ADVANCES IN GLOBAL EDUCATION AND RESEARCH

GLOCER'21

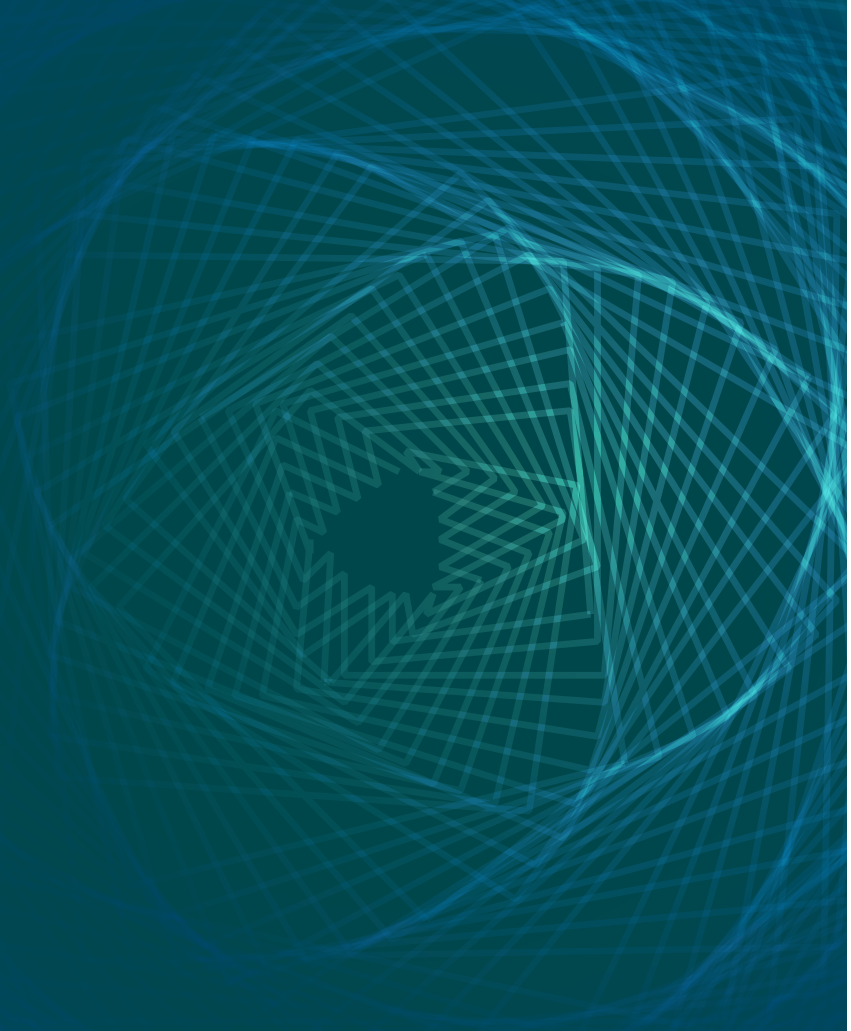
VOLUME 4

Editors:

Dr. Waynne B. James

Dr. Cihan Cobanoglu

Dr. Muhittin Cavusoglu



Co-Editors

Dr. Waynne James**,** University of South Florida, USA
Dr. Cihan Cobanoglu, University of South Florida, USA
Dr. Muhittin Cavusoglu, Northern Arizona University, USA

ADVANCES IN GLOBAL EDUCATION AND RESEARCH: VOLUME 4

ISBN 978-1-955833-04-2

Authors are fully responsible for corrections of any typographical, copyrighted materials, technical and content errors.

https://digitalcommons.usf.edu/m3publishing/vol3/iss2021/49
DOI: https://www.doi.org/10.5038/9781955833042

Co-Editors

Dr. Waynne James, University of South Florida, USA
Dr. Cihan Cobanoglu, University of South Florida, USA
Dr. Muhittin Cavusoglu, Northern Arizona University, USA

ISBN 978-1-955833-04-2

This imprint is published by USF M3 Publishing, LLC
The registered company address is University of South Florida, 8350 N Tamiami Tr, Sarasota, FL 34243 USA.

Assistant Editor

Editor Assistants

****Authors are fully responsible for corrections of any typographical, copyrighted materials, technical and content errors.***

https://digitalcommons.usf.edu/m3publishing/vol3/iss2021/49
DOI: https://www.doi.org/10.5038/9781955833042

Metacognitive Functions of Solving Routine and Non-Routine Problems

Yasemin Katrancı

Faculty of Education
Kocaeli University, Turkey

Abstract

In this study, the main aim was to analyze the role of metacognition while solving routine and non-routine problems. The study was conducted with a qualitative approach. "Routine and Non-routine Problems Form" and "Metacognitive Skills Questionnaire" were used as the data collection instruments. The study group was determined according to a convenience sampling. Within this regard, 66 pre-service middle school mathematics teachers participated in the study. In order to fill out the data collection instruments, 60 minutes were given to the pre-service teachers. The collected data were analyzed descriptively. As a result, it was seen that the pre-service middle school mathematics teachers use their metacognitive skills in solving routine and non-routine problems. It was found that the pre-service teachers faced some difficulties in this process and all the difficulties are about to solve non-routine problems. In addition, it was seen that the pre-service teachers made some mistakes while solving the problems. The last result of the study is that the pre-service teachers paid attention to some points while solving problems. As the main result, although the pre-service teachers use their metacognitive skills while solving routine and non-routine problems, they face some difficulties and make mistakes.

