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Quantitative Reasoning in the Contemporary World, 2: Focus Questions for the Numeracy Community

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Quantitative Reasoning in the Contemporary World, 2: Focus Questions for the Numeracy Community

Abstract

Numerous questions about student learning of quantitative reasoning arose as we developed, taught and assessed the Quantitative Reasoning in the Contemporary World course described in the companion paper in this issue of Numeracy. In this paper, we present some of those questions and describe the context in which they arose. They fall into eight general problem areas: learning that is context-bound and does not easily transfer (i.e., situated learning); the need for a productive disposition regarding mathematics; the connection between QL and mathematical proficiency; the persistence of students, despite our efforts, for using the wrong base for percents; the inconsistent and sometimes incorrect language in media articles on percent and percent change; the need for students to possess quantitative benchmarks in order to comprehend the size of large quantities and to know when their answers are unreasonable; students' avoidance of using the algebra they learned in the prerequisite course; and conflation of "bigger" and "better". We offer these questions as products of our experience with this course in order to encourage future research on issues that affect teaching similar courses that develop QR skills in undergraduate students.

Keywords

quantitative reasoning, student learning, quantitative literacy, research questions

Cover Page Footnote

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Introduction

During six years of creating, expanding, and researching a one-semester undergraduate course in quantitative reasoning (QR) at the University of Arkansas, we have observed numerous facets of student QR skills and tendencies, while many other questions have surfaced. We have studied certain aspects of some of these questions, but we believe much remains to be learned about promoting and developing QR in the setting of a one-semester undergraduate course.

In this paper, we detail some of the questions that have stemmed from our work and report some observations in the area of each of these questions. We refer to the course as QRCW, the title acronym of the National Science Foundation grant that currently supports expansion of the course, Quantitative Reasoning in the Contemporary World (DUE-0715039). The development of QRCW, including the successes and challenges we faced in the process, is described in some detail in a companion article in this issue of *Numeracy* (Dingman and Madison, 2010) (a sample case study of a media article used in the QRCW course can be found in Appendix A[1]). The information and data reported in this paper come from sections of QRCW classes led by the authors, from pre- and post-tests and attitude surveys conducted with students during the Fall 2007, Spring 2008, and Fall 2008 semesters (these tests, surveys, and some results are Appendices B and C), and from think-aloud sessions of QRCW students conducted by the authors.

Below, we highlight eight areas related to the teaching and learning of QR that we believe require further study and examination. For each area, we articulate questions whose answers could shed important light on students' abilities to reason quantitatively. In considering these questions, one feature of the QRCW course is especially important. Recognizing that QR is a habit of mind and likely will not be developed in one single-semester course of 30 class meetings, a major goal of QRCW is to establish a venue for students to continue to practice QR beyond the course and beyond school (see Dingman and Madison, 2010, for further discussion).

Before presenting our observations and questions, we wish to acknowledge three caveats:

- We realize that others have recognized many, if not all, of the questions we raise here.

[1] Appendices are available as supplemental files accessible from the cover page.

- We know that these questions have many aspects, and sharper, more-detailed questions will likely be required for research designs to provide answers.
- We recognize that applicable research literature is available on some, if not all, of these questions, but that literature is spread over a vast academic landscape of K-12 education and multiple disciplines where education for QR resides.

Relative to the above caveats, it is likely that studies of mathematical reasoning—geometric, multiplicative, proportional, algebraic, statistical, and quantitative—in younger students (e.g., Harel and Confrey, 1994; Steffe and Nesher, 1996; Thompson, 1988; Thompson and Saladanha, 2003; Smith and Thompson, 2007) may also be applicable in the undergraduate setting, but we have not explored the possibilities. Further, we have not exploited research on mainstream undergraduate mathematics education (e.g., Dubinsky, 1991; Carlson and Rasmussen, 2008) or work in the undergraduate social sciences (e.g., Caulfield and Persell, 2006; Howery and Rodriguez, 2006); Hunt, 2004; Wilder, 2009) in search of answers to these questions.

Our goal is to document our thoughts and observations to these questions stemming from our work with QRCW. It is our hope that by articulating these questions, we can spur on the conversation and promote the mission of *Numeracy*—advancing education in quantitative literacy (QL)[2]—which in our case is focused at the collegiate level.

