2009

Transport of Momentum and Scalar in Turbulent Flows with Anisotropic Dispersive Waves

Semion Sukoriansky
Ben-Gurion University of the Negev

Nadejda Dikovskaya
Ben-Gurion University of the Negev

Boris Galperin
University of South Florida, bgalperin@usf.edu

Transport of momentum and scalar in turbulent flows with anisotropic dispersive waves

Semion Sukoriansky,[1] Nadejda Dikovskaya,[1] and Boris Galperin[2]

Received 7 May 2009; revised 16 June 2009; accepted 22 June 2009; published 29 July 2009.

[1] Most geophysical flows encompass turbulence and internal and/or Rossby waves. We demonstrate that these two different classes of waves cause remarkably similar anomalies in the turbulent transport. While all scales in both types of flows contribute to the momentum diffusion, the vertical (diapycnal) scalar diffusion in stratified flows and lateral diffusion in β-plane turbulence can be carried out only by turbulent eddies whose size is smaller than the thresholds of turbulence anisotropization. Beyond these thresholds, both flows become dominated by waves that provide no contribution to the scalar diffusion. Stably stratified flows exhibit enhanced isopycnal diffusion of both momentum and scalar. These results shed new light on the Osborn mixing model, diapycnal and isopycnal viscosity and diffusivity, absence of the critical gradient Richardson number, and large scale meridional transport. **Citation:** Sukoriansky, S., N. Dikovskaya, and B. Galperin (2009), Transport of momentum and scalar in turbulent flows with anisotropic dispersive waves, *Geophys. Res. Lett.*, *36*, L14609, doi:10.1029/2009GL038632.

1. Anisotropic Waves and Turbulence

[2] It is well known that anisotropic dispersive waves play a fundamental role in systems where they coexist with turbulence. As elucidated by *Smith and Waleffe* [2002], in each of the cases of stable stratification, system rotation or a β-effect, the large-scale flows become anisotropic and a slow manifold develops in a lower-dimensional subspace normal to the wavevector corresponding to the direction with zero frequency. The resulting flows would appear, respectively, as a system of vertically decorrelated horizontal layers [*Lilly*, 1983; *Fernando*, 2000], large-scale cyclonic vortical columns [*Smith and Waleffe*, 1999], or stable systems of alternating zonal jets [*Rhines*, 1975; *Huang et al.*, 2001]. Since most geophysical flows encompass various combinations of the internal, inertial and Rossby waves, understanding of their effects on turbulent transport is of paramount importance.

[3] This Letter focuses on the transport processes in flows with stable stratification or a β-effect thus extending studies of the analogies between linear internal and Rossby waves [e.g., *Bühler*, 2005]. Stratified flows are three-dimensional (3D) and will be studied analytically using the quasi-normal scale elimination (QNSE) turbulence theory [*Sukoriansky et al.*, 2005]. Flows with a β-effect are two-dimensional (2D) and will be investigated numerically using the barotropic vorticity equation on the surface of a rotating sphere [*Sukoriansky et al.*, 2007]. We shall see that in both cases, the scales associated with the turbulence-wave transitions also control anisotropy thresholds and transport processes.

2. Diffusion in Stably Stratified Flows

[4] In neutral, isotropic, homogeneous 3D turbulence, the *effective* viscosity and diffusivity at a scale k^{-1} can be described by the Richardson law [*Lesieur*, 1997],

$$\nu_n = C_\nu \epsilon^{1/3} k^{-4/3}, \quad \kappa_n = C_\kappa \epsilon^{1/3} k^{-4/3}, \qquad (1)$$

where ϵ is the rate of the viscous dissipation and C_ν, $C_\kappa = O(1)$. One can view ν_n and κ_n as the *eddy* viscosity and diffusivity that accumulated at a scale k^{-1} as a result of contributions from all eddies smaller than k^{-1}. The scaling equation (1) with $C_\nu/C_\kappa = O(1)$ leads to the Reynolds analogy constituting a similarity between momentum and heat transfer.

