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Quantitative Reasoning in the Contemporary World, 1: The Course and Its Challenges:

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Quantitative Reasoning in the Contemporary World, 1: The Course and Its Challenges:

Abstract

The authors describe successes and challenges in developing a QL-friendly course at the University of Arkansas. This work is part of a three-year NSF project Quantitative Reasoning in the Contemporary World (QRCW) that supported the expansion of the course. The course, MATH 2183, began experimentally in Fall 2004 as a section of finite mathematics known informally as “News Math” for 26 students from arts and humanities disciplines. Over the past six years, the course has evolved and now MATH 2183 is approved to satisfy the College of Arts and Sciences mathematics requirement for the Bachelor of Arts degree. In 2009-2010, it was offered in 16 sections to about 500 students. The course,, which is designed so that students work collaboratively in groups of three to four to discuss and answer questions related to quantitative information found in newspaper and other media articles, has encountered a variety of challenges that exemplify broader questions confronting interactive teaching of mathematics in context. Many students possess deeply held views regarding mathematics and struggle with the departure from traditional, lecture-driven mathematics classes. Available curricular materials that engage undergraduate students to reason in real-world settings are limited. Solving new problems on quizzes and examinations is challenging and uncomfortable for students, but necessary as QL requires “authentic” tasks. The variety of contexts in which QR is needed tests the instructor’s flexibility and knowledge. Many of the challenges have been ameliorated by putting together *Case Studies for Quantitative Reasoning: A Casebook of Media Articles* which bases sets of questions upon quantitative content derived from media articles. Learning gains measured by pre- and post-course tests are modest (two percentage points for the mean), but still larger than in other control groups. Faculty advisors’ attitudes about the course are overall positive. Persistence beyond the course, as measured by a survey of 300 former students, seems positive; for example, 29 of the 42 respondents state that their confidence with QR had increased since the course. Similar courses are being taught at Central Washington University and Hollins University by Stuart Boersma and Caren Diefenderfer, respectively, who are also co-PIs on the NSF QRCW project.

Keywords

Quantitative Literacy, Curriculum, Instructional Practices, Research, College Mathematics

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Cover Page Footnote

Shannon Dingman is an assistant professor in the Department of Mathematics at the University of Arkansas. His research specialty is mathematics education, including such topics as the impact of policy on mathematics curriculum.

Bernie Madison is professor and former Chair of the Department of Mathematics, University of Arkansas, and former Dean of its Fulbright College of Arts and Sciences. He was the founding president of the National Numeracy Network and is a frequent contributor to this journal.

Introduction

The quantitative reasoning (QR) demands imposed by today's society are enormous and growing, as documented in numerous sources including works edited and authored by Madison and Steen (Steen, 2001; Madison and Steen, 2003, 2008b; Steen, 2004). With the increased accessibility of data, news, and commentary through media and the Internet, attentive citizens face a multitude of quantitative statements and arguments to process and understand.

The increased need to reason and understand quantitative information has prompted calls to reform and enhance the undergraduate curriculum by including quantitative literacy (QL)¹ in order to fulfill students' needs to be informed and educated citizens. These calls have spurred initiation of a wide variety of efforts across the country focused on developing undergraduates' abilities to analyze, interpret, and comprehend quantitative information (Madison and Steen, 2008a). These efforts range from comprehensive efforts across the college curriculum to single one-semester courses. This paper is a report of a single one-semester course with features aimed at extending its effects beyond the course and beyond school, in recognition of QL being a habit of mind and therefore requiring continuing practice.

These efforts to enhance the QR of college students are taking place with little guidance from learning research results and tested curricular materials. Various kinds of mathematical reasoning—geometric, multiplicative, proportional, algebraic, statistical, and quantitative—have received considerable attention and have been well researched in younger students (e.g., Harel and Confrey, 1994; Steffe and Nesher, 1996; Thompson, 1988; Thompson and Saladanha, 2003; Smith and Thompson, 2007). Research on undergraduate mathematics education,² growing in recent years, has focused largely on the mainstream content areas such as algebra and calculus (e.g., Dubinsky, 1991; Carlson and Rasmussen, 2008). Studying the development of reasoning in college students during a one-semester course offers particular challenges, including relatively brief student contact (in our case, thirty 80-minute class periods), ingrained student learning habits, and fragmented student attention and commitment. The collegiate social sciences have faced the challenges of QL for quite some time, and the increased availability and complexity of data and data analyses have accelerated attention of educators, researchers, and professional

¹ We use the terms quantitative reasoning (QR) and quantitative literacy (QL) interchangeably.

