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Optimal Design of An Accelerated Degradation Experiment with Reciprocal Weibull Degradation Rate

by

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A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering
Department of Industrial and Management Systems Engineering
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ABSTRACT

To meet increasing competition, get products to market in the shortest possible time, and satisfy heightened customer expectations, products must be made more robust and fewer failures must be observed in a short development period. In this circumstance, assessing product reliability based on degradation data at high stress levels becomes necessary. This assessment is accomplished through accelerated degradation tests (ADT). These tests involve over stress testing in which instead of life product performance is measured as it degrades over time. Due to the role these tests play in determining proper reliability estimates for the product, it is necessary to scientifically design these test plans so as to save time and expense and provide more accurate estimates of reliability for a given number of test units and test time. In ADTs, several decision variables such as inspection frequency, the sample size, and the termination time at each stress level are important.

In this research, an optimal plan is developed for the design of accelerated degradation test with a reciprocal Weibull degradation data using the mean time to failure (MTTF) as the minimizing criteria. A non linear integer programming problem is developed under the constraint that the total experimental cost does not exceed a predetermined budget. The optimal combination of sample size, inspection frequency and the termination time at each stress level is found. A case example based on Light Emitting Diode (LED) example is used to illustrate the proposed method. Sensitivity analyses on the cost parameters and the parameters of the underlying probability distribution are performed to assess the robustness of the proposed method.

CHAPTER 1

INTRODUCTION

1.1 Background

Today’s manufacturers are facing new pressures to develop highly sophisticated products to match rapid advances in technology, intense global competition and increasing customer expectations. As a result manufacturers must produce components in record time, while improving productivity, reliability, and overall quality of the component. It is a significant challenge to design, develop, test, and manufacture highly reliable products within short turn around times and remain within the stringent conditions, imposed by both internal and external circumstances.

Estimating the time-to-failure distribution or long-term performance of components of high reliability products is particularly difficult. Most modern products are designed to operate without failure for several years. Thus few of such units will fail or degrade to a significant amount in a test of any practical length based on normal use conditions. For example, during the design and construction of a communication satellite, there may be only 6 months available to test the components which are meant to be in service for 15 to 20 years. The components used in submarine cables are often required to operate for 25 years under the sea. Very few test units are available that will actually reflect the life profiles of these components. For these reasons, Accelerated tests (ATs) are used widely in manufacturing industries, particularly to obtain timely information on the reliability of products.

1.2 ADTs Versus Other Testing Methods

Traditionally, reliability assessment of new products has been based on accelerated life tests (ALTs) that record failure and censoring times of products subjected to elevated stress. However, this approach may offer little help for highly reliable products which are not likely to fail during an experiment of reasonable length. An alternative approach is to assess the reliability from the changes in performance (degradation) observed during the experiment, if there exists a quality characteristic of the product whose degradation over time can be related to reliability. Accelerated degradation tests compared to other tests have the advantage of analyzing performance before the material or the component fails. Degradation tests determine how much life there is left in a material or in components, and such knowledge enables life extension. Extrapolating performance degradation to estimate when it reaches failure level enables analysis of degradation data. However, such analysis is correct only if a good model for extrapolation of performance degradation and a suitable performance failure have been established.

Some of the general assumptions of accelerated degradation models are

- Degradation is not reversible.
- A model applies to a single degradation process mechanism or failure mode. If there are simultaneous degradation processes and failure modes, each requires its own model.
- Degradation of specimen performance before the test starts is negligible.
- Performance is measured with negligible random error.

- The failure process at high stress levels are the same as at the design or use stress levels.

Accelerated degradation tests are usually very expensive and thus it is essential to plan them carefully. Good test plans yield better results for a given cost and time parameters, on the other hand poor test plans waste time and resources, and may not even yield the desired information. When conducting an ADT, the following issues are usually of interest. “How long should the test be run?” or “How many units should be tested at each stress level?” Thus to address the issues, a scientific plan is needed to make the most efficient use of test resources and ultimately to obtain an accurate estimate of the life profile of an entity under the normal use conditions.

1.3 Reliability Analysis

Most things have a life span, defined in some form or another. These life times when measured, present us with data sets that are used for scientific or other purposes. It is natural to study the life time distribution of an entity through a set of measured data. An area of research, which is still vary much active, is the theory of reliability. A generic definition of reliability is:

Reliability is the probability that a product or a system will perform its intended function without failure for a specified period of time under specific operating conditions.

To express this relationship mathematically we define the continuous random variable to be the time to failure of the component or system. Thus reliability at time t can be expressed as:

$$R(t) = \Pr\{T \geq t\}; T \geq 0,$$

$$Where: \quad R(t) \geq 0, R(0) = 1, \ Lim_{t \to \infty} R(t) = 0$$

In reliability analysis, the major issue is the probability distribution of the life times of the entity under study. For this purpose, the standard method is to take a set of observed life times T_1, T_2... T_n, or censored sometimes, where we assume that the observed life time is a function of an unknown parameter θ which can be expressed as $T_i \sim F(.; \theta)$. Where $F(.; \theta)$ is the probability density function of an unknown parameter θ. From the likelihood function constructed from this sample, we can make an inference with respect to the unknown parameter θ .

When the form of $F(.; \theta)$ is known and the complete distribution of F is determined by a finite dimensional parameter θ, then we have a classical parametric model; if F is completely unknown except for some qualitative descriptions such as continuity or smoothness, then we have a non-parametric model; finally, if F is unknown but the parameter θ has some structure to explore, then we have a semi-parametric model.

When the parameter θ exhibits some structure, we will naturally embed our inference problem into traditional, and time-tested models for statistical analysis. These include techniques such as experimental design, regression, logistic regression, accelerated life testing, etc. These methods incorporate various situations that may be encountered in practice. There is no need, however, to restrict the inference to the classical frequentists' parametric setup. We can, if the situation requires, use the Bayesian method or even the empirical Bayes techniques.

1.4 Applications

Applications of accelerated degradation tests include light emitting diodes, logic integrated circuits, power supplies, etc.

1.4.1 Light Emitting Diodes (LED)

Light emitting diodes are widely used in many fields ranging from consumer electronics to optical fiber transmission systems. The LED has many features such as less power consumption, small volume, good visual effects and long life. Nowadays they are used as electronic boards on highways and streets, and as smoke censors on ceilings, etc. Because of their high reliability, it is difficult to obtain the product life time information under normal stress levels in a relatively short time. Thus, ADTs are used to obtain the reliability information of LED products [6, 7].

1.4.2 Logic Integrated Circuits

Some logic integrated circuits are used as components of submarine cables. The important parameter in determining the reliability information of logic integrated circuits is propagation delay [8]. The logic integrated circuits might not function if the propagation delay of a logic gate increases (degrades). For example, a logic circuit which is designed to have a maximum propagation delay of 10 nsec from input to the output requires that the combined propagation delay of the individual logic gates in the critical path does not exceed 10 nsec. These logic integrated circuits are required to operate for at least 25 years under the sea without failure. So, accelerated degradation tests are employed to predict the life of logic integrated circuit and to study the associated propagation delay.

1.4.3 Power Circuits

For power supplies, failures are common due to low DC output. Power supply units show downward drift in their DC output. Accelerated degradation tests are used to measure the DC output and to monitor the device for reliability information.

Nelson [3, pp 521-548] lists other applications of accelerated degradation tests which include: metals, semi conductors and micro electronics, dielectrics and insulations, food and drugs, and plastics and polymers.

1.5 Motivation for This Research

An Accelerated Degradation test is a mechanism designed to shorten the life of products by subjecting the test units to higher levels of stresses that are more severe than the normal use stress levels. The information from high stress levels is extrapolated through a reasonable statistical method to obtain estimates of life, or long-term performance, at the normal use stress level. Traditional approaches are based on life tests that record only time-to-failure. Such analyses have been extensively studied and developed over the past few decades and many articles have been published in this area. Due to the fact that traditional life testing of highly reliable products does not give good reliability estimates, reliability assessment using degradation data has become increasingly important.

In the literature, most degradation models assumes that the degradation paths or transformed degradation paths are linear, and are developed for only the normal use stress level [3-8]. Most of the literature focuses on estimating the parameters in the linear degradation model and the life distribution. Research about accelerated test planning is also reported. But, when carrying out the accelerated degradation tests several decision variables such as inspection frequency, the sample size, and the termination time at each stress level are important.

The primary objective for this research is to determine the optimal parameters of an ADT with respect to products whose degradation rates follow the reciprocal weibull distribution. This is accomplished by taking the MLE's of the variance of the confidence interval of the MTTF under the constraint that the total experimental cost does not exceed a pre-determined budget. A non linear programming problem is developed to determine the optimal value of the decision variables such as sample size, inspection frequency and the termination time at each stress level.

1.6 Organization of the Thesis

In Chapter 2, a review of the literature is discussed. Chapter 3 discusses the problem statement, assumptions made and notations used. An optimization problem is proposed. In Chapter 4, an optimal plan for solving the optimization problem is presented. To illustrate the optimal plan, an example of LED degradation is presented in chapter 5.Finally, the conclusions of the study and suggestions for further research are presented in chapter 6.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

During the 1990's Nelson[3], (chapter 11) provided a fairly thorough survey on ADT, which included areas of applications, statistical models, describes basic ideas on ADT models, and, using a specific example, shows how to analyze a type of degradation data. Carey and Koenig[4] (1991) have described a data-analysis strategy and a model-fitting method to extract reliability information from observations on the degradation of integrated logic devices that are components in a new generation of submarine cables. Most failures can be traced to an underlying degradation process. Meeker and Escobar (1998) gave some examples of three general shapes for degradation curves in arbitrary units of degradation and time: linear, convex, and concave which are shown in fig .2.1. The dashed horizontal line represents the degradation level at which failure would occur.

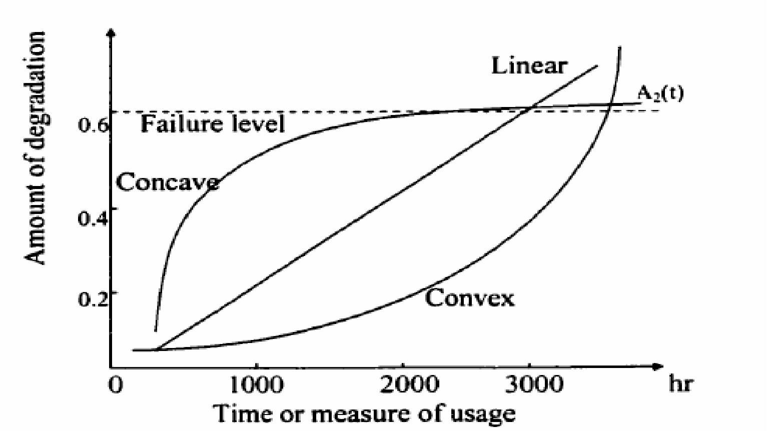


Figure 2.1 Possible Shapes of Degradation Curves

2.2 Degradation Models

2.2.1 Linear Degradation

Meeker and Hamada [17] uses linear degradation in some simple wear processes like automobile tire wear.

Let D (t) represents the amount of automobile tire tread at time t, and

wear rate $\frac{dD(t)}{dt}$ = C,

then

D (t) = D (0) + C t.

The parameters D (0) and C could be taken as constant for individual units, but random from unit-to-unit.

2.2.2 Convex Degradation

The convex degradation approach is used in models for which the degradation rate increases with the level of degradation such as in modeling the growth of fatigue cracks.

Let a (t) denote the size of a crack at time t.

The Paris model[18] is given as

$$\frac{da(t)}{dt} = C[\Delta k(a)]^m , \tag{2.21}$$

Where

a = size of the crack,

C and m = material properties,

and

$\Delta k(a) =$ stress-intensity factor, is often used to describe the growth of fatigue cracks. Lu and Meeker [18] use this model in which $\Delta k(a) = a$ for describing the growth rate of fatigue cracks within a certain size range. Then,

$$a(t) = \frac{a(0)}{\{1-[a(0)]^{m-1}C(m-1)t\}^{1/(m-1)}}, \tag{2.22}$$

where a (0) = 0.90 inches is the initial crack length at t = 0.

Dividing both sides of Eq. (2.2) by a (0) yields

$$a(t)/a(0) = 1/\{1-[a(0)]^{m-1}C(m-1)t\}^{1/(m-1)} \tag{2.22'}$$

2.2.3 Concave Degradation

Meeker and LuValle [19] describe models for growth of failure-causing conducting filaments of chlorine-copper compounds in printed-circuit boards. They consider degradation from a first-order chemical reaction. These filaments cause failure when they reach from one copper-plated through-hole to another.

Let $A_1(t)$ be the amount of chlorine available for reaction at time t, and $A_2(t)$ be the amount of failure- causing chlorine-copper compound at time t. Under appropriate conditions, copper combines with chlorine A_1 to produce the chlorine-copper compound A_2 with a constant rate k.

