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An Inventory Model With Two Truckload Transportation and Quantity Discounts

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by

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Dedication

I dedicate this thesis to my Mom, Dad, Grand Mother, Jayashree, Karthik and Vaishnavi

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An Inventory Model with Two Truckload Transportation and Quantity Discounts

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ABSTRACT

Transportation plays a vital role in the movement of raw materials and finished goods from one place to another. Trucks play a vital role in the movement of materials and are indispensable part of almost every shipment, both domestic and international. On the average, thirty-nine percent of the total logistics cost is spent on transportation. Therefore reducing the transportation cost may significantly reduce the total logistics cost.

The total annual logistics cost considered in this research includes ordering cost, material cost, transportation cost and inventory holding cost. The main objective of this research is to develop algorithms for finding the optimal ordering quantity that minimizes total annual logistics cost, when the suppliers offer

- No quantity discounts
- All-unit quantity discounts
- Incremental quantity discounts

This research considers truckload transportation where two truck sizes are available. The algorithm developed in this research will identify the optimum ordering quantity and the optimum number of trucks required to ship the ordering quantity.

MATLAB programming of the algorithm will analyze the factors that affect that the total annual logistics cost.

Chapter 1 Introduction

This chapter gives a brief idea of what a supply chain is and discusses some important links that forms the chain. This chapter also discusses the importance of transportation in a supply chain. The current problem under investigation is explained in detail and the initial ideas about the proposed solution are specified in this chapter. This chapter explains the goals of the thesis and the last section of this chapter gives an overview of the layout in which the thesis is organized.

1.1 What is a Supply Chain

All the processes involved from procurement of a raw material, transportation of raw materials to the facilities, transformation of raw materials into finished goods, transportation of finished good to the retailers, and finally to the consumers form the basis of a supply chain. For example let us consider an apparel manufacturing firm, the primary source for cloth is cotton, the cotton is obtained from cotton fields, raw cotton thus obtained is then transported to the cotton gin to remove burs and leaf trash, the processed cotton fibers are then sent to the thread making facility, where these cotton's are made in to bundles of thread. These threads are transported to dying industry where threads are dyed in to different colors. Colored threads are then transported to knitting facility, where the threads of different colors are knitted to form a cloth. The cloth thus obtained is shipped to distribution center, which in turn ships the cloth to various retailers, based on the demand. Cotton fields, threading facility, dying facility, knitting facility, distribution centers (DC's), and retailers, are referred to as the stages of the supply chain. Each and every stage has multiple suppliers and consumers, which can be clearly viewed from the above example.

If you look at the dying industry, their suppliers include dye manufacturing, and these dye manufacturing industry have chemical industry as their suppliers. Therefore each and every stages of the supply chain have multiple suppliers and multiple consumers. It is easy to identify a supply chain in a manufacturing enterprise, but the complexity of the chain may vary from industry to industry or even company to company. A simple schematic representation of a supply chain is shown in Figure 1.1 [30].

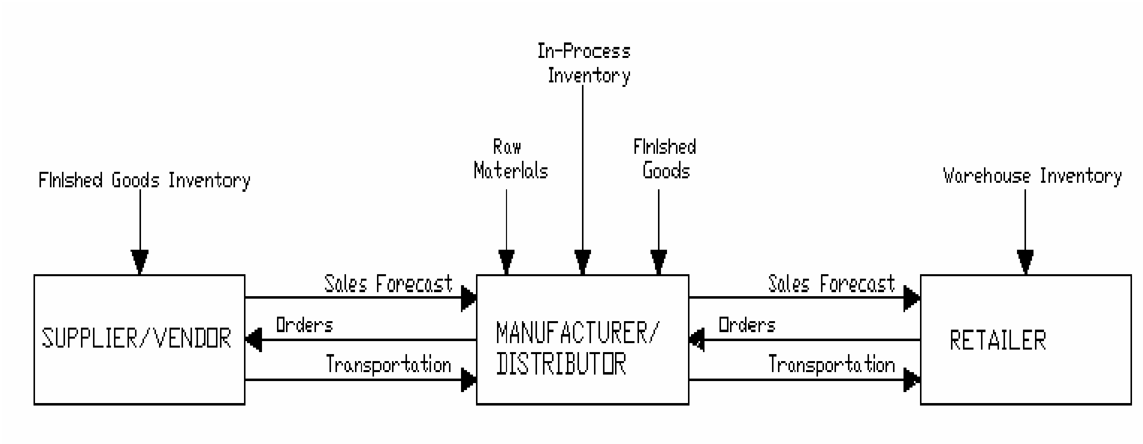

Figure 1.1 Example of a Supply Chain

1.2 Objective of a Supply Chain

The objective of every supply chain is to maximize the overall "value" generated. The difference between, the cost incurred for the final product by the customer, and the cost incurred by all stages of the supply chain in fulfilling the customer's request is called the "value" of the supply chain. For example, a customer purchasing a watch from a showroom pays $2,500, to purchase it. This $2,500 the customer pays, represents the revenue the supply chain receives. The manufacturer of the watch and the other stages of the supply chain incur cost to produce components, convey information, store them, transport them and transfer funds and so on. The difference between the $2,500 that the customer paid and the sum of all costs incurred by the supply chain to produce and distribute the watch represents the supply chain profitability. Supply chain profitability is the total profit to be shared across all stages of the chain. The higher the supply chain profitability, the more successful the supply chain. Supply chain success should be measured in terms of supply chain profitability and not in terms of the profits at an individual stage [9].

Co-ordination within the stages of the chain by, optimization of the resources needed to fulfill the customer's request increases the supply chain profitability. Optimization promises to improve a company's supply chain performance in a variety of areas:

- Reduced supply costs
- Improved product margins
- Increase production
- Better return assets

1.3 Supply Chain Drivers

Drivers are the compelling forces that facilitate the movement of material, information and resources in the supply chain. The four major drivers of supply chain are inventory, transportation, facilities and information. These drivers not only determine the supply chain's performance in terms of responsiveness and efficiency, they also determine whether strategic fit is achieved across the supply chain [24]. Let us define each driver and its impact in the performance of the supply chain.

1.3.1 Inventory

Inventory is all raw materials, work in process, and finished goods within a supply chain. Inventory is an essential aspect of materials management, knowledge about in-hand inventory and demand rate of the material is important for successful handling of the inventory. Improper inventory management will reduce the responsiveness and efficiency of the supply chain.

1.3.2 Transportation

In today's fast moving world, products are produced in one region and consumed in other, due to cheap production cost, availability of cheap labor, etc. The product produced in one region have to reach its consumers who are located in different regions, because of this reason transportation plays a major role in supply chain for transporting raw materials and finished goods from suppliers to retailer. Transportation choices have a large impact on supply chain responsiveness and efficiency.

1.3.3 Facilities

A facility is a general term for a fixed location where the logistics activities are carried out. The two major types of facilities are manufacturing locations and warehouses. Decisions regarding location, capacity and flexibility of facilities have a significant impact on the performance of the supply chain.

1.3.4 Information

Though in most cases these drivers are not clearly visible, they are potentially the biggest driver that drives the supply chain. Synchronization of all the other drivers is made possible with the help of information exchange. Efficiency of the supply chain will decrease without proper communication between the stages of the chain. The concept of supply chain management would have been impossible without information exchange between the stages of the chain. Responsiveness and efficiency of any supply chain depends on how good the information is transferred, any small miscommunication may lead to heavy loss. Measures have to be made for synchronized transfer of information from one stage to the other, while designing the supply chain.

1.4 Different Modes of Transportation in Supply Chain

There are various modes of transportation available in the supply chain to ship raw materials and finished goods.

- Air
- Truck
- Rail
- Water
- Pipeline

Each mode incorporates specific advantages and disadvantages that determine its usefulness within any given industry. There is no “best” mode for a given firm, the shipper can select one, all or any combination of the above mentioned modes based on their preferred choice.

The preferred choice is a function of many factors such as

- The type of industry
- The location of the firm
- Location and distribution of suppliers
- Marketing Area
- Availability of various transportation modes

The following figures gives information about volume of freight shipped and the revenue earned from each of the above mentioned modes.

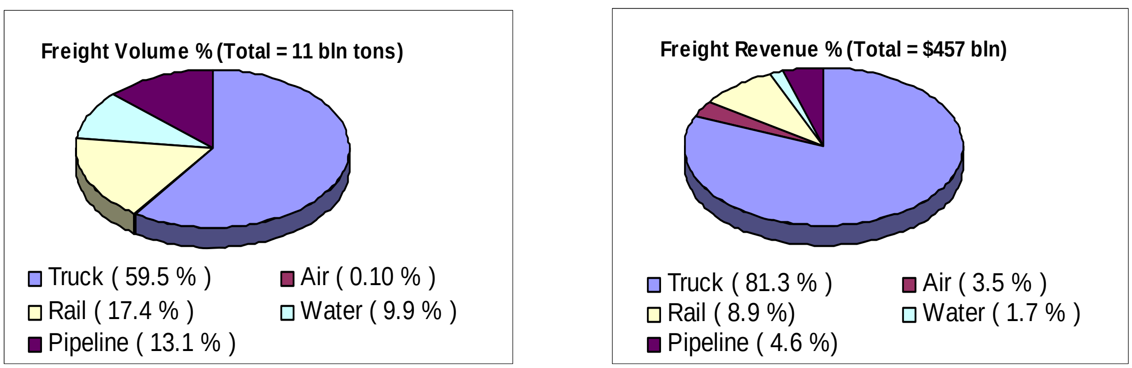

Figure 1.2 Domestic Primary Freight Market by Mode

From the figure above it is clearly visible that trucks are the most preferred mode of transportation [30]. Since the majority of freight's are shipped through trucks, the trucks yield more revenue than any other modes of transportation.

Trucks however, possess significant advantages over other modes.

- The capital cost of vehicles is relatively small
- High relative speed of vehicles

- Flexibility in route choice and flexibility in loading and unloading of shipments
- Door-to-Door delivery is made possible with the help of trucks

1.5 Road Transportation

Road transportation has gained its popularity due to the following reasons. It gives very high reliability in delivering the goods on-time to the doors of the customer's with less damage to the shipment, by giving more options and flexibility in shipping, at a moderate cost.

Road transportation mainly uses trucks as carriers. The role of trucking varies depending on the region. In large, sparsely populated areas where railroads are well developed, trucks would be used for local delivery and defer to the rail for long distance trips. In areas where the railroad is not so well developed or the market area is heavily populated, trucks become more useful.

1.6 Truckload Transportation and Less-than-Truckload Transportation

There are two different truck shipments, namely truckload (TL) and less-than-truckload (LTL) shipments. Shipment that is charged by its maximum capacity, either by weight or cube is called a TL shipment. Trucking company which dedicates trailers to a single shipper's cargo is called a TL carrier. TL carriers charge for the full truck irrespective of the quantity shipped by the shipper. Carriers give a rate reduction for shipping a TL size shipment and the rates vary with distance. The general rule, which influence the transportation cost for any mode of transportation is higher the quantity shipped, lower will be the transportation cost [24]. The quantity of freight required in filling a trailer to truck load capacity is usually more than 10,000 pounds. In less-than-truckload the cost of the freight will usually depend upon the weight of the freight shipped and the cost of shipment varies with distance. Trucking company that consolidates LTL cargo for multiple destinations on one vehicle is called LTL carriers [17]. Unit shipping cost is less for TL if the truck is filled to its maximum capacity. The unit shipping cost of LTL is bit high when compared to TL. TL is more profitable for long distance shipment. The volume of shipment and the revenue obtained from the TL and LTL is shown in the below figure [30]. Many large companies use their own trucks for transportation and it is indicated as "Private" in the below figure.

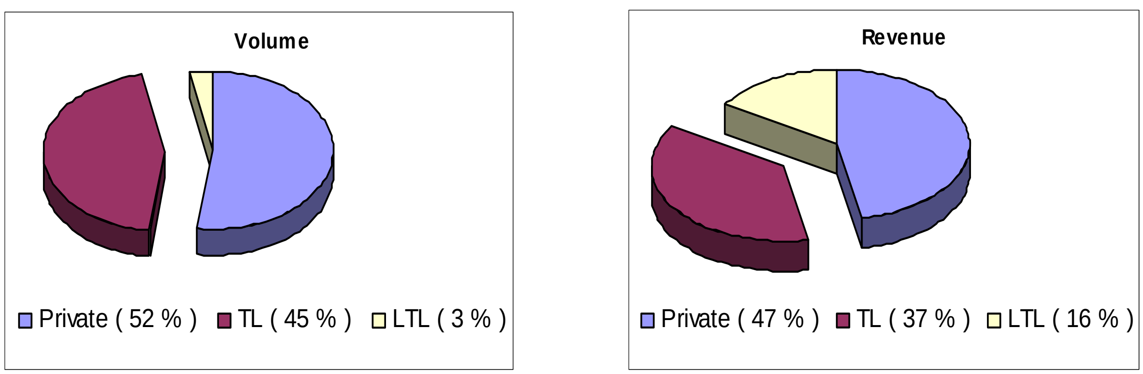

Figure 1.3 General Freight Shipments by Carrier Type

1.7 Research Objective

The objective of this research is to formulate an algorithm for finding the optimal ordering quantity, that minimizes the total annual logistics cost. The total annual logistics cost includes ordering cost, material cost, transportation cost and inventory holding cost. The ordering cost is fixed for each order, irrespective of the quantity ordered. The material cost depends upon the ordering quantity. The transportation cost varies depending upon the transportation choices made for shipping the freight. The ordering quantity will determine the average inventory for which an inventory carrying cost will be charged.

This research considers two types of transportation choices, they are

- Large truckload
- Small truckload

Both the transportation choices offer fixed transportation cost. The carrying capacity of the large truckload is greater than the carrying capacity of the small truckload. The unit transportation cost for using a large truckload is less than that of small truckload.

This research also analyzes the effect of price discounts on the ordering quantity. The price discounts considered in this research are

All-units quantity discounts: A one time price reduction for all the units ordered, based on the number of units ordered or the size of the order are called all-units quantity discounts

Incremental quantity discounts: The first set of ordered quantities will be given at particular price and the remaining quantities are given at a reduced price

1.8 Thesis Layout

In Chapter 2, a brief review on inventory management and impact of transportation cost and quantity discounts on inventory decision is provided. Chapter 3, presents a methodology to find the optimal ordering quantity that minimizes the total annual logistics cost, with transportation cost consideration but no quantity discounts. In Chapter 4, an algorithm for finding the order quantity that minimizes the total annual logistics cost, when (a) all-unit quantity discounts, and (b) incremental quantity discounts are offered by the supplier. Chapter 5 presents the results based on the methodology used. Chapter 6 contains the concluding remarks and outlines the potential research extensions.

Chapter 2 Literature Review

A brief review of literature associated with this research is presented in this chapter. An overview of different authors those who have researched on inventory models is discussed in section 2.1 and section 2.2 discusses the literature involved in the field of transportation systems and quantity discounts.

2.1 Inventory Systems

Many authors have researched inventory models, considering various assumptions and solved the inventory problem by different methods. In 1913 Harris [11] addresses a practical industrial problem of, finding the lot size of each order such that the overall costs associated with manufacturing a unit of the product is minimized. The formula developed by Harris forms the basis of all economic order quantity models. Vassian [34] finds an optimal inventory policy for periodic inventory models to satisfy the requirement of a particular management. Morse [25] extended the work done by Vassian [34] by assuming the system to be stochastic and analyzed the effect changes in the inventory policy. Morse [25] discusses periodic review inventory model and uses Markov's process for solving following situations,

- When the size of the replenishment order is equal to the number of demands arriving at the last period
- When the order is 'quantized' in multiples of some lot-size "q"

- When the replenishment order is delivered within the next period
- When the replenishment order is delayed for one or more periods

The author analyzes the inventory policy when the replenishment order is delayed for one or more periods. Hadley and Whitin [12] extended the work done by Morse [25] by assuming a stock-out cost for any replenishment order which is delayed for one or more periods. The stock-out cost considered by Hadley and Whitin [12] is a sum of fixed cost per unit for the quantity that is out of stock and a variable cost which is proportional to the time period for which that particular stock-out quantity. They also find the total cost expression, by considering, Poisson demand with fixed and gamma lead times.

Veinott Jr. [35] considers the same model as Hadley and Whitin [12], but makes a assumption that the demand in each period are independent and identically distributed random variables and the lead time is constant. Veinott Jr. [35] considered a policy in which the inventory is reviewed at the beginning of each period. If the stock on-hand and on-order is less than the fixed inventory level k, and a quantity Q is ordered which will bring the combination of the on-hand inventory and on-order inventory to a level greater than or equal to inventory level k. No order will be placed if the stock on-hand and on-order is greater than the fixed inventory level k during the review made at the start of the each period. The author named this policy as (k, Q) policy and proved that the (k, Q) policy is optimal for the finite and infinite models.

Lippman [18] finds a optimal inventory policy for a discrete review, single product, dynamic inventory model by assuming the ordering cost as a multiple set-up cost. Lippman [19] assumes that the holding cost in each period i is a non-decreasing, left continuous function of the inventory level at the end of period of i and the ordering cost function of the inventory level at the period i is neither concave nor convex. Lippman [19] establishes the existence of an optimal production schedule for each period and also studies the stationary, infinite horizon version of the multiple set-up cost problem.

Lippman [20] extends Lippman [19] to a case in which the holding cost is proportional to the inventory level while the ordering cost constitutes of a cost proportional to the amount ordered and a set-up cost independent of the ordering quantity. Iwaniec [16] modifies the ordering cost assumptions made by Lippman [18] in finding the optimal ordering quantity. The author also derives a solution algorithm for the case of periodic review inventory policy, and assumes that the ordering cost consists of linear purchase cost and a fixed cost (truckload cost) for each vehicle used in shipping the quantity. The author examines a policy, if the stock level at the beginning of period n, does not exceed a critical amount T_n then order a the smallest number of full vehicle loads which will raise the inventory level just above the critical amount T_n. No order is placed if the stock level is above the critical level T_n.

Since the solution algorithm developed by Iwaniec [16] is difficult to understand and use, Aucamp [1] derives an easy to use algorithm for the multiple set-up cost by using continuous review inventory policy. The application of the algorithm extends beyond the situation of fixed carload charges as given by [16].

Extension of the classical economic order quantity model to economies of scale is performed by Buffa and Miller [6], McClain and Thomas [26], Silver and Peterson [33]. While Lee [21] considers freight discount instead of quantity discount, he considers the practical situation where the freight cost is discounted whenever a large shipment is placed. The assumptions made by Lee [21] are same as the classical economic order quantity model developed by Harris [11], except for the set-up cost structure. The author considers the set-up cost a sum of fixed cost and freight cost with discounts, which implies that the set-up cost depends upon the quantity ordered. The higher the quantity the lower will be the set-up cost.

The authors of [21] and [13] discuss about the individual impact of price discount and freight discount on inventory policy. Hwang, Moon, and Shinn [14] discuss the combined effect of price and freight discount on an inventory policy. Whenever an order is placed price discount is offered, so it is profitable to buy large quantities of the product, at the same time the freight cost also decreases due to large shipment size. Providing price and freight discount will increase the ordering quantity, there by having a heavy impact on inventory policy.

