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## On the Indemonstrability of the Principle of Contradiction

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On the Indemonstrability of the Principle of Contradiction

by

Elisabeta Sarca

A thesis submitted in partial fulfillment  
of the requirements for the degree of  
Master of Arts  
Department of Philosophy  
College of Arts and Sciences  
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## **On the Indemonstrability of the Principle of Contradiction**

Elisabeta Sarca

### **ABSTRACT**

In this thesis I examine three models of justification for the epistemic authority of the principle of contradiction. Aristotle has deemed the principle “that the same attribute cannot at the same time belong and not belong to the same subject and in the same respect” the most certain and most prior of all principles, both in the order of nature and in the order of knowledge, and as such it is indemonstrable. The principle of contradiction is involved in any act of rational discourse, and to deny it would be to reduce ourselves to a vegetative state, being incapable of uttering anything with meaning. The way we reach the principle of contradiction is by intuitive grasping (*epagoge*) from the experience of the particulars, by recognizing the universals in the particulars encountered, and it is different from simple induction, which, in Mill’s view, is the process through which we *construct* a general statement on the basis of a limited sample of observed particulars. Hence, the principle of contradiction, being a mere generalization from experience, through induction, loses its certainty and necessity. Even though it has a high degree of confirmation from experience, it is in principle possible to come across a counter-example which would refute it. Mill’s account opens the path to the modern view of the principle of contradiction. In *Principia Mathematica*, Russell and Whitehead

contend that the principle of contradiction is still a tautology, always true, but it is derived from other propositions, set forth as axioms. Its formulation, ' $\sim (p \ \& \ \sim p)$ ' is quite different from Aristotle's, and this is why we are faced with the bizarre situation of being able to derive the law of contradiction in a formal system which could not have been built without the very principle of which the law is an expression of. This is perhaps because the principle of contradiction, as a principle, has a much larger range of application and is consequently more fundamental than what we call today the law of contradiction, with its formal function.

## Chapter One: Aristotle, The Classical Approach

### *Introduction*

It has been claimed that Aristotle's treatment of the principle of contradiction, especially his justification of it, is not anymore pertinent to serve the purposes of modern logic and even that the philosopher engaged in a circular demonstration of the principle, contrary to what he himself had deemed possible or reasonable. Before we embark upon the enquiry into Aristotle's account of the principle of contradiction, it would be very useful to first understand his metaphysics and his conception of logic<sup>1</sup> and science, since I believe that these will provide us with valuable clues about the basis of Aristotle's views on what he called "the most certain of all principles"<sup>2</sup>. This is because of two reasons: (i) the principle of contradiction is an ontological principle, and as such we must understand *what* we are talking about and to *what* exactly the principle is applied; and (ii) the principle of contradiction is also a logical principle and as such we must understand *how* to formulate it and to speak about it correctly, in Aristotle's view.

As a consequence, we shall first examine the Aristotelian theory of categories, attempting to understand, on the one hand, what counts as existent and in what sense, and on the other hand, in view of this, what counts as correct predication and in what sense. Then we will explore the link with the theory of science and with metaphysics (first

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<sup>1</sup> Aristotle himself did not use the term 'logic' to refer to what we now so call, nor did he thus name his treatises, which he called 'analytics'.

<sup>2</sup> *Met.* Γ3, 1005b17. Unless otherwise specified, throughout this thesis I am using Richard McKeon, *The Basic Works of Aristotle*, Random House, New York, © 1941.

philosophy), whose subject matter are the first principles of reality and knowledge, because the principle of contradiction is one such principle; in fact it is the most prior and certain of them. All the sciences, in addition to their special principles, have at the basis and indeed at the most fundamental level, the metaphysical principles, which pertain to all being and are involved in all discourse. Of course, the thorough assessment of these matters, if at all possible, would require an enormous amount of work. I will only point out to those issues that I appreciate would be indispensable if we want to grasp the Aristotelian conception of the principle of contradiction. By doing that, I am running the risk of not doing justice to the wonderful complexity of Aristotelian philosophy, but that is something which is evidently not within the scope of this thesis.

Following the exposition of the general theoretical frame relevant to the discussion of the principle of contradiction, I will focus my attention on the nature of the principle of contradiction, its special status in the edifice of knowledge, its indemonstrability as a result of its priority and the elenctic proof (or the proof by refutation) that Aristotle brings in support, not of the principle itself, but of the claim that the principle of contradiction is the firmest of all principles and nobody can seriously claim to disbelieve it.

### *I. Ontological background*

The basic Aristotelian ontological structure of the world, as outlined in *Categories*, is partitioned into the ten genera of being. The first genus of being, *ousia*, is of two types: primary and secondary. The primary *ousiai* are the individual things, and they are the fundamental units of reality, in fact the units with most reality. The secondary *ousiai* are the classes of things, or the genera and species of things. The other



nine genera of being (quantity, quality, relation, place, time, position, state, action, affection) are properties inherent in the primary *ousiai*, which means that they cannot be without a certain *ousia*; they are, in other words, co-incidentals (they occur *with* the things - *symbebekota*). It is not, of course, necessary that a certain *ousia* have a certain co-incidental property (*e.g.* a certain place or a certain shape etc.), nor that it have all of them. In fact it can have all the nine *types* of co-incidental attributes, but it cannot have all the particular co-incidental attributes, in virtue of the principle of contradiction itself, because that would mean that it admits of attributes and their negations – but we will see about that later.

What is necessary is that a particular thing cannot *lack* all co-incidental properties. If we were to remove all co-incidental properties from an *ousia*, provided that such an action is possible, we are left with nothing: the *ousia* cannot be without them: it must have at least one attribute. We will see later that this is important for understanding the notion of contradictory terms: an attribute and its negation cover the entire universe of discourse and, as such, it is absurd to claim that they can both be false of an *ousia* – this is in virtue of the principle of the excluded middle.

For Aristotle, this picture of the world is reflected in the way we can predicate things: the primary *ousiai* can be only in the subject position of a sentence (categorical predication), whereas everything else can be only in the predicate position of a sentence. These other types of being are either only predicable of the subject (and they are the universals, or classes of things), or only present in the subject (being attributes that are inherent to things and could not exist without being tied to these), or both predicable of and present in the subject (these are attributes that in certain respects are inherent to

things, but in other respects can also form classes of things). The primary entities, or *ousiai*, the individual, concrete things, are the ultimate subjects of any discourse and of course they are neither present in, nor predicable of, a subject<sup>3</sup>. For example, ‘man’ is predicable of a particular human (which is an *ousia*), but it is not inherent in the particular human, since it is a class of things, of the type that Aristotle calls “secondary *ousia*”, white is a property that is inherent in a particular thing, but it does not form a class of things, and colour is inherent in a particular thing – because it is a quality, and that is one of the nine inherent properties – but also ‘colour’ is predicable of any particular colour, since it forms a class of things (*i.e.* the class of all colours).

Aristotle contends that if we plan to say anything with sense, we must use the model of predication sketched in the *Categories* and in *De Interpretatione*. The individual things are the fundamental elements of reality and whatever we know, we must know through them. Now, this does not mean Aristotle argues that knowledge is of the individuals, but, on the contrary, he claims that knowledge is of the universals: “what knowledge apprehends is universals”<sup>4</sup>; “scientific knowledge is about things that are universal and necessary”<sup>5</sup>. We shall investigate that in the next section of the chapter, but first let us make notice of the status of secondary *ousiai*.

As we have mentioned, the secondary *ousiai* are the classes of individual things, and they are next in importance and reality only to the primary *ousiai*. There is, though, a hierarchy among them: “the species is more truly substance [*ousia*] than the genus, being more nearly related to primary substance [*ousia*]”<sup>6</sup>. Whatever is predicable of the genus is

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<sup>3</sup> *Cat.* 1, 2, 1<sup>a</sup>20-1<sup>b</sup>9.

<sup>4</sup> *De An.* II5, 417<sup>b</sup>22.

<sup>5</sup> *Nic. Eth.*, VI6, 1140<sup>b</sup>31.

<sup>6</sup> *Cat.* 5, 2<sup>b</sup>7.

predicable of the species, and whatever is predicable of the species, is predicable of the individual – this is how, in fact, scientific knowledge proceeds through syllogism, which is an inference from universals to the particulars. The secondary *ousiai* form what we will see later to be essential predication and they are the object of knowledge since, as we saw, according to Aristotle, knowledge is of the universals.

## *II. Scientific background*

In *Topics*<sup>7</sup>, Aristotle enumerates the four possible types of predicables: (i) definition, (ii) property, (iii) genus and (iv) accident.

(i) A definition is “a phrase signifying a thing’s essence”<sup>8</sup> – and a thing’s essence is what makes it be what it is, it’s “what it is said to be *proper se*”<sup>9</sup> (in virtue of itself), making it impossible to be anything else. As such, definitions are necessary predications: what they affirm of a thing belongs necessarily to that thing. For example, ‘rational animal’ is a phrase of the particular kind that Aristotle coins as definitions: it indicates both the genus and the species, and it states the essence of man, which belongs to every member of the class of men with necessity.

(ii) A property is “a predicate which does not indicate the essence of a thing, but yet belongs to that thing alone, and is predicated convertibly of it”<sup>10</sup>. Aristotle’s example of a property is ‘capability to learn grammar’, which belongs to humans only. If something can learn grammar, then it is a human, and if something is human, then it has

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<sup>7</sup> *Top.* I5, 101<sup>b</sup>38.

<sup>8</sup> *Ibid.* I5, 101<sup>b</sup>39.

<sup>9</sup> *Met.* Z4, 1029<sup>b</sup>14.

<sup>10</sup> *Top.* I5, 102<sup>a</sup>17-18.

the capability to learn grammar. This is also an instance of necessary predication, and that is what accounts also for its convertibility<sup>11</sup>.

(iii) A genus is “what is predicated in the category of essence of a number of things exhibiting differences in kind”<sup>12</sup>. For example, ‘animal’ is predicated of humans as part of their essence, but it can also be predicated of other things: elephants, cats, fish, amoebae. It is necessary predication since it is part of the essential predication and the respective things cannot but be in those categories (it is part of what they are).