The equations for the rate for this process are

$$\frac{dA_1}{dt} = -kA_1$$

and

$$\frac{dA_2}{dt} = -kA_2$$

Let c and $A_2(0)$ be the initial amounts. Assuming $A_2(0) = 0$, we get

$$A_1(t) = A_2(\infty)[1-\exp(-kt)]. \quad (2.23)$$

This function is illustrated by the concave curve in Fig.2.1. The asymptote at $A_2(\infty) = A_1(0)$ reflects the amount of chlorine available for the reaction producing compounds . LuValle and Meeker [19] also suggest other more elaborate models for this failure process. Carey and Koenig [4] use similar models to describe degradation of electronic components.

2.3 General Degradation Path Model

Lu & Meeker (1993) use the following model [Eq 2.31] for the analysis of degradation data at a fixed level of stress (i.e., no acceleration) and to estimate a time-to-failure distribution. They denote the true degradation path of a particular unit(a function of time) by D(t), t > 0. In applications, values of D(t) are sampled at discrete points in time,t_1,t_2,….The observed sample degradation path for unit i at time t_{ij} is a unit's actual degradation path plus error and is given by

$$y_{ij} = D_{ij} + \in_{ij}, \quad i = 1,\ldots., n \qquad j = 1, 2,\ldots\ldots,m_i \quad (2.31)$$

Where $D_{ij} = D(t_{ij}, \beta_i)$ is the actual path of the ith unit at time t_{ij}(the times need not be the same for all units),

$\in_{ij} \sim N\left(0, \sigma_{\in}^2\right)$ is the deviation from the assumed model for unit i at time t_{ij} ,

$\beta_i = (\beta_{1i}, \ldots. \beta_{ki})$ is a vector of k unknown parameters for unit i.

The deviations are used to describe the measurement error. The total number of inspections on unit i is denoted by m_i. Time t could be real-time, operating time, or some surrogate like miles for automobile tires or loading cycles in fatigue tests. Typically small paths are described with a model that has up to four points (i.e., k=1, 2, 3, 4). Some of the parameters in β could be random from unit-to-unit and some of them could be modeled as constant across all units.

The scales of y and t can be chosen to simplify the form of D(t, β). The choice of a degradation model requires not only the specification of the form of D(t, β) function, but also the specification of which of the parameters in β are random and which are fixed as well as the joint distribution of the random components in β. Lu & Meeker (1993) describe the use of a general family of transformations to a multivariate normal distribution with mean vector μ_β and covariance matrix $\sum \beta$.

It is generally reasonable to assume that the random components of the vector β are independent of the $\in_{ij}$. We also assume that $\in_{ij}$ are independent and identically distributed. Because the y_{ij} are taken serially on a unit, however, there is potential for autocorrelation among the $\in_{ij}$. Especially if there are many closely spaced readings. In many practical applications involving inference on the degradation of units from a population or process, however, if the model fit is adequate and if the testing and measurement processes are in control, then the autocorrelation is typically weak and , moreover, it is dominated by the unit-to-unit variability in the β values and thus can be ignored. Also, it is well known that point estimates of regression models are not seriously affected by autocorrelation, but ignoring autocorrelation can result in standard errors that are seriously biased. This however is not a problem when confidence intervals are constructed by using an appropriate simulation-based bootstrap method. In more complicated situations it may also happen that $\sigma_\in$ will depend on the level of the acceleration variable. Often, however, appropriate modeling (for variance stratification, e.g., transformation of the degradation response) will allow the use of a simpler model based on constant σ_{ϵ}.

2.4 Degradation and Failure Types

2.4.1 Soft Failures: Specified Degradation Level

For some products there is a gradual loss of performance (e.g., decreasing light output from a fluorescent light bulb). Then failure would be defined in an arbitrary manner at a specified level of degradation such as 60 % of initial output. Tseng, Hamada, and Chiao (1995) explain this with an example and defined this as “soft failure”.

A fixed value of D_f is used to denote the critical level for the degradation path above (or below) which failure is assumed to have occurred. The failure time T is defined as the time when the actual path D(t) crosses the critical degradation level c and t_c is used to denote the planned stopping time in the experiment. Inferences are made on the failure-time distribution of a particular product or material. For soft failures it may be possible to continue observation beyond D_f.

2.4.2 Hard Failures: Joint Distribution of Degradation and Failure Level

For some products, a failure event is defined as when the product stops working (e.g., when the resistance of a resistor deviates too much from its nominal value, causing the oscillator in an electronic circuit to stop oscillating or when an incandescent light bulb burns out). These are called “hard failures”. In general with hard failures, failure times correspond to a particular level of degradation. But, the level of degradation at which failure occurs is random from unit to unit or from time to time. This could be modeled by using a distribution to describe unit-to-unit variability in D_f or, more generally, the joint distribution of β and the stochastic behavior of D_f.

2.5 Constant Stress Degradation Models

Nelson (1990) briefly describes some basic degradation models for constant stress. The following are the most widely used constant stress degradation models.

2.5.1 The Arrhenius Rate Model

The Arrhenius rate relationship is widely used for temperature-accelerated degradation. This model is mostly used in pharmaceuticals, insulations, dielectrics, plastics, polymers, Adhesives, battery cells, and incandescent lamp filaments.

Arrhenius law: According to the Arrhenius rate law, the rate of a simple (first-order) chemical reaction depends on temperature as follows

$$rate = A' \exp[-E/(kT)] \tag{2.51}$$

where:

E is the activation energy of the reaction, usually in electron-volts.

k is Boltzmann's constant, 8.671×10^{-5} electron-volts per ^{0}C.

T is the absolute Kelvin temperature; it equals the temperature in Centigrade plus 273.16 degrees; the absolute Rankine temperature equals the Fahrenheit temperature plus 459.7 Fahrenheit degrees.

A' is a constant that is characteristic of the product failure mechanism and the test conditions.

The product is assumed to fail when some critical amount of the chemical has reacted (or diffused);

Critical amount = (rate) × (time to failure)

or,

Time to failure = (critical amount) / (rate)

Therefore, the nominal time τ to failure ("life") is inversely proportional to the rate. This yields the Arrhenius life relationship $\tau = A\exp[E/(kT)]$

2.5.2 Inverse Power Relationship

The inverse power relationship is widely used to model product life as a function of an accelerating stress. This is mostly used in electrical insulations, dielectrics in voltage-endurance tests, ball and roller bearings, incandescent lamps and flash lamps etc. The relationship is sometimes called the inverse power law or simply the power law.

Suppose that the accelerating stress variable V is positive. The inverse power relationship between "nominal" life τ of a product and V is

$$\tau(V) = A/V^{\gamma}; \qquad (2.52)$$

Here A and γ are parameters characteristic of the product, specimen geometry and fabrication, the test method, etc., The parameter γ is called the power or exponent.

2.5.3 Eyring Relationship

An alterative to the Arrhenius relationship for temperature acceleration is the Eyring relationship.

The Eyring relationship for "nominal" life τ as a function of absolute temperature T is

$$\tau = (A/T)\exp[B/(kT)]; \qquad (2.53)$$

here A and B are constants that are characteristic of the product ad test method, and k is the Boltzmann's constant.

2.6 Acceleration Model

To obtain timely information from laboratory tests, sometimes it is required to use some form of acceleration. In some failure mechanisms such as the weakening of an adhesive mechanical bond or the growth of a conducting filament through an insulator, the chemical or other degradation process can be accelerated by increasing the level of acceleration variables like temperature, humidity, voltage, or pressure. If an adequate physically-based statistical model is available to relate failure time to levels of accelerating variables, the model can be used to estimate lifetime or degradation rates at product use conditions. Lu, Meeker & Escobar [5] mentioned the following acceleration models.

2.6.1 Elevated Temperature Acceleration

The Arrhenius model, which describes the effect of temperature on the rate of a simple first-order chemical reaction is

$$R(temp) = \gamma_0 \exp\left[\frac{-E_a}{k_B \times (temp + 273.15)}\right] = \gamma_0 \exp\left(\frac{-E_a \times 11605}{temp + 273.15}\right) \qquad (2.61)$$

Where temp is temperature in ^{0}C and k_B= 1/11605 is the Boltzmann's constant in units of electron volts per ^{0}C. The pre-exponential factor γ_0 and the reaction activation energy E_a in units of electron volts are characteristics of the particular chemical reaction. Taking the ratio of the reaction rates at temperatures temp and $temp_U$ cancels γ_0 giving an Acceleration Factor

$$AF(\text{temp}, temp_U, E_a) = \frac{R(temp)}{R(temp_U)}$$

2.6.2 Non-linear Degradation Path and Reaction-rate Acceleration

The simple chemical degradation path model with a temperature acceleration factor affecting the rate of reaction is given by

$$D(t; temp) = D_\infty \times \{1 - \exp[-R_U \times AF(temp) \times t]\} \qquad (2.62)$$

Here R_U is the rate reaction at use temperature $temp_U$, $R_U \times AF$(temp) is the rate reaction at temperature temp, and D_∞ is the asymptote. When degradation is measured on a scale decreasing from zero, $D_\infty < 0$ then the failure occurs at the smallest t such that $D(t) \leq D_f$. Equating D(T ; temp) to D_f and solving for T gives the failure time at temperature temp as

$$T(temp) = \frac{-\frac{1}{R_U}\log\left(1 - \frac{D_f}{D_\infty}\right)}{AF(temp)} = \frac{T(temp_U)}{AF(temp)}$$

Where $T(temp_U) = -(1/R_U)\log(1-D_f/D_\infty)$ is failure time at use conditions.

Here, the life/temperature model induced by the simple degradation process and the Arrhenius-acceleration model results in a Scale Accelerated Failure Time (SAFT) model. Under the SAFT model, the degradation path of a unit at any temperature can be used to determine the degradation path that the same unit would have had at any other specified temperature by scaling the time axis by the acceleration factor AF (temp)

2.6.3 Linear Degradation Path Reaction-rate Acceleration

Consider the model with nonlinear degradation path and reaction rate acceleration along with the critical level D_f. When D(t) is small relative to D_∞,

$$D(t;temp) = D_\infty \times \{1 - \exp[-R_U \times AF(temp) \times t]\} \tag{2.63}$$

$$\approx D_\infty \times R_U \times AF(temp) \times t = R_U^+ \times AF(temp) \times t$$

If failure occurs when $D(T) \leq D_f$, then $D(T;temp) = D_f$ and the failure time is given as

$T(temp) = \frac{D_f}{R_U^+} \times \frac{1}{AF(temp)} = \frac{T(temp_U)}{AF(temp)}$ where $T(temp_U) = D_f / R_U^+$ is failure time at use conditions. This is also an SAFT model.

2.6.4 Degradation with Parallel Reactions

A more complicated degradation path model with two parallel one-step failure causing chemical reactions is given by :

$$\begin{aligned} D(t;temp) &= D_{1\infty} \times \{1 - \exp[-R_{1U} \times AF_1(temp) \times t]\} \\ &+ D_{2\infty} \times \{1 - \exp[-R_{2U} \times AF_2(temp) \times t]\} \end{aligned} \tag{2.64}$$

Where R_{1U} and R_{2U} are the use-condition rates of the two parallel reactions contributing to failure. This degradation model does not lead to an SAFT model because the temperature affects the degradation processes differently, inducing nonlinearity into the acceleration function relating times at two different temperatures.

2.7 Estimation of Accelerated Degradation Model Parameters

Lu and Meeker (1993) use a two-stage method to estimate the parameters of the mixed-effects accelerated degradation model.

Stage 1- For each unit, fit the degradation model to the sample path and obtain the estimate of the model parameters of each unit.

Stage 2- Combine the estimate of the model parameters of each unit in the first stage to produce estimates of the population parameters.

In another research Lu, Meeker & Escobar [5] suggest that, in some cases, an approximate maximum likelihood (ML) is faster than n nonlinear least squares estimations required for the two-stage method. ML estimation also has the advantages of desirable large-simple properties and easy to use sample paths for which all of the parameters cannot be estimated. The two-stage estimation is useful for obtaining starting values for the ML approach for modeling, especially when another distribution other than a joint normal distribution for the random effects is being considered.

Meeker and Escobar (1993) have given an updated literature survey on various approaches used to assess reliability information from degradation data. Boulanger and Escobar[7] address the problem of designing a class of ADTs. They assume that each unit is subjected to an elevated constant stress level over the duration of the experiment. The performance degradation of each test unit at a stress level could be described by a growth curve which levels off to a plateau (maximum degradation) after a period of time. Figure 2.2 shows the degradation amount over time.

The model is give by:

$$y(t) = \alpha[1-\exp(-(\beta t)^{\gamma})] + \epsilon(t),$$

Where

$y(t)$ = observed change of propagation delay up to time t,

α = plateau where the degradation will level off,

β = random coefficient,

γ = a constant, which is equal to 0.5,

and

$\epsilon(t)$ = measurement error.

