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Computing for Numeracy: Kiyoo Mogi and the Nature of Volcanoes

Charles Connor

University of South Florida, cbconnor@usf.edu

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Computing for Numeracy: Kiyoo Mogi and the Nature of Volcanoes

Abstract

Dramatic volcanic eruptions occurred in the Spring of 2021 in Iceland and St. Vincent. This column explores the use of a numerical model to understand the giant displacements of the Earth's surface that result from such volcanic activity. The model used was development by Japanese geophysicist Kiyoo Mogi to explain a much older eruption, the 1914 eruption of Sakurajima volcano, located in Kyushu, Japan. Mogi's model was so successful, and is still widely used today, because he took a step-by-step approach to solving this complicated problem, making simplifying assumptions where he could, and using data to the maximum extent possible to estimate a reasonable solution.

Keywords

volcano, Mogi, deformation, integration, solid of revolution, volcanic eruption, Iceland, St. Vincent, Japan, python, javascript

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Cover Page Footnote

C. Connor is a professor of geology at the University of South Florida, former Chair of the Geology Department and former Associate Dean of Research for the College of Arts and Sciences. He studies volcanoes mostly, especially how to forecast volcanic eruptions and their hazards. Connor teaches students about quantitative methods in geology and about writing computer code to solve geology and geophysics problems. He is a Fellow of the American Association for the Advancement of Science.

Computers for Numeracy

Numeracy, computing and computers, go hand-in-hand. This column explores topics in science, technology, engineering and mathematics (STEM!) that are of social importance by taking an algorithmic approach to problem solving. Each column defines a problem, develops a recipe for solving the problem, and implements a solution, with an example in computer code, often provided as an accompanying website.

Introduction

In this column, I approach STEM questions from the perspective of quantitative literacy. Questions that arise in the natural sciences are important to us all, but a few specific, if idealized readers come to mind in writing the column. This particular column includes a real-world scientific application of the calculus of solids of revolution. Perhaps calculus teachers and their students will be interested in how solids of revolution help us understand dangerous volcanic eruptions. The column is also written for STEM researchers, including graduate and undergraduate students, with the idea that natural sciences can advance significantly by considering questions, in this case geophysical questions, using a pragmatic, numerate approach. My hope is that many numerate people who are not scientists will enjoy contemplating topics in natural sciences by utilizing the increasingly familiar tools of numeracy.

2021 was a busy year for the Earth's crust. In the fall, global attention turned to spectacular eruptions on the island of La Palma, Spain. Before that, a new volcanic eruption started in Iceland in March, exactly where the Mid-Atlantic Ridge makes landfall on the southwest side of that island, only 30 km from the capital city of Reykjavík. Fortunately 30 km is far enough away, so far, for the population and infrastructure to avoid damage, unlike on La Palma. Since the Iceland eruption is in a very sparsely settled area, it has evolved into a celebration of the dynamism of our planet, with tens of thousands of people driving out from the city to watch the lava flows progress across a barren landscape (Ives and Peltier 2021), and to watch remotely-operated drones dive to and fro among the erupting vents, sometimes falling into them (Machemer 2021).

The Caribbean island of St. Vincent was not so fortunate with its 2021 volcanic eruption. Explosive eruptions occurred from La Soufrière, the most active volcano on the island, in April. These explosions were presaged by unrest that began in December, 2020, with the formation of a new lava dome within the crater of La Soufrière. This lava dome is a blister of viscous magma that erupted slowly at the

surface, literally constructing an ellipsoidal dome of rock more than 600×200 m in diameter and tens of meters tall. This dome-building activity in the La Soufrière crater garnered considerable attention among islanders because of the volcano's violent history of eruptions. An eruption in 1902 killed approximately 1,600 people, only months after the eruption on Mt. Pelée on the nearby island of Martinique killed more than 20,000. In April, 2021, the new eruption turned explosive, with eruption columns reaching at least 16 km into the atmosphere and widely distributing volcanic ash across the island and the nearby islands of The Grenadines and Barbados.¹ This volcanic activity resulted in evacuation of more than 16,000 people, and it will be long before the island will return to normal. Unlike the Icelandic eruption, this disturbance of Earth's crust is no cause for celebration. Instead, it is a challenge, at best, to contend with an explosive volcanic eruption.