2.2 Transportation Systems and Price Discounts

The literature in this area discusses mainly about the truckload inventory models, less-than-truckload inventory models, economies of scale in quantity price, and discounted freight cost.

Lancaster [22] and Quandt [31], discuss about various transportation choices, their advantages, disadvantages and common practices in the shipping industry. Baumol and Vinod [7] explains the importance of transportation choices made by shippers, whereby order quantity and transportation alternative can be jointly determined. The optimal choice of mode is shown to involve a trade-off among freight rates, speed, dependability, and en-route losses. They also prove that faster and more dependable service simply reduces the safety stock and the in-transit inventory for a shipper or receiver. The authors develop a model which will help in statistical comparison of the different modes of transportation by using the four attributes mentioned below, thereby the ordering quantity and the transportation alternative is jointly determined. The four attributes used in the development of the model are:

- Shipping cost per unit (including freight rate, insurance, etc.)
- Mean shipping time
- Variance in shipping time
- Carrying cost per unit of time while in transit (interest on capital, pilferage and deterioration)

Wehrman [36] finds the minimum total cost including, freight cost, ordering cost, inventory carrying cost and material cost. Freight cost has significant impact on total cost incurred in procuring a material. The author constructs the model for freight cost, which is substituted in the total cost function to find the minimum total cost for various quantities.

Larson [23] makes changes to the work done by Baumol and Vinod [7] by considering the safety stock equal to the in-transit inventory, in finding the economic transportation quantity. Das [10] presents a model for finding the economic order quantity when the supplier offers quantity discounts while formulating the inventory holding cost. The author considers only the cost incurred due to the in-transit inventory and cycle inventory and does not consider the cost incurred due to safety stock. This model is also studied from the supplier's point of view by Monahan [28], who designs the procedures for determining the optimum discount schedule for the supplier. Model developed by both Das and Monahan assumes that the demand rate for the product is known and constant. Abad [2] assumes the demand to be stochastic and develops a model for determining the optimal selling price and lot-size, while all-unit quantity discounts are offered by the supplier. The author also analyzes the problem for incremental quantity discounts and discusses in Abad [3].

Abdelwahab and Sargious [4] consider both the mode of transportation and the shipment size in determining the optimal shipment size. The author also extends the selection of transportation mode [4], to the selection of transportation carrier within the mode and discusses in [5]. Abdelwahab and Sargious [5] also examine the nature of dependency between the unit freight charge and shipment size.

Benton and Park [8] give a classification literature on all the research done in the field of quantity discounts. Munson and Rosenblatt [29] analyze thirty nine firms that receives/offers quantity discounts. The result of the study indicates that eighty three percent of the buyers receive quantity discounts for most of the items they purchase, which illustrates the prevalence and importance of quantity discounts in practice.

Rieksts and Ventura [32] determines the optimal inventory policy with two modes of freight transportation. The author considers two transportation choices, i.e. truckload (TL) and less-than-truckload (LTL) transportation. In truckload transportation, there is fixed cost per load up to given capacity irrespective of the quantity shipped. For quantities that are less than a full truckload are shipped using LTL transportation with the cost of shipment depends upon the quantity shipped. Mendoza and Ventura [27] extends the work by Rieksts and Ventura [32] to all-unit quantity discounts and finds the optimum ordering quantity that minimizes the total annual cost.

Chapter 3 No Unit Quantity Discounts

3.1 Introduction

The problem addressed in this chapter can be described as follows. There is an annual demand R of a single product. The various cost associated with each ordering quantity Q are fixed ordering cost K, and a transportation cost that varies with Q. The Q will also determine the average inventory for which an inventory carrying cost is charged. What is to be determined is the ordering quantity Q that yields minimum total logistics cost. The total annual logistics cost includes material cost, ordering cost, transportation cost and inventory holding cost. In particular, this chapter considers the following transportation scenario. There are two truck sizes: large and small. A large truck has a capacity of W_L and charges a fixed price of C_L, regardless of actual quantity loaded. Similarly, a small truck has a capacity of W_S and charges a fixed price of C_S, regardless of actual load (not exceeding its capacity). Depending upon the ordering quantity Q, it is necessary to use a combination of J_L large trucks and J_S small trucks, for some $J_L \geq 0$ and $J_S \geq 0$. It is assumed that $\frac{C_L}{W_L} < \frac{C_S}{W_S}$ (i.e., if both large and small trucks are fully loaded, the unit shipping cost for a large truck is smaller than that for a small truck).

The main objective of this chapter is to present an algorithm that identifies an ordering quantity Q that minimizes the total annual logistics cost.

3.2 Transportation Cost

This section describes the transportation cost. Let $\lceil Y \rceil$ indicate the least integer that is greater than or equal to Y, and $\lfloor Y \rfloor$ the largest integer that is lesser than or equal to Y. We now derive the optimal J_L^* and J_S^*, for a given order size Q.

Let $J_L^*(Q)$ and $J_S^*(Q)$ be the optimal number of large and small trucks used to transport quantity Q. Then the corresponding transportation cost $T\left(Q, J_L^*(Q), J_S^*(Q)\right) = J_L^*(Q)\,C_L + J_S^*(Q)\,C_S$. It is clear that $J_L^*(Q) \le \left\lceil \frac{Q}{W_L} \right\rceil$. Since $\frac{C_L}{W_L} < \frac{C_S}{W_S}$, it must be true that $J_S^*(Q) \le \left\lceil \frac{W_L}{W_S} \right\rceil$, and $J_L^*(Q) \ge \left\lfloor \frac{Q}{W_L} \right\rfloor$. Define

$$A = \left\lfloor \frac{Q}{W_L} \right\rfloor, \tag{1}$$

$$B = \left\lceil \frac{Q - AW_L}{W_S} \right\rceil, \text{ and} \tag{2}$$

$$n = \left\lfloor \frac{C_L}{C_S} \right\rfloor. \tag{3}$$

Then it can be shown that the following must be true.

$$J_L^*(Q) = \begin{cases} A, & \text{if } B \le n, \\ A+1, & \text{if } B > n. \end{cases} \tag{4}$$

$$J_S^*(Q) = \begin{cases} B, & \text{if } B \le n, \\ 0, & \text{if } B > n. \end{cases} \tag{5}$$

And the optimal transportation cost is

$$T\left(Q, J_L^*(Q), J_S^*(Q)\right) = \begin{cases} AC_L + BC_S, & \text{if } B \le n, \\ (A+1)C_L, & \text{if } B > n. \end{cases} \tag{6}$$

That is, it is optimal to fill A large trucks, and the remaining part $Q - AW_L$ will be shipped either by B small trucks if no more than n small trucks are needed, or by another large truck, otherwise. This optimal transportation cost is depicted in Figure 3.1.

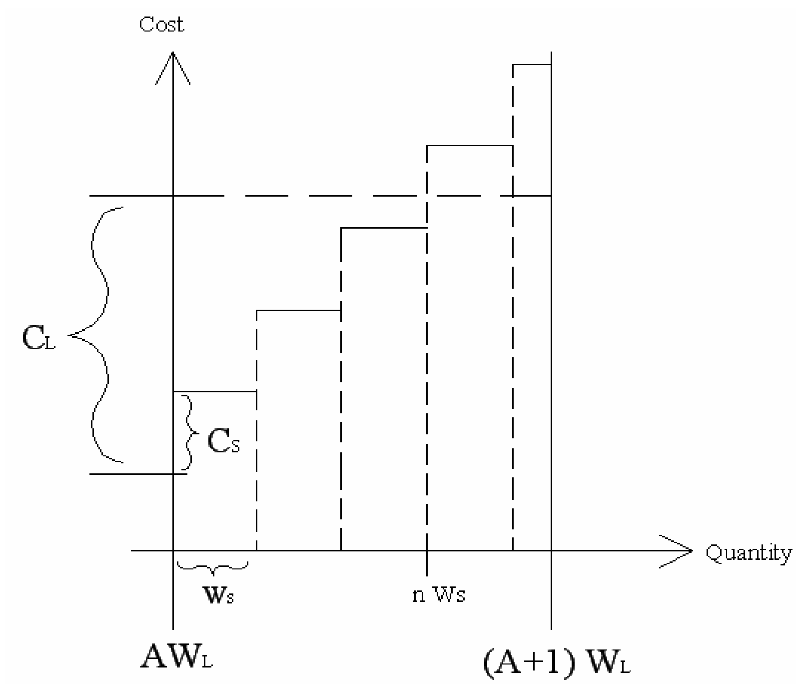

Figure 3.1 Transportation Choice Selections

3.3 Total Annual Logistics Cost

The total annual logistics cost is the sum of ordering cost, holding cost, material cost and transportation cost. Given order quantity Q, let $TC(Q)$ denote the corresponding total annual logistics cost. Then

$$TC(Q) = \frac{R}{Q}K + \frac{hC}{2}Q + RC + \frac{R}{Q}T(Q) \tag{7}$$

Substituting Eq. (6) in to Eq. (7), yields Eq. (8)

$$TC(Q, J_L^*, J_S^*) = \begin{cases} \frac{R}{Q}K + \frac{hC}{2}Q + RC + \frac{R}{Q}(AC_L + BC_S), & \text{if } B \le n, \\ \frac{R}{Q}K + \frac{hC}{2}Q + RC + \frac{R}{Q}(A+1)C_L, & \text{if } B > n. \end{cases} \tag{8}$$

3.4 Development of an Optimal Algorithm

The total cost given by (8) is a function of ordering quantity Q. Let the derivative $TC'(Q) = 0$. Then we can get the following ordering quantity Q.

$$Q = \begin{cases} \sqrt{\frac{2R(K + AC_L + BC_S)}{hC}}, & \text{if } B \le n, \\ \sqrt{\frac{2R(K + AC_L + C_L)}{hC}}, & \text{if } B > n. \end{cases} \tag{9}$$

In Eq. (9), however, A and B are also functions of Q, as defined by (1) and (2). Therefore, we cannot use Eq. (9) to find the true optimal Q^* that minimizes the total cost given by (8).To find the true optimal Q^*, we will consider all combinations of J_L and J_S. That is, for any given J_L and J_S, we consider all $Q \in \left(J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S\right]$ to find the optimal Q^* (J_L, J_S) in the range. It can be shown that $TC'(Q) = 0$ leads to the following.

$$Q\ (J_L, J_S) = \begin{cases} \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC}}, & \text{if } J_S \le n, \\ \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC}}, & \text{if } J_S > n. \end{cases} \tag{10}$$

The corresponding optimal Q^* can be determined by Eq. (11) and Eq. (12) presented below.

If $J_S \le n$, then the optimal Q^* is given by

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + (J_S - 1)W_S + 1, & if \;\; Q \le J_L W_L + (J_S - 1)W_S; \\ Q, & if \;\; Q \in \left(J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S\right]; \\ J_L W_L + J_S W_S, & if \;\; Q > J_L W_L + J_S W_S. \end{cases} \tag{11}$$

If $J_S > n$, then the optimal Q^* is given by

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + nW_S + 1, & if \;\; Q \le J_L W_L + nW_S; \\ Q, & if \;\; Q \in \left(J_L W_L + nW_S, (J_L + 1)W_L\right]; \\ (J_L + 1)W_L, & if \;\; Q > (J_L + 1)W_L. \end{cases} \tag{12}$$

Note that if $J_S > n$, then it is optimal to use J_L +1 large trucks by Eq. (6). Consequently, the corresponding optimal cost can be determined by

$$TC\left(Q^*(J_L, J_S)\right) = \begin{cases} \dfrac{R}{Q^*(J_L, J_S)}K + \dfrac{hC}{2}Q^*(J_L, J_S) + RC + \dfrac{R}{Q^*(J_L, J_S)}(J_L C_L + J_S C_S), & if \; J_S \le n, \\ \dfrac{R}{Q^*(J_L, J_S)}K + \dfrac{hC}{2}Q^*(J_L, J_S) + RC + \dfrac{R}{Q^*(J_L, J_S)}(J_L + 1)C_L, & if \; J_S > n. \end{cases} \tag{13}$$

The optimal order size $Q^* = Q^*(J_L^*, J_S^*)$ and the corresponding optimal total cost is TC (Q^*), where

$$(J_L^*, J_S^*) = \text{argmin} \left\{ TC\left(Q^*(J_L, J_S) \middle| \, J_L = 0, 1, .., \left\lceil \frac{R}{W_L} \right\rceil; J_S = 1, \; 2, .., n+1 \right) \right\} \tag{14}$$

If $Q^* > R$ the optimal number of trucks required can be determined by

$$(J_L^*, J_S^*) = \text{argmin} \left\{ TC\left(Q^*(J_L, J_S) \middle| J_L = t\left\lceil \frac{R}{W_L} \right\rceil, t\left\lceil \frac{R}{W_L} \right\rceil + 1, .., (t+1)\left\lceil \frac{R}{W_L} \right\rceil; J_S = 1, \; 2, .., n+1 \right) \right\} \tag{15}$$

The discussion in section 3.2 indicates that if $J_S^* > n$, then it is optimal to use $J_L^* + 1$ large trucks. Therefore the optimal order size Q^* is given by

$$Q^*\ (J_L^*,\ J_S^*) = \begin{cases} \sqrt{\dfrac{2R(K + J_L^* C_L + J_S^* C_S)}{hC}}, & \text{if } J_S^* \le n, \\ \sqrt{\dfrac{2R(K + J_L^* C_L + C_L)}{hC}}, & \text{if } J_S^* > n. \end{cases} \tag{16}$$

And the optimal cost is given by

$$TC\left(Q^*(J_L^*, J_S^*)\right) = \begin{cases} \dfrac{R}{Q^*(J_L^*, J_S^*)} K + \dfrac{hC}{2} Q^*(J_L^*, J_S^*) + RC + \dfrac{R}{Q^*(J_L^*, J_S^*)} (J_L^* C_L + J_S^* C_S), & \text{if } J_S^* \le n, \\ \dfrac{R}{Q^*(J_L^*, J_S^*)} K + \dfrac{hC}{2} Q^*(J_L^*, J_S^*) + RC + \dfrac{R}{Q^*(J_L^*, J_S^*)} (J_L^* + 1) C_L, & \text{if } J_S^* > n. \end{cases} \tag{17}$$

3.5 Optimal Ordering Quantity Algorithm

The algorithm given below, gives a step-by-step approach for finding the optimum ordering quantity that minimizes the total annual logistics cost.

Algorithm A

START

For $J_L = 0, 1, 2, \ldots, \left\lceil \dfrac{R}{W_L} \right\rceil$.

For $J_S = 1, 2, \ldots, n, n+1$.

Compute $Q(J_L, J_S)$ by Eq. (10); and $Q^*(J_L, J_S)$ by Eq. (11), or (12)

Compute $TC\left(Q^*(J_L, J_S)\right)$ by Eq. (13);

End

End

Determine the optimal number of large trucks J_L^*, and small trucks J_S^* by Eq. (14), Q^* by (16) and $TC(Q^*)$ by (17).

STOP

3.6 Optimal Ordering Quantity Algorithm when $Q^* > R$

Algorithm A gives an optimal ordering quantity that is no more than the annual demand. In some cases, this may not be true. This section provides an optimal ordering quantity that may be more than R.

Algorithm B

Step 1: START

Step 2: Initialize t =0

Step 3: *For* $J_L = t\left\lceil \frac{R}{W_L} \right\rceil, t\left\lceil \frac{R}{W_L} \right\rceil + 1, \ldots\ldots, (t+1)\left\lceil \frac{R}{W_L} \right\rceil$

For $J_S = 1, 2, \ldots, n, n+1$.

Compute $Q(J_L, J_S)$ by Eq. (10); and $Q^*(J_L, J_S)$ by Eq. (11), or (12)

Compute $TC(Q^*(J_L, J_S))$ by Eq. (13);

End

End

Determine the optimal number of large trucks J_L^*, and small trucks J_S^* by Eq. (15), Q^* by (16) and $TC(Q^*)$ by (17).

Step 4: *If* $J_L^* = (t+1)\left\lceil \frac{R}{W_L} \right\rceil$ and $J_S^* = n+1$, go to Step 5, *Else* go to Step 6

Step 5: Increment *t* by 1 and go to Step 3

Step 6: STOP

Chapter 4 Quantity Discounts

4.1 Introduction

When specifying the material cost, the Algorithm A presented in Chapter 3 assumes that the unit cost is constant regardless of the quantity ordered. However, there are many instances in which the pricing schedule yields economies of scale, with prices decreasing as lot size is increased. This form of pricing is very common in business-to-business transactions.

Quantity discounts are generally provided for:

- Increasing the sales of the product
- Reducing the in-hand inventory, by increasing the sales
- Better production planning
- Lower order processing cost
- Reducing the transportation cost, by making use of the discounts offered by the trucking industry

A discount is lot size-based if the pricing schedule offers discounts based on the quantity ordered in a single lot. A discount is volume-based if the discount is based on the total quantity purchased over a given period (e.g. a year), regardless of the number of lots purchased over that period. Chapter 3 discusses the case of volume-based discount (i.e.

the annual demand R determines the unit price C). This chapter deals with the case of lot size-based discount.

Two commonly used lot-size based discount schemes are the following.

- All-unit quantity discounts
- Incremental quantity discounts

This chapter analyses the effect of lot size-based quantity discounts on the inventory policy. The main objective of this chapter is to present algorithms that identifies an ordering quantity Q that minimizes the total annual logistics cost, when all-unit quantity discounts and incremental quantity discounts are offered.

4.2 All-unit Quantity Discounts

In all-unit quantity discounts, the entire order is charged with the same unit price, which is, however, a function of the actual ordering quantity Q. In particular, the pricing schedule contains specified break points M_0, M_1, M_2, ..., M_{k-1}, M_k with $0 = M_0 < M_1 < M_2 < M_3 \ldots\ldots < M_k$, such that

$$Unit\ \ price\, C(Q) = \begin{cases} C_0, & if\ M_0 < Q \le M_1 \\ C_1, & if\ M_1 < Q \le M_2 \\ C_2, & if\ M_2 < Q \le M_3 \\ . & . \quad . \quad . \\ . & . \quad . \quad . \\ C_{k-1}, & if\ M_{k-1} < Q \le M_k \\ C_k, & if\ M_k < Q \end{cases} \tag{18}$$

where $C_0 > C_1 > \ldots\ldots > C_k$. In this case the material cost for ordering M_j+1 units may be smaller than that for ordering M_j units. In general, material cost of purchasing M_j units is greater than that of purchasing M_j+1 units, if $M_j > \frac{C_j}{C_{j-1} - C_j}, \forall j \geq 1$. If the ordering quantity falls in the interval (M_j, M_{j+1}], then the unit cost is C_j.

4.2.1 Optimal Ordering Quantity Algorithm for All-unit Quantity Discounts

The transportation cost and the total annual logistics cost remains the same as discussed in Chapter 3. For convenience the transportation cost is restated here.

$$T(Q) = \begin{cases} J_L C_L + J_S C_S, & \text{if } J_S \leq n, \\ (J_L + 1) C_L, & \text{if } J_S > n. \end{cases} \tag{19}$$

The total annual logistics cost is computed by

$$TC(Q) = \frac{R}{Q} K + \frac{hC(Q)}{2} Q + RC(Q) + \frac{R}{Q} T(Q) \tag{20}$$

To find the optimal Q^*, we will consider all combinations of J_L and J_S. If, for the given J_L and J_S, there exists a single j ($0 \leq j \leq k$) such that $M_j \in (J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]$, then the unit material cost for any ordering quantity in the range $(J_L W_L + (J_S - 1) W_S, M_j]$ is set as C_{j-1}, and the unit material cost for any ordering quantity in the range $(M_j, J_L W_L + J_S W_S]$ is set as C_j.