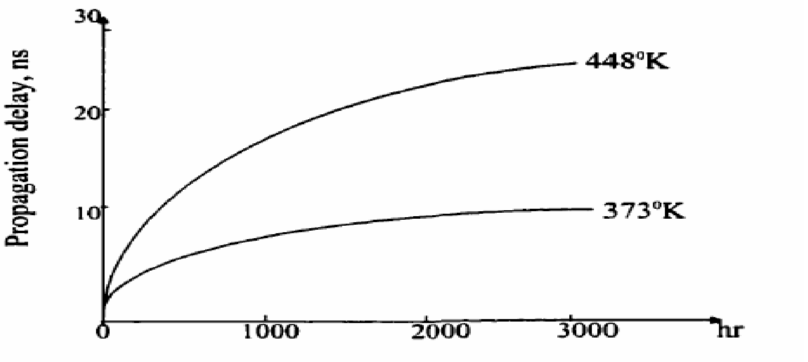


Figure 2.2 Propagation Delay Growth Curve with a Plateau (Maximum Degradation)

The authors consider α as lognormally distributed and stress dependent. The design problem they consider is to minimize the variance of the estimate of the mean of the logarithm, of the plateau, ln(α) at use condition. Their objective here is to provide some guidelines in designing a useful plan for a special class of accelerated degradation tests. They first determine the optimal stress levels and the proportion of units allocated to each stress level, and then determine optimal times to measure the performance degradation of units at each stress level. The test stress levels are chosen to be 448^0K and 373^0K, which are the highest temperatures the plastic package can withstand, and the minimum temperature the measurement equipment can detect any degradation at the end of the experiment, respectively. Equal log-spacing plan, shown in Fig.2.3., is used for measurement because the process shows a great deal of degradation at the early stage and then stabilizes.

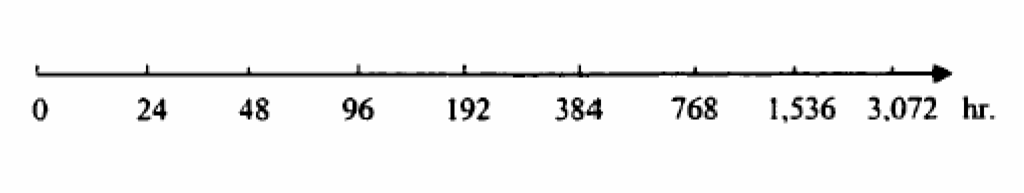


Figure 2.3 Equal Log-Spacing Plan for Measurement

Although the result is interesting, it is not practical since an appropriate termination time for an experiment is usually not known in advance.

In the literature most of the degradation models are linear, or can be transformed to linear models. Also most of the literature focuses on estimating the parameters in the degradation model and the life distribution. Yu and Tseng[8] proposed an intuitively on-line and real-time rule to determine an appropriate termination time for an ADT. Park and Yum[9] develop optimal accelerated degradation test plans under the assumptions of destructive testing and the simple constant rate relationship between stress and product performance. The authors determine stress levels, the proportion of test units allocated to each stress level and measurement times such that the asymptotic variance of the maximum likelihood estimator of the MTTF at the use condition is minimized. Yu and Tseng [1, 2, 12-14] describe a method for conducting a degradation experiment efficiently considering several factors, such as the inspection frequency, termination time and the sample size. They consider a typical degradation path of an LED, which is

$$\ln(-\ln(y(t))) = \ln(\alpha) + \beta \ln(t) + \epsilon(t),$$

Where

$y(t)$ = standardized light intensity of an LED device at time t,

α , β = parameters of the degradation path,

and

$\epsilon(t)$ = measurement error of the device at time t.

Based on data ($t_{i,k}$, $y_{i,j}(t_{i,k})$), i is the for stress level, k = 1,2,………,m, where k is the index that represents the m^{th} measurement for unit j , the degradation parameters α , β for unit j can be obtained.

The failure time of unit j can be found using

$$\tau = \left(\frac{-\ln(D)}{\alpha} \right)^{1/\beta},$$

assuming D is the critical value for the standardized light intensity when an LED fails. In these papers, they deal with the optimal design for a degradation experiment under the constraint that the total experimental cost does not exceed a predetermined budget. The optimal decision variables are obtained by minimizing the variance of the estimated 100pth percentile of the lifetime distribution. But, these three decision variables have a great deal of influence upon the experimental cost and the precision of selecting the most reliable product using degradation data. A nonlinear integer programming problem is developed to determine the optimal combination of sample size, inspection frequency and termination time.

As is evident from the above review, an extensive literature on the design of degradation tests and accelerated degradation tests exists. But when designing a degradation test, the distribution of the degradation rate of the product/component at which it degrades is very important. The Weibull and lognormal distributions are two most popular lifetime models in reliability analysis that have been used for this purpose. An incorrect choice of the distribution may lead to serious bias.

CHAPTER 3

PROBLEM STATEMENT

With regard to highly reliable products, it is important to consider the issues of how to plan tests that provide the most efficient use of resources especially as it relates to conducting an accelerated degradation test. The problem is to minimize the expected value of the Mean-Time-To-Failure of a product subject to the constraint that the total cost of the test does not exceed a predetermined test budget.

3.1 The Optimization Problem

The following decision variables are important in conducting an ADT efficiently (Yu & Chiao [1, 2]). These variables not only affect the experimental cost but also affect the precision of estimating the MTTF (ϕ_0), which can be defined as the expected or the mean value of the failure time. The pertinent questions regarding the test plan are

- How is an appropriate inspection frequency (f_i) determined?
- How many times (l_i) should the product's performance be inspected at each stress level?
- How many devices (sample size, n) should be taken for testing at each stress level?

In order to measure the precision of estimating the MTTF, the expected width of the confidence interval values of the Mean-Time-To-Failure(ϕ_0) is computed. The expected value of a real-valued random variable gives the mean or central tendency of the distribution of the variable. The unbiased maximum likelihood estimator of the MTTF can be achieved by estimating the asymptotic variance. The asymptotic variance can be obtained by minimizing either the mean square error or the expected value of the range of MTTF ($\text{MTTF}_{\max}$-$\text{MTTF}_{\min}$) . Since we do not have a prior estimate of the mean square error , we will estimate the asymptotic variance by minimizing the MTTF.

Thus, the optimal decision problem based on the expected range of the MTTF is formulated as follows:

$$\text{Minimize } E\left[\overline{\phi}_0 - \underline{\phi}_0\right] \tag{3.11}$$

$$\text{Subject to } TC\left(\left\{f_i, l_i\right\}_{i=1}^{m}, n\right) \le C_b \tag{3.12}$$

$$f_i, l_i, \text{ n } \epsilon \text{ N} = \{1, 2, 3\ldots\} \tag{3.13}$$

$$\text{i} = 1, 2, 3 \ldots.\text{m}$$

Where as,

$TC\left(\left\{f_i, l_i\right\}_{i=1}^{m}, n\right)$ denote the total cost of conducting an ADT.

$\hat{\phi}_0\left(\left\{f_i, l_i\right\}_{i=1}^{m}, n\right)$ denote an estimator of ϕ_0 based on a test plan $\left(\left\{f_i, l_i\right\}_{i=1}^{m}, n\right)$

$\left[\underline{\phi}_0, \overline{\phi}_0\right]$denote a 100(1-p) % CI of ϕ_0 from the test plan $\left(\left\{f_i, l_i\right\}_{i=1}^{m}, n\right)$

$E\left[\overline{\phi}_0 - \underline{\phi}_0\right]$ denotes the expected width of the $100(1-p)\%$ CI of ϕ_0

C_b is the total cost of the budget.

p is the percentile of the life time distribution of the product at normal use condition.

CHAPTER 4

METHODOLOGY

4.1 Degradation Model with Random Coefficient

Let $\eta(t)$denote the quality characteristic (degradation path) of the product at time t.

Assume that there exists a suitable function $\omega(.)$ such that

$$\omega(\eta(t)) = -\beta t^{\alpha}, t \geq 0, \tag{4.11}$$

Where α > 0 is a fixed and known constant; β>0 is a random coefficient that varies from unit to unit.

4.2 Assumptions

- The ADT uses m stress levels, S_0, S_1, S_2............ S_m, satisfying ($S_0 \leq$) $S_1 \leq S_2$ ≤……… S_m, where S_0 is the use condition.
- Due to the measurement errors, the actual degradation path cannot be observed directly. Let y_{ij} ($t_{i,k}$) denote the sample degradation path of the j_{th} device at time $t_{i,k}$ under the stress level S_i .The path can be expressed as follows:

$$\omega(y_{ij}(t_{i,k})) = -\beta_{ij} t_{i,k}^{\alpha} + \in_{ij} (t_{i,k}) \tag{4.21}$$

- The units put into test are randomly selected from the samples, and are randomly assigned to test stress levels. At each stress level, n devices are randomly selected for testing.

- Suppose that, under stress level S_i , the inspections are made l_i times for every f_i units of time (e.g. f_i hours or f_i days) until time $t_{i,l_i} = f_i l_i t_u$,where l_i is a positive integer and t_u is a unit of time.
- Assume that β_{ij} follows a reciprocal weibull- distribution then β^{-1} follows a weibull distribution with scale parameter θ and shape parameter δ (which is denoted by $\beta^{-1} \sim Weibull\,(\theta, \delta)$)
- The shape parameter δ does not depend on the stress level and the scale parameter or the characteristic life θ is a function of transformed levels of stress:

$$\ln(\theta) = \gamma_0 + \gamma_1 X_0 \tag{4.22}$$

Where γ_0 and γ_1 are unknown parameters to be estimated from the data, $X_i = X(S_i)$ and X (.) is a suitable transformation. Two familiar examples for X (.) are as follows:

$X(S_i) = 1/S_i$, if an Arrhenius model is assumed

$= \ln S_i$, if an inverse-power model is assumed

Some other relationships which are commonly used are mentioned in the literature review in chapter 2.

In order to solve the optimization problem the MTTF has to be computed first.

4.3 The Mean-Time-To-Failure

The product life time (τ) is suitably defined as the time when η crosses the critical level D. From Eq (3.11), τ can be expressed as

$$\tau = \left[-\frac{\omega(D)}{\beta} \right]^{1/\alpha}$$

Taking natural logarithm on both sides

$$\ln(\tau) = \frac{1}{\alpha}\left[\ln(-\omega(D)) - \ln\beta\right] \tag{4.31}$$

Since β follows reciprocal weibull distribution, -ln β follows the extreme value distribution with scale parameter u, and location parameter b (which is denoted by -ln β ~ *Extreme* (u, b) and in equation (4.11) with α fixed, it can be shown that τ follows the Weibull distribution with scale parameter $(\theta * (-\omega(D)))^{1/\alpha}$ and shape parameter $\alpha\delta$.

Let τ_0 denote the product's lifetime under S_0. Thus, we have

$\tau_0 = Weibull\left((\theta * (-\omega(D)))^{1/\alpha}, \alpha\delta\right)$

The MTTF, ϕ_0 of the product's lifetime distribution under S_0 is

$$\phi_0 = \left(\theta_0 * \left(-\omega(D)\right)\right)^{1/\alpha} \Gamma\left(1 + \frac{1}{\alpha\delta}\right)$$

$$\phi_0 = \left(-\omega(D)\right)^{1/\alpha} * \exp\left(\frac{u_0}{\alpha}\right)\Gamma\left(1 + \frac{b}{\alpha}\right) \tag{4.32}$$

Where $u_0 = \gamma_0 + \gamma_1 X(S_0)$

Here, the problem is to design an efficient ADT such that ϕ_0 can be estimated precisely. The optimization problem can be solved by using the following steps.

4.4 The Computation of $\left[\underline{\phi_0} - \overline{\phi}_0\right]$

For 1≤ j ≤ n and 1 ≤ i ≤ m, based on the observations $\left\{\left(t_{i,k}, y_{ij}(t_{i,k})\right)\right\}_{k=1}^{l_i}$, the least-squares estimator (LSE) $\hat{\beta}_{ij}$ of β_{ij}, conditional on β_{ij}, can be computed by minimizing

$$LS(\beta_{ij}) = \sum_{k=1}^{l_i} \left\{\omega\left(y_{ij}\left(t_{i,k}\right)\right) + \beta_{ij} t_{i,k}^{\alpha}\right\}^2$$

Thus, we obtain

$$\hat{\beta}_{ij} = -\frac{\sum_{k=1}^{l_i} \omega\left(y_{ij}\left(t_{i,k}\right)\right) t_{i,k}^{\alpha}}{\sum_{k=1}^{l_i} t_{i,k}^{2\alpha}} \tag{4.41}$$

and $\sigma_{\in}^{2}$ can be estimated by

$$\sigma_{\in}^{2}=\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1}{mn(l_i-1)}LS\left(\hat{\beta}_{ij}\right) \tag{4.42}$$

By considering the first-order Taylor series expansion about 1 of $\ln\frac{\hat{\beta}_{ij}}{\beta_{ij}}$,we can obtain the following approximate formula for $\ln\hat{\beta}_{ij}$:

$$\ln\hat{\beta}_{ij}\approx\ln\beta_{ij}+\left(\frac{\hat{\beta}_{ij}}{\beta_{ij}}-1\right) \tag{4.43}$$

where conditional on β_{ij}, $E\left[\frac{\hat{\beta}_{ij}}{\beta_{ij}}-1\right]=0$ and $Var\left[\hat{\beta}_{ij}\middle|\beta_{ij}\right]=\frac{\sigma_{\in}^{2}/\sum_{k=1}^{l_i}t_{i,k}^{2\alpha}}{\beta_{ij}^{2}}$.Hence, it is seen that

$$\left(\frac{\hat{\beta}_{ij}}{\beta_{ij}}-1\right)\rightarrow 0, as\sum_{k=1}^{l_i}t_{i,k}^{2\alpha}\rightarrow\infty \tag{4.44}$$

From equation (4.44), it is seen that the asymptotic distribution of un-conditional $-\ln\hat{\beta}_{ij}$ follows an extreme value distribution with $\left(\hat{u}_i,\hat{b}_i\right)$ Thus $\left(\hat{u}_i,\hat{b}_i\right)$, the conventional maximum likelihood estimators (MLEs) of $\left(u_i,b_i\right)$, can be obtained directly (Lawless,1982) by:

$$e^{\hat{u}_i}=\left[\frac{1}{n}\sum_{j=1}^{n}\exp\left(\frac{x_{ij}}{\hat{b}_i}\right)\right]^{\hat{b}_i},$$

and

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \exp\left(\frac{x_{ij}}{b_i}\right)}{\sum_{i=1}^{m}\sum_{j=1}^{n} \exp\left(\frac{x_{ij}}{b_i}\right)} - \hat{b}_i - \frac{1}{n}\sum_{j=1}^{n} x_{ij} = 0,$$

where $x_{ij} = -\ln \hat{\beta}_{ij}, 1 \le i \le m, 1 \le j \le n.$ Here $\hat{u}_i$ and $\hat{b}_i$ can be solved by using some numerical methods(e.g., Newton's method) with an iterative procedure.

