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Fuzzy C-Means Clustering Approach to Design a Warehouse Layout

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by

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ABSTRACT

Allocation of products in a warehouse is done by various storage policies. These are broadly classified into three main categories: dedicated storage, randomized storage, and class-based storage. In dedicated storage policy a product is assigned a designated slot while in random storage policy incoming product is randomly assigned a storage location close to the input/output point. Finally, the class-based storage is a mixed policy where products are randomly assigned within their fixed class. Dedicated storage policy is most commonly used in practice. While designing large warehouse layout, the product information in terms of throughput and storage level is either uncertain or is not available to the warehouse designer. Hence it is not possible to locate products on the basis of the throughput to storage ratio method used in the above mentioned storage location policies. To take care of this uncertainty in product data we propose a fuzzy C-means clustering (FCM) approach.

This research is mainly directed to improve the efficiency (distance or time traveled) by designing a fuzzy logic based warehouse with large number of products. The proposed approach looks for similarity in the product data to form clusters. The obtained clusters can be directly utilized to develop the warehouse layout. Further, it is investigated if the FCM approach can take into account other factors such as product size, similarity and/or characteristics to generate layouts which are not only efficient in terms of reducing distance traveled to store/retrieve products but are effective in terms of retrieval time, space utilization and/or better material control.

CHAPTER 1
INTRODUCTION

Warehousing is a complex of facilities and activities in support of the manufacturing enterprise with impact throughout the corporation. Within the supply chain, warehousing is an important activity in the distribution of materials from raw materials, and work in process, to the finished goods. Hence, warehouse system design has acquired lot of importance in the supply chain cycle. The goal of warehouse design is to minimize the existing cost of establishing and operating a warehouse. The key goal of a Warehouse system is to maintain and store stock of parts ready for distribution, so that at all times the demand for items is met. Another important goal of a warehouse is to assemble product batches prior to delivery, to stockpile critical parts, and to facilitate regional distribution network for quick and cost efficient delivery. The following sections are an attempt to explain the key feature and importance of a warehouse facility.

1.1 Functions of Warehouse

A warehouse environment may serve any of the following requirements. Figure 1.1 gives a brief idea of roles of a warehouse in distribution network.

1. Buffer- it holds inventory that is used to balance and buffer the variation between production schedules and demand. For this application, the warehouse is located close to the manufacturing facility. A warehouse that serves these demands is replenished on monthly to quarterly basis.
2. Consolidation – A warehouse may be used to accumulate and consolidate products from various areas of manufacturing within a single firm or from many firms. It facilitates combined shipment to common customers. This type of warehouse may be located central to the production location or the customer base. This type of facility responds to regular weekly or monthly orders.

3. Rapid Response- Rapid response in a warehouse is an important aspect to shorten transportation distances to permit easy access to customer's demand.

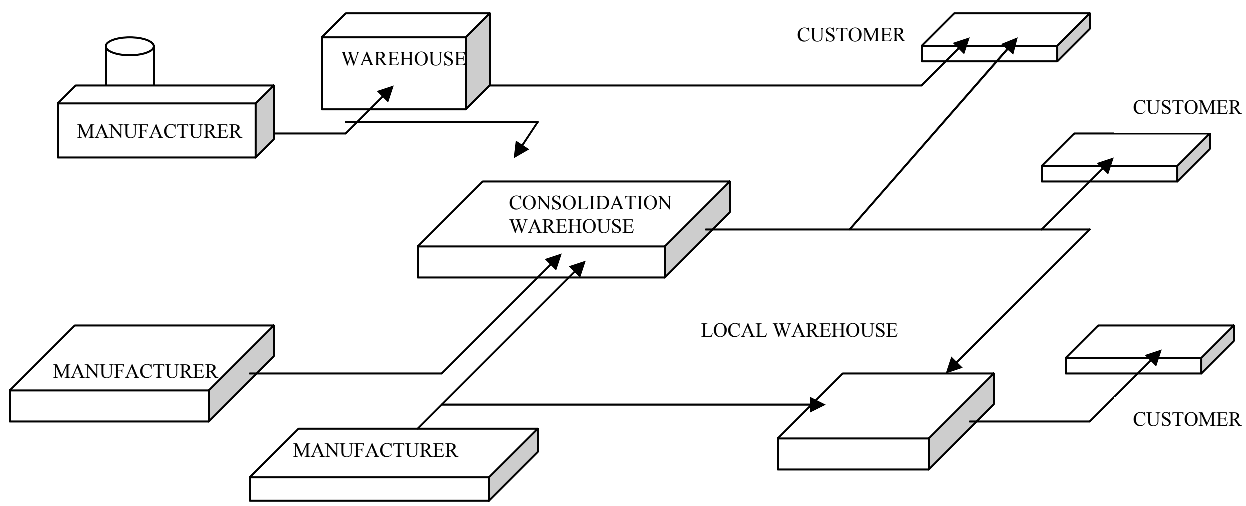

Figure 1.1 Warehouse Roles in the Distribution Network

1.2 Activities in Warehouse

As a part of product storage there are many activities that occur in the process of getting material into and out of the warehouse. Discussed are some of the important activities involved in a warehouse. Refer figure 1.2 for a block diagram of activities in a warehouse.

1. Receiving- begins with advance notification of arrival of the goods to the warehouse. Conceptually, it is a collection of activities that involve the orderly receipt of all materials in to the warehouse. This activity provides the assurance that the quantity and quality are according to the order, and helps to disburse material to storage or other organizational functions needing them. Products arrive in large pallet loads and so labor requirement are not high. Hence receiving accounts for a low operating cost in a warehouse.

2. Prepackaging- in a warehouse when products are received in bulk from a supplier then packaging is performed which is packaging the products subsequently single

packages or in combination of other products to form kits or assortments. When packaging greatly increases the storage cube size requirements or when a part is common to several assortments, either then the entire receipt of merchandise is processed at once, or a portion is kept in bulk form to be processed later.

3. Put Away- the process or act of placing merchandise in storage that includes transportation and placement is commonly known as put-away. Before product can be put away an appropriate storage location must be determined. The importance of this task is that it determines how quickly and at what cost the item is later retrieved for the customer. When a product is put away, the storage location should also be scanned to record where the product has been stored. Put-away generally requires a large amount of labor as the products are required to be moved a large distance to their respective locations.

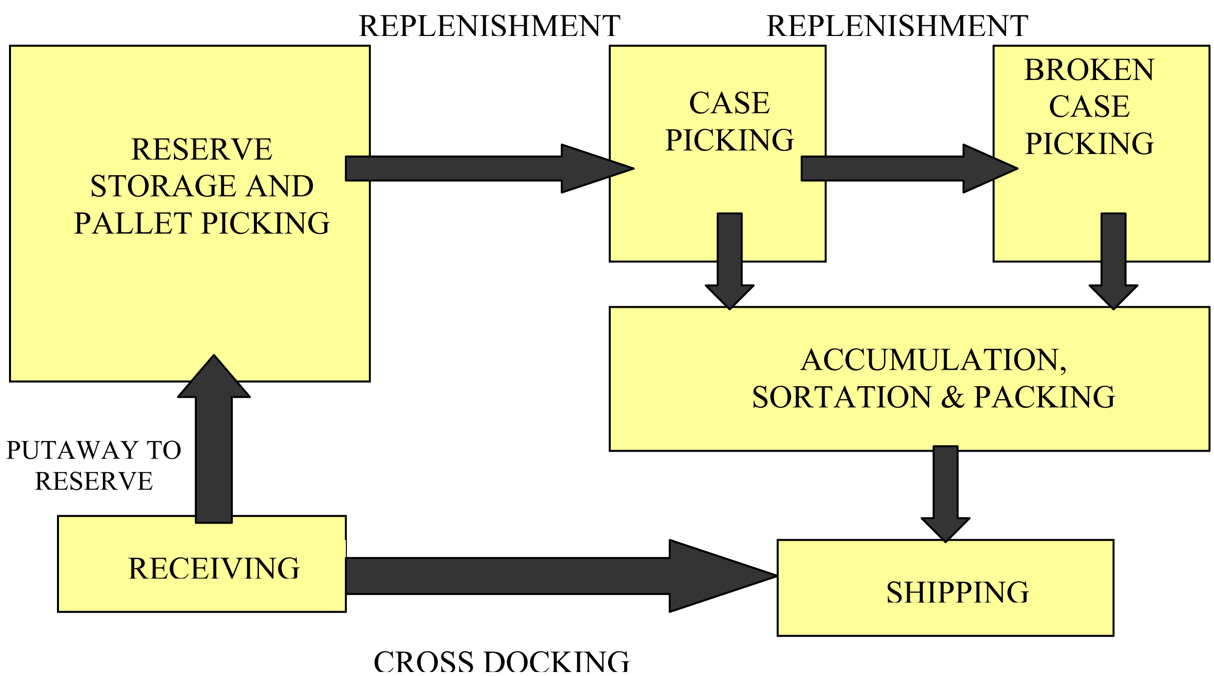

Figure 1.2 Activities in Warehouse

4. Storage-While merchandise is waiting for demand; the physical containment of that merchandise is called as Storage. Various forms of storage depend on the size and quantity of the items in inventory and the handling characteristics.
5. Order Picking- Order Picking is the service that the warehouse provides for the customers. It is the process or act of removing merchandise items from storage to meet a specific demand. It is one of the most important activities as it is the function around which most of the warehouses are designed.
6. Sortation-When an order has more than one item and the accumulation is not done as the picks are made; then the Sortation of batch picks into individual orders and accumulation of distributed picks into orders must be done.
7. Packing and Shipping- It is a combination of various activities following order picking and package. Some key aspects are as mentioned below.
    A. Packaging of items in appropriate shipping containers.
    B. Preparation of shipping documents
    C. Checking orders for completeness and weighing to calculate charges.
    D. Accumulating orders by outbound carriers.
    E. Loading trucks may or may not be a part of it as in many cases this is carrier's responsibility.
8. Cross Docking- cross-docks are high-speed warehouses. If an arriving item has been requested by a customer there is no need to store it as anticipated inventory, instead the items can move directly from receiving to shipping, without intermediate storage and retrieval. Thus, the item can move much more quickly through the facility and the costly part of warehouse labor can be avoided.
9. Replenishment primary locations from the reserve storage location.

1.3 Types of Warehouse

1. Factory Warehouse- a factory warehouse interfaces production with wholesalers. Such warehouses have following important features.
    A. A comparatively small number of orders are picked up on daily basis.

B. For a factory warehouse advance information about the order composition is required.
C. Focus on cost and order accuracy is also high.
D. The responsiveness depends heavily on production schedules.

2. Retail Distribution warehouse- it serves a number of captive retail units. Following are the main features of a retail distribution warehouse.
    A. Advance info about order composition is needed.
    B. Carton and item picking is done from a forward area.
    C. More orders per shift than consolidation/shipping lanes
    D. It focuses on cost, accuracy and fill rate of the packages.
    E. Responsiveness depends heavily on truck routing schedules
    F. The only critical point is that if the retail units are not captive, then responsiveness becomes a crucial issue.
3. Catalog Retailer warehouse- this type of warehouse deals with filling orders from catalog sales. Main features are as follows.
    A. A large number of small; frequently single-line orders are picked up.
    B. Item and, sometimes, carton picking
    C. Daily compositions of orders are usually unknown.
    D. Only statistical information available.
    E. Like factory warehouse and retail warehouse, the emphasis is on cost and response time.
4. Support of manufacturing operations warehouse- this type of warehouse serves the purpose of a stock room providing raw material and work-in-process items to manufacturing operations. The main features of this type of warehouse are as mentioned.
    A. Contains many small orders.
    B. Only statistical information available about order composition.
    C. Stringent time requirements for response time.
    D. Focus on response time but also accuracy and cost.

1.4 Material Handling

Material Handling is defined as "... providing the right amount of the right material, in the right condition, at the right place, at the right time, in the right position, in the right sequence, and for the right cost, by using the right methods." (Tompkins, 1996) In a warehouse, it is the material handling system that makes possible the materials flow specified in the layout of the facility. Material handling system tasks facilitate distribution of material to the plant cell; implementation of planned flow paths in the layout and controls the flow of parts within and between the departments. The three major activities in material handling that include rest other sub activities are receiving, order picking, and shipping. Material handling has several key functions; the important amongst them is the setting up of directed flow paths among carriers and buffering between the staging area and storage area. In addition, material handling facilitates the continuous flow without excessive congestion or idleness in the warehouse. Further, it helps in maintaining safety and good housekeeping in the warehouse.

1.4.1 Material Handling Equipment

Material Handling Equipments are broadly classified into two categories namely order picking equipment and receiving/shipping equipment. There is a variety of equipment to reduce labor cost and to increase space utilization. This equipment's are discussed in details below.

1.4.2 Order Picking Equipment

As with the picking methods, the picking equipment used will also depend on a variety of factors. Below mentioned is a consolidated list of material handling equipment's and its application in various picking environment.

1. Static shelving**-** the most common equipment for storage in piece pick operations, static shelving is designed with depths from 12" to 24". Product is placed either directly on the shelving or in corrugated or plastic parts bins. Static shelving is economical and is the best method where there are few picks per *SKU* or where parts are very small.

2. Carton Flow Rack- Carton flow rack is similar to static shelving with the exception that rather than shelves there are small sections of gravity conveyor mounted at a slight angle. Product is stocked from the rear of the flow rack and picking is done from the face. Product can be stocked in cartons or small totes or bins, as a carton or tote is emptied, it is removed from the rack, and another one will roll into place. Carton flow rack is most useful where there are a very high number of picks per *SKU*.
3. Carousels**-** *Horizontal Carousels* are a version of the same equipment used by dry cleaners to store and retrieve clothing; they have racks hanging from them that can be configured to accommodate various size storage bins. Generally, an operator will run 2 to 4 carousels at a time avoiding the need for the operator to wait while one unit is turning. Picking is usually performed in batches with orders downloaded from the host system to the carousel software. Horizontal carousels are most common in picking operations with very high number of orders, low to moderate picks per order, and low to moderate picks per *SKU*. Horizontal carousels provide very high pick rates as well as high storage density. Pick-to-light systems are often integrated into carousels. *Vertical Carousels* are frequently used in laboratories and specialty manufacturing operations and are rarely used in regular order picking operations.
4. Automatic Storage and Retrieval Systems (AS/RS) **-** An AS/RS is a system of rows of rack, each row having a dedicated retrieval unit that moves vertically and horizontally along the rack picking and putting away loads. ASRS systems are available in Mini-load types that store and transfer product on some type of tray or in bins and Unit-load types that transfer and store pallet loads. In addition to the automation features, AS/RS units can provide extremely high storage density with capabilities to work in racking up to 100 feet high. The high costs of AS/RS equipment and the length of the retrieval times make it difficult to incorporate into a piece picking operation.
5. Automatic Picking Machines- Fully automated picking machines (such as A-frames) are rare and are used only where very high volumes of similar products

are picked, or where high volume in combination with high accuracy requirements exists.

