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Functional Gait Asymmetries Achieved Through Modeling and Understanding the Interaction of Multiple Gait Modulations

Fatemeh Rasouli
University of South Florida

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Functional Gait Asymmetries Achieved Through Modeling and Understanding the Interaction of Multiple Gait Modulations

by

Fatemeh Rasouli

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering
Department of Mechanical Engineering
College of Engineering
University of South Florida

Major Professor: Kyle B. Reed, Ph.D.
Seok Hun Kim, Ph.D.
Craig Lusk, Ph.D.
Tansel Yucelen, Ph.D.
William E. Lee III, Ph.D.

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Dedication

I dedicate this dissertation to four amazing people in my life whom their love and support never wavered.

To my parents, for raising me to believe I can achieve anything I set my mind to.

To my brother, for giving me endless support and encouragement.

To my sister, for teaching me how to love fully.
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Abstract

Walking is an important determinant of human functionality. Gait disabilities affect millions of people worldwide every year. Investigating the science of walking advances recovery techniques and assistive devices for gait rehabilitation. A functional gait promotes productivity, independence, and quality of life. Human gait, like any other moving mechanism, is a dynamic system. Understanding and analyzing the dynamic aspects of gait improves the recovery methods to fundamentally affect and interact with lower limbs.

This dissertation aims to fill the gaps in mechanical simulations of gait and dynamic analysis of rehabilitation techniques. The solutions consider kinematic, kinetic, and spatiotemporal parameters of gait as a whole system. The studies analyze the asymmetric gait through both theoretical and experimental means. Asymmetry is one of the most common indexes of walking impairment. The goals of this dissertation are to develop techniques and gait models that elevate asymmetric walking performance, create walking patterns with dynamic symmetry, and design rehabilitation therapies for hemiparetic gait. This dissertation aims to revamp gait models and simulations, develop pioneering rehabilitation methods, and enhance therapeutic and assistive outcome measures.

The studies take three different but cohesive approaches to achieve these aims. The three main segments of this dissertation include a gait-altering assistive device, dynamic simulations of gait, and multiple rehabilitation interventions. The first segment introduces the Kinetic Crutch Tip (KCT), an assistive invention that enhances the performance of crutch walking. The study of KCT
indicates the kinetic shape of the device creates dynamic alteration in gait by increasing positive forces and assistive range of rotation. The second segment formulates a novel method for dynamic gait modulation using double pendulums. This new model can derive equations of motion for dissimilar systems with kinematic and kinetic symmetry. The theoretical results and simulations demonstrate a new possibility for modeling asymmetric gait with symmetric outcomes. In the final segment, a multi-rehabilitation technique is developed for gait recovery training. The technique combines two prominent physical therapies; split-belt treadmill and rhythmic stimulations. The study proposes a gait response model that measures the kinematic and kinetic performance of gait under multiple stimuli.

Enhancing the science of gait biomechanics contributes to the improvement of rehabilitation techniques. The research studies in this dissertation seek to increase the existing knowledge of functional gait asymmetries and introduce dynamic approaches for alteration and modulation of gait.
Chapter 1: Introduction

Gait disability is a global health concern. 15% of the world’s population [1] and 25% of adults in the US [2] suffer from a form of disability that impacts their quality of lives. Mobility is the most common type of disability in the US, and 1 in every 7 adults have a type of walking disorder. This number increases by almost threefold in the age range 65 and older [2]. 4.2% of women and 3.4% of men in Europe, and around 2.8 million people in Canada have a type of walking disorder [3], [4]. Walking is a determining factor for physical activity level, independence, and wellbeing. Gait dysfunction leads to a 20 - 39 percent decrease in employment rate between people with and without disabilities [3], [5]. The number of individuals using assistive devices for mobility is growing more rapidly than the general population [6], [7]. Improving our understanding of the walking mechanism, advancing the design of ambulatory assistive devices, and developing new rehabilitation interventions are complementary approaches for solving mobility issues.

Gait interventions and alterations can be divided into two categories; assistive or rehabilitative. Assistive approaches aim to provide support for walking, decrease pain and weight-bearing in lower limbs, and compensate for lack of muscle strength and balance in patients. These approaches include ambulatory assistive devices (AAD) such as walking frames, canes, crutches, prosthetics, and orthoses. The objective of these mobility aids is to improve walking efficiency and decrease energy consumption by providing assistance during walking. Rehabilitative approaches, on the other hand, seek to restore the physical ability to walk normally and train patients for independent walking. These methods consist of adaptation periods, and their goal is to improve gait performance post-adaptation. The training can incorporate techniques such as
treadmill training, auditory cueing, or resistive training, and they take advantage of multiple sensory feedbacks and the somatosensory system. The objective of rehabilitation training is for the patient to become independent of the approach afterward, while with assistive methods, the patient depends on the device for aid.

This dissertation investigates two sets of experimental studies; one with an assistive aid and one with rehabilitative interventions. The Kinetic Crutch Tip (KCT) is a gait-altering assistive tip that is capable of improving kinetic and kinematic outcomes of asymmetric crutch walking, including positive horizontal forces and the range of rotation. The study in this dissertation explores KCT performance by conducting experimental tests and analyzing human and physical data. For the rehabilitative intervention, a new combined method is introduced. The method examines the effect of applying simultaneous physical therapies and establishes a linear model for the gait response under split-belt treadmill (SBT) and rhythmic auditory stimulations (RAS).

A better understanding of lower limbs’ dynamics and the interworking of the human musculoskeletal system is also required for increasing the efficiency of rehabilitation techniques. Gait modulations are physics-based methods that simulate the biomechanics of gait through dynamic models. They include a wide variety of elements such as rigid bodies, point masses, links, or dampers, and mimic human walking sequences by logical algorithms. The theoretical work in this dissertation breaks down the effective parameters in the dynamic motion of a gait cycle by remodeling dissimilar double pendulums that resemble asymmetric limbs. The study creates a system of coefficients for identifying kinetic and kinematic symmetry in the human walking system. Specifically, the study examines the possibility of achieving kinematic and kinetic symmetry in asymmetric limbs.
Figure 1.1 demonstrates a concept map of all the three areas of research presented in this dissertation. All displayed research areas work on different but synergetic goals. These studies incorporate both experimental and theoretical aspects of functional asymmetric gait. They all follow dynamic-altering approaches (devices, interventions, and modulations) that aim to understand and improve forces, motions, angles, and spatiotemporal measurements of impaired lower limbs. This dissertation creates and analyzes recovery methods for both assistive and non-assistive asymmetric gaits. Mathematical equations represent conceptual gait trajectories under various human-like events. Linear systems are also used to model and estimate gait response patterns under therapy.

![Concept map of research studies](image)

Figure 1.1: Concept map of the research studies in this dissertation
Engineering can contribute a critical and unique perspective in solving the challenges faced in gait disability and its recovery. Dynamic gait alterations and interventions improve forces and motions of gait outcome measures. They affect kinetic, kinematic, spatial, and temporal parameters of gait structure and generate new walking patterns. In addition, mechanical modulation of walking and deriving equations of motion for the human system enhances our understanding of the interactions in gait elements and how to manipulate them. This dissertation looks at the asymmetry of an impaired gait through the lens of mechanical dynamics improvement.

Asymmetry is among the most common walking disorders. Asymmetric gait leads to reduced walking speed, increased energy consumption, and impaired balance control [8]. Therefore, an important goal for gait recovery is improving symmetry indexes (for spatiotemporal, kinetic, or kinematic parameters) between the left and right sides. Past studies have sufficiently explored the stability and comfort of assistive devices and recovery methods [9], [10]. However, the dynamic improvements of these techniques for a systematic alteration of gait mechanism require more attention. This dissertation reformulates the theoretical modeling of asymmetric gait and creates a new perspective on achieving symmetric performance in an impaired gait. It also experimentally assesses the dynamic outcomes of an existing device for assisted (asymmetric) walking and combines two rehabilitation ideas for hemiparetic gait training.

Another challenge for gait recovery is the customization of the methods. A wide range of diseases and health problems with congenital or acquired origin such as stroke, knee and hip osteoarthritis, amputation of a limb, or Ataxia can cause gait impairments. Individual differences between patients, such as physical capabilities, age, or posture, also contribute to the effectiveness of the recovery approach. For these reasons, gait recovery requires customizable solutions that can take these individual differences into account. This dissertation pioneers an original multimethod
rehabilitation technique that can be tailored to the patient’s strengths and weaknesses. The developed technique combines two distinctive physical therapy approaches and evaluates gait response under various ratios of each therapy. The study demonstrates the benefits of personalization of rehabilitation by calculating coefficients specific to each participant.

1.1 Specific Aims

The goal of this dissertation is to enhance the outcome measures of functional asymmetric walking through dynamic alteration of gait. The research pursues this goal through three specific aims that incorporate assistive gait devices, physical gait modulation, and rehabilitative gait interventions (Figure 1.1).

1.1.1 Aim 1: Gait-Altering Assistive Device

The aim of Chapter 2 is the improvement of assistive crutch gait through a dynamic-altering crutch tip. The research investigates the following hypotheses by designing and examining different experimental tests.

- **Hypothesis I**: Kinetic Crutch Tip (KCT) design produces better positive assistive forces and a bigger range of rotations in the crutch walking compared to the conventional tip.
- **Hypothesis II**: Different stiffnesses of KCT affect the performance of crutch walking.
- **Hypothesis III**: Standard tip rotates backward with small backward angles (closer to the vertical line) compared to KCT.

1.1.2 Aim 2: Dynamic Gait Modulation

The aim of Chapter 3 is to achieve theoretical gait symmetry with dynamic modulation of two limbs that have different physical characteristics. The following hypothesis is tested by simulation of groups of dissimilar double pendulums.
• *Hypothesis:* Asymmetric dynamic models of double pendulums can be kinematically and kinetically matched.

1.1.3 Aim 3: Gait Rehabilitative Interventions

The aim of Chapter 4 is to introduce an innovative approach for personalizing gait training for unilateral stroke therapy. The study also represents a model for gait response mechanisms under simultaneous rehabilitation therapies. An extensive experimental study examines the following hypotheses:

• *Hypothesis I:* Asymmetric rhythmic auditory stimulation (ARAS) can train the left and right sides independently.

• *Hypothesis II:* Superposition principle applies to gait response (gait linearly combines the effects of two rehabilitation methods applied at the same time)

1.2 Dissertation Contributions

This research provides practical solutions to the problems of gait impairment. In this dissertation, I analyze, evaluate, and create modules and interventions for the enhancement of functional gait asymmetries.

Crutches are used as assistive walking devices for a broad demographic. They help gait disabilities ranging from short-term use such as sprained ankles, to lifelong use as in the case of Multiple Sclerosis or amputation. The assistive devices help people become functional in the community and navigate through life independently. Any improvement in the crutch efficiency and assistance can considerably benefit a continuous user. The study of Chapter 2 indicates that the kinetic shape of a crutch tip improves the horizontal assistive forces and rotation angles of the crutch. These results benefit temporary and permanent crutch users who depend on the device for their daily tasks. Two sets of experiments are designed and conducted in the study.
experiment tests the performance of KCT with eight participants walking with swing-through crutch gait. An additional set of experiments investigates the pure effect of the design by replacing the human factor with a physical setup. The study highlights the dynamic advantage of kinetic outer edge for crutch tips.

The complexity of the musculoskeletal system has long been a challenge for the design of mechanical aids and robotic limbs. The asymmetry of an impaired gait adds additional difficulty to the design. Finding the most advantageous structure in every gait impairment is an essential goal for rehabilitation. The conceptual model of legs in Chapter 3 recompiles the equations of motion and regroups the parameters of the lower limbs. It introduces groups of parallel double pendulums with different physical attributes but similar kinematic or kinetic behavior. The simulation of these parallel systems unlocks a new opportunity for creating dynamically matched left and right sides in lower limbs.

Millions of people suffer from post-stroke disabilities worldwide each year. Asymmetric walking is a common side effect of unilateral stroke. Effective recovery methods are vital for achieving independence and improving the quality of life for stroke survivors. Understanding the gait response mechanism helps develop better physical therapy techniques. Chapter 4 introduces a new innovative technique for asymmetric gait recovery. The multiple rehabilitation technique combines treadmill training and rhythmic stimulations together. An extensive experimental study investigates the effect of the method on gait response. A model for the gait response under simultaneous stimuli defines the gait parameters’ performance during and shortly after training. The superposition principle describes the linear behavior of gait and how it combines the effects of both interventions simultaneously.
Chapter 2: Walking Assistance Using Kinetic Crutch Tip

2.1 Introduction

Crutches are one of the most common ambulatory assistive devices. Using crutches encourages more physical activity than many other assistive devices, which has long-term health benefits. Recent advances have led to improvements in performance, but using crutches remains slower than normal walking. Modifications to crutch designs should consider not only comfort and stability but also the efficiency of walking. Crutch gait is inherently more energy consuming than regular walking, and new devices need to consider designs that alter kinetic and kinematic interaction between the user, the device, and the environment.

This chapter studies the effect of an assistive crutch tip on improving the dynamics of the user. The Kinetic Crutch Tip (KCT) is a gait altering design that enhances the performance of the crutch gait by taking advantage of the ground interaction with the tip. The curvature in KCT design restructures the forces in the system to benefit the user during gait. It creates force couples that could be used in assistive or resistive modes, allowing more control in level or uneven grounds.

Another benefit of the outer curvature is to enhance the range of rotation and dynamic balance of the crutch. Compared to conventional designs, KCT shape tends to roll up even when released from backward angles. The combined advantage of the assistive force couple and an increase in the range of rotation can improve the dynamic of crutch walking.

1 The work presented in this chapter has been published in [138]–[140]. Copyright permissions have been obtained from all publishers (Appendix A).
The current research studies the kinetic and kinematic advantages of KCT design using different tests with human participants and mechanical frameworks. Section 2.2 reviews important crutch designs and modifications throughout the past century. Section 2.3 specifies the main contributions of the current study. Section 2.4 explains the gait-altering features of the KCT and the specifications of the devices used in the experiments. Section 2.5 develops the experimental designs and setups for this study. Section 2.6 compares the results of gait parameters, horizontal ground reaction forces (GRF), and crutch angles between KCTs and a conventional tip. Section 2.7 concludes the outcomes of this research.

2.2 Background

A walking crutch is a type of gait assistive device that transfers weight bearing from the lower limbs to the upper body to relieve stresses on the lower body while also promoting stability. It is often used by people who cannot use their legs to support their weight for reasons ranging from short-term injuries (less than 6 months) to lifelong chronic disabilities. Crutch users also vary from children, teens, adults, and the elderly. Because of this wide range of applicability, there are variations in the design of a crutch.

Crutches are typically categorized as axillary and nonaxillary designs [11], [12]. Figure 2.1 shows some of the significant crutch advances over the past century. Forearm and axillary crutches are the most common types of crutches.

Axillary crutches were first used during the Greek era in Egypt [13] and generally include an underarm pad and have more trunk support than other crutch types [12]. Weight-bearing should only happen on the handle, and the axilla pad should only be used for stability.
Figure 2.1: Timeline of crutch evolution through design modifications. The designs on the left-hand side focus on remodeling one part of the crutch (tips, shafts, or handles), while the right-hand side is the designs that include major alteration to the whole crutch structure.
Non-axillary crutches have many shapes and configurations. One of the most known is the forearm crutch, where the underarm pad is replaced with an armband around the forearm to transfer body weight and reduce pressure from the underarm [14]. They are suitable for patients with weak hip abductors and extensors. They also have faster walking speeds compared to axillary crutches [12]. The Lofstrand crutch, introduced in 1948 [15], [16], includes a handgrip and an attached cane (Figure 2.1). The forearm clamp is at an angle so the arm can slip easier into the cuff. There have been other patented crutches with various modifications of the Lofstrand design [17], [18].

The Canadian crutch is another frequently used non-axillary crutch that combines features of both the forearm and axillary crutch [19]–[21]. It creates more stability than a forearm crutch while having less underarm pressure than axillary crutches. A triceps Canadian has two cuffs around the forearm and above the elbow joint, which could be full-arm or half-arm cuffs. Although half-arm gives better freedom of hand movements, full-arm can be more assistive in patients with less strength in their triceps [11].

Although crutches are beneficial for those that need them, studies have shown that crutch gait is slower [20], [22], has a higher energy cost [22], and increases heart rate [20]. It also changes joint displacement [23], plantar foot pressure [24], and vertical and horizontal ground reaction forces [25]. Many recent advances and modifications have improved crutch performance to address these needs. These modifications in crutch designs can be categorized into three different parts of a crutch, as shown in Figure 2.2.

a) Handle (upper side of the crutch including cuffs, forearm, and underarm pads): This section focuses on optimizing the weight-bearing on upper extremities. New design suggestions include elbow cuffs, fixating wrist position, and repositioning handle and underarm pads
to distribute pressure. Modifications to the handle aim to reduce the risk of neurovascular damage to upper limbs.

**b) Shaft (the middle section connecting the upper side to the tip):** The majority of modifications in this section contain shock absorbers to reduce ground impact force to upper extremities as well as storing energy to improve efficiency. Shafts also include additional mechanisms to adjust the length.

**c) Tip (lower section including the foot of the crutch):** Designs involving the tip of the crutch deal with the stability of walking and GRF. However, the crutch tip has a great potential to alter crutch gait dynamics. Crutch tips are the primary contact points with the ground and can produce assistive forces during the gait cycle.

---

**Figure 2.2: Three parts of the crutch: Handle, shaft, and tip; their functions and capabilities**

- **Handle**
  - Weight transfer through upper extremities
  - Distribution of pressure between wrists, arms, shoulders
  - Preventing distal movements

- **Shaft**
  - Adjusting length to users
  - Shock absorption from ground impact
  - Storage of mechanical energy

- **Tip**
  - Improving stability and balance
  - Altering gait dynamics and interaction with ground
  - Generation of forward/backward progression forces

---

2.2.1 **Handle or Grip**

Chronic crutch users often sustain injuries on their upper extremities because crutch gait partially transfers body weight to the upper limbs. Long-term use of crutches generates higher pressures on the palms, wrists, forearms, axilla, and shoulders. These forces applied to the handle
can indicate improper crutch use [26]. Research has indicated case reports of ulnar tunnel syndrome after even short-term use of bilateral forearm crutches [27]. Designing proper handles can reduce the risk of injury.

The design of the handles in crutches can affect weight bearing on upper limbs. A handle in a forearm crutch with an extended elbow has fewer forces and momentum on the shoulder compared to a flexed elbow design [28]. Research [29] has also shown that smaller handles with 20mm diameters increase flexor digitorum muscle activity as detected by surface electromyography (sEMG) when compared to 40mm handles. Higher activity in this muscle was also accompanied by higher perceived exertion on users. Comparing cylindrical and wide shaped handles in forearm crutches indicated a similar pattern. Maximum vertical forces were found in the distal radial and middle palm for cylindrical and palm’s proximal ulnar region for the wide handle. However, researchers concluded they both showed similar pressure distribution pattern because 60–70% of average palmar loads were in the radial side for both. Therefore, one cannot be recommended over the other [30].

One of the inventions attempting to reduce extra loading on upper extremities is a pneumatic sleeve for Lofstrand crutches [31]. This research suggested a sleeve to fixate the posture of the wrists and decrease pressure on them by redirecting forces to cuffs (Figure 2.1). This design includes a modified actuator in helical form and two half-cylindrical splints. Using the pressure applied to the crutch tip, the length of the helical actuator decreases, hence holding the wrists in place and preventing distal movements. Removing the tip depressurizes the actuator.

Another design for axillary crutches [32] created a space in between the underarm pads and the rest of the crutch by adding a spring in between (Figure 2.1). This patent aimed to decrease
the risk of pressure on the axilla by absorbing the shock through the spring and generates more flexibility with a pivoting handle.