Keywords: metacognition, problem-solving, routine and non-routine problems

Recommended Citation: Katranci, Y. (2021). Metacognitive functions of solving routine and non-routine problems. In W. B. James, C. Cobanoglu, & M. Cavusoglu (Eds.), *Advances in global education and research* (Vol. 4, pp. 1–14). USF M3 Publishing.
https://www.doi.org/10.5038/9781955833042

Introduction

When someone says "problem", mathematical problems based on four operations come to mind (Heddens & Speer, 1997). In general, a problem is a situation that attracts the attention of an individual and this individual does not have sufficient knowledge to solve this situation (Blum & Niss, 1991). Katrancı (2014) defines the problem as a situation that obscures and challenges an individual's beliefs as well as the individual doesn't have a memorized or determined rule to solve it. The problems are classified as routine and non-routine. The routine problems require improving four operations skills and to solve them it is necessary to use these skills. The non-routine problems are known as questioned forms of the expression of daily life (Altun, 2014). In learning and teaching mathematics, using the routine and non-routine problems is a need for both students and teachers (Chapman, 2002). If there is a problem, it should be solved to clarify the individual's mind. Thus, this requires problem-solving.

Problem-solving is to know what to do when it is not known what to do (Altun, 2014). Besides getting an outcome, it is to get rid of an obstacle (Polya, 1957). According to Silver (1994), problem-solving is a situation that removing confusion in an individual's mind. Altun (2014)

mentions that problem-solving has a systematic although it doesn't have rules. Within this systematic, problem-solving occurs in four different stages. These stages are as follows; i) Understanding the problem, ii) Devising a plan, iii) Carrying out the plan, and iv) Looking back (Polya, 1957). Polya expresses that an individual have to understand the problem. The individual has to ask the following questions at the first stage; "What is the unknown?", "What are the data?", "What is the condition?" etc. At the second stage, s/he should find the connection between the data and the unknown. S/he should obtain eventually a plan of the solution at this stage. Third, s/he should carry out this plan for the solution. Then, s/he should ask that can s/he prove whether it is correct or not. Finally, s/he should examine the solution. In order to do this, s/he has to ask the following questions; "Can s/he check the results?", "Can s/he see it at a glance?", "Can s/he derive the results differently?", etc. As it is seen, these stages create a process.

Problem-solving is a dynamic process that is affected by some factors identified by Jonassen (2000). These factors are as follows; i) familiarity, ii) subject matter knowledge and structural knowledge, iii) epistemological beliefs, iv) affective variables, v) general problem-solving skills, vi) cognitive controls, and vii) metacognition. The individuals with high cognitive flexibility and cognitive complexity are good problem solvers. It could be ensured that the students understand the problems, devising a plan, and eliminate the situations that prevent problem-solving with the development of metacognitive skills. For this reason, metacognition was discussed in this study.

Metacognition refers to the planning, observing, and evaluating of an individual's learning. The individual with metacognitive skills has both the knowledge of explaining cognition and the skill of organizing this knowledge (Flavel, 1979). Metacognition is an individual's knowledge about her/his cognitive processes and products (Selçuk, 2000). Besides, metacognition is the knowledge that individuals have about their own thinking processes, strategies, ability to monitor, and regulate these processes (McCormick, Miller, & Pressley, 1989). It is about the metacognition of planning, understanding, monitoring how to approach a given task, and evaluating to complete the task or making changes according to new situations that arise (Panaoura & Philippou, 2005; Pugalee, 2001).

The individuals who learn with metacognition are aware of their skills and ability. Therefore, they reach the solution by applying appropriate strategies in problem situations. It is important that the individual has knowledge about her/his own learning and cognitive processes or develop an awareness of this context to achieve in learning processes (Şen, 2012). Additionally, the individual solves problems, is aware of strategies that are used, and uses the processes of thinking and evaluating effectively (Costa, 1984). In problem-solving, metacognition comes to the forefront in the processes of constructing appropriate representations about the given problem and providing the solutions obtained (Schoenfeld, 1983; Verschaffel, 1999). The importance of metacognition in problem-solving was emphasized in many studies (Artz & Armour-Thomas, 1992; Aşık, 2015; Desoete & Veenman, 2006; Mihalca, Mengelkamp, & Schnotz, 2017). In this regard, it was aimed to analyze the role of metacognition while solving routine and non-routine problems in this study. In addition to this main aim, there are three sub-aims. These are as follows; i) observing metacognitive behaviors of pre-service mathematics teachers, ii) determining the importance of metacognition in problem-solving, and iii) determining typical errors, challenges, and important points to solve routine and non-routine problems.

Methods

Research Design

This study was designed as a case study which is a qualitative one. In this approach, the aim is to find out the results related to a specific situation (Yıldırım & Şimşek, 2008). Within this scope, the results regarding metacognitive aspects in the process of solving routine and non-routine problems were determined in detail in this study.

Study Group

The study group was determined according to a convenience sampling. In this sampling, the study group had been created by starting from the most accessible respondents until a sufficient size of the members was created. Then, the study was conducted with these members (Ravid, 1994). In this regard, 66 pre-service middle school mathematics teachers participated in this study. All pre-service teachers were juniors at the university where the researcher works. For this reason, they were the convenience samples. 49 (%74.24) of them were female and 17 (%25.76) of them were male.

Data Collection Instruments and Collecting of Data

In this study, two different data collection instruments were used. These are as follows; i) Routine and Non-routine Problems Form (RNPF) and ii) Metacognitive Skills Questionnaire (MSQ). RNPF includes two problems. The first problem is a routine problem while the second problem is a non-routine problem. This form is shown below.

Table 1. Routine and Non-Routine Problems Form (RNPF)

1.	Two vehicles from two cities with 320 km between each other depart towards each other at the same time. While the speed of one is 36 km/h, the speed of the other is 44 km/h, after how many hours will they meet?
2.	Everyone shakes hands with each other in a meeting attended by 20 people. How many handshakes will there be?