6. Pick to light- Pick to light systems consists of lights and LED displays for each pick location. The system uses software to light the next pick and display the quantity to pick. Pick-to-light systems have the advantage of not only increasing accuracy, but also increasing productivity. Since hardware is required for each pick location, pick-to-light systems are costly and are suitable where very high picks per *SKU* occur. Carton flow rack and horizontal carousels are good applications for pick to light. In batch picking, pick-to-light is also incorporated into the cart or rack that holds the cartons or totes that you are picking in to. The light will designate which order you should be placing the picked items in.
7. Voice Directed Picking- Voice technology has come of age in recent years and is now a very viable solution for piece picks, case picks, or pallet picks operations.
8. Automated Conveyor and Sorting Systems- Automated Conveyor systems and sorting systems are an integral part of any large-scale piece picks operation. The variety of equipment and system designs is enormous.

1.4.3 Receiving/Shipping Equipment

Also known as the material transport equipment they differ from the other category of material handling equipment by their primary function namely material transport. Some of the widely used materials handling equipment's are listed below.

1. Conveyors- conveyors are used when material is to be moved more often between locations. They are mostly used for a fixed path traverse. Hence there must be a sufficient volume of product movement to implement conveyor type handling. Conveyors are mainly classified based on the product type and the location of the conveyor. Some common types are chute, belt, roller, wheel, and chain and trolley type conveyors.
2. Industrial Vehicles- is the simplest mode of material transport in a warehouse. The main advantage that they provide in a warehouse is maneuvering and transportation. Industrial vehicles are broadly classified as hand trucks, pallet jacks, and powered industrial trucks.

3. Automated Storage and Retrieval Machines-this type of storage system uses a fixed path storage and retrieval machines running on rails between storage racks. These systems handle loads in excess of 1000 pounds.
4. Automated Guided Vehicles- as the name suggests these are driverless industrial trucks and they follow a predefined path in an aisle. The path followed is a simple loop or a complex network with many designated load/unload stations.

1.5 Receiving /Shipping Dock Design

The most valuable area in a warehouse is the receiving/shipping dock area. Every item in the warehouse comes some time or the other through the dock. Everything that leaves the warehouse goes out across the dock. Hence, no activity in a warehouse is complete without the dock area. Unfortunately, loading docks generally receive less thought and foresight in the layout and design efforts. The key factors while designing dock area is the selection of right number of docks, location of the docks, and the type of dock to be used. Other important factors considered are productivity of the docks, safety features, and dimensions of the receiving/shipping areas.

1.5.1 Number of Docks

The number of docks required is determined by a combination of factors namely; number of receipts and shipments, type of loading and unloading, types and sizes of vehicles, number and timing of carriers, and different areas in which materials will be utilized, stored or prepared for shipment. Based upon the various characteristics one dock position should be allowed for each seven hours of planned activity per shift. The greater the number of operating shifts for shipping/receiving lower the total number of doors that are required.

1.5.2 Location of Docks

Traditionally docks were located in the rear of a facility and out of sight. Generally receiving and shipping docks were all located in the same area, in order to reduce the need for duplicate supervision. In some larger facilities, shipping would be at one end of the building and receiving at the other, in order to create a straight through material movement. However, today, given the move to reduced inventories and the tendency for shipments to be in close proximity to the manufacturing location, more facilities are being constructed with multiple shipping and receiving docks. These multiple docks drastically reduce the flow of materials within a facility.

1.5.3 Types of Docks

Various types of docks are used as per the application for which they are designed. Saw tooth docks are useful when a site does not have sufficient exterior area to maneuver vehicles in and out of the docks. They optimize the amount of distance from the edge of the building to the end of the property line or the end of the paved area. This reduces the number of docks and interior space that can be used around the docks because of the saw-toothed pattern. Straight docks, on the other hand, optimize interior space. Open docks are impractical in most environments, and even where they can be used, they need to be evaluated as to their benefit versus potential theft and malicious damage. Interior docks provide protection from the external conditions and protect products from potential loss. Interior docks, however, come at a considerable cost of lost space and increased energy consumption.

1.5.4 Dock Productivity

Warehouse productivity, which includes dock productivity, is a very complicated issue because of the differences of activities, types of receiving and shipping units, types of material handled, sizes of individual received/shipped items, and types of loading and unloading equipment's. Even in comparing two warehouses in the same company, productivity will differ due to the differences in volumes and types of activity in each facility. The only accurate productivity benchmarks are those developed specifically for that operation based upon the applicable activities, type, volume, and equipment used. Thus, dock space forms an important aspect of warehouse design and must be carefully designed for getting the maximum cost benefit and throughputs.

1.6 Summary

This chapter discussed the basics of warehouse management principals, types of material handling equipment used for various activities in the warehouse, factors related to dock design. The rest of the thesis is organized as follows. In chapter 2 we discuss the research in the field of warehouse design and research in product allocation policies. We also discuss the storage location models for warehouse design. Chapter 3 deals with the concept of fuzzy data sets and data clustering algorithms. Fuzzy C-means (FCM) clustering approach forms the basis of solving the warehouse layout problem dealt in this thesis. Chapter 4 explains an existing dedicated storage model in detail with an example problem. We also explain the steps to follow for solving the warehouse design problem by FCM approach. Chapter 5 reviews the results and analysis of the warehouse layout problem solved by FCM. In chapter 6 we conclude and summarize the results obtained to achieve the goal of FCM approach to design a Fuzzy logic based warehouse.

CHAPTER 2
LITERATURE REVIEW

In this literature review, we discuss the various approaches to warehouse design and the framework for classification of warehouse design problems. The problems encountered during the design of warehouse and a systematic approach for warehouse design is stressed upon. We explain the storage location policies in brief. Later the review focuses upon the research done in the field of assignment of products with three assignment policies namely; dedicated, randomized and class based storage policy. The papers reviewed helped in understanding the current approaches to warehouse design.

2.1 Warehouse Design: Problems and Methodologies

This section discusses factors that must be kept in mind while designing a warehouse. Rounwenhorst, *et al* (2000) discuss a reference framework and classification of warehouse design and control problems. The authors emphasize a need for design-oriented studies as opposed to strong analysis oriented research on isolated sub problems that are dominant in current papers. Bartholdi and Hackman (1998) analyze problems that are encountered during the design of a warehouse and its subsystem. A design-oriented approach primarily aims at a synthesis of a large number of both technical systems and planning and control procedures. The authors develop a methodology for systematic warehouse design. The paper broadly discusses the concept of three different axes along which warehouses may be viewed upon namely process, resources, and organization. Further, they discuss performance criteria and process of warehouse design on a strategic, tactical, and operational level. The problem in each area and suggestion for improvement in the problem areas is made.

2.2 Warehouse Design Research Classification

Warehouse design problems can be posed in a number of ways. In this literature review, two major categories of design problems are studied. The first category addresses the overall design problem and concentrates on the formulation of top-down; iterative, optimization-based approaches (Ashayeri and Gelders, 1985 and Gray, *et al*., 1992). The overall design problem is a complex problem and has many aspects to it. These models provide a basic conceptual framework for the design problem. Even in the case when a proposed design procedure is applied to a case study as in Gray *et al*. (1992), it is not always clear how results can be validated beyond the case study structure.

The second category addresses specific design problems like design of a storage system or an order-picking system. The papers that discuss this issue are Bozer and White (1996); Goetschalckx (1992); Jarvis and McDowell (1991); Rosenblatt, *et al*. (1993) and Yoon and Sharp (1996). The models discussed in these papers are useful but it is difficult to integrate models for different problems into an overall design procedure due to different assumptions or data representations. In reality expert practitioners rarely use the results of the extensive research done in the warehouse design area. Rather, they rely on their experience and expertise.

2.3 Information Systems in Warehouse Design

Information is the key in designing warehouses with large number of product data. In the warehouse design field, this information, experience and knowledge may be applied by the use of a well-established design procedure such as Systematic Layout Planning, Muther (1973). In any case, over time an expert develops methods for decision-making, as well as specific information requirements that are integral to design decisions. Information is the key to the design decision-making process (Hazelrigg, 1996). In the warehouse design domain, today's computerized information systems provide the designer with large historical datasets that can be used in the design process. The authors seek to formalize the decision-making process and sequence, the information used, the criteria applied, and the evaluation methods utilized. Green (1992) outlines a number of relevant attributes possessed by experts namely supplying context, ordering decisions, abstracting parameters, and classifying heuristics.

2.4 Type of Storage Policy

Type of Storage policy decides how to allocate the various storage locations of a uniform storage medium to a number of *SKU*. These are broadly classified in to three main categories mentioned as below.

2.4.1 Dedicated Storage

Also referred to as fixed slot storage, involves the assignment of specific storage locations for each product stored. As storage location is assigned or dedicated to a specific product, the term "dedicated storage" is used. Every *SKU* is assigned a particular number of storage locations, exclusively allocated to it. The number of storage locations allocated reflects its maximum storage needs and is determined through inventory activity profiling. Two variations in dedicated storage policy are commonly used. Part number sequence storage is frequently used due to its simplicity. In this type, the storage location of a product is based entirely on the part number assigned to it. Low part numbers are assigned to the best location in the warehouse. Hence, a part with a large part number gets a poor storage location. This type of storage policy does not take in to account the activity level of the parts. Throughput based dedicated storage is an alternative to part number sequence storage. Such a storage policy gives a thought to activity levels and storage requirements among products to be stored. Throughput-based storage is preferred to the part number sequencing storage when there are significant differences in either the activity level or the inventory level for products being stored. With dedicated storage, the number of storage locations assigned to product must be capable of satisfying the maximum storage requirement for the product. With multi-product storage, the storage space required is the sum of the maximum storage requirements for each of the product.

2.4.2 Randomized Storage

Also referred to as floating slot storage, allows the storage location for a particular product to change over a period. When a product arrives for storage it is placed in the closest location available and retrieval occurs on a first in first out basis. For warehouses with more than one I/O points, the storage location selected is the one nearest to the I/O point. With randomized storage, products can be stored in any available storage location. Hence, the storage space requirement will be equal to the maximum of the aggregate storage requirements for the products. Due to the dynamic conditions in the replenishment of products, it is very difficult to forecast the exact storage requirements for this storage policy. Hence, storage capacity levels are decided by treating inventory levels of the products as random variables. The randomized storage model is explained in brief as below.

For given *n* storage spaces required we have to determine the storage space layout that minimizes the total expected travel distance between each storage space and *m* I/O points. The sum of the distances of storage space j from each I/O point is given by the equation $\sum_{k=1}^{m} d_{kj}$. Arrange the spaces in ascending order of the sum of these distances, and pick the *n* closest storage spaces. Here *n* depends on the inventory levels of all the items, so the total number of spaces *n* is less than that required under the dedicated policy. The basic assumptions for the randomized storage model are that every empty slot is equally likely to be selected for storage and each unit of a particular product is equally likely to be retrieved when multiple storage locations exist, Heragu (1997) and Francis (1992).

2.4.3 Class Based Storage

This storage location policy is midway between dedicated storage policy and the randomized storage location policy. The class based storage policy is based on Pareto's law with respect to storage and retrieval (S/R) activity level generated by different items. "In a warehouse 80% of the S/R activity is directed at 20% of the items, 15% at 30% of the items and the remaining 5% of the S/R activity at 50% of the items," Heragu (1997). Incoming items are thus classified into three classes as A, B, and C, based on the level of

S/R activity (from high to low) they generate. Thus to minimize the time/distance spent in storage and retrieval, Class A items must be stored closest to the input/output point, Class B next closest and Class C the farthest.

2.5 Product Allocation Using Different Policies

In this section we will discuss the research done in the field of dedicated and class based storage allocation policies, that forms the basis of the existing approach in this thesis.
In dedicated storage policy products are assigned to storage /retrieval locations in an attempt to minimize the time/distance required to perform the storage and retrieval operations. For dedicated storage to be feasible there should be sufficient number of storage slots to be dedicated to the products. The basic aim is to minimize the distance traveled to store and retrieve the assigned products. Tompkins (1996) suggest the T/S approach where T is the throughput of products to be placed and S is the storage level. However for large warehouses, the input data available is fuzzy and so T/S approach cannot be used. Also this approach does not consider the product characteristics that are important to a warehouse designer.

Malmborg and Bhaskaran (1990) evaluate the Cube per Order Index storage policy for different kinds of warehouses, based on analytical expressions for the maximum throughput. Park and Webster (1989) derive analytical expressions for the maximum throughput of multiple three-dimensional storage systems with the cubic-in-time storage policy. Kaylan and Medeiros (1988) evaluate storage policies for a miniload system with multiple I/O points and suggest storage algorithms for the Deep Lane Storage System that minimizes the number of relocations. Further, Hausman et al. (1976) analyze class-based storage in an AS/RS, assuming single commands. They develop analytical methods to determine the optimal dimensions of the zones, considering storage capacity and maximum throughput.

Goetschalckx and Ratlif (1990) evaluate storage policies for block storage through an analytical study. Jarvis and McDowell (1991) propose a heuristic for the storage policy in a conventional warehouse. Roll and Rosenblatt (1983) analyze the storage capacity of conventional warehouses with alternative storage policies, using simulation techniques. Wilhelm and Shaw (1996) present an empirical study concerning the storage policy of an AS/RS. This research was helpful in understanding the selection of appropriate storage policy and the various approaches to implement them.

2.6 Summary

In this chapter we reviewed the varied research done in the field of warehouse layout design. We also learnt the different storage location models and how they help design a warehouse layout. The dedicated storage approach uses crisp information of the input data namely throughput and storage level, that in most cases is not available to the warehouse designer for large number of parts that are stored even in a small sized warehouse. We would like to incorporate the fuzzy logic approach to design a warehouse layout where the input data is large and fuzzy in nature. To our knowledge, no research effort has been reported in the literature for designing warehouse layouts based on fuzzy logic approach.

In chapter 3 we would discuss the fuzzy C-means clustering approach (FCM) and how it helps to solve the fuzzy nature of the input variables. An attempt will be made in this thesis to use a fuzzy logic based method which will hopefully give good results in comparison with existing T/S method.

CHATER 3
CLUSTERING ALGORITHMS

This chapter introduces the basic definitions and concepts of fuzzy data sets and Fuzzy c-means clustering technique that will be needed in the further chapters. Gradually the chapter shifts to Fuzzy c-means clustering, which forms the basis for solving the warehouse layout problem.

3.1 Classical Sets

A classical set is a set that has a crisp boundary. For example, a classical set X of real numbers greater than 6 is expressed as

$$A = \{x \mid x > 6\}$$

In this set of real numbers there is a clear unambiguous boundary 6 such that if x is greater than this number. In this case x either belongs to this set *'A'* or it does not belong to this set. These types of sets are called Classical Sets and the elements in this set are a part of the set or they are not a part of the set. Classical sets are an important tool in mathematics and computer science but they do not reflect the nature of human concepts and thought.