What do these different volcanic eruptions have in common? They are caused by magma rising through the Earth's crust and erupting at the surface. Volcano scientists in Iceland monitored the effusion rate of magma from the volcano, which actually consists of a series of volcanic vents distributed along one or more igneous dikes. These dikes are plane-like sheets of magma that literally crack the Earth as they ascend, like a knife forced into a hard cheese wheel. Monitoring the eruption, Icelandic scientists found that for much of the eruption lavas were emitted at a rather steady rate of about 6 cubic meters per second.² Given a magma density of 2800 kg m^{-3} (kilogram per cubic meter) that is a mass flux of about 16,000 kg per second. During its dome-building phase of the eruption, La Soufrière was effusing magma at a comparable rate, so these two volcanoes together were emitting roughly 30,000 kg per second.

For comparison, the global rate of steel production is about 57,000 kg per second (Statistica 2021). In other words, in March 2021, these two volcanoes alone were pumping out mass at a rate about 1/2 of the rate of Earth's total steel production. But all of these numbers are dwarfed by the mass flow rate during explosive La Soufrière eruptions in April. To achieve a vertical column of volcanic ash extending 16 km into the Earth's atmosphere requires a mass flow rate on the order of 2×10^7 kg per second (Wilson et al. 1980; Woods and Self 1992). During April, some ash plumes were reported to have been sustained throughout the day, with plume heights varying from 11–16 km. Obviously there is considerable uncertainty in these mass flow numbers, but it is clear that in a single day, April 10, 2021, mass output from La Soufrière exceeded the annual production of steel in all of Earth's factories combined.

Where does this mass come from? Scientists have long contemplated “the

¹Arlene Laing, Coordinating Director of the Caribbean Meteorological Organization, personal communication

²Magnus T. Gudmundsson, Professor, University of Iceland, personal communication

volume problem”. New magma erupts at the surface, but where was it originally in the subsurface? Does the Earth change shape when magma moves from the inside to the outside, and by how much? It turns out the answer to the second question is yes, the Earth does change shape. To learn how much it changes shape requires precise measurements and a numerate approach to interpreting these precise data. The first and perhaps best known application of numeracy to solve the problem of volcano deformation started in Japan, after the 1914 eruption of another dangerous volcano.

A numerate approach to a volcanic eruption

In 1958, Kiyoo Mogi published a paper in which he described remarkable change in land elevation of Sakurajima volcano during and following its 1914 eruption (Mogi 1958). Sakurajima is a volcano located in Kagoshima Bay at the southern end of the island of Kyushu. The bay itself was formed by giant volcanic eruptions in the very recent geologic past, only about 22,000 years ago (Aramaki 1984). The Sakurajima volcano grew from the seafloor of the bay after these giant eruptions. It is one of the most active volcanoes in Japan.

In 1914, Sakurajima was an island in the bay. Its eruption started with explosive volcanic eruptions, much like those that rocked St. Vincent in the Spring of 2021. The Sakurajima explosive eruption is thought to have sent volcanic ash up to 17 km above the volcano and hundreds of kilometers downwind (Todde et al. 2017). It was the largest explosive eruption to occur in Japan in the 20th century. Even so, most of the magma erupted in 1914 was effused from the volcano as lava flows. These lava flows were so voluminous they created a new isthmus, linking the island to the mainland (Ishihara et al. 1981).