Our Questions and How They Arose

Situated Learning

One of the critical specific learning goals of QRCW is to enhance students' ability to transfer knowledge and cognitive processes to solve problems in a variety of contexts. This goal gives rise to many questions about pedagogical strategy. Instances of situated learning (i.e., learning that is context-bound and does not easily transfer) seem to occur in QRCW, and we are very sensitive to having the learning in a case study not being bound to just that specific case study. Consequently, we work to see that we reinforce learning and promote transfer by encountering concepts and procedures in multiple contexts (Halpern and Hakel, 2003).

We do encounter instances where learning seems to be context-bound. For example, a pre- and post-test item used early in QRCW asked for the total bill at

[2] We use the terms quantitative reasoning (QR) and quantitative literacy (QL) interchangeably.

McDonald's for a soda, burger and fries (prices given) with a 5% sales tax (see Appendix B and C for the test items and student results). Almost all students answered this problem correctly; however, in other instances such as a problem involving a credit card bill where the component tasks were indeed similar, some 40% of students gave incorrect answers on a pre-test. We encounter numerous instances where knowledge does not easily transfer from the traditional mathematics classroom to reasoning about case studies that comprise the QRCW curriculum (Madison et al., 2009). Some examples that are discussed below illustrate situations where students do not recall operations and representations of linear and exponential equations (functions), topics they have studied in middle grades, secondary courses, and the pre-requisite college algebra. As Anderson, Reder and Simon (2000) point out in their discussion of four claims about situated learning in the National Research Council (1994) report, the design of the classroom instruction rather than abstract instruction may be at fault in the lack of transfer from the classroom.

We utilize abstract instruction and study of authentic situations in the QRCW case studies; however, we place the abstractions (always using contextual illustrations) where they will be immediately applied to an authentic media article, and we extend abstraction only when we find we need it. For example, one case study from the QRCW course has students investigate the balance in an account stemming from saving and investing $2.50 a day for 25 years, where the savings can be deposited and interest compounded yearly, monthly, daily, etc. In this case study, we develop the formula for the sum of a geometric series only after we have exceeded the power of our calculators to find the balance in an installment savings account with frequent compounding, such as compounding daily over a period of 25 years where the sum would have over 9000 terms. We try to decompose larger tasks associated with a case study into assessable component skills that we address in warm-up exercises and assessments.

Sometimes in QRCW there are what Anderson, Reder, and Simon (2000) call specialized subprocedures, such as ignoring cents when calculating income taxes. For example, the Dow Jones Industrial Average (DJIA) is found by summing 30 stock prices and dividing, not by 30, but by a divisor that carries a history of stock splits as well as the changes over time in the 30 companies listed on the DJIA. The current DJIA divisor in 2010 is approximately 0.13. In the QRCW course, we compute a model average similar to the DJIA of a few stocks, say four, by dividing the sum of four stock prices by a number less than 4 after one of the stocks has split. Okrent (2005) writes that such an average is "mathematically preposterous." This kind of average is not likely to occur in other contexts, and learning about this calculation procedure is important only to understand the highly visible and widely cited DJIA. It is an interesting use of a number (or changes in that number) to document economic history.

Questions:

- What is the proper mix of abstract learning and learning in authentic situations for achieving quantitative literacy (QL)?
- What instructional techniques and tasks best promote transfer of knowledge and skills to a variety of contexts?

Productive Disposition

The National Research Council study *Adding It Up* (Kilpatrick, Swafford, and Findell, 2001) contains a model of mathematical proficiency that has five intertwined strands: procedural fluency, conceptual understanding, adaptive reasoning, strategic competence, and productive disposition. QRCW aims primarily at the last three, with the final one – productive disposition – seemingly critical for the QRCW students. As described in *Adding It Up*, productive disposition is the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy." (p. 5) This seems to be the weakest of the five strands of mathematical proficiency for QRCW students. As evidence, we asked students in pre- and post-QRCW attitude surveys during the Spring 2008 semester to respond with strongly agree, agree, neutral, disagree, or strongly disagree to the statements shown in Table 1. Table 1 also illustrates the percentage of QRCW students who agreed or strongly agreed with these statements on the surveys. Additionally, in a pre- and post-QRCW attitude survey given during the Fall 2007 semester, we asked students to agree or disagree with the statement, "I am good at mathematics." On the pre-QRCW survey, 46% of students disagreed with this statement, while on the post-QRCW survey, only 27% disagreed. The percent that agreed with the statement moved from 18% on the pre-survey to 24% on the post-survey, indicating a movement from disagreeing with the statement to being neutral.