(iv) An accident is “(1) something which, though it is none of the foregoing – neither a definition nor a property nor a genus – yet belongs to the thing; (2) something which may possibly either belong or not belong to any one and the same-self thing”<sup>13</sup>. This is a case of non-necessary, non-essential predication, where the predicate does belong to the thing, but it might have not belonged to it. Even if sometimes an accident belongs only to the thing in question, we can call that a temporary or relative property (in the sense of ‘property’ above), but not an absolute property, since it may change and either it will cease to belong to the thing or other things will also have it. We will come back to this distinction later in this chapter, but let us note for now that for Aristotle propositions are formed by predicating a term of another<sup>14</sup>, the latter being an *ousia* and the former one of the aforementioned types. Propositions and their terms are combined in certain ways to form syllogisms<sup>15</sup>, which are the tools of inference in all sciences, and

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<sup>11</sup> We should note though that in this case, the mentioned property is a consequence of the thing’s essence: capability to learn grammar is a consequence of the rationality of humans, which is their essence. It is an interesting question to ask whether it is so in all cases of predicating property.

<sup>12</sup> *Top.* 15, 102<sup>a</sup>32.

<sup>13</sup> *Ibid.*, 102<sup>b</sup>4-7.

<sup>14</sup> See *De In.* 4, 16<sup>b</sup>26-17<sup>a</sup>8.

<sup>15</sup> *Pr. An.* 123, 41<sup>a</sup>5.

together with *epagoge* (intuition or induction<sup>16</sup>) they form the apparatus for all knowledge: “for every belief comes either through syllogism or from induction”<sup>17</sup>.

We must mention at this point the Aristotelian treatment of the scientific method. The Aristotelian conception of science is closely linked with the theory of predication sketched above. The sciences are differentiated<sup>18</sup> into three types: “all thought is either practical or productive or theoretical”<sup>19</sup>. Theoretical sciences are the ones that have knowledge as end; practical sciences are the ones that have action as end; and productive activity (*techne* – “art”) has the making of things as end:

For the end of theoretical knowledge is truth, while that of practical knowledge is action (for even if they consider how things are, practical men do not study the eternal, but what is relative and in the present).<sup>20</sup>  
Art, then, as has been said, is a state concerned with making, involving a true course of reasoning.<sup>21</sup>

Scientific inquiry starts from the particulars and works its way up, through increasing levels of abstraction and generality, to knowledge of universals and eventually to the formulation of ultimate principles. The scientific endeavor is one in which, even though we start from the things that are readily available to our experience, *i.e.* the particulars, the ultimate goal is to reach knowledge of the universals, but this does not entail that we reach such a level of abstraction that we are removed from what is, to deal with some kind of contraption of the mind. We only start from what we can perceive because this is what is prior and better known to us, *i.e.* what is more readily available to

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<sup>16</sup> We'll come back to the meaning and role of *epagoge* in the next section of this chapter.

<sup>17</sup> *Pr. An.* II23, 68<sup>b</sup>14.

<sup>18</sup> *Met.* E1, 1025<sup>b</sup>1-1026<sup>a</sup>33.

<sup>19</sup> *Ibid.*, 1025<sup>b</sup>25.

<sup>20</sup> *Met.* α1, 993<sup>b</sup>20-23.

<sup>21</sup> *Nic. Eth.*, VI 4, 1140<sup>a</sup>19-23.

us. What we intend to reach, though, is an understanding of what is prior and better known in nature, *i.e.* the universals.

What science is searching for, then, is the one in the many and as such it must proceed, at least in part, inductively. This process is connected, in addition to the demonstration, with the theory of causes. The discovery of causes provides the first principles of the particular sciences and the necessary connections for scientific demonstration. For Aristotle, any complete explanation of anything would have to include an account of the four types of causes of the thing in question – but this concerns the methods the particular sciences and, while extremely interesting and important, we will not include a more comprehensive account of these matters. A scientific proof consists of a demonstration that something is based on a more fundamental principle, how is it based on that principle, and what is that principle. We do not perceive the universals or the first principles directly, *i.e.* before anything else, but once we reach them, we understand them and we know them better than the particulars or the experiences which occasioned their apprehension:

Thus it is clear that we must get to know the primary premises by induction; for the method by which even sense-perception implants the universal is inductive. [...] Primary premises are more knowable than demonstrations [...] and since except intuition nothing can be truer than scientific knowledge, it will be intuition that apprehends the primary premises. [...] Intuition will be the originative source of scientific knowledge. And the originative source of science grasps the original basic premise.<sup>22</sup>

What Aristotle means by intuition, we will see later in the next section of this chapter.

What is important now is that the first principles (or first premises) are more knowable and truer than experience or scientific knowledge.

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<sup>22</sup> *Post. An.*, II 19, 100<sup>b</sup>4-15, *passim*.

As we said, the means, or tool, for advancement in knowledge is the syllogism, if applied correctly. For Aristotle, there are two types of (correct) reasoning: demonstrative and dialectical:

(a) it is a ‘demonstration’, when the premises from which the reasoning starts are true and primary, or are such that our knowledge of them has originally come through premises which are primary and true; (b) reasoning, on the other hand, is ‘dialectical’ if it reasons from opinions that are generally accepted<sup>23</sup>.

The analytic (demonstrative) syllogism is the one used in particular sciences and it starts from previously attained knowledge, or given true premises, inferring the conclusion, which must also be true, if the syllogism is correctly applied. In the demonstrative procedure the premises are true and certain and the inference must be valid, with no traces of probability lurking around. Its purpose is to preserve truth from the premises to the conclusion: if the premises are true, the conclusion must be true also. The dialectical arguments are of two types: induction (“a passage from individuals to universals”<sup>24</sup>), which is “more convincing and clear”<sup>25</sup>, and reasoning (in the sense described above), which is “more forcible and effective against contradictory people”<sup>26</sup>.

It is a type of elenctic inference, or proof by refutation, where all that one does is taking the opponent’s opinion and showing it wrong or absurd, without putting forth any thesis of one’s own:

The demonstrative premise differs from the dialectical, because the demonstrative premise is the assertion of one of two contradictory statements (the demonstrator does not ask for his premise, but lays it down), whereas the dialectical premise depends on the adversary’s choice between two contradictories.<sup>27</sup>

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<sup>23</sup> *Top.*, I1, 100<sup>a</sup>27-31.

<sup>24</sup> *Ibid.*, I12, 105<sup>a</sup>14.

<sup>25</sup> *Ibid.*, I12, 105<sup>a</sup>17.

<sup>26</sup> *Ibid.*, I12, 105<sup>a</sup>19.

<sup>27</sup> *Pr. An.*, I1, 24<sup>a</sup>21-26.

But, Aristotle says later, “this will make no difference to the production of a syllogism in either case; for both the demonstrator and the dialectician argue syllogistically after stating that something does or does not belong to something else”<sup>28</sup>. The reason why it is important to understand the Aristotelian distinction between demonstrative and dialectical reasoning is because this is how he will avoid the charge of inconsistency and circularity when he does provide his proof with regard to the primacy of the principle of contradiction. He does not give a demonstration of the principle of contradiction, since he had declared the attempt impossible and a sign of ignorance, because the principle, being a basic premise and prior in nature and logic to any other knowledge, cannot be the conclusion of any demonstration. It cannot be derived from anything else, because then it wouldn't be a *first* principle anymore: the things from which it would be derived would be more basic and better known than it.

### ***The Nature of the Principle of Contradiction***

#### *I. Archai and the science of metaphysics*

Even though the sciences are separated due to the different subject matters and kinds of things they explain, there are certain principles which are pervasive of all the sciences and are prior to all the other principles in the particular sciences. They are so fundamental, that they are implicit in all our demonstrations and in fact all our instances of meaningful speech or thought. They are so basic, that the particular sciences do not concern themselves with their study, taking them as given and using them in their syllogistic and other inferential activities. Therefore, we need to have a separate science,

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<sup>28</sup> *Ibid.*, 24<sup>a</sup>26-28.



which would deal with these principles, and this science is Metaphysics, or First Philosophy:

There is a science which investigates being as being and the attributes which belong to this in virtue of its own nature. Now this is not the same as any of the so-called special sciences; for none of these treats universally of being as being. They cut off a part of being and investigate the attribute of this part.<sup>29</sup>

In *Metaphysics* Book E chapter 1, Aristotle makes a distinction among the theoretical sciences, according to their objects of investigation:

If all thought is either practical, or productive or theoretical, physics must be a theoretical science, but it will theorize about such being as admits of being moved, and about substance-as-defined for the most part only as not separable from matter. [...] Mathematics also, however, is theoretical; but whether its objects are immovable and separate from matter, is not at present clear; [...] some parts of mathematics deal with things which are immovable but presumably do not exist separately, but as embodied in matter; while the first science deals with things which both exist separately and are immovable. [...] There must, then, be three theoretical philosophies: mathematics, physics and what we may call theology. [...] And the highest science must deal with the highest genus.<sup>30</sup>

This science is, as we saw, what has come to be called metaphysics, and what Aristotle himself calls first philosophy or theology (because its objects are like divine things, immovable, separable from matter, necessary: “it is obvious that if the divine is present anywhere, it is present in things of this sort”<sup>31</sup>).

Metaphysics pertains to the most abstract, but at the same time the most generally applicable principles of reality and knowledge. Since the universe is for Aristotle ultimately intelligible, if these are principles of reality, they must also be principles of knowledge, and if they are principles of knowledge, they must also be principles of reality. This is the science that studies being *qua* being, and it is knowledge for the sake

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<sup>29</sup> *Met.* Γ1, 1003<sup>a</sup>21-25.

<sup>30</sup> *Met.* E1, 1025<sup>b</sup>25-1026<sup>a</sup>21, *passim*.

<sup>31</sup> 1026<sup>a</sup>19.

of knowledge. It is a quest for the most general traits of existent things and its results must be universally applicable, *i.e.* to everything that there is:

But if there is an immovable substance, the science of this must be prior and must be first philosophy, and universal in this way, because it is first. And it will belong to this to consider being *qua* being – both what it is and the attributes which belong to it *qua* being.<sup>32</sup>

Among the principles (*archai*) investigated by metaphysics are cause, substance, the ten genera of being, logical principles, potentiality and actuality, essence, change and process etc, in other words, everything that pertains to everything. These ontological and epistemological principles are used in all other sciences, taken as granted, but it is not within their scope to study them.