² See <http://www.maa.org/features/rumec.html> for information on research on undergraduate mathematics education (RUMEC).

organizations (e.g., Caulfield and Persell, 2006; Howery and Rodriguez, 2006; Hunt, 2004; and Wilder, 2009).

In this article, we describe our effort to better understand QL for college students through a six-year curriculum development and a parallel formative research project. We discuss the successes in creating and implementing a QL-friendly course at the University of Arkansas as well as the challenges encountered throughout the process, including how students' prior experiences in traditional mathematics and statistics courses have shaped their views and approaches to learning. Based on our experiences teaching this QL course and researching students' strengths and weaknesses when working in settings requiring quantitative literacy, we offer some conclusions from our study, and, in a companion article in this issue of *Numeracy*, we suggest areas where further research is needed as well as questions that have materialized from this work (Madison and Dingman, 2010).

Description of the Undergraduate Quantitative Reasoning Course

The source of most of the information discussed in this article stems from the development of a QL-friendly course that originated and has evolved over the past six years at the University of Arkansas. For this report, we will refer to this course as QRCW, the title acronym of the National Science Foundation project (DUE-0715039, Quantitative Reasoning in the Contemporary World) that currently supports expansion of the course. This course was initiated by Bernard L. Madison as an experiment in the Fall 2004 semester to 26 students who were majoring in various arts and humanities disciplines. The experimental course used as its curricular guide newspaper and other public media articles and graphics collected by the instructor or submitted by students that contained quantitative information or analyses of data. Initially, the course was offered as a section of a standard course in finite mathematics with the unofficial title of "News Math," partly because the second and third sections of the course had mostly journalism students, for whom the course had a professional dimension. After three semesters, the department, college, and university faculties approved the course as MATH 2183, with college algebra as a pre-requisite.

Over the past six years, the course has evolved into its current form. The course now meets the mathematics requirement for the Bachelor of Arts degree in the College of Arts and Sciences and serves as the terminal mathematics or statistics course for many students. Additionally, it is attracting increasing numbers of education and business-related majors who need mathematics elective courses to complete their degree program. During the 2009–2010 academic year, 16 sections of QRCW were offered to approximately 500 students. The source

material for the course has also evolved, beginning as a collection of newspaper and media articles that eventually provided the foundation for the development of a non-traditional textbook *Case Studies for Quantitative Reasoning: A Casebook of Media Articles* (2nd edition) written by the authors of this paper, Stuart Boersma, and Caren Diefenderfer (2009). The textbook contains 24 case studies of media articles, with each case study having warm-up exercises and study questions pertaining to the quantitative information in the article. One of the more mathematically intensive case studies can be found in Appendix A.³ Almost all assessment and study questions are contextual items from far-ranging settings. The major learning goal of the course is to prepare students to answer questions analogous to the study questions about unpredictable media articles or quantitative situations they encounter in everyday life. Each assessment administered in the course, whether a quiz or summative examination, contains questions on articles that the students have not seen prior to the assessment. Versions of QRCW have been developed at other institutions, including by Stuart Boersma at Central Washington University and Caren Diefenderfer at Hollins University.

Context of the Course

The class is conducted with as much interaction with and between students as can be elicited. The course is designed so that students work collaboratively in groups of 3–4 discussing and answering questions related to the media articles under study. There are several standard activities apparent in most QRCW classes: mini-lectures over concepts as they are needed, group work on case studies of media articles, and discussion of homework assignments. Feedback on student understandings and misunderstandings is fairly constant. One critical component of the course is called “News of the Day.” Students are awarded bonus points for bringing in newspaper or magazine articles that have quantitative content and that the student has found interesting. The articles are projected on a viewing screen by a document camera, and the student summarizes the article and explains the quantitative content or poses questions stemming from the article that he/she did not understand. Other class members are encouraged to join the discussion. This activity keeps the class content fresh and allows for student interests to surface. The discussions venture into social, economic, and political issues and create connections that some students had not previously considered. The News of the Day feature is aimed at extending QR beyond the course and beyond school by providing a venue for continuing practice and leveraging student interest.

³ Appendices are available as supplemental files linked individually on the electronic cover page for this article.