In contrast to a classical set, a fuzzy set is a set without crisp boundaries. That is, the process of an element "belongs to a set" to "does not belong to a set" is gradual. This transition is decided by the membership function of a fuzzy dataset. Real life problems have data which most of the time has a degree of "trueness" or "falseness" that is the data cannot be expressed in terms of classical set. A good example of this is; the same set A is a set of tall basketball players. According to the classical set logic a player 6.01 ft tall is considered to be tall whereas a player 5.99 ft tall is considered to be short.

3.2 Fuzzy Sets and Membership Function

Membership functions give fuzzy sets the flexibility in modeling commonly used linguistic terms such as "the water is hot" or "the temperature is high." Zadeh (1965) points out that, this imprecise data set information plays an important role in human approach to problem solving. It is important to note that fuzziness in a dataset comes does not come from the randomness of the elements of the set, but from the uncertain and imprecise nature of the abstract thoughts and concepts.

If X is a collection of objects denoted by x, then a fuzzy set 'A' in 'X' is defined as a set of ordered pairs

$$\boldsymbol{A = \{(x, \mu A\,(x)) \mid x \in X\},}$$

Where $\mu A\,(x)$ is called the membership function (MF) for the fuzzy set A. The membership function maps each element of X to a membership grade between 0 and 1. If the value of the membership function is restricted to either 0 or 1, then A is reduced to a classical set and $\mu A\,(x)$ is the characteristic function of A. Usually X is referred to as the universe of discourse and may consist of discrete objects or continuous space.

3.3 Data Clustering Algorithms

Clustering of numerical data forms the basis of various classification and system modeling algorithms. The purpose of clustering is to identify natural groupings of data from a large data set to produce a concise representation of a system's behavior. Clustering algorithms are not only used to organize and categorize data, but are helpful in data compression and model construction. Clustering partitions the data set into several groups such that the similarity within a group is larger than among the groups. To achieve such partitions it is essential to have a similarity metrics that takes two input vectors and returns a value reflecting their similarity. As most of the similarity metrics are sensitive to the range of elements in the input vectors, each of the input variables must be normalized or scaled down. Clustering techniques are broadly classified as hard clustering and fuzzy clustering.

3.3.1 K means Clustering Algorithm

The K-means clustering, also known as C-means clustering, has been applied to variety of areas, including image and speech data compression. This technique is based on randomly choosing *k* initial cluster centers, or means. These initial cluster centers are updated in such a way that after a number of cycles they represent the clusters in the data as much as possible. A drawback of the *k*-means algorithm is that the number of clusters is fixed; once *k* is chosen it always remains *k* cluster centers. The K-means algorithm circumvents the problem by removing the redundant clusters. Whenever a cluster centre is not assigned enough samples, it may be removed. In this way one is left with a more or less optimal number of clusters. The problem of choosing the initial number of clusters still remains unsolved, but by taking *k* large enough this will usually not be a problem.

1. The algorithm starts out with initializing C_i this is achieved by randomly selecting *C* points from among all the data points.
2. Determine the membership matrix U, where the element u_{ij} is 1 if the j^{th} data point x_j belongs to the group *I* and 0 otherwise.
3. Compute the cost function by the equation given below. Stop if the value of cost function is below a certain threshold value.

$$J = \sum_{i=1}^{c} Ji = \sum_{i=1}^{c} \Big(\sum_{k, Xk \in Gi} \| X_k - Ci \|^2 \Big)$$

4. Update the clusters center centers *Ci* and determine the new *U* matrix.

The K-means algorithm is mainly iterative, and hence hard to predict its convergence to optimum solution. The performance of the K-means algorithm depends on the initial position of the cluster centers. Hence, initial clusters centers are predicted by a front-end tool, which generates cluster centers iteratively.

3.3.2 Hierarchical Clustering Algorithm

In hierarchical clustering the data is not partitioned into a particular cluster in a single step. Instead, a series of partitions takes place that run from a single cluster containing all objects to *N* clusters each containing a single object. Hierarchical clustering is further classified as agglomerative method, which proceed by series of fusions of the *N* objects into groups, and divisive method, which separate *N* objects successively into finer groupings. Hierarchical clustering may be represented by a two-dimensional diagram known as dendrogram, which illustrates the fusion or divisions made at each successive stage of analysis. Given a set of N items to be clustered, and an NxN distance matrix, the basic process of Johnson's (1967) hierarchical clustering is briefly explained below.

1. The algorithm starts by assigning each item to its own cluster, such that for N items, we have N clusters, each containing just one item. Let the distances between the clusters equal the distances between the items they contain.
2. Find the closest (most similar) pair of clusters and merge them into a single cluster, so that we have one less cluster.
3. Compute distances between the new cluster and each of the old clusters.
4. Repeat steps 2 and 3 until all items are clustered into a single cluster of size N.

Step 3 can be done in different ways, which is what distinguishes single-link from complete-link and average-link clustering. In single-link, clustering (also called the connectedness or minimum method); we consider the distance between one cluster and another cluster to be equal to the shortest distance from any member of one cluster to any member of the other cluster. If the data consist of similarities, we consider the similarity between one cluster and another cluster to be equal to the largest similarity from any member of one cluster to any member of the other cluster. In complete-link, clustering (also called the diameter or maximum method); we consider the distance between one cluster and another cluster to be equal to the longest distance from any member of one cluster to any member of the other cluster. In average-link clustering, we consider the distance between one cluster and another cluster to be equal to the average distance from any member of one cluster to any member of the other cluster.

3.3.3 Fuzzy C-means Clustering Algorithm

Fuzzy C-means clustering (FCM) algorithm, also known as fuzzy Isodata, is a data clustering algorithm in which each data point belongs to a cluster to a degree specified by a membership grade. Bezdek proposed this algorithm in 1973 as an improvement to K-

means algorithm also known as the hard C-means algorithm. Hard k-means algorithm executes a sharp classification, in which each object is either assigned to a class or not. The application of fuzzy clustering to the dataset function allows the class membership to have several classes at the same time but with different degrees of membership function ranging from 0 to 1. Fuzzy c-means (FCM) is a method of clustering which allows one piece of data to belong to two or more clusters.
It is based on minimization of the following objective function

$$J_m = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^{m} \left\| x_i - c_j \right\|^2$$

Where *m* the fuzzy factor is any real number greater than 1, *j* is the number of cluster decided by the user, u_{ij} is the degree of membership of x_i in the cluster *j*, x_i is the *i*th of d-dimensional measured data namely throughput, storage level and volume, c_j is the d-dimension center of the cluster, and $\left\| x_i - c_j \right\|^2$ is any norm expressing the similarity between the measured data (throughput, storage level and volume) and the center. Fuzzy partitioning is carried out through an iterative optimization of the objective function shown above, with the update of membership matrix u_{ij} and the cluster centers c_j by,

$$u_{ij} = \frac{1}{\sum_{k=1}^{C} \left(\frac{\left\| x_i - c_j \right\|^2}{\left\| x_i - c_k \right\|^2} \right)^{2/m-1}} \quad \text{and,} \quad c_j = \frac{\sum_{i=1}^{N} u_{ij}^{m} . x_i}{\sum_{i=1}^{N} u_{ij}^{m}}$$

This iteration will stop when $\max u_{ij}, \left\{ \left| u_{ij}^{(k+1)} - u_{ij}^{(k)} \right| \right\} \leq \varepsilon$, where ε is a termination criterion between 0 and 1 and usually set to 0.02 (Zimmermann, (1990) whereas *k* is the iteration steps. This procedure converges to a local minimum or a saddle point of J_m. The algorithm is composed of the following steps mentioned below.

1. Initialize $U=[u_{ij}]$ matrix, $U^{(0)}$
2. At k-step, calculate the centers vectors $C^{(k)}=[c_j]$ with $U^{(k)}$

$$c_j = \frac{\sum_{i=1}^{N} u_{ij}^m . x_i}{\sum_{i=1}^{N} u_{ij}^m}$$

3. Update $U^{(k)}$, $U^{(k+1)}$

$$u_{ij} = \frac{1}{\sum_{k=1}^{C} \left(\frac{\left\| x_i - c_j \right\|^2}{\left\| x_i - c_k \right\|^2} \right)^{2/m-1}}$$

4. If $|| U^{(k+1)} - U^{(k)} || < \varepsilon$ then STOP, otherwise return to step 2.

As indicated earlier, the value of ε lies between 0 and 1 and 0.02 is the commonly used value. The number of iterations for the algorithm to reach a local minimum will be decided by the value of ε.

3.3.4 Fuzzy Factor

The fuzzy factor 'm' was introduced by Bezdek (1974) and is also known as 'fuzzifier' As the value of m approaches 1 the clusters formed tend to be hard and as the value of m tends to infinity the obtained clusters tend to go in a the fuzziest state. There is no theoretical justification on the value of 'm' but is usually set to 2 and in a more generalized form tends to be between 1.5 and 3 (Zimmermann, 1990).

3.3.5 Ideal Number of Clusters 'c'

From the research on decision of ideal number of clusters for the FCM algorithm, we find out that there is nothing called as an ideal number of clusters (Zimmermann, 1990). The number of clusters for a certain type of data will vary based on the data partition desired. The number of clusters can vary between 2 to infinity. In Chapter 5 we will discuss the effect of varying the number of clusters on the total expected distance traveled.

3.3.6 Significance of Membership Function in Cluster Analysis

As discussed in the earlier section, data are bound to each cluster by means of a membership function, which represents the fuzzy behavior of this algorithm. To do that, we build an appropriate matrix named U whose factors are numbers between 0 and 1, and represent the degree of membership between data and centers of clusters. In the FCM approach, instead, the same given datum does not belong exclusively to a well-defined cluster, but it can be placed in a middle way. In the case of FCM, the membership function follows a smoother line to indicate that every datum may belong to several clusters with different values of the membership coefficient.

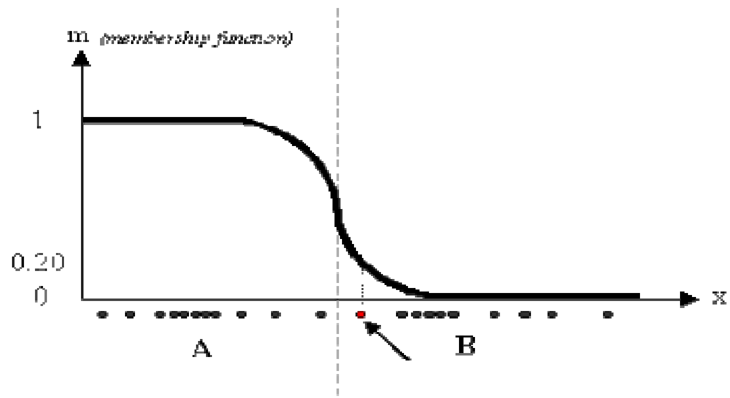

Figure 3.1 Membership Function for FCM Algorithm

In figure 3.1 (George and Yuan, 1995), the datum shown as a red marked spot belongs more to the cluster B rather than the cluster A. The value 0.2 of membership function indicates the degree of membership to A for such datum. Now, instead of using a graphical representation, we introduce a matrix U_{NxC} whose factors are the ones taken from the membership functions. The number of rows and columns depends on how many data and clusters we are considering. Here C (columns) is the total number of clusters and N (rows) is the total data points.

$$U_{NxC} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \\ 0.6 & 0.4 \\ . & \\ 0.9 & 0.1 \end{bmatrix}$$

3.4 Fuzzy C-means Clustering Application in Facilities Design

Cell formation, one of the most important problems faced in designing cellular manufacturing systems, is to group parts with similar geometry, function, material and process into part families and the corresponding machines into machine cells. There has been an extensive amount of work in this area and, consequently, numerous analytical approaches have been developed. One common weakness of these conventional approaches is that they assume that disjoint part families exist in the data; therefore, a part can only belong to one part family. In reality, it is clear that some parts belong to more than one part family. Chu (1991) and Unde (2003) propose a fuzzy c-means clustering algorithm to formulate the problem. This approach offers a special advantage over conventional clustering. It not only shows the specific part family that a part belongs to, but also provides the degree of membership of a part associated with each part family. This information allows users flexibility in determining to which part family a part should be assigned so that the workload balance among machine cells can be taken into consideration. The author develops computer program to simplify the implementation and to study the impact of the model's parameters on the clustering results.

3.5 Summary

The concept of hard and fuzzy clustering algorithms was introduced in this chapter. The chapter also dealt with basic information on fuzzy data sets. Further the fuzzy c-means algorithm was explained in detail. This FCM algorithm will form the basis for solving the warehouse layout problem with fuzzy product data of throughput and storage requirement.

CHAPTER 4
PROBLEM DEFINITION

In this chapter, we discuss the existing T/S approach to dedicated storage location problem in a warehouse and the assumptions made to solve this problem. We also discuss the fuzzy c-means approach to solve the warehouse storage location problem. Further we introduce the concept of fuzzy data also known as linguistic variables. A step by step approach to designing a fuzzy based warehouse layout is also given. We solve a small warehouse problem with T/S method and FCM method and compare the results for the total expected distance traveled in the warehouse.

4.1 Design Model for Dedicated Storage Policy by (T/S) Approach

Francis (1992) suggests a generalized model for dedicated storage policy. A warehouse has *m* I/O points through which *n* items enter and leave the warehouse. The items are stored in one of *s* storage spaces or locations. Each location requires the same storage space, and it is known that item *j* requires S_j storage spaces. T_j is the throughput requirement level for product *j* in number of storage/retrieval per unit time and $p_{i,j}$ is the percent of storage/retrieval trips for product *j* from I/O point *i*. The distance traveled between I/O point *I* and storage/retrieval locations *k* is given by $d_{j,k}$. Hence we can express *f(x)* as the expected distance traveled between storage location *k* and the I/O point *i* required to fulfill the throughput requirement for the warehouse facility.

$$\min f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s}\frac{T_j}{S_j}\left[p_{i,j}d_{i,k}x_{j,k}\right]$$

Subject to $\sum_{j=1}^{n} x_{j,k} = 1$, for k = 1, 2… s and,

$$\sum_{i=1}^{s} x_{ij} = S_j \text{ , for } j = 1, 2\ldots n$$

$x_{j,k}$ = (0, 1) for all j and k

$$\min f(x) = \sum_{j=1}^{n} \frac{T_j}{S_j} \sum_{k=1}^{s} x_{j,k} \sum_{i=1}^{m} \left(p_{i,j} d_{i,k} \right)$$

Based on this formulation, Tompkins (1996) gives a T/S method to minimize the total expected distance traveled approach mentioned as below.

1. Rank the products in the descending order of their T_j / S_j
2. Compute the distance traveled $f(x)$ for all the slots in the warehouse.
3. Assign the products with the highest T/S ratio to the slot with the least $f(x)$ and so on.

4.2 Motivation for Research

The main motivation for this research was to assist in the development of efficient warehouse layout in the absence of precise information about the throughput and storage levels for large number of products found in a modern warehouse. Furthermore, it was of value to investigate if in addition to throughput and storage, other product attributes such as product similarity, characteristics or volume could be taken into account while developing the layout which will give good result for the expected distance criterion and at the same time generate a layout which will reduce storage/retrieval time, improve space utilization or yield better material control.