Now, what it means for these objects of study to be immovable is that they are not subject to change: every existing thing exhibits these features and cannot but exhibit them; and what it means for them to be separate is not that they are completely removed from the world of experience, but that they are studied in general, and not in connection with any particular embodied thing. For example, process, which Aristotle treats as a first principle, is not studied in connection with any particular thing: metaphysics does not study the processes that horses undergo in their lives, or the processes that a falling body on an incline undergoes, but it studies process in general, *i.e.* the characteristics that all processes have in common, *e.g.* that all processes involve change of an attribute or set of attributes, but not change of substance. Because the study of metaphysics is at this level of generality, its objects do not regard any particular material thing, but are addressed in principle.

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<sup>32</sup> 1026<sup>a</sup>29-33.

## *II. Knowledge of the first principles*

The most fundamental principles of this kind are the axioms: the principle of identity, the principle of the excluded middle, and the principle of contradiction. They are at the basis of any meaningful discourse and, as such, cannot be demonstrated, because direct demonstration would require either an even more fundamental principle to rely upon in our demonstration, and then the burden of proof would rely on that one, and so on to infinity, or it would require that we rely on the very principles we wish to demonstrate, which would be, unmistakably, a *petitio principii*. Aristotle himself points to this difficulty in his preliminary discussion of the principle of contradiction, considering it a sign of ignorance to aim to prove the axioms in a direct manner, for the reasons explained earlier: “some demand that even this shall be demonstrated, but this they do through want of education, for not to know of what things one should demand demonstration, and of what one should not, argues want of education”<sup>33</sup>.

Now, the question arises, how do we come to know these fundamental principles? Aristotle says, “every belief comes either through syllogism or induction (*epagoge*)”<sup>34</sup>. The term ‘*epagoge*’ is translated sometimes as induction, other times as intuition. It has a larger meaning than both, because it is a sort of intuitive induction, or inductive intuition, that, while it allows one to grasp certain things, it is not out of nowhere, or on a “hunch”, but also on the basis of experience; and while it allows one through extrapolating from sufficient particular cases to reach a generalization, it is also an immediate, direct understanding of the general principle. (I will use the Greek term whenever required, to

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<sup>33</sup> *Met.* Γ4, 1006<sup>a</sup>5-8.

<sup>34</sup> *Pr. An.*, II23, 68<sup>b</sup>14.

avoid ambiguity, with the understanding that it is meant in the more general sense described here).

Since, as we saw earlier, the first principles cannot be known through syllogism, they must be grasped through *epagoge*: “It is consequently impossible to come to grasp universals except through induction”<sup>35</sup>. One might object that this is not the most certain way in which we can attain knowledge, especially knowledge of things as important and universal as the first principles of being and knowledge. Aristotle himself points out to this relative uncertainty of the inductive method: “In the order of nature, syllogism through the middle term [*i.e., proper syllogistic demonstration*] is prior and better known, but syllogism through induction is clearer to us”<sup>36</sup>. We would want the basis of our knowledge to be somewhat more reliable, because we can very well imagine, and in fact it happens all the time, that what *we* see more clearly is not necessarily what is the case, our intuitions are wrong and inductions can always be proved wrong through one single counter-example.

But *epagoge*, as mentioned, does not refer only to generalization from particulars. In *Posterior Analytics* Book II chapter 19, Aristotle gives us a more detailed account of the way we know basic truths, like universals and first principles (which in fact are cases of universals):

When one of a number of logically indiscriminable particulars has made a stand, the earliest universal is present in the soul: for though the act of sense-perception is of the particular, its content is universal. [...] A fresh stand is made among these rudimentary universals, and the process does not cease until the indivisible concepts, the true universals, are established: *e.g.* such and such a species of animal is a step towards the genus animal, which by the same process is a step towards a further generalization. Thus it is clear that we must get to know the

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<sup>35</sup> *Post. An.*, I18, 81<sup>b</sup>5.

<sup>36</sup> *Pr. An.*, II23, 68<sup>b</sup>35-36.

primary premises by induction; for the method by which even sense-perception implants the universal is inductive.<sup>37</sup>

Knowledge indeed starts from sense perceptions of particulars, but the apprehension of universals it is not a mere generalizing conjecture on the basis of the particulars observed; rather, for Aristotle, it is a grasping of the universal that is immanent in those particulars. The different sense-perceptions of the particulars simply occasion the recognition of the universal in them by the *nous* (intuitive reason):

If, then, the states of mind by which we have truth and are never deceived about things invariable or even variable are scientific knowledge, practical wisdom, philosophic wisdom and intuitive reason, and it cannot be any of the three (*i.e.* practical wisdom, scientific knowledge or philosophic wisdom), the remaining alternative is that it is *intuitive reason (nous)* that grasps the first principles.<sup>38</sup>

But again, one might ask, what confers to this process of grasping of universals and first principles its indubitability? This is the paradox of axiomatic systems, and it may very well be the reason why Aristotle felt the need to offer another type of *justification* for the principle of contradiction, which is not by any means a demonstration, but a series of reasons for accepting the principle, not on the basis of other premises, but on the basis of its undeniability. We will see more about that in the next section of this chapter; for now, let us note that Aristotle doesn't leave the justification for accepting the principle of contradiction to *epagoge* alone: that is only the explanation of the way we arrive at the principle; he further addresses the matter in an attempt to avoid the sense of uncertainty or unreliability one might get from the inductive-intuitive account.

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<sup>37</sup> *Post. An.*, II 19, 100<sup>a</sup>15-<sup>b</sup>5.

<sup>38</sup> *Nic. Eth.*, VI 6, 1141<sup>a</sup>3-8.

### *III. Contraries and contradiction*

Aristotle gives his direct and most cited formulation of the principle of contradiction at *Met.* Γ 3, 1005<sup>b</sup>19-20: “The same attribute cannot at the same time belong and not belong to the same subject and in the same respect”, and he declares it the most certain of all principles. Before we deal with the principle itself and the justification Aristotle gives for proving its certainty, I would like to see if and in what way the principle of contradiction is connected with his theory of contraries, as well as whether and in what way they differ from one another.

According to Aristotle, there are many ways in which we can say that things are contrary:

The term ‘contrary’ is applied (1) to those attributes differing in genus which cannot belong at the same time to the same subject, (2) to the most different things in the same genus, (3) to the most different of the attributes in the same recipient subject, (4) to the most different of the things that fall under the same faculty, (5) to the things whose difference is greatest either absolutely or in genus or in species. The other things that are called contrary are so called, some because they possess contraries of the above kind, some because they are receptive of such, some because they are productive of or susceptible to such, or are producing or suffering them, or are the losses or acquisitions, or possessions or privations, of such.<sup>39</sup>

As we can see, attributes and things can be contraries, and usually contrariety has to do with the two ends of a certain spectrum, with the exception of the first case, where the contraries have nothing in common, the respective attributes being said to belong to different genera altogether. This is where the contradictories are: they are the ones that cannot belong to the same subject at the same time. So, contradictory attributes are a subset of contraries. A pair of contraries cannot be both true at the same time, but they

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<sup>39</sup> *Met.* Δ10, 1018<sup>a</sup>25-37.

can be both false: for example, generation and destruction cannot happen at the same time to a subject, but it can be that neither generation, nor destruction is happening at one time to a subject. Contraries are defined in terms of difference, and this is why they cannot be true at the same time, of the same subject. But because the differences, with the noted exception, belong to the same genus, and are in the same spectrum of attributes or things, namely they are the *most* different in the respective spectrum, it is quite possible for the subject in question to lack both the extremes and have any of the other attributes in between, therefore making the contraries false at the same time, of the same subject.

Contradictory attributes, on the other hand, cannot both be true and cannot both be false of the same subject at the same time. They cannot both be true in virtue of the principle of contradiction and in virtue of the fact that they are a type of contraries. They cannot be both false in virtue of the fact that contradictory attributes don't share the same spectrum. For example, 'white' and 'non-white' are not the two extremities within the colour range like, say, 'white' and 'black', and they don't admit, like these, other attributes of the same kind between them. 'White' and 'non-white' are complementary attributes, their conjunction exhausts the entire universe of discourse, so nothing can lack both of them, because then it wouldn't be in the universe of discourse anymore.

As Aristotle notes, in the case of contraries we have an underlying subject that is, the "locus of process"<sup>40</sup>, which means that in every process (or change) a subject, and the same subject, is going from having one attribute to having its contrary, and this is possible only because there is a substratum for these changes:

But all things which are generated from their contraries involve an underlying subject; a subject, then, must be present in the case of contraries, if anywhere. All

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<sup>40</sup> John P. Anton, *Aristotle's Theory of Contrariety*, University Press of America, Lanham, 1987, p. 51 *passim*.

contraries, then, are always predicable of a subject, and none can exist apart, but just as appearances suggest that there is nothing contrary to substance, argument confirms this.<sup>41</sup>

On the other hand, there are limits to the processes a subject can undergo, since nothing can change its essential properties. If it did, we wouldn't have the same subject anymore. According to J. Anton, the principle of contradiction is “the logical formulation of the principle of contrariety”<sup>42</sup>, in virtue of which the individual *ousiai* “possess, as loci [...] a set of determinations that mark the boundaries of its process, affording thus the grounds for a generic contrariety, the metaphysical contrariety, which in turn sustains the law of non-contradiction”<sup>43</sup>.

One might ask then, whether the notion of contraries is not more fundamental than the principle of contradiction. An attempt for an answer might be that, since the principle is expressed through a proposition, it would be quite absurd to ask that it have precedence over the meanings of the very concepts it uses. The principle is most certain and most prior in the sense that it cannot be *derived* from other propositions and it is involved in anything we utter and think, but not in the sense that we can grasp it before we even understand what the words contained in it signify.

It can be concluded that the principle of contradiction, inasmuch as it reflects a special subset of contrary attributes, is based on the notion of contrariety, but ultimately the notion of contraries is much larger and differs in essential aspects from the notion of contradictories.

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<sup>41</sup> *Met.* N1, 1087<sup>a</sup>35-1087<sup>b</sup>3.

<sup>42</sup> J. P. Anton, *Ibid.*, p. 100.