Table 1

Results from Pre- and Post-QRCW Attitude Surveys, Spring 2008

Statement	**% of QRCW Students Agreeing or Strongly Agreeing**	
	pre-QRCW	**post-QRCW**
I am comfortable talking about mathematics.	26%	33%
I enjoy doing mathematics.	18%	26%
Lots of things I do everyday involve mathematics.	26%	32%
I expect to find mathematics important in my future occupation and everyday life.	38%	42%

In a follow-up survey of students who had completed QRCW earlier, forty-two students responded, and approximately three-fourths of those reported that both the usefulness of QR and their confidence in their QR ability had increased because of QRCW. More details of this survey are reported in Dingman and Madison (2010). The survey questions and response data are in Appendix D.

These survey results suggest that QRCW students are reluctant to claim that they are good at mathematics. The relevance of mathematics to these students is more complex than the issue of mathematics alone. Students do not necessarily see understanding government and political discourse as relevant to their lives. More immediately relevant issues are matters of parking, credit card costs, and sports statistics. They are somewhat suspicious of quantitative arguments, stemming in part, no doubt, from a lack of understanding and difficulty in discerning the correctness of the arguments. These students' past experiences with mathematics have left them skeptical of the subject. In fact, one of the advisers for QRCW suggested that the word "mathematics" in the attitude surveys may be influencing student responses and might be replaced with "quantitative reasoning."

Questions:

- How can productive disposition be strengthened, particularly among students majoring in non-mathematical fields of study?
- Does the use of the word "mathematics" rather than "quantitative reasoning" or some other word such as "statistics" skew responses to attitude questions?

QL and Mathematical Proficiency

Conventional wisdom is that students in mathematically intensive disciplines eventually become sufficiently proficient in mathematics or in mathematics and statistics to successfully engage quantitative situations that occur in everyday life as a consumer and citizen—that is, to achieve QL. However, QL is neither an immediate nor automatic consequence of mathematical or statistical fluency. We have tested some of the ideas from the QRCW classes in different domains: with senior mathematics majors; with middle and secondary mathematics and science teachers in summer workshops; with instructors assigned to teach QRCW sections; and with mathematics and statistics tutors. In all four domains, work was required for these more mathematically proficient individuals to recognize the utility of known mathematics as applied to the contextual situations under study in QR.

However, fluency with mathematics or statistics does equip individuals to become quantitatively literate, particularly because the tools we frequently utilize for QR are traditional mathematical or statistical structures. Although likely there are other approaches to QR, the tools of such approaches are not part of traditional educational systems. To better assist new instructors, especially graduate teaching assistants, we have produced password-protected sample answers to the study questions that can be downloaded from the QRCW web site. These sample solutions have proved to be indispensable, particularly for those instructors teaching the course for the first time. The wide-ranging contexts—political, social, economic, health and medicine, sports, etc.—presented a challenge to all of these groups, and this is a major challenge in increasing the number of course offerings of QRCW. Issues such as the subtle and confusing language (e.g., the percentage of working mothers phrases cited below in the section on language of percentages) puzzle many mathematically accomplished people who are accustomed to precision of language. In fact, the fuzzy world of quantitative media articles is rather far removed from the pristine, sterile world of abstract mathematics.

Questions:

- How can QL be achieved by way of arithmetic and proportional reasoning?
- Are there approaches to QL that reduce dependence on formal mathematics and statistics?

A Persistent Wrong Base for Percent

One of the most common and recurring mistakes we have observed students make in the QRCW class involves using the wrong base for a percent. Even though the topic of percents emerges in the K-12 mathematics curriculum in later elementary and middle grades, students in our course still struggle with this concept, particularly in finding a quantity that has changed by a stated percent to a given value. For example, the following item was used in a pre- and post-test during the Spring 2008 semester for 95 QRCW students as well as 83 students in a mathematics course using *For All Practical Purposes* (CoMAP, 1988) as a textbook. The two groups of students were similar, namely all arts, humanities or social sciences majors. The students were posed the following question: *The Fall 2007 enrollment of 18,200 was an increase of 4% over the Fall 2006 enrollment. What was the Fall 2006 enrollment?*

