Calculus: the Board Game

By

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CERTIFICATE OF APPROVAL

Honors Thesis

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Abstract

This thesis explores the creation and evolution of an original calculus board game. To chart the evolution of the game, participant feedback was gathered over multiple developmental game phases. To see how well the board game could teach calculus, pre and post test data was gathered during later game development phases to see if individual participants showed any improvements in their abilities to solve calculus problems. The results showed minor improvements in calculus abilities regardless of prior calculus knowledge. Further game testing needs to be done to fully evaluate the effectiveness of this calculus board game.

Introduction

When it comes to learning higher level math, students are easily intimidated. Teaching out of a textbook can add to these fears and further perpetuate the perception that learning higher level math is difficult. This doesn't have to be the case, as learning can be accomplished through many different formats. The goal of this research project was to remove these negative beliefs and introduce a new way to learn higher level math by creating a calculus board game.

The idea for this calculus board game started back in 2014 just before the Fall semester. Over the summer I had been playing a board game called "Stratego" which was a two player strategy game. Each player has an army of 40 game pieces ranked one through nine, with a rank of one being the strongest and nine being the weakest. The goal of Stratego is to capture the opponent's flag, hidden on the other side of the game board. I enjoyed playing the game so much that I decided to make a four player version of the game, using grid paper and designing my own game board. I took this a step further and encouraged everyone who played my four player version of Stratego to make their own four player gameboard, which I would help them sketch up and create on a large sheet of grid paper. After helping create a few of these various four player gameboards, I decided to go another step further and create my own version of the game rules, changing stationary pieces to movable ones and adding special abilities and items. I didn't get to test all of these ideas and games rules, but the mere ideation and creation of potentially new versions of the game was a fun and creative challenge. My summer game ideation extended into my fall semester when I took a course that asked students to create a new business concept. The course was flexible and allowed students to create and build a business of their choosing. I decided to take advantage of this opportunity by extending the game ideas I had started over the summer into a gaming business. I started ideating and expanding on the game rules and ideas I had altered from Stratego, and I decided I would develop one of these game ideas into a creative project for my thesis.

At the beginning of the Fall semester, I started tutoring students in calculus. During my tutoring sessions, I realized that students seemed to struggle with some of the basic concepts of calculus. Seeing this, I wanted to find a way to make learning calculus less intimidating for the students. While a full-fledged calculus game was not created until Spring semester, I realized that the game I wanted to develop for my thesis could be calculus based. I did research on the internet, and found that calculus games were scarce to non-existent. This further inspired me to create the calculus game which was developed for this research project.

Literature Review

The idea of utilizing games to teach students learning concepts has grown in popularity over the past few years (Connolly, Boyle, MacArthur, Hainey, & Boyle, 2012; Clark, Tanner-Smith, Killingsworth, & Bellamy, 2013; Walker de Felix & Johnson, 1993; Silvern, 1986; Mayer & Harris, 2010). Supplementing classroom teaching with learning games is beneficial for teachers and students in numerous ways. There is huge diversity in ways games can be played and multiple theories about games and their benefits to learning. Looking at various game formats and theories can help shed some light on how supplemental games are beneficial to the learning process.

Video Games

Why are video games so appealing to kids and young adults? According to researchers Walker de Felix and Johnson (1993) "…the games provide a series of problem-solving situations that require quick decisions and reflexes…. The key to the video game structure appears to be, first, to entice students with a thought- or feeling-provoking idea, then, to allow a way to develop it on their own."

These researchers further explain that video games are visually active and interactive, allowing players to be personally involved while keeping them interested in the game. This initial interest is maintained through the presence of a goal that players must work towards (Walker de Felix & Johnson, 1993). On top of having a goal, video games should also strive to challenge players, peak their curiosity, and have an element of fantasy to maintain continuous attention and motivation of players (Malone, 1981).

Video games require players to learn the rules of the game through trial and error, which are subject to change as players advance from one level to the next (Silvern, 1986; Walker de Felix & Johnson, 1993). Players come to realize how "solving one problem gives players the tools to proceed to the next problem" (Walker de Felix & Johnson, 1993). This is similar to how in math courses, a student first learns addition and subtraction, then multiplication and division, then square roots and exponents, all the way to derivatives and integrals by the time they reach calculus. Problem solving skills in video games have the potential to be transferred into classroom settings. To make this work, teachers should consider using educational video games within their course structure.

Computer Games

Online computer games that are designed for educational purposes, referred to as Digital Game-Based Learning (DGBL) by some researchers, have "been recognized for [their] capability to motivate and engage learners" (Huang, Johnson, & Han, 2013, p.58). This is because DGBL games use a wide variety of game features and designs that engage players and motivate them to want to keep playing. Huang et al. (2013) came up with the following list of game features to be used in DGBL: challenge, competition, enforced rules, goal-oriented tasks, simulating a fantasy world, telling a story, engaging learners, allowing role-playing, supporting learner autonomy, and utilizing multimedia representations. These game features can be organized under three game feature headings. The first is game structure, consisting of game rules, goals, tasks, and instructions. The second is game involvement, consisting of a simulated fantasy world, role-playing, and curiosity. The third is game appeal, which consists of the aesthetic qualities of graphics, animations, and game audio (Huang et al., 2013).

The game features within each of these categories can impact the motivation of players. Game structure features can greatly influence player/learner confidence depending on the complexity of the learning goals. Game involvement features can affect player attention and sense of relevance, as players can either feel fully immersed in a game or lose interest because the game play is just like any other assignment. Game appeal features can also greatly impact attention, as a game can seem real and dynamic or scripted and fake based on the quality of interactive media (Huang et al., 2013). An ideal balance of all three of these game feature categories would ideally increase intrinsic motivation in players by making the game complex enough to stimulate interest but not complex to the point of being cognitively overwhelming. Caution should be used when implementing such game features though, as excessive use of game features can "overload [a] learners' cognitive and motivational processing capacities [and] thus disrupt the intended learning" (Huang et al., 2013, p.59).

To help balance game complexity, Huang et al., (2013) looked at two different models of motivation to consider when designing a DGBL game. The first is the ARCS Motivational

Design Model, which looks at four distinct components that contribute towards a learner's invested efforts in completing learning goals (Huang et al., 2013). The first of these components, attention, looks at how well curiosity is sparked in players during game play. The second component, relevance, looks at how useful and valuable players deem the overall learning experience of the game. Confidence is the third component, which looks at how players perceive their abilities to complete the required goals of the game. The fourth and final component, satisfaction, compares a learner's invested mental efforts against the perceived benefits of their mental efforts (Huang et al., 2013). Taken together, these four motivational components determine how likely a player is to continue playing a DGBL game while working towards the educational goals inside the game. Ultimately, the ARCS Motivational Design Model helps game developers create DGBL games that students will want to play from start to finish.

The second model considered by Huang et al., (2013), is the integrative theory of motivation, volition, and performance (MVP), developed by J. M. Keller (Keller, 2008), which builds off of the ARCS Model of Motivation. Keller's MVP theory "proposes that the learning process is initiated by motivational processing, [which] enable[s] initial goal setting" (Huang et al., 2013, p.60). The MVP model utilizes the four motivational components of the ARCS model to initiate motivation in the learner. Once motivated, the learner moves into the "volitional processing" stage, where their motivated intentions turn into actions as they strive to complete the stated learning goal of the DGBL game. This striving to complete the learning goal leads into the "cognitive learning" stage, where the player actually starts completing the learning tasks and begins achieving the learning goal. Once the learning goal has been obtained, the player/learner moves into the final stage, "outcome processing," where the player/learner mentally decides if their mental efforts were worth the perceived learning benefits of the game, almost identical to the satisfaction component of the ARCS model (Keller, 2008; Huang et al., 2013).The MVP model acts as an extension of the ARCS model, and helps explain what motivates a game player to initially play a game and what keeps a player wanting to keep advancing in the same game.