Based on the asymptotically efficient property of maximum likelihood estimate (Lawless, 1982), the joint density of $\hat{u}_i$ and $\hat{b}_i$ follows an asymptotically bivariate normal distribution as follows.

$$\begin{pmatrix} \hat{u}_i \\ \hat{b}_i \end{pmatrix} \sim N\left(\begin{pmatrix} u_i \\ b_i \end{pmatrix}, \Sigma\right), \tag{4.45}$$

where

$$\Sigma = \begin{bmatrix} Var[\hat{u}_i] & Cov[\hat{u}_i, \hat{b}_i] \\ Cov[\hat{u}_i, \hat{b}_i] & Var[\hat{b}_i] \end{bmatrix} = \mathrm{I}^{-1}(\mathrm{u}_{\mathrm{i}}, \mathrm{b}_{\mathrm{i}})$$

denotes the covariance matrix of $\hat{u}_i$ and $\hat{b}_i$. The fisher information matrix $\mathrm{I}(\mathrm{u}_{\mathrm{i}}, \mathrm{b}_{\mathrm{i}})$ can be expressed as follows:

$$I(u_i, b_i) = \begin{bmatrix} E\left[\frac{-\partial^2 \ln L(u_i, b_i)}{\partial {u_i}^2}\right] & E\left[\frac{-\partial^2 \ln L(u_i, b_i)}{\partial u_i \partial b_i}\right] \\ E\left[\frac{-\partial^2 \ln L(u_i, b_i)}{\partial u_i \partial b_i}\right] & E\left[\frac{-\partial^2 \ln L(u_i, b_i)}{\partial {b_i}^2}\right] \end{bmatrix},$$

where

$$L(u_i, b_i) = \prod_{j=1}^{n} \frac{1}{b_i} \exp\left(\frac{x_j - u_i}{b_i} - \exp\left(\frac{x_j - u_i}{b_i}\right)\right)$$

By using the technique of integration by parts, Var [$\hat{u}_i$], Cov [$\hat{u}_i, \hat{b}_i$], and Var [$\hat{b}_i$] can be obtained as follows.

$$Var[\hat{u}_i] = \frac{b_i^2}{n} * \frac{6}{\pi^2} * \left[\frac{\pi^2}{6} + (1-\gamma)^2 \right], \tag{4.46}$$

$$Cov[\hat{u}_i, \hat{b}_i] = \frac{b_i^2}{n} * \frac{6}{\pi^2} * (\gamma - 1), \tag{4.47}$$

$$Var[\hat{b}_i] = \frac{b_i^2}{n} * \frac{6}{\pi^2} \quad , \tag{4.48}$$

Where Γ (x) is the gamma function and γ = 0.5772… is the Euler's constant.

In a real situation, the experiment is only conducted up to time t_l .Thus, the parameters (u_i, b_i) can be slightly calibrated by the conditional expectation technique. Assuming (u_{li}, b_{li}) denotes the parameters after refined calibration, the approximate relations between (u_{li}, b_{li}) and (u_i, b_i) can be expressed (Hong Fwu Yu, [20]) as follows:

$$u_{li} \approx u_i + \gamma * (b_{li} - b_i)$$

where:

$$b_{li} \approx \left[b_i^{\,2} + \frac{6\sigma_\in^2 \theta_i^2 \Gamma\left(1 + \frac{2}{\delta_i}\right)}{\Pi^2 \, t_u^{2\alpha} f_i^{2\alpha} \sum_{k=1}^{l_i} t_{i,k}^{2\alpha}} \right]^{\frac{1}{2}}$$

To assure that $\sum_{k=1}^{l_i} t_{i,k}^{2\alpha_i}$ is sufficiently large, it is reasonable to set

$$\left[\frac{6\sigma_\in^2 \theta_i^2 \Gamma\left(1 + \frac{2}{\delta_i}\right)}{\Pi^2 \, t_u^{2\alpha} f_i^{2\alpha} \sum_{k=1}^{l_i} t_{i,k}^{2\alpha}} \right]^{1/2} \cong s, 1 \le i \le m, 0 < s < 1 \tag{4.49}$$

This equation indicates that the slower the quality characteristic of a product degrades, the longer the degradation test should last. Thus, $\hat{b}$ can be further estimated as follows:

$$\hat{b} = \frac{1}{m}\sum_{i=1}^{m}\hat{b}_i \tag{4.410}$$

Based on these estimators, $\{\hat{u}_i\}_{i=1}^{m}$, the LSEs $(\hat{\gamma}_0, \hat{\gamma}_1)$ of (γ_0, γ_1) in Equation (4.22) can be obtained as follows (Lawless [16]):

$$\begin{pmatrix} \hat{\gamma}_0 \\ \hat{\gamma}_1 \end{pmatrix} = \left(X^T X\right)^{-1} X^T Y$$

where $X^T = \begin{bmatrix} 1 & 1 & \dots & 1 \\ X(S_1) & X(S_2) & \dots & X(S_m) \end{bmatrix}$ and

$Y^T = \left(\hat{u}_1, \hat{u}_2, \dots\dots\dots \hat{u}_m\right)$. Thus, u_0 can be estimated by

$$\hat{u}_0 = \hat{\gamma}_0 + \hat{\gamma}_1 X(S_0) \tag{4.411}$$

The approximate distribution of $\hat{u}_0$ and $\hat{b}$ is as follows

$$\hat{u}_0 \sim N\left(u_0, \left(\frac{b^2}{n} * \frac{6}{\Pi^2}\right) * \left(\frac{\Pi^2}{6} + \gamma(\gamma - 2)\right) * \frac{H}{G}\right) \tag{4.412}$$

where $H = \sum_{i=1}^{m} X(S_i)^2 + mX(S_0)^2 - 2X(S_0)\sum_{i=1}^{m} X(S_i)$

and $G = m\sum_{i=1}^{m} X(S_i)^2 - \left(\sum_{i=1}^{m} X(S_i)\right)^2$

$$\hat{b} \sim N\left(b, \frac{b^2}{n} * \frac{6}{\Pi^2}\right) \tag{4.413}$$

The approximate $100(1 - p_1)$ % and $100(1 - p_2)$ % confidence interval (CI) of u_0 and b can be obtained as follows:

$$\left[\underline{u}_0, \bar{u}_0\right] = \left[\frac{\hat{u}_0}{\exp\left(\frac{Z_{1-p_1/2}\sqrt{\operatorname{var}(\hat{u}_0)}}{\hat{u}_0} \right)}, \hat{u}_0 \exp\left(\frac{Z_{1-p_1/2}\sqrt{\operatorname{var}(\hat{u}_0)}}{\hat{u}_0} \right) \right]$$

and

$$\left[\underline{b}, \bar{b}\right] = \left[\frac{\hat{b}}{\exp\left(\frac{Z_{1-p_2/2}\sqrt{\operatorname{var}\left(\hat{b}\right)}}{\hat{b}} \right)}, \hat{b} \exp\left(\frac{Z_{1-p_2/2}\sqrt{\operatorname{var}\left(\hat{b}\right)}}{\hat{b}} \right) \right]$$

Where:

$Z_{1-p_1/2}$ is the $100(1-p_1)th$ percentile of standard normal distribution and $Z_{1-p_2/2}$ is the $100(1-p_2)th$ percentile of standard normal distribution.

p_1 and p_2 are the percentile values for scale(u_0) and shape(b) parameters respectively.

Now, substituting $\left(\bar{u}_0, \underline{b}\right)$ and $\left(\underline{u}_0, \bar{b}\right)$ into Equation (4.32) we obtain an approximate $100(1-p_1)(1-p_2)\%$ CI for ϕ_0 as follows:

$$\left[\bar{\phi}_0 - \underline{\phi}_0\right] = \left[\left(-\omega(D)\right)^{\frac{1}{\alpha}} * \exp\left(\frac{\bar{u}_0}{\alpha}\right) * \Gamma\left(1 + \frac{\underline{b}}{\alpha}\right), \left(-\omega(D)\right)^{\frac{1}{\alpha}} * \exp\left(\frac{\underline{u}_0}{\alpha}\right) * \Gamma\left(1 + \frac{\bar{b}}{\alpha}\right) \right] \quad (4.414)$$

4.5 The Computation of $E\left[\bar{\phi}_0 - \underline{\phi}_0\right]$

By taking the natural logarithm of both sides of Eq (4.32), we have

$$\ln \hat{\phi}_0\left(f_i, l_i, n\right) = \frac{1}{\alpha}\left[\ln\left(-\omega(D)\right) + u_0\right] + \ln\Gamma\left(1 + \frac{b}{\alpha}\right) \quad (4.51)$$

The asymptotic distribution of $\ln \hat{\phi}_0(f_i, l_i, n)$ follows a normal distribution $N(m, \nu^2)$,

where

$$m = \frac{1}{\alpha}\left[\ln(-(\omega(D))) + \hat{u}_0\right] + E\left[\ln\Gamma\left(1 + \frac{b}{\alpha}\right)\right]$$

and

$$\nu^2 = \frac{1}{\alpha^2} * \operatorname{var}(\hat{u}_0) + E\left[\ln\Gamma\left(1 + \frac{b}{\alpha}\right)\right]^2 - \left[E\left[\ln\Gamma\left(1 + \frac{b}{\alpha}\right)\right]\right]^2 + 2 * \frac{1}{\alpha} * \frac{\left[\ln\Gamma\left(1 + \frac{b}{\alpha}\right)\right]^2}{n} * \frac{6}{\pi^2} * (\gamma - 1)$$

Hence, the asymptotic mean of $\hat{\phi}_0\left(\{f_i, l_i\}_{i=1}^{m}, n\right)$ can be expressed as follows:

$$E\left[\hat{\phi}_0\left(\{f_i, l_i\}_{i=1}^{m}, n\right)\right] \approx \exp\left(m + \frac{\nu^2}{2}\right) \tag{4.52}$$

Therefore,

$$E\left[\overline{\phi}_0 - \underline{\phi}_0\right] \approx \exp\left((m_1 - m_2) + \frac{{\nu_1}^2 - {\nu_2}^2}{2}\right)$$

where

$$m_1 - m_2 = \frac{1}{\alpha}\left[\ln(-(\omega(D))) + \left(\overline{\hat{u}}_0 - \underline{\hat{u}}_0\right)\right] + E\left[\ln\Gamma\left(1 + \frac{\overline{b}}{\alpha}\right)\right] - E\left[\ln\Gamma\left(1 + \frac{\underline{b}}{\alpha}\right)\right]$$

and

$$\begin{aligned} {\nu_1}^2 - {\nu_2}^2 = {} & \frac{1}{\alpha^2} * \left[\operatorname{var}\left(\overline{\hat{u}}_0\right) - \operatorname{var}\left(\underline{\hat{u}}_0\right)\right] + E\left[\ln\Gamma\left(1 + \frac{\overline{b}}{\alpha}\right)\right]^2 - E\left[\ln\Gamma\left(1 + \frac{\underline{b}}{\alpha}\right)\right]^2 \\ & - \left[E\left[\ln\Gamma\left(1 + \frac{\overline{b}}{\alpha}\right)\right]\right]^2 + \left[E\left[\ln\Gamma\left(1 + \frac{\underline{b}}{\alpha}\right)\right]\right]^2 + 2 * \frac{1}{\alpha} * \frac{6}{\pi^2} * (\gamma - 1) * \left[\frac{\left[\ln\Gamma\left(1 + \frac{\overline{b}}{\alpha}\right)\right]^2}{n} - \frac{\left[\ln\Gamma\left(1 + \frac{\underline{b}}{\alpha}\right)\right]^2}{n}\right] \end{aligned}$$

4.6 The Cost Function $TC\left(\{f_i, l_i\}_{i=1}^{m}, n\right)$

The total cost $TC\left(\{f_i, l_i\}_{i=1}^{m}, n\right)$ of conducting an ADT is divided into three parts (Yu and Tseng [1, 2]).