Source. Altun, 2014

MSQ that consists of 14 questions defining cognitive and metacognitive behaviors was adopted from Biryukov (2004). MSQ can be seen in the findings section. First, RNPF was distributed to the pre-service mathematics teachers. In order to fill out this form, 50 minutes were given to the pre-service teachers. After the pre-service teachers had solved the problems, MSQ was distributed to them. 10 minutes were given to complete this form. Totally, the study was completed in one hour. RNPF and MSQ were taken back from the pre-service teachers at the end of this time. These forms were coded as PST1, PST2, …, PST66 (PST=Pre-service teacher). From here, these codes are going to used to express them.

Data Analysis

The data that were collected after the application were analyzed descriptively. In the descriptive analysis, the data could be presented according to dimensions or questions used (Yıldırım & Şimşek, 2008). Within the scope of this explanation, first, the solutions of the given problems in RNPF were analyzed. Then, the items of MSQ which were filled out regarding the solutions of the problems by pre-service teachers were analyzed one by one. The aim of this kind of analysis

is to present the findings as edited and interpreted (Yıldırım & Şimşek, 2008). Therefore, the findings were presented as frequencies (f) and percentages (%).

Findings

First, RNPF was distributed to the pre-service mathematics teachers to solve the problems. When the solutions were examined, it was seen that most of the solutions were done according to the problem-solving stages. It is really important point of the study because no explanation about solving problems was shared with the PSTs. They were free while solving the problems and filling out the MSQ. Then, the following findings were gained after the PSTs' responses had been analyzed;

- The first problem which is routine was solved by all the PSTs.
- The second problem which is non-routine was solved by 55 (%83.33) of the PSTs.
- In addition to these 55 PSTs, six PSTs tried to solve the second problem, but they couldn't find out the correct solution.
- Though another four PSTs did the first and the second stages of problem-solving related to the second problem's solution, they couldn't complete the solving.
- Only one PST couldn't solve the second problem. It was seen that this PST even couldn't try to solve this problem.

Immediately after the PSTs had solved the problems given in the RNPF, they filled out MSQ which defined their cognitive and metacognitive behaviors while solving routine and non-routine problems. The findings can be seen in Table 2 below.

Table 2. The Answers of MSQ

Statements	Yes	No	Not Sure
1. I read the problem more than once.	30	29	07
2. I checked what the problem was asking to me.	61	05	00
3. I evaluated how much time I need for solving this kind of problem.	33	20	13
4. I showed the problem schematically.	59	01	06
5. I tried to recall whether I solved a similar problem before or not.	56	07	03
6. I developed a strategy for solving the problem.	60	01	05
7. I did not know where to start.	02	51	13
8. I faced some difficulties while solving problem (If your answer is 'yes', define the features of the difficulty).	20	43	03
9. I found one of my mistakes while solving the problem and corrected it (If your answer is yes, define the mistake).	09	54	03
10. I thought how the solution was going on.	62	01	03
11. I tried different approaches to solve the problem.	36	20	10
12. I asked myself whether my answers were meaningful or not.	59	03	04
13. I checked my calculations to be sure that they were correct.	61	05	00
14. I thought whether there was something that I should especially pay attention or not in the information given in the problem (If 'yes', define it).	32	21	13

The findings of the analysis of MSQ are presented below in detail.

Table 3. 1st Item of the MSQ

1st Item of the MSQ	Yes	No	Not Sure
I read the problem more than once.	30 (%45.45)	29 (%43.94)	07 (%10.61)

%45.45 of the PSTs answered the first item as 'Yes'. All of these PSTs solved the first problem according to the problem-solving stages and got the correct result. 26 of these PSTs solved the second problem in a similar way; they got the correct result, as well. Although four PSTs couldn't find the correct result for the second problem, they followed the problem-solving stages while solving it. They did the first two stages of problem-solving; however, they found the incorrect result.

All of the PSTs who answered this item as 'No' solved the first problem correctly. 24 of these PSTs solved the second problem correctly, as well. It was seen that four PSTs couldn't gain the correct answer for the second problem. One PST completed the first two stages of problem-solving, but this PST didn't do the other stages.

%10.61 of the PSTs answered this item as 'Not sure'. All of them solved the first problem correctly according to problem-solving stages. Six of these PSTs also solved the second problem correctly in a similar way. Only one PST couldn't find the correct result for the second problem.

Table 4. 2nd Item of the MSQ

2nd Item of the MSQ	**Yes**	**No**	**Not Sure**
I checked what the problem was asking to me.	61 (%92.42)	05 (%07.58)	00 (%00.00)

As seen above, %92.42 of them answered this item as 'Yes'. All of them solved the first problem correctly. Only eight of them didn't follow the problem-solving stages. 51 of them solved the second problem correctly. Nine of them didn't follow the problem-solving stages. 10 of them reached either the wrong result or the lacking solution.

None of the PSTs answered this item as 'Not sure'. %07.58 of them answered this item as 'No'. All of these PSTs solved both the first and the second problems correctly. Only one of them didn't follow the problem-solving stages for both problems.

Table 5. 3rd Item of the MSQ

3rd Item of the MSQ	**Yes**	**No**	**Not Sure**
I evaluated how much time I need for solving this kind of problem.	33 (%50.0)	20 (%30.3)	13 (%19.70)

This item is about the time needed to solve problems. %50.00 of the PSTs answered this item as 'Yes'. All of these PSTs solved the first problem correctly, but only two of them didn't follow the problem-solving stages. 28 of them solved the second problem correctly. Three of them didn't follow the problem-solving stages. Five of them reached either the wrong result or the lacking solution.

