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Research Article

Abundant Lump-Type Solutions and Interaction Solutions for a Nonlinear (3+1) Dimensional Model

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We explore dynamical features of lump solutions as diversion and propagation in the space. Through the Hirota bilinear method and the Cole-Hopf transformation, lump-type solutions and their interaction solutions with one- or two-stripe solutions have been generated for a generalized (3+1) shallow water-like (SWL) equation, via symbolic computations associated with three different ansatzes. The analyticity and localization of the resulting solutions in the (x, y, z, and t) space have been analyzed. Three-dimensional plots and contour plots are made for some special cases of the solutions to illustrate physical motions and peak dynamics of lump soliton waves in higher dimensions. The study of lump-type solutions moderates the visuality of optics media and oceanography waves.

1. Introduction

It is very important to control the physical mechanisms of rough waves and interaction waves specially with lump-type waves. The significance of nonlinear waves of these types appears from natural disasters. Many physical phenomena need analytical approaches to classify the physical dynamics of nonlinear evaluation equations. The Darboux transformation (DT) and the Lie symmetry (LS) method [1–3] are efficient approaches to obtaining closed-form solutions. However, some problems occur in applying those methods, such as how to find Lax pairs in the DT method and how to carry out the back-substitution procedure in the LS method. There are also new types of closed-form solutions, for example, positions and complexions [4–8], and even new collision phenomena including fissions and fusions [9–14]. The Hirota bilinear method plays an influential role in discovering all the mentioned types of solutions to overcome a lot of analytic problems. Most studies apply the Hirota method to completely integrability nonlinear problems as in [10, 15–23]. We would like to demonstrate that the Hirota method can be used to explore various types of closed-form solutions: interaction solutions of lumps with solitons, kinks, line-solitons, resonance solutions, and one- or two-stripe solitons; and two classes of breather solutions (time periodic or space periodic solutions). Our analysis will show that those solutions can predicate the characteristics and physical significance of nonlinear problems.

Consider the following generalized (3+1) SWL equation [24, 25]:

$$u_{xxxy} + 3u_{xx}u_y + 3u_xu_{xy} - u_{yt} - u_{xz} = 0. \tag{1}$$

There are a few studies on this equation. For example, Tian et al. in [26] generated a traveling wave solution via the tanh method. In 2010, Zayed [27] used *the* G'/G method to obtain some traveling wave solutions by reducing the independent variables using the linear D'lambert transformation.

In what follows, we investigate lump soliton solutions and their dynamics and the susceptibility of their interactions with other types of solutions using the Hirota method for (1). By using the singular manifold method (SMM) with two-term truncated series, one derives the same ansatz in [24, 25]

$$u(x,y,t,z) = 2(\ln(\psi(x,y,t,z))_x. \quad (2)$$

This is called the Cole-Hopf transformation, where ψ is an auxiliary or test function that will be determined later. Starting by substituting (2) into (1), one gets

$$\begin{aligned}
&8\psi_{xxxy}\psi_x\psi - 4\psi_{xxx}\psi_{xy}\psi + 2\psi_{xxxx}\psi_y\psi - 12\psi_{xxy}\psi_x{}^2 \\
&- 2\psi_{xt}\psi_y\psi - 2\psi_{xy}\psi_t\psi - 2\psi_{yt}\psi_x\psi - 2\psi_{xx}\psi_z\psi \\
&- 4\psi_{xz}\psi_x\psi + 4\psi_x{}^2\psi_z + 2\psi_{xxz}\psi^2 - 2\psi_{xxxxy}\psi^2 \quad (3)\\
&+ 2\psi_{xyt}\psi^2 - 4\psi_{xxx}\psi_x\psi_y + 12\psi_{xx}\psi_x\psi_{xy} \\
&+ 4\psi_x\psi_t\psi_y = 0.
\end{aligned}$$

The transformation increases the nonlinearity but allows us to work with the test function. In [24], Zhang used Bell polynomial theories to generate lump-kink solutions, lumps with one-stripe solitons and lumps with two-stripe solitons for (1), but he supposed that $z = x$ to minimize the number of independent variables and so studied the equation in a (2+1)-dimensional domain.