- The cost of conducting an ADT is $C_s \max_{1\le i\le m}\{f_i l_i\} + C_p \sum_{i=1}^{m} f_i l_i$, where C_s denotes the operator's salary per unit of time and C_p denotes the unit cost of power of the testing equipment.
- The measurement cost is $C_m n \sum_{i=1}^{m} l_i$,where C_m denotes the unit cost of measurement.
- The cost to test the devices is $C_d mn$,where C_d denotes the unit cost per device.

Therefore, the total cost of the experiment is

$$TC\left(\{f_i, l_i\}_{i=1}^{m}, n\right) = \mathrm{C_s}\max_{1\le \mathrm{i} \le \mathrm{m}}\{f_i, l_i\} + \mathrm{C_p}\sum_{i=1}^{m} f_i, l_i + C_m n \sum_{i=1}^{m} l_i + C_d mn \tag{4.61}$$

4.7 The Optimization Model

From the foregoing results, the optimization problem can be expressed as follows

$$Min \quad \exp\left((m_1 - m_2) + \frac{(\nu_1 - \nu_2)^2}{2} \right) \tag{4.71}$$

$$s.t. \quad C_s \max_{1\le i\le m}\{f_i, l_i\} + C_p \sum_{i=1}^{m} f_i, l_i + C_m n \sum_{i=1}^{m} l_i + C_d mn \le C_b \tag{4.72}$$

$$t_u^{2\alpha} f_i^{2\alpha} \sum\nolimits_{k=1}^{l_i} k^{2\alpha} \approx \frac{6\sigma_\varepsilon^2 \theta^2 \Gamma\left(1 + \frac{2}{\delta}\right)}{\prod^2 * s} \tag{4.73}$$

$$(1 - p_1)(1 - p_2) = 1 - p$$

$$where:$$

$$0 < p_1, p_2 < p, f_i, l_i, n \in N, n \ge 2$$

$$i = 1,2,3,\ldots\ldots\ldots, m$$

4.8 Algorithm

Due to the complexity of the objective function, it is difficult to find an efficient method to solve the optimization model in equation (4.71). The objective function can be expressed as a function of p1 and $\left(\{f_i, l_i\}_{i=1}^{m}, n\right)$. Hence, with simplicity structure of the constraint and the integer restriction on the decision variables, an approximate solution can be obtained by the following steps.

- Let $\Delta\left(p_1, \left(\{f_i, l_i\}_{i=1}^{m}, n\right)\right) = E\left[\overline{\phi_0} - \underline{\phi_0}\right]$.
- Partition the interval $(0, p_1)$ equally into l (say, l=100) subintervals.
- Set $p_1(k) = k * \frac{p}{l}$, k = 1,2,…………,(l-1). For each $p_1(k)$, the corresponding optimal combination $\left(\{f_i(k), l_i(k)\}_{i=1}^{m}, n(k)\right)$ can be obtained as follows.
    - Given $\{f_i\}_{i=1}^{m}$.
    - Determine the corresponding $\{l_i\}_{i=1}^{m}$ by Equation (4.49).
    - Determine the corresponding n.
    - Compute $V\left(\{f_i\}_{i=1}^{m}\right) = E\left[\overline{\phi_0} - \underline{\phi_0}\right]$ from the test plan $\left(\{f_i, l_i\}_{i=1}^{m}, n\right)$
    - The optimal solution $\left(\{f_i^*(k), l_i^*(k)\}_{i=1}^{m}, n^*(k)\right)$ can be determined if

      $\{f_i^*\}_{i=1}^{m}$ satisfy $$V\left(\{f_i^*\}_{i=1}^{m}\right) = \min_{1 \le f_1 \le f_b^{(1)}} \cdots\cdots \min_{1 \le f_m \le f_b^{(m)}} V\left(\{f_i\}_{i=1}^{m}\right), \qquad (4.81)$$

      Where

      $$f_b^i = \left[\frac{1}{t_u^{2\alpha}} \frac{6\sigma_{\in}^2 \theta^2 \Gamma\left(1 + \frac{2}{\delta}\right)}{\pi^2 * s} \right]_G, 1 \le i \le m,$$

and $\lceil x \rceil_G$ denotes the largest integer not greater than x. In the minimization process of (4.61), any $\left(\{f_i, l_i\}_{i=1}^{m}, n\right)$ that does not satisfy the cost constraint would not be taken into consideration.

- Finally, an approximate optimal solution $\left(p_1^*, \left(\{f_i^*, l_i^*\}_{i=1}^{m}, n^*\right)\right)$ can be determined if

$$\Delta\left(p_1^*, \left(\{f_i^*, l_i^*\}_{i=1}^{m}, n^*\right)\right) = \min_{1 \le k \le l-1} \Delta\left(p_1(k)\right)\left(\{f_i^*(k), l_i^*(k)\}_{i=1}^{m}, n^*(k)\right)$$

CHAPTER 5

EXAMPLE

The reliability of electronic devices is of a critical concern especially for military, aerospace and communication applications. LEDs (light emitting diodes) are considered a good light source for optical links with good temperature dependence, small power consumption and high reliability. Since LEDs are designed to be in service for several years without failure, it is hard to observe failures under normal operating conditions in a short time. The reliability performance of LEDs (Light Emitting Diodes) has nearly always been superior to that of incandescent, neon and other type lamps. In addition, today's LED's have much higher reliabilities than early LED devices. Improved assembly, growth methods and structures along with new materials have allowed for the development and mass production of extremely reliable high brightness LEDs in all colors including white.

The expected useable lifetime of an LED is usually estimated by the extrapolation of measured data or by estimating the value from accelerated testing. Accelerated testing involves subjecting the LED to more extreme conditions (i.e.: higher temperature and/or higher currents) than would be expected under normal operating conditions. This is necessary since it is often difficult and impractical to actually test an LED for 100,000 hours or over 10 years. The main concern with accelerated testing of LEDs is understanding how to accurately translate these results to normal operating conditions.

The lifetime of an LED is defined as the time it takes for the light output to reach 50% of its initial value. The average lifetime specified by LED manufacturers is 100,000 hours. This does not mean that the LED will cease to operate after 100K hours; in fact, most LEDs will function for thousands of hours beyond the specified lifetime value. It means that after 100,000 hours, the LED will be half as bright as its initial luminosity level. In this chapter, the applicability of the proposed model is demonstrated by a numerical example.

5.1 Simulation Experiment

The purpose of the simulation experiment is to generate the data that would be used to estimate the reliability of LEDs (type GaAlAs) at normal operating condition with temperature $S_0 = 278$ K (5^0 C), by using the degradation data obtained at the three accelerated stress levels, $S_1 = 298$ K (25^0 C), $S_2 = 338$ K (65^0 C), $S_3 = 378$ K (1050^0 C). The data for twenty five LEDs were simulated at each of these three temperatures. The duration of each cycle (Simulation run) is 336 hours and the total number of cycles is 29. Each cycle represents an inspection interval.

Let $\omega_{ij}(t)$ denote the observed standardized light intensity of the jth LED at time t under S_i. The data is simulated in Matlab by assuming the random variable β_{ij} follows a reciprocal Weibull distribution. By using the Arrhenius relationship between temperature and time, the degradation data was generated at the three stress levels S_1, S_2, and S_3. The data represents the standardized light intensity of each component at a particular time $t^{0.65}$ .The resulting data is given in tables 5.1-5.3. Figure 5.1 shows the simulated sample degradation paths of 25 LEDs. Figure 5.2 is the plots of $\omega_{ij}(t)$ versus $t^{0.65}$ under S_1, S_2 and S_3. It is seen from the figure that there exists a linear relationship between $\omega_{ij}(t)$ and $t^{0.65}$ is given by:

$$\omega_{ij}(t) = -\beta_{ij} t^{0.65} + \in_{ij}(t) \tag{5.1}$$

Where $\in_{ij}(t)$ is the error term.

Table 5.1: The Simulated Standardized Sample Degradation Paths at Stress Level S_1

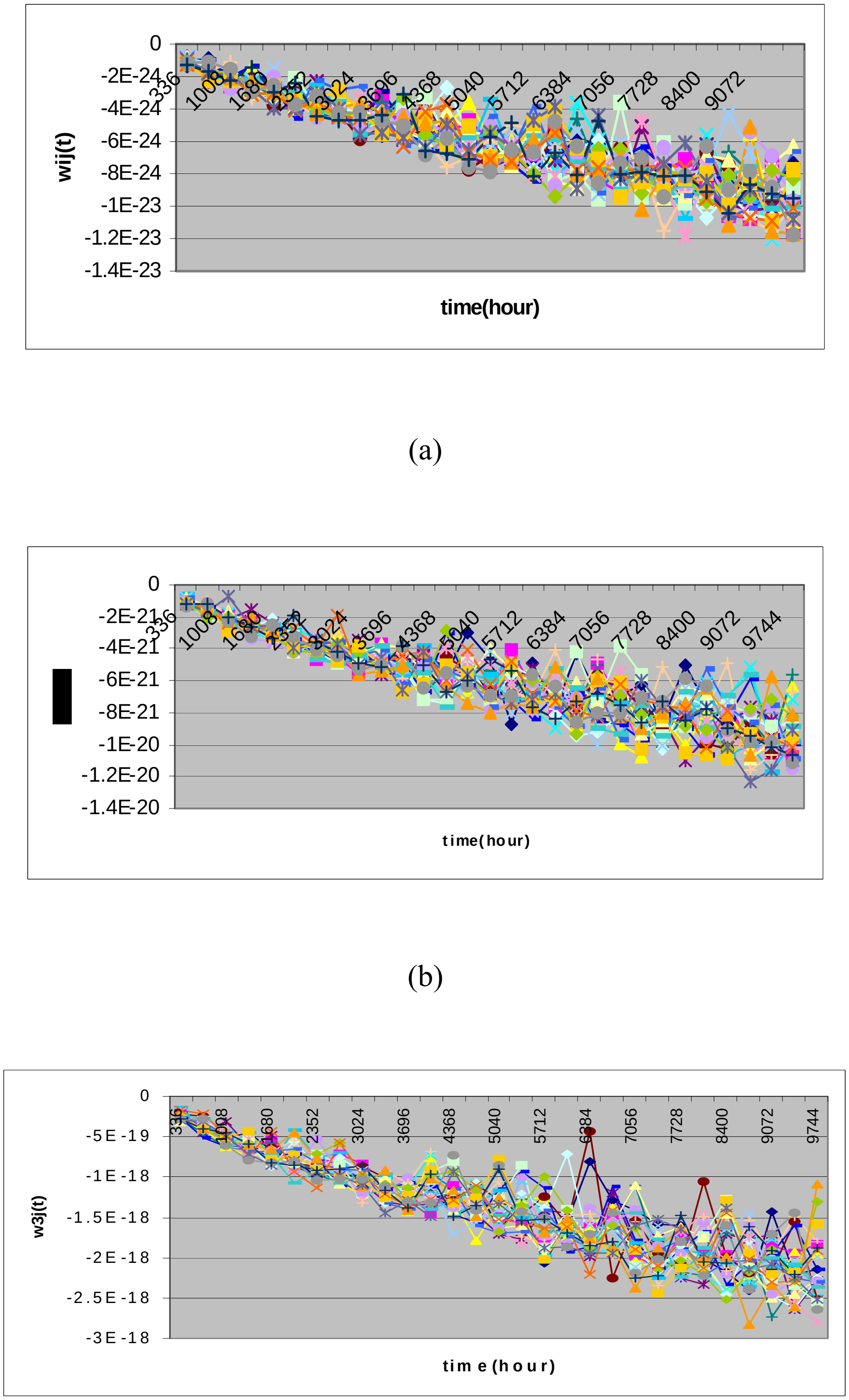
Time (hr)	$W_{11(t)}$	$W_{12(t)}$	$W_{13(t)}$	$W_{14(t)}$	$W_{15(t)}$	$W_{16(t)}$	$W_{17(t)}$	$W_{18(t)}$	$W_{19(t)}$	$W_{110(t)}$
18.3303	1.07935	1.106374	1.117466	1.133543	1.125094	1.100233	1.102839	1.101366	1.09907	1.04425
25.92296	1.12622	1.060427	1.066732	1.073545	1.050061	1.065074	1.077524	1.116496	1.06451	1.080337
31.74902	1.05546	1.047119	1.050312	1.053935	1.055861	1.07712	1.054351	1.058936	1.05386	1.05477
36.66061	1.04718	1.05395	1.085407	1.057926	1.050733	1.057619	1.051191	1.043303	1.06295	1.04842
40.9878	1.04072	1.04013	1.044154	1.037704	1.039239	1.035047	1.039479	1.049797	1.05707	1.038244
44.89989	1.03421	1.03016	1.035172	1.038326	1.037716	1.047078	1.038366	1.029211	1.03972	1.040597
48.49742	1.03518	1.04309	1.045905	1.04116	1.058704	1.03477	1.032937	1.042499	1.05391	1.037875
51.84593	1.03825	1.03873	1.037903	1.035722	1.031493	1.033257	1.029529	1.030294	1.03322	1.036445
54.99091	1.03545	1.03778	1.039389	1.029118	1.032946	1.0245	1.028547	1.028461	1.02274	1.03316
57.96551	1.02726	1.04442	1.026739	1.028678	1.027194	1.027911	1.040484	1.0491	1.02794	1.034701
60.79474	1.02656	1.02771	1.029465	1.030429	1.024977	1.026736	1.030009	1.030573	1.02604	1.025327
63.49803	1.03437	1.02395	1.027438	1.026053	1.045125	1.026988	1.030341	1.022703	1.02139	1.033662
66.09085	1.02136	1.02001	1.029822	1.028299	1.023832	1.029282	1.03289	1.023057	1.02026	1.055558
68.58571	1.02611	1.03215	1.042356	1.025861	1.022144	1.019716	1.025547	1.024775	1.02294	1.0276
70.99296	1.02397	1.02572	1.026756	1.025015	1.02026	1.022398	1.023347	1.028618	1.04982	1.021252
73.32121	1.02803	1.02109	1.026131	1.025391	1.02193	1.023118	1.023271	1.020993	1.02496	1.024933
75.57777	1.02224	1.02199	1.029381	1.033507	1.026498	1.019545	1.019913	1.018471	1.02639	1.020203
77.76889	1.02379	1.03999	1.022724	1.020332	1.022085	1.028844	1.022826	1.027802	1.01634	1.032612
79.89994	1.02235	1.02775	1.021768	1.043594	1.022109	1.021503	1.034442	1.02003	1.02471	1.024816
81.97561	1.02592	1.01094	1.019459	1.027154	1.021445	1.019957	1.020114	1.020628	1.01807	1.026998
84	1.02772	1.01702	1.021561	1.022158	1.01766	1.025087	1.019705	1.02286	1.02441	1.01834
85.97674	1.01259	1.02928	1.023346	1.018301	1.031948	1.022337	1.024057	1.025693	1.01135	1.020277
87.90904	1.02393	1.02974	1.018872	1.026758	1.019456	1.020292	1.017483	1.021769	1.02504	1.018883
89.79978	1.02129	1.02205	1.019215	1.0169	1.017946	1.017259	1.017112	1.016943	1.01328	1.018143
91.65151	1.02126	1.01931	1.019346	1.029531	1.026679	1.024926	1.016801	1.019856	1.02488	1.015501