[5] To analyze stably stratified flows, we use the QNSE theory formulated for a horizontally homogeneous turbulent flow in steady state with imposed constant stabilizing temperature gradient [*Sukoriansky et al.*, 2005]. The theory explicitly accounts for the turbulence-wave interaction and flow anisotropy. In the neutral case, it yields $C_\nu = 0.46$, $C_\kappa = 0.64$, $Pr_t = \nu_n/\kappa_n = 0.72$, Pr_t being the turbulent Prandtl number, thus quantifying the Richardson law (1).

[6] An important characteristic of the stratified flows is the Ozmidov wavenumber, $k_O = (N^3/\epsilon)^{1/2}$, that arises from equating the turbulence turnover time scale, $[k^3 E(k)]^{-1/2}$, where $E(k)$ is the isotropic Kolmogorov turbulence spectrum, and the internal wave period, N^{-1}, N being the Brunt-Väisälä frequency. Thus, k_O characterizes the turbulence-wave transition. The QNSE theory provides expressions for the horizontal and vertical (isopycnal and diapycnal) viscosities and diffusivities, ν_h, ν_z, κ_h and κ_z, respectively, normalized with ν_n, as functions of k/k_O. These expressions have been implemented and tested in various planetary boundary layer and weather prediction models [*Sukoriansky et al.*, 2006; *Sukoriansky and Galperin*, 2008].

[7] In the limit of strong stratification, $k/k_O \to 0$, the QNSE theory yields $\nu_h \simeq 1.3\nu_n$ and $\kappa_h \simeq 3\nu_n$, both are larger than their respective values in the neutral case. The Richardson laws for ν_h and κ_h are consistent with the Kolmogorov $-5/3$ scaling of the horizontal kinetic and potential energies spectra in strongly anisotropic stably stratified turbulence [*Riley and Lindborg*, 2008]. For the diapycnal mixing coefficients, the QNSE asymptotic

[1] Department of Mechanical Engineering and Perlstone Center for Aeronautical Engineering Studies, Ben-Gurion University of the Negev, Beer-Sheva, Israel.

[2] College of Marine Science, University of South Florida, St. Petersburg, Florida, USA.

0094-8276/09/2009GL038632

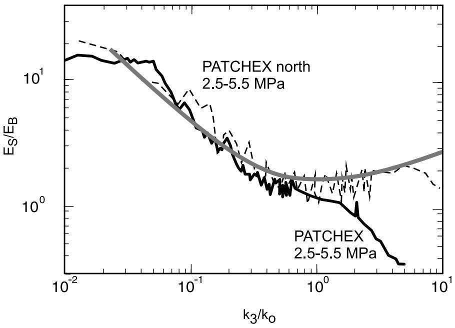

Figure 1. Comparison of the spectrum of the vertical shear in the ocean, $E_S(k_3/k_O)/E_B \equiv 2k_3^2 E_1(k_3)/E_B$, with the theoretical QNSE expression derived from equation (4) (grey thick line). The data is from *Gregg et al.* [1993], the normalization with $E_B = (\epsilon N)^{1/2}$ is after *Gargett et al.* [1981]. Note that for $k_3/k_O > 1$ the PATCHEX data belong in the viscous range.

expressions are $\nu_z/\nu_n \to \text{const} \simeq 0.2$ and $\kappa_z/\nu_n \to 0$, in good agreement with the atmospheric data [e.g., *Mauritsen and Svensson*, 2007]. The consequence of this result is the growing vertical turbulent Prandtl number, $Pr_t = \nu_z/\kappa_z$, and the failure of the Reynolds analogy. Observational, laboratory and numerical data [e.g., *Zilitinkevich et al.*, 2008; *Huq and Stewart*, 2008; *Yagüe et al.*, 2006] provide quantitative support to these conclusions. One can infer that waves can mix momentum but not scalar, a paradigm well accepted in geophysical fluid dynamics. These results point to the absence of the critical gradient Richardson number, Ri_{cr} [*Galperin et al.*, 2007], a conclusion of fundamental significance with important implications for numerical modeling of geophysical and planetary systems. The absence of Ri_{cr} has recently been adopted in Reynolds stress modeling [*Zilitinkevich et al.*, 2007; *Canuto et al.*, 2008; *L'vov et al.*, 2008; *Violeau*, 2009].