4.3 Proposed Fuzzy c-Means Model

The proposed approach involves the application of the FCM algorithm to solve the layout design problem for dedicated storage location problem and the class based storage location problem. As discussed in Chapter 2 class based storage operates as a dedicated storage for the formation of the classes and randomized storage within the formed class. The approach as formulated earlier in this chapter works fine when the input information is crisp. But when the throughput and storage requirement information is fuzzy, i.e. in the form of “High” and “low”, the T/S method does not work. Hence, for a warehouse with fuzzy input information of product data, fuzzy c-means algorithm generates clusters of similar data. These clusters can be used as groups or classes for a storage policy. The fuzzy c- means algorithm was discussed in details in Chapter 3. Thus, the obtained cluster information helps in designing the warehouse layout. The generated layout should result in values comparable to T/S method for the total distance/time traveled. The validity of this method lies in the fact that, clustering tries to identify the relationships among patterns in a data set by organizing the patterns into a number of clusters, where the patterns in each cluster show a certain degree of closeness or similarity.

4.4 Use of Linguistic Variables for Uncertain Data

The set of data that are defined on the set of 'R' real numbers are known as fuzzy sets. Membership functions of these numbers have a quantitative meaning and are viewed as fuzzy numbers or fuzzy intervals. These fuzzy numbers are numbers that are close to a real number. The concept of fuzzy numbers helps in characterizing many applications like states of fuzzy control, decision-making, approximate reasoning, optimization, and statistics with imprecise probabilities, (Klir and Yuan, 1995). When the fuzzy numbers represent linguistic concepts like very small, small, medium, and high and so on. These variables are set as per the user's discretion and are known as 'Linguistic Variables'. Each linguistic variable is defined in terms of a base variable the values of which are real numbers within a specific range. A base variable is a variable in the classical sense, like in our case throughput, storage requirement and Volume. This concept of linguistic variables will be used to solve the warehouse layout problem.

4.5 Step by Step Methodology for the Warehouse Layout

The steps to be followed for running the FCM algorithm for calculation of the total expected distance traveled in the warehouse are as given below.

1. *Input.* The user enters the number of clusters, throughput levels in terms of number of input/output trips per unit of time, storage and volume requirement. The input data for storage, throughput and volume is fuzzy and in levels such as very low, low, medium low, medium, medium high, high and very high. The user, based on the product data size, decides the levels of the fuzzy variables. Each level of fuzzy data for storage and throughput has a fixed range and divided in to equal intervals. The user also decides the number of clusters based on the size of

product data. The number of iterations is achieved by running the FCM algorithm till it achieves the condition $|| U^{(k+1)} - U^{(k)} || < \varepsilon$ (refer section 3.3.3).

2. *Distance Calculations for Storage Slots*. Based on the dimensions of the warehouse and the probability of throughput for each port the rectilinear distance traveled in the warehouse for each slot is calculated (refer section 4.5).
3. *Conversion of Linguistic Categories to Numeric Values.* The program for linguistic to numerical data converter converts the linguistic data to numerical data. The data is randomly generated with a fixed range for each linguistic variable of throughput (T) and storage (S). This numerical data is the input to the FCM algorithm. Several replications for generating random data is done to see the effect of change in data on the total expected distance
4. *Normalization of Storage Requirements.* The values of randomly generated data of storage levels are normalized to equal the total number of available storage bays.
5. *Cluster Generation.* Clusters are generated by the FCM algorithm. After obtaining the clusters they are ranked in the descending order based on the ratio of cluster center distance.
6. *Cluster Ranking.* The cluster with highest rank gets the closest slots to the I/O port and within the cluster the product with highest T/S ratio is placed first and so on in. (For generating layout with 3 features, *e.g.* throughput, storage and volume the cluster with highest weight is identified based on the cluster center information for each cluster. Within the cluster the products are ranked in the descending order of T/S ratio.)
7. *Total Distance Calculation.* Based on the obtained layout, the total expected distance traveled for storage/retrieval in the warehouse is calculated (Refer section 4.5).

4.6 Example 1: A Small Warehouse

This problem has been taken from Francis (1992). The problem deals with a small warehouse with only four different products. The warehouse has separate I/O ports for receiving and shipping items with variable amount of activity from these ports. There is small variation in terms of activity levels of different products. The storage requirements of these products, however, vary greatly.

Problem Data

1. Warehouse dimensions are 20ft x 20ft.
2. Total number of slots is 50.
3. Receiving ports are port numbers 4 and 5.
4. Shipping ports are 1, 2, and 3 with the middle port more likely to be used.
5. Probability of activity level from each port p_1= 0.15, p_2= 0.20, p_3= 0.15, p_4= 0.25 and p_5= 0.25
6. Number of products is 4 namely A, B, C and D.
7. Throughput information for the 4 products is 60, 70, 80 and 90 trips per day.
8. Storage requirement for the 4 products is 20, 10, 15 and 5 bays.

Assumptions

1. Assume rectilinear travel at constant speed within the warehouse and is assumed to originate at the centroid of the bay.
2. Full units are assumed to be Received/Shipped and the number of loads received equals the number of loads shipped.

Sample Distance Calculations for Storage Slots to I/O Points

Rectilinear distance traveled in the warehouse for slot number 50 is calculated below

$$f(50) = p_1(f_1) + p_2(f_2) + p_3(f_3) + p_4(f_4) + p_5(f_5)$$

Where,

p_i is the probability of products entering through port *i*.

f_i is the distance of a slot from port *i*.

The total distance of slot 50 is given as,

$$f(50) = 0.15(7x20) + 0.20(5x20) + 0.15(3x20) + 0.25(2x20) + 0.25(3x20)$$

$$f(50) = 75 \text{ ft.}$$

Figure 4.1 shows the distance calculations for all the storage slots in a warehouse with 5 I/O ports. From the figure we observe that the distance for slots closer to the I/O ports is less than that for slots away from the I/O ports for example the distance for slot 40 is 75 ft and that for slot 1 is 205 ft The distance calculations are done from each I/O port to the centroid of the storage slot.

4.6.1 Example 1 Solved by T/S Method

T/S Ratio for the 4 products A, B, C and D is 3, 7, 5.3 and 18. Arranging the products in descending order of T/S the new sequence is D, B, C and A. The product with highest T/S gets the closest slot available. In this manner all the products are placed in the warehouse. Figure 4.2 shows a layout for a dedicated storage warehouse. From the figure we see that product D, which has the highest T/S ratio is allocated the closest slot in the warehouse. Similar allocation is followed for all the products. The products can be shifted to the next best slot in order to obtain a rectangular or 'L' shaped pattern for similar products. However, this will affect the total expected distance (TED) traveled in the warehouse.

1 205	2 185	3 165	4 148	5 134	6 124	7 118	8 115	9 115	10 115
11 185	12 165	13 145	14 128	15 114	16 104	17 98	18 95	19 95	20 95
21 170	22 150	23 130	24 113	25 99	26 89	27 83	28 80	29 80	30 80
31 165	32 145	33 125	34 108	35 94	36 84	37 78	38 75	39 75	40 75
41 165	42 145	43 125	44 108	45 94	46 84	47 78	48 75	49 75	50 75

IO 4, p4=0.25

IO 5, p5=0.25

IO 1, p1=0.15 IO 2, p2=0.2 IO 3, p3=0.15

Figure 4.1 Rectilinear Distance Traveled: Example 1

3 A	3 A	3 A	3 A	3 A	3 A	3 A	2 C	2 C	2 C
3 A	3 A	3 A	3 A	2 C	2 C	2 C	2 C	2 C	2 C
3 A	3 A	3 A	2 C	2 C	2 B	2 B	2 B	2 B	2 B
3 A	3 A	3 A	2 C	2 C	2 B	2 B	2 B	1 D	1 D
3 A	3 A	3 A	2 C	2 C	2 B	2 B	1 D	1 D	1 D

IO 4, p4=0.25

IO 5, p5=0.25

IO 1, p1=0.15 IO 2, p2=0.2 IO 3, p3=0.15

Note: The numbers in top right corner denote the cluster number and the numbers in the left bottom corner denote the product assigned to that slot.

Figure 4.2 Layout by T/S Method

Total Expected distance traveled in the Warehouse per day

$$f(x) = 5(75)\frac{90}{5}$$

$$+(75+78+78+80+80+80+83+84+84+89)\frac{70}{10}$$

$$+(94+94+95+95+95+98+99+104+108+108+113+114+115+115+115)\frac{80}{15}$$

$$+\begin{pmatrix}118+124+125+125+128+130+134+145+145+145+148\\+150+165+165+165+165+170+185+185+205\end{pmatrix}\frac{60}{20}$$

$$= 29{,}824\ ft/day$$

4.6.2 Example 1 Solved by FCM Method Using Crisp Data

In order to check the results for the FCM method that has been developed in this thesis, the example problem would be run with the crisp values given earlier. The distance comparison would then be indicative of the performance of the FCM method. The cluster output for crisp product data for example 1 is given in table 4.1. This cluster information is used to design the warehouse layout. Total number of clusters for this problem is set to 3. The steps given in section 4.4 are followed except for steps 4 and 5 which deal with converting linguistic values to numerical data. For this small problem both methods result in same layout (as shown in figure 4.2) and hence the distance traveled will be the same. Table 4.2 shows the expected distance traveled for each of the 3 clusters.

Table 4.1 Cluster Output for Crisp Data – Cluster 1, 2 and 3

Cluster 1

Product	Throughput	Storage
D	90	5

Cluster 2

Product	Throughput	Storage
B	70	10
C	80	15

Cluster 3

Product	Throughput	Storage
A	60	20

Table 4.2 Total Expected Distance by FCM Method for Crisp Data: Example 1

Cluster Number	Expected Distance Traveled in ft/day
1	6,750
2	14,008
3	9,066
Total	29,824

4.6.3 Example 1 Solved by FCM Method

We use this approach to design a fuzzy based warehouse, where the input information is in the form of fuzzy data. The data for the throughput and storage were converted as high, medium and low as given in table 4.3. To generate numerical values for the converted data, a range for each linguistic variable is set. Within this range a randomly generated numerical value is our input to the FCM algorithm.

Table 4.3 Fuzzy Data: Example 1

Product	Throughput	Storage
A	M	H
B	H	M
C	H	H
D	H	L

The cluster output for fuzzy product data for example 1 is given in table 4.4. This cluster information is used to design a warehouse layout with fuzzy data. Total number of clusters for this problem is 3. The steps 1 through 7 to be followed for layout generation are given in section 4.4. Total Expected Distance traveled per day is 27,308 ft/day shown in table 4.5. Layout for this problem is shown in figure 4.3. Comparing this layout with the one developed by T/S method (figure 4.2), we see that due to changes in the number of storage slots calculated by FCM method for linguistic categories, the resulting layout is somewhat different. However, the relative location of the products in both the layouts is same.

Table 4.4 Cluster Output for Fuzzy Data – Cluster 1, 2 and 3

Cluster 1

Product	Throughput	Storage
D	87	6

Cluster 2

Product	Throughput	Storage
B	77	13
C	63	11

Cluster 3

Product	Throughput	Storage
A	49	20

Table 4.5 Comparison of Results: Example 1

Cluster Number	Expected Distance Traveled in ft/day
1	6,525
2	13,379
3	7,404
Total	27,308

3 A	3 A	3 A	3 A	3 A	3 A	3 A	2 C	2 C	2 C
3 A	3 A	3 A	3 A	2 C	2 C	2 C	2 B	2 B	2 C
3 A	3 A	3 A	2 C	2 C	2 B	2 B	2 B	2 B	2 B
3 A	3 A	3 A	2 C	2 B	2 B	2 B	1 D	1 D	1 D
3 A	3 A	3 A	2 C	2 B	2 B	2 B	1 D	1 D	1 D

IO 4, p4=0.25

IO 5, p5=0.25

IO 1, p1=0.15 IO 2, p2=0.2 IO 3, p3=0.15

Figure 4.3 Layout by FCM Method Fuzzy Data

4.7 Summary

In this chapter we discussed the concept of T/S method to design a warehouse layout for a small problem. We explained the FCM approach to the warehouse layout problem. Also the concept of linguistic variables, which forms the basis for converting the crisp data to fuzzy data was explained. We solved a small warehouse problem by T/S and FCM method and compared the results for total expected distance traveled . The results obtained were same by both the methods for this small problem.

In our attempt to designing an efficient practical method for warehouse layout, in Chapter 5 we will follow the FCM method for two larger problems one with 20 products and 250 storage locations and other with 50 products and 700 locations by both the methods. We will try to explore if FCM method can include a third feature e.g. product size (volume) to increase space utilization and/or have better material control. The addition of the third feature can not be incorporated in T/S method.

CHAPTER 5
RESULTS AND ANALYSIS

In this chapter, we will discuss the results obtained by running the FCM algorithm namely cluster output, expected distance traveled in the warehouse and effect of number of clusters on the expected distance traveled with the help of two additional problems. We will follow the same presentation format for these two problems, namely, first we will solve the problem using T/S method, followed by using FCM method with the same numerical (crisp) data. This will be done to compare the efficiency of the FCM method in terms of the expected distance traveled. Finally, we will use the FCM method on a linguistic data used for the throughput and storage levels and comment on the quality of the generated layout. As mentioned earlier we will explore if FCM method can include information such as product size, similarity or characteristics to be able to increase space utilization, lower storage/retrieval time and/or have better material control. This fact can not be incorporated in T/S method. We will further do a sensitivity analysis for the effect of number of clusters on the total expected distance traveled.

5.1 Example Problem 2: A Medium Warehouse

This problem deals with a medium warehouse with 20 different products consisting of 250 storage slots. The warehouse has 4 separate I/O ports for receiving and shipping items with variable amount of activity from these ports. There is large variation in terms of activity levels, storage requirements in this problem.

Problem Data

1. Warehouse dimensions are 10ft x 10ft.
2. Total number of slots is 250.
3. Receiving ports are port numbers 1 and 2, both equally likely to be used
4. Shipping port are 3 and 4, both equally likely to be used.

5. Probability of throughput from each port $p_1 = 0.25$, $p_2 = 0.25$, $p_3 = 0.20$ and p4 = 0.30 respectively.
6. Number of products is 20 namely 1 through 20
7. Throughput and storage information for the 20 products is as shown in table 5.1 below. These values are given as crisp values. However, in real life the numerical data for large number of products may not be available. We will convert this data into fuzzy linguistic data and solve it using FCM method.

