

2015

## Quantitative Literacy and Co-Construction in a High School Math Course

Mark Russo

Montclair State University, russomp@gmail.com

Follow this and additional works at: <https://scholarcommons.usf.edu/numeracy>



Part of the [Curriculum and Instruction Commons](#), and the [Science and Mathematics Education Commons](#)

---

### Recommended Citation

Russo, Mark. "Quantitative Literacy and Co-Construction in a High School Math Course." *Numeracy* 8, Iss. 1 (2015): Article 8. DOI: <http://dx.doi.org/10.5038/1936-4660.8.1.8>

Authors retain copyright of their material under a [Creative Commons Non-Commercial Attribution 4.0 License](#).

---

## Quantitative Literacy and Co-Construction in a High School Math Course

### Abstract

This article reports some of the key findings from a practitioner-action research study that analyzed the impact of co-construction on students' quantitative literacy (QL) and attitudes towards mathematics. Co-construction is a process where students work alongside their teachers to plan units, lessons, and assessments, and this approach was chosen because of its potential to help students advocate for the specific mathematical contexts that would best develop their QL. This yearlong study took place in a public high school, with forty-five students in two different classes participating. Students formally contributed to the development of the course by completing written questionnaires and participating in large- and small-group discussions, and they contributed informally through their participation in class, performance on assessments, and reflections on various assignments, which I considered in field notes and a research journal. I used the constant comparative method to analyze these data, and I arrived at three key themes: (1) co-construction recast traditional roles in the classroom, while still serving as an important form of instruction in itself; (2) developmentally appropriate assignments triggered students' situational interest, while individualized co-construction proved somewhat effective at developing individual interest; and (3) the co-construction process challenged the way I used to think about mathematics teaching, as students and I reconsidered the content, technology, and classroom practices that would best develop students' QL. This study fills an important gap in the QL and co-construction literature, and it also has important implications for mathematics practitioners, and for practitioners who want to give students more of a voice in the classroom.

### Keywords

Quantitative Literacy, co-construction, high school

### Creative Commons License



This work is licensed under a [Creative Commons Attribution-Noncommercial 4.0 License](https://creativecommons.org/licenses/by-nc/4.0/)

### Cover Page Footnote

Mark Russo is the supervisor of mathematics for the Pascack Valley Regional High School District, where he works to support effective instruction in mathematics. Previously he served as a high school mathematics teacher, where he focused on developing students' quantitative reasoning skills and improving their attitudes towards mathematics.

## Introduction

As a high school mathematics teacher, I have taught many students who loved mathematics and achieved at a high level, but I have also worked with students who did not learn much in my classes, who achieved far below their potential, and who boasted a strong dislike, or even hatred, for mathematics. It was extremely challenging to work with these latter students, because I would not only struggle to help them understand the material, but I would struggle even more to justify why they had to learn these concepts at all. It is difficult enough to teach a low-performing high school senior polynomial division and Descartes' rule of signs, but it is far more difficult to explain to them why these things are worth learning in the first place. Year after year, I tried different ways to teach these courses, but I began to consider that the problem might not be my pedagogy; rather, the problem might be the content. These students were struggling not only because of a lack of understanding, though that was definitely part of it; rather, they might have been underperforming because of a lack of interest or a lack of relevance. What if I stopped trying to prepare students for a calculus course that they might never take and, instead, focused on material that would capture their interests and relate to their lives or their future professions? Perhaps this change could help students enjoy mathematics and achieve at a higher level, while also equipping them with the knowledge and skills they would need to be educated consumers of quantitative information. I decided to implement this change in two of my senior elective classes, by designing courses with an emphasis on content and skills that would develop students' quantitative literacy (QL).

In my experience, many students in senior elective classes do not consider mathematics class to be useful for their lives, nor do they learn much while they are enrolled in the course. I set out to learn whether a QL course could influence students' attitudes towards and understanding of mathematics. In order to do this, I decided to construct a course around students' interests and future ambitions, because QL focuses on skills that are necessary to "engage effectively in quantitative situations arising in life and work" (ILSS 2000). This proved to be difficult, though, because, as Appleton and Lawrenz (2011: 150) suggested, "mathematics teachers' ideas of what is real world or practical are not the same as their students." In order to better understand where students are coming from, Appleton and Lawrenz (2011: 151) suggested that "mathematics teachers could productively spend more class time discussing with students what their ideas of real world or practical issues are and how these relate to classroom mathematics." Fredricks et al. (2004: 73) also argued for the importance of student voices, as they described a positive relationship between student engagement in high school and "voluntary choice, ...student participation in school policy and management,

opportunities for staff and students to be involved in cooperative endeavors, and academic work that allows for the development of products.”

One way to increase students’ participation in the decision-making process is through the practice of *co-construction*. Unlike more-traditional techniques where teachers incorporate students’ feedback into their lesson planning, co-construction is a process where students work alongside their teachers to plan units, lessons, and assessments. This technique can address the problem presented by Appleton and Lawrenz (2011), because students are the best judges of what is important and relevant to them. Consequently, co-construction could be a valuable tool in a QL classroom, because it increases the likelihood that course content will be pertinent and meaningful to students’ lives.

In order to make mathematics class more meaningful, I conducted a practitioner-action research project in which I invited students to co-construct the course with me. I reported the findings in great detail in my dissertation (Russo 2014), where I analyzed not only how a QL course impacted students’ attitudes and understandings of mathematics, but also how the process of co-construction impacted the development of the course. Specifically, I attempted to answer the following research questions:

1. How does the ongoing co-construction of a QL course between my students and me affect the evolution and development of the course?
2. How does participating in this course affect students’ QL and attitudes about mathematics?
3. Through these experiences, what do I as a teacher learn about teaching for QL and making mathematics more relevant to my students?

This article will summarize some of the key findings from my dissertation, with a particular emphasis on question 3 and the lessons I learned about teaching for QL at the high school level. Because this article is a modified, streamlined, and truncated version of my original study, I was unable to include many of the raw data that helped me answer my research questions. In the original study, I included transcriptions of small-group discussions, field notes, questionnaires, surveys, and student responses on various assessments. Since each of these data helped me to arrive at my findings, I would encourage readers who want more information to read the study in its entirety.<sup>1</sup>

## **Definition of Terms**

In this study, I utilize the definition of QL that comes from the International Life Skills Survey (ILSS 2000): “an aggregate of skills, knowledge, beliefs,

---

<sup>1</sup> Russo (2014) is available through Dissertations and Theses – Grad works (Proquest) at <http://gradworks.umi.com/36/19/3619421.html> (accessed Oct. 21, 2014).

dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work.” I chose the term QL over QR (quantitative reasoning) because I believe that the former clearly emphasizes concrete mathematical knowledge and skills, as well as many of the skills associated with the latter.