2.2.2 Shaft or Rod

Researchers have developed various methods for adjusting the length of the crutch based on the subject’s height, length of the axillary fold to the heel, or arm span [33]. Although the methods differ, crutch height can vary by 2.5cm, with no significant difference in energy cost [34].

Shock absorption is most frequently added to the shaft by adding springs or dampers, but other methods include creating an S-shape compliant shaft [35]. Measuring shock wave amplitude at the time of the crutch strike indicated a decrease of 22% in the spring-loaded axillary crutch [36]. Maximum forces on wrists and hands were also reduced by 24%. Seemingly contrary research [37] has found an increase in the maximum GRF when using spring-loaded crutches. While the large GRF is measured at the tip below the spring, a reduced force to the hand and wrist is measured at the handle above the spring. Thus, the decrease in the shock wave and maximum forces on hands and wrists indicates that the energy storage capacity of spring-loaded crutches can result in less fatigue and pain on the upper extremities. Shortell et al. [35] provided a method based on body weight to choose an appropriate stiffness for the desired result.

Additionally, a reduced rate of GRF could help reduce the abrupt change of forces and the consequent shocks applied to upper limbs. Comparing spatiotemporal parameters [37] indicated lower velocity, higher stride time, and higher crutch stance time in spring-loaded axillary crutches; although, stride length remained the same. One explanation is that the flexible structure of the spring-loaded axillary crutch makes this design more difficult to control. Therefore, gaining balance comes at the expense of making users' feet land closer to the crutch [37], [38].
Although metabolic cost and mechanical cost decreased in spring-loaded axillary crutches, the ratio of mechanical cost to metabolic cost showed a significant increase in the spring-loaded axillary setup [38]. This increase in mechanical efficiency can be related to the additional energy storage capacity during the crutch stance phase.

Springs have also been incorporated into non-axillary crutches [39], [40]. One design, the pogo crutch [40], integrates a telescopic spring into a forearm crutch (Figure 2.1). The design reduces the center of mass fluctuation, and stores impact forces at crutch strike as mechanical energy used for toe-off. Experiments showed changes in vertical forces, impact forces at initial contact were reduced by 50%, and toe-off peaks were eliminated. This result demonstrates that mechanical energy stored in springs could assist with the push-off. However, there is a tradeoff between energy storage capacity and stability.

Other designs have used an elastomeric system (damper) to eliminate the sense of instability due to height changes in springs. They found no significant difference in GRF, spatiotemporal parameters [41], and peak vertical forces [42]. However, a mechanical test indicated a reduction in peak GRF when a large force (greater than the human range) is applied [41]. Also, experiments within a male group showed larger propulsive and smaller braking forces. This can be an indication of potential changes in the forward momentum of walking [42].

2.2.3 Tip or Foot

The invention of the crutch tip goes back to the 18th century [43]. The first tip was a screw-on rubber foot with a metallic thread attached to the crutch bar and an elastic cover at the foot. Crutch tip designs have seen less development compared to the handle or shaft. However, as the only point of contact with the ground during the swing phase, they contain a lot of potential for improving walking dynamics and efficiency.
Most of the advances in crutch tips have focused on improving stability, balance, and traction. The first patent in the 20th century [44] details an elastic socket with a ring-shaped flange made of vulcanized rubber (Figure 2.1). Other patented designs have added rubber cushions to prevent slipping [45], a suction grip to compress air for a better hold on the ground [46], and inserting rigid materials to transfer forces to the sides [47].

Most of the crutch tip designs have incorporated the same idea of a rubber tip, using a flat surface to increase stability on different grounds. However, few designs have developed curved tips that create a rolling effect for improving assistance during walking. The roller crutch [48] was similar to an axillary crutch at the top, but the two vertical bars diverged at the bottom and were connected to a curved hardwood with an attached rubber called “the roller” (Figure 2.1). Experimental testing of the rolling design reported improved stability, a 16% increase in step length, and an increase in the effective supporting angle by 16 degrees. The rolling crutch did not see any improvements for 80 years until the SureGait and Rocker-bottom axillary crutches were evaluated, but there were no differences between the Sure-Gait (Figure 2.1) and standard axillary crutches [49], [50]. Although both crutches required significantly more energy expenditure than normal gait, they had similar oxygen uptake and heart rate [49], [50].

Several industrial crutch tips have recently been developed for popular crutch designs. The Aventure™ tip is a pivoting tip designed for uneven and slippery terrain. The Tornado™ tip is a gel-infused crutch tip that dampens crutch ground strike impacts. The SandPad™ has a large pivoting tip that allows it to be used on the sand. Although all these crutch tips vary in weight, compliance, and ground traction, they always mimic a constant radius. A pointed tip mimics a constant radius when rolled over and cannot change the user dynamics during swinging or rolling over the crutch tip, and all forward progression forces are generated by the user pushing themselves
forward over the crutch. Since the crutch tip is the ground contact point, it can play a critical role in improving the dynamics of crutch users by redirecting applied forces.

The KCT [51] utilizes a variable radius kinetic shape in order to passively assist the user’s forward motion during the swing phase of the gait. The bottom left box in Figure 2.1 displays a graphic of the KCT design. The idea behind this crutch tip is that objects with uneven radius tend to roll in the direction of decreasing radius [52], [53]. This occurs because the force the user applies through the crutch is offset from the contact point of the ground due to the curvature of the KCT. The moment generated from this offset can help propel the user forward when walking up a hill or on level ground. If the KCT is turned around, it can also resist forward motion when walking downhill and help to keep the person in control during the descent.

2.3 Main Contributions

A previous study with the KCT and standard tip indicated an increase in horizontal assistive forces for KCT among a small number of subjects [54]. However, the experiment only tests one KCT and includes four participants. The present study expands the investigation of KCT performance by:

- Doubling the number of participants walking with the crutch gait and increasing the number of trials (with different crutch tips) per participant.
- Designing and conducting a new set of experiment with a physical setup in addition to the human experiment that eliminates the effects of the user and directly compares the KCT design with conventional design in different slopes.
- Analyzing parameters containing both kinematic as well as kinetic aspects of the motion.
- Comparing the effect of stiffness on KCT performance.
2.4 Kinetic Crutch Tip: A Gait-Altering Approach to Assistive Devices

The kinetic shape of KCT with a varying radius fundamentally changes the ground-device-user dynamics. The exterior shape generates a distance between forces at the ground contact and creates an additional moment by coupling and redirecting vertical forces. The user steps up onto the larger portion of the KCT radius and rolls down to a smaller radius in order to generate a forward-forcing moment.

The KCT uses its curvature to create assistive or resistive force couples in the direction of the movement. Figure 2.3 shows a general explanation of the KCT with a comparison to a standard crutch tip standing over level ground, an uphill, and a downhill surface. Figure 2.3a shows the lack of assistive momentum in the free body diagram of a standard tip. Both ground reaction and applied vertical weight are aligned. However, the misalignment of forces in Figure 2.3b for KCT has created a force couple that propels the user forward.

![Figure 2.3: Kinetic Crutch Tip (KCT) and standard rubber tip over inclined and flat surfaces](image)

In the case of walking on an uphill surface, the point of contact with the ground moves further in the standard tip. Figure 2.3c indicates that the shift creates a backward (negative) momentum in a standard tip causing a resistive moment in the opposite direction of walking. This
resistive moment makes walking uphill a challenging and energy-consuming task for a crutch user. For a KCT on an uphill ground, the opposite force couple has been eliminated and replaced by an assistive (positive) momentum (Figure 2.3d). This positive momentum stays even if the slope increases until both the reaction force and vertical weight are aligned.

Figure 2.3e and Figure 2.3f compare the two crutch tip designs on a downhill slope. The design of the standard tip creates a misalignment between the applied weight and GRF, causing a positive momentum that makes controlling the crutch gait difficult. The KCT shape is rotated around to reverse the assistance force and provide a more controlled descent down a hill by reducing the user’s momentum, although this aspect will not be examined in this study. Note that these illustrations are for a crutch in the vertical position, but similar benefits are generated at other angles of a tilted crutch as well.

The design of KCT slightly increases the crutch height. However, previous studies have shown that the effects of increasing the crutch height by the height of the KCT at first contact have no significant effect on crutch walking energy cost, cadence, velocity, peak and mean elbow and shoulder moments and impulses [34], [55]. The assistance of KCT is provided passively, so no motors or power supplies are required. The assistance force helps the individual use less energy while moving forward over level ground and when walking uphill.

Two important measurements indicating better assistive walking are the positive horizontal force, and the maximum backward angle demonstrated in Figure 2.4. The positive horizontal force is the component of GRF in the frontal plane and in the direction of walking. A bigger positive horizontal force means more assistive forces are created during walking. In other words, if the proportion of positive forces (relative to negative forces in the opposite direction) increases, the more assistance is provided from the device.
The crutch always lands at a backward angle during walking (Figure 2.5) and rotates from a negative angle $-\alpha$ to positive $+\beta$ by the user. The more the crutch is stable during backward angles, the easier for the user to swing forward. The maximum backward angle for a crutch is the farthest angle on the left side of this range that the crutch is stable or moves forward without any external push. The bigger the maximum backward angle of a crutch design is, the bigger range of rotation (without resistance) it provides. Figure 2.4 shows both the horizontal force and the rotation angles of the crutch during swinging of the body.

Figure 2.4: Backward and forward rotation angles and the GRF of the crutch in the frontal plane

This research studies and compares the maximum backward angle and the positive horizontal forces for crutches using KCT compared to the standard tip. This study hypothesizes that KCT has better positive assistive forces and bigger maximum backward angles compared to a conventional standard tip.
Various parameters can affect the motion of a crutch tip. Finding a design with optimum material and characteristics that could improve walking parameters as well as increasing comfort is important to optimize the benefit. The present study considers different stiffnesses for KCTs.

The stiffness of a crutch can influence the reaction forces applied to the crutch user. The amount of energy absorbed by the crutch structure could have an indirect effect on the comfort of walking and stability. The hardness of the material in the crutch tip is one of the key characteristics that could enhance both the dynamic motion and reaction forces during walking.

Shore A hardness scale is a measurement of stiffness used for rubber molds and semi-rigid plastics. It ranges from 0 to 100, with zero indicating very soft and hundred indicating hard with almost no flexibility. In the following study, six different KCTs and a standard rubber tip as a baseline are compared. Five of the KCTs have a Shore A hardness ranging from 40 to 80, and the sixth KCT is 3D printed with reinforced carbon fiber. Table 2.1 indicates the features and the name associated with the crutch tips used in this research.

Table 2.1: Crutch tips’ features and specifications

<table>
<thead>
<tr>
<th>Name</th>
<th>Standard</th>
<th>KCT_{40}</th>
<th>KCT_{50}</th>
<th>KCT_{60}</th>
<th>KCT_{70}</th>
<th>KCT_{80}</th>
<th>KCT_{3D}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness*</td>
<td>~</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>~</td>
</tr>
<tr>
<td>Shape</td>
<td><img src="image1.png" alt="Shape" /></td>
<td><img src="image2.png" alt="Shape" /></td>
<td><img src="image3.png" alt="Shape" /></td>
<td><img src="image4.png" alt="Shape" /></td>
<td><img src="image5.png" alt="Shape" /></td>
<td><img src="image6.png" alt="Shape" /></td>
<td><img src="image7.png" alt="Shape" /></td>
</tr>
</tbody>
</table>

*stiffness was measured using the Durometer ASTM D2240 (Shore A hardness scale)
A softer material can have more cushion during impact and absorb shocks during ground contact but also creates a feeling of instability. A harder material, while more stable, can transfer bigger GRF and discomfort to underarms. This study hypothesizes that different stiffnesses alter the performance of the crutch tips with the same design.

2.5 Experimental Design and Procedures

This research studies how the dynamic design of KCT affects the beneficial outcomes in assistive forces and the range of motion of the crutch. The experimental section studies these features in two different sets of experiments to compare the performance of the crutch tips in human gait and analyze the absolute effect of the design on dynamic balance.

In the human subject experiment, eight healthy participants complete crutch walking trials that help measure forces at the crutch tip during full gait cycles. The study tests the hypothesis of improved assistive forces in KCT by comparing the results of walking with different crutch tips in Table 2.1 among subjects.

The physical setup experiment studies the pure dynamic movement of a crutch by eliminating the human factor. This setup compares different crutch tips without the effect of the crutch user’s attributes (i.e., height, weight, specific motions, etc.) and walking style. It compares the unadulterated effect of crutch tip curvature on its range of rotation. This experiment studies the effect of the shape of the tip on the crutch tendency to pivot forward (maximum backward angle).

Both experiments were conducted in the Computer Assisted Rehabilitation ENvironment (CAREN) system. CAREN system includes a split-belt treadmill surrounded by ten motion capture cameras. Handrails on each side, a safety harness, and multiple emergency buttons close to the subject and the operator provide fully sufficient safety measures for the experiments. Motion capture software (Vicon) tracks the movement of reflective markers during trials, and the force
platforms underneath the treadmill record the kinetic interaction between the subject and the ground.

2.5.1 Human Subject Experiment

Eight healthy individuals (six male) between the ages of 20 to 30 were recruited. All participants had zero to medium experience walking with crutches. Written informed consent was obtained from each subject prior to participation with a protocol approved by the Western Institutional Review Board. Each subject had time to get used to the crutches before starting the experiment. The crutches were adjusted to the participant’s height according to measurements in [33]. All participants walked with crutches using a non-weight bearing swing-through pattern. In this crutch pattern, the subject lifts one leg (assumed impaired side), positions both crutches in front of the body, and swings past the crutches (Figure 2.5). The swing through crutch gait was chosen for this experiment since it is the easiest and most common crutch walking gait for temporary users. Figure 2.6 shows this crutch walking pattern. The participants were given the option to use their preferred leg when walking with crutches; the other leg was kept in the air to simulate a common usage of crutches, such as when a foot is broken and unable to bear weight.

![Figure 2.6: Non-weight bearing swing-through crutch gait](image)

A trial run was conducted to adjust the treadmills’ speed to the participant’s preferred walking speed. The experiment started when the subject felt comfortable walking with crutches on
the treadmill. Each trial with a crutch tip lasts for 2 minutes long. Subjects walked with the crutch tips demonstrated in Table 2.1 in random order.

This study conducted a total of 56 human experiments (8 subjects with 7 trials each). 18 reflective markers tracked the position of the limb joints and the crutches (9 on each side). Figure 2.7a demonstrates the location of the markers for one side. Figure 2.7b shows a screenshot of one of the experiments when the subject is moving the crutch to a position in front of the body.

This experiment analyzes the effect a crutch tip has on two factors during dynamic walking: (1) the total angle crutch swings through (range of rotation) and (2) the duration of assistive forces presents per gait cycle.

Figure 2.7: Human experiment setup and the location of the reflective markers. a) Schematic of the crutch walking. 18 markers (9 left and 9 right) were located on the subject, and the crutch to track movements. The location of the markers for one side is demonstrated. b) Participant walking with the crutch on the treadmill during one of the trials
2.5.2 Physical Setup Experiment

This experiment analyzes the motion of a crutch freefalling without the influence of human variability. Eliminating the human factor enables the study to analyze and compare different types of crutch tips consistently. The objective of this experiment is to analyze the free motion of a weighted crutch. The weight serves to simulate the loading experienced on the crutch during walking. This experiment obtains the maximum backward angle from the vertical axis that the crutch can fall forward without any external push and compares the performance of different crutch tips in Table 2.1.

Figure 2.8 shows the setup designed for this experiment. Two Pins are connected to the handrail with clamps and hold the crutch in a fixed position. The crutch tip is located at different tilted positions. The structure prevents the crutch from moving sideways. Each trial starts by removing the pins at the same time for each position, and the backward or forward motion of the crutch for that angle is recorded. Starting at the biggest negative angle and gradually increasing toward zero (vertical position), the first angle that the crutch rolls up forward is the maximum backward angle for that crutch tip. Considering the curvature of the KCTs, it is expected that this angle would considerably increase for KCTs compared with the standard tip. The negative sign of $\alpha$ indicates the backward position of the crutch. The study hypothesizes that small negative angles with the vertical axis would cause the standard tip to rotate backward. However, the kinetic shape of KCTs would redirect downward forces into forwarding motion, and the rotation angle is shifted backward, so it almost looks like the crutch is rolling up.

A follow-up experiment tested different inclined angles for the platform: 0°, 3°, and 6° (Figure 2.8). These angles were chosen to cover the angles typically allowed by the Americans with Disabilities Act for elevations greater than 6 inches [56]. Specifically, a slope of 1:10 (5.7°)
is allowed. Only four of the crutch tips were tested in this experiment: Standard, KCT_{60}, KCT_{70}, and KCT_{3D}. The other three were eliminated since they did not indicate any significant changes based on the previous experiments (more on section 2.6). For each trial, the weighted crutch was held at various angles and released. The test was repeated 15 times for all 12 configurations (four different crutch tips on three different slopes). The goal is to compare the change in maximum backward angles when the ground is inclined. Repeated trials were conducted to account for any variability in releasing of the crutch.

Figure 2.8: A schematic of the setup for Crutch Free Fall Experiment

For each trial, the crutch was held at a balance in level or uphill direction. Then, the crutch was released, and motion direction (Forward or Backward) was recorded. All data, including initial position and time of release, was gathered with the CAREN system. Each trial was repeated 15 times, and each time random (positive and negative) angles with vertical axes for the initial position were tested to cover the transitional angle area.
2.6 Results and Discussion

2.6.1 Gait Parameters

The human subject experiment records and compares the joints moving as well as force reactions during crutch gait. The aim is to compare the effect of the standard tip with KCTs as well as comparing KCTs with various stiffness. Figure 2.9 shows the average gait parameters for all eight subjects, including step time and step length between all the crutch tips in Table 2.1.

The KCT$_{80}$ and KCT$_{60}$ had longer crutch step lengths in comparison to the softer crutch tips such as KCT$_{40}$ and KCT$_{50}$. The KCT$_{3D}$ had almost the same results as the standard tip. However, the magnitudes of differences in both step time and step length are relatively small among all crutch types. Gait parameters are not affected by the different crutch tips during a short period of walking.

![Figure 2.9: Averaged step time and step length during human subject experiment](image-url)
2.6.2 Assistive Horizontal Force

The ratio of total positive to negative horizontal forces is a useful metric for measuring the performance of assistive walking. This quantity indicates the proportion of horizontal forces used in forwarding motion. This metric indicates the percentage of the time during the stance phase that the horizontal force assisted the forward motion. Figure 2.10 shows the average result of assistive forces for all subjects between all the crutch tips.

Figure 2.10 demonstrates that KCT$_{70}$ and KCT$_{80}$ (with the highest stiffness) have the highest ratio of positive horizontal forces with 60.62% and 61.39%, respectively. Three of the kinetic crutch tips have higher values than the standard tip, with 55.6%. Crutch tips with the low stiffness values (KCT$_{40}$, KCT$_{50}$, and KCT$_{60}$) displayed lower percentages than crutch tips with higher values (KCT$_{70}$ and KCT$_{80}$).

![Figure 2.10: Percentage of time during stance that the horizontal force is forward (assistive) for different crutch tips.](image)

This result indicates hard crutch tips with higher durometer performed better than softer ones. Furthermore, the KCT$_{3D}$ printed crutch tip with 52.25% had the lowest percentage of assistive forces. The reason for the low performance of KCT$_{3D}$ could be due to a mismatch in the angle between the crutch tip and the crutch shaft. After attaching KCT$_{3D}$ to the crutch, the crutch
tip seemed to get loose during movements, and the plastic slid on the crutch shaft. As a result, KCT_{3D} tended to turn sideways during the experiments, and this could have affected the results.

Assistive forces that are present for a longer time help increase efficiency since they require less effort from the user. The crutch tips that offered a longer period of assistive forces were the ones with the highest durometer rating. This is likely because the stiffer crutch tips deflect less when the load of the subject is applied through them. Maintaining the KCT’s kinetic shape could contribute to the longer period of assistive forces. Additionally, a less stiff crutch tip could dampen the assistive force generated by the KCT, making it act for a shorter period.