There is one other theory that should be considered when discussing DGBL and any other educational games offered online, and that is cognitive load theory, or CLT. Created by Chandler and Sweller (1991), CLT aims to clarify how much of a person's working memory and processing is taken up by mental efforts involved with accomplishing and completing mental tasks, and in this context, learning tasks (Chandler & Sweller, 1991; Huang et al., 2013). As described by Huang et al., (2013), CLT can be broken down into three distinct cognitive loads that add up to total cognitive load. The three cognitive load components are- the intrinsic load, extraneous load, and germane load (Huang et al., 2013). The basic principle behind the theory is that the total cognitive load cannot surpass the full capacity of a player's working memory and mental processing. The ideal use of a person's full working memory capacities involves minimizing the amount of extraneous, or tedious, cognitive load while maximizing germane, or task-relevant, cognitive load (Van Gerven, Paas, Van Merriënboer, & Schmidt, 2006; Huang et al., 2013). Essentially, this means eliminating repetitive, tedious tasks from games to allow players to focus their mental energy on the main learning objectives.

One study looked at what makes DGBL and other online education games so effective. Based on information from Pannese and Carlesi, (2007), the meaningful learning produced by these online educational games derives from the in-game opportunities to learn by doing, utilizing a trial and error approach to the games. Referring back to the ARCS motivational design model, instructional board games were compared to DGBL games to see which one was a better tool for learning outcomes. In a study by Klein and Freitag, (1991) (as cited in Huang W.-

H. , 2011), while instructional board games did show a positive influence on levels of player attention, relevance, confidence, and satisfaction, the context of the board games was less complex and interactive in comparison to the online gaming environments in DGBL. DGBL games have also provided evidence for both the ARCS and MVP motivational models, showing how motivational processes tie in with cognitive processing (Huang W.-H. , 2011; Huang et al., 2013).

Board Games

Instructional board games can be just as useful as online learning games. Currently, school libraries are considering getting more involved with offering board games to supplement course curriculums (Mayer & Harris, 2010). According to Mayer and Harris, (2010), board games, or "*designer games*" help support student learning by "offering authentic experiences,… engaging students on a familiar level,…. reinforc[ing] critical social and life skills,… [and] promoting informational literacy" (Mayer & Harris, 2010). The authors further explain how "designer games" can be used to help students build skills and competencies in all the major curricular areas, such as English and Language Arts (ELA), social studies, math, and science, thus helping students achieve state and national learning standards (Mayer & Harris, 2010, p.60). Beyond core curriculum use, board games help drama students with improvisation, help psychology students study how games affect the brain, or help in AP Language and Composition classes, using the game "Apples to Apples" to help students review literary terms (Crews, 2011). Libraries and school staff in the k-12 system are pushing for increased funding for even more board games to be added for educational use (Crews, 2011; Mayer & Harris, 2010). Even though board games may not be as popular as other game formats, they shouldn't be discounted or neglected when searching for educational games.

Game Topics

All game formats can be used for educational purposes. According to one meta-analysis of game studies, the majority of educational games were digital or simulation games on the computer, usually in a rough beta state or created on a research budget (Clark, Tanner-Smith, Killingsworth, & Bellamy, 2013). The subject areas of the games consisted of (from most popular to least popular) literacy (30%), psychology (26%), math (15%), science (14%), general knowledge (12%), social sciences (1%), and engineering/computer science (1%), out of around 80 game articles included in the meta-analysis (Clark et al., 2013).

According to a different meta-analysis consisting of 129 educational game articles, the most popular game format was simulation, with action games being the second most popular and puzzle games the third (Connolly, Boyle, MacArthur, Hainey, & Boyle, 2012). Simulation and puzzle games alone accounted for 80% of all educational games. Looking into the studied subject areas from this meta-analysis, "games for learning are being used across a wide range of subject disciplines with health (21) the most popular followed by games about social issues (14), science (11) and business (8)" (Connolly et al., 2012, p.666).

Also included in the group were language and math subjects at 4 games each, followed by geography and military/war games with 3 games each, 2 in history, and only 1 for statistics and computing subject areas (Connolly, Boyle, MacArthur, Hainey, & Boyle, 2012). The overall summary of both meta-analyses is that educational games can enhance learning (Clark et al., 2013), and produce numerous positive impacts in players (Connolly et al., 2012).

Outside of meta-analyses, individual research articles point to a few other subject areas for learning through educational games. Economics seems to be a prevalent subject (Dobrescu, Greiner, & Motta, 2015; Huang W.-H. , 2011; Huang et al., 2013). One article by Brauer and Delemeester (2001) even acts as a meta-analysis for one website (Delemeester & Brauer, 2008) listing over 170 non-computerized macro- and micro-economics games. There is a related website that includes a database of macro- and micro- economics experiments that can be used in place of educational games as well (Delemeester & Neral, 2008).

Research has also been conducted on business games. These educational games, usually simulations, are slowly branching out into business schools and even MBA programs (Tao, Yeh, & Hung, 2012). One business school offers students a business simulation game that counts for course credit, no teacher or instructor needed (Lerner, 2014). Even history courses are utilizing sophisticated educational games to make the learning material more meaningful and interesting to students. A current game series being used is called "Reacting to the Past" (Lang, 2014). These games

> …assign students roles in historical-simulation games in order to encourage intensive reading of complex texts, help students develop core intellectual skills (writing, speaking, thinking), and motivate them to take a deep approach to their learning. Although these games were initially developed for history courses, they now span the disciplines, in fields as varied as political science and chemistry. (Lang, 2014, p.B34)

"Reacting to the Past" games immerse students in a historical context, allowing students to play out the roles of key figures in history to get a sense of why events happened as they did and how things could have happened differently. The immersive nature of these games is what really captivates students and keeps their attention during the learning experience.

Educational games have also branched out into the math and science subject areas. As far as research articles go, game studies have been created for biology (Gutierrez, 2014; Spraggins & Rowsey, 1986), math (Beserra, Nussbaum, Zeni, Rodriguez, & Wurman, 2014; Castellar, Van Looy, Szmalec, & De Marez, 2014; Jimenez-Silva, White-Taylor, & Gomez, 2010; Oleson, Surprenant, Carbone, & Blair, 2011; Polycarpou, et al., 2010) and engineering (Ross, Fitzgerald, & Rhodes, 2014). Looking at references outside of research articles, there are a wide variety of educational games spanning mathematics, biology, chemistry, astronomy, geology, physics, health science, computer science, and more (Andersen, Play and Learn, 2014).

Out of all the education game subjects previously listed, math games are by far one of the most prevalent. There are easily over four dozen different kinds of digital computer math games (Andersen, Collection of Math Games, 2011; Andersen, Playing to Learn Math, 2011), and over 180 math puzzle and paper-based games (Andersen, Collection of Math Games, 2011; Muschla & Muschla, 2011). However, when looking into the field of calculus, there seems to be a shortage of educational games in comparison to other subject areas (Andersen, Playing to Learn Math, 2011; Andersen, Play and Learn, 2014). Currently about half a dozen paper cut-out puzzle games exist for calculus (Andersen, Calculus, n.d.). These puzzle games, while efficient in teaching calculus, seem like jumbled arrangements of calculus problems rather than games. On the digital side there is a computer game called "*Waker*" that teaches about position on a curve and velocity, a physics concept related to calculus (Waker, 2009). The game is designed "to teach students about position and velocity curves" by having players create curving lines that allow their character to travel over large gaps in the game levels (Andersen, Calculus, n.d.). This digital game is more immersive and engaging for players than the puzzle games but it only touches on one specific calculus topic, whereas the puzzle games cover the core concepts and principles of calculus among the various puzzle games (Andersen, Calculus, n.d.). These limitations leave plenty of room for growth for future calculus games.