Table 5.1 Product Data for Example 2

Product	Throughput (T)	Storage (S)	T/S	Rank	Product	Throughput (T)	Storage (S)	T/S	Rank
1	2	3	0.67	17	11	60	6	10	2
2	7	2	3.5	9	12	70	15	4.67	6
3	10	30	0.33	19	13	90	25	3.6	8
4	15	7	2.14	13	14	5	21	0.24	20
5	4	9	0.44	18	15	50	8	6.25	5
6	8	12	0.67	16	16	55	1	55	1
7	20	14	1.43	15	17	80	11	7.27	4
8	28	17	1.65	14	18	75	10	7.5	3
9	35	8	4.38	7	19	68	23	2.96	10
10	44	19	2.32	12	20	25	9	2.78	11

Rectilinear Distance Traveled

The rectilinear distance traveled in the warehouse is shown in the figure 5.1 below. The sample calculations for distance calculations are done in section 4.5 . Note, the numbers on the upper right corner denote warehouse slot number and the numbers on the lower left corner denote the rectilinear distance traveled for that slot.

IO 1, p1=0.25 IO 2, p2=0.25

1 167	2 157	3 147	4 137	5 127	6 117	7 107	8 97	9 87	10 82	11 83	12 84	13 85	14 86	15 87	16 88	17 96	18 106	19 112	20 118	21 128	22 138	23 148	24 158	25 168
26 167	27 157	28 147	29 137	30 127	31 117	31 107	33 97	34 87	35 82	36 83	37 84	38 85	39 86	40 87	41 88	42 96	43 106	44 112	45 118	46 128	47 138	48 148	49 158	50 168
51 167	52 157	53 147	54 137	55 127	56 117	57 107	58 97	59 87	60 82	61 83	62 84	63 85	64 86	65 87	66 88	67 96	68 106	69 112	70 118	71 128	72 138	73 148	74 158	75 168
76 167	77 157	78 147	79 137	80 127	81 117	82 107	83 97	84 87	85 82	86 83	87 84	88 85	89 86	90 87	91 88	92 96	93 106	94 112	95 118	96 128	97 138	98 148	99 158	100 168
101 167	102 157	103 147	104 137	105 127	106 117	107 107	108 97	109 87	110 82	111 83	112 84	113 85	114 86	115 87	116 88	117 96	118 106	119 112	120 118	121 128	122 138	123 148	124 158	125 168
126 167	127 157	128 147	129 137	130 127	131 117	132 107	133 97	134 87	135 82	136 83	137 84	138 85	139 86	140 87	141 88	142 96	143 106	144 112	145 118	146 128	147 138	148 148	149 158	150 168
151 167	152 157	153 147	154 137	155 127	156 117	157 107	158 97	159 87	160 82	161 83	162 84	163 85	164 86	165 87	166 88	167 96	168 106	169 112	170 118	171 128	172 138	173 148	174 158	175 168
176 167	177 157	178 147	179 137	180 127	181 117	182 107	183 97	184 87	185 82	186 83	187 84	188 85	189 86	190 87	191 88	192 96	193 106	194 112	195 118	196 128	197 138	198 148	199 158	200 168
201 167	202 157	203 147	204 137	205 127	206 117	207 107	208 97	209 87	210 82	211 83	212 84	213 85	214 86	215 87	216 88	217 96	218 106	219 112	220 118	221 128	222 138	223 148	224 158	225 168
226 167	227 157	228 147	229 137	230 127	231 117	232 107	233 97	234 87	235 82	236 83	237 84	238 85	239 86	240 87	241 88	242 96	243 106	244 112	245 118	246 128	247 138	248 148	249 158	250 168

IO 3, p3=0.20 IO 4, p4=0.30

Figure 5.1 Rectilinear Distance Traveled for Example 2

5.1.1 Layout by T/S Method

The layout obtained by T/S method for example 2 with crisp product data is shown in figure 5.2. The numbers in the warehouse slot indicate the product number and the arrows denote the position of the I/O ports with probabilities of activity level for each port. The products are allocated in the descending order of their T/S ratio. The products have to be rearranged to obtain a modular layout. Rearranging the products will affect the total expected distance traveled in the warehouse. In normal practice the similar products are arranged so as to form a rectangular or ‘L’ shaped layout, which is normally desired.

The total expected distance traveled in the warehouse by T/S method is 70,818 ft/day. The distance calculations are done by arranging the products in the descending order of the T/S ratio. As the data of products is large the distance calculations are performed by implementing a ‘c’ code.

5.1.2 Example 2 Solved by FCM Method Using Crisp Data

From the cluster output for the product data as given in table 5.2, it can be seen that product having similarity in throughput and storage are grouped together. This cluster information is used to design a warehouse layout with crisp data. Total number of clusters chosen for this problem is 5.

The allocation of products is done by following steps followed for example 1. The steps are given in detail in section 4.4. Layout obtained by FCM method for the product data is shown in the figure 5.3. Note the numbers in the slot indicate the product number. From the layout we can see that the product allocation similar to T/S method is disjointed. This is due to the fact that the products in similar cluster try to occupy the least available distance in the warehouse. This problem can be solved by making classes of products and allocating product of similar clusters to the respective class. This will however affect the total expected distance traveled in the warehouse.

I/O 1, p1=0.25

I/O 2, p2=0.25

14	3	5	7	8	10	20	19	12	16	18	17	15	12	13	13	13	19	10	4	8	6	3	3	14
14	3	5	7	8	10	20	19	9	11	18	17	15	12	13	13	13	19	10	4	7	6	3	3	14
14	3	5	7	8	10	20	19	9	11	18	17	15	12	13	13	13	19	10	4	7	6	3	3	14
14	3	5	7	8	10	20	19	9	11	18	17	15	12	13	13	13	19	10	4	7	6	3	3	14
14	3	5	7	8	10	20	19	9	11	18	17	15	12	13	13	2	19	10	8	7	6	3	3	14
14	3	5	1	8	10	20	19	9	11	18	17	15	12	13	13	2	19	10	8	7	6	3	3	14
14	3	5	1	8	10	20	19	9	11	18	17	12	12	13	13	19	19	10	8	7	6	3	3	14
14	3	5	1	8	4	20	19	9	18	17	17	12	12	13	13	19	19	10	8	7	6	3	3	14
14	3	5	6	8	4	10	19	9	18	17	15	12	12	13	13	19	19	10	8	7	6	3	3	14
14	3	5	6	8	4	10	19	13	18	17	15	12	12	13	13	19	20	10	8	7	6	3	14	14

I/O 3, p3=0.20

I/O 4, p4=0.30

Figure 5.2 Layout by T/S Method

Table 5.2 Cluster Output for Crisp Data: Example 2

Cluster 1		
Product	Throughput (T)	Storage (S)
1	2	3
2	7	2
4	15	7
5	4	9
Cluster 2		
3	10	30
14	5	21
Cluster 3		
13	90	25
19	68	23
Cluster 4		
6	8	12
7	20	14
8	28	17
9	35	8
10	44	19
20	25	9
Cluster 5		
11	60	6
12	70	15
15	50	8
16	55	1
17	80	11
18	75	10

The total expected distance traveled in feet per day in the warehouse is calculated. The expected distance is calculated for each cluster and the sum of these distances for the 5 clusters is the total expected distance traveled as shown in table 5.3.

Table 5.3 Total Expected Distance Traveled Per Day

Cluster Number	Expected Distance Traveled in ft/day
1	32,572
2	14,267
3	18,142
4	3,879
5	2,367
Total	71,228

I/O 1, p1=0.25

I/O 2, p2=0.25

14	3	5	6	8	10	20	19	12	16	18	17	15	12	13	13	19	9	10	8	7	4	3	3	14
14	3	5	6	8	10	20	19	13	11	18	17	15	12	13	13	19	9	10	8	7	4	3	3	14
14	3	5	6	7	10	20	19	13	11	18	17	15	12	13	13	19	9	10	8	7	4	3	3	14
14	3	5	6	7	10	20	19	13	11	18	17	15	12	13	13	19	9	10	8	7	4	3	3	14
14	3	5	6	7	10	20	19	13	11	18	17	15	12	13	13	19	9	10	8	7	4	3	3	14
14	3	5	6	7	8	20	19	13	11	18	17	15	12	13	13	19	9	10	8	7	4	3	3	14
14	3	5	6	7	8	10	19	13	11	18	17	12	12	13	19	19	9	10	8	6	4	3	3	14
14	3	5	6	7	8	10	19	13	18	17	17	12	12	13	19	19	20	10	8	6	1	3	3	14
14	3	5	2	7	8	10	19	13	18	17	15	12	12	13	19	19	20	10	8	6	1	3	3	14
14	3	3	2	7	8	10	9	13	18	17	15	12	12	13	19	19	20	10	8	6	1	3	14	14

I/O 3, p3=0.20

I/O 4, p4=0.30

Figure 5.3 Layout by FCM Method Crisp Data

5.1.3 Comparison of Layout and Total Expected Distance

By observing the layout by T/S and FCM we can see that there are some minor changes in the layout while the relative locations of different products is more or less unchanged. This is due to the difference in allocation techniques used by the two methods. In the T/S method, the product with the highest T/S ratio gets the closest slot in the warehouse. Where as in the FCM method the clusters with relatively similar product data are clustered together and then they are ranked within the cluster. The results for the total expected distance traveled for both the above mentioned cases is given in table 5.4. The percentage increase in total expected distance by FCM method is 0.58 %. Here we can see that the percentage increase in distance traveled by FCM method is negligible.

Table 5.4 Comparison of Results: Example 2

Total Exp. Distance Traveled by T/S method	70,818 ft/day
Total Exp. Distance Traveled by FCM Method	71,228 ft/day

5.1.4 Example 2 Solved by FCM Method Using Fuzzy Data

The crisp product information given earlier was coded into five fuzzy levels namely, very low, low, medium, high and very high. The category ranges were found by dividing the highest value of throughput and storage into number of levels. For this example for throughput it will result in 90/5 = 18 and for storage it will be 30/5 = 6. Therefore the values for throughput in the increasing order for very low to very high will be 1-18, 19-37, etc. Similarly, values for very low to very high for storage will be 1-6, 7-13, etc. Table 5.5 gives the fuzzy values for the throughput and storage levels shown earlier in table 5.4. Total number of clusters for this problem is 5. The output of FCM algorithm generates clusters that are used to design a warehouse layout with fuzzy data. From FCM cluster output below we see that products with similar pattern of data are clustered together, for example products 1 and 2 with very low values of throughput and storage. Similarly products 17 and 18 with very high throughput and low storage are clustered together and so on.

Cluster 1- 1, 2, 4, 5, 6, 7, 8, 9 and 20

Cluster 2- 3 and 14

Cluster 3- 17 and18

Cluster 4- 10, 12, 13 and 19

Cluster 5- 11, 15 and 16

Table 5.5 Fuzzy Product Data for Example 2

Product	Throughput (T)	Storage (S)	Product	Throughput (T)	Storage (S)
1	VL	VL	11	H	VL
2	VL	VL	12	H	M
3	VL	VH	13	VH	VH
4	VL	L	14	VL	H
5	VL	L	15	M	L
6	VL	L	16	H	VL
7	L	M	17	VH	L
8	L	M	18	VH	L
9	L	L	19	H	H
10	M	H	20	L	L

5.1.5 Layout for Fuzzy Data by FCM Method

Layout obtained by FCM method for the product data is shown in figure 5.4. Note the numbers in the slot indicate the product number. The allocation of products is done by following steps 1 through 7 in section 4.4. From the layout we can see that the product allocation is disjoint. This is due to the fact that the products in one cluster occupy the least available distance in the warehouse.

Comparing the layout with the T/S layout we can see that the pattern of product allocation does not vary much. The small change in expected distance justifies this claim. However, due to the difference to the random generation of storage data the number of products allocated are different. The total expected distance traveled in feet per day in the warehouse is calculated. The expected distance is calculated for each cluster and the sum of these distances for the 5 clusters is the total expected distance traveled (Refer table 5.6).

Table 5.6 Total Expected Distance Traveled

Cluster Number	Expected Distance Traveled in ft/day
1	13,873
2	12,940
3	21,825
4	18,705
5	4,328
Total	71,671

5.2 Principles for Warehouse Design

The FCM algorithm generates clusters for data point in a n-dimensional space. This aspect of FCM can be used to add more features to design a warehouse layout. This can be done by using critical principles that really have an impact on the design of a warehouse. A warehouse designer gets the flexibility to use these principles to the warehouse that the T/S approach does not have. We brief the main principles that play an important role in warehouse design (Tompkins, 1996)

1. Popularity- The popularity of products is derived from 'Pareto's law' that suggests to place15% of the products closer to the I/O ports to minimize the distance traveled..
2. Similarity- the items received/ shipped together should be stored together. By doing this travel times for order receipt and order picking can be minimized.
3. Size- locating products based on the size or bulk and the space it utilizes. It is a common practice to locate all the bulky items close to the I/O ports to minimize the traveling and maneuvering time.
4. Characteristics- of products like perishable materials, crushable items, hazardous material, security items and compatibility of products.
5. Space Utilization- Some important factors to be considered are conservation of space, limitations of space and accessibility of products.

I/O 1, p1=0.25 I/O 2, p2=0.25

3	14	6	5	7	8	10	13	19	11	15	17	12	12	19	13	13	10	20	7	9	4	14	14	3
3	14	6	5	7	8	10	13	19	16	15	17	12	12	19	13	13	10	20	7	9	4	14	3	3
3	14	6	5	7	8	10	13	19	16	18	17	12	12	19	13	13	10	20	7	9	4	14	3	3
3	14	6	5	7	8	10	13	19	16	18	17	12	12	19	13	13	10	2	7	9	4	14	3	3
3	14	6	5	7	8	1	10	19	16	18	17	12	12	19	13	13	10	2	7	9	4	14	3	3
3	14	6	5	9	8	1	10	19	15	18	17	12	19	13	13	13	10	2	7	9	4	14	3	3
3	14	6	4	9	8	20	10	19	15	18	17	12	19	13	13	13	10	2	7	9	4	14	3	3
3	14	14	4	9	8	20	10	19	15	18	17	12	19	13	13	13	10	8	7	7	4	14	3	3
3	14	14	4	9	8	20	10	19	15	18	12	12	19	13	13	13	10	8	7	7	6	14	3	3
3	14	14	4	9	8	20	10	19	15	18	12	12	19	13	13	13	10	8	7	7	6	14	3	3

I/O 3, p3=0.20 I/O 4, p4=0.30

Figure 5.4 Layout by FCM Method Fuzzy Data

5.3 Example 2 with Volume Information

The product information for throughput, storage and volume is fuzzy and is defined in five fuzzy levels namely, very low, low, medium, high and very high (refer table 5.7). The ranges for throughput and storage are same as that for the 2 feature problem. In this case the fuzzy data for volume is generated by considering 1000 as the maximum volume and dividing it in to 5 levels namely 1000/5= 200. So the ranges for the data starting from very low to high will be 1-200, 201-400, etc. Total number of clusters for this problem is 5. The output of FCM algorithm generates clusters that are used to design a warehouse layout with fuzzy data.