The standard tip has a flat surface that makes it more resistive during crutch stance motion. As a result, the standard tip will have larger negative horizontal forces with a smaller percentage of positive (forward) forces. In contrast, KCTs have a greater rotation radius that can augment the positive portion of horizontal forces. Four of the Kinetic Crutch Tips had better results than the standard tip.

2.6.3 Maximum Backward Angle

Out of the eight recorded subjects, subject three had to be excluded from this result due to an error with the motion capture data collection. For the rest of the subjects, the angle of the crutch with respect to the vertical axis was calculated throughout their gait cycles. The average range of rotation of the crutch that a subject swung through during their trial on a given crutch tip was found. This angle was averaged between all valid subjects to find the total range for each crutch tip. The results of these angles have been provided in Table 2.2.

These results show a nearly identical behavior between each crutch tip. The curvature of the KCT should allow for a greater range of motion to be swung through when compared to the flat face of the standard tip. One reason for this discrepancy could be due to the experience level
of the subjects. Walking on crutches at a constant pace on a treadmill can be a difficult task for people who do not use crutches regularly. Being concerned about maintaining balance and control during gait could prevent a subject from taking advantage of the ability to swing through a larger angle in order to cover more distance.

Table 2.2: Range of rotation (total swinging angle) between crutch tips for the human subject experiment

<table>
<thead>
<tr>
<th>Crutch tip</th>
<th>Standard</th>
<th>KCT_{40}</th>
<th>KCT_{50}</th>
<th>KCT_{60}</th>
<th>KCT_{70}</th>
<th>KCT_{80}</th>
<th>KCT_{3D}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of rotation* (mean)</td>
<td>31.74</td>
<td>31.47</td>
<td>31.69</td>
<td>31.27</td>
<td>32.97</td>
<td>32.52</td>
<td>32.7</td>
</tr>
<tr>
<td>Range of rotation (STD)</td>
<td>6.731</td>
<td>6.681</td>
<td>6.674</td>
<td>6.72</td>
<td>7.03</td>
<td>6</td>
<td>7.004</td>
</tr>
</tbody>
</table>

*All values are in degrees.

The physical setup experiment gives measurements of the maximum backward angle that a crutch could fall forward without any external force (setup shown in Figure 2.8). This angle was calculated for a motion that takes out the human effects. This experiment enables us to perceive the net motion of the crutch without a user. The result of this experiment can be used to directly compare various crutch tip designs. The experiment was done for eight different angles for each crutch tip. The mean value of the maximum angle that each crutch fell forward, and the minimum angle that it fell backward was calculated. The results are shown in Figure 2.11. As expected, the standard crutch tip has the lowest value angle (−1.23°) and closer to the vertical line, which means it tended to fall back more easily. The surface of the standard tip is not perfectly flat, which caused the rotation angle to not be zero. However, this angle is still small and indicates that standard tips have the lowest tendency to lean forward and subsequently have the least assistance in aiding the users to move forward. On the other hand, the KCT_{80} has the highest value angle (−3.64°), which means it tends to move forward for a larger interval. Stiffness did not seem to make a significant
difference for the angle among KCT designs, but this could be partially due to the smaller amount of weight used compared to the weight of a person.

![Figure 2.11: Maximum backward rotation angle for different crutch tips on level ground](image)

The assistance effect of this change is noticeable when it is translated into a distance at the top of the crutch, which is where the person is attached to the crutch. This angle is equivalent to approximately four inches at the armpit for a six-foot-tall person. This distance could be of significant importance for a crutch user with temporary or permanent injuries. The importance of this angle can be seen at the leg swing period of a crutch user’s gait cycle.

The human subject experiment and the first physical setup experiment indicate that the stiffness of KCTs does not change the range of rotation and maximum backward angle significantly. Therefore, only four different crutch tips were tested for the follow-up experiment on the mechanical setup. The crutch tips were compared on three surfaces with fifteen drop tests on all twelve combinations of slopes and crutch tips, and the mean value of each transitional angle is shown in Figure 2.12. F indicates forward motion and B backward motion of the crutch. Since
the crutch is being dropped at different angles, there is a small range of angles (highlighted bars) between forward and backward motion where the direction of movement was not determined since no drops were performed in that area or the crutch did not move when the pins were removed. In other words, that angle is very near the equilibrium point. The middle of this range is defined as the transitional angle in which the freefall motion changes from resisting (i.e., backward) to assisting (i.e., forward).

As was expected, the maximum backward angles are the farthest on the left (negative side) for the 0° (flat) surface, and they gradually move toward the right (positive side) as inclination increases. The transitional angle is around $\sim[-3,-2]$ degrees for KCTs while it is close to zero or even positive values for the standard tip. The standard tip’s transition close to the vertical line is as expected since there is no moment generated in this design. All the KCTs were able to generate forward motions even when they started at farther backward angles and on uphill surfaces.

Figure 2.12: Transitional angle for the switch between forward and backward motions for four crutch tips (F: forward falling B: backward falling). All values are in degree.
As the surface inclination increased, the maximum backward angle for the forward motion decreased, meaning moving upward gets more difficult. The KCTs still provide some assistance. On all three surfaces, KCTs (KCT_{3D}, KCT_{60}, KCT_{70}) indicate a larger maximum backward angle than the standard tip. As the slope increases, a clear reduction in the transitional angles is visible for the standard tip. This means that the standard tip cannot tolerate backward angles with vertical axes in the uphill ground as much as KCTs can. In the two bottom right graphs of Figure 2.12 (standard tip in 3° and 6°), the transition area (vertical black lines) have shifted to the positive side. These areas indicate that the standard crutch tip needed to have a forward angle on 3- and 6-degrees inclined surfaces in order to move forward, meaning the standard tip requires more external force (from the user) in uphill grounds to rotate. Figure 2.13 demonstrates the exact transition angles (middle of the bars) for the crutch tips. While all KCT design performed relatively similar in all slopes, the standard design has a notably poor performance in transitioning to the onward direction.

Figure 2.13: Average transitional angle for each surface with the four different crutch tips
2.7 Conclusion

This research studies the effect of the design and hardness of KCTs on the crutch gait cycle. Two different sets of experiments evaluate the efficiency of KCTs relative to a standard tip as well as testing a range of stiffness for KCT design.

The human subject experiment studies the step length, step time, range of rotation, and percentage of positive horizontal forces over all the eight subjects for seven crutch tips. The results show an increase in the ratio of positive horizontal forces for four of the KCTs. Positive (forward) horizontal forces are assistive during movement, while negative (backward) ones are resistive. The increase in the percentage indicates that the augment in backward rotation angle for KCTs has caused the forward (assistive) horizontal forces to increase. While the results of KCT’s performance with human subjects are not fully consistent with the hypothesis, a trend of better performance in gait parameters, range of rotation, and positive horizontal forces for the KCT with higher stiffness can be observed.

The physical setup experiment purely compares the differences between crutch tips. This study includes the calculation of the maximum backward rotation angle that the crutch was able to move forward without any external forces. By using a mechanism for releasing a crutch, the effect of human characteristics is eliminated. This elimination helped directly compare the effect of crutch tips’ kinetic profile on crutch movements.

An increase in the maximum backward angle was observed for KCTs. The results show that transitional angles for KCTs are farther to the left side than the standard tip. This means that KCTs are able to move forward in steeper planes and give the crutch more stability as a result. It was shown that a KCT is able to move forward with a negative angle of -3.64 deg while a standard tip can only move forward with a negative rotation angle of -1.23 deg. This augments in the
backward rotation angle were caused due to the surface kinetic shape of KCTs and confirmed the hypothesis that KCT design has a bigger range of rotation and improves the assistive forward forces of the crutch. However, various stiffnesses for the KCTs did not indicate any significant difference. Experiments with longer periods are required to test the durability and the effect of material hardness on gait parameters.
Chapter 3:  Mathematical Modeling and Simulation of Dissimilar Double Pendulums

While the experimental approach and analysis of human data have considerably influenced our understanding of human walking, developing mathematical models can also play a critical role in improving these discoveries, finding new possibilities for gait rehabilitation, and testing many configurations that may not be possible in experiments. Dynamic models consist of connected linkages with masses that imitate limb motions in space. They provide a simple yet flexible fundamental platform to study the effect of every parameter of the system on kinetic and kinematic outcomes. Dynamic models have also been extensively useful in designing robotic limbs and manipulators.

Figure 3.1: Dynamic model of an asymmetric human gait with double pendulums

There are wide ranges of designs for dynamic models from Passive Dynamic Walkers (PDW) to motorized inverted pendulums. In most cases, the system is first developed and solidified, and then the resultant kinematic and kinetic performance is simulated. Adjustments to the system are only limited to external factors such as external torques, the slope of the ground, or
initial conditions. Moreover, the majority of the dynamic models focus on symmetric systems that only imitate healthy gait. Dynamic models have a vast potential to improve our understanding of impaired walking patterns. Furthermore, studying the effects of internal parameters of dynamic models can help answer unknown aspects of disabled gait and improve designs of prosthetics or rehabilitation techniques.

The two legs of an impaired gait can be considered as two dissimilar double pendulum systems (Figure 3.1), where each system has a different trajectory leading to an asymmetric pattern between the steps. For two different double pendulums to have similar motion trajectories and kinetic outcomes, do all the physical parameters need to be the same? Or can two dissimilar systems have similar kinematic and kinetic outputs? If yes, what are the possible conditions for that to happen? This research studies the requirements for different double pendulums to have similar motion trajectories and forces. This research aims to answer the question of whether inherently asymmetric (dissimilar) double pendulum systems can be kinetically and kinematically matched?

To answer these questions, the study reforms and extends the dynamic model of a double pendulum that imitates various walking events. The model introduces a series of coefficients determining the kinetic and kinematic outcomes of any system. The effects of changing physical parameters (such as weight, link length, and center of mass) on these coefficients will develop a grouping of systems with similar coefficients and, therefore similar kinematics or kinetic performance. Changing the internal parameters of a dynamic model can have considerable benefits in the rehabilitation of impaired walking. Many rehabilitation methods use added weight to one side in order to retrain gait in a symmetrical pattern [57], [58]. Another example is finding the
distribution of weight in the design of a prosthetic leg that creates similar motion as the healthy side [59], [60].

3.1 Introduction

Human walking is a complex dynamical system. It involves a lot of internal forces and torques that are applied through multiple muscles and joints to create a desired walking trajectory. Gait is affected by both internal and external factors. The internal factors include parameters such as limb lengths and distribution of mass in the musculoskeletal system as well as control parameters of the neural system. The external factors can also significantly affect walking performance, such as the slope of the ground, assistive devices, or external stimuli.

Dynamic models are mechanisms that convert kinetic and potential energy to each other in order to create movement. This mechanism is similar to human walking, where the locomotion is the result of moving the Center of Gravity (CG) of the body while conserving the maximum amount of energy and with minimum displacement of CG in vertical or lateral direction [61]. Dynamic models have been developed in multiple shapes and forms. Their flexible structure could include different point masses, rigid bodies, additional links, dampers, and joints, making them a versatile tool in modeling human limb movements as well as robotic limbs. They typically include two or more connected links or pendulums with point masses and are capable of creating a periodic motion. Compensating for the loss of energy during walking and achieving dynamic stability are two critical issues in the simulation of dynamic models.

The most significant point of energy loss in walking is during ground impact [62]. Heel strike with the ground is an (almost completely) inelastic collision, meaning that all or most of the kinetic energy is lost when the swing leg changes to stance position. In human walking, torques created at the hip and the ankle push-off force compensate for this loss of energy and lift the stance
foot into the swing motion. Dynamic models need to compensate for the heel collision by either adding external forces or torques to the system or taking advantage of gravitational forces. Therefore, dynamic models can be divided into two categories of passive or active systems. In the passive dynamic models, the system of at least two inverted pendulums is put on a small slope, and the energy loss in every leg collision is followed by the potential energy (due to the inclination) converting to kinetic energy in the other leg. On the other hand, active systems walk over the ground, and an external input such as an impulse to the ankle or hip torque compensate for the energy loss in every step. These external parameters also act as the control parameters to the system to adjust the motion or increase the speed.

The other important topic in dynamic models is achieving dynamic stability. While a single pendulum has a stable motion, double pendulum or inverted pendulum can be highly unstable or have unpredictable, chaotic movements. The double pendulum has been shown to be chaotic in nature [63], which means that minimal variations in the initial conditions will cause the system to change the trajectory exponentially. While the double pendulum has highly nonlinear equations of motion, for small oscillations, it can be approximated with a linear set of equations. Therefore, with very small initial conditions, the system is stable, and the trajectory is predictable. This is a system that is highly sensitive to initial values, namely the two initial angles and two initial angular velocities. For relatively large angles, the double pendulum can become unpredictable and chaotic. In chaotic systems, long time predictions of outcomes are not possible. In terms of mathematics, if \( A \) is the matrix representing the change in trajectory between two identical systems with small variations in initial conditions, if the biggest eigenvalue of \( A \), \( \lambda \), is positive, the systems will grow exponentially different and are chaotic [63].
Reviewing the field of simulating human walking reveals different physical-based methods that have been developed to better understand walking dynamics, optimize the performance of bipedal robotics, and create mechanisms (such as controllers or rehabilitation techniques) that improve walking. Xiang et al. [64] summarized all these methods into five categories, including inverted pendulums, passive dynamic walkers, zero-momentum-point, optimization-based, and control-based. The authors derived the conclusion that the first three methods can be more efficiently used for robotic motions while the two latter can better simulate the complexity of human walking [64].

However, double pendulum systems have shown great potential for simulating human motions [65]–[70]. Simple models with even linear equations of motion can create trajectories similar to human walking and with reasonable accuracy [65]. The presence of gait parameters in the equations improves our understanding of physical movements. Dynamic models also provide a flexible foundation for simulating human walking, where a model outcome can improve through easily upgrading the original equations by adding extra masses to the links or putting external forces to different joints.

This study expands the modeling of double pendulums to include human-like events such as ground strike and knee-lock. The developed model introduces coefficients that represent kinematic and kinetic symmetry. Analysis of these coefficients introduces possibilities of dissimilar double pendulums with similar coefficients (and therefore similar kinetics or kinematics). The results of various simulations will demonstrate the outcome trajectories and forces of multiple double pendulums under various human-like events such as surface collision, tripping, and knee-lock. The following section provides an overview of the groundbreaking dynamic models developed in the past four decades.
3.2 Background

One of the earliest dynamic models developed for simulating human walking is the ballistic model [71]. Ballistic walking has three massless links (one for the stance leg and two for the swing), point masses for shank, thighs, and the hip. It used a passive mechanism over the ground. To compensate for the energy expenditure, the authors added an impulse during the knee-lock incident and torque at the hip so the system can walk over the ground. However, they applied these inputs as constant values to balance energy loss and without any feedback. Therefore, the ballistic walking stayed passive. The authors created a projectile similar to walking over the ground by adding constraints such as a limit to knee flexion [71]. Later, the ballistic model was improved by the addition of knee flexion to the stance leg and heel rising. The ballistic model of a biped system with knees was able to match the experimental data closely. These changes, while creating faster velocities and more accurate swing time, failed to develop accurate vertical forces from the ground. The addition of a constant pelvic rotation was also considered as a way to increase the velocity [72].

The idea of using PDW over an inclined surface was first introduced by McGeer [73]. PDWs move passively and without external force input, using only inertia and gravitational forces. The equations of motions provide a system that could be analyzed and compared to the physical world and are capable of creating cyclic patterns that are similar to the human gait. The author used linearized equations with the assumption of small angles. Knee mechanism was also added in the follow-up model [74] using nonlinear equations and numerical analysis. The key component for PDW stability is choosing the right initial values. The model starts with an initial condition for the angles and velocities and records the final values at the end of each cycle. Then the algorithm simply uses the final output as the initial condition for the next step until it reaches where initial
input and output are equal in value and opposite in direction. Repetitive cycles and stability are two important features that the authors achieved in PDW design. The authors also tested the robustness of the model by adding a small perturbation to the system. They found that perturbations added to the swing leg will dissipate within one gait cycle. However, perturbations added to the stance leg cause more disturbance, and the excitation will take several cycles to dissipate [74].

PDWs have been extensively developed afterward. The model has been simulated using collision events and switches between two-link and three-link systems [75]. The compass gait model [65], PDW motion in 3D [76], the addition of upper body and arm movements [77], and the addition of actuators [78] are some of the PDW developments.

Research has also modeled walking on a treadmill using inverted pendulums [79]. The study developed a model by deriving equations of motion for two inverted pendulums attached at the hip. Simulation over a flat treadmill requires an external force to compensate for the loss of kinetic energy at heel strike. The authors added a torque to the swing leg representing the rotational torque applied from the hip. They also modeled a push-off force during double support by adding an impulse to the stance leg. The spring coefficient for the torque input and the impulse value acted as the control parameters to stabilize the system and changed gait variables such as step length, step time, and velocity. These studies show the great flexibility of dynamic models and how well they can be expanded and adapted to simulate different walking mechanisms.

While the behavior of a healthy symmetric gait has been studied through symmetric dynamic models, very few of them have been developed to simulate pathological gait trajectory with asymmetry [80]–[83]. One of the limited studies in this area is Asymmetric Passive Dynamic Walkers (APDW) [82], [83]. The two-link APDW simulation indicated stable and periodic behavior up to 5% change in the mass and length ratios between the two links [83]. The APDW,
with the addition of the knee, was able to walk with a stable pattern over a slope by up to 15% asymmetry between the two links [82]. The follow-up research demonstrated a simulation of lighter prosthetic legs with an asymmetric proportion from the healthy leg that produced a symmetric walking trajectory [59]. Comparing the simulation result with human experiments with asymmetric gait (adding extra weight to one leg) indicated a similar pattern and GRF between the model and human data [84].

However, achieving a symmetric trajectory within an asymmetric dynamic model is not possible with every configuration. Handžić et al. [85] demonstrated that having only one mass on each link of a double pendulum system couples the moments of inertia. Therefore, any change in the system parameters would mean a change in the motion trajectory or force reactions since not enough parameters (masses, moments, or moments of inertia) exist to compensate for the change. However, adding a second point mass to each link creates an underdetermined system that would allow for a change in parameters without a change in kinematics. The authors derived Kinematically Matched Coefficients (KMC) that determine the system trajectory.

Dissimilar systems yielding similar motion can have a hugely beneficial effect in our understanding of pathologic limb movements and consequently, the rehabilitation of asymmetric gait. Prosthetic limb users, crutch walkers, and stroke survivors all have asymmetrical movements and can benefit from alternative gait pattern that produces symmetric motions. In this research, I analyze the motion of double pendulum systems under different conditional constraints and study both the kinetic and kinematic outcomes.

3.3 Main Contributions

The previous research work [85] introduced the idea of replacing groups of physical parameters in the double pendulum system with coefficients. However, the authors only
considered an eventless motion and did not study the system under collisions. They also did not include the kinetic equations. The study also did not solve the system of coefficient equations for parallel solutions and only tested one configuration.

In this research, I develop on the KMC concept and extend it to include human-like events and collisions. I also investigate the kinetic outcomes of dissimilar systems and introduce new sets of coefficients for a comprehensive analysis. The goal of the current research is to answer the research question of whether “kinematic and kinetic symmetry can be achieved within an inherently asymmetric system?” My contributions to this area include:

- Extending the application of coefficients to contain single pendulum, knee-lock collision, and multiple types of collision with external surfaces.
- Introducing new coefficients to define the collisions and kinetic measurements at the hip and knee joints
- Developing a logical algorithm for double pendulum trajectory motion
- Numerically solving the system of coefficients and finding group solutions for both humanly and physically realizable data sets.