Procedures

First, the game idea was created. This was done by brainstorming different ways to embed calculus into a game, preferably having the game rules be calculus principles. Initially a single player video game idea was considered, with some potential game levels being built around performing derivatives as attacks, and eventually multiplying, dividing and performing derivatives within derivatives. While this was a feasible option, the idea of creating a board game was also considered. Again, the board game's rules focused around performing derivatives, multiplying and dividing derivatives, and performing derivatives within derivatives. The major difference between these two game platforms was that the video game was better suited for a single player, while the board game was better suited for two players.

An important consideration when deciding whether to create a calculus video game or board game was the utility and social aspect of the game. While both game platforms had a similar utility value, the board game offered players a way to interact with other people also learning calculus. In a classroom setting, this would turn the calculus game into a social activity rather than a solo assignment to be completed. These social interactions between players would also provide an opportunity for reciprocal peer tutoring, a type of collaborative learning where students help teach one another through alternating tutor and tutee roles (De Backer, Van Keer, & Valcke, 2012). Seeing this greater potential for social interactions and the opportunity to incorporate reciprocal peer tutoring with the board game is what lead to the ultimate decision to make the calculus game a board game instead of a video game.

The original board game outline and set-up consisted of a two-player game with six game pieces per player on a small 3x6 inch grid gameboard. The game pieces consisted of a mathematical variable such as “X”, “Y”, or “Z” (except for the first level which only included “X” variables) representing the main game piece, and a smaller exponent game piece that would rest on top of the main game piece. Players would set up their game pieces on the two back rows of their side of the board. The players could either move or attack once per turn, and each attack consisted of performing a derivative, thereby reducing the attacking piece’s exponent by one, as X^3’s derivative would be $3X^2$. The game piece being attacked would reduce its exponent by the number in front of the variable after a derivative had been performed, as a derivative that came out to $3X^2$ would do 3 damage to the piece it attacked. After attacking, the number in front of the variable would be used up, turning $3X^2$ into X^2 once the attack was finished. Once a game piece reached an exponent of zero, it was removed from the board. Players would keep taking turns until one player lost all of their game pieces. After finishing each game, players would then advance to the next level, where both players would get two additional game pieces and would gain another column on the gameboard to use for setting up their game pieces. This leveling up pattern was meant to slowly expand the gameboard and the number of game pieces with each new level. Along with the expansion of the gameboard and game pieces, a new calculus rule was introduced at each level. After the first four levels, players would have learned four different calculus rules, thereby allowing them to play a fifth and final level of the game that combined all of these rules.

Ultimately the goal of the game was to walk players through the basics of calculus, starting with how to perform simple derivatives with no numbers in front of the variable, then move on to multiplying derivatives, then dividing derivatives, and finally, performing derivatives within derivatives. After being walked through each of these specialized tutorial levels, the fifth and final level would combine all of the game rules from all of the four previous game levels, acting as a review of what all the previous levels had covered. This fifth level was meant to be the main level of the game as it would involve strategically using all the basic calculus rules to try to win the game. The entire game experience was meant to be fun and engaging.

Each of the first four game levels were meant to be played in 20 minutes or less, for a total of 80 minutes or less of tutorial game play. The fifth level was meant to be the main game and was estimated to take no more than 30 minutes to an hour, with the hopes that the entire game could be played in under three hours. While this was the ultimate goal of the original board game, this goal was never reached by a single participant who played the game, as it took longer to complete each specialized tutorial level than was originally expected.

After sketching out the various game levels and board set-ups, the second step was to create the board game. There needed to be main game pieces that could represent different mathematical variables, and exponent pieces that could change during game play. While there were many materials and options to choose from, for the sake of time and energy, small wooden cubes were used as the main game pieces, and even smaller wooden cubes with sticks were used to represent the exponents. To keep the small exponent cubes on the larger game piece cubes, adhesive magnets were attached to the top of the main game pieces and metal pins/findings were stuck to the bottom of the exponent cubes. The outer faces of these exponent cubes were numbered one through 4 so that a player could rotate the cube to change their piece's exponent during game play.

Letter stamps and colored inks were used to put different mathematical variables (X,Y, and Z) on the main game pieces. Small flat wooden rectangles were used as guards on the game pieces to hide game piece exponents from opponents. A wooden slab with tree bark still in-tact on the sides was used as the gameboard, and thin, multi-colored taped was used to create and place a grid design on the gameboard. A few other materials were used for each specialized calculus rule, such as using small plastic bins to represent multiplied game pieces, using small flat wooden squares to represent the bottom half of a fraction for division, and using small plastic jars and paper cups to represent a derivative being performed within a derivative. Finally, all of the constructed game pieces were placed in a plastic bin so that the game pieces could travel in one portable container.

Once all the needed game components had been constructed, the next step was to test out the game and gather feedback. This step was broken down into multiple game phases, as the game continually evolved and changed during this process. The first phase of the game was started while the gameboard and game pieces were still under construction. The wooden gameboard's grid design had not yet been taped down, so a large sheet of graph paper with a penciled-in 6x8 inch grid pattern was used as the gameboard during this phase. The first two levels had all the pieces needed for game play, but the third and fourth levels were missing some of the specialized pieces needed to represent fractions in division and vehicles for derivatives within derivatives. As the game was still under construction during the first game phase, all the rules and explanations on how to set-up the gameboard for each level were given audibly to each participant.

Phase One

During game phase one, a total of five people tested out the game. All of the participants were USFSP students; two were calculus tutors, one had taken a calculus course, and two had never taken calculus before. The first two participants played the game on the same day, and one was able to play the first three levels of the game. Both of them enjoyed the game's concept and gave helpful feedback on how to improve loopholes and issues with the game. After playing through the first three levels and realizing how long it took to play (over two hours), the fifth level of the game was deemed to be an extraneous level that would rarely be reached by players in one sitting. As a result, the focus of the game shifted to minimizing the amount of time needed to play through the first four specialized tutorial levels.

Feedback on the other three participants was collected after game pieces for the third and fourth specialized game levels were fully created. All of the participants from phase one enjoyed the unique strategic element of the game, even though only one reached the fourth level. However, players noted that the game favored the attacker, and that the specialized rules for multiplying, dividing, and performing a derivative within a derivative had complex and confusing attack procedures, including the placement of newly formed and duplicated game pieces. Recommendations were given for ways to reduce the level of complexity while still adhering to the rules of calculus for each specialized level. These recommendations were then applied to the game in preparation for the second phase of testing. For further details into participant feedback for phase one, please see Appendix A.

Phase Two

After game phase one, two additional tutorial levels were added between the original first and second levels to help reduce game complexity. The first new level (now level two) allowed "X" and "Y" variable game pieces to combine into a stronger Z variable piece (instead of incorporating calculus rules for multiplying derivatives) and also allowed game pieces to perform double derivative attacks. The second new level (now level three) incorporated logarithms being used as traps to cancel out "X" and "Y" variables. For example, a logarithm with a base of X, when applied to an X variable, would reduce to 1 ($\log_X(X) = 1$), and a logarithm with a base of Y would reduce Y variables to 1 ($\log_Y(Y) = 1$). By having these two new levels come before the other specialized levels (now levels four, five, and six), players would feel more engaged with the game early on and by the time they reached the specialized game levels it would be easier to understand the specialized rules. The only other major rule change made was that any game piece with an exponent of one could now move and attack on the same turn, helping to balance out the game.