There is no M_j such that $M_j \in (J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]$, for the given J_L and J_S. Then the unit material cost C_j for all $Q \in (J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]$, is determined by finding the largest value of j that satisfies the condition $M_j \leq J_L W_L + (J_S - 1) W_S$.

The third case is that there are more than one M_j that belongs to $(J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]$. Without loss of generality, assume M_j, M_{j+1}, …., M_{j+g} belong to $(J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]$, where $g \geq 1$. Then each interval $(J_L W_L + (J_S - 1) W_S, M_j]$, $(M_j, M_{j+1}]$,

$(M_{j+1}, M_{j+2}]$, ….,$(M_{j+g}, J_L W_L + J_S W_S]$ is considered separately and the corresponding unit material cost for each interval is C_{j-1}, C_j, C_{j+1}, …., C_{j+g}, respectively.

The procedure for obtaining the optimal ordering quantity remains the same as Algorithm A, except for the unit cost structure. The following algorithm identifies the unit material cost C_j for any given J_L and J_S, and then finds the optimal ordering quantity.

Algorithm C

START

For $J_L = 0, 1, 2, ..., \left\lceil \frac{R}{W_L} \right\rceil$.

For $J_S = 1, 2, ..., n, n+1$.

If $J_S \le n$

If $M_j \notin (J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S]$, for all $j = 1, 2, …., k$.

Find the largest j such that $M_j \le J_L W_L + (J_S - 1)W_S$ and set unit price as C_j and compute $Q_j(J_L, J_S)$, $Q^*(J_L, J_S)$ and $TC(Q^*(J_L, J_s))$, by

$$Q_j(J_L, J_s) = \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC_j}} .$$

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + (J_S - 1)W_S + 1, & \text{if } Q_j(J_L, J_S) \le J_L W_L + (J_S - 1)W_S; \\ Q_j(J_L, J_S), & \text{if } Q_j(J_L, J_S) \in (J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S]; \\ J_L W_L + J_S W_S, & \text{if } Q_j(J_L, J_S) > J_L W_L + J_S W_S. \end{cases}$$

$$TC(Q^*(J_L, J_s)) = \frac{R}{Q^*(J_L, J_s)} K + \frac{hC_j}{2} Q^*(J_L, J_s) + RC_j + \frac{R}{Q^*(J_L, J_s)} (J_L C_L + J_S C_S).$$

Else there exists some $j \ge 1$ and $g \ge 0$ such that $M_j, M_{j+1}, …., M_{j+g} \in (J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S,]$, $M_{j-1} \le J_L W_L + (J_S - 1)W_S$, and $M_{j+g+1} > J_L W_L + J_S W_S$.

Then consider each interval $(J_L W_L + (J_S - 1)W_S, M_j]$, $(M_j, M_{j+1}]$, ..., $(M_{j+g}, J_L W_L + J_S W_S]$ separately as follows.

For all $Q \in (J_L W_L + (J_S - 1)W_S, M_j]$, set unit price as C_{j-1} and compute $Q_{j-1}(J_L, J_S)$, $Q^*_{j-1}(J_L, J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC_{j-1}}} .$$

$$Q_{j-1}^*(J_L,J_S)=\begin{cases} J_LW_L+(J_S-1)W_S+1, & \text{if } Q_{j-1}(J_L,J_S)\le J_LW_L+(J_S-1)W_S; \\ Q_{j-1}(J_L,J_S), & \text{if } Q_{j-1}(J_L,J_S)\in (J_LW_L+(J_S-1)W_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L,J_S)>M_j. \end{cases}$$

$$TC_{j-1}(Q^*)=\frac{R}{Q_{j-1}^*(J_L,J_S)}K+\frac{hC_{j-1}}{2}Q_{j-1}^*(J_L,J_S)+RC_{j-1}+\frac{R}{Q_{j-1}^*(J_L,J_S)}(J_LC_L+J_SC_S).$$

For all $Q\in(M_i, M_{i+1}]$, $i = j, j+1, \ldots, j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L,J_S)$, $Q_i^*(J_L,J_S)$ and $TC_i(Q^*)$, by

$$Q_i(J_L,J_S)=\sqrt{\frac{2R\,(K+J_LC_L+J_SC_S)}{hC_i}}\ .$$

$$Q_i^*(J_L,J_S)=\begin{cases} M_i+1, & \text{if } Q_i(J_L,J_S)\le M_i; \\ Q_i(J_L,J_S), & \text{if } Q_i(J_L,J_S)\in (M_i,M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L,J_S)>M_{i+1}. \end{cases}$$

$$TC_i(Q^*)=\frac{R}{Q_i^*(J_L,J_S)}K+\frac{hC_i}{2}Q_i^*(J_L,J_S)+RC_i+\frac{R}{Q_i^*(J_L,J_S)}(J_LC_L+J_SC_S).$$

For all $Q\in(M_{j+g}, J_LW_L+J_SW_S]$, set unit price as C_{j+g} and compute $Q_{j+g}(J_L,J_S)$, $Q_{j+g}^*(J_L,J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L,J_S)=\sqrt{\frac{2R(K+J_LC_L+J_SC_S)}{hC_{j+g}}}\ .$$

$$Q_{j+g}^*(J_L,J_S)=\begin{cases} M_{j+g}+1, & \text{if } Q_{j+g}(J_L,J_S)\le M_{j+g}; \\ Q_{j+g}(J_L,J_S), & \text{if } Q_{j+g}(J_L,J_S)\in (M_{j+g},J_LW_L+J_SW_S]; \\ J_LW_L+J_SW_S, & \text{if } Q_{j+g}(J_L,J_S)>J_LW_L+J_SW_S. \end{cases}$$

$$TC_{j+g}(Q^*)=\frac{R}{Q_{j+g}^*(J_L,J_S)}K+\frac{hC_{j+g}}{2}Q_{j+g}^*(J_L,J_S)+RC_{j+g}+\frac{R}{Q_{j+g}^*(J_L,J_S)}(J_LC_L+J_SC_S).$$

Find u = *argmin* $\{TC_i(Q^*) \mid i=j-1, j, j+1, \ldots, j+g\}$, and set $Q^*(J_L,J_S)=Q_u^*(J_L,J_S)$ and $TC(Q^*(J_L,J_S))=TC_u(Q^*)$.

Else If $J_S > n$

If $M_j \notin (J_LW_L+nW_S, (J_L+1)W_L]$, for all $j = 1, 2, \ldots, k$.

Find the largest j such that $M_j \le J_LW_L+nW_S$ and set unit price as C_j and compute $Q_j(J_L,J_S)$, $Q^*(J_L,J_S)$ and $TC(Q^*(J_L,J_S))$, by

$$Q_j(J_L,J_S)=\sqrt{\frac{2R(K+J_LC_L+C_L)}{hC_j}}\ .$$

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + nW_S + 1, & \text{if } Q_j(J_L, J_S) \le J_L W_L + nW_S; \\ Q_j(J_L, J_S), & \text{if } Q_j(J_L, J_S) \in (J_L W_L + nW_S, (J_L + 1)W_L]; \\ (J_L + 1)W_L, & \text{if } Q_j(J_L, J_S) > (J_L + 1)W_L. \end{cases}$$

$$TC(Q^*(J_L, J_S)) = \frac{R}{Q^*(J_L, J_S)} K + \frac{hC_j}{2} Q^*(J_L, J_S) + RC_j + \frac{R}{Q^*(J_L, J_S)} (J_L + 1)C_L.$$

Else there exists some $j \ge 1$ and $g \ge 0$ such that M_j, M_{j+1}, …., $M_{j+g} \in$ $(J_L W_L + n W_S$, $(J_L +1) W_L]$, $M_{j-1} \le J_L W_L + nW_S$,and $M_{j+g+1} > (J_L + 1)W_L$.

Then consider each interval $(J_L W_L + nW_S, M_j]$, $(M_j, M_{j+1}]$, …., $(M_{j+g}, (J_L+1)W_L]$ separately as follows.

For all $Q \in$ $(J_L W_L + nW_S, M_j]$, set unit price as C_{j-1} and compute $Q_{j-1}(J_L, J_S)$, $Q^*_{j-1}(J_L, J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_{j-1}}}.$$

$$Q^*_{j-1}(J_L, J_S) = \begin{cases} J_L W_L + nW_S + 1, & \text{if } Q_{j-1}(J_L, J_S) \le J_L W_L + nW_S; \\ Q_{j-1}(J_L, J_S), & \text{if } Q_{j-1}(J_L, J_S) \in (J_L W_L + nW_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L, J_S) > M_j. \end{cases}$$

$$TC_{j-1}(Q^*) = \frac{R}{Q^*_{j-1}(J_L, J_S)} K + \frac{hC_{j-1}}{2} Q^*_{j-1}(J_L, J_S) + RC_{j-1} + \frac{R}{Q^*_{j-1}(J_L, J_S)} (J_L + 1)C_L.$$

For all $Q \in (M_i, M_{i+1}]$, $i = j, j+1, …., j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L, J_S)$, $Q^*_i(J_L, J_s)$ and $TC_i(Q^*)$, by

$$Q_i(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_i}}.$$

$$Q^*_i(J_L, J_S) = \begin{cases} M_i + 1, & \text{if } Q_i(J_L, J_S) \le M_i; \\ Q_i(J_L, J_S), & \text{if } Q_i(J_L, J_S) \in (M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L, J_S) > M_{i+1}. \end{cases}$$

$$TC_i(Q^*) = \frac{R}{Q^*_i(J_L, J_S)} K + \frac{hC_i}{2} Q^*_i(J_L, J_S) + RC_i + \frac{R}{Q^*_i(J_L, J_S)} (J_L + 1)C_L.$$

For all $Q \in (M_{j+g}, (J_L + 1)W_L]$, set unit price as C_{j+g} and compute $Q_{j+g}(J_L, J_S)$, $Q^*_{j+g}(J_L, J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_{j+g}}}.$$

$$Q^*_{j+g}(J_L,J_S)=\begin{cases} M_{j+g}+1, & \text{if } Q_{j+g}(J_L,J_S)\le M_{j+g};\\ Q_{j+g}(J_L,J_S), & \text{if } Q_{j+g}(J_L,J_S)\in \left(M_{j+g},(J_L+1)W_L\right];\\ (J_L+1)W_L, & \text{if } Q_{j+g}(J_L,J_S)>(J_L+1)W_L. \end{cases}$$

$$TC_{j+g}(Q^*)=\frac{R}{Q^*_{j+g}(J_L,J_S)}K+\frac{hC_{j+g}}{2}Q^*_{j+g}(J_L,J_S)+RC_{j+g}+\frac{R}{Q^*_{j+g}(J_L,J_S)}(J_L+1)C_L.$$

Find u = $argmin\{TC_i(Q^*)\,|\,i=j-1,j,j+1,....,j+g\}$, and set $Q^*(J_L,J_S)$ = $Q^*_u(J_L,J_S)$ and $TC\left(Q^*(J_L,J_S)\right)$ = $TC_u(Q^*)$.

End

End

Optimal (J_L^*,J_S^*) = $argmin\left\{TC\left(Q^*(J_L,J_S)\right)\Big|J_L=0,1,....,\left\lceil\frac{R}{W_L}\right\rceil,J_S=1,2,....,n,n+1\right\}$, $Q^*=Q^*\left(J_L^*,J_S^*\right)$ and $TC(Q^*)=TC\left(Q^*(J_L^*,J_S^*)\right)$.

STOP

4.2.2 Optimal Ordering Quantity Algorithm for All-unit Quantity Discounts when $Q^* > R$

Algorithm C may not be true if optimal ordering quantity is more than the annual demand *R*. This section provides an optimal ordering quantity that may be more than *R*, when all-unit quantity discounts is offered.

Algorithm D

Step 1. START

Step 2. Initialize t = 0.

Step 3. *For* $J_L=t\left\lceil\frac{R}{W_L}\right\rceil,t\left\lceil\frac{R}{W_L}\right\rceil+1,.......,(t+1)\left\lceil\frac{R}{W_L}\right\rceil$.

For J_S = 1, 2, ..., *n*, *n*+1.

If $J_S\le n$

If $M_j\notin\left(J_LW_L+(J_S-1)W_S,J_LW_L+J_SW_S\right]$, for all j = 1, 2, …., *k*.

Find the largest j such that $M_j \le J_L W_L + (J_S - 1) W_S$ and set unit price as C_j and compute $Q_j(J_L, J_S)$, $Q^*(J_L, J_S)$ and $TC(Q^*(J_L, J_S))$, by

$$Q_j(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC_j}} .$$

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + (J_S - 1) W_S + 1, & \text{if } Q_j(J_L, J_S) \le J_L W_L + (J_S - 1) W_S; \\ Q_j(J_L, J_S), & \text{if } Q_j(J_L, J_S) \in (J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]; \\ J_L W_L + J_S W_S, & \text{if } Q_j(J_L, J_S) > J_L W_L + J_S W_S. \end{cases}$$

$$TC(Q^*(J_L, J_S)) = \frac{R}{Q^*(J_L, J_S)} K + \frac{hC_j}{2} Q^*(J_L, J_S) + RC_j + \frac{R}{Q^*(J_L, J_S)} (J_L C_L + J_S C_S).$$

Else there exists some $j \ge 1$ and $g \ge 0$ such that $M_j, M_{j+1}, \ldots, M_{j+g} \in (J_L W_L + (J_S - 1) W_S, J_L W_L + J_S W_S]$, $M_{j-1} \le J_L W_L + (J_S - 1) W_S$, and $M_{j+g+1} > J_L W_L + J_S W_S$.

Then consider each interval $(J_L W_L + (J_S - 1) W_S, M_j]$, $(M_j, M_{j+1}]$, ..., $(M_{j+g}, J_L W_L + J_S W_S]$ separately as follows.

For all $Q \in (J_L W_L + (J_S - 1) W_S, M_j]$, set unit price as C_{j-1} and compute $Q_{j-1}(J_L, J_S)$, $Q^*_{j-1}(J_L, J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC_{j-1}}} .$$

$$Q^*_{j-1}(J_L, J_S) = \begin{cases} J_L W_L + (J_S - 1) W_S + 1, & \text{if } Q_{j-1}(J_L, J_S) \le J_L W_L + (J_S - 1) W_S; \\ Q_{j-1}(J_L, J_S), & \text{if } Q_{j-1}(J_L, J_S) \in (J_L W_L + (J_S - 1) W_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L, J_S) > M_j. \end{cases}$$

$$TC_{j-1}(Q^*) = \frac{R}{Q^*_{j-1}(J_L, J_S)} K + \frac{hC_{j-1}}{2} Q^*_{j-1}(J_L, J_S) + RC_{j-1} + \frac{R}{Q^*_{j-1}(J_L, J_S)} (J_L C_L + J_S C_S).$$

For all $Q \in (M_i, M_{i+1}]$, $i = j, j+1, \ldots, j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L, J_S)$, $Q^*_i(J_L, J_S)$ and $TC_i(Q^*)$, by

$$Q_i(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC_i}} .$$

$$Q^*_i(J_L, J_S) = \begin{cases} M_i + 1, & \text{if } Q_i(J_L, J_S) \le M_i; \\ Q_i(J_L, J_S), & \text{if } Q_i(J_L, J_S) \in (M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L, J_S) > M_{i+1}. \end{cases}$$

$$TC_i(Q^*) = \frac{R}{Q^*_i(J_L, J_S)} K + \frac{hC_i}{2} Q^*_i(J_L, J_S) + RC_i + \frac{R}{Q^*_i(J_L, J_S)} (J_L C_L + J_S C_S).$$

For all $Q \in (M_{j+g}, J_L W_L + J_S W_S]$, set unit price as C_{j+g} and compute $Q_{j+g}(J_L, J_S)$, $Q^*_{j+g}(J_L, J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + J_S C_S)}{hC_{j+g}}} .$$

$$Q^*_{j+g}(J_L, J_S) = \begin{cases} M_{j+g} + 1, & \text{if } Q_{j+g}(J_L, J_S) \le M_{j+g}; \\ Q_{j+g}(J_L, J_S), & \text{if } Q_{j+g}(J_L, J_S) \in (M_{j+g}, J_L W_L + J_S W_S]; \\ J_L W_L + J_S W_S, & \text{if } Q_{j+g}(J_L, J_S) > J_L W_L + J_S W_S. \end{cases}$$

$$TC_{j+g}(Q^*) = \frac{R}{Q^*_{j+g}(J_L, J_S)} K + \frac{hC_{j+g}}{2} Q^*_{j+g}(J_L, J_S) + RC_{j+g} + \frac{R}{Q^*_{j+g}(J_L, J_S)} (J_L C_L + J_S C_S).$$

Find $u = argmin \{TC_i(Q^*) | i = j-1, j, j+1, ..., j+g\}$, and set $Q^*(J_L, J_S) = Q^*_u(J_L, J_S)$ and $TC(Q^*(J_L, J_S)) = TC_u(Q^*)$.

Else If $J_S > n$

If $M_j \notin (J_L W_L + nW_S, (J_L + 1)W_L]$, for all $j = 1, 2, ..., k$.

Find the largest j such that $M_j \le J_L W_L + nW_S$ and set unit price as C_j and compute $Q_j(J_L, J_S)$, $Q^*(J_L, J_S)$ and $TC(Q^*(J_L, J_S))$, by

$$Q_j(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_j}} .$$

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + nW_S + 1, & \text{if } Q_j(J_L, J_S) \le J_L W_L + nW_S; \\ Q_j(J_L, J_S), & \text{if } Q_j(J_L, J_S) \in (J_L W_L + nW_S, (J_L + 1)W_L]; \\ (J_L + 1)W_L, & \text{if } Q_j(J_L, J_S) > (J_L + 1)W_L. \end{cases}$$

$$TC(Q^*(J_L, J_S)) = \frac{R}{Q^*(J_L, J_S)} K + \frac{hC_j}{2} Q^*(J_L, J_S) + RC_j + \frac{R}{Q^*(J_L, J_S)} (J_L + 1)C_L.$$

Else there exists some $j \ge 1$ and $g \ge 0$ such that $M_j, M_{j+1}, ..., M_{j+g} \in (J_L W_L + n W_S, (J_L + 1) W_L]$, $M_{j-1} \le J_L W_L + nW_S$, and $M_{j+g+1} > (J_L + 1)W_L$.

Then consider each interval $(J_L W_L + nW_S, M_j]$, $(M_j, M_{j+1}]$, ..., $(M_{j+g}, (J_L+1)W_L]$ separately as follows.

