NUMERICAL SIMULATION OF SPRING HYDROGRAPH RECESSION CURVES FOR EVALUATING BEHAVIOR OF THE EAST YORKSHIRE CHALK AQUIFER

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Abstract

The Cretaceous Chalk aquifer is the most important in the UK for the provision of water to public supply and agriculture. The Chalk has both matrix and fracture porosity and is thus best considered as a dual porosity aquifer system. Although the matrix porosity is large, typically around 0.35 in the study area of East Yorkshire, UK (ESI, 2010), pore diameters are typically very small, and the water contained in them is virtually immobile. The high permeability fracture network is responsible for the ability of water to drain; spatial variations in fracture network properties mean conventional approaches to aquifer characterization such as borehole pumping tests are of limited utility. Hence this study attempts to better understand the flow system and characterise aquifer properties from the recession response seen at springs during the spring/summer period when recharge is minimal. This approach has the advantage that spring hydrographs represent the sum of the response from entire catchments.

This paper reports numerical modeling for simulating aquifer and spring responses during hydrological recession. Firstly, available geological and hydrogeological information for the study area was used to develop hydrogeological conceptual models. Three different numerical models have been constructed representing three possible scenarios that could represent the aquifer in the selected area. These are: single reservoir aquifer, double reservoir aquifer, and single reservoir aquifer containing tunnel shaped highly permeable zone at the spring elevation respectively. The sensitivity of spring recession response to various external and internal parameter values was investigated, to understand relations between spring recession, hydrological inputs (recharge) and aquifer

structure. Spring hydrographs from the real aquifer were compared with the hydrographs generated from models, in order to estimate aquifer properties. The work aims to identify the utility of spring hydrographs in eliciting aquifer permeability structure, as well as identifying the conceptual scenario which best represents the Chalk Aquifer in East Yorkshire, UK.

Introduction

The Chalk is the most significant aquifer in Britain; it underlies much of eastern and southern England. Groundwater from the Chalk aquifer of Yorkshire is an important resource for public supply, agriculture and industry.

Two types of porosity systems have been recognized in the chalk rocks: primary and secondary porosity. The primary porosity is pore spaces formed between rock grains during rock formation processes, simply termed “matrix porosity”. Secondary porosity exists in the form of fractures which were produced by dissolution and tectonic activity (Singhal and Gupta, 2010). This characteristic of dual porosity in the Chalk aquifer was confirmed by many studies (Foster and Crease, 1974; Wellings and Bell, 1980; Price, 1987; Price et al., 1993; Downing, et al., 2005; Mathias et al., 2005).

The role of the porosity systems within the Chalk aquifer are as follows: the fracture system has very low porosity but high permeability which makes it dominate the flow system, while the matrix has very high porosity but low permeability so seldom contributes (Allen et al., 1997; Gale and Rutter, 2006). The storage co-efficient (specific yield) is also likely to derive from drainage of fracture space, rather than matrix porosity (MacDonald and Allen, 2001).

Increasing overburden with depth gave the Chalk a significant feature which is developing permeability toward the top remarkably. Overburden affects the permeability in two ways, first reducing the fracture and aperture size. Second, because of lack of groundwater circulation it prevents processes of fracture enhancement due to dissolution (Foster and Milton, 1974; Foster and Robertson, 1977; Price et al., 1977).

Hydrographs are graphical representations of the time series flow rate, generally consisting of three segments, rising limb, peak and falling limb, respectively. The falling limb, which is also known as a recession curve, is that part of a hydrograph that comes after peak flow. Studying hydrograph recession curves of springs may provide hydrogeological information especially where fracture or conduit flows are significant. This approach is preferred over other geological and geophysical methods (Dreiss, 1982; Bakalowicz, 2005) because the spring drains water from large areas of aquifer, so the discharge is governed by accumulative effect from the flow systems that exist in the aquifer. This contrasts with other geological and geophysical methods that only represent the aquifer locally at the investigation points.

Factors affecting hydrograph shape essentially grouped into two groups, external and internal factors. External factors include physiography, climate and vegetation which control recharge, while internal factors are the hydrogeological properties of the aquifer rocks, such as transmissivity (product of aquifer thickness and hydraulic conductivity). Precipitation intensity, duration and distribution over the catchment influence shape of the hydrograph; intensity and duration of rainfall strongly affect the peak flow. Temperature and humidity influence evapotranspiration and effective rainfall. Catchment size, shape, slope and morphology (surface depressions can act as natural water storage ponds) are important external factors.