There was more game construction in preparation for game phase two, as items and item tiles were added to the game to better randomize when specialized rules could be used in the game. The grid design for the gameboard was finished midway through phase two, so the game was played on large grid paper for the first half of this phase and on a wooden gameboard during the second half of this phase.

As these changes were being made to the game, more feedback was gathered on this second phase of the game. During this phase, six participants tested out the game, one of which was a previous participant from phase one who wanted to try the game with the new changes. The repeat participant really enjoyed the new changes to the game and felt the game was much less complex than the phase one version. Other participants also agreed that the game was fun to play, and recommended tweaks that would make the game better. Some of these recommendations consisted of using calculus problems to decide who goes first each level, reducing the power of “Z” variable game pieces, adding diagonal moves and attacks, and changing how items are used and displayed on the gameboard. Two other valuable recommendations from participants were recommendations to add pre- and post- tests to the game testing and to create attack sheets that players could use to write out their derivative attacks to visually see what the derivatives looked like on paper. All the feedback and recommendations were considered and some were even tested within this phase of the game (such as using calculus problems to see who goes first and changing how items are used and set-up). While not every recommendation was added to the game, a good majority of the recommendations were added into the third game phase. For more details on participant feedback for game phase two, please see Appendix B.

Phase Three

The recommendations from phase two changed the game in numerous ways for phase three. First, item tiles went from being one-time use disposable game pieces to being permanent game spots on the board that, when stepped on, would allow players to grab an item from an item card deck. These item tiles were placed in a checkerboard fashion in the middle of the board, compared to the old item tiles that were placed on every space in the middle of the board (see Figures B1 and B2 from Appendix B). New items had to be created and designed in order to have a randomized item card deck for each level of the game. Some of these items were based off of previous recommendations, like adding a shield item and a move/attack diagonally item, while others were added for variety or to speed up the game. Item sheets were also created and printed off explaining what each item does. Second, attack sheets were added to the game to help players visually see and practice writing out derivatives. The attack sheets covered all specialized derivative attacks, with a visual step-by-step guide for performing each type of derivative and a fill-in-the-blank format for keeping track of the changing exponents and duplicate variables for each specialized derivative attack. Third, a rule sheet was created and printed out for players to read before playing the game. Minor changes were also made, such as allowing the X and Y health blocks to be vulnerable to attack by any variable game piece instead of only by variables that matched the variable of the health block.

The only recommendation that was still being created during this phase of the game was the pre- and post- test to see how much participants learned from playing the calculus game. A few different versions of the pre- and post- test were given to participants during this third game phase, and ultimately the last version of the pre- and post- test created ended up being used to collect data during the final phase of the game. During game phase three, game feedback and recommendations from each participant were added into the game as quickly as possible so that the next participant could benefit from the improvements. As a result, the rules for game phase three slowly changed and evolved to match the needed improvements recommended by each participant. Those recommendations which were not added into this game phase were usually added in during the final game phase.

There were a total of four participants who tested out game phase three, each with their own helpful recommendations to improve the game. One of the participants recommended creating two separate rule sheets, one for calculus students, and one for non-calculus students who are less familiar with math terminology. Another participant recommended allowing “X” and “Y” variable pieces to do half damage to “Z” variable pieces instead of no damage, which would then allow double derivative attacks to be useful items for “X” and “Y” variables attacking “Z” variables. A third participant recommended improving the item ratios in the item card deck so that math function items were more prevalent items than non-math function items, as well as adding an item to break up grouped pieces during the multiplication level, adding better visuals to the attack sheets, and laminating the attack sheets so that the same two attack sheets could be re-used after each game. The last participant recommended revising and improving the explanation for double derivative attacks, agreed that better photo visuals should be added to attack sheets, also felt the item card deck ratios could be improved, felt items should be able to be placed on the board as traps, and most importantly, recommended merging the logarithm item level with the product rule level. After applying the level merger recommendation to the calculus game, phase three needed to be updated in order to keep accurate data on which levels participants actually reached. As a result, phase three transitioned into phase four to properly update the game levels and item card decks. For further details on feedback from game phase three, please see Appendix C.

Phase Four

The final game phase, phase four, was the only game phase to collect pre- and post- test calculus data on all participants. This was due to the fact that different pre- and post- tests were created and given out during game phase three, and the pre- and post- test given to the last participant from phase three ended up being the best format for recording calculus improvements. As a result, this last version of the pre- and post- test was used for all remaining game participants in game phase four (this final pre- and post- test can be found in Appendix E, Questions and Answers). In order to keep the testing conditions and formats the same for all participant, all pre-and post- tests were hand written on paper, with the pre-test on the front (given before playing the game) and the post-test on the back (given after playing the game). Along with the addition of the pre- and post- test, all the feedback and recommendations from phase three were added into game phase four, with the exception of laminated attack sheets with photo visuals and an item for splitting up groups in the multiplication level.

In game phase four, a total of nine participants played the calculus game, providing further feedback, recommendations, and pre- and post- test data. Some of the feedback was similar to previous game phases, such as participants enjoying the game for its unique use of strategy and asking that explanations for specialized levels be clarified further with simpler examples and explanations. Two participants recommended using numerical examples on attack sheets instead of examples with an "n" to make the rules more intuitive and simple to understand. There was also feedback on the new rule about items being placed down on the board as traps. It was unclear if the double derivative attack, when set as a trap on the board, would force a player to carry out a double derivative attack and lose the resulting coefficient or if the resulting coefficient could be held on the side of the game piece to be used for a future attack. It needs to be clearer which items can and cannot be placed as traps on the board, and what the resulting effect is once a trap item is activated. Another recommendation was to change the attack sheets so that each level had its own attack sheet and could be used to keep track of each individual game piece. Other recommendations included making the game more applicable to all K-12 and college students, adding in more color options beyond green and red pieces to choose from, and removing the word "calculus" from the tentative name "Calculus Wars." Due to time constraints, no further feedback was collected on the calculus game. For further details on feedback from game phase four, please see Appendix D.

Pre-tests and Post-tests

Pre- and post- test data was collected on all participants in game phase four, with pre-tests given before playing the calculus game and post-tests given after playing through the calculus game. Both the pre-test and the post-test consisted of nine questions that covered all the calculus material from all five levels of the game, including derivatives with and without numbers in front of the variable, performing double derivatives, multiplying and dividing derivatives, and performing derivatives within derivatives. The questions follow the same outline as the game levels, starting with a simple derivative and ending with a derivative within a derivative. The pre- and post- test questions maintain the same math formats and only differ in the numbers and exponents used to prevent participants from recalling pre-test answers on the post-test questions. All participants were given as much time as they needed to complete the pre-test and the post-test.

