Numeracy
Advancing Education in Quantitative Literacy

Volume 7 | Issue 2 Article 6

2014

Review of *The Joy of x: A Guided Tour of Math, from One to Infinity* by Steven Strogatz

Michael T. Catalano
Dakota Wesleyan University, micatala@dwu.edu

Recommended Citation

Catalano, Michael T.. "Review of *The Joy of x: A Guided Tour of Math, from One to Infinity* by Steven Strogatz." *Numeracy* 7, Iss. 2 (2014): Article 6. DOI: http://dx.doi.org/10.5038/1936-4660.7.2.6

Review of *The Joy of x: A Guided Tour of Math, from One to Infinity* by Steven Strogatz

Abstract

Strogatz, Steven. *The Joy of x: A Guided Tour of Math, from One to Infinity,* (New York, NY, Houghton Mifflin Harcourt, 2012). 316 pp. ISBN 978-0-547-51765-0

The *Joy of x: A Guided Tour of Math, from One to Infinity*, by Steven Strogatz, is an engaging and example-filled argument for mathematics as a valuable and enjoyable activity. The thirty chapters are divided into six parts, entitled Numbers, Relationships, Shapes, Change, Data, and Frontiers. The discussion ranges from intuitive explanations of basic concepts such as place value, the four arithmetic operations, percentage increase and decrease, and solving equations, to "higher" levels of mathematics such as calculus, probability and statistics, group theory, and the nature of infinity. As in John Allen Paulos' work, *Beyond Numeracy*, the chapters are short and punchy, and they can be read independently. While the book is not specifically devoted to numeracy, several chapters, especially those in Part Five on Data, address ideas and examples relevant to quantitative literacy.

Keywords

Mathematics, Mathematics History, Steven Strogatz, Book Review

Cover Page Footnote

Mike Catalano is the new Book Review Editor for this journal. He is a professor of mathematics and chair of the department at Dakota Wesleyan University. He is a former member of the Board of Directors of NNN and has a particular interest in incorporating numeracy into college algebra through the use of examples related to social issues.

Introduction

In this review, I will provide a fairly detailed summary, part by part, of *The Joy of x*, emphasizing points which might be more directly related to quantitative literacy. I will also give some explanation for the inclusion in *Numeracy* of a review of this book, which addresses itself to mathematics in general, not numeracy specifically. Is there a place in *Numeracy* for expository articles on mathematics that do not actually have much to say about skills, applicability, confidence, and other aspects that most of us would agree are a part of numeracy? I argue that there should be, and that even if some readers disagree, *The Joy of x* will be worth their time.

Preface

Although he is a Professor of Applied Mathematics at Cornell, Steven Strogatz is something of a generalist, with an appreciation for other disciplines, their values and viewpoints. In his preface, he describes the book as an attempt to explain mathematics from the ground up to people like artists, whose experiences with mathematics may have been unfriendly, unsuccessful, and uninteresting. However, this is not a book for remediation, but rather an overview of what mathematics is all about and how it can be beautiful, not to mention useful.

The book grew out of a sequence of fifteen articles entitled "The Elements of Math" written for the *New York Times* (Strogatz 2010). Based on the positive feedback he received from readers, Strogatz felt there was a "profound but little-recognized hunger for math," at least among a sizable enough population to merit this book, "an introduction to math's most compelling and far-reaching ideas." To measure this hunger in the spirit of *How to Measure Anything* (Hubbard 2010), reviewed in the last issue of *Numeracy* (i.e., find a quick and easy measure, without obsessing about accuracy), I note that Amazon has *Joy* ranked the 8th best seller in the category of Mathematics History, 77th in Mathematics Applied, and 12,835th overall, with 170 submitted reviews. This is behind Hubbard's most recent 2014 edition (8,793rd overall), but ahead of *Guesstimation* (Weinstein and Adam 2008), *Guesstimation 2.0* (Weinstein 2012), *Math on Trial* (Colmez and Schneps 2013), and *Turning Numbers into Knowledge*, (Koomey 2008) also covered in my last review.

The style is informal, and occasionally facetious. Strogatz certainly seeks to make the book accessible. A few of the allusions did seem oddly dated. In a 2013 book, why would you refer to Michael Jordan as your example for a high-flying dunker, rather than Lebron James, Kevin Durant, or even Kobe Bryant? Perhaps this is an indication of the target demographic. Still, even younger readers should find the book engaging. The book does include copious notes,

some of which include more technical discussion of mathematical details that are glossed over in the text.

