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An Argument for Expanding Research on Quantitative Pedagogical Content Knowledge

Abstract

Pedagogical content knowledge (PCK) traditionally has been used in research concerning prospective and practicing classroom teachers. This essay argues that PCK is also relevant to other professions including those advancing quantitative reasoning (QR). To illustrate, the case of PCK for teaching QR is considered. Those in fields such as public health, journalism, meteorology, and government increasingly find themselves responsible for helping the public understand an ever-growing amount of quantitative information that has a bearing on societal well-being. Several examples illustrate how such professionals' responsibilities require knowing prevalent QR patterns in society, strategies for fostering sound reasoning, and the general nature of school curricula. Professional organizations in education, colleges of education, and educational researchers would benefit from expanding PCK research to encompass multiple professions. These expanded research and development efforts would simultaneously address urgent societal needs.

Keywords

pedagogical content knowledge, adult education, statistics, probability

Cover Page Footnote

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Introduction

Pedagogical content knowledge (PCK) is one of the foundational concepts in teacher education. Shulman (1987) originally characterized PCK as a "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (8). In the decades after Shulman's seminal work, much scholarship has been dedicated to conceptualizing and debating the essential components and structure of PCK (Neumann et al. 2019). Less work has been done regarding the scope of its applicability and its utility in various professional fields. Drawing motivation from current events, this article questions whether it is still productive to consider PCK to be "uniquely the province of teachers" or if it is a type of knowledge needed across multiple professions. The issue is taken up for the specific case of PCK needed to foster quantitative reasoning in the public; it is argued that it would be fruitful for educational researchers to give this type of PCK focused study beyond just formal classroom teacher preparation.

In recent years, it has become increasingly clear that the task of teaching quantitative reasoning is not limited to school classrooms. This point was vividly illustrated during the COVID-19 pandemic. Health officials and journalists attempted to help the public understand and act upon quantitative information from graphs, statistics, models, infection rates, and results of clinical vaccine trials. Although their attempts attained some degree of success, well-documented problems revealed needed areas for improvement when interacting with the public about quantitative matters (Barchas-Lichtenstein et al. 2021; Batova 2021; Malik et al. 2021). In some cases, it was clear that those interacting with the public needed better knowledge of common quantitative preconceptions and misunderstandings, and strategies for addressing them (Groth 2021; Kollosche & Meyerhöfer 2021). Developing this sort of knowledge is a well-established part of the preparation to teach school mathematics (Hill et al. 2008); efforts to educate the public during the pandemic proved that professionals who communicate with the public need similar types of knowledge.

Bakker et al. (2021) pointed out that the pandemic magnified the fact that the teaching of mathematics and quantitative reasoning is inevitably at times carried out by those who are not formally prepared as classroom teachers. Public health and journalism are just two of the many professions with responsibilities that sometimes overlap with the teaching of mathematics. Those in government, data science, meteorology, and many other fields at times have responsibilities that include educating the public about quantitative matters and their impact on society. Accordingly, this commentary argues for expanding research on PCK beyond its traditional focus on classroom teachers. The Hill et al. (2008) conceptualization of PCK is used as a framework for discussion; it offers three primary PCK components to consider: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and curriculum knowledge. The relevance of the three components for multiple professions is considered next.

Knowledge of Content and Students

KCS can be defined as "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (Hill et al. 2008, 375). Furthermore, "KCS is used in tasks of teaching that involve attending to both the specific content and something particular about learners, for instance, how students typically learn to add fractions and the mistakes or misconceptions that commonly arise during this process" (ibid.). Just as mathematical difficulties prevent students from functioning optimally in mathematics classrooms, they present challenges to many adults as they navigate increasingly complex quantitative aspects of society (Steen 2001).

Table 1
Examples of Prevalent Misunderstandings of Quantitative Ideas and Professionals in Positions to Address Them

Quantitative Knowledge Needed in Society	Sample Misunderstandings	Potential Remediators
Interpreting probabilities expressed as percentages to understand likelihood and risk	Interpreting a low percentage to indicate that an event will not occur (e.g., 10% chance of adverse side effects meaning there will absolutely be no side effects) or a high percentage to indicate that it will (70% chance of a weather event meaning it absolutely will happen; 80% probability that a document was written by artificial intelligence (AI) meaning it was written by AI)	Meteorologists, doctors, pharmacists, government officials, data scientists
The relationship between statistical study design and the certainty with which conclusions can be drawn	Equating randomized, controlled trials of a medication or vaccine with observational studies or anecdotes	Public health officials, drug companies, journalists
Combinatorial reasoning and its role in computing probabilities	Overestimating one's chances of winning the jackpot in a lottery or casino game	Government officials, intervention programs
Distinguishing between random and other forms of sampling	Believing that stories selected for individual newsfeeds by algorithms are representative in the same way as news stories drawn at random from a comprehensive collection	Social media companies, journalists
Recognizing that the mean is usually more sensitive to outliers than the median	Not realizing that averages are often selectively chosen to support a given argument (e.g., using mean home price in an area that has one very expensive property rather than median during negotiations to argue for a higher selling price)	Real estate agents

