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A Linear Analysis of Equatorial Atlantic Ocean Thermocline Variability

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ABSTRACT

Observed variations in the Atlantic Ocean's equatorial thermocline are compared at four locations with simulations using an analytical reduced-gravity model. The comparison shows the essential features of the seasonal wind-forced thermocline response to be accounted for by a linear superposition of equatorial long waves, evolving basinwide, tending to bring the zonal pressure gradient into balance with the wind stress. A frequency response function is derived whose properties provide a basis for discussing the large scale features of the equatorial Atlantic Ocean's seasonal cycle—for example, its evolution along the equator, the maximum upwelling region observed in the Gulf of Guinea and the secondary upwelling season also observed there. Clarification is also given to the issue of remote versus local forcing for these features.

1. Introduction

Tropical air-sea interactions are recognized for their role in global climate (e.g., Bjerknes 1969). The tropical ocean (atmosphere) responds primarily to sea surface wind stress (temperature) perturbations and the coupled system is thought to control the El Niño/Southern Oscillation and related phenomena. Fundamental to tropical ocean and atmosphere response theories is the equatorial waveguide, since it communicates information across entire ocean basins on time scales comparable to or shorter than the phenomena in question. A recent review of oceanographic equatorial waveguide studies is provided by Eriksen and Katz (1987).

For the Atlantic Ocean the Seasonal Response of the Equatorial Atlantic (SEQUAL) Experiment and the Français Ocean et Climat dans l'Atlantique Equatorial (FOCAL) Program have provided time series showing how the thermocline evolves through its annual cycle at four locations along the equator [Katz et al. (1989) provide a SEQUAL/FOCAL Program overview]. Under the premise that these observations derived from a basinwide forced response, Weisberg and Tang (1985, 1987, hereafter referred to as WT85 and WT87, respectively) sought to determine if the thermocline evolution could be accounted for by linear equatorial wave modes tending to bring the basinwide

pressure gradient into balance with the wind stress. For this purpose they chose the simplest applicable model, using a reduced-gravity specification in the vertical and the sum of directly forced and reflected equatorial long waves in the horizontal. The model was adapted from Cane and Sarachik (1976, 1977) and modified to include damping and to allow for long-term integrations of easterly wind stress variations similar to what was observed during the SEQUAL/FOCAL Programs. The present paper extends this line of investigation with three additional objectives. First, to complete the comparison between the thermocline variations observed over the entire 2.7 years of recorded data with the thermocline variations simulated by the linear, single vertical mode, equatorial long-wave model. Second, to generalize the model formulation by deriving a frequency response function applicable to arbitrary distributions of easterly wind stress along the equator using a linear systems analysis. Third, to discuss the properties of the frequency response function as pertaining to the basinwide features of the equatorial Atlantic Ocean's seasonal cycle; specifically, to account for the character of the response observed along the equator with maximum annual and semiannual upwelling occurring in the Gulf of Guinea and to resolve the issue of remote versus local forcing for these features. The original model simulation and its comparison with the data are given in section 2. Section 3 then develops the linear system's analysis and discusses the frequency response function's properties and section 4 summarizes the results.

2. Equatorial wave simulation

Within the hierarchy of models applied to the equatorial Atlantic Ocean's seasonal response (e.g., Busa-

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Iacchi and Picaut 1983; Du Penhoat and Gouriou 1987; Philander and Pacanowski 1986; for examples of linear single vertical mode, linear multiple vertical modes and primitive equation models, respectively) a linear, single vertical mode, reduced-gravity, equatorial long-wave model is the simplest one that includes the direct effects of basinwide wind stress forcing and the subsequent boundary reflections. The equations of motion are

$$\partial u / \partial t - yv + \partial h / \partial x = -\epsilon u + \tau \quad (1)$$

$$\partial v / \partial t + yu + \partial h / \partial y = -\epsilon v \quad (2)$$

$$\partial h / \partial t + \partial u / \partial x + \partial v / \partial y = -\epsilon h \quad (3)$$