2. Lump Soliton Solutions

To generate single-lump solutions, we suppose that

$$\begin{aligned}
\psi &= \beta^2 + \gamma^2 + \alpha_{11}, \\
\beta &= \alpha_1 x + \alpha_2 y + \alpha_3 t + \alpha_4 z + \alpha_5, \quad (4)\\
\gamma &= \alpha_6 x + \alpha_7 y + \alpha_8 t + \alpha_9 z + \alpha_{10}.
\end{aligned}$$

where $\alpha_i, i = 1\dots.11$, are real unknowns that will be found subsequently. We carry out a direct substitution of (4) into (3) and gather the coefficients of the resulting polynomial in x, y, t, and z, to obtain a nonlinear algebraic system in α_k. By solving this system of nonlinear algebraic equations with the aid of Maple, we acquire some sets of solutions for the parameters. Avoiding the redundancy, we surpass one studying case as follows:

$$\begin{aligned}
\alpha_1 &= \alpha_1, \\
\alpha_2 &= \alpha_2, \\
\alpha_3 &= -\frac{1}{\alpha_6\alpha_{11}(\alpha_1\alpha_7 - \alpha_2\alpha_6)}\left(-\alpha_1^2\alpha_7\alpha_8\alpha_{11}\right. \\
&\quad + \alpha_2\alpha_1\alpha_6\alpha_8\alpha_{11} + 3\alpha_6^5 a_7 + 3\alpha_1^5\alpha_2 + 6\alpha_1^3\alpha_6^2\alpha_2 \\
&\quad \left. + 3\alpha_1^4\alpha_6\alpha_7 + 6\alpha_1^2\alpha_6^3\alpha_7 + 3\alpha_6^4\alpha_1\alpha_2\right), \\
\alpha_4 &= \frac{1}{\alpha_6\alpha_{11}(\alpha_1\alpha_7 - \alpha_2\alpha_6)}\left(3\alpha_1^4\alpha_2^2 + 6\alpha_1^3\alpha_2\alpha_6\alpha_7\right. \\
&\quad + 3\alpha_6^2\alpha_1^2\alpha_2^2 + 3\alpha_6^2\alpha_1^2\alpha_7^2 - \alpha_2\alpha_1\alpha_7\alpha_8\alpha_{11} + 6\alpha_6^3\alpha_1\alpha_2\alpha_7 \\
&\quad \left. + \alpha_2^2\alpha_{11}\alpha_6\alpha_8 + 3\alpha_6^4\alpha_7^2\right), \\
\alpha_5 &= \alpha_5, \\
\alpha_6 &= \alpha_6, \\
\alpha_7 &= \alpha_7, \\
\alpha_8 &= \alpha_8, \\
\alpha_9 &= \frac{\left(3\alpha_1^3\alpha_2 + 3\alpha_1^2\alpha_6\alpha_7 + 3\alpha_6^2\alpha_1\alpha_2 - \alpha_7\alpha_8\alpha_{11} + 3\alpha_6^3\alpha_7\right)}{\alpha_6\alpha_{11}}, \\
\alpha_{10} &= 0, \\
\alpha_{11} &= \alpha_{11}.
\end{aligned} \quad (5)$$