To illustrate, consider human reasoning related to statistics and probability. Misunderstandings in these areas have been documented extensively by those doing research in mathematics and statistics education (Shaughnessy 2007) and in other fields such as psychology, biology, medicine, economics, and various social sciences (Petocz et al. 2018). Misunderstandings of statistics and probability often have deleterious effects on well-being at an individual and societal level. Table 1 gives examples of some prevalent misunderstandings and how they manifest themselves in different everyday contexts. Table 1 also identifies some professionals in society who are in positions to potentially help address each misunderstanding.

In some cases, the professionals listed in the third column of Table 1 do not have the KCS needed to anticipate prevalent misunderstandings of those with whom they interact. For example, during a press conference held in the early stages of the COVID-19 pandemic, public health officials expressed a need for randomized, controlled vaccine trials. Comments and questions from those in attendance reflected lack of understanding of how such trials differ from other types of trials (Groth 2021), suggesting that public health officials were caught off-guard by their audience's lack of understanding of this matter. As another example, meteorologists and government officials who seek to help the public assess personal risk need to know that the outcome approach to probabilistic thinking (Konold 1989) is prevalent. Individuals using the outcome approach often think deterministically about probability statements; they may interpret a high probability to indicate that an event absolutely will occur and a low probability to indicate that it will not. This often leads to poor decisions that involve risk-benefit analyses, such as deciding whether to evacuate an area as a hurricane approaches or whether to fasten one's seatbelt when driving. Individuals may use a statement of low probability of harm to predict an absolute outcome of no harm, and hence not take even minimal effort to avoid danger. In such cases, professionals need KCS for statistics to better understand the audiences with whom they are attempting to communicate.

In other cases, professionals have knowledge of prevalent misunderstandings but purposefully leverage the misunderstandings for their own gain. For example, it is in the self-interest of casinos and lotteries to de-emphasize or avoid mentioning the low probabilities of winning large sums of money in favor of taking advantage of many individuals' propensities to overestimate their chances of winning. As another example, a real estate agent selling a home has a built-in profit motive to advertise the mean, rather than median, value of homes in the neighborhood when there are outlying home values that inflate the potential selling price. Such a decision may be based on knowledge of individuals' difficulties characterizing typical values of data sets (Garfield & Ben-Zvi 2007). Pharmaceutical companies have also been accused of using misleading advertisements that intentionally exploit lack of understanding of statistics and probability. Lexchin (2010) found that such advertisements at times use misleading graphs and charts, make misleading claims about risk reduction, and omit important information such as power and confidence intervals. To compound the problem, professionals who could potentially help the public interpret such advertisements, including doctors and journalists, at times have the same weaknesses in statistical reasoning that prevent others from critically evaluating the advertising claims (Gigerenzer et al. 2007; Voiklis et al. 2022).

The preceding examples suggest some important and intriguing research questions. One such question pertains to how KCS and content knowledge can be developed in tandem in multiple professions, to avoid situations such as doctors having some of the same statistical misunderstandings as patients. Existing educational research on helping teachers develop statistics PCK alongside statistics content knowledge may provide starting points for this work (Groth 2012; Green & Blankenship 2013). Such research suggests that KCS need not be developed in isolation from content knowledge; learning about common statistical misconceptions may help an individual diagnose whether they are falling prey to misconceptions as they develop their own statistical reasoning. Another important and intriguing set of questions pertains to what could be called the weaponization of KCS. This has not been an issue in PCK research up to this point, as classroom teachers have no profit motive for exploiting the misunderstandings of others. Professionals in other fields, however, can encounter such conflicts of interest. Investigating misuses of KCS and developing ethical frameworks to mitigate them are among the areas of investigation awaiting educational researchers who extend the study of PCK to other fields.

Knowledge of Content and Teaching

KCS provides a foundation for development of knowledge of content and teaching (KCT) (Groth 2013). KCT allows one to “evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally” (Ball et al. 2008, 401). As one illustration of the relevance of this type of knowledge beyond traditional classroom settings, consider the “cone of uncertainty” representation government officials and meteorologists use to try to help the public assess their personal risk from a hurricane. The cone of uncertainty representation is meant to foster sound probabilistic thinking among citizens in potentially dangerous situations. Using such representations to teach the public about risk is an important endeavor. Hence, knowledge of the cone of uncertainty representation can be considered an aspect of KCT important to many meteorologists and government officials.