The student population for QRCW has been quite diverse with about equal numbers of men and women, students from various majors, honors students, student athletes, non-traditional students, students with learning disabilities, and students who consider themselves “bad” at mathematics. Most students come from schooling in mathematics that is heavy on algebra and evidently weaker on proportional reasoning, but the algebra is minimally accessible to them. They have very limited sets of personal quantitative benchmarks and weak productive disposition toward mathematics (Kilpatrick, Swafford, and Findell, 2001). We cite evidence of these characteristics later in this paper and expand on them in the companion paper (Madison and Dingman, 2010).

Most of the students are majors in arts, humanities, or social sciences. Many students have written about the encouragement the course has given them to deal with quantitative issues. The relevance to their everyday lives—for all students—promises to keep all students’ interests and motivate them to succeed. Student evaluations of QRCW have been very positive, most of all, because the content is obviously relevant to everyday lives. One student put it this way: “This course takes off the table once and for all the question of where will I ever use this.”

Throughout the six-year development of the QRCW course, a number of successes as well as trials have been confronted. In the next section, we describe some of the broad challenges that we have met in developing and modifying the QRCW course. It is important to note that the information and data used here stem from work conducted at the University of Arkansas. Some of the information comes from QRCW class sessions led by the authors; some comes from pre- and post-course tests and attitude surveys conducted with students during the Fall 2007 (Appendix B), Spring 2008 (Appendix C), and Fall 2008 semesters for QRCW; and some comes from think-aloud sessions conducted by the authors. The think-aloud sessions used 33 volunteer QRCW students at three different points during the 2008 calendar year (one session during the spring semester, two during the fall). Students were grouped around tables, most with four students, and were presented with fresh media articles containing quantitative information and a set of questions pertaining to the quantitative content of the articles. The audio and video of the think-aloud sessions were recorded and the audio was transcribed for study. This report is not a complete analysis of the transcripts of the think-aloud sessions, which are much richer in detail than we can report here.

Challenges in the Development of the Course

In a previous paper based on an early version of QRCW, Madison (2006) outlined 17 individual challenges that were apparent in developing a QL course and in working with undergraduate students to construct and enhance their quantitative

reasoning skills. Many of these challenges are still evident. Here we provide four broad areas that we have confronted in the development and modification of the QRCW course and describe some of the issues we have faced within each area.

Traditional Mathematics' Impact on the Student

For many of the students in our QRCW classes, success in mathematics has been elusive. Many of them have been taught mathematics for much of their academic careers in traditional, lecture-driven classes, where mastering mathematics is seen as memorizing facts and formulas or learning how to use a particular procedure to solve problems that typically are void of context. This traditional background colors the way these students view mathematics and how it should be taught and learned. Therefore the format of the QRCW course (e.g., collaborative learning involving group work) and the content focus (e.g., contextual situations requiring reasoning about what mathematical ideas to use) provide a dramatic shift in their vision of how a mathematics class should be conducted.

There are certain traits that follow students from their traditional mathematics education. Many students struggle with the requirement that they reason in order to determine how to solve a problem. If the students are unsure of how to solve a problem, they generally will ask an instructor what method they should use (e.g., in working with percentages, students will inquire whether they should multiply or divide in order to get their answer) rather than spend time investigating what method would be most useful in the given situation. Additionally, some students wish to be shown a template problem that would model what they should do in similar problems. In general, many of our students are more focused on obtaining the correct answer than understanding the process that led to that answer. Unfortunately, these students often lack the knowledge and skills needed to determine on their own whether their answer is correct or even reasonable.

Because many of the problems in the QRCW course typically do not suggest mathematical strategies that should be used, students in our classes request certain cues regarding what they should do to solve problems. For example, in a unit concerning percents and percentage change, students must judge whether problems are requiring the determination of an absolute change (e.g., the population of a town growing from 5,000 people to 10,000 people is an increase of 5,000 people) or a relative change (e.g., the population increased 100%). On a number of occasions, we have been pressed to help students determine whether an absolute or relative change is warranted by focusing on what language cues are apparent in the problem that will help them determine a solution method.

The push for a correct answer is also an issue in certain situations where abstracting general patterns would be much more valuable to solving QL problems. For example, one case study in our QRCW class has students examine how long it would take to break even on the cost of buying a traditional gasoline

automobile versus a hybrid automobile, where the hybrid vehicle costs more and achieves better fuel efficiency (Appendix A). In this situation, an algebraic modeling strategy would examine the pattern for costs over several years to operate each vehicle in order to determine a mathematical model or models (e.g., a linear equation). Many of our students pursue a tabular or numerical method to solve the problem, as one will compute the cost of operating each car for one year, for two years, etc., until they reach the point where the cost of the hybrid is less than the cost of the gas version. In this instance, the student is more concerned with determining the costs after each individual year than determining the overall pattern of changing costs. The push for the right answer (e.g., the number of years at which the cost of the hybrid car is less than the cost of the gas version), therefore, trumps methods that would extend understanding of the process of comparing costs.