## **Literature**

The literature includes many studies that describe the prevalence and impact of QL initiatives in our schools. Some of these studies looked at how colleges address QL in a comprehensive manner (Jordan and Haines 2003; Richardson and McCallum 2003; Brakke and Carothers 2004; Diefenderfer et al. 2004; Steel and Kilic-Bahi 2010), while others analyzed the impact of specific undergraduate courses that were designed to develop students’ QL (Briggs et al. 2004; Madison 2006; Catalano 2010; Dingman and Madison 2010; Van Peurse et al. 2010; Boersma and Klyve 2013). While each of these studies offered important insights into teaching for QL, they were limited to courses and programs at the undergraduate level. By contrast, at the secondary level, the literature has been largely limited to curricular suggestions (Hoachlander 1997; Madison 2006; Lesh et al. 2008; Madison and Steen 2008; Gillman 2010; Steen 2012; Lusardi and Wallace 2013), with very little published research on QL courses or QL initiatives that practitioners have actually tried (Steen 2001a; Madison and Steen 2007). Steen (2001a: 115) attempted to offer a rationale for why: “Numeracy in secondary schools is harder to detect and describe, primarily because, as an interdisciplinary enterprise, it must live in an atmosphere dominated by pressure for disciplinary standards and, recently, by vigorous arguments about competing mathematics curricula.” In an educational landscape dominated by the Common Core State Standards, research that considers alternative approaches has never been more important. My study fills an important gap, then, by describing the impact of a QL course at the high school level.

In addition to filling a gap in the QL literature, my study also considered how the process of co-construction can be utilized to help students develop their QL. I was able to find only two examples in the literature that investigated a style of co-construction that was similar to the one employed by my study. The first examined how a responsive teaching environment impacted scientific inquiry in one teacher’s fifth-grade classroom over two consecutive years (Maskiewicz and Winters 2012), while the second examined the co-construction of midterm exams at the undergraduate level (Ahn and Class 2011). The study reported here thus adds to the co-construction literature by analyzing the impact of a full-year course that was fully co-constructed by students and the teacher. In addition, the study suggests a potentially important relationship between QL and co-construction. Because QL focuses on knowledge and skills that are important for “quantitative

situations arising in life and work,” co-construction provides a mechanism for teachers to learn the contexts that are important to individual students.

## Methods

I chose practitioner-action research as the design of my study because it allowed me to intentionally and methodically study some of the major challenges that I faced in my classroom. Herr and Anderson (2005: 3) defined action research as “inquiry that is done *by* or *with* insiders to an organization or community, but never *to* or *on* them.” In particular, I engaged in action research using qualitative methods, which was appropriate because I wanted to understand how the course evolved over time, as well as how my students and I were impacted by the course. The data collection in the study had two main components: data that I gathered from students, and my own reflections and observations. In terms of the data that I gathered from students, I collected surveys and questionnaires, examined major assessments, and analyzed transcriptions from large- and small-group discussions. In addition, I gathered my reflections and observations in field notes, and I kept a research journal.

Anderson et al. (2007: 3) described action research as “an ongoing series of cycles that involve moments of planning actions, acting, observing the effects, and reflecting on one’s observations.” This process served as the backbone of the project as I planned and implemented units of study, observed my students, reflected on my observations and those of my students, and then planned the next unit. Since “all qualitative data analysis is inductive and comparative in the service of developing common themes or patterns or categories that cut across the data” (Merriam 2009: 269), I utilized the constant comparative method employed by Glaser and Strauss (1967). This method was appropriate for this research project because it allowed me to make sense of the development of the course, as well as its relationship to students’ QL and attitudes about mathematics.

As is common with practitioner-action research, I chose to utilize purposeful sampling, which is based on the idea that “the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned” (Merriam 2009: 77). The criteria I used to select my sample included the following: 1) select a course that I could develop and teach, because I wanted to learn more about a problem that I noticed in my own teaching; 2) select an upper-level elective course, because these are the courses where students had tended to underperform and demonstrate negative attitudes towards mathematics; 3) select a course without a predefined curriculum, where I would have the freedom to design and revise the course as I go. After discussing these criteria with my supervisor, we decided that I could teach two sections of a new, upper-level elective course, Discrete Math.

## The Course

In previous years, students had the opportunity to enroll in AP Calculus, AP Statistics, precalculus or college algebra, but there was no option for students who either had not been very successful in previous mathematics courses, or who were not interested in abstract mathematics. In response, a new Discrete Math course was proposed, with the idea that topics like graph theory, probability and statistics, polling, game theory, and combinatorics could better motivate this specific group of students. After speaking with my supervisor, we agreed that I would teach this course, and while I was asked to consider some of the traditional topics covered in a Discrete Math course, I was given the flexibility to respond to the particular needs of my students and emphasize topics that would best develop students' QL. My goal for this course was not only to motivate and engage students, but also to equip them with the skills they would need to meet the quantitative demands of everyday life. Additionally, I wanted to give students an opportunity to co-construct the course with me, with the idea that students should have input into the content they study, because they are the best judges of their skills, interests, and future aspirations.

In practice, Discrete Math functioned much like a project-based course, as students regularly worked together on assignments that built towards an end-of-unit project. Group work was a key component of almost every lesson, as students were asked to rely on each other, as well as outside resources and technology, to help them grapple with new ideas and engage with relevant mathematics. The truly unique aspect of this course was the fact that students were actively involved in its planning. Students contributed to the course in the following ways:

(1) Several times per unit, students completed written questionnaires that informed me about their interests and opinions regarding the course. I asked students to respond to a few questions, and I shared general themes with students after I collected and analyzed the data.

(2) For the first two units, all students participated in large- or small-group discussions, and afterwards, students were invited to participate in voluntary small-group discussions. These discussions were a critical part of the co-construction process, because they gave students an opportunity to reflect on previous units of study and plan for the next. These discussions were semi-structured, as I created guided questions based on the results of the questionnaires, while also leaving space for students to brainstorm and consider new directions. These discussions informed me about students' attitudes and developing QL, but they were also the primary mechanism for me to co-construct the course with my students. During these discussions, we brainstormed various units of study, but since we were unable to reach consensus every time, I often allowed students to explore different subtopics that they wanted to study.

(3) I gave students the option to speak with me individually. I thought that this would be important early on in the course, particularly if a student did not feel comfortable voicing his or her opinion in a large-group setting, but I found that some students felt much more comfortable talking to me several months into the course.

(4) Finally, students were sometimes given a say in the type of assessment they wanted to produce. These decisions were sometimes made during our discussions, but more often they were made once students learned a bit more about each unit.

My role, then, was very different from previous courses, as I facilitated discussions, set parameters, and helped students develop the confidence and skills to consider the mathematics that was valuable to their lives, even though they were still novices in the field.