To prevent obscuring improvements in calculus abilities, all pre- and post- tests were scored three times with three different grading criteria. The most stringent grading criteria only assigned credit for answers that were 100% correct. The second grading criteria assigned two points to fully correct answers, and 1 point to partially correct answers (i.e. doing the top half of the quotient rule or only one side of the product rule) and answers where a derivative was correctly performed but not in accordance with the specialized calculus rule (i.e. not performing a product rule to multiply derivatives or quotient rule to divide derivatives). The third and most liberal grading criteria assigned 4 points for fully correct answers, 2 points to partially correct answers, and 1 point to a correct normal derivative in place of the correct specialized calculus rule. All three grading criteria were used to calculate improvements for each participant from pre- to post- test to see how well participants improved their calculus knowledge as a result of playing the calculus game. Please see Appendix E, Results and Appendix E, Statistics for more details into participant performance on pre- and post- tests.

Results

Statistics run on the pre- and post- test data using all three scoring methods shows nearly identical averages for test improvements. Across all three grading criteria, test performance increased by an average of 15% from pre-test to post-test (see Figure 1 below).

Figure 1

All Students n = 10				95% Confidence Intervals	
Grading Method	**Pre-test average % questions answered correctly**	**Post-test average % questions answered correctly**	**Change in average % questions answered correctly**	**Lower Limit**	**Upper Limit**
Strict	30%	44%	**14%**	6%	22%
Partial Credit	44%	60%	**16%**	8%	23%
Liberal	40%	54%	**14%**	7%	22%

To put these percentages into context, an 11% increase represents getting one more question correct for full credit, a 16.6% increase represents getting one more question right plus partial credit on another question, and a 22% increase would represent getting two more questions correct (33%, 3 more questions, 44%, 4 more questions, etc.) from pre-test to post-test.

The statistics showed minimal differences between participants who had prior calculus knowledge and those who did not know calculus. The two noticeable differences between these two groups was that participants who had taken or were taking calculus showed slightly more improvement when using the most stringent scoring criteria (15% vs 11%), but when using the second scoring criteria that gave half credit for partial and simple derivative answers, participants without prior calculus knowledge showed slightly more improvement over participants with prior calculus knowledge (19% vs 14%), as shown in Figures 2 and 3 below.

Figure 2

Non-Calculus Students
n = 3

Grading Method	Pre-test average % questions answered correctly	Post-test average % questions answered correctly	Change in average % questions answered correctly
Strict	7%	19%	**11%**
Partial Credit	13%	31%	**19%**
Liberal	11%	26%	**15%**

Figure 3

Calculus Students
n = 7

Grading Method	Pre-test average % questions answered correctly	Post-test average % questions answered correctly	Change in average % questions answered correctly
Strict	40%	56%	**16%**
Partial Credit	58%	72%	**14%**
Liberal	53%	67%	**14%**

There were also statistics run on the rating of the calculus game on a scale from 1 (low) to 10 (high). The calculus game rating averages fluctuated between a low of 7.4 to a high of 8.5 over the various game phases. Actual participant game ratings fluctuated between a low of 6 and a high of 10 over all of the game phases. The overall average game rating was identical for calculus and non-calculus participants, a 7.8 out of 10. All participants who played the calculus game a second time at a later game phase either increased their original rating or maintained their original max rating (10 out of 10). Please see Appendix F for further game rating information.

Discussion

The pre- and post- test results showed an average improvement of about 15%, or about one and a half more questions correctly answered from pre-test to post-test, for both calculus and non-calculus students. The reason for this is likely due to the fact that participants did not have enough time to play through all of the game levels, and the pre- and post- tests covered knowledge from every game level. The furthest any of the tested participants got in the game was up to the multiplication level with product rule. None of the tested participants reached the quotient rule or chain rule levels, and none of the participants from any phase of the game ever got to try the final version of the game where all of the calculus rules were combined into one big calculus strategy game. This limited sampling of only the first few levels of the game would explain why the average tested participant only improved by one and a half questions. This reveals that the current board game has the potential to teach players calculus, but it may not be possible to play every level of the game in a single sitting. One of the last participants even noted that in one sitting, a player cannot get through all of the game levels, and recommended players should only do two levels per sitting for the sake of time. For future research and studies, all of the game levels should be spread out over the course of a week. The pre-test should be given at the beginning of the week, and one level of the game should be played each day of the week, with the post-test given at the end of the week. This would allow participants to try out all the levels of the game and give feedback on each specialized level and the final version of the game with all the rules combined.

Another option to consider for future testing is condensing down the play time for the specialized levels and having one comprehensive tutorial game that goes through each and every kind of specialized rule. With a condensed tutorial level, all the specialized rules of the calculus game could be learned in a single day and the final version of the game could also be played in the same day. This would be a better option for quickly getting feedback on the calculus game, but it may not be the best learning approach, as rushing through all of the tutorial levels risks mentally overwhelming players with too much information.

The results could also be due to the fact that data was only collected on ten participants, most of which were calculus students. This is a limitation that needs to be addressed in future research, as the game should be tested with a wider range of participants and strive to have a more equal number of calculus and non-calculus students in the sample for more accurate results.

Even though there is still room for improvement, the idea behind this calculus board game has many potential applications in the education field. Teachers can use board games similar to this one to teach other calculus principles and concepts, giving them and their students another learning tool outside of traditional textbook problems and class lectures. Seeing how this calculus board game received positive feedback and reactions from all participants is a good indication that this learning format has the potential to be more inviting and less intimidating than traditional learning formats. This calculus game could also be used as a resource for reciprocal peer tutoring. The format of the game could easily be adjusted to allow pairs of students to switch between tutor and tutee roles as they play through the game, giving students another way to learn calculus. Tutors could even incorporate the board game into their teaching practices and utilize reciprocal peer tutoring to give students the opportunity to both learn the material and try to teach it back to the tutor. The best part is that this game can be changed and modified to fit the learning outcomes of other math and science subjects beyond calculus, helping to further break down any perceived difficulties in any and all of the higher level math courses. This game board learning format would make higher level math courses more accessible and open to students who shy away from textbook and lecture learning formats. These future applications should be further studied and researched as well.

Conclusion

Looking back over all the game feedback and data, some interesting conclusions can be drawn. One of the overarching themes among nearly all of the game participants was that the calculus board game was a fun and unique strategy game that kept the attention of participants. The game board format seemed to be very inviting to students, helping them overcome negative perceptions about math. Another theme that emerged was that some of the specialized game rules seemed very complex and needed to be simplified. Reviewing how far each player got in the game, it seems that within a single game play session, two to three game levels is a good stopping point. Trying to fit all of the game levels into a single game play session can take hours to accomplish and might be too much information for a player to handle all at once. The game was able to teach all players how to perform simple derivatives (such as X^3 becoming $3X^2$) regardless of prior calculus knowledge. There was not enough data gathered on specialized game levels to determine if the game can teach students how to properly multiple derivatives, divide derivatives, and perform a derivative within a derivative. Further testing should be done to determine if this calculus board game is an effective learning tool for teaching students the principles of calculus.

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Appendices

Appendix A-

Phase One Feedback

Brandon- USFSP student, calculus tutor, played level 1 of the game. Was a little confused initially due to how the instructions were given verbally, but understood by the end of the game and really liked the potential of the game.

Sidney- USFSP student, has not taken calculus, played levels 1, 2 and 3 of the game. Rated the game a 10 out of 10. Really liked the game and that it was a strategy based board game like chess. She suggested adding number dials to the flat/underground pieces to better keep track of the exponents.

Tess- USFSP student, calculus tutor, played level 1 and 2 of the game. Rated the game an 8 out of 10.

Malachi- USFSP student, has taken business calculus, played levels 1 and 2 of the game. Rated the game a 6.5 out of 10. He noted that the game favors the attacker, needs to find a better attack and defense balance. Said there should be a tutorial explanation that is relatively simple so that players do not have to level up to the more advanced rules. Suggested adding a rule book. He also suggested that the XY combo pieces in bins should be able to attack all pieces (X,Y, or Z) and any variable piece can attack the XY combo bins. Noted that it was a small playing field.