3.4 Extended Modeling of Double Pendulum

This section will develop and reorganize Lagrange’s equations for a double pendulum system trajectory. The human-like events such as knee joint and ground impact will be added during the motion to replicate similar collisions as in human walking. New coefficients will be introduced to replicate similar trajectories among dissimilar systems. The analysis of the system of the coefficients will provide a path toward developing symmetric motions using asymmetric systems.
Figure 3.2: Double pendulum with four masses

The double pendulum has four masses (two per link) with hinge joints at the top and between the two links. As mentioned in previous research [85], only having one mass per link will create restrictive motions with an overdetermined system in the double pendulum that would not allow for sufficient flexibility. Therefore, this study uses a double pendulum with four masses. Figure 3.2 shows the double pendulum parameters, including mass locations, links lengths, and the angle of each link at a given time. Two local polar coordinate systems were attached at the hinge of each link.

Section 3.4.1 develops the kinematics for the double pendulum and calculates the angular displacement, angular velocity, and angular acceleration of each link. Two sets of equations of motion describe models for a bent leg (two links) and a straight leg (one link). Section 3.4.2 derives the equations for multiple collisions and impact events. These two sections together demonstrate the trajectory of the double pendulum in 2D space. Section 3.4.3 calculates the forces at the top (hip), and the middle (knee) hinge at any given time. At each section, new coefficients reorganize the equations and replace the physical parameters of the system.
3.4.1 Lagrange’s Equations

In order to drive the equations of motion using Lagrangian mechanics, total kinetic, and potential energies of the system are calculated. $\theta_1$ and $\theta_2$ are the angle of the first link with vertical line and the angle of the second link with the first link, respectively. $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular velocities with respect to time. $\ddot{\theta}_1$ and $\ddot{\theta}_2$ are the angular accelerations. The velocity of each mass in Figure 3.2 is:

\[ \vec{V}_{1a} = \vec{\omega}_{1a} \times \vec{r}_{1a} = \dot{\theta}_1 \hat{k}_1 \times l_{1a} \hat{r}_1 = l_{1a} \dot{\theta}_1 \hat{\theta}_1 \]  
\[ \vec{V}_{1b} = \vec{\omega}_{1b} \times \vec{r}_{1b} = \dot{\theta}_1 \hat{k}_1 \times (l_{1a} + l_{1b}) \hat{r}_1 = (l_{1a} + l_{1b}) \dot{\theta}_1 \hat{\theta}_1 \]  
\[ \vec{V}_{2a} = V_1 + (\dot{\theta}_1 + \dot{\theta}_2) \hat{k}_2 \times l_{2a} \hat{r}_2 = l_1 \dot{\theta}_1 \hat{\theta}_1 + l_2 (\dot{\theta}_1 + \dot{\theta}_2) \hat{\theta}_2 \]  
\[ \vec{V}_{2b} = V_1 + (\dot{\theta}_1 + \dot{\theta}_2) \hat{k}_2 \times (l_{2a} + l_{2b}) \hat{r}_2 = l_1 \dot{\theta}_1 \hat{\theta}_1 + (l_{2a} + l_{2b}) (\dot{\theta}_1 + \dot{\theta}_2) \hat{\theta}_2 \]

Here, a vector is differentiated from a scalar by the arrow sign on top. Unit vectors of the local coordinates are noted by a hat sign whenever the magnitude and the direction of a vector are separated. It is important to note that the two local systems of the links can be transferred using Equations (3.5)-(3.7).

\[ \hat{r}_2 = \sin \theta_2 \hat{\theta}_1 + \cos \theta_2 \hat{r}_1 \]  
\[ \hat{\theta}_2 = \cos \theta_2 \hat{\theta}_1 - \sin \theta_2 \hat{r}_1 \]  
\[ \hat{k}_2 = \hat{k}_1 \]

Therefore, the last two velocities in the Equations (3.3)-(3.4) can be rewritten as:

\[ \vec{V}_{2a} = l_1 \dot{\theta}_1 \hat{\theta}_1 + l_{2a} (\dot{\theta}_1 + \dot{\theta}_2) \hat{\theta}_2 \]  
\[ = [l_1 \dot{\theta}_1 + l_{2a} \cos \theta_2 (\dot{\theta}_1 + \dot{\theta}_2)] \hat{\theta}_1 - [l_{2a} \sin \theta_2 (\dot{\theta}_1 + \dot{\theta}_2)] \hat{r}_1 \]
Using the velocities and the height of the masses at time $t$, the total kinetic and potential energy of the system can be derived at any time as depicted in Equations (3.10) and (3.11):

$$T_{tot} = \frac{1}{2} m_{1a} V^2_{1a} + \frac{1}{2} m_{1b} V^2_{1b} + \frac{1}{2} m_{2a} V^2_{2a} + \frac{1}{2} m_{2b} V^2_{2b}$$

$$= \frac{1}{2} \{ m_{1a} l_1^2 \dot{\theta}_1^2 + m_{1b} (l_{1a} + l_{1b})^2 \dot{\theta}_1^2$$

$$+ m_{2a} [l_1^2 \dot{\theta}_1^2 + l_2^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2 \dot{\theta}_1 \dot{\theta}_2) + 2 l_1 l_2 a \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) \cos \theta_1]$$

$$+ m_{2b} [l_1^2 \dot{\theta}_1^2 + (l_{2a} + l_{2b})^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2 \dot{\theta}_1 \dot{\theta}_2)$$

$$+ 2 l_1 (l_{2a} + l_{2b}) \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) \cos \theta_2] \}$$

$$U_{tot} = -m_{1a} g l_{1a} \cos \theta_1 - m_{1b} g (l_{1a} + l_{1b}) \cos \theta_1 - m_{2a} g (l_1 \cos \theta_1$$

$$+ l_{2a} \cos (\theta_1 + \theta_2)) - m_{2b} g (l_1 \cos \theta_1 + (l_{2a} + l_{2b}) \cos (\theta_1 + \theta_2))$$

where, $g$ is the gravity constant. Forming the Lagrangian $L = T_{tot} - U_{tot}$, the modeling can derive the equation of motion under the two variables of the system, namely $\theta_1$ and $\theta_2$. Then, the Lagrange's equations with no external force or torque would be:

$$\frac{d}{dt} \left( \frac{\partial T_{tot}}{\partial \dot{\theta}_i} \right) - \frac{\partial T_{tot}}{\partial \theta_i} + \frac{\partial U_{tot}}{\partial \dot{\theta}_i} = 0$$

(3.12)

where $i = \{1,2\}$. Calculating the Lagrange's equations for the variables of the system and reorganizing the order of parameters will result in two equations of motion as Equations (3.13)-(3.14):
\[
\begin{align*}
\mathcal{M}_{1a} l_{1a}^2 + m_{1b} (l_{1a} + l_{1b})^2 + m_{2a} (l_1^2 + l_{2a}^2 + 2l_1 l_{2a} \cos \theta_2) + m_{2b} (l_1^2 + (l_{2a} + l_{2b})^2 + l_1 (l_{2a} + l_{2b})) \\
&+ (l_{2a} + l_{2b})^2 + 2l_1 (l_{2a} + l_{2b}) \cos \theta_2) ] \dot{\theta}_1 \\
&+ [m_{2a} (l_{2a}^2 + l_{1a} l_{2a} \cos \theta_2) + m_{2b} ((l_{2a} + l_{2b})^2 + l_1 (l_{2a} + l_{2b}) \cos \theta_2)] \dot{\theta}_2 \\
- 2[m_{2a} l_{1a} l_{2a} \sin \theta_2 + m_{2b} l_{1a} (l_{2a} + l_{2b}) \sin \theta_2 ] \dot{\theta}_1 \dot{\theta}_2 \\
- [m_{2a} l_{1a} l_{2a} \sin \theta_2 + m_{2b} l_{1a} (l_{2a} + l_{2b}) \sin \theta_2 ] \dot{\theta}_2^2 \\
+ [m_{1a} l_{1a} + m_{1b} (l_{1a} + l_{1b}) + m_{2a} l_{1a} + m_{2b} l_{1a}] g \sin \theta_1 \\
+ [m_{2a} l_{2a} + m_{2b} (l_{2a} + l_{2b})] g \sin(\theta_1 + \theta_2) = 0 \\
\end{align*}
\]

Nine out of ten physical parameters \((m_{1a}, l_{1a}, m_{1b}, l_{1b}, m_{2a}, l_{2a}, m_{2b}, l_{2b}, l_1, l_2)\) are used in the derivation of Equations (3.13) and (3.14). However, when the equations are reorganized, only a limited number of combinations of these parameters show up. Here, the first five coefficients for deriving dissimilar systems with similar motions will be introduced using the kinematically matched coefficients developed in [85]. These combinations are defined as the coefficients shown in Equations (3.15)-(3.19). \(a, b, d, e\) and \(f\) represents the new combination of parameters. Letter \(c\) is not used to avoid confusion with damping ratio.

\[
\begin{align*}
a &= m_{2a} l_{2a}^2 + m_{2b} (l_{2a} + l_{2b})^2 \\
b &= [m_{2a} l_{2a} + m_{2b} (l_{2a} + l_{2b})] \times l_1 \\
d &= m_{2a} l_{2a} + m_{2b} (l_{2a} + l_{2b})
\end{align*}
\]
Replacing Equations (3.13) and (3.14) with coefficients of Equations (3.15)-(3.19) demonstrates that these parameters need to be matched between two systems to have the same motion and not the full set of all physical parameters. Defining the vector \( \theta = [\theta_1 \theta_2] \), the equations of motions can be rewritten as:

\[
\begin{bmatrix}
 f + a + 2b \\
 a + b
\end{bmatrix} \ddot{\theta} - \begin{bmatrix}
 0 & b \sin \theta \\
 b \sin \theta & 0
\end{bmatrix} \dot{\theta}^2 - \begin{bmatrix}
 b \sin \theta & b \sin \theta \\
 0 & 0
\end{bmatrix} \begin{bmatrix}
 \dot{\theta}_1 \\
 \dot{\theta}_2
\end{bmatrix}
\]

\[+ \begin{bmatrix}
 g(e \sin \theta_1 + d \sin(\theta_1 + \theta_2)) \\
 g d \sin(\theta_1 + \theta_2)
\end{bmatrix} = 0 \tag{3.20}
\]

Equation (3.20) represents the kinematic motion of a double pendulum with four point-masses at any given time. If one wants to calculate the motion for a single pendulum system (the straight leg with knee hinge locked), only \( \theta_2 \) and the corresponding derivatives (\( \dot{\theta}_2, \ddot{\theta}_2 \)) need to be set to zero:

\[
(f + a + 2b) \ddot{\theta}_1 + g(e + d) \sin \theta_1 = 0 \tag{3.21}
\]

Equations (3.20) and (3.21) indicate kinematics for a double and single pendulum with four masses and no external force, respectively. In the following section, the collisions that occur during the movement are introduced. The modeling in this research will extend these coefficients to define kinematic events beyond the equations of motion. It will also introduce new kinetic coefficients to describe the internal forces of the joints.

### 3.4.2 Collisions

Collision or impact is the sudden change of motion between two or more bodies that happens due to colliding of the objects and alters internal forces within them. Two types of
collisions can be defined in dynamic models that mimic human walking. The first one is the knee-lock event and happens when the bent leg straightens out. In terms of dynamic models, during the knee-lock event, the double pendulum switches to a single pendulum by locking the hinge between the two links and preventing the second link from moving toward the opposite side. The second event is the contact with outside surfaces such as hitting the ground during heel strike or impact with an obstacle along the way. There are multiple ways to imitate collisions with a surface in dynamic models. Section 3.4.2.2 will discuss these events in further detail.

Collisions can be defined from perfectly elastic to perfectly inelastic depending on the loss of kinetic energy and the coefficient of restitution (e) [86]. A collision with no loss of kinetic energy and e=1 is a perfectly elastic collision, and a collision with maximum kinetic energy loss, and e=0 is perfectly inelastic. No matter of the collision type, angular momentum during collision is always conserved if no external force or impulse is applied. During impact (collision with external force), on the other hand, an external impulse applies to the system. The change in angular momentum before and after an impact will be equal to the angular impulse applied during the time of contact with the external object.

Including collision in the development of the double pendulum model enhances the accuracy of predicting human walking events. In deriving the equations, new and existing coefficients will replace the physical parameters of the system. The goal is to find all the coefficients required for achieving symmetry between two dissimilar systems.

3.4.2.1 Knee-Lock

As a result of a knee-lock event, equations of motion for the system switches from Equation (3.20) to (3.21). However, angles and angular velocities change due to this collision. The new values can be calculated based on the before collision condition. In a leg with two links, $\theta_2$ is
always bigger or equal to zero; meaning the knee cannot bend backward. Therefore, at $\theta_2 = 0$, the middle hinge (knee) is locked, and the double pendulum is turned into a single pendulum. The dynamic model right before and after the knee collision is depicted in Figure 3.3.

The before and after conditions are shown with $+$ and $-$ superscripts, and since no external force is involved, conservation of angular momentum is applicable:

$$\Delta P = \Sigma P^+ - \Sigma P^- = 0$$  \hspace{1cm} (3.22)

$$\sum_{i=0}^{n} \vec{r}_i^- \times m_i \vec{V}_i^- = \sum_{i=0}^{n} \vec{r}_i^+ \times m_i \vec{V}_i^+$$ \hspace{1cm} (3.23)

The total angular momentums before and after knee-lock are shown in Equations (3.24) and (3.25).

$$\Sigma P^- = [m_{1a}l_{1a}^2 + m_{1b}(l_{1a} + l_{1b})^2 + (m_{2a} + m_{2b})l_1^2 + m_{2a}l_{2a}^2$$

$$+ m_{2b}(l_{2a} + l_{2b})^2 + 2l_1 \cos \theta_2^- (m_{2a}l_{2a} + m_{2b}(l_{2a} + l_{2b}))] \dot{\theta}_1^-$$

$$+ [m_{2a}l_{2a}^2 + m_{2b}(l_{2a} + l_{2b})^2 + l_1 \cos \theta_2^- (m_{2a}l_{2a} + m_{2b}(l_{2a} + l_{2b})] \dot{\theta}_2^-$$ \hspace{1cm} (3.24)
\[ \Sigma P^+ = [m_1 a l_1^2 + m_1 b (l_1 + l_1 b)^2 + m_2 a (l_1 + l_2 a)^2 + m_2 b (l_1 + l_2 a + l_2 b)^2] \hat{\theta}_1^+ \quad (3.25) \]

Applying the conservation of angular momentum will result in a transfer matrix \( Q \), as shown in Equation (3.26). After the knee-lock, the second link only moves due to the movement of the first link. Therefore, the second row of the transfer matrix is zero. After reorganizing the equations of angular momentum, all the parameters in the first row of transfer matrix can be replaced using the same kinematic coefficients \((a, b, d, e, f)\) introduced in Equations (3.15)-(3.19):

\[
\hat{\theta}^+ = Q \times \hat{\theta}^- 
\]

\[
Q = \begin{bmatrix} q_{11} & q_{12} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} f + a + 2 b \cos \theta_2^- & a + b \cos \theta_2^- \\ f + a + 2 b & f + a + 2 b \end{bmatrix} \quad (3.27)
\]

So far, both equations of motion for the double pendulum and single pendulum as well as the knee-lock collision, were modeled and reformed by the same coefficients (Equations (3.15)-(3.19)). The following section studies the double pendulum movement under collision with external surfaces.

3.4.2.2 External Surfaces

Lower limbs repeatedly interact with the ground during walking. Each step ends with the corresponding foot hitting the ground. The ground collision (heel-strike) stops the motion of the leg and acts as the cue for initiating the movement of the other leg. Ground collisions almost always are modeled as a perfectly inelastic collision, meaning that the maximum loss of kinetic energy happens during contact, and the two collided bodies (ground and the foot) stay attached without any slipping. However, in reality, more scenarios such as partially elastic heel-strike, tripping over an object, or slipping could happen. Modeling similar events in the double pendulum system can
create an improved dynamic model that simulates similar behavior to human walking. Moreover, modeling collision with objects in the workspace is beneficial for robotic limb motions.

Here, the case of the double pendulum hitting different external surfaces on the trajectory path is considered. Similar to PDWs and other dynamic models, it is assumed that the contact happens during knee-lock, but whether the system stays in that position will depend on the condition of the collision. This study, contrary to other models that assume only perfectly inelastic collision, will consider multiple possible outcomes of the collision. Figure 3.4 shows the different predicted outcomes that are going to be discussed in the following. Regardless of the collision type, the obstacle or the wall are located at a distance $\lambda$ and the condition for the contact to happen will be:

$$(l_1 + l_2) \sin \theta_1 > \lambda$$

(3.28)

Therefore, for the collision to happen at the same time for two different double pendulums, the total length of the systems ($l_1 + l_2$) need to be the same. An additional new kinematic coefficient defines this condition:

$$h = l_1 + l_2$$

(3.29)
3.4.2.2.1 Perfectly Inelastic

In a perfectly inelastic collision, the maximum amount of kinetic energy is lost. It is also assumed that the wall has infinite mass. The angular velocity immediately after the collision is zero (Figure 3.4a). The pendulum is completely stopped by the wall, and all the kinetic energy of the system is lost. It is interesting to note that even in a perfectly inelastic collision, the angular momentum of the system is conserved. The assumption is that the external surface (the wall) has infinite mass, the velocity of the colliding system after collision becomes zero as shown in Equations (3.30)-(3.32).

\[ \Sigma P^− = \Sigma P^+ \] (3.30)

\[ \sum_{\text{All pendulum masses}} r_i \times m_i v_i + M_{\text{wall}} \times 0 = (\sum m_i + M_{\text{wall}})V^+ \] (3.31)

\[ V^+ = \lim_{M_{\text{wall}} \to \infty} \frac{\sum_{\text{All pendulum masses}} r_i \times m_i v_i}{\sum m_i + M_{\text{wall}}} = 0 \] (3.32)

However, the double pendulum is not at the equilibrium position during the collision and will start to move due to gravity. The potential energy will start to convert to kinetic energy. This collision does not add any new coefficient.

3.4.2.2.2 Perfectly Elastic

For a perfectly elastic collision, the angular momentum and kinetic energy are conserved (Equation (3.33)). It is also assumed that the knee joint is not activated (leg does not bend) because of the collision.

\[ \Delta K = 0 \& \Delta P = 0 \] (3.33)

\[ \Sigma P^− = \vec{r}_{1a} \times m_{1a} \vec{v}_{1a} + \vec{r}_{2a} \times m_{2a} \vec{v}_{2a} + \vec{r}_{1b} \times m_{1b} \vec{v}_{1b} + \vec{r}_{2b} \times m_{2b} \vec{v}_{2b} \]

\[ = \left[ m_{1a} l_{1a}^2 + m_{1b} (l_{1a} + l_{1b})^2 + m_{2a} (l_1 + l_{2a})^2 + m_{2b} (l_1 + l_{2a} + l_{2b})^2 \right] \dot{\theta}_1 (−\vec{k}) \] (3.34)
\[ \Sigma P^+ = [m_{1a}l^2_{1a} + m_{1b}(l_{1a} + l_{1b})^2 + m_{2a}(l_1 + l_{2a})^2 \\
+ m_{2b}(l_1 + l_{2a} + l_{2b})^2]\dot{\theta}_1^+ (\dot{k}) \quad (3.35) \]

After calculating the sum of angular momentum and replacing the parameters with kinematic coefficients of Equations (3.15)-(3.19), Equation (3.36) is derived.

\[ \Sigma P^- = \Sigma P^+ \rightarrow [f + 2b + a]\dot{\theta}_1^- = -[f + 2b + a]\dot{\theta}_1^+ \quad (3.36) \]

\[ \dot{\theta}_1^+ = -\dot{\theta}_1^- \quad (3.37) \]

Based on Equation (3.37), the magnitude of the velocity of the system does not change, but the direction will be reversed. The result is similar to other perfectly elastic collisions, such as a ball bouncing back from an infinite wall. Since the magnitude of the velocities and the masses stay without change before and after impact, conservation of kinetic energy also applies.