According to Mogi, a geodesist named Ōmori visited Kagoshima Bay shortly after the 1914 eruption. Ōmori re-surveyed railway lines around the bay, following survey transects that were first made in 1895. Remarkably, he found that the post-eruption 1914 land surface was deformed compared to 1895, even though the changes were so gradual they could not be observed by the eye. The change in topography was only revealed by the precise leveling measurements. Twenty kilometers from the volcano the 1914 ground surface had subsided by approximately 20 cm compared with the elevations measured in 1895. Subsidence increased as the survey transect approached the volcano from any direction, reaching almost 1 m of subsidence approximately 7 km from the volcano. The 1895 survey approached no closer to the volcano, so Ōmori could not measure change in topography closer in. Much of this area was covered by new lava flows and volcanic ash anyway, so it might have been difficult to measure the sagging of the landscape precisely.

At any rate, Ōmori had documented tremendous deformation of the land around

the volcano. Consider a change land elevation of $\Delta h = 20$ cm spread over a circle of radius, $r = 20$ km, and centered on the volcano. The change in volume represented by this change in elevation is:

$$\Delta V = \pi r^2 \Delta h. \quad (1)$$

Converting to SI units and substituting Ōmori's measurements:

$$\Delta V = \pi \times (20000 \text{ m})^2 \times 0.2 \text{ m} \approx 8 \times 10^7 \text{ m}^3 \quad (2)$$

and of course Δh continues to increase toward the volcano. Ōmori ascribed this volume change to the magma erupted onto the Earth's surface during the eruption.

Mogi wanted to know more about the process by which this change in volume occurred. He realized the pattern of deformation of the ground surface provided insight about the change in pressure beneath the volcano during the eruption. New magma rising in the Earth's crust causes excess pressure, which causes the Earth's surface to bulge over the magma. When the magma erupts, the excess pressure is removed and the surface deflates. Mogi reasoned that magma was already present during the 1895 topographic survey, pressurizing the Earth like a loaded water balloon near its elastic limit. Ōmori observed deflation when he re-surveyed the area after the eruption because the eruption relieved this excess pressure by erupting the magma onto the Earth's surface.

Mogi's first step in solving this problem was to visualize the deformation observed by Ōmori. Mogi did not know the geometry of the magma body, so he simplified. He considered the magma body to be a sphere located at some depth in the Earth. As long as the sphere was deep, which means its radius is much less than its depth, then the magma body can be approximated as a point source. With this symmetric source, Mogi could consider the deformation as a function of radial distance from the volcano, because the magnitude of deformation should be the same in any direction.

Developing a nomenclature, the point S is a distance r from the center of the pressure source, O :

$$r = \sqrt{x^2 + d^2}.$$

x is the radial distance at the Earth's surface and d is the depth to the center of the sphere, where depth is measured positive downward (Fig. 1).

The vertical change in elevation produced by change in excess pressure within a spherical magma in cylindrical coordinates (displacement as a function of radial distance from the center of the sphere) is given by:

$$w = \frac{\Delta P(1 - \nu)}{G} \frac{a^3 d}{(x^2 + d^2)^{\frac{3}{2}}} \quad (3)$$