Table 5.2: The Simulated Standardized Sample Degradation Paths at Stress Level S_2

Time (hr)	$w_{21(t)}$	$w_{22(t)}$	$w_{23(t)}$	$w_{24(t)}$	$w_{25(t)}$	$w_{26(t)}$	$w_{27(t)}$	$w_{28(t)}$	$w_{29(t)}$	$w_{210(t)}$
18.3303	1.083064	1.093911	1.088369	1.090598	1.094499	1.092493	1.079407	1.096592	1.158085	1.097408
25.92296	1.063451	1.063121	1.081652	1.067255	1.07079	1.06906	1.061419	1.080809	1.069296	1.062061
31.74902	1.055711	1.064438	1.056714	1.068797	1.055978	1.049084	1.052435	1.053248	1.060183	1.046967
36.66061	1.054208	1.056095	1.048199	1.040888	1.080672	1.046832	1.050334	1.047707	1.048236	1.049045
40.9878	1.047902	1.044018	1.044924	1.041242	1.055953	1.04147	1.041537	1.047809	1.051648	1.059726
44.89989	1.04339	1.043106	1.04219	1.048251	1.041538	1.03495	1.04126	1.040572	1.046628	1.033962
48.49742	1.03519	1.029806	1.037786	1.040816	1.03423	1.033533	1.035362	1.03259	1.032558	1.036929
51.84593	1.035142	1.031449	1.045936	1.038761	1.031519	1.030909	1.032876	1.030529	1.028562	1.03026
54.99091	1.02913	1.026032	1.032026	1.028897	1.041802	1.029795	1.035486	1.030287	1.031439	1.033077
57.96551	1.031875	1.040095	1.036833	1.034542	1.025785	1.031013	1.038359	1.029498	1.034757	1.027613
60.79474	1.030573	1.026684	1.038962	1.026103	1.026781	1.034867	1.026424	1.03087	1.026432	1.030975
63.49803	1.036283	1.034469	1.029039	1.027316	1.034058	1.025011	1.022115	1.02264	1.039253	1.021025
66.09085	1.027908	1.025532	1.032858	1.021379	1.029767	1.033671	1.028401	1.050284	1.026274	1.020501
68.58571	1.05173	1.02511	1.022872	1.025943	1.02736	1.029152	1.030734	1.024586	1.025578	1.028771
70.99296	1.033309	1.024035	1.024598	1.023856	1.022682	1.023191	1.028628	1.022958	1.027683	1.020977
73.32121	1.017947	1.039156	1.02294	1.031268	1.026811	1.021873	1.023791	1.02178	1.031946	1.027648
75.57777	1.03217	1.024348	1.022624	1.020449	1.030024	1.026106	1.028379	1.018991	1.023837	1.026298
77.76889	1.020475	1.020105	1.02263	1.017911	1.022745	1.0221	1.020493	1.02317	1.022181	1.019669
79.89994	1.037412	1.022764	1.018497	1.022932	1.022541	1.020505	1.020961	1.023444	1.025072	1.017131
81.97561	1.023667	1.032842	1.02225	1.022134	1.027271	1.028145	1.032541	1.019202	1.023479	1.017526
84	1.023412	1.02272	1.016485	1.021718	1.025872	1.022958	1.020715	1.018847	1.021911	1.020042
85.97674	1.026491	1.016062	1.015382	1.019268	1.021383	1.023284	1.02077	1.016924	1.017539	1.020003
87.90904	1.01957	1.01933	1.020496	1.020545	1.019898	1.018306	1.016381	1.020434	1.019204	1.016082
89.79978	1.033029	1.02061	1.016696	1.018849	1.015128	1.020583	1.02069	1.018512	1.021257	1.023701
91.65151	1.022242	1.017528	1.021078	1.027367	1.017643	1.016869	1.020708	1.015785	1.02264	1.022233

Table 5.3: The Simulated Standardized Sample Degradation Paths at Stress Level S_3

Time (hr)	$w_{31(t)}$	$w_{32(t)}$	$w_{33(t)}$	$w_{34(t)}$	$w_{35(t)}$	$w_{36(t)}$	$w_{37(t)}$	$w_{38(t)}$	$w_{39(t)}$	$w_{310(t)}$
18.3303	1.116713	1.116424	1.07611	1.085523	1.080313	1.108905	1.091278	1.091449	1.168591	1.093037
25.92296	1.061079	1.075182	1.065665	1.077985	1.059366	1.109148	1.056661	1.05238	1.068154	1.063144
31.74902	1.054674	1.052407	1.04596	1.052782	1.088781	1.047037	1.044986	1.045865	1.06488	1.052765
36.66061	1.064789	1.047319	1.039176	1.057227	1.047726	1.045485	1.047774	1.044076	1.051149	1.048209
40.9878	1.046079	1.05674	1.044139	1.040924	1.062397	1.048351	1.052361	1.041278	1.037227	1.035244
44.89989	1.039816	1.037031	1.033922	1.038509	1.033496	1.037023	1.035839	1.043566	1.074323	1.055403
48.49742	1.032178	1.036642	1.037947	1.034645	1.039146	1.037357	1.032525	1.041278	1.041859	1.036732
51.84593	1.033359	1.045759	1.030183	1.034775	1.040358	1.036747	1.030882	1.029637	1.047175	1.043319
54.99091	1.033793	1.039719	1.027402	1.029583	1.030977	1.038164	1.027943	1.034262	1.027894	1.035869
57.96551	1.032444	1.028069	1.029241	1.030035	1.026042	1.033548	1.028464	1.02935	1.033495	1.025466
60.79474	1.02756	1.02863	1.023949	1.030439	1.02529	1.027118	1.034097	1.032199	1.027059	1.032003
63.49803	1.028833	1.023803	1.045537	1.047665	1.026124	1.029232	1.028564	1.03101	1.029018	1.027949
66.09085	1.026727	1.032092	1.027556	1.025928	1.02998	1.027416	1.029107	1.023995	1.024048	1.029652
68.58571	1.032206	1.030197	1.019919	1.032557	1.024254	1.032864	1.02894	1.022671	1.026592	1.033131
70.99296	1.021881	1.024141	1.027005	1.02323	1.021114	1.027875	1.022182	1.031182	1.030673	1.02383
73.32121	1.026763	1.023206	1.028572	1.034996	1.020536	1.024918	1.028537	1.038958	1.024194	1.030105
75.57777	1.017785	1.020032	1.029161	1.024041	1.025808	1.029499	1.021735	1.032376	1.018548	1.026491
77.76889	1.023826	1.022353	1.021229	1.021767	1.025663	1.023262	1.022358	1.019112	1.02278	1.051391
79.89994	1.046034	1.022948	1.022963	1.019117	1.018748	1.020358	1.020731	1.019956	1.021942	1.020262
81.97561	1.02918	1.024729	1.027092	1.01972	1.031339	1.01666	1.023544	1.033503	1.024197	1.023429
84	1.026764	1.019094	1.018628	1.023794	1.020413	1.024631	1.034146	1.021094	1.020807	1.019964
85.97674	1.023956	1.0179	1.019372	1.020533	1.021607	1.019568	1.019145	1.017828	1.016844	1.017577
87.90904	1.023876	1.022272	1.019764	1.021695	1.017011	1.018452	1.017567	1.020193	1.019967	1.020708
89.79978	1.019046	1.020171	1.01824	1.017358	1.016558	1.036252	1.017322	1.022877	1.019805	1.018959
91.65151	1.017778	1.019853	1.016095	1.01622	1.022731	1.019442	1.022878	1.016684	1.023761	1.015779

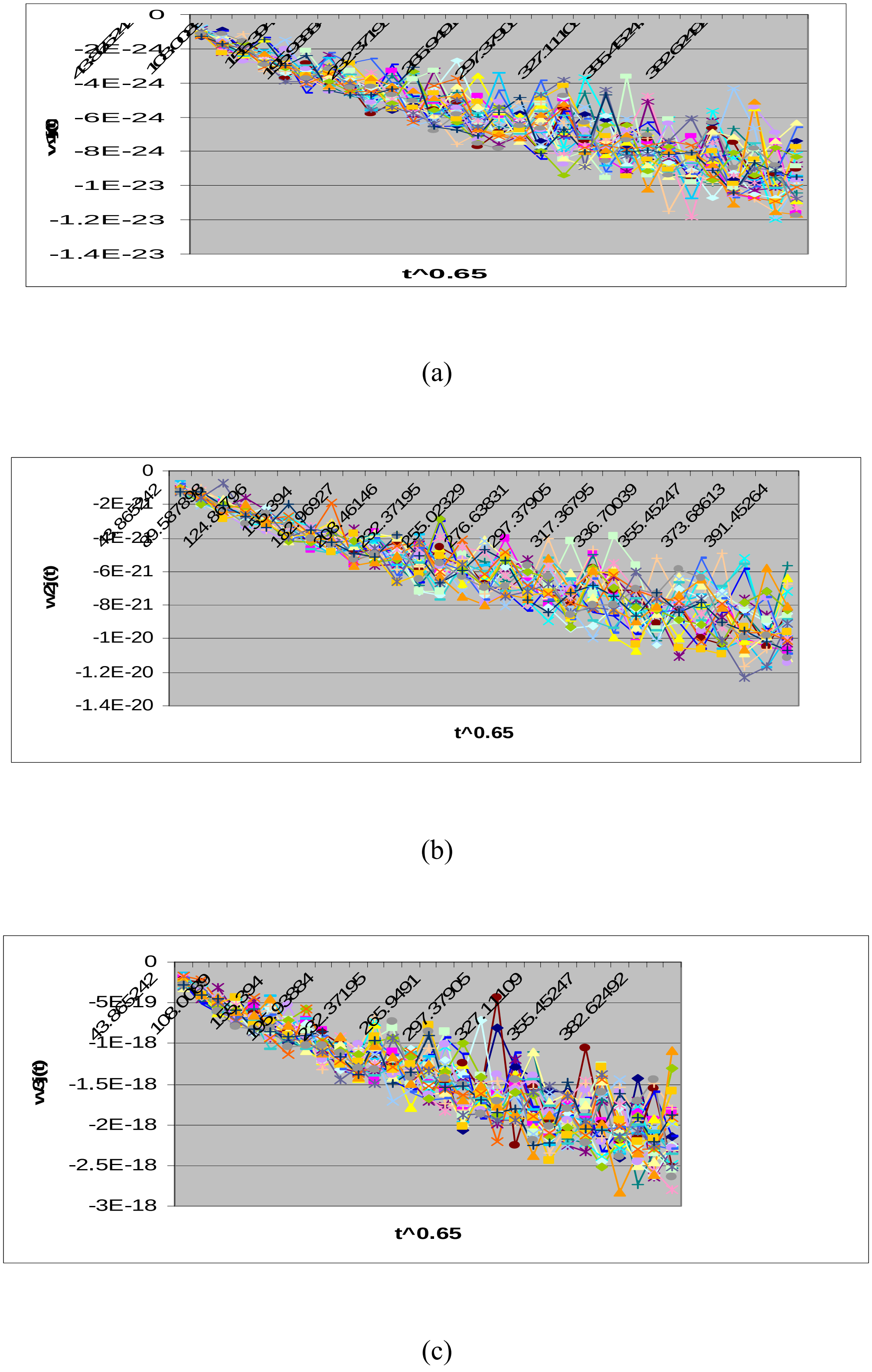


(a)