## Participants

The study took place in a small high school in New Jersey, in a middle- to upper-middle-class community. Thirty-seven students enrolled in the course in the beginning of the year, but by the time I ended data collection, 45 students were enrolled. Of the 45 students, 28 were boys and 17 were girls, 43 were seniors and 2 were juniors, and I had taught 10 of the students in previous years. Most of the students had come from our school's lower mathematics track, and, not surprisingly, many did not consider mathematics to be their favorite or best class.

I was also a participant in the research study, both as a teacher and a researcher. My multiple roles had an important impact on the study, in both positive and negative ways. As a classroom teacher, I had insider information that an outsider might not have access to. I was familiar with the culture of the school; students recognized me and hopefully trusted me; and I even had some students in class before. On the other hand, this insider status could have obscured my ability to see the data objectively, which is why I relied heavily on critical friends to challenge my preconceptions and help me see my data from a different perspective. Additionally, my role as a teacher may have prevented students from being fully honest with me. I attempted to create a classroom environment that was open and honest, but I was cognizant of this limitation when performing my data analysis.

## Findings

In the original study (Russo 2014), I presented a narrative that described the evolution of the class, my students, and my own thoughts and goals regarding co-construction and teaching for QL. The narrative documented some of the key

events that took place during the year, particularly with regards to the co-construction of the course, students' attitudes towards mathematics and their developing QL, and some of the things I learned as both teacher and researcher. The narrative also provided context for some of the major decisions I made during this process, both by highlighting the factors that led to those decisions and describing the impact of those decisions on students, on me, and on the course as a whole. That narrative included a significant amount of data that highlighted students' evolving attitudes and QL and my own reflections and changing opinions, and these data enabled me to arrive at the key themes of my study. In this section, I will give a brief synopsis of my first two themes – the ramifications of co-construction and student interest – and I will describe my third theme, teaching for QL, in greater detail.

### ***Theme 1: The Ramifications of Co-construction***

Taken as a whole, these data helped me to understand some of the challenges and benefits involved when including students in the co-construction process. I learned about the tensions that might arise in the context of shared decision making, particularly in regards to power relations and classroom roles. I also found that co-construction can be a type of instruction, because students have an opportunity to think more deeply about mathematics and develop important analytical skills as they examine and evaluate the merits of different content and classroom structures.

**Shared decision making.** I initiated the process of co-construction with the literature on democratic mathematics education in mind, because it stressed the importance of having students participate in decisions that affect them and their classroom (Ellis and Malloy 2007). Though I imagined that this process could be difficult for students, I found that co-construction was also trying for me, and it forced me to rethink the way I planned for instruction and organized my classroom. Because my students and I were learning how to co-construct together, they worked alongside me and offered feedback and suggestions for new directions as I struggled to find my footing. Students informed me when my assignments were too tedious or too difficult, when my directions were too vague or too painstaking, and when the projects were too advanced or too irrelevant. It was informative and humbling to hear students' opinions, but it was a necessary byproduct of giving students a true voice in how our classroom would function. After engaging in the co-construction process for six months with two different classes, I found the traditional classroom roles of teacher and student were challenged as we negotiated a new balance of power. This renegotiation of power took into account students' experiences and beliefs about mathematics and mathematics class, my own objectives, limitations, and beliefs about education, and our ability to work together within the confines of our school's norms.

**Co-construction as instruction.** In addition to its benefits for democratic mathematics education, I found co-construction to be an effective form of instruction in its own right. Many of our small-group discussions provided me with opportunities to develop students' QL and clear up misconceptions about mathematics or topics of interest in students' lives. Secondly, the process of co-construction helped students to be reflective, to consider how they learned best, and to contemplate the value of their education both at the time and for their future. Thirdly, co-construction helped students to practice other important life skills, such as decision making and the ability to work independently.

## ***Theme 2: Student Interest***

In order to better understand the relationship between teaching for QL and students' interest, I utilized Hidi and Renninger's (2006) theoretical model of interest development. The basic premise of their model is that situational interest, or "focused attention and the affective reaction that is triggered in the moment by environmental stimuli," can develop into individual interest, or "a person's relatively enduring predisposition to reengage particular content over time" (Hidi and Renninger 2006: 113). In my study, I analyzed any external stimuli that triggered students' situational interest, and I documented any classroom structures that supported students' development of individual interest.

**Triggering situational interest.** In their model, Hidi and Renninger (2006: 113) state that situational interest "is triggered in the moment by environmental stimuli" and not by the students themselves. A budget simulation (Appendix A, see Additional Files) was an excellent example of an assignment that triggered students' situational interest, because it intersected with students at a pertinent place in their development. In this assignment, I asked students to use Microsoft Excel to run a budget simulation for their 20s, where they were asked to research the starting salary for their dream job, calculate yearly 3% raises, and factor in the salary of a potential spouse as well. My data showed that unlike some other assignments, a critical mass of students really invested in this project, as students who were "not usually interested were discussing what house they would live in, car they would drive, etc." (Russo 2014: 147). This is consistent with my understanding of Vygotsky's *zone of proximal development* (ZPD; Vygotsky 1978), because I was able to construct an assignment that began with students' interests, but led to valuable mathematical understandings. The majority of my students were in the midst of the college application process at the time, so they were thinking in earnest about what they wanted to make of their lives. So even though the assignment asked students to consider their financial lives in their 20s, it may have resonated with some of the questions and concerns that were at the forefront of students' minds.

**Developing deeper interest.** I found that the development of individual interest required a more-differentiated approach, and further, that this differentiated approach was already available to us through the co-construction process. The promotion of individual interest was more difficult than that of situational interest, because this type of interest is often self-generated, rather than triggered by external stimuli (Hidi and Renninger 2006). In order to determine if any students had developed this level of interest, I analyzed my data using Hidi and Renninger's indicators to see if anyone took advantage of "the opportunity to reengage tasks related to his or her emerging individual interest and [opted] to do these if given a choice," generated "his or her own 'curiosity' questions about the content of an emerging individual interest," or exceeded "task demands in their work with an emerging individual interest" (Hidi and Renninger 2006: 115). These indicators reminded me immediately of certain students in my classes, and particularly those who initiated what I will call an "individualized co-construction process." On certain occasions throughout the year, students felt that the regular class assignments were not effectively developing their QL, so I asked them to propose alternative assignments that were more appropriate for their individual interests and life goals. While I was initially reluctant to allow students this flexibility, I eventually determined that if I wanted to develop students' QL, then I would have to give them the freedom to study contexts that were meaningful to them. While fewer than ten students took advantage of this opportunity, I found that many of the students who did demonstrated higher levels of individual interest than their peers.

