Time-Distance Diagrams: A Powerful Tool for Service Planning and Control

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Abstract

Graphical scheduling is an old technique that has been neglected, or never acquired, in many North American transit agencies. It retains its advantages in basic schedule design and analysis as it eases the solution to problems that are difficult to solve analytically. Even information about simple routes is enhanced by the detailed operating characteristics inherent in detailed vehicle trajectories and by the relative ease with which accelerated service and service recovery strategies can be investigated. It also can be used to confirm and refine solutions that are generated by analytic methods. The methodology is reviewed in the context of such planning applications. Graphical scheduling has additional advantages in operational control with the advent of modern ITS technologies. By movement of the cursor on a terminal screen, detailed information about all activity along a route becomes available. It is possible to link the altering of trajectories through clicking and dragging to the automatic issuance of control commands and updates of passenger information. These and other possible uses of the technique in an operational context are presented.
Introduction

Development and use of graphical schedules for planning and supervision of transit systems operations is by no means a new subject. It is a time-proven method used for both development and analysis of schedules. Yet, its use is far from universal. While many transit and railway systems use graphical schedules in their daily operations for multiple purposes, the entire concept and technique is virtually unknown in most North American transit systems, including the largest ones. The latest dispatching/control software packages in the United States that monitor buses in real-time through Automatic Vehicle Location (AVL) do not use it either. However, this software does display GIS maps showing vehicle location along streets, as well as checkpoint data in spreadsheet format.

While the spreadsheet format is useful, the data display format that for many purposes reveals the most information—the time-distance diagram—should also be available. Actually, the graphical method is superior to the numerical ones for many applications. The purpose of this paper is to broaden the knowledge about this technique in those parts of the transportation community where it is not used and, often, where it is not even known.

This paper is organized as follows. The following section describes the concept of the time-distance diagram and explains how it is designed and interpreted. The third describes several applications in scheduling and the advantages this technique provides. The fourth section describes its advantages in real-time operational control, an area of vital interest with the advent of highly capable Intelligent Transportation Systems (ITS) that provide precise vehicle locations and numerous communications options. The final section is a concluding summary.

Definition of Time-Distance Diagram

A time-distance diagram, as the name implies, is used to plot motion of a vehicle or train—henceforth referred to as Transit Unit or TU—with time on the horizontal and distance on the vertical axis. The distance axis usually has a length equal to the route length, with the terminals defining the end points. Each TU has an individual trajectory describing its position and movement over time. The family of trajectories...
form a time-distance diagram representing a complete schedule for one, sometimes for several, lines.

The detail at which the trajectory of a TU is presented should reflect the purpose of the analysis. In some cases, particularly when the distances between planned stops are very long, the trajectory need be no more than a straight line connecting the departing terminal at the dispatch time to the other terminal at the arrival time, usually in minutes. In other cases it may be necessary to provide very detailed trajectory showing all accelerations, decelerations, and station dwell time periods. These detailed TU trajectories are commonly plotted in seconds.

Using a rapid transit line as an example of a detailed trajectory, the line is modeled as a series of interstation spacings where the train accelerates with constant rate $a$, cruises at speed $v$, and decelerates at rate $b$. Each station $i$ has a dwell time $t_{di}$. Travel time on any interstation spacing is composed of the time intervals required for accelerating to cruising speed, running at cruising speed, decelerating into the station, and the station dwell time. This kind of trajectory, shown in Figure 1, may be constructed for individual interstation spacings, or for entire lines.

For certain analyses, it is necessary to compute the incremental time lost by stopping at station $i$, or $T_i$. This is the time that TU needs for travel with stopping at one station, as compared to the travel time on the same section while moving at cruising speed, without stopping at the station. This incremental time consists of additional time due to deceleration in entering station $i$, $t_{bi}$, dwell time at station $i$, $t_{di}$, and incremental time for acceleration while departing station $i$, $t_{ai}$. Figure 2 shows a straight-line approximation of time lost for each stop, referred to as $T_i$, instead of exact acceleration and deceleration paths. This straight line simplification is convenient for plotting, and yet sufficiently accurate for most scheduling purposes. Each incremental time lost is connected to the next by a straight-line with slope equal to the cruising speed between them. Therefore, if the cruising speeds on different spacings are equal, the sloped lines are parallel. On line sections where cruising speeds vary, the slopes also change among interstation spacings.