%30.30 of the PSTs answered this item as 'No'. All of them solved the first problem correctly. While 16 of them followed the problem-solving stages, only four PSTs didn't follow these stages. 17 of them solved the second problem correctly. Only four of them didn't follow the problem-solving stages. Three of them reached either the wrong result or the lacking solution.

%19.70 of the PSTs answered this item as 'Not sure'. All of them solved the first problem correctly and followed the problem-solving stages. While three of them got the correct result, they didn't follow the problem-solving stages. 11 of them solved the second problem correctly.

While eight of them followed the problem-solving stages, three of them didn't follow these stages. Besides, two of them solved this problem incorrectly.

Table 6. 4th Item of the MSQ

4th Item of the MSQ	Yes	No	Not Sure
I showed the problem schematically.	59 (%83.39)	01 (%01.52)	06 (%09.09)

%83.39 of the PSTs answered this item as 'Yes'. All of these 59 PSTs solved the first problem correctly. While 51 of them followed the problem-solving stages, eight of them didn't follow the stages. 49 of these PSTs solved the second problem correctly. While 40 of them solved this problem according to problem-solving stages, nine of them didn't solve the problem according to the stages. 10 of them reached either the wrong result or the lacking solution.

Only one PST answered this item as 'No'. This PST solved both problems correctly, but s/he didn't follow the problem-solving stages for both of them. Six PSTs who answered this item as 'Not sure' solved both problems correctly and they followed the problem-solving stages.

Table 7. 5th Item of the MSQ

5th Item of the MSQ	Yes	No	Not Sure
I tried to recall whether I solved a similar problem before or not.	56 (%84.85)	07 (%10.61)	03 (%04.54)

%84.85 of the PSTs answered this item as 'Yes'. All of these pre-service teachers solved the first problem successfully. While 49 of them solved the problem according to problem-solving stages, seven of them didn't. 46 of them solved the second problem correctly. While 36 of these PSTs solved the problem according to problem-solving stages, eight of them didn't consider these stages. 10 of 46 PSTs reached either the wrong result or the lacking solution.

All pre-service teachers (%10.61) who answered this item as 'No' solved both problems correctly. For both problems, the two same PSTs didn't pay attention to the problem-solving stages. Indeed, five PSTs solved the both problems according to problem-solving stages.

%04.54 of the pre-service teachers answered this item as 'Not sure'. It was seen that all of these PSTs solved both problems correctly and considered the problem-solving stages.

Table 8. 6th Item of the MSQ

6th Item of the MSQ	Yes	No	Not Sure
I developed a strategy for solving the problem.	60 (%90.91)	01 (%01.52)	05 (%07.57)

All of these 60 (%90.91) PSTs who answered this item as 'Yes' solved the first problem successfully. While 52 of them considered problem-solving stages, eight of them didn't. 50 of these 60 pre-service teachers solved the second problem correctly. While 42 of them used the problem-solving stages, eight of them didn't. 10 of these 60 pre-service teachers found out either the wrong result or the lacking solution.

Only one pre-service teacher said 'No' for this item and this PST solved both problems correctly using problem-solving stages. Five (%07.57) PSTs said 'Not sure' for this item and all of them solved both problems correctly. It was determined that one of these five PSTs didn't use the

problem-solving stages for both problems and another PST didn't use the stages for just the second problem.

Table 9. 7th Item of the MSQ

7th Item of the MSQ	Yes	No	Not Sure
I did not know where to start.	02 (%03.03)	51 (%77.27)	13 (%19.70)

For this item, two pre-service teachers said 'Yes'. These PSTs solve both problems successfully, but one of them didn't consider the problem-solving stages for both problems. %19.70 of pre-service teachers answered this item as 'Not sure'. All these PSTs solved the first problem. While 12 of them solved the problems using the problem-solving stages, only one of them didn't. 11 of them solved the second problem correctly, but two of them didn't find the correct result for this problem. Two of these 11 PSTs didn't use the problem-solving stages.

%77.27 of the pre-service teachers said 'No' for this item and solved the first problem correctly. While 44 of these 51 PSTs solved this problem using the problem-solving stages, seven of them didn't use these stages. 43 of these 51 pre-service teachers solved the second problem successfully. While 36 of them used the problem-solving stages, seven of them didn't. Eight of these PSTs reached either the wrong result or the lacking solution.

Table 10. 8th Item of the MSQ

8th Item of the MSQ	Yes	No	Not Sure
I faced some difficulties while solving problem (If your answer is 'yes', define the features of the difficulty).	20 (%30.30)	43 (%65.15)	03 (%04.55)

For this item, %30.30 pre-service teachers said 'Yes'. All these PSTs solve the first problem correctly. Only two of them didn't follow the problem-solving stages. 15 of these 20 PSTs solved the second problem successfully. While 12 of them solved this problem using the problem-solving stages, three of them didn't use the stages. Five of them found either the incorrect result or the lacking solution for this problem. Since they said 'yes', they had to write the feature of the difficulty. When looking at their explanations of this item, it was seen that there were the following comments;

> Exactly, I had a hard time creating a relationship. (PST4)
> The second problem is a difficult one. (PST5)
> The second problem is beyond the routine. The pattern was difficult to mentally assess. (PST11)
> I had difficulty about reasoning at the second problem. (PST12)
> I didn't know how to find a solution for the second problem. (PST19)
> I tried to remember a formula to solve the problems. In fact I could have created a formula by myself. (PST20)
> I tried to remember a formula for the second problem. Then, I created the formula. (PST21, PST33, PST34, PST59)
> Whether the solution is understandable or not, it was hard for me. (PST23)
> I couldn't find how to solve the second problem. (PST24)
> I have a prejudice about the second problem, so that I feel discomfort when I face this kind of problems. (PST25)
> Until I found the formula of the second problem, I had thought for a while. (PST31)
> I had a difficulty about understanding the second problem. (PST32)
> I couldn't find the result. (PST39)
> I couldn't remember the reasoning of the problem and how to create the formula. (PST49)
> I tried to remember how I would evaluate the result of the problem. (PST57)

%65.15 of the PSTs answered this item as 'No'. All of them solved the first problem successfully. While 36 of them used the problem-solving stages for the solution, seven of them didn't. 39 of 43 PSTs solved the second problem correctly. While 31 of them used the problem-solving stages, eight of them didn't. Four of 39 PSTs reached either the wrong result or the lacking solution.