### ***Theme 3: Teaching for QL***

While co-construction and student interest ended up being important goals in their own right, each of these themes was initially a mechanism through which I hoped to accomplish my primary objective for the course: to develop students' QL. I found that over the year I gained a great deal of insight into the relationship between QL and mathematics, and I also learned some strategies for teaching QL in a classroom setting.

**Relationship between QL and mathematics.** The QL literature presented differing viewpoints on the relationship between QL and mathematics, and my experiences with Discrete Math only reinforced this conflict. Two tensions that emerged between my students and me involved our challenge to reconsider mathematics from the perspective of QL and our struggle to define exactly whose mathematics should be paramount.

**Learning to see mathematics in a new light.** Though we initially struggled to re-conceptualize our preconceptions about school mathematics, I found that over the course of the year, some students and I became increasingly capable of seeing the value of mathematics from a QL perspective. This was certainly not the

case for all students, though, as my data show that some continued to think of mathematics as being completely separate from our work in Discrete Math. At the beginning of the year, I was not surprised that students espoused some of the more-stereotypical conceptions of mathematics class, such as the idea that you “don’t have to read or write essays” or the misconception that your success “depends on how well you retain the information and spit it down to the paper” (Russo 2014: 91). I was more surprised to find that some students retained these beliefs more than halfway through the year. For example, one student stated that she liked “how math has legitimate answers...how there is no in between,” while another remarked that “math is all numbers, so it doesn’t need explanation in words.” I wondered if these students considered our course to be something other than mathematics, because every assignment we did involved some sort of ambiguity or “in between” and required lengthy explanations “in words.” My suspicion increased with a third student’s comments, as she admitted that she “could learn to enjoy math like what we’re learning” but that our course was “not ‘Math,’ it’s life.” The premise that students didn’t believe our course to be mathematics was further supported by two of my highest-achieving students, who not only received outstanding grades throughout the year, but also asked meaningful questions, made connections between the assignments and their lives, and mentioned on several occasions that they enjoyed the work we were doing together. When I asked them if they felt like they were good at mathematics, one responded “No, I don’t think I am very good at math,” while the other commented that “No, I suck at math.” It would seem that even as students’ attitudes towards my class began to improve, they still considered mathematics primarily in terms of Algebra I, Geometry, and Algebra II. I concluded that it would likely require a more-comprehensive effort to expand students’ understanding of what constituted mathematics, rather than one course at the end of their high school careers.

Students were not the only ones who had trouble re-conceptualizing mathematics, as I found myself struggling to understand the exact relationship between mathematics and QL. As a mathematics teacher for almost a decade, I thought I knew my subject really well, but my exposure to QL made me question the very nature of mathematics. My data show that at several points throughout the year, I questioned whether a certain topic or project involved mathematics, QL, or something entirely different. For example, when I first read through some students’ suggestions from one of our small-group discussions, I wrote comments such as “Is this math?” and “What is math, really? Is this math? Is it not?” (Russo 2014: 237). I documented some of my meanderings in my dissertation, where I eventually concluded that mathematical reasoning was the key to understanding “how quantitative arguments, approaches, and tools are used in everyday life.” While mathematics and QL certainly share some content, the more-meaningful connection involves the reasoning skills that help individuals be successful in

both areas. I found that many students saw a relationship in terms of shared content, but I was unable to say for certain whether students saw a connection with reasoning skills as well.

When searching through my data, I found that students were much more comfortable reflecting on content, rather than reasoning, when describing their impressions of the course. For example, several students found that QL offered “a more-practical form of math” that could actually be useful for their lives, and that math was “not all hard equations” (Russo 2014: 237-238). It was much more difficult to ascertain students’ understanding of mathematical reasoning, but two examples from my data show that even if they were not aware of it, students were applying these reasoning skills to QL situations, particularly towards the end of my data-collection period. The first example took place during an assignment on caloric expenditure, where I provided students with different and sometimes contradictory approaches for calculating the number of calories they burned during various activities. I observed students’ interactions and wrote the following in my field notes (Russo 2014: 238):

*I was happy to see that students were really questioning the formula I gave, as well as the scale factors that they were using for various activities. Students had to perform various conversions... [but they were also] grappling with the reasons why formulas looked a certain way.*

I believe that students were utilizing mathematical reasoning skills in this instance and, further, that they were using them to consider a topic that was important to their lives. The second example comes from our fifth unit, when many students relied on Google Maps to help them calculate driving times for a road trip assignment (Appendix B, see Additional Files). I pushed students to question the veracity of these calculations, and students were able to ask insightful questions related to Google’s methods for calculating time, the impact of stops on Google’s algorithms, and the influence of time of day, traffic, and individual drivers’ characteristics on each of these calculations. So even if they didn’t articulate it, students had begun to employ mathematical reasoning to help them solve QL problems. The next step, then, would be to help students become aware of this association, so that they could use mathematics more purposely to solve problems in their everyday lives.

**Whose mathematics?** Despite an increase in most students’ appreciation for the value of mathematics, I found that students and I still struggled to determine which aspects of mathematics, or more precisely, whose mathematics was the most worthwhile. This back-and-forth was one of the key themes for the entire study, because it underscored the link between the co-construction process and QL. It was only through the co-construction process that I discovered the tension between students’ preferences for mathematical content and my own. The data show that this tension was further amplified by our age differences, because

things that students thought were significant were often different from what mattered to me, and they were often nothing like what was covered in textbooks. As I mention in my dissertation, I emphasized content such as interest rates, retirement savings, life insurance, and the government shutdown, and while students were able to state the headlines, I found that they hadn't "internalized the meanings, or applied those meanings to their lives" (Russo 2014: 239). I wanted to prepare students to be knowledgeable about the quantitative elements of society as a whole, but what I failed to understand is that some of these concepts might have to wait, because they were not especially relevant to students' lives at that particular time.

If the previous tension was not enough, there was the added challenge of navigating the mathematics that students thought was important and the mathematics that had a significant impact on the world around them. My data show that these issues surfaced primarily during the co-construction process, because students' primary task during these discussions was to evaluate the mathematical topics that would be included in our course. To provide an example, a segment from a longer dialogue mentioned in the dissertation is pertinent, because students debated the value of studying important, but in some students' opinions, irrelevant mathematics: (Russo 2014: 240).