It has been reported from comparison between the spring hydrograph recession curve of different springs, that the recession curves steepness and shape (i.e., recession coefficients) are mainly governed by the intensity and geometry of fracture system (Kovács et al., 2005). Based on the analytical curve fitting method based on the Maillet exponential model, it has been suggested that the recession of spring hydrographs from fractured rock aquifers decomposes into several

segments, each segment reflecting different flow system in the aquifer (Kovács and Perrochet, 2008; Liu and Li, 2012). However, the analysis of spring recession curves simulated by numerical modeling revealed that multiple segments do not necessarily reflect the presence of multiple flow systems (Baedke and Krothe, 2001; Kovács, 2003). In our study, we investigate the extent to which recession curve shape can provide information about the permeability structure and characteristics, using numerical simulations of flow in conceptual permeability scenarios based on those potentially found within the case-study aquifer.

Site Location and Characterization

The field study area is located at northern part of Yorkshire Wolds of East Yorkshire, it occupies an area about 250 km² (Figure 1A). Two gauging stations exist in the study area, one located at Kirby Grindalythe village in the NW of the study area and second one in Driffield town in the SE of the study area. This paper focuses on the Kirby Grindalythe catchment as this is closer to the topographic divide (Figure 1B), so the catchment boundary conditions are easier to constrain.

The Cretaceous Chalk crops out across the study area and is overlain by glacial sediments to the East. Chalk rocks rest unconformably on Jurassic rocks of the Penarth group (largely argillaceous) and Lias Group (mudstones and thin silty limestone). A schematic diagram of the Geological cross section in the area is illustrated in Figure 1B.

The Gypsey Race is the most significant surface water course in the area, it rises through a series of springs just upstream of Kirkby Grindalythe village and runs eastwards to Bridlington. The Kirby Grindalythe gauging station measures the discharge in the upper reaches of the Gypsey Race, just downstream from these springs.

The unconfined Chalk aquifer is covered by a shallow lime-rich sandy soil on the interfluvies and by a lime-rich loamy soil along the water drainages and dry valleys. Both soil types allow the water to freely drain. Figure 1A illustrates location of the study area.

Methodology

To investigate factors that govern groundwater flow in the aquifer, we analyze that part of recession curve representing water discharge in the absence of recharge,

Figure 1. A. Location of study area and surface geological map. B. Geological cross section through the study catchments.

i.e., the recession curve. Actual evapotranspiration (AE) and soil moisture deficit (SMD) information from the UK Meteorological Office Rainfall and Evapotranspiration Calculation System (MORECS) database have been used to identify date of the cessation of recharge and hence the start of flow recession. Figure 2 shows hydrograph recession curves from the Kirby Grindalythe gauging station for selected hydrological years between 1998 and 2014.

As it appears in the figure starting time and length of the recession period was different from year to year, but generally began between February 15 to April 15

(except 2000 when recession started middle of June), and recession ended on early to late September.

Recession curves show variation in the peak flow at starting recession period, starting date and length of recession period between different water years. To understand relation between this variation in recession curves from same sources and rainfall the total annual effective rainfall has been calculated from climate data (from UK MORECS data) for the years between 2010 to 2014 and then plotted simultaneously with hydrograph for same years. Figure 3 is graphically showing relation between annual total effective rainfall and spring hydrograph.

To overcome the problem of variation which exists between recession curves from different years a master recession curve MRC technique was used for constructing a mean recession curve. Several approaches can be used for constructing a master recession curve: e.g., matching strip, correlation and tabulation method (Brownlee, 1960; Toebe and Strang, 1994; Hall, 1968; Toebe, 1969; Brutsaert and Nieber, 1977; Sugiyama, 1996). In this study the tabulation method was used as it is the most appropriate technique for constructing a MRC for a range of years. In the tabulation method the recession data at regular intervals of time are tabulated in columns, each recession in separate column. The columns are adjusted vertically until the discharge values approximately agree horizontally (Figure 4). Finally,

Figure 2. Hydrograph recession curves from Kirby Grindalythe gauging station for selected years 1998 to 2014.

Figure 3. A is annual total effective rainfall for years between 2010 to 2014 over Kirby Grindalythe and Driffield catchments. B. Hydrograph for Kirby Grindalythe station. C. Hydrograph for Driffield station

the average discharges are calculated, representing the master recession curve. Figure 5 shows construction of a master recession curve for the Kirby Grindalythe station.