For all $Q \in (J_L W_L + nW_S, M_j]$, set unit price as C_{j-1} and compute $Q_{j-1}(J_L, J_S)$, $Q^*_{j-1}(J_L, J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_{j-1}}} .$$

$$Q^*_{j-1}(J_L, J_S) = \begin{cases} J_L W_L + nW_S + 1, & \text{if } Q_{j-1}(J_L, J_S) \le J_L W_L + nW_S; \\ Q_{j-1}(J_L, J_S), & \text{if } Q_{j-1}(J_L, J_S) \in (J_L W_L + nW_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L, J_S) > M_j. \end{cases}$$

$$TC_{j-1}(Q^*) = \frac{R}{Q^*_{j-1}(J_L, J_S)} K + \frac{hC_{j-1}}{2} Q^*_{j-1}(J_L, J_S) + RC_{j-1} + \frac{R}{Q^*_{j-1}(J_L, J_S)} (J_L + 1)C_L.$$

For all $Q \in (M_i, M_{i+1}]$, $i = j, j+1, \ldots, j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L, J_S)$, $Q^*_i(J_L, J_s)$ and $TC_i(Q^*)$, by

$$Q_i(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_i}}.$$

$$Q^*_i(J_L, J_S) = \begin{cases} M_i + 1, & \text{if } Q_i(J_L, J_S) \le M_i; \\ Q_i(J_L, J_S), & \text{if } Q_i(J_L, J_S) \in (M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L, J_S) > M_{i+1}. \end{cases}$$

$$TC_i(Q^*) = \frac{R}{Q^*_i(J_L, J_S)} K + \frac{hC_i}{2} Q^*_i(J_L, J_S) + RC_i + \frac{R}{Q^*_i(J_L, J_S)} (J_L + 1)C_L.$$

For all $Q \in (M_{j+g}, (J_L + 1)W_L]$, set unit price as C_{j+g} and compute $Q_{j+g}(J_L, J_S)$, $Q^*_{j+g}(J_L, J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L, J_S) = \sqrt{\frac{2R(K + J_L C_L + C_L)}{hC_{j+g}}}.$$

$$Q^*_{j+g}(J_L, J_S) = \begin{cases} M_{j+g} + 1, & \text{if } Q_{j+g}(J_L, J_S) \le M_{j+g}; \\ Q_{j+g}(J_L, J_S), & \text{if } Q_{j+g}(J_L, J_S) \in (M_{j+g}, (J_L + 1)W_L]; \\ (J_L + 1)W_L, & \text{if } Q_{j+g}(J_L, J_S) > (J_L + 1)W_L. \end{cases}$$

$$TC_{j+g}(Q^*) = \frac{R}{Q^*_{j+g}(J_L, J_S)} K + \frac{hC_{j+g}}{2} Q^*_{j+g}(J_L, J_S) + RC_{j+g} + \frac{R}{Q^*_{j+g}(J_L, J_S)} (J_L + 1)C_L.$$

Find u = *argmin* $\{TC_i(Q^*) \mid i = j-1, j, j+1, \ldots, j+g\}$, and set $Q^*(J_L, J_S)$ = $Q^*_u(J_L, J_S)$ and $TC(Q^*(J_L, J_S))$ = $TC_u(Q^*)$.

End

End

Optimal (J^*_L, J^*_S) = *argmin* $\left\{ TC(Q^*(J_L, J_S)) \middle| J_L = 0, 1, \ldots, \left\lceil \frac{R}{W_L} \right\rceil, J_S = 1, 2, \ldots, n, n+1 \right\}$, $Q^* = Q^*(J^*_L, J^*_S)$ and $TC(Q^*) = TC(Q^*(J^*_L, J^*_S))$.

Step 4. *If* $J^*_L = (t+1)\left\lceil \frac{R}{W_L} \right\rceil$ and $J^*_S = n+1$, go to Step 5, *Else* go to Step 6.

Step 5. Increment t by 1 and go to Step 3.

Step 6. STOP

4.3 Incremental Quantity Discounts

In the incremental quantity discounts, the unit material cost is incremental and varies with the break point quantities. As for the all-unit discount case, let M_j (j = 0, 1, …,k) represent the j-th break point in the pricing schedule, with $0 = M_0 < M_1 < M_2 .... < M_k$. Then, the incremental quantity discounts can be depicted as Figure 4.1. If the ordering quantity $Q \leq M_1$, the entire order is charged with unit price C_0; if $M_1 \leq Q \leq M_2$, then the unit price is C_0 for the first M_1 units and C_1 for the rest of the order; and so on in general, $C_0 > C_1 > ...... > C_k$.

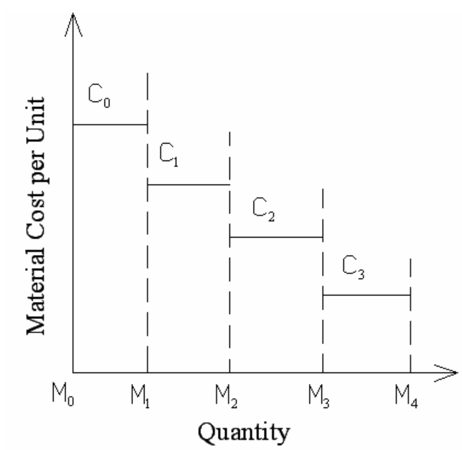

Figure 4.1 Unit Price with Incremental Quantity Discounts

For an order size of $Q \in (M_j, M_{j+1}]$, the material cost is $V_j + (Q - M_j)\, C_j$ where $V_j = V_{j-1} + (M_j - M_{j-1})\, C_{j-1}$, j = 1,2, ….,k., with $V_0 = 0$. Therefore the annual material cost, annual holding cost and annual logistics cost are given by, respectively,

$$\text{Annual material cost} = \frac{R}{Q}\left[V_j + \left(Q - M_j\right)C_j\right]. \tag{21}$$

Annual holding cost = $\left[V_j + \left(Q - M_j\right)C_j\right]\frac{h}{2}$. (22)

Annual logistics cost, $TC(Q) = \frac{R}{Q}K + \left[V_j + \left(Q - M_j\right)C_j\right]\frac{h}{2} + \frac{R}{Q}\left[V_j + \left(Q - M_j\right)C_j\right] + \frac{R}{Q}T(Q)$ (23)

substituting Eq. (19) in Eq. (23) yields

$$TC(Q) = \begin{cases} \frac{R}{Q}K + \left[V_j + \left(Q - M_j\right)C_j\right]\frac{h}{2} + \frac{R}{Q}\left[V_j + \left(Q - M_j\right)C_j\right] + \frac{R}{Q}(J_L C_L + J_S C_S), & \text{If } J_S \le n, \\ \frac{R}{Q}K + \left[V_j + \left(Q - M_j\right)C_j\right]\frac{h}{2} + \frac{R}{Q}\left[V_j + \left(Q - M_j\right)C_j\right] + \frac{R}{Q}(J_L + 1)C_L, & \text{If } J_S > n. \end{cases} \quad (24)$$

4.3.1 Optimal Ordering Quantity Algorithm for Incremental Quantity Discounts

The algorithm given below, gives a step-by-step approach for finding the optimal ordering quantity, that minimizes the total annual logistics cost, when incremental quantity discounts is offered by the supplier.

Algorithm E

START

For $J_L = 0, 1, 2, \ldots, \left\lceil \frac{R}{W_L} \right\rceil$.

For $J_S = 1, 2, \ldots\ldots, n, n+1$.

If $J_S \le n$

If $M_j \notin \left(J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S\right]$, for all $j = 1, 2, \ldots, k$.

Find the largest j such that $M_j \le J_L W_L + (J_S - 1)W_S$ and compute $Q_j(J_L, J_S)$, $Q^*(J_L, J_S)$ and $TC\left(Q^*(J_L, J_S)\right)$, by

$$Q_j\left(J_L, J_S\right) = \sqrt{\frac{2R(K + V_j - M_j C_j + J_L C_L + J_S C_S)}{hC_j}}.$$

$$Q^*(J_L,J_S)=\begin{cases} J_LW_L+(J_S-1)W_S+1, & \text{if } Q_j(J_L,J_S)\le J_LW_L+(J_S-1)W_S; \\ Q_j(J_L,J_S), & \text{if } Q_j(J_L,J_S)\in(J_LW_L+(J_S-1)W_S, J_LW_L+J_SW_S]; \\ J_LW_L+J_SW_S, & \text{if } Q_j(J_L,J_S)> J_LW_L+J_SW_S. \end{cases}$$

$$TC(Q^*(J_L,J_S))=\frac{R}{Q^*(J_L,J_S)}K+\left[V_j+\left(Q^*(J_L,J_S)-M_j\right)C_j\right]\frac{h}{2}+$$
$$\frac{R}{Q^*(J_L,J_S)}\left[V_j+\left(Q^*(J_L,J_S)-M_j\right)C_j\right]+\frac{R}{Q^*(J_L,J_S)}(J_LC_L+J_SC_S).$$

Else there exists some $j\ge 1$ and $g\ge 0$ such that M_j, M_{j+1}, …., $M_{j+g}\in$ $(J_L\ W_L+(J_S-1)\ W_S, J_L\ W_L+J_S\ W_S,]$, $M_{j-1}\le J_LW_L+(J_S-1)W_S$, and $M_{j+g+1}>J_LW_L+J_SW_S$.

Then consider each interval $(J_LW_L+(J_S\text{-}1)W_S, M_j]$, $(M_j, M_{j+1}]$, …., $(M_{j+g}, J_L\ W_L+J_S\ W_S]$ separately as follows.

For all $Q\in$ $(J_LW_L+(J_S\text{-}1)W_S, M_j]$, compute $Q_{j-1}(J_L,J_S)$, $Q^*_{j-1}(J_L,J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L,J_S)=\sqrt{\frac{2R(K+V_{j-1}-M_{j-1}C_{j-1}+J_LC_L+J_SC_S)}{hC_{j-1}}}.$$

$$Q^*_{j-1}(J_L,J_S)=\begin{cases} J_LW_L+(J_S-1)W_S+1, & \text{if } Q_{j-1}(J_L,J_S)\le J_LW_L+(J_S-1)W_S; \\ Q_{j-1}(J_L,J_S), & \text{if } Q_{j-1}(J_L,J_S)\in(J_LW_L+(J_S-1)W_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L,J_S)> M_j. \end{cases}$$

$$TC_{j-1}(Q^*)=\frac{R}{Q^*_{j-1}(J_L,J_S)}K+\left[V_{j-1}+\left(Q^*_{j-1}(J_L,J_S)-M_{j-1}\right)C_{j-1}\right]\frac{h}{2}+$$
$$\frac{R}{Q^*_{j-1}(J_L,J_S)}\left[V_{j-1}+\left(Q^*_{j-1}(J_L,J_S)-M_{j-1}\right)C_{j-1}\right]+\frac{R}{Q^*_{j-1}(J_L,J_S)}(J_LC_L+J_SC_S).$$

For all $Q\in(M_i, M_{i+1}]$, $i=j, j+1$, …., $j+g\text{-}1$, set unit price as C_i. Note that this case disappears if $g=0$. Compute $Q_i(J_L,J_S)$, $Q^*_i(J_L,J_S)$ and $TC_i(Q^*)$, by

$$Q_i(J_L,J_S)=\sqrt{\frac{2R(K+V_i-M_iC_i+J_LC_L+J_SC_S)}{hC_i}}.$$

$$Q^*_i(J_L,J_S)=\begin{cases} M_i+1, & \text{if } Q_i(J_L,J_S)\le M_i; \\ Q_i(J_L,J_S), & \text{if } Q_i(J_L,J_S)\in(M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L,J_S)> M_{i+1}. \end{cases}$$

$$TC_i(Q^*)=\frac{R}{Q^*_i(J_L,J_S)}K+\left[V_i+\left(Q^*_i(J_L,J_S)-M_i\right)C_i\right]\frac{h}{2}+$$
$$\frac{R}{Q^*_i(J_L,J_S)}\left[V_i+\left(Q^*_i(J_L,J_S)-M_i\right)C_i\right]+\frac{R}{Q^*_i(J_L,J_S)}(J_LC_L+J_SC_S).$$

For all $Q\in(M_{j+g}, J_LW_L+J_SW_S]$, compute $Q_{j+g}(J_L,J_S)$, $Q^*_{j+g}(J_L,J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L,J_S)=\sqrt{\frac{2R(K+V_{j+g}-M_{j+g}C_{j+g}+J_LC_L+J_SC_S)}{hC_{j+g}}}.$$

$$Q^*_{j+g}(J_L,J_S)=\begin{cases}M_{j+g}+1, & \text{if } Q_{j+g}(J_L,J_S)\le M_{j+g};\\ Q_{j+g}(J_L,J_S), & \text{if } Q_{j+g}(J_L,J_S)\in (M_{j+g},J_LW_L+J_SW_S];\\ J_LW_L+J_SW_S, & \text{if } Q_{j+g}(J_L,J_S)> J_LW_L+J_SW_S.\end{cases}$$

$$TC_{j+g}(Q^*)=\frac{R}{Q^*_{j+g}(J_L,J_S)}K+\left[V_{j+g}+\left(Q^*_{j+g}(J_L,J_S)-M_{j+g}\right)C_{j+g}\right]\frac{h}{2}+$$
$$\frac{R}{Q^*_{j+g}(J_L,J_S)}\left[V_{j+g}+\left(Q^*_{j+g}(J_L,J_S)-M_{j+g}\right)C_{j+g}\right]+\frac{R}{Q^*_{j+g}(J_L,J_S)}(J_LC_L+J_SC_S).$$

Find u = *argmin* $\{TC_i(Q^*)|\, i=j-1,j,j+1,....,j+g\}$, and set $Q^*(J_L,J_S)$ = $Q^*_u(J_L,J_S)$ and $TC(Q^*(J_L,J_S))=TC_u(Q^*)$.

Else If $J_S>n$

If $M_j\notin(J_LW_L+nW_S,(J_L+1)W_L]$, for all j = 1, 2, ….., k.

Find the largest j such that $M_j\le J_LW_L+nW_S$ and compute $Q_j(J_L,J_S)$, $Q^*(J_L,J_S)$ and $TC(Q^*(J_L,J_S))$, by

$$Q_j(J_L,J_S)=\sqrt{\frac{2R(K+V_j-M_jC_j+J_LC_L+C_L)}{hC_j}}.$$

$$Q^*(J_L,J_S)=\begin{cases}J_LW_L+nW_S+1, & \text{if } Q_j(J_L,J_S)\le J_LW_L+nW_S;\\ Q_j(J_L,J_S), & \text{if } Q_j(J_L,J_S)\in (J_LW_L+nW_S,(J_L+1)W_L];\\ (J_L+1)W_L, & \text{if } Q_j(J_L,J_S)> (J_L+1)W_L.\end{cases}$$

$$TC(Q^*(J_L,J_S))=\frac{R}{Q^*(J_L,J_S)}K+\left[V_j+\left(Q^*(J_L,J_S)-M_j\right)C_j\right]\frac{h}{2}+\frac{R}{Q}\left[V_j+\left(Q^*(J_L,J_S)-M_j\right)C_j\right]+$$
$$\frac{R}{Q^*(J_L,J_S)}(J_L+1)C_L.$$

Else there exists some $j\ge 1$ and $g\ge 0$ such that M_j, M_{j+1}, …., $M_{j+g}\in(J_L W_L + n W_S, (J_L + 1) W_L]$, $M_{j-1}\le J_LW_L+nW_S$, and $M_{j+g+1}>(J_L+1)W_L$.

Then consider each interval $(J_LW_L + nW_S, M_j]$, $(M_j, M_{j+1}]$, …., $(M_{j+g}, (J_L+1)W_L]$ separately as follows.

For all $Q\in(J_LW_L + nW_S, M_j]$, compute $Q_{j-1}(J_L,J_S)$, $Q^*_{j-1}(J_L,J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L,J_S)=\sqrt{\frac{2R(K+V_{j-1}-M_{j-1}C_{j-1}+J_LC_L+C_L)}{hC_{j-1}}}.$$

$$Q_{j-1}^*(J_L,J_S)=\begin{cases} J_LW_L+nW_S+1, & \text{if } Q_{j-1}(J_L,J_S)\le J_LW_L+nW_S; \\ Q_{j-1}(J_L,J_S), & \text{if } Q_{j-1}(J_L,J_S)\in (J_LW_L+nW_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L,J_S)>M_j. \end{cases}$$

$$TC_{j-1}(Q^*)=\frac{R}{Q_{j-1}^*(J_L,J_S)}K+\left[V_{j-1}+\left(Q_{j-1}^*(J_L,J_S)-M_{j-1}\right)C_{j-1}\right]\frac{h}{2}+$$
$$\frac{R}{Q_{j-1}^*(J_L,J_S)}\left[V_{j-1}+\left(Q_{j-1}^*(J_L,J_S)-M_{j-1}\right)C_{j-1}\right]+\frac{R}{Q_{j-1}^*(J_L,J_S)}(J_L+1)C_L.$$

For all $Q\in(M_i, M_{i+1}]$, $i = j, j+1, \ldots, j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L,J_S)$, $Q_i^*(J_L,J_S)$ and $TC_i(Q^*)$, by

$$Q_i(J_L,J_S)=\sqrt{\frac{2R\,(K+V_i-M_iC_i+J_LC_L+C_L)}{hC_i}}\,.$$

$$Q_i^*(J_L,J_S)=\begin{cases} M_i+1, & \text{if } Q_i(J_L,J_S)\le M_i; \\ Q_i(J_L,J_S), & \text{if } Q_i(J_L,J_S)\in (M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L,J_S)>M_{i+1}. \end{cases}$$

$$TC_i(Q^*)=\frac{R}{Q_i^*(J_L,J_S)}K+\left[V_i+\left(Q_i^*(J_L,J_S)-M_i\right)C_i\right]\frac{h}{2}+$$
$$\frac{R}{Q_i^*(J_L,J_S)}\left[V_i+\left(Q_i^*(J_L,J_S)-M_i\right)C_i\right]+\frac{R}{Q_i^*(J_L,J_S)}(J_L+1)C_L.$$

For all $Q\in(M_{j+g},(J_L+1)W_L]$, compute $Q_{j+g}(J_L,J_S)$, $Q_{j+g}^*(J_L,J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L,J_S)=\sqrt{\frac{2R(K+V_{j+g}-M_{j+g}C_{j+g}+J_LC_L+C_L)}{hC_{j+g}}}\,.$$

$$Q_{j+g}^*(J_L,J_S)=\begin{cases} M_{j+g}+1, & \text{if } Q_{j+g}(J_L,J_S)\le M_{j+g}; \\ Q_{j+g}(J_L,J_S), & \text{if } Q_{j+g}(J_L,J_S)\in (M_{j+g},(J_L+1)W_L]; \\ (J_L+1)W_L, & \text{if } Q_{j+g}(J_L,J_S)>J_LW_L+J_SW_S. \end{cases}$$

$$TC_{j+g}(Q^*)=\frac{R}{Q_{j+g}^*(J_L,J_S)}K+\left[V_{j+g}+\left(Q_{j+g}^*(J_L,J_S)-M_{j+g}\right)C_{j+g}\right]\frac{h}{2}+$$
$$\frac{R}{Q_{j+g}^*(J_L,J_S)}\left[V_{j+g}+\left(Q_{j+g}^*(J_L,J_S)-M_{j+g}\right)C_{j+g}\right]+\frac{R}{Q_{j+g}^*(J_L,J_S)}(J_L+1)C_L.$$

Find $u = argmin\{TC_i(Q^*)\,|\, i=j-1, j, j+1, \ldots, j+g\}$, and set $Q^*(J_L,J_S) = Q_u^*(J_L,J_S)$ and $TC(Q^*(J_L,J_S)) = TC_u(Q^*)$.