Using the aggregation equation (4), one can represent the auxiliary function as

$$\begin{aligned}
\psi &= \left(\alpha_1 x + \alpha_2 y - \frac{1}{\alpha_6\alpha_{11}(\alpha_1\alpha_7 - \alpha_2\alpha_6)}\left(-\alpha_1^2\alpha_7\alpha_8\alpha_{11}\right.\right. \\
&\quad + \alpha_2\alpha_1\alpha_6\alpha_8\alpha_{11} + 3\alpha_6^5 a_7 + 3\alpha_1^5\alpha_2 + 6\alpha_1^3\alpha_6^2\alpha_2 \\
&\quad \left. + 3\alpha_1^4\alpha_6\alpha_7 + 6\alpha_1^2\alpha_6^3\alpha_7 + 3\alpha_6^4\alpha_1\alpha_2\right)t \\
&\quad + \frac{1}{\alpha_6\alpha_{11}(\alpha_1\alpha_7 - \alpha_2\alpha_6)}\left(3\alpha_1^4\alpha_2^2 + 6\alpha_1^3\alpha_2\alpha_6\alpha_7\right. \\
&\quad + 3\alpha_6^2\alpha_1^2\alpha_2^2 + 3\alpha_6^2\alpha_1^2\alpha_7^2 - \alpha_2\alpha_1\alpha_7\alpha_8\alpha_{11} + 6\alpha_6^3\alpha_1\alpha_2\alpha_7 \quad (6)\\
&\quad \left.\left. + \alpha_2^2\alpha_{11}\alpha_6\alpha_8 + 3\alpha_6^4\alpha_7^2\right)z + \alpha_5\right)^2 + \left(\alpha_6 x + \alpha_7 y + \alpha_8 t\right. \\
&\quad + \frac{\left(3\alpha_1^3\alpha_2 + 3\alpha_1^2\alpha_6\alpha_7 + 3\alpha_6^2\alpha_1\alpha_2 - \alpha_7\alpha_8\alpha_{11} + 3\alpha_6^3\alpha_7\right)}{\alpha_6\alpha_{11}} \\
&\quad \left.\cdot z\right)^2 + \alpha_{11},
\end{aligned}$$

where

$$\Delta = \begin{vmatrix}\alpha_1 & \alpha_2 \\ \alpha_6 & \alpha_7\end{vmatrix} \neq 0, \quad \alpha_6\alpha_{11} \neq 0. \quad (7)$$

By using (2), the solution of (1) has the form

$$u = 4\frac{\alpha_1\beta + \alpha_6\gamma}{\psi} \quad (8)$$

Incorporating (6) and (5) into (8), one gets a class of lump solutions of (1) depicted in Figure 1.

3. Interaction Solutions

3.1. Lump Solitons with One-Stripe Waves. Suppose that the test function is a confederation of a quadratic function withan