[8] Dimensional analysis reveals that in horizontally homogeneous stratified flows, κ_z can only depend on k, ϵ and N. In the case of strong stratification, the QNSE asymptotics indicates that κ_z becomes k-independent yielding

$$\kappa_z = \Gamma \epsilon N^{-2}, \quad (2)$$

where the "mixing" coefficient Γ needs to be determined. Equation (2), known in oceanography as the Osborn model [*Osborn*, 1980; *Lindborg and Brethouwer*, 2008], plays a major role in estimations of vertical diffusion in oceanic and atmospheric flows.

[9] Using the results from *Sukoriansky et al.* [2005] and equation (2), the asymptotic expressions for the diapycnal viscosity and diffusivity can be rearranged in a form congruent with the Richardson diffusion law,

$$\nu_z = 0.1\epsilon^{1/3}k^{-4/3}, \quad \kappa_z = \Gamma\epsilon^{1/3}k_O^{-4/3}. \quad (3)$$

One infers that while the diapycnal viscosity obeys the Richardson law, only the scales up to a fraction of k_O^{-1} contribute to the diapycnal diffusivity. The latter result provides a new interpretation to the Osborn model. Expressions in equation (3) can also be viewed as a statement about the absence of the critical Richardson number, Ri_{cr}.

[10] Using equation (2), it is easy to show that $R_f = \Gamma/(\Gamma + 1)$ where R_f is the flux Richardson number [*Pardyjak et al.*, 2002]. At strong stratification the QNSE theory predicts the limiting value of R_f in the range between 0.5 and 0.3 [*Sukoriansky et al.*, 2006] which yields Γ between 1 and 0.4. These values are somewhat larger than the widely accepted in the oceanographic literature estimate of 0.2 [*Osborn*, 1980; *Gregg et al.*, 2003; *Wunsch and Ferrari*, 2004]. They are well in line with the atmospheric data, however, where Γ was found in the range between 0.33 and 1 [*Lilly et al.*, 1974; *Pardyjak et al.*, 2002; *Bishop et al.*, 2004; *Clayson and Kantha*, 2008].

[11] What determines the maximum scale of the eddies that contribute to the diapycnal diffusivity? Consider one-dimensional vertical (along k_3) energy spectrum of a horizontal velocity component derived analytically from the QNSE theory [*Sukoriansky et al.*, 2005],

$$\begin{aligned} E_1(k_3) &= 0.626\,\epsilon^{2/3}\,k_3^{-5/3} + 0.214\,N^2 k_3^{-3} \\ &= 0.626\,\epsilon^{2/3}\,k_3^{-5/3}\left[1 + 0.34(k_3/k_O)^{-4/3}\right]. \end{aligned} \quad (4)$$

The coefficient 0.214 with the $N^2k_3^{-3}$ term agrees well with the tropospheric, stratospheric, mesospheric and thermospheric data [*Smith et al.*, 1987]. Figure 1 compares equation (4) recast in terms of the vertical shear and normalized after *Gargett et al.* [1981] with the data by *Gregg et al.* [1993]. Good agreement extends into the range of strong stratification. For $k_3 \lesssim 0.5k_O$, the vertical shear spectrum attains the slope k_3^{-1} which can be associated with the transition to the internal wave dominated flow regime [*Gregg et al.*, 1993].