End

End

Optimal $\left(J_L^*, J_S^*\right) = argmin \left\{TC\left(Q^*(J_L, J_S)\right) \middle| J_L = 0, 1, ..., \left\lceil \frac{R}{W_L} \right\rceil, J_S = 1, 2, ..., n, n+1\right\}$, $Q^* = Q^*\left(J_L^*, J_S^*\right)$ and $TC(Q^*) = TC\left(Q^*(J_L^*, J_S^*)\right)$.

STOP

4.3.2 Optimal Ordering Quantity Algorithm for Incremental Quantity Discounts when $Q^* > R$

Algorithm E may not be true for some cases where optimal ordering quantity is more than the demand. This section provides an optimal ordering quantity that may be more than R, when incremental quantity discounts is offered.

Algorithm F

Step 1. START

Step 2. Initialize t =0.

Step 3. *For* $J_L = 0, 1, 2, ..., \left\lceil \frac{R}{W_L} \right\rceil$.

For $J_S = 1, 2, ..., n, n+1$.

If $J_S \le n$

If $M_j \notin \left(J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S\right]$, for all $j = 1, 2, ..., k$.

Find the largest j such that $M_j \le J_L W_L + (J_S - 1)W_S$ and compute $Q_j(J_L, J_S)$, $Q^*(J_L, J_S)$ and $TC\left(Q^*(J_L, J_S)\right)$, by

$$Q_j(J_L, J_S) = \sqrt{\frac{2R(K + V_j - M_j C_j + J_L C_L + J_S C_S)}{hC_j}}.$$

$$Q^*(J_L, J_S) = \begin{cases} J_L W_L + (J_S - 1)W_S + 1, & if\ Q_j(J_L, J_S) \le J_L W_L + (J_S - 1)W_S; \\ Q_j(J_L, J_S), & if\ Q_j(J_L, J_S) \in \left(J_L W_L + (J_S - 1)W_S, J_L W_L + J_S W_S\right]; \\ J_L W_L + J_S W_S, & if\ Q_j(J_L, J_S) > J_L W_L + J_S W_S. \end{cases}$$

$$TC\left(Q^*(J_L, J_S)\right) = \frac{R}{Q^*(J_L, J_S)}K + \left[V_j + \left(Q^*(J_L, J_S) - M_j\right)C_j\right]\frac{h}{2} + \frac{R}{Q^*(J_L, J_S)}\left[V_j + \left(Q^*(J_L, J_S) - M_j\right)C_j\right] + \frac{R}{Q^*(J_L, J_S)}(J_L C_L + J_S C_S).$$

Else there exists some $j \geq 1$ and $g \geq 0$ such that M_j, M_{j+1}, …., $M_{j+g} \in$ (J_L W_L + (J_S – 1) W_S, J_L W_L + $J_S W_S$], $M_{j-1} \leq J_L W_L + (J_S - 1)W_S$, and $M_{j+g+1} > J_L W_L + J_S W_S$.

Then consider each interval ($J_L W_L$ + (J_S-1)W_S, M_j], (M_j, M_{j+1}], ..., (M_{j+g}, J_L W_L +J_S W_S] separately as follows.

For all $Q \in$ ($J_L W_L$ + (J_S-1)W_S, M_j], compute $Q_{j-1}(J_L, J_S), Q^*_{j-1}(J_L, J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L, J_S) = \sqrt{\frac{2R(K + V_{j-1} - M_{j-1}C_{j-1} + J_L C_L + J_S C_S)}{hC_{j-1}}} .$$

$$Q^*_{j-1}(J_L, J_S) = \begin{cases} J_L W_L + (J_S - 1)W_S + 1, & \text{if } Q_{j-1}(J_L, J_S) \leq J_L W_L + (J_S - 1)W_S; \\ Q_{j-1}(J_L, J_S), & \text{if } Q_{j-1}(J_L, J_S) \in (J_L W_L + (J_S - 1)W_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L, J_S) > M_j. \end{cases}$$

$$TC_{j-1}(Q^*) = \frac{R}{Q^*_{j-1}(J_L, J_S)} K + \left[V_{j-1} + \left(Q^*_{j-1}(J_L, J_S) - M_{j-1}\right)C_{j-1}\right]\frac{h}{2} + \frac{R}{Q^*_{j-1}(J_L, J_S)}\left[V_{j-1} + \left(Q^*_{j-1}(J_L, J_S) - M_{j-1}\right)C_{j-1}\right] + \frac{R}{Q^*_{j-1}(J_L, J_S)}(J_L C_L + J_S C_S).$$

For all $Q \in (M_i, M_{i+1}]$, $i = j, j+1, \ldots, j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L, J_S), Q^*_i(J_L, J_S)$ and $TC_i(Q^*)$, by

$$Q_i(J_L, J_S) = \sqrt{\frac{2R(K + V_i - M_i C_i + J_L C_L + J_S C_S)}{hC_i}} .$$

$$Q^*_i(J_L, J_S) = \begin{cases} M_i + 1, & \text{if } Q_i(J_L, J_S) \leq M_i; \\ Q_i(J_L, J_S), & \text{if } Q_i(J_L, J_S) \in (M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L, J_S) > M_{i+1}. \end{cases}$$

$$TC_i(Q^*) = \frac{R}{Q^*_i(J_L, J_S)} K + \left[V_i + \left(Q^*_i(J_L, J_S) - M_i\right)C_i\right]\frac{h}{2} + \frac{R}{Q^*_i(J_L, J_S)}\left[V_i + \left(Q^*_i(J_L, J_S) - M_i\right)C_i\right] + \frac{R}{Q^*_i(J_L, J_S)}(J_L C_L + J_S C_S).$$

For all $Q \in (M_{j+g}, J_L W_L + J_S W_S]$, compute $Q_{j+g}(J_L, J_S), Q^*_{j+g}(J_L, J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L, J_S) = \sqrt{\frac{2R(K + V_{j+g} - M_{j+g}C_{j+g} + J_L C_L + J_S C_S)}{hC_{j+g}}} .$$

$$Q^*_{j+g}(J_L, J_S) = \begin{cases} M_{j+g} + 1, & \text{if } Q_{j+g}(J_L, J_S) \leq M_{j+g}; \\ Q_{j+g}(J_L, J_S), & \text{if } Q_{j+g}(J_L, J_S) \in (M_{j+g}, J_L W_L + J_S W_S]; \\ J_L W_L + J_S W_S, & \text{if } Q_{j+g}(J_L, J_S) > J_L W_L + J_S W_S. \end{cases}$$

$$TC_{j+g}(Q^*) = \frac{R}{Q^*_{j+g}(J_L,J_S)}K + \left[V_{j+g} + \left(Q^*_{j+g}(J_L,J_S) - M_{j+g}\right)C_{j+g}\right]\frac{h}{2} + \frac{R}{Q^*_{j+g}(J_L,J_S)}\left[V_{j+g} + \left(Q^*_{j+g}(J_L,J_S) - M_{j+g}\right)C_{j+g}\right] + \frac{R}{Q^*_{j+g}(J_L,J_S)}(J_L C_L + J_S C_S).$$

Find u = *argmin* $\{TC_i(Q^*) | i = j-1, j, j+1, ..., j+g\}$, and set $Q^*(J_L, J_S)$ = $Q^*_u(J_L, J_S)$ and $TC(Q^*(J_L, J_S))$ = $TC_u(Q^*)$.

Else If $J_S > n$

If $M_j \notin (J_L W_L + nW_S, (J_L+1)W_L]$, for all j = 1, 2, ….., k.

Find the largest j such that $M_j \le J_L W_L + nW_S$ and compute $Q_j(J_L, J_S)$, $Q^*(J_L, J_S)$ and $TC(Q^*(J_L, J_S))$, by

$$Q_j(J_L,J_S) = \sqrt{\frac{2R(K + V_j - M_j C_j + J_L C_L + C_L)}{hC_j}}.$$

$$Q^*(J_L,J_S) = \begin{cases} J_L W_L + nW_S + 1, & \text{if } Q_j(J_L,J_S) \le J_L W_L + nW_S; \\ Q_j(J_L,J_S), & \text{if } Q_j(J_L,J_S) \in (J_L W_L + nW_S, (J_L+1)W_L]; \\ (J_L+1)W_L, & \text{if } Q_j(J_L,J_S) > (J_L+1)W_L. \end{cases}$$

$$TC(Q^*(J_L,J_S)) = \frac{R}{Q^*(J_L,J_S)}K + \left[V_j + \left(Q^*(J_L,J_S) - M_j\right)C_j\right]\frac{h}{2} + \frac{R}{Q}\left[V_j + \left(Q^*(J_L,J_S) - M_j\right)C_j\right] + \frac{R}{Q^*(J_L,J_S)}(J_L+1)C_L.$$

Else there exists some $j \ge 1$ and $g \ge 0$ such that M_j, M_{j+1}, …., $M_{j+g} \in (J_L\ W_L + n\ W_S, (J_L + 1)\ W_L,]$, $M_{j-1} \le J_L W_L + nW_S$,and $M_{j+g+1} > (J_L+1)W_L$.

Then consider each interval $(J_L W_L + nW_S, M_j]$, $(M_j, M_{j+1}]$, …., $(M_{j+g}, (J_L+1)W_L]$ separately as follows.

For all $Q \in (J_L W_L + nW_S, M_j]$, compute $Q_{j-1}(J_L, J_S)$, $Q^*_{j-1}(J_L,J_S)$ and $TC_{j-1}(Q^*)$, by

$$Q_{j-1}(J_L,J_S) = \sqrt{\frac{2R(K + V_{j-1} - M_{j-1}C_{j-1} + J_L C_L + C_L)}{hC_{j-1}}}.$$

$$Q^*_{j-1}(J_L,J_S) = \begin{cases} J_L W_L + nW_S + 1, & \text{if } Q_{j-1}(J_L,J_S) \le J_L W_L + nW_S; \\ Q_{j-1}(J_L,J_S), & \text{if } Q_{j-1}(J_L,J_S) \in (J_L W_L + nW_S, M_j]; \\ M_j, & \text{if } Q_{j-1}(J_L,J_S) > M_j. \end{cases}$$

$$TC_{j-1}(Q^*) = \frac{R}{Q^*_{j-1}(J_L,J_S)}K + \left[V_{j-1} + \left(Q^*_{j-1}(J_L,J_S) - M_{j-1}\right)C_{j-1}\right]\frac{h}{2} + \frac{R}{Q^*_{j-1}(J_L,J_S)}\left[V_{j-1} + \left(Q^*_{j-1}(J_L,J_S) - M_{j-1}\right)C_{j-1}\right] + \frac{R}{Q^*_{j-1}(J_L,J_S)}(J_L+1)C_L.$$

For all $Q \in (M_i, M_{i+1}]$, $i = j, j+1, \ldots, j+g-1$, set unit price as C_i. Note that this case disappears if $g = 0$. Compute $Q_i(J_L, J_S)$, $Q_i^*(J_L, J_S)$ and $TC_i(Q^*)$, by

$$Q_i(J_L, J_S) = \sqrt{\frac{2R(K + V_i - M_i C_i + J_L C_L + C_L)}{hC_i}}\ .$$

$$Q_i^*(J_L, J_S) = \begin{cases} M_i + 1, & \text{if } Q_i(J_L, J_S) \le M_i; \\ Q_i(J_L, J_S), & \text{if } Q_i(J_L, J_S) \in (M_i, M_{i+1}]; \\ M_{i+1}, & \text{if } Q_i(J_L, J_S) > M_{i+1}. \end{cases}$$

$$TC_i(Q^*) = \frac{R}{Q_i^*(J_L, J_S)} K + \left[V_i + \left(Q_i^*(J_L, J_S) - M_i\right)C_i\right]\frac{h}{2} + \frac{R}{Q_i^*(J_L, J_S)}\left[V_i + \left(Q_i^*(J_L, J_S) - M_i\right)C_i\right] + \frac{R}{Q_i^*(J_L, J_S)}(J_L + 1)C_L.$$

For all $Q \in (M_{j+g}, (J_L + 1)W_L]$, compute $Q_{j+g}(J_L, J_S)$, $Q_{j+g}^*(J_L, J_S)$ and $TC_{j+g}(Q^*)$, by

$$Q_{j+g}(J_L, J_S) = \sqrt{\frac{2R(K + V_{j+g} - M_{j+g} C_{j+g} + J_L C_L + C_L)}{hC_{j+g}}}\ .$$

$$Q_{j+g}^*(J_L, J_S) = \begin{cases} M_{j+g} + 1, & \text{if } Q_{j+g}(J_L, J_S) \le M_{j+g}; \\ Q_{j+g}(J_L, J_S), & \text{if } Q_{j+g}(J_L, J_S) \in (M_{j+g}, (J_L + 1)W_L]; \\ (J_L + 1)W_L, & \text{if } Q_{j+g}(J_L, J_S) > J_L W_L + J_S W_S. \end{cases}$$

$$TC_{j+g}(Q^*) = \frac{R}{Q_{j+g}^*(J_L, J_S)} K + \left[V_{j+g} + \left(Q_{j+g}^*(J_L, J_S) - M_{j+g}\right)C_{j+g}\right]\frac{h}{2} + \frac{R}{Q_{j+g}^*(J_L, J_S)}\left[V_{j+g} + \left(Q_{j+g}^*(J_L, J_S) - M_{j+g}\right)C_{j+g}\right] + \frac{R}{Q_{j+g}^*(J_L, J_S)}(J_L + 1)C_L.$$

Find u = *argmin* $\{TC_i(Q^*) \mid i = j-1, j, j+1, \ldots, j+g\}$, and set $Q^*(J_L, J_S)$ = $Q_u^*(J_L, J_S)$ and $TC(Q^*(J_L, J_S))$ = $TC_u(Q^*)$.

End

End

Optimal (J_L^*, J_S^*) = *argmin* $\left\{TC(Q^*(J_L, J_S)) \middle| J_L = 0, 1, \ldots, \left\lceil \frac{R}{W_L} \right\rceil, J_S = 1, 2, \ldots, n, n+1\right\}$, $Q^* = Q^*(J_L^*, J_S^*)$ and $TC(Q^*) = TC(Q^*(J_L^*, J_S^*))$.

Step 4. *If* $J_L^* = (t+1)\left\lceil \frac{R}{W_L} \right\rceil$ and $J_S^* = n+1$, go to Step 5, *Else* go to Step 6.

Step 5. Increment t by 1 and go to Step 3.

Step 6. STOP

Chapter 5 Numerical Study

5.1 Introduction

This chapter deals with the numerical analysis of the algorithms developed in Chapters 3 and 4. MATLAB programming of the algorithms is performed to show the effect of demand, unit material cost, ordering cost, and truck capacity on the ordering quantity and total logistics cost. This chapter analyses several random instances in which the values of K, R, W_L, and C varies for each instances and Q^*, TC (Q^*), J_L^* and J_S^* are computed for all the instances, considering the following three cases

- when no quantity discounts are offered
- when all-unit quantity discounts are offered
- when incremental quantity discounts are offered

If the quantity discounts are not offered, the unit material cost C is arbitrarily assumed. Whenever quantity discounts are offered, the unit price of the material depends upon the break point quantities. In all the instances, we consider four break points M_1, M_2, M_3, and M_4 following a continuous discount quantity schedule. In the continuous discount quantity schedule the break points are assumed to be continuous just to make it simple i.e. if the first break point starts at M_0 and ends at M_1 then the next break point starts at M_1 + 1, and ends at M_2 and so on. The corresponding unit material cost for any ordering quantity in the range [M_0, M_1], (M_1, M_2], (M_2, M_3], (M_3, M_4], and (M_4, ∞] is C_0, C_1, C_2, C_3, and C_4, respectively. Values of C_0, C_1, C_2, C_3, and C_4 are chosen as percentages of C as shown in Table 5.1. We consider four scenarios of discount percentage. In all these scenarios the difference in the discount percentage at each break point quantity is 1%, 2%, 3% or 4%.

Table 5.1 Quantity Discount Structure

Break-point Quantities	% Discount			
0 ≤ Q ≤ 400	0%	0%	0%	0%
400 < Q ≤ 800	1%	2%	3%	4%
800 < Q ≤ 1200	2%	4%	6%	8%
1200 < Q ≤ 1600	3%	6%	9%	12%
1600 < Q ≤ ∞	4%	8%	12%	16%

5.2 Optimal Solution by MATLAB

MATLAB programming of Algorithm A, B, C, D, E and F are done. All the instances considering three different cases are executed in the program and the results obtained are discussed in the following sections.

5.2.1 Impact of Discount Percentage and Annual Demand on the Optimal Ordering Quantity

In this section, we will analyze the impact of discount percentage and annual demand on the optimal ordering quantity. We consider three demands: 4000, 8000, and 12000 units/year. In addition, we consider K = 500, h = 25%, W_L = 800, W_S = 600, C_L = 820, C_S = 700, and C = 20. Table 5.2 shows the optimal solutions obtained from MATLAB, along the actual total unit cost, computed by $TC\ (Q^*)/R$, for all theses cases: no quantity discounts, all-unit quantity discounts and incremental quantity discounts.

Table 5.2 Impact of *R* and Discount % on Ordering Quantity

R		Discount %								
		0%	1%		2%		3%		4%	
		N-QD	A-QD	I-QD	A-QD	I-QD	A-QD	I-QD	A-QD	I-QD
4000	Q^*	800	1400	800	2200	2400	2200	2400	2200	2400
	$TC(Q^*)$	88600	86766	88190	83824	86920	80404	84913	76984	82907
	JL^*	1	1	1	2	3	2	3	2	3
	JS^*	0	1	0	1	0	1	0	1	0
	$TC(Q^*)/R$	22.15	21.69	22.05	20.96	21.73	20.10	21.23	19.25	20.72
	% decrease	0 %	2.1 %	0.5 %	5.4%	1.8 %	9.2 %	4.1 %	13.0 %	6.4 %
8000	Q^*	1600	2200	2400	2400	2400	2400	3200	2400	4000
	$TC(Q^*)$	174700	169210	171990	162590	168120	155950	163590	149310	158800
	JL^*	2	2	3	3	3	3	4	3	5
	JS^*	0	1	0	0	0	0	0	0	0
	$TC(Q^*)/R$	21.84	21.15	21.50	20.32	21.01	19.49	20.45	18.66	19.85
	% decrease	0 %	3.2 %	1.5 %	6.9 %	3.8 %	10.7 %	6.3 %	14.5 %	9.1 %
12000	Q^*	1600	2400	2400	2400	3200	2400	4000	2400	4800
	$TC(Q^*)$	260050	250960	255060	241120	248535	231280	241300	221440	233630
	JL^*	2	3	3	3	4	3	5	3	6
	JS^*	0	0	0	0	0	0	0	0	0
	$TC(Q^*)/R$	21.67	20.91	21.25	20.09	20.71	19.27	20.11	18.45	19.47
	% decrease	0 %	3.5 %	2.0%	7.3 %	4.4 %	11.0 %	7.2 %	14.8 %	10.1 %

N-QD = No Quantity Discounts; A-QD = All-unit Quantity Discounts; I-QD = Incremental Quantity Discounts;
% decrease = % decrease in $TC(Q^*)$ with respect to $TC(Q^*)$ at 0% discount rate.