There were five possible choices for an answer. One was the correct answer of 17,500, since raising 17,500 by 4% produces an enrollment of 18,200, while another possible solution was the result of reducing 18,200 by 4%, or 17,472. On the pre-test, 60% of the 178 students chose the incorrect 17,472, with 25% choosing the correct answer of 17,500. The other 15% of the 178 students chose one of the other three incorrect options. On the post-test, 56% of the students chose the incorrect 17,472, while 38% chose the correct answer. The QRCW students fared better than the other group with regards to this item on the post-test, but still only 44% of QRCW students chose the correct answer after similar items were considered several times in the QRCW course. In fact, on a number of occasions during the course, we have warned students about these problems and how students often will find incorrect solutions, and yet this error remains prevalent. Why is this mistake so persistent? From our observations, we believe that it is a combination of two issues: reluctance to compute with an unknown (i.e., algebra) and the tendency to multiply rather than divide, especially where percents are involved. However, this question warrants further attention, including examining what types of interventions are successful in remedying this problem.

Questions:

- What types of instructional or curricular interventions best assist students in determining the correct base for a percent?
- Is multiplication more natural than division for these students? If so, when and how is this tendency developed?

Language of Percentages in Media

In using media articles as the source for students to examine quantitative arguments and hone their QL skills, we have noticed that the language used in media articles on percent and percent change is inconsistent, often difficult to parse, and sometimes incorrect. Instances of language use are central issues in several of the case studies. For example, students (and others!) have difficulty in understanding the difference between the following subjects of the lead sentences in the print version and the online version of an article in the *New York Times*:

(1) The percentage of women in the workforce with babies under 1 year old.
(2) The percentage of women with babies under 1 year old who work.

The language for absolute and relative change is often incomplete or strained. Particularly difficult are relative changes in rates, such as determining the percentage change in two percents. Measuring this change has given rise to the special language of "an increase of percentage points" as cue language to indicate an absolute increase in a rate as a percent.

For example, a letter to the editor was the source of an assessment item in a QRCW section where the writer was correcting a newspaper article that indicated that an increase from 1 percent proficient to 3 percent proficient was an increase of 300 percent. As part of the assessment, students were posed the following questions:

Is the letter writer correct or incorrect when he states, "going from 1 percent proficient to 3 percent proficient is an increase of 200 percent?" Why?

Most students who answered the problem indicated that the writer was correct, but a very perceptive student gave the following response: "The letter writer is incorrect in making that statement due to a misuse of wording. The letter writer made an error in saying 'increase of 200%,' when he should have said 'it's a *percent increase* of 200%.'" (Emphasis is from student.)

In other areas of measurement, different terms are used to help distinguish the actual measurements being discussed. For example, percent change in percent is analogous to average rate of change in a velocity (or speed), which we call average acceleration. In the case of percent change in percent, use of a different word is partly done with the phrase change "in percentage points," in which case one abandons relative for absolute change (i.e., a change from 4% to 6% is an increase of two percentage points; in terms of a relative change, it is a 50% increase in percent). Clearly, the lack of standard language to use when distinguishing between various measurements with respect to percents complicates the interpretation of what is being measured.

Question:

- How can the language for absolute and relative increases be standardized to avoid misinterpretations, especially in cases where the quantity changing is a rate (a percent)?

Quantitative Benchmarks and Determining the Reasonableness of an Answer

Quantitative reasoning is more about relationships among quantities rather than the quantities themselves (Smith and Thompson, 2007). Understanding large (or small) quantities such as $1.2 trillion calls upon the experience of the person trying to understand the magnitude of such large quantities. Few people have personal experience with such a large amount of money, so comparisons with amounts that have personal meaning are necessary.

As part of the previously described think-aloud sessions with QRCW students, we posed the question of understanding $1.2 trillion (an estimate of the cost of the war in Iraq in one newspaper column) to five groups of four students each. Specifically, we asked: "What measure of $1.2 trillion would help you understand its magnitude?" After seeing how many $100 purses or sports shoes one could buy and arriving at a rather large 12 billion, most groups settled on buying houses. However, some students were not sure how much a house costs, so the average cost of houses ranged from $30,000 to $250,000 in different groups. Using $150,000 as the cost of a house, one group arrived at an answer of 8 billion houses, illustrating the million-billion confusion that often occurs. Even the correct answer of 8 million required further breakdown to something like three houses for every resident of Arkansas or six houses for every resident of Dallas, TX.