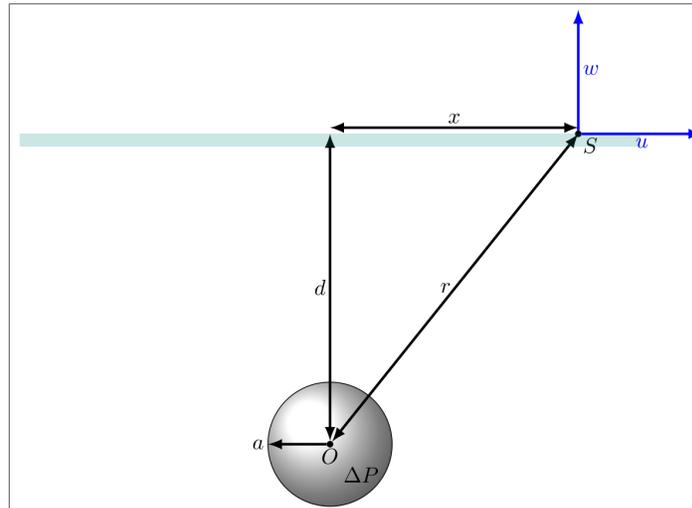


Figure 1. The geometry of Mogi's problem. The spherical magma source, centered on point O , has radius a and is located at depth d , where $a \ll d$. The displacement of the Earth's surface is calculated at point S , located a radial distance x from O . The point-like pressure source has excess pressure, ΔP , which results in displacement at the Earth's surface (blue shaded line), which is described by the two vectors w and u , for vertical and horizontal displacement. If the pressure is in excess, the ground surface inflates and $\Delta P > 0$. If pressure is removed, by a volcanic eruption, the pressure is reduced and $\Delta P < 0$.

where w is the vertical (z) component of surface displacement (m) at point $S = (x, 0, 0)$. Notice the right-hand side of this equation is shown in two parts. The first fraction has ΔP in the numerator, that is the excess pressure or, if negative, the decrease in pressure inside the Earth following the eruption. The fraction also has two variables, v and G , which are defined below.

Consider the second fraction on the right-hand side of the equation. In the numerator, d is the depth (m) to the center of the spherical pressure source, and a is its radius (m). The numerator has the dimension of length raised to the fourth power (m^4). The denominator is the cube of the radial distance of the observation point, S , from the center of the sphere, so the unit of this entire fraction is also length (m). This is the unit of the displacement that we want to explain, w . Since the units of displacement are accounted for, it follows that the first fraction:

$$\frac{\Delta P(1 - v)}{G} \quad (4)$$

is a dimensionless ratio. Volcano scientists use these kinds of ratios to predict the magnitude of the response to some force. In this case, ΔP is the applied pressure change (MPa) because of the magma. The SI units of pressure are Pascal (Pa), and since there is a big pressure change in this problem, mega-Pascal is used (MPa). The

variable G is called the shear modulus, but can be thought of as the Earth's rigidity or strength, and has the same units (MPa). If the Earth is particularly strong and rigid, G is large and the ratio shows that there will be less surface displacement for a given excess pressure. If G is relatively small, the Earth is relatively weak and easily bent, then the displacement will be relatively large for a given excess pressure. The remaining variable in this fraction is v , which is called Poisson's ratio. This variable attempts to account for the compressibility of the Earth and varies between 0 and 1. If the Earth is highly compressible, Poisson's ratio is large. This means the rocks will squeeze together, changing volume in response to the excess pressure. For highly compressible rocks, v is large so the displacement observed at the surface will be less for a given excess pressure. If the rocks are not particularly compressible they retain their original shape when the pressure is applied. In this case, with small Poisson's ratio, the excess pressure will create more displacement at the surface.

You can see that the problem of how much displacement will occur at the surface due to the volcanic eruption is divided into two parts. For the geometric part, Mogi knew the radial distance from the volcano, but he did not have any information about the radius of the spherical magma, a , or about the depth of the center of the sphere, d . Mogi also did not have information about the excess pressure, ΔP , and he had access to scant experimental information about the shear modulus, G and Poisson's ratio, v .

That is, he had one equation and many unknowns—a common problem faced by many scientists in using equations to explore the natural world. Nevertheless, Mogi had Ōmori's data. So he set about experimenting with different sets of variables to solve the equation for vertical displacement and compared the results with the observed change in topography. One solution is shown in Figure 2.

Although we cannot be certain what the depth or radius of the magma body was prior to the 1914 eruption, or what the values of the parameters in the fraction

$$\frac{\Delta P(1 - v)}{G} \quad (5)$$

are precisely, we can see that some combination of these parameters fits the data nicely. The advantage of using this model is that it predicts what the deformation may have looked like closer to the volcano, where Ōmori could not measure it. Notice (Fig. 2) that Mogi's model forecasts deformation reaching about 1.5 m directly over the pressurized magma source. If we can make educated guesses about some model parameters, like G and v , then we can begin to deduce values for the other parameters in the equation.