(1) No quantity discounts. As annual demand increases from 4,000 to 8,000, optimal ordering quantity also doubles. However, Q^* remains the same as R increases from 8,000 to 12,000. From Table 5.2, one can see that TC (Q^*)/R decreases as R increases, this is because the unit ordering cost and the unit holding cost decreases and the unit material cost and unit transportation cost remains the same as R increases.

(2) All-unit quantity discounts. We can see from Table 5.2 that at the discount rate of 1%, the optimal ordering quantity increases by 800 units and 200 units, when R increases from 4000 to 8000 and from 8000 to 12000, respectively. For the discount rate of 2%, 3% and 4%, the optimal ordering quantity increases by 200 units as R increases from 4000 to 8000 and remains the same as R increases from 8000 to 12000. From Table 5.2, one can see that for R = 4000, 8000 and 12000 the total annual logistics cost decreases by more than 2%, 3% and 3.5%, respectively, for every 1% increase in discount rate. Figure 5.1 depicts Q^* for all discount rates and R considered. One can see that for R =12000, Q^* increases as discount rate changes from 0 to 1%, but does not change as discount rate further increases. For both R = 8000 and R = 4000, Q^* increases as the discount rate increases up to 2%, and remains the same as the discount rate further increases.

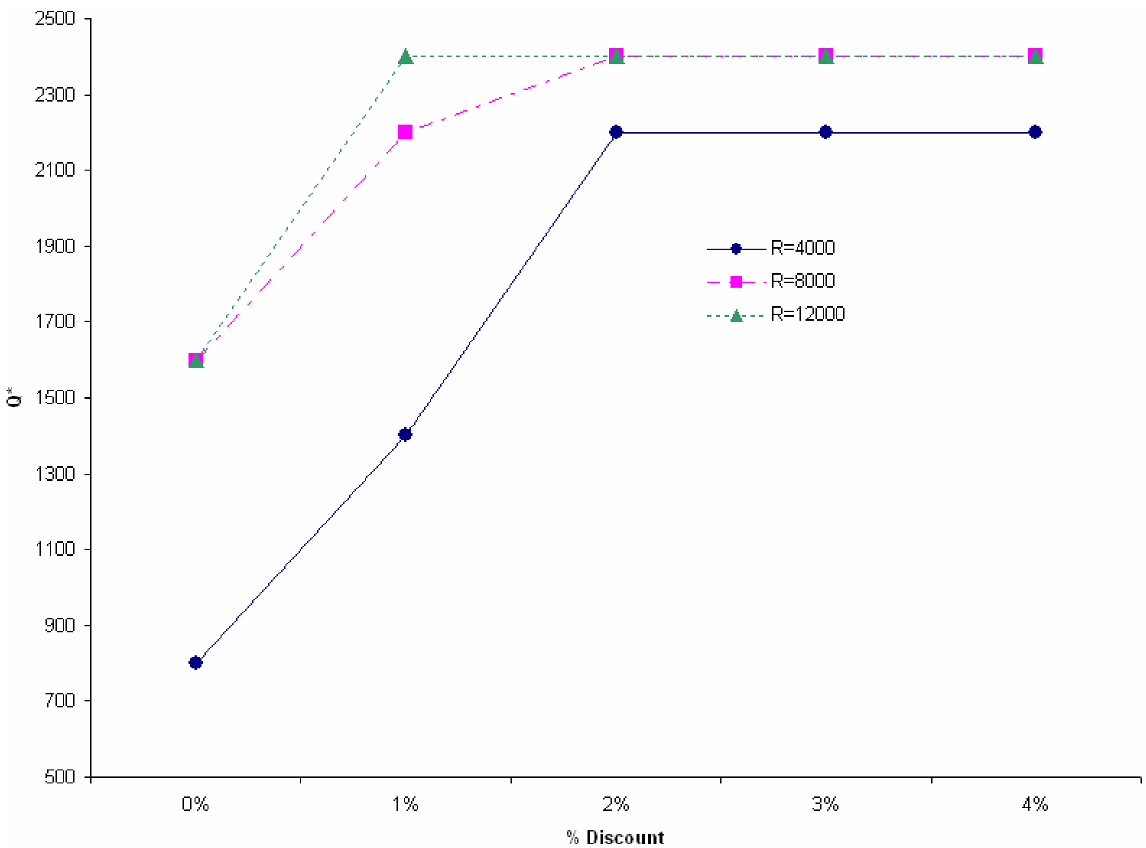

Figure 5.1 Impact of *R* on Q* for All-unit Quantity Discounts

Figure 5.2 depicts $TC(Q^*)/R$ for all discount rates and R considered. The unit total cost decreases almost linearly as discount rate increases, for all R studied. This indicates that, the impact of all-unit quantity discount on the total actual cost per unit decreases as demand increases.

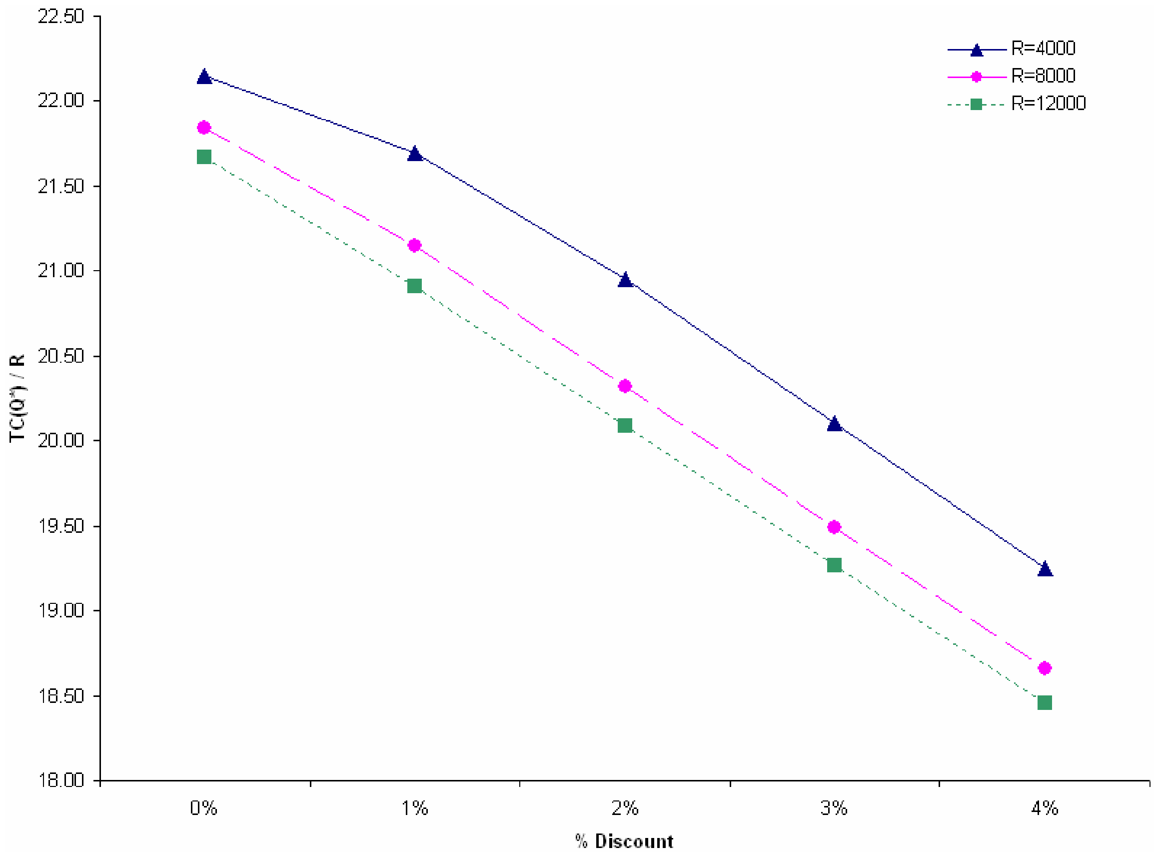

Figure 5.2 *TC (Q*)/R* vs. % Discount for All-unit Quantity Discounts

(3) Incremental quantity discounts. We can see from Table 5.2 that at the discount rate of 1%, the optimal ordering quantity increases by 1600 units as R increases from 4000 to 8000. However, the optimal ordering quantity does not increase as R increases from 8000 to 12000. For the discount rate of 2%, the optimal ordering quantity remains the same as R increases from 4000 to 8000, but the optimal ordering quantity increases by 800 units as R increases from 8000 to 12000. When a discount rate of 3% is offered, the optimal ordering quantity increases by 800 units as R increases from 4000 to 8000 and from 8000 to 12000. For a discount rate of 4%, the optimal ordering quantity increases by 1600 units and 800 units, when R increases from 4000 to 8000 and from 8000 to 12000, respectively. From Table 5.2, one can see that for R = 4000, the total annual logistics cost

decreases by more than 0.5% for every 1% increase in discount rate. For R = 8000, the total annual logistics cost decreases by more than 1.5% for every 1% increase in the discount rate. For R = 12000, the total annual logistics cost decreases by more than 2% for every 1% increase in the discount rate.

Figure 5.3 depicts Q^* for all discount rates and R considered. One can see that when R = 12000, Q^* increases linearly as discount rates increases. We can also see that Q^* increases linearly as the discount percentage increases beyond 2% for R = 8000. For R = 4000, Q^* increases linearly until discount rate of 2% and does not change as the discount rate increases further.

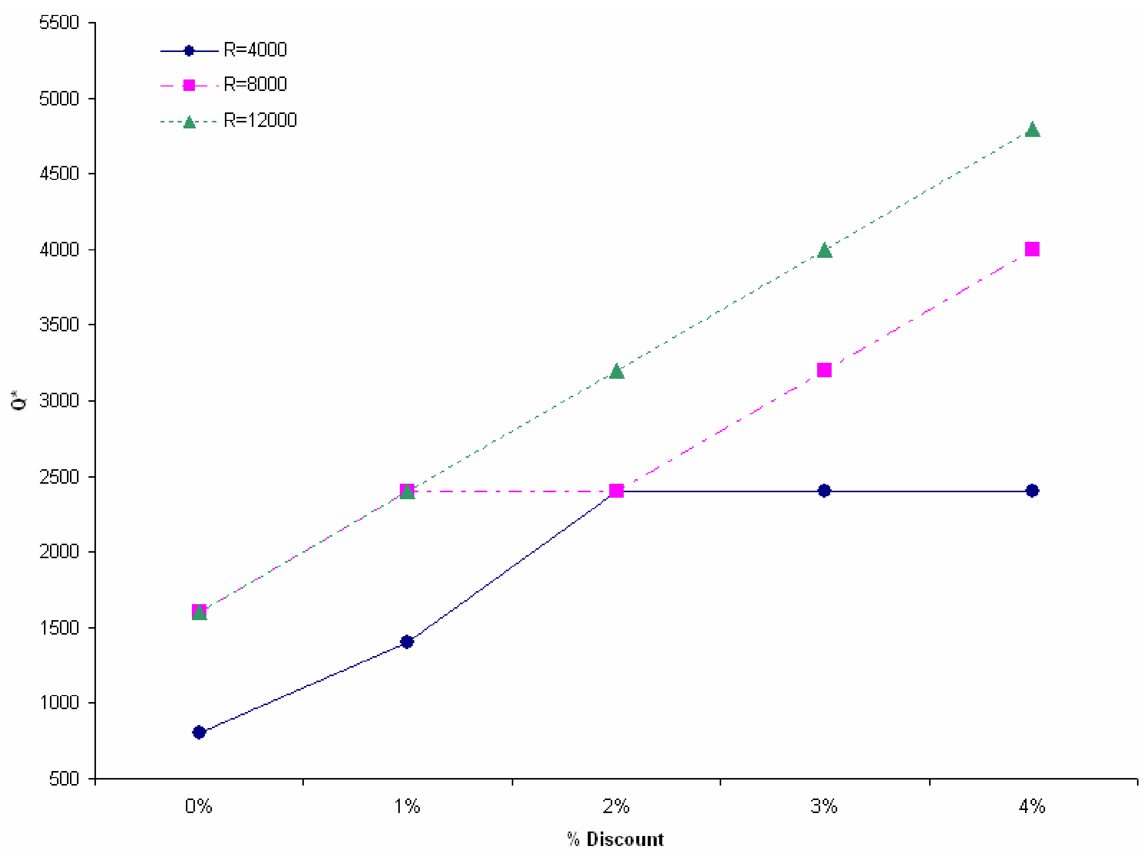

Figure 5.3 Impact of *R* on Q* for Incremental Quantity Discounts

Figure 5.4 depicts *TC* (*Q**)/*R* for all discount rates and *R* considered. The unit total cost decreases almost linearly as discount rate increases, for all *R* studied. This indicates that, the impact of incremental quantity discount on the total actual cost per unit decreases as demand increases.

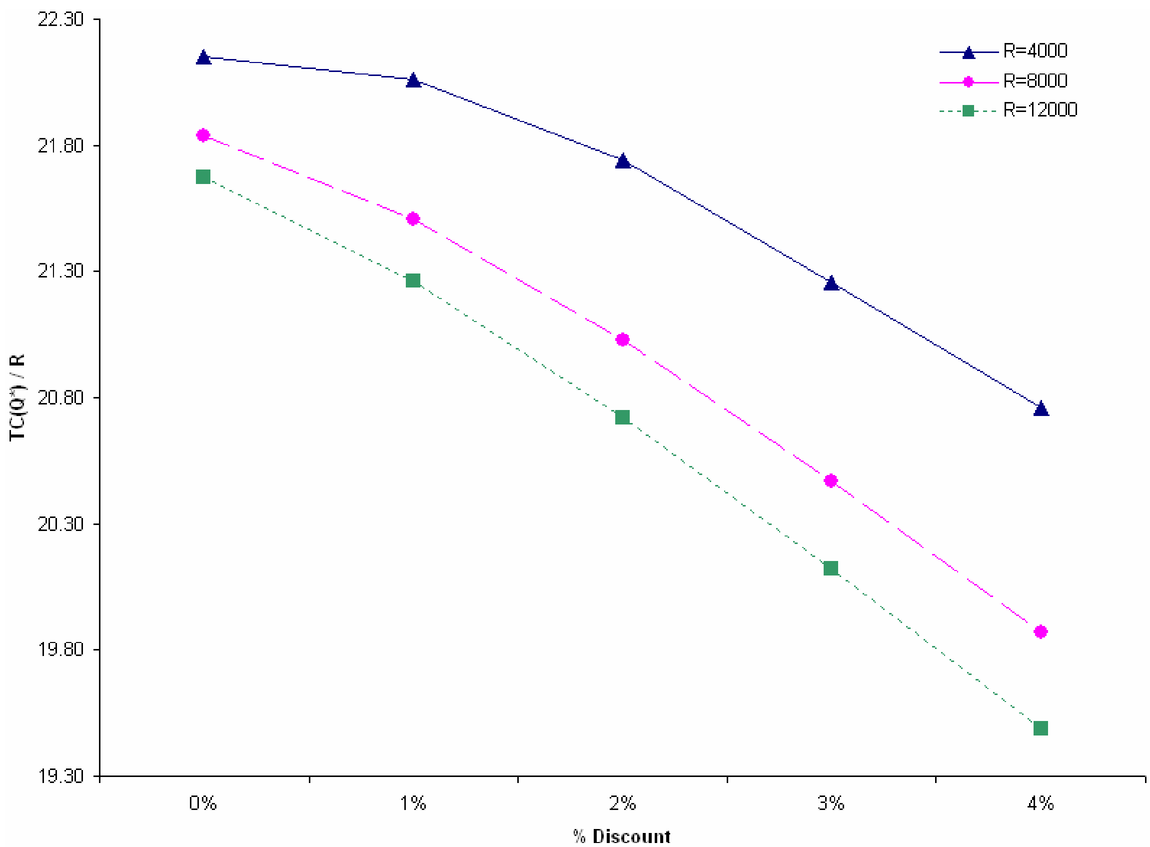

Figure 5.4 *TC* (*Q)/*R* vs. % Discount for Incremental Quantity Discounts**

5.2.2 Impact of *K* on the Ordering Quantity

In this section we will analyze the impact of K on the optimal ordering quantity. Let the values of h, W_L, W_S, C_L, C_S, and C be the same as given in Section 5.2.1, consider R = 8,000 and K = 300, 500, and 700. Table 5.3 presents the computational results.

Table 5.3 Impact of *K* on the Ordering Quantity

K		Discount %								
		0%	1%		2%		3%		4%	
		N-QD	A-QD	I-QD	A-QD	I-QD	A-QD	I-QD	A-QD	I-QD
300	*Q**	800	2200	2400	2200	2400	2200	3200	2200	4000
	TC (*Q**)	173200	168480	171330	161860	167450	155240	163090	148620	158400
	*JL**	1	2	3	2	3	2	4	2	5
	*JS**	0	1	0	1	0	1	0	1	0
	TC (*Q**) / *R*	21.65	21.06	21.42	20.23	20.93	19.40	20.38	18.57	19.80
	% decrease	0 %	2.7 %	1.1 %	6.5 %	3.2 %	10.4 %	5.7 %	14.2 %	8.4 %
500	*Q**	1600	2200	2400	2400	2400	2400	3200	2400	4000
	TC (*Q**)	174700	169210	171990	162590	168120	155950	163590	149310	158800
	*JL**	2	2	3	3	3	3	4	3	5
	*JS**	0	1	0	0	0	0	0	0	0
	TC (*Q**) / *R*	21.83	21.15	21.49	20.32	21.01	19.49	20.45	18.66	19.85
	% decrease	0 %	3.1 %	1.5 %	6.9 %	3.7 %	10.7 %	6.3 %	14.5 %	9.0 %
700	*Q**	1600	2400	2400	2400	3200	2400	3200	2400	4000
	TC (*Q**)	175700	169890	172660	163250	168710	156610	164090	149970	159200
	*JL**	2	3	3	3	4	3	4	3	5
	*JS**	0	0	0	0	0	0	0	0	0
	TC (*Q**) / *R*	21.96	21.23	21.58	20.40	21.08	19.57	20.51	18.74	19.90
	% decrease	0 %	3.3 %	1.7 %	7.1 %	3.9 %	10.9 %	6.5 %	14.7 %	9.3 %

N-QD = No Quantity Discounts; A-QD = All-unit Quantity Discounts; I-QD = Incremental Quantity Discounts;
% decrease = % decrease in *TC* (*Q**) with respect to *TC* (*Q**) at 0% discount rate.

(1) No quantity discounts. As the ordering cost increases from 300 to 500, the optimal ordering quantity almost doubles. However Q^* remains the same as K increases from 500 to 700. TC (Q^*) increases by 0.8% and 0.6% as K increases from 300 to 500 and 500 to 700, respectively. This increase in TC (Q^*) is due to the increase in the ordering cost. The total annual logistics cost and the total unit cost increases as the ordering cost increases.