3.4.2.2.3 Tripping

The pendulum hits an object while the knee is locked. The knee immediately unlocks (bends) after the impact, as shown in Figure 3.5. In other words, the single pendulum switches to double pendulum due to an impulse applied to the foot of the pendulum. Therefore, the conservation of angular momentum does not apply. Still, the change in the momentum before and after the event will be equal to the angular impulse applied to each hinge.

Angular momentums before and after impact around the first hinge (hip) can be derived using the velocities:

\[ \Sigma \vec{P}_{H1}^- = \vec{r}_{1a} \times m_{1a} \vec{v}_{1a}^- + \vec{r}_{1b}^- \times m_{1b} \vec{v}_{1b}^- + \vec{r}_{2a}^- \times m_{2a} \vec{v}_{2a}^- + \vec{r}_{2b}^- \times m_{2b} \vec{v}_{2b}^- \]

\[ = [m_{1a}l^2_{1a} + m_{1b}(l_{1a} + l_{1b})^2 + m_{2a}(l_1 + l_{2a})^2 \\
+ m_{2b}(l_1 + l_{2a} + l_{2b})^2]\dot{\theta}_1^- (-\dot{k}) = [f + a + 2b]\dot{\theta}_1^- (-\dot{k}) \quad (3.38) \]
\[ \Sigma \dot{p}_{H1}^+ = \dot{r}_{1a}^+ \times m_{1a} \ddot{v}_{1a}^+ + \dot{r}_{1b}^+ \times m_{1b} \ddot{v}_{1b}^+ + \dot{r}_{2a}^+ \times m_{2a} \ddot{v}_{2a}^+ + \dot{r}_{2b}^+ \times m_{2b} \ddot{v}_{2b}^+ \]
\[ = [m_{1a} l_{1a}^2 + m_{1b}(l_{1a} + l_{1b})^2 + m_{2a}(l_1 + l_{2a} \cos \theta_2)^2 \]
\[ + m_{2b}(l_1 + (l_{2a} + l_{2b}) \cos \theta_2)^2 + m_{2a}l_{2a}^2 \sin^2 \theta_2 \]
\[ + m_{2b}(l_{2a} + l_{2b})^2 \sin^2 \theta_2] \dot{\theta}_1^+ (-\dot{\theta}) \]
\[ + [m_{2a}(l_{2a})^2 + m_{2b}(l_{2a} + l_{2b})^2 + m_{2a}l_1 l_{2a} \cos \theta_2 \]
\[ + m_{2b} l_1(l_{2a} + l_{2b}) \cos \theta_2] \dot{\theta}_2^+ (\dot{\theta}) \]
\[ = [f + a + 2b \cos \theta_2^+] \dot{\theta}_1^+ (-\dot{\theta}) + [a + b \cos \theta_2^+] \dot{\theta}_2^+ (\dot{\theta}) \]

Figure 3.5: Pendulum impact with an obstacle

The impulse applied during impact can be modeled as an averaged force applied during a short time such as \( \Delta t \):
\[ I = \int_{t_{\text{start}}}^{t_{\text{end}}} f(t)dt = \bar{F}\Delta t \] (3.40)

Then, the change in angular momentum around the first hinge will be equal to the angular impulse (\(\bar{r}\Delta t\)) as calculated below:

\[ \Delta \vec{P}_{H1} = (\vec{F}. r_{\text{arm}1})\Delta t = \bar{r}\Delta t \] (3.41)

where, \(r_{\text{arm}1}\) is the vertical distance from the first hinge to the point of contact with the obstacle (Figure 3.5). Therefore, the first equation for the change in angular momentum will be:

\[-[f + a + 2b \cos \theta_2^+] \dot{\theta}_1^- + [a + b \cos \theta_2^+] \dot{\theta}_2^- + [f + a + 2b] \dot{\theta}_1^- = (\bar{F}\Delta t)[(l_1 + l_2) \cos \theta_1^-] \] (3.42)

In this equation \(\theta_1^-\) and \(\dot{\theta}_1^-\) are the state of the system before the impact and are known values. To calculate the state of the system after the impact (\(\theta_1^+, \theta_2^+, \dot{\theta}_1^+, \dot{\theta}_2^+\)), three more equations are needed. The change in angular momentums before and after impact around the second hinge (knee) can be driven similar to the first hinge:

\[ \Sigma \vec{P}_{H2}^- = \vec{r}_{2a}^- \times m_{2a} \vec{v}_{2a}^- + \vec{r}_{2b}^- \times m_{2b} \vec{v}_{2b}^- \]
\[ = [l_{2a} \hat{r}_1^- \times m_{2a}(l_1 + l_{2a}) \hat{\theta}_1^- (-\hat{\theta}_1^-)] \]
\[ + [(l_{2a} + l_{2b}) \hat{r}_1^- \times m_{2b}(l_1 + l_{2a} + l_{2b}) \hat{\theta}_1^- (-\hat{\theta}_1^-)] \] (3.43)
\[ = (a + b) \hat{\theta}_1^- (-\hat{\theta}_1^-) \]

\[ \Sigma \vec{P}_{H2}^+ = \vec{r}_{2a}^+ \times m_{2a} \vec{v}_{2a}^+ + \vec{r}_{2b}^+ \times m_{2b} \vec{v}_{2b}^+ \]
\[ = [m_{2a} l_{2a}^2 + m_{2b}(l_{2a} + l_{2b})^2 + m_{2a} l_1 l_{2a} \cos \theta_2^+] \hat{\theta}_1^+ (-\hat{k}) \]
\[ + [m_{2a} l_{2a}^2 + m_{2b}(l_{2a} + l_{2b})^2] \hat{\theta}_2^+ (\hat{k}) \] (3.44)
\[ = (a + b \cos \theta_2^+) \hat{\theta}_1^+ (-\hat{k}) + a \hat{\theta}_2^+ (\hat{k}) \]
\[ \Delta \vec{P}_{H2} = (\vec{F}, \vec{r}_{arm2}) \Delta t = \bar{r} \Delta t \]  

(3.45)

where, \( r_{arm2} \) is the perpendicular moment arm between the second hinge and the impulse force (Figure 3.5).

\[ -(a + b \cos \theta_2^+) \dot{\theta}_1^+ + a \dot{\theta}_2^+ + (a + b) \dot{\theta}_1^- = \bar{F} \Delta t l_2 \cos \theta_1^- \]  

(3.46)

Two mathematical equations based on the physical location of the pendulum can be derived; assuming the double pendulum moves along the radial direction during the impact:

\[ l_1 \dot{\theta}_1^+ = -l_2 \dot{\theta}_2^+ \]  

(3.47)

\[ l_1 \sin(\theta_1^+ - \theta_1^-) = l_2 \sin(\theta_2^+ - \theta_1^- - \theta_1^-) \]  

(3.48)

The four equations (3.42), (3.46), (3.47), and (3.48) will form the state of the pendulum immediately after the impact. It is important to note that the coefficients described in (3.15)-(3.19) were able to replace all the system parameters in these impact equations.

### 3.4.2.2.4 Slipping

The last scenario for the double pendulum and external surface colliding is the foot of the second link slipping on the surface. This introduces the stick-slip mechanism to the system. Calculating the velocity of the second pendulum over the wall will depend on the static and kinetic friction coefficients between the wall and the pendulum at the contact point. This research does not model this type of collision between the system and wall because this situation requires a lot of information and therefore assumptions about the material of the surface, the shape of the second link foot, the static and kinetic friction coefficients between the two objects, and so on.

### 3.4.3 Kinetic Measurements

Figure 3.6 shows the internal forces at the two hinges of the double pendulum. The forces are indicated in the 2-dimensional plane and in the direction of the local coordinate of each link.
Newton’s second law is used to calculate the forces at any given time during the motion of the double pendulum.

For the first hinge ($H_1$):

\[
F_{H1r} = -[m_{1a} + m_{1b} + m_{2a} + m_{2b}]g \cos \theta_1 - [m_{2a}l_{2a} + m_{2a}(l_{2a} + l_{2b})] \sin \theta_2 \ddot{\theta}_2
\]

\[
F_{H1\theta} = -[m_{1a} + m_{1b} + m_{2a} + m_{2b}]g \sin \theta_1
\]

\[
+ [m_{1a}l_{1a} + m_{1a}(l_{1a} + l_{1b}) + (m_{2a} + m_{2b})l_{1}] \ddot{\theta}_1 + [m_{2a}l_{2a} + m_{2a}(l_{2a} + l_{2b})] \cos \theta_2 \ddot{\theta}_2
\]

For the second hinge ($H_2$):

\[
F_{H2r} = -(m_{2a} + m_{2b})g \cos(\theta_1 + \theta_2)
\]

\[
F_{H2\theta} = (m_{2a} + m_{2b})g \sin(\theta_1 + \theta_2) + [m_{2a}l_{2a} + m_{2a}(l_{2a} + l_{2b})] \ddot{\theta}_2
\]

Replacing the parameters of the system with the coefficients (3.15)-(3.19) and introducing two new coefficients for kinetic forces:
\[ k = m_{1a} + m_{1b} + m_{2a} + m_{2b} \]  
(3.53)

\[ s = m_{2a} + m_{2b} \]  
(3.54)

The forces are rewritten as:

\[
F_{H1r} = -kg \cos \theta_2 - d \sin \theta_2 \ddot{\theta}_2
\]  
(3.55)

\[
F_{H1\theta} = kg \sin \theta_2 + e \dot{\theta}_1 + d \cos \theta_2 \ddot{\theta}_2
\]  
(3.56)

\[
F_{H2r} = -sg \cos(\theta_1 + \theta_2)
\]  
(3.57)

\[
F_{H2\theta} = sg \sin(\theta_1 + \theta_2) + d\ddot{\theta}_2
\]  
(3.58)

Six coefficients for the kinematic models (Equations (3.15)-(3.19) and (3.29)) define the equations of motion, knee-lock collision, and multiple types of collision with external surfaces. Achieving kinetic symmetry in addition to kinematic symmetry, introduces two more kinetic coefficients (Equations (3.53) and (3.54)).

The combined eight equations for the kinematic and kinetic coefficients include all ten physical parameters of the double pendulum \((m_{1a}, m_{1b}, l_{1a}, l_{1b}, l_1, m_{2a}, m_{2b}, l_{2a}, l_{2b}, l_2)\). If all the eight coefficients between two different double pendulums are equal, they are kinematically and kinetically symmetric. Since there are ten variables within the coefficients, the system of equations is underdetermined \((n = 8 \text{ equations and } m = 10 \text{ variables with } n < m)\). The underdetermined system creates the possibility of systems with different physical parameters but similar coefficients. However, the coefficient equations are quadratic and nonlinear. Therefore, it is important to look for answers that are physically and humanly realizable. Negative or complex solutions are not physically acceptable values. Positive values outside of anthropometric data will not be humanly realizable models.

Section 3.5 analyzes and numerically solves the undetermined system of coefficients. The analysis includes the case of achieving only kinematic symmetry \((n = 6)\) and both kinematic and
kinetic symmetry \((n = 8)\). The numerical method considers instances for physically realizable and humanly realizable variables.

### 3.5 Underdetermined System of Coefficients

The coefficients’ equations can introduce grouping of systems with different physical parameters but dynamic symmetry. There are multiple analytical approaches studied for solving underdetermined systems such as interval method or Kipnis-Patarin-Goubin [87], [88]. However, a numerical approach is beneficial for deriving solutions that are physically and humanly acceptable. The underdetermined system is solved for cases of only kinematic symmetry, and both kinematic and kinetic symmetry.

#### 3.5.1 Kinematic Symmetry

Two kinematically matched double pendulum systems would move in unison, given they start at the same initial conditions. However, the internal forces within the joints will not necessarily be the same. As a reminder, the coefficients used for the kinematic modeling (equations of motion and collisions) were:

\[
\begin{align*}
a &= m_{2a}l_{2a}^2 + m_{2b}(l_{2a} + l_{2b})^2 \\
b &= [m_{2a}l_{2a} + m_{2b}(l_{2a} + l_{2b})] \times l_1 \\
d &= m_{2a}l_{2a} + m_{2b}(l_{2a} + l_{2b}) \\
e &= m_{1a}l_{1a} + m_{1b}(l_{1a} + l_{1b}) + (m_{2a} + m_{2b})l_1 \\
f &= m_{1a}l_{1a}^2 + m_{1b}(l_{1a} + l_{1b})^2 + (m_{2a} + m_{2b})l_1^2 \\
h &= l_1 + l_2
\end{align*}
\]

Here the underdetermined system has \(n = 6\) equations and \(m = 10\) variables. The \(a, b, d, e, f, h\) represent parameters that stay constant between the parallel systems. Equations (3.16) and (3.17) indicate that \(l_1\) won’t change between the systems:
\[ b = d \times l_1 \rightarrow l_1 = \frac{b}{d} \]

From Equation (3.29), \( l_2 \) will be constant as well:

\[ l_2 = h - l_1 \]

With \( l_1 \) and \( l_2 \) constant, a reduced underdetermined system with \( n = 4 \) equations ((3.15), (3.17), (3.18), and (3.19)) and \( m = 8 \) variables \((m_{1a}, m_{1b}, l_{1a}, l_{1b}, m_{2a}, l_{2a}, m_{2b}, l_{2b})\) remains.

These four equations \((a, f, d, e)\) represent moments and moments of inertia for the second link and the entire pendulum. Numerical approach is used to solve for two instances, physically realizable and humanly realizable data. The two samples are chosen for a simple normalized physical system and a set of average anthropometric data from [89]. Table 3.1 shows the data for the sample systems.

Table 3.1: Physically and humanly realizable sample systems for the numerical solution

<table>
<thead>
<tr>
<th></th>
<th>( m_{1a} )</th>
<th>( l_{1a} )</th>
<th>( m_{1b} )</th>
<th>( l_{1b} )</th>
<th>( l_1 )</th>
<th>( m_{2a} )</th>
<th>( l_{2a} )</th>
<th>( m_{2b} )</th>
<th>( l_{2b} )</th>
<th>( l_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sys A</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Sys B</td>
<td>4.0</td>
<td>0.2</td>
<td>3.5</td>
<td>0.2</td>
<td>0.6</td>
<td>2</td>
<td>0.1</td>
<td>1.5</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

*All masses are in kilogram (kg) and lengths are in meter (m).

Now, the numerical method can solve for all possible combination of physical parameters that result in the same coefficient for each system in Table 3.1.

Starting with System A, \( l_1 \) and \( l_2 \) will stay without change and equal to one for all the parallel systems based on the beginning discussion. However, the point masses and their locations on each link can vary between the systems. If all variables can change without limits, an infinite number of possible combinations of parameters exist depending on the precision. However, not all the combinations have the same coefficients as System A, nor are all physically meaningful. The numerical method applies the four equations of the reduced system of coefficients \((a, d, e, f)\) as well as physical restrictions such as \( l_{1a} + l_{1b} \leq l_1 \) and \( l_{2a} + l_{2b} \leq l_2 \). The range for each variable is set between zero and one, and the precision for length values is set to 0.1.
A total of 243 systems parallel to System A were derived. Each system in this grouping is different from others in physical parameters, but all of them have the same kinematic coefficients values, meaning all the kinematic equations will result in the same output. Figure 3.7 demonstrates the whisker plot for all the changes in the physical parameters of systems kinematically parallel to System A.

Figure 3.7: Change of physical parameters for parallel systems to System A

Figure 3.8 shows four examples of these systems compared to System A. The variations in the physical parameters indicate the possibilities of interchangeable designs that could be achieved with the grouping of systems with similar coefficients.

For example, let’s say a robotic manipulator or a crane can be modeled as a double pendulum with the same physical parameters as System A. The ratio of the total mass of the first link to the second link in the original System A is \( \frac{M_{1st}}{M_{2nd}} = 1 \). If one wants to decrease this ratio and design a similar device (like a robotic arm) with a lighter first link and heavier second link that
would create the same kinematics, the kinematic coefficients offer different kinematically parallel systems with up to 60% decrease in total mass ratios. Conversely, a machine such as a wrecking ball can benefit from maximizing this ratio. Figure 3.9 shows the system with minimum and maximum $\frac{M_{1st}}{M_{2nd}}$ from the group of 243 derived systems.

![Figure 3.9](image)

Figure 3.8: Examples of systems with similar kinematics as System A

The same process can help with various other design alternatives, such as designing the lightest or heaviest possible device without affecting the motion of the double pendulum. The system of coefficients represents a lot of potential for optimizing the design without sacrificing the desired outcomes.

This benefit gets more important when designing rehabilitation devices for disabled people. Creating assistive devices such as prosthetic legs that are mechanically capable of creating a symmetric gait pattern and are lighter in weight could potentially improve the walking experience and efficiency for amputees, crutch users, and stroke survivors.
Figure 3.9: Kinematically parallel system to System A with lightest and heaviest links’ mass ratios

System B from Table 3.1 demonstrates a sample of anthropometric data for lower limbs. Similar to the procedure for System A, \( l_1 \) and \( l_2 \) stay without change and equal to 0.6 and 0.5 meters respectively for all solutions. The accuracy of variables’ change is set to 0.1 for a finite number of solutions. Moreover, the numerical method assumes that each total link mass can change up to two times the original total mass for the same link in System B. Physical restrictions also prevent the total length values on each link from being bigger than the link length.

The numerical method resulted in 48 kinematically parallel systems with System B after applying all the physical and human restrictions. All the parameters for the 48 humanly realizable answers are indicated in Appendix B. Figure 3.10 shows two examples of these systems compared to System B. The first example indicates the lightest possible double pendulum design that creates the same kinematic outcome as System B. The second example demonstrates the lowest ratio of the first link total mass to the second link. All these dissimilar double pendulum models create the same trajectory of motion, angular positions, velocities, and acceleration for both links.
Figure 3.10: Examples of parallel systems to System B with the lowest total mass and the lowest first link to the second link mass ratio. All masses are in kg.

Figure 3.11 demonstrates an application of dissimilar double pendulums using the systems in Figure 3.10. If an amputee has the same anthropometric data as System B on their healthy leg (Figure 3.11a), designs of a prosthetic leg that can have similar kinematic outcomes could benefit from the 48 derived systems with similar kinematic coefficients.

Figure 3.11b shows the lightest prosthetic leg design with a 9% decrease in total mass. Figure 3.11c demonstrates a design that minimizes the ratio of total mass between the two links by 40%. Similar kinematics between a healthy leg and a prosthetic leg can assist with the improvement of symmetric gait patterns where both sides are able to walk with similar step times and step lengths.
3.5.2 Kinetic Symmetry

While systems with kinematic symmetry have similar trajectories, the forces generated in the hip or knee joint might not be the same. To achieve kinetic symmetry along with kinematic symmetry, all the eight coefficients (Equations (3.15)-(3.19), (3.29), (3.53), and (3.54)) between two dissimilar systems should be the same. Section 3.5.1 numerically solves the first six kinematic equations. The final two kinetic equations of the coefficients are:

\[ k = m_{1a} + m_{1b} + m_{2a} + m_{2b} \]  

(3.53)

\[ s = m_{2a} + m_{2b} \]  

(3.54)

Equation (3.53) is the condition for forces at the hip joint and indicates if the total mass of the double pendulum system remains unchanged, the internal forces at this joint will be identical.
Equation (3.54) is for the knee joint forces and has the condition for the total mass of the second link to remain unchanged.

For System A, a group of 243 systems with identical kinematics were derived. Applying both equations of kinetic symmetry among them returns zero results; meaning that there is no dissimilar double pendulum that can have kinematic and kinetic symmetry with System A within these numerical solutions. However, 13 systems out of this group comply with Equation (3.53), meaning they have the same total mass as System A. These systems generate similar force reactions at the hip joint based on Equations (3.55) and (3.56) since both \( k \) and \( d \) coefficients match. They have the same motion trajectory, angular velocities, and angular accelerations, but the forces generated at the knee joint will be asymmetric. Applying only Equation (3.54) returned 14 different systems with similar forces at the knee joint. No cross-over exists between the results of the last two coefficients, indicating that full kinematic and kinetic symmetry between dissimilar systems, in the range that simulates human realizable constraints, is not possible.