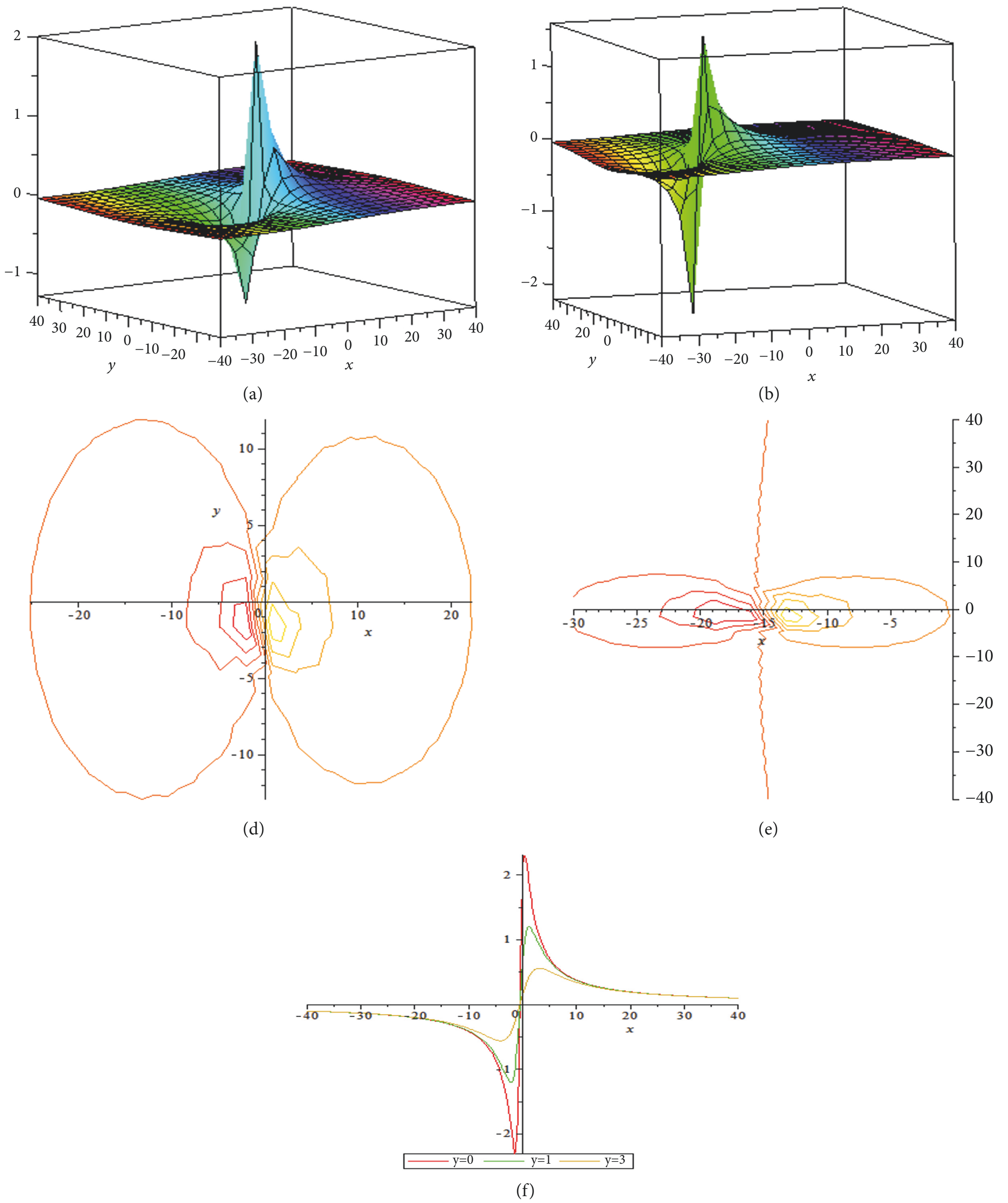

Figure 1: Proportion sight of the solution equation (8) with (6) and (5). For arbitrary constant values, $\alpha_2 = \alpha_1 = 1, \alpha_7 = -1, \alpha_6 = 1, \alpha_{10} = 0, \alpha_{11} = 1$, and $\alpha_8 = 5$. (a-b) 3D plots for $t = 0$, and 3, respectively. (d-e) Consistent contour plot of (a), (b). 2D plot present in (f) for various values of y.

exponential function as follows:

$$\begin{aligned} \psi &= \beta^2 + \gamma^2 + \alpha_{11} + \lambda, \\ \beta &= \alpha_1 x + \alpha_2 y + \alpha_3 t + \alpha_4 z + \alpha_5, \\ \gamma &= \alpha_6 x + \alpha_7 y + \alpha_8 t + \alpha_9 z + \alpha_{10} \\ \lambda &= e^{k_1 x + k_2 y + k_3 t + k_4 z + k_5}. \end{aligned} \tag{9}$$

where $\alpha_i, i = 1 \dots .11$ and $k_j, j = 1..5$, are real unknown constants that will be determined subsequently. Using the ansatz in (2),

$$u = 2\frac{2\alpha_1\beta + 2\alpha_6\gamma + k_1 e^{k_1x+k_2y+k_3t+k_4z+k_5}}{\psi}. \tag{10}$$