<sup>43</sup> *Ibid.*



## *The Justification of the Principle of Contradiction*

Aristotle's formulation of the principle of contradiction, again, is: "The same attribute cannot at the same time belong and not belong to the same subject and in the same respect"<sup>44</sup>. It is the most certain of all principles and, while he admits of the impossibility of giving a direct proof of it, he engages in a negative proof, or a proof by refutation (*elenchus*), which is directed against any one who might declare he denies the principle of contradiction. Aristotle sets out to prove that, even though one might *say* the principle of contradiction isn't true, one cannot really *believe* that. After that, he is analyzing the consequences that follow from the philosophical views of Protagoras and Heraclitus, which are in their turn derived from the denial of the principle of contradiction, and shows that they are highly incoherent.

Again, let us emphasize that his *elenchus* is not aimed at proving the principle of contradiction is true, nor at proving the denial of the principle is false, but at proving that nobody can really disbelieve it, and therefore that this is the firmest of all principles. In this, he employs the dialectical method of science, which is to start from a certain given opinion and show the consequences that result from it. If the consequences, correctly inferred, are unacceptable, then the opinion which served as a basis for them is also unacceptable.

### *I. The argument*

The crux of the argument is as follows:

Suppose somebody were to really believe that the principle does not hold (we will call such a person 'the opponent'). He will have to choose between two options: either

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<sup>44</sup> *Met.* Γ 3, 1005<sup>b</sup>19-20

refrain from meaningful speech (or, for that matter, from speech at all), or say something with sense, even if it is the smallest unit of meaningful discourse (as a matter of fact, it should be the smallest unit of meaningful discourse, since Aristotle doesn't want to compel the opponent into anything he wouldn't agree to, lest the argument lose its force).

In the first case, says Aristotle, the opponent is no better than a vegetable and there is no need, and indeed it would be quite ridiculous, to argue against his supposed view. Besides, if he even declares his view, it must be done through a meaningful act of speech. If he doesn't, we are fine and we need not worry about any opposition.

In the second case, he will have to say something with sense. In order to understand what this implies, we must examine Aristotle's theory of meaning. Let us turn our attention to *De Interpretatione*. He declares there that the smallest units of significant speech are nouns (or names) and verbs:

By a noun we mean a sound significant by convention, which has no reference to time, and of which no part is significant apart from the rest.<sup>45</sup>

A verb is that which, in addition to its proper meaning, carries with it the notion of time. No part of it has any independent meaning and it is a sign of something said of something else.<sup>46</sup>

It is important to note that he doesn't restrict signification to names, which are words for *ousiai*, and this will have a bearing on our later attempt to reject the objection that the elenctic proof of the principle of contradiction rests upon Aristotle's ontological theory. Further, he proclaims that any utterance of a significant word is accompanied by a thought both in the hearer and in the speaker. In addition to the simple atoms of signification, Aristotle admits of complex units, formed from the simple ones, and these are the sentences. All sentences have signification, but only propositions are true or false:

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<sup>45</sup> *De Int.*, 2, 16<sup>a</sup>19-20.

<sup>46</sup> *Ibid.*, 16<sup>b</sup>6-7.

“every sentence has meaning, [...] by convention. Yet every sentence is not a proposition; only such are propositions as have in them either truth or falsity”<sup>47</sup>.

Propositions can be either affirmative or negative. The isolated words by themselves, even if they have meaning, become an affirmation or a denial only when they are combined with others to form propositions. Aristotle suggests that truth and falsity belong in fact to the thoughts corresponding to propositions and, as William and Martha Kneale observe, “truth or falsity of the spoken word is derivative”<sup>48</sup>. Aristotle says, “as there are in the mind thoughts which do not involve truth or falsity, and also those which must be either true or false, so it is in speech. For truth and falsity imply combination and separation”<sup>49</sup>.

Now the most important characteristic of the theory of meaning, from the point of view of our discussion, is that any signification is *definite*. Whenever we utter a significant word, we pick out something and eliminate other things. In the case of definite names, it is clear that we pick a particular thing and eliminate all the others, *e.g.* by uttering ‘Socrates’ we pick out a particular individual and eliminate all the others from our signification. In the case of classes of things, we pick out the kinds of things which fall under the respective concept, *e.g.* by uttering ‘man’ we isolate all the particulars of which ‘man’ can be truly predicated and rule out all the other ones. Now, one might say that negative names have an indefinite signification, since, for example, ‘not-man’ picks out an infinite number of things, in fact everything except the finite set of men. Aristotle himself considered the expression ‘not-man’ indefinite: “the expression ‘not-man’ is not a noun. There is indeed no recognized term by which we may denote such an expression,

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<sup>47</sup> *Ibid.*, 4, 17<sup>a</sup>1-4.

<sup>48</sup> William Kneale and Martha Kneale, *The Development of Logic*, Clarendon Press, Oxford, 1964, p.45.

<sup>49</sup> *De Int.*, 1, 16<sup>a</sup>9-13.

for it is not a sentence or a denial. Let it then be called an indefinite noun<sup>50</sup>. But, as C.A. Whitaker points out<sup>51</sup>, even negative names have a definite meaning, in virtue of the fact that, when we say ‘not-man’, we pick out, in a definite manner, the property of being a man and we say that we mean those things to which this definite property does not apply. It doesn’t matter that the number of referents may be infinite: our meaning is still definite. Indeed, the only way such an expression can signify is by first picking out a definite meaning, ‘man’ and without this definite meaning the expression ‘not-man’ wouldn’t have a meaning, either.

The importance of the definiteness of meaning is revealed in the next step of the argument, which is simple, but clever: by admitting that he had said a *meaningful* word, the opponent is forced to concede that he meant something and not its negation. If he doesn’t, he will not have signified. Now, one could ask why would this conclusion bother the opponent of the principle of contradiction. All that Aristotle has shown is that the opponent contradicted himself; but if the interlocutor really believes in the denial of the principle of contradiction, this should not create a problem for him, since he will be very willing to admit that his word has meaning and at the same time does not have meaning. This would have the consequence that all words mean anything and nothing at the same time, which would also be fine by the opponent, but Aristotle defines something ‘meaningful’ as being meaningful to both the speaker and the hearer<sup>52</sup>. It is conceivable that the opponent will contend this definition, but by doing so, he practically waives his right to rational discourse, since, if he doesn’t care about being understood, whatever he

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<sup>50</sup> *Ibid.* 2, 16<sup>a</sup>29-31.

<sup>51</sup> C.A. Whitaker, *Aristotle’s De Interpretatione. Contradiction and Dialectic*, Clarendon Press, Oxford, 1966, p. 192.

<sup>52</sup> *Met.* Γ 4, 1006<sup>a</sup>21-23.

says from now on will seem like blabber to the rest of us and he will inevitably fall into the category of a vegetable.

## *II. The argument*

Two questions arise, though: the first question is with regard to the apparent relativity of the theory of meaning. If meaning is described as dependent on whether what is uttered is understood by both parties (*i.e.* the speaker and the hearer), then could we say, for example, that a foreigner with no knowledge of English, if placed in a room full of people who speak only English, could not engage in rational discourse? Yes, his words will seem like gibberish to the others, but that doesn't mean he actually speaks gibberish. What if the opponent of the principle of contradiction is in the same situation? Are we entitled to infer that he is not capable of reason only from the fact that we, who accept the principle of contradiction, don't understand? And, even more importantly, is what gives the epistemological and logical ineluctability to the principle of contradiction simply the disarming power of the overwhelming majority? On the other hand, one can argue, if the majority is so overwhelming that it includes *everybody*, then that should be enough. Still, that doesn't explain on a theoretical level the logical necessity and primacy which we attribute to this principle – and this is what we want to understand: *why* do we all agree with the principle of contradiction and its privileged position in the system of knowledge, not the *fact* that we so unanimously accept it.

The second question that arises is with regard to the status of the theory of meaning. It seems that Aristotle's elenctic proof rests upon his theory of meaning. Even if we leave aside his considerations about *ousia*, we still have to deal with the question that, if the proof of the firmness of the principle of contradiction depends on the theory of

meaning, should we conclude that in fact this theory has primacy over the principle, and that it is in this sense “prior”? But his would be against what we are trying to prove. And if it is not, then the theory of meaning, on which we base the proof, is dependent on the principle of contradiction, so we can be charged with *petitio principii*. In response to the charge of *petitio principii*, or begging the question, many commentators<sup>53</sup> made the observation that, in fact, the proof works only if we accept the principle of contradiction, which is fine, since the proof is addressed to us (and we are assumed to adhere to it), not directly to the ‘opponent’ – in fact, according to Aristotle, such an opponent doesn’t even exist, since no one can really, sincerely disbelieve the principle of contradiction. What the proof is trying to achieve is to show that whoever claims to disbelieve it, “while disowning reason, he listens to reason”<sup>54</sup>, and this should be primarily convincing to us, not to the one who might make these claims.

The question still remains whether, if we don’t accept Aristotle’s metaphysics and theory of meaning, we could still accept his account of the principle of contradiction. The Polish logician Jan Łukasiewicz claimed in his famous article<sup>55</sup> that in fact the principle of contradiction is dependent upon the Aristotelian conceptions of ontology, logic and psychology. He identified three corresponding (ontological, logical and psychological) formulations of the principle and analyzed each of them in turn, losing from sight the connections Aristotle held between what we now differentiate as logical, ontological and psychological facts. On the basis of the distinctions he imposed on the Aristotelian system, he attempted to refute Aristotle’s account of the principle of contradiction, on the

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<sup>53</sup> Among them, C.A. Whitaker, *op. cit.* and Jonathan Lear, *Aristotle and Logical Theory*, Cambridge University Press, Cambridge, 1985, esp. chapter 6.

<sup>54</sup> *Met.* Γ 4, 1006<sup>a</sup>27-28.

<sup>55</sup> Jan Łukasiewicz, “On the Principle of Contradiction in Aristotle”, translated by Vernon Wedin, in *Review of Metaphysics*, 1971, 24, 485-509.

charge of self-contradiction, and circularity. But I tend to disagree with Łukasiewicz's interpretation, because it seems that he fails to see the distinction between demonstration and refutation and he misinterprets Aristotle's intentions and goals in providing the proof against the alleged disbelievers in the principle of contradiction.. Of course, a proof by refutation is still a proof, and as such must make use of the principles of logic like any other proof, but we must not lose sight of what Aristotle is trying to achieve: to point to us the difficulties and absurdities in which we may fall if we try to deny a certain general principle, which we already take as true. This is the meaning of a *dialectical* demonstration.