Only three PSTs answered this item as 'Not sure' and all of them solved the first problem correctly using the problem-solving stages. Two of them solved the second problem successfully using the stages, as well. One of them reached the incorrect result.

Table 11. 9th Item of the MSQ

9th Item of the MSQ	Yes	No	Not Sure
I found one of my mistakes while solving the problem and corrected it (If your answer is yes, define the mistake).	09 (%13.64)	54 (%81.82)	03 (%04.55)

%13.64 of the PSTs answered this item as 'Yes'. Therefore, these PSTs found some mistakes while they were solving problems. All these PSTs solved the first problem correctly using the problem-solving stages. Six of them also solved the second problem successfully using the problem-solving stages. Three of them found either the incorrect result or the lacking solution. When the PSTs' explanations were examined, it was found that there were the following findings.

> I made a calculator error. (PST19, PST26, PST64)
> I tried leaving "t" alone at the first problem. Then, I noticed that it was wrong. (PST27)
> I drew the diagram incorrectly for the second problem. (PST30)
> I made a few coding mistakes. (PST35)
> The result was wrong and there was a reasoning mistake. (PST39)
> I had thought all the people made 19 handshakes. Then I rethought, I noticed that if someone made a handshake one time, the second one wouldn't be necessary. (PST63)

%81.82 of the pre-service teachers answered this item as 'No' and all 54 PSTs solved the first problem correctly. While 46 of them used the problem-solving stages, eight of them didn't. 47 of these 54 PSTs solved the second problem successfully. While 38 of them followed the problem-solving stages, nine of them didn't. Seven of them got either the wrong result or the lacking solution. Only three pre-service teachers who said 'Not sure' for this item solved both problems. While two of them used the problem-solving stages for both problems, one PST didn't.

Table 12. 10th Item of the MSQ

10th Item of the MSQ	Yes	No	Not Sure
I thought how the solution was going on.	62 (%93.94)	01 (%01.52)	03 (%04.54)

%93.94 of the PSTs answered this item as 'Yes' and all 62 PSTs solved the first problem correctly. While 53 of them followed the problem-solving stages, nine of them didn't follow the stages for this problem. 53 of these 62 PSTs solved the second problem successfully. 43 of them used the problem-solving stages when they were solving this problem. 10 of them didn't use the stages. Nine of 53 PSTs found either the incorrect result or the lacking solution.

Only one PST said 'No' for this item and s/he solved both problems correctly using the problem-solving stages. Three of the PSTs answered this item as 'Not sure' and all solved the first

problem successfully using the stages. Two of them solved the second problem correctly using the problem-solving stages. Only one of them got the wrong result for the second problem.

Table 13. 11th Item of the MSQ

11th Item of the MSQ	Yes	No	Not Sure
I tried different approaches to solve the problem.	36 (%54.55)	20 (%30.30)	10 (%15.15)

Almost half of the PSTs answered this item as 'Yes'. Similarly, all these PSTs solved the first problem correctly. While 32 of them used the problem-solving stages during the solving process, four of them didn't. 31 of these 36 PSTs solved the second problem successfully. While 26 of them followed the stages of problem-solving, five of them didn't. Five of these 36 PSTs reached either the incorrect result or the lacking solution.

%30.30 of the PSTs said 'No' for this item and all PSTs solved the first problem correctly. While 15 of these 20 PSTs used the stages of problem-solving, five of them didn't. 16 of 20 PSTs teachers solved the second problem successfully. Four of them didn't find the correct result. While 11 of 16 PSTs followed the stages of problem-solving during the solving section of the second problem, five PSTs did not.

%15.15 of the PSTs said 'Not sure' for this item and all these PSTs solved the first problem correctly using the stages of problem-solving. Except for one pre-service teacher, the others solved the second problem successfully using the stages, as well.

Table 14. 12th Item of the MSQ

12th Item of the MSQ	Yes	No	Not Sure
I asked myself whether my answers were meaningful or not.	59 (%89.40)	03 (%04.54)	04 (%0.06)

%89.40 PSTs said 'Yes' for this item and all 59 PSTs solved the first problem correctly. While 53 of them followed the stages of problem-solving, six PSTs didn't. 49 of 59 PSTs solved the second problem successfully. While 42 of them used the stages, seven of them didn't. 10 of 49 PSTs found out either the incorrect result or gave the lacking answer.

Merely three PSTs said 'No' for this item and they solved both problems correctly. However, only one of them followed the stages of problem-solving for both problems. It was seen that the others didn't follow the stages. Four of 66 PSTs said 'Not sure'. It was found that all these PSTs solved both problems successfully. While three of them used the stages of problem-solving for both problems, one of them didn't use the stages for both problems.

Table 15. 13th Item of the MSQ

13th Item of the MSQ	Yes	No	Not Sure
I checked my calculations to be sure that they were correct.	61 (%92.42)	05 (%07.58)	00 (%00.00)

%92.42 PSTs said 'Yes' for this item and all 61 PSTs solved the first problem successfully. While 55 of 61 PSTs followed the stages of problem-solving, six of them didn't. 51 of 61 PSTs solved the second problem correctly. While 44 of them used the stages of problem-solving, seven of them didn't during solving the second problem. 10 of these PSTs reached either the incorrect result or the lacking solution.

