>
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<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>10.0</td>
<td>2.4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>23.0</td>
<td>5.6</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>18.0</td>
<td>3.2</td>
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</table>

Table 15: Four Classes Data

<table>
<thead>
<tr>
<th>Class</th>
<th>p(j)</th>
<th>μ(j)</th>
<th>σ(j)</th>
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<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>5.0</td>
<td>2.3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>10.0</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>23.0</td>
<td>5.8</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>13.0</td>
<td>4.6</td>
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</table>

Table 16: EMSR-a for Three Classes

<table>
<thead>
<tr>
<th>j</th>
<th>k = 2</th>
<th>k = 1</th>
<th>y(j)</th>
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<tbody>
<tr>
<td>2</td>
<td>19.83</td>
<td>9.66</td>
<td>29.50</td>
</tr>
<tr>
<td>1</td>
<td>8.16</td>
<td>8.16</td>
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</table>

Table 17: EMSR-b for Three Classes

<table>
<thead>
<tr>
<th>j</th>
<th>μ</th>
<th>σ</th>
<th>P(j)</th>
<th>y(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>33.0</td>
<td>6.09</td>
<td>7.61</td>
<td>30.53</td>
</tr>
<tr>
<td>1</td>
<td>10.0</td>
<td>2.40</td>
<td>9</td>
<td>8.16</td>
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Table 18: EMSR-a for Four Classes

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<th>j</th>
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<th>y(j)</th>
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<td>3</td>
<td>19.72</td>
<td>9.65</td>
<td>5.26</td>
<td>34.63</td>
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<td>2</td>
<td>8.09</td>
<td>4.20</td>
<td>12.29</td>
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<td>1</td>
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<td>2.91</td>
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### Appendix G: (Continued)

Table 19: EMSR-b for Four Classes

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<th>$\mu$</th>
<th>$\sigma$</th>
<th>$P(j)$</th>
<th>$y(j)$</th>
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<tbody>
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<td>3</td>
<td>38.00</td>
<td>6.72</td>
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<td>2</td>
<td>15.00</td>
<td>3.40</td>
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<td>1</td>
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