Personal quantitative benchmarks (sometimes called anchors) are extremely important in understanding or determining quantities. In *Stat-spotting: A Field Guide for Identifying Dubious Data*, Joel Best (2008) offers some helpful statistical benchmarks for thinking about U.S. social statistics. The basic benchmarks he lists are the U.S. population (about 300 million), the annual number of births (about 4 million), and the annual number of deaths (about 2.4 million). We have found that few of our students know even a reasonable approximation of the latter two, but, on discussion and reflection during the term, they find all three of these benchmarks useful and enlightening.

In addition to possessing quantitative benchmarks, understanding if a quantity is a reasonable answer to a question requires some frame of reference. Arriving at an answer of 400 billion for the population of Louisiana or an estimate that 30 million U.S. high school graduates of 2010 will enter college in the fall are easily seen to be unreasonable if one has Best's benchmarks from above as comparative measures. Often, reasonableness of answers is not quite so direct to see; benchmarks may depend on the context of the problem as seen in the example below.

In a 2010 QRCW class session, warm-up exercises for a case study on installment savings were under discussion. One exercise asked the amount of money that would result from placing $12,000 in an account earning 6.4% per annum when the interest was compounded annually. The next part asked for the amount if the interest was compounded quarterly, and the third part asked for the amount if the interest was compounded monthly. The discussion proceeded smoothly through the first two parts, but the third part became somewhat thorny when one student used 0.53% (for 6.4% ÷12) as the monthly interest rate. That resulted in an amount that was less than the amount for quarterly compounding, which did not seem reasonable to some students. Seeing this error required possessing knowledge about what should be expected (e.g., the balance compounding monthly should be slightly more than the amount compounding quarterly) to see that rounding 0.5333… to 0.53 creates an error that produces an "unreasonable" result. When asked the expected question, "How will I know on a test whether to use 0.53, 0.533, or 0.5333?" the instructor answered that the choice depended on the context of the question, and this context said that one would not use 0.53. It did not point to a reasonable choice among the other possibilities. The situation did highlight the need to examine one's solution to determine whether the answer seems reasonable within the context of the problem or the larger setting.

Questions:

- How can students develop reasonable collections of quantitative benchmarks, both absolute (such as Best's three) and contextual (such as the compound interest problem above)?
- How can one determine the content of a reasonable and usable collection of quantitative benchmarks?

Algebra Issues

Although college algebra is a prerequisite for QRCW, any use of algebra in the course —particularly linear and exponential expressions and equations—requires at least a review of these algebraic concepts. Rarely do students recognize algebraic methods as the route to solving problems, and they often do not recognize manipulation methods that they have surely seen before. Some instances are:

- A very good student overall was totally stumped on how to solve an equation of the form $500/k = 20$. This type of equation typically arises while illustrating how the Dow Jones Industrial Average is computed.
- Students do not recognize cost equations such as $C = 793.08t + 25000$ as linear equations. As we all know, linear equations are more like $y = 3x - 7$ or $2x + 3y = 5$ (i.e., written in *x* and *y*).
- Students are puzzled as to why $x + 0.06x = 1.06x$.

Why does the algebra that these students know have such limited utility?

Our work points to several possibilities. These students' understandings of linear and exponential equations/functions are restricted, often to a single equation that can be evaluated at specific points and whose expression can be manipulated. This limited conception is what Dubinsky (1991) and Asiala et al. (1996) have termed the action conception, which is the first stage of a mental framework termed Action-Process-Object-Schema, or APOS. This limited view interferes with students' thinking of a cost equation as being a process, and this limitation emerged strongly when we asked students to compare the savings over time from spending more money to purchase an automobile with greater fuel efficiency versus placing the extra cost in an account earning compounding interest. This comparison is best done, we believe, by observing the graphs of a linear cost equation (function) and an exponential equation (function) over the considered number of years. Our students' approaches usually result in something like evaluating both functions at a fixed time(s), and reporting how the savings compare at some fixed number of years.