This is an instance where students possess what Dubinsky (1991) calls a recurring limited conception of models of quantitative circumstances. In this example, students' conceptions fall at a very early stage, called action in the action-process-object-schema (APOS) model. As stated in Asiala et al. (1996),

a student who is unable to interpret a situation as a function unless he has a single formula for computing values is restricted to an action concept of function. In such a case, the student is unable to do very much with this function except to evaluate it at specific points and to manipulate the formula. (p. 9)

One of the positive changes we have seen is the modest shift in the students' views regarding the relevance of the mathematics in their everyday life. By placing the mathematical and statistical topics in real-world contexts, the connections to their life are much more real and apparent than their past experiences in learning mathematics. Interestingly, in our experiences with students, we have noted differences between the younger, traditional college students (e.g., students 18–22 years old) and older, non-traditional students. The non-traditional students seem more receptive to the applicability of the content under study to their everyday life, presumably due to their greater life experiences, in comparison to that of their younger classmates. However, we have also seen evidence that these non-traditional students are less receptive to the cooperative learning environment where reasoning is required. In one specific instance, a non-traditional student articulated her displeasure over the structure of the course, stating that the course “needs lectures and not so much group work” and that the professor should take a more active role in the class by directly explaining more of the content. This difference may be due to the fact that, with the increase in collaborative work in K-12 schooling over the past several decades, the younger students may have more experience with and are more accustomed to these learning environments than non-traditional students.

Curriculum Materials and Content

A second challenge in developing the QRCW course was defining the scope of the mathematical content to study and finding curricular materials that would facilitate student learning. The availability of high-quality curricular materials that engage undergraduate students to reason in real-world settings is rather limited. In our case, this limitation prompted creation of the textbook *Case Studies for Quantitative Reasoning: A Casebook of Media Articles* (2nd edition) (Madison et al., 2009), which bases a set of questions upon the quantitative content derived from media articles (see Appendix A for a sample case study).

Creating case studies surrounding the mathematical content found in media articles presented a set of challenges. Although the real-world contexts provide authentic settings for enhancing students' quantitative reasoning skills, the challenge was to create questions that were accessible to the skill levels of the students. In essence, we needed to create questions stemming from the articles that were challenging enough to warrant the students' time and effort but not so challenging that students could not be successful. In several of the contexts, this challenge was heightened by the students' general lack of problem-solving skills and tools to tackle the problems.

Inherent in using media articles as the basis for investigation is the challenge of keeping the tasks fresh and engaging. In fact, in several of the case studies in the textbook, the realities of the present world setting are quite different from what is described in an article written less than a decade ago. For example, in a 2001 article, the author makes the claim that foregoing a \$2.50 cup of coffee each day for 25 years and investing the savings at a "modest" annual rate of 8% would yield \$72,800 for one's sacrifice. Some students scoff at this claim, not only for the amount that is yielded (which is accurate using installment savings on a daily basis) but also for the fact that finding an 8% rate of return in 2010 is difficult if not impossible. Additionally, a case study surrounding a 2003 graph examining the political debate regarding whether or not the \$304 billion dollar federal deficit in 2003 was a record now seems trivial in an era of trillion dollar deficits.

The challenge in keeping material fresh has been lessened by the requirement in our course that students participate in bringing "News of the Day" to share with the class. This option in our course pushes students not only to exercise heightened awareness to the mathematical content that is found everyday in media articles but also works to bring new, fresh examples of the topics that have been investigated. In several cases, the articles that have been presented by students in class have formed the basis for later questions on quizzes and examinations as well as ideas and references for further case study development.

Although much of the mathematical and statistical content encountered in the course is generally taught in middle to early secondary grades, the embedding of the content in real-world contexts and the use of reasoning to determine solution

strategies elevates the degree of sophistication for many students. As described in the previous section, students have come to view mathematics as something completely different from what is presented in the QRCW course. As many have stated to us, they feel that they understand the concepts but struggle when they have to apply them in real-world settings where a procedure to solve the problem is not readily evident. This problem is not unique to the students involved in the course. In fact, students have complained on several occasions that the on-campus tutors, who provide assistance for students in courses spanning college algebra to the calculus sequence, struggle in helping them with homework from the QRCW course, primarily because the problems are unlike ones the tutors have ever experienced in their mathematical preparation and that the problems require using mathematical processes that are not foci of traditional courses.