[12] The following picture of the diapycnal mixing in stably stratified flows is now emerging. The small scales are practically unaffected by stratification and obey the Richardson law. On larger scales, up to a transitional scale $\simeq 0.5k_O^{-1}$, vertical mixing becomes progressively less efficient compared to the Richardson law. The latter scale provides the maximum size of a turbulent eddy that can contribute to the diapycnal diffusivity. Larger scales are wave dominated and do not contribute to the scalar mixing. The diapycnal diffusivity on those scales becomes scale independent and is determined by the Ozmidov wavenumber according to equation (3). The transitional scale also demarcates the region in which the buoyancy contribution, $N^2k_3^{-3}$, dominates the vertical energy spectrum.

[13] Earlier we have seen that unlike κ_z, κ_h is scale dependent. The striking contrast between κ_z and κ_h distinguishes dispersion processes in geophysical systems. For instance, studies of the mixing in the oceanic pycnocline by *Ledwell et al.* [1998] reveal that the diapycnal diffusivity remains approximately constant and largely scale independent on the time scales from 6 to 24 months. The isopycnal diffusivity, on the other hand, was substantially scale-dependent increasing from 2 m^2 s^{-1} to 10^3 m^2 s^{-1} on scales from about 1 to about 300 km. These numbers are in good quantitative agreement with the obtained earlier QNSE expressions assuming that $\epsilon \simeq 10^{-9}$ m^2 s^{-3}.

[14] The results of this section pertain to relatively small-scale flows with large-scale forcing. Large-scale oceanic flows are forced by the baroclinic instability and consid-

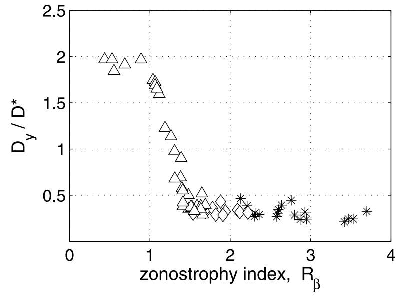

Figure 2. The meridional diffusivity, D_y, normalized with $D^* = \epsilon^{1/3} n_E^{-4/3}$ in the friction-dominated regime (open triangles) and with $D^* = \epsilon^{1/3} n_\beta^{-4/3}$ in the transitional and zonostrophic regimes (open diamonds and asterisks), respectively.

erations of the inverse energy cascade and a β-effect become important [*Ollitrault et al.*, 2005; *LaCasce*, 2008]. In the next section we shall show that important features of turbulent diffusion in stably stratified 3D flows with internal waves and direct energy cascade are similar to those in 2D flows with a β-effect, Rossby-Haurwitz waves and inverse energy cascade.

3. Diffusion in 2D Flows With a β-Effect

[15] Small-scale forced homogeneous and isotropic 2D flows feature the inverse energy cascade in the energy subrange. Such flows exhibit the negative Laplacian viscosity phenomenon [*Starr*, 1968]. *Sukoriansky et al.* [1996] showed that these flows can be successfully simulated using the stabilized negative viscosity (SNV) method combining flow-dependent negative Laplacian and stabilizing biharmonic viscosities. The turbulent diffusivity in such flows is positive and, similarly to 3D flows, obeys the Richardson law (1), where ϵ is the inverse cascade rate [*LaCasce*, 2008]. Clearly, the Reynolds analogy does not apply to 2D flows.

[16] We consider turbulent diffusion in a 2D flow on the surface of a rotating sphere. The dynamics of this system is governed by the barotropic vorticity equation (see *Sukoriansky et al.* [2007] for details). Linearized equation supports the Rossby-Haurwitz waves (RHWs). The inverse cascade becomes anisotropic on large scales. When a mechanism of the large-scale dissipation is included, the balance between the forcing and the dissipation gives rise to a steady state. In our simulations, this dissipation was emulated by the linear drag that affects all wave numbers up to a friction wave number, n_{fr}.