The analytical model suggested by Maillet (1905) (Toebes and Strang, 1994; Tallaksen, 1995; Stella, 2013; Eslamian, 2014; Hingray, et al., 2015) was used for initial interpretation of recession curves. This method is the most widely used approach for describing the flow depletion during recession period. The model is expressed by the equation:

$$Q_t = Q_0 \exp(-\alpha t)$$

Where Q_t and Q_0 are flow [L^3/T] at time t [T] and the start of recession, and α is the recession coefficient [$1/T$].

time (day)	Q (m ³ /day)				Q average
	Q 2010	Q 2011	Q 2012	Q 2013	MRC
36	4406.4				
37	4579.2				
38	4492.8				
39	4492.8		4147.2		
40	3888		3974.4		
41	3801.6		3715.2		
42	3715.2		3542.4		
43	3715.2		3542.4		
44	3628.8		3542.4		
45	3542.4	3542.4	3456		
46	3542.4	3110.4	3283.2		
47	3456	3024	3283.2		
48	3369.6	3024	3283.2		
49	3628.8	3024	3196.8		
50	3369.6	2937.6	3110.4		
51	3196.8	3024	3110.4		
52	3110.4	3283.2	3110.4		
53	3110.4	3196.8	3110.4		
54	3024	3110.4	3024	3076.2	3058.65
55	3369.6	3110.4	2937.6	3024	
56	3283.2	3110.4	2937.6	3009.6	
57	3283.2	3110.4	2851.2	2937.6	
58	2851.2	3110.4	2764.8	2937.6	
59	3024	3110.4	2764.8	2887.2	
60	2851.2	3110.4	2764.8	2851.2	
61	2764.8	3110.4	2592	2813.4	

Figure 4. Calculation of MRC using the tabulation method.

Figure 5. MRC and recession curves from 2000 – 2014 at Kirby Grindalythe gauging station.

The MRC was fitted with the Maillet recession equation by plotting the recession hydrograph on semi-log graph, discharge plotted on the log axes and time on the ordinary axes. It shows a good fit with a single segment, with recession coefficient (0.017 day^{-1}) (Figure 6).

This paper next examines how recession curves relate to the aquifer permeability structure. Numerical modeling was used to investigate the response of the recession curve to different aquifer permeability scenarios. The models aimed to simulate the spring drainage for the real Chalk aquifer catchments in the area. Both Kirby Grindalythe and Driffild catchments were simulated, but only the former are presented here. Saturated thicknesses of the aquifer, boundary conditions given by catchment water divides, and geological information from previous studies were used in formulation of the conceptual model for each catchment.

Figure 7 shows a conceptual model for the Kirby Grindalythe catchment. Catchment boundaries were based on topography. The conceptual model was then translated into a numerical simulation grid. Figure 8 illustrates a schematic diagram of the 3D model grid.

Figure 6. Analysis of MRC depending on Maillet model. (A) semi-log graph, the R^2 between the MRC and fitted recession line is 0.99 and recession coefficient 0.017 day^{-1} . (B) Black curve is MRC from observed discharge; red represents fitted curve to MRC which was calculated based on Maillet equation.

Figure 7. Conceptual model of Kirby Grindalythe catchment area. The red dashed line represents the groundwater divide, which was assumed to correspond to the topographic divide.

Groundwater Flow Model

A transient three dimensional numerical model was developed using Groundwater Vistas to simulate water drainage via a spring (Figure 8). The model was discretized into a uniform grid of finite-difference cells consisting of 70 rows by 45 columns of $100 \text{ m} \times 100 \text{ m}$ cells and vertically with 15 layers of cells of 2 m thickness. To represent aquifer drainage via a spring during the recession period, no rainfall recharge was added; instead the model was run from an initial head representing that at the start of the recession period.

The aquifer was modeled as unconfined; water depletes from the aquifer through a spring freely under the influence of gravity. The spring was simulated using a drain cell located at the level of the base of the model with very high hydraulic conductivity so as not to mask the conductivity in the aquifer. The modeled catchment was surrounded by no-flow boundaries representing the catchment divide. The soil zone was not explicitly represented in the model, because soil permeability is high enough to allow rainfall infiltration at all times.

Four targets (representing monitoring wells) were placed along the mid-plane of the model containing the drain cell. One of the targets was located at the drain cell for the purpose of recording the flow during recession while the other three targets were located at different distances upstream from the drain cell (100 m , $1,200 \text{ m}$, and $2,500 \text{ m}$) for monitoring hydraulic head.