The cone of uncertainty example also illustrates that representations used to educate the public need to be carefully studied for their pedagogical efficacy and continuously refined and improved through systematic research. Evans et al. (2022) documented several misinterpretations of the cone of uncertainty representation; people sometimes think that it shows the size of a storm or that it deterministically shows where damage will occur. As another example, misunderstandings of quantitative representations intended to help the public understand the dynamics of

COVID-19 (e.g., “flattened” curves) have been well-documented (Chan et al. 2021). As such misunderstandings of quantitative representations occur, it is important to revise the original representation or develop alternative representations. KCT needs to be a continuously evolving aspect of PCK, with educational representations and methods steadily improving as new research-based insights are gained about their effectiveness. Educational researchers are positioned to engage in this type of research and development with methods such as design-based research (Bakker & van Eerde 2015).

Educational researchers are also positioned to help other professionals move beyond extensive reliance upon one-way, univocal (Otten et al. 2015), transmission-oriented methods of educating the public. In the twentieth century, large-scale educative communication efforts often featured public service announcements delivered via one-way means such as television and print media. In the twenty-first century, it is still common for professionals in various fields to focus on one-way transmission of information via public service announcements, press releases, educational television, videos, and websites. For example, the Centers for Disease Control and Prevention (CDC) published a “Clear Communication Index” in 2019. It advised health professionals to include a main message at the top of the page to be shared with the public, include visual supports for text, explain unfamiliar terms, and use headings and chunked text. It offered guidelines for helping the public understand some specific quantitative issues, such as the following advice on the concept of risk:

> When you write, talk about or show images to convey risk, be sure the meaning of risk you intend is clear from the context and topic of the material. You should not use only qualitative descriptors, such as high and low or large and small, by themselves to describe risk because people may not interpret these words the same way. A large risk to one person may be a small risk to someone else (CDC 2019, 25).

Although this advice is based on KCS about human conceptions of risk, the underlying KCT is naïve in assuming that content can be directly transmitted to an audience if it is presented correctly. The excerpt is representative of the entire CDC (2019) document in that it focuses on trying to help professionals “convey” an idea as they “write,” “talk,” and “show images.” Limitations of teaching methods that rely extensively on such univocal transmission-oriented pedagogy have been well-documented in educational research (Sfard 2020).

In the twenty-first century, mass public communication is no longer limited to professional experts conveying messages via one-way means. Information disseminated via univocal means catalyzes interactions among individuals in online spaces. Such interactions are often dialogic (Otten et al. 2015) in that they involve back-and-forth interaction among individuals to construct shared meanings. The meanings collectively generated about matters of public concern in online spaces often become pervasive, for better or worse. So the pressing question for twenty-first century professionals who seek to educate the public is not *if* dialogic discourse should occur, but rather *how* they might position themselves in it and steer it in productive directions. Improving one-way, large-scale univocal teaching about quantitative matters can still be beneficial, but it is not adequate. Professionals with well-developed KCT would be skeptical of public education efforts that rely too heavily on univocal teaching methods.

Educational research has a great deal to offer regarding how educators can position themselves to steer dialogic discourse in productive directions. It contains findings about effective teacher discourse moves (Cirillo et al. 2014; Jacobs & Spangler 2017) and conditions under which productive dialogic interactions occur (Otten et al. 2015; Herbel-Eisenmann et al. 2017). Insights from such research do not necessarily translate seamlessly to the dynamics of large public online discourse spaces, yet they provide a basis for empirically grounded conjectures about the optimal positioning of professionals within such spaces. Educational researchers and professionals from other fields could collaborate to create and test conjectures about experts' positioning in public online discourse and refine approaches considering results. Progressively making, testing, and refining conjectures in this manner could lead to robust strategies for fostering productive dialogic teaching about specific quantitative ideas on a large scale. Ultimately, these strategies could be codified into aspects of KCT shared across multiple professions.