The inaccessibility of algebra for our QRCW students was evident in their approaches used in think-aloud sessions concerning a ratio problem situated in a context concerning college football. During the three think-aloud sessions conducted with QRCW students, we gave seven different groups of 3–4 students a November 2006 news article containing a table listing the twelve football teams in the Southeastern Conference, the number of passes attempted thus far during the 2006 season, the number of those passes that had resulted in a touchdown, and the ratio of pass attempts per touchdown. Arkansas had the smallest ratio, 11.8, with 224 pass attempts and 19 touchdown passes. Mississippi State had the largest ratio, 36.2, with 326 attempts and 9 touchdowns. In each group, we posed the following questions:

> 1. Suppose that Mississippi State goes on a streak so that every pass attempt goes for a touchdown. How many pass attempts would they need to lower their ratio and overtake Arkansas to be first in the conference in attempts/TDs ratio?
>
> 2. Suppose instead that Mississippi State goes on a pace so that for every three pass attempts they make, they throw for a TD. How many attempts would Mississippi State need to overtake Arkansas for first in the conference in attempts/TD ratio?

All seven groups were able to decide that the answer to the first question is 21, and most groups found 75 as the answer to the second question. Here are typical explanations by students:

> Student: *I am just plugging in the numbers. I went ahead and gave them 20 touchdowns per 20 attempts so that would be 346 divided by 29 and that is 11.9 so that is pretty close. We just need to add like one or two more.*
>
> Student: *I saw 326 for nine TD passes and at first I guessed 10 more so I did 336 divided by 19 and that was 11.5 so I did 10 more and I mean it was 17.6 so I did 10 more which I said 346 divided by 29 and that is 11.9 and then I just added one more because I was pretty close.*
> Professor: *So what is your strategy?*
> Student: *Um my strategy? Guessing and using common sense.*

In all, only one group out of the seven used algebra and/or tables generated by a calculator to find the solution. In fact, in this single group, the method was suggested and implemented by just one student.

We have made several observations regarding the students' thinking about this football article and these questions. First, the students were able to understand the context, football. Some were not familiar with the game, but there was ample help from other students. Second, when asked, the students did not consider guess-and-check as a valid and sometimes effective problem-solving strategy, even though it was the most prevalently used strategy on this problem. Third, they had an action view (Dubinsky, 1991) of the ratios, resorting to checking for particular values rather than looking at graphical or symbolic representations that reveal overall behavior. In fact, algebra was ignored by almost all of these students as a tool to use.

The challenges of QRCW and similar efforts are more daunting because many students are not fluent in mathematics or statistics, even in the algebra for which they have college credit. In addition to a lack of recall, the algebraic knowledge of these students is organized in ways that reduces likelihood of transfer. Rather than being organized around core concepts, such as linear functions, the students' algebra is more likely a fragmented collection of methods and operations. This kind of fragmentation is known to reduce transfer, and hence usability of the knowledge (Bransford et al., 2000). As Smith and Thompson (2007) stated:

> For many students, … algebra is a set of rituals involving strings of symbols and rules for rewriting them instead of being a useful and powerful way to reason about situations and questions that matter to them. Consequently, many students limit their engagement with algebra and stop trying to understand its nature and purpose. In many cases, this marks more or less the end of their mathematical growth. (p. 97)

One basic premise of QRCW is that we can move students who have weak formal mathematical skills toward stronger QL, although perhaps not growing as Smith and Thompson mean. There are many examples of individuals—tradesmen of all sorts—who are very adept at some sophisticated quantitative reasoning, yet have no formal algebra skills, but the adeptness may be situated and not easily transferred. In QRCW we work to avoid such situation-bound reasoning.

We have expanded some of the studies of articles when we use them with in-service middle and secondary teachers to illustrate how the content can be differentiated across various grade levels and for varying levels of mathematical abilities. For example, for the TDS/ATTEMPTS article discussed above, we have added the following questions when working with in-service teachers:

- How many pass attempts would Mississippi State need to overtake Arkansas if their pace changed to a TD pass for every 5 pass attempts? Every 10 pass attempts? Every 25 pass attempts?
- Suppose we change this to every *n* pass attempts. For what values of *n* will Mississippi State eventually overtake Arkansas?
- Examine your solutions to the previous questions. How can you solve these questions using: guess-and-check? an equation (symbolically)? a table (numerically)? a graph (graphically)?

This example illustrates how media articles and the accompanying case studies can be adapted to different audiences. In this example, middle grade audiences can focus on problems that highlight the concepts of ratio and the use of equations and tables in order to solve problems, while upper secondary audiences can examine the asymptotic behavior of the functions that model the scenarios as well as study the concept of limits within the context of the problem. As a result of our piloting the materials with QRCW students as well as our knowledge of the general QRCW student, the case studies provided in the QRCW text (Madison et al., 2009) are written with students with weak formal mathematics skills in mind.