where u , v , and h are the velocity components in the x and y directions and the upper layer thickness perturbation, respectively; t is time; τ is the zonal component of wind stress and ϵ is a damping parameter. These equations have been nondimensionalized using the equatorial Rossby radius of deformation and time scale, $L = (c/\beta)^{1/2}$ and $T = (c\beta)^{-1/2}$, where $c = (g'H)^{1/2}$ is the reduced-gravity baroclinic wave speed corresponding to the reduced-gravity g' and the undisturbed upper layer thickness H and β is the gradient in planetary vorticity. These equations are solved analytically using a long-wave approximation with parameter values, $c = 160 \text{ cm s}^{-1}$ and $\epsilon = 0.01$ (an e -folding time of 192 days), as described in WT87.

The model domain relative to the equatorial Atlantic Ocean, the moored instrument locations and the zonal distribution of the easterly wind stress used to force the model are shown in Fig. 1. Dimensionally, the model basin extends from 46°W to 8.6°E for a total width of 6000 km. The easterly wind stress is distributed linearly along the equator with maximum amplitude at the western boundary and a zero crossing at 5°W . Nondimensionally, this forcing function is given by

$$\tau(x, y, t) = \gamma \exp(-y^2/2)X(x)F(t) \quad (4)$$

with

$$F(t) = H(t)t/T_1 + \sum_{i=1}^I H(t-T_i)\{S_i(t-T_i)/T_i\} \quad (5)$$

and

$$X(x) = -x\{H(x-L_2) - H(x+L_1)\} \quad (6)$$

where the H are step functions and the parameters of the wind stress distribution are γ and S_i , specifying the magnitude and temporal variations; T_i , specifying the duration of these variations and L_1 and L_2 , specifying the fetches to the west and east of the origin, respectively. With γ equal to 0.0068 the maximum dimensional amplitude of the easterly wind stress at the western boundary is 0.67 dyn cm^{-2} .

The simplifications in x , y and t and their effects upon the model solutions have been discussed in Weisberg and Tang (1983), WT85, and WT87. The zonal distribution, being linear with a nodal point at 5°W , follows from both the climatology and the synoptic observations, including the limited Seasat mission; the meridional Gaussian distribution greatly simplifies the computation without significantly affecting the thermocline response on the equator and the sequence of linear changes in time are a convenient way of representing the observed wind stress variations. With these simplifications the equations of motion can be solved analytically in terms of explicit equatorial wave modes which may be traced as they evolve and reflect over the model domain.

Daily averaged easterly wind stress values obtained from St. Peter and St. Paul Rocks (SPPR) at 1°N , 29°W (Garzoli and Katz, personal communication),

FIG. 1. The model domain and the spatial wind stress distributions used to force the model in relationship to the equatorial Atlantic Ocean and the in situ measurement locations. Squares denote mooring locations and the cross denotes the wind station at SPPR. X_W and X_E are at 46°W and 8.6°E ; the approximate locations, on the equator, of the 200 m isobaths at the western and eastern boundaries, respectively.

the model representation of this time series and the model simulated responses for h and u on the equator at 28°W , 24°W , 15°W and 4°W are shown in Fig. 2. This extends the previous results of WT87 over the full 2.7 years of recorded data at 28°W . Most of the discussion will focus upon h because of the model limitations for u due to nonlinearities and dissipation (e.g., Cane 1980; Philander and Pacanowski 1986), but u is included for completeness and for noting several relevant points later on. The response evolves as an interference pattern between the directly forced Kelvin and first meridional mode Rossby waves that are generated with each change in the wind stress, plus the additional Kelvin and Rossby waves that are generated by sequential reflections at the western and eastern boundaries, respectively. Owing to the short adjustment time of the basin, the relative amplitudes of the forced and reflected waves and the prescribed dissipation, the overall character of the response in any given year depends primarily upon the wind stress variations occurring during that year. Consequently, distinct response patterns emerge with each large scale change in wind stress and the evolution of these patterns differs along the equator due to wave interference. Nowhere within the forced region (which for the equatorial Atlantic includes the entire basin) can the response be considered as either local or remote. It develops as the

integral of the wind stress over the entire basin and its character at each specific point is determined by the relative proportions of the directly forced and reflected waves there.