Curriculum Knowledge

Some quantitative reasoning difficulties among the public may be plausibly traced to experiences with school curricula. For example, traditional mathematics courses that focus mainly on decontextualized problems may lack opportunities to help students see how quantitative reasoning can be used to make sense of the world (Dingman & Madison 2010). Additionally, in US mathematics curricula, statistical study design and subjective probability have historically been under-emphasized (Shaughnessy 2007; Langrall et al. 2017), which may help explain some of the reasoning difficulties among the public discussed earlier. The nature of school curricula in a country or region is useful to consider as a predictor of the type of quantitative reasoning support adults in the public will need (Shaughnessy 2007; Krause et al. 2021). Professionals with strong curriculum knowledge are better positioned to anticipate quantitative reasoning strengths and difficulties among the public. This type of knowledge can inform efforts to design teaching strategies and representations intended for public use. As professionals in various fields become more conversant with educational researchers and curriculum developers about typical curricular patterns, school curricula and attempts to engage adults in quantitative reasoning can improve simultaneously. Ideally, during interdisciplinary collaborative research and development, Pre-K–12 curricula

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would be optimized to meet the quantitative reasoning needs of future adults and attempts to engage adults in quantitative reasoning would become more responsive to experiences the adults generally had in school.

Several phenomena suggest that development of quantitative reasoning will always need to be conceptualized as an ever-evolving lifelong learning progression rather than one that culminates with the completion of the study of school mathematics curricula. One reason for this is that the relative importance of different quantitative ideas to daily life constantly shifts with societal trends. Increased legalization of gambling suggests a need for greater public knowledge of combinatorics and odds, the COVID-19 pandemic suggested a need for greater emphasis on understanding the nature of statistical models, the emergence of social media algorithms to disseminate news stories illustrates a need for more understanding of how random and non-random samples differ, and so on. Even if it were possible to predict the increasing importance of certain quantitative ideas associated with future social trends, an extensive body of research on the context-dependency of human learning illustrates that people often do not seamlessly transfer learning from school mathematics curricula to related quantitative tasks outside of school settings (National Research Council 2000). Integrating adult education and Pre-K–12 curriculum research more closely could enhance knowledge of Pre-K–12 curricula across multiple professions while improving efforts to educate both children and adults.

Synergistic Benefits of Expanding PCK Research

Several societal benefits of expanding research on PCK to multiple professions have been conjectured up to this point. Some of the key infrastructure needed to attain these societal benefits resides in professional organizations and schools of education. Given the membership declines in professional organizations (e.g., Larson 2016; Stevens et al. 2023) and declining enrollments and funding for many schools of education (American Association of Colleges for Teacher Education 2022; Herbst 2023), it may be difficult for these infrastructure elements to meet the challenge. However, taking up the challenge of extending PCK research to multiple professions can strengthen this infrastructure as greater societal benefits are realized (as shown in Figure 1). Next, some ways in which interdisciplinary PCK research can produce the synergies illustrated in Figure 1 are considered.

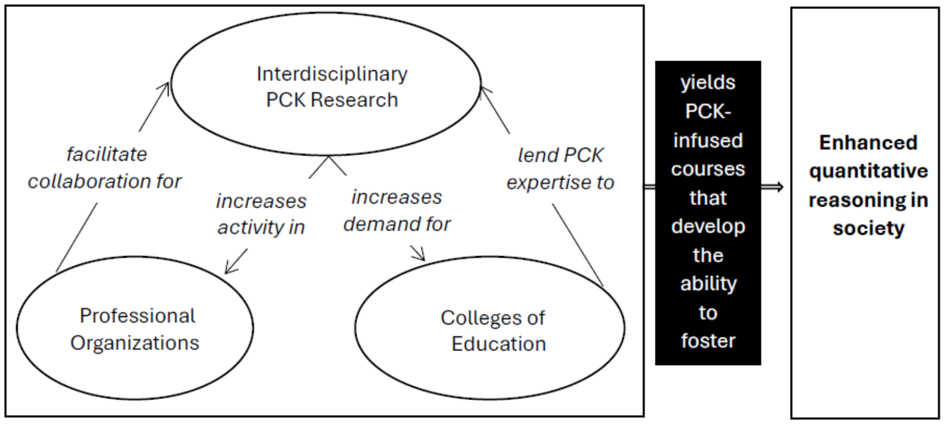

Figure 1. Potential synergistic relationships related to expanding PCK research to multiple disciplines

Synergy Involving Professional Organizations

Fostering the kinds of PCK considered in this article across a broad array of professions will require improvements to courses various professionals take during their postsecondary preparation. Ideally, such improvements would be informed by those with expertise in each professional domain and by those with expertise in quantitative reasoning and teacher education. Professional organizations could play important roles in bringing such professional experts together. Consider, for example, the PCK infrastructure that might be developed through a hypothetical collaboration between the National Council of Teachers of Mathematics (NCTM) and the Society for Public Health Education (SOPHE). SOPHE seeks to "build knowledge and skills to help health educators respond to public health emergencies" and to help public health workers (SOPHE 2023). NCTM "provides guidance and resources for the implementation of research-informed and high-quality teaching" (NCTM 2023). These SOPHE and NCTM goals overlap in the case of helping health professionals learn to communicate effectively with the public about quantitative matters. Working together within this shared space could yield incisive scholarship on the mathematics PCK health professionals need and how to develop it. Similar interdisciplinary collaborations could begin to address the same issue for other professions while strengthening multiple disciplinary fields and professional organizations that support educational research in the process.