## Chapter Two: The Modern Approach

### *The Principle of Contradiction Induced: Mill*

Sharing his empiricism with Aristotle, John Stuart Mill has an importantly different twist regarding the status of the principle of contradiction. He maintained partly the Aristotelian view that logic does tell us about the world and it does add to our knowledge, because it is not merely an enclosed system of rules, independent of reality or truth, but one that reflects the way the world is and the way we actually think. He maintained that our knowledge starts from the particulars, given to us by observation, from which we infer truths about the universals. He defined logic as “the science of the operations of the understanding which are subservient to the estimation of evidence: both the process itself of advancing from known truths to unknown and all other intellectual operations in so far as auxiliary to this”<sup>56</sup>. But the consequences of his view, specifically as far as the principle of contradiction is concerned, are quite different from Aristotle’s and they mark one of the beginnings of the modern standpoint on the matter.

In this section of the chapter I will analyze the context of Mill’s general theory of knowledge, which will provide us with the necessary apparatus for investigating his account of the way we come to know the most fundamental principles, in particular the principle of contradiction, then we will make a few remarks on the charge of logical

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<sup>56</sup> John Stuart Mill, *A System of Logic, Ratiocinative and Inductive*, in *Collected Works of John Stuart Mill*, vol. VII, University of Toronto Press, Toronto, 1978, p. 12.



psychologism often directed against Mill, whether it is well founded and what such a position would entail, and finally we will evaluate the shift that has started to take place with Mill from the Aristotelian logic towards the modern approach with regard to the status and grounds of the principle of contradiction. One preliminary remark here should be about the shift in terminology. Aristotle's *archai* (principles) are, as the term itself suggests, "the first point from which a thing either is, or comes to be or is known"<sup>57</sup> therefore prior in nature and knowledge, most certain and most general. Mill, while sometimes maintaining the expression 'principle of contradiction', other times calls it, together with the other principles, a 'law' or an 'axiom'. The 'law of contradiction' is not anymore an inherent principle of reality, but it becomes a law of logic, *i.e.* a law of the way we think. We shall see later in the chapter what that means for Mill.

### *I. Epistemological background*

Mill's picture of knowledge is a radically empiricist one: it is a structure stemming from the data of immediate consciousness and developing from those by inference. The logician's task is to assist the quest for truth by providing a systematic account of the conditions under which inferences are correctly performed. There are, he says, two ways in which we can know truths: through immediate intuition, or consciousness, and through logical inference:

Truths are known in two ways: some are known directly, and of themselves; some through the medium of other truths. The former are the subject of Intuition, or Consciousness; the latter, of Inference. The truths known by intuition are the original premises from which all others are inferred. Our assent to the conclusion being grounded on the truth of the premises, we never could arrive at any knowledge by reasoning, unless something could be known antecedently to all reasoning.<sup>58</sup>

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<sup>57</sup> *Met.*, Δ1, 1013<sup>a</sup>19.

<sup>58</sup> *System of Logic.*, p. 6-7.

In the first category he admits “our own bodily sensations and mental feelings”, but nothing beyond that level of sensory and mental immediacy<sup>59</sup>; in the second category he includes all reasoning, even those “rapid inferences” that we usually take as immediate intuition: “a truth, or supposed truth, which is really the result of a very rapid inference, may seem to be apprehended intuitively”<sup>60</sup>. For example, the judgment of the distance between two objects is an instance of knowledge which comes from inference, because, associated with the “bodily sensation” of seeing patches of light and colour of different sizes and shapes, we train ourselves from experience how to interpret the data and draw conclusions about certain properties, like the distance between them; but we learn to do that so early in our mental development, that the process of reasoning through which we originally reached the conclusion is forgotten and we and take this habitual, quick, automatic inference to be an immediate intuition.

Mirroring this division of the sources of truth, Mill also distinguishes two types of logic: the Logic of Consistency (or Formal Logic), which is concerned with the correctness of our inferences, without worrying about their connection with reality, and the Logic of Truth, which includes the Logic of Consistency and is concerned with both the correctness of our reasoning and the connection with reality. These both are, according to Mill, indispensable components of the great endeavour which strives for knowledge. The ultimate basis, though, for this endeavour is the immediate evidence of the senses.

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<sup>59</sup> This resembles very much Quine’s reference to “observation sentences”, which provide infallible and certain knowledge, as long as it remains on the strict level of observation. Perhaps in this way also Mill meant that these “bodily sensations” are a source of knowledge.

<sup>60</sup> *Ibid.*, p. 7.

This view would make Mill, given the sense we give today to the word, a straightforward, if radical, empiricist, but he himself didn't want to be called so, since he took an empiricist to be one who merely collects observed data and draws conclusions directly and unselectively from them. Mill declared himself an "experientialist"<sup>61</sup>, since according to him experience does much more than collecting data: it organizes them, it stores memories and uses them later to evaluate outcomes and calculate probabilities, it helps one learn from one's empirical data, not just hoard them blindly. Knowledge still comes from experience, but it takes account of the connections between those experiences and performs scientific, safe generalizations, not the direct, piecemeal ones of empiricism. It is questionable that the empiricists were as unsophisticated in their epistemology as Mill seems to suggest, but what should seize our attention is the way he describes his own theory of knowledge, no matter how close or how far he is from what is generally identified as the empiricist view.

One might legitimately ask then, how does the inference from empirical data take place, *i.e.*, how does one go from the raw contents of the senses to universal and abstract notions, which are seemingly unlinked with anything that one can observe directly? Mill's answer is that we form these ideas by means of induction from particular cases. For example, the only grounds for admitting that all men are mortal is that *every* man we encountered or know of until now was mortal. The proposition 'All men are mortal' is an abbreviation for the indefinitely long conjunction 'Socrates is a man and mortal & Alexander is a man and mortal & Mill is a man and mortal & Smith is a man and mortal & ...', which means that all propositions are about particulars, and therefore all reasoning

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<sup>61</sup> See R.P. Anschutz, "The Logic of J.S. Mill", in *Mill: A Collection of Critical Essays*, ed. J.B. Schneewind, Anchor Books, Garden City, New York, 1968, pp.46-83.

is about particulars, too. Syllogisms, which for Aristotle were inferences from universals to particulars, become in Mill's view inferences that can be only from particulars to particulars, and the premises are, ultimately, the particular fact from which, by induction, we derive a generalization.

But in that case, one might again observe, the very process of induction needs a general criterion that would allow us to make the correct, scientific generalizations that Mill is striving for. How do we establish that general criterion? If it is by generalization from experience, like all other universal propositions, how can we account for the criterion of *that* generalization? If it is by some other process, it follows that the most basic start for all knowledge is *not* raw experience, at least not by itself, but in addition to this other type of process, whatever it may be. William and Martha Kneale<sup>62</sup>, in an attempt to provide a solution to this problem, suggest that, even though Mill never expressed it clearly, he took as implicit a distinction between "first order generalizations" and "second order generalizations". The former are the usual generalizations from experience, while the latter are "principles about the use of those non-formal principles of inference which we accept on inductive grounds"<sup>63</sup> and regarded by the Kneales in the same way in which Mill regarded the quick inferences that appear to be immediate intuition, but in reality they are generalizing inferences whose processes are long forgotten: "we have learnt them by finding them implicit in rules of language which we have unconsciously adopted during the course of our experience and without which we could not practise induction deliberately as a scientific policy"<sup>64</sup>.

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<sup>62</sup> William Kneale and Martha Kneale, *The Development of Logic*, Clarendon Press, Oxford, 1964, p. 377.

<sup>63</sup> *Ibid.*, p. 377.

<sup>64</sup> *Ibid.*, p. 377.

It is not certain, though, that this explanation, be it implicit in Mill's account, would solve the difficulty presented above, of finding a legitimate criterion for induction in the context of exclusive empiricism, and even if it does solve that one, it threatens to create troubles elsewhere: the solution suggests that second order generalizations, if they are generalizations from observed data, they must be of observed patterns of language and thought, which would be fine by Mill's empiricism (or experientialism). But the problem is that at the basis of our inferential rules we would find a series of observations about particular mental states and patterns, which is something that Mill was vehemently against: logic, in Mill's view, should not be a science descriptive of how we are usually thinking, or a science descriptive of particular, individual ideas of people, but he wanted logic to be a prescriptive science, to tell us how we *must* think, if we want to think correctly: "Logic is not the theory of thought as thought, but of valid thought; not of thinking, but of correct thinking"<sup>65</sup>. The reason why it is important for us to understand the status of these criteria for induction and inference, or "second-order generalizations", is because, as we shall see in the next section, the principle of contradiction will turn out to be, at least in a certain sense, one of them.

## *II. Knowledge of the axioms and psychologism*

Mill's treatment of the principle of contradiction, whose formulation is given in the *System of Logic*, as "an affirmative proposition and the corresponding negative proposition cannot both be true"<sup>66</sup>, is not very lengthy, but it is dense:

I consider it to be, like other axioms, one of our first and most familiar generalizations from experience. The 'original foundation' of it I take to be, that

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<sup>65</sup> John Stuart Mill, *An Examination of Sir Hamilton's Philosophy*, in *Collected Works of John Stuart Mill*, vol. IX, University of Toronto Press, Toronto, 1978, p. 359.

<sup>66</sup> *Idem*, p. 277.

Belief and Disbelief are two different mental states, excluding one another. This we know by the simplest observation of our own minds. And if we carry our observation outwards, we also find that light and darkness, sound and silence, motion and quiescence, equality and inequality, preceding and following, succession and simultaneousness, any positive phenomenon whatever and its negative, are distinct phenomena, pointedly contrasted, and the one always absent where the other is present. I consider the maxim in question to be a generalization from all these facts<sup>67</sup>.

In other words, the principle of contradiction is an axiom among others, *e.g.* the axioms of mathematics, and they are, on the basis of Mill's empiricism, taken to be generalizations from experience, learnt very early through processes that now have become automatic. This is why the axioms have given the impression (*e.g.* to the rationalists and intuitionists, against whom Mill is arguing) that they are somehow grasped intuitively, that they are known *a priori*, or that they are innate ideas. The process through which these generalizations are realized is induction from particular cases and the material of these generalizations comes from two sources: (a) external “phenomena”; and (b) our own mental states.