Figure 2 also shows trajectory of another TU, following the first one. The horizontal separation between trajectories represents the headway between TUs, $h$. The
minimum allowable headway depends on vehicle performance, route alignment, and control system characteristics (e.g., manually driven on street, discrete block, moving block, etc.). The vertical separation at any point in time represents the distance separation between corresponding points on successive TUs, or their spacing, s. The minimum spacing, which corresponds to the minimum headway, consists of three components: the TU length, the distance passed during the driver's reaction time, and actual braking distance. The minimum safe spacing is often also presented as a continuous line in front of the TU trajectory, and it is known as the TU shadow. There are
Figure 2. Graphical schedule of two successive trains.

actually different regimes or degrees of safety, and the reader interested in this de­tailed analysis is referred to Vuchic (1981). For TUs that travel on the same path, such as a rail track or bus lane, once any minimum is violated, the schedule is infeasible. In practice, there should be separation well beyond the minimum to ensure schedule reliability. When trajectories reflect actual operations instead of the intended schedule, as soon as any revised trajectory can be projected to violate the minimums, delays can be anticipated, unless corrective action is taken to re-separate them.
Graphical schedules have a visual clarity that numerical schedules can never provide. Tables with numerical values of TU departures at different points along a line may contain errors that are not immediately noticeable. When plotted graphically, every incorrect time is immediately conspicuous. Uneven travel speeds, TUs traveling at less than the minimum headway, or any conflicts in TU travel paths are also easy to detect on graphical schedules.

Graphical schedules can also be useful to present actual running of trains where some skip different stations along the line. An example is shown in Figure 3 for the CalTrain commuter railroad serving communities south of San Francisco. The plot is based on scheduled departures of every train at every station, and shows different headways during the a.m. peak and midday periods, as well as the relationship of local and partial express peak period trains.

In addition to schedules for TUs traveling on the same track in the same direction, graphical analysis can also be used to find meetings of TUs moving in opposite directions on the same track or path. Figure 4 shows train scheduling for a single

![Figure 3. Graphical presentation of a.m. peak and midday schedule for CalTrain, San Francisco Bay Area.](image-url)
track section A-B of a double track line. If the trajectories cross each other along the shared section, as shown by the dashed line in the figure, the operation is infeasible. A feasible schedule is shown by the solid lines, which intersect on a double track section.

As another example, this graphical method was used to verify a temporary schedule involving single-tracking during reconstruction of the Market-Frankford
rapid transit line in Philadelphia, as described by Bruun and Salpeas (1991). The full peak period schedule was plotted using the incremental time-lost format to ensure that no trajectories crossed each other on the single-track section. For clarity and size, only part of the schedule is reproduced as Figure 5; the two solid horizontal lines show the single-track section.

Applications in Scheduling

The most common application of graphical schedules is for single lines. However, there are a number of cases where several lines that merge, diverge, intersect, or form a triangle can also be scheduled graphically, i.e., with application of time-distance diagrams. Several typical cases of different types of graphical schedules are described here.

Single Line Operation

Once average operating speed from terminal to terminal is known and desired headways are specified, development of the basic schedule for single routes is alge-
braically and graphically simple. Its development involves only basic preliminary calculations, followed by adjustments to terminal times to maintain the integer constraint on fleet size. Even in this simple case, a time-distance diagram showing the detailed trajectory of each individual TU is helpful, because it reveals information on each individual terminal time, dwell times at different stations, average running speeds, etc. It is a simple method to clearly display deadheading, short turns, individual runs and fleet size on the line. All pull-ins and pull-outs, short-term storage on sidings, and other details can be shown. The diagram also is helpful for planning transitions between service plans during the day.

**Special Operations**

Once a basic pattern for the schedule has been established, variants can be easily plotted and analyzed. These include short turn, skip-stop, and partial express (also called zonal) operation. The modeling of each will be explained briefly.

Short turns can be treated as if they simply were intermediate terminals, although, for trains, the minimum terminal time may have to include allowances for maneuvering time shorter or longer than at the outer terminals, depending upon the length of line headways, short-turn cycle time and track layout. The simplest operation is when line headways are long and timing of a short-turning train such that the short-turning train can reverse at a station with center platform without conflict with regular trains passing in either direction, as shown in Figure 6a. In another case, the train has stopped and had sufficient dwell time to discharge passengers, it must wait before reversing. In this case, there is enough time for the operator to change ends and accept passengers, departing in reverse direction and crossing over to the opposite track prior to arrival of a following train in either direction, so that no delays occur.