As discussed in WT85 and WT87 a sequence of upwelling followed by downwelling occurs in response to increasing easterlies and conversely for decreasing easterlies. These sequences have the appearance of propagating eastward but they continually change their form. If a given wind state has long enough duration then an initially generated pattern repeats with lesser amplitude by reflection. This occurs in the 1983 simulation as a secondary period of upwelling at 4°W but not in the 1984 simulation since the duration of the strong easterly wind stress season in that year was shorter. Hence, intra-annual variability can be related to the annual cycle via reflections and interannual variability can occur as a modulation of the annual cycle.

In each year there are two periods in which upwelling and downwelling are pronounced. These follow the major changes in wind stress, from the low wind season to the high wind season in boreal spring and conversely in boreal fall. For example, in the Gulf of Guinea (4°W) upwelling begins in wintertime as the latter portion of the response to the easterly wind stress relaxation and then continues in springtime as the initial

FIG. 2. Time series of daily averaged wind stress observed at 1°N , 29°W (courtesy of Garzoli and Katz); the model representation of this time series and the model simulated responses to this representation for h and u on the equator at 28°W , 24°W , 15°W and 4°W .

FIG. 3. The simulated thermocline interface along the equator as a function of longitude and time. The vantage point is at the western boundary looking eastward.

response to the easterly wind stress intensification. Similarly, downwelling begins in summertime as the latter portion of the response to easterly wind stress intensification and then continues in wintertime as the initial response to easterly wind stress relaxation. The anomalously large downwelling that was observed in the Gulf of Guinea during the winter of 1984 (e.g., Hisard et al. 1986; Philander 1986; Weisberg and Colin 1986) may thus be accounted for as a modulation of the annual cycle. This downwelling occurs every year but it was more pronounced in 1984 due to a more pronounced, rapidly decreasing and lengthy low wind season during that year.

The basinwide evolution of the simulated thermocline interface is shown in perspective view in Fig. 3. The vantage point is from the western boundary looking eastward along the equator with time increasing to the right. A longitude of minimum variability lies at approximately 24°W consistent with the pivot point suggested by Merle's (1980) climatological data analysis. This minimum is bracketed by maxima at the western boundary and at 5°W . The 5°W maximum

FIG. 4. A comparison between the vertically integrated temperature observed on the equator at 28°W , 24°W , 15°W and 4°W (4°W data courtesy of C. Colin) and the model simulated thermocline. The time series have been normalized by their standard deviations and superimposed using a linear least squares fit with respect to the ordinate. The units on the ordinate are standard deviations.

coincides with the region of maximum upwelling observed in the Gulf of Guinea (Houghton 1983; Weisberg and Tang 1983). The shoaling of the interface during the primary upwelling season beginning in boreal spring evolves all along the equator with amplitude and phase increasing to the east. A secondary upwelling season is also observed during the 1983 boreal fall. Then during boreal winter, after the wind stress relaxes, the interface flattens and even reverses slope. As shown in WT87 the pressure gradient variations implied by these interface slope changes mirror those computed from the moored temperature data.

The simulated thermocline variability can be compared directly with the observations at four longitudes: 28°W, 24°W, 15°W and 4°W. Figure 4 shows the vertically integrated temperature from the moorings at each of these locations and the model simulated thermocline displacement. The magnitudes of these two quantities have been normalized by their standard deviations and a linear least squares fit has been made with respect to the ordinate. Visually, most of the large scale features of the observations are accounted for by the model. As a quantitative comparison, Fig. 5 shows the coherence squared and phase between the model simulation and the data at 28° and 4°W. The autocovariance functions for both the model and the data have zero crossings occurring at lags of around 50 days. An appropriate bandwidth for the coherence analysis is thus 0.01 day^{-1} , resulting in approximately 20 (11) degrees of freedom at 28°W (4°W) and the 90% significance levels on the null hypothesis as shown. At both locations high and in-phase coherence is observed between the data and the model simulation over the lowest independent frequency band (time scales longer than 2 months). Since most of the variance occurs within this band the analysis shows that 70 to 80 percent of the observed thermocline variability on basin-wide seasonal time scales can be accounted for by the simplest applicable linear model.