Figure 8. Schematic illustration of model grid for simulating spring recession in Chalk catchments in the study area (Note: cells are shown larger than actual size relative to catchment dimensions for clarity).

To investigate the effect of hydraulic conductivity heterogeneity on spring recession three scenarios were tested. All simulations had the same boundary and initial conditions. Figure 9 schematically shows the scenarios tested.

First Scenario (Figure 9A): homogenous and isotropic aquifer.

Second Scenario (Figure 9B): heterogeneous aquifer, consisting of two parallel horizontal reservoirs, with different hydraulic properties. The lower reservoir represents a high permeability zone, corresponding to zone just below the level of water table fluctuation, where the maximum flow occurs. This zone is recognized to have very high hydraulic conductivity in chalk aquifers because of fracture enhancement due to calcite dissolution. The upper reservoir represents cumulative effect of the matrix, small fractures with lower permeability; this zone has been subjected to less water flow so fracture solution enhancement is less well developed.

The low permeability zone which is symbolized by K1 occupied 22m of the total model thickness and the high permeability zone symbolized by K2 occupied the 8 m thickness of the model.

$$K1 < K2$$

Third Scenario (Figure 9C): A relatively low permeability aquifer contains a longitudinal-tunnel shaped high permeability zone at the drain cell level. This geometry represents a high permeability major fracture zone or solution conduit. The highly permeable zone works as the transporting medium and the less permeable surrounding rock as a storage reservoir.

Figure 9. (A) Single reservoir aquifer. (B) Double reservoir aquifer, parallel reservoirs model. (C) Double reservoir, tunnel model.

The high permeability zone is an elongated cuboid with the plan dimensions of 2,000 m x100 m, and thickness of 8 m, located at the base of the model and at the level of drain cell.

Hydraulic Conductivity Sensitivity Test

Sensitivity tests for hydraulic conductivity (K) have been accomplished for all models. All the other conditions and parameters stayed unchanged. The models were run with zero recharge and initial head of 30 m above model base. This thickness is based on the water table map of the area provided by the British Environment Agency. Storage coefficient and specific yield were set to fixed values of 0.0001 and 0.01 respectively (Allen et al., 1997; Gale and Rutter, 2006; ESI, 2010).

Table 1 summarizes input values used for testing sensitivity to hydraulic conductivity; K represents the hydraulic conductivity in homogenous single reservoir aquifer model, K1 and K2 are hydraulic conductivity of low permeability and high permeability reservoirs respectively in the double reservoir aquifer models.

Note that the hydraulic conductivity of the low permeability zones (K1) remained constant while conductivity value of high permeable zones (K2) were changed; this is because the high permeability zones have more significant impact on the recession curve.

The last stage of development was calibration of the models against recession data from field measurements. Calibration was accomplished by using the trial-and-error method (Anderson and Woessner, 1992). For the Kirby Grindalthe catchment model, both single reservoir and double reservoir simulations were calibrated against field

Table 1. Values of hydraulic conductivity in m/day used for sensitivity test.

test #	1	2	3	4	5	6
single reservoir	K					
	1	10	50	100	125	150
test #	1	2	3	4	5	
2 parallel reservoirs & Tunnel model	K1	K2				
		1	10	50	100	125

data. Figure 10 demonstrates results of calibration between observed MRC and the recession curves obtained from the numerical models.

Results

The recession curves from tunnel and double reservoir models reveal that at the early stage of the recession period the flow rate falls rapidly then flattens off (Figure 10). This pattern of recession for the tunnel model appears more clearly when the contrast between hydraulic conductivity of the block and tunnel zone is larger. The steep initial recession curve arises from rapid hydraulic head depletion within the high permeability zone; the slower recession later reflects drainage behavior from the low permeability zone in the model.

Recession curves from single and parallel horizontal reservoir models are shown in Figure 11; both models behave similarly where the thickness of the high permeability zone within the parallel horizontal reservoir model was about 25% or more of the total aquifer thickness (black and green curves in Figure 11).

The high permeability zone clearly has a dominant impact when its size is sufficient such as to force the

Figure 10. Result of calibration between MRC and recession curve deduced from the tested numerical models (s – single porosity model; p – parallel reservoir model; t – tunnel model; numbers are K2; K1 = 1m/day in all the models shown).

aquifer to behave as a single reservoir aquifer with the same permeability as the high K layer. Where the highly permeable zone is thinner, e.g. representing only 10% of the model thickness, flow rates are reduced.