The Book

Strogatz really does start from the "ground up" in Part One, "Numbers" (p. 3–42). Throughout the book, he draws from existing explanations he finds compelling, and his first one comes from a *Sesame Street* video. Humphrey is serving a roomful of penguins, and calls out the order to the kitchen: "Fish, fish, fish, fish, fish, fish." Ernie launches into an explanation of the number six. Using numbers makes things more efficient for Humphrey, but at the price of abstraction. Numbers, after all, are "ethereal," and Strogatz even refers to them as existing in "some sort of Platonic realm," one over which we have no control once meanings are assigned. Perhaps this is introducing the esoteric in a context that pre-school children should be able to master, but Strogatz is writing for an adult population that may not have thought about such basics for many years.

Subsequent chapters go on to concretely explain the four basic operations, including intuitive "proofs" for why a negative times a negative is a positive, or why multiplication is commutative. He includes concrete examples of these laws, including how double-negatives relate to the firing of neurons, and how multiplication with negatives can help analyze the shifting national alliances leading up to World War One.

Part Two, "Relationships" (p. 43–81), brings us into the realm of algebra. Formulas are analogous to relationships, albeit between quantities rather than people. Understanding and manipulating formulas is, in the author's view, a combination of art and science. One should make use of common sense, while realizing common sense can also be deceived. Strogatz describes a teaser presented by a friend of his, an educational consultant. If the length of an object is y when measured in yards, and f when measured in feet, what equation relates y and f? In the consultant's experience, most students wrongly answer $y = 3f$. Numeracy instructors often advocate that students check their units, a great help in situations like this one.

In the next chapter, "Finding Your Roots," Strogatz goes from famous problems from Greek antiquity involving roots, through the basics of complex arithmetic (taking a geometric approach), to fractals and chaos. The following chapter opens with another numeracy example. If the cold water faucet can fill the tub in a half-hour, and the hot in an hour, how long will it take to fill the tub with both faucets on? A ten-year old Strogatz had guessed 45 minutes and was then schooled in some common-sense argument from example by his Uncle Irv, the book-keeper, who first noted that two faucets must be faster than one, so the correct answer must be less than a half-hour. Again, this may seem like a simple

example, but it is one that would trip up many college students, especially if the context and the numerical values were changed.

Subsequent chapters in this part discuss the quadratic formula, and several families of functions including power, exponential, and logarithmic. Strogatz outlines the example of high school junior Britney Gallivan, who derived a formula relating the number of times, *n*, a piece of paper could be folded in half lengthwise with the length of the paper, L. She then used roughly 3000 feet of toilet paper to smash the world record, folding it in half 12 times. The moral of the story is that algebra can be predictive.

As you might guess, the section "Shapes" (p. 83–128) is devoted to geometry. The first two of the five chapters include two different proofs of the Pythagorean Theorem, one based on area, which is rather more intuitive than the other, which is based on ratios and algebra. Strogatz cites this as an example of how one proof can be more beautiful than another. The second chapter goes on to discuss deductive thinking, a la Euclid, and links it with Thomas Jefferson and the Declaration of Independence's reference to self-evident truths. Subsequent chapters discuss conics and how their properties can be applied to sound and light, trigonometry, and the idea of limits as illustrated by Archimedes' method of exhaustion. This last chapter on limits has a very nice, intuitive proof that the area of a circle is πr^2.

Part Four, "Change" (p. 129–171), focuses on calculus. A bit of history is included here, and the concepts of derivative and integral are described intuitively as relating to how fast a quantity is changing, and how it accumulates. Velocity is emphasized, of course, illustrated by references to Michael Jordan dunks and viral epidemics. Some traditional max-min and geometric volume problems appear in the chapter on integrals. There is a chapter on e and its relation to exponential growth, as well as a probability example. It turns out that if you seat couples in a movie theater at random, specifying only that they sit together, that no couple will move seats to accommodate other couples, and that seating continues until there are no free adjacent pairs of seats, the proportion of single unoccupied seats to the total number of seats will be $1/e^2$. A second probability problem related to finding your perfect mate follows (you should pick a mate in the first $1/e$ of your prospective dating life, and this gives you a $1/e$ chance of finding your perfect mate). The next chapter covers differential equations, using a whimsical love-hate relationship as the context, but going on to discuss some of Newton's accomplishments and the open three-body problem. The final chapter in this section highlights Maxwell's equations on electro-magnetism.