Student 1: *Who cares about 0s and 1s (referring to binary)?*

Student 2: (smacks table) *0s and 1s are what run your life.*

Student 3: *Really, they do.*

Student 4: *Yeah, but it's not going to change anything.*

This idea that "it's not going to change anything" is so interesting, because it highlights the conflict between mathematics as a discipline in itself and mathematics as a tool that can help an individual student. As I mentioned earlier, these tensions boil down to the question of whose mathematics is important. The students'? The teacher's? The mathematics community's? The free market's? This question is further complicated by the concept of QL, because on the one hand, students should be studying things that are relevant to their lives, but on the other, they do not necessarily know what types of quantitative situations they will face once they graduate high school. Furthermore, unlike in traditional mathematics courses where students needed to learn the language and practices of the mathematics community, QL courses require no such conformity, because knowledge depends on the individual and things that matter to him or her, rather than anyone else. Therefore, there was no way to resolve this tension. Nevertheless, I believe that our repeated attempts were still important, primarily because they helped students understand the importance of mathematics in their own lives, in the lives of their peers, and in society as a whole.

**Teaching for QL in a classroom setting.** In addition to the previous challenges, I also struggled to effectively teach for QL in a classroom setting. In particular, I found that teaching for QL required a form of authentic instruction that was extremely difficult to implement in my classroom, and presumably most other classrooms like it. In addition, teaching for QL was often more successful when I relied on an individualized approach, which added several layers of complexity to an already demanding course.

**Authentic instruction.** I found that my Discrete Math course was a perfect case study for the planning paradox of Ainley et al. (2006), which suggests that teachers face a tradeoff between clear learning objectives and engaging tasks. My data showed that many of my less-successful lessons fell victim to this planning paradox, either because the tasks were unrewarding due to “tightly focused learning objectives” (i.e., such as the extra paperwork I required in a sampling activity) or because the learning was “less focused [and] difficult to assess” (i.e., my experiment with a three-week fantasy football league) when I focused on engaging activities (Ainley et al. 2006: 24). I wrote in my research journal that I experienced “a legitimate struggle between authenticity and pedagogical structure,” but thankfully I was able to implement at least a couple of authentic assignments, or assignments that closely resembled experiences that students might face outside of the classroom, without rendering them completely hollow (Russo 2014: 241).

Two assignments that stand out are the budget simulation (Appendix A) and the road trip assignment (Appendix B), because both experiences required nothing more than a computer and some time, which I made available to students in both situations. For an authentic assignment to be successful, students needed the proper tools for the task, and for topics like codes (special encryption and decryption machines) and nutrition (monitors that measure physical activity), this was simply impossible. I found that while authentic instruction was difficult, it was not impossible, provided that the assignment lent itself well to the tools available in the classroom. What made the implementation of authentic assignments especially difficult was the availability of new technologies, which made me reconsider the amount of emphasis to place on some traditional mathematical techniques.

Technology had a huge impact on the authenticity of my classroom, as various applications had supplanted the need for certain types of mathematics. Two instances from my data are especially pertinent, and they convinced me that I would have to let students use the proper tools if I wanted to develop their QL. The first example involves a student who questioned why he had to calculate nutritional information when a website could do it for him. The second example took place a couple weeks earlier in our nutrition unit, when students informed me that an app was capable of doing everything I had asked them to do in their

assignment. I wrote in my field notes that “if an app can do it, and do it much better, then why should students have to do it?” (Russo 2014: 242). I realized that I could no longer censor technology, because teaching for QL had to prepare students to solve real world problems with real world tools, not with pencil and paper in a mathematics classroom. This does not imply that mathematics is not necessary, though, and this point is essential to my study. Mathematical computation might not be as important as it once was, but, as I mentioned earlier, mathematical reasoning takes on even more significance. Students now have the responsibility of evaluating these new technologies. So rather than shunning them, I encouraged students to use them, understand them, and then evaluate their validity and precision (as with the road trip assignment of Appendix A and Google Map of Appendix B). Returning to an earlier point, I found that authentic assignments were much more successful if I provided students with the tools they would use in real life, and technology had become an important part of this equation. This is one of the reasons why the budget simulation was such a success, because I provided students with the exact technological tool (Excel) they would use if they were performing this task in their adult lives.

**Individualization.** In addition to the importance of authenticity, I found that individualization was an equally significant component of a QL classroom. This finding is an extension of my previous two themes, because, while individualization became a key component of the co-construction process and served to increase students’ interest, it also turned out to be an important stimulus for developing students’ QL. The co-construction process was meant to incorporate students’ ideas in the planning of the course, which I felt was a necessary prerequisite to develop a course for students’ QL. Unfortunately, only a fraction of each class participated in the small-group discussions, and, of that fraction, only a handful exhibited the eloquence and the persuasiveness to successfully advocate for their interests. The result was a situation where certain students fell between the cracks, and one student in particular demonstrated the harmful impact that could result (Russo 2014: 244):

*Personally, I did not find that the co-construction process improved my experience in this course because although the topics we studied were interesting and useful, I felt that I was interested in different things than the rest of the class.*

This comment spoke to the fact that since QL emphasizes mathematics in context, and different students care about different contexts, I would have to take students’ individual interests and experiences into account. This particular student wanted to major in engineering in college, so topics like nutrition and road trips may have been interesting to her, but they would not be as useful for developing her QL as would other, more-advanced mathematics topics. I wrote in my research journal that if I want “to develop students’ QL skills, some of those will be general (reading the newspaper, understanding sampling, financial literacy), but some will

be more specific, depending on students' interests and career goals" (Russo 2014: 244). This task would require individualized co-construction, because it would be nearly impossible to plan a program based on the unique interests and career goals of every student, particularly when these interests might be at odds with one another or change over the course of the year. As an extension, these assignments would have to develop students' interests, and in particular, their individual interests. Then, students would have an opportunity to construct new knowledge about a topic of real value to them, and, as a result, they would develop the skills and knowledge they would need "to engage effectively in quantitative situations arising in life and work" (ILSS 2000).

## Discussion and Lessons Learned

### *Implications for the Literature*

This research study contributes to the literature on QL primarily by filling two important gaps. To my knowledge, this study (Russo 2014) is the first to examine a full-scale QL course at the high school level, and it is one of just a few (Madison 2006; Catalano 2010; Madison and Dingman 2010) that studied a QL course using qualitative techniques. Therefore, this study represents an important new direction for the QL literature, which has focused largely on undergraduate courses and quantitative methodologies. Furthermore, the study offers a distinctive approach to teaching for QL by employing the process of co-construction, which can invite dialogue between the QL literature and the literature on democratic mathematics education. In addition to these contributions, my findings also dialogue with some other important themes in the QL literature.

I found that several of the literature's curricular suggestions for high school were appropriate, particularly the recommendations to include financial literacy (Lusardi and Wallace 2013), to organize education around different students' interests (Hoachlander 1997), and to highlight QL tasks in a senior elective (Madison and Steen 2008). My findings suggest that QR is the key link between school mathematics and QL, which adds to the dialogue on this important issue (Cobb 1997; Porter 1997; Ellis, Jr. 2001; Hughes-Hallett 2001; Manaster 2001; Orrill 2001; Steen 2001b; Madison and Steen 2009). Additionally, this study can respond to the following four difficulties in teaching QL that have been discussed in the literature.