Interdisciplinary PCK research collaborations supported by professional organizations might take many different forms. In some cases, they might involve recruiting new members to a given professional organization. Bringing new members into a professional organization, however, is not a trivial matter. It may require re-thinking the core goals of the organization. For example, if NCTM were to consider recruiting health professionals to their organization, they would have to decide whether they could best serve society by continuing to have a core focus on classroom mathematics teachers or if it would be beneficial to broaden the definition of "mathematics teacher" to include others in society who need to be able to foster quantitative reasoning. Such judgments are ideally made by the organization's members. If broadening the scope of such organizations is not deemed feasible, partnerships with other professional organizations could be pursued. The hypothetical NCTM-SOPHE collaboration discussed earlier might be one example of this. In any case, professional organizations can become stronger by expanding their horizons to include diverse professional perspectives. Strengthened professional organizations help ensure that professions continue to define, support, and regulate themselves rather than being overly constrained by questionable regulations devised by others.

Synergy Involving Colleges of Education

As noted, the need to teach the public about quantitative matters has rapidly spread beyond the bounds of traditional classrooms, so PCK-infused coursework would be valuable in many professional preparation programs. Typically, PCK development is a primary goal in teacher education. Accordingly, the construct of PCK has primarily been used and developed by educational researchers. Hence, much of the expertise needed to design and develop PCK-infused courses resides in colleges of education. Infrastructure that resides in colleges of education should therefore be leveraged by research teams that seek to develop PCK-infused courses and professional development. Professionals in various fields who gain these enhanced educational experiences would be in better positions to foster sound quantitative reasoning in society.

Along with the broader societal benefits of enhanced PCK across professions, the infrastructure for doing educational research can benefit from interdisciplinary work. Such work can lead educational researchers to tap new and under-utilized external funding sources. For example, development of PCK courses for health professionals might be funded by the National Institutes of Health (NIH) and similar agencies whose funding practices and norms would be new territory for many educational researchers. In such cases, interdisciplinary collaborations could help educational researchers navigate unfamiliar terrain. Agencies that are more familiar to many educational researchers also have funding programs that could be used for similar purposes. The National Science Foundation (NSF), for example, has a solicitation entitled Advancing Informal STEM Learning (AISL). The solicitation encourages proposals that focus on public engagement with and understanding of STEM (NSF 2023). This funding stream may be overlooked by many educational researchers because it specifically excludes "formal elementary, middle, or high school, or undergraduate or graduate education" (NSF 2023). However, AISL funds could be used to learn more about the quantitative reasoning patterns of the public in everyday settings and subsequently help professionals who interact with them develop stronger KCS. As educational researchers focus on teaching and learning beyond traditional classrooms, possibilities for funding their studies expand.

Participating in the development and teaching of PCK-infused courses across disciplines can also increase the demand for educational researchers. Universities have experienced declining participation in undergraduate teacher preparation programs in recent decades (American Association of Colleges for Teacher Education 2022). Declining enrollment in colleges of education negatively impacts the number of university faculty members doing educational research. It can lead to the hiring of clinical faculty members rather than tenure-track faculty expected to do research as part of their work (Herbst 2023). Declining enrollment in colleges of education also prompts reduced financial support from universities and closures of smaller programs. Closure of programs can result in loss of tenure-track faculty members from disciplines in which teacher shortages are the most pronounced, such as mathematics and science education. Expanding the roles of colleges of education within universities beyond formal classroom teacher preparation could help turn the tide on such existential threats by increasing the demand for faculty with educational research expertise.

Conclusion

Teaching that fosters quantitative reasoning and societal well-being is needed beyond school settings. This has become increasingly clear in the twenty-first century with the explosion of quantitative information related to societal well-being. Given the quantitative reasoning demands of being a citizen of the modern world and the nature of human learning, education is best viewed as a lifelong endeavor rather than one that ends at the conclusion of formal schooling. Professionals who need to educate the public about urgent matters can benefit from having strong PCK to do so effectively. Educational researchers can help clarify and address such PCK needs by leveraging and building upon extant professional organizations, methods, and findings in educational research. The current moment in history provides an opportunity to build these educational research infrastructure elements while simultaneously addressing the urgent societal need for widespread, sound quantitative reasoning.

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