Part Five, "Data" (p. 173–198), will be the most interesting for many *Numeracy* readers. Some readers may wish there were more than three chapters here. The first of the three is The New Normal, and it begins by noting the increasing prevalence of statistics in the modern world, which is now "teeming

with data" related to everything from the Internet to the Human Genome Project. Strogatz makes the case for increasing emphasis on this area in education. Here are two quotations, the first from an economist and the second from a well-known conservative pundit.

> Learn some statistics. . . High school mathematics curriculums spend too much time on traditional topics like Euclidean geometry and trigonometry. For a typical person, these are useful intellectual exercises but have little applicability to daily life. Students would be better served by learning more about probability and statistics. (Mankiw 2010 [Strogatz, p. 175])

> Take statistics. Sorry, but you'll find later in life that it's handy to know what a standard deviation is. (Brooks 2006 [Strogatz, p. 176])

Strogatz goes on to argue that understanding what a distribution is may be even more important. He specifically discusses the normal distribution, as well as other kinds of distributions such as power law distributions (sometimes referred to as Zipf distributions, although Strogatz does not use that term) which are skewed or have heavy tails. The distribution of city sizes, sizes of 2003 Bush tax cuts, and daily percentage change in the Dow Jones Industrial Average are used as examples. I would have liked to see more examples and additional types of distributions discussed here.

The next chapter delves into conditional probability, including Bayes' Theorem. Strogatz notes that students often have an easier time understanding this theorem when it is couched in terms of relative or natural frequencies, rather than probabilities expressed in decimal form. He refers to *Calculated Risks* (Gigerenzer 2003), which I highly recommend. Gerd Girgerenzer, a cognitive psychologist, conducted a study of how well doctors in Germany understood relative risks. The problem posed had to do with communicating the results of positive mammogram tests to women whose histories indicated a low risk of breast cancer.

> The probability that one of these women has breast cancer is 0.8 percent. If a woman has breast cancer, the probability is 90 percent that she will have a positive mammogram. If a woman does not have breast cancer, the probability is 7 percent that she will still have a positive mammogram. Imagine a woman who has a positive mammogram. What is the probability that she actually has breast cancer? (Girgerenzer 2003 [Strogatz, p. 185])

In a group of 24 German doctors, estimates ranged from 1 percent to 90 percent, with many doctors exhibiting anxiety about their estimates. Girgerenzer found that if the information was presented in terms of frequencies (e.g., 8 out of 1000 instead of 0.8 percent), anxiety decreased and the estimates improved. Strogatz then asks the reader to contemplate whether the sacrifice in rigor represented by this approach is justified by the increase in clarity and understanding. This is certainly a relevant question for those of us involved in promoting numeracy.

The chapter concludes with discussion of conditional probabilities related to the O.J. Simpson trial.

The final chapter in this part is "Untangling the Web." Without getting into details (some of these are in the notes), Strogatz discusses how Google uses linear algebra to develop page rankings for searches based on information on links between pages. The discussion is fairly intuitive, giving readers the basic idea in the context of three linked pages.

Part Six is entitled "Frontiers" (p. 199–256). Here, Strogatz presents some modern examples related to each of the previous five chapters. The Prime Number Theorem, with a nod to the twin-prime conjecture, comes first. Group Theory is illustrated geometrically, and through the problem of deciding what sequence of flips and rotations you should use in turning a mattress to provide for even wear. Mobius strips come next, and Strogatz includes references to some fun videos available online that students of all ages should find interesting. One video was produced by Vi Hart (n.d.) and features characters Wind and Mr. Ug. Geodesics on a variety of surfaces are considered next, and there are more video references related to geodesics on tori, produced by Konrad Polthier et al. (n.d.). Infinite sums, including Fourier series, and other paradoxes of infinity, illustrated by the famous Hilbert Hotel and Cantor's diagonalization argument, close out the book.