It was seen that five of these PSTs answered this item as 'No' and all these pre-service teachers solved both problems correctly. While two of them followed the problem-solving stages for both problems, three of them didn't. As it is seen, there are no PSTs who said 'Not sure' about this item.

Table 16. 14th Item of the MSQ

14th Item of the MSQ	Yes	No	Not Sure
I thought whether there was something that I should especially pay attention or not in the information given in the problem (If 'yes', define it).	32 (%48.48)	21 (%31.82)	13 (%19.70)

%48.48 PSTs answered as 'Yes'. It was seen that they thought about paying attention. All 32 PSTs solved the first problem correctly. While 29 of them used the stages of problem-solving, three of them didn't. 28 of these 32 PSTs solved the second problem successfully, as well. While 25 of them used the stages of problem-solving, three of them didn't. It was found that four of them got either the incorrect result or the lacking solution. It was found that when examining the explanations of the PSTs that said 'Yes' to this item, there were the following findings.

> In the second problem, it is important to pay attention that the same two people did not shake hands for the second time. (PST1, PST11, PST28, PST49)
> I had paid attention to whether pose a problem or solve a problem. (PST3)
> I paid attention to the units such as km, hour. (PST5, PST18, PST23, PST54, PST63, PST65)
> It is necessary to understand the reasoning of the problem. (PST7, PST51)
> I thought that how the incidents given in the problems happened. (PST10)
> It is important to pay attention that the distance of the way is the same for the first problem. (PST12)
> I thought that whether I wrote the data correctly. (PST13)
> I kept the important points of the problems in my mind. (PST14)
> Sometimes, these kinds of problems could be misleading. So, I checked again. (PST16, PST42)
> I followed the stages of problem-solving. (PST22)
> I thought that whether there is a keyword or a different emphasis. (PST25, PST35, PST53, PST56)
> I noticed that the vehicles are coming towards each other in the first problem. (PST34)
> I examined the information carefully to draw the suitable schema and to choose the right plan. (PST38)
> I paid attention to whether I had used the information that was given. (PST44, PST62, PST64)
> I paid attention to reading the problems carefully. (PST66)

%31.82 PSTs said 'No' to this item and all 21 PSTs solved the first problem correctly. While 17 of them used the stages of problem-solving, four of them didn't. 16 of these PSTs solved the second problem successfully. While 11 of them followed the stages of problem-solving, five of them didn't. Besides, five of them found either the wrong result or got the lacking solution.

%19.70 of PSTs answered this item as 'Not sure' and all these 13 PSTs solved the first problem correctly. Except for two of them, the others used the problem-solving stages for both problems. While 12 of them solved the second problem successfully, one of them solved this problem incorrectly.

Conclusions, Discussion, and Suggestions

In this study, it was aimed to determine the role of metacognition while solving routine and non-routine problems. Within the scope of this, 56 (%84.85) of the pre-service middle school mathematics teachers solved both problems correctly. This result could mean that the PSTs are successful in solving routine and non-routine problems. While 47 of them followed the stages of

problem-solving when they were solving the first problem, nine of them didn't. Similarly, while 46 of them used the stages for solving the second problem, 10 of them didn't.

When looking at MSQ, it can be seen that the 1st, 2nd, 4th, 5th, and 14th items are related to understanding a problem. The second item of the MSQ shows metacognitive self-regulation behavior. At the same time, it verifies whether the problem was understood. An example of metacognitive behavior is to create models by individuals with their own experiences and it can be seen in the 4th item of MSQ. In order to achieve targets, metacognitive control and analysis provide strategy development. The 6th item means this. 30 (%45.45), 61 (%92.42), 59 (%83.39), 56 (%84.85), and 32 (%48.48) PSTs answered these items as 'Yes', respectively. It was found that either almost all of them or almost half of them said 'Yes'. This shows that the PSTs pay attention to understanding a problem. The 6th and 11th items of MSQ are related to generating alternative solutions. 60 (%90.91) and 36 (%54.55) pre-service teachers answered these items as 'Yes'. This result shows that almost all PSTs generated an alternative solution. It means that they really paid attention to this stage of problem-solving. The 9th, 12th, and 13th items are about the stage of evaluation of a problem. The important point is to check whether the answers are meaningful or not while solving mathematical problems. This behavior shows metacognition and it can be seen in the 12th item of MSQ. When we looked at the answers of these items, it was seen that nine (%13.64), 59 (%89.40), and 61 (%92.42) PSTs said 'Yes' to these items, respectively. Especially the answers of the 12th and 13th items show that the PSTs focused on the evaluation stage of problem-solving.

All of these results above show that most of the pre-service middle school mathematics teachers use their metacognitive skills in solving routine and non-routine problems. This result also shows the relationship between metacognition and problem-solving. According to this result, this study coincides with many studies (Culaste, 2011; Flavel, 1976; Goldbergvfgc & Bush, 2003; Özcan & Gümüş, 2019; Özsoy, 2011; Pennequin, Sorel, Nanty, & Fontaine, 2010; Schurter, 2002; Susilo & Retnawti, 2018; Şengül & Katrancı, 2012; 2015). In these studies, it was seen that there is a positive significant relationship between mathematical problem-solving and metacognition level. Pennequin, Sorel, Nanty, and Fontaine (2010) found that there is a relationship between metacognitive knowledge and mathematical problem-solving achievement. Aşık and Erktin (2019) reached the same result in their study. Besides, Özcan, İmamoğlu, and Bayraklı (2017) mentioned that metacognition has an important role in the process of problem-solving. Similarly, Lee, Yeo, and Hong (2014) specified that the metacognitive approach makes a difference in the problem-solving process. However, Aydın, Dinç, Sezgin-Memnun, and Muyo-Yıldırım (2020) didn't find a significant relationship between students' metacognitive awareness and their success in solving non-routine problems. As it is seen, while this study shows parallelism with some studies, it doesn't coincide with another study. This may be due to the problems chosen or the participants of these studies. For this reason, similar studies could be executed about different mathematics topics with different participants.