Prior to the development of the textbook, a number of students would give up trying to solve the case study problems due to not knowing where to begin. In the development of our textbook, we conceded somewhat to the students' struggles in working with the pilot materials by providing warm-up exercises for the case studies that would focus on the mathematical concepts that could be used in further work (see Appendix A for a sample). By offering this piece of scaffolding for students, we have seen in general that students have been more successful in solving the case study questions surrounding the content found in the media articles.

Assessment

In his keynote address to the 2007 Wingspread conference on QL and its implications for teacher education (Madison and Steen, 2008b), Richard Shavelson (2008) took an assessment approach to QL. He stated:

If we seek to enhance the quantitative reasoning of the American public, not only do we need to say clearly what quantitative reasoning is, we also need to know how to teach for it and how to measure progress toward our QR goal. (p. 27)

We teach for QL in QRCW and we assess student progress in developing their QL skills, but a one-semester course is far too limited to achieve or assess our students' QL in the larger setting. This limited view does not significantly lessen the challenges of assessment of QL in QRCW.

Assessment of QL offers significant challenges, both for the QRCW course and the more comprehensive assessment of QL as an outcome of undergraduate education. The major challenge in assessing QL concerns the central goal of transfer of knowledge and cognitive processes to contexts that are unpredictable and of unbounded variation. The assessments we have utilized in QRCW classes consist of homework assignments for most class meetings, quizzes, and two examinations: a mid-term and a final. Almost all problems and exercises are contextual, and each quiz and examination contains questions stemming from at

least one media article that is new to the students. This challenges our students to use their existing knowledge in new and sometimes unfamiliar settings. Again, many of our students' traditional views of mathematics learning hinders them, as they are used to preparing for assessments by studying existing problems that have already been worked. The requirement of solving new problems on quizzes and examinations is therefore challenging and uncomfortable to these students.

Another challenge we faced at the beginning of the QRCW course development was the lack of clearly defined learning goals and the associated levels of development or performance standards for QL. Because much of QL stems from mathematical processes such as reasoning, problem solving, and communication as well as overall skills regarding critical reading and interpretation, assessing students' QL abilities is rather difficult. Recently, there have been recommendations published that have assisted us in thinking about QL assessment. The Association of American Colleges and Universities (AAC&U) addressed the broader issue of QL as an outcome of undergraduate education as part of its Valid Assessment in Undergraduate Education (VALUE) project (AAC&U, 2009). The VALUE rubric for QL lays out the challenge as follows:

Individuals with strong QL skills possess the ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate).

The QL VALUE rubric has four developmental benchmarks in six competency areas: interpretation, representation, calculation, application/analysis, assumptions, and communication. These competency areas align very well with our conception of a canonical QL situation (Madison, 2006) in QRCW, consisting of five steps: encountering; interpreting; gleaning and assuming; modeling and solving; and reflecting. In addition to the broad competency areas, assessment of QL requires using authentic tasks, which separates it from traditional assessment in the mathematical sciences and most all of undergraduate education. Even though "authentic" is sometimes difficult to define, QRCW's model of using case studies of media articles makes authenticity easier to discern, namely, quantitative analyses of media articles.

We have made two attempts at a pre- and post-course test instrument, with mostly multiple-choice items, to assess the growth in our students' QL abilities across a semester of QRCW. We used pre- and post-course testing for three semesters of the QRCW course in 2007 and 2008 (see Appendices B and C for the tests and student results for Fall 2007 and Spring 2008 respectively), with each testing showing modest gains made by students. However, we are not satisfied with the instruments and are planning to make another attempt during the

2010–2011 school year. We believe in-depth probing will be required to measure performance in the six competency areas above, and scoring the results of such probing for more than a few students is not practical. We have considered the possibility of developing an in-depth assessment process paired with a multiple-choice instrument so that the results have high enough correlation to use the multiple-choice instrument as a proxy for the in-depth process. This hypothesis presents some interesting research questions. Can a multiple-choice assessment instrument serve as a valid proxy for an in-depth assessment process? Are analyses of quantitative media articles sufficiently authentic to measure QR/QL?