(b)

(c)

Figure 5.1. The Standardized Sample Degradation Paths under (a) S_1, (b) S_2 and (c) S_3



(a)

(b)

(c)

Figure 5.2. The Plots of $\omega_{ij}(t)$ versus $t^{0.65}$ under (a) S_1, (b) S_2 and (c) S_3

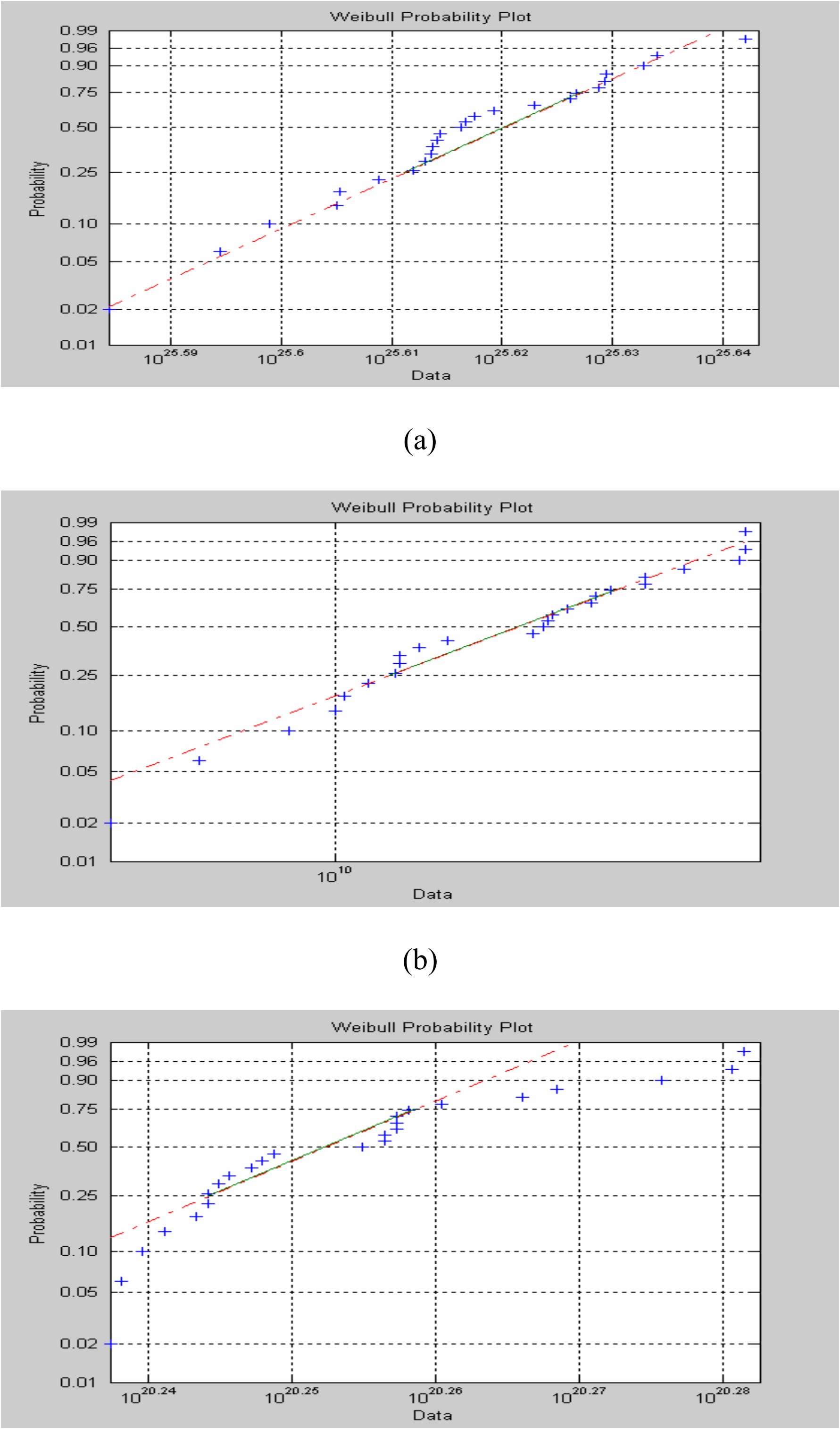
Based on the observations $\{(t_{i,k}, \omega_{ij}(t_{i,k}))\}_{k=1}^{30}$, the LSEs $\hat{\beta}_{ij}$ and $\hat{\sigma}^2_{\in ij}$ can be computed. To make sure of the appropriateness of the Weibull-distribution, Weibull probability plots were constructed for each higher stress level (Figure 5.3). All of the trends appear linear about the reference line. Figure 5.4 shows the normal probability plots for the residuals under S_1, S_2, and S_3. The plots indicate that the distribution assumptions for β and $\in(t)$ are reasonable.

From Equations 4.21, 4.22, 4.210 and 4.211, we have $\hat{\sigma}^2_{\in} = 2.12683 \times 10^{-16}$, $\hat{b} = 0.0268$, and the Arrhenius relationship is given by:

$$\hat{u}_i = \hat{\gamma}_0 + \frac{\hat{\gamma}_1}{S_i + 273.16} \tag{5.2}$$

Where $\hat{\gamma}_0 = 0.9977$ and $\hat{\gamma}_1 = -62.3124$.

To obtain the optimal test plan for the ADT of LED, we need the actual values of $\sigma^2_{\in}$, b, and (γ_0, γ_1). For convenience, these estimates are treated as the true values to evaluate the optimal test plan of LED data.

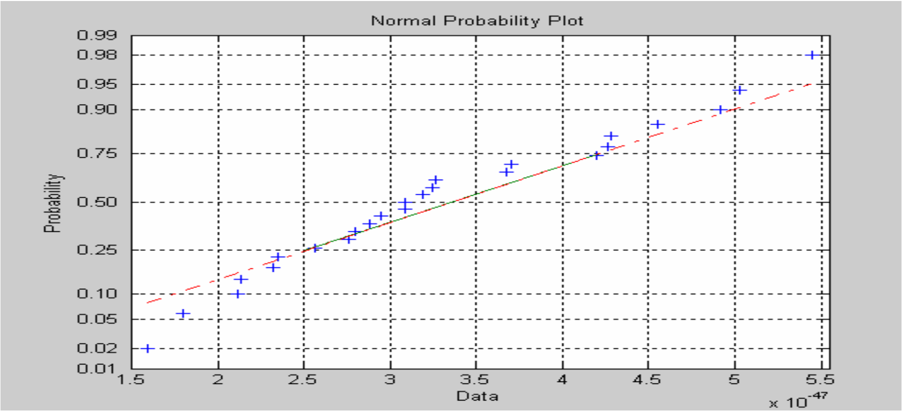


(a)

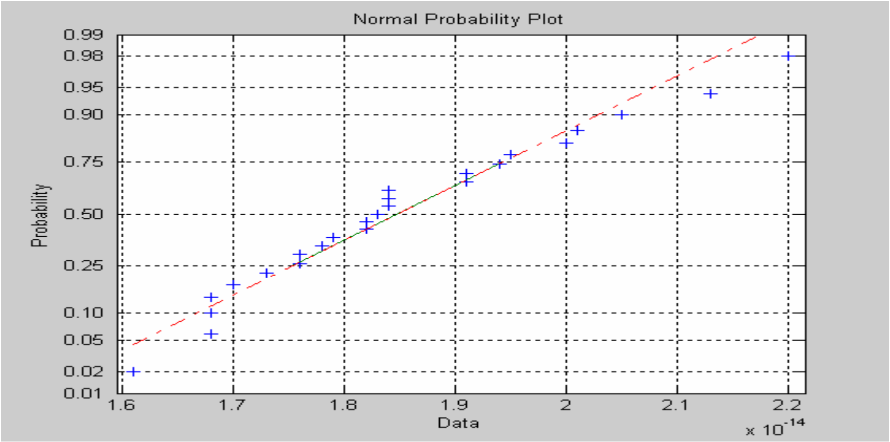
(b)

(c)

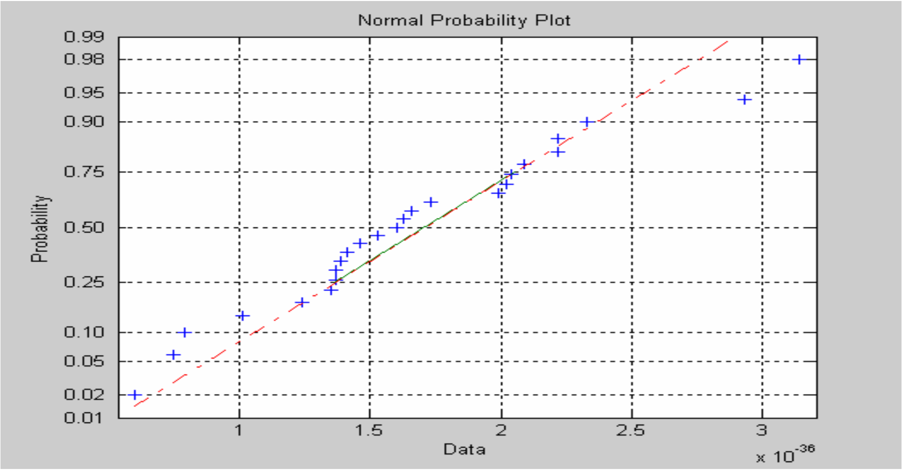
Figure 5.3 The Weibull Probability Plot for $\{\hat{\beta}_{ij}\}_{j=1}^{25}, 1 \le i \le 3$



(a)



(b)



(c)

Figure 5.4 The Normal Probability Plots for Residuals under (a) S_1, (b) S_2 (c) S_3

5.2 Optimal Test Plan Based on the LED Data

We now solve for the optimal parameters of the objective function at $t_u = 24$ hours and $l = 100$. In solving the objective function we compute the following:

- Sample Size
- Inspection Frequency
- Termination Time and
- CIs of the parameters involved in the MTTF's expression

The optimal combination of the CIs of the parameters involved in the MTTF's expression is found such that the expected width of a 100(1-p) % CI of the MTTF is minimized. The CI width of the MTTF is $p\left[\Delta_L \le E\left(\overline{\phi_0} - \underline{\phi_0}\right) \le \Delta_U\right] = 100(1 - p_1^*)(1 - p_2^*)$ .Where Δ_L and Δ_U are the lower and upper limits of $E\left(\overline{\phi_0} - \underline{\phi_0}\right)$ respectively. The optimal test plans for the p = 0.01, 0.05, 0.10, 0.20, 0.30, 0.40 and 0.50 under different cost conditions (C_s, C_p, C_m, C_d, C_b) are listed in Table 5.4. Here p is the percentile of the lifetime distribution of the product at the normal use condition.

Table 5.4: Optimal Degradation Test Plans under Various Cost Conditions (C_s, C_p, C_m, C_d, C_b)

p	(C_s, C_p, C_m, C_d, C_b)	$E\left[\overline{\phi}_0 - \underline{\phi_0}\right]$	$\left(p_1^*, p_2^*, f_1^*, f_2^*, f_3^*, l_1^*, l_2^*, l_3^*, n^*\right)$
0.01	(16.5,4.05,0.65,35,10000)	697298.6	(0.0087,0.00142,2,4,7,62,29,14,38)
0.05	(16.5,4.05,0.65,35,10000)	685465.5	(0.0430,0.00731,1,5,8,100,42,10,44)
0.10	(16.5,4.05,0.65,35,10000)	575512.6	(0.086,0.0153,2,4,7,62,30,14,38)

(Table 5.4 continued)

0.20	(16.5,4.05,0.65,35,10000)	547801.8	(0.086,0.0153,2,4,7,62,29,15,38)
0.30	(16.5,4.05,0.65,35,10000)	503478.3	(0.087,0.0213,2,4,9,65,30,15,39)
0.40	(16.5,4.05,0.65,35,10000)	499734.1	(0.091,0.0309,3,4,9,62,29,14,39)
0.50	(16.5,4.05,0.65,35,10000)	448923.4	(0.085,0.0415,2,4,8,60,28,14,38)

For example, with p=0.1 and (C_s, C_p, C_m, C_d, C_b) = (16.5, 4.05, 0.65,35, 10000), the optimal combination of the percentile values (p_1, p_2) is (p_1^*, p_2^*)= (0.086, 0.0153); the optimal inspection intervals equal f_1^* *24 = 2*24= 48 hours, f_2^* *24 = 4*24= 96 hours, and f_3^* *24= 7*24= 168 hours under S_1, S_2 and S_3 respectively. The optimal sample size is n^* = 38; and the optimal termination times are $t^*_{1,l_1^*} = 2*62*24 = 2976$, t^*_{2,l_2^*} = 4*30*24 = 2880, t^*_{3,l_3^*} = 7*14*24 = 2352 hours under S_1, S_2 and S_3 respectively. The CI width of the MTTF in this case is: $p\left[\Delta_L \le E\left(\overline{\phi_0} - \underline{\phi_0}\right) \le \Delta_U\right]$= 100(1-0.0087) (1-0.00142) = 98.98 %.