[17] As discussed by *Sukoriansky et al.* [2007], the transitional spherical wavenumber $n_\beta = 0.5(\beta^3/\epsilon)^{1/5}$, where $\beta = \Omega/R$, Ω and R being the angular velocity and the radius of the sphere, respectively, defines a scale at which the time scale of turbulence equilibrates with the time scale of RHWs and is analogous to the Ozmidov wavenumber in flows with stable stratification. Larger scales are marked by progressive anisotropy and steepening of the zonal spectrum. The energy flux is directed towards the zonal modes and the flow morphs into a system of stable alternating zonal jets typical of zonostrophic turbulence [*Galperin and Sukoriansky*, 2008]. In this regime, n_{fr} becomes close to the Rhines's wavenumber, $n_R = (\beta/2U)^{1/2}$, where U is rms of the total velocity fluctuation. A *zonostrophy index*, $R_\beta \equiv n_\beta/n_R$, measures the width of the zonostrophic inertial range [*Galperin and Sukoriansky*, 2008]. Different flow regimes can be classified in terms of R_β: the ranges $R_\beta \lesssim 1.5$, $1.5 \lesssim R_\beta \lesssim 2$, and $R_\beta \gtrsim 2$ correspond to the friction-dominated, transitional and zonostrophic regimes, respectively.

[18] *Sukoriansky et al.* [2007] demonstrated the presence of RHWs in the friction-dominated regime. The zonostrophic regime harbors not only RHWs but also a new class of *nonlinear* waves, or zonons [*Sukoriansky et al.*, 2008]. The large-scale flow dynamics in both regimes are progressively dominated by waves.

[19] The zonostrophic turbulence is similar to flows with strong stable stratification in three aspects: the steepening of the spectrum along the zero-frequency direction; increasing large-scale anisotropy, and domination of waves. The spectral thresholds of these phenomena can be expressed in terms of the transitional and Ozmidov wave numbers, n_β and k_O, respectively.

[20] One can now inquire whether or not the diffusion processes in these two regimes also bear similar traits. This inquiry was addressed in a series of simulations with the barotropic vorticity equation coupled with the diffusion equation. The flow was in a steady state; R_β varied between 0 and 4 thus covering all three flow regimes. A tracer was released from thin zonal rings located at different latitudes and from the polar caps. Such a release eliminated complications due to the zonal advection while allowing for testing of the effect of the zonal jets on the meridional diffusion. The tracers' evolution was followed until they reached high latitudes from which they could escape via polar regions. The meridional diffusivity, D_y, was calculated using three different methods, from the meridional spread, σ_y^2, according to the well known expression, $\sigma_y^2 = 2D_y t$, where t is the time elapsed since the tracer release; from the Lagrangian trajectories, and from the Reynolds averaging, by dividing the meridional turbulent tracer flux by the mean meridional tracer gradient. The results obtained from all three methods were very similar; they are summarized in Figure 2.

[21] In the friction-dominated regime, for $R_\beta \lesssim 1$ the diffusion is nearly isotropic and obeys the Richardson law, $D_y \simeq 2\epsilon^{1/3} n_E^{-4/3}$. Here, $n_E \propto n_{fr}$ is the wavenumber of the mode with the maximum kinetic energy. While still in the friction regime, $1 \lesssim R_\beta \lesssim 1.5$, D_y undergoes the transition to $D_y \simeq 0.3\epsilon^{1/3} n_\beta^{-4/3}$. Afterwards, D_y keeps this value throughout the transitional and zonostrophic ranges, $R_\beta \gtrsim 1.5$. This result is rather remarkable as it demonstrates that the scales where the β-effect is important and which are dominated by RHWs make no contribution to the meridional diffusion. The effective transport on those scales is determined by quasi-isotropic turbulent eddies with scales not exceeding a fraction of n_β^{-1}. This result extends the analogy between the cross-jet diffusion in 2D turbulence with a β-effect and diapycnal diffusion in 3D flows with strong stable stratification.