(a) In the case of external phenomena, the contention is that we repeatedly observe that “pointedly contrasted” attributes or physical states or processes never appear to us together, that they exclude one another, and we learn through induction from these innumerable particular observations to conclude, through inductive generalization, that contradictories can never co-exist. Two observations are in order: first, the examples Mill uses for contradictory “phenomena” are better characterized as what Aristotle called ‘contraries’, rather than ‘contradictories’. In a strict sense, contradictories are a thing and its direct *negation*. For example, motion has as contradictory non-motion and not quiescence, which is actually its contrary. Contradictory terms are, technically, any term

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<sup>67</sup> *Ibid.*, pp. 277-278.

and its direct negation and as such they cannot be both true of a thing at the same time, or both false at the same time, of the same thing. Contrary terms also cannot be both true of a thing at the same time, but they can be both false, since they don't exhaust the entire universe of discourse. They are two possible terms (usually from the two extremes of a certain range) among other ones, and they don't cover the entire range. To go back to our example: on the other hand, quiescence is usually defined as lack of motion, or, in other words, as non-motion; so, if we view 'quiescence' as simply a substitute for 'non-motion', we indeed have contradictory states (although not contradictory terms). This doesn't hold in all cases, though: 'preceding' and 'following' are merely contrary states, not contradictory, because the contradictory of 'preceding', 'non-preceding', could stand for both 'following' and 'concurrent' and as such, 'preceding' and 'following', or 'preceding' and 'concurrent', or, for that matter, 'following' and 'concurrent', being contraries, cannot both be true when predicated of the same thing in the same respect, but can both be false, whereas 'preceding' and 'non-preceding', being contradictories, cannot both be true, nor both be false (of the same object in the same respect<sup>68</sup>).

The second observation is that Mill's criterion for determining contradictory "phenomena" seems to be the fact that they are never encountered together, or, better, that the presence of one of them is always accompanied by the absence of the other. But, there are many things that are never encountered together and are not contradictories, and that holds for all contrary or even alternative attributes, states or processes, as noted above: whenever something is preceding, it is not also concurring and it is not also

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<sup>68</sup> Even though Mill does not include these qualifications, I believe we must give him enough credit and not succumb to the temptation to criticize him on the ground that, in fact, something can be preceding and following at the same time, if it is with regard to different things (*e.g.* 5 precedes 6, but follows 4). It is, I think, safe to assume that this is what Mill intended.

following. We would now have to determine which one of them, ‘following’ or ‘concurrent’, is contradictory to ‘preceding’? If we decide to choose one of them, the criterion must obviously be different from the one suggested by Mill, because that criterion yielded both of them. If we decide that both are contradictories, then we must admit that they are so only in virtue of them being non-preceding, that is, in virtue of them being something that preceding is not, together with the rest of the non-preceding things. Geoffrey Scarre suggests<sup>69</sup> that in order to do that we must already have a notion of negation and, to have that, we must use the principle of contradiction, because in defining the logical operator of negation, through a truth-table, we employ the principle of contradiction. It is not at all clear how one would apply or even have a grasp of the principle of contradiction *before* one has a notion of negation (since one supposedly uses the principle of contradiction to define negation), but the gist of the matter is that once we grasp the notion of negation, we must grasp and accept the fact that what is preceding is not what preceding is not, if not in virtue of the principle of contradiction, at least in virtue of the notion of negation we have just acquired.

(b) In the case of our mental states being the observed data for the generalization that yields the principle of contradiction, Mill's contentions is that the basis for this generalization is that we repeatedly observe that the mental states of belief and disbelief are exclusive of one another and we can never exhibit them at the same time. This is one claim for which Mill has been declared a propounder of logical psychologism, which is the view that logic is a part of general psychology, and thus a descriptive discipline, which is concerned with observations of the way we think, of the mental states

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<sup>69</sup> See Geoffrey Scarre, *Logic and Reality in the Philosophy of John Stuart Mill*, Kluwer Academic Publishers, Dordrecht, 1989, p. 118.



and the actual, individual acts of thinking involved in reasoning. In such a view it is practically impossible to justify logical necessity and the prescriptive role of logical rules. As a consequence, logical correctness is replaced with a precarious regularity of behaviour. J. Richards<sup>70</sup> distinguishes two types of identifying claims for this position: the methodological claim, that logical laws have a descriptive role towards experience and they are to be arrived at through observation; and the epistemological claim, that logical laws are empirical generalizations from the experience of the subject, which makes them *a posteriori* and, very importantly, not necessary. He argues that Mill is a logical psychologist on both accounts<sup>71</sup>. It will be our contention that, according to this definition of logical psychologism, Mill does not maintain the methodological claim, but does agree with the epistemological one.

The methodological claim of logical psychologism has two points: the first one is that logical laws are reached through observation, and Mill readily admitted that point, adding though that we also use a kind of safe, scientific induction to order these observations. We saw above that there are certain difficulties with the justification of that addition, but what is important at this time is to note that, methodologically, it is not through observation alone that we arrive at the logical laws, according to Mill. The second point of the methodological claim is that the logical laws are descriptive of experience, but Mill was very explicitly opposed to this view, at least in principle<sup>72</sup>:

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<sup>70</sup> John Richards, "Boole and Mill: Differing Perspectives on Logical Psychologism", in *History and Philosophy of Logic*, 1, 1980, 19-36.

<sup>71</sup> G. Scarre noted (*ibid.*, p. 113) that in fact, these claims are not characteristic of logical psychologism, but rather of empiricism. If we have in mind that in 'experience' and 'observation' Mill includes observation of one's own states of mind, we could grant that, even if the logical laws are not *about* mental states, his explanation of the way we attain logical principles depends, at least in part, on descriptions of internal states and hence in a sense it is still open to the charge of psychologism.

<sup>72</sup> He is surprisingly close on this matter with Frege, who was a dedicated foe of psychologism: "the laws of logic ought to be guiding principles for thought in the attainment of truth" (Gottlob Frege, *The Foundations*

The sole object of Logic is the guidance of one's own thoughts.<sup>73</sup>  
Logic is the common judge and arbiter of all particular investigations. It does not undertake to find evidence, but to determine whether it has been found. Logic neither observes, nor invents, nor discovers; but judges.<sup>74</sup>

Even so, he leaves unexplained the way in which we are to fill the gap between the observations of empirical data and the desired norms of good reasoning. The gap comes from the fact that the way we *must* reason, the prescriptive rules for correct thinking can never be grasped through observation, no matter how much of it we undergo. Induction doesn't help in this process either because, although different from observation, its sole, ultimate aim is, like the aim of observation, description of reality, be it internal or external. If we rely exclusively on sense experience, it is impossible to account for the derivation of normative principles, be they moral, logical or of any other kind.

The epistemological claim is that logical laws are empirical generalizations from experience, and Mill has definitely proposed and defended this view. To understand better what Mill means by that, it would be helpful to turn to his much more detailed account of mathematical axioms; since he counted the principle of contradiction among “the other axioms”, I think it is safe to assume he would admit, *mutatis mutandis*, of similar explanations. The example he uses<sup>75</sup> is the geometrical axiom that two straight lines cannot enclose a space and in his defense of it as a truth arrived at through empirical generalization he is compelled to give an account of the way in which, by observing any number of pairs of straight lines, we reach the conclusion that they *cannot* enclose a space, since our observation cannot follow them to infinity (just like, in the case of the

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*of Arithmetic*, Blackwell, Oxford, 1953, p. 12); “always to separate sharply the psychological from the logical, the subjective from the objective” (*ibid*, p. x).

<sup>73</sup> J. S. Mill, *System of Logic*, p. 6.

<sup>74</sup> *Ibid.*, p. 10.

<sup>75</sup> *System of Logic*, Book II, Chapter V, § 4-6.

principle of contradiction, by observing the occurrence of belief and disbelief and contradictory pairs, we reach the conclusion that they *cannot* coexist solely from the fact that they *do not* coexist).

This is where Mill introduces the idea of mental experiments and explains that we follow the lines to infinity in our minds, but not through sheer imagination, which can take us anywhere, but in a scientific manner. When we learn the meaning of straight lines, we learn their properties as well and this ensures that the mental image we form is identical with an actual straight line. If we imagine the line to bend at some point, which is quite possible, we wouldn't say anymore that we imagine a straight line, but a bent one; once we establish we are picturing a straight line, the line pictured must behave like an actual straight line, through and through. Scarre notes<sup>76</sup> that this kind of mental experimentation is different from *a priori* knowledge, since the latter deals with analytic truths, purely intellectual, dependent on meaning of terms and propositions and the former works with images, expanded mentally. It was indeed important for Mill to differentiate himself from the *a priori* theorists, but it is questionable whether this idea of mental experimentation was a satisfactory solution: after all, it does involve, in a primordial way, the meanings of terms (like with the analytic truths he is fighting against) and perhaps it doesn't matter how one pictures things to oneself, or what one does afterwards with the knowledge of the meaning of the words, because the ulterior expanded mental image will be limited in virtue of the very meaning of the words. The sense of the words 'straight line' dictates the way the imagined line will behave when we follow it to infinity. The imagined line will not surprise us one day by curving at some

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<sup>76</sup> *Idem*, p. 133.

point, in spite of the meaning we had assigned to its name, forcing us to change it: it is not a matter of discovery, but ultimately a matter of semantics.

If we apply Mill's idea to the principle of contradiction, it follows that we experiment on mental contents of belief and disbelief and find, or *discover*, that propositions cannot be accepted together with their negations. But, one might ask, isn't the disposition to believe or disbelieve something dependent upon the acceptance of the principle of contradiction? If the principle of contradiction weren't prior, one could believe or disbelieve anything, randomly, including propositions in conjunction with their negations. It is because of the principle of contradiction that we are forced to believe only one of two contradictories and we are forced to disbelieve the other. In conclusion, no matter how much he tried to avoid that, Mill's explanation seems to depend ultimately on meanings of words: it is only when we learn the meaning of 'straight line' that we can proceed with the mental experiment. But that means that the knowledge of the axiom that two straight lines cannot enclose a space is based ultimately on the sense of the term 'straight line', and not on the mental observation, as Mill suggests. In the same way, the principle of contradiction depends on the meaning of negation, as we saw previously.