(1) One of the primary benefits of teaching for QL is that students will learn the importance of mathematics, but Ma (1997: 228) wrote that "successful efforts in bringing students to a better awareness of the importance of mathematics may not automatically improve other attitudinal aspects." My findings were consistent with this assertion, as students' beliefs in the importance of different topics did

not necessarily correspond to their feelings of difficulty or enjoyment with those same topics.

(2) I found Jurdak's (2006: 296) assertion that situated problem solving that is "constrained by school rules, norms, and expectations" is much different from decision making in real life to be somewhat true, but much less so when I allowed students to use any and all modes of technology to help them complete assignments (such as the "My Fitness Pal" app, Excel, etc.). In particular, when students were presented with tools that were similar to those they would use in a real-life scenario (such as when they used Google Maps during the road trip assignment), I found that they were much less constrained by "school rules, norms, and expectations."

(3) Similarly, Ainley et al. (2006: 27) suggested that "the use of 'authentic' settings...will fail to resolve the planning paradox," but I found that lessons with correct and appropriate technology (like the budget simulation) were able to have both "clear mathematical focus" and "socially meaningful contexts."

(4) Finally, I found difficulties similar to those in the literature regarding assessment, and while I found the QLAR rubric of Boersma et al. (2011) to be generally helpful, I did not find it specific enough to highlight key elements in many of my QL assignments.

### ***Implications for Practitioners***

The study carries implications for practitioners, particularly for those who work with traditionally low-achieving students, students who are not on a calculus track, or seniors who might be lacking in motivation. I believe that some of the lessons I learned might have value because, like many teachers, I faced challenges as a result of tracking, scheduling difficulties, and other aspects of school culture that were out of my control. I realized that students wouldn't learn in these contexts if I didn't operate within their ZPD, not only in terms of the "actions the child can actually carry out," but also in terms of "what the child wants" (Van Oers 1996: 97). I found that this shift in focus helped me to engage a good number of students, and I did this through whole-class co-construction, as well as through co-construction of individual assignments. In addition, I found that my study has additional implications for teachers in a QL classroom.

**Whole-class co-construction.** When learning objectives focused on general skill development (such as QR skills), I found whole-class co-construction to be a powerful technique to promote participation, interest, and ownership. Co-constructing a course with an entire class of students might seem excessive to some practitioners, but at least in my context, it proved to be largely effective. I found that by using this approach, many students participated in varying degrees over the course of the year: I was able to trigger situational interest for most and promote individual interest in some: and many students felt empowered by the

opportunity to study material that was relevant to their lives and their futures. It might not have been the easiest technique, and I certainly made my fair share of mistakes along the way, but when I compare it to previous experiences with similar populations, I found this approach to promote higher levels of engagement for a greater number of students. My findings may be valuable to practitioners who want to attempt this approach, because they can learn about some of the effective practices (i.e., small-group discussions, written and verbal co-construction, careful attention to unit categories, leaving oneself enough time) that took me some time to discover (for more information, see Russo 2014). In addition, practitioners can consider how individualized co-construction can augment the whole-class co-construction process, and while this method might not be appropriate for every teacher or every classroom, it provides some insight into practices that at least for me, elicited greater participation and buy-in to the course.

**Co-construction of individual assignments.** Alternatively, if practitioners are not comfortable with whole-class co-construction but still want to increase engagement by giving students more control over the curriculum, then my findings regarding the use of well-crafted assignments that allow for self-pacing might be of value (Russo 2014). I created a handful of lessons that engaged a good number of students (notably the budget simulation and the road trip assignment), and my findings regarding these lessons point to the importance of incorporating the following techniques:

(1) These lessons met students at a developmentally appropriate place in their lives. As opposed to some of my failures with catchier ideas (codes, fantasy football), lessons like the budget simulation appealed to students because they related to issues that students cared about deeply, such as their lives after college.

(2) I found that self-paced assignments with clear instructions were most effective for my group of students, because they allowed me to circulate around the room and work with students in small groups; they mitigated tensions that could have resulted if I had to reprimand students to get their attention; and it reduced my anxiety, because I could focus on helping students who wanted to learn rather than constantly correcting students who didn't.

(3) These lessons incorporated appropriate technology, which added to the authenticity and relevance of the assignments for many students.

(4) Finally, each of these assignments allowed for a degree of differentiation or individualization, which promoted ownership and limited cheating. I found that each of these techniques improved the quality of these assignments, and I have been able to apply them not only in Discrete Math, but in my other classes as well.

I do not believe that this approach worked for every student, but I found it to be much more successful than whole-group approaches that I attempted in the past. These self-paced assignments gave the more-interested students a chance to engage, and they freed me up to support and encourage those who were on the fence. While this approach may not have engaged the most-difficult students every time, it did prevent them from holding the rest of the class back, since each student could continue working at his or her own pace. I found that it was nearly impossible to engage every student on any given day, and I hope this offers some encouragement to teachers who work with similar groups of students. As I mention in the dissertation, I found a disconnect between students' macro-feelings about mathematics class and their attitudes on a day-to-day basis, which helped me realize that even if I could improve students' attitudes towards mathematics, I would never be able to control how students felt on any given day. The recognition that every student would not be at their best every day was incredibly liberating to me, and it helped me to focus more on a learning trajectory over the course of the year, rather than on any daily checklist to measure participation, interest, or engagement.

**Teaching for QL.** The study also has implications for mathematics teachers who want to teach for QL. Teachers and students can develop overly rigid conceptions of what constitutes mathematics, and I found that teaching for QL enabled us to rethink the way that a mathematics classroom should look, particularly in terms of what we studied and how we studied it. My findings suggest that whether teachers choose contexts (as suggested by some of the literature) or teachers and students co-construct contexts (as shown in this study), QL classrooms should focus primarily on QR. As opposed to units that focus primarily on content, QR skills can be applied to many diverse situations, so they are more likely to appeal to a broader range of students. In order to help students develop these skills, I found that "authentic" assignments that utilized current and appropriate technology were the most effective. By giving students the tools they would use outside of school to complete the same task inside the classroom, it reinforced the relevance and utility of these skills while also increasing student interest and engagement. Additionally, I found that the co-construction process helped me to more effectively teach for QL, because it enabled me to differentiate the course based on students' interests and individual lives. I believe that the co-construction process enabled students to develop ownership over their work and cultivate skills that were uniquely appropriate for them and their lives. While whole-class co-construction was one way to achieve this end, I believe that teachers could also allow students to co-construct their own assignments, assign projects with various types of choices, or give students assignments that invite them to show their unique personalities and interests.