It can happen, however, that the short-turning train cannot travel back immediately because the other track is occupied by a train passing in the opposite direction, or immediate turning would create irregular headways in the opposite direction. Then terminal time must be extended until a train in the reverse direction has passed before the short-turning train can travel back, as shown in Figure 6b.
As headways become shorter, it is likely that the TU cannot wait long enough to reverse without blocking traffic in its initial direction. In this case, either a different reversing location must be used, or an extra track for reversal must be provided. This situation is shown in Figure 6c. The short-turning train must be moved promptly into the center track for reversing, so as to allow at least $h_{\text{min}}$ for passing of the train that is following it in its initial direction. The center track then allows the reversing train to wait for the desired time to depart in the returning direction. Thus, any possibility of delays is eliminated and regular headway can be easily maintained.

Figure 6a. Simple short turn.
This example shows that a situation that is very complex to model algebraically due to several constraints and solution possibilities can be quickly analyzed graphically to see what type of solution(s) are actually feasible for a given basic headway, cycle time and track layout.

*Skip-stop operations*, described in Vuchic (1976), are such that at some stations, only alternating trains stop (typically called A and B stops). This is done generally only where the basic headway is short, as the stations where only alternating trains
stop have doubled headways. Skip-stop service reduces operating time, so that either terminal times or dispatching times must be changed. Either possibility can be readily plotted and compared to regular operation. The horizontal separation between trajectories of trains that stop at a given station gives the headway passengers experience at that station. The diagram in that case shows that joint stations retain the same average headway as in regular, all-stop operation, while the A and B stations, being served by alternate trains, have twice longer headways. Since the joint stations have shorter headways, they remain critical for the line capacity. The diagram for skip-
stop operation can also illustrate the increase in operating speed on the line achieved by skipping several stations.

*Express operations* on two-track lines require careful scheduling of local and express trains. Again, graphical scheduling is greatly superior to numerical methods, because inserting the trajectory of an express train between regularly scheduled train

![Diagram of train synchronization](image)

**Figure 7. Synchronization of local and express trains.**
runs is visually very simple. As shown in Figure 7, the express, shown by trajectory E, should be scheduled so that it overtakes a local train in a station with siding; the initial headway between the local and express trains $h_1$, which is needed for this operation is easy to obtain graphically. If there are no stations with sidings, so that overtaking is not possible, trajectory of the express train is "slid" to the right until it reaches the dashed-line position on the right, $E'$, where it "catches up" with the minimum headway behind the local train on the last interstation spacing. If it is important to utilize maximum line capacity achievable with this service regime, the express can be followed at the beginning of the line by another local train after a minimum headway, as shown on the diagram.

In general, express running is practical only when average headway on the line is considerably longer than the minimum one, so that the express train can skip several stations. The limits on express runs are imposed either by line capacity requirements or by the maximum acceptable headway.

**Trunk and Branch Operation**

Scheduling for a trunk line that divides into two or more branches is much more complicated than scheduling for a single line because it involves divergence and convergence of trains. The sequencing of TUs arriving from the various branches and the resulting regularity of the headway along the trunk section become important considerations. Algebraical coordination of schedules becomes quite complex, involving constraints on terminal times and merging sequences with multiple feasibilities, but not equally efficient solutions. Even when solved analytically, it may be difficult to visualize these solutions.

By contrast, using a graphical approach, feasible solutions can quickly be identified and compared. The method is to use the point of branch divergence $X$ as a reference. The trajectories for all routes (each consisting of a trunk and a branch section) are shown together on the trunk section, while the trajectories along each branch section are shown on individual, vertically separated but synchronized time-distance diagrams. A vertical dashed line is drawn between corresponding diverging and merging points for each TU trajectory, as shown in Figure 8.
To explain in more detail, a graphical diagram is started by plotting trajectories of successive TUs at given headways on the trunk line. At point X, branch A is continued without interruption, while the following TUs, going to branches B and C, are transferred vertically to the two respective diagrams. Terminal times on each

Figure 8. Time-distance diagram of a trunk with three branches.
branch must be determined so that TUs merge at point X with equal headways. Graphically, these terminal times are obtained easily, and they can be visualized much more clearly than in numerical schedule tables. This solution, which provides even headways along the trunk, can be compared to alternate ones that would minimize terminal times and fleet size for each route, but, instead, have uneven headways along the trunk.