Concluding Comments

The Joy of x is similar in some ways to *Beyond Numeracy* (Paulos 1992). The chapters are short and punchy, can be read independently, and are meant to give readers a general overview and appreciation of the subject matter. It is more an expository book on mathematics than a numeracy book. However, some of the discussions have numeracy relevancy. In addition, to the degree the book is successful in getting readers to see the value of math, or feel more confident in their understanding of math, it can contribute to numeracy. Certainly, the attitude that much of mathematics is next to useless, held by many among the general public and our students, is a significant obstacle to increasing numeracy. Consider the following quotation, from Dorothy Wallace's Parts of the Whole column, "Only Connect" in the last issue of *Numeracy*.

> To promote numeracy effectively we must simultaneously make it relevant to our students, our university colleagues, and the public at large. Relevant, here, means establishing connections in the minds of individuals between numeracy and other things they already consider truly important. (Wallace 2014)

I would add that making mathematics enjoyable cannot hurt either.

A second advantage to this book is the extent to which it provides clear and intuitive explanations for why certain mathematical statements are true. These do not often rise to the level of rigorous proof, but for a general population, I think the sacrifice in rigor is worth the benefit in clarity and understanding. Understanding "why," even if that explanation is somewhat incomplete, can help in retention, in linking isolated concepts into a more complete whole, and in applying mathematics in a variety of contexts.

Finally, there is a trade-off between direct numerical understanding and trusting the experts. Certainly one contributing factor to innumeracy is a distrust or devaluing of what numerical experts claim. Yes, the populace should have enough quantitative literacy to engage in reasoned discussion, but there is a place for respecting the claims of experts even when one is not able to substantively evaluate those claims. Skepticism needs to be informed and reasonable, not blanket and dismissive. Strogatz's work, in being accessible and engaging, has the capacity to reduce the irrational skepticism and fear that many have concerning mathematics and quantitative information.

The *Joy of x* is not a "how to teach numeracy" book. However, I think many readers of *Numeracy* would find it interesting and useful, and I think the same is true for the general public.

References

Brooks, David. 2006. Harvard-bound? Chin up. *New York Times*, March 2. http://query.nytimes.com/gst/fullpage.html?res=9C06EFDB1731F931A35750C0A9609C8B63 (accessed June 24, 2014)

Colmez, Coralie, and Leila Schneps. 2013. *Math on trial*. New York: Basic Books Publishing.

Girgerenzer, Gerd. 2003. *Calculated risks*. New York: Simon & Schuster

Hart, Vi. n.d. Mobius story: Wind and Mr. Ug. Recreational math: Doodling in math and more. Khan Academy. https://www.khanacademy.org/math/recreational-math/vi-hart/mobius-strips/v/m-bius-story--wind-and-mr--ug (accessed June 20, 2014).

Hubbard, Douglas W. 2010. *How to measure anything: Finding the value of "intangibles" in business*. Hoboken, NJ: John Wiley and Sons, Inc. http://dx.doi.org/10.1002/9781118983836

Koomey, Jonathan G. 2008. *Turning numbers into knowledge: Mastering the art of problem solving*. Oakland, CA: Analytics Press.

Mankiw, Gregory. 2010. Economic view: A course load for the Game of Life. *New York Times*, September 4.

http://www.nytimes.com/2010/09/05/business/economy/05view.html?_r=1& (accessed June 24, 2014)

Paulos, John Allen. 1992. *Beyond numeracy*. New York: Vintage Books, Random House, Inc.

Polthier, Konrad, Markus Schmies, Martin Steffens, and Christian Teitzel. n.d. The video "Geodesics and Waves" http://page.mi.fu-berlin.de/polthier/video/Geodesics/index.html (accessed June 24, 2014)

Strogatz, Steven. 2010. The Elements of Math, *New York Times*. http://topics.nytimes.com/top/opinion/series/steven_strogatz_on_the_elements_of_math/index.html (accessed June 21, 2014)

———. 2012. *The Joy of x*. Boston, MA, Houghton Mifflin Harcourt.

Wallace, Dorothy. 2014. Parts of the whole: Only connect. *Numeracy* 7(1): Article 9. http://dx.doi.org/10.5038/1936-4660.7.1.9 (accessed June 20, 2014)

Weinstein, Lawrence. 2012. *Guesstimation, 2.0*. Princeton, NJ, Princeton University Press.

——— and John A. Adam. 2008. *Guesstimation*. Princeton, NJ, Princeton University Press.