Glenn- USFSP student, has taken up to pre-calculus, played all levels (1-4) of the game. Rated this version of the game a 5.5 out of 10. He noted that while he felt he had gained a strategic experience from the game, he did not feel he had learned any calculus. He thought the game had a lot of complexity with little depth. Thought the game was too complex with the spawning pieces in levels 2, 3, and 4, suggested having new pieces spawn at the back of the board instead of next to attacking piece.

Also suggested having any variable piece raised to the 1st power [(X,Y,Z)^1], should be able to move and attack on the same turn to make X^1 and Y^1 pieces more valuable in the game. Suggested having a selection of calculus problems and having players solve one of the problems to determine who goes first. Further recommended using integrals to allow pieces to regenerate health. A piece could take health from the X or Y variable health blocks and use it to increase their exponent by one (i.e. an X^2 could take 3 health from the X variable health block in order to become an X^3, a Y^4 could take 5 health from the Y variable health block and become Y^5 [3x^2 integrated is x^3, 5Y^4 integrated is Y^5]). From these recommendations, Glenn suggested having three phases to the game, in the first phase, a player can regenerate health using the integral rule stated above. During the second phase, players could utilize any items they possessed. During the third phase, players would move or attack (or both if raised to the 1st power).

The last two suggestions Glenn gave were to remove the spawning pieces altogether (eliminate product rule, quotient rule, and chain rule from the game due to duplicate pieces being spawned) to make the game less complex, and have Zs on the board disappear when either the X or the Y health blocks reach or fall below zero. Zs disappear and turn into whichever variable’s health block is still above zero, and the exponent of the previous Z piece is cut in half, rounded down as needed (i.e. a Z^7 would become an X^3 if the Y variable health block reached or fell below zero). This would further show that Zs consist of both an X and a Y piece to exist.

Appendix B-

Phase Two Feedback

Glenn (2^{nd} time)- played through the new levels 1, 2 and 3. New rating of the game is an 8 out of 10. With this new version of the game, says there is high depth to the game with low complexity. Says he has never seen a game like this before, creates a unique experience. He genuinely wants to keep playing and trying out the calculus game as it evolves and grows. According to Glenn, this new version of the game “Achieves the maximum amount of entertainment and strategy with minimal complexity.”

Carrie- USF Tampa student, has taken calculus, played levels 1 and 2 of the game. Rated the game a 7 out of 10. She said this was a good strategy game. Suggested either having an attack cube that gets launched at the opponent, or making little flat squares representing attack markers that you get to throw at your opponent’s piece (each attack marker could count for 1 damage). Also recommended adding calculus questions to be asked at the end of each level to test calculus knowledge of students.

William- USFSP student, taken up to pre-calculus, played levels 1, 2, and 3 of game. Rated the game a 7 out of 10. He thought the game was very creative, not too complicated, simple and easy to play.

William felt the condition for winning the game needed to be more specific. He also suggested allowing the general X and Y health to be attacked by any variable instead of being variable specific to make the health blocks more vulnerable, creating a need to be guarded.

Eddie- USFSP student, taught himself calculus but does not have good memory of calculus knowledge, played levels 1, 2, and 3 of the game. Ranked the game a 6.5 out of 10. He liked that the game made him think strategically, and he felt it was more fun than he had thought it would be. He likes that there is room for expansion.

He felt that the Z piece should be reduced in power to better balance the game. He suggested adding diagonal attacks and moves, making a post test to see how well participants learned calculus rules, and he recommended players be able to pick where X and Y health blocks should be located. Plus, consider having players be able to set their own traps anywhere on the board, instead of discovering them like items in the middle of the board.

Ashton- USFSP student, has taken calculus, played levels 1 and 2 of the game. He rated the game a 7.5 out of 10. He liked the calculus theme and the items in the game. He felt that players should have to explore the board more to find items, should be more sprawled out. He also recommended a switch in item use, as shown below:

Figure B6

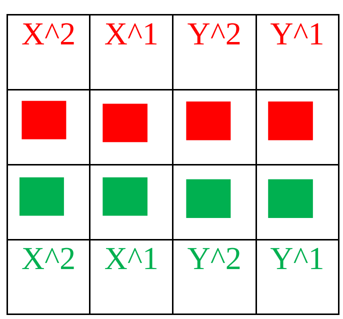

Figure B7

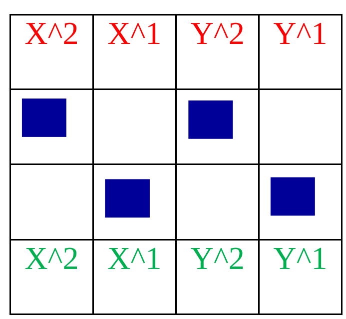

Figure B6 shows how items are currently used, with all the items and traps being placed on the board. Traps with the opposite color of the player must be applied, but traps with the same color as the player can be applied or picked up and used as an item. Once an item is picked up, that item tile is gone from the board. Figure B7 shows how Ashton recommends items be used, with item tiles appearing in a checkerboard pattern in the middle of the field. Each time an item tile is stepped on, a card is drawn from an item & trap card pile, and that player is then given that item or trap (or the trap is used on the player if applicable). This leaves the item tiles on the board to be used for the entire game length and gives more randomness to the possible items and traps that can be drawn.

Ashton further recommended adding a wider variety of items, such as attacks that can be launched until they hit an opponent and using dud/blank items as one-time shields that nullify an attack. He also suggested letting pieces be able to swap items with one another if they are adjacent to each other. He feels that item use or item swapping should be the 1st phase of a player's move, and that the 2nd phase should be attacking or moving pieces, so that a player could use an item and move or attack in the same round.

Another recommendation was to make a flowchart, or an "input/output tree", that represents all possible game play options and outcomes so that a player knows what to do in any situation during game play. He further suggested that players start with only X variable pieces, and once a player gets his or her piece to the opposite side of the board, it should become a Y variable piece. If it goes back to the other side of the board, it can then become a Z. Also suggested improving the aesthetics of the game, including color scheme of the game board, and possibly using dice inside of cups to represent a piece's health/exponent instead of using exponent cubes.

Kayla- USFSP student, currently taking a calculus course, played levels 1 and 2 of the game (with the new item layout from Ashton, Figure B7). She rated the game an 8.5 out of 10, and said she would probably rate the more advanced levels a 9.5 as she believes the game would be more fun with the more advanced calculus rules.

She really enjoyed the game and was curious to try the more advanced levels of the game (product, quotient, and chain rule levels). Based upon advice from my thesis director, I had Kayla write out each derivative attack she did on paper before applying it on the board. She liked that the game made her write out the derivatives for each attack, as it helped reinforce simple derivatives in her mind.

She also felt that there should be a derivative attack sheet pre-written out sheet on paper. The attack sheet would show the proper format for all derivative attacks from each level of the game. The attack sheet would only need to be filled out with different numbers to make it easier to write out and visually see what the derivatives looked like on paper. This would also make it easier to reintroduce product rule, quotient rule, and chain rule back into the game as levels 4, 5, and 6 of the game.

To better implement product, quotient, and chain rule, instead of having new pieces spawn on the board with each attack, have players travel across a series of tiles on the board that represent the product rule, quotient rule, and chain rule broken down into simple steps, one step completed at each tile along the series.