Figure 9. San Francisco BART network and lines (1996).
In some complicated networks, analytic solutions can also become intractable. A good example is the Bay Area Rapid Transit (BART) network, which, until recently, consisted of three branches plus a "cross-connection" between two of the branches. Thus, the operation represents a triangle plus an additional trunk-and-branch line. The line pattern is shown in Figure 9. Although very difficult to model analytically, this network was successfully analyzed using a graphical method to arrive at several practical candidate scheduling solutions. This work was presented in several reports by Vuchic, Bruun and Krstanoski (1995-97).

Although space does not allow discussion of a complicated network like BART, application of graphical analysis to a network consisting of a trunk with two branches and cross-line between the branches will be explained here. This network also can be interpreted as a three-leg network with lines between all three terminals, as shown in Figure 10.

Graphical presentation for this case, shown in Figure 11, is the same as for a trunk line with two branches: the main diagram shows the A-X-B section, while the X-C branch is shown above it. A-B and A-C trains are shown as in Figure 8. If the B-C trains are shown as inbound trains on the B-X section, then, when transferring to the X-C diagram, they are shown as outbound trains. Once understood, this diagram is very helpful in coordinating the schedules of the three branches. Because of the interdependency, such schedules usually cannot provide regular headways on all lines, and the graphical schedule can be very useful in making adjustments that make the best candidate sets of headways for the three branches.
Use of Math Program Generated Schedules

Several computer-based packages, using math-programming methods, have been in use for more than 20 years for scheduling transit services and crews. However, there are occasionally constraints that cannot be expressed well mathematically that may affect the practicality of the generated solutions. Examples include the operation of different rail services on shared trackage that do not fully cooperate in scheduling and dispatching, or transit lines that require large amounts of slack time in order to operate reliably. Interconnected lines, such as the above-discussed BART network, may also be more conducive to graphical than numerical scheduling and analysis.

Plotting the results generated by the program as a time-distance diagram will allow visualization of the latitude that is available for deviations from the schedule.
before a recovery strategy must be used to reposition TUs due to loss of slot in the schedule. This is done by checking the horizontal separation between the non-agency-controlled vehicles and its own. The history of schedule adherence and delays along the shared section should provide some insight into the likelihood of delays serious enough to lose a slot as a function of the time slack available. Should the schedule appear too tight for reliable operation, the program can be rerun with changed constraints to include additional slack or “cushions” in the schedule.

An example application where a math-program-based schedule should be further analyzed is the Southeastern Pennsylvania Transportation Authority (SEPTA) Regional Rail Division. This is a regional rail network that includes not only SEPTA-controlled tracks, but also major portions where Amtrak controls all train dispatching. By using a time-distance diagram, it is possible to see how tight the gaps are between the Amtrak and SEPTA trains at crucial locations along the network. Crucial locations are those where SEPTA trains are likely to be held when there are delays. Adjustments can be made to the schedule to provide recovery time or, if possible, to avoid tight scheduling in the first place. Such a diagram also allows quick visualization of the quality of connections between the various interconnected lines at key transfer points.

**Applications in Operations Control**

So far, the discussion has been in a planning context. Graphical analysis is useful in an operational context as well.

**Real-Time Oversight and Reporting**

Real-time oversight of train operations with elaborate control centers has been around for many decades, although, in some cases, the positional resolution is poor when signal blocks are long. Oversight of buses through Automatic Vehicle Location and other Intelligent Transportation Systems (ITS) technologies is a much more recent capability with generally high accuracy. In both cases, numerical schedules in spreadsheet format provide information with which to monitor adherence in real-time by displaying the numerical deviation, perhaps highlighted by a blinking or color-changing display when deviations become sufficiently large. For trains, a sys-
tem schematic with lights indicating block occupancy is common. For buses, a GIS map showing vehicles of different colors (yellow—early, green—on time, red—late, for example) can also be used to highlight current status.

Spreadsheet-type displays have, however, some definite limitations. The bigger picture that might reveal how any deviations might be interrelated and where deviations are likely to propagate is not readily apparent with spreadsheets, schematics, or maps. A time-distance diagram where trajectories, or portions of trajectories change colors, used, for example, in the Italian Italtel bus dispatching/monitoring system, can readily reveal not only the current situation but impending deviations and conflicts. Even more importantly, as discussed in the next section, it provides some insight into when and where recovery strategies must be employed.