Another result of this study is that there are some difficulties in solving routine and non-routine problems. These difficulties are as follows; i) reasoning a non-routine problem, ii) not knowing how to find a result for a non-routine problem, iii) creating a relationship, iv) trying to remember a formula to solve a non-routine problem, v) having a prejudice about a non-routine problem, vi) understanding a non-routine problem, and vii) remembering how would evaluate the problems. As it is seen, most of these difficulties are about solving non-routine problems. This result

coincides with some studies (Şengül & Katrancı, 2012; 2015). Şengül and Katrancı (2012) searched the metacognitive aspect of solving function problems. In this study, they mentioned two examples of the difficulties about the topic. These are as follows; i) The pre-service teachers couldn't remember some things and ii) They couldn't remember whether the signs had changed or not. In their other study (2015), they aimed to examine what metacognitive aspects of solving indefinite integral problems were. They determined some of the difficulties that the pre-service teachers faced, as well. These difficulties are as follows; i) remembering what I know, ii) not being able to decide from where to start solving the problems, iii) not being able to solve the problem with the strategy, and iv) not being able to remember the features of integral. It could be said that within the scope of these studies, the PSTs face some difficulties using their metacognitive skills while solving problems regarding different mathematics topics. Blakey and Spence (1990) offer some strategies to improve metacognitive skills. These strategies are as follows;

- Identifying what you know or not,
- Talking about what you think,
- Keeping a thought dairy,
- Debriefing the thinking process,
- Self-evaluation.

In order to overcome these difficulties, studies related to improving metacognitive skills could be executed with the pre-service teachers about different mathematics topics. In these studies, these strategies could be used in order to improve the metacognitive skills.

Another result of this study is that the pre-service teachers made mistakes while solving problems. These mistakes are; i) calculator errors, ii) coding mistakes, iii) reasoning mistakes, and iv) thinking wrongly about a non-routine problem. Şengül and Katrancı (2012) mentioned that the pre-service teachers faced some mistakes. For instance, one PST had realized that s/he was following the wrong strategy. In their other study (2015), they found that the mistakes which the PSTs faced such as i) operational mistakes and ii) choosing an incorrect strategy. The mistakes faced are almost similar such as calculator errors. It could be said that these results could be key points to increase problem-solving achievement. If we pay attention to the stages of problem-solving, we can see our mistakes in this process. Then, we can fix these errors, so that can be successful in problem-solving. As seen that the PSTs who found any mistakes fix the error and got the correct answer in this study. Thus, it could be evidence of the importance of problem-solving stages. In addition, Biryukov (2004) expresses that metacognition includes an individual's ability to control the process of problem-solving. Therefore, it could be said that the PSTs used their metacognitive skills while they were solving problems in this study.

The last result of the study is that the PSTs paid attention to some points while they were solving problems. These are; i) paying attention to units such as km, hour, etc., ii) understanding of the reasoning of the problem, iii) looking for a keyword or emphasized point, and iv) correctly reading and writing the data that were given. These are the important points of solving routine and non-routine problems for the PSTs. Şengül and Katrancı (2012) found that the PSTs paid attention to absolute value sign. They (2015) also found out that the PSTs paid attention to how they would find the solution, finding a correct strategy, and looking for a formula. In this

context, it could be said that when a PST solves problems about any kind of mathematics topic, s/he pays attention to some key points such as units, a strategy, etc.

As the main result, although the pre-service middle school mathematics teachers use their metacognitive skills while solving routine and non-routine problems, they face some difficulties and make mistakes. In future studies, first, the PSTs could be informed about the stages of problem-solving and metacognitive skills. Then, the metacognitive aspects of the different topics of mathematics could be searched. In these studies, different data collection instruments could be used such as a problem-solving achievement test. After all, obtained findings could be compared and results could be reported.

References

Altun, M. (2014). *Ortaokullarda (5, 6, 7 ve 8. sınıflarda) matematik öğretimi*. Bursa: Alfa Akademi.

Artz, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction, 9*(2), 137-175.

Aşık, G. (2015). A design study on metacognitive traning in problem solving. Unpublised doctoral dissertation, Marmara University, İstanbul.

Aşık, G., & Erktin, E. (2019). Metacognitive experiences: Mediating the relationship between metacognitive knowledge and problem solving. *Education and Science, 44*(197), 85-103. doi: 10.15390/EB.2019.7199

Aydın, B., Dinç, E., Sezgin-Memnun, D., & Muyo-Yıldırım, M. (2020). Metacognitive skills and non-routine problem solving achievements of eighth and ninth grade students: Kosovo and Turkey samples. *International Journal of Science and Education, 3*(2), 154-172. doi: 10.47477/ubed.818879

Biryukov, P. (2004). Metacognitive aspects of solving combinatorics problems. *International Journal for Mathematics Teaching and Learning, 74*, 1-19. Retrieved from https://www.cimt.org.uk/journal/biryukov.pdf on 05/26/2021.

Blakey, E., & Spence, S. (1990). Developing metacognition. Retrieved from https://eric.ed.gov/?id=ED327218 on 05/26/2021. ERIC Identifier: ED327218

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to subjects-state, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37-68.