(2) All-unit quantity discounts. We can see from Table 5.3 that at the discount rate of 1%, the optimal ordering quantity remains the same as K increases from 300 to 500. However, the optimal ordering quantity increases by 200 units as K increases from 500 to 700. For the discount rate of 2%, 3% and 4%, the optimal ordering quantity increases by 200 units as K increases from 300 to 500 and remains the same as K increases from 500 to 700. From Table 5.3, one can see that for $K = 300$, the total annual logistics cost decreases by 2.7% as the discount rate increases from 0% to 1%. As the discount rate increases beyond 1% the total annual logistics decreases by almost 4% for every 1% increase in discount rate. For $K = 500$ and 700, as the discount rate increases from 0% to 1% the total annual logistics decreases by 3.1% and 3.3%, respectively. For both $K = 500$ and 700 as the discount rate increases beyond 1% the total annual logistics cost decreases steadily by 3.8% for every 1% increase in the discount rate.

Figure 5.5 depicts Q^* for all discount rates and K considered. One can see that for K =500, Q^* increases almost linearly as discount rate increases up to 2%, but does not change as discount rate further increases. For both $K = 300$ and $K = 700$, Q^* increases as the discount rate increases up to 1%, and remains the same as the discount rate further increases.

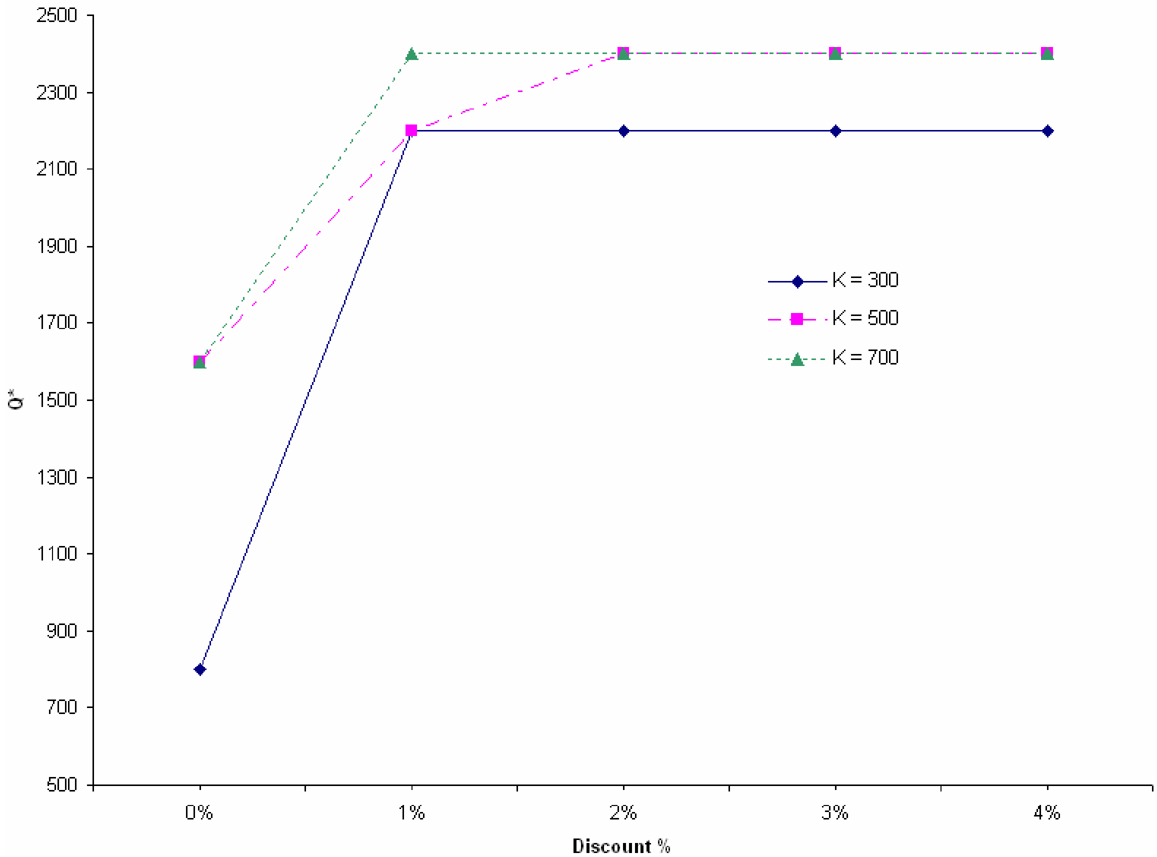

Figure 5.5 Impact of *K* on *Q for All-unit Quantity Discounts**

Figure 5.6 depicts $TC(Q^*)/R$ for all discount rates and K considered. The unit total cost decreases almost linearly as discount rate increases, for all K studied. This indicates that, the impact of all-unit quantity discount on the total actual cost per unit increases as ordering cost increases.

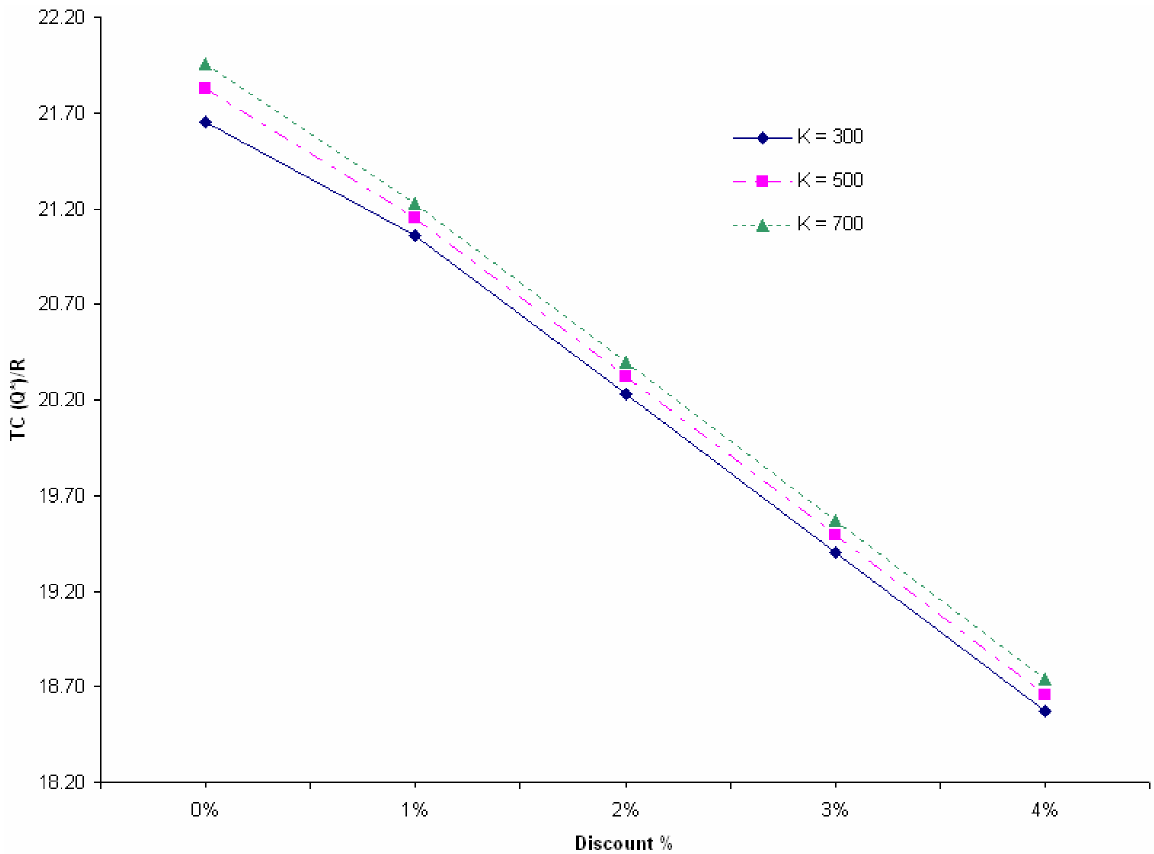

Figure 5.6 *TC* (*Q)/*R* vs. % Discount for All-unit Quantity Discounts**

(3) Incremental quantity discounts. We can see from Table 5.3 that at the discount rate of 1%, 3% and 4%, the optimal ordering quantity remains the same for all *K* studied. For the discount rate of 2%, the optimal ordering quantity remains the same as *K* increases from 4000 to 8000, but the optimal ordering quantity increases by 800 units as *K* increases from 8000 to 12000. From Table 5.3, one can see that for *K* = 300, the total annual logistics cost decreases by 1.1% as the discount rate increases from 0% to 1%. As the discount rate increases beyond 1% the total annual logistics decreases by more than 2% for every 1% increase in discount rate. For *K* = 500, as the discount rate increases from 0% to 1% the total annual logistics decreases by 1.5%. As the discount rate increases beyond 1% the total annual logistics cost decreases by more than 2.0% for every 1% increase in the discount rate. For *K* = 700, as the discount rate increases from 0% to 1%

the total annual logistics decreases by 1.7%. As the discount rate increases beyond 1% the total annual logistics cost decreases by more than 2.0% for every 1% increase in the discount rate.

Figures 5.7 depicts *Q** for all discount rates and *K* considered. One can see that for *K* = 300 and 500, *Q** increases as the discount rate increases from 0% to 1%. The optimal ordering quantity remains the same as the discount rate increases from 1% to 2% and increases linearly as the discount rate increases beyond 2%. For *K* = 700, the optimal ordering quantity increases linearly as the discount rate increases up to 2%. *Q** remains the same as the discount rate increases from 2% to 3% and the optimal ordering quantity increases for further increase in discount rate.

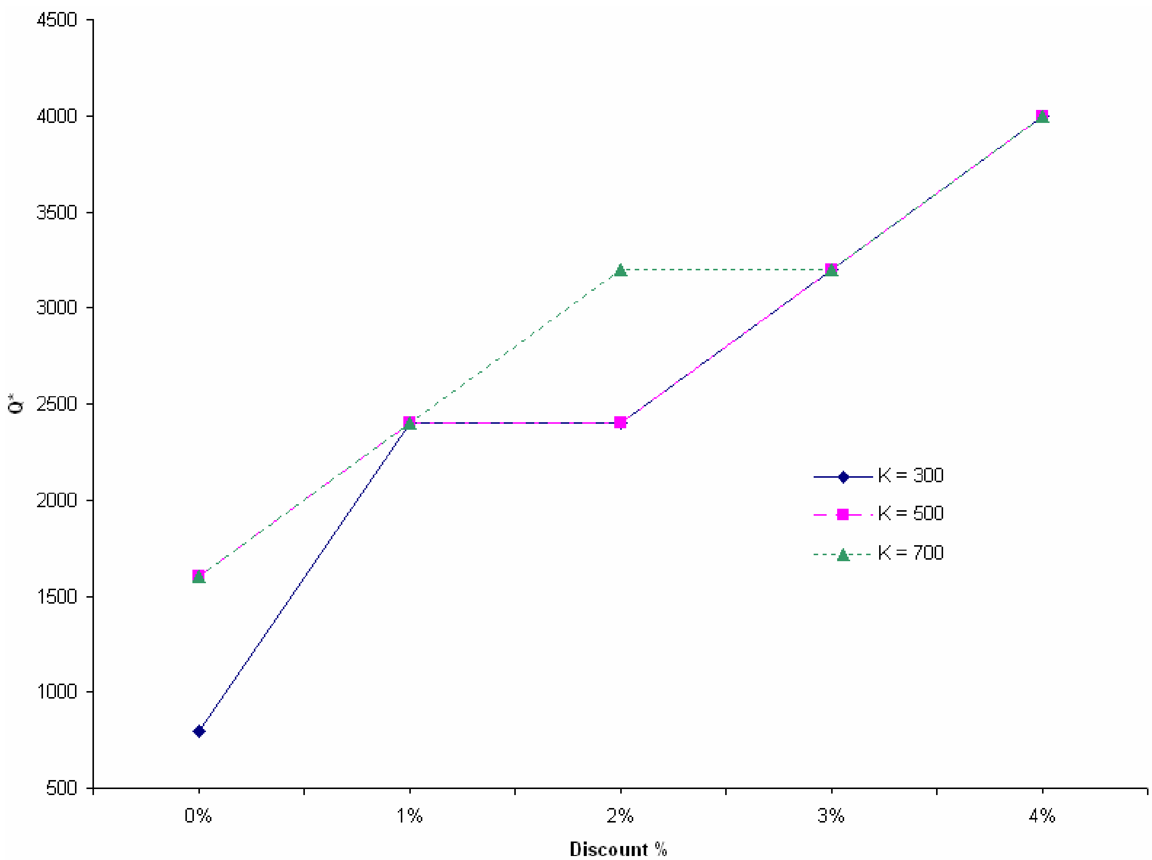

Figure 5.7 Impact of *K* on *Q for Incremental Quantity Discounts**

Figure 5.8 depicts *TC* (Q^*)/*R* for all discount rates and *K* considered. The unit total cost decreases almost linearly as discount rate increases, for all *K* studied. This indicates that, the impact of all-unit quantity discount on the total actual cost per unit increases as ordering cost increases

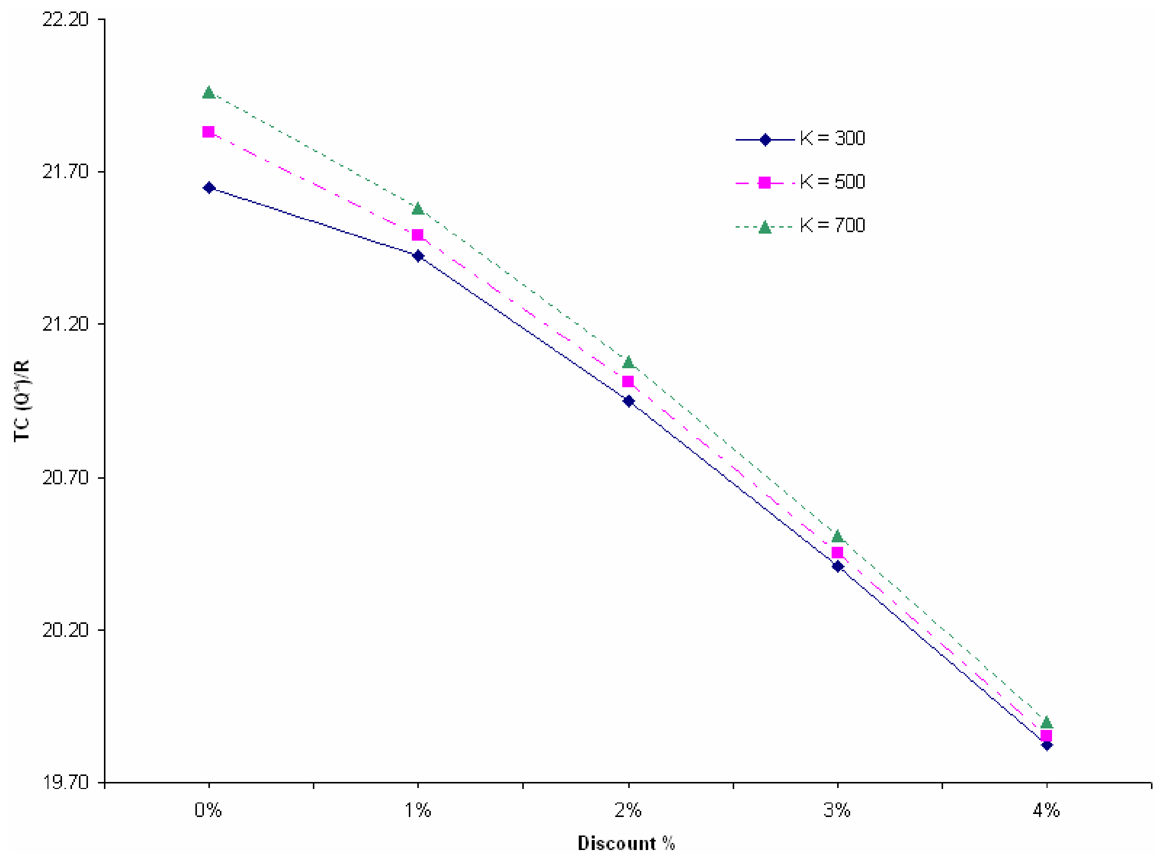

Figure 5.8 ***TC*** **(Q^*)/*R* vs. % Discount for Incremental Quantity Discounts**

5.2.3 Impact of W_L on the Optimal Ordering Quantity and Number of Trucks

In this section we will analyze the impact of W_S / W_L on the optimal ordering quantity, for a given W_S. Let the values of h, W_S, C_L, C_S, and C be the same as given in Section 5.2.1. Consider R=8000, discount rate of 1%, and W_S / W_L as given in Table 5.4. Note that the ratio of W_S to W_L cannot be greater than 1 because of the assumption $W_S < W_L$. Table 5.1 also provides W_L values, computed by $W_S / (W_S / W_L)$, where $W_S = 600$.

Table 5.4 Ratio of Capacity of Small Truck to Capacity of Large Truck

W_S / W_L	0.65	0.70	0.75	0.80	0.85
W_L	923	857	800	750	706

Table 5.5 shows the computational results for all three cases: no quantity discounts, all-unit quantity discounts and incremental quantity discounts.

Table 5.5 Impact of W_L on the Total Annual Logistics Cost

		W_S / W_L				
		0.65	0.70	0.75	0.80	0.85
No Quantity Discounts	Q^*	923	1714	1600	1500	1306
	$TC(Q^*)$	173750	174270	174700	175160	175640
	J_L^*	1	2	2	2	1
	J_S^*	0	0	0	0	1
	$TC(Q^*) / R$	21.72	21.78	21.84	21.89	21.95
All-Unit Quantity Discounts	Q^*	1846	1714	2200	2100	2012
	$TC(Q^*)$	167300	167700	169210	169460	169720
	J_L^*	2	2	2	2	2
	J_S^*	0	0	1	1	1
	$TC(Q^*) / R$	20.91	20.96	21.151	21.18	21.21
Incremental Quantity Discounts	Q^*	1846	1714	2400	2250	2118
	$TC(Q^*)$	170870	171540	171990	172740	172990
	J_L^*	2	2	3	3	3
	J_S^*	0	0	0	0	0
	$TC(Q^*) / R$	21.36	21.44	21.50	21.59	21.62

(1) No quantity discounts. As W_S / W_L increases from 0.65 to 0.70, the optimal ordering quantity increases by 791 units. The optimal ordering quantity decreases by 114 units as W_S / W_L increases from 0.70 to 0.75. As W_S / W_L further increases the optimal ordering quantity decreases. This decrease in the optimal ordering quantity is due to the decrease in the capacity of the large truck as W_S / W_L increases. From Table 5.5, one can

also see that the total annual logistics cost increases as W_S / W_L increases. J_L^* increases from 1 to 2 as W_S / W_L increases from 0.65 to 0.70. This is increase in J_L^* is due to the increase in the optimal ordering quantity. For W_S / W_L = 0.70, 0.75, and 0.80 the optimum number of large trucks required to ship the quantity remains the same. As W_S / W_L increases from 0.80 to 0.85, J_L^* decreases by 1. This is due to the decrease in the optimal ordering quantity. J_S^* remains the same as for all values of W_S / W_L, considered, except for W_S / W_L = 0.85. J_S^* increases from 0 to 1 as W_S / W_L increases from 0.80 to 0.85. This increase in J_S^* is due to reduction in capacity of the large truck.