3. Linear systems analysis

Given the fidelity between the model simulation and the data, a linear systems analysis is used to provide a framework for discussing the model results relative to the data and for generalizing the model. For a linear system the upper-layer thickness perturbation may be expressed as the convolution between an impulse response function and an external forcing function:

$$h(x, y, t) = \int_{-\infty}^{\infty} g(x, y, t - \xi) F(\xi) d\xi, \quad (7)$$

where the impulse response function, $g(x, y, t - \xi)$, is the response at time t to a unit impulse delivered at time ξ and $F(\xi)$ represents the time dependence of the external forcing. According to the convolution theorem this integral may be evaluated in the frequency domain as

FIG. 5. The coherence squared and phase between the model simulated thermocline and the vertically integrated temperature observed on the equator at 28°W and 4°W. The spectra were averaged over a bandwidth of 0.407×10^{-4} cph for approximately 20 (11) degrees of freedom at 28°W (4°W) and the associated 90% significance levels for the null hypothesis are given by the horizontal lines.

$$\tilde{h}(x, y, \sigma) = \tilde{g}(x, y, \sigma) \tilde{F}(\sigma), \quad (8)$$

where the tilde denotes the Fourier transform, from which

$$h(x, y, t) = \int_{-\infty}^{\infty} \tilde{h}(x, y, \sigma) \exp(i\sigma t) d\sigma. \quad (9)$$

Note that the spatial dependence of the external forcing has been incorporated in the impulse response function. A Gaussian y dependence is kept for simplicity as previously discussed and two specific impulse response functions are developed with regard to x . The first maintains the linear distribution of wind stress treated in section 3 and the second employs a zonally uniform distribution. This second distribution is quite general since any piecewise continuous distribution can be approximated as the sum of zonally uniform seg-

ments thereby accommodating complicated zonal wind stress distributions by the principle of linear superposition.

The properties of the linear system's response are defined by its frequency response function, $\tilde{g}(x, y, \sigma)$, consisting of contributions from both directly forced and subsequently reflected wave modes. Figure 6 shows the amplitude and phase of the portion of the frequency response function due to the directly forced Kelvin waves only. The left-hand plates result from zonal winds that are uniformly distributed across the entire

basin and the right-hand plates result from zonal winds that are linearly distributed as in Fig. 1. Here (and in subsequent frequency response function plots) both dimensional and nondimensional ordinates and abscissas are given with amplitude and phase contour intervals of $3.0 \text{ m per dyn cm}^{-2}$ and 0.4 rad .

Consider first the uniform wind stress case. The response is an integral along characteristics and since Kelvin waves propagate eastward the impulse response will appear as a boxcar function with eastward increasing duration. Its Fourier transform is a sinc function

FIG. 6. The amplitude and phase of the frequency response function for h due to the directly forced Kelvin wave part of the solution. The left-hand plates are for easterly wind stress distributed uniformly across the basin and the right hand plates are for easterly wind stress distributed linearly with maximum value on the western boundary and a nodal point at 5°W . Contour intervals are $3.0 \text{ m per dyn cm}^{-2}$ and 0.4 rad .

with amplitude increasing linearly to the east at zero frequency and falling off at higher frequency. Phase variations increase with frequency according to the dispersion relationship. At any given frequency the phase appears to increase uniformly to the east with only a slight departure from this owing to the small damping parameter.