For the purpose of identifying the most representative models, all the recession curves (from MRC and models) have been analyzed using the Maillet formula, and then the results were compared (Table 2). The double reservoir (parallel reservoirs) and tunnel models all require three segments during the curve fitting process, each segment with different recession coefficient. However, the recession curve from the single reservoir model could be fitted with a single segment (single recession coefficient). This led the authors to conclude that the single reservoir model is more likely to be representative of the real aquifer in the area.

The coefficient of regression (R-squared) between recession curve from the best fitting single reservoir

Figure 11. Effect of size of high permeability zone on the shape of recession curve. **Black dashed line** is from single reservoir aquifer, with hydraulic conductivity = 100m/day. **Solid green line** is from parallel reservoir model when the high permeability zone represents about 25% of total model volume. **Solid red line** is from parallel reservoir model when high permeability zone represents about 10% of total model volume. **Solid blue line** is from tunnel model when high permeability zone represents about 1% of total model volume. **Solid purple line** is from tunnel model when high permeability zone represents about 0.3% of total aquifer volume. Note: in all double reservoirs models $K_1 = 1$ m/day and $K_2 = 100$ m/day

Table 2. Recession coefficient from models and MRC recession curves.

Model	recession coefficient (days ⁻¹)		
	α_1	α_2	α_3
K 2-100,t	0.080	0.013	0.003
K 2-150,t	0.058	0.010	0.003
K 2-50,t	0.065	0.013	0.003
K 2-50,p	0.058	0.008	0.003
K 2-100,p	0.067	0.011	0.004
K 2-150,p	0.103	0.019	0.006
K 125,s	0.013		
MRC	0.017		

model and MRC for the period between 2000 to 2014 was 0.79 – the fit is not perfect because the model curve falls rather more steeply than the MRC initially and later flattens off more. Nevertheless, the model curve does fall within the range of behavior seen in the recession curves for individual years.

The results from calibration tests for Kirby Grindalythe catchment suggest that the recession curve which was produced from the single reservoir model with a calibrated K value of 125 m/day show best agreement to the field recession curve (Figure 10). Given the initial model saturated thickness of 30 m this indicates a maximum model transmissivity value of 3,750 m²/day.

The mean transmissivity value in the Yorkshire Chalk is about 1,250 m²/day obtained from borehole measurements (Gale and Rutter, 2006). A pumping test in a Low Mothrope borehole close to the catchment area shows a transmissivity value of 450 m²/day (Figure 12), whereas a pumping test at Etton south of the study area shows transmissivity values of 1,000-2,200 m²/day (Gale and Rutter, 2006). The above data suggest that the spring recession-derived T values may be higher than those likely to be observed in pumping tests. The recession derived K value of 125 m/day agrees better with K found by calibrating numerical simulations (e.g., 4 to 170 m/day, University of Birmingham, 1978 from Allen et al. 1997; Jones et al. 2000). This result suggests that spring hydrograph analysis can be a better choice for deriving hydraulic properties representative of the catchment scale than pumping tests, where complex fracture system are responsible for the permeability. In these cases, borehole tests may not be as representative, as they offer information only at and near the drilling site.

Figure 12. Transmissivity distribution (in m^2/day) measured from pumping tests (data supplied by Environment Agency of England and Wales). From Parker (2009).

Conclusions

In this study we used numerical modelling to simulate Chalk aquifer drainage by springs for the purpose of aquifer characterization from hydrograph recession. Daily mean discharge data from the Chalk aquifer in East Yorkshire were used with climate data (SMD and AE) for drawing hydrograph recession curves. Hydrogeological and geological information were used to construct numerical models of the targeted catchments. Three different scenarios were simulated to represent the aquifer in the study area. Recession curves from numerical models were calibrated against Master Recession Curves based on measured discharge. The trial-and-error method was used to accomplish the calibration between observed and model recession curve. In addition, the recession curves from the observed

discharge and model discharge were analysed using the analytical exponential model of Maillet, to identify the aquifer permeability scenario that best matched the field data.

This study confirms that the highly permeable fracture system dominates flow in the Chalk aquifer in the study area. Moreover, it revealed that in such complex fractured aquifers, the hydraulic parameters measured through borehole tests may not be representative; transmissivity values obtained from model calibration are higher than those from the borehole tests in the area.

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