Pedagogical Issues

A final area of challenges centers on the instructor's role in directing the QRCW course. One of the powerful aspects of quantitative literacy is that there is a wide range of contexts in which this type of reasoning is needed—from business and economics to social and political arguments; from understanding weather forecasts to watching and playing sports; from reading the newspaper or watching the local news to visiting the local marketplace. These are just some of the settings of opportunities for students to examine quantitative arguments.

The variety of contexts in which quantitative reasoning is needed tests the flexibility of the QRCW instructor's knowledge and understanding of working in these contexts. In particular, students have brought in “News of the Day” topics from a wide array of far-ranging topics. The extemporaneous nature of this feature of the QRCW course (the instructor generally does not have any fore-notice regarding the content or context of these student presentations) forces the instructor to think on the spot—not only to understand what the student is presenting but also to discern what mathematical content is being discussed and whether or not it is correct. The adaptable nature of the course is quite challenging, particularly for faculty accustomed to having greater control over what is discussed and covered in the classroom.

From time to time, students will share news that is grounded in arenas that have the potential to lead into delicate and sensitive areas. On occasion, students have presented articles concerning political or social issues that have led to tangential forays away from mathematics and into their opinions regarding the current state of affairs. Although students are urged to focus on the quantitative aspects of the article and refrain from delving into opinion-based speeches, it is a challenge for the instructor to steer the discussion back to the mathematics when, from the students' vantage point, the underlying context provides for an interesting debate and discussion.

The use of student-centered learning environments through collaborative group activities can also challenge new instructors not used to this type of classroom environment. Because students have flexibility in how they come to an

answer to a problem, instructors must also be adaptable to these strategies and understand whether the student has been successful or not as well as whether or not the strategy is valid on a wider array of problems. The nature of this type of classroom directly confronts instructors' ideas regarding how mathematics is taught—a manner completely different from one that many have experienced in a traditional, lecture-driven classroom format.

Because students tend to struggle with QRCW problems, a final pedagogical challenge is fighting the urge as an instructor to step in and show the student how to work a problem. Several key components of learning to reason quantitatively are learning to examine various strategies to know which one(s) would be appropriate in the given situation as well as the ability to apply one's knowledge across a number of contexts. Although students may be well served to observe how a knowledgeable other solves a problem, showing students how to solve a specific problem generally does not help the students solve other problems and thereby has only helped them with the roadblock in front of them. Examining strategies that students have attempted and building from their existing knowledge to push them along, without giving them the answer, is more helpful to students. Although difficult to do, this type of scaffolding can better serve students over the long-term in assisting them to understand how to solve problems and reason quantitatively.

Evaluation of the QRCW Course

Evaluation of the QRCW course is in progress and will be our focus for the next year or two, but we give some preliminary information here, in the appendices, and in the companion paper. Our evaluation plan focuses on four areas: student and faculty advisors' attitudes, student learning in QRCW, student persistence beyond the QRCW course, and student success rates in QRCW as measured by grades. To measure student attitudes before and after QRCW we surveyed students during the Fall 2007, Spring 2008, and Fall 2008 semesters. In Fall 2007, we asked students to respond to ten attitude statements with one of five choices: strongly agree, agree, neutral, disagree, and strongly disagree. For the Spring and Fall 2008 semesters, we reduced the number of attitude items from ten to five. Along with the attitude surveys, we administered a pre- and post-course test to attempt to measure learning gains over semester. For Fall 2007, the test consisted of 15 multiple-choice items, while for Spring and Fall 2008, the test had 17 multiple-choice items and three student-generated response items (see Appendices B and C for these tests with attitude surveys).

We have compared the attitude survey responses, the pre- and post-test scores, and score changes of QRCW students to the same data of a comparison group of students from two other courses—a survey of calculus course and a

general education course taught using *For All Practical Purposes* (CoMAP, 1988) as a textbook. For QRCW students, the mean of the responses to each pre- and post-attitude item moved in the direction we wanted, but none of the improvements were dramatic. In each pre- and post-testing the mean scores (out of 15 in 2007 and 20 in 2008) for QRCW students improved by approximately two points, and in each semester, the QRCW students' gains exceeded those of the other student groups. These results are positive but not dramatic (Madison and Dingman, 2010).