Among all the 48 solutions of kinematically parallel systems to System B, none has complete kinetic symmetry (numerically solving for both kinetic coefficients return zero results). Moreover, no system has the same total mass as System B, and therefore, the forces at the hip joint will not be similar to System B in any of the systems. However, 8 out of the 48 systems comply with Equation (3.54) and have the same total mass on the second link as System B. The data for these systems are highlighted in Appendix B. These systems have partial kinetic symmetry and generate similar internal forces at the knee joint while the hip joint forces will be different, confirming that fully kinetic and kinematic symmetry between dissimilar double pendulums is not possible within the range of physically or humanly realizable solutions.
3.5.3 Full Symmetry

The underdetermined system of coefficients can have an unlimited number of solutions if the human and physical constraints are removed. This section briefly overviews the answers that satisfy all the coefficients’ equations but are not humanly or physically realizable systems.

Solving for all possible answers without any constrains returns more than a thousand solutions to the underdetermined system of equations. The precision was set to 0.1 for mass values and 0.01 for length values to keep the numerical results at a finite number. However, the constrains for only positive real values, and range restriction for mass locations was removed. Less than one percent (~0.8%) of the $10^5$ tested combinations match all the coefficients of System A. Table 3.2 shows two examples of these answers compared to System A. Each solution has imaginary or negative values that are physically unrealizable. Therefore, while there are mathematical solutions to the underdetermined system of coefficients, they are not meaningful physical systems.

Table 3.2: Sample answers for Sys A satisfying all coefficients without physical constraints

<table>
<thead>
<tr>
<th></th>
<th>$m_{1a}$</th>
<th>$l_{1a}$</th>
<th>$m_{1b}$</th>
<th>$l_{1b}$</th>
<th>$l_1$</th>
<th>$m_{2a}$</th>
<th>$l_{2a}$</th>
<th>$m_{2b}$</th>
<th>$l_{2b}$</th>
<th>$l_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sys A</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Ans I</td>
<td>4</td>
<td>0.75+ 0.22i</td>
<td>-3</td>
<td>0.07i</td>
<td>1</td>
<td>-2</td>
<td>0.75 - 0.31i</td>
<td>3</td>
<td>0.102i</td>
<td>1</td>
</tr>
<tr>
<td>Ans II</td>
<td>-0.5</td>
<td>0.75 - 0.43i</td>
<td>1.5</td>
<td>0.29i</td>
<td>1</td>
<td>-1.9</td>
<td>0.75 + 0.31i</td>
<td>2.9</td>
<td>-0.11i</td>
<td>1</td>
</tr>
</tbody>
</table>

Solving with the same process for System B returns more than a thousand solutions (~0.01% of $10^5$ systems). Table 3.3 demonstrates two samples of answers with all eight coefficients identical to System B. All answers contain humanly unrealizable values. Ans I has negative values for the locations of the first masses on each link. While this answer is physically achievable (by attaching masses on massless links extended outside of the system), Ans I is not a humanly realizable solution. Ans II has negative mass values and complex values for lengths that are physically and humanly unrealizable.
Table 3.3: Sample answers for Sys B satisfying all coefficients without human constraints

<table>
<thead>
<tr>
<th></th>
<th>(m_{1a})</th>
<th>(l_{1a})</th>
<th>(m_{1b})</th>
<th>(l_{1b})</th>
<th>(l_1)</th>
<th>(m_{2a})</th>
<th>(l_{2a})</th>
<th>(m_{2b})</th>
<th>(l_{2b})</th>
<th>(l_2)</th>
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</thead>
<tbody>
<tr>
<td>Sys B</td>
<td>4.0</td>
<td>0.2</td>
<td>3.5</td>
<td>0.2</td>
<td>0.6</td>
<td>2</td>
<td>0.1</td>
<td>1.5</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>Ans I</td>
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<td>-0.08</td>
<td>7</td>
<td>0.40</td>
<td>0.6</td>
<td>0.5</td>
<td>-0.06</td>
<td>3</td>
<td>0.28</td>
<td>0.5</td>
</tr>
<tr>
<td>Ans II</td>
<td>-3.5</td>
<td>0.29+ 0.18i</td>
<td>11</td>
<td>-0.12i</td>
<td>0.6</td>
<td>1</td>
<td>0.03</td>
<td>2.5</td>
<td>0.22</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The numerical method solves the underdetermined system of coefficients for a subset of possible solutions. Increasing the precession and range of the variables can return more answers for the datasets. Analytical solutions can determine whether full kinetic and kinematic solutions are possible. In the following section, simulations of the derived parallel systems for the human dataset (Table 3.1) examine the accuracy of the coefficients to predict kinematic and kinetic symmetry.

3.6 Simulations

This section provides a proof of concept for the model of dissimilar double pendulums by simulating the results of kinematic and kinetic outcomes. Full kinetic and kinematic symmetry between dissimilar systems was not achieved for physically or humanly realizable data. However, the partial kinetic similarity in one of the joints was possible between a number of systems. The simulations demonstrate the outcomes of the anthropometric data of a healthy leg, and two of the systems derived based on the matching coefficients, one with only kinematic symmetry and one with kinematic symmetry and partial kinetic symmetry at the knee joint.

The simulation results compare the angle, angular velocity, angular acceleration, and the reaction forces at both joints. The modeling considers the different cases of plastic or elastic collision and impact with external surfaces. The goal of this section is to show that the new coefficients introduced in this research can accurately predict similarity in kinematic and kinetic outcomes of dissimilar double pendulums.
3.6.1 Algorithm Logic

Figure 3.12 introduces the algorithm logic for simulating the dynamics of the system. The programming was written in MATLAB R2019a. The double pendulum starts with an initial condition as an input. At every increment of time, the angular positions, velocities, and accelerations of the two links are calculated. The system can be locked at the hinge as not to allow the second link to move toward the opposite side. Therefore, if $\theta_2$ becomes zero, knee is locked, and the double pendulum switches to a single pendulum. During single pendulum mode and before reaching the external surface ($l_1 + l_2 \sin \theta_1 < \lambda$), a pseudo-double pendulum tracks the angular position and angular velocity of the unlocked second link at every time step. The moment $\theta_2$ in the pseudo-double pendulum turns positive, the real system is unlocked and will switch back to the double pendulum mode.

When the system reaches the external surface ($l_1 + l_2 \sin \theta_1 = \lambda$), three different types of collisions (described in section 3.4.2) can happen. For a locked knee after the collision, if the contact is perfectly elastic, an elastic collision determines the change in velocities and positions of the system afterward. If the contact is perfectly inelastic, the plastic collision dictates the change afterward. For an unlocked knee, an external impulse is applied to the system, and the state of the double pendulum is calculated based on the discussion in section 3.4.2.2.3 for tripping.

Based on the condition of the knee joint and elasticity of the collision, the results of a perfectly elastic collision, perfectly inelastic, or an impulse impact are demonstrated in the following section. Three sets of models for anthropometric data discuss and compare the kinematic and kinetic similarity between dissimilar double pendulums.
Figure 3.12: Algorithm flowchart for simulation of a double pendulum system with knee-lock and different collisions with external surfaces. Yellow diamonds indicate the conditional statements and make decisions depending on the answer to the questions. Red circles are instance changes in the state of the system under a sudden collision. Blue rectangles are the processors for calculating the state of the system (angles, angular velocities, angular accelerations).

3.6.2 Results

The simulation results compare three dissimilar double pendulums, as indicated in Table 3.4. System 1 has the same physical parameters as the anthropometric data in Table 3.1. Both System 2 and 3 are chosen from the group of 48 parallel systems (Appendix B) derived based on similar kinematic coefficients. Systems 2 is the data set with only kinematic symmetry to System 1 and System 3 has kinematic symmetry and partial kinetic symmetry with System 1 at the knee.
Table 3.4: Physical parameters for simulation of dissimilar double pendulums

<table>
<thead>
<tr>
<th></th>
<th>(m_1a)</th>
<th>(l_{1a})</th>
<th>(m_{1b})</th>
<th>(l_{1b})</th>
<th>(l_1)</th>
<th>(m_{2a})</th>
<th>(l_{2a})</th>
<th>(m_{2b})</th>
<th>(l_{2b})</th>
<th>(l_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sys 1</td>
<td>4.0</td>
<td>0.2</td>
<td>3.5</td>
<td>0.2</td>
<td>0.6</td>
<td>2</td>
<td>0.1</td>
<td>1.5</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>Sys 2</td>
<td>12.0</td>
<td>0.1</td>
<td>2.33</td>
<td>0.5</td>
<td>0.6</td>
<td>2</td>
<td>0.2</td>
<td>0.83</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>Sys 3</td>
<td>9.5</td>
<td>0.1</td>
<td>2.5</td>
<td>0.4</td>
<td>0.6</td>
<td>2</td>
<td>0.1</td>
<td>1.5</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The simulation includes various human-like events. The kinematic measurements include angular positions, velocities, and acceleration of each link. The kinetic measurements are radial and angular forces at the hip joint and knee joint. The study hypothesizes that all systems in Table 3.4 have kinematic symmetry, but the only kinetic symmetry happens between System 1 and 3 at the knee joint.

The simulation can include contact with three different external surfaces: perfectly elastic, perfectly plastic, and tripping impact. As all the results confirm the hypothesis, the case of perfectly elastic and tripping are demonstrated here, and the results of the perfectly plastic collision are indicated in Appendix C for more clarity.

Figure 3.13 shows the kinematic outcomes of the three systems with an elastic collision along the way. The colliding surface is located at 0.4 \(m\) vertical distance to the left of the hip joint. The frequency of the simulation is set to 1 \(ms\) and the result are depicted for 5 seconds of pendulum movements. The initial condition is set to \([\theta_1 \ \theta_2 \ \dot{\theta}_1 \ \dot{\theta}_2] = [0.6 \ 0.4 \ 0 \ 0]\). All the kinematic outcomes in Figure 3.13 match between the three systems. This result confirms the hypothesis that systems with kinematically matched coefficients move along the same trajectory and in unison.
Figure 3.13: Kinematics of three dissimilar double pendulums under elastic collision. a) angular positions. b) angular velocities. c) angular accelerations.

Figure 3.14 demonstrates the kinetic outcomes at the hip joint and knee joint for forces in the radial and angular directions. As expected, the forces at the hip joint are different for all three systems. However, System 1 and 3 have matching forces at the knee joint.
Figure 3.14: Kinetic outcomes of three dissimilar double pendulums under elastic collision. a) forces at the hip joint. b) forces at the knee joint.
Figure 3.15 and Figure 3.16 indicate the kinematic and kinetic outcomes of the same systems with modeling of a tripping incident along the way. The obstacle is located at the same 0.4 m distance from the hip and a small impulse force is applied during 1 ms contact with an average force of 5 N.

Figure 3.15: Kinematics of three dissimilar double pendulums under tripping impact. a) angular positions. b) angular velocities. c) angular accelerations.
Figure 3.16: Kinetic outcomes of three dissimilar double pendulums under tripping impact. a) forces at the hip joint. b) forces at the knee joint.
3.7 Discussion

Dynamic models offer a simple yet flexible approach for understanding the mechanics of walking. Double pendulums can closely model human walking patterns. Each leg consists of two links (thigh and calf) connected with a hinge (knee). Dissimilar double pendulums can model asymmetric gaits where the physical characteristic of each side is different from the other. Understanding the influence of physical parameters of legs on kinematic and kinetic performance can help develop assistive devices and rehabilitation techniques capable of generating symmetric gait between inherently asymmetric sides.

This research redefines the kinematic and kinetic equations describing double pendulums, including equations of motion, collision events, and internal forces at the joints. A set of new coefficients replaces all physical parameters of the system in the kinematic and kinetic equations. The approach indicates that not all parameters individually affect the outcome performance of the double pendulum; rather, various combinations of them collectively influence the kinematic and kinetic measurements. This discovery creates flexibility with choosing system parameters that can create a desired motion or force reaction. This flexibility can significantly help in creating symmetric walking in an asymmetric gait. For instance, a prosthetic leg does not need to have the exact physical characteristics as the unimpaired side to have a similar step time and step length.

The model in this study introduces six coefficients for kinematic symmetry and two coefficients for kinetic symmetry. The double pendulum model has ten different physical parameters, and all of them are used in the coefficients’ equations. Therefore, the coefficients shape an undetermined system, meaning there are more physical parameters in the system than the required conditions for symmetry. However, the solution needs to find only physically and humanly meaningful answers.
Two sets of physically realizable and humanly realizable data sets were considered. The system of coefficients is numerically solved for acceptable dissimilar answers. For kinematic symmetry, 243 dissimilar systems were found for the physical data set and 48 for the anthropometric data set. Solving for fully kinematic and kinetic symmetry returned unrealizable results for both data sets. However, partial kinetic symmetry (along with kinematic symmetry) in one of the joints (hip or knee) were found among several systems. Simulation of the sample systems for the anthropometric data confirmed the accuracy of the coefficients in achieving kinematic symmetry and partial kinetic symmetry. While these data sets did not return any dissimilar system with complete dynamic symmetry, further analysis with different data sets can reveal otherwise.

This research introduces a new idea to redefine the modeling of dynamic systems. The introduction of coefficients acts as a connection between the physical parameters of the system and multipliers defining the dynamic equations. In a system with fewer coefficients than parameters, a range of flexibility in the design can create asymmetric systems with matching dynamic outcomes. These innovative coefficients can potentially help improve our understanding of asymmetric gait and optimize the techniques and devices that promote symmetry.
Chapter 4: A Multi-Rehabilitation Technique for Post-Stroke Walking

4.1 Introduction

Gait rehabilitation therapies entrain walking patterns by modifying the environment or providing additional controlled feedback or sensory input. Asymmetric walking is a common gait abnormality, especially among stroke survivors. Interventions incorporating various adaptation techniques (such as treadmill training, auditory stimulation, visual biofeedback, etc.) have been developed to retrain gait toward symmetry. However, single rehabilitation techniques come up short of affecting all aspects of gait performance. Multiple-rehabilitation therapy applies simultaneous stimuli to affect a wider range of gait parameters and create flexible training regimens. This research investigates how gait response combines the effect of multiple stimuli applied simultaneously.

Understanding gait responses to individual and jointly applied stimuli is important for developing improved and efficient therapies. In this study, 16 healthy subjects participated in an experiment with four sessions to test the hypothesis of a linear model for gait response. Each session consisted of two stimuli, treadmill training and auditory stimulation, with symmetric or asymmetric ratios. We found that the superposition principle largely applies to the gait response under two simultaneous interventions. The linear models developed in this study fit the actual data from experiments with the r-squared values of 0.96 or more.

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2 A study was published based on the preliminary results in [141]. Copyright permission was acquired in Appendix A.
4.2 Background

Human walking is a complex yet flexible mechanism. It involves hundreds of muscles and bones controlled by elaborate pathways and interconnections of the neuromusculoskeletal system. Damage to this system can significantly decrease functional performance and quality of life [90], [91]. Stroke is the third cause of disability worldwide [90], with 80% of the survivors suffering from walking dysfunction [91]. Hemiparesis (hemiplegia), defined as muscle weakness (paralysis) on one side of the body, are common conditions post-stroke [92]. Hemiparesis causes asymmetric walking that considerably decreases gait functionality and daily task performance [93], [94]. Addressing these problems requires a better understanding of the gait reaction mechanism to different external stimuli. Identifying patterns and principle components of human walking response to changes in environment and sensory inputs is critical for developing appropriate rehabilitation therapies.

Motor adaptation is the temporary storage of a gait pattern immediately after a period of training [95]. The essential goal of motor adaptation is to minimize the error between the brain’s predicted outcome and the actual observed outcome while considering the cost of body movement [94]. Motor adaptation is a short-term effect and is de-adapted when training is removed (post-adaptation). Motor learning is the long-term learning of a new or relearnt pattern that can be easily switched to without requiring training beforehand [96]. While motor adaptation and motor learning interaction have not been fully understood yet, it is highly suggested that repeated motor adaptation and de-adaptation processes over time (days, weeks, or months) can lead to motor learning [94], [95], [97]; meaning that a temporary walking pattern can become permanent through continuous training. Patients with unilateral cerebral stroke have strongly demonstrated short-term
motor adaptation capabilities [98], which then can turn to long-term motor learning through repeated training [97].

Gait rehabilitation therapies post-stroke can be divided into two main categories: neurophysiology and motor learning approach [96]. The neurophysiological approach uses a developed technique (such as Bobath [99]) to reconfigure the incorrect gait pattern, and the patients take a passive role in the process. Motor learning techniques, on the other hand, require active participation from patients where they get training (with the help of the physical therapist) to improve their gait performance gradually. These training techniques incorporate different setups, modalities, and sensory inputs to entrain gait using approaches such as strength training [5], [100], treadmill walking [101]–[104], rhythmic auditory stimulation (RAS) [105]–[107], robot-assisted training [108], [109], and visual biofeedback [110]. However, Current rehabilitation therapies are unable to control and retrain all the gait parameters at the same time. In most cases, the gait performance as a whole might not improve long-term due to rehabilitation therapies that do not last long-term [98], [111], or patients are not able to transfer trained patterns from a rehabilitation facility to their home and everyday life [111]. Other reasons for ineffective therapies include techniques that lack consideration for individual strengths and weaknesses of the survivors [112] or therapies that are not entraining pathologic gait for normal walking [113].

A common training approach for symmetric gait is treadmill walking because it enforces adjusted speed to the user. Inadequate walking speed is a major contributing factor for inefficient gait among stroke survivors [114]. When matching speed on a treadmill, the spatiotemporal differences between post-stroke and healthy matched subjects were reduced [92], [93], [115], [116]. While tied-belt treadmill walking (same speed on both sides) is able to improve walking speed (and increase efficiency as a result), it is not effective in improving symmetry long-term.
Exaggerating asymmetry of an impaired gait (error augmentation) using a split-belt training (SBT) has shown a better post-training aftereffect [98], [111]. This aftereffect can be linked to the compensation mechanism in gait that is applied and stored through proprioceptors (sensory receptors within the muscles) during training [117], [118]. For instance, imagine you are skateboarding with wheels tilted toward the left. You need to constantly adjust your legs and posture to compensate for the tilted wheel and stay in a straight line. After a while, if you try a new skateboard, you will notice your gait is tilted toward the right even though your new skateboard is unflawed. This is the effect of error augmentation training, and it is used for augmenting the asymmetry in stroke survivors with the goal of reaching symmetric gait post-training. While using an SBT has improved gait speed and interleg parameters such as step length [98], [111], it does not show any aftereffect on intraleg parameters such as stride duration [98]. It is possible that the lack of sensory feedback during training may account for the limited effects. In addition, accessing an SBT is not easy or cheap, which decreases the chances of carrying over the adapted patterns to the ground and into the everyday home environment.

Rhythmic Auditory Stimulation (RAS) is another common training approach for gait symmetry that can affect a different range of gait parameters with a different impact than SBT. Research has tested the effect of RAS with various features, ranging from isochronic [119], [120] to biologically varied [113] and from metronomes [121] to music-based beats [122]. Walking parameters such as gait speed and stride length significantly decrease in post stroke. For rehabilitation it is important that improvement of walking speed does not sacrifice the stride length. Therefore, an increase in walking speed is a successful outcome when it is accompanied by no decrease in stride length. Research has shown that RAS is effective in significantly increasing speed and stride length post-stroke. Improvements in cadence and symmetry have also been
reported. A month-long training with RAS of 30 minutes for 4 times per week have demonstrated significant improvement in speed by 0.23 m/s and stride length by 0.21 m and a trend of improvement in symmetry and cadence post stroke compared to training without RAS. Clinical training suggests that an improvement of at least 0.16 m/s in gait speed is the minimal clinically important difference (MCID) for post-stroke. So addition of RAS to already exciting trainings can be a helpful tool in improving the outcomes [110].