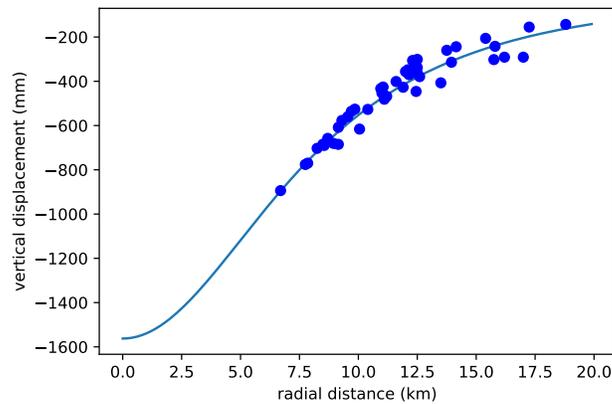


Figure 2. The calculated vertical displacement (blue line) as a function of distance from the center of the model spherical magma body at some depth within the Earth. The displacement data collected by Ōmori are shown as solid blue circles. Changing the model parameters changes the fit between the Mogi model and Ōmori’s observations, but more than one set of parameters reproduces the shape of the curve shown here.

Volume reduction in the subsurface following the 1914 eruption

The Mogi model provides a means of estimating the volume of material “missing” from the subsurface following the 1914 eruption and comparison with the volume of magma erupted. The Mogi model is fit to the observed displacements (Fig. 2), then these parameters are used to calculate the change in radial distance from the volcano as a function of vertical displacement. This function defines a solid of revolution, so integration over the range from zero displacement to the maximum displacement (at $x = 0$) gives the volume of the solid, in this case the volume of material missing as reflected in the observed radially symmetric ground deformation (Fig. 3). That is, by fitting a physical model to the data, Mogi was able to forecast ground displacement even where it was not observed. If we believe the model is reasonable, then we can calculate the total volume of material missing from the relatively few observations that could be made.

First, create a lumped parameter to simplify the algebra:

$$\zeta = \frac{a^3 |\Delta P| (1 - \nu)}{G}$$

Then, recast Mogi’s equation as the radial distance in terms of magnitude of vertical displacement, $|w|$:

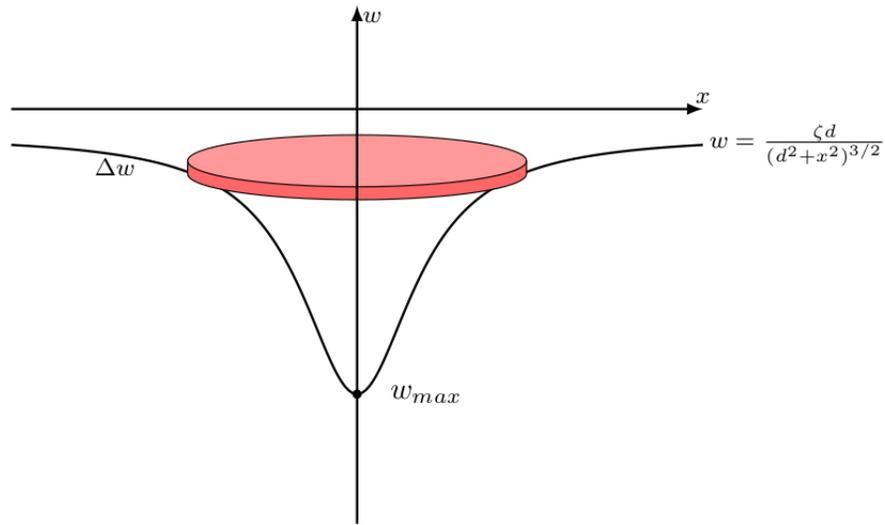


Figure 3. Because Mogi’s model is radially symmetric about the volcano pressure source, we can think of the radial distance, x , as a function of the displacement. That is, the total volume is the sum of a group of horizontal disks of thickness Δw . One of these disks is shown in red. The radii of these disks is a function of w . The volume missing from the subsurface after the volcanic eruption is the sum of the volumes of these disks, or the integral of the function describing the change in disk radius with displacement.