**Challenges of teaching for QL using co-construction.** It is also important to understand the challenges that may emerge from this process, particularly for teachers who want to attempt something similar in their own classrooms. As for the co-construction process, teachers should be prepared to invest a significant amount of time, because students (and teachers) will likely need time to adjust to their new roles in the classroom. Similarly, teachers will need to carefully manage interactions between students, so that all students, and not just the most-outgoing or persuasive ones, feel encouraged to participate. In terms of teaching for QL, my findings point to the challenges that teachers can face when managing multiple assignments in the classroom. Differentiation is an important part of a QL classroom, but it can also be extremely difficult, particularly for teachers who are more comfortable implementing a single lesson plan.

Additionally, there are unique challenges for teachers who want to combine teaching for QL and co-construction. For one, many students have never studied QL topics in a mathematics course, and they might be unfamiliar with any mathematics topics that are relevant to their lives. As a result, students may not be able to provide much input about what they want to study, particularly in the beginning of the year. In addition, school culture has the potential to impede students' willingness to participate in the co-construction of a QL course, and this needs to be considered before attempting this type of project. If possible, it may be helpful to consider this type of project in collaboration with other teachers or administrators, so that students receive a unified message about how they should participate in their courses. Hopefully, my study has provided some guidance on how to handle these challenges, so that teachers who attempt to teach for QL using co-construction can be prepared when faced with similar challenges in their own classrooms.

## Concluding Remarks

There are some important limitations that need to be considered in a case study such as this one. First and foremost, my study was not intended to be generalizable, so practitioners and researchers should consider their own contexts when evaluating the findings. Secondly, students and I learned how to co-construct as the year progressed, so the process, and students' participation in the process, underwent several important changes over the course of the year. Thirdly, the study attempted to develop students' QL solely in a mathematics classroom, so students did not have opportunities to apply their knowledge in different disciplines or work with content experts from different fields. There were also methodological limitations to the study, which include, but are not limited to: Institutional Review Board recommendations that students write their names on all questionnaires and surveys; varying degrees of participation from

students during the course of the year; and conflicts that arose as a result of my dual roles as teacher and researcher. There were several additional logistical and methodological limitations to this study, and these are documented more extensively in the dissertation.

Future research may be able not only to address some of these limitations, but also to expand on this study in several important ways. I co-constructed this course with my students, but it would be interesting to see whether our newly constructed curriculum would be effective with a future cohort of students, or if the course would need to be co-constructed each year to get similar results. My study utilized co-construction to develop a course from the ground up, but future research should consider how co-construction could play a role in a course that is limited by a set curriculum or content standards. This would be particularly timely, because most teachers are now required to align their courses with the Common Core State Standards for Mathematics. It would also be interesting to study the relationship between individualized co-construction and individual interest. My data showed that students who participated in individualized co-construction often demonstrated signs of individual interest, but future research should examine whether this is a causal relationship, a reverse causal relationship, or something else. Similarly, I focused primarily on the impact of individualized co-construction on students' individual interest, but less on the impact of students on one another. Future research could consider how students support or oppose one another in a co-construction setting, particularly in terms of each student's unique ZPD.

Further research could also enhance our understanding of teaching for QL. Although this was the first study that I am aware of to take an in-depth look at QL in high school, additional research could consider alternative approaches at the high school level. For example, researchers could consider the efficacy of a high school QL course that does not utilize co-construction. Alternatively, researchers could examine how QL topics could be infused into a course with a predetermined curriculum. A third option would be to examine assessment in a QL course at the high school level. In addition, it would be interesting to study the impact of a QL course on students' attitudes towards mathematics, as well as their achievement in future mathematics courses. Finally, it would be valuable to consider how QL can be promoted in an interdisciplinary setting, with teachers and experts from various disciplines contributing to students' educations.

## **Acknowledgments**

I want to thank my dissertation committee, and especially Mika Munakata, for their time, energy, and wisdom.

## References

- Ahn, R., and M. Class. 2011. Student-centered pedagogy: Co-construction of knowledge through student-generated midterm exams. *International Journal of Teaching and Learning in Higher Education* 23(2): 269–281.
- Ainley, J., D. Pratt, and A. Hansen. 2006. Connecting engagement and focus in pedagogic task design. *British Educational Research Journal* 32(1): 23–38. <http://dx.doi.org/10.1080/01411920500401971>
- Anderson, G. L., K. Herr, and A. S. Nihlen. 2007. *Studying your own school: An educator's guide to practitioner action research*. Thousand Oaks, CA: Corwin Press. <http://dx.doi.org/10.4135/9781483329574>
- Appleton, J. J., and F. Lawrenz. 2011. Student and teacher perspectives across mathematics and science classrooms: The importance of engaging contexts. *School Science and Mathematics* 111(4): 143–155. <http://dx.doi.org/10.1111/j.1949-8594.2011.00072.x>
- Boersma, S., C. Diefenderfer, S. W. Dingman, and B. L. Madison. 2011. Quantitative reasoning in the contemporary world, 3: Assessing student learning. *Numeracy* 4(2): Article 8. <http://dx.doi.org/10.5038/1936-4660.4.2.8>
- Boersma, S., and D. Klyve, D. (2013). Using a media-article approach to quantitative reasoning as an honors course: An exploratory study. *Numeracy* 6(1): Article 5. <http://dx.doi.org/10.5038/1936-4660.6.1.5>
- Brakke, D. F., and D. C. Carothers. 2004. Multiple approaches to improving quantitative reasoning skills at James Madison University. *Peer Review* 6(4): 19–21.
- Briggs, W. L., N. Sullivan, and M. M. Handelsman. 2004. Student engagement in a quantitative literacy course. *The AMATYC Review* 26(1), 18-28..
- Catalano, M. T. 2010. College algebra in context: A project incorporating social issues. *Numeracy* 3(1): Article 7. <http://dx.doi.org/10.5038/1936-4660.3.1.7>
- Cobb, G. W. (1997). Mere literacy is not enough. In *Why numbers count: Quantitative literacy for tomorrow's America*, ed. L. A. Steen, 75–90. New York, NY: The College Board.
- Diefenderfer, C. L., R. A. Doan, and C. Salowey. 2004. The quantitative reasoning program at Hollins University. *Peer Review* 6(4), 13–15.
- Dingman, S. W., and B. L. Madison. 2010. Quantitative reasoning in the contemporary world, 1: The course and its challenges. *Numeracy* 3(2): Article 4. <http://dx.doi.org/10.5038/1936-4660.3.2.4>
- Ellis, M., and C. Malloy. 2007. Preparing teachers for democratic mathematics education. In *Proceedings of the ninth international conference: Mathematics education in a global community*, ed. D. Pugalee, A. Rogerson, and A. Schinck, 160–164). Charlotte, NC.
- Ellis Jr., W. 2001. Numerical common sense for all. In *Mathematics and democracy: The case for quantitative literacy*, ed. L. A. Steen, 61–65). Washington, DC: National Council on Education and the Disciplines.