The frequency response function for the Kelvin waves generated by the linearly distributed zonal wind stress has significantly different properties. Amplitude contours no longer increase uniformly to the east. Their

gradient is largest on the western side of the basin and a region of maximum response appears just to the west of the nodal point in the Gulf of Guinea. The variations in phase with longitude are also nonuniform. WT85 noted that this results by pulse shaping within the forced region. The present plot shows that this effect upon the longitudinal gradient in phase (and hence upon the apparent propagation speed) is pronounced, even for the case of a single wave mode. Propagation speeds estimated from time lags between pulses observed at different locations within a forced region should there-

FIG. 7. The amplitude and phase of the frequency response functions for h (left-hand plates) and u (right-hand plates) due to sum of the directly forced Kelvin wave and the directly forced Rossby wave parts of the response to easterly wind stress distributed linearly with maximum value on the western boundary and a nodal point at 5°W . Contour intervals are $3.0 \text{ m per dyn cm}^{-2}$ and 0.4 radians, respectively for h and $6.0 \text{ cm s}^{-1} \text{ per dyn cm}^{-2}$ and 0.4 rad for u .

fore differ from the theoretical phase speed of any wave mode.

Addition of the directly forced Rossby wave response results in the frequency response function for the directly forced part of the solution. The amplitude and phase for h and u are shown in Fig. 7 for the case of linearly distributed wind stress (the contour interval for the amplitude of u is 6.0 cm s^{-1} per dyn cm^{-2}). The amplitude of h now contains a minimum near the western side of the basin and a maximum in the Gulf of Guinea. The phase distribution is extremely complicated even though this response function consists of

only two wave modes. The phase gradient for h is eastward with rapid variation on the western side of the basin and more gradual variation in the east. The u -component response shows increasing amplitude to the west where the wind stress is the largest and its phase gradient is generally small and directed eastward (westward) on the eastern (western) side of the basin.

Further addition of the reflected wave modes results in the total frequency response function. The amplitudes and phases for h and u are shown in Fig. 8 for the case of linearly distributed wind stress. The reflected waves add more structure in both zonal position and

FIG. 8. The amplitude and phase of the frequency response functions for h (left-hand plates) and u (right-hand plates) in response to easterly wind stress distributed linearly with maximum value on the western boundary and a nodal point at 5°W . Contour intervals are $3.0 \text{ m per dyn cm}^{-2}$ and 0.4 radians, respectively for h and 6.0 cm s^{-1} per dyn cm^{-2} and 0.4 rad for u .

frequency. At low frequency the amplitude of h shows maximum values on the western boundary, a minimum to the west of center (at approximately 24°W) and a local maximum in the Gulf of Guinea. At semi-annual periodicity maxima appear near both the eastern and western boundaries. These maxima are part of a sequence of zonally distributed maxima and minima that repeat in frequency at intervals of approximately $c/4L_b$ where L_b is the basin length. This frequency interval corresponds to the resonant behavior discussed by Cane and Sarachik (1981) in relation to the eastern boundary. Resonant peaks also appear within the basin since the zonal wavenumber structure for this behavior increases with frequency; for example, at approximately 3-month periodicity a relative maximum occurs at the center of the basin as well as on the eastern and western boundaries.

The distribution of phase shows that at low frequency the western and eastern sides of the basin vary out of phase separated by a large eastward directed phase gradient region to the west of the basin's center. In contrast to this most of the eastern half of the basin appears to vary in phase.

The amplitude of the u -component frequency response function at low frequency shows a maximum on the western side of the basin and a minimum to the east of center. At higher frequency an interference pattern develops with maximum values shifting to the east and with relative maxima repeating at the same frequency interval as noted for h ; for example, at semi-annual period a maximum appears to be located near

the center of the basin. These patterns are very similar to those described by Schopf et al. (1981) by tracing Rossby wave rays emanating from an eastern boundary. In the present approach the infinite sum of Rossby waves required at the eastern boundary to match the incoming Kelvin waves separate as they propagate westward resulting in the pattern. The phase distribution for the u -component is also much different than that for h . At low frequency the phase gradient is more uniformly distributed across the basin and it is directed westward as opposed to eastward.