Table 5.7 Fuzzy Throughput, Storage and Volume Data for Example 2

Product	Throughput (T)	Storage (S)	Volume	Product	Throughput (T)	Storage (S)	Volume
1	VL	VL	VL	11	H	VL	VL
2	VL	VL	L	12	H	M	H
3	VL	VH	M	13	VH	VH	M
4	VL	L	VH	14	VL	H	L
5	VL	L	VH	15	M	L	M
6	VL	L	VH	16	H	VL	M
7	L	M	VL	17	VH	L	L
8	L	M	H	18	VH	L	H
9	L	L	VL	19	H	H	VH
10	M	H	H	20	L	L	L

5.3.1 Layout Based on Fuzzy Data by FCM Method

Layout obtained by FCM method for the product data is shown in the figure 5.5. Note the numbers in the slot indicate the product number. The allocation of products is done by following steps 1 through 7 in section 4.4. Due to the addition of the third feature the layout changes. This change is due to the change in cluster formation and the impact 'volume' has on the cluster formation. From the layouts for fuzzy 2 feature and 3 feature , we observe that due to the addition of the third feature (volume) there is change in the

I/O 1, p1=0.25

I/O 2, p2=0.25

7	14	14	3	17	18	13	10	19	6	4	4	8	8	19	12	10	13	13	15	3	3	14	2	9
7	20	14	3	17	18	13	10	19	6	4	5	8	8	19	12	10	13	13	15	3	3	14	2	9
7	20	14	3	17	18	13	10	19	6	4	5	8	8	19	12	10	13	13	15	3	3	14	7	9
7	20	14	3	17	18	13	10	19	6	4	5	8	19	12	12	10	13	13	15	3	3	14	7	9
7	20	14	3	17	18	13	10	19	6	4	5	8	19	12	12	10	13	13	16	3	3	14	7	9
7	20	14	3	17	18	13	10	19	6	4	5	8	19	12	12	10	13	13	16	3	3	14	7	9
7	20	14	3	11	18	13	10	19	6	4	5	8	19	12	12	10	13	13	16	3	3	14	7	9
9	20	14	3	3	15	13	10	19	6	4	5	8	19	12	12	10	13	13	16	3	14	14	7	9
9	2	14	3	3	15	13	10	19	6	4	5	8	19	12	12	10	13	13	17	3	14	14	7	9
9	2	14	3	3	15	13	10	19	4	4	5	8	19	12	12	10	13	18	17	3	14	14	7	1

I/O 3, p3=0.20

I/O 4, p4=0.30

Figure 5.5 Layout by FCM Method Fuzzy Data with 3 Features

product allocation. For example product 7, 13, 17 and 18 move away from the I/O ports. This is due to the impact on cluster formation by the comparatively low values of third feature. On the contrary, products 4, 5, 6 and 8 move closer to the I/O ports due to the high values of the third feature.

The expected distance is calculated for each cluster and the sum of these distances for the 5 clusters is the total expected distance traveled (Refer table 5.8). The expected distance is larger (by 19.8 %) in this case compared to that for 2 feature data. This is the tradeoff made by including the third feature in cluster formation.

Table 5.8 Total Expected Distance Traveled

Cluster Number	Expected Distance Traveled in ft/day
1	4,307
2	22,869
3	38,936
4	3,932
5	15,902
Total	85,945

5.4 Example Problem 3: A Large Warehouse

This problem has been taken from Francis (1992). The problem deals with a large warehouse with 50 different products. The warehouse has 3 separate I/O ports for receiving and shipping items with variable amount of activity from these ports. There is large variation in terms of activity levels, storage and volume requirement for this problem.

Problem Data

1. Warehouse dimensions are 10ft x 10ft.
2. Total number of slots is 700.
3. Receiving ports are port numbers 1 and 2, both equally likely to be used
4. Shipping port is 3.

5. Probability of throughput from each port p_1= 0.25, p_2= 0.25 and p_3= 0.50 respectively.
6. Number of products is 50 namely 1 through 50.
7. Throughput and storage requirement for the 50 products is mentioned in table 5.9.

Rectilinear Distance Traveled

The rectilinear distance traveled in the warehouse is shown in the figure 5.6. The sample calculations for distance calculations are done in section 4.5 . Note, the numbers on the upper right corner denote warehouse slot number and the numbers on the lower left corner denote the rectilinear distance traveled for that slot.

Table 5.9 Crisp Product Data for Example 3

Product	Throughput (T)	Storage (S)	T/S	Rank	Product	Throughput (T)	Storage (S)	T/S	Rank
1	4	8	0.5	30	26	3	2	1.5	12
2	5	12	0.42	35	27	10	16	0.63	26
3	9	4	2.25	7	28	3	6	0.5	31
4	7	8	0.86	22	29	8	4	2	9
5	3	8	0.36	36	30	15	13	1.15	17
6	9	5	1.8	10	31	10	9	1.11	19
7	3	10	0.3	38	32	7	5	1.4	14
8	30	24	1.25	16	33	5	6	0.83	23
9	2	28	0.07	44	34	15	13	1.15	18
10	34	12	2.83	4	35	30	8	3.75	2
11	12	12	1	20	36	3	4	0.75	24
12	13	10	1.3	15	37	10	4	2.5	6
13	1	25	0.04	50	38	6	4	1.5	13
14	9	10	0.9	21	39	4	9	0.44	32
15	4	2	2	8	40	10	6	1.67	11
16	11	20	0.55	29	41	3	7	0.43	33
17	3	5	0.6	28	42	5	15	0.33	37
18	13	19	0.68	25	43	50	16	3.13	3
19	2	40	0.05	48	44	10	45	0.22	39
20	17	4	4.25	1	45	4	18	0.22	40
21	1	18	0.06	46	46	56	20	2.8	5
22	8	19	0.42	34	47	3	15	0.2	41
23	1	15	0.07	45	48	4	25	0.16	42
24	3	50	0.06	47	49	1	20	0.05	49
25	1	10	0.1	43	50	20	32	0.63	27

I/O 3, p3=0.50

1 **263**	2 **253**	3 **243**	4 **233**	5 **223**	6 **213**	7 **203**	8 **193**	9 **183**	10 **175**	11 **170**	12 **165**	13 **160**	14 **155**	15 **150**	16 **145**	17 **145**	18 **150**	19 **155**	20 **160**	21 **165**	22 **170**	23 **175**	24 **180**	25 **185**	26 **190**	27 **198**	28 **208**	29 **218**	30 **228**	31 **238**	32 **248**	33 **258**	34 **268**	35 **278**
36 **263**	37 **253**	38 **243**	39 **233**	40 **223**	41 **213**	42 **203**	43 **193**	44 **183**	45 **175**	46 **170**	47 **165**	48 **160**	49 **155**	50 **150**	51 **145**	52 **145**	53 **150**	54 **155**	55 **160**	56 **165**	57 **170**	58 **175**	59 **180**	60 **185**	61 **190**	62 **198**	63 **208**	64 **218**	65 **228**	66 **238**	67 **248**	68 **258**	69 **268**	70 **278**
71 **263**	72 **253**	73 **243**	74 **233**	75 **223**	76 **213**	77 **203**	78 **193**	79 **183**	80 **175**	81 **170**	82 **165**	83 **160**	84 **155**	85 **150**	86 **145**	87 **145**	88 **150**	89 **155**	90 **160**	91 **165**	92 **170**	93 **175**	94 **180**	95 **185**	96 **190**	97 **198**	98 **208**	99 **218**	100 **228**	101 **238**	102 **248**	103 **258**	104 **268**	105 **278**
106 **263**	107 **253**	108 **243**	109 **233**	110 **223**	111 **213**	112 **203**	113 **193**	114 **183**	115 **175**	116 **170**	117 **165**	118 **160**	119 **155**	120 **150**	121 **145**	122 **145**	123 **150**	124 **155**	125 **160**	126 **165**	127 **170**	128 **175**	129 **180**	130 **185**	131 **190**	132 **198**	133 **208**	134 **218**	135 **228**	136 **238**	137 **248**	138 **258**	139 **268**	140 **278**
141 **263**	142 **253**	143 **243**	144 **233**	145 **223**	146 **213**	147 **203**	148 **193**	149 **183**	150 **175**	151 **170**	152 **165**	153 **160**	154 **155**	155 **150**	156 **145**	157 **145**	158 **150**	159 **155**	160 **160**	161 **165**	162 **170**	163 **175**	164 **180**	165 **185**	166 **190**	167 **198**	168 **208**	169 **218**	170 **228**	171 **238**	172 **248**	173 **258**	174 **268**	175 **278**
176 **263**	177 **253**	178 **243**	179 **233**	180 **223**	181 **213**	182 **203**	183 **193**	184 **183**	185 **175**	186 **170**	187 **165**	188 **160**	189 **155**	190 **150**	191 **145**	192 **145**	193 **150**	194 **155**	195 **160**	196 **165**	197 **170**	198 **175**	199 **180**	200 **185**	201 **190**	202 **198**	203 **208**	204 **218**	205 **228**	206 **238**	207 **248**	208 **258**	209 **268**	210 **278**
211 **263**	212 **253**	213 **243**	214 **233**	215 **223**	216 **213**	217 **203**	218 **193**	219 **183**	220 **175**	221 **170**	222 **165**	223 **160**	224 **155**	225 **150**	226 **145**	227 **145**	228 **150**	229 **155**	230 **160**	231 **165**	232 **170**	233 **175**	234 **180**	235 **185**	236 **190**	237 **198**	238 **208**	239 **218**	240 **228**	241 **238**	242 **248**	243 **258**	244 **268**	245 **278**
246 **263**	247 **253**	248 **243**	249 **233**	250 **223**	251 **213**	252 **203**	253 **193**	254 **183**	255 **175**	256 **170**	257 **165**	258 **160**	259 **155**	260 **150**	261 **145**	262 **145**	263 **150**	264 **155**	265 **160**	266 **165**	267 **170**	268 **175**	269 **180**	270 **185**	271 **190**	272 **198**	273 **208**	274 **218**	275 **228**	276 **238**	277 **248**	278 **258**	279 **268**	280 **278**
281 **263**	282 **253**	283 **243**	284 **233**	285 **223**	286 **213**	287 **203**	288 **193**	289 **183**	290 **175**	291 **170**	292 **165**	293 **160**	294 **155**	295 **150**	296 **145**	297 **145**	298 **150**	299 **155**	300 **160**	301 **165**	302 **170**	303 **175**	304 **180**	305 **185**	306 **190**	307 **198**	308 **208**	309 **218**	310 **228**	311 **238**	312 **248**	313 **258**	314 **268**	315 **278**
316 **263**	317 **253**	318 **243**	319 **233**	320 **223**	321 **213**	322 **203**	323 **193**	324 **183**	325 **175**	326 **170**	327 **165**	328 **160**	329 **155**	330 **150**	331 **145**	332 **145**	333 **150**	334 **155**	335 **160**	336 **165**	337 **170**	338 **175**	339 **180**	340 **185**	341 **190**	342 **198**	343 **208**	344 **218**	345 **228**	346 **238**	347 **248**	348 **258**	349 **268**	350 **278**
351 **263**	352 **253**	353 **243**	354 **233**	355 **223**	356 **213**	357 **203**	358 **193**	359 **183**	360 **175**	361 **170**	362 **165**	363 **160**	364 **155**	365 **150**	366 **145**	367 **145**	368 **150**	369 **155**	370 **160**	371 **165**	372 **170**	373 **175**	374 **180**	375 **185**	376 **190**	377 **198**	378 **208**	379 **218**	380 **228**	381 **238**	382 **248**	383 **258**	384 **268**	385 **278**
386 **263**	387 **253**	388 **243**	389 **233**	390 **223**	391 **213**	392 **203**	393 **193**	394 **183**	395 **175**	396 **170**	397 **165**	398 **160**	399 **155**	400 **150**	401 **145**	402 **145**	403 **150**	404 **155**	405 **160**	406 **165**	407 **170**	408 **175**	409 **180**	410 **185**	411 **190**	412 **198**	413 **208**	414 **218**	415 **228**	416 **238**	417 **248**	418 **258**	419 **268**	420 **278**
421 **263**	422 **253**	423 **243**	424 **233**	425 **223**	426 **213**	427 **203**	428 **193**	429 **183**	430 **175**	431 **170**	432 **165**	433 **160**	434 **155**	435 **150**	436 **145**	437 **145**	438 **150**	439 **155**	440 **160**	441 **165**	442 **170**	443 **175**	444 **180**	445 **185**	446 **190**	447 **198**	448 **208**	449 **218**	450 **228**	451 **238**	452 **248**	453 **258**	454 **268**	455 **278**
456 **263**	457 **253**	458 **243**	459 **233**	460 **223**	461 **213**	462 **203**	463 **193**	464 **183**	465 **175**	466 **170**	467 **165**	468 **160**	469 **155**	470 **150**	471 **145**	472 **145**	473 **150**	474 **155**	475 **160**	476 **165**	477 **170**	478 **175**	479 **180**	480 **185**	481 **190**	482 **198**	483 **208**	484 **218**	485 **228**	486 **238**	487 **248**	488 **258**	489 **268**	490 **278**
491 **263**	492 **253**	493 **243**	494 **233**	495 **223**	496 **213**	497 **203**	498 **193**	499 **183**	500 **175**	501 **170**	502 **165**	503 **160**	504 **155**	505 **150**	506 **145**	507 **145**	508 **150**	509 **155**	510 **160**	511 **165**	512 **170**	513 **175**	514 **180**	515 **185**	516 **190**	517 **198**	518 **208**	519 **218**	520 **228**	521 **238**	522 **248**	523 **258**	524 **268**	525 **278**
526 **263**	527 **253**	528 **243**	529 **233**	530 **223**	531 **213**	532 **203**	533 **193**	534 **183**	535 **175**	536 **170**	537 **165**	538 **160**	539 **155**	540 **150**	541 **145**	542 **145**	543 **150**	544 **155**	545 **160**	546 **165**	547 **170**	548 **175**	549 **180**	550 **185**	551 **190**	552 **198**	553 **208**	554 **218**	555 **228**	556 **238**	557 **248**	558 **258**	559 **268**	560 **278**
561 **263**	562 **253**	563 **243**	564 **233**	565 **223**	566 **213**	567 **203**	568 **193**	569 **183**	570 **175**	571 **170**	572 **165**	573 **160**	574 **155**	575 **150**	576 **145**	577 **145**	578 **150**	579 **155**	580 **160**	581 **165**	582 **170**	583 **175**	584 **180**	585 **185**	586 **190**	587 **198**	588 **208**	589 **218**	590 **228**	591 **238**	592 **248**	593 **258**	594 **268**	595 **278**
596 **263**	597 **253**	598 **243**	599 **233**	600 **223**	601 **213**	602 **203**	603 **193**	604 **183**	605 **175**	606 **170**	607 **165**	608 **160**	609 **155**	610 **150**	611 **145**	612 **145**	613 **150**	614 **155**	615 **160**	616 **165**	617 **170**	618 **175**	619 **180**	620 **185**	621 **190**	622 **198**	623 **208**	624 **218**	625 **228**	626 **238**	627 **248**	628 **258**	629 **268**	630 **278**
631 **263**	632 **253**	633 **243**	634 **233**	635 **223**	636 **213**	637 **203**	638 **193**	639 **183**	640 **175**	641 **170**	642 **165**	643 **160**	644 **155**	645 **150**	646 **145**	647 **145**	648 **150**	649 **155**	650 **160**	651 **165**	652 **170**	653 **175**	654 **180**	655 **185**	656 **190**	657 **198**	658 **208**	659 **218**	660 **228**	661 **238**	662 **248**	663 **258**	664 **268**	665 **278**
666 **263**	667 **253**	668 **243**	669 **233**	670 **223**	671 **213**	672 **203**	673 **193**	674 **183**	675 **175**	676 **170**	677 **165**	678 **160**	679 **155**	680 **150**	681 **145**	682 **145**	683 **150**	684 **155**	685 **160**	686 **165**	687 **170**	688 **175**	689 **180**	690 **185**	691 **190**	692 **198**	693 **208**	694 **218**	695 **228**	696 **238**	697 **248**	698 **258**	699 **268**	700 **278**

I/O 1, p1=0.25

I/O 2, p2=0.25

Figure 5.6 Rectilinear Distance Traveled in Warehouse: Example 3

5.4.1 Layout by T/S Method

The layout obtained for the numerical data of throughput and storage by T/S is shown in figure 5.7. The numbers in the lower right corner of the warehouse slot indicate the product number and the arrows denote the position of the I/O ports with probabilities of throughput for each port. The products are allocated in the descending order of their T/S ratio. As indicated in the previous problem, the products have to be rearranged to obtain a modular layout. Rearranging the products will affect the total expected distance traveled in the warehouse.