- Fredricks, J. A., P. C. Blumenfeld, and A. H. Paris. 2004. School engagement: Potential of the concept, state of the evidence. *Review of Educational Research* 74(1): 59–109. <http://dx.doi.org/10.3102/00346543074001059>
- Gillman, R. 2010. Reorganizing school mathematics for quantitative literacy. *Numeracy* 3(2): Article 7. <http://dx.doi.org/10.5038/1936-4660.3.2.7>
- Glaser, B. G., and A. L. Strauss. 1967. *The discovery of grounded theory: Strategies for qualitative research*. New Brunswick, NJ: Aldine Transaction.
- Herr, K., and G. L. Anderson. 2005. *The action research dissertation: A guide for students and faculty*. SAGE Publications, Inc.
- Hidi, S., and K. A. Renninger. 2006. The four-phase model of interest development. *Educational Psychologist* 41(2), 111–127. [http://dx.doi.org/10.1207/s15326985ep4102\\_4](http://dx.doi.org/10.1207/s15326985ep4102_4)
- Hoachlander, G. 1997. Organizing mathematics education around work. In *Why numbers count: Quantitative literacy for tomorrow's America*, ed. L. A. Steen, 122–136). New York, NY: The College Board.
- Hughes-Hallett, D. 2001. Achieving numeracy: The challenge of implementation. In *Mathematics and democracy: The case for quantitative literacy*, ed. L. A. Steen, 93–98). Washington, DC: National Council on Education and the Disciplines.
- ILSS. See International Life Skills Survey.
- International Life Skills Survey. 2000. Policy Research Initiative. Statistics Canada.
- Jordan, J., and B. Haines, B. 2003. Fostering quantitative literacy: Clarifying goals, assessing student progress. *Peer Review* 5(4): 16–19.
- Jurdak, M. E. 2006. Contrasting perspectives and performance of high school students on problem solving in real world situated, and school contexts. *Educational Studies in Mathematics* 63(3): 283–301.
- Lesh, R., J. A. Middleton, E. Caylor, and S. Gupta. 2008. A science need: Designing tasks to engage students in modeling complex data. *Educational Studies in Mathematics* 68(2): 113–130. <http://dx.doi.org/10.1007/s10649-008-9118-4>
- Lusardi, A., and D. Wallace. 2013. Financial literacy and quantitative reasoning in the high school and college classroom. *Numeracy* 6(2): Article 1. <http://dx.doi.org/10.5038/1936-4660.6.2.1>
- Ma, X. 1997. Reciprocal relationships between attitude toward mathematics and achievement in mathematics. *Journal of Educational Research* 90: 221–229. <http://dx.doi.org/10.1080/00220671.1997.10544576>
- Madison, B. L. 2006. Pedagogical challenges of quantitative literacy. *Proceedings of the ASA Section on Statistical Education*: 2323–2328.
- , and S. W. Dingman. 2010. Quantitative reasoning in the contemporary world, 2: Focus questions for the numeracy community. *Numeracy*, 3(2): Article 5. <http://dx.doi.org/10.5038/1936-4660.3.2.5>
- Madison, B. L., and L. A. Steen. 2007. Evolution of numeracy and the National Numeracy Network. *Numeracy* 1(1): Article 2. <http://dx.doi.org/10.5038/1936-4660.1.1.2>
- . 2008. *Calculation vs. context: Quantitative literacy and its implications for teacher education*. Washington, DC: Mathematical Association of America.

- . 2009. Confronting challenges, overcoming obstacles: A conversation about quantitative literacy. *Numeracy* 2(1): Article 2. <http://dx.doi.org/10.5038/1936-4660.2.1.2>
- Manaster A. B. 2001. Mathematics and numeracy: Mutual reinforcement. In *Mathematics and democracy: The case for quantitative literacy*, ed., L. A. Steen, 67–72. Washington, DC: National Council on Education and the Disciplines.
- Maskiewicz, A. C., and V. A. Winters. 2012. Understanding the co-construction of inquiry practices: A case study of a responsive teaching environment. *Journal of Research in Science Teaching* 49(4), 429–464. <http://dx.doi.org/10.1002/tea.21007>
- Merriam, S. B. 2009. *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.
- Orrill, R. (2001). Preface: Mathematics, numeracy, and democracy. In *Mathematics and democracy: The case for quantitative literacy*, ed. L. A. Steen, xiii–xx. Washington, DC: National Council on Education and the Disciplines.
- Porter, T. M. 1997. The triumph of numbers: Civic implications of quantitative literacy. In *Why numbers count: Quantitative literacy for tomorrow's America*, ed. L. A. Steen, 1–10. New York, NY: The College Board.
- Richardson, R. M., and W. G. McCallum. 2003. The third R in literacy. In L. A. Steen (Ed.), *Quantitative literacy: Why numeracy matters for schools and colleges*, ed. L. A. Steen, 99–106. Washington, DC: National Council on Education and the Disciplines.
- Russo, M. F. (2014). *Quantitative literacy and high school mathematics: The evolution of a collaboratively constructed course and its impact on students' attitudes and numeracy*. Ed.D. dissertation, Montclair State University, 293 pp.
- Steele, B., and S. Kilic-Bahi, S. 2010. Quantitative literacy: Does it work? Evaluation of student outcomes at Colby-Sawyer College. *Numeracy* 3(2): Article 3. <http://dx.doi.org/10.5038/1936-4660.3.2.3>
- Steen, L. A. 2001a. Embracing numeracy. In *Mathematics and democracy: The case for quantitative literacy*, ed. L. A. Steen, 107–116. Washington, DC: National Council on Education and the Disciplines.
- . 2001b. Mathematics and numeracy: Two literacies, one language. *The Mathematics Educator* 6(1), 10–16.
- . 2012. Reflections on mathematics and democracy. Presented at a joint AMS-MAA special session at MathFest 2012 in Madison, Wisconsin.
- Van Oers, B. 1996. Learning mathematics as a meaningful activity. In *Theories of mathematical learning*, ed. L.P. Steffe, P. Nesher, P. Cobb, G.A. Goldin and B. Greer, 91–113. Mahwah, NJ: Erlbaum.
- Van Peurse, D., C. Keller, D. Pietrzak, C. Wagner, and C. Bennett. 2012. A comparison of performance and attitudes between students enrolled in college algebra vs. quantitative literacy. *Mathematics and Computer Education*, 46(2), 107–118.
- Vygotsky, L. S. 1978. *Mind in society: The development of higher psychological processes*. Cambridge MA: Harvard University Press.