Appendix C-

Phase Three Feedback

A.J.- Played levels 1 & 2, no prior calculus knowledge, rated the game a 7 out of 10. He enjoyed the strategy aspect of the game. His advice to improve the game was to make a basic rule sheet and a calculus student/advanced rule sheet (i.e., coefficient, exponent, and variable terminology vs power and attack numbers).

Stephan- Played levels 1, 2, and 3, no prior calculus knowledge, rated the game a 9 out of 10. He liked the multiplier item, thought it was very strategic. He felt the Z pieces were too strong, recommended that X and Y attacks get cut in half instead of having no effect on Z pieces. Also suggested leaving the board more open allowing for two free blocks during set up so pieces are not as confined.

Sidney (2nd time)- played levels 1,2,3, and 4 (older version), took pre-calculus, wanted to rate the game above a 10 out of 10. She enjoyed the overall gaming experience and liked how duplicate pieces spawned in the back row of the game board. She really enjoyed the different item decks for each level, felt the variety kept the game interesting. She thought the zero multiplier item was good and necessary to help speed the game up. She felt that the item ratios could be done better so that people could get and perform double derivative attacks. Suggested adding a group-splitter item for the product rule level, as well as photo visuals for the attack sheets. She recommended having a manual of the game and level set-ups. She also recommended using bigger square tiles for future boards and using laminated attack sheets with dry erase markers to save trees.

Jim- played levels 1,2, & 3, finished calculus 3, rated the game an 8 out of 10. Thought the game was fun, liked the strategy. Recommended double derivatives be better defined and add visuals to aid explanations. Felt coefficients could be more visually represented in the game. Felt items should be able to be planted on the ground once you have them, and once stepped on the item activates on that player. Also found out that the multiplier effects from $\log_X(\ )$ and $\log_Y(\ )$ items can create exponents greater than 12, which need a better visual representation. During the game, a piece of paper kept track of the abnormally large exponent instead of an exponent cube. He also felt that the item ratios for math items could be increased so there would be better odds of practicing the math principles.. Also recommended merging the product rule level with the log item level to save time and focus more on calculus principles.

Appendix D-

Phase Four Feedback

Rachel- played levels 1, 2, & 3, has taken calculus 2, rated the game an 8 out of 10. She felt the game would be great for math teachers and had potential to be used in classrooms. Suggested the use of real numbers instead of "n" when giving explanations and examples of how the items work mathematically on attack sheets. She also suggested making the product rule explanation less confusing in regards to how duplicates are created and grouped together.

Austin- played levels 1 and 2, is currently taking calculus 1, rated the game a 9 out of 10. He felt the game helped reinforce simple derivatives while utilizing an enjoyable game board format. He said that the rules were unclear; didn't know if coefficient went away due to attacking or due to performing a derivative. Said attack sheets need to be more intuitive, using real numbers in the examples. He also felt that the rules were vague in regards to dropping items. For example, if a double derivative attack item is dropped/placed on the board, does it simulate a full derivative attack with the coefficient vanishing or does it merely charge up the game piece and allow it to hold on to it's coefficient until it attacks? Also suggested keeping track of each individual game piece with the attack sheets to reduce confusion (i.e. attack columns for x^1,x^2,x^3,and x^4 for the first level, and x^1 through y^4 for the second level, etc.).

Eddie (2nd time)- Played levels 1 and 2, has learned calculus. He failed to give additional feedback or a second rating, as he felt the original feedback was fine and left before giving an updated rating.

Shannon- Played levels 1 and 2, never taken calculus, rated the game a 7.5 out of 10. She was tired during game play and declined to give feedback at the time.

Alicia- Played levels 1 and 2, has taken calculus 1, rated the game an 8 out of 10. She liked that the game used math skills in a strategic way that reminded her of battleship. She enjoyed the hands-on feel of the game and liked the current prototype game board and pieces. She said the explanations of items and rules needs to be improved and clarified through visuals of all the items being used in action.

Kamila- Played levels 1 and 2, took AP Calculus AB in high school (equivalent of calculus 1), rated the game an 8 out of 10. She liked that the game was different and unique. She felt it was fun and educational and would be willing to play it again. She thought the game was good practice for doing simple derivatives in your head. She also suggested the same improvements that Alicia made, as they both played the game and gave feedback at the same time.

Victoria- Played levels 1 and 2, has not taken calculus, rated the game a 7 out of 10. She felt this is a good game for students who are ambitious and want to learn calculus. Felt the game concept limits the available audience it can reach, applicable only to students in middle school up to college. She suggested making a version that is playable even at an elementary school level to broaden audience to all k-12 + college students. Felt there should be different themes for the board and pieces. Suggested using a 3D board with terrain in the future.

Jennifer- Played levels 1 and 2, has not taken calculus, rated the game a 6 out of 10. She liked that the game was educational and had strategy. She enjoyed playing the game and would be willing to try the other levels of the game. She felt there should be a limit on how many times game pieces can swap items. She recommended adding more color options for players to choose from (specifically blue and yellow, in addition to the current red and green colors). Also recommended that the name calculus be removed from the potential game name, “Calculus Wars”.

Malachi (2nd time)- Played levels 1 and 2, has taken business calculus, rated the game an 8 out of 10. He liked that he won the game against Jennifer and felt that item switching should remain the same. He liked the strategy element of the game and enjoyed playing. He also liked how the constrained color borders added to the strategy of the game. He agreed with Jennifer that the word calculus should be removed from the possible game name. He recommended being discreet about the fact that the game teaches calculus and derivatives. He suggested a name like "Exponential Growth the Game." He also stated that all the levels of the game cannot be played through in one sitting, and that a good stopping point is two levels per game play.

Appendix E-

Pre- and Post-test Data

Questions and Answers

Pre-test

Questions	Answers
1) X^3 becomes ……………………………….	$3X^2$
2) $3X^2$ becomes ……………………………..	$6X$
3) $4X^1$ becomes ……………………………….	4
4) $F(X^4)''$ becomes …………………………….	$12X^2$
5) X^2*Y^2 becomes …………………………….	$X^2*2Y + Y^2*2X$ or $2X^2Y + 2Y^2X$
6) Y^3*X^1 becomes …………………………..	$3Y^2X + 1X^0Y^3$ or $3Y^2X + Y^3$
7) $\frac{X^3}{Y^3}$ becomes ……………………………….	$\frac{X^3 3Y^2 - Y^3 3X^2}{(Y^3)^2}$
8) $\frac{X^4}{X^1}$ becomes ……………………………….	$X^3 = 3x^2$
9) $(Y^3)^3$ becomes …………………………….	$3(Y^3)^2 *3Y^2$ or $Y^9 = 9Y$

Post-test

Questions	Answers
1) X^4 becomes …………………………..	$4X^3$
2) $3X^3$ becomes …………………………….	$9X^2$
3) $2X^1$ becomes …………………………….	2
4) $F(X{\wedge}^{3})''$ becomes ……………………….	$3X^2 = 6X$
5) X^3*Y^2 becomes ………………………….	$2YX^3 + 3X^2Y^2$
6) X^4*Y^1 becomes ………………………….	$4X^3Y + X^4$
7) $\frac{X^4}{Y^2}$ becomes …………………………..……	$\frac{4X^3Y^2 - 2YX^4}{Y^4}$
8) $\frac{Y^2}{Y^5}$ becomes ………………….………	$Y^{-3} = -3Y^{-4}$
9) $(X^2)^4$ becomes ………………………….	$X^8 = 8x^7$

Results

(**n=**4, n=2, n=1)