Multiplication of the frequency response function with the Fourier transform of the forcing function's time dependence yields the Fourier transform of the total response. Figure 9 shows the response amplitudes of h and u as a function of frequency and zonal position. For h (with the exception of the zero frequency peak due to the inclusion of a mean value) the largest response amplitude is centered upon the annual cycle. Within this annual band the response is largest on the western boundary, smallest near the center of the basin at approximately 24°W and locally maximum again in the Gulf of Guinea. The separation from the eastern boundary of this local maximum follows from the nodal point of the linearly distributed wind stress (located at 5°W in the model) as shown by the systematic discussion of the solution components developed in Figs. 6–9. The pattern of the annual cycle response is duplicated with lesser amplitude near semiannual periodicity. This results from the reflections of the waves that give rise to the annual cycle (recall that h has its

FIG. 9. The Fourier transform amplitude for the response of h (top) and u (bottom) to easterly wind stress distributed linearly with maximum value on the western boundary and a nodal point at 5°W , with time variations as given in Fig. 2. Contour intervals are $1.0 \text{ m per cycle per day}$ for h and $1.0 \text{ cm s}^{-1} \text{ per cycle per day}$ for u .

largest frequency response near semiannual periodicity).

For the u -component (with the exception of the mean value induced peak at zero frequency) the annual cycle again has the largest amplitude. However, unlike h , the u -component response decreases monotonically from maximum values at the western boundary to zero at the eastern boundary. At higher frequencies a series of local maxima evolve according to the interference pattern of the frequency response function. The first and largest of these occurs near the center of the basin around semiannual periodicity. This result was also noted in the numerical model simulation of the climatological seasonal cycle by Philander and Pacanowski (1986). In general, the spectrum of the u -component is more structured near the eastern boundary than it is near the western boundary due to the superposition of high mode Rossby waves at the former.

The time evolution of the response at any zonal position may now be obtained by taking the inverse Fourier transform of the frequency domain response using Eq. (9). As a check on the frequency response function formulation the time evolutions, thus obtained, have been compared with those in section 2 and they were found to be identical. Thus, the frequency response function developed here provides a general way of calculating the linear response of an equatorial ocean basin to zonal wind stress forcing. The y dependence remains Gaussian for simplicity but for x and t any arbitrary functions satisfying the Dirichlet conditions may be analyzed.

As a final set of analyses the coherence squared and phase between the model response and the wind stress used to force the model were calculated as

$$\gamma^2(x, 0, f) = \{ \langle \tilde{F}^*(\sigma) \tilde{h}(x, 0, \sigma) \rangle \}^2 / \langle \tilde{F}^*(\sigma) \tilde{F}(\sigma) \rangle \langle \tilde{h}^*(x, 0, \sigma) \tilde{h}(x, 0, \sigma) \rangle \quad (10)$$

$$\phi(x, 0, f) = \tan^{-1} [\text{Im} \{ \langle \tilde{F}^*(\sigma) \tilde{h}(x, 0, \sigma) \rangle \} / \text{Re} \{ \langle \tilde{F}^*(\sigma) \tilde{h}(x, 0, \sigma) \rangle \}] \quad (11)$$

where the angle brackets denote a frequency band average. These quantities are shown for h and u in Fig. 10 where averaging has been performed over a bandwidth of $4.7 \times 10^{-3} \text{ day}^{-1}$ for approximately 10 degrees of freedom. Two regions of high coherence, the western side of the basin and the Gulf of Guinea, separated by a region of low coherence centered about 24°W , are observed for h . The phase within the high coherence regions is constant while it varies rapidly across the low coherence region with the western side of the basin varying out-of-phase with the Gulf of Guinea. The distribution of coherence explains why the Gulf of Guinea has the appearance of responding remotely to forcing on the western side of the basin when in fact its response is to the integral of the forcing all along the equator. This point is further borne out by the drop off in coherence at the eastern boundary which is again a basinwide effect. Within the region of rapid phase change

the gradient is directed eastward implying a Kelvin wave predominance; however, this results from the superposition of all of the waves in the model.