Chapman, O. (2002). Teaching word problems: What high school teachers value. Retrieved from https://files.eric.ed.gov/fulltext/ED471769.pdf on 05/27/2021.

Costa, A. L. (1984). Mediating the metacognitive. *Educational Leadership, 11*, 57-62.

Culaste, I. C. (2011). Cognitive skills of mathematical problem solving of grade 6 children. *International Journal of Innovative Interdisciplinary Research, 1*, 120-125.

Desoete, A., & Veenman, M. (2006). Metacognition in mathematics: Critical issues on nature, theory, assessment, and treatment. In A. Desoete and M. Veenman (Ed.), *Metacognition in mathematics education* (pp. 1-10). New York: Nova Science Publishers, Inc.

Flavel, J. H. (1976). Metacognitive aspects of problem solving. In L. Resnick (Ed.), *The nature of intelligence. Hillsdale*, NJ: Lawrence Erlbaum.

Flavel, J. H. (1979). Metacognition and cognitive monitoring. *American Psychologist, 34*(10), 906-911.

Goldberg, P. D., & Bush, W. S. (2003). Using metacognitive skills to improve 3rd graders' math problem solving. *Focus on Learning Problems in Mathematics, 5*(10), 29-48.

Heddens, J. W., & Speer, W. R. (1997). *Today's mathematics*. New Jersey: Prentice Hall.

Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational Technology: Research and Development, 48*(4), 63-85. doi: 10.1007/BF02300500

Katrancı, Y. (2014). The effect of problem posing studies on mathematical understanding and problem solving achievement in cooperaitve learning environment. Unpublished doctoral dissertation, Marmara University, İstanbul.

Lee, N. H., Yeo, D. J. S., & Hong, S. E. (2014). A metacognitive-based instruction for primary four students to approach non-routine mathematical word problems. *ZDM Mathematics Education, 46*, 465-480. doi: 10.1007/s11858-014-0599-6

McCormick, C. B., Miller, G. E., & Pressley, M. (1989). *Cognitive strategy research: From basic research to educational applications*. New York: Springer-Verlag.

Mihalca, L., Mengelkamp, C., & Schnotz, W. (2017). Accuracy of metacognitive judgments as a moderator of learner control effectiveness in problem-solving tasks. *Metacognition and Learning, 12*(3), 357-379.

Özsoy, G. (2011). An investigation of the relationship between metacognition and mathematics achievement. *Asia Pacific Education Review, 12*, 227-235.

Özcan, Z. C., & Gümüş, A. E. (2019). A modeling study to explain mathematical problem-solving performance through metacognition, self-efficacy, motivation, and anxiety. *Australian Journal of Education, 63*(1), 116-134. doi: 10.1177/0004944119840073

Özcan, Z. C., İmamoğlu, Y., & Bayraklı, V. K. (2017). Analysis of sixth grade students' think-aloud processes while solving a non-routine mathematical problem. *Educational Sciences: Theory & Practice, 17*(1), 129-144. doi: 10.12738/estp.2017.1.2680

Panaoura, A., & Philippou, G. (2005). The measurement of young pupils' metacognitive ability in mathematic: The case of self-representation and self-evaluation. Sant feliu de guixols: Cerme 4. Retrieved from https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.120.3575&rep=rep1&type=pdf on 05/27/2021.

Pennequin, V., Sorel, O., Nanty, I., & Fontaine, R. (2010). Metacognition and low achievement in mathematics: The effect of training in the use of metacognitive skills to solve mathematical word problems. *Thinking & Reasoning, 16*(3), 198-220. doi: 10.1080/13546783.2010.509052

Polya, G. (1957). *How to solve it*. Princeton, NJ: Princeton University Press.

Pugalee, D. K. (2001). Writing, mathematics, and meta-cognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics, 101*(5), 236-245.

Ravid, R. (1994). *Practical statistics for educators*. New York: University Press in America.

Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh and M. Landau (Ed.), *Acquisition of mathematics concepts and processes* (pp. 345-395). New York: Academic Press.

Schurter, W. A. (2002). Comprehension monitoring: An aid to mathematical problem solving. *Journal of Developmental Education, 26*(2), 22-33

Selçuk, Z. (2000). *Gelişim ve öğrenme*. Ankara: Nobel Yayın Dağıtım.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Susilo, M. B., & Retnawati, H. (2018). An analysis of metacognition and mathematical self-efficacy toward mathematical problem solving ability. Journal of Physics: Conference Series, 1097. IOP Publishing.

Şen, Ş. H. (2012). Ortaöğretim öğrencilerinin bilişüstü yetileri kullanma durumlarının bazı değişkenler açısından incelenmesi. *Journal of Educational and Instructional Studies in the World, 2*(1), 2146-7463.

Şengül, S., & Katrancı, Y. (2012). Metacognitive aspects of solving function problems. *Procedia-Social and Behavioral Sciences, 46*, 2178-2182. doi: 10.1016/j.sbspro.2012.05.450

Şengül, S., & Katrancı, Y. (2015). Meta-cognitive aspects of solving indefinite integral problems. *Procedia-Social and Behavioral Sciences, 197*, 622-629. doi: 10.1016/j.sbspro.2015.07.205

Verschaffel, L. (1999). Realistic mathematical modeling and problem solving in the upper elementary school: Analysis and improvement. In J. H. M. Hamers, J. E. H. Van Luit and B. Csapo (Ed.), *Teaching and learning thinking skills: Context of learning* (pp. 215-240). Lisse: Swets & Zeitlinger.

Yıldırım, A., & Şimşek, H. (2008). *Sosyal bilimlerde nitel araştırma yöntemleri*. Ankara: Seçkin Yayıncılık.