Moreover, when a real-time time-distance diagram is viewed on a high-resolution graphics terminal with a cursor ball, detailed information is instantly available. The horizontal cursor shows time information for any position along a route or common section. Every point where a trajectory crosses this cursor can be labeled with the time a vehicle passed or is projected to pass, which run number, vehicle, crew, etc. The vertical cursor provides position information at any particular time. Every point where the cursor crosses a trajectory can be labeled with the position coordinates (the precision of display depends upon the locating technology and update rate), run number, vehicle, distance separation from leading and following vehicles, etc. If there is more than one vehicle at a terminal, this is also readily shown automatically with more than one label.

Depending upon the ITS technologies in use, additional information about vehicle loading on a particular run, vehicle condition and other system attributes also can be instantly available. Thus, a graphical schedule is a means not only to track vehicles, but to monitor the entire operational status along a line or in a network, if desired. The data also can be stored, either temporarily or permanently. In the case of an incident a record is available, analogous to air-traffic control tapes or black boxes on airliners. Interesting records can be used for training purposes by replaying them to investigate control decisions that were taken.
Schedule Recovery Strategy Design

A numerical format provides little assistance in designing a delay recovery strategy. A GIS display map on a computer terminal provides some visualization of the situation, but it does not provide an accurate picture of headway between vehicles as it shows distance between vehicles. Distance is not necessarily proportional to headway in urban situations where speeds vary widely over the course of a route. By comparison, an experienced dispatcher can quickly assess the status of an entire fleet operating on a route, or assess an entire sector in the case where dispatching responsibility is divided by region instead of by route.

Several recovery techniques can be tested on the time-distance diagram, alone or in combination. Ideally, likely delay scenarios should be studied beforehand so dispatchers can be trained in effective strategies, but it is also possible to investigate alternatives in real-time, at the cost of a few minutes of elapsed time (and the risk that the situation may deteriorate further).

The basic recovery technique is to simply delay the dispatching time of a following TU, which is done by "sliding" an entire trajectory. Another technique is to extend the dwell time at a station. By extending a dwell time, part of a trajectory can be shifted to represent holding a TU at a station to restore its headway separation from the first delayed one. In turn, the next TU can be held at another station to restore its headway separation. This process is repeated until scheduled headways are re-established. An example of a recovery strategy involving holding two TUs following delayed ones is shown in Figure 12. Other techniques include improvised short-turns, express running, and insertion of extra TUs. These concepts were already discussed, so it suffices to say that these techniques can also be readily tested on the diagram as recovery strategies as well as basic scheduling strategies.

Real-Time Control

Once the capability for oversight and for design of recovery strategies is in place, the capability is almost in hand to actively control operations. Instructions can, of course, be sent manually through verbal, either oral or written, messages. But it is also possible to automate the instructions, thereby shortening the response time and reducing the layers of supervision required. An example follows.
Figure 12. Schedule recovery by delaying two following trains.

Assume that a tentative schedule recovery strategy has been tried on the computer screen by clicking and dragging the dwell time of a particular train to prolong its stay at a station. The result is a tentative revised trajectory. A verification prompt could ask if the dispatcher would like to implement this plan. If answered in the affirmative, the command can automatically be issued to the in-vehicle control panel as instructions to the operator in the case of manually operated vehicles, or to the Automatic Train Operation (ATO) unit in the case of automatic operation. Further-
more, the delay information can be automatically forwarded to the Passenger Information System and to the control centers for connecting modes. In turn, decisions can be automatically made with preset criteria whether to hold or dispatch connecting services, if desired.

**Conclusions**

Graphical scheduling is an old technique that has been neglected, or never acquired, in many North American transit agencies. Introduction of diversified and complex transit network operations in many cities creates potential for increasing applications of graphical methods in developing schedules, as well as in operations control. Actually, with modern computer graphics and ITS technologies it is more powerful than ever and deserves consideration where it is not currently being used. However, the latest dispatching/control software offered by transit ITS vendors in the U.S. does not take advantage of this approach.

Graphical scheduling retains its advantages in basic schedule design and analysis. It allows the solution of problems that are quite difficult to solve analytically. Even information about simple routes is enhanced by the detailed operating characteristics inherent in detailed vehicle trajectories and by the relative ease with which accelerated service and service recovery strategies can be investigated. It also can be used to confirm and refine solutions that are generated by analytic methods.

Graphical scheduling has additional advantages in operational control with the advent of modern ITS technologies. By movement of the cursor on a terminal screen, detailed information about any position along a route, or about all activity along a route at any particular moment, becomes available. An experienced dispatcher can gain a perspective on an entire route or geographic sector and anticipate problems at an early stage. In addition, it is possible to link the altering of trajectories through clicking and dragging to the automatic issuance of control commands and updates of passenger information.

**References**


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