[22] The analogue of equation (2) in zonostrophic turbulence can be derived by recasting the expression for D_y in terms of β and ϵ,

$$D_y \simeq 0.8\epsilon^{3/5}\beta^{-4/5}. \quad (5)$$

[23] A similar expression was obtained by *Smith* [2005] in numerical simulations. *Lapeyre and Held* [2003] derived

this scaling theoretically assuming that the inverse energy cascade is halted by a β-effect on scales of the order of n_β^{-1} and, thus, that D_y depends on ϵ and β only. Here, we give this equation an alternative explanation as *Sukoriansky et al.* [2007] demonstrated that a β-effect does not halt the inverse cascade and only causes its anisotropization on scales $O[n_\beta^{-1}]$ and larger.

[24] *Galperin et al.* [2004] argued that the deep large-scale oceanic circulation is marginally zonostrophic in which case one could expect a transition from the Richardson law to the scale-independent diffusivity equation (5). Such transition was indeed observed in some experiments on relative dispersion of subsurface floats in the North Atlantic [*Ollitrault et al.*, 2005; *LaCasce*, 2008]. The scale of this transition can be estimated as n_β^{-1}. With $\epsilon \sim 10^{-10}$ m^2 s^{-3} [*Ollitrault et al.*, 2005; *Galperin et al.*, 2004] and $\beta \sim 10^{-11}$ m^{-1} s^{-1}, n_β^{-1} is appraised at approximately 100 km, in reasonable agreement with the observations.

[25] To better understand the effects of the turbulence forcing and waves on the diffusion processes in 2D flows, we ran twin forced simulations on rotating and non-rotating spheres. After a steady state was established, the forcing and the large-scale drag were turned off simultaneously in both cases. On the non-rotating sphere, D_y did not change much while in the other case, D_y decreased dramatically and was soon extinguished. Further analysis of the rotating case revealed that after the forcing and the drag are turned off, the energy spectrum undergoes rapid reorganization. The energy is re-distributed to the large-scale zonal and nearly-zonal, wave dominated modes at the expense of other modes while the total energy remains nearly unchanged. Since the most energetic modes in the new flow configuration do not contribute to the meridional transport, D_y decreases dramatically. These results suggest that the variability of the turbulence forcing has a strong impact upon lateral transport via changes in the spectral dynamics and energy transfer to the wave dominated modes. It is an important challenge to investigate whether or not similar processes are at work during the formation and dissolution of the mixing barriers in the atmosphere and the ocean.

[26] Concluding we emphasize that internal and planetary waves have a profound effect on turbulent diffusion and that the analogies between the transport of momentum/vorticity and scalar are not always warranted in turbulent flows.

[27] **Acknowledgments.** Partial support of this research by the ARO grants W911NF-05-1-0055 and W911NF-09-1-0018, ONR grant N00014-07-1-1065, and the Israel Science Foundation grant 134/03 is gratefully appreciated.

References

Bishop, R. L., M. F. Larsen, J. H. Hecht, A. Z. Liu, and C. S. Gardner (2004), TOMEX: Mesospheric and lower thermospheric diffusivities and instability layers, *J. Geophys. Res.*, *109*, D02S03, doi:10.1029/2002JD003079.

Bühler, O. (2005), Wave-mean interaction theory, in *Nonlinear Waves in Fluids*, *CISM Courses Lect.*, *483*, edited by R. Grimshaw, pp. 95–133, Springer, Vienna.

Canuto, V., Y. Cheng, A. Howard, and I. Esau (2008), Stably stratified flows: A model with no Ri(cr), *J. Atmos. Sci.*, *65*, 2437–2447.

Clayson, C., and L. Kantha (2008), On turbulence and mixing in the free atmosphere inferred from high-resolution soundings, *J. Atmos. Oceanic Technol.*, *25*, 833–852.

Fernando, H. (2000), Aspects of stratified turbulence, in *Developments in Geophysical Turbulence*, *Fluid Mech. Appl.*, *58*, 81–92, Kluwer Acad., Dordrecht, Netherlands.