The total expected distance traveled in the warehouse by T/S method is 80,936 ft/day. The distance calculations are done by arranging the products in the descending order of the T/S ratio. As the data of products is large the distance calculations are performed by implementing a 'c' code.

5.4.2 Example 3 Solved by FCM Method Using Crisp Data

The cluster output for the problem data based on 10 clusters is given in table 5.10. This cluster information is used to design a warehouse layout with crisp data. Layout obtained by FCM method is shown in the figure 5.8. The appropriate steps for allocation of products is followed as given in section 4.4.

The total expected distance traveled in the warehouse is calculated. The expected distance is calculated for each cluster and the sum of these distances for the 10 clusters is the total expected distance traveled (Refer table 5.11).

I/O 3, p3=0.50

19	21	24	9	48	45	44	5	1	50	33	31	8	40	46	20	20	46	40	8	31	36	50	16	41	22	42	44	47	25	23	24	19	49	13
19	21	24	9	48	45	44	5	1	50	36	31	8	40	46	20	20	46	40	8	31	36	50	16	41	22	7	44	47	25	23	24	19	49	13
19	21	24	9	48	45	44	5	1	50	36	31	8	40	46	35	35	46	26	8	11	18	50	16	41	22	7	44	47	25	23	24	19	49	13
19	21	24	9	48	45	44	5	1	50	18	11	8	26	46	35	35	46	38	8	11	18	50	16	41	22	7	44	47	25	23	24	19	49	13
19	21	24	9	48	45	44	5	1	50	18	11	8	38	46	35	35	46	38	8	11	18	50	16	41	22	7	44	47	9	23	24	19	49	13
19	21	24	9	48	45	44	5	28	50	18	11	30	38	46	35	35	46	32	30	11	18	50	16	41	22	7	44	47	9	23	24	19	49	13
19	21	24	9	48	45	44	42	28	50	18	11	30	32	46	43	43	46	32	30	11	18	50	16	41	2	7	44	47	9	23	24	19	49	13
19	21	24	9	48	45	44	42	28	50	18	11	30	32	46	43	43	46	32	30	11	18	50	16	22	2	7	44	47	9	24	24	19	49	13
19	21	24	9	48	45	44	42	28	27	18	11	30	12	46	43	43	46	12	30	14	18	27	16	22	2	7	44	47	9	24	24	19	49	13
19	21	24	9	48	45	44	42	28	27	18	14	30	12	46	43	43	46	12	30	14	18	27	16	22	2	7	44	48	9	24	24	19	49	13
19	21	24	9	48	45	44	42	28	27	18	14	30	12	37	43	43	37	12	30	14	18	27	16	22	2	7	44	48	9	24	24	19	49	13
19	21	24	9	48	45	44	42	39	27	18	14	30	12	37	43	43	37	12	34	14	18	27	16	22	2	44	44	48	9	24	24	19	49	13
19	21	24	23	48	45	44	42	39	27	50	14	34	12	3	43	43	3	12	34	14	50	27	16	22	2	44	44	48	9	24	24	19	49	13
19	21	24	23	48	45	44	42	39	27	50	14	34	8	3	43	43	3	8	34	4	50	27	16	22	2	44	44	48	9	24	24	19	49	13
19	21	24	23	25	47	44	42	39	27	50	4	34	8	15	10	10	15	8	34	4	50	27	16	22	2	44	44	48	9	24	24	19	49	13
49	19	24	23	25	47	44	42	39	27	50	4	34	8	29	10	10	29	8	34	4	50	27	16	22	2	44	44	48	9	24	24	19	13	13
49	19	24	23	25	47	44	42	39	17	50	4	34	8	29	10	10	29	8	34	4	50	17	16	22	2	44	45	48	9	24	24	19	13	13
49	19	24	23	25	47	44	42	39	17	50	4	34	8	6	10	10	6	8	34	33	50	17	1	22	2	44	45	48	9	24	21	19	13	13
49	19	24	23	25	47	44	42	39	17	50	33	31	8	6	10	10	6	8	31	33	50	16	1	22	5	44	45	48	9	24	21	19	13	13
49	19	24	23	25	47	44	42	39	16	50	33	31	8	6	10	10	40	8	31	33	50	16	1	22	5	44	45	48	9	24	21	19	13	13

I/O 1, p1=0.25

I/O 2, p2=0.25

Figure 5.7 Layout by T/S Method

Table 5.10 Cluster Output for Crisp Data: Example 3

Product	Throughput (T)	Storage (S)
Cluster 1		
10	34	12
35	30	8
Cluster 2		
15	4	2
17	3	5
26	3	2
28	3	6
33	5	6
36	3	4
38	6	4
Cluster 3		
8	30	24
50	20	32
Cluster 4		
9	2	28
13	1	25
48	4	25
Cluster 5		
16	11	20
21	1	18
22	8	19
23	1	15
42	5	15
45	4	18
47	3	15
49	1	20
Cluster 6		
19	2	40
24	3	50
44	10	45
Cluster 7		
43	50	16
46	56	20
Cluster 8		
1	4	8
2	5	12
4	7	8
5	3	8
7	3	10
25	1	10
39	4	9
41	3	7
Cluster 9		
11	12	12
12	13	10
14	9	10
18	13	19
27	10	16
30	15	13
30	15	13
Cluster 10		
3	9	4
6	9	5
20	17	4
29	8	4
31	10	9
32	7	5
37	10	4
40	10	6

Table 5.11 Total Expected Distance Traveled Per Day

Cluster Number	Expected Distance Traveled in ft/day
1	9,280
2	15,594
3	12,110
4	13,878
5	4,517
6	8,619
7	5,436
8	6,717
9	3,492
10	1,841
Total	81,484

5.4.3 Comparison of Layout and Total Expected Distance

Comparing the layouts for both the cases we observe that there is small change in the product placement. This is due to the fact that in T/S approach we locate products in descending order of the T/S ratio, where as with FCM method we rank the clusters and then allocate the products in that cluster. The results for the total expected distance traveled for both the above mentioned cases is given in table 5.12. The percentage increase in total expected distance by FCM method is 0.68 %. Here we can see that the percentage increase in distance traveled by FCM method is negligible.

Table 5.12 Comparison of Results: Example 3

Total Exp. Distance Traveled by T/S method	80,936 ft/day
Total Exp. Distance Traveled by FCM Method	81484 ft/day

I/O 3, p3=0.50

48	19	24	44	49	23	42	16	41	8	33	18	30	40	46	35	35	46	40	30	18	33	8	4	5	25	22	47	21	44	24	24	19	9	13
48	19	24	44	49	23	45	16	41	8	33	18	30	40	46	35	35	46	32	30	18	36	50	4	5	25	22	47	21	44	24	24	19	9	13
48	19	24	44	49	23	45	16	41	50	36	18	30	32	46	35	35	46	32	30	18	36	50	4	5	25	22	47	21	44	24	19	48	9	13
48	19	24	44	49	23	45	16	41	50	36	18	30	32	46	35	35	46	32	30	18	17	50	1	5	25	22	47	21	44	24	19	48	9	13
48	19	24	44	49	23	45	16	41	50	17	18	30	31	46	10	10	46	31	30	18	17	50	1	5	25	22	47	21	44	24	19	48	9	13
48	19	24	44	49	23	45	16	41	50	17	18	30	31	46	10	10	46	31	30	18	28	50	1	5	25	22	47	21	44	24	19	48	9	13
48	19	24	44	49	23	45	16	41	50	17	18	30	31	46	10	10	46	31	11	18	28	50	1	5	25	42	47	21	44	24	19	48	9	13
9	19	24	44	44	23	45	22	2	50	28	27	11	31	46	10	10	46	31	11	27	28	50	1	7	16	42	47	49	44	24	19	48	9	13
9	19	24	44	44	23	45	22	2	50	28	27	11	31	20	10	10	20	12	11	27	8	50	1	7	16	42	47	49	44	24	19	48	9	13
9	19	24	44	44	21	45	22	2	50	28	27	11	12	20	10	10	20	12	11	27	8	50	1	7	16	42	47	49	44	24	19	48	9	13
9	19	24	44	44	21	45	22	2	50	8	27	11	12	37	43	43	37	12	11	27	8	50	1	7	16	42	47	49	44	24	19	48	9	13
9	19	24	44	44	21	45	22	2	50	8	27	11	12	37	43	43	37	12	11	27	8	50	39	7	16	42	47	49	44	24	19	48	9	13
9	19	24	24	44	21	45	22	2	50	8	27	11	12	3	43	43	3	12	14	27	8	50	39	7	16	42	47	49	44	24	19	48	9	13
9	19	24	24	44	21	45	22	2	50	8	27	11	12	3	43	43	3	30	14	27	8	50	39	7	16	42	47	49	44	24	19	48	9	13
9	19	24	24	44	21	45	22	2	50	8	27	14	30	29	43	43	29	30	14	27	8	50	39	7	16	42	23	49	44	24	19	48	9	13
9	19	24	24	44	21	45	22	2	50	8	15	14	30	29	43	43	29	30	14	15	8	50	39	7	16	42	23	49	44	24	19	48	13	13
9	19	24	24	44	21	45	22	2	50	8	26	14	30	6	43	43	6	30	14	26	8	50	39	7	16	42	23	49	44	24	19	48	13	13
9	19	24	24	44	21	45	22	2	50	8	38	14	30	6	43	43	6	30	18	38	8	4	39	25	16	42	23	49	44	24	19	48	13	13
9	19	24	24	44	21	45	22	2	4	8	38	18	30	6	46	46	40	30	18	38	8	4	39	25	16	42	23	49	44	24	19	48	13	13
9	19	24	24	44	21	47	22	5	4	8	33	18	30	40	46	46	40	30	18	33	8	4	39	25	16	42	23	49	44	24	19	48	13	13

I/O 1, p1=0.25

I/O 2, p2=0.25

Figure 5.8 Layout by FCM Method Crisp Data

5.4.4 Example 3 Solved by FCM Method Using Fuzzy Data

The product information given earlier (table 5.9) was converted into seven fuzzy levels namely, very low, low, medium, medium low, medium high, high and very high following the procedure explained in example 2. The highest value for the throughput was 50 and the largest storage requirement was 50 as well. Table 5.13 shows the fuzzy product data for the problem. Total number of clusters assumed for this problem is 10. The output of FCM algorithm generates clusters that are used to design a warehouse layout with fuzzy data.

Table 5.13 Fuzzy Product Data for Example 3

Product	Throughput	Storage Level	Product	Throughput	Storage Level
1	VL	L	26	VL	VL
2	VL	L	27	L	ML
3	L	VL	28	VL	VL
4	VL	L	29	L	VL
5	VL	L	30	L	L
6	L	VL	31	L	L
7	VL	L	32	VL	VL
8	M	M	33	VL	VL
9	VL	M	34	L	L
10	MH	L	35	M	L
11	L	L	36	VL	VL
12	L	L	37	L	VL
13	VL	M	38	VL	VL
14	L	L	39	VL	L
15	VL	VL	40	L	VL
16	L	ML	41	VL	VL
17	VL	VL	42	VL	L
18	L	ML	43	VH	ML
19	VL	H	44	L	H
20	ML	VL	45	VL	ML
21	VL	ML	46	VH	ML
22	L	ML	47	VL	L
23	VL	L	48	VL	M
24	VL	VH	49	VL	ML
25	VL	L	50	ML	MH

From the cluster output results shown below we can see that products 15, 17, 26 and 28 with very low product data are in one cluster. Cluster 3 has only one product 8 with medium values, this is due to the large number of clusters there is fine data partition.

Cluster 1- 10 and 35; *Cluster 2*- 15, 17, 26, 28, 32, 33, 36, 38, 41; *Cluster 3*- 8
Cluster 4- 50; *Cluster 5*- 9, 13, 21, 48 and 49; *Cluster 6*- 19, 24 and 44
Cluster 7- 43 and 46; *Cluster 8*- 1, 2, 4, 5, 7, 23, 25, 39, 42, 45 and 47
Cluster 9- 14, 16, 18, 16, 27, 30, 31 and 34; *Cluster 10*- 3, 6, 11, 12, 20, 29, 37 and 40

5.4.5 Layout for Fuzzy Data by FCM Method

Comparing the layout in figure 5.9 with T/S layout we can see that the pattern of product allocation in both the layout is similar. The small change in expected distance traveled justifies this claim. The slots occupied however are different due to the random generation of fuzzy storage data for this problem. The products have to be rearranged to obtain a rectangular layout of similar product type. This however will affect the total expected distance traveled in the warehouse. The total expected distance traveled per day in the warehouse is calculated (Refer sample calculation section 4.5). The expected distance for each of the 10 clusters is calculated and the total expected distance is the sum of the expected distances of the 10 clusters. Refer table 5.14 for the total expected distance traveled by FCM method.