Jim: Pre-test (**1,2,3,4,7,9**): 24pts /

Post-test (**1,2,3,4,5,6,7,8**,9): 34pts

Rachel: Pre-test (**1,2**,5,6,7,9): 13pts /

Post-test (**1,2**,5,6,7,8,9): 14pts

Austin: Pre-test (**1,2,3,4,5**,6,7,8,**9**): 29pts /

Post-test (**1,2,3,4,5,6**,7,**8,9**): 34pts

Shannon: Pre-test (): 0 pts /

Post-test (**1**,6,7): 6pts

Alicia: Pre-test (**1,2**,5,6,7,9): 13pts /

Post-test (**1,2**,5,6,7,8,9): 14pts

Kamila: Pre-test (**1,2**,4,5,6,7,8): 16pts /

Post-test (**1,2,3,4,5,6**,7,8,9): 30pts

Eddie: Pre-test (**1,2,3**,5,6,7,**8**,9): 21pts /

Post-test (**1,2,3**,5,6,7,**8,9**): 23pts

Victoria: Pre-test (): 0pts /

Post-test (**1**,5,6,7,9): 9pts

Jennifer: Pre-test (**1,2**,5,8,9): 12pts /

Post-test (**2,3,4**,5): 13pts

Malachi: Pre-test (**1,2,3**,5,6,7): 16pts /

Post-test (**1,2,3,4**,5,7,8):19pts

(**n=**2, n=1, n=1)

Jim: Pre-test (**1,2,3,4,7,9**): 12pts /

Post-test (**1,2,3,4,5,6,7,8**,9): 17pts

Rachel: Pre-test (**1,2**,5,6,7,9): 8pts /

Post-test (**1,2**,5,6,7,8,9): 9pts

Key:

n = Exact correct answer

n = Partial but incomplete answer (i.e. $[x^2]^4 = x^8$ instead of $8x^9$)

n = successful simple derivative but wrong answer (i.e, $x^3y^4 = 3x^2 4y^3$ instead of $3x^2y^4 + 4y^3x^3$)

_ = Performance differences, best result

Austin: Pre-test (**1,2,3,4,5**,6,7,8,**9**): 15pts /

Post-test (**1,2,3,4,5,6**,7,**8,9**): 17pts

Shannon: Pre-test (): 0 pts /

Post-test (**1**,6,7): 4pts

Alicia: Pre-test (**1,2**,5,6,7,9): 8pts /

Post-test (**1,2**,5,6,7,8,9): 9pts

Kamila: Pre-test (**1,2**,4,5,6,7,8): 9pts /

Post-test (**1,2,3,4,5,6**,7,8,9): 15pts

Eddie: Pre-test (**1,2,3**,5,6,7,**8**,9): 12pts /

Post-test (**1,2,3**,5,6,7,**8,9**): 13pts

Victoria: Pre-test (): 0pts /

Post-test (**1**,5,6,7,9): 6pts

Jennifer: Pre-test (**1,2**,5,8,9): 7pts /

Post-test (**2,3,4**,5): 7pts

Malachi: Pre-test (**1,2,3**,5,6,7): 9pts /

Post-test (**1,2,3,4**,5,7,8):11pts

(**n=**1, n=0, n=0)

Jim: Pre-test (**1,2,3,4,7,9**): 6pts /

Post-test (**1,2,3,4,5,6,7,8**,9): 8pts

Rachel: Pre-test (**1,2**,5,6,7,9): 2pts /

Post-test (**1,2**,5,6,7,8,9): 2pts

Austin: Pre-test (**1,2,3,4,5**,6,7,8,**9**): 6pts /

Post-test (**1,2,3,4,5,6**,7,**8,9**): 8pts

Shannon: Pre-test (): 0 pts /

Post-test (**1**,6,7): 1pts

Alicia: Pre-test (**1,2**,5,6,7,9): 2pts /

Post-test (**1,2**,5,6,7,8,9): 2pts

Kamila: Pre-test (**1,2**,4,5,6,7,8): 2pts /

Post-test (**1,2,3,4,5,6**,7,8,9): 6pts

Eddie: Pre-test (**1,2,3**,5,6,7,**8**,9): 4pts /

Post-test (**1,2,3**,5,6,7,**8,9**): 5pts

Victoria: Pre-test (): 0pts /

Post-test (**1**,5,6,7,9): 1pts

Jennifer: Pre-test (**1,2**,5,8,9): 2pts /

Post-test (**2,3,4**,5): 3pts

Malachi: Pre-test (**1,2,3**,5,6,7): 3pts /

Post-test (**1,2,3,4**,5,7,8):4pts

Statistics

(4,2,1)	Pre-test	Post-test	Pt Increase	% increase
Jim	24	34	10	27.78
Rachel	13	14	1	2.78
Austin	29	34	5	13.89
Shannon	0	6	6	16.67
Alicia	13	14	1	2.78
Kamila	16	30	14	38.89
Eddie	21	23	2	5.56
Victoria	0	9	9	25.00
Jennifer	12	13	1	2.78
Malachi	16	19	3	8.33
		Average:	5.2	14.44

(2,1,1)	Pre-test	Post-test	Pt Increase	% increase
Jim	12	17	5	27.78
Rachel	8	9	1	5.56
Austin	15	17	2	11.11
Shannon	0	4	4	22.22
Alicia	8	9	1	5.56
Kamila	9	15	6	33.33
Eddie	12	13	1	5.56
Victoria	0	6	6	33.33
Jennifer	7	7	0	0.00
Malachi	9	11	2	11.11
		Average:	2.8	15.56

(1,0,0)	Pre-test	Post-test	Pt Increase	% increase
Jim	6	8	2	22.22
Rachel	2	2	0	0.00
Austin	6	8	2	22.22
Shannon	0	1	1	11.11
Alicia	2	2	0	0.00
Kamila	2	6	4	44.44
Eddie	4	5	1	11.11
Victoria	0	1	1	11.11
Jennifer	2	3	1	11.11
Malachi	3	4	1	11.11
		Average:	1.3	14.44

Appendix F-

Game Rating Data

Phase One	Rating	Calc or Non-calc	
Brandon		Calc	
Sidney-1	10		Non-calc
Tess	8	Calc	
Malachi-1	6.5	Calc	
Glenn-1	6		Non-calc
Average:	7.6		

Phase Two	Rating	Calc or Non-calc	
Glenn-2	8		Non-calc
Carrie	7	Calc	
William	7		Non-Calc
Eddie-1	6.5	Calc	
Ashton	7.5	Calc	
Kayla	8.5	Calc	
Average:	7.4		

Phase Three	Rating	Calc or Non-calc	
A.J.	7		Non-calc
Stephan	9		Non-calc
Sidney-2	10		Non-calc
Jim	8	Calc	
Average	8.5		

Phase Four	Rating	Calc or Non-calc	
Rachel	8	Calc	
Austin	9	Calc	
Eddie-2		Calc	
Shannon	7.5		Non-calc
Alicia	8	Calc	
Kamila	8	Calc	
Victoria	7		Non-calc
Jennifer	6		Non-calc
Malachi-2	8	Calc	
Average	7.7		

Non-calculus

Name	Rating
Sidney-1	10
Glenn-1	6
Glenn-2	8
William	7
A.J.	7
Stephan	9
Sidney-2	10
Shannon	7.5
Victoria	7
Jennifer	6
Average	7.8

Calculus

Name	Rating
Tess	8
Malachi-1	6.5
Carrie	7
Eddie	6.5
Ashton	7.5
Kayla	8.5
Jim	8
Rachel	8
Austin	9
Alicia	8
Kamila	8
Malachi-2	8
Average	7.8