(2) All-unit quantity discounts. We can see that from Table 5.5 that the optimal ordering quantity decreases by 132 units as W_S / W_L increases from 0.65 to 0.70. When W_S / W_L increases from 0.70 to 0.75 the optimal ordering quantity increases by 486. As the W_S / W_L increases from 0.75 to 0.80 and 0.80 to 0.85, the optimal ordering quantity decreases by 100 units and 88 units, respectively. From Table 5.5, one can also see that the total annual logistics cost increases as W_S / W_L increases. This increase in total annual logistics cost is due to the decrease in the capacity of the large truck. J_S^* remains the same for W_S / W_L = 0.65 and 0.70, however, J_S^* increases by 1 as W_S / W_L increases from 0.70 to 0.75. In this case, reduction in capacity of the large trucks forces the increase in J_S^*. The unit total cost increases by less than 1% for every 0.05 increase in the ratio of W_S to W_L.

(3) Incremental quantity discounts. From Table 5.5, one can see that the optimal ordering quantity decreases by 132 units as W_S / W_L increases from 0.65 to 0.70. As W_S / W_L increases from 0.70 to 0.75 the optimal ordering quantity increases by 686 units. As W_S / W_L increases from 0.75 to 0.80 and 0.80 to 0.85, the optimal ordering quantity increases by 750 units and 250 units, respectively. The number of large trucks required

increases from 2 to 3 as W_S / W_L increases from 0.70 to 0.75. The number of large trucks required increases due to the decrease in capacity of the large truck as W_S / W_L increases.

Figure 5.9 depicts the Q* for all W_S / W_L considered. From Figure 5.9, one can see that as W_S / W_L increases beyond 0.75, Q^* decreases almost linearly as W_S / W_L increases. For $W_S / W_L = 0.70$, the optimum ordering quantity remains the same for all the three cases considered.

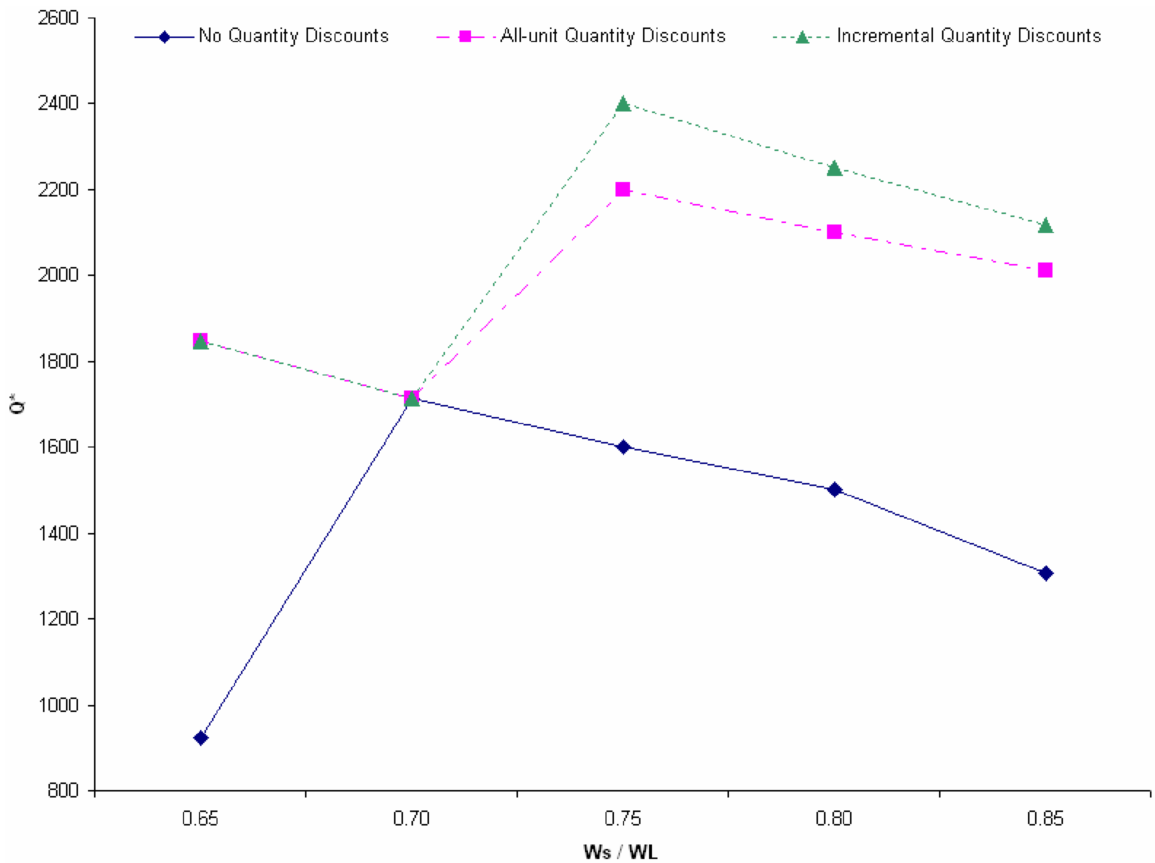

Figure 5.9 W_S / W_L vs. Q^*

5.2.4 Impact of *C* on the Optimal Ordering Quantity

In this section we will analyze the impact of C on the optimal ordering quantity. Let the values of h, W_L, W_S, C_L, C_S, and K be the same as given in Section 5.2.1, consider $R = 8000$ and $C = 15$, 20, and 25. Table 5.6 presents the computational results.

Table 5.6 Impact of *C* on the Ordering Quantity

		Discount %								
		0%	1%		2%		3%		4%	
		N-QD	A-QD	I-QD	A-QD	I-QD	A-QD	I-QD	A-QD	I-QD
C = 15	*Q**	1600	2400	2400	2400	3200	2400	3200	2400	4000
	TC (*Q**)	133700	129390	131460	124410	128520	119430	125055	114450	121400
	*JL**	2	3	3	3	4	3	4	3	5
	*JS**	0	0	0	0	0	0	0	0	0
	TC (*Q**) / *R*	16.71	16.17	16.43	15.55	16.06	14.93	15.63	14.31	15.17
	% decrease	0 %	3.2 %	1.6 %	7.0 %	3.8 %	10.7 %	6.4 %	14.4 %	9.2 %
C = 20	*Q**	1600	2200	2400	2400	2400	2400	3200	2400	4000
	TC (*Q**)	174700	169210	171990	162590	168120	155950	163590	149310	158800
	*JL**	2	2	3	3	3	3	4	3	5
	*JS**	0	1	0	0	0	0	0	0	0
	TC (*Q**) / *R*	21.83	21.15	21.50	20.32	21.01	19.49	20.45	18.66	19.85
	% decrease	0 %	3.1 %	1.5 %	6.9 %	3.7 %	10.7 %	6.3 %	14.5 %	9.0 %
C = 25	*Q**	800	2200	2400	2200	2400	2200	3200	2200	4000
	TC (*Q**)	215700	208930	212525	200650	207680	192380	202125	184100	196200
	*JL**	1	2	3	2	3	2	4	2	5
	*JS**	0	1	0	1	0	1	0	1	0
	TC (*Q**) / *R*	26.96	26.12	26.55	25.08	25.96	24.05	25.26	23.01	24.52
	% decrease	0 %	3.1 %	1.4 %	7.0 %	3.6 %	10.8 %	6.3 %	14.6 %	9.0 %

N-QD = No Quantity Discounts; A-QD = All-unit Quantity Discounts; I-QD = Incremental Quantity Discounts;
% decrease = % decrease in *TC* (*Q**) with respect to *TC* (*Q**) at 0% discount rate.

(1) No quantity discounts. As unit material cost increases from 20 to 25, optimal ordering quantity decreases by 50%. However, *Q** remains the same as *C* increases from 15 to 20. The total annual logistics cost increases by 23% and 19% as *C* increases from 15 to 20 and 20 to 25, respectively. This increase in total annual logistics cost is due to the increase in material cost and inventory holding cost.

(2) All-unit quantity discounts. We can see from Table 5.6 that at the discount rate of 1%, the optimal ordering quantity increases by 200 units when *C* increase from 15 to 20. However, *Q** remains the same as *C* increases from 20 to 25. For the discount rate of 2%, 3% and 4%, the optimal ordering quantity decreases by 200 units as *C* increases from 20 to 25 and remains the same as *C* increases from 15 to 20. From Table 5.6, one can see that for *C* = 15, 20 and 25 the total annual logistics cost decreases by more than 3.2%, 3.1% and 3.2%, respectively, for every 1% increase in discount rate. Figure 5.10 depicts *Q** for all discount rates and *C* considered. One can see that for *C* = 15 and 25, *Q** remains the same as discount rate increases beyond 1%. For *C* =20 the value of *Q** increases linearly until the discount percentage of 2% and remains the same as the discount percentage further increases.

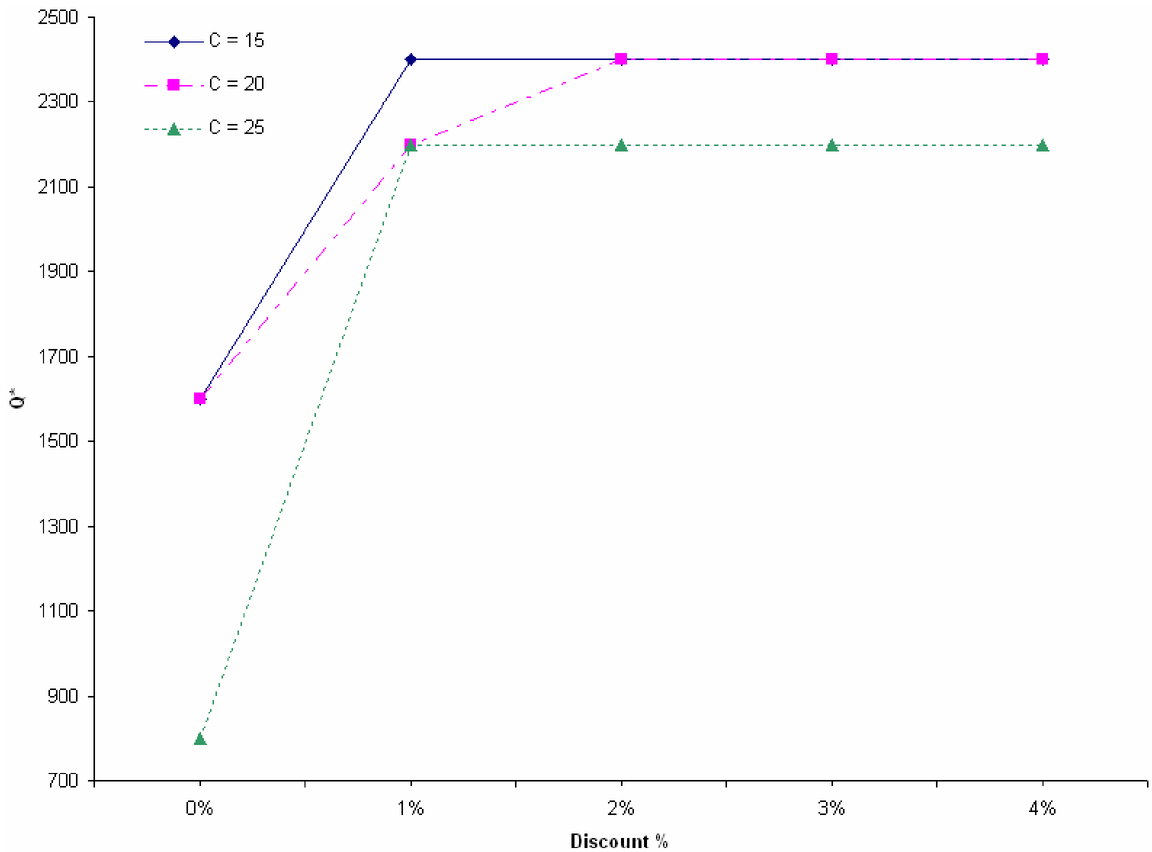

Figure 5.10 Impact of *C* on Q* for All-unit Quantity Discounts

(3) Incremental quantity discounts. We can see from Table 5.6 that at the discount rate of 1%, 3% and 4%, the optimal ordering quantity remains the same for all *C* studied. For the discount rate of 2%, the optimal ordering quantity remains the same as *C* increases from 20 to 25, but the optimal ordering quantity remains the same as *C* increases from 15 to 20. From Table 5.6, one can see that for *C* = 15, the total annual logistics cost decreases by 1.6% as the discount rate increases from 0% to 1%. As the discount rate increases beyond 1%, the total annual logistics decreases by more than 2% for every 1% increase in discount rate. For *C* = 20, as the discount rate increases from 0% to 1% the total annual logistics decreases by 1.5%. As the discount rate increases beyond 1% the total annual logistics cost decreases by more than by 2.2% for every 1% increase in the discount rate. For *C* = 25, as the discount rate increases from 0% to 1% the total annual logistics

decreases by 1.4%. As the discount rate increases beyond 1% the total annual logistics cost decreases by more than 2.0% for every 1% increase in the discount rate.

Figures 5.11 depicts Q^* for all discount rates and C considered. One can see that for C = 15, Q^* increases linearly as the discount rate increases from 0% to 2%. The optimal ordering quantity remains the same as the discount rate increases from 2% to 3%. For C = 20 and 25, the optimal ordering quantity increases as the discount rate increases from 0% to 1%. Q^* remains the same as the discount rate increases from 1% to 2%. We can also see that, for C = 20 and 25, Q^* increases linearly as the discount rate increases beyond 2%.

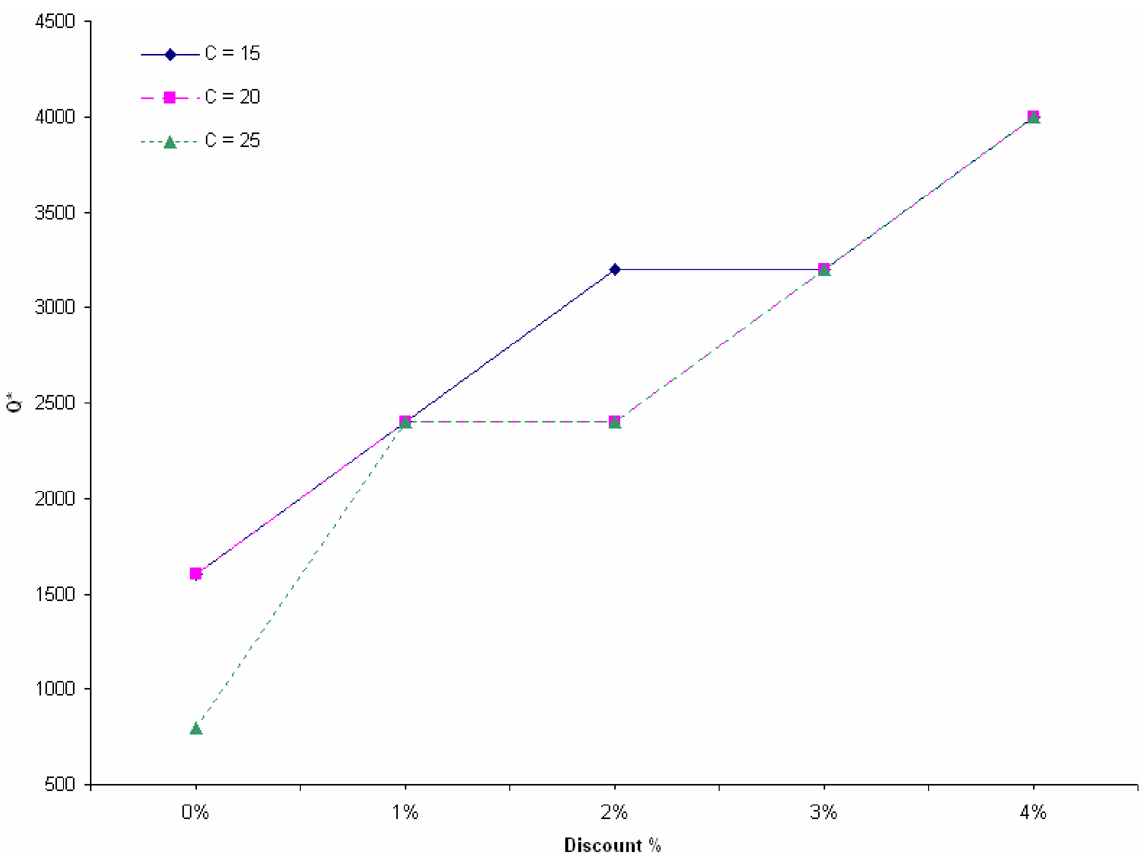

Figure 5.11 Impact of *C* on Q* for Incremental Quantity Discounts

Chapter 6 Conclusions and Future Directions

6.1 Conclusions

The main objective of this thesis was to develop algorithms for finding the optimal ordering quantity that minimizes total annual logistics cost, when the suppliers offer

- No quantity discounts
- All-unit quantity discounts
- Incremental quantity discounts

The total annual logistics cost considered in this research includes ordering cost, material cost, inventory holding cost and transportation cost. We have considered a fixed ordering cost, the unit price of an item will depend upon the ordering quantity and quantity discounts, the inventory holding cost is charged based on the average inventory of the system and the transportation cost depends upon the ordering quantity of each order.

This research considers the following transportation scenario. There are two truck sizes: large and small. A large truck has a capacity of W_L and charges a fixed price of C_L, regardless of actual quantity loaded. Similarly, a small truck has a capacity of W_S and charges a fixed price of C_S, regardless of actual load (not exceeding its capacity). Depending upon the ordering quantity Q, it is necessary to use a combination of J_L large trucks and J_S small trucks, for some $J_L \geq 0$ and $J_S \geq 0$. It is assumed that $\frac{C_L}{W_L} < \frac{C_S}{W_S}$ (i.e., if both large and small trucks are fully loaded, the unit shipping cost for a large truck is smaller than that for a small truck).

MATLAB programming of the algorithm is done. Numerical analysis of various factors that affect the ordering quantity and the total cost are analyzed in Chapter 5. The factors that are considered in the numerical analysis are the annual demand, ordering cost, unit price and capacity of the truck. Discount rates of 1%, 2%, 3% and 4% are also considered in determining the impact of quantity on discounts on the ordering quantity.

6.2 Summary of Contributions

- Developed an optimal ordering quantity algorithm that considers only truckload transportation for shipments.
- Extended the optimal ordering quantity algorithm for all-unit quantity discounts and incremental quantity discounts.

6.3 Future Directions

This thesis has presented an inventory system assuming the demand to be a constant. It would be interesting to formulate an algorithm assuming the annual demand to be stochastic. The algorithm presented in this research considers only two trucks sizes for transportation. It would also be interesting to formulate an algorithm when there are 3 trucks sizes namely, large, medium and small are available.

Even though quantity discounts play a vital role in today's buyer-shipper relationship, there are other factors like speed of delivery, service, and quality should be considered before purchasing an item. By demanding a larger discount, for example, a retailer may have to agree to accept a longer lead time from the supplier. Future research could examine these interactions more closely and explore the role and power of quantity discounts as a bargaining chip in the overall buyer-supplier negotiation process.

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