Inserting (9) into (3), gathering the coefficients of the resulting polynomial in x, y, t, and z, and equaling these coefficients to zero, we explore a complicated algebraic system on the unknown constants. We then solve the obtained system using Maple and snaffle the following assortment of solutions:

$$\begin{aligned}
\alpha_1 &= \alpha_1,\\
\alpha_2 &= \frac{k_2\left(\alpha_1^2+\alpha_6^2\right)}{k_1\alpha_1},\\
\alpha_3 &= 3k_1^2\alpha_1,\\
\alpha_5 &= \alpha_5,\\
\alpha_6 &= \alpha_6,\\
\alpha_7 &= 0,\\
\alpha_8 &= -3\frac{k_2\left(\alpha_1^2\right)}{\alpha_6},\\
\alpha_9 &= \frac{3k_1k_2\left(\alpha_1^2+\alpha_6^2\right)}{\alpha_6},\\
\alpha_{10} &= 0,\\
\alpha_{11} &= \frac{\left(\alpha_1^2+\alpha_6^2\right)}{k_1^2},\\
k_1 &= k_1,\\
k_2 &= k_2,\\
k_3 &= k_1^3,\\
k_4 &= 0,\\
k_5 &= k_5.
\end{aligned} \tag{11}$$

To avoid the singularity and promote the wave to localize in all directions, the following stipulation must be taken into consideration:

$$k_1 a_1 a_6 \neq 0. \tag{12}$$

Substituting (11) into (9), we obtain

$$\begin{aligned}
\psi = {} & \left(\alpha_1 x + \frac{k_2\left(\alpha_1^2+\alpha_6^2\right)}{k_1\alpha_1} y + 3k_1^2\alpha_1 t + \alpha_5\right)^2 \\
& + \left(\alpha_6 x - 3\frac{k_2\left(\alpha_1^2\right)}{\alpha_6} t + \frac{3k_1k_2\left(\alpha_1^2+\alpha_6^2\right)}{\alpha_6} z\right)^2 \\
& + \frac{\left(\alpha_1^2+\alpha_6^2\right)}{k_1^2} + e^{k_1x+k_2y+k_1^3t+k_5}.
\end{aligned} \tag{13}$$

Introducing (13) into (10), we generate a class of interaction solutions with stripe soliton (solitary wave) solutions. The results have been plotted in Figure 2 for different values of times.

3.2. Lump Solitons with Tough Waves (Two-Stripe Solitons). We suppose that the new ansatz is a combination of a quadratic function and a hyperbolic function as follows:

$$\begin{aligned}
\psi &= \beta^2 + \gamma^2 + \alpha_{11} + \delta,\\
\beta &= \alpha_1 x + \alpha_2 y + \alpha_3 t + \alpha_4 z + \alpha_5,\\
\gamma &= \alpha_6 x + \alpha_7 y + \alpha_8 t + \alpha_9 z + \alpha_{10},\\
\delta &= \cosh\left(k_1 x + k_2 y + k_3 t + k_4 z + k_5\right).
\end{aligned} \tag{14}$$

Substituting (17) into (2), we snaffle an assortment of solutions for (1) as follows:

$$u = 2\frac{2\alpha_1\beta + 2\alpha_6\gamma + k_1 \sinh\left(k_1 x + k_2 y + k_3 t + k_4 z + k_5\right)}{\psi}. \tag{15}$$

More complicated calculations have been done using Maple, to acquire the unidentified constants. Substituting (14) into (3), equaling the coefficients of x, y, t, and z to zero, and solving the resulting nonlinear algebraic system (up to 150 equations), we explore the following solution cases of the constant parameters. In each case, we do back substitution in (14).