To examine the faculty advisors' attitudes toward QRCW, the evaluator for our NSF grant that supports expansion of the course conducted a survey of 18 faculty advisors who advise University of Arkansas students about course selection. We are interested in the views of these advisors because they not only influence course selection but also student attitudes about learning in the courses selected. Nine of the 18 responded to the survey, and the QRCW evaluator determined that seven responses were substantive in addressing the questions posed. The evaluator summarized the results:

The seven respondents were unanimous in saying that they have not encountered resistance to the QRCW project. In fact, the respondents used very positive words when describing the reaction of students, faculty, and staff regarding QRCW, including 'glad,' 'excited,' 'enthusiastically embraced,' and 'fantastic option.' This finding stands in contrast to a number of other reform efforts within mathematics.

The third area of evaluation—persistence beyond the QRCW course—provides the greatest challenge in obtaining information regarding our efforts in the QRCW class. During the Fall 2009 semester, we sent email questionnaires to approximately 300 students who had finished the QRCW course in previous semesters and whose university email addresses were still active. In the survey, we asked:

1. How often have you practiced analyzing quantitative content of public media (newspaper, magazine, advertising flyer, online material, etc) articles since you finished the Mathematical Reasoning course? Never, Rarely, Regularly, or Often.
2. How has your confidence with quantitative reasoning changed since the course? Decreased, Stayed the same, or Increased.
3. How has your view of the importance to you of quantitative reasoning changed since the course? Decreased, Stayed the same, or Increased.
4. Any other comment?

Forty-two former QRCW students responded to the survey. From their responses, we found that 69% (29 of 42) of respondents stated their confidence with QR had increased since the course and that 76% (32 of 42) held an increased importance

to QR, although 55% (23 of 42) replied they rarely practiced analyzing the quantitative content in public media. Many respondents commented with positive reflections on the course and its utility and impact on their everyday life. The survey, summary of responses, and comments regarding the course are located in Appendix D.

With regard to student success rates in QRCW, the course is organized in such a way that 60% of a student's grade is based on daily work, with this work generally being collected at each of the two weekly class meetings for grading. This has had the effect of keeping most students continually engaged, reducing withdrawals, and raising grades. Consequently, student retention and grades are significantly better in QRCW than that of other introductory mathematics courses. During the first five years of QRCW, 83% of the students earned grades of A, B, or C; another 7% earned D's; 7% withdrew with a mark of W; and 3% received a failing grade of F.

Conclusion

Designing and implementing experiences for undergraduate students to enhance their QL abilities can be complex and challenging. The challenges are amplified by the fact that students are often only immersed in QL for a single semester-long course. Because students possess deeply held beliefs and ideas regarding how mathematics is taught and learned, QL-friendly courses must overcome these conceptions by focusing on the applicability of the skills and the power of the mathematical processes that are to be developed during a semester-long course as well as practiced and refined in the student's life after the QL-course and post-college. Therefore, getting students to buy into the importance of gaining and strengthening the ability to reason quantitatively is a fundamental component they must gain from their experiences.

The foregoing describes a six-year course development and research effort at a major state university. In our companion article, we expand on these observations and the many questions that have arisen. Specifically in the context of college QL-friendly courses, much remains to be learned about how students develop quantitative reasoning skills and how this can best be supported through curricular materials, high-quality assessment tasks, and by instructors knowledgeable about the subject and how it is best learned. Research is needed that sharpens and examines important questions to provide guidance for curricular development in universities across the country. The goal is ambitious, but the stakes are high and rising—to ready and equip all graduates with the understanding and skills needed to be productive members of the workforce and knowledgeable citizens in our democracy.