Often, quantitative reasoning, as illustrated in the pass attempts per touchdown example above, should lead to algebraic reasoning, giving evidence supporting the premise that algebraic reasoning should derive from quantitative reasoning rather than as a generalization of arithmetic (Thompson, 1988; Smith and Thompson, 2007). Algebraic models often outstrip student understanding. Too often, once the model is found, thinking tends to end. Reflection on the solution to the model is a step often not taken by QRCW students.

Some QRCW students who profess to be weak at formal mathematics do show strong QR skills. For example, students were given the task of computing the amount of the 2001 annual budget from a statement in an article that reported that $1 billion per month was a mere one-half of one percent of the federal budget. A history major who professed to be "bad" at mathematics gave this answer: "Ok, if one-half of one percent is $12 billion per year then 1 percent is $24 billion and 100% is $2400 billion, or $2.4 trillion." Additionally, a political science major, who was a top performer in his QRCW class and who reported that he was not good at mathematics, reacted negatively to using unit analysis in unit conversions. He said he could just reason each conversion out without using a memorized method that he did not understand—and he showed repeatedly that he could do as he claimed. Of course, because we de-emphasize algorithms and formulas in QRCW, we admired and accepted his insistence on reasoning.

Questions:

- Is fluency in algebra necessary for achieving QL?
- Can (weak understanding of) algebra hinder achieving QL?
- How can (weak understanding of) algebra be used more effectively to achieve QL?
- Can QL be developed solely from arithmetic and proportional reasoning?
- To what extent does mathematical fluency contribute to QL?
- How can QL "courses" best be adapted to a mathematically proficient audience?

Bigger is Better

One student approached the football *ATTEMPTS/TDs* situation above as follows:

> Student: *I tried to go at it from a different angle over here. I did 19 divide by 225 and that is .085 and I went to Mississippi State and I said nine divide by 326 and I saw that that was .028 and I knew we had to get that .028 up to .085. How? I don't know.*

That approach is of interest because the table in the article actually had the ratio column mis-labeled as TDS/ATTEMPT rather than what was actually computed in the column ATTEMPTS/TD. We have observed that students tend to believe that increasing is preferred over decreasing, so getting ".028 up to .085" is likely to improve understanding. In fact, our Hollins University co-PI, Caren

Diefenderfer, has reported that in a QRCW writing assignment on a news article dealing with statistics from the war in Iraq, students believed that increasing numbers identified a favorable change in that category, regardless of what categories the numbers were measuring.

This concept emerges in one of the QRCW case studies about hospitals. At one point, the article discusses "better than average death rates." Students are asked what this statement means and whether or not it is a good thing for a hospital to have "better than average death rates." Does "bigger" or "better than average" always mean a good thing? Additionally, another instance of this phenomenon occurred in a think-aloud session that examined a graph that represented the returns from stocks in the energy sector. In the 2007 graphic, Williams Companies' stocks had returned 39.5%, Chevron 23.5% and ExxonMobil about 19% over the past year. When asked what stock one would have preferred to own over the past year, one student argued for ExxonMobil:

> Student: *Cause Exxon stocks worth much higher than Chevron stock, it's worth more, so even though they're not gaining as much as Williams, but it doesn't say how much their stock is worth, so yeah, they gained plus 39.5. What's their stock worth?*

This is a case of confusing the magnitude of a quantity and its relative change. The student was more concerned about how much a single share of stock in each of the companies cost, assuming that Exxon's stock was worth more than the other companies because Exxon is a larger company (as represented on the graph). The student neglected to notice that the rate of return would actually be of importance in determining which stock to buy.

Questions:

- Is the belief that "bigger is better" developed in school mathematics? If so, how can it be countered?
- How can language in QR be adjusted so that bigger (more) has less of an advantage over smaller (less)?

Conclusion

The foregoing observations and questions are based on our work in developing, expanding, and researching a one-semester undergraduate course in quantitative reasoning (QR) at the University of Arkansas. The course and the challenges it has presented are described in a companion paper in this issue of *Numeracy* (Dingman and Madison, 2010). We present both of these papers in the interest of informing others about our experiences and observations. In so doing we hope to focus knowledge and research efforts on issues that we consider important in developing QR skills in undergraduate students. We have particular interests in the effects of undergraduate courses in QR beyond individual courses and beyond school—effects that frame QL as a habit of mind that will be part of our student's disposition throughout their life.

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