$$x = \sqrt{\left[\frac{\zeta d}{|w|}\right]^{2/3} - d^2}.$$

Now, find the volume of the solid of revolution, rotating about the vertical axis:

$$\Delta V = \pi \int_0^{|w_{max}|} \left[\left[\frac{\zeta d}{|w|}\right]^{2/3} - d^2 \right] d|w|$$

where $|w_{max}|$ is the maximum amplitude of vertical displacement at $x = 0$. Solving:

$$\Delta V = 3\pi\zeta^{2/3}d^{2/3}|w_{max}|^{1/3} - \pi d^2|w_{max}|.$$

For displacements following the 1914 Sakurajima eruption, one reasonable model uses depth, $d = 10000$ m, excess pressure, $\Delta P = -400$ MPa, rock shear strength, $G = 30000$ MPa, pressure source radius, $a = 2500$ m, and Poisson’s ratio, $\nu = 0.25$. The maximum vertical displacement calculated for these parameters at $x = 0$ m is $|w_{max}| = 1.5625$ m.

These parameters yield a change in volume following the eruption, $\Delta V = 0.98$ km³. This is the volume missing, represented by the deflation of the ground around the volcano following the eruption.

The volume missing due to deflation is not quite the same as the volume of the magma chamber, because the surrounding rock is compressible ($v < 0.5$). Long after Mogi made his original calculations, Delaney and McTigue (1994) showed that for a point-like pressure source:

$$\frac{\Delta V_{surface}}{\Delta V_{chamber}} = 2(1 - v)$$

In the above calculations, it is assumed that Poisson's ratio is $v = 0.25$. Given $\Delta V_{surface} = 0.98 \text{ km}^3$, $\Delta V_{chamber} = 0.65 \text{ km}^3$. That is, Mogi's model forecasts that the magma chamber excess pressure corresponding to a volume of magma of about one-half a cubic kilometer prior to the eruption.

The volume of lava reported to have erupted in the months of the 1914 eruption is reported to be 0.5 to 1.5 km^3 (Ishihara et al. 1981). A relatively small volume of this material was erupted as volcanic ash: $0.09 \pm 0.03 \text{ km}^3$ (Todde et al. 2017). Most of the magma erupted as lava flows. These lava flows built an isthmus connecting Sakurajima island with the mainland (Ishihara et al. 1981).

Although there are large uncertainties in the volume estimates, it appears that the volume erupted agrees well with the volume needed to account for the observed ground deflation following the 1914 eruption. This correlation between deflation and volume erupted suggests there can also be correlation between magnitude of inflation and available magma to erupt, which is why deformation data and models, like the Mogi model, are widely used to monitor active volcanoes today.

Do the calculations yourself

The calculations in Mogi's model are complex for most of us. I developed a webpage to accompany this column that presents these calculations using computer code.³ The webpage gives a solution to the deformation model using javascript, a computer language that is very useful for illustrating numeracy concepts in a dynamic web environment. The solution to the solids-of-revolution problem is presented on the webpage using the python computer language. The code used to draw Figure 1 is also presented on the webpage. Hopefully, these materials will provide a pathway for further exploration of the concepts introduced in this column.

³https://gscommunitycodes.usf.edu/geoscicommunitycodes/public/numeracy/numeracy_mogi/mogi.html

Concluding thoughts

Mogi's research is well-known today among geoscientists primarily because he developed a mathematical model to describe the change in pressure within the Earth necessary to explain the observed radial pattern of deformation. His quantitative approach is elegant in its simplifying assumptions and useful because it explains deformation associated with a variety of processes that affect us. These additional processes include the subsidence of the ground surface when we pump large amounts of water from the ground (Tan et al. 2016), and pressurization prior to some earthquakes (Wang and Manga 2010). Whether forecasting volcanic activity or an entirely different process, one would do well to emulate Kiyoo Mogi's numerate approach.