This brings us to the second part of the above-mentioned epistemological claim, that the laws of logic are not necessary. Mill addresses this important point, again with reference to mathematical axioms, by denying necessity altogether. He would have to do so, if he is to maintain his empiricist claim that all knowledge comes from observation, since one cannot observe necessity or certainty anywhere in the actual world: "This character of necessity, ascribed to the truths of mathematics, and even (with some reservations to be hereafter made) the peculiar certainty attributed to them, is an

illusion”<sup>77</sup>. Even if one observes extremely high regularities, that is all they will ever be; if one relies only on observation, one must abandon the idea of necessity. Therefore, the only pseudo-certainty and pseudo-necessity the principle of contradiction will ever be able to exhibit will simply be its high degree of confirmation in experience, but nothing more. This view is one of the gates that opened, as we shall see, towards Russell’s denying any special character to the principle of contradiction, except as a mere theorem of logic, among many others, and derivable from others.

### *III. Comments*

In one sense, Aristotle and Mill have a similar picture of knowledge, inasmuch as they both contend that any investigation, any knowledge starts from empirical observation. They are even similar in the claim that the principles of logic, including the principle of contradiction, reflect characteristics of reality. But what accounts for a great part of the dissimilarities is the small, but crucial fact that Mill’s induction from empirical fact does not allow for anything reminiscent of Aristotle’s *epagoge*, which provided a direct knowledge of the principles, through *nous*. The admission of this direct understanding, even though hard to justify in Mill’s construction, would have allowed him to solve at least some of the difficulties that arise from his kind of empiricism. But that would have meant for him to lose ground to the contemporary intuitionists and rationalists, because such a view entails, as it did for Aristotle, that the principles are characteristics of the intelligibility of the world and that we have a special capacity or faculty for capturing their meaning. It could not be otherwise, on this analysis, since for Aristotle the principles, as we saw earlier, are prior, they are the starting points and thus

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<sup>77</sup> *System of Logic*, p. 224.

cannot be arrived at through other means, but only intuited directly and with most certainty.

For Mill, on the other hand, because of this important detail (the lack of the comprehension of the principles by *nous*, through *epagoge*) the shift in their significance has begun already: they must lose their privileged place in the construction of knowledge and reality, due to the loss of necessity and certainty provided by the accompanying metaphysical system<sup>78</sup>. The principles are only truths with a very high degree of confirmation from experience, but there is nothing to stop us from supposing that it could always happen that our experiences will not corroborate them anymore. Necessity and certainty are metaphysical traits that can never be experienced, or at least they can never be experienced through the senses, and that is why the principle of contradiction must, in Mill's account, be demoted to a mere empirical generalization. He asks, “where then is the necessity for assuming that our recognition of these truths has a different origin from the rest of our knowledge, when its existence is perfectly accounted for by supposing its origin to be the same?”<sup>79</sup>.

But it is not only the existence of these truths or the existence of our recognition of these truths that we want to justify and explain. The more important task is to account for the special place the principles, including the principle of contradiction, have in the edifice of human knowledge, and the denial of this privileged position will not do. But with Mill we started to talk about something quite different from Aristotle's principle of contradiction. Even though he sometimes goes back to the principle of contradiction as a

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<sup>78</sup> It is an interesting question to ask whether, had Mill incorporated a concept similar to the Aristotelian *epagoge*, it would have meant that he conceded territory to the *a priori* theorists, and if so, whether such a compromise would have benefited him.

<sup>79</sup> *Ibid.*, p. 232.

trait of reality that we somehow discover, he also treats it as a law of logic, as a formal property of propositions, which is what modern logicians refer to as the law of contradiction. As such, the principle of contradiction in the Aristotelian sense is not quite the same thing as the law of contradiction in the modern sense.

### ***The Principle of Contradiction Derived: Principia Mathematica***

#### *I. Logicist background*

The logicist project of Frege and Russell was to introduce a language of logic, in which the ambiguities and inaccuracies of ordinary language would be eliminated; Russell and Whitehead, in their *Principia Mathematica*<sup>80</sup> attempted to prove that mathematics is a discipline whose propositions can be exclusively expressed through the propositions of logic, because according to logicism, mathematical terms can be defined by means of logical terms and mathematical truths derived from logical axioms. Their project was conceived as a reaction to three traditions in the philosophy of logic and mathematics: psychologism, empiricism and formalism, and these reactions were reflected in the three principles of philosophical logic, which Frege formulated in the “Introduction” of *The Foundations of Arithmetic*:

“In the enquiry that follows, I have kept to three fundamental principles: always to separate sharply the psychological from the logical, the subjective from the objective; never to ask for the meaning of a word in isolation, but only in the context of a proposition; never to lose sight of the distinction between concept and object.”<sup>81</sup>

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<sup>80</sup> Alfred North Whitehead and Bertrand Russell, *Principia Mathematica* to \*56, Cambridge University Press, New York, 1993.

<sup>81</sup> Gottlob Frege, *The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number*, Basil Blackwell Publisher, 1956, p. x. Cited in Milton K Munitz, *Contemporary Analytic Philosophy*, MacMillan Publishing Co., New York, 1981, p. 75.

The first principle is most clearly an attack on psychologist theories, and Frege insisted on the distinction between the thought as a subjective mental act and the objective idea expressed in it, which is independent of different individuals who might have that thought. In Frege's view, the objective thoughts are not physical objects in space and time, nor ideas in the minds of individual persons, but some kind of abstract objects, and they are therefore objective, in the sense of intersubjectivity (what is expressed in a proposition can be accessed by many people, whereas the individual, subjective thought is accessible solely to the individual who has it). The thought is what is asserted in a judgment, and has no truth-value until it is asserted. As we saw in the previous section of this chapter, the views of Mill and Frege are surprisingly similar in this aspect, of recommending logic to be concerned not with thoughts or ideas in the subjective, psychological sense, but rather with what is expressed or contained in these individual acts of thinking. The role of logic with regard to thoughts is a normative one, as opposed to psychological descriptions of ideas in the mind. The last two principles do not relate to our theme directly and are mentioned here only for the sake of completeness<sup>82</sup>.

The project in *Principia Mathematica* was, and still is, extremely influential in the subsequent developments of logic and philosophy of logic, even after the strike it

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<sup>82</sup> The second principle states that, since only propositions as wholes have truth-values, then ideas, concepts and words have a meaning only in the context of a proposition. If, with the empiricists, our judgments were only a juxtaposition or conjugation of those elements, it is inexplicable how they acquire a truth-value when combined. Therefore, it must be the whole of the sentence that confers the constituents their meaning. The third principle comes as a reaction to formalism, according to which mathematics and logic are simply a play on symbols which don't have to correspond to anything: mathematical proofs can be construed or modeled as the following of mechanical rules on sequences of typographic characters, while the formulae may as well be meaningless. One aim of formalism is to avoid commitment to a presumably dubious ontology, while providing a tractable method for mathematics, but Frege insist that we deal with words and symbols in virtue of them being signs of real entities, and their manipulation reflects the real nature of these entities.



received with Kurt Gödel's undecidability (or incompleteness) theorem, which provided a proof<sup>83</sup> that any axiomatic system of arithmetic is incomplete, since necessarily it contains one proposition that is unable to be proved true or false within the system.

Russell and Whitehead were continuing Frege's logicist program and set out to constructing a system based on a few logical axioms (eleven, of which one is a definition), from which they derived, much later in the course of the work, the concepts and axioms of mathematics. What we are concerned with here is the status of what they call the 'law of contradiction' within this construction, what is the significance of the shift in status, compared with the models we saw previously, and what conclusions we can draw from the comparison.

## *II. The law of contradiction*

Russell has not addressed the status of the law of contradiction but sparingly, and even then it was only to reject its special place in the machinery of logic. In what follows, we will try to untangle his position and some of the underlying motivations for sustaining it. The law of contradiction is the eightieth *derived* proposition in *Principia Mathematica*, and is formulated so:

$$*3.24. \vdash \cdot \sim (p \cdot \sim p)^{84}$$

It is further demonstrated from previous propositions, themselves established on the basis of the axioms or propositions derived from the axioms. In fact, as G. Von Wright points

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<sup>83</sup> Kurt Gödel, *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, Dover Publications Inc., New York, 1992. While this proof is tremendously important from the point of view of the foundations of mathematics, our focus here is quite different, so we will not go into the quite technical and marginal to our topic details of this work.

<sup>84</sup> *Principia Mathematica*, p. 111.

out, this is a rather “sloppy mode of expression”<sup>85</sup>, because what is meant by the formula above is that the assertion is logically true in all instances of substitution of the propositional variable expressed by ‘ $p$ ’. Therefore, what the law of contradiction, as stated above, asserts is: the conjunction of a proposition and its negation is false, or in other words, it is not the case that a proposition and its negation are both true. We must note that, unlike Aristotle’s and even Mill’s principle of contradiction, which was about properties of objects in the actual world, the law of contradiction is a statement about propositions and as such it is a law of logic.

Russell declares that logic is concerned only with formal reasoning, not with actual things and properties, which is something that traditional logic has, according to him, been able only to intimate. His treatment of traditional (Aristotelian) logic is as follows: starting from an example of syllogism (“All men are mortal, Socrates is a man, therefore Socrates is mortal”), he changes that into a hypothetical statement (“If all men are mortal and Socrates is a man, then Socrates is mortal”) and the validity of this argument depends now on its form, and not on the particular terms that occur in it. On that basis, he goes one step further and eliminates also the particular properties that occur in it, replacing them with variables (“no matter what possible values  $x$  and  $\alpha$  and  $\beta$  may have, if all  $\alpha$ ’s are  $\beta$ ’s and  $x$  is an  $\alpha$ , then  $x$  is a  $\beta$ ” or “the propositional function ‘if all  $\alpha$ ’s are  $\beta$  and  $x$  is an  $\alpha$ , then  $x$  is a  $\beta$ ’ is always true”).