The current study introduces a new approach for testing the effectiveness of asymmetric trainings, the Asymmetric Rhythmic Auditory Stimulation (ARAS) in which the cue duration differs between the left and right legs. The adaptation of each leg is independent of the other side [123]. Therefore, I hypothesized that ARAS is able to train each leg to their cue duration independently. Auditory stimulation is more accessible than SBT and can be easily accessed in any environment (like home) through any sound playing device. However, the effectiveness of auditory stimulation varies significantly among patients depending on their rhythmic skills [113]. While some patients have shown significant positive results, others show no or negative results [105], [113].

Rehabilitation therapies using a single intervention (such as auditory stimulation or treadmill training alone) are not capable of creating widespread training that is effective for various patients who have different aspects of asymmetric gait. The combination of rehabilitation therapies has shown better results than a single rehabilitation therapy [96], [114], giving more control over parameter changes and the gait outcome performance as a result. Multiple-rehabilitation therapy includes two or more interventions applied simultaneously with the purpose of enhancing performance by engaging more sensory feedback and increasing the control over adjusting gait parameters.
Single rehabilitation therapy can entrain gait to improve some parameters while making no change or worsening others. A multi-modality approach can increase the outcomes of clinical training and focus on the individual’s needs and capabilities. These combined rehabilitation therapies have recently started to get more attention [110], [121], [124]–[127]. Combinations of tied-belt treadmill training and symmetric RAS have been shown to enhance gait performance more than symmetric RAS overground [125], [127]. This research asks how gait patterns combine the sum of these two rehabilitation therapies applied simultaneously.

While multiple-rehabilitation therapy is starting to get more attention, the mechanism of gait response to multiple external stimuli is not completely understood yet. This study focuses on the interworking of gait performance, and the linearity of two rehabilitation therapies applied simultaneously: treadmill training and rhythmic stimulation.

Each training uses a distinct port of sensory feedback and walking modality targeting different gait parameters to various degrees. The research hypothesis of this study is that the superposition principle [128] applies to gait response; the net gait response caused by two or more stimuli (rehabilitation therapies) is the sum of the gait responses that would have been caused by each stimulus individually.

Figure 4.1 demonstrates the predicted behavior of the neuromusculoskeletal system under this hypothesis. Each parameter gets affected differently under different interventions (single rehabilitation therapy). However, if multiple-rehabilitation therapy (combining multiple interventions) is applied, the hypothesis predicts that gait combines the response with a linear model. The black box on the right is the hypothetical behavior of gait under multiple-rehabilitation therapy with therapies 1 and 2 applied simultaneously.
4.2.1 Interlimb vs Intralimb

There are two type of mechanisms that control gait performance: interleg and intraleg. Gait parameters can be divided into two categories based on this control mechanism: Intralimb and Interlimb parameters. Intralimb refers to parameters where the measurements only depend on one leg while interlimb parameters require both legs for measurement. For instance, stride length is an intralimb parameter and can be defined as the traveled distance (in anterior/posterior direction) between foot strikes on one side to the next foot strike on the same side. However, step length is an interlimb measurement of anterior/posterior distance between the two consecutive foot strikes.

Reisman et al. [98] observed interlimb and intralimb parameters show different behavior through split-belt training. Intralimb parameters changed immediately at the beginning of the adaptation period. These parameters have a feedback mechanism and show no aftereffect during post-adaptation. The top schematic of Figure 4.2 shows the general behavior. Experiments have shown only a significant difference between the baseline and early adaptation periods in intralimb parameters [98], [111].

However, interlimb parameters behave differently. After the initial increase in asymmetry, they gradually adapt to the training and return to the baseline values by readjusting their
performance (the interlimb communication). Once the adaptation condition is changed back to normal, these parameters show a robust aftereffect in the opposite direction of the training (over-correction). These parameters have significant differences between baseline and early adaptation, between early and late adaptation, and between baseline and early post-adaptation. This behavior indicates two key results that are important for rehabilitation training. First, interlimb parameters are capable of learning a new pattern for an existing developed task (walking) through a period of training (adaptation). Second, they are capable of storing the pattern at-least for a short-term period.

**Figure 4.2: Changes in interlimb and intralimb parameters during training**

Similar behavior in interlimb parameters compared to split-belt training has been observed in other experiments. For instance, in an experiment with robotic-applied resistance [129], a restrain was attached to one leg. Hip and knee flexion were measured before, during, and after walking with the robotic restrain. The result demonstrated similar patterns as interlimb parameters on Figure 4.2. Other studies such as weight increase on one leg [130] have shown similar results.

Both experiments showed an over-correction mechanism. During the adaptation, the interlimb parameters adjusted flexor muscle activity, foot placement, and timing. As a result, hip and knee flexion, single-limb support time, and step length got closer to their baseline values.
When the condition of restraining or extra weight is removed, they continue to produce the adapted pattern which creates an opposite aftereffect for a short time [95].

4.3 Main Contributions

This research creates and evaluates a new approach for rehabilitation of asymmetric gait. Previous studies investigated the effects of SBT and RAS individually. Recent studies also indicated the benefits of applying multiple symmetric rehabilitation therapies at the same time. However, I evaluate the gait performance under two simultaneous asymmetric techniques, SBT and ARAS, for the first time. The contributions of this study include:

- Developing and analyzing a new method of asymmetric therapy (ARAS) by creating auditory cues with unequal durations for each side.
- Applying two simultaneous stimuli for adaptation of gait parameters under asymmetric therapies
- Formulating the gait response model during adaptation and post-adaptation for step time, step length, and peak vertical forces.
- Introducing a personalized approach for gait rehabilitation by developing individual coefficients for each participant.

4.4 Method

4.4.1 Rationale

To investigate the linearity of the gait response, the experiment design will evaluate two properties of linear systems: (1) is the gait asymmetry under multiple-rehabilitation therapy the sum of the gait asymmetries under single therapies (additivity), and (2) if the proportion of asymmetries applied in each therapy are transferred (homogeneity). Evidence from existing research has given glimpses into the related walking mechanisms. Previous research has shown
spatial and temporal gait parameters are accessed through distinct neural pathways [103] and right and left legs as well as forward and backward walking are controlled through independent networks [123]. However, previous research has not yet studied (as of July 2020) the relationship between how gait responds under single and multiple-rehabilitation therapy.

This study proposes a combination of SBT and ARAS. Each method creates a type of asymmetry in the gait parameters. The study hypothesizes that each of these therapies will affect the resulting gait independently of the other. It is also important to be aware of individual differences between each subject and provide personalized models for the multiple-rehabilitation therapy that takes these differences into account to allow for future customization.

4.4.2 Experimental Design

A prospective cohort study was conducted to test our hypothesis. Each subject completed four different trials. Trials were at least 24 hours apart to make sure all residual effects from the previous trial had washed out. Multiple-rehabilitation therapy (combining treadmill training and auditory stimulation) was applied in all trials, using two out of the four interventions depicted in Figure 4.3a. Figure 4.3a shows types of therapeutic interventions and the proportion of left and right sides that were used in the trials. The two interventions (treadmill and auditory) included two different proportions (1:1 and 2:1). For both asymmetric interventions, a 2 to 1 ratio was applied between left and right to be consistent with previous research [98], [111]. In trials 1 and 2, only one of the therapies had an asymmetric 2:1 pattern (blue shapes) while the other stayed symmetric (green shapes). In the other two trials (3 and 4), both therapies were applied with an asymmetric ratio of 2:1. In trial 3, the asymmetric ratios were matched congruently where the faster belt was on the same side of the longer cue. In trial 4, the asymmetric ratios were matched incongruently, where the faster belt was on the same side of the shorter cue. Figure 4.3b shows the experimental
design of each trial. The study does not include a trial with two symmetric (green) therapies, as this has been explored in previous research [125], [127] and would not provide additional information for modeling the gait response.

![Figure 4.3: Experimental design. a) type and proportion of the interventions. b) structure of the four trials in the study](image)

4.4.3 Participants

Twenty healthy subjects were recruited initially in the study. Two subjects did not complete all the visits/trials, and their data were removed for consistency. One subject was also removed due to errors in the recording from the force plates. Another subject was not included because there were problems with marker drop during the experiments more than one time, and I refrained from redoing the experiments more than once due to the learning effect. In total, 16 healthy subjects (5 females; mean weight=70.5 kg, SD=13.0; mean height=170.9 cm, SD=9.6) with no prior history of gait impairment and no gait injury in the past 12 months formed the final group. All experimental protocols and the consent form were approved by the University of South Florida’s Institutional Review Board. Written informed consent was obtained from all the subjects prior to participation. The subjects were divided into two equal groups. Each completed all four trials (Figure 4.3b) in random order. The side of SBT in trial 2 was switched with the left belt faster for group A in trial 2 and the right belt for group B.
4.4.4 Definitions, Data Collection, and Procedure

In this study, the term ‘tied-belt’ is used when the treadmill belts have the same speed and ‘split-belt’ when each belt has a different speed. Percent asymmetry is defined as ‘left - right’ divided by the mean. I chose this definition over ‘slow - fast’ because the side of asymmetry in the trials is important for the purpose of this study. Each time window is the average of 25 left steps and 25 right steps (50 consecutive steps) located within 10 steps of the start or stop of each phase to make sure gait reaches stability (first averaged the steps within each subject, then averaged the overall group). Yellow rectangular shapes in Figure 4.4 show the approximate location of time windows used for analysis.

![Timeline of the experiment and duration of each phase. BL: baseline, EA: early adaptation, LA: late adaptation, EP: early post-adaptation, LP: late post-adaptation](image)

All experiments and data recording were conducted using the Computer Assisted Rehabilitation ENvironment (CAREN) system. This study used 11 markers on the lower body to keep track of lower limb movements, as depicted in Figure 4.5. Markers on the subject are MT (metatarsal), LM (lateral malleolus of the ankle), LEK (lateral epicondyle of the knee), ASIS (anterior superior iliac spine), and XIPH (xiphoid process of the sternum). Data were recorded at 100 Hz frequency. Prior to the first trial, subjects practiced on the tied-belt treadmill until they felt comfortable (approximately 3 minutes). During this time, they were asked to choose their comfortable speed while the operator changed the speed of the treadmill by increments of 0.1 m/s. Then the stride time of the tied-belt walking was calculated by averaging 10 consecutive steps on
the treadmill with the set comfortable speed. The fast belt was calculated by $4/3$ of the comfortable speed, and the slow belt was half of the fast belt ($2/3$ of the comfortable speed) to keep the same average speed with a 2:1 ratio. The same process was implemented for the duration of asymmetric cues in ARAS. The cues (metronomes) are spaced isochronously based on the step time. For instance, if the average step time of a subject is 750 ms, a 2:1 ratio of ARAS has a 1000 ms cue on one side and a 500 ms cue on the other. Each trial took 23 minutes to complete (Figure 4.4) and had three phases: 3 minutes of baseline (tied-belt and no sound), 15 minutes of adaptation using one of the trials 1 through 4 combinations, and 5 minutes of post adaptation (tied-belt and no sound).

![Marker setup and leg angles for the rehabilitation experiments](image)

**Figure 4.5: Marker setup and leg angles for the rehabilitation experiments**

### 4.5 Results

A complete understanding of the effect of rehabilitation therapies on gait requires a comprehensive analysis that includes all aspects of walking including spatiotemporal, kinetic, and kinematic parameters. The parameters measured in this experiment include leg angles, peak leg
flexion and extension, interlimb phase value, cross-correlation of leg angles, step length, step time, and peak vertical force. In this study, I analyze the effect of the multi-rehabilitation therapy on different aspects of gait and compare the effect of each combination of interventions on gait parameters. I will also develop a new linear model for estimating the behavior of gait under multiple simultaneous stimuli and examine the accuracy of the hypothesis by applying superposition principle to the gait parameters.

4.5.1 Lower Limb Kinematics

Gait kinematics is an important aspect of analysis for symmetric gait. Stroke survivors commonly suffer from joint impairments. Leg angles, range of motion, flexion and extension in hip, knee, or ankle can significantly be affected post-stroke. This effect includes a reduction on knee flexion during swing phase on the paretic side and less hip extension at the end of stance phase [131]. Therefore, rehabilitation therapies need to take kinematics into consideration. Leg angles, interlimb phase values, peak flexion and extension, and phase cross-correlation are among the important measurements for lower limb kinematics.

4.5.1.1 Leg Angles

In this study, leg angles are measured and compared, similar to other studies in the field [98], [111]. Moreover, secondary parameters such as interlimb phase values and signal cross-correlation can be calculated based on the leg angles. Leg angles are intralimb parameters since they depend on measurements from only one leg. Left (right) leg angle is defined as the angle of the left (right) hip to left (right) toe marker with the vertical line. Figure 4.5 indicates these angles. Leg angles similar to other intralimb parameters are reactive in nature; meaning they rapidly change during the start of the adaptation, maintain the change through training (adaptation), and immediately return back to baseline values in post adaptation. Intralimb parameters respond to the
immediate need of walking adjustments during training [95], [98]. An example of a subject change in leg angles throughout an entire trial is depicted in Figure 4.6. Red and blue are the right and left leg angle, respectively, at any given time. Each sub plot is a time window comprising of 12 steps. Figure 4.6 shows that peak flexion and extension of each leg drastically change during adaptation periods (early and late adaptation). Maximum and minimum points of leg angles are indicated by diamond and star shapes, respectively.

![Figure 4.6: Example of leg angles during time windows and peaks for a sample subject during trial 4 (incongruent)](image)

In this example, left belt speed is two times faster than right and left cues are two times longer than right cues. Immediately after the start of the adaptation (early adaptation), right leg angle range of motion decreases considerably and left leg angle range of motion increases. This change indicates that the subject is lifting their right foot at a smaller angle (almost close to vertical line) and left foot at a much bigger negative angle. While the maximum positive angle of the right foot stays at the same approximate range during early adaptation, it decreases at late adaptation. During early post-adaptation, a reverse change especially in right leg peak negative values can be
observed. Most of the changes are washed out during post-adaptation, as it can be seen at the late post-adaptation window.

4.5.1.2 Flexion and Extension

Peak leg flexion and extension on the sagittal plane can be defined based on the leg angles. Leg flexion is the bending of the knee joint where the bones as well as the toe and hip markers get closer and leg extension is straightening the knee and leg. Therefore, leg flexion here is the positive leg angles on the sagittal plane and leg extension is the negative angles. Peak flexion and extension can indicate how limb trajectory gets affected under each trial.

Figure 4.7: Average peak limb flexion and extension for group A. Solid vertical lines indicate start of each phase. Each point represents the average of 25 left and 25 right steps for all subjects during that time window.
As it can be seen in Figure 4.6, peak flexion and extension considerably change during each phase. Therefore, it is important to look at the average behavior of subjects and compare them between different trials.

Figure 4.7 shows the result for group A. Peak leg extension is affected considerably more than peak flexion in all trials. Moreover, all trials indicate the overcorrection aftereffect during early post-adaptation for peak extension. While peak flexion does not change during early adaptation, it slightly alters during late adaptation toward increased asymmetry.

4.5.1.3 Interlimb Phase Values

Interlimb parameters are better indicators of residual learning during post adaptation. A common interlimb parameter measured during split-belt treadmill for post stroke is phase value. Left and right leg phase values are defined as the duration from peak left leg flexion to peak right leg extension and peak right leg flexion to peak left leg extension, respectively [123]. These durations are shown by light blue and pink shaded areas in Figure 4.8 for the same sample subject. Limb trajectories for three consecutive steps are shown in this figure for three time windows: baseline, early adaptation, and late post-adaptation. During baseline trajectory, both left and right phase values are very close to each other (light blue and pink shaded areas are the same size). When the adaptation starts, right phase values get smaller (pink shaded areas become thinner) while the left phase value gets bigger (blue shaded areas become thicker). During early post-adaptation, a change in the opposite direction is observed when right phase values become slightly bigger and left phase values become smaller than their initial amount during baseline.
Figure 4.8: Example of limb trajectories with shaded areas indicating phase value windows during trial 4 (incongruent)

If these interlimb phase values are calculated for the entire experiment, the result can be used to compare the interlimb mechanism under the four trials. An example of the same sample subject has been indicated in Figure 4.9. During congruent and incongruent conditions, phase values indicate the most change. At the beginning of adaptation, phase values jump to opposite directions but as the training continues, the interlimb mechanism adjusts to the new pattern and the interlimb phases go back to symmetric values. During post adaptation, overcorrection of the phase values in opposite directions happens as the result of removing the adaptation training.
Figure 4.9: Interlimb phase values for a sample subject under different trials. Vertical dashed lines indicate start of each phase.
4.5.1.4 Cross-Correlation

Cross-correlation is a measurement for quantifying the displacement of two signals by producing a third signal that compares the two time-series and find the location of best match. Here, the cross-correlation function for left and right leg angles in all the four trials for a sample subject have been compared. The result shows how trial 3 (congruent) and trial 4 (incongruent) have created the most dissimilarity between leg angles (Figure 4.10). For trial 4 (incongruent) cross-correlation in Figure 4.10, during early adaptation, peak correlation value shifts forward in the gait cycle from 49.14% in the baseline to 53.45%. However, as the interlimb phase value adjusts to the training condition, they slowly reverse back to the baseline values (48.28%). As a result of this adjustment, during early post adaptation, the peak correlation shifts backward (overcorrection) and happens at 43.96% of gait cycle. After 5 minutes of post adaptation, most of the peak correlation overshoot is washed out, however, it does not completely return to the symmetric phase correlation (at 50% of the gait cycle) and stays phase lagged at 46.55%. None of the results of trial 1 (tiedbelt+ARAS) and trial 2 (SBT+RAS) indicate as big of a change as the trial 3 (congruent) and 4 (incongruent).

Averaging the lag location for the cross-correlation function for all the subjects in group A is indicated in Figure 4.11. Peak cross-correlation for a symmetric walking is at 50 percent of the gait cycle (as it can be seen in the baseline values of Figure 4.11). Trial 2 (split-belt+RAS) and trial 3 (congruent) are phase advanced during early adaptation while trial 4 (incongruent) is phase lagged. However, all of them reverse to opposite direction during early post-adaptation. Trial 1 (tied-belt+ARAS) does not change significantly and stays close to 50 percent gait cycle.
Figure 4.10: Cross-correlation function between limb trajectories for a sample subject under the four trials.

Figure 4.11: Averaged peak cross-correlation location for all subjects in group A during the four trials.
4.5.2 Spatiotemporal and Kinetic Parameters

Subjects had an average comfortable speed of 0.91 m/s (SD = 0.21) with average stride time of 1.29 s (SD = 0.18). Spatiotemporal parameters, as well as ground reaction forces (GRF), were calculated. Figure 4.12 indicates the average of percent asymmetry for step length, step time, and peak vertical force for group A during the three phases of the experiments (baseline, adaptation, post-adaptation).

Trial 1, using only one asymmetric training (ARAS), has a larger effect on step time compared to step length asymmetry. Auditory sensory feedback has been shown to influence largely the temporal parameters in the gait [132]. Trial 2, using another asymmetric training (split-belt), affects both step length and step time (step length slightly more than step time).

This result is also in accordance with previous findings that SBT engages interlimb parameters [111]. Trials 3 and 4, using two asymmetric trainings at the same time, indicate a combination of both SBT and ARAS effects. Figure 4.12c with the congruent combination is indicating a sum effect of trial 1 (tied-belt+ARAS) and 2 (split-belt+RAS), while Figure 4.12d with the incongruent combination is indicating the difference of trial 1 (tied-belt+ARAS) minus trial 2 (split-belt+RAS). In all trials, there is an opposite change in asymmetry during early post-adaptation compared to early adaptation, which indicates the neural system has temporarily stored the applied asymmetry [95].