Acknowledgment

The idea for this manuscript stems from my conversation with volcano scientist Shigeo Aramaki at the office of the International Atomic Energy Commission sometime in the mid-1990s. Discussion of the Mogi model with Rocco Malservisi and Peter La Femina was valuable. Webpage implementation of the Mogi model was facilitated by John Coonan. Comments from editors Len Vacher and Nathan Grawe improved this manuscript. Errors and omissions are mine.

References

- Aramaki, S. 1984. "Formation of the Aira Caldera, Southern Kyushu, 22,000 Years Ago." *Journal of Geophysical Research: Solid Earth* 89(B10): 8485–8501. <https://doi.org/10.1029/JB089iB10p08485>
- Delaney, P. T., and D. F. McTigue 1994. "Volume of Magma Accumulation or Withdrawal Estimated from Surface Uplift or Subsidence, with Application to the 1960 Collapse of Kilauea Volcano." *Bulletin of Volcanology* 56(6): 417–424. <https://doi.org/10.1007/BF00302823>
- Ishihara, K., T. Takayama, Y. Tanaka, and J. Hirabayashi. 1981. "Lava Flows at Sakurajima Volcano I: Volume of the Historical Lava Flows." *Annals of Disaster Prevention Research Institute, Kyoto University* 24: 1–10.
- Ives, M., and E. Peltier 2021. "A Volcano Erupted in Iceland. Björk was Sooo Excited." *The New York Times*. <https://www.nytimes.com/2021/03/20/world/europe/iceland-volcano-eruption.html>
- Machemer, T. 2021. "New Fissure in Iceland Volcano Prompts Evacuation of Tourists." *The Smithsonian Magazine*.

- <https://www.smithsonianmag.com/smart-news/new-fissure-iceland-volcano-prompts-evacuation-tourists-180977433/>
- Mogi, K. 1958. "Relations Between the Eruptions of Various Volcanoes and the Deformations of the Ground Surfaces Around Them." *Earthquake Research Institute, University of Tokyo* 36: 99–134.
- Statista 2021. "World Crude Steel Production." *Statistica Research Department*. <https://www.statista.com/statistics/267264/world-crude-steel-production/>
- Tan, W., D. Dong, J. Chen, and B. Wu. 2016. "Analysis of Systematic Differences from GPS-measured and GRACE-modeled Deformation in Central Valley, California." *Advances in Space Research* 57(1): 19–29. <https://doi.org/10.1016/j.asr.2015.08.034>
- Todde, A., R. Cioni, M. Pistolesi, N. Geshi, and C. Bonadonna. 2017. "The 1914 Taisho Eruption of Sakurajima Volcano: Stratigraphy and Dynamics of the Largest Explosive Event in Japan During the Twentieth Century." *Bulletin of Volcanology* 79(10): 1–22. <https://doi.org/10.1007/s00445-017-1154-4>
- Wang, C.-Y., and M. Manga 2010. "Hydrologic Responses to Earthquakes and a General Metric." *Geofluids* 10(1-2): 206–216. <https://doi.org/10.1002/9781444394900.ch14>
- Wilson, L., R. S. J. Sparks, and G. P. L. Walker. 1980. "Explosive Volcanic Eruptions – IV. The Control of Magma Properties and Conduit Geometry on Eruption Column Behaviour." *Geophysical Journal International* 63(1): 117–148. <https://doi.org/10.1111/j.1365-246X.1980.tb02613.x>
- Woods, A. W., and S. Self 1992. "Thermal Disequilibrium at the Top of Volcanic Clouds and its Effect on Estimates of the Column Height." *Nature* 355(6361): 628–630. <https://doi.org/10.1038/355628a0>