Galperin, B., and S. Sukoriansky (2008), Zonostrophic turbulence, *Phys. Scr. T*, *132*, 014034, doi:10.1088/0031-8949/2008/T132/014034.

Galperin, B., H. Nakano, H.-P. Huang, and S. Sukoriansky (2004), The ubiquitous zonal jets in the atmospheres of giant planets and Earth's oceans, *Geophys. Res. Lett.*, *31*, L13303, doi:10.1029/2004GL019691.

Galperin, B., S. Sukoriansky, and P. Anderson (2007), On the critical Richardson number in stably stratified turbulence, *Atmos. Sci. Lett.*, *8*, 65–69.

Gargett, A., P. Hendricks, T. Sanford, T. Osborn, and A. Williams III (1981), A composite spectrum of vertical shear in the upper ocean, *J. Phys. Oceanogr.*, *11*, 1258–1271.

Gregg, M., D. Winkel, and T. Sanford (1993), Varieties of fully resolved spectra of vertical shear, *J. Phys. Oceanogr.*, *23*, 124–141.

Gregg, M., T. Sanford, and D. Winkel (2003), Reduced mixing from the breaking of internal waves in equatorial waters, *Nature*, *422*, 513–515.

Huang, H.-P., B. Galperin, and S. Sukoriansky (2001), Anisotropic spectra in two-dimensional turbulence on the surface of a rotating sphere, *Phys. Fluids*, *13*, 225–240.

Huq, P., and E. J. Stewart (2008), Measurements and analysis of the turbulent Schmidt number in density stratified turbulence, *Geophys. Res. Lett.*, *35*, L23604, doi:10.1029/2008GL036056.

LaCasce, J. (2008), Statistics from Lagrangian observations, *Prog. Oceanogr.*, *88*, 1–29.

Lapeyre, G., and I. M. Held (2003), Diffusivity, kinetic energy dissipation, and closure theories for the poleward eddy heat flux, *J. Atmos. Sci.*, *60*, 2907–2916.

Ledwell, J. R., A. J. Watson, and C. S. Law (1998), Mixing of a tracer in the pycnocline, *J. Geophys. Res.*, *103*, 21,499–21,529.

Lesieur, M. (1997), *Turbulence in Fluids*, *Mech. Fluid Transp. Processes*, vol.*6*, Kluwer Acad., Dordrecht, Netherlands.

Lilly, D. (1983), Stratied turbulence and mesoscale variability in the atmosphere, *J. Atmos. Sci.*, *40*, 749–761.

Lilly, D., D. Waco, and S. Adelfang (1974), Stratospheric mixing estimated from high-altitude turbulence measurements, *J. Appl. Meteorol.*, *13*, 488–493.

Lindborg, E., and G. Brethouwer (2008), Vertical dispersion by stratified turbulence, *J. Fluid Mech.*, *614*, 303–314.

L'vov, V., I. Procaccia, and O. Rudenko (2008), Turbulent fluxes in stably stratified boundary layers, *Phys. Scr. T*, *132*, 014010, doi:10.1088/0031-8949/2008/T132/014010.

Mauritsen, T., and G. Svensson (2007), Observations of stably stratified shear-driven atmospheric turbulence at low and high Richardson numbers, *J. Atmos. Sci.*, *64*, 645–655.

Ollitrault, M., C. Gabillet, and A. C. de Verdiére (2005), Open ocean regimes of relative dispersion, *J. Fluid Mech.*, *533*, 381–407.

Osborn, T. (1980), Estimates of the local rate of vertical diffusion from dissipation measurements, *J. Phys. Oceanogr.*, *20*, 83–89.

Pardyjak, E., P. Monti, and H. Fernando (2002), Flux Richardson number measurements in stable atmospheric shear flows, *J. Fluid Mech.*, *459*, 307–316.

Rhines, P. (1975), Waves and turbulence on a beta-plane, *J. Fluid Mech.*, *69*, 417–443.