$$\begin{aligned}
\alpha_1 &= 0,\\
\alpha_2 &= \frac{12\left(k_2^2\alpha_4\alpha_6^2\right)}{9k_2^2\alpha_6^2k_1^2 + 4\alpha_4^2},\\
\alpha_3 &= \frac{-k_1\alpha_4}{k_2},\\
\alpha_4 &= \alpha_4,\\
\alpha_5 &= 0,\\
\alpha_6 &= \alpha_6,\\
\alpha_7 &= \frac{\alpha_6 k_2\left(4\alpha_4^2 - 9k_2^2\alpha_6^2k_1^2\right)}{k_1\left(4\alpha_4^2 + 9k_2^2\alpha_6^2k_1^2\right)},\\
\alpha_8 &= \frac{3\left(k_1^2\alpha_6\right)}{2},\\
\alpha_9 &= \frac{3\left(k_1k_2\alpha_6\right)}{2},\\
\alpha_{10} &= 0,\\
\alpha_{11} &= \frac{9k_2^2\alpha_6^2k_1^2 + 4k_1^4\alpha_4^2 + 16\alpha_4^2\alpha_6^2 - 36\alpha_6^2k_1^2k_2^2}{16\alpha_4^2\alpha_6^2k_1^2},\\
k_1 &= k_1,
\end{aligned}$$