5.2.1 Optimal Parameters Based on the ADT Experiment

In the table 5.4, as the percentile (p) value changes from 0.1 to 0.5, the expected width changes from 697298.6 to 448923.4. Correspondingly, the percentage change in the width is 35 % whereas the percentage change in the percentile value p_1 is 9 %. Thus, there is more significant reduction in the variation (CI width) than in the precision (p). Based on these results the optimal parameters for the design of this ADT experiment are:

- The percentile value p = 0.5
- The optimal value of the expected width of the MTTF = 448923.4

- The optimal combination of the percentile values for the expected width $\left(p_1^*, p_2^*\right)$ = (0.085, 0.0415)
- The optimal inspection intervals $\left(f_1^*, f_2^*, f_3^*\right)$ = (2, 4, 8)
- The optimal termination times $\left(t_{1,l_1^*}^*, t_{2,l_2^*}^*, t_{3,l_3^*}^*\right)$ = (60, 28, 14)
- The optimal sample size n* = 38
- The optimal CI width of the MTTF: $p\left[\Delta_L \leq E\left(\overline{\phi_0} - \underline{\phi_0}\right) \leq \Delta_U\right]$= 100(1-0.085) (1-0.0415) = 87.70 %.

5.3 Sensitivity Analysis

In practical situation, some parameters of the LED problem (e.g., u, b, $\sigma_\in^2$, the cost parameters, etc) in the previous section are not well known. Thus, it is important to investigate the effects of these parameters on the optimal test plan. Considering the following cases for the above LED data, the effects of these parameters can be found.

5.3.1 Test Plans under a Variety of C$_b$

The effect of C$_b$ on the test plan can be assessed by computing $\left(\left\{f_i^*, l_i^*\right\}_{i=1}^3, n^*\right)$ for various values of C$_b$ with p = 0.1, and the cost condition (*C$_s$, C$_p$, C$_m$, C$_d$,*) = (16.5, 4.05, 0.65, 35). The results are given in Table 5.5. It can be seen that all n* and $\left\{f_i^*\right\}_{i=1}^3$ and $\left\{l_i^*\right\}_{i=1}^3$ are sensitive to the moderate change of C$_b$.

Table 5.5: The Optimal Test Plans for Some Values of C$_b$

p	C$_b$	$E\left[\overline{\phi}_0 - \underline{\phi_0}\right]$	$\left(f_1^*, f_2^*, f_3^*, l_1^*, l_2^*, l_3^*, n^*\right)$
0.1	7000	55134.47	(1,2,2,92,44,30,21)

(Table 5.5 Continued)

	7500	52388.07	(1,2,2,92,44,30,23)
	8000	48945.37	(1,2,3,92,44,24,26)
	8500	46104.25	(2,4,4,62,29,20,29)
	9000	43707.46	(2,4,6,62,29,16,32)
	9500	41649.93	(2,4,7,62,29,14,35)
	10500	38785.14	(2,3,7,62,35,14,40)
	11000	37326.47	(2,3,7,62,35,14,43)
	11500	35615.43	(3,7,9,49,21,12,47)
	12000	34474.94	(3,6,9,49,23,12,50)

5.3.2 Test Plans for Different Values of m and a Variety of Combinations of $\{S_i\}_{i=1}^{m}$

The number of stress levels and the choice of $\{S_i\}_{i=1}^{m}$ in an ADT would affect the optimal test plan and the estimation precision. To assess the effects of the number of the stress levels and the choice of $\{S_i\}_{i=1}^{m}$ on the test plan, we assume that the scale parameter u satisfies Eq (5.2). Table 5.6 gives the optimal solutions for a variety of values of m and various combinations of $\{S_i\}_{i=1}^{m}$ with p = 0.1 and ℓ=100 under the cost condition (C_s, C_p, C_m, C_d, C_b) = (16.5, 4.05, 0.65, 35, 10000).

The results indicate that

- The estimation precision is better when the stress levels are father away from each other and from the use condition S_0

- There is only a moderate change in the inspection frequency and termination time when the number of stress levels varies, but there is a drastic change in the sample size.

Table 5.6: The Optimal Test Plans for Various Values of m and Combinations of Stress Levels

m	S_1	S_2	S_3	S_4	$E\left[\overline{\theta_0} - \underline{\theta_0}\right]$	f_1^*,	f_2^*,	f_3^*,	f_4^*	l_1^*,	l_2^*,	l_3^*,	,l_4^*	n^*
2	25	65	--	--	667891.74	2	4	--	--	62	14	--	--	92
	30	105	--	--	889389.42	2	10	--	--	59	19	--	--	89
	35	105	--	--	982861.30	4	12	--	--	33	9	--	--	54
	40	65	--	--	872079.37	1	3	--	--	32	5	--	--	44
	55	80	--	--	938174.21	2	8	--	--	46	9	--	--	32
	60	80	--	--	1248977.80	2	8	--	--	27	14	--	--	34
3	25	65	105	--	685629.87	1	4	8	--	82	12	9	--	72
	30	65	100	--	896120.78	1	5	8	--	86	12	12	--	69
	35	65	95	--	907847.52	3	9	12	--	86	12	19	--	72
	40	65	90	--	918733.29	2	5	8	--	75	10	22	--	67
	55	80	105	--	948237.57	2	6	7	--	69	14	23	--	58
	60	80	100	--	1187768.31	3	9	14	--	89	14	24	--	62
4	25	45	65	105	727863.67	2	4	7	7	89	56	45	14	48
	30	50	75	95	896389.20	5	9	10	11	78	45	38	7	56
	35	50	75	95	896782.11	3	7	9	9	67	36	27	15	44
	40	60	80	100	947820.29	3	9	13	18	64	57	29	9	56
	50	65	80	95	1092948.72	3	8	9	9	61	44	32	11	29

5.3.3 Sensitivity Analysis of Mis -specifying u, b, and $\sigma_{\in}^{2}$

Let $\varepsilon_1, \varepsilon_2$, and ε_3 denote the predicted errors for u, b, and $\sigma_{\in}^{2}$, respectively. Table 5.4 shows the optimal solution $\left(\{f_i^*, l_i^*\}_{i=1}^{3}, n^*\right)$ for various combinations of $\left((1+\varepsilon_1)u, (1+\varepsilon_2)b, (1+\varepsilon_3)\sigma_{\varepsilon}^{2}\right)$ under the cost condition (C_s, C_p, C_m, C_d, C_b) = (16.5, 4.05, 0.65, 35, 10000). Here the values of $\{\varepsilon_i\}_{i=1}^{3}$ are changed over the ranges $\pm 2.5\%, \pm 5\%, and\ \pm 10\%$. The optimal test plans are listed in Table 5.7, 5.8 & 5.9 respectively.

Table 5.7: The Optimal Solution $\left\{\left(f_i^*, l_i^*\right)_{i=1}^{3}, n^*\right\}$ for the Case that $\{\varepsilon_i\}_{i=1}^{3}$ are Changed over the Ranges $\pm 2.5\%$

ε_1	ε_2	ε_3	$(f_1^*,$	$f_2^*,$	$f_3^*,$	$l_1^*,$	$l_2^*,$	$l_3^*,$	$n)$
-2.5%	-2.5%	-2.5%	1	3	5	38	19	12	44
-2.5%	0	0	1	3	6	39	24	14	43
-2.5%	+2.5%	+2.5%	2	3	6	40	24	14	42
0	-2.5%	0	2	4	7	39	19	14	42
0	0	+2.5%	1	3	8	38	18	9	42
0	+2.5%	-2.5%	1	3	8	38	19	10	42
+2.5%	-2.5%	+2.5%	2	4	7	40	18	9	43
+2.5%	0	-2.5%	2	4	7	39	19	9	40
+2.5%	+2.5%	0	3	5	7	38	19	10	40

Table 5.8: The Optimal Solution $\left\{\left(f_i^*, l_i^*\right)_{i=1}^3, n^*\right\}$ for the Case that $\{\varepsilon_i\}_{i=1}^3$ are Changed over the Ranges $\pm 5\%$

ε_1	ε_2	ε_3	$(f_1^*,$	$f_2^*,$	$f_3^*,$	$l_1^*,$	$l_2^*,$	$l_3^*,$	$n)$
-5%	-5%	-5%	4	6	8	68	32	14	45
-5%	0	0	4	6	8	67	30	14	44
-5%	+5%	+5%	3	6	9	62	29	14	39
0	-5%	0	3	6	8	62	30	14	39
0	0	+5%	3	6	9	62	30	11	38
0	+5%	-5%	4	7	9	59	31	14	39
+5%	-5%	+5%	3	6	8	60	32	11	44
+5%	0	-5%	4	6	8	59	34	12	40
+5%	+5%	0	2	7	9	58	32	16	41

Table 5.9: The Optimal Solution $\left\{\left(f_i^*, l_i^*\right)_{i=1}^3, n^*\right\}$ for the Case that $\{\varepsilon_i\}_{i=1}^3$ are Changed over the Ranges $\pm 10\%$

ε_1	ε_2	ε_3	$(f_1^*,$	$f_2^*,$	$f_3^*,$	$l_1^*,$	$l_2^*,$	$l_3^*,$	$n)$
-10%	-10%	-10%	4	7	9	45	32	18	38
-10%	0	0	5	7	9	64	32	16	39
-10%	+10%	+10%	5	7	9	63	32	16	38
0	-10%	0	5	6	9	48	32	19	39
0	0	+10%	4	6	8	63	32	18	40

(Table 5.9 Continued)

0	+10%	-10%	4	5	7	62	33	18	39
+10%	-10%	+10%	4	7	8	63	32	17	39
+10%	0	-10%	4	6	9	63	32	17	38
+10%	+10%	0	4	6	9	62	31	16	39

From the results above, it is clear that the test plan is quite robust for a moderate deviation from the assumed values of b and σ_{ε}^{2} .On the other hand, if the true value of u has a moderate change from the assumed value, then the test plan will also be changed moderately.

CHAPTER 6

CONCLUSIONS AND FUTURE RESEARCH

In this thesis, we developed an optimal plan for the design of accelerated degradation test (ADT) using reciprocal Weibull degradation data based on the minimization of the mean time to failure (MTTF) criterion.

A nonlinear integer programming problem was developed under the constraint that the total experimental cost does not exceed a pre-determined budget. The optimal combination of sample size, inspection frequency and the termination time at each stress level was found.

An LED example was used to illustrate the proposed method by a simulation experiment to estimate the reliability of LEDs at normal operating condition with temperature S_0 = 278 K (5^0 C), by using the degradation data obtained at three accelerated stress levels, S_1 = 298 K (25^0 C), S_2 = 338 K(65^0 C), S_3 = 378 K (105^0 C). By solving the optimization model developed in chapter 4, we determined the optimal values of the sample size, inspection frequency and the termination time. In addition the expected width of a 100(1-p) % CI of the MTTF was also minimized. From the results the optimal parameters for the design of ADT are:

- The percentile value p = 0.5
- The optimal value of the expected width of the MTTF = 448923.4
- The optimal combination of the percentile values for the expected width $\left(p_1^*, p_2^*\right)$ = (0.085, 0.0415)

- The optimal inspection intervals $\left(f_1^*, f_2^*, f_3^*\right)$ = (2, 4, 8)
- The optimal termination times $\left(t^*_{1,l_1^*}, t^*_{2,l_2^*}, t^*_{3,l_3^*}\right)$ = (60, 28, 14)
- The optimal sample size n* = 38
- The CI width of the MTTF $p\left[\Delta_L \le E\left(\overline{\phi_0} - \underline{\phi_0}\right) \le \Delta_U\right] =$ 100(1-0.085) (1-0.0415) = 87.70 %.

Sensitivity analysis was performed by varying the cost and by varying the test plan with different combinations of stress levels to find the effect of different parameters on the optimal plan. The results from the sensitivity analysis indicate that the parameters sample size, inspection frequency and the termination time are sensitive to the moderate change in the cost. The estimation precision was better when the stress levels are father away from each other and from the use condition S_0. There was only a moderate change in the inspection frequency and termination time when the number of stress levels varies, but there was a drastic change in the sample size.

6.1 Future Research Directions

- In this research, we developed the optimal test plan for an ADT under the assumption that the sample sizes for each stress level are equal. However this may not be practical because unequal allocation is much more common. Therefore research into the problem of unequal allocation is an important area that should be explored.
- Although ADT is an efficient life test method, it may not be applicable in some cases. For a newly developed product, it is very difficult to have many units to be put for testing at each stress level. In that case a Step-Stress Accelerated

Degradation Test (SSADT), where a sample of tested devices is subjected to successively higher levels of stress can be used.

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