The coherence squared and phase was also calculated between the response at 5°W and the responses at any other position along the equator by replacing $\tilde{F}(\sigma)$ with $\tilde{h}(5^\circ\text{W}, 0, \sigma)$ in Eqs. (10) and (11). The results, given in Fig. 11, are very similar to those of Fig. 10 showing high and out-of-phase coherence between the Gulf of Guinea and the western side of the basin and relatively low coherence between the Gulf of Guinea and the central portion of the basin. This accounts for the different appearance between observations in the central portion of the basin and those in the Gulf and Guinea and thus the perception of remote forcing despite the causal influence of forcing all along the equator.

The u -component is generally less coherent with the forcing function than h . At low frequency the western and eastern sides of the basin are the regions of highest coherence; however, the extent of these regions is smaller than for h . The central portion of the basin extending into the Gulf of Guinea shows lower coherence. This relative behavior between u and h results from a broader effective zonal wavenumber bandwidth for u since the ratio of reflected Rossby wave amplitudes between u and h on the equator is $2n + 1$ (nondimensionally), where n is the meridional mode number. Thus, the variations in h on the equator can be described by fewer Rossby waves than the variations in u . Also in contrast to h , the phase variations for u occur more uniformly across the basin and with a westward directed gradient. While Rossby waves appear to dominate the u -component phase changes this pattern again arises by the superposition of all of the waves present and can not be attributed to any single wave mode.

4. Summary and conclusions

This paper has continued the work of WT85 and WT87 describing the seasonal evolution of the equatorial thermocline in the Atlantic Ocean. The simplest applicable model, consisting of a reduced-gravity specification in the vertical and the sum of directly forced and reflected equatorial long-waves in the horizontal, has been used with three new objectives. First, to complete a quantitative comparison between the modelled and observed thermocline variations over the entire 2.7 years of data recorded during the SEQUAL/FOCAL Programs. Second, to generalize the analytical model formulation to accommodate arbitrary distributions of easterly wind stress both in x and t using a linear systems analysis. Third, to discuss the properties of the resulting frequency response function as pertaining to the basinwide features of the equatorial Atlantic Ocean's response.

Quantitative comparisons were made between the model results and the thermocline variations observed on the equator at 28°W and 4°W by computing the coherence squared and phase over the record lengths

of 2.7 years and 1.5 years, respectively. At time scales longer than two months some 70 to 80 percent of the observed variance is accounted for by the model showing that the essential aspects of the thermocline's seasonal response can be attributed explicitly to linear equatorial waves.

The wave model was generalized by calculating the Fourier transform of the response to impulsive forcing. The resulting frequency response function formulation accommodates any distribution of easterly wind stress in x and t that satisfies the Dirichlet conditions. It does retain a Gaussian meridional distribution for simplicity but this does not substantially affect the equatorial response for h . As a check, the results from the new for-

mulation were found to be the same as those obtained previously.

The properties of the frequency response function provide a basis for discussing the seasonal variations in the equatorial Atlantic Ocean's thermocline. For this purpose easterly wind stress distributed linearly along the equator with a nodal point at 5°W was chosen as being representative of the basinwide distribution. At low frequency the thermocline shows maximum variability on the western boundary, minimum variability to the west of the basin's center and a region of local maximum variability in the Gulf of Guinea. At higher frequencies the basin approaches resonance near semiannual and quarter-annual periodicities with in-

FIG. 10. The coherence squared and phase between the model response and the wind stress used to force the model (linear distribution along the equator with a maximum on the western boundary and a node at 5°W , with time dependence as given in Fig. 2) for h (left-hand plates) and u (right-hand plates). The contour intervals are 0.1 for coherence squared and 0.4 radians for phase. The spectra were averaged over a bandwidth of $4.7 \times 10^{-3} \text{ day}^{-1}$ for 10 degrees of freedom.