Riley, J., and E. Lindborg (2008), Stratified turbulence: A possible interpretation of some geophysical turbulence measurements, *J. Atmos. Sci.*, *65*, 2416–2424.

Smith, K. (2005), Tracer transport along and across coherent jets in two-dimensional turbulent flow, *J. Fluid Mech.*, *544*, 133–142.

Smith, L., and F. Waleffe (1999), Transfer of energy to two-dimensional large scales in forced, rotating three-dimensional turbulence, *Phys. Fluids*, *11*, 1608–1622.

Smith, L., and F. Waleffe (2002), Generation of slow large scales in forced rotating stratified turbulence, *J. Fluid Mech.*, *451*, 145–168.

Smith, S., D. Fritts, and T. VanZandt (1987), Evidence for a saturated spectrum of atmospheric gravity waves, *J. Atmos. Sci.*, *44*, 1404–1410.

Starr, V. (1968), *Physics of Negative Viscosity Phenomena*, McGraw-Hill, New York.

Sukoriansky, S., and B. Galperin (2008), Anisotropic turbulence and internal waves in stably stratified flows (QNSE theory), *Phys. Scr. T*, *132*, 014036, doi:10.1088/0031-8949/2008/T132/014036.

Sukoriansky, S., A. Chekhlov, S. Orszag, B. Galperin, and I. Staroselsky (1996), Large eddy simulation of two-dimensional isotropic turbulence, *J. Sci. Comput.*, *11*, 13–45.

Sukoriansky, S., B. Galperin, and I. Staroselsky (2005), A quasinormal scale elimination model of turbulent flows with stable stratification, *Phys. Fluids*, *17*, 085107, doi:10.1063/1.2009010.

Sukoriansky, S., B. Galperin, and V. Perov (2006), A quasi-normal scale elimination model of turbulence and its application to stably stratified flows, *Nonlinear Processes Geophys.*, *13*, 9–22.

Sukoriansky, S., N. Dikovskaya, and B. Galperin (2007), On the "arrest" of the inverse energy cascade and the Rhines scale, *J. Atmos. Sci.*, *64*, 3312–3327.

Sukoriansky, S., N. Dikovskaya, and B. Galperin (2008), Nonlinear waves in zonostrophic turbulence, *Phys. Rev. Lett.*, *101*, 178501, doi:10.1103/PhysRevLett.101.178501.
Violeau, D. (2009), Explicit algebraic Reynolds stresses and scalar fluxes for density-stratified shear flows, *Phys. Fluids*, *21*, 035103, doi:10.1063/1.3081552.
Wunsch, C., and R. Ferrari (2004), Vertical mixing, energy, and the general circulation of the oceans, *Annu. Rev. Fluid Mech.*, *36*, 281–314.
Yagüe, C., S. Viana, G. Maqueda, and J. Redondo (2006), Influence of stability on the flux-profile relationships for wind speed, ϕ_m, and temperature, ϕ_h, for the stable atmospheric boundary layer, *Nonlinear Processes Geophys.*, *13*, 185–203.
Zilitinkevich, S., T. Elperin, N. Kleeorin, and I. Rogachevskii (2007), Energy and flux-budget (EFB) turbulence closure model for stably stratified flows. Part I: Steady-state, homogeneous regimes, *Boundary Layer Meteorol.*, *125*, 167–191.
Zilitinkevich, S., T. Elperin, N. Kleeorin, I. Rogachevskii, I. Esau, T. Mauritsen, and M. Miles (2008), Turbulence energetics in stably stratified geophysical flows: Strong and weak mixing regimes, *Q. J. R. Meteorol. Soc.*, *134*, 793–799.

N. Dikovskaya and S. Sukoriansky, Perlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University, PO Box 653, Beer-Sheva 84105, Israel. (nadejd@bgu.ac.il; semion@bgu.ac.il)

B. Galperin, College of Marine Science, University of South Florida, 140 7th Avenue South, St. Petersburg, FL 33701, USA. (bgalperin@marine.usf.edu)