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References

- Asiala, M., A. Brown, D. DeVries, E. Dubinsky, D. Mathews, and K. Thomas. 1996. A framework for research and curriculum development in undergraduate mathematics education. In *Research in college mathematics education*, ed. A.H. Schoenfeld, J. Kaput, and E. Dubinsky, 1–32. CBMS Issues in Mathematics Education, vol. 6. Providence, RI: American Mathematical Society.
- Association of American Colleges and Universities (AAC&U). 2009. Valid assessment of learning in undergraduate education. <http://www.aacu.org/value/index.cfm> (accessed May 28, 2010).
- Carlson, M., and C. Rasmussen. 2008. *Making the connection: Research and teaching in undergraduate mathematics*. Washington, DC: Mathematical Association of America.
- Caulfield, S. L., and C.H. Persell. 2006. Teaching social science reasoning and quantitative literacy. *Teaching Sociology*, 34(1): 39–53. <http://dx.doi.org/10.1177/0092055X0603400104>
- Consortium for Mathematics and Its Applications (CoMAP). 1988. *For all practical purposes: Introduction to contemporary mathematics*. New York, NY: W. H. Freeman and Company.
- Dubinsky, E. 1991. Reflective abstraction in advanced mathematical thinking. In *Mathematical thinking and problem solving*, ed. A. Schoenfeld, 221–243. Hillsdale, NJ: Erlbaum.
- Harel, G., and J. Confrey. 1994. *The development of multiplicative reasoning in the learning of mathematics*. Albany, NY: State University of New York Press.
- Howery, C. B., and H. Rodriguez. 2006. Integrating data analysis (IDA): Working with sociology departments to address the quantitative literacy gap. *Teaching Sociology*, 34(1): 23–38. <http://dx.doi.org/10.1177/0092055X0603400103>
- Hunt, K. 2004. The challenges of integrating data literacy into the curriculum in an undergraduate institution. *International Association for Social Sciences Information Service and Technology Quarterly*, Summer/Fall, 12–15.
- Kilpatrick, J., J. Swafford, and B. Findell, B. 2001. *Adding it up: Helping students learn mathematics*. Washington, DC: National Academy Press.

- Madison, B. L. 2006. Pedagogical challenges of quantitative literacy. ASA Section on Statistical Education. <http://www.statlit.org/pdf/2006MadisonASA.pdf> (accessed May 28, 2010).
- Madison, B. L., S. Boersma, C.L. Diefenderfer, and S.W. Dingman. 2009. *Case studies for quantitative reasoning: A casebook of media articles*, 2nd ed. New York, NY: Pearson.
- Madison, B. L., and S.W. Dingman. 2010. Quantitative reasoning in the contemporary world, 2: Focus questions for the numeracy community. *Numeracy*, 3(2). <http://dx.doi.org/10.5038/1936-4660.3.2.5>
- Madison, B. L., and L.A. Steen. 2003. *Quantitative literacy: Why numeracy matters for schools and colleges*. Princeton, NJ: National Council on Education and the Disciplines. <http://www.maa.org/ql/qltoc.html> (accessed May 28, 2010).
- . 2008a. Evolution of numeracy and the National Numeracy Network. *Numeracy*, 1(1), Article 2. <http://dx.doi.org/10.5038/1936-4660.1.1.2> (accessed May 28, 2010).
- , eds. 2008b. *Calculation vs. context: Quantitative literacy and its implications for teacher education*. Washington, DC: Mathematical Association of America. <http://www.maa.org/Ql/calcvcontext.html> (accessed May 28, 2010).
- Shavelson, R. J. 2008. Reflections on quantitative reasoning: An assessment perspective. In *Calculation vs context: Quantitative literacy and its implications for teacher education*, ed. B. L. Madison and L. A. Steen, 27–44. Washington, DC: Mathematical Association of America. (see <http://www.maa.org/Ql/calcvcontext.html>, accessed May 28, 2010).
- Smith, J., and P.W. Thompson. 2007. Quantitative reasoning and the development of algebraic reasoning. In *Algebra in the early grades*, ed. J. Kaput, D. Carraher, and M. Blanton, 95–132). New York, NY: Erlbaum.
- Steen, Lynn Arthur, ed. 2001. *Mathematics and democracy: The case for quantitative literacy*. Washington, DC: Woodrow Wilson National Fellowship Foundation. <http://www.maa.org/ql/mathanddemocracy.html> (accessed May 28, 2010).
- . 2004. *Achieving quantitative literacy: An urgent challenge for higher education*. Washington, DC: Mathematical Association of America.
- Steffe, L. P., and P. Neshier, P., eds.. 1996. *Theories of mathematical learning*. Mahwah, NJ: Erlbaum.
- Thompson, P. W. 1988. Quantitative concepts as a foundation for algebra. In *Proceedings of the annual meeting of the North American chapter of the international group for the psychology of mathematics education*, Vol. 1, ed. M. Behr, 248–254. Montreal, Canada: University of Quebec, Montreal.

- Thompson, P. W., and L. Saladanha. 2003. Fractions and multiplicative reasoning. In *Research companion to the principles and standards for school mathematics*, ed. J. Kilpatrick, G. Martin, and D. Schifter, 95–114. Reston, VA: National Council of Teachers of Mathematics.
- Wilder, E. I. 2009. Responding to the quantitative literacy gap among students in sociology courses. *Teaching Sociology*, 37(2): 151-170. <http://dx.doi.org/10.1177/0092055X0903700203>