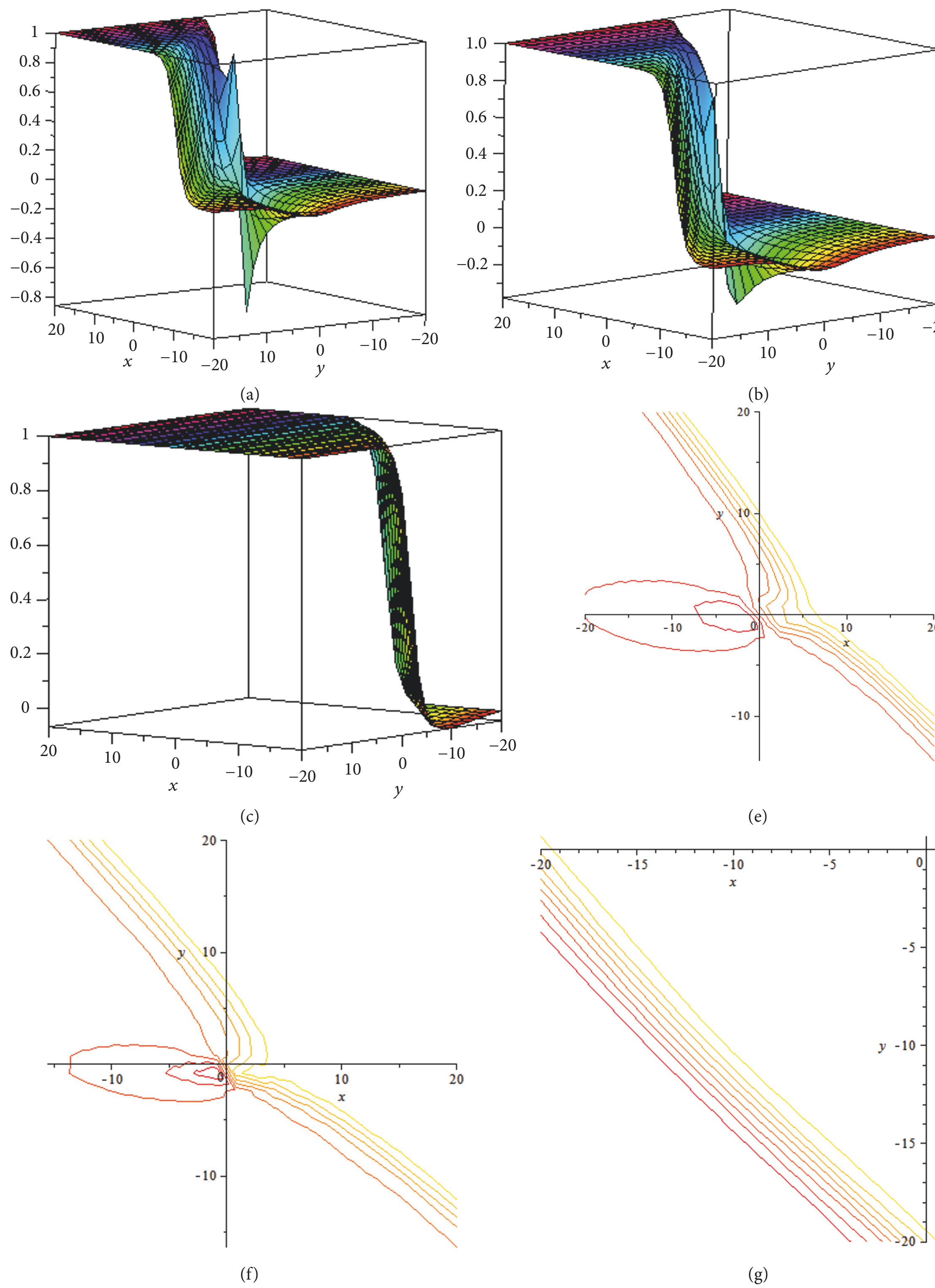

Figure 2: Proportion scene of the solution equation (10) with (13) and (11) for the values of arbitrary constants is $\alpha_1 = 1, \alpha_4 = 0, \alpha_6 = 3, \alpha_5 = 1, \alpha_{10} = 0$, and $k_1 = k_2 = k_5 = 1$. (a-c) represent 3D plots for (10) at $t = 0, 2$, and 30, $z = 0$, respectively. (e-g) Consistent contour plot of (a, b, c), respectively.