Group B confirms the similar behavior of combined effect during trial 3 (congruent) and 4 (incongruent) and storage of adaptation through overcorrection mechanism. Group B switch asymmetry compared to group A within trial 2 (split-belt+RAS) since the fast and slow belts sides were switched in this trial for group B (Figure 4.13).
Figure 4.12: Percent asymmetry average of step length, step time, and peak vertical force for group A. Solid vertical lines indicate the start and stop of each phase. a) trial 1: tied-belt + ARAS b) trial 2: SBT + RAS c) trial 3: SBT + ARAS congruently d) trial 4: SBT + ARAS incongruently. In trials 1 and 2 (top row) only one intervention was applied asymmetrically while both interventions were asymmetric in trials 3 and 4 (bottom row).
Figure 4.13: Percent asymmetry average of step length, step time, and peak vertical force for group B. Solid vertical lines indicate start and stop of each phase. a) trial 1: tied-belt + ARAS b) trial 2: SBT + RAS c) trial 3: SBT + ARAS congruently d) trial 4: SBT + ARAS incongruently. In trial 1 and 2 (top) only one intervention was applied asymmetric while both interventions were asymmetric in trial 3 and 4 (bottom).

4.5.3 Linear Model

The superposition hypothesis proposes that the resultant change in gait under two stimuli is the linear combination of changes under each stimulus applied individually. To test the applicability of this hypothesis for gait response under asymmetric interventions, I developed a linear model for the three measurements indicated in Figure 4.12. Two asymmetric stimuli, SBT and ARAS, were applied both individually and combined. Trial 1 (tied-belt+ARAS) and 2 (split-
belt+RAS) included only one asymmetric stimulus, while trials 3 (congruent) and 4 (incongruent) applied SBT and ARAS simultaneously. The fitted model estimates the gait response for the two later trials based on trial 1 and 2 gait responses. Equation 4.1) shows the linear model equations:

\[ \text{Trial 3 estimate} = C_1(Trial 1) + C_2(Trial 2) \]
\[ \text{Trial 4 estimate} = C_1(Trial 1) - C_2(Trial 2) \]  

\[(4.1)\]

C\(_1\) and C\(_2\) are constant coefficients of the model and stay the same in both equations since the same cohort of subjects completed all the trials. The two coefficients were calculated by minimizing the root mean square error of the data at the same time. Trial 3 is the congruent combination because both directions of asymmetry in ARAS and SBT guide the gait toward the same asymmetric side. Trial 4 is the incongruent combination since the asymmetric direction of SBT and ARAS guide the gait toward opposite sides of asymmetry. Looking at three parameters (step length, step time, and peak vertical force) during only one time window (early post-adaptation) in Figure 4.14 shows that the linear model can largely estimate the behavior under two simultaneous stimuli for congruent and incongruent combination based on the Equation 4.1). Figure 4.14 and Figure 4.15 compare the result of the fitted model to the actual data for both groups A and B. This linear model estimates the behavior of three gait parameters in spatial, temporal, and kinetic areas over the three phases of the experiments for two combinations of asymmetric interventions.

Comparing Figure 4.1 and Figure 4.14 indicates superposition hypothesis was largely applied during early post-adaptation by calculating corresponding coefficients. Expanding the linear model estimates to all 5 time windows and all subjects can indicate that gait is capable of linearly combining two asymmetric interventions for both during adaptation and during short-term learning post-adaptation. Figure 4.15 shows the expanded linear model for both group A and B.
Figure 4.14: Three parameters estimated linear model compared to real values during one time window for group A.

The coefficient values indicate the relative effectiveness of each intervention in the combined trials. A value of one for $C_1$ or $C_2$ indicates the full effect is realized, and zero would indicate an intervention has no effect.
Figure 4.15: Linear model predicting the asymmetric performance of step length, step time, and peak vertical force compared to real data during trial 3 and 4. The predictions are the dashed red lines, and the actual results from experiments are solid blue lines. a) Group A actual data and model with the coefficients $C_1 = 0.85$ and $C_2 = 0.91$, b) Group B actual data and model with the coefficients $C_1 = 0.62$ and $C_2 = 0.90$. The top row in each group represents the linear model and real data for the congruent combination (trial 3), and the bottom row represents the linear model and real data for the incongruent combination (trial 4).
[C1, C2] values were [0.85, 0.91] for group A and [0.62, 0.9] for group B. C1 and C2 values for both groups are higher than 0.6 (higher than 0.85 in three of the four values) and less than 1, which indicates that about 40 percent (15 percent in three cases) of each therapy is lost when the combination of asymmetries is applied simultaneously. For example, C1 = 0.85 and C2 = 0.91 for group A means that 85% of the ARAS effect and 91% of the SBT effect were demonstrated across step time, step length, and peak vertical force during asymmetric combinations of ARAS and SBT. The overall r-squared values for group A and B models were 0.99 and 0.96, respectively. While C2 values in both groups were close (0.91 and 0.90), the C1 value for group B was 0.62, indicating less effect of ARAS in group B compared to group A. After close examination, two subjects in group B showed close to zero or negative gait response under trial 1 (tied-belt+ARAS). Running the model for group B without the two subjects gives values closer to group A: C1 = 0.77 and C2 = 1.00.

4.5.4 Personalized Multi-Rehabilitation Method

Our linear model of the gait response applied the superposition principle to asymmetric gait behavior under two simultaneous rehabilitation stimuli for the averaged results of all subjects. While the model presents a great fit for the averaged data supporting the hypothesis, it does not provide the optimal model for each individual. Personal strengths and weaknesses, as well as the level of impairment can change the effectiveness of different interventions between individuals. For this reason, it is important to look at the variability of the model among individuals.

I calculated personalized coefficients for each subject by minimizing the root mean square error of the linear model, Equation 4.1), for each person. Figure 4.16 indicates the whisker plot of the coefficient values for the optimal personalized linear models. The exact values for all subjects have also been represented in Table 4.1. A coefficient of more than 1 for either C1 or C2 would
mean addition of another intervention has emphasized the corresponding intervention and acted as a catalyst. Coefficients of 1 for both \( C_1 \) and \( C_2 \) would mean that the subject’s gait response combined the exact effects of the two interventions under the simultaneous application of them. A coefficient of zero (or close to zero) for either \( C_1 \) or \( C_2 \) would mean that the subject was not able to respond to the corresponding intervention or the effect of the corresponding intervention was lost or overlapped with the other intervention during the combined ARAS and SBT trials.

![Best fitted linear model among all subjects](image)

**Figure 4.16: Whisker plot of coefficients for individual linear models**

| Table 4.1: Personalized coefficients for the best fit linear model of all subjects. |
|---|---|---|
| **Group A** | **Group B** |
| **Sub #** | **C_1** | **C_2** | **Sub #** | **C_1** | **C_2** |
| 1 | 0.48 | 0.57 | 9 | 0.79 | 1.34 |
| 2 | 0.54 | 1.07 | 10 | 0.65 | 0.84 |
| 3 | 1.14 | 0.75 | 11 | 0.72 | 0.71 |
| 4 | 0.87 | 0.54 | 12 | 0.96 | 0.76 |
| 5 | 0.64 | 0.7 | 13 | 0.19 | 0.48 |
| 6 | 0.68 | 0.68 | 14 | 0.88 | 1.06 |
| 7 | 0.6 | 0.75 | 15 | -0.1 | 0.86 |
| 8 | 0.61 | 1.23 | 16 | 0.66 | 0.69 |

Two subjects in group B had close to zero or negative \( C_1 \) values for their best fit models (subjects 13 and 15 in Table 4.1). Training based on the auditory sensory feedback have shown
none or even negative responses from participants depending on their rhythmic synchronization capabilities [113]. As a result, the first quartile of $C_1$ in group B was stretched to near zero.

4.6 Discussion

Reciprocal movements such as walking contain three stages: initiation, cyclic pattern, and termination of the movement. Central nervous system (CNS) involvement has been suggested to vary between these stages [133]. While initiation and termination are regulated at the supraspinal level, cyclic patterns are generated at the spinal level by central pattern generators (CPGs) and motor neurons [133]–[135]. Therefore, the involvement of the supraspinal level in pattern generation is minimal and indirect. While there is strong direct evidence for the existence of CPGs in other vertebrates such as cats [135], there is indirect yet strong evidence supporting the existence of CPGs in humans [133]–[135]. Moreover, research has shown that the brain does not control every detail of every movement. Instead, it only controls the endpoint, in this case, the final placement of the foot. It combines groups of muscles into modules for controlling tasks such as balance or walking [114]. Modules simplify control of the coordination of movement by the nervous system and create flexibility within the muscles, sensory feedback, and neurons [99]. In patients suffering from stroke, damage to the brain causes these modules to integrate together in order to make them easier to control [114]. Therefore, the default walking module changes. The default walking module is the preferred walking style when no intervention is applied, whether a person has a healthy or an impaired gait. Now we know that due to the interconnections of the neuromusculoskeletal system, damage to the brain does not mean that the ability to walk normally and create new modules is completely lost [95]. Cerebellar damage limits gait adaptation [136], but damage to the cerebral cortex has indicated the capability for rehabilitation and gait retraining [94]. Therefore, stroke survivors with cerebral damage are capable of (re)learning symmetric
walking patterns. (Re)training the muscles and restoring the pathways of neural control of movement through rehabilitation therapy have shown promising results [97]. However, single rehabilitation therapy comes up short of complete gait restoration.

There are indications that there is not a single method that is most effective for gait retraining [96]. Rather, multimodal approaches are likely to provide better outcomes [114], and studies have demonstrated the benefits of combined therapies [99], [124]–[127], [137]. The benefits of multimodal approaches could likely stem from the capability of the neural system to independently control different aspects of gait. For example, Kozlowska et al. [103] found that different neural circuits are responsible for spatial and temporal locomotion control. Choi and Bastian [123] found that walking adaptations are stored independently for each leg. While most of the multiple-rehabilitation therapies result in better overall performance in participants, there needs to be a framework on how adaptations merge together in the neuromusculoskeletal system.

Two interventions were applied with 1:1 or 2:1 ratio at the same time in four different trials, measuring gait response each time. By combining two rehabilitation therapies, this study hypothesized that superposition principle applies to the gait response under combined treadmill training and rhythmic stimulation. Using the ARAS technique, an innovative method developed for the first time; the experiment design matched the 2:1 asymmetry in split-belt with the same asymmetry in sound ratios. Auditory stimulation (RAS or ARAS) has a feedforward mechanism where gait parameters adjust in anticipation of the upcoming cue. In this rehabilitation therapy, the ability to minimize the error between the predicted cue and the actual cue plays an essential role. Treadmill training (split-belt or tied-belt) has a feedback mechanism where the subject reacts to the change of speed or asymmetry, activating interleg control [103], which adjusts the gait parameters to bring the pattern back to the default walking pattern. While the treadmill trains gait
by engaging muscles and proprioceptors into a different walking pattern, auditory stimulation requires conscious attention to auditory sensory feedback.

Modeling gait response can lead to a better understanding of the neuromusculoskeletal system performance under multiple stimuli. A gait response model can also help with developing personalized multiple-rehabilitation therapies that encompass multitudinous aspects of individual capabilities. This study proposed a linear model for the gait response under simultaneous stimuli (interventions) and calculated the percentage of contribution of each stimulus to the final gait asymmetry in each subject.

The linear model of the averaged results indicated coefficients ranging between 0.6 and 1. The high r-squared values confirmed that the suggested model estimates the gait response under two simultaneous stimuli with high accuracy. While this model shows the average response of all subjects, it might not represent the best estimate for each subject individually. Rhythmic capability, muscle strength, physique, and other personal aspects can affect the gait response of individuals under asymmetric interventions. It is vital for the improvement of personalized therapies that individual characteristics, especially individual gait response to various inputs from different therapies, to be taken into consideration when prescribing a multiple-rehabilitation therapy.

Calculating personalized coefficients from gait responses of 16 healthy subjects showed more than 70 percent of the coefficient values are between 0.54 and 1, meaning that the majority of the healthy subjects demonstrated more than 50 percent to complete the effect of the interventions when combined. Five subjects had a coefficient ranging between 1.06 and 1.34. This could indicate that the addition of a therapy method has made the other method more impactful. Two subjects indicated close to zero or negative coefficients during trial 1 (ARAS+tied-belt). This is not an unexpected outcome in therapies that are based on rhythmic auditory feedback. Previous
research has shown that while some people have a positive response to rhythmic stimulation, others might have none or even negative response [105], [113]. The performance of rhythmic stimulation has been connected to the rhythmic skills of people [113].

However, if a minimum of 40 percent is set as a considerable demonstration of asymmetries in gait response, more than 90 percent of the subjects were able to considerably show the effects of asymmetries from each intervention during both combined trials. While the exact coefficients for each subject might vary depending on their strengths and weaknesses, both the fitted linear model on the averaged data (Figure 4.15) and the close range of the whisker plots for individuals (Figure 4.16) indicate that neuromusculoskeletal system can linearly combine the effects of two simultaneous rehabilitation stimuli on gait asymmetric response. Experimental results showed that the additivity principle was met; however, the coefficients not being one indicates that the homogeneity was not fully applicable. Therefore, the superposition principle was largely applied. This result can lead to a better and improved combination of therapies that accommodate the needs of patients as well as leveraging their strengths for better outcomes.
Chapter 5: Conclusions

5.1 Dissertation Contributions

This dissertation presents various gait modulations and interventions for functional asymmetric walking. Improving assistive aids and rehabilitation techniques and simulation of gait models are collective routes for better understanding and enhancing gait performance.

Chapter 2 tests the hypothesis of improving crutch walking forces and angles through the dynamic-altering design of KCT. Crutch users depend on their devices regularly to function every day. Any small enhancement in assistance can potentially promote more prolonged crutch use and better health. The varying-radius of KCT is assumed to enhance horizontal assistive forces and range of rotation of the crutch. The research findings indicate the maximum backward angles of KCTs are up to three times bigger than Standard tip; therefore, they provide a greater range of rotation without resistance. While horizontal assistive forces were not significantly bigger in KCT, a trend of improved forces and step parameters was observed in KCTs with harder material. Testing with a physical setup and human trials provided an opportunity to compare and analyze KCT performance and possible improvement areas for users.

Chapter 3 models lower limbs’ motion with double pendulum systems. The model incorporates a new approach for redefining combinations of physical parameters in dynamic equations with coefficients. The process discovers that coefficients required to match dynamic equations are less than physical parameters in the system (8 coefficients and 10 parameters). Therefore, if two different systems (with different characteristics) have identical coefficients, they
will have dynamic symmetry in their motions and force reactions. The study numerically solves for two sets of physically and humanly meaningful data. While full symmetry was not achieved, multiple answers for partial symmetry were found. More than 200 parallel systems for physical data and 48 parallel systems for human data were found to have kinematic symmetry. Multiple systems in each parallel group had partial kinetic symmetry (matching forces at the hip or the knee joint) along with full kinematic symmetry. Simulation of the systems using a logical algorithm for motion indicated the accuracy of the coefficients in predicting kinematic and kinetic symmetry.

Chapter 4 introduces a multi rehabilitation technique consisting of a split-belt treadmill (SBT) and a newly developed auditory-based method named asymmetric rhythmic auditory stimulation (ARAS). The research can model gait behavior under training and post-training by applying the superposition principle to step time, step length, and peak vertical forces. This research achieved two discoveries in the path of understanding gait response. First, ARAS is able to adapt different sides of gait by applying asymmetric cues with a 2:1 ratio, meaning that auditory sensory feedback can independently access each side of lower limbs. Second, gait response can be mainly modeled as a linear system with the possibility of quantifying the contribution of various stimuli at the same time.

5.2 Limitations and Future Works

Chapter 2 only studies the gait parameters of healthy participants. Future studies can test the effect of KCT design with temporary and permanent crutch users. The experiment includes axillary crutches with swing-through gait, but further research can expand the experiment to non-axillary crutches and different crutch gait patterns. Also, the length of the experiment with each crutch is short (2 minutes) due to the difficulty of walking with crutches on a treadmill; therefore, the long-term effect of KCT is not studied. Further study can indicate the benefits of longer
duration use of KCT. KCT has an advantageous design for walking uphill or downhill. Future studies can expand the same experimental setups for walking on a slope with human subjects. A misalignment in the design of 3D printed tip (KCT$_{3D}$) with the crutch decreased the performance of this product. KCT$_{3D}$ is one of the lightest compositions of the tip. Rebuilding this crutch tip and testing the new design can lead to better performance of KCT$_{3D}$.

Chapter 3 only investigates the performance of the double pendulum theoretically and does not include experimental confirmation of the models. The model does not study slipping. The underdetermined system of coefficients is only solved numerically and for specific sets of data because of the nonlinearity in the equations and constraints with inequalities. Future research can add the modeling of the foot to the system. The addition of extra point masses with extra moments and moments of inertia can introduce new coefficients and add more flexibility to the design. Moreover, the modeling of the external surfaces can be extended to include slipping events and collisions during the unlock mode of the double pendulum.

There are limitations in the study of Chapter 4 that further research can expand on. First, the interventions employed in this research (split-belt and rhythmic stimulation) engage two different mechanisms to train gait. Split-belt engages mechanoreceptors (proprioceptors) in lower limbs and affects interlimb control while rhythmic stimulation takes advantage of auditory sensory feedback. As a result, there is little interference in their mechanism to create an asymmetric response in gait when implementing both stimuli simultaneously. Applying interventions that engage similar sensory receptors or motor control could create non-linearity of feedback mechanism in gait. Extending the model to other therapies using tactile or somatosensory feedback as well as muscle retraining or robot-assisted training is needed to understand the gait feedback mechanism better. Another limitation of the current study is that asymmetric interventions were
only applied with 2:1 or 1:2 ratios (consistent with the standard of previous research with successful outcomes [97], [111]). Further experiments are needed to study the effect of different asymmetric ratios. Also, recruitment included only healthy subjects since this study wanted to test the hypothesis without the influence of asymmetries in an impaired gait. However, future studies will test the hypothesis among stroke survivors since the ultimate goal is to develop multiple-rehabilitation therapies that can retrain an impaired gait for the long-term. Moreover, incorporating three or more simultaneous interventions can extend the modeling of gait response to more than two stimuli.

5.3 Conclusions

Symmetry is the salient index of normal walking. Achieving symmetry in an asymmetric gait, whether through training or assistive aids, has been a major goal of gait recovery techniques. Developing effective experimental methods, along with formulating gait dynamics, can transform the future of rehabilitation. This dissertation attempts to bridge the existing gaps of biomechanics and response mechanism of gait by creating new dynamical models as well as state-of-the-art experimental therapies. Both theoretical and experimental approaches hint at the possibility of achieving symmetry in an inherently asymmetric gait. However, plenty more steps remain for reaching symmetry after gait impairment. I develop gait simulations with asymmetric systems that achieved full kinematic and partial kinetic symmetry. Pioneering experimental methods for walking rehabilitation revealed a linear model for the gait response mechanism. Dynamic-altering assistive aids resulted in better kinetic and kinematic performance. All these gait interventions and modulations are synergetic steps for achieving functional gait asymmetries.
References


[52] I. Handžić and K. B. Reed, “Kinetic Shapes: Analysis, Verification, and Applications,” 


[87] A. Neumaier, “The Enclosure of Solutions of Parameter-Dependent Systems of


with post-stroke hemiparesis and non-disabled controls at matched speeds,” *Gait Posture*,


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### Appendix B: Numerical Solutions of Underdetermined System of Coefficients

Table B.1: Group of parallel systems with matching kinematic coefficients to System B (human dataset)

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* Highlighted systems have kinetic symmetry at knee joint with System B
Appendix C: Simulation of Perfectly Plastic Collision

Figure C.1: Kinematics of three dissimilar double pendulums under plastic collision. a) angular positions. b) angular velocities. c) angular accelerations.
Figure C.2: Kinetic outcomes of three dissimilar double pendulums under plastic collision. a) forces at hip joint. b) forces at knee joint