Table 5.14 Total Expected Distance Traveled Per Day

Cluster Number	Expected Distance Traveled in ft/day
1	8,845
2	15,459
3	14,106
4	4,694
5	14,834
6	3,500
7	4,307
8	6,850
9	3,324
10	2,812
Total	78,730

I/O 3, p3=0.50

13	49	19	24	44	44	5	7	33	27	31	30	8	40	43	10	10	43	40	8	30	31	27	38	42	23	25	45	44	24	19	49	9	21	48
13	9	19	24	44	44	5	7	33	27	31	30	8	40	43	10	10	43	40	8	30	31	27	38	42	23	25	45	44	24	19	49	9	21	48
13	9	19	24	44	44	5	7	33	27	31	16	8	40	43	10	10	43	11	8	16	31	50	32	42	23	25	45	44	24	19	49	9	21	48
13	9	19	24	44	44	5	7	33	50	31	16	8	11	43	10	10	43	11	34	16	31	50	32	42	23	25	45	44	24	19	49	9	21	48
13	9	19	24	24	44	5	7	41	50	31	16	34	11	43	10	10	43	11	34	16	31	50	32	42	23	2	45	44	24	19	49	9	21	48
13	9	19	24	24	44	5	4	41	50	18	16	34	11	43	35	35	43	11	34	16	18	50	32	42	39	2	45	44	24	19	49	9	21	48
13	9	19	24	24	44	5	4	41	50	18	16	34	11	43	35	35	43	12	34	16	18	50	32	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	5	4	15	50	18	16	34	12	43	35	35	43	12	34	16	18	50	17	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	1	4	15	50	18	16	34	12	43	35	35	43	12	34	16	18	50	17	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	1	4	15	50	18	16	34	12	20	46	46	20	12	14	16	18	50	17	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	1	4	15	50	18	16	14	12	37	46	46	37	12	14	16	18	50	17	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	1	4	15	50	18	16	14	12	37	46	46	29	8	14	16	18	50	17	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	1	4	28	50	18	16	14	8	29	46	46	29	8	14	16	18	50	17	42	39	2	45	44	24	19	49	13	21	48
13	9	19	24	24	44	1	4	28	50	18	16	14	8	29	46	46	6	8	14	16	18	50	17	42	7	2	45	44	24	19	49	13	21	48
13	9	19	19	24	44	1	4	28	50	27	16	14	8	6	46	46	6	8	14	16	27	50	17	23	7	2	45	44	24	19	49	13	21	48
21	9	19	19	24	44	1	25	28	50	27	16	14	8	6	46	46	6	8	14	16	27	50	17	23	7	2	45	44	24	19	49	13	48	48
21	9	19	19	24	44	1	25	28	50	27	16	14	8	3	46	46	3	8	30	16	27	50	17	23	7	2	45	44	24	19	49	13	48	48
21	9	19	19	24	44	1	25	28	50	27	16	30	8	3	46	46	3	8	30	16	27	50	33	23	7	5	44	44	24	19	49	13	48	48
21	9	49	19	24	44	1	25	42	50	27	31	30	8	3	46	46	3	8	30	31	27	36	33	23	7	5	44	44	24	19	49	13	48	48
21	9	49	19	24	44	1	25	42	36	27	31	30	8	40	43	43	40	8	30	31	27	36	33	23	7	5	44	44	24	19	49	13	48	48

I/O 1, p1=0.25

I/O 2, p2=0.25

Figure 5.9 Layout by FCM Method Fuzzy Data

5.5 Example Problem by FCM for Fuzzy Data: 3 Features

The product information for throughput, storage and volume is fuzzy and is defined in seven fuzzy levels namely, very low, low, medium, medium low, medium high, high and very high (refer table 5.15). The fuzzy data for volume in this example is generated in the same way as example 2. Total number of clusters for this problem is 10. The output of FCM algorithm generates clusters that are used to design a warehouse layout with fuzzy data.

Table 5.15 Fuzzy Product Data for Example 3

Product	Throughput	Storage Level	Volume	Product	Throughput	Storage Level	Volume
1	VL	L	VL	26	VL	VL	H
2	VL	L	L	27	L	ML	VH
3	L	VL	VL	28	VL	VL	M
4	VL	L	ML	29	L	VL	MH
5	VL	L	VL	30	L	L	VH
6	L	VL	L	31	L	L	H
7	VL	L	ML	32	VL	VL	ML
8	M	M	M	33	VL	VL	L
9	VL	M	L	34	L	L	VH
10	MH	L	L	35	M	L	ML
11	L	L	VL	36	VL	VL	M
12	L	L	ML	37	L	VL	L
13	VL	M	ML	38	VL	VL	L
14	L	L	VL	39	VL	L	VL
15	VL	VL	L	40	L	VL	ML
16	L	ML	M	41	VL	VL	M
17	VL	VL	L	42	VL	L	H
18	L	ML	ML	43	VH	ML	VH
19	VL	H	M	44	L	H	VL
20	ML	VL	L	45	VL	ML	VL
21	VL	ML	L	46	VH	ML	VL
22	L	ML	L	47	VL	L	VL
23	VL	L	M	48	VL	M	L
24	VL	H	MH	49	VL	ML	ML
25	VL	L	ML	50	ML	MH	M

5.5.1 Layout for Fuzzy Data by FCM Method for 3 Features

The layout obtained by FCM method for 3 features is shown in the figure 5.10. From the layout we can see that the products with high values of volume data are grouped together and are placed in locations close to the I/O ports (for example product 26, 27, 30 and 31). Similarly the products with low values of volume data (product 5, 35, 47 and 49) are placed away from the I/O ports. This justifies the effect of third feature on the layout obtained.

The total expected distance traveled per day in the warehouse is calculated. The expected distance for each of the 10 clusters is given in table 5.16. The total expected distance (TED) traveled in the warehouse is the sum of expected distance for each cluster. Due to the third feature the cluster output changes and this causes the increase in the distance traveled. The distance traveled for this problem is more than that for fuzzy data with 2 features and the increase is 26.8%. The sacrifice in distance is the gain in better space utilization with 3 features.

Table 5.16 TED by FCM Method for 3 Features: Example 3

Cluster Number	Expected Distance Traveled in ft/day
1	6,903
2	4,525
3	15,241
4	2,612
5	14,758
6	2,744
7	4,839
8	21,067
9	21,185
10	5,923
Total	99,797

I/O 3, p3=0.50

45	33	44	21	48	19	2	7	40	16	50	24	24	23	26	30	30	31	23	24	24	50	16	49	12	22	25	19	9	48	44	46	14	47	39
45	33	44	21	48	19	2	7	40	16	50	24	24	23	31	30	30	31	23	24	24	50	16	49	12	22	25	19	9	48	44	46	14	47	39
45	15	44	21	48	19	2	7	18	16	50	24	24	23	31	30	30	31	23	24	24	50	16	49	12	22	25	19	9	48	44	46	14	1	39
45	17	44	21	48	19	2	7	18	16	50	8	24	23	31	30	30	31	23	24	8	50	16	49	12	22	25	19	9	21	44	46	14	1	39
45	6	44	21	48	19	19	7	18	16	50	8	24	23	31	30	27	31	23	24	8	50	16	49	12	22	25	19	9	21	44	46	14	1	39
45	6	44	44	48	19	19	7	18	16	50	8	24	23	31	27	27	31	23	24	8	50	16	49	12	22	25	19	9	21	44	46	14	1	39
45	6	44	44	48	9	19	4	18	13	50	8	24	43	31	27	27	42	43	24	8	50	13	49	12	22	25	19	9	21	44	46	3	1	39
45	37	44	44	48	9	19	4	18	13	50	8	24	43	42	27	27	42	43	24	8	50	13	49	12	22	25	19	9	21	44	46	3	1	39
45	37	44	44	48	9	19	4	18	13	50	8	24	43	42	27	27	42	43	24	8	50	13	49	12	22	25	19	9	21	44	46	45	1	5
47	11	10	44	48	9	19	4	18	13	50	8	24	43	42	27	27	42	43	24	8	50	13	49	35	22	25	19	9	21	44	46	45	1	5
47	11	10	44	48	9	19	4	18	13	50	8	24	43	42	27	27	42	43	24	8	50	13	49	35	22	25	19	9	21	44	46	45	1	5
47	11	10	44	48	9	19	4	18	13	50	8	24	43	42	27	27	42	43	24	8	50	13	49	35	22	38	19	9	21	44	46	45	1	5
47	11	10	44	48	9	19	4	18	13	50	8	24	43	29	27	34	29	43	24	8	50	13	49	35	22	38	19	9	21	44	46	45	1	5
47	11	10	44	48	9	19	4	18	13	50	8	24	43	29	34	34	29	43	24	8	50	13	49	35	22	38	19	9	21	44	46	45	1	5
47	11	10	44	48	9	19	4	18	13	50	8	24	43	36	34	34	36	43	24	8	50	13	49	35	22	38	19	9	21	44	46	45	39	5
47	11	10	44	48	9	19	4	18	13	16	8	24	43	28	34	34	28	43	24	8	16	13	49	35	22	2	19	48	21	44	20	45	39	5
47	11	10	44	48	9	19	32	18	13	16	8	24	43	28	34	34	28	24	24	8	16	13	40	35	7	2	19	48	21	44	33	45	39	5
47	14	10	44	48	9	19	32	18	13	16	8	24	24	28	34	34	41	24	24	50	16	13	40	22	7	2	19	48	21	44	33	45	39	5
47	14	46	44	48	9	19	32	12	49	16	50	24	24	41	34	26	41	24	24	50	16	49	40	22	7	2	19	48	21	44	33	45	39	5
47	14	46	44	48	9	19	32	12	49	16	50	24	24	41	26	26	41	24	24	50	16	49	40	22	7	2	19	48	21	44	33	45	39	5

I/O 1, p1=0.25

I/O 2, p2=0.25

Figure 5.10 Layout by FCM Method Fuzzy Data with 3 Features

5.6 Sensitivity of Generated Layouts

One of the steps of the FCM method involves assuming range for the linguistic variables used and then randomly generating values for throughput and storage. The random data was generated for five replications for medium warehouse Example 2. The result of the replications indicated a small variation in the expected distance traveled between the layouts (largest difference of 5.5%, see table 5.17) and hence very small changes in the layout . On the basis of this problem we can ascertain that the FCM method performs well and is not very sensitive to the random generation of product data.

Table 5.17 Effect of Random Product Data on Expected Distance

Replications	1	2	3	4	5
Exp. Distance Traveled ft/day	74,757	74,483	73,316	76,669	72,658

5.7 Effect of Number of Clusters on Total Expected Distance

There is no significant research done to study the effect of number of clusters on the cluster formation. The analysis we have done to see the effect of number of cluster on total expected distance (TED) in shown in table 5.18. We can observe that there is no significant change in the total expected distance with the change in number of clusters. From table we see that for 10 clusters the TED is the least. To decide the ideal number of clusters for running the FCM algorithm start with 3 clusters for products less than 10. For product data ranging from 50 and above use of 10 clusters should give good results.

Table 5.18 Analysis of Number of Clusters on Total Expected Distance

Cluster No	Total Exp. Distance 20 products		Total Exp. Distance 50 Products	
	2 Features	3 Features	2 Features	3 Features
3	74,698	83,385	79,116	102,602
4	71,648	87,009	81,174	105,936
5	71,671	85,945	79,583	104,167
6	71,949	84,985	78,981	107,058
7	71,179	84,985	78,918	104,696
8	71,179	84,895	79,054	102,700
9	71,156	81,891	78,898	101,352
10	71,503	81,999	78,730	99,797

5.8 Research Contributions

The current state of the warehouse layout techniques use exact information about the product data which may not be available for large number of products found in today's warehouse. Furthermore, the existing approaches can only take into account the throughput and storage information to yield a layout that will minimize the expected distance traveled. This research effort in our opinion has resulted into the following two contributions in the field of warehouse design.

1. A fuzzy logic based warehouse with uncertain information of product data was developed that gave excellent results for the layout generated as measured by the total expected distance traveled.
2. It was shown (with the help of two warehouse examples) that it is possible to incorporate, in addition to throughput and storage, another product feature such as volume to generate a layout that will have added flexibility.

5.9 Summary

In this chapter we developed layouts for a medium and a large warehouse. We have analyzed the results obtained for total expected distance traveled for both the cases with crisp data and fuzzy data. Also the results for total expected distance for the fuzzy data of 3 feature problem is solved and analyzed. Further we analyze the effect of number of clusters on the total expected distance traveled in the warehouse for the two problems with fuzzy data. From the results obtained we can say that the FCM method gives good results for total expected distance traveled. The introduction of the third factor increases the total expected distance but generates a layout that clusters products with more features and that is user friendly. We will summarize the work done and draw appropriate conclusions in the next chapter. Some of the logical extensions to the problem will also be presented.

CHAPTER 6
SUMMARY AND CONCLUSIONS

6.1 Summary and Conclusions

In a warehouse environment, layout design is one of the important aspects. This is due to the fact that the cost involved in storage/retrieval (S/R) of products is high. A variety of research has been done to minimize the distance traveled for S/R activities. In most of the large warehouses the product information of throughput (T) and storage (S) is not exact and typically available in the form of categorical data such as low medium and high. This is an ideal environment for exploring the use of fuzzy logic based method, which looks out for a pattern in the data to get a cluster of similar data. The existing T/S method for layout design needs exact information of throughput and storage to rank the product on the basis of T/S ratio. A fuzzy c-means (FCM) clustering method was implemented in this thesis to design a warehouse layout. Both the existing T/S method as well as FCM method was explained with the help of a small warehouse problem. Comparison of expected distance traveled – the performance measure used to judge how good a generated layout was – by both the methods showed that FCM algorithm resulted in very good layouts.

Two more problems, a medium and a large warehouse, were solved by both the methods and the results obtained showed an insignificant (less than 1%) increase in the total expected distance traveled in the warehouse. The problem was also solved by using linguistic variables of product data to design a fuzzy based warehouse using this additional information. Further, the problem was solved by using fuzzy data with 3 features (product volume information was added) to obtain a layout. The resulting layout did honor the third feature in layout generation, however, with a tradeoff in the expected distance traveled.

The conversion of linguistic variables to numeric values was done by drawing random values within the range assigned for the variable. The results of several replications indicated very insignificant difference in the effectiveness of the generated layout as measured by expected distance traveled. Finally, the sensitivity of the FCM method to the number of clusters was investigated. It was observed that the method was not sensitive to the number of clusters used. However, it is recommended that with larger number of products more clusters should be used which will reduce the need for deciding the allocation of products within a cluster. Thus the overall goal of designing a fuzzy logic based warehouse layout was achieved.

6.2 Scope for Future Research

In this research an attempt was made to apply a fuzzy logic based cluster formation technique to develop an efficient warehouse layout in the absence of precise information regarding throughput and storage levels of large number of products stored in a modern warehouse. The method was validated using several examples taken from literature. There are several research extensions (mentioned below) to the approach developed which will improve the applicability of the method even further.

1. In this thesis a third feature, product volume, was used to see if a layout can be developed using information in addition to throughput and distance which could be helpful in increased space utilization. However, it will be interesting to see if the method could use other attributes such as product similarity and see its effect on the generated layout. This will widen the applicability of the FCM method for designing warehouse layouts in such areas as retail and pharmaceutical industry.
2. The sensitivity of the FCM approach to the number of clusters was investigated in this thesis. Sensitivity of the FCM method to the number of classes of linguistic variables perhaps can also impact the layout which will be helpful in providing guidance while collecting throughput and storage information.
3. The developed method was tested using several problems. The largest warehouse problem was fairly large having 50 products and 700 locations. However, in comparison to real life warehouse it was still small. The real proof of the applicability of the FCM method will be using it for a real life warehouse layout.

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