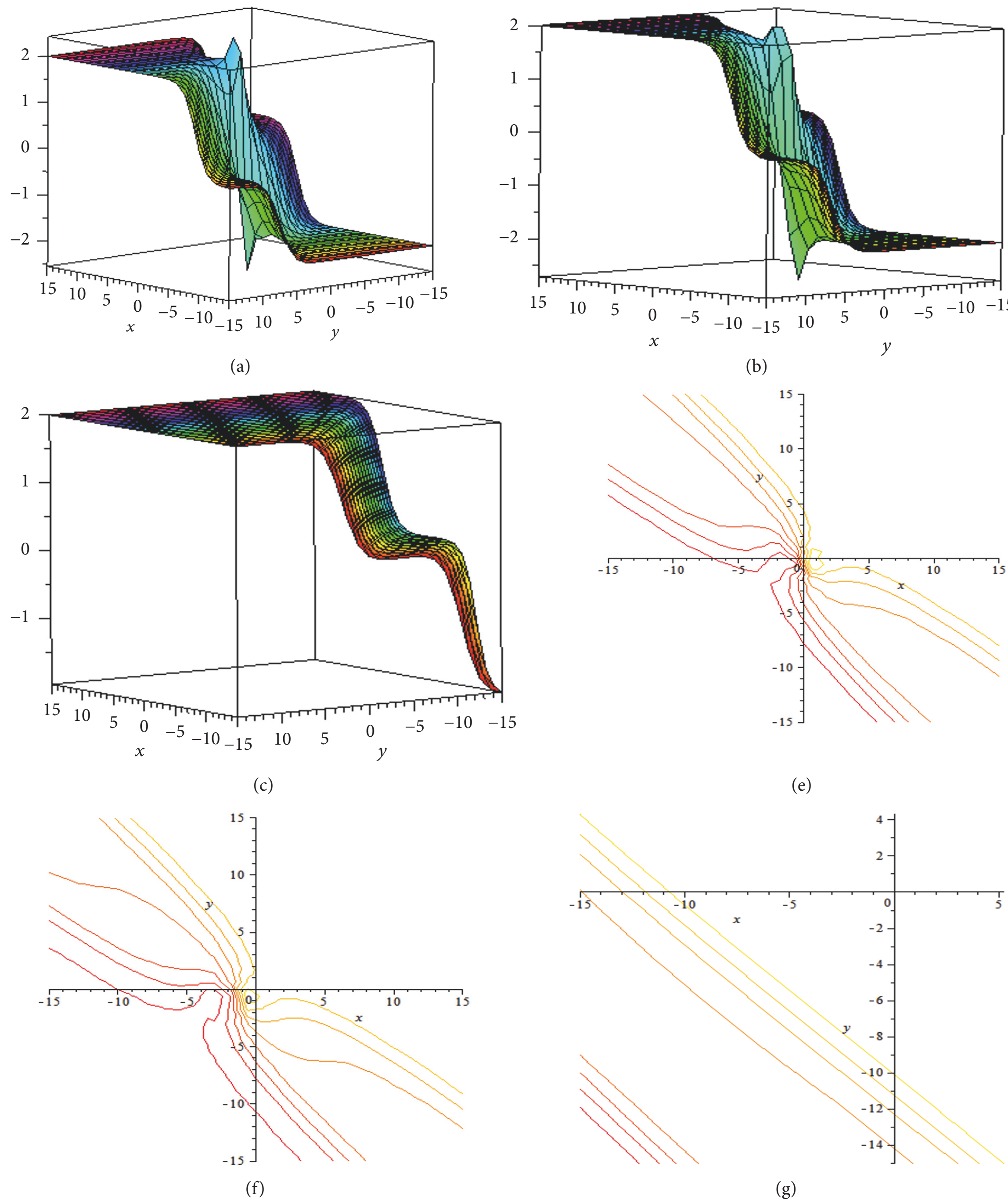

Figure 3: Proportion scenes of the solution equation (15) with (16) and (17) for the values of arbitrary constants are for $\alpha_2 = -1, \alpha_6 = 1, \alpha_7 = 2, \alpha_5 = 1, \alpha_{10} = 0, \alpha_8 = 1, \alpha_{11} = 1, k_1 = 1, k_2 = 1, k_5 = 1$. (a), (b), and (c) represent 3D plots for (10) at $t = 0, 5, 18$ and $z = 0$, respectively. (e), (f), and (g) Consistent contour plot of (a), (b), and (c).

$$
\begin{aligned}
k_2 &= k_2, \\
k_3 &= k_1^3, \\
k_4 &= 0, \\
k_5 &= k_5.
\end{aligned}
\tag{16}
$$

Substituting (16) into (14), we obtain

$$
\psi = \left(\alpha_1 x + \frac{12\left(k_2^2 \alpha_4 \alpha_6^2\right)}{9k_2^2\alpha_6^2k_1^2 + 4\alpha_4^2} y - \frac{k_1\alpha_4}{k_2} t + \alpha_4 z\right)^2 + \left(\alpha_6 x + \frac{\alpha_6 k_2\left(4\alpha_4^2 - 9k_2^2\alpha_6^2k_1^2\right)}{k_1\left(4\alpha_4^2 + 9k_2^2\alpha_6^2k_1^2\right)} y + \frac{3\left(k_1^2\alpha_6\right)}{2} t\right.
$$

$$+\frac{3\left(k_1 k_2 \alpha_6\right)}{2} z\Bigg)^2 + \frac{9k_2^2\alpha_6^2k_1^2 + 4k_1^4\alpha_4^2 + 16\alpha_4^2\alpha_6^2 - 36\alpha_6^2k_1^2k_2^2}{16\alpha_4^2\alpha_6^2k_1^2} + \cosh\left(k_1 x + k_2 y + k_1^3 t + k_5\right). \tag{17}$$

Through the same procedure, we get a class of solutions of (1) and plot a special solution in Figure 3.

4. Conclusions

Starting from the Cole-Hopf transformation, investigated in the SMM with a two-term truncated series, we derived novel lump solitons, lump-kinks, interacted lumps with one-stripe solitons or kinks, and interacted lumps with two-stripe solitons or kink waves, after some complicated calculations using the Maple software. The presented three-dimensional plots of the interaction solutions show that the lump solitons are coalesced or spliced up by the stripe solitons. To the best of our knowledge, those types of solutions for (1) are presented for the first time. Our solutions are localized in the four dimensional space (x, y, z, and t), but in [24], the authors assumed that $z = x$ and generated only one lump solution.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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