creasing zonal wavenumber structure. The local maximum in the Gulf of Guinea corresponds to the region where annual upwelling is observed to be a maximum and separate from the African coast. The position of this maximum and its separation from the African coast in the frequency response function both follow from the nodal point of the easterly wind stress distribution. For the zonal pressure gradient to balance the easterly wind stress component as frequency approaches zero the pressure distribution (thermocline displacement) must have an extremum where the stress is zero. The near resonant semiannual peak corresponds to the secondary upwelling season also observed in the Gulf of Guinea. This arises in the frequency response function by boundary reflections (a repetition

of the primary upwelling season). Consequently the semiannual response in the Gulf of Guinea does not require a distinctive semiannual peak in the forcing function. The results also suggest that a semiannual response should occur near the western boundary.

The phase gradient for the thermocline response at low frequency is directed eastward with maximum values west of the basin's center, separating the western and eastern sides which tend to vary out of phase. The SEQUAL/FOCAL moored measurements spanned this phase gradient region thereby showing the basin-wide evolution of the response. Had these observations been limited to just the western and eastern sides of the basin they would simply have shown out-of-phase behavior with an implied nodal point as described by

COHERENCY BETWEEN 5°W & OTHER LOCATIONS

FIG. 11. The coherence squared and phase between the model response at 5°W and the model response at any other position along the equator for h (left-hand plates) and u (right-hand plates). The contour intervals are 0.1 for coherence squared and 0.4 radians for phase. The spectra were averaged over a bandwidth of $4.7 \times 10^{-3} \text{ day}^{-1}$ for 10 degrees of freedom.

climatology. From the distribution of phase it is clear that individual waves with uniform phase speed can not be traced across the basin. This point is strengthened by consideration of the forced Kelvin wave element of the response alone. Due to the pulse shaping by the zonal variations in wind stress the phase gradient, even for a single evolving wave packet, is nonuniform.

The frequency response function and the coherence analyses clarify the issue of remote versus local forcing in the Gulf of Guinea. For the wind stress distribution used here, the entire basin is forced albeit with largest values at the western boundary and minimum (zero at 5°W) values in the Gulf of Guinea. Obviously the Gulf of Guinea is responding to remote forcing; however, it is the localized distribution of wind stress within the Gulf of Guinea that determines the location of the maximum upwelling region there, and interannual variations of this distribution can affect the location of maximum upwelling as occurred between 1983 and 1984. While the model coherence structure suggests remote forcing along a similar vein as Servain et al. (1982), the coherence structure itself derives from wave interference. It is the wave interference that distinguishes the central portion of the basin from the Gulf of Guinea despite the fact that upwelling in the latter causally evolves through the former. For these reasons the response must be considered as the integrated effect of the forcing over the entire zonal extent of the basin.

The u -component was also examined for completeness although nonlinearities and dissipation would render this variable less appropriate for linear analysis near the equator than for h . At low frequency, the u -component response is largest on the western side of the basin, consistent with climatology. At higher frequencies near resonant interference patterns appear as with h and the peak predicted for the central portion of the basin at semiannual periodicity agrees with the ship drift observations of Richardson and Walsh (1986). At all frequencies, and distinctly different from h , the u -component phase gradient is generally westward due to the predominance of Rossby waves in the u -component amplitude on the equator.

In summary, the essential features of the seasonal response of the equatorial Atlantic Ocean thermocline can be accounted for by a linear superposition of equatorial long waves, of single reduced-gravity vertical mode, evolving basinwide so as to bring the zonal pressure gradient into balance with the wind stress. A linear systems analysis of the response to impulsive forcing describes the properties of the solution in time and space. The frequency response function derived by this analysis can be used for calculating equatorial ocean responses to generalized easterly wind stress given an arbitrary number of zonal segments within which local wind stress measurements are available. The resulting explicit equatorial wave constructions provide influence and coherence functions for comparisons with ocean data and for assistance in interpreting more complete but complex numerical model simulations.

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