Here at last we have a proposition of logic – the one which is only *suggested* by the traditional statement about Socrates and men and mortals. It is clear that, if *formal* reasoning is what we are aiming at, we shall always arrive ultimately at statement like the above, in which no actual things or properties are mentioned;

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<sup>85</sup> Georg Henrik Von Wright, *Truth, Negation and Contradiction*, in *Synthese* 66, 1986, p. 3.

this will happen through the mere desire not to waste our time proving in a particular case what can be proved generally.<sup>86</sup>

But this is precisely what Aristotle set out to do with his theory of syllogism: he wanted to find the valid *forms* of syllogism, regardless of the particular terms and properties involved in it. He did more than “suggest” that the original *particular* syllogism about Socrates and men and mortals was valid in virtue of the valid *form*: ‘All *A* are *B*. All *B* are *C*. Therefore, all *A* are *C*’. We might note that the form of Aristotle’s syllogism does not have the hypothetical form that it acquired with Russell. But that is only because he was not interested in false premises. For him, syllogism is, as we saw in the previous chapter, the tool of scientific knowledge, whose goal is to attain truth about the world. If you start from false premises, while your syllogism might be valid and your conclusion might be true, its truth is not necessary, since, according to the definition of validity, the conclusion of a valid argument with false premises may be true or may be false. If you start from true premises, validity will guarantee the truth of the conclusion, and this is what Aristotle is looking for.

In Russell’s case, all he did by transforming the syllogism into a hypothetical statement was to include the cases where the antecedent (the conjunction of the premises) is false, while retaining a valid form. By doing that, the focus is not on knowledge anymore, as he himself pointed out, but on the formal properties of propositions. Validity is, like with Aristotle, a formal property of arguments, and this is all that Russell is concerned with, as opposed to Aristotle, whose main goal is the attainment of knowledge,

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<sup>86</sup> Bertrand Russell, *Introduction To Mathematical Philosophy*, Dover Publications Inc., New York, 1993, p. 197.

and who, in consequence, will not investigate those trails that don't lead with necessity and certainty to truth.

Because he is interested only in the formal properties of propositions, the law of contradiction becomes, according to Russell, just a proposition that can be deduced from other ones, which are obviously seen as more fundamental:

The law of contradiction is merely one among logical propositions; it has no special pre-eminence; and the proof that the contradictory of some proposition is self-contradictory is likely to require other principles of deduction besides the law of contradiction.<sup>87</sup>

One example of these more fundamental principles, mentioned by Russell, is the case of the principles of inference, which he considers more obvious and prior to any others:

“some at least of these principles must be granted before any argument or proof becomes possible”<sup>88</sup>.

Russell's account of the way we arrive at the knowledge of these principles is at times surprisingly similar to Aristotle's: we start from observing the particular instances and then, by recognizing the generality in them, we come to grasp the principle:

In all our knowledge of general principles, what actually happens is that first of all we realize some particular application of the principle, and when we realize that the particularity is irrelevant, and that there is a generality which may equally truly be affirmed.<sup>89</sup>

It is not certain that Aristotle would contend that particulars are irrelevant, at least not literally, but if what is meant by the claim is that in grasping the principles we move to a level of generality where we don't deal with particulars anymore, then, as we saw in the previous chapter, he would gladly concede this point. Let us note also that Russell's

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<sup>87</sup> *Ibid.*, p. 203.

<sup>88</sup> Bertrand Russell, *The Problems of Philosophy*, Dover Publications Inc., Mineola, New York, 1999, p. 50.

<sup>89</sup> *Ibid.*, pp. 49-50.

account differs from Mill's, first in that the principles are not generalizations from experience and they are not the result of induction, and second, as a consequence of the first, in that they are as certain as the data of immediate experience:

Some of these principles have even greater evidence than the principle of induction, and that the knowledge of them has the same degree of certainty as the knowledge of the existence of sense-data.<sup>90</sup> Logical principles are known to us, and cannot be themselves proved by experience.<sup>91</sup>

Among the abovementioned principles of logic, Russell includes the principle of contradiction, although he does suggest that it is not among the first. To be sure, Russell does not maintain that the law of contradiction is a contingent statement, in the sense that it can be false; no: he still recognizes it as a proposition that is always true (or as what he called a 'tautology'). But it is not anymore, in his view, the most basic and most general principle of reality, and not even of logic. Speaking of principle of logic in general, he says:

When some of them have been granted, others can be proved, though these others, so long as they are simple, are just as obvious as the principles taken for granted. For no very good reason, three of these principles have been singled out by tradition under the name of 'Laws of Thought'.<sup>92</sup>

Among those principles is the law of contradiction, which in Russell's formulation is: "nothing can both be and not be". This formulation is confusing on at least two accounts: first, it is unclear whether 'be' and 'not be' refers to an existential claim, in which case it would quite limit the scope of the principle, or they refer to being in certain ways, *i.e.* having properties (where the qualifications have been removed for the sake of suggesting generality), in which case the second confusion arises: whether

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<sup>90</sup> *Ibid.*, p. 49.

<sup>91</sup> *Ibid.*, p. 52.

<sup>92</sup> *Ibid.*, p. 51.

‘nothing’ refers to objects or to propositions. At any rate, we will not consider this the most representative position of Russell’s regarding the principle of contradiction, but we will turn to *Principia Mathematica* for further clues.

We mentioned before that the law of contradiction was derived in *Principia Mathematica* from a set of eleven basic propositions. The first one of those is a definition, namely the definition for implication:

$$p \supset q . = . \sim p \vee q$$

What is meant by it is, if  $p$  and  $q$  are propositional variables, then ‘ $p$  implies  $q$ ’ is identical with ‘either  $p$  is false, or  $q$  is true’. The form of it is an identity and it very plain to see that in order to be able to pose an identity one must employ the principle of contradiction. We could not assert that  $A = B$  without presupposing that it is impossible for  $A$  to be at the same time not- $A$ , because then  $B$  would also be not- $A$  and everything else would be both  $A$  and not- $A$ . When we identify  $B$  as  $A$ , we specifically indicate that  $B$  is not not- $A$ . But if  $A$  is not- $A$ , then, by Leibniz’ law of the substitutivity of identicals,  $B$  will also be not- $A$ . It follows that we do, inevitably, employ the principle of contradiction in the very first of the propositions of the *Principia Mathematica*. What is derived later, at \*3.24, is only a proposition that follows from the formal conventions that act as rules of combining logical symbols, according to axioms put forth.

We find ourselves now in the odd situation of deriving a proposition, (*i.e.*, proving it a tautology) in a formal system which could not have in fact been built without assuming at its core the very principle of which the formal proposition is an expression.

### Chapter Three: Conclusion

The basic theme of my thesis was the indemonstrability of the principle of contradiction and the ways of justifying its primacy in human knowledge, as treated in the works of Aristotle, Mill and Russell. In Aristotle's view, this is a metaphysical principle, and it is the most certain of all, its denial being impossible, because it is unintelligible. Being a principle (*arche*), it is a starting point for every rational discourse, as every utterance that is not meaningless, every proposition and every argument must presuppose it, even though it need not always be explicit. Because it is prior to any knowledge, it cannot be derived from other premises, as that would make those premises more fundamental. What has been often misinterpreted as Aristotle's demonstration for the principle of contradiction is in fact a dialectical proof to reject the alleged view that would deny this principle. Aristotle himself declared the principle indemonstrable and the request for such a demonstration an act of the uneducated.

There must be another way in which we reach them, since Aristotle denies the fact that they are innate, and that way is through *epagoge*, which is a sort of inductive intuition. The intuitive intuition (*nous*) grasps the universals and the first principles (which are cases of universals) directly from particular experiences. Simple induction creates the universals and the laws of logic on the basis of generalizations from those particular experiences. *Epagoge* is the immediate recognition of the one in the many, and as such it starts from the preliminary acquaintance, or encounter, with the particulars, just

like simple induction, but the latter is not immediate and is not an act of recognition or grasping, but an act of generalizing from particular observations.

The one who sustained the view that the principles are reached at through induction is Mill. In his conception, the principle of contradiction, together with other “axioms” of logic and mathematics are mere generalizations from sensory experience, and since all our knowledge is based on data from sensory observations, it becomes impossible to maintain the certainty and necessity of these axioms, including the principle of contradiction. With Mill, they become highly probable generalizations, but nothing more. Necessity is a metaphysical trait that cannot be an object of sense experience. Sense data can only tell us what there *is*, but never can we derive only from it what there *must* be. Analogously, sense data cannot be the sole grounds for certainty, since certainty is a result of necessity: one is certain of things that are, or at least are deemed, necessary. If one cannot, on whatever grounds, concede necessity, then one cannot claim certainty.

This demotion in the status of the principle of contradiction, together with the other axioms, heralds the transformation that took place in modern logic, especially with the *Principia Mathematica* of Russell and Whitehead. The name of the principle is changed to the ‘law of contradiction’, to signal that it is not a starting point anymore and it is not the most fundamental piece of knowledge. As a law of contradiction it is still a tautology, always true and universally applicable, although the objects to which it applies are now propositions, not actual things. The law of contradiction is a formal property of propositions, be it of all propositions. It refers to language and its syntactical properties, rather than objects and the semantical properties of language. In *Principia Mathematica*,



the principle of contradiction is the eightieth derived proposition, on the basis of eleven more fundamental axioms, which are accepted without proof. But the *principle* of contradiction is employed in the very first definition of the system and indeed throughout the proofs, including the proof through which the *law* of contradiction is given, which is an oddity that is still present in standard logic textbooks. The fact that the principle is so fundamental and that it is evident on other grounds than the proof that is provided for it is the very reason why, in everyday practice of logic, this oddity is overlooked and does not normally create worries about circularity. But in philosophy this precarious accord is not satisfactory and a more accurate explanation, if and as much as possible, is required.

The principle of contradiction does indeed seem to have a privileged place among the other beliefs and I contend, in spite of various philosophical attempts to prove otherwise, that it is indeed impossible to deny. But when we raise the question of the grounds for its extra-ordinary certainty, we run into all sorts of difficulties, as we have seen in this thesis. I am inclined to take the basic Aristotelian position. I would agree that we employ the principle in every act of thought, no matter if it is as simple or as fundamental as uttering a word with sense. While the denial of the principle of contradiction is impossible and in fact unintelligible, the exact reasons why this is so escape us and it seems that whenever we try to explain anything about it, we must